A WEIGHT VECTOR LMS ALGORITHM FOR ADAPTIVE BEAMFORMING

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ABSTRACT
The conventional adaptive LMS algorithm has been utilized in array antenna beamforming to direct the radiated power towards the desired signal and null the multipath signals. In this paper, we present two new weight-vector adaptive LMS algorithms (WV-LMS) for minimum mean square error (MMSE) beamforming adaptive algorithm. Rather than using a fixed convergence parameter $\mu$ in the conventional LMS algorithm, the two proposed algorithms exploit the useful information in the forward prediction of the weights vector. We used the forward prediction of the weights vector to dynamically change the convergence parameter $\mu_n$. Both algorithms have been tested for beamforming using a narrow-band FM signal and have been shown to provide better MMSE performance than the conventional LMS.

1. INTRODUCTION

Wireless signals sent from the transmitter experience not only propagation losses of the electromagnetic wave power, but also the geometrical effect that results in multipath components and angle spread [3, 5, 6]. These paths result from the reflection, diffraction or scattering of its signal from objects that lie in the environment, like buildings or natural scatterers. In other words, the signal received at the antenna will contain the transmitted signal as well as some unwanted signals at different angles of arrival. By using beamforming techniques, we can direct the radiated power towards the desired signal and nulling the interfering signals [3, 6]. This will result in the increase of signal to interference ratio (SIR) and lead to better signal estimation.

The LMS algorithm is a well-known adaptive estimation and prediction technique. It has been extensively studied in the past. Despite its simplicity, the LMS algorithm is capable of achieving satisfactory performance which converges to the optimum Wiener solution [1, 2, 3]. In beamforming, the LMS algorithm provides the optimum solution for the antenna weights vector and results in minimizing the mean-square error (MSE) between the beamforming output and the desired signal.

In this paper, we propose two structures for the adaptive LMS algorithm, the fixed forward prediction WV-LMS (FWV-LMS) and the updated forward prediction WV-LMS (UWV-LMS), and apply them in a beamforming technique. It is known that the performance of the conventional LMS algorithm is dependent on the signal condition and the chosen convergence parameter $\mu$. The parameter $\mu$ in the conventional LMS algorithm is fixed, while the proposed algorithms dynamically change the convergence parameter $\mu_n$, depending on the forward prediction of the weights vector. In section 2, we present the conventional LMS algorithm, and in section 3 we present the WV-LMS algorithms. In section 4, we describe the MMSE adaptive beamforming system. Section 5 provides the simulation results under different conditions.

2. LEAST-MEAN SQUARE ADAPTIVE ALGORITHM

In the conventional adaptive LMS algorithm, the weights vector coefficients $w(n)$ for the FIR filter are updated according to the following formula [1, 2]:

$$w(n) = w(n) + \mu e(n)y(n)$$ (1)

where $w(n) = [w_0(n) \ w_1(n) \ ... \ w_M(n)]$ ($M + 1$ being the filter length), $\mu$ is the convergence parameter (sometimes referred to as step-size), $e(n) = d(n) - z(n)$ is the output error ($z(n)$ being the filter output), and $d(n)$ is the reference signal. Note that $z(n) = w(n)y^T(n) = \hat{x}(n)$, where $\hat{x}(n)$ is the original signal and $y(n) = [y(n) \ y(n-1) \ ... \ y(n-M)]$ is the filter input signal.

3. WEIGHTS VECTOR LMS ALGORITHM

The weights vector in the beamforming algorithm control the beam radiation direction. In the conventional LMS beamforming algorithm, we utilize the feedback information ($e(n)$ the output error) to forward predict the new weights vector $w(n + 1)$. These new weights vector tell the antenna array where the beam should be directed in time $n + 1$. Hence, the system should improve if the new weights vector are feedback to the system for further prediction. The two proposed algorithms used this forward predicted weights vector and
current weights vector to generate a new parameter $\epsilon_n$ using the difference between the norm of both weights vectors as shown below:

$$
\epsilon_n = \frac{\| w(n+1) - w(n) \|}{\| w(n+1) \|}
$$

(2)

the $\epsilon_n$ is then passed to the alpha filter to generate the new convergence parameter $\mu_{n+1}$ that will be used to predict the weights vector $w(n+1)$:

$$
\mu_{n+1} = \begin{cases} 
\alpha_{\mu} / n + \delta_{\mu} & , \text{if } 0 < \mu_{n+1} < \mu_{\text{max}} \\
\mu_{\text{max}} & , \text{otherwise}
\end{cases}
$$

(3)

where $\mu_{\text{max}}$ is the maximum convergence parameter that can be used for LMS algorithm, where $\mu_{\text{max}}$ is defined as $[1, 2]$:

$$
\mu_{\text{max}} < \frac{2}{\lambda_{\text{max}}}
$$

(4)

where $\lambda_{\text{max}}$ is the largest eigenvalue of the correlation matrix parameter $\mu$ can be a fixed value (like LMS algorithm) named as fixed forward WV-LMS (FWV-LMS) in this paper or using the updated dynamic convergence parameter $\mu_n$, named as updated forward prediction WV-LMS (UWV-LMS).

3.1. Fixed Forward Prediction WV-LMS (FWV-LMS)

In the fixed forward prediction WV-LMS, a conventional LMS with a fixed convergence parameter $\mu$ is used to forward prediction of the weights vector $w(n+1)$. New convergence parameter $\mu_{n+1}$ is then calculated from the $w(n+1)$. The weights vector $w(n+1)$ will be recalculated using the new convergence parameter $\mu_{n+1}$. An outline of the FWV-LMS algorithm is shown in the following steps:

$$
\begin{align*}
  z(n) & = w(n)y^T(n) \\
  e(n) & = d(n) - z(n) \\
  w(n+1) & = w(n) + \mu e(n)y(n) \\
  \epsilon_n & = \frac{\| w(n+1) - w(n) \|}{\| w(n+1) \|} \\
  \mu_{n+1} & = \begin{cases} 
\alpha_{\mu} / n + \delta_{\mu} & , \text{if } 0 < \mu_{n+1} < \mu_{\text{max}} \\
\mu_{\text{max}} & , \text{otherwise}
\end{cases} \\
  w(n+1) & = w(n) + \mu_{n+1} e(n)y(n)
\end{align*}
$$

where $^T$ indicates matrix transposition.

3.2. Updated Forward Prediction WV-LMS (UWV-LMS)

The updated forward prediction WV-LMS works the same as the FWV-LMS. However, it will forward predict the weights vector using the updated $\mu_n$. An outline of the UWD-LMS algorithm is shown in the following steps:

$$
\begin{align*}
  z(n) & = w(n)y^T(n) \\
  e(n) & = d(n) - z(n) \\
  w(n+1) & = w(n) + \mu e(n)y(n) \\
  \epsilon_n & = \frac{\| w(n+1) - w(n) \|}{\| w(n+1) \|} \\
  \mu_{n+1} & = \begin{cases} 
\alpha_{\mu} / n + \delta_{\mu} & , \text{if } 0 < \mu_{n+1} < \mu_{\text{max}} \\
\mu_{\text{max}} & , \text{otherwise}
\end{cases} \\
  w(n+1) & = w(n) + \mu_{n+1} e(n)y(n)
\end{align*}
$$

where $^T$ indicates matrix transposition.

4. ADAPTIVE BEAMFORMING

In a uniformly spaced linear antenna array, the desired signal arrives at the antenna array with an angle $\theta_0$ and the $i$th interfering signals $u_i(t) | i = 1, ..., N_u$ arrive with an angle $\theta_i$. $N_u$ is the number of interfering signals. The output of the linear antenna array can be formulated as follows:

$$
\mathbf{x}(t) = \mathbf{s}(t)\mathbf{v} + \mathbf{u} = \mathbf{s} + \mathbf{u}
$$

(5)

where $\mathbf{v}$ is the array propagation vector for desired signal:

$$
\mathbf{v}^T = [1, e^{j2\pi \sin \theta_0 / \lambda}, ..., e^{j(K-1)2\pi \sin \theta_0 / \lambda}]
$$

(6)

$K$ is the number of elements in the antenna array. $\mathbf{u}$ represents the sum of all interfering signal vectors $[3]$:

$$
\mathbf{u} = \sum_{i=1}^{N_u} u_i(t)\gamma_i
$$

(7)

$\gamma_i$ is the array propagation vector for the $i$th interfering signal,

$$
\gamma_i^T = [1, e^{j2\pi \sin \theta_i / \lambda}, ..., e^{j(K-1)2\pi \sin \theta_i / \lambda}]
$$

(8)

Fig. 1 shows the generic adaptive beamforming system. The beamforming output $\delta(t) = \mathbf{w}^H \mathbf{x}(t)$ (referred as $z(n)$ in Sections II and III) is optimized (by minimizing the difference with the desired signal) using the LMS adaptive algorithm.
5. SIMULATION RESULTS

For study purposes, we assume the desired signal \( s(t) \) is known to the receiver, hence reference signal \( d(n) = s(n) \). The interference signals at the receiver are assumed to be a Rayleigh fading type. The weight coefficient for the antenna array will be the weights coefficient of the FIR filter in the adaptive algorithm. The weights are updated using the equation in Sections II and III. The relation \( \mu_{\text{max}} = \mu_{\text{fix}} = 3\mu \) is used in this simulation. The mean squared error (MSE) is used as a comparison criterion for the performance of the beamforming system in the simulation and is calculated as follows:

\[
MSE = \frac{1}{N} \sum_{n=0}^{N} [s(n) - \hat{s}(n)]^2
\]  

(9)

As many real-life and synthetic information-bearing signals can be frequency-modulated (linear, non-linear, or combinations of both), we perform our simulations using a non-linear FM signal as follows:

\[
s(t) = \cos(\omega_0 t + \gamma t^2/2 + \beta t^3/3) \Pi_T(t-T)
\]  

(10)

where \( \omega_0 = 2\pi f_0 \) is a constant (initial frequency), \( T \) is the signal duration, and \( \gamma \) and \( \beta \) are the modulation indices which determine the bandwidth of the quadratic frequency modulated (QFM) signal. Similar simulation results can be obtained using linear FM signals.

The bandwidth \( BW \) of this QFM signal can be adjusted by varying the parameters \( \gamma \) and \( \beta \) as numerically shown using the following relationships [4]:

\[
f_m = \frac{1}{2\pi} \int_0^\infty \omega |X(\omega)|^2 d\omega
\]  

(11)

\[
BW = \frac{1}{2\pi} \int_0^\infty (\omega - \omega_m)^2 |X(\omega)|^2 d\omega
\]  

(12)

where \( X(f) \) is the Fourier transform of \( x(t) \) and \( f_m \) is its mean frequency.

Fig. 2 shows the spectrum of a QFM narrow-band signal with \( f_0 = 100 \text{ Hz} \), \( \gamma = 0.5 \), and \( \beta = 0.37 \).

Fig. (3) and Fig. (4) show the MSE performance of the LMS, FWV-LMS and UWV-LMS beamforming for 4 antennas using a non-linear FM signal in a low SIR (-10dB) environment. The graphs show that the performance is dependent on the choice of algorithm. WV-LMS algorithms show that they provide better MMSE performance and will always converge close to MMSE. The UWV-LMS also proved to be more dynamically adapted to environment change, while the FWV-LMS algorithm have better MSE performance than the conventional LMS algorithm.
Fig. 6. MSE performance for different adaptive beamforming algorithms using a non linear FM signal (4 antenna, SIR=[0, 0], d = \lambda / 2, \delta t = [-30, 50], SNR = 10, BW = 200Hz, \alpha = 0.90, \delta = 0.0003).

Fig. 7. MSE performance for different adaptive beamforming algorithms using a non linear FM signal (2 antenna, SIR=[-10, -10], d = \lambda / 2, \delta t = [-30, 50], SNR = 10, BW = 200Hz, \alpha = 0.95, \delta = 0.00003).

Fig. 8. MSE performance for different adaptive beamforming algorithms using a non linear FM signal (2 antenna, SIR=[0, 0], d = \lambda / 2, \delta t = [-30, 50], SNR = 10, BW = 200Hz, \alpha = 0.90, \delta = 0.0005).

Fig. (7) and Fig. (8) show the MSE performance of the LMS, FWV-LMS and UWV-LMS beamforming with 2 antennas using a non-linear FM signal. Under both high and low SIR conditions, the graphs show identical advantage for 2 antenna system. Both of the proposed algorithms provide better MMSE performance.

In general, Fig. (3), Fig. (4), Fig. (5), Fig. (6), Fig. (7) and Fig. (8) show that the UWV-LMS has better performance when a larger initial convergence parameter \( \mu_0 \) is used. The parameters \( \alpha \) and \( \delta \) are used to control the WV-LMS algorithms performance and should be carefully selected.

6. CONCLUSIONS

In this work we presented two modifications for the least mean-squared (LMS) algorithm using the forward prediction of the weights vector to dynamically modify the convergence parameter. The study concentrated on the performance of these algorithms in beamforming applications. In mobile communication systems, the physical size of the device is limited. As a result, only 2 and 4 antennas had been studied in our simulations. The proposed algorithms (called FWV-LMS and UWV-LMS) showed to provide better minimum mean-squared error (MMSE) than the conventional LMS algorithm under different SIR conditions. Simulation results showed that the performance of FWV-LMS is parallel to that of the conventional LMS algorithm, whereas the UWV-LMS is more dynamically adaptable depending on the predicted convergence parameter. Hence, we have more control over the FWV-LMS algorithm than the UWV-LMS algorithm. However, UWV-LMS will perform better under dynamic channel environment. Further study is needed to understand this algorithm performance in other channel environments.

7. REFERENCES