Adaptive Receiver Beamforming for Diversity Coded OFDM Systems: Maximum SNR Design

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Abstract—Over the recent years, advance multiantenna transmission schemes have attracted considerable interest due to their potential benefits in improving the system capacity and error-rate performance. As a result, space-time coding and transmit beamforming have emerged as the two most promising techniques. Because of limited space at the mobile station (MS) and the fact that download intensive services are to be introduced in the next generation of cellular systems, most of research efforts have been pouring on transmit diversity techniques. In this paper, we focus on adaptive uplink transmission and reception techniques for wireless communications and introduce a new frequency-time encoding scheme that can be used to exploit frequency diversity branches for broadband OFDM systems with only one antenna at the MS. By incorporating this with receive beamforming at the base station (BS), the instantaneous signal-to-noise ratio (SNR) is maximized and the system error-rate performance is then further improved. Numerical results showed that systems employed the proposed transceiver structure have a 4-dB improvement over the conventional space-time coding scheme when two receive antennas are used.

I. INTRODUCTION

Signal transmission in multi-input multi-output (MIMO) systems that employs more than one antennas at the transmitter and the receiver has shown to be effective in exploiting spatial diversified paths of wireless channels [1]-[2] and increasing both system capacity and error-rate performance. In particular, space-time coding includes both space-time block coding [3]-[4] and trellis coding [5] had gained a significant attention due to their superior performance and simplicity of transceiver design over other known techniques. However, their performance improvements are based on the assumption that the arriving multipath signals are sufficiently uncorrelated. In cellular communications, due to close spacing between antenna elements at the base station (BS), signal paths are often correlated to some degree. As a consequence, coherent deep fade between propagation signal paths is unavoidable and studies have shown that signal correlation can degrade the system performance significantly [6], [7].

The application of space-time coding to orthogonal frequency division multiplexing (OFDM) systems was first introduced in [8]. Motivated by the presence of additional multipath diversity offered by frequency-selectivity in broadband wireless channels, space-frequency (SF) [7] and space-time-frequency (STF) coding [9] were introduced. Extending from their work, the combination of diversity coding schemes with transmit beamforming was investigated for broadband OFDM systems [10]-[11]. However, all of these transmission schemes are for downlink application with multiple transmit antennas.

In this paper we propose a new frequency-time (FT) encoding scheme and combine it with receiver beamforming (denote by FT-Beam) to maximize the received signal-to-noise (SNR) for OFDM systems with single transmit antenna. By knowing subchannel gains at the mobile station (MS), we utilize the concept of subchannel grouping in [12] and perform FT encoding of existing space-time codes across OFDM subcarriers to achieve transmit diversity in the frequency domain. With the effective use of beamforming at the multiantennas base station (BS) receiver, the optimal adaptive beam-mapping weights is applied to maximize the instantaneous SNR, and thus, system error-rate performance during uplink transmission in a single-input multi-output (SIMO) channel is further enhanced.

Notation used: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ are complex conjugate, vector transposition, and Hermitian transposition, respectively. $\| \cdot \|_F$ is the Frobenius norm; $\sqrt{\mathbf{A}}$ stands for Hermitian square root of matrix $\mathbf{A}$; $\det(\cdot)$ denotes the determinant; $\mathbb{E}\{\cdot\}$ is the expectation operator. Finally, capital (small) bold letters represent matrices (vectors).

II. SYSTEM MODEL

Consider an uplink cellular communication scenario employing an $N_r$ frequency tone OFDM system with a single transmit antenna at the MS and $N_r$ receive antennas at the BS over a frequency-selective fading channel. It is assumed that the channel coherent bandwidth is larger than the bandwidth of each subcarrier; we thus consider the corresponding subchannel to be frequency-flat. In Fig. 1, we depicted a general structure of this OFDM system and combined with the proposed adaptive transceiver structure. In this work, it is also assumed that the system operates in a typical cellular environment where the BS antennas are placed at the building roof-top in an unobstructed manner. It is stated in [13] that signal transmission in such an environment over a multipath channel will lead to partially correlated signal paths in the spatial domain arriving at the BS. Next, assume that a uniform linear array (ULA) configuration is used for $N_r$ BS antennas with a spacing of $d$ meters. The normalized correlation matrix that specifies the correlation between antenna elements is...
defined in [14] as

\[ \mathbf{R}_r = \frac{1}{L} \sum_{\ell=1}^{L} a(\theta_\ell) a^H(\theta_\ell) \] (1)

where \( L \) denotes the number of dominant resolvable paths and \( a(\theta_\ell) := [1, e^{j2\beta}, \ldots, e^{j(N_c-1)\beta}]^T \) is the array propagation vector for the \( \ell \)th tap with an angle of arrival (AoA) of \( \theta_\ell, \beta = [2\pi \cdot d \cdot \sin(\theta_\ell)]/\lambda, \lambda \) being the carrier frequency wavelength. In general, \( R_r \) is a nonnegative-definite Hermitian matrix and the eigenvalue-decomposition (EVD) of \( R_r \) can be expressed as \( \mathbf{VR}_r \mathbf{V}^H = \mathbf{\Delta} \), where \( \mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_{N_c}] \) is a unitary matrix with columns that are the eigenvectors and \( \mathbf{\Delta} = \text{diag}[\delta_1, \ldots, \delta_n, \ldots, \delta_{N_c}] \) is a diagonal matrix contains the corresponding eigenvalues. Without loss of generality, we assume that \( \delta_1 \)'s are ordered in a non-increasing fashion: \( \delta_1 \geq \delta_2 \geq \cdots \geq \delta_{N_c} \geq 0 \).

Let us denote the correlated SIMO channel frequency response vector for the \( k \)th subcarrier as \( \mathbf{h}_k \in \mathbb{C}^{1 \times N_r} \). The \( j \)th element, which represents the subchannel gain between the transmit and the \( j \)th receive antenna, is defined as \( h_k(j) := g_{j}(1), \ldots, g_{j}(L-1) \) is the channel impulse response vector with independent circularly symmetric complex Gaussian random variables from \( \mathcal{CN}(0, \sigma^2) \) and \( \mathbf{f}_k = [1, e^{-j2\pi(k-1)/N_c}, \ldots, e^{-j2\pi(k-1)N_c/N_c}]^T \) is the corresponding discrete Fourier transform coefficients. According to [13], the channel frequency response vector can also be expressed as \( \mathbf{h}_k \approx \mathbf{R}_r \mathbf{h}_k \), where \( \mathbf{h}_k \) can be thought as a prewhitened channel vector. Furthermore, quasi-static fading is also assumed throughout the duration of one FTBC codeword length but fading may vary from one block to another.

**A. Subchannel Grouping & Frequency-Time Encoding**

The concept of subchannel grouping, sometimes referred as subcarrier grouping, was originally used in adaptive modulation scheme in [10] to reduce processing complexity by grouping subcarriers or subchannels that are within one channel coherent bandwidth and having a similar fading gain.

In [9] and [12], subcarrier grouping is used for grouping frequencies that are approximately one coherent bandwidth apart to perform STF coding in OFDM systems to exploit both spatial and multipath diversity. In this work, we utilize the concept of subchannel grouping in [12] by treating subcarriers that are having different fading gains as additional antennas at the MS. By doing so, we can then directly apply the space-time codes in [3]-[5], [15] in our system by spreading symbol energy across OFDM frequencies instead of antennas. An illustration of this subchannel grouping concept is shown in Fig. 2, where the channel coherent bandwidth is assumed to be equivalent to three frequency tones and subchannels that are having different fading gains are grouped together. Thus, the number of subchannels (subcarriers) that are in one group depends on the spatial dimension of the original space-time code.

Let us denote \( N_g \) as the total number of groups as a result of this sub-channel grouping process. If the well-known Alamouti’s space-time block code in [3]

\[ \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}, \]

(which has a spatial dimension of 2) is used for the FT encoding process, then \( N_g = N_r/2 \). An example of this FT encoding output is shown in Fig. 3, where baseband modulated data symbols \( s_1 \) and \( s_2 \) are FT-encoded across two subcarriers \( f_1 \) and \( f_4 \) (c.f. Fig. 2) as well as two OFDM symbol periods \( n = 0 \) and \( n = 1 \) \( (n \) being the time index in mod-2 sense). Similarly, data symbols \( s_3 \) and \( s_4 \) are encoded in group 2, while \( s_5 \) and \( s_6 \) are encoded in group 3. Note that other STBC matrices in [4] and STTC technique in [5] can be applied in the same way, but with different subchannel grouping sizes.

**B. Receiver Beamforming**

At the receiver, discrete Fourier transformation is applied to the noisy samples of SIMO signals arriving at \( N_r \) antennas. Assume ideal symbol-time sampling and carrier synchronization, the discrete time baseband equivalent form of system input-output equation can be expressed as

\[ \mathbf{r}_k = x_k \mathbf{R}_r + \mathbf{e}_k, \]

where \( x_k \) is an FT encoded data symbol transmitted on the \( k \)th subcarrier, and \( e_k \) is an additive white Gaussian noise vector with each element having zero mean and \( \sigma^2 \) variance.
Estimation of the channel fading gains is carried out at the adaptive processor (AP) by correlating pilot tones embedded in the transmitted signal. Results are then used for generating the antenna weighting matrix to maximize the received SNR. The antenna weight mapping process is performed across the antenna weighting matrix to maximize the received SNR.

The objective of this Section is to maximize the received SNR at the adaptive processor (AP) by correlating pilot tones embedded in different subcarriers that are within the same subchannel group. Similarly, the above MLD expressions can be easily extended and used for decoding of other space-time block codes in [4].

### D. Signal-to-Noise Ratio (SNR)

Given that the average transmitted energy during one OFDM-symbol interval is

$$
E\{x_k\} = E\{|s_n|^2\} = \varepsilon_s,
$$

the received SNR at the $k^{th}$ subcarrier for the detection of $x_k$ has the form:

$$
\gamma_k = \frac{\varepsilon_s \|\hat{U}_k \sqrt{R_r} W_k\|^2_F}{\sigma_k^2 \|W_k\|^2_F}.
$$

Denote $|s_n - \hat{s}_n|$ as the minimum distance between the underlying constellation symbols, then the symbol energy for both QAM and PSK modulation schemes are given in [16] as

$$
\varepsilon_s = \frac{(M - 1)|s_n - \hat{s}_n|^2_{QAM}}{6}, \quad \text{for QAM}
$$

$$
\varepsilon_s = \frac{|s_n - \hat{s}_n|^2_{PSK}}{4\sin^2(\pi/M)}, \quad \text{for PSK}.
$$

### III. OPTIMAL ANTENNA WEIGHTING MATRIX: MAXIMUM SNR DESIGN

The objective of this Section is to maximize the received SNR in order to improve the system error-rate performance. Effectively, this amounts choosing the weight mapping matrix that maximize (8) by solving the following cost function:

$$
\max_{W_k} J = \det(I_{N_r} + \varepsilon_s \|\hat{U}_k \sqrt{R_r} W_k\|^2_F / \sigma_k^2 \|W_k\|^2_F)
$$

subject to: $\|W_k\|^2_F = N_r$.  

Equivalently, this can be re-written as

$$
\max_{W_k} J = \det\left(I_{N_r} + \frac{\varepsilon_s \|\hat{U}_k \sqrt{X} \Psi \sqrt{X}^H \hat{U}_k^H\|^2_F}{N_r \sigma^2_k}\right),
$$

where $h_i(n)$ and $y_i(n)$ denotes the $j^{th}$ entry of $\mathbf{R}_i \mathbf{w}_i(n)$ and $\mathbf{y}_i(n)$, respectively. The decision matrix

$$
\begin{bmatrix}
\sum_{j=1}^{N_r} \left(y_{i,j}(n)h_{i(1)}^{*}(j) + y_{i,j}(n+1)h_{i(2)}^{*}(j)\right) - s_1^2 \\
\sum_{j=1}^{N_r} \left(y_{i,j}(n)h_{i(2)}^{*}(j) - y_{i,j}(n+1)h_{i(1)}^{*}(j)\right) - s_2^2
\end{bmatrix}
$$

is used for decoding $s_2$. Similarly, the above MLD expressions can be easily extended and used for decoding of other space-time block codes in [4].
where $\Phi_k = V^H \sqrt{\sum_k W_k H_k^H} \sqrt{\sum_k W_k H_k^H}$. By using Hadamard inequality, the optimization problem (12) can be rewritten as

$$J \leq \det \left( I_{N_r} + \frac{\varepsilon_s N_t}{N_r \sigma_k^2} (\Phi_k V^H \Phi_k^H) \right)$$

(13)

and the equality is achieved if and only if $\Phi_k$ is a diagonal matrix. Assume $\Phi_k = \text{diag} [\phi_{1,k}, \phi_{2,k}, \ldots, \phi_{N_r,k}]$, hence (13) becomes

$$J = \prod_{j=1}^{N_r} \left( 1 + \frac{\varepsilon_s N_t}{N_r \sigma_k^2} |\tilde{h}_k(j)|^2 \delta_j \phi_{j,k} \right),$$

(14)

where $\tilde{h}_k(j)$ is the $j^{th}$ element of $\tilde{h}_k$. Note that our optimization problem has a similar form to that in [17]. Although the water-filling strategy was originally used for enhancing the channel capacity, in this work we utilize it for maximizing the instantaneous received SNR. Following [17], we arrive at an initial solution for $\phi_{j,k}$’s as $\phi_{j,k} = \xi - \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(j)|^2 \delta_j}$. However, depending on the channel fading gains and the receiver noise variance, this solution may not satisfy the constraint $\phi_{j,k} \geq 0, \forall j$ due to the constraint $\sum_{j=1}^{N_r} \phi_{j,k} = N_r$. Thus we introduce a special notation $(\cdot)^+$ denoting $\max(\cdot, 0)$. Now $\phi_{j,k}$ will be

$$\phi_{j,k} = \left( \xi - \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(j)|^2 \delta_j} \right)^+$$

(15)

Recall that $\delta_j$’s are arranged in a non-increasing order, for now, we assume $\phi_{j,k}$’s are also arranged in the same order, $\phi_{1,k} \geq \phi_{2,k} \geq \cdots \geq \phi_{N_r,k}$, as long as $|\tilde{h}_k(1)|^2 \geq |\tilde{h}_k(2)|^2 \geq \cdots \geq |\tilde{h}_k(N_r)|^2$. Let $B_k$ represent the number of non-zero $\phi_{j,k}$’s, then $\Phi_k = \text{diag} [\phi_{1,k}, \ldots, \phi_{B_k,k}, 0_{N_r-B_k}, \ldots, 0_{N_r,k}]$, where $B_k \leq N_r$. Next, based on the power splitting constraint, we know that $\xi$ is chosen so that

$$\sum_{b=1}^{B_k} \phi_{b,k} = \sum_{b=1}^{B_k} \xi - \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(b)|^2 \delta_b} = N_r.$$  

(16)

Inverting (16), expressing it as a function of $\xi$, and substituting into (15) we get

$$\phi_{j,k} = \left[ \frac{1}{B_k} + \frac{1}{B_k} \sum_{b=1}^{B_k} \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(b)|^2 \delta_b} - \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(j)|^2 \delta_j} \right]^+.$$  

(17)

Up to this point, we still need to find a value for $B_k$. To find the optimal value for $B_k$, we set $\tilde{h}_k(j)$ and $\delta_j$ to $\tilde{h}_k(B_k)$ and $\delta_{B_k}$, respectively, then test the following inequality

$$\frac{1}{B_k} + \frac{1}{B_k} \sum_{b=1}^{B_k} \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(b)|^2 \delta_b} - \frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(B_k)|^2 \delta_{B_k}} > 1$$

(18)

for $B_k = 1, \ldots, N_r$. Thus, the optimum value for $B_k$ is the largest value that satisfies the inequality, and signal transmission utilizing a number of beams that is greater than $B_k$ will incur a loss in the potential performance gain.

Now the optimization of (13) with respect to $\Phi_k$ for a given $\Omega_k$ and subject to a power constraint can be written as

$$J \leq \prod_{b=1}^{B_k} \left( 1 + \frac{\varepsilon_s N_t}{N_r \sigma_k^2} |\tilde{h}_k(b)|^2 \delta_{b,\phi_{b,k}} \right)$$

with equality achieved if and only if $\Phi_k$ is chosen as $\Phi_k = V \Phi_k V^H$, where $W = V$ and $U_k = \Phi_k$. Hence, $\mu_{j,k} = \sqrt{\phi_{j,k}}$. This shows that signal reception should be in the eigen-modes of the channel covariance matrix and effectively transforms the SIMO channel configuration into a set of $B_k$ parallel and independent subchannels with the $j^{th}$ subchannel having a gain of $|\tilde{h}_k(j)|^2 \delta_{j,\phi_{j,k}}$. In case when the previous assumption $|\tilde{h}_k(1)|^2 \geq |\tilde{h}_k(2)|^2 \geq \cdots \geq |\tilde{h}_k(N_r)|^2$ does not hold, then $\frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(j)|^2 \delta_j}$ may not be valid. Hence, (16)-(18) will no longer be applicable since $b$ is indexing $|\tilde{h}_k(j)|^2$’s in a non-descending order. Without going to the extend of rewriting all (16)-(18) for this case, we can first re-arrange $\frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(j)|^2 \delta_j}$’s in a non-descending order. This guarantees that $\phi_1 \geq \phi_2 \geq \cdots \geq \phi_{N_r}$. Use $b$ to index $\frac{N_r \sigma_k^2}{\varepsilon_s |\tilde{h}_k(j)|^2 \delta_j}$’s that are less than $\xi$ (as shown in Fig. 1) instead of $\delta_j$’s, such that $\phi_{j,k}$’s and the optimum $B_k$ can still be found by using (17) and (18), respectively. Then signal reception is now in the directions of $B_k$ eigen-beams (not necessary corresponding to the first $B_k$ eigenvalues) that give highest instantaneous received SNR gain.

### IV. Numerical Results

In this Section we provide bit-error-rate (BER) and symbol-error-rate (SER) curves for the proposed transmission schemes in broadband frequency-selective channels. In our simulation, the following parameters and assumptions were adopted: the spatial channel correlation is modelled using the space-time channel with hyperbolically distributed scatterers in [18], $N_c = 512$, $N_t = 1$ (except for ST-OFDM, where two transmit antennas were used ), and QPSK baseband modulation is employed for both Figs. 5 and 6. In Fig. 5, we plot simulation results for systems with two receive antennas and different diversity coding schemes, i.e., OFDM with Alamouti’s
time block code (ST-OFDM), FT encoding with Alamouti’s space-time block code (FT-OFDM), and no coding (uncoded-OFDM). Comparing the curves that correspond to these three schemes, it is clear that both ST-OFDM and FT-OFDM systems give significant error-rate improvement over the uncoded-OFDM. Even though the FT-OFDM gives an approximately 1-dB performance degradation with respect to the ST-OFDM scheme, it remains as an attractive scheme for hand-held terminals with only one antenna.

In Fig. 6, we showed performance curves of the proposed adaptive transceiver structure (FT-Beam) for OFDM systems with different number of receive antennas. By comparing the results for systems with \( N_r = 2 \) in Fig. 6 to the results in Fig. 5, it is clear that the error-rate performance of FT-encoded OFDM system has improved as a result of adding the receive beamforming scheme, where it is now quite comparable to ST-OFDM systems with two transmit antennas. Amongst the three sets of curves in Fig. 6, we see that the performance gain from adding an additional receive antenna is larger for systems with \( N_r = 2 \) than systems with \( N_r = 3 \). However, it is observed that better performance curves can be obtained in channels with higher spatial correlations for FT-Beam structure.

V. CONCLUSIONS

An adaptive transceiver structure that combines a new diversity coding scheme and receiver beamforming for uplink SIMO-OFDM transmission is investigated. By utilizing the concept of subchannel grouping, FT coding that provides frequency diversity in the broadband wireless channel has proven to be an effective means of signal transmission for MS with a single antenna. It is shown that adaptive eigenbeamforming at the receiver (to handle uplink signals in the eigen-modes of the correlation matrix) maximizes the received SNR and improves the system error-rate performance of the FT-coded OFDM system.

REFERENCES