The Enhancement of Atmospheric Drag Prediction Using Space-Tracking Data for Accurate Debris Surveillance and Collision Warning

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

Bobby Lut Yin Wong

B.Geomatic Engineering / B.Planning and Design (Property and construction) (Hons)

School of Mathematical and Geospatial Sciences
College of Science Engineering and Health
RMIT University

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged.

Bobby Lut Yin Wong
February 2012
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Abstract

This research investigates the temporal variation of the coefficient of atmospheric drag value ($C_D$) and its subsequent effects on orbit prediction. Atmospheric drag is one of the most dominant forces exerted to space objects at altitudes below approximately 1500 km in the Low Earth Orbit (LEO). Its accuracy is governed primarily by the accuracy of the atmospheric mass density modelling, area-to-mass ratio and the $C_D$ value.

In the near-earth environment, there has been an exponential growth in the amount of space debris in the past half a century and this number is anticipated to further escalate due to the large number of planned space missions in the future. Hence, space surveillance and collision warning are now becoming integral elements for providing a safe space environment. Traditionally, the tracking of space objects is conducted using the radar method and the optical approach. Recently, Satellite Laser Ranging (SLR) technique is being developed to track space debris. However, the ability and capacity of the space surveillance and collision warning services are still limited by the most fundamental problem of unable to accurately predict the motion of space objects, which is largely due to the insufficient accuracy of determining the atmospheric drag.

The focus of this research is to investigate viable approaches to enhance the prediction of the $C_D$ value for higher accuracy prediction of orbits of space objects. The conventional $C_D$ value prediction approaches, i.e., the fix 2.2 $C_D$ and variable $C_D$ methods, are investigated. The more accurate variable $C_D$ approach has presented a repetitive cyclical change in the estimated $C_D$ values over the study period from 2004 to 2006 using Stella as the experimental satellite. This suggests a different scenario to the fixed value of 2.2 approach commonly adopted by the space industry. Therefore, optimal approaches to enhance the prediction for the $C_D$ value are explored.
Due to the repetitive cycle of the $C_D$ variations, Fourier series are selected to fit the estimated $C_D$ values over the study period. The fitting function is extrapolated to predict $C_D$ values for 2007, which are subsequently applied to the orbit prediction process using the fix $C_D$ value method. This implies that the predicted $C_D$ values are pre-determined prior to the orbit determination and prediction, similar to the fix 2.2 method that adopts a fixed value of 2.2. The orbit prediction results using the fitting function have demonstrated significant improvements over the traditional fixed 2.2 $C_D$ value method. The fitting function approach is also verified by performing the same experiments to satellites Starlette and ERS-2, where noticeable improvements in the orbit predictions are also achieved.

For orbit prediction, the fixed value method has shown to be more computationally efficient since approximate 20% reduction in data processing time is achieved compared to the more accurate variable $C_D$ approach. This is one of the fundamental reasons for the space industry to adopt the fixed value method, especially when timely prediction of orbits is the primary goal to many orbit applications.

This research has presented the fitting function approach for $C_D$ value prediction and the results have demonstrated that higher accuracy orbit predictions without degradation to the efficiency are achieved compared to the fix 2.2 $C_D$ method. Thus, this research will provide a valuable performance assessment of the conventional and the fitting function $C_D$ value estimation/prediction approaches for atmospheric research. In addition, it will also offer constructive guidance to minimise the limitations currently confronted by the space debris tracking, specifically atmospheric drag prediction.
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Acronyms

ADM Atmospheric Density Model(s)
ADR Active Debris Removal
BIPM Bureau International des Poids et Mesures
\( C_D \) Coefficient of Drag
CIS Conventional Inertial System
CNES Centre National d'Etudes Spatiales
COSPAR Committee on Space Research
CTS Conventional Terrestrial Reference
DISCOS Database and Information System Characterising Objects in Space
DTM Drag Temperature Model
ENVISAT Environmental Satellite
EOP Earth Orientation Parameters
ERS European Remote Sensing
ESA European Space Agency
Fix2.2 Fixed coefficient of drag value of 2.2
GOME Global Ozone Monitoring Equipment
GORID Geostationary Orbit Impact Detector
GPS Global Positioning System
HASDM High Accuracy Satellite Drag Model
HEO High Earth Orbit
IADC Inter-Agency Space Debris Coordination Committee
IERS International Earth Rotation and Reference Systems Services
ILRS International Laser Ranging Services
ISS International Space Station
ITRF International Terrestrial Reference Frame
LEGEND LEO-to-GEO Environment Debris model
LEO Low Earth Orbit
LLR Lunar Laser Ranging
MASTER Meteoroid and Space Debris Terrestrial Environment Reference
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>MSIS</td>
<td>Mass-Spectrometer-Incoherent-Scatter</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>PMD</td>
<td>Postmission Disposal</td>
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<tr>
<td>POD</td>
<td>Precise Orbit Determination</td>
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<tr>
<td>PRARE</td>
<td>Precise Range and Range-rate Equipment</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RRA</td>
<td>Retroreflector Arrays</td>
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<tr>
<td>SLR</td>
<td>Satellite Laser Ranging</td>
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<tr>
<td>SSN</td>
<td>Space Surveillance Network</td>
</tr>
<tr>
<td>TAI</td>
<td>Temps Atomique International (International Atomic Time)</td>
</tr>
<tr>
<td>TIRV</td>
<td>Tuned Inter-Range Vector</td>
</tr>
<tr>
<td>USSTRATCOM</td>
<td>United States Strategic Command</td>
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<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
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<tr>
<td>VaryC$_D$</td>
<td>Variable Coefficient of Drag</td>
</tr>
<tr>
<td>VLBI</td>
<td>Very Long Baseline Interferometry</td>
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Chapter 1 - Introduction

1.1. Background

Space debris surveillance and situation awareness is one of the integral components for a safe space mission. It involves providing accurate and timely debris information to facilitate precise monitoring and prediction of the space environment. In the near-earth space, there are over 10,000 space debris greater than one centimetre and countless smaller debris that cannot be tracked due to the limitations of the current space technologies (Greene, 2002; Wu et al., 2011). This poses an increasingly significant threat to the space applications, because satellites are vulnerable to collisions with space debris if the debris are not precisely monitored (Kessler, 1990; Swinerd et al., 1997; Anselmo et al., 1999; Johnson, 2004; Swinerd et al., 2004; Wei and Yan, 2007; Flohrer et al., 2009; Xu et al., 2009). Furthermore, this problem is anticipated to be more severe in the near future due to the escalating numbers of planned space missions because a proportional amount of debris generated from the missions is expected (Krag et al., 2000; Walker and Martin, 2004; Anselmo and Pardini, 2008). A recent near-miss event was recorded in late March 2011 where the International Space Station (ISS) was due to collide with a fragment from another collision that occurred in February 2009 between two satellites, Cosmos 2251, a Russian communication satellite and U.S. Iridium 33 communications satellite. The ISS was forced to perform an evasive manoeuvre to avoid collision between the ISS and the fragment as the collision risk was predicted to exceed the 1 in 10,000 threshold. This demonstrates the importance of space debris surveillance for collision avoidance (NASA, 2011a).
Continuous efforts to improve existing and identify new approaches for mitigating of space debris collisions have been attempted by the space industry, such as using breakup models and future estimate models to predict the movement and population of space debris, innovative material designs to improve shielding for satellites, collision avoidance manoeuvring and de-orbiting of unused space objects (Dietrich, 1997; Johnson et al., 2001; Pardini and Anselmo, 2001b; Alarcon-Rodrıiguez et al., 2004; Taylor, 2006; Chobotov et al., 2009). One of the core space debris researches is to track and predict movements of the space objects at a certain level of accuracy to mitigate debris-to-space-vehicle collisions (Sang and Smith, 2011).

Traditionally, radar and optical approaches are the two most commonly used methods to track space debris, but even with the advent of the Satellite Laser Ranging (SLR) technology, space debris surveillance is still constrained by the difficulties of accurate space object orbit prediction due to inaccurate atmospheric drag determination (Lee and Alfriend, 2000; Willis et al., 2005; Wong et al., 2007). Other tracking techniques are also often used for space object tracking, however, some of these techniques require additional devices mounted on the subject under surveillance (which cannot be applied for space debris application), such as using accelerometers (Zhang et al., 2006; Menvielle et al., 2007; Helleputte et al., 2009). Accelerometers are devices that measure the actual acceleration of the subject in relation to the actual orbit path. Orbital path with additional accelerometer data are often compared with the intended path from other Precise Orbit Determination (POD) approaches, the differences between the intended path and actual measured path using accelerometers are possibly caused by other atmospheric events, such as, thermospheric densities and/or thermospheric wind, density model corrections and other short period temporal variations (Doornbos et al., 2007; Tapley et al., 2007; Cheng et al., 2008; Volkov et al., 2008).
The most commonly used method for determining the motion or orbit of space objects is based on measurements and estimations of the perturbing forces encountered by the space objects, such as the solar radiation pressure, the ocean and Earth tides, the geopotential force and the atmospheric drag (Walterscheid, 1989; Harwood and Swinerd, 1997; Knowles et al., 2001; Lean et al., 2003; Forbes et al., 2007). Due to the insufficient accuracies of the force estimates, especially the atmospheric drag, it causes great difficulties to predict orbits of space objects accurately (Kechichian, 1990; Marcos et al., 2003; Bezděk and Vokrouhlický, 2004).

Atmospheric drag is one of the strongest non-gravitational forces affecting the motion of space objects, particularly for Low Earth Orbit (LEO) space objects (Sang and Zhang, 2010). Modelling the drag force uses three prominent variables: the atmospheric mass density, the coefficient of drag (the $C_D$ value) and the area-to-mass ratio of the space object. Hence, the inaccuracy of atmospheric drag determination is predominantly caused by the errors from all these three contributing factors (Bowman et al., 2008).

Over the years, researches have demonstrated a high correlation between the $C_D$ value, area-to-mass ratio and the air mass density (Williams and Trw Systems Group, 1972; Hedin, 1992; McLaughlin et al., 2011). However, due to the complexity of the relationship between these variables, no definite solution has been developed to accurately determine the $C_D$ value for different space objects (Brundin, 1963; Batyr et al., 1993; Prasad et al., 1995). Traditionally, the $C_D$ value is considered in two senses, the first is to absorb errors associated with the area-to-mass ratio and the mass density and thus, errors in the mismodelling of these two factors are reflected in the $C_D$ value (Nicholas et al., 2003; Xavier James Raj and Sharma, 2006; Pardini and Anselmo, 2008).
To calculate the atmospheric mass density, Atmospheric Density Models (ADM) have been established by researchers over the world. The ADM are established based on a number of factors, such as solar activity, atmospheric constituents, altitudes, temperature, time, location, geomagnetic indices and other factors (Bowman, 2001). These factors are formulated into a diffusion equation as shown by Jacchia (Jacchia et al., 1968; Jacchia, 1977). Many of the currently available ADM take account most of these factors, e.g., the commonly adopted MSIS-86 and DTM-78 models (Mayr et al., 1985; Marcos, 1990; Hedin, 1991; Berger et al., 1998; Doornbos et al., 2002; Picone, 2002; Akins et al., 2003; Bruinsma et al., 2003). However, there is a 15% or more error in the developed ADM, which will subsequently contribute to inaccuracy of orbit determination and prediction (Nicholas et al., 2000; Yurasov et al., 2005; Santoni et al., 2010; Volkov et al., 2011). Therefore, different techniques were developed to improve the accuracy of ADM, such as modelling the errors of ADM using space tracking data and incorporating the error model to the ADM to improve the accuracy of atmospheric predictions (Shum et al., 1986; Nazarenko et al., 1991; Sang et al., 2012). This approach compares the intended orbital path against the space tracking data to obtain corrections for the atmospheric mass model (Granholm, 2000; Picone et al., 2005; Yurasov et al., 2008). The differences between the intended orbit and the tracking data, or the errors, are assumed to be caused mainly by atmospheric density, this assumption is heavily reliant on highly accurate determination of the ballistic coefficient (Roberts, 2000; Bergstrom et al., 2001; Walker, 2001; Bock, 2003; Yurasov et al., 2004; Doornbos et al., 2005; Graziano, 2007).

The ballistic value is a measure of the $C_D$ value and the area-to-mass ratio of the satellite. The area refers to the cross-sectional area of the space object in its direction of motion where it is subject to most of the drag force. However, accurate determination of the ballistic coefficient is, at times, complicated, due to the variation of the cross-sectional area while the space object rotates in orbit (Anz-Meador et al., 1995; Cefola et al., 1999b; Raitses et al., 1999). In general, it is a common practice to assume the $C_D$ to be a fixed value of 2.2, but this figure may only be valid for spherical objects since the area-to-mass ratio is consistently
known (Cook, 1965; Bowman, 2002). Thus, it is also important to consider the relationship between the shape of satellites and the $C_D$ value to enhance the determination of the ballistic coefficient (Moe et al., 1998; Wu et al., 2011).

The $C_D$ value can also be determined as the interaction between the atmospheric particles and the impinging surface materials through estimating the physical properties of the atmosphere (Harrison and Swinerd, 1995; Harrison and Swinerd, 1996). This includes accounting for the air molecules striking the surface and leaving the surface of the space object, modelled by the energy accommodation coefficient (Williams, 1975; Collins and Knox, 1994; Gaposchkin, 1994; Moe et al., 1996). The accommodation coefficient has shown to increase with altitude, i.e., energy is absorbed at lower altitudes such as 200 km and diffused at higher altitudes, which suggests drag force being more dominant at lower altitudes (Koppenwallner, 2008; Pardini et al., 2010).

The ability to represent atmospheric conditions is one of the crucial factors affecting accurate orbit determination and prediction. However, without sufficient spatial data, our currently achievable orbit prediction accuracy is still insufficient to satisfy some of our space industry's requirements.

### 1.2. Project and Motivation

This research is sponsored by the Australian Research Council (ARC) and industry partner EOS Space Systems through ARC Linkage Scheme. The motivation of this research is to enhance the ability to track objects in space, especially space debris. This is due to the escalating amount of space missions causing an exponential growth in the generated space debris and as a result, immensely increasing the possibilities of debris-to-space-vehicle collisions (Klinkrad et al., 2001; Lyndon, 2006; Carrico et al., 2008). This poses a significant threat to the multi-billion dollars space industry and more importantly,
it endangers the lives of astronauts. Therefore, it is an on-going task to seek improvements in approaches for accurate determination and prediction of orbits of space objects.

Nowadays, the dynamic orbit determination method is commonly used for orbit determination and prediction. In this method, it accounts for all knowingly significant forces affecting the motion of space objects. Atmospheric drag is one of the most dominating forces in the LEO hindering the accuracy of space object orbit trajectory estimation and it is also one of the most difficult forces to be predicted accurately (Santora, 1975; Saad et al., 2008).

1.3. Research Aims, Scopes and Questions

The aim of this research is to investigate the variations in the coefficient of drag ($C_D$ value) with time (caused primarily by the inaccuracies of ADM modelling) and explore approaches to enhance the prediction for the $C_D$ value, which is ultimately adopted for accurate orbit prediction for space objects. The research work is separated into two distinct parts. The first part is to assess the current techniques for the estimation of the $C_D$ value, which is the commonly adopted constant $C_D$ value of 2.2 and the variable $C_D$ value approaches. The second part is to identify new methods for $C_D$ value prediction. The performance of the new approach is assessed by comparisons of orbit prediction against the traditional approaches used for satellite orbit prediction.

The objectives of this research are: 1) to investigate the variation in the $C_D$ value over a study period, 2) to assess the effects of the variation in the $C_D$ value on orbit prediction, 3) to establish a more accurate approach for the prediction of the $C_D$ value and 4) to validate and evaluate the new approach using the predicted orbits from different satellites.
The following questions are formulated to achieve the above objectives of this research:

1. What is the accuracy of orbit prediction using the commonly adopted fixed $C_D$ value of 2.2?

2. How does the $C_D$ value vary with time? Is there an obvious trend in the variation?

3. What effects do Atmospheric Density Models (ADM) have on the $C_D$ value and subsequently, the orbit prediction?

4. How does the constantly varying area-to-mass ratio of a satellite affect the $C_D$ value? What are the subsequent effects on the orbit prediction?

5. Are there other superior approaches to enhance the prediction of the $C_D$ value? If so, how much improvement is achievable against the fixed value method (when $C_D = 2.2$)?

6. Can the fitting functions, from the new approach, derived using data from one satellite be applied to other satellites for their orbit predictions? If so, what is the achievable accuracy?

7. What are the accuracies when other satellites adopt the new fitting function approach using their own data? Are there major differences in the predicted $C_D$ values and orbit prediction when the new approach is adopted to spherical satellites against non-spherical satellites?
1.4. Research Contributions

The enhancement of the prediction of the $C_D$ value for accurate determination of the atmospheric drag is of great significance to the improvements of orbit prediction. The $C_D$ value is also one of the most challenging research areas in atmospheric drag due to the nature of the parameter, which is primarily used for absorbing errors in the atmospheric drag function. Hence, the focus of this research is to investigate new approaches for enhancing the prediction of the $C_D$ value. The significant contributions of this research are summarised as follows:

- The differences in the currently available approaches for estimating the $C_D$ value are evaluated, specifically comparisons of the $C_D$ estimates from the variable $C_D$ approach and the fixed value method (when $C_D = 2.2$). The magnitude of error is identified for each of the approaches when measured against an assumed "true" value from an independent source of measurement (i.e., International Laser Ranging Services (ILRS)).

- The relationship between the constantly varying area-to-mass ratio and the $C_D$ value is assessed by comparing the estimated $C_D$ values and orbit predictions from satellites in spherical and non-spherical shapes.

- The effects of ADM on the $C_D$ estimates are evaluated by comparing two of the commonly used ADM in space atmospheric research, MSIS-86 and DTM-78. The performances of the two models are evaluated in a comparison of the relative effects on the $C_D$ estimates and orbit prediction when the two ADM are adopted.

- Approaches to enhance the prediction of the $C_D$ value are investigated using Stella as an experimental satellite. Different functions are fitted to the datasets of $C_D$ estimates over a period of three years and the fitting functions are used to extrapolate $C_D$ values for the year following the study period. The optimal fitting function is selected based on prediction of the $C_D$ values closest to the $C_D$ estimates from the more accurate variable $C_D$ approach.
• The $C_D$ values predicted by the optimal fitting function are validated by orbit prediction, where the performance of the function is assessed against the assumed "true" values. Two other satellites Starlette and ERS-2 are also selected to test the optimal fitting function, based on $C_D$ estimates from Stella, to evaluate the applicability of the function to other satellites that are at similar altitudes.

• The fitting functions to enhance the prediction of the $C_D$ value are further investigated by applying the same experiment to two different satellites, Starlette and ERS-2. The approach to adopt ideal fitting functions to enhance the prediction of the $C_D$ value and ultimately improve the orbit prediction is validated.

1.5. Thesis Outline

This thesis consists of seven chapters and the outline of these chapters is as follows:

**Chapter 1 - Introduction** - presents an overview of the research topics outlining the motivation, aim and objectives, and contributions of this research.

**Chapter 2 - Space debris and satellite selections** - provides an analysis of the current status in space debris research, as well as different types of tracking techniques and their associated error sources. The satellites selected for testing and the limitations of this research are also discussed.

**Chapter 3 - Motion of space objects and perturbing forces** - explores the theory of motion in space and the associated perturbing forces. The effects of atmospheric drag are explored in details.
Chapter 4 - Methodology for $C_D$ value modelling - presents the framework and procedures for a number of case studies aimed at resolving the objectives outlined in Chapter 1. The limitations and assumptions of the case studies as well as the procedures for the orbit determination and prediction software are also outlined.

Chapter 5 - Characterising the $C_D$ estimates - case studies - investigates the trend of the $C_D$ estimates over the study periods. Orbit prediction results are assessed against results from an independent organisation, the ILRS. The impact of different ADM and the Ballistic value to the orbit determination process is studied and analysed. The results and other factors for consideration are also discussed.

Chapter 6 - The fitting function approach for the prediction of the $C_D$ value - provides the procedures for establishing the fitting function approach, selection of the optimal functions for $C_D$ value prediction and assessment of the orbit predictions. The fitting function approach is also validated using other satellites and the results, discussions and analyses are presented.

Chapter 7 - Summary, conclusions and recommendations - presents a summary of the findings, conclusions for each topic and recommendations for the future work.
Chapter 2 - Space Debris

2.1. Introduction

In this chapter, an overview of the capabilities and limitations of the current space debris surveillance systems and the observation techniques is presented. The methods for improving the accurate tracking of space debris and the mathematical models for the determination of motion of space objects are also investigated and analysed. This includes the examination of some of the most dominating parameters affecting the accuracy of debris tracking.

2.2. Space Debris Environment

Gravity is the strongest force of attraction among all the forces in nature. It is described as the mutual attraction force between two bodies of mass. This force is, in particular, dominant for space debris, due to the two-body interaction by attraction from the gravity of the Earth.

Some of the orbiting objects, such as satellites and space-vehicles/stations, have a designated orbital path with a rotation period and life expectancy. However, the functional satellites and space vehicles are constantly under the threat of collision with space debris due to the fact that a large number of space debris are orbiting around the Earth (Barrows et al., 1996; Anselmo et al., 1999; Mehrholz et al., 2002; Walker et al., 2002; Wright, 2007; Olmedo et al., 2009).
Space debris is defined as useless objects without a designated path of orbit around the Earth. Space debris consists of two types: natural and artificial. Natural space debris is mainly a collection of meteoroids from the solar system, which are natural objects attracted to orbit around the Earth due to gravity. In contrast, artificial space debris, also called man-made debris, is composed of disintegrated fragments from previous launches, such as disintegration of space-vehicle surfaces or wreckages from previous collisions and/or explosions. There are approximately hundreds of thousands of space debris floating around the near Earth surface, some of which are expected to stay in orbits for tens of years or even hundreds of years.

Conventionally, space debris can be divided into three groups in terms of sizes. The following three groups/categories of space debris are classified in terms of the effective cross-section area of the debris:

**Small** - diameter of debris < 1 mm. Satellites are designed with a protection shield on the surface to withstand debris in this category, but it may still cause noticeable damages and indestructible impairment to the unprotected components, such as the satellite's solar panels and antennas.

**Medium** - in the range of 1 - 100 mm in cross-sectional diameter. It is estimated that there are over 120,000 objects in this category (Greene, 2002). Likewise, with small sized debris, it can only be tracked by in-situ sampling, that is, by measuring the amount of degradation on the sensors retrieved from space, which will be discussed later on in this chapter. Debris in this category can cause damages to the protected components of the satellites, subsequently possibly abolishing partial capabilities of the satellite's function.

**Large** - cross-sectional diameter > 100 mm. Current available tracking techniques are capable of tracking debris in this category, some of
which are under the surveillance of the space debris tracking agencies. Collision with these large sized debris can cause significant damages to the satellite and in the worst case, it may lead to possible abortion of satellite mission.

2.3. Space Surveillance Agencies

In the past decades, the number of launched satellites has increased exponentially and the amount of space debris has also escalated proportionally. As a result, the accurate tracking of space debris has become one of the most critical issues for space safety. However, due to the fact that the tracking of space debris is an intricate and complex task, only space agencies with high-level expertise and technologies have the ability to undertake this never ending task. For example, during the early eras of the space race, only countries with such capabilities, such as the former Soviet Union and the United States of America, could undertake such an impervious task. Nowadays, more countries and industry sectors are increasingly reliant on the use of satellites. Many space agencies around the world, such as the European Space Agency (ESA) and the National Aeronautics and Space Administration (NASA) of the United States of America, have proven a success in contributing to this complicated task (Johnson, 2004).

ESA maintains one of the most comprehensive catalogues of all Earth orbiting space objects and their relevant characteristics using a database known as the Database and Information System Characterising Objects in Space (DISCOS). Figure 2-1 illustrates the progression in the numbers of space objects over the period of 1957 to 2008. From this figure, an exponential growth in the number of tracked space objects during this period is visible. By 2009, more than 12,000 objects had been tracked and maintained by the DISCOS catalogue (ESA, 2009c). This has demonstrated the strong correlation between the escalating number of debris in space and the increasing number of satellite missions. Consequently, this
implies an escalating danger for space missions due to the hazardous space environment, which leads to the demand for accurate tracking and prediction of the orbits of space objects (Chobotov et al., 2009).

![Figure 2-1: Catalogued space objects in the near Earth orbit by ESA (ESA, 2009c)](image)

The United States Strategic Command (USSTRATCOM) maintains a similar system to DISCOS, known as the Space Surveillance Network (SSN). The SSN is also renowned for its capability to maintain a database of space objects. The USSTRATCOM uses engineering models to simulate the space debris environment in the LEO where altitudes and areas of high risk are identified. Figure 2-2 demonstrates the density of space objects with respect to altitudes catalogued by NASA. It is evident from the figure that there is a high density of space objects in orbit at the approximate altitude of 750 km to 1000 km above the surface of the Earth. This is due to the high numbers of satellites launched to such altitudes, where a number of high-intensity explosions or breakups of the
spacecraft have also contributed to the debris population in this environment (Mcknight et al., 1993).

![Figure 2-2: The mass density of catalogued space objects with respect to altitudes (Liou, 2011b)](image)

The SSN is supported by more than 20 stations across the world. The stations are installed with different instruments, of which some are dedicated and others are collateral or contributing. The dedicated sensors are primarily designed for space surveillance missions and the stations that host the sensors can detect, track and collect information of the space objects of concern. The information collected by the dedicated sensors includes the accumulation and combination of data such as, the shape, size and orientation of the tracked space objects. In contrast, the collateral sensors assigns space object tracking as second priority, where the sensors are predominantly assigned with other primary tasks. The contributing sensors are owned by other space agencies, but can provide support upon request. Hence, the collateral and contributing sensors are known as auxiliary sensors. Only the dedicated sensors are the primary sensors that perform majority of the space tracking.
2.4. Methods for Space Debris Surveillance

A successful space surveillance system is underpinned by the ability to accurately track, monitor and even predict the orbit of space debris to a required level of accuracy. However, to accurately track space debris is an arduous and intricate task as many factors limit the ability of space objects tracking, especially space debris tracking. Despite the combination of the current available ground-based and space-based techniques, only a limited number of space objects, especially small sized space debris, can be tracked and catalogued due to the limitations of the existing space object tracking technology. Furthermore, the diminishing size and the exponential growth in the population of space fragments increase the complexity of the problem. In addition, the expensive operating and maintenance cost to upkeep the system is also another critical factor that limits the ability to track space objects.

Currently, the main available techniques for space debris tracking are ground-based and space-based. These two techniques will be discussed below.

2.4.1. Space-Based Techniques

The space-based technique refers to the system that uses sensors that are placed in space to observe the flux of space objects (Flohrer et al., 2011). The sensors positioned in space can observe space objects much closer than ground-based sensors without the need to observe through the atmosphere. It simplifies the complexity of characterising the tracked space debris, especially for debris in a high orbit. In addition, the commonly used space-based sampling method is the only approach that makes direct contact between the sensors and the tracked debris, so the structure and composition of the debris particle are recorded (NASA,
Other types of observations and approaches cannot perform this type of data recording.

The space-based technique has the following drawbacks: 1) the limited flexibility due to the inability to make adjustments after the sensors are launched into space. 2) the very high cost associated with the development and deployment of the sensors, including the significantly long planning process. 3) the complexity to transfer large volumes of data from the sensors to the Earth for the sensors with on-board processors. This poses an even more significant problem for the sensors without on-board processors for data processing. Thus, the ability to minimise the volume of data and storage of data is critical for this technique.

Despite the aforementioned disadvantages, this technique is still commonly adopted in space debris tracking. The reason for this is that some of the capabilities cannot be achieved by ground-based sensors, e.g., the recording of the actual impact of space debris to the sensors. Moreover, the space-based approach can observe objects in the High Earth Orbit (HEO) environment with less atmospheric interference as opposed to the ground-based observations (Flohrer et al., 2011).

Impact sampling is a technique whereby sensors are launched into the near Earth orbit. As the sensors orbit the Earth, space debris are expected to collide with sensors that are specially designed with exposed surfaces. Thus, the results are obtained by measuring the effects of the impact from the collisions and consequently, the hypervelocity impact and the debris particles can be resolved. This technique can track smaller sized debris, e.g. even debris with a diameter at the millimetre level, which cannot be tracked by the conventional ground-based methods (NASA, 1995). As shown in Figure 2-3, an example of an impact sensor used by ESA is called the Geostationary Orbit Impact Detector (GORID). The GORID sensor is a plasma type of detector launched to the geostationary orbit to detect the long-term temporal variation of the debris flux (Graps et al., 2005).
There are two types of impact sampling sensors, passive sensors and active sensors and they will be discussed below.

![Exposed surface inside the GORID sensor for detecting space debris (ESA, 2009a)](image)

**Figure 2-3:** Exposed surface inside the GORID sensor for detecting space debris (ESA, 2009a)

*Passive sensors*

Passive sensors refer to the typical observation method for this technique. The sensors are launched into space, typically at the altitudes of approximately 300 to 600 km, and after a specified period of exposure to space, the sensors are returned to the Earth (NASA, 1995). Thus, the data is only available after the return. The challenges of this approach lies in the requirement to calibrate the returned samples. Moreover, the returned data contain no reference to the time, velocity or the type of debris relevant to each of the collision. Hence, only an average number of collisions over the exposed period can be determined. Furthermore, as the sensors are expected to return to the Earth, the altitude of the orbit of the sensors are restricted, in other words, these sensors can only be deployed at a limited range of altitudes.
Active Sensors

Active sensors can overcome some of the limitations of passive sensors, such as the restrictions to the operating altitudes and inability to resolve the time discrepancy of the collision. In theory, the active sensors adopt the approach similar to that of passive sensors. However, the active sensors do not require the re-entry for data collection. The data retrieval process relies on the transmission from the on-board transmitters to the Earth. Hence, active sensors can measure the debris flux over time and over a diverse range of altitudes, but it is limited by the power of the signals from the sensors for data transmission to the Earth.

A diverse range of detectors can be fitted to measure debris collision, such as the comparatively cheaper and simple pressurised cells, or the more expensive and complicated plasma detectors. These detectors have the ability to record data (collision) with reference to time, velocity and particle constituents. Thus, they are very useful to monitor the debris environment migration with respect to time. As a consequence, the volume of data needed to transmit back to the Earth receivers increases, which poses a big problem for the detectors that do not have on-board analysers to process such data prior to the transmission to the Earth.

Although the cheaper passive sensors and the more complex active sensors can be used in combination, there are still some limitations. For instance, a sizeable amount of instruments adds significant weight to the detection system and the returned data need in-depth calibrations. Moreover, even with the ability to differentiate the type of materials in the impinging debris and the impact velocity, the relevant pre-impact information, such as the pre-impact orbit cannot be recorded. Furthermore, the collisions can only be captured by confronting debris, which is limited by the cross-sectional area of the exposed surface. Hence, neither near-miss data nor its relevant information can be recorded.
2.4.2. Ground-Based Techniques

The ground-based technique uses ground-based sensors, i.e., ground-based observing instruments built and used on the Earth for the observation of space debris. Generally, this technique requires a network of sensors spread over the surface of the Earth to become effective. Since the early ages of space debris tracking, there are two traditional types of observation methods adopted, the optical and the radar techniques. In the recent years, rapid development in the new laser technologies permits laser monitoring as a valid technique for space debris surveillance. The two traditional types of ground-based observation methods will be elaborated in the following sections.

Optical observations
Telescopes are established across the world as a network array as data collection points for the optical ground-based approach. The principle of the technique is to observe the number of space objects, typically space debris fragments that passes cross the field of view of the telescope set in a particular observing direction. The objects can only be visualised when light is reflected from the sun onto the debris and in most cases, only 10% of the light is reflected from the debris. This means that in a dark environment, space objects are less visible, which is an enormous restriction or limitation on the effective operating hours of the optical sensors (United, 1999; Africano et al., 2004).

Missions from the past have identified that, in general, the optical technique can observe debris size down to approximately 10 cm in the LEO altitude (Schildknecht et al., 2004). However, one of the major shortcomings of this technique is its inability to differentiate the light reflected from the debris and light from meteors entering the atmosphere (Kessler and Jarvis, 2004). In order to address this issue, the parallax approach is adopted. In this approach, two telescopes are used to monitor the same objects so that the angular velocity and the altitude of the object can be resolved.
One of the important advantages of the optical approach is the comparatively lower operation cost in comparison to other ground-based techniques such as the radar method.

*Radar observations*

The radar observation theory is similar to that of the optical system. It measures the amount of debris that passes through its field of view. The radar beam is transmitted at the direction of interest and the observation is made when the signal is returned from the reflection of the subject (Goldstein et al., 1998). The governing parameters of the radar approach are the field of view and the power of the transmitter, as it limits the range between the transmitter and space debris. Moreover, this technique is hindered by the altitude and the shape of the subject because irregularities in the shape of the subject dramatically cause decreases in the reflectance values.

The main advantage of this technique is that it makes continuous observation possible and the observations are not restricted by the need for the reflecting light from the observed subjects, which is a pre-requisite for the optical approach. Continuous range observations over time enables increase in the volume of data, consequently, it improves the statistical confidence of the data collected, which in our research interest, the debris flux at a particular altitude (Stansbery et al., 1995). However, the continuous observations may mean high operation and maintenance costs, which includes a considerable monetary input to develop and deploy powerful sensors.

Both radar and optical techniques are often used in the ground-based technique for space object tracking. In fact, the space-based approaches can also use radar and optical observations, but it is costly and intricate, due predominately to the complication of the technology and the instruments involved. Moreover, it is a demanding task to calibrate the instrument and transmit the observed data back on to the Earth. Furthermore, there are significant limitations, for example, space-
based telescopes are capable of observing debris in LEO at a close range, but due to the closeness of the debris travelling at a high velocity in the near Earth environment, a significant noise factor may be introduced. Thus, the reduction or mitigation of the effects of noise may be a demanding task. However, the problem of the noise factor in the observation is more prominent for telescopes with a narrow degree of view.

2.4.3. Breakup Models

The precise monitoring and prediction of possible space objects including space debris surrounding the intended orbits of a space mission is very critical prior to space vehicle launches. The prediction should not be limited to existing space objects, but also for new debris fragments from potential collisions of current space debris (Johnson et al., 2001). Therefore, simulating the behaviour of debris during its collision with other objects and predicting the population of debris after the collision are vital (Loftus, 1989).

Several methods such as the Poisson Distribution, Distance of Closest Approach and Weibull Distribution can be used to assess the probability of the collision between a satellite and an object in space (Chobotov and Mains, 1999). However, in order to better estimate the vulnerability of collision of space debris to space vehicle, it is important to understand the characteristics of space debris itself. The characterisation of space debris, such as the mass, velocity and the ballistic coefficients are important parameters for breakup models (Barrows et al., 1996).

In fact, it is important to understand the process of the breakup and more importantly, the upshot of the debris created by the breakup model. The period after break-up is an essential parameter for the breakup modelling. The period is usually divided into two categories, short-term and long-term. The short-term implies immediately, in terms of days, after the breakup, whereas long-term refers
to the continual orbit of the debris and its future movements after the breakup (Wang, 2010). The basis of the breakup modelling is governed by some of its primary parameters, such as the cause and the process of the breakup and the predicted movements of the debris fragments newly generated from the breakup. However, due to the lack of experimental data for validation, refinements and improvements for the breakup models are only achievable by adding historical testing data.

An example of an incident that has caused an increase in space debris due to a breakup occurred on 21st April 2002, a diminutive space debris crashed into a 30-year old satellite at the altitude of approximately 1370 km, which created debris large enough to be tracked by the SSN (NASA, 2002). However, the debris was not catalogued until the 6th May that year. It was believed that it travelled at a velocity much greater than expected, at approximately 19 m/s since the collision. In addition, the debris originated at an altitude of 1370 km and within four weeks of the collision, the debris was in an eccentric orbit with an apogee of 1895 km and perigee of 750 km. On the 3rd June 2002, the debris de-orbited and re-entered the atmosphere due to atmospheric drag. This demonstrates the importance of understanding and identifying the cause of space debris, because the cause of the incident is the major determining factor for the direction, the velocity and the life expectancy of the newly created space debris. Therefore, in order to reduce the chance/risk of collisions with space debris, closely monitoring of orbit debris and the prediction of its future movement are a significant task for safety of space missions.

2.4.4. Future Debris Population Estimation

Historically, there is a minor chance in space debris collision with space vehicles. However, an exponential growth of space missions has proportionally increased the space debris population, as shown in the catalogued space debris in Figure 2-1
and Figure 2-2. Thus, it is a reasonable prediction to expect a rapid and continual growth in space debris (Contant, 2000). The large population of debris poses a high risk for space vehicles. Hence, accurate models for estimating or predicting the growth of future space debris are needed for a safe operating space environment (Rossi et al., 1997; Bendisch et al., 2004; Martin et al., 2004; Stabroth et al., 2006).

The prediction of debris population requires not only the knowledge of the current debris population but also the estimated future launches, the rate of debris inter-collisions and the accuracy of the break up models (Loftus, 1989). In addition, the accuracy of the future debris population growth is affected by the following components:

- Future constellation deployments
- Rate of future space vehicle traffic developments
- De-orbit control
- Dynamicity of the evolution of space debris under various perturbing forces

NASA has produced a prediction of the number of objects that will orbit around the Earth for the next two centuries using the orbital debris prediction software package, LEGEND and EVOLVE (Eichler and Reynolds, 1997; Krisko et al., 2001; Krisko, 2004). A similar prediction was also conducted by ESA using the prediction software package called MASTER (Klinkrad et al., 1995; Klinkrad and Sdunnus, 1997; Krag et al., 2001; Lewis et al., 2001; Bendisch et al., 2002). The prediction results from NASA using the LEGEND software as illustrated in Figure 2-4 predict that the total amount of space debris is still increasing even with post-mission disposal (PMD) and active debris removal (ADR) implemented, although a much slower rate is evident. The space industry realises the catastrophic impact of debris collisions and the very limited active approaches to minimise space debris. Consequently, immense efforts are invested to more accurately monitor the space debris environment and developing space debris
mitigation methods (Klinkrad et al., 2004). Furthermore, the advancements in technology also contribute to minimising space debris because more preventive measures are possible, such as higher accuracy space debris modelling to avoid collision and using stronger materials to reduce impact and fragmentation for space vehicles.

Figure 2-4: Prediction of the number of space debris by NASA's LEGEND program (Liou, 2011a)

2.4.5. Space Debris Mitigation Measures

Space debris tracking is an essential part of surveillance of the space environment and the tracking information is vital for future mission planning. However, it is an intensive and costly operation to find, track and maintain a catalogue of space debris. Thus, it is important to investigate methods or measures to address this ongoing problem (Walker et al., 2001; Finkleman, 2005). Although the current space debris population may not be at a highly dangerous level, but the escalating
number of space debris adds potential risks to future collisions. The recognition of this problem has encouraged space agencies to investigate mitigation measures for safe space operation for the future (Reynolds et al., 1997; Gottlieb et al., 2001; Lewis et al., 2001; Alby et al., 2004).

Investigations into the debris mitigation measures have been conducted and are still on going to minimise the exponential growth of space debris over time. The Inter-Agency Debris Coordination Committee (IADC) is one of the world leading technical agencies, with members from various countries, aimed to mitigate space debris (IADC, 2002; IADC, 2010). It assembles the key users of the space environment over the world and focuses on collaborating information in space debris research and space debris mitigation measures. Recommendations and guidelines are also established to minimise and control the progression of the growth of space debris.

Over the years, many methods for the mitigation of space debris have been proven successful. However, there are still some limitations in these methods. For instance, the natural removal of debris by atmospheric drag is the only natural mitigation method that has proven to be successful, but the time involved in the process is relatively long and the relocation of debris to different altitudes is required (Pardini et al., 2007). Several other measures for space debris mitigation are discussed below.

*Improvement in designs and materials*

The improvement in the design of space vehicles to withstand or reduce possible debris impact can reduce the disintegration of space vehicle fragments during debris collision. For example, minimising unprotected areas of the satellite, especially large size antennas and solar panels, without diminishing the functionality of the satellite should be considered in the design phase. Nowadays, new materials are used in modern satellites designs to shield against space debris and it has proven to be a successful measure to reduce the damages caused by debris collisions. Intensive testings are carried out by ESA for the characterisation
and testing of the new materials, which includes improving the experimental facilities to replicate the actual orbital environment.

Figure 2-5 illustrates a controlled laboratory test conducted by ESA to test materials used to shield against space debris contact, this particular test was conducted to understand the effects of separating layers of thin materials at particular distance apart to reduce the impact of space debris. Such design is lightweight and is currently adopted by many of the new satellites.

![Thin materials layered apart to counteract space debris collisions (ESA, 2009b)](image)

**Figure 2-5:** Thin materials layered apart to counteract space debris collisions (ESA, 2009b)

*Orbit Manoeuvring*

Manoeuvring the orbit of unused satellites or rocket bodies can also reduce the amount of waste in an operational orbit (Graziano, 2007). The re-orbiting of unused satellites to higher altitudes that are currently unused is a short-term solution to the problem until the currently unused orbit becomes an operational orbit, where the re-orbited satellites or rocket bodies need to be dealt with again. Therefore, an innovative approach, i.e., the approach of de-orbiting unused
saturates using a solar sail type of device is currently under investigation. For example, NASA deployed a 100-square foot sail device in the experimental satellite NanoSail-D launched on the 20th January 2011 as shown in Figure 2-6. The satellite is designed to deploy the sail for de-orbiting to re-enter the Earth's atmosphere and totally burning up during the re-entry. The purpose of the experiment by NASA is to demonstrate the capability of de-orbit for space objects with low mass but large surface area ratio, where this approach can be applied by small satellites and space debris. This approach is one of the very limited methods for eliminating space debris (Newton, 2011).

Collision avoidance manoeuvring
Collision avoidance manoeuvring, also called evasive manoeuvring, is another common practice in the orbit manoeuvring technique. It involves collision warning systems that provide warnings to a near approach of other space objects. This technique needs an extensive catalogue of space debris with continuous monitoring of space debris. An example of this type of orbital manoeuvring is the one conducted by the Earth Remote Sensing Satellite (ERS) in June 1997, after the collision warning of a near-miss distance of 130 metres was detected, the
The satellite was manoeuvred to 4 km above the near-miss location and re-orbited to its original orbit the day after the event (NASA, 1997). However, this technique can only be employed by satellites that are capable of being manoeuvred without causing any risks to the spacecraft or the space mission's objectives.

2.4.6. Mathematical models

Traditionally, the amount of space debris is estimated using tracking techniques to monitor debris flux over a period of time in a specified area. This data is then stochastically modelled to predict the space environment. With the use of other numerical models, e.g., the breakup models, the predictions can be improved. This approach is particularly precise when there are only nominal amount of debris breakup and the prediction area is very limited. Nonetheless, when the whole debris environment surrounding the near Earth orbit is predicted, the amount of computation is vastly amplified due to the increasing amount of debris breakup and fragmentation. To overcome the problem of the excessive computation, simplified breakup models are developed and used in combination with traditional tracking techniques.

In contrast to implementing the combination of tracking data and breakup models, continuum mechanics are adopted to the prediction of the whole debris environment, such approach is adopted by the Russians in their debris prediction program. This method has three major components: spatial distribution of debris over time, different types of fragment scenarios and information of possible future launches. This approach requires accurate information of the characteristics of the debris in order to model it more accurately using continuum mechanics. However, it is difficult to obtain such information, especially with the limited knowledge of smaller sized debris or debris too small to be tracked.
The theory of continuum mechanics is based on the prediction of mass exchange caused by either growth or reduction of debris from events, such as new space launches or debris collision. The formula of mass exchange can be expressed as (Smirnov, 2002):

\[ I_j = \sum_{k=1}^{N} K_{jk} + M_{j\text{op}} + M_{j\text{ex}} - \mu_j \]  

(2-1)

where,

- \( I_j \) - the mass exchange of the j-th phase
- \( K_{jk} \) - the mass flux from k-th to j-th phase caused by collision breakup of particles
- \( M_{j\text{op}} \) - the mass contribution due to new space launches
- \( M_{j\text{ex}} \) - the mass flux caused by disintegration of large objects in space
- \( \mu_j \) - the mass decrease

As illustrated above, the principle of continuum mechanics is the measurement of the distribution of particles. These particles however, are constantly under the pressure from other perturbing forces, so they are constantly in motion. Hence, it is equally as important to understand the motion of the particles within the frames of transition (Smirnov, 2002):

\[ \frac{\partial (\bar{\rho}_j \vec{v}_j)}{\partial t} + \text{div} (\bar{\rho}_j \vec{v}_j \otimes \vec{v}_j) = \vec{F}_j + \vec{R} + \vec{F}_{dj} + \vec{K}_j \]  

(2-2)

where,

- \( \bar{\rho}_j \) - distributed mass density of debris particles of the j-th phase
- \( \vec{v}_j \) - local velocity of the j-th phase
- \( \vec{F}_j \) - the mass force
- \( \vec{F}_{dj} \) - the atmospheric drag
- \( \vec{P}_j \) - the solar radiation pressure
\( \vec{K}_j \) - the momentum flux to the j-th phase due to the mass exchange \( I_j \)

It is evident from Equation (2-2), that the mass force, atmospheric drag, solar radiation pressure are three major forces that determine the movements of space objects, including space debris.

The momentum flux, \( \vec{K}_j \), is expressed in Equation (2-3), which defines the mass exchange (Smirnov, 2002):

\[
\vec{K}_j = \sum_{k=1}^{N_p} K_{jk} \vec{v}_{jk} + M_{j \text{op}} \vec{v}_j + M_{j \text{ex}} \vec{v}_j - \mu_j \vec{v}_j
\]  

(2-3)

where,

- \( K_{jk} \) - the mass flux from k-th to j-th phase caused by collision breakup of particles
- \( \vec{v}_{jk} \) - the velocity of the particles from the k-th to j-th phase
- \( M_{j \text{op}} \) - the mass contribution due to new space launches
- \( M_{j \text{ex}} \) - the mass flux caused by disintegration of large objects in space
- \( \mu_j \) - the mass decrease

The mean value for the term of the volumetric mass force term from Equation (2-2) can be expressed as follows, where \( g(\vec{x}) \) is the acceleration due to gravity and \( \vec{e}_r \) is the radial basis vector in a spherical system of coordinates (Smirnov, 2002):

\[
\vec{F}_j = -\rho_j g(\vec{x})\vec{e}_r
\]  

(2-4)

The solar radiation pressure term \( \vec{P}_j \) from Equation (2-2) is composed of two components, the light emission travelling at a speed of light, \( c = 3 \times 10^8 \text{ m/s} \), and solar wind at \( \omega = 4 \times 10^5 \text{ m/s} \). These two terms are also affected by the altitude of
the debris, which can also be described as the distance from the sun (Smirnov, 2002).

\[
\vec{p}_j = n_j \frac{\pi d_j^2}{4} H(-\vec{R}_H \cdot \vec{R}) \left[ (p_r + p_r^b) \frac{\vec{R}_H}{|\vec{R}_H|} - \left( \frac{p_r}{c} + \frac{p_r^b}{\omega} \right) \vec{v}_j^H \right]
\]

(2-5)

where,

- \( n_j \) - number density of objects per volume unit
- \( d_j \) - size of debris phase
- \( R_H \) - distance from the Sun
- \( \vec{R}_H \) - radius-vectors of a particle in heliocentric coordinate system
- \( \vec{R} \) - radius-vectors of a particle in geocentric coordinate system
- \( p_r \) - the pressure from photon radiation
- \( p_r^b \) - the pressure from solar wind

\[ \cong p_r \frac{\vec{v}_j^H}{c} \] - the aberration dynamics Poynting-Robertson effect of photon radiation

\[ \cong p_r^b \frac{\vec{v}_j^H}{\omega} \] - the aberration dynamics effect of a solar wind

\( \vec{v}_j^H \) is a heliocentric velocity of particles

The atmospheric drag term \( \vec{F}_{dj} \) from Equation (2-2) can be described by (Smirnov, 2002):

\[
\vec{F}_{dj} = -\frac{1}{2} c_f \rho_a(\vec{x}, t) |\vec{v}_j| \left[ \frac{3}{2} \frac{\alpha_j}{d_j} \right]
\]

(2-6)

where,

- \( \rho_a(\vec{x}, t) \) - the spatial density distribution in the upper atmosphere which is highly dependent on altitude
- \( c_f \) - coefficient of drag
- \( \alpha_j \) - distributed volume density of debris objects
The drag coefficient $c_i^j$ in Equation (2-6) can be expressed as:

$$c_i^j = \frac{2 e^{-\beta_j^2}}{\sqrt{\pi \beta_j^2}} (2 \beta_j^2 + 1) + \frac{\text{erf}(\beta_j)}{\beta_j^4} (4 \beta_j^4 + 4 \beta_j^2 - 1) + \frac{4 k_{ac} \sqrt{\pi}}{\beta_{jw}}$$

(2-7)

where,

$$\beta_j = v_j \sqrt{\frac{m_a}{2kT_a}} ; \beta_{jw} = v_j \sqrt{\frac{m_j}{2kT_j}} ; \text{erf}(\beta_j) = \frac{2}{\sqrt{\pi}} \int_0^{\beta_j} e^{-x^2} dx$$

(2-8)

where,

- $m_a$ - the mean molar mass of gases
- $k$ - the Boltzmann constant, $8.617 \times 10^{-5}$ eV K$^{-1}$
- $T_a$ - the gas temperature
- $T_j$ - the temperature of surface particles
- $k_{ac}$ - the accommodation coefficient

### 2.5. Satellite Laser Ranging Technique

In the recent years, the Satellite Laser Ranging (SLR) technique has been widely adopted for tracking space objects with retroreflectors and success is evident for the tracking of space debris. SLR is capable of operating remotely in an unmanned environment. Moreover, researches and experiments on SLR conducted in the past have shown that SLR can provide instantaneous range measurements at millimetre accuracy (Moe and Moe, 2005). This research adopts SLR measurements for satellite orbit determination and prediction for studying the atmospheric condition. The intention of this research is to use SLR...
measurements to satellites and measure the differences between the intended orbit and the actual satellite orbit under different atmospheric conditions.

The fundamental of SLR is a distance measurement from the ground surface on the Earth to a subject of concern located in space, e.g., either satellites fitted with retro-reflectors or space debris. In theory, a laser pulse is fired from a transmitter to a subject of concern, the laser pulse is then reflected from the subject back to the receiver located on the Earth. The time difference is multiplied by the speed of light, which in fact is the two-way distance from the Earth to the subject. The SLR observations are provided by a number of SLR monitoring stations distributed around the world, as illustrated in Figure 2-7.

![Figure 2-7: ILRS monitoring stations across the world (ILRS, 2010)](image)

The renowned success of SLR has enabled reliable studies of the Earth and the ocean system, e.g., the temporal variations in the ocean system and gravity field.
Furthermore, SLR studies have provided advantageous contributions to the following areas of research (NASA, 2009):

- The precise monitoring of movements in the network of ground-based stations with respect to the geo-centre of the Earth, the reliability of this information can strongly support other on-going geodetic observation systems, such as GPS, PRARE and VLBI;
- The accurate mapping of the temporal motion of ice-sheets and the ice-levels;
- The monitoring of sea-level and sea-surface variations.

However, extensive experiments are currently conducted by many organisations to improve the accuracy and ability to monitor space debris. The future of SLR technology concentrates on the refinement of a fully automated operating system, with proposed improvements to further enhance the accuracy of POD. In addition, innovative approaches to enabling continuous 24-hour operations and minimising the operation and maintenance costs are being investigated.

Although SLR range measurements have claimed millimetre accuracy, there are some minor shortcomings to this technique. The strength of the signal is weakened as it travels through the different types of mediums of the atmosphere. In addition, the signal is drastically weakened, depending on the reflectance of the different types of surface on the subject under surveillance. The significance of this is illustrated in Figure 2-8, where the protective tube at the bottom of the image is the laser pulse at the time of firing and the smaller tube is the beam of laser on return. For example, the surface of the satellites may be equipped with retro-reflectors that supports signal reflection, whereas space debris do not have such equipment and so the capability to reflect laser signals is reduced. Furthermore, the accurate geocentric position of the SLR stations at the time of observation must be known, as the accuracy of the position has a direct impact on the accuracy of orbit determination for the space object (Schillak and Wnuk, 2003; Lejba et al., 2007; Wilkinson and Appleby, 2011). Hence, accurate SLR station
coordinates need to be in a well defined terrestrial reference frame (the International Terrestrial Reference Frame, or ITRF). The effects on SLR or other types of tracking stations, such as, plate tectonics, solid Earth tides, ocean tide loading, atmosphere loading and other modelling need to be corrected before performing the POD.

![Image](image.png)

**Figure 2-8:** The protective tube that the laser beam travels through at the time firing and on return

The SLR observations can be best described as the two-way measurements of the laser signals from the time of firing to the time of returning with the addition of corrections and biases terms as shown in Equation (2-9). This function is the basic form of the SLR observation equation and needs to be linearised for solving its unknown parameters, including force parameters and state vector of the space subject at the observation epoch. Based on the linearised observation equations of all epochs and Least Squares estimation method, the optimal estimates for the unknown parameters can be obtained.
\[ O_{\text{SLR}} = r_{t_s,t_r} + r_{t_s,t_f} + \text{Correction} + \Delta O_{\text{SLR-BIAS}} + \left( \dot{r}_{t_s,t_r} + \dot{r}_{t_s,t_f} \right) \cdot \Delta t_{\text{BIAS}} \]  

(2-9)

where,

- \( t_f \) - epoch when the laser signal is transmitted or fired
- \( t_s \) - epoch when the laser signal arrives at the satellite
- \( t_r \) - epoch when the laser signal returns to the receiver

- \( r_{t_s,t_r} \) - is \( |\vec{r}_{t_s} - \vec{r}_{t_r}| \)
- \( r_{t_s,t_f} \) - is \( |\vec{r}_{t_s} - \vec{r}_{t_f}| \)

\( \dot{r}_{t_s,t_r} \) - is computed from \( \frac{(\vec{r}_{t_s} \cdot \vec{r}_{t_r}) \cdot (\vec{r}_{t_s} \cdot \vec{r}_{t_r})}{r_{t_s,t_r}} \)

\( \dot{r}_{t_s,t_f} \) - is computed from \( \frac{(\vec{r}_{t_s} \cdot \vec{r}_{t_f}) \cdot (\vec{r}_{t_s} \cdot \vec{r}_{t_f})}{r_{t_s,t_f}} \)

Correction - system corrections for SLR observation, including: atmospheric refraction, centre of mass correction

\( \Delta O_{\text{SLR-BIAS}} \) - is the measurement bias

\( \Delta t_{\text{BIAS}} \) - is the timing bias

**ILRS**

The International Laser Ranging Services (ILRS) is an organisation as a part of the NASA programs to maintain a platform for distributing SLR and Luna Laser Ranging (LLR) information (Gurtner et al., 2005; Pearlman et al., 2005). The organisation aims to provide satellite information, including orbits of satellites, at its highest quality for geodetic and geophysical research. It also supports the collection, archiving, transmission and distribution of the SLR data. ILRS also generates a number of data products, such as centimetre accuracy satellite ephemerides. This research uses the normal point file and the state vector from ILRS.
2.6. Summary

This chapter has provided an overview of the core components of the space debris surveillance system. The number of space objects, especially space debris fragments, has exponentially increased for the past decades. This number will continue to escalate in the future due to the factors such as the space vehicle-to-debris collisions, debris-to-debris collisions and disintegrated parts of space vehicles from future launches. Thus, it is crucial to investigate and develop new approaches to reducing the risk of collisions with space debris for a safe space environment.

The accurate monitoring of the space debris environment is a complex task. It involves both refinement of the observation techniques and space debris mitigation measures. The following tasks are required to aid the safety of the space environment. The first is the improvement in space-vehicle designs to reduce exposure of the unprotected components of the vehicle and the construction of space vehicles with strengthened and lightened materials to increase durability. The second is advancement in technology to permit continuous and unmanned tracking of small sized debris using traditional tracking techniques, which will inevitably enable a more extensive space debris catalogue. The third is the development of space debris reduction measures to eliminate space debris rather than re-orbit debris into a currently unused orbit.

In short, the successful avoidance of debris collision is reliant on timely and accurate information of the debris environment. This is underpinned by cataloguing space debris and estimating the risks of collision through orbit determination and prediction of space objects. In the next chapter, we will introduce the principle of determining the motion of space objects.
Chapter 3 - Motion of Space Objects and Perturbing Forces

3.1. Introduction

An accurate approach for satellite orbit determination is based on estimating the forces exerted to the satellite in orbit. This requires comprehensive knowledge of each of the contributing forces as the position of the satellite is determined by all the forces. This chapter provides an overview of the fundamentals of motion of space objects in space and the different forces acting on the space objects in orbit. The atmospheric drag is examined in more detail because it is one of the dominant forces affecting the motion of satellites especially in the LEO.

3.2. Motion of Space Objects

The current theory on the motion of objects in space could be dated back centuries ago. However, one of the important milestones in describing the motion of planets is based on the work of astronomer Johannes Kepler (1571 - 1630). The three laws developed by Kepler that describes the planetary physical motion are:

1. All planets revolve in elliptical orbits with the Sun at one focus
2. A line that connects the planet and the Sun sweeps out equal areas in equal lengths of time
3. The cube of the planet’s mean distance to the Sun is proportional to the square of the period of the planet

These three laws approximate the kinematics of the planetary system. In Kepler's first law, the elliptical orbit is described to have a closest point of approach to the Sun at its focus and it can be described by Keplerian's six orbital elements. Up
until today, these six Keplerian elements are still widely adopted in celestial mechanics. The terms of the six Keplerian elements are described below and illustrated in Figure 3-1:

- a - semi-major axis
- e - eccentricity
- i - inclination
- $\Omega$ - the longitude of ascending node
- $\omega$ - argument of perigee
- $\nu$ - mean anomaly

![Figure 3-1: Six Keplerian orbit elements for an elliptical orbit (Beutler, 2005)](image)

The Kepler's three laws of motion have shown great success in the approximation of the planetary kinematics, however, the reason that supports the theory was unresolved until another astronomer Issac Newton (1643 - 1727) provided an explanation to Kepler's laws on planetary motion. Newton developed three principles that describes the dynamics of motion, they were:

1. Every body remains in uniform motion or its state of rest, unless acted upon by an external force
2. The net force is equal to the change in momentum, where the momentum is the product of the object's mass and its change in velocity, commonly known as, \( \text{Force} = \text{Mass} \cdot \text{Acceleration} \)

3. The force exerted on an object is always subject to an equal force in the opposite and collinear direction

Furthermore, Newton has developed a law that describes the forces of two bodies of masses acting on each other at a particular distance apart. This theory can be applied to any two-body problems, where an orbiting element revolves around a bigger body of mass in one of its focuses. Two-body is the simplest form for describing body motions and it can be further developed by adding other body of masses, consequently forming a three-body motion or a multi-body motion (Vallado and Mcclain, 2001). The law of gravitation can be applied to solve for two-body motion where the attraction force, \( F \), can be expressed as:

\[
F = -\frac{GMm}{r^2}
\]

where,

\[ G = 6.673 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \] the gravitational constant

\[ M = \text{the mass of centre object} \]

\[ m = \text{the mass of orbiting object} \]

\[ r = \text{the distance between the two masses} \]

The equation describing the law of gravitational attraction, as shown in Equation (3-2), is the basis to all celestial mechanics with the Earth as its centre of mass, where \( r \) is the geocentric position vector of the space object. A transformation of the equation into vector form:

\[
\vec{r} = -\frac{GM}{r^3} \cdot \vec{r}
\]
In Equation (3-2), the mass of the space object, such as satellite, is neglected, due to its small mass compared with the Earth. Moreover, it assumes that the only force acting on the space object is from the Earth's gravity and disregards all other forces. This assumption introduces errors into this equation due to the negligence. Hence, it is vital to add a parameter to accommodate such perturbing forces, $K_s$, in the equation of motion as described in Equation (3-3) (Seeber, 1993).

$$\ddot{r} = -\frac{GM}{r^3} r + K_s$$

(3-3)

The additional term $K_s$ in Equation (3-3), is composed of all other perturbing forces acting on the space object as it travels in its orbit other than the centrifugal force. This includes:

- $\ddot{r}_E$ - the acceleration due to the non-spherically mass distribution of the earth
- $\ddot{r}_s, \ddot{r}_m$ - the accelerations due to the Sun and the Moon respectively
- $\ddot{r}_o, \ddot{r}_o$ - the accelerations due to the Earth and oceanic tides respectively
- $\ddot{r}_D$ - the acceleration due to atmospheric drag
- $\ddot{r}_{SP}, \ddot{r}_A$ - the accelerations due to direct and the earth-reflected solar radiation pressures respectively

Thus,

$$K_s = \ddot{r}_E + \ddot{r}_s + \ddot{r}_m + \ddot{r}_e + \ddot{r}_o + \ddot{r}_D + \ddot{r}_{SP} + \ddot{r}_A$$

(3-4)

### 3.2.1. Geopotential Force

The strongest force of nature is the geopotential force from the Earth and it maintains objects to revolve around the Earth, with the addition of other forces. This force also strongly affects for space objects in the Low Earth Orbit (LEO) and subsequently, orbital objects in the LEO must travel at a higher velocity to counteract this natural phenomenon compared with orbital objects in the High
Earth Orbit (HEO). This conforms to two of the Kepler's laws, the second law that states a line that connects the planet and the Sun sweeps equal areas in equal lengths of time and the third law that describes the distance and period relationship.

The Earth is often assumed a sphere with uniformly distributed mass. However, the same cannot be applied where high accuracy applications such as for POD. In fact, the Earth is an oblate sphere with variable mass distribution and it is crucial to model for such variations in mass (Lemoine et al., 2007). There are four parameters that describes the physical characteristics of the solid Earth (Vallado and Mcclain, 2001):

- The eccentricity of the Earth;
- The rotational velocity of the Earth;
- The equatorial radius of the Earth; and
- The gravitational parameters of the Earth.

Models have been developed to simulate the mass variation of the Earth using spherical harmonics. Where the $C_{nm}$ and $S_{nm}$ harmonics coefficient represents the integral of mass and the mass distribution within the Earth, as shown in Equation (3-5) (Seeber, 1993).

$$V = \frac{GM}{r} \left( 1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{a_e}{r} \right)^n (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\cos \theta) \right)$$

(3-5)

where,

- $a_e$ - equatorial radius
- $P_{nm}$ - associated Legendre functions
- $\frac{GM}{r}$ - the potential of the Earth;
A commonly adopted gravity model, EIGEN-CG01C, is considered a precise representation of the Earth's gravity field. It is compiled from 0.5 degree by 0.5 degree surface data in combination with gravity data from the CHAMP and GRACE satellites. As a result, the EIGEN-CG01C can achieve accuracy approximately 20 cm and 5 mgal for geoid height and gravity anomalies respectively (Reigber et al., 2006). Therefore, this model is widely adopted for many applications, such as POD and ocean tide measurements (Daho et al., 2008).

### 3.2.2. Direct and Indirect Solar Radiation Pressure

In the LEO environment, space objects are primarily subject to gravity and atmospheric drag. However, when the altitude exceeds approximately 1000 km, the solar effects from the Sun becomes increasingly dominant (Knowles et al., 2001; Belehaki et al., 2009). The sun emits radiation particles that impact directly with the surfaces of the space objects and as a result, the force from the Sun causes the space object to deviate from the intended orbit (King-Hele, 1987). Hence, the acceleration caused by this perturbing force must be accurately modelled as a function of the reflective surface of the space object, the reflectivity of the space object surface to the solar flux and the distance between the space object, e.g., satellite, and the Sun as expressed in Equation (3-6) (Seeber, 1993).

\[
\begin{align*}
\vec{a}_{SP} &= \frac{P_s}{m} \frac{C_e O}{(AU)^2} \frac{r - r_s}{|r - r_s|^3} \\
\text{(3-6)}
\end{align*}
\]

where,

- \(P_s\) - sun-constant (quotient of solar flux and velocity of light in the Astronomical units)
- \((AU)\) - Astronomical Unit (1.5 x 10^8 km)
- \(\frac{0}{m}\) - surface to mass ratio of the satellite
$r, r_s$ - position vectors of the satellite and of the sun in the space-fixed equatorial system respectively

$C_r$ - factor of reflectivity for the satellites surface ($C_r=1.95$ for aluminium)

$\upsilon$ - shadow function; $\upsilon = 0$, satellite in the earth shadow; $\upsilon = 1$, satellite in sunlight; $0 < \upsilon < 1$, satellite in half-shadow

The reflection of the solar radiation from the Earth's surface, albedo, also contributes to the accuracy of determining the effects of solar radiation for a space object. However, it is more dominant for space objects in the LEO as the strength of reflections from the Earth is proportional to the distance between the space object and the Earth's surface. Furthermore, to accurately determine albedo requires detailed knowledge of the distribution of the Earth surface's properties, such as land, sea and ice, due to different reflectivity coefficients for different surface properties. The relationship between the vector of the Sun from the Earth and the orbiting body is illustrated in Figure 3-2, where $a_e$ refers to the semi-axis of the shadow generating body (the Earth). $D$ is greater than 0 in sunlight and $D$ is less than 0 in shadow. When the satellite is in shadow, it can be described as:

$$|S_c| = |r' - Dr'| < a_e$$

(3-7)

**Figure 3-2:** Relative positions of the satellite, the Earth and the Sun define the effects of solar radiation (Seeber, 1993)
3.2.3. Perturbing Forces Caused by the Sun and Moon

The gravitational force of the Sun and Moon causes unpredictable acceleration to the space objects in orbit. Hence, this force must be accounted for in the space object's, e.g., satellite's, orbit determination process (Vallado and Mcclain, 2001). A common approach adopted by the space industry to resolve the perturbing force by the Sun and the Moon is to assume they are individual masses, as illustrated in Figure 3-3.

![Figure 3-3: Perturbing force of the Sun and the Moon is governed by the relative position of the orbiting celestial bodies and the satellite (Seeber, 1993)](image)

The acceleration caused by the perturbing force of the mass of the Earth, $m_m$, and the mass of the Moon, $m_E$, on the space object can be described as (Seeber, 1993):

$$\ddot{r}_0 = G \left( -\frac{m_E}{|\mathbf{r}|^3} \mathbf{r} + \frac{m_m}{|\rho_m|^{3} \rho_m} \right)$$  \hspace{1cm} (3-8)
The acceleration of the mass of the Earth, $m_E$, caused by the acceleration of the mass of the Moon, $m_m$, is expressed as:

$$\ddot{r}_{0E} = G \frac{m_m}{|r_m|^3} r_m$$

(3-9)

Hence, the relative acceleration of the space object with respect to the Earth is (Seeber, 1993):

$$\ddot{r}_0 - \ddot{r}_{0E} = \ddot{r} = G \left( \frac{m_m}{|\rho_m|^3} \rho_m - \frac{m_m}{|r_m|^3} r_m - \frac{m_E}{|r|^3} r \right)$$

With $|\rho| = \rho$, $|r| = r$ and $m_E = M$, we find:

$$\ddot{r} = \frac{-GM}{r^3} r + Gm_m \left( \frac{\rho_m}{\rho_m^3} - \frac{r_m}{r^3} \right)$$

(3-10)

The initial term in the equation defines the acceleration due to the Earth and the additional term defines the accelerations caused by the gravitational forces of the Moon acting on the space object (Seeber, 1993):

$$\ddot{r}_m = Gm_m \left( \frac{r_m - r}{(r_m - r)^3} - \frac{r_m}{r_m^3} \right)$$

(3-11)

In addition, the acceleration caused by the Sun, $\ddot{r}_s$, can be defined as (Seeber, 1993):

$$\ddot{r}_s = Gm_s \left( \frac{r_s - r}{(r_s - r)^3} - \frac{r_s}{r_s^3} \right)$$

(3-12)
3.2.4. **Solid Earth Tide and Ocean Tide**

The force of gravity from external celestial bodies, namely the Sun and the Moon, induces a force that instigates minor deformations to the shape of the Earth (King-Hele, 1987). Consequently, this force triggers variations in the gravitational potential of the Earth and ultimately creates an additional acceleration force to the orbiting space object, which is expressed in Equation (3-13) (Seeber, 1993). As previously stated, the gravitational potential force is the strongest for space objects in the LEO, which implies that the force from solid Earth tides has more influence for space objects in the LEO. For this reason, satellites Starlette and Stella were employed partly for modelling solid Earth tides.

\[
\ddot{r}_e = \frac{k_2 \, Gm_d \, a_e^5}{2 \, r_d^3 \, r^4} \left[ 3 - 15 \cos^2 \theta \right] \frac{r}{r} + 6 \cos \theta \frac{r_d}{r_d} 
\]

(3-13)

where,

- \( m_d \) - mass of the perturbing body (the Sun and the Moon)
- \( r_d \) - geocentric position vector of the perturbing body
- \( \theta \) - angle between the geocentric position vector \( r \) of the satellite \( r_d \)
- \( k_2 \) - Love number describing the elasticity of the earth body

In addition, the induced forces from the external celestial bodies, mainly the moon, cause variations in the ocean surface, this attraction causes the ocean surface to bulge at the nearside and underside of the Earth closest to the Moon. Moreover, the bulge causes not only the Earth’s crust to deform due to the mass of water, but also the peaks and troughs of the ocean currents. As a result, the induced forces from the solid Earth tide and ocean tides will create additional accelerations force to the orbiting space object.

Ocean tide models have been developed to model the movement of the ocean and the induced mass variations caused by the variations in the ocean movements. CSR3.0 is a widely used ocean model. This empirical model is referenced to an
earlier model FES94.1, it is estimated from normal points over 2.4 years on the TOPEX/Poseidon data.

### 3.2.5. Atmospheric Drag

Atmospheric drag is one of the most dominant non-gravitational perturbing forces that a space object endures in the LEO. The air drag represents the contact between the particles in the atmosphere and the surface of the space object. This interaction produces a frictional force that resists mainly the forward movement of the space object in orbit, implying that it is more dominant in the along-track direction (King-Hele, 1987). The acceleration due to this drag can be calculated by (Seeber, 1993):

\[
\ddot{r}_D = -\frac{1}{2} C_D \rho(r, t) \frac{A}{m_s} (\ddot{r} - \dot{r}_a) |\ddot{r} - \dot{r}_a|
\]

(3-14)

where,
- \(m_s\) - the mass of the space object
- \(A\) - the effective cross-sectional area of the space object
- \(C_D\) - the coefficient of drag (space object specific)
- \(\rho(r, t)\) - the density of the atmosphere at the space object
- \(r, \dot{r}\) - the position and velocity vector of the space object
- \(\dot{r}_a\) - the velocity of the atmosphere near the space object

An integral part of determining and predicting positions and velocities of objects in space is the precise knowledge of atmospheric conditions, especially atmospheric drag. However, atmospheric conditions are highly variable and so it is an intricate and complex task to accurately model the actual atmospheric conditions. Predominately, the induced aerodynamic force on the space object is governed by four major parameters (Vallado and Mcclain, 2001; Tewari, 2007):
• the shape of the space object;
• the attitude of the space object during its orbital path;
• the velocity of the space object and;
• the atmospheric density surrounding the space object.

The acceleration due to drag acting on the satellite can be accurately determined only when all these parameters in the drag equation (see Equation (3-14)) are precisely known. These parameters are the coefficient of drag, atmospheric density and area-to-mass ratio of the space object (King-Hele, 1987). However, there are uncertainties in all the parameters so models are developed to better predict the effects of these parameters. These parameters will be discussed in the following sections.

3.2.5.1. Atmospheric Density

Atmospheric density can be defined as the concentration of the atmospheric particles, which includes the molecular structure of the particles that interact with the surface of the space object during its orbital path (Jacchia, 1971; Rajendra and Kuga, 2001; Vallado and Finkleman, 2008). The atmospheric density varies with time, geographical location and height (Gaposchkin, 1986; Doornbos, 2011). Therefore, it is very difficult to estimate the variations of these parameters. Table 3-1 presents the approximate values of atmospheric density at various altitudes and it is evident that the atmosphere is less dense at higher altitudes.
Table 3-1: Atmospheric density values at different altitudes (Seeber, 1993)

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Density (g/km$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>497,400</td>
</tr>
<tr>
<td>200</td>
<td>255 - 316</td>
</tr>
<tr>
<td>300</td>
<td>17 - 35</td>
</tr>
<tr>
<td>400</td>
<td>2.2 - 7.5</td>
</tr>
<tr>
<td>500</td>
<td>0.4 - 2.0</td>
</tr>
<tr>
<td>600</td>
<td>0.08 - 0.64</td>
</tr>
<tr>
<td>700</td>
<td>0.02 - 0.22</td>
</tr>
<tr>
<td>800</td>
<td>0.007 - 0.08</td>
</tr>
<tr>
<td>900</td>
<td>0.003 - 0.04</td>
</tr>
<tr>
<td>1000</td>
<td>0.001 - 0.02</td>
</tr>
</tbody>
</table>

The variations in the density presented in the above table are due to the concentration of different atmospheric constituents at different altitudes above the surface of the Earth. There are also other factors contributing to the variation of the atmospheric density, such as, the emissions from the Sun. The $F_{10.7}$ index is commonly used to measure the solar flux from the Sun constantly heating up the Earth’s upper atmosphere (Fuller-Rowell, 1981; Tobiska, 2001; Tobiska et al., 2008). The geomagnetic storms also affects the activity of the particles in the upper atmosphere and the Ap index is often used to measure the strength of the magnetic storms (Burns et al., 2004; Fuller-Rowell et al., 2004).

Currently, there are many atmospheric density models (ADM) established to calculate the atmospheric environment, such as the High Accuracy Satellite Drag Model, HASDM, (Casali and Barker, 2002; Bowman et al., 2004; Gabor, 2005; Storz et al., 2005). The errors of the HASDM range from 8% at the altitude of 200 km to approximately 24% at 800 km and similar percentage of errors are also evident for other ADM (Cefola et al., 1999a; Bergstrom et al., 2002; Bowman and
Hence, if the ADM are presumed accurate, the errors in the density models will be reflected by the $C_D$ value (Gaposchkin and Coster, 1988).

Two of the commonly used models for atmospheric research have been selected for the testings of this research, they are described below.

**Mass-Spectrometer-Incoherent-Scatter (MSIS)-86**

This model is established based on the work of A.E. Hedin and other researchers to formulate an atmospheric model that describes the temperature and densities of the commonly found atmospheric constituents, such as He, O, H and others (Hedin, 1988; Rajendra and Kuga, 2001). The data sources are from the combination of measurements to satellites and rockets with additional incoherent scatter radar data. This model is commonly adopted in space physics research and it is a refinement to a previous model MSIS-83, with the improvements of additional composite data from more satellites, namely Explorer satellite, to improve polar region estimations.

**Drag Temperature Model (DTM)-78**

The primary purpose of the DTM is to model the temperature, density and composition of the gases in the upper atmospheric region. It is modelled using combined data from the atmospheric densities derived from satellite data and measurements of exospheric temperatures. The performance of this model is limited to the lower boundary of approximately 120 km, thus this model is not suitable for calculating of the density and temperature below such an altitude (Barlier *et al.*, 1978; Berger *et al.*, 1998; Pardini and Anselmo, 2001a).
3.2.5.2. 'B' Value

The definition of the Ballistic value 'B' is the projectile of an object. It is determined by the product of the coefficient of drag, \(C_D\), and the ratio of the cross sectional area in the along-track direction and the mass of the space object, as shown in Equation (3-15) (Pardini and Anselmo, 2000; Tewari, 2007).

\[
B = C_D \frac{A}{M}
\]

where,
- \(B\) - ballistic coefficient
- \(C_D\) - coefficient of drag
- \(A\) - cross sectional area in along track direction of the orbital path (m\(^2\))
- \(M\) - mass of space object (kg)

It is evident from Equation (3-15) that there is a strong correlation between the three contributing factors of the ballistic value, the \(C_D\) value, the area and the mass of the space object. They will be discussed in the following section.

Relationship between cross-sectional area and the mass of space object, area-to-mass ratio

Generally, if the space object is a satellite, the mass is known relatively more accurate than the cross-sectional area. The area refers to the cross sectional area of the satellite at the along-track direction of orbit, this is heavily reliant on the instantaneous attitude information of the satellite. This poses a significant problem because satellites rotate in orbit. Historically, it was a difficult task to determine the attitude of satellites, but with the current space-aged technology, the ability to monitor the manoeuvre and the attitude of the satellite has dramatically improved. Different techniques are applied to monitor the attitude of satellites, e.g., the attitude of the COSMIC satellite is obtained via the monitoring the outputs from a magnetometer, earth sensor and sun sensor (Hwang et al., 2009).
Such technique could provide highly accurate attitude information of the satellite to users but at a high cost and require tremendous effort although it is one of the most accurate approaches.

To reduce the effort required to determine the instantaneous attitude of the satellite, spherical shapes are desired for satellites because the cross-sectional area is consistent in orbit regardless of the rotation angle. However, for various reasons and considerations different satellites may need different designs, such as, the addition of antennas and solar panels. Consequently, it adds complexity to accurately predict the atmospheric drag exerted to non-spherical satellite as the cross-sectional area of the satellite varies in orbit. Hence, it introduces an uncertainty to the ballistic value (Nicholas et al., 2003).

A research conducted by Moe and his collaborators demonstrated the differences in the $C_D$ value caused by the different shapes of the satellite, specifically, differences between spherical shape and cylindrical shape (with a ratio of one in the length and diameter of the cylinder) (Moe et al., 1993; Moe and Moe, 2007). The results have shown that under the same atmospheric conditions, the shape of the satellite has significant influence on the $C_D$ value, ranging from 2.123 units for a spherical satellite to 3.253 units for a long cylindrical shaped satellite as shown in Figure 3-4. Moreover, the change in the $C_D$ value is also governed by the change in atmospheric constituents at different altitudes as illustrated in Figure 3-5.

![Figure 3-4: Effects of satellite’s shape on $C_D$ value (Moe et al., 1998)](image-url)
Figure 3-5: Change in $C_D$ value over a range of altitudes caused by different shapes (Moe and Moe, 2005)

$C_D$ value
The term $C_D$ is a parameter that represents the coefficient of drag. For spherical satellites, a generic value of 2.2 is generally accepted by the space industry (Moe et al., 2004). But in some cases, the $C_D$ value variation can reach up to and possibly exceed 4.0 units above 1500 km altitude caused by the different constituents segregation over different altitudes, which is significantly different to the commonly adopted value of 2.2 (Bowman, 2002).

The $C_D$ value is widely used in two distinct applications. In the first application, atmospheric density models solving for $\rho$ have a general 15% error in the accuracy for density prediction (Marcos et al., 1998; Pardini et al., 2006). As $\rho$ and the $C_D$ value are factored together as shown in Equation (3-14), the $C_D$ value will absorb the errors of the atmospheric density. Similarly, the $C_D$ value can also absorb the uncertainties in the instantaneous cross-sectional area of the satellite. In
the second application, the $C_D$ value is used to determine the true atmospheric interaction with the satellite surface. It relates to the amount of drag caused by the forces of air molecules of the atmospheric constituents acting on the satellite where the along-track direction of the orbit is subject to most of this force (Moe et al., 1993).

When the coefficient of drag is used to determine the true particle interaction, the term $C_D$ is described as a sum of two forces acting on the satellite, they are (Moe et al., 1993):

- Air molecule making contact with the surface and;
- Air molecule as they leave the surface.

Air molecule striking the surface
As a satellite travels in its orbit, the first of the two forces described above is created, it is caused by air molecules striking on the surface of the satellite. This force is dependent on the cross sectional area of the satellite in the along-track direction of the orbit, therefore the accuracy of the attitude of the satellite is the critical factor to the determination of the amount of force acting on the satellite at one particular moment.

Air molecule leaving the surface
The second force is created when the air molecules leave the surface of the satellite. This force is either a positive or a negative and dependent on the accommodation coefficient for the surface material of the satellite (Murad, 1996).

Accommodation coefficient - $\alpha$
The accommodation coefficient can be defined as a coefficient that measures the accommodation factor of the surface material that makes contact with the molecules. Due to the difficulty to measure the accommodation coefficient in space, usually, experiments are conducted in controlled environments to model the performance of the molecule during impact and reflection. With the assistance of satellite measurements by mass spectrometer and pressure gauge measurements
adsorbed gas on satellite surfaces, it enhances the knowledge of the interactions between the molecules and the satellite surface.

To accurately determine the accommodation coefficient, the accurate knowledge of some of its confining parameters is required. This includes:

**Incident molecule:**
- Velocity
- Angle of strike
- Molecular composition

**Satellite surface:**
- Molecular composition
- Surface conditions

The accommodation coefficient $\alpha$ can be expressed as (Moe et al., 1998):

$$\alpha = \frac{(E_i - E_r)}{(E_i - E_w)}$$

where,
- $\alpha$ - accommodation coefficient
- $E_i$ - kinetic energy carried to the surface by incident molecules
- $E_r$ - kinetic energy carried away by reemitted molecules
- $E_w$ - kinetic energy that fully accommodated molecules would carry away

The altitude is the major factor that determines the accommodation coefficient, as the content of the atmosphere changes with respect to altitude. At 200 km, the dominant constituents are Nitrogen, N$_2$, and Oxygen, O. As altitude increases to approximately 600 km, the densities of hydrogen, H, and helium, He, also increases. At higher altitudes, hydrogen becomes the major constituents. Moreover, a layer of adsorbed gas and/or other surface contaminants that is difficult to remove covers the surface layer of the satellite. The amount and type of gas adsorbed on this surface determines the reaction of the striking molecule
and that the different constituents at the different altitudes strongly affect the accommodation coefficient, as it determines the amount of adsorbed molecules on the surface of the satellite.

### 3.2.6. Coordinate Systems

A properly defined coordinate reference system is needed for dealing with motion of objects in space. Traditionally, the reference system used for satellite geodesy is a space-fixed inertial reference frame, also known as the Conventional Inertial System (CIS). The inertial system suggests that the reference system is non-accelerating in a rectilinear motion or in a state of rest (Vallado and Mcclain, 2001). Moreover, the Newton's equation applied for describing the motion of satellites around the Earth is only valid with an inertial reference system. The inertial reference system is defined as:

- the Z-axis is directed to the North Pole normal to the equator at a given epoch;
- the X-axis is directed at the equinox, and;
- the Y-axis is from a right-handed system with the Z-axis and the X-axis.

In contrast, for terrestrial measurements or Earth-based applications, such as determining positions of ground observation stations, an Earth-fixed reference system known as the Conventional Terrestrial System (CTS) is adopted. This is due to the fact that the Earth is in constant rotation with varying direction of the rotating axis. Therefore, it is inconvenient to implement a space-fixed system on rotating surfaces. The terrestrial system can be defined as:

- the Z-axis is directed to the North Pole normal to the equator at a given epoch;
- the X-axis is directed at the true reference meridian; and
- the Y-axis is from the right-handed system with the Z-axis and the X-axis.
For accurate space geodesy applications, it is essential to perform transformation between the CIS and CTS coordinate reference frame. This is because there are constant variations between the position and the orientation with Earth. In the transformation process, the following three major issues needs to be considered for the transformation (Vallado and Mcclain, 2001):

*The Earth's rotation* - the measurement of the rotation of the Earth through actual observations. The observations are conducted by International Earth Rotation Services (IERS), using the laser ranging techniques, GPS and Very Long Baseline Interferometry (VLBI);

*Precession* - the measurements of the variations in the orientation of the rotation axis of the Earth;

*Nutation* - the irregularities in the precessions of the axis of the Earth.

In contrast to the Earth's rotation, the precession and nutation of the Earth are usually modelled. The causes of such movements are due to the gravitational forces from the Sun and Moon.

### 3.2.7. Time Systems

For accurate prediction of motion in space, properly defined time systems are required (El-Rabbany, 2002). The following time systems are used for orbit prediction:

**Sidereal time**

A sidereal time system is a system devised by astronomers to maintain a record of the direction of the Earth. Measurements are made at a nominated fixed point on the Earth to a fixed reference point in space. A full revolution is measured when the fixed point on the Earth faces the reference point each time it passes by. This system adopts stars as fixed reference points because the distance between the Earth to the stars is much greater than the distance from the Earth to the Sun.
Hence, Earth's movements around the Sun are considered minimal. A full rotation of the Earth is approximately 23 hours 56 minutes and 4 seconds.

**Solar time**

The solar time system is based on the same theory as sidereal time, however, the solar time adopts the Sun as the reference point rather than stars. This suggests that a full rotation of the Earth is measured in consideration of two aspects, i.e. the rotation of the Earth itself and the rotation of the Earth around the Sun. Consequently, the Earth rotates a full revolution with addition of one degree of arc over one day around the Sun. This system is maintained by the Royal Observatory in Greenwich and the Greenwich meridian is used as the fixed reference point on Earth. A full solar day is 24 hours.

**Atomic time**

The atomic time system or the International Atomic Time, i.e. Temps Atomique International (TAI) is based on an average of two hundred atomic clocks over fifty laboratories that measure the transition of electrons. It is the most accurate measure of time. This system is maintained by the Bureau International des Poids et Mesures (BIPM), also known as International Bureau of Weights and Measures.

**Dynamical time**

In the equation of motion describing satellites around the Earth, it is important to have time as an independent variable. This variable is only a measure of the elapsed time over reference points.
3.3. Summary

The perturbing forces affecting the motion of space objects have been thoroughly covered in this chapter. The precise orbit determination of space objects using the dynamic method is primarily based on the accuracy of the calculated forces acting on the space objects with reference to time and location. The forces are constantly varying due to the relative movements around the rotating Earth. The forces are calculated based on empirical models but due to the variation of some of the atmospheric parameters over time, empirical models may not always offer highly accurate predictions, especially for atmospheric drag.

It is difficult to model the atmospheric drag encountered by a space object as it is dependent on a number of factors, such as, the atmospheric density, the instantaneous cross-sectional area of the space object in the along-track direction of the orbit and the $C_D$ value. The $C_D$ value can be treated as a parameter to absorb the uncertainties in the ballistic value and the errors associated with the density models. Moreover, the $C_D$ value can also be determined as the forces from the interaction of the particles in the atmosphere and the surfaces of the satellites. This research focuses on evaluation of the performance of the $C_D$ values and their effects on orbit prediction when the $C_D$ value is determined as a parameter primarily to absorb errors in the atmospheric drag density model in the atmospheric drag equation.
Chapter 4 - Methodology for C_D Value Modelling

4.1. Introduction

For space debris surveillance and safety of space operations, systems to catalogue the movements of space objects have been established and maintained by many space agencies, such as ESA and NASA. However, the expediency of the surveillance systems is underpinned by the ability to provide a fast and accurate prediction of orbits of space objects.

Atmospheric drag is the most dominant non-gravitational force at LEO altitudes and is also one of the most challenging parameters to model accurately. This has instigated the commencement of this research, which is to improve orbit prediction for orbits of space objects in the LEO environment through the enhancement of the C_D value. This research focuses on quantifying the effects of the two conventional C_D value estimation approaches on orbit prediction, specifically the fixed C_D and the variable C_D methods. Moreover, approaches to improving the efficiency and effectiveness of the prediction of C_D values are also explored.

In this research, the primary concern of space objects is space debris, but due to the unknown geometrical and physical properties of space debris, it introduces great difficulties for the validation of the results. Hence, satellites are selected for testing, although the final application is intended for space debris.
4.2. Satellite Selection Criteria

The initiative for this research is to enhance atmospheric drag prediction for accurate space debris tracking. However, due to the uncertainties of the properties of space debris, satellites are adopted for testing. Three factors considered in the selection of suitable satellites for the testing of this research are:

- The accurate knowledge of the ballistic value;
- Altitude of 800 km above the Earth's surface; and
- SLR capability.

The accurate knowledge of the ballistic value
The atmospheric drag encountered by a satellite is strongest in the along-track direction and it is heavily dependent on the ballistic value. The rotation of the satellite in orbit causes changes to the cross-sectional area in the along-track direction to the orbital path and this poses a substantial problem when resolving the atmospheric drag, as it is an extremely complex task to determine the instantaneous cross-sectional area/attitude of the satellite. Thus, using spherical satellites are ideal to overcome such problem, because the cross-sectional area is always constant.

Altitude of 800 km above the Earth's surface
Generally, the effects of atmospheric drag on a satellite increases as the altitude declines, since the atmosphere is denser at lower altitudes. To limit the scope of the research to a manageable level, one of the highest usage altitudes is selected for this research, which is at the altitude of approximately 800 km above the surface of the Earth, as previously discussed in Chapter 2.3 Space Surveillance Agencies. A number of important satellite constellations orbit at this altitude, e.g. the Iridium constellation with total of seventy-three satellites and the COSMIC's constellation with six microsatellites.
**SLR capability**

The primary purpose of this research is to improve the atmospheric drag prediction for accurate prediction of orbit of space objects using space tracking data, in particular the SLR data. The rationale for using SLR data is that the SLR technology currently offers one of the highest precision ranging measurements. Thus, it enables high accuracy tracking of satellites and space objects.

**4.3. Satellite Descriptions**

As discussed in the preceding section, altitudes between 750 km to 1000 km above the surface of the Earth are altitudes with highest density of space objects. Hence, monitoring such an altitude is vital. An active approach is to enhance the prediction of space debris flux or orbits for satellites, so that a more accurate knowledge of the orbital path is available for collision warning.

The satellite selection criteria for the case studies of this research are based on the shape and altitude of the satellite. The amount of atmospheric force exerted on a satellite is dependent on the cross-sectional area in the along-track direction of the orbital path. This poses a significant problem for non-spherical shaped satellites because the cross-sectional area of the satellite is constantly changing as the satellite rotates in orbit. This problem can be overcome by selecting a spherical satellite, where a consistent amount of drag force is exerted on the satellite during the satellite's rotation. For this reason, it is a common approach to adopt spherical satellites for atmospheric research.
Stella and Starlette are selected in this research due to their spherical shape, circular orbit and similar orbital altitudes. It can be anticipated that the performance of the two satellites are similar because of their similar satellite properties. Hence, the experiment conducted on Stella can be verified by applying the same procedures to Starlette. ERS-2 is also chosen for this research due to its circular orbit at approximately 800 km (see Table 4-1). The non-spherical shape of ERS-2 implies the constant variation in the ballistic value due to the changing area-to-mass ratio, this draws similarities to space debris.

**Table 4-1:** Properties of satellites Starlette, Stella and ERS-2.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Starlette</th>
<th>Stella</th>
<th>ERS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country</td>
<td>France</td>
<td>France</td>
<td>ESA</td>
</tr>
<tr>
<td>Launch Year</td>
<td>1975</td>
<td>1993</td>
<td>1995</td>
</tr>
<tr>
<td>Application</td>
<td>Gravity Field</td>
<td>Gravity Field</td>
<td>Earth Sciences</td>
</tr>
<tr>
<td>RRA Shape</td>
<td>Spherical</td>
<td>Spherical</td>
<td>Hemi-spherical</td>
</tr>
<tr>
<td>SLR Capabilities</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Perigee</td>
<td>812 km</td>
<td>800 km</td>
<td>785 km</td>
</tr>
<tr>
<td>Orbital Path</td>
<td>Circular</td>
<td>Circular</td>
<td>Circular</td>
</tr>
<tr>
<td>Orbital Period</td>
<td>104 minutes</td>
<td>101 minutes</td>
<td>100 minutes</td>
</tr>
<tr>
<td>Inclination</td>
<td>49.83 degrees</td>
<td>98.60 degrees</td>
<td>98.5 degrees</td>
</tr>
</tbody>
</table>

**Starlette and Stella**

Starlette is a France sponsored satellite launched on the 6th February 1975. The perigee of the satellite is approximately at 800 km above the surface of the Earth, with the inclination of 49.83 degrees. This satellite is specifically designed to be compact and spherical for the purpose of gravity field recovery. It has a diameter and mass of 240 mm and 48 kg respectively. Such a small size, compared to its mass, makes it more sensitive to the gravitational attraction than the forces acting on the surface from radiation pressure and the residual atmosphere at the satellite.
The passive satellite is also equipped with 60 laser retro-reflectors (see Figure 4-1) on the surface of the satellite that reflect laser signals back to the receivers located on the Earth for the purpose of tracking using Satellite Laser Ranging (SLR) technique.

Over the years, the data retrieved from Starlette have proven a great success in establishing the foundation for studies of the Earth's gravity (Schutz et al., 1989; Lejba and Schillak, 2011). However, the retrieved SLR data is reliant on the coverage of the satellite's orbital path and the spread of ground observation stations. In order to increase the coverage of the orbit of the satellite, the French government agency Centre National d'Etudes Spatiales (CNES), a key participant of the European space program, enlarged the scope of the gravity field research project, with intention to amplify the volume of satellite data. This has led to the launch of Stella, a sister satellite to Starlette, on 26th September 1993, focused to accomplish such task. Stella was designed to travel in a circular orbit with an inclination of 98.6 degrees, with properties that are virtually identical to Starlette.

Figure 4-1: Image of satellite Starlette (ILRS, 2009)
ERS-2

The European Remote Sensing (ERS) satellites include two Earth orbiting remote sensing satellites, ERS-1 and ERS-2, which are primarily aimed at monitoring the environmental progressions, such as the ocean surface temperature and sea winds. Initially, when ERS-1 was launched in 1991 to an altitude of approximately 800 km by the European Space Agency (ESA), the satellite is in a non-spherical shape (see Figure 4-2) and approximately 11.8 m in height with additional 11.7 m by 2.4 m solar panels. It was equipped with a synthetic aperture radar, a wind scatterometer, a radar altimeter, a microwave sounder, an along track scanning radiometer, the Precise Range and Range-Rate Equipments (PRARE) and retro-reflector arrays (Zandbergen et al., 1995; Zandbergen et al., 1997; Andersen et al., 1998). However, the PRARE system failed shortly after the launch, therefore the precise orbit of the satellite can only be determined by the SLR technique.

Despite the malfunction of the components in the on-board systems on ERS-1, ERS-2 was launched a few years later on the 21st April 1995. The ERS-2 system was designed to carry out the same tasks as ERS-1, thus the design is nearly identical, but ERS-2 also carried an additional Global Ozone Monitoring Experiment (GOME) instrument on-board to measure the ozone content of the atmosphere.

The ERS system was originally designed as an Earth observing satellite and it was one of the most complex designs in that era. The near polar orbit at an inclination of 98.5 degrees enabled great coverage of the whole Earth. This also made collecting precious data on the surface of the Earth possible, such as data of the ocean current systems and the marine gravity field. The data retrieved from the satellites were so reliable that the satellites even conducted a superfluous task to its original design, which is to monitor natural disasters in the event of earthquakes and flooding.
Although the designed life expectancy of the two satellites are three to four years, both satellites have served far longer. In March of 2000, the ERS-1’s computer and gyro control failed and so it has ended the operation of ERS-1. On 4th July 2011, the ERS-2 mission was decommissioned and the satellite was de-orbited to an altitude of 570 km to and burning up on re-entry (ESA, 2011b). Environmental satellite ENVISAT-1 was designed as a follow-on mission from ERS-2.

![Image of satellite ERS-2 and its payloads](image)

**Figure 4-2: Image of satellite ERS-2 and its payloads (ESA, 2011a)**

### 4.4. Structure of Experiments

For orbit determination and prediction, the most common and easiest approach for determining the $C_D$ value is the fixed $C_D$ method (Fix2.2). This implies that the recommended $C_D$ value is a constant figure of 2.2 for spherical shaped space objects, specifically satellites, despite time, location and the actual atmospheric environment (Jacchia, 1971). In contrast, the variable $C_D$ approach (Vary$C_D$) sets the $C_D$ value as an unknown parameter to account for the constantly varying atmosphere. This unknown parameter is resolved during the orbit determination
process by estimating an optimal value most suitable for the coefficient of drag, where the estimated value is used in the orbit prediction process.

This research sets out to evaluate the differences between the $C_D$ values from the Fix2.2 method and the Vary$C_D$ approach. The accuracy of the estimated $C_D$ value are validated by the orbit prediction results derived from the two methods, which are then compared against the orbit predictions by ILRS, the assumed "true" value for this research. The approach that offers orbit prediction with the least difference to the prediction by ILRS is assumed the more accurate approach.

The experiments conducted cover a period of four years from 2004 to 2007. The data from the first three years were used to investigate the variation of the $C_D$ values estimated using the Vary$C_D$ approach (the $C_D$ estimates). The fourth year data were used for validation of the orbit prediction results derived from the new enhanced approach developed through the case studies.

In this research, the data processing segment consists of two major sections: 1) Compare current methods for the estimation of the $C_D$ value, 2) Establish fitting function approach for the prediction of the $C_D$ value, as shown in Figure 4-3.
Characterising the Variation of the $C_D$ Estimates

Case Study-1
Determine Variation in the $C_D$ Estimates Over a One-Year Period

Case Study-2
Determine Variation in the $C_D$ Estimates Over Three Consecutive Years

Case Study-3
Assess the Effects of the Ballistic Value on Orbit Prediction

Case Study-4
Assess the Effects of Different ADM on Orbit Prediction

The Fitting Function Approach for the Prediction of the $C_D$ Value

Determination of the Optimal Fitting Function Using the $C_{ij}$ Estimates Derived From Stella (2004 to 2006)

Applicability Testing of the $C_D$ Values Predicted from the Optimal Fitting Functions to Orbit Prediction

Investigation of the Applicability of the Fitting Function Approach to Starlette and ERS-2

Figure 4-3: Flow chart of the case studies
Characterising the variation of the $C_D$ estimates

The purpose of this section is to compare the currently available methods of prediction for the $C_D$ value, namely the Fix2.2 method and the Vary$C_D$ approach, and their orbit prediction results using Stella as the experimental satellite. The differences in the $C_D$ estimates by the two methods are examined to seek whether the $C_D$ value varies with time, i.e. during the observation period from 2004 to 2006. In the succeeding phase, the orbit prediction using the $C_D$ estimates from the two different methods are compared against orbit prediction by ILRS, where the latter is assumed "true" value in this research, to quantify and assess the performance of different approaches.

Similarly, the same experiments are also conducted for two other satellites, Starlette and ERS-2, to confirm the findings from Stella. A separate case study is dedicated to quantify the effects of the predicted $C_D$ value and predicted orbits caused by the different shapes of Stella and ERS-2. In addition, another case study is assigned to measure the differences in the $C_D$ estimates and the resultant orbit prediction caused by using two different ADM, namely the MSIS-86 and DTM-78.

The fitting function approach for the prediction of the $C_D$ value

The rationale for this section is to utilise the data obtained from the previous section, which is the variation in the $C_D$ estimates over the study period from 2004 to 2006 from Stella, to establish fitting functions for enhancing prediction for the $C_D$ value. Ultimately, it is aimed using the predicted $C_D$ values for achieving higher accuracy orbit prediction and/or reducing data processing time for orbit prediction.

Fitting functions were used in this research due to the fact that the historical $C_D$ estimates from 2004 to 2006 had shown a similar yearly cyclical variation trend. Thus using a function that best fit the $C_D$ data of the past year to predict the current year’s $C_D$ values is meaningful because, generally, any prediction must be based on the trend reflected by a time series historical data. The resulting predicted $C_D$ values are very useful for a fast and for a long-term prediction of
orbit, especially for the cases when recent $C_D$ estimates are not available for orbit prediction. The optimal function is selected based on the accuracy of the predicted $C_D$ values for 2007 compared against the $C_D$ estimates from the Vary$C_D$ approach for the same period, where the $C_D$ estimates from the latter approach are deemed more accurate.

The $C_D$ values predicted using the optimal fitting functions are applied to the orbit prediction process using the fixed value approach, the same orbit prediction process as the Fix2.2 method. The orbit predictions from the three $C_D$ estimation approaches, Fix2.2, Vary$C_D$ and the fitting function are compared for their performance assessment.

The fitting function approach is also verified by performing the same experiment to Starlette and ERS-2, where the predicted $C_D$ values and orbit predictions are compared. Furthermore, the advantages and disadvantages of these three approaches are analysed.

### 4.4.1. Characterising the Variation of the $C_D$ Estimates

**Case Study-1: Determination of variations in the $C_D$ estimates over a one-year period**

The purpose of this test is to observe the variation in the $C_D$ estimates (for satellite Stella) over one year, in 2006, and compare the orbit prediction results derived using the two $C_D$ value estimation approaches. The magnitude of the differences in the predicted $C_D$ estimates and their orbit prediction identifies the effects of adopting the two different approaches (see Table 4-2). To reduce the amount of data processing, a fixed date from each month is selected for testing. The monthly intervals over one year identify a general yearly trend of the $C_D$ values.
Table 4-2: Parameters selected for Case study-1

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Stella</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach for determining the $C_D$ value</td>
<td>Fix2.2</td>
</tr>
<tr>
<td>Study period</td>
<td>2006, monthly intervals</td>
</tr>
</tbody>
</table>

This process is a decisive experiment to identify the necessity of this research on the atmospheric drag coefficient. This is because if the variation in the $C_D$ estimates from the more accurate Vary$C_D$ approach over the one-year period is moderately flat, i.e., it approximates to 2.2, then it denotes that the current approach adopted by the industry of using a constant value of 2.2 to represent atmospheric drag coefficient is sufficient. In contrast, noticeable variation in the $C_D$ estimates implies a dynamic change in the atmospheric drag over the year thus a constant value of 2.2 cannot reflect the fluctuation of the actual change in the atmospheric drag coefficient.

Furthermore, the orbit predictions using the Fix2.2 and the Vary$C_D$ approaches are compared against orbit prediction by ILRS, as the ILRS results are treated as the "true" value in this research. The approach that results in the least difference from predictions by the ILRS is considered more accurate. Moreover, RMS values are calculated to represent the differences in the total position and velocity measured against predictions by ILRS. In addition, the comparison of the orbit predictions demonstrates the extent of problem caused by adopting a less accurate $C_D$ value estimation approach, the Fix2.2 method.

**Case Study-2: Determination of variations in the $C_D$ estimates over three consecutive years**

This is an extensive experiment for observing the variation in the $C_D$ estimates over three consecutive years from 2004 to 2006 using denser intervals as shown in Table 4-3. It is anticipated that a noticeable variation in the $C_D$ estimates will be
present. The results from the three-year study period permits a more extensive dataset for observing the variation in the C\textsubscript{D} estimates and most importantly, it will indicate the trend of the variation.

**Table 4-3: Parameters selected for Case study-2**

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Stella</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Approach for determining the C\textsubscript{D} value</strong></td>
<td>Fix2.2</td>
</tr>
<tr>
<td><strong>Study period</strong></td>
<td>2004 - 2006, weekly intervals</td>
</tr>
</tbody>
</table>

The orbit predictions derived using C\textsubscript{D} estimates from both the Fix2.2 and the VaryC\textsubscript{D} approaches are measured against ILRS, similar to that conducted in case study-1. The comparison in the orbit predictions again reveals the magnitude of the problem with a fixed value of 2.2 to represent the atmospheric drag coefficient.

**Case Study-3: Assessment of the effects of the ballistic value on orbit prediction**

This experiment is similar to case study-2 but using two different satellites, Starlette and ERS-2. The two main purposes for this are: the first is to observe the differences in the C\textsubscript{D} estimates that are possibly caused by the different satellite properties, namely the satellite shape and the orbit inclination in this case, as described in Chapter 4.2 Satellite Selection. The second is to verify the existence of the variation in the C\textsubscript{D} estimates using different satellites and to examine similarities in the predicted C\textsubscript{D} estimates, e.g., similar peaks and troughs or frequency, using Starlette and ERS-2 compared with Stella from the previous case study.
Table 4-4: Parameters selected for Case study-3

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Stella, Starlette and ERS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Approach for determining the $C_D$ value</strong></td>
<td>Fix2.2</td>
</tr>
<tr>
<td><strong>Study period</strong></td>
<td>2004 - 2006, weekly intervals</td>
</tr>
</tbody>
</table>

Similar performances, in terms of the $C_D$ estimates and the orbit predictions, from Stella and Starlette are expected due to their near identical satellite properties. In contrast, the performance of ERS-2 is unknown due to the uncertainty caused by the constantly varying cross-sectional area.

Case Study-4: Assessment of the effects of different ADM on orbit prediction

The atmospheric density models are usually calculated by empirical models and the errors of the atmospheric density model will be reflected in the $C_D$ value. This experiment sets out to assess the differences in orbit prediction when different ADM are applied, specifically the MSIS-86 that is used throughout the case studies in this research and the DTM-78 (see Table 4-5). The two ADM are commonly used in space physics and atmospheric research, hence, similar performance are expected. Two spherical satellites are selected for testing for this research, Stella and Starlette, because it is anticipated that the performance of the two satellites to be similar. Thus, the results form Stella can be verified using Starlette.

Table 4-5: Parameters selected for Case study-4

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Stella and Starlette</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Approach for determining the $C_D$ value</strong></td>
<td>Fix2.2</td>
</tr>
<tr>
<td><strong>Study period</strong></td>
<td>2004 - 2006, weekly intervals</td>
</tr>
<tr>
<td><strong>ADM</strong></td>
<td>MSIS-86</td>
</tr>
</tbody>
</table>
4.4.2. The Fitting Function Approach for the Prediction of the C_D Value

_Determination of the optimal fitting function using the C_D estimates derived from Stella (2004 to 2006)_

The discrete C_D estimates from the VaryC_D approach over the three consecutive years from 2004 to 2006 obtained in case study-2 are adopted in this experiment. The C_D estimates are divided into three datasets: 1) _1-year dataset _using C_D estimates from 2006 only, 2) _2-years dataset _using C_D estimates from 2005 to 2006, and 3) _3-years dataset _using C_D estimates from 2004 to 2006.

The three datasets are fitted with two different types of functions, i.e., Fourier series and sine functions, because a repetitive sinusoidal variation is evident from the C_D estimates through visual observation. An optimal function is selected from each dataset based on the accuracy of the predicted C_D values for 2007. The accuracy is measured by comparing the differences between the fitting function predicted C_D values and the C_D estimates from the VaryC_D approach because the latter approach is considered more accurate. The three selected functions from the three datasets are adopted for the succeeding case study for application of the functions.

Three datasets are adopted in this experiment for assessment of the effects of adding more historical data to the fitting functions. The additional data refers to data that span further back in time. This is for examining if more historical data will be beneficial to the improvement in the accuracy of the predicted C_D values from the fitting function. Based on its results, the optimal time span of the sample data that should be used for the fitting function can be determined. This is important because theoretically, it is difficult to generalise the optimal length of time span of the sample data for the fitting function for the purpose of C_D value prediction.
Applicability testing of the $C_D$ values predicted using the optimal fitting functions to orbit prediction

The $C_D$ values predicted using the three fitting functions from the previous experiment (Determination of the optimal fitting function using the $C_D$ estimates derived from Stella) are applied to orbit prediction. Identical to the Fix2.2 method, the fitting function approach has the advantage of pre-determining the $C_D$ value prior to orbit determination dissimilar to the Vary$C_D$ approach that estimates an optimal $C_D$ value at the time of orbit determination. The pre-determined values suggest it can provide faster orbit prediction but less accurate compared to the Vary$C_D$ approach. The predicted orbits using the fitting function approach are compared with that using the Fix2.2 method. It is anticipated that the fitting function approach will offer higher accuracy orbit prediction compared with that of the Fix2.2 method.

Moreover, the comparison of orbit prediction using the three fitting functions against predictions by ILRS offers the optimal length of the time span of historical sample data among data period of one year, two years and three years. The predicted $C_D$ values, derived from the fitting function based on $C_D$ estimates from Stella, are also applied to the orbit predictions for Starlette and ERS-2 to examine the applicability of the functions derived from one satellite applied to other satellites at similar altitudes.

Investigation of the applicability of the fitting function approach to Starlette and ERS-2

The fitting functions derived from the $C_D$ estimates over a three-year study period from Stella are evaluated in the aforementioned applicability test, where an optimal type of fitting function for prediction of the $C_D$ values is identified. The rationale for this experiment is to validate the fitting function approach by applying the same procedures (conducted for Stella) to satellites Starlette and ERS-2, i.e., the determination of the fitting function and application of the $C_D$ values predicted using the fitting function to orbit prediction.
4.5. Data Used

The SLR data used in this research is downloaded from the International Laser Ranging Service (ILRS) website. The ILRS is an organisation supported by NASA aimed at collecting, merging, archiving and distributing SLR and Lunar Laser Ranging (LLR) data. The primary purpose for ILRS is supporting geodetic and geophysical areas of research. The two types of files used in this research are the normal point file and the state vector file.

Normal Point File
The normal point data or the quick-look data is a type of SLR observation processed from raw SLR data, where normal point data is a sample SLR observation within an interval that contains multiple raw data points. Via the compression, the size of the file is significantly reduced and can be quickly transferred to data centres for further processing. This is the reason the data is called "quick-look" data. The reason for the use of the normal point file in this research is that it can reduce time for compilation, transferring and processing.

State Vector File
The state vector file contains position and velocity vectors for a selected satellite at given epochs. The state vector is given in the Tuned Inter-Range Vectors format (TIRV), where it uses a pseudo-body fixed reference frame, which denotes that the X-axis is directed at the Greenwich meridian and the Z-axis is directed to the True of Date axis.

There are two main purposes for adopting the state vector file: the first is used for the satellite's initial position for orbit determination and prediction. Secondly, it is used as a "true" value for comparison and validation for case studies conducted in this research. This is because the prediction of orbits by ILRS is assumed to be at an accuracy that is higher than the predictions obtained in this research.
4.6. Limitations of This Research

As previously stated, the state vector and the normal point files from ILRS are adopted in this research. This has posed problems in the data processing phase when the files were not available from ILRS, e.g., no data were available for Starlette, Stella and ERS-2 from the 9th May 2007 to the 22nd May 2007. This implies that during this period orbit prediction process cannot be performed due to missing observation files for the satellite. Consequently, this also affects the accuracy of the fitting function because the fitting functions are heavily reliant on the $C_D$ estimates and without observations to the satellites, the estimation of the $C_D$ during orbit determination cannot be performed.

The processing power is another limited resource for this research. It restricted the volume of data processed for the duration of the research period. This is also partly due to the operating platform for the software and the design of the software itself. The operating system and the design of the software are limited only one process to operate at any given time, which implies no simultaneous data processing and as a result, the data processing time significantly increases. This issue is resolved by adopting virtual systems, which runs multiple operating systems simultaneously. However, the operating of multiple virtual systems is memory consuming for computers, which significantly reduces the processing power for the orbit prediction software.

The computer used for this research that simultaneously runs two operating systems in the virtual machines is the most efficient setup to overcome the limitations of the insufficient computational resources. If available, super computers or computer clusters would significantly improve the processing power and processing time. By using such computer resources, larger volume of data could be processed, such as increasing the density of the data within the three years or extension to the current study period from three years to more.
The same computer is used for all the data processing in this research for comparing the time required for data processing for the different approaches adopted in the experiments, e.g. the fixed value method and the VaryC\(D\) approach. This is to eliminate different processing time caused by different hardware and software setups for different computers. Using one computer throughout the research, the volume of data processed by the computer is very limited.

### 4.7. Precise Orbit Determination Software

The main objective of most POD software is to provide accurate position and velocity information of the observed objects to the users. However, the processing and the calculation of position and velocity can differ depending on the requirements of the users. Different softwares are designed with different intended purposes, such as the high speed but low accuracy orbit predictions or vice-versa. Hence, different performances are expected from different POD softwares. This POD software is to provide timely positional information for users, it is divided into three sections, and they are described below.

**Initialisation stage**  
*Setup time and date for period of orbit determination and prediction*  
The selection of dates for orbit determination and prediction and setup of the interval step length for the prediction. In this research, seven days are used for orbit determination and the first day of the orbit prediction is used for orbit comparison.

**Observation file (normal point file) & state vector file**  
*Normal point file*  
Normal point file for the selected dates are downloaded from the ILRS website. It contains the SLR measurements from the selected satellites.
State vector file

The TIRV for selected dates and selected satellites are downloaded from the ILRS website. The information contained within the file is used as an initial status of the satellite at a particular time for the POD process. Integration used in the POD processing stage relies on this initial status of the satellite. The TIRV state vectors are given at 00 hours 00 minutes and 00 seconds of each day (midnight).

Satellite and station information files

These files contain information for the selected satellites and the SLR stations. They are used in the software to identify measurements made from relevant SLR stations.

The typical satellite information file contains:

- Satellite name
- Satellite identification number in different forms, e.g., COSPAR, SIC, NORAD
- Mass of satellite
- Retro-reflector offset from mass centre
- Satellite geometry
- Size parameters

The typical SLR station file contains:

- Station name
- Station number
- Survey data for the station

Setup of force model parameters

The selection of force models and parameters and the setup of data processing are maintained consistent throughout the experiments, except when specially noted to test a particular parameter, e.g., ADM. This is for avoiding differences in the POD results caused by different force models. The parameters and force models used in this research are outlined in Table 4-6.
Table 4-6: Parameters used by the orbit determination software package for this research

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Approaches/Models used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$ value estimation method</td>
<td>Fixed value Vs Variable $C_D$</td>
</tr>
<tr>
<td>ADM</td>
<td>MSIS-86</td>
</tr>
<tr>
<td>Gravity model</td>
<td>EIGEN</td>
</tr>
<tr>
<td>Gravity max degree</td>
<td>100</td>
</tr>
<tr>
<td>Ocean tide model</td>
<td>CSR 3.0, Topex</td>
</tr>
<tr>
<td>Solar and geomagnetic data</td>
<td>Ap and $F_{10.7}$</td>
</tr>
<tr>
<td>Coordinate system</td>
<td>ITRF 2000</td>
</tr>
<tr>
<td>Time system</td>
<td>UTC</td>
</tr>
<tr>
<td>Leap seconds</td>
<td>Tai-UTC</td>
</tr>
<tr>
<td>General relativity</td>
<td>Applied</td>
</tr>
<tr>
<td>Planetary ephemeris</td>
<td>DE200</td>
</tr>
<tr>
<td>EOP</td>
<td>Bulletin B</td>
</tr>
</tbody>
</table>

**Processing Stage**

*Cowell integration and Least Squares*

Cowell integrator was adopted to propagate the satellite orbit and its variational equations. Weighted Least Squares is used to estimate the unknown parameters. Cowell integration is a multi-step numerical method using addition and linear combination of accelerations from previous epoch to predict then correct the state at current epoch. The correction at the current epoch may not be necessary for short step lengths or if only low accuracy results are required. In this research, the correction step is applied to ensure highest accuracy results are achieved.
Solve equations of Least Squares

Least squares solution is performed for best estimates of the parameters.

Least Squares solution convergence

Determine whether a least squares solution has converged from the iteration process. If the solution has not converged, then processing is reiterated again.

Removal of outliers

When the solution has converged, detection of outlier is performed. This eliminates any bad data that is contained in the observations for reliable orbit prediction. If there are outliers, the corresponding observations will be removed then the POD process will start again.

Orbit prediction

The software can predict orbit for more than ten days. However, only the prediction for the first day is used for this research.

Output Stage

Orbit File

The output file containing the orbit determination and predictions is provided for the three components of position and velocity in ITRF 2000. The file also contains the $C_D$ value estimated from the orbit determination, depending on the initial setup, the value is either pre-determined for the fixed $C_D$ value method or estimated during orbit determination for the variable $C_D$ value approach, the latter is called Vary$C_D$ approach in this research.
Figure 4-4: Data processing flowchart of the precise orbit determination software package
4.8. Summary

Atmospheric drag is difficult to be predicted due to its dynamic nature and this is one of the main limiting factors for precise orbit determination and prediction. The focus of this research is to investigate and identify approaches for improving the prediction of the drag coefficient, which is ultimately for enhancing the accuracy and efficiency of orbit prediction for space objects. This is because the $C_D$ value may vary significantly with altitude, location and time, thus the commonly used fixed value of 2.2 is not reasonable to represent the dynamicity of the atmospheric condition.

The case studies in this research are designed to assess and identify the differences in the $C_D$ estimates from the two commonly used methods, the Vary$C_D$ and the Fix2.2 approaches. Based on the variation trend shown in the $C_D$ estimates over the study period, the optimal fitting functions that are best fitted to the $C_D$ estimates over the study period will be identified and established. The optimal fitting functions used to predict the $C_D$ values are ultimately for improving the accuracy and efficiency of orbit predictions. The three satellites, Stella, Starlette and ERS-2 are selected for the experiments for this research. The experiment results will be discussed and analysed in the next two chapters.
Chapter 5 - Characterising the $C_D$ Estimates - Case Studies

5.1. Introduction

The accuracy of atmospheric drag will significantly affect the accuracy of orbit determination and prediction of space objects, which is especially true for space objects in the LEO. The atmospheric drag in POD is primarily governed by three parameters, the atmospheric density, $C_D$ value and area-to-mass ratio of the space object.

The main objective of this research is to seek feasible approaches to improve the accuracy of atmospheric drag prediction, specifically, through the enhancement of prediction for the $C_D$ value. This chapter contains four case studies for assessing the effects of the aforementioned three atmospheric drag parameters. The orbit prediction results derived from the case studies are compared with the orbit predictions from ILRS as the ILRS results are assumed to be more accurate. The four case studies are described below.

- Case study-1 and case study -2: The objectives of these two case studies are to compare and quantify the $C_D$ value estimated using the two methods widely adopted by the space industry, known as the Fix2.2 method and Vary$C_D$ approach in this research. The Fix2.2 method assigns a fixed figure of 2.2 to the $C_D$ value in all the orbit determination and prediction throughout the whole study period and this method is commonly adopted due to its timely orbit prediction. In contrast, the Vary$C_D$ approach estimates an optimal $C_D$ value when performing POD, referred to as $C_D$ estimates. The reason for using this approach is that the
actual $C_D$ value may vary with time according to the characteristics of the atmosphere.

- Case study-3: The ballistic value of a satellite is determined by the product of the $C_D$ value and the area-to-mass ratio of the satellite. Therefore, different satellites may have different ballistic values depending on the cross-sectional area in the along-track direction of orbit. This case study focuses on assessing the relationship between the $C_D$ value and area-to-mass ratio of the satellite by comparing three different shaped satellites with similar properties.

- Case study-4: Numerous ADM have been developed in the past and in most cases, the ADM are developed purposely to suit particular situations thus their performances are expected to vary between different ADM. This case study is for assessing the effects of two commonly adopted ADM, namely MSIS-86 and DTM-78, on the $C_D$ estimates and orbit predictions.

5.2. Case Study-1: Determination of Variations in the $C_D$ Estimates Over a One Year Period

Introduction
This experiment is a feasibility study to investigate the fluctuations of the $C_D$ values and their effects on the orbit prediction over a one-year period at a monthly interval. The differences between the two $C_D$ estimation methods, i.e., the Fix2.2 and Vary$C_D$ approaches are compared. The experimental subject is the spherical-shaped satellite, Stella, because it is widely accepted by the space industry that a fixed value of 2.2 can be used as the $C_D$ value for spherical shaped objects. Thus, it is anticipated that the $C_D$ estimates from the Vary$C_D$ approach would approximate to 2.2 units throughout the study period, if the atmospheric conditions were stable.
Results and analyses

To reduce the amount of data processing for this feasibility study, the same day was selected from each month for the testing, that is, the last Monday of each month from January 2006 to December 2006. The monthly interval implies that any variation in the CD values within the one-month interval will not be reflected or detected. However, it is sufficient to identify a general yearly trend that may exist in the CD values.

Figure 5-1 shows the CD estimates obtained from the VaryCD approach over 2006 and their differences from the fixed CD value of 2.2. This graph indicates a significant variation in the CD estimates at monthly intervals over the one-year period, from a minimum of 1.92 to a maximum of 2.68. This implies a significant variation in the atmospheric condition because the area-to-mass ratio of this spherical satellite is a constant, which suggests that the variation in the CD estimates is caused primarily by the variation in the atmospheric density.

Moreover, the differences in the CD estimates illustrated in the figure also suggest that the fixed CD value of 2.2, the commonly adopted approach for orbit determination and prediction by the space industry, may not be reasonable and hence may result in degradation in the accuracy of the orbit determination and prediction. Another significant finding from this case study (see Figure 5-1) is the trend of the variation of the CD estimates over the one-year period, it suggests a likelihood of a sinusoidal trend because it is apparent that the trend presents two peaks in February and August and two troughs in June and November.
To further validate the finding from Figure 5-1, i.e., the fluctuation in the \( C_D \) values over the one-year period, their resultant orbit predictions derived using the two approaches are compared against the prediction from ILRS. The approach whose orbit results have minimal differences from the ILRS is considered the more accurate and more preferred approach.

Table 5-1 presents the comparison results of the predicted overall position and velocity of the two \( C_D \) estimation approaches against predictions by ILRS. The overall position and velocity differences are the vector or the displacements from the three directions, i.e., the vector or the displacement of the along-track, cross-track and radial directions. This table demonstrates that the Vary\( C_D \) approach outperforms the Fix2.2 method as its RMS value of position differences is 1.45 m, which is significantly lower than that of the Fix2.2 achieved 3.48 m. Although the velocity results of the Vary\( C_D \) approach show no apparent improvement compared to the Fix2.2 results.
It is evident from Figure 5-1 and Table 5-1 that a large magnitude in the differences between the $C_D$ estimates from the VaryC$_D$ and the Fix2.2 approaches are reflected in the overall predicted positions. For example, in February 2006, the $C_D$ estimate from the VaryC$_D$ approach is 2.67, which is 0.47 units different from fixed $C_D$ value 2.2 and the respective orbit prediction results are 0.54 m and 6.97 m respectively. This indicates that inaccurate $C_D$ estimates can cause metre-level differences in the overall position prediction.

**Table 5-1:** Comparison of differences in the predicted position and velocity using the $C_D$ estimates from the two approaches against the ILRS results for satellite Stella at a monthly interval in 2006

<table>
<thead>
<tr>
<th>Overall Position Difference (m)</th>
<th>Fix2.2</th>
<th>VaryC$_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>8.13</td>
<td>2.95</td>
</tr>
<tr>
<td>Feb</td>
<td>6.97</td>
<td>0.54</td>
</tr>
<tr>
<td>Mar</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td>Apr</td>
<td>0.39</td>
<td>0.79</td>
</tr>
<tr>
<td>May</td>
<td>0.60</td>
<td>0.89</td>
</tr>
<tr>
<td>Jun</td>
<td>0.60</td>
<td>2.29</td>
</tr>
<tr>
<td>Jul</td>
<td>2.93</td>
<td>0.55</td>
</tr>
<tr>
<td>Aug</td>
<td>2.04</td>
<td>1.22</td>
</tr>
<tr>
<td>Sep</td>
<td>1.56</td>
<td>0.95</td>
</tr>
<tr>
<td>Oct</td>
<td>3.56</td>
<td>0.41</td>
</tr>
<tr>
<td>Nov</td>
<td>0.13</td>
<td>2.39</td>
</tr>
<tr>
<td>Dec</td>
<td>0.78</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>RMS</strong></td>
<td><strong>3.48</strong></td>
<td><strong>1.45</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Overall Velocity Difference (m/s)</th>
<th>Fix2.2</th>
<th>VaryC$_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.0051</td>
<td>0.0094</td>
</tr>
<tr>
<td>Feb</td>
<td>0.0134</td>
<td>0.0168</td>
</tr>
<tr>
<td>Mar</td>
<td>0.0058</td>
<td>0.0058</td>
</tr>
<tr>
<td>Apr</td>
<td>0.0117</td>
<td>0.0119</td>
</tr>
<tr>
<td>May</td>
<td>0.0149</td>
<td>0.0144</td>
</tr>
<tr>
<td>Jun</td>
<td>0.0085</td>
<td>0.0096</td>
</tr>
<tr>
<td>Jul</td>
<td>0.0059</td>
<td>0.0044</td>
</tr>
<tr>
<td>Aug</td>
<td>0.0087</td>
<td>0.0094</td>
</tr>
<tr>
<td>Sep</td>
<td>0.0106</td>
<td>0.0100</td>
</tr>
<tr>
<td>Oct</td>
<td>0.0075</td>
<td>0.0100</td>
</tr>
<tr>
<td>Nov</td>
<td>0.0062</td>
<td>0.0038</td>
</tr>
<tr>
<td>Dec</td>
<td>0.0049</td>
<td>0.0056</td>
</tr>
<tr>
<td><strong>RMS</strong></td>
<td><strong>0.0100</strong></td>
<td><strong>0.0092</strong></td>
</tr>
</tbody>
</table>
The results are further analysed in a comparison of the position and velocity in the three directions presented in Table 5-2 and Table 5-3 for VaryC\(_D\) and Fix2.2 approaches respectively. The results generally show that superior results are achieved using the VaryC\(_D\) approach in all three directions compared to the Fix2.2 method for the twelve months of 2006. For example, in February 2006, the differences in the predicted along-track, cross-track and radial position using the VaryC\(_D\) approach measured against the ILRS results are 0.32 m, 0.19 m and 0.39 m respectively compared to Fix2.2 method achieved 2.22 m, 3.22 m and 5.77 m. The velocity results for the three directions in February have also shown comparable findings. This has demonstrated higher accuracy orbits are achieved using the VaryC\(_D\) approach compared to the Fix2.2 method. In addition, this reinforces that the C\(_D\) estimates from the VaryC\(_D\) approach is more accurate than a generic fixed value of 2.2 for representing the coefficient of drag.

**Table 5-2:** Comparison of the differences in the predicted position and velocity in the three directions using the C\(_D\) estimates from the VaryC\(_D\) approach against the ILRS results for satellite Stella at monthly intervals

<table>
<thead>
<tr>
<th>VARYC(_D)</th>
<th>Position (m)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Along-Track</td>
<td>Cross-Track</td>
</tr>
<tr>
<td>Jan</td>
<td>0.30</td>
<td>0.58</td>
</tr>
<tr>
<td>Feb</td>
<td>0.32</td>
<td>0.19</td>
</tr>
<tr>
<td>Mar</td>
<td>0.28</td>
<td>0.74</td>
</tr>
<tr>
<td>Apr</td>
<td>0.60</td>
<td>0.48</td>
</tr>
<tr>
<td>May</td>
<td>0.32</td>
<td>0.02</td>
</tr>
<tr>
<td>Jun</td>
<td>0.65</td>
<td>1.35</td>
</tr>
<tr>
<td>Jul</td>
<td>0.25</td>
<td>0.49</td>
</tr>
<tr>
<td>Aug</td>
<td>0.58</td>
<td>0.66</td>
</tr>
<tr>
<td>Sep</td>
<td>0.90</td>
<td>0.20</td>
</tr>
<tr>
<td>Oct</td>
<td>0.30</td>
<td>0.24</td>
</tr>
<tr>
<td>Nov</td>
<td>0.82</td>
<td>1.68</td>
</tr>
<tr>
<td>Dec</td>
<td>0.34</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>RMS</strong></td>
<td>0.52</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Table 5-3: Comparison of the differences in the predicted position and velocity in the three directions using the $C_D$ estimates from the Fix2.2 method against the ILRS results for satellite Stella at monthly intervals

<table>
<thead>
<tr>
<th></th>
<th>Fix2.2</th>
<th>Position (m)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Along-Track</td>
<td>Cross-Track</td>
<td>Radial</td>
</tr>
<tr>
<td>Jan</td>
<td>1.03</td>
<td>0.94</td>
<td>8.01</td>
</tr>
<tr>
<td>Feb</td>
<td>2.22</td>
<td>3.22</td>
<td>5.77</td>
</tr>
<tr>
<td>Mar</td>
<td>0.32</td>
<td>0.80</td>
<td>0.47</td>
</tr>
<tr>
<td>Apr</td>
<td>0.16</td>
<td>0.36</td>
<td>0.02</td>
</tr>
<tr>
<td>May</td>
<td>0.21</td>
<td>0.28</td>
<td>0.49</td>
</tr>
<tr>
<td>Jun</td>
<td>0.54</td>
<td>0.05</td>
<td>0.26</td>
</tr>
<tr>
<td>Jul</td>
<td>1.43</td>
<td>1.81</td>
<td>1.81</td>
</tr>
<tr>
<td>Aug</td>
<td>0.82</td>
<td>0.36</td>
<td>1.83</td>
</tr>
<tr>
<td>Sep</td>
<td>0.42</td>
<td>0.43</td>
<td>1.43</td>
</tr>
<tr>
<td>Oct</td>
<td>1.70</td>
<td>3.09</td>
<td>0.44</td>
</tr>
<tr>
<td>Nov</td>
<td>0.05</td>
<td>0.01</td>
<td>0.12</td>
</tr>
<tr>
<td>Dec</td>
<td>0.38</td>
<td>0.42</td>
<td>0.54</td>
</tr>
<tr>
<td>RMS</td>
<td>1.02</td>
<td>1.46</td>
<td>2.99</td>
</tr>
</tbody>
</table>

**Summary**

The results from this experiment have highlighted the critical need for estimating the atmospheric drag values, particularly the $C_D$ value. The results have shown that the Vary$C_D$ approach can provide more accurate orbit prediction, compared to the Fix2.2 method. The $C_D$ estimates obtained from the more accurate Vary$C_D$ approach has also demonstrated the existence of significant fluctuations in the $C_D$ estimates, which suggests a different scenario to the commonly adopted Fix2.2 method adopted by the industry. In the next case study, data from a longer period (three years) and denser sample interval (weekly) will be used for a more detailed investigation on the variations in the $C_D$ estimates over the study period.
5.3. **Case Study-2: Determination of Variations in the $C_D$ Estimates Over Three Consecutive Years**

**Introduction**

The results from the feasibility study in case study-1 have identified the apparent variations in the $C_D$ value over the one-year period. The purpose for this case study is to further investigate the variations over a longer period with a higher resolution, i.e., using data from three years from 2004 to 2006 at a weekly interval (Monday of every week is the date for orbit prediction comparison). It is anticipated that a significant sinusoidal trend in the $C_D$ estimates will be evident throughout the three-year study period.

**Results and analyses**

The $C_D$ estimates, for 2004, 2005 and 2006 are shown in Figure 5-2. From the results shown in Figure 5-2, the significant variations in the $C_D$ value over the three-year period are obvious. However, the amplitudes and the frequencies of the variation over each single year do slightly differ. For example, the $C_D$ estimates range from approximately 1.9 to 2.8 units over the one-year period in 2006, but a bigger fluctuation is apparent in 2004 and 2005. This is reasonable due to the dynamic nature of the atmosphere. Moreover, it is critical to acknowledge the number of occurrences that the $C_D$ estimates fall and rise to its minimum and maximum in the duration of each year. This figure shows approximately two peaks and two troughs in each year, this conforms to the conclusion from case study-1. The trend of the repetitive variation in the $C_D$ values at a yearly cycle is clearly visible over the three-year period. This is also the rationale for the use of sample data to establish fitting functions for the prediction of the $C_D$ value, which will be discussed in the next chapter.
The variation of the $C_D$ estimates are identified (see Figure 5-2) over the three-year study period and the cause of the variation is possibly due to the dynamic nature of the atmosphere. Figure 5-3 is a comparison of the $C_D$ values from Figure 5-2 and the mass density from MSIS-E-90 (NASA). It is evident that the trends between the two values are similar, e.g., the peak at approximately the 13th month and the trough at approximately the 20th week. Furthermore, the comparison between the $C_D$ values and the Ap values in Figure 5-4 also presents a correlation, i.e., the peaks of the $C_D$ values is at the Ap value's minimum and vice versa, this similarity is evident throughout the three-year period.

**Figure 5-2:** Fluctuation of $C_D$ estimates over three consecutive years from Vary$C_D$ approach for Stella
Figure 5-3: Comparison of the $C_D$ values and the mass density values

Figure 5-4: Comparison of $C_D$ values and $Ap$ values
Similar to the experiment for case study-1, the results of the orbit predictions can be used to validate if the VaryC\textsubscript{D} approach outperforms the Fix2.2 method by comparing them with the ILRS prediction results. If the accuracy of orbit prediction using the VaryC\textsubscript{D} approach is higher than that achieved by the Fix2.2 method, then it implies the C\textsubscript{D} estimates using the VaryC\textsubscript{D} approach are more accurate representation of the C\textsubscript{D} value.

The test results for the differences in the predicted position and velocity in the along-track, cross-track and radial directions are presented in Table 5-4. It is evident that all the RMS values of the orbit prediction based on the VaryC\textsubscript{D} approach are significantly lower than the Fix2.2 method. For example, in 2006, the position RMS value in the along-track direction using the Fix2.2 method is 1.36 m whereas the resultant value for the VaryC\textsubscript{D} approach is 0.76 m, which is an equivalent to a 44\% improvement. Significant improvements are also apparent in the cross-track and radial directions, i.e., 56.08\% and 58.46\% respectively, and as a result, a 55.10\% improvement in the overall position (or displacement) is achieved. Over the whole three-year period, a 69.25\% improvement in the predicted overall position is obtained using the VaryC\textsubscript{D} approach compared to the Fix2.2 method.

The velocity results in Table 5-5 are similar to the position results, where the VaryC\textsubscript{D} approach has demonstrated to be a superior method than the Fix2.2 approach, except in the along-track direction. This table indicates the improvements in the velocity predictions are significant in the cross-track and radial directions but the magnitude of improvements are not as large as the position prediction. For example, in 2006, the improvements using the VaryC\textsubscript{D} approach in the along-track, cross-track and radial results are 0.18\%, 9.47\% and 25.34\% respectively and the improvements in the overall velocity over the three-year period is 29.57\%.
### Table 5-4: Comparison of the position differences of the Fix2.2 and VaryC\textsubscript{D} approaches against ILRS results

<table>
<thead>
<tr>
<th>Year</th>
<th>Fix2.2 Along-Track</th>
<th>Fix2.2 Cross-Track</th>
<th>Fix2.2 Radial</th>
<th>Fix2.2 Overall Position</th>
<th>VaryC\textsubscript{D} Along-Track</th>
<th>VaryC\textsubscript{D} Cross-Track</th>
<th>VaryC\textsubscript{D} Radial</th>
<th>VaryC\textsubscript{D} Overall Position</th>
<th>Improvements</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>1.63</td>
<td>10.97</td>
<td>8.11</td>
<td>13.74</td>
<td>0.84</td>
<td>4.14</td>
<td>2.50</td>
<td>4.91</td>
<td>48.73%</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>64.28%</td>
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<tr>
<td>2005</td>
<td>2.69</td>
<td>7.77</td>
<td>7.24</td>
<td>10.96</td>
<td>0.61</td>
<td>1.48</td>
<td>1.39</td>
<td>2.12</td>
<td>77.24%</td>
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<tr>
<td>2006</td>
<td>1.36</td>
<td>2.13</td>
<td>2.39</td>
<td>3.48</td>
<td>0.76</td>
<td>0.94</td>
<td>0.99</td>
<td>1.56</td>
<td>44.03%</td>
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<tr>
<td>3 years</td>
<td>2.00</td>
<td>7.81</td>
<td>6.42</td>
<td>10.30</td>
<td>0.74</td>
<td>2.55</td>
<td>1.73</td>
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<td></td>
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<td></td>
<td>69.25%</td>
</tr>
</tbody>
</table>

### Table 5-5: Comparison of the velocity differences of the Fix2.2 and VaryC\textsubscript{D} approaches against ILRS results

<table>
<thead>
<tr>
<th>Year</th>
<th>Fix2.2 Along-Track</th>
<th>Fix2.2 Cross-Track</th>
<th>Fix2.2 Radial</th>
<th>Fix2.2 Overall Velocity</th>
<th>VaryC\textsubscript{D} Along-Track</th>
<th>VaryC\textsubscript{D} Cross-Track</th>
<th>VaryC\textsubscript{D} Radial</th>
<th>VaryC\textsubscript{D} Overall Velocity</th>
<th>Improvements</th>
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<tbody>
<tr>
<td>2004</td>
<td>0.0078</td>
<td>0.0090</td>
<td>0.0118</td>
<td>0.0167</td>
<td>0.0082</td>
<td>0.0039</td>
<td>0.0051</td>
<td>0.0105</td>
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<td></td>
<td></td>
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<td>37.57%</td>
</tr>
<tr>
<td>2005</td>
<td>0.0081</td>
<td>0.0080</td>
<td>0.0088</td>
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<td>0.0030</td>
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</tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>2006</td>
<td>0.0074</td>
<td>0.0058</td>
<td>0.0035</td>
<td>0.0100</td>
<td>0.0074</td>
<td>0.0053</td>
<td>0.0026</td>
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<td></td>
<td></td>
<td></td>
<td>5.95%</td>
</tr>
<tr>
<td>3 years</td>
<td>0.0078</td>
<td>0.0077</td>
<td>0.0087</td>
<td>0.0140</td>
<td>0.0081</td>
<td>0.0043</td>
<td>0.0037</td>
<td>0.0098</td>
<td>-3.66%</td>
</tr>
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<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>57.37%</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>29.57%</td>
</tr>
</tbody>
</table>
Summary
The results from this experiment have evidently presented the variation in the \( C_D \) estimates over the three year period and the variation range was between 1.9 to 3.4 units. The results have also demonstrated a yearly repetitive cyclical trend in the variation of the \( C_D \) estimates. The orbit prediction results indicated the improvement of approximately 70% and 30% in position and velocity respectively, using the Vary\( C_D \) approach compared to the Fix2.2 method. This validates the significance of using the Vary\( C_D \) approach for estimating the \( C_D \) value, instead of using the commonly adopted Fix2.2 method. The results presented in this case study have similar correlation with results conducted by past researchers, where density models have demonstrated to contain RMS error of nearly 10-30% depending on the atmospheric condition (Bergstrom et al., 2001; Bowman et al., 2006). Therefore, the variations in the \( C_D \) estimates are possible reflectance of the errors in the density model. Furthermore, researches conducted by Bowman and collaborators have also demonstrated a variation in the \( C_D \) value from 2.10 to 2.40 caused by the changing ballistic value for satellites at various altitudes (Bowman, 2002).

5.4. Case Study-3: Assessment of the Effects of the Ballistic Value on Orbit Prediction

Introduction
The rationale for this case study is to use different satellites, i.e., Starlette and ERS-2, to validate the existence of the variations in the \( C_D \) estimates over the three-year study period and also to quantify the effects of the ballistic value on orbit prediction. The ballistic value refers to the product of the \( C_D \) value and the area-to-mass ratio. For non-spherical satellites, specifically ERS-2 in this case, the area-to-mass ratio is constantly changing due to the rotation of the satellite in orbit and as a result, the drag force encountered by the satellite is more difficult to model.
For comparison of the results from this case study to those from case study-2, this experiment is conducted under the same conditions as the case study-2, i.e. all variables and parameter settings in the software were consistent with those used in case study-2. The same density data over the same period are adopted for testing and the Monday of every week are used for the comparison of orbit prediction. It is anticipated that the variation in the $C_D$ estimates from Starlette to be similar to that achieved by Stella, whereas the $C_D$ estimates from ERS-2 may be significantly different. This is due to the near identical properties of the spherical satellites Stella and Starlette (their main differences are in the inclination and the nominal mass), where ERS-2 is a non-spherical satellite with a changing area-to-mass ratio.

**Results and analyses - Starlette**

The test results from Starlette shown in Figure 5-5 indicate the variation in the $C_D$ estimates over the three-year study period, ranging from 1.8 to 3.4 units, is similar to that obtained from Stella in case study-2. In addition, the frequencies of the peaks and troughs in the $C_D$ estimates obtained from the two satellites are also comparable due to the similarities in the properties of the Stella and Starlette, e.g. the shape, size and altitude.
The orbit prediction results for Starlette, shown in Table 5-6 and Table 5-7, have supported that the \( C_D \) estimates, derived from the Vary\( C_D \) approach, represent the actual coefficient of drag more accurately than the constant value of 2.2. For example, from the position results of 2006 in Table 5-6, the accuracy of orbit predictions have improved by 42.43%, 45.38% and 41.82% in the along-track, cross-track and radial directions respectively. The improvement is evident throughout the three-year study period. The velocity prediction in Table 5-7 demonstrates that using the Vary\( C_D \) approach, a 2.84%, 5.58% and 3.87% improvements are evident for the along-track cross-track and radial respectively. This improvement is similar to that achieved by Stella in Table 5-5 from case study-2.
Table 5-6: Comparison of the differences in predicted position for the Fix2.2 and VaryC\textsubscript{D} approaches against ILRS results for Starlette

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>3.71</td>
<td>4.82</td>
<td>4.57</td>
<td>7.61</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>2.23</td>
<td>3.15</td>
<td>3.05</td>
<td>4.92</td>
</tr>
<tr>
<td>Improvements</td>
<td>39.85%</td>
<td>34.72%</td>
<td>33.18%</td>
<td>35.33%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>2.12</td>
<td>2.48</td>
<td>2.46</td>
<td>4.09</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>0.68</td>
<td>0.68</td>
<td>0.83</td>
<td>1.27</td>
</tr>
<tr>
<td>Improvements</td>
<td>67.93%</td>
<td>72.56%</td>
<td>66.41%</td>
<td>68.97%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.18</td>
<td>1.16</td>
<td>1.61</td>
<td>2.31</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>0.68</td>
<td>0.63</td>
<td>0.94</td>
<td>1.32</td>
</tr>
<tr>
<td>Improvements</td>
<td>42.43%</td>
<td>45.38%</td>
<td>41.82%</td>
<td>42.86%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>2.52</td>
<td>3.14</td>
<td>3.09</td>
<td>5.08</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>1.37</td>
<td>1.85</td>
<td>1.86</td>
<td>2.96</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>45.48%</td>
<td>41.27%</td>
<td>39.75%</td>
<td>41.70%</td>
</tr>
</tbody>
</table>

Table 5-7: Comparison of the differences in predicted velocity for the Fix2.2 and VaryC\textsubscript{D} approaches against ILRS results for Starlette

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0054</td>
<td>0.0064</td>
<td>0.0056</td>
<td>0.0100</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>0.0047</td>
<td>0.0046</td>
<td>0.0055</td>
<td>0.0086</td>
</tr>
<tr>
<td>Improvements</td>
<td>12.31%</td>
<td>27.79%</td>
<td>0.59%</td>
<td>14.24%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0050</td>
<td>0.0044</td>
<td>0.0046</td>
<td>0.0081</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>0.0043</td>
<td>0.0039</td>
<td>0.0043</td>
<td>0.0073</td>
</tr>
<tr>
<td>Improvements</td>
<td>14.83%</td>
<td>10.61%</td>
<td>0.29%</td>
<td>10.74%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0037</td>
<td>0.0037</td>
<td>0.0042</td>
<td>0.0067</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>0.0036</td>
<td>0.0034</td>
<td>0.0041</td>
<td>0.0065</td>
</tr>
<tr>
<td>Improvements</td>
<td>2.84%</td>
<td>5.58%</td>
<td>3.87%</td>
<td>4.06%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0048</td>
<td>0.0049</td>
<td>0.0048</td>
<td>0.0084</td>
</tr>
<tr>
<td>VaryC\textsubscript{D}</td>
<td>0.0042</td>
<td>0.0040</td>
<td>0.0047</td>
<td>0.0075</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>11.35%</td>
<td>18.24%</td>
<td>3.49%</td>
<td>10.89%</td>
</tr>
</tbody>
</table>
Results and analyses - ERS-2

The $C_D$ estimates from ERS-2 presented in Figure 5-6 demonstrate a similar trend, with comparable frequency of the peaks and troughs, over the three-year period with those from Stella and Starlette. However, the $C_D$ estimates fluctuate over bigger amplitudes with tentatively larger values, ranging from 2.6 to 5.2 units. The large fluctuation is most likely due to the variation of the cross-sectional area of the satellite in orbit because any uncertainties in the computed cross-sectional area are reflected in the $C_D$ value, as discussed in Chapter 3.2.5.2 Ballistic Value. Although this phenomenon is anticipated, it is important to acknowledge the sinusoidal trend in the $C_D$ estimates despite the differences in the shape of the satellite and the effect of the large fluctuation of $C_D$ estimates on orbit prediction.

![Figure 5-6: Fluctuation of $C_D$ estimates over three consecutive years from Vary$C_D$ approach for ERS-2](image)

The comparisons of the orbit prediction results for ERS-2, shown in Table 5-8 and Table 5-9, have revealed similar findings to those from Stella and Starlette. Although the accuracy of the ERS-2 orbit prediction results are significantly
lower than the other two spherical satellites. From the comparison of the two C_D approaches, significant percentage improvements in both the position and velocity prediction results are achieved by the VaryC_D approach. The large differences in orbit predictions between this satellite and the other two spherical satellites, Starlette and Stella, are likely due to the variation of the area-to-mass ratio of ERS-2. This is because the cross-sectional area of a spherical satellite in orbit is consistent, but for non-spherical satellites, such as ERS-2, the drag force is affected by any pitch, roll or yaw in the along-track direction of the orbit, therefore, it leads to difficulty to represent the uncertainties by the C_D estimates.

Table 5-8: Comparison of the differences in predicted position for the Fix2.2 and VaryC_D approaches against ILRS results for ERS-2

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Along-Track</td>
<td>Cross-Track</td>
<td>Radial</td>
<td>Overall Position</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>226.06</td>
<td>104.03</td>
<td>170.60</td>
<td>301.71</td>
</tr>
<tr>
<td>VaryC_D</td>
<td>21.86</td>
<td>9.83</td>
<td>13.72</td>
<td>27.62</td>
</tr>
<tr>
<td>Improvements</td>
<td>90.33%</td>
<td>90.55%</td>
<td>91.96%</td>
<td>90.85%</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>171.76</td>
<td>79.05</td>
<td>182.48</td>
<td>262.77</td>
</tr>
<tr>
<td>VaryC_D</td>
<td>15.59</td>
<td>6.39</td>
<td>21.37</td>
<td>27.21</td>
</tr>
<tr>
<td>Improvements</td>
<td>90.92%</td>
<td>91.92%</td>
<td>88.29%</td>
<td>89.64%</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>113.06</td>
<td>46.20</td>
<td>128.16</td>
<td>177.03</td>
</tr>
<tr>
<td>VaryC_D</td>
<td>12.44</td>
<td>4.11</td>
<td>16.48</td>
<td>21.05</td>
</tr>
<tr>
<td>Improvements</td>
<td>89.00%</td>
<td>91.11%</td>
<td>87.14%</td>
<td>88.11%</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>168.29</td>
<td>75.99</td>
<td>160.22</td>
<td>244.47</td>
</tr>
<tr>
<td>VaryC_D</td>
<td>16.33</td>
<td>6.73</td>
<td>17.83</td>
<td>25.10</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>90.30%</td>
<td>91.14%</td>
<td>88.87%</td>
<td>89.73%</td>
</tr>
</tbody>
</table>
Table 5-9: Comparison of the differences in predicted velocity for the Fix2.2 and VaryC\(D\) approaches against ILRS results for ERS-2

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.1776</td>
<td>0.0765</td>
<td>0.2565</td>
<td>0.3212</td>
</tr>
<tr>
<td>VaryC(D)</td>
<td>0.0135</td>
<td>0.0097</td>
<td>0.0243</td>
<td>0.0294</td>
</tr>
<tr>
<td>Improvements</td>
<td>92.42%</td>
<td>87.28%</td>
<td>90.54%</td>
<td>90.84%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.1799</td>
<td>0.0821</td>
<td>0.1939</td>
<td>0.2769</td>
</tr>
<tr>
<td>VaryC(D)</td>
<td>0.0208</td>
<td>0.0150</td>
<td>0.0169</td>
<td>0.0307</td>
</tr>
<tr>
<td>Improvements</td>
<td>88.46%</td>
<td>81.78%</td>
<td>91.30%</td>
<td>88.93%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.1235</td>
<td>0.0612</td>
<td>0.1250</td>
<td>0.1860</td>
</tr>
<tr>
<td>VaryC(D)</td>
<td>0.0163</td>
<td>0.0116</td>
<td>0.0140</td>
<td>0.0244</td>
</tr>
<tr>
<td>Improvements</td>
<td>86.76%</td>
<td>81.11%</td>
<td>88.83%</td>
<td>86.88%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.1596</td>
<td>0.0732</td>
<td>0.1897</td>
<td>0.2585</td>
</tr>
<tr>
<td>VaryC(D)</td>
<td>0.0175</td>
<td>0.0125</td>
<td>0.0180</td>
<td>0.0280</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>89.05%</td>
<td>82.89%</td>
<td>90.50%</td>
<td>89.15%</td>
</tr>
</tbody>
</table>

The C\(D\) estimates from the three satellites over the three-year period are presented in Figure 5-7. It should be noted that in this figure, the C\(D\) estimates from both Stella and Starlette are on the same scale, but the results of ERS-2 is on a different scale (right side in the figure). This figure suggests not only the existence of the variation in the C\(D\) estimates from all the three satellites over the three-year period but also the similar variation from Stella and Starlette due to their similar satellite properties. It is also evident that the rise and fall of the C\(D\) estimates occur at a similar frequency for the three satellites.
Summary
The results from this case study using two different satellites, Starlette and ERS-2, further validates and supports the results achieved in case study-2, where Stella was adopted as the experimental satellite. There are three important findings from the results of this case study. The first is the presence of the variation in the $C_D$ estimates from the Vary$C_D$ approach over the three-year period, the higher accuracy orbit prediction results obtained based on the Vary$C_D$ approach implies that the $C_D$ estimates derived from this approach represents the actual coefficient of drag better than constant values. Secondly, the trends in the variation of the $C_D$ estimates from the three experimental satellites have displayed similarities in the frequency of the peaks and troughs despite the differences in the shape of the satellite. Finally, the irregular shape of the satellite has a direct impact on the $C_D$ estimates as shown in the results from ERS-2 and as a result, it causes uncertainties in the ballistic value and the uncertainties are reflected in the orbit prediction results.
5.5. Case Study-4: Assessment of the Effects of Different ADM on Orbit Prediction

Introduction
It is well known that the atmospheric drag modelling plays an important role in the accuracy of orbit prediction. Atmospheric drag exerted on a space object is determined by two parameters, the atmospheric density and the ballistic value. The effects from the two parameters are closely coupled, which leads to difficulties to distinguish the effects contributed by the ADM and the ballistic value in orbit prediction. Case study-3 has focused on assessing the effects of ballistic value on orbit prediction. Whereas, the focus of this case study is to compare the differences in the orbit predictions caused by two different ADM, specifically, the MSIS-86 and the DTM-78 models.

The MSIS-86 has been adopted in all the aforementioned case studies. The objective of this experiment is to assess the differences in the $C_D$ estimates and the effects on orbit prediction caused by the two selected ADM, in this case DTM-78, which is also another commonly adopted ADM in atmospheric studies, compared with MSIS-86 used in case study-1 and case study-2. In order to compare the orbit prediction and the $C_D$ estimates with that from case study-2, the global processing environment variables for the data processing for the two case studies were set to be consistent.

Results and analyses - Stella
The scatter plot from Figure 5-8 represents the $C_D$ estimates using the Vary$C_D$ approach for Stella when the DTM-78 model is adopted. The cyclical trend suggests the fluctuation in the $C_D$ values over the three-year study period, from approximately 1.7 to nearly 3.2. The fluctuation in the $C_D$ estimates has again reconfirmed the variations in the $C_D$ value over time, although there are other possible contributing factors to the magnitude of fluctuations, such as the shape of the satellite as identified in case study-3.
The comparison of the prediction results for the position and velocity between the VaryC<sub>D</sub> approach and the Fixed 2.2 method is illustrated in Table 5-10 and Table 5-11 respectively. The results indicate that the VaryC<sub>D</sub> approach to be a superior method for orbit prediction than Fix2.2 method. For example, the RMS values of the three years total improvement of position for the along-track, cross-track and radial directions are 54.07%, 65.81% and 69.48% respectively. Improvements are also evident for the velocity prediction except in the along-track direction. However, an improvement of 26.13% over the three years for the overall velocity is achieved. This has shown comparable orbit prediction results with the MSIS-86 as shown in Table 5-4 in case study-2.
Table 5-10: Comparison of the differences in the predicted position using DTM-78 against ILRS results for Stella

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.76</td>
<td>10.73</td>
<td>8.33</td>
<td>13.70</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.81</td>
<td>3.90</td>
<td>2.36</td>
<td>4.63</td>
</tr>
<tr>
<td>Improvements</td>
<td>54.06%</td>
<td>63.68%</td>
<td>71.71%</td>
<td>66.23%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>2.25</td>
<td>6.92</td>
<td>5.27</td>
<td>8.98</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.70</td>
<td>1.70</td>
<td>1.50</td>
<td>2.38</td>
</tr>
<tr>
<td>Improvements</td>
<td>68.75%</td>
<td>75.37%</td>
<td>71.50%</td>
<td>73.53%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.01</td>
<td>1.52</td>
<td>1.81</td>
<td>2.57</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.90</td>
<td>1.19</td>
<td>1.23</td>
<td>1.94</td>
</tr>
<tr>
<td>Improvements</td>
<td>10.67%</td>
<td>21.64%</td>
<td>31.74%</td>
<td>24.56%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.76</td>
<td>7.36</td>
<td>5.74</td>
<td>9.50</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.81</td>
<td>2.52</td>
<td>1.75</td>
<td>3.17</td>
</tr>
<tr>
<td>Improvements</td>
<td>Total</td>
<td>54.07%</td>
<td>65.81%</td>
<td>69.48%</td>
</tr>
</tbody>
</table>

Table 5-11: Comparison of the differences in the predicted velocity using DTM-78 against ILRS results for Stella

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0078</td>
<td>0.0091</td>
<td>0.0116</td>
<td>0.0167</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.0083</td>
<td>0.0037</td>
<td>0.0048</td>
<td>0.0103</td>
</tr>
<tr>
<td>Improvements</td>
<td>-5.62%</td>
<td>58.86%</td>
<td>58.29%</td>
<td>38.38%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0082</td>
<td>0.0064</td>
<td>0.0076</td>
<td>0.0129</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.0086</td>
<td>0.0033</td>
<td>0.0030</td>
<td>0.0097</td>
</tr>
<tr>
<td>Improvements</td>
<td>-5.28%</td>
<td>47.60%</td>
<td>60.31%</td>
<td>24.41%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0072</td>
<td>0.0058</td>
<td>0.0029</td>
<td>0.0097</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.0074</td>
<td>0.0054</td>
<td>0.0028</td>
<td>0.0095</td>
</tr>
<tr>
<td>Improvements</td>
<td>-2.83%</td>
<td>7.04%</td>
<td>6.04%</td>
<td>1.39%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0078</td>
<td>0.0072</td>
<td>0.0081</td>
<td>0.0133</td>
</tr>
<tr>
<td>VaryC₀</td>
<td>0.0081</td>
<td>0.0042</td>
<td>0.0036</td>
<td>0.0099</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>-4.70%</td>
<td>41.03%</td>
<td>55.28%</td>
<td>26.13%</td>
</tr>
</tbody>
</table>
The comparison of the differences in the $C_D$ estimates caused by the differences of DTM-78 and MSIS-86 is shown in Figure 5-9. This figure demonstrates the similarity in the trend of the fluctuations of the $C_D$ estimates, e.g., the occurrences of the frequency and amplitude are comparable, although the MSIS-86 trend has a slightly larger fluctuation. For example, a trough is apparent just after week 26 and this trough is visible through the trend of $C_D$ estimates from both the DTM-78 and MSIS-86. This similarity is evident for all the peaks and troughs throughout the three-year study period, where both the $C_D$ estimate trends are comparable. For this reason, it is evident that there are similarities in the trend of the $C_D$ estimates using the DTM-78 and MSIS-86 models over the study period.

![Figure 5-9: Comparison of the $C_D$ estimates using MSIS-86 and DTM-78 over the three-year period](image)

Table 5-12 shows the comparison of orbit prediction using the Vary$C_D$ approach and the two different density models, MSIS-86 and DTM-78. The RMS value of the overall position prediction for both density models are 3.17 m and the predicted overall velocity is also very similar with differences of 0.0001 m/s for
the RMS value. This implies there are insignificant differences in the orbit predictions over the three years caused by the two different ADM, MSIS-86 and DTM-78.

**Table 5-12:** Comparison of the differences in the predicted position and velocity over the three-year period using the VaryC\(_D\) approach for MSIS-86 and DTM-78

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RMS of Position Difference for Stella (m)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VaryC(_D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DTM-78</td>
<td>0.81</td>
<td>2.52</td>
<td>1.75</td>
<td>3.17</td>
</tr>
<tr>
<td>MSIS-86</td>
<td>0.74</td>
<td>2.55</td>
<td>1.73</td>
<td>3.17</td>
</tr>
<tr>
<td><strong>RMS of Velocity Difference for Stella (m/s)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VaryC(_D)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DTM-78</td>
<td>0.0081</td>
<td>0.0042</td>
<td>0.0036</td>
<td>0.0099</td>
</tr>
<tr>
<td>MSIS-86</td>
<td>0.0081</td>
<td>0.0043</td>
<td>0.0037</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

**Results and analyses - Starlette**

The same experiment using DTM-78 is conducted for Starlette and the comparison of the C\(_D\) estimates using MSIS-86 and DTM-78 for Starlette is illustrated in Figure 5-10. Similar sinusoidal trend from the two ADM are identified, e.g., the peaks and troughs occur at approximately the same frequency. The C\(_D\) estimates based on the MSIS-86 model have larger fluctuations compared to that of DTM-78, this scenario is similar to Stella when MSIS-86 is adopted. The results have also indicated the cyclical trend in the time series of the CD estimate is evident regardless of using MSIS-86 or DTM-78.
The orbit prediction results in Table 5-13 and Table 5-14 are comparisons of the predicted position and velocity differences respectively. The percentage of improvements in the position prediction using the VaryC\textsubscript{D} approach is noticeable over the Fix2.2 method. For instance, noticeable improvements of 54.44\%, 50.43\% and 55.21\% using the VaryC\textsubscript{D} approach in the along-track, cross-track and radial in the predicted position over the three years are evident. However, less significant improvements are apparent in the velocity comparison with a total improvement of 8.71\% over the three years for the overall velocity. Comparable results are achieved using Stella as shown in Table 5-10 and Table 5-11, where similar percentage of improvements are evident.
Table 5-13: Comparison of the differences in the predicted position using DTM-78 against ILRS results for Starlette

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>3.28</td>
<td>3.25</td>
<td>2.67</td>
<td>5.34</td>
</tr>
<tr>
<td>VaryC0</td>
<td>1.47</td>
<td>1.59</td>
<td>1.19</td>
<td>2.47</td>
</tr>
<tr>
<td>Improvements</td>
<td>55.21%</td>
<td>51.19%</td>
<td>55.61%</td>
<td>53.77%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.47</td>
<td>1.19</td>
<td>1.87</td>
<td>2.66</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.63</td>
<td>0.54</td>
<td>0.65</td>
<td>1.06</td>
</tr>
<tr>
<td>Improvements</td>
<td>56.85%</td>
<td>54.41%</td>
<td>65.36%</td>
<td>60.30%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.10</td>
<td>0.95</td>
<td>1.37</td>
<td>2.00</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.64</td>
<td>0.65</td>
<td>0.87</td>
<td>1.26</td>
</tr>
<tr>
<td>Improvements</td>
<td>42.14%</td>
<td>31.33%</td>
<td>36.29%</td>
<td>36.83%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>2.13</td>
<td>2.03</td>
<td>2.04</td>
<td>3.58</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.97</td>
<td>1.00</td>
<td>0.91</td>
<td>1.67</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>54.44%</td>
<td>50.43%</td>
<td>55.21%</td>
<td>53.36%</td>
</tr>
</tbody>
</table>

Table 5-14: Comparison of the differences in the predicted velocity using DTM-78 against ILRS results for Starlette

<table>
<thead>
<tr>
<th></th>
<th>Along-Track</th>
<th>Cross-Track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0041</td>
<td>0.0049</td>
<td>0.0054</td>
<td>0.0084</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.0035</td>
<td>0.0039</td>
<td>0.0048</td>
<td>0.0071</td>
</tr>
<tr>
<td>Improvements</td>
<td>14.96%</td>
<td>20.37%</td>
<td>11.69%</td>
<td>15.35%</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0042</td>
<td>0.0040</td>
<td>0.0037</td>
<td>0.0069</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.0037</td>
<td>0.0037</td>
<td>0.0039</td>
<td>0.0065</td>
</tr>
<tr>
<td>Improvements</td>
<td>10.01%</td>
<td>7.30%</td>
<td>-3.44%</td>
<td>4.93%</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0037</td>
<td>0.0037</td>
<td>0.0040</td>
<td>0.0066</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.0038</td>
<td>0.0035</td>
<td>0.0041</td>
<td>0.0066</td>
</tr>
<tr>
<td>Improvements</td>
<td>-2.46%</td>
<td>3.43%</td>
<td>-2.81%</td>
<td>-0.80%</td>
</tr>
<tr>
<td>3 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0040</td>
<td>0.0042</td>
<td>0.0044</td>
<td>0.0073</td>
</tr>
<tr>
<td>VaryC0</td>
<td>0.0037</td>
<td>0.0036</td>
<td>0.0042</td>
<td>0.0066</td>
</tr>
<tr>
<td>Total Improvement</td>
<td>9.51%</td>
<td>12.28%</td>
<td>5.00%</td>
<td>8.71%</td>
</tr>
</tbody>
</table>
Summary
The orbit prediction results from this case study have again highlighted the variation in the $C_D$ estimates over the study period, similar to that achieved in case study-2 and case study-3. The $C_D$ values estimated using the Vary$C_D$ approach and DTM-78 suggests a comparable variation in the $C_D$ estimates to that found using MSIS-86, i.e., similar frequencies of the rise and fall are present. Although the CD estimates based on different ADM are visible throughout the three-year period, their resultant orbit prediction has no significant differences.

5.6. Discussion
The four case studies discussed in this chapter have identified the variation in the $C_D$ estimates over the three-year study period and a cyclical repetition of the trend in the $C_D$ estimates is presented. The accuracy of the $C_D$ estimates is validated by comparisons of the orbit prediction results based on the Vary$C_D$ approach and the Fix2.2 method, measured against the ILRS results. The comparisons conducted so far have concentrated on the $C_D$ estimates and their orbit predictions. However, it is also important to address the issue of time consumption for the data processing for the different $C_D$ value estimation methods.

Compared to the Fix2.2 method, the Vary$C_D$ approach has proven capable of achieving higher accuracy orbit prediction. However, the processing time needed for orbit prediction using the Fix2.2 is on average 20% to 25% faster than the Vary$C_D$ approach. This is important because fast processing speed means faster orbit prediction, which is the ultimate goal to high-speed orbit prediction applications. Thus, the two $C_D$ estimation approaches have different advantages and disadvantages, i.e., the Vary$C_D$ approach is more accurate for $C_D$ value prediction, but obtaining the optimal $C_D$ estimates consumes more time for processing orbit predictions. In contrast, the Fix2.2 method is less accurate but can provide faster orbit prediction results. To ensure accurate measurement of the
time taken for orbit prediction, the hardware and software used for the data processing are maintained consistent for the relevant experiments. This includes using the same computer, the same software and hardware and the same operating system under the same environmental conditions.

5.7. Conclusion

The case studies in this chapter investigated and assessed the differences between the two approaches for the estimation of the coefficient of drag currently adopted by the industry, specifically the Fixed 2.2 method and the VaryC

\[ C_D \]

approach. The results from the two approaches from case study-1 and case study-2 have shown significant differences in the \( C_D \) estimates over the three-year study period and a noticeable repetitive cyclical variation in the \( C_D \) estimates that are obtained from the more accurate VaryC\( C_D \) approach can be observed. The comparison of RMS values of the orbit prediction from satellite Stella has validated that the VaryC\( C_D \) as the more accurate approach because nearly 70% and 30% improvements in the predicted position and velocity respectively over the three years are achieved.

The results from case study-3 have demonstrated the effects of the ballistic value have on orbit prediction. The \( C_D \) values estimated from ERS-2 fluctuated with a larger amplitude and also tentatively larger \( C_D \) estimates, which is most likely caused by the constant variation of the area-to-mass ratio of the satellite. This causes large errors in the orbit prediction.

The results from case study-4 suggests insignificant differences in orbit predictions caused by different ADM, MSIS-86 and DTM-78, the two commonly adopted atmospheric density models for atmospheric research. The results also indicate that regardless of which ADM is used, a repetitive cyclical variation in the \( C_D \) value is present, although the magnitude of the \( C_D \) estimates from the use of different ADM are different.
In addition, although the Fix2.2 method has demonstrated lower accuracy orbit prediction, but the time required for the data processing is approximately one-fifth less than that needed by the VaryC<sub>D</sub> approach. This can be an important consideration because timely orbit prediction is the ultimately goal for many POD applications. The issue of providing efficient and effective approaches for C<sub>D</sub> value prediction using optimal fitting functions will be discussed in the succeeding chapter.
Chapter 6 - The Fitting Function Approach for the Prediction of the $C_D$ Value

6.1. Introduction

This chapter focuses on improving the accuracy of orbit predictions through enhancing the prediction of $C_D$ values and analyses the subsequent effects on the speed of orbit determination and prediction, because timely orbit prediction is also another vital aspect of orbit prediction. The case studies from the preceding chapter have investigated the differences in the two methods for estimating the $C_D$ value and assessed their effects on orbit prediction. The results suggest that a fixed value of 2.2 is not always the optimal value to represent the atmospheric drag coefficient and more importantly, a repetitive cyclical trend of the variation in the $C_D$ estimates over the three-year study period is presented.

The objective of this chapter is to investigate different types of functions to fit the $C_D$ estimates derived from previous case studies from 2004 to 2006 and identify the optimal function for $C_D$ value prediction. The orbit prediction using predicted $C_D$ values from the fitting functions are assessed and the possibilities of applying the fitting function approach to other satellites are also evaluated. The rationale for using the fitting function is to enhance the accuracy of the prediction of the $C_D$ value, which will ultimately facilitate higher accuracy orbit prediction. The performance of this technique will be compared against the Fix2.2 method because the latter is an approach widely adopted by the space industry and also proven to achieve lower accuracy estimation of the $C_D$ value as presented in Chapter 5. The two orbit processing methods are similar by assigning pre-determined $C_D$ values prior to orbit prediction, rather than estimated at the time of orbit determination as adopted by the Vary$C_D$ approach.

**Introduction**

The case studies from Chapter 5 have demonstrated that the $C_D$ estimates indicated a sinusoidal trend over the three-year study period. The objective of this experiment is to use the three-years of $C_D$ estimates from Stella (from case study-2 in Chapter 5) and establish an optimal fitting function for the prediction of $C_D$ values. The optimal function is determined by comparing the predicted $C_D$ values against $C_D$ estimates from the more accurate Vary$C_D$ approach. The purpose of establishing the function is to enhance accuracy for prediction of the $C_D$ value, which is ultimately used for orbit prediction.

**Methodology and selection of datasets**

The $C_D$ estimates derived using the Vary$C_D$ approach from the previous case studies covered a period of three-years from 2004 to 2006. Hence, this experiment will cover the following three sections:

- **1-year-dataset** - 2006;
- **2-years-dataset** - 2005 to 2006;
- **3-years-dataset** - 2004 to 2006.

The rationale for the selection of these three datasets is to assess the effects and performance using different lengths of historical data to the fitting functions because the optimal length of time span of the data that should be used for the fitting function is unknown. The results from these three datasets will provide an indication of the optimal time span.

The method for establishing the fitting functions are as follows. The first is that the historical $C_D$ estimates will be plotted in a scatter plot for each dataset. Secondly, based on the trend of each scatter plot functions are fitted to the trend of the $C_D$ estimates. Finally, using the established fitting functions to calculate (i.e., extrapolate) the predicted $C_D$ values, which is then adopted as pre-determined
values for orbit prediction using the fixed value approach. The results of the predictions can be further validated against the independent high-accuracy predictions from ILRS that will be conducted in the following experiments.

Data pre-processing
Prior to adopting the historical $C_D$ estimates to investigate the optimal fitting functions, data pre-processing is needed to identify and remove outliers or bad quality data, if any, from the dataset. In this research, the criterion of three times of the standard deviation is adopted for identifying the outliers, which also means statistical confidence level of 99.7%. If a $C_D$ estimate falls outside the criterion, it is regarded as an outlier, thus, it will be excluded from the dataset in the process of searching for the fitting function. The removal of outlier and bad quality data is important as the accuracy of the fitting functions to be established is heavily reliant on the quality of the dataset.

Selection of fitting function
Based on the cyclical variation nature of the $C_D$ value identified through previous case studies (see Figure 5-2), a sinusoidal function is considered the best-fit to the dataset. The basic sinusoidal function can be expressed as follows (see Equation (6-1)):

$$y(t) = a \cdot \sin(bt + c)$$

(6-1)

where,

- $y$ - $C_D$ value at $t$
- $t$ - epoch
- $a$ - amplitude
- $b$ - angular frequency
- $c$ - phase

However, the basic form of sinusoidal function in Equation (6-1) has one set of fixed amplitude, angular frequency and cycle phase for a period of the dataset, which reduces the flexibility of the curve. Instead, additional terms are added to
the function to increase the flexibility, as expressed in Equation (6-2), where the value, n, denotes the number of terms added to the function to increase the flexibility of the fit to the dataset:

\[ y(t) = \sum_{i=1}^{n} a_i \sin(b_i t + c_i) \]  

(6-2)

where,
- \( a_i \) - amplitude
- \( b_i \) - angular frequency
- \( c_i \) - phase
- \( n \) - number of terms in the sinusoidal series, \( 1 \leq n \leq 7 \)

Similarly, for more generic cases, Fourier series are also commonly used to simulate cyclical variations in datasets. It is a sum of two functions, sine and cosine, which enables flexible manipulation of the function to best fit the dataset.

Fourier series can be expressed as:

\[ y(t) = a_0 + \sum_{i=1}^{n} (a_i \cos(nwt) + b_i \sin(nwt)) \]  

(6-3)

where,
- \( a_0 \) - constant
- \( a_i \) - amplitude
- \( b_n \) - angular frequency
- \( n \) - number of terms in the series, \( 1 \leq n \leq 7 \)
- \( w \) - frequency
Establish fitting functions for each dataset

The sine and Fourier series are selected as an optimal type of fitting functions for the dataset, which are then used for extrapolating \( C_D \) values for 2007. The optimal fitting function from each dataset is selected based on the minimal differences in the predicted \( C_D \) values using the fitting function compared against the \( C_D \) estimates from the Vary\( C_D \) approach for 2007. The latter is regarded as the closest to "true" as the \( C_D \) estimates are the best estimates at the time of orbit determination and proven a more accurate approach in previous case studies. The following six figures demonstrate the Fourier series and sine functions fitted to 1-year-dataset, 2-years-dataset and 3-years dataset on the left hand side column of the plots and the right hand side column of the plot are the extrapolated \( C_D \) values for 2007. The following table demonstrates the legend and the axis details for the following six figures.

<table>
<thead>
<tr>
<th>Fitting function applied to ( C_D ) estimates from Stella</th>
<th>Predicted ( C_D ) values against ( C_D ) estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Legend:</strong></td>
<td><strong>Legend:</strong></td>
</tr>
<tr>
<td>♦ - ( C_D ) estimates predicted using the Vary( C_D ) approach</td>
<td>♦ - ( C_D ) values predicted using the Vary( C_D ) approach</td>
</tr>
<tr>
<td>Solid line - fitting function</td>
<td>■ - ( C_D ) values predicted by the fitting function</td>
</tr>
<tr>
<td><strong>Axies:</strong></td>
<td><strong>Axies:</strong></td>
</tr>
<tr>
<td>X - Weekly intervals for the study period (dataset)</td>
<td>X - Weekly intervals in 2007</td>
</tr>
<tr>
<td>Y - ( C_D ) values</td>
<td>Y - ( C_D ) values</td>
</tr>
</tbody>
</table>

The maximum number of terms in the sine functions and the Fourier series is set to seven (\( n = 7 \)). This is because it is evident from the following figures that more additional terms does not necessarily improve the predicted \( C_D \) values for 2007 and this is further discussed in the following section.
<table>
<thead>
<tr>
<th></th>
<th>Study period - 2006</th>
<th>Prediction period - 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier1, n=1</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Fourier2, n=2</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Fourier3, n=3</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>Fourier4, n=4</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>Fourier5, n=5</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
<tr>
<td>Fourier6, n=6</td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
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<tr>
<td>Fourier7, n=7</td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Figure 6-1:** Fourier series fitted to 1-year-dataset (2006) and $C_D$ value prediction for 2007
<table>
<thead>
<tr>
<th>Sine Function</th>
<th>Study period - 2006</th>
<th>Prediction period - 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sin1, n=1</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Sin2, n=2</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Sin3, n=3</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>Sin4, n=4</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>Sin5, n=5</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
<tr>
<td>Sin6, n=6</td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
</tr>
<tr>
<td>Sin7, n=7</td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Figure 6-2:** Sine functions fitted to 1-year-dataset (2006) and $C_D$ value prediction for 2007
<table>
<thead>
<tr>
<th>Fourier, n</th>
<th>Study period - 2005 to 2006</th>
<th>Prediction period - 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier1, n=1</td>
<td><img src="image1" alt="Plot" /></td>
<td><img src="image2" alt="Plot" /></td>
</tr>
<tr>
<td>Fourier2, n=2</td>
<td><img src="image3" alt="Plot" /></td>
<td><img src="image4" alt="Plot" /></td>
</tr>
<tr>
<td>Fourier3, n=3</td>
<td><img src="image5" alt="Plot" /></td>
<td><img src="image6" alt="Plot" /></td>
</tr>
<tr>
<td>Fourier4, n=4</td>
<td><img src="image7" alt="Plot" /></td>
<td><img src="image8" alt="Plot" /></td>
</tr>
<tr>
<td>Fourier5, n=5</td>
<td><img src="image9" alt="Plot" /></td>
<td><img src="image10" alt="Plot" /></td>
</tr>
<tr>
<td>Fourier6, n=6</td>
<td><img src="image11" alt="Plot" /></td>
<td><img src="image12" alt="Plot" /></td>
</tr>
<tr>
<td>Fourier7, n=7</td>
<td><img src="image13" alt="Plot" /></td>
<td><img src="image14" alt="Plot" /></td>
</tr>
</tbody>
</table>

**Figure 6-3:** Fourier series fitted to 2-years-dataset (2005 to 2006) and $C_D$ value prediction for 2007
Study period - 2005 to 2006  
Prediction period - 2007

<table>
<thead>
<tr>
<th>Sine Function</th>
<th>Study Period</th>
<th>Prediction Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sin1, n=1</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Sin2, n=2</td>
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<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Sin3, n=3</td>
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<td><img src="image6" alt="Graph" /></td>
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<tr>
<td>Sin4, n=4</td>
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<td><img src="image8" alt="Graph" /></td>
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<tr>
<td>Sin5, n=5</td>
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<td><img src="image10" alt="Graph" /></td>
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<tr>
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<td><img src="image12" alt="Graph" /></td>
</tr>
<tr>
<td>Sin7, n=7</td>
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</table>

**Figure 6-4:** Sine functions fitted to 2-years-dataset (2005 to 2006) and $C_D$ value prediction for 2007
<table>
<thead>
<tr>
<th>Fouriers</th>
<th>Study period - 2004 to 2006</th>
<th>Prediction period - 2007</th>
</tr>
</thead>
<tbody>
<tr>
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<td><img src="image1.png" alt="Image" /></td>
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<tr>
<td>Fourier4, n=4</td>
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<td>Fourier5, n=5</td>
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<td><img src="image10.png" alt="Image" /></td>
</tr>
<tr>
<td>Fourier6, n=6</td>
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<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>Fourier7, n=7</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
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</tbody>
</table>

**Figure 6-5:** Fourier series fitted to 3-years-dataset (2004 to 2006) and $C_D$ value prediction for 2007
<table>
<thead>
<tr>
<th>Sine Label</th>
<th>Study period - 2004 to 2006</th>
<th>Prediction period - 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sin1, n=1</td>
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<td><img src="image2" alt="Graph" /></td>
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<tr>
<td>Sin2, n=2</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Sin3, n=3</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>Sin4, n=4</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>Sin5, n=5</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
<tr>
<td>Sin6, n=6</td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
</tr>
<tr>
<td>Sin7, n=7</td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Figure 6-6:** Sine functions fitted to 3-years-dataset (2004 to 2006) and $C_D$ value prediction for 2007
Selection of optimal fitting function for each dataset

The three-year coverage of the $C_D$ values and their relevant fitting functions has been separated into three datasets as shown above. The RMS values in Table 6-2 is derived from measuring the differences between the $C_D$ values predicted using the fitting function against the $C_D$ estimates from the Vary$C_D$ approach over the specified data period as the estimates from the latter approach is assumed more accurate. The lowest RMS value in each section represents the minimal difference from the $C_D$ estimates, which is relatively more accurate. The comparison of the RMS values demonstrates that Fourier1, i.e. Fourier series without additional harmonic terms, achieves the minimum RMS values among all the Fourier series and sine functions for each of the three datasets. Therefore, this suggests Fourier1 is the optimal type of fitting function for the prediction $C_D$ values for 2007. It is also evident (see Table 6-2) that increasing the additional terms to the fitting functions decreases the RMS values for prediction of the $C_D$ values.

### Table 6-2: Comparison of the RMS values of the differences between the $C_D$ values predicted by the fitting function against the Vary$C_D$ approach for 2007

<table>
<thead>
<tr>
<th>RMS</th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier1</td>
<td>0.14</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>Fourier2</td>
<td>0.17</td>
<td>0.16</td>
<td>0.23</td>
</tr>
<tr>
<td>Fourier3</td>
<td>0.15</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td>Fourier4</td>
<td>0.33</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>Fourier5</td>
<td>0.36</td>
<td>0.36</td>
<td>0.34</td>
</tr>
<tr>
<td>Fourier6</td>
<td>0.37</td>
<td>0.40</td>
<td>0.31</td>
</tr>
<tr>
<td>Fourier7</td>
<td>0.44</td>
<td>0.42</td>
<td>0.38</td>
</tr>
<tr>
<td>Sin1</td>
<td>0.33</td>
<td>0.28</td>
<td>0.26</td>
</tr>
<tr>
<td>Sin2</td>
<td>0.14</td>
<td>1.25</td>
<td>0.53</td>
</tr>
<tr>
<td>Sin3</td>
<td>2.12</td>
<td>1.34</td>
<td>2.05</td>
</tr>
<tr>
<td>Sin4</td>
<td>6.82</td>
<td>3.53</td>
<td>0.49</td>
</tr>
<tr>
<td>Sin5</td>
<td>6.83</td>
<td>8.76</td>
<td>0.45</td>
</tr>
<tr>
<td>Sin6</td>
<td>4.73</td>
<td>6.64</td>
<td>0.30</td>
</tr>
<tr>
<td>Sin7</td>
<td>11.44</td>
<td>7.10</td>
<td>0.32</td>
</tr>
</tbody>
</table>
Optimal fitting function for the 1-year-dataset

The fitting function for the 1-year-dataset illustrated in Figure 6-7 is expressed in Equation (6-4), named "Fourier1_1year_Stella" for this research. The fitting function demonstrates a close fit to the $C_D$ estimates. Thus, it is anticipated that if there is similar and persistent trend in the $C_D$ values from 2006 to 2007, the fitting function can accurately predict $C_D$ values comparable to the $C_D$ estimates by the Vary$C_D$ approach.

\[ f(x) = 2.258 + -0.04274 \cdot \cos(x \cdot 0.231) + 0.2196 \cdot \sin(x \cdot 0.231) \]  

(6-4)

The $C_D$ values predicted using Fourier1_1year_Stella is illustrated in Figure 6-8 and it is evident that the predicted $C_D$ values using the Fourier1_1year_Stella is similar to that estimated using the Vary$C_D$ approach. This is noticeable due to the small displacement between the $C_D$ values predicted using the fitting function and the $C_D$ estimates for 2007. The frequency of peaks and troughs estimated by the Vary$C_D$ approach, which have possible relations to the atmospheric factors (see Figure 5-3 and Figure 5-4), are closely matched by the fitting function. Hence, it can be deduced that the Fourier1_1year_Stella is a more suitable function for $C_D$.
value prediction, which is also reflected in the nominal RMS value presented in Table 6-2.

![Figure 6-8: CD values predicted by Fourier1_1year_Stella for 2007 compared against CD estimates](image)

**Optimal fitting function for the 2-years-dataset**

The Fourier series fitted to the 2-year-dataset (see Figure 6-9) is expressed in Equation (6-5), named "Fourier1_2years_Stella" for this research. The fitting function has demonstrated noticeably larger displacements between the function and the CD estimates compared with Fourier1_1year_Stella from Figure 6-7. This is evident particularly during the larger fluctuations in the CD estimates at approximately the 10th week and 35th week. Consequently, it is anticipated that the CD values predicted for 2007 using function Fourier1_2years_Stella captures the peaks and troughs but at a lower level of accuracy than Fourier1_1year_Stella.
The predicted $C_D$ values for 2007 using Fourier1_2years_Stella (see Figure 6-10) have demonstrated similarities to the $C_D$ estimates. The peaks at week 10 and at approximately week 35 of 2007 have been acknowledged by the Fourier1_2years_Stella function. However, it is evident that the prediction of the $C_D$ values by Fourier1_1year_Stella (refer to Figure 6-8) fits closer to the $C_D$ estimates than the $C_D$ values predicted by the Fourier1_2years_Stella function. Therefore, it is expected that the orbit prediction to be less accurate using the predicted $C_D$ values from Fourier1_2years_Stella, this will be further discussed in the next experiment.

$$f(x) = 2.39 + -0.1132 \cdot \cos(x \cdot 0.2402) + 0.1901 \cdot \sin(x \cdot 0.2402)$$

(6-5)

**Figure 6-9:** Fourier1_2years_Stella fitted to $C_D$ estimates from 2005 to 2006
Optimal fitting function for the 3-years-dataset

The optimal fitting function for the 3-years-dataset illustrated in Figure 6-11 is expressed in Equation (6-6), named "Fourier1_3years_Stella". It is evident from Figure 6-11 that Fourier1_3years_Stella is a more generic function, suggesting that there are large displacements between $C_D$ values predicted by the fitting function and the $C_D$ estimates. For example, the Fourier1_3years_Stella has recognised the general frequency of the variations, but large differences are evident in the amplitude of the variations, especially in the first 60 weeks. It is also apparent that the amplitude of the variations in the $C_D$ estimates is diminishing over the three years and a higher frequency of the cyclical variation towards 2006. However, this phenomenon is not precisely accounted for by the fitting function.

Figure 6-10: $C_D$ values predicted by Fourier1_2years_Stella for 2007 compared against $C_D$ estimates
The $C_D$ values predicted by Fourier1_3years_Stella for 2007 have demonstrated similarities to the $C_D$ estimates as illustrated in Figure 6-12. The general trend of the $C_D$ values has been recognised, however there are noticeable differences in the amplitude of the variation. For instance, Fourier1_3years_Stella has recognised the trough at approximately the 25th week, but it is not accurately predicted because the actual $C_D$ estimates is approximately 1.8 units compared to the approximate 2.1 predicted by Fourier1_3years_Stella. This difference is likely to be reflected in the orbit predictions, where large errors are expected.

$$f(x) = 2.48 + -0.1641 \cdot \cos(x \cdot 0.264) + -0.1242 \cdot \sin(x \cdot 0.264)$$
As previously stated, additional terms may be added to both sine and Fourier series to improve the accuracy of the fitting functions. However, the increasing number of terms added to the function may introduce vibrations to the function fitted to the dataset. This is clearly demonstrated when a sine function with seven terms is fitted to the 3-years-dataset as illustrated in Figure 6-13. Furthermore, this vibration will result in poor prediction of $C_D$ values in 2007 as demonstrated in Figure 6-14.

**Figure 6-12:** $C_D$ values predicted by Fourier1_3years_Stella for 2007 compared against $C_D$ estimates

**Figure 6-13:** $C_D$ values fitted to the 3-years-dataset using Sine function ($n = 7$)
**Selection of the best time span for optimal prediction**

Theoretically, from the least squares perspective for establishing a fitting function, the precision of the solution for an empirical model may be improved by increasing the number of the observations. However, in cases where additional observations that span further back in time are used for solving the fitting function, it may not offer improved prediction results and may even result in degradation to the accuracy of prediction results. This is due to the temporal variation of the atmospheric conditions.

Table 6-3 demonstrates the statistical prediction results from the fitting functions for Fourier1, Fourier2 and Fourier3. The comparison of the RMS value from each row represents the accuracies of the predicted $C_D$ values from the three different time span datasets. It is evident that, except the value of 0.16 from Fourier2 in the 2-years-dataset, dataset with a longer time span decreases the accuracy of the prediction. This scenario suggests that using the one-year-dataset, i.e., the data
closest to the time of the orbit to be predicted achieves a higher accuracy $C_D$ value prediction compared with using data that span further back in time.

**Table 6-3:** Comparison of the RMS of the differences between the $C_D$ values predicted using Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella for 2007 compared against the $C_D$ estimates

<table>
<thead>
<tr>
<th>RMS</th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier1</td>
<td>0.14</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>Fourier2</td>
<td>0.17</td>
<td>0.16</td>
<td>0.23</td>
</tr>
<tr>
<td>Fourier3</td>
<td>0.15</td>
<td>0.17</td>
<td>0.22</td>
</tr>
</tbody>
</table>

**Summary**

This section of the chapter has determined the optimal fitting functions using $C_D$ estimates from Stella. The three times of standard deviation criterion adopted in this experiment has successfully eliminated the bad quality data through data pre-processing, i.e. the poor quality $C_D$ estimates from the dataset. Various Fourier series and sine functions have been examined and tested for each of the three datasets selected in this experiment. The results show that Fourier series without additional harmonic terms is the ideal function for predicting $C_D$ values for 2007 because it achieves the lowest RMS values in the comparison between the predicted $C_D$ values and the $C_D$ estimates. Thus, the three functions are established, Fourier1_1year_stella, Fourier1_2years_Stella and Fourier1_3years_Stella. In addition, the results from the three datasets with different lengths of time span indicate that 1-year-datasets offers best $C_D$ value prediction compared to 2-years and 3-years dataset.
6.3. Applicability testing of the $C_D$ Values Predicted from the Optimal Fitting Functions to Orbit Prediction

**Introduction**

The experiment in the previous section (Chapter 6.2) has established three fitting functions that provide optimal prediction of $C_D$ values for 2007. They were established through fitting Fourier series to the three datasets and the optimal function for each dataset was identified based on the minimal RMS value derived from comparing the fitting function predicted $C_D$ values against $C_D$ estimates. The objective of this section is to test and evaluate the performance of the three functions by applying the predicted $C_D$ values to orbit prediction. The orbit prediction results from the fitting functions, the Fix2.2 and the Vary$C_D$ approaches are assessed and compared against orbit predictions from ILRS.

This test is separated into two parts. In the first part, the $C_D$ values predicted using the fitting function from Stella are applied to orbit prediction for Stella for 2007. In the second part, the predicted $C_D$ values from Stella are applied to orbit predictions for two other satellites, i.e., Starlette and ERS-2, to evaluate the performance of the fitting function derived from one satellite applied for other satellites at similar altitudes (i.e. applicability study).

**Comparison of orbit prediction for Stella**

The $C_D$ values predicted from the fitting functions are used in the orbit prediction process as a fixed value input, where the $C_D$ values are pre-determined prior to the orbit determination and prediction process. This is the same processing technique as the Fix2.2 method, except that the Fix2.2 method adopts a constant value of 2.2. However, this approach is different to the Vary$C_D$ method, where the $C_D$ values are estimated to best-fit the actual atmospheric environment at the time of the data processing through the process of orbit determination. Hence, it is anticipated that the Vary$C_D$ approach achieves better accuracy for orbit prediction, even though it is a less efficient method (due to the longer data processing time required). The
The main emphasis of this test is to measure the accuracy of the fitting function technique against the Fix2.2 method through the comparison of their orbit prediction results. The reason for that is the Fix2.2 method is widely adopted by the industry due to its short data processing time compared with the VaryC\textsubscript{D} approach.

The comparison of the predicted position and velocity derived using the different approaches of C\textsubscript{D} value estimation/prediction against predictions from ILRS are illustrated in Table 6-4. As expected, the VaryC\textsubscript{D} approach achieved the most accurate orbit prediction. The comparison of the fitting functions against the Fix2.2 method has demonstrated that the fitting function approach achieved lower RMS values for all three established functions, which implies higher accuracy orbit prediction. The Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella functions established based on different time spans of historical C\textsubscript{D} estimates achieved RMS values 2.47 m, 2.54 m and 3.00 m respectively, against Fix2.2 accomplished 3.81 m for the predicted overall position, which equates to 35.03%, 33.26% and 21.22% improvement in the accuracy of the predicted position respectively. Enhancements to the predicted overall velocity are also evident with 5.61%, 5.89% and 4.50% improvements using the three respective fitting functions.

The comparison of orbit prediction results from the three datasets with different lengths of time span, i.e., 1-year, 2-years and 3-years, indicates that shorter data time span offers higher accuracy orbit prediction. This suggests that adding more C\textsubscript{D} estimates data by extending the time span to the fitting function for orbit prediction may not necessarily improve the accuracy of prediction due to the dynamic and temporal variation of the atmosphere.
Table 6-4: Comparison of the predicted position and velocity from the different approaches for $C_D$ value prediction/estimation for 2007

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RMS of Position Difference (m) - Stella</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vary$C_D$</td>
<td>0.76</td>
<td>0.61</td>
<td>1.01</td>
<td>1.40</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>1.94</td>
<td>1.86</td>
<td>2.70</td>
<td>3.81</td>
</tr>
<tr>
<td>Fourier1_1year_Stella</td>
<td>1.47</td>
<td>1.31</td>
<td>1.50</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>Fourier1_1year_Stella Vs Fix2.2 (%)</td>
<td>24.33%</td>
<td>29.75%</td>
<td>44.28%</td>
</tr>
<tr>
<td>Fourier1_2years_Stella</td>
<td>1.53</td>
<td>1.40</td>
<td>1.47</td>
<td>2.54</td>
</tr>
<tr>
<td></td>
<td>Fourier1_2years_Stella Vs Fix2.2 (%)</td>
<td>21.31%</td>
<td>24.77%</td>
<td>45.37%</td>
</tr>
<tr>
<td>Fourier1_3years_Stella</td>
<td>1.73</td>
<td>1.59</td>
<td>1.86</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>Fourier1_3years_Stella Vs Fix2.2 (%)</td>
<td>10.59%</td>
<td>14.68%</td>
<td>30.91%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RMS of Velocity Difference (m/s) - Stella</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vary$C_D$</td>
<td>0.0059</td>
<td>0.0057</td>
<td>0.0030</td>
<td>0.0087</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0059</td>
<td>0.0063</td>
<td>0.0042</td>
<td>0.0096</td>
</tr>
<tr>
<td>Fourier1_1year_Stella</td>
<td>0.0059</td>
<td>0.0058</td>
<td>0.0037</td>
<td>0.0091</td>
</tr>
<tr>
<td></td>
<td>Fourier1_1year_Stella Vs Fix2.2 (%)</td>
<td>-0.58%</td>
<td>8.82%</td>
<td>11.04%</td>
</tr>
<tr>
<td>Fourier1_2years_Stella</td>
<td>0.0060</td>
<td>0.0057</td>
<td>0.0037</td>
<td>0.0090</td>
</tr>
<tr>
<td></td>
<td>Fourier1_2years_Stella Vs Fix2.2 (%)</td>
<td>-1.56%</td>
<td>9.98%</td>
<td>12.06%</td>
</tr>
<tr>
<td>Fourier1_3years_Stella</td>
<td>0.0060</td>
<td>0.0058</td>
<td>0.0038</td>
<td>0.0092</td>
</tr>
<tr>
<td></td>
<td>Fourier1_3years_Stella Vs Fix2.2 (%)</td>
<td>-2.22%</td>
<td>8.29%</td>
<td>9.73%</td>
</tr>
</tbody>
</table>
Application of the fitting function predicted $C_D$ values from Stella to Starlette and ERS-2

The objective of this study is to evaluate whether $C_D$ values predicted by the three fitting functions using historical $C_D$ estimates from a reference satellite, in this case Stella, are suitable for application to other space objects, i.e., Starlette and ERS-2. There are limitations to this application, because the space objects to be tested must be from a similar altitude and ideally, the same spherical shape as the reference satellite. However, the fitting function predicted $C_D$ values are also applied to the non-spherical shaped satellite, i.e., ERS-2, to investigate whether improvements to the orbit prediction are possible.

The comparison of orbit prediction for Starlette is presented in Table 6-5. The results suggest that the three established Fourier series based on data from Stella, significantly outperforms the Fix2.2 method. Approximate 20% and 1.5% improvements in the predicted overall position and overall velocity respectively are achieved for all three fitting functions. The results also demonstrate that the percentages of improvements are generally similar in the three directions. These results also imply that the predicted $C_D$ values from the fitting function based on the three different spans of data period from Stella offers higher accuracy than the Fix2.2 method.
Table 6-5: Comparison of the RMS values of orbit prediction for Starlette using fitting function predicted $C_D$ values derived from Stella against the two conventional approaches

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vary$C_D$</strong></td>
<td>0.68</td>
<td>0.67</td>
<td>0.86</td>
<td>1.29</td>
</tr>
<tr>
<td><strong>Fix2.2</strong></td>
<td>1.27</td>
<td>1.46</td>
<td>1.71</td>
<td>2.58</td>
</tr>
<tr>
<td><strong>Fourier1_1year_Stella</strong></td>
<td>0.94</td>
<td>1.16</td>
<td>1.42</td>
<td>2.06</td>
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<tr>
<td>Fourier1_1year_Stella Vs Fix2.2 (%)</td>
<td>25.44%</td>
<td>20.91%</td>
<td>17.13%</td>
<td>20.27%</td>
</tr>
<tr>
<td><strong>Fourier1_2years_Stella</strong></td>
<td>0.93</td>
<td>1.08</td>
<td>1.37</td>
<td>1.98</td>
</tr>
<tr>
<td>Fourier1_2years_Stella Vs Fix2.2 (%)</td>
<td>26.78%</td>
<td>25.99%</td>
<td>19.95%</td>
<td>23.46%</td>
</tr>
<tr>
<td><strong>Fourier1_3years_Stella</strong></td>
<td>1.00</td>
<td>1.12</td>
<td>1.39</td>
<td>2.04</td>
</tr>
<tr>
<td>Fourier1_3years_Stella Vs Fix2.2 (%)</td>
<td>20.93%</td>
<td>23.45%</td>
<td>19.01%</td>
<td>20.87%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vary$C_D$</strong></td>
<td>0.0048</td>
<td>0.0044</td>
<td>0.0048</td>
<td>0.0081</td>
</tr>
<tr>
<td><strong>Fix2.2</strong></td>
<td>0.0047</td>
<td>0.0048</td>
<td>0.0050</td>
<td>0.0084</td>
</tr>
<tr>
<td><strong>Fourier1_1year_Stella</strong></td>
<td>0.0048</td>
<td>0.0048</td>
<td>0.0048</td>
<td>0.0083</td>
</tr>
<tr>
<td>Fourier1_1year_Stella Vs Fix2.2 (%)</td>
<td>-1.22%</td>
<td>-0.03%</td>
<td>5.06%</td>
<td>1.40%</td>
</tr>
<tr>
<td><strong>Fourier1_2years_Stella</strong></td>
<td>0.0047</td>
<td>0.0047</td>
<td>0.0048</td>
<td>0.0082</td>
</tr>
<tr>
<td>Fourier1_2years_Stella Vs Fix2.2 (%)</td>
<td>-0.71%</td>
<td>0.22%</td>
<td>5.23%</td>
<td>1.71%</td>
</tr>
<tr>
<td><strong>Fourier1_3years_Stella</strong></td>
<td>0.0047</td>
<td>0.0047</td>
<td>0.0049</td>
<td>0.0083</td>
</tr>
<tr>
<td>Fourier1_3years_Stella Vs Fix2.2 (%)</td>
<td>0.14%</td>
<td>0.82%</td>
<td>3.42%</td>
<td>1.54%</td>
</tr>
</tbody>
</table>
The experiment results of the orbit prediction for satellite ERS-2 are listed in Table 6-6. It is evident that the accuracies of the orbit predictions using the three established functions Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella based on data from Stella have presented improvements of approximately 5.41%, 14.25% and 14.20% respectively for the overall positions. These small percentage improvements using the Stella derived fitting functions are anticipated because the results from case study-3 have identified a significant fluctuation in the $C_D$ estimates and tentatively larger values for ERS-2. Thus, the relatively small fluctuation in the predicted $C_D$ values for 2007 is insufficient to be effective on the rather large changes in amplitude expected for ERS-2, which is a possible indicator for insufficient accuracy for the satellite ballistic value estimation.

The effects of the constantly varying area-to-mass ratio are evident in the comparison between the percentages of improvements for the RMS values between ERS-2 and Starlette. For instance, the RMS value from Fourier1_1year_Stella in the along-track direction for ERS-2 is 92.86 m compared to Fix2.2 derived 96.67 m, which is a 3.94% improvement. However, the corresponding result for Starlette has improved the along-track prediction by 25.44% when compared against the Fix2.2 method as illustrated in Table 6-5. The difference between the two derived RMS values for the two satellites is likely due to the inclination differences and the non-spherical shape of ERS-2. The results derived from ERS-2 have demonstrated that the $C_D$ values predicted using fitting functions derived using $C_D$ estimates from Stella has improved orbit prediction for non-spherical satellite ERS-2, although, the predicted $C_D$ values are preferably for application to spherical shaped satellites.
Table 6-6: Comparison of the RMS values of orbit prediction for ERS-2 using fitting function predicted $C_D$ values derived from Stella and the two conventional approaches

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vary $C_0$</td>
<td>73.18</td>
<td>25.90</td>
<td>30.04</td>
<td>83.24</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>96.67</td>
<td>40.29</td>
<td>105.13</td>
<td>148.39</td>
</tr>
<tr>
<td>Fourier1_1year Stella</td>
<td>92.86</td>
<td>40.32</td>
<td>97.24</td>
<td>140.37</td>
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<tr>
<td>Fourier1_1year Stella Vs Fix2.2 (%)</td>
<td>3.94%</td>
<td>-0.06%</td>
<td>7.50%</td>
<td>5.41%</td>
</tr>
<tr>
<td>Fourier1_2years Stella</td>
<td>79.22</td>
<td>34.50</td>
<td>93.41</td>
<td>127.25</td>
</tr>
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<tr>
<td>Fourier1_2years Stella Vs Fix2.2 (%)</td>
<td>18.05%</td>
<td>14.38%</td>
<td>11.15%</td>
<td>14.25%</td>
</tr>
<tr>
<td>Fourier1_3years Stella</td>
<td>79.58</td>
<td>33.25</td>
<td>93.67</td>
<td>127.32</td>
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<tr>
<td>Fourier1_3years Stella Vs Fix2.2 (%)</td>
<td>17.68%</td>
<td>17.48%</td>
<td>10.90%</td>
<td>14.20%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vary $C_0$</td>
<td>0.0211</td>
<td>0.0268</td>
<td>0.0791</td>
<td>0.0861</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.0999</td>
<td>0.0518</td>
<td>0.1082</td>
<td>0.1561</td>
</tr>
<tr>
<td>Fourier1_1year Stella</td>
<td>0.0930</td>
<td>0.0488</td>
<td>0.1043</td>
<td>0.1480</td>
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<td></td>
</tr>
<tr>
<td>Fourier1_1year Stella Vs Fix2.2 (%)</td>
<td>6.87%</td>
<td>5.79%</td>
<td>3.60%</td>
<td>5.17%</td>
</tr>
<tr>
<td>Fourier1_2years Stella</td>
<td>0.0893</td>
<td>0.0452</td>
<td>0.0895</td>
<td>0.1343</td>
</tr>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Fourier1_2years Stella Vs Fix2.2 (%)</td>
<td>10.62%</td>
<td>12.78%</td>
<td>17.24%</td>
<td>13.98%</td>
</tr>
<tr>
<td>Fourier1_3years Stella</td>
<td>0.0893</td>
<td>0.0458</td>
<td>0.0891</td>
<td>0.1342</td>
</tr>
<tr>
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</tr>
<tr>
<td>Fourier1_3years Stella Vs Fix2.2 (%)</td>
<td>10.56%</td>
<td>11.64%</td>
<td>17.59%</td>
<td>13.99%</td>
</tr>
</tbody>
</table>
Summary

The test conducted in this section has demonstrated the applicability of the fitting function approach (using $C_D$ estimates from Stella) on orbit predictions for Stella, Starlette and ERS-2. The Fourier series fitting functions used in this test were derived using predicted $C_D$ values from Stella, Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella from the previous experiment in Chapter 6.2. Their predicted orbits for 2007 are compared against the predicted orbits using the Fix2.2 method, due to their identical processing method.

The results from this test have demonstrated that the established fitting functions are effective methods for predicting $C_D$ values due to their achieved higher accuracy orbit predictions. The predicted $C_D$ values were adopted to Stella for the orbit prediction, where the results display significant enhancement such as, the 35.03%, 33.26% and 21.22% improvements in the predicted overall position using Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella respectively. In addition, the gradual changes in the percentage of improvements confirm that additional sample data that span further back in time may not necessarily improve the accuracy of predicted $C_D$ values and subsequently, the orbit predictions.

This test has also demonstrated that the $C_D$ values predicted by Stella are suitable for the application for Starlette and ERS-2. This is supported by their comparably higher accuracy orbit predictions achieved using the fitting functions against Fix2.2 method. However, the application is limited to space objects with similar altitude of orbit and ideally the same spherical shape as Stella. The orbit prediction results from ERS-2 have also presented 5% to 15% improvements when the three fitting functions derived from Stella are used compared with the Fix2.2 method.
6.4. Investigation of the Applicability of the Fitting Function Approach to Starlette and ERS-2

Introduction
The previous section (Chapter 6.2 and 6.3) have applied Fourier series fitting to three datasets of C\textsubscript{D} estimates covering a three-year period, where the fitting functions are extrapolated to predict the C\textsubscript{D} values. This approach was proven successful for achieving higher accuracy orbit predictions compared to the Fix2.2 method. In addition, it was evident that using data closest to the time of orbit prediction offered higher accuracy than data with longer time span. This was presented in the results of Chapter 6.3, when higher accuracy orbits were achieved using C\textsubscript{D} values predicted from Fourier1_1year_Stella. The objective of this experiment is to apply the same approach of establishing Fourier series fitting functions for Starlette and ERS-2 and evaluate the orbit prediction derived using the predicted C\textsubscript{D} values from the established fitting functions. However, only C\textsubscript{D} estimates from 2006 will be tested in this experiment based on the results that the optimal time span of sample data for accurate C\textsubscript{D} value prediction is one year.

Methodology and selection of data
The two datasets selected for this experiment are from C\textsubscript{D} estimates of one-year period derived from the VaryC\textsubscript{D} approach for Starlette and ERS-2 in Case study-3. Subsequently, there are two parts for this experiment and each part is dedicated for experiments for each satellite. The procedures for testing both satellites are identical to the procedures for the tests of Stella in Section 6.2. The data pre-processing tests are conducted to the datasets to eliminate outliers or bad quality data and the threshold value of three times of the standard deviation is applied to the C\textsubscript{D} estimates from each satellite. Fourier series without additional harmonic terms are solved based on the C\textsubscript{D} estimates from 2006 for the selected satellites. Finally, the predicted C\textsubscript{D} values from the established functions are applied to the orbit prediction as fixed value input. Finally, predicted orbits for 2007 are
compared for assessment and validation of the improvements of the fitting function approach.

**Optimal fitting function for Starlette**
The Fourier series fitted to the dataset of $C_D$ values in 2006 for Starlette is illustrated in Figure 6-15. It is named "F1_1yr_Starlette" and is expressed in Equation (6-7). It is evident that the fitting function is closely fitted to the data, which implies accurate $C_D$ value predictions are expected for 2007, if the variation in the atmospheric conditions is similar.

![Figure 6-15: Fourier series fitted to the dataset of $C_D$ estimates in 2006 for Starlette](image)

$$f(x) = 2.45 + 0.2996 \cdot \cos(x \cdot 0.2329) + 0.2854 \cdot \sin(x \cdot 0.2329)$$

(6-7)

The predicted $C_D$ values by the F1_1yr_Starlette function for 2007 presented in Figure 6-16 are compared with the $C_D$ estimates. From this figure, it is evident that the predictions are more accurate in the first half of the time series, which is reasonable according to the temporal variation of the atmosphere. This suggests that the $C_D$ values predicted closer to the time of prediction is more accurate than $C_D$ values predicted further away from the time of prediction. This conforms to
the hypothesis from the test in Section 6.2 where historic data closest to the time of prediction offers higher accuracy prediction.

![Figure 6-16: Comparison of the $C_D$ values predicted by F1_1yr_Starlette function and $C_D$ estimates for Starlette](image)

The orbit prediction using the F1_1yr_Starlette as depicted in Table 6-7 reveals enhanced accuracy is attained compared with the Fix2.2 method, where a 13.38%, 25.33% and 21.99% improvement in the RMS value in the predicted along-track, cross-track and radial directions respectively are achieved, when measured against predictions by the ILRS. This equates to a 20.86% improvement in the overall position and similar to the 20.27% achieved using Fourier1_1year_Stella from Table 6-4. The improvement of 1.54% is also evident in the predicted overall velocity. The similar percentages of improvements in the orbit prediction achieved for Starlette using the fitting functions derived from both Starlette and Stella further confirms that $C_D$ values predicted by one spherical satellite is applicable to another spherical satellite at the same altitude.
Optimal fitting function for ERS-2

It is evident from the $C_D$ estimates of ERS-2, as illustrated in Figure 6-17, that the fluctuation is more significant than Stella and Starlette (see Figure 6-7 and Figure 6-15), with the amplitude ranging from approximately 2.8 to over 4.2. The $C_D$ estimates from ERS-2 are tentatively larger than the $C_D$ estimates derived from the spherical satellites, which is due likely to the uncertainties caused by lack of accurate determination of the cross-sectional area of the satellite in orbit. This uncertainty is reflected in the $C_D$ estimates. Hence, the function fitted to the data, named "F1_1yr_ERS2" for this research, is expected to predict larger $C_D$ values with bigger amplitude than F1_1yr_Starlette and Fourier1_1year_Stella for 2007. The F1_1yr_ERS2 fitting function is expressed in Equation (6-8).
Figure 6-17: Fourier series fitted to the dataset of C_D estimates in 2006 for ERS-2

\[ f(x) = 3.419 + 0.05335 \cdot \cos(x \cdot 0.2462) + 0.2978 \cdot \sin(x \cdot 0.2462) \]  \hspace{1cm} (6-8)

Figure 6-18 illustrates the results of the predicted C_D values for 2007 using F1_1yr_ERS2 function compared with the C_D estimates. The predicted C_D values have evidently matched the peaks and troughs of the C_D estimates. Moreover, the accuracy of the predicted C_D values in the first 20 weeks offers better predictions than further on in the prediction period. This is the same as the results from Starlette and Stella. In practical applications, it is not advisable to use the fitting function for long-term prediction due to the temporal variation of the atmospheric conditions.
Figure 6-18: Comparison of the $C_D$ values predicted by F1_1yr_ERS2 function and $C_D$ estimates for ERS-2

The accuracy of orbit prediction based on the F1_1yr_ERS2 function is significantly higher than that achieved using the Fix2.2 method and is also comparable to the Vary$C_D$ approach predicted orbits as illustrated in Table 6-8. A significant enhancement of 80.68% and 80.38% for the predicted position and velocity respectively is evident from the comparison of orbits using the fitting function and Fix2.2 method. There are similar percentages of improvements in all three directions for position and velocity. Therefore, the $C_D$ values predicted by the F1_1yr_ERS2 function are more accurate representation of the coefficient of drag than a fixed value of 2.2.

The significant enhancement for the orbit prediction of ERS-2 is possibly a consequence of the ability of the F1_1yr_ERS2 function to account for the effects caused by the varying cross-sectional area of the satellite. The reason for this is due to the $C_D$ estimates used for the fitting function, because the Vary$C_D$ approach estimates an optimal $C_D$ value at the time of orbit determination taking into account the effects of the area-to-mass ratio. As a result, the function fitted to the $C_D$ estimates has already accounted for, to an extent, the uncertainties caused by the changing of the area-to-mass ratio.
Table 6-8: Comparison of the RMS values of orbit predictions from the three approaches, VaryC_D, Fix2.2 and F1_1yr_ERS2

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaryC_D</td>
<td>11.46</td>
<td>5.53</td>
<td>6.88</td>
<td>14.47</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>96.97</td>
<td>40.88</td>
<td>106.49</td>
<td>149.72</td>
</tr>
<tr>
<td>F1_1yr_ERS2</td>
<td>15.35</td>
<td>6.55</td>
<td>23.31</td>
<td>28.67</td>
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<tr>
<td>F1_1yr_ERS2 Vs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2 (%)</td>
<td>84.12%</td>
<td>83.75%</td>
<td>77.82%</td>
<td>80.68%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Along-track</th>
<th>Cross-track</th>
<th>Radial</th>
<th>Overall Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaryC_D</td>
<td>0.0084</td>
<td>0.0064</td>
<td>0.0134</td>
<td>0.0171</td>
</tr>
<tr>
<td>Fix2.2</td>
<td>0.1012</td>
<td>0.0524</td>
<td>0.1087</td>
<td>0.1575</td>
</tr>
<tr>
<td>F1_1yr_ERS2</td>
<td>0.0228</td>
<td>0.0114</td>
<td>0.0170</td>
<td>0.0306</td>
</tr>
<tr>
<td>F1_1yr_ERS2 Vs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fix2.2 (%)</td>
<td>77.21%</td>
<td>77.98%</td>
<td>84.28%</td>
<td>80.38%</td>
</tr>
</tbody>
</table>

Summary

This experiment has focused on verifying the validity of the fitting function approach by using Starlette and ERS-2 covering the same study period, which is year 2006. Identical procedures were carried out as the experiment conducted in Chapter 6.2 and only Fourier series without additional harmonic terms were used in the experiments in this section, due to its high accuracy orbit prediction achieved as shown in the test results in Chapter 6.3. Consequently, two functions were established, the F1_1yr_Starlette function that was developed by fitting the Fourier series to C_D estimates of Starlette from 2006 and F1_1yr_ERS2 function for ERS-2 covering the same period of C_D estimates of ERS-2.

The results strongly support the fitting function approach because higher accuracies were achieved compared with the Fix2.2 method. The enhancement of the predicted overall position for Starlette and ERS-2 were 20.86% and 80.68% respectively. The improvements were evident in all directions for position and
velocity for both satellites. Hence, this experiment has successfully verified that higher accuracy orbit prediction is achieved using the fitting function approach compared to the Fix2.2 method.

6.5. Discussion

The application of the fitting functions derived from \( C_D \) estimates from one satellite (Stella) to another satellite (Starlette or ERS-2) shown in the experiment from Chapter 6.3 is heavily reliant on the satellite properties because it may limit the suitability of application. For example, the orbit of the experimental satellite should be circular and at an altitude of approximately 800 km similar to the reference satellite, Stella. In addition, the \( C_D \) estimates used to establish the fitting functions were based on a spherical satellite, therefore, it is also ideal for the \( C_D \) values from the fitting functions to be applied for other spherical satellites. Although it can also be adopted for non-spherical satellites as shown in the results of ERS-2 in Chapter 6.3, but the improvement is less evident. This suggests that the properties of the satellite must be carefully considered prior to applying the fitting function derived from one satellite to other satellites.

The dataset from Stella covered a period of three years, this imply that variations in the \( C_D \) value caused by the changes in the atmospheric density within the three years are accounted for by the fitting function. However, long-term trends in the \( C_D \) values cannot be identified by this method. For example, the 11-year solar cycle affects the dynamicity of the atmospheric constituents and ultimately affecting atmospheric drag, but due to the extended time span of its cycle, the use of a three-year dataset cannot detect such a long-term variation.

The fitting functions are established based on \( C_D \) estimates at weekly intervals. Consequently, any unexpected spikes that occur within a one-week interval of 2006 are not recognised by this approach. Thus, it is important to understand the
possibility of any short-terms, i.e., events that could occur in less than one week, variations in the $C_D$ values when applying the fitting function.

The previous chapter has revealed that using the fixed value approach, it reduces the data processing time by approximately one-fifth when compared to the Vary$C_D$ approach, which is particularly important for timely orbit predictions. The fitting function and the Fix2.2 approaches adopt pre-determined $C_D$ values for orbit prediction and there are insignificant time differences when the two methods are adopted for orbit processing. Furthermore, the fitting function method is capable of predicting $C_D$ values for any moment in the future, especially for long-term predictions, e.g. a few months away, compared with the Vary$C_D$ approach that estimates $C_D$ in the process of orbit determination and the estimated $C_D$ values are usually used for orbit prediction for a short-term, e.g. several days away. This is the main advantage of using fitting function to predict $C_D$.

### 6.6. Conclusion

This chapter has confirmed that the fitting function approach is a viable solution for enhancing the accuracy of predicted $C_D$ values, which ultimately improves orbit prediction. Three functions, Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella, were established based on $C_D$ estimates from 2004 to 2006 and from Stella. These three functions have shown to improve the accuracy of the overall position prediction by 35.03%, 33.26% and 21.22% respectively when compared with the Fix2.2 method. The predicted $C_D$ values using these three functions were also applied to two other satellites, Starlette and ERS-2, and the results have also presented improvements to the accuracy of orbit prediction. Furthermore, the results have evidently shown that using the 1-year-dataset achieves the most accurate orbit prediction compared to the 2-years-dataset and 3-years-dataset. This suggests that the one-year dataset is the optimal time span of sample data for the fitting function for prediction of the $C_D$ values.
The results from Chapter 6.2 and Chapter 6.3 have demonstrated the accomplishment of the fitting function approach and they were reinforced by the results from Chapter 6.4, where the fitting function approach was applied to Starlette and ERS-2. The orbit prediction presented significant improvement of 20.86% and 80.68% for the predicted overall position for Starlette and Stella respectively, when compared to the Fix2.2 method. The results from this chapter have demonstrated the fitting function approach to be a viable method to enhance the accuracy of orbit prediction and a more superior approach than the Fix2.2 method because it can achieve higher accuracy orbits without reducing the efficiency of the fixed $C_D$ value processing method. In addition, the orbit results from ERS-2 have shown that the fitting function approach is likely to have accounted for the uncertainty caused by the area-to-mass ratio to some extent. This is because the $C_D$ estimates in which the function is fitted to are optimal estimations of the $C_D$ value at the time of orbit determination taking into consideration the changing of the area-to-mass ratio. The application of this approach could likely be extended to other space objects and possibly space debris.
Chapter 7 - Summary, Conclusions and Recommendations

7.1. Summary

This thesis has investigated variations in the $C_D$ value over time and its impact on satellite orbit prediction. New methods for $C_D$ value prediction for the purpose of orbit prediction of space objects are studied. The main contributions of this research are summarised as follows:

- An extensive assessment of the differences between the $C_D$ values estimated using the Vary$C_D$ approach and the Fix2.2 method and their subsequent effects on the accuracy of orbit prediction.
- The identification of the cyclical sinusoidal trend of the variations in the $C_D$ estimates over the study period and the possible correlation with other atmospheric parameters, e.g., Ap values and the mass density.
- The differences in the estimated $C_D$ values and the orbit predictions using different atmospheric density models, i.e. MSIS-86 and DTM-78, used in the orbit prediction process.
- The effects due to the changes in the area-to-mass ratio of satellite in orbit on the $C_D$ estimates and orbit prediction.
- The development of the fitting function approach to improve the accuracy of prediction of the $C_D$ value for satellite orbit prediction without degradation to the efficiency when compared to the Fix2.2 method.
- The selection of optimal fitting functions, i.e., Fourier series, for the prediction of the $C_D$ value to facilitate higher accuracy orbit prediction for satellites Stella, Starlette and ERS-2.
7.2. Conclusions

7.2.1. Variation in the $C_D$ Value

This research has demonstrated temporal variation in the $C_D$ values, which suggests the commonly adopted fixed figure of 2.2 may not be sufficient to accurately represent the coefficient of drag, especially in the LEO environment where the atmosphere is more dynamic. The experiments conducted verified the changes in the $C_D$ value over time and quantified the effects of the variations in the $C_D$ value on orbit prediction.

A feasibility test has presented significant variations in the $C_D$ value over a study period of one year (2006) using Stella as the experimental satellite. The $C_D$ estimates, derived from the more accurate Vary$C_D$ approach, fluctuated from a minimal of 1.9 to a maximum of 2.7. A more comprehensive experiment, case study-2, using dataset covering three-years (from 2004 to 2006) at a weekly interval was dedicated to measure the variations in the $C_D$ estimates and assess their subsequent effects on orbit prediction in comparison with the fix 2.2 $C_D$ method.

The results from case study-1 and case study-2 indicated significant fluctuations in the $C_D$ estimates over the three-year period for Stella, ranging from the minimal of 1.9 to the maximum of 3.4. This posed a significant difference from the fixed value of 2.2. A comparison of the RMS values of orbit predictions from the two traditional approaches against the ILRS predictions demonstrated that a 70% improvement in the overall position prediction was achieved using the Vary$C_D$ approach compared to the Fix2.2 method. This supports that accounting for the variation in the $C_D$ values, i.e., using the $C_D$ estimates from the Vary$C_D$ approach, offer more accurate orbit prediction results. Thus, the Vary$C_D$ approach is a more accurate representation of the actual coefficient of drag than any constant fixed
value. The cause of the variation is possibly correlated to other atmospheric parameters, such as the mass density and Ap values as presented in case study-2.

The results from Stella were verified using both satellites Starlette and ERS-2. The primary reason for selecting these three satellites is due to their circular orbit at approximately 800 km altitude because it is identified that this altitude is one of the most densely used altitude. Hence, the possibility of satellite-to-debris collision is higher than less densely used altitudes. In addition, both Stella and Starlette are spherical in shape, which implies a constant area-to-mass ratio. The $C_D$ estimates from Starlette and ERS-2 have also supported the variations in the $C_D$ value over the study period, and more importantly, the $C_D$ estimates have presented a similar yearly cyclical trend. Furthermore, the Vary$C_D$ approach has improved the orbit prediction accuracy by 41.70% and 89.73% for Starlette and ERS-2 respectively when compared to the Fix2.2 method. This further reinforces the importance of considering the variations in the $C_D$ value for orbit prediction.

### 7.2.2. Ballistic Value’s Effects on Orbit Prediction

The comparison of the $C_D$ estimates and orbit predictions from Stella and Starlette against the non-spherical shaped ERS-2 in case study-3 has demonstrated the effects of the varying area-to-mass ratio in the ballistic value on orbit prediction. The $C_D$ estimates for ERS-2 fluctuated at a much larger amplitude with larger $C_D$ values ranging from the minimum of 2.6 to the maximum of 5.2, compared to the $C_D$ values from Stella and Starlette estimated 1.9 to 3.4. The similarities in the $C_D$ estimates for Stella and Starlette were anticipated because of their identical satellite properties, especially their spherical shape, which denotes a constant area-to-mass ratio. The differences in the $C_D$ estimates for ERS-2 are likely caused by the variations in the area-to-mass ratio due to the rotation of the satellite in orbit. Even though the magnitude of the $C_D$ estimates derived from ERS-2 was significantly different to that derived from both Stella and Starlette, but the trends
in the variation of the $C_D$ estimates by the three satellites were similar, e.g., similar frequencies in the peaks and troughs of the cyclical changes were detected.

7.2.3. Establish Fitting Functions for the Prediction of the $C_D$ Value

The Vary$C_D$ approach has proven to be capable of achieving higher accuracy of orbit prediction than the Fix2.2 method. However, using the Fix2.2 method to predict orbit is faster thus is more efficient, i.e., it could reduce the orbit processing time by approximately 20%, based on the test results in this thesis. This is because the fixed value approach assigns pre-determined $C_D$ values, in contrast to the Vary$C_D$ approach that estimates an optimal $C_D$ value at the time of orbit determination. Taking into consideration the accuracy and efficiency of the resultant orbit prediction and the cyclical variation trend in the $C_D$ estimates presented in the test results, fitting functions for the prediction of the $C_D$ value were established. The predicted $C_D$ values are also assigned as the pre-determined $C_D$ values when performing orbit prediction.

Three fitting functions were established based on the three datasets of $C_D$ estimates from Stella from 2004 to 2006. The functions from each associated dataset were developed based on two types of optimal approximation functions: Sine functions and Fourier series, with various numbers of terms in the functions. The results indicated that the predicted $C_D$ values (for 2007) from the Fourier series without additional harmonic terms were most comparable to the $C_D$ estimates from the Vary$C_D$ approach. Therefore, this type of fitting function is identified as the optimal fitting function for the $C_D$ prediction of this research. Consequently, three functions, Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella based on one-year, two-year and three-year datasets respectively from Stella were established and assessed. The RMS values measuring the differences in the $C_D$ values predicted by the fitting functions
against the $C_D$ estimates have shown that the fitting function using data closest to the time of prediction offers the highest accuracy $C_D$ value prediction. Hence, the one-year sample data is considered as the optimal time span for the fitting function approach.

7.2.4. Application of the Optimal Fitting Functions from Stella

An extensive evaluation of the Fourier series fitting functions was conducted by applying the $C_D$ values predicted by using Fourier1_1year_Stella, Fourier1_2years_Stella and Fourier1_3years_Stella functions to perform orbit predictions. The results have demonstrated improvements to the accuracy of the orbit predictions, e.g., the overall positional improvements are 35.03%, 33.26% and 21.22% for the aforementioned fitting functions respectively compared with the Fix2.2 method. This has presented that Fourier1_1year_Stella is the optimal fitting function for the prediction of $C_D$ values and again validated that short time span (1-year) of sample data and data closest to the time of prediction offer highest accuracy $C_D$ value prediction. In addition, when compared to the Fix2.2 method, the predicted $C_D$ values from the three established functions, using $C_D$ estimates from Stella, applied to Starlette and ERS-2 have also demonstrated improvements to the accuracy of orbit prediction. This implies the $C_D$ values predicted by Stella are applicable to both Starlette and ERS-2 satellites.

7.2.5. Verification of the Fitting Function Approach

The approach of applying Fourier series without additional terms to fit $C_D$ estimates from Stella has proven to be able to enhance the accuracy of the $C_D$ value prediction and orbit prediction. The same approach was also tested and evaluated for different satellites, i.e., Starlette and ERS-2, to further validate the
performance and applicability of the fitting function approach. The comparison of orbit predictions measured against predictions by ILRS has demonstrated accuracy improvements of 20.86% for Stella and 80.68% for ERS-2 in the overall positions respectively and the accuracy improvement was evident in all three directions for position and velocity. The enhanced results from Starlette were expected due to its similar geometrical and physical properties with Stella. However, a noticeable improvement for ERS-2 was possibly due to the ability of Fourier series (fitted to the $C_D$ estimates) to account for the effects caused by the varying area-to-mass ratio. The results from satellites Starlette and ERS-2 have confirmed that the fitting function approach achieves higher accuracy orbit prediction than the commonly adopted Fix2.2 method for both spherical and non-spherical shaped satellites.

7.2.6. Implications and Significance of this Research

The accuracy of atmospheric drag prediction is one of the largest limitations for high accuracy orbit prediction of space objects using the dynamic method. The coefficient of drag is usually predicted either by a pre-determined fixed value such as 2.2, or by an optimal estimation from the POD process. The two approaches for the $C_D$ value prediction have their limitations on either accuracy or efficiency of orbit prediction results. In order to overcome these limitations, the fitting function approach proposed and developed in this research has presented significant improvements in these regards. Test results have demonstrated that the fitting function approach is applicable to spherical and non-spherical satellites and can achieve higher accuracy $C_D$ value prediction and orbit prediction compared to the Fix2.2 method without degradation to the efficiency. In addition, the predicted $C_D$ values based on data from spherical satellites are also applicable to non-spherical satellites. This is significant for fast and high accuracy prediction of orbits of space objects, possibly space debris, at similar altitudes because the $C_D$ value
predicted by the fitting function is more accurate than any fixed values including 2.2, which will result in more accurate orbit prediction.

7.3. Recommendations

This research has demonstrated significant variations in $C_D$ value over the study period. The extensive case studies and experiments examined the possibilities to enhance the prediction of $C_D$ values that consequently improves the prediction of atmospheric drag and ultimately, to facilitate higher accuracy orbit prediction of space objects with and without consistent cross-sectional area. There are numerous research topics for the enhancement of atmospheric drag, in particular the $C_D$ value, which are worth exploring. Some recommendations are given below:

**Denser and longer time span sample data for more satellites**

This research has tested data from three satellites covering a period of three years. For more conclusive results, testing should be conducted using more satellites and more data with higher sampling intervals. This task is very important, especially for the identification and determination of the yearly cyclical variation trend of the $C_D$ estimates. A long-term study of the variations in the $C_D$ value would provide valuable information for ADM development. Furthermore, the continual monitoring of the $C_D$ value for a particular satellite is beneficial for measuring the effects of other atmospheric occurrences on the $C_D$ value, such as solar maximum and solar minimum year.

**Vertical profile of the $C_D$ value**

This research has emphasised the importance and rationale for studying the variations of atmospheric drag at the altitude of 800 km above the surface of the Earth. The same methodology and performance assessment could be applied for testing satellites at different altitudes to aid accurate orbit prediction for space objects at other altitudes. Combining all the results from various altitudes, a
vertical profile of the $C_D$ values could become readily available and the $C_D$ values for a specific altitude can be predicted based on the nature of the spatial correlation of the atmosphere. This will be very useful for the prediction of space objects, in particular space debris, at various altitudes.

**Elliptical orbit**

A study on developing a model to integrate the effects of time, the vertical profile of the $C_D$ value and the eccentricity of the orbit would provide a significant contribution to timely orbit prediction, because the effects of the $C_D$ value can be calculated by a selection of parameters in the orbit prediction process. Such research is capable of predicting $C_D$ values at higher resolution, location and altitude for satellites.

As a final remark, this research is an exclusive study on the variations of $C_D$ values and the subsequent effects on orbit prediction. Undoubtedly, there are future developments in algorithms or empirical models to further enhance the predictions of the $C_D$ value. The predictions could be in the form of faster or higher accuracy predictions, which will inevitably provide significant contributions to orbit prediction for satellites and space debris.
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