Psychological Pricing in Mergers & Acquisitions

A thesis submitted in fulfillment of the requirements for the degree of Masters in Applied Science

Nipun Agarwal
DBusAdmin MAppFin BBus

School of Mathematics & Geospatial Sciences
RMIT University
July 2013
Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Nipun Agarwal

Date 24th July 2013
I would especially like to thank my supervisor, Professor Panlop Zeephongsekul, for his outstanding advice and support. I have really appreciated his guidance through my candidature and he has taught me many excellent things through my time with him. Finally, I would really like to thank my wife, Ashima, for her tolerance and support while I have been writing this thesis. I would also like to thank my father and mother, who have sacrificed so much for me throughout my life. I would also like to thank my son, Aarav, who has a very special place in my life!
Conference Presentation

Chapter 4 was presented at the Modelling and Simulation 2011 conference:


Journal Publications

Chapter 4 has been published in the Studies in Economics and Finance journal:


Chapter 5 has been published in the Corporate Finance Review journal:


Chapter 6 has been published in the Journal of Self-governance and Management Economics:

Table of Contents

1. INTRODUCTION .................................................................................................................. 7
   1.1. INTRODUCTION ............................................................................................................. 7
   1.2. RESEARCH QUESTIONS AND THEIR SIGNIFICANCE ................................................. 8
   1.3. AN ORIGINAL CONTRIBUTION TO KNOWLEDGE ..................................................... 8
   1.4. OUTLINE OF THE THESIS’ ARGUMENT ................................................................... 9

2. LITERATURE REVIEW & METHODOLOGY ...................................................................... 10
   2.1. INTRODUCTION ............................................................................................................ 10
   2.2. GAME THEORY AND NEGOTIATION ........................................................................... 10
   2.3. COGNITIVE, SOCIAL AND BEHAVIOURAL PSYCHOLOGY IN NEGOTIATION .......... 28
   2.4. PROSPECT THEORY ................................................................................................... 31
   2.5. CONCLUSION ............................................................................................................... 33

3. PSYCHOLOGICAL PRICING OF MERGERS & ACQUISITIONS USING GAME THEORY  ................................................................................................................................. 34
   3.1. INTRODUCTION ............................................................................................................ 34
   3.2. TWO-PERSON M&A MODEL WITH COMPLETE INFORMATION ............................... 38
   3.3. TWO-PERSON M&A MODEL WITH INCOMPLETE INFORMATION .............................. 44
   3.4. CONCLUSION ............................................................................................................... 48

4. PSYCHOLOGICAL PRICING OF MERGERS & ACQUISITIONS USING PROSPECT THEORY ...................................................................................................................................................... 49
   4.1. INTRODUCTION ............................................................................................................ 49
   4.2. TWO-PERSON M&A MODEL USING PROSPECT THEORY ......................................... 50
   4.3. OPTIMAL BEHAVIOUR OF ACQUIRER AND TARGET ............................................... 54
   4.4. CONCLUSION ............................................................................................................... 59

5. PSYCHOLOGICAL PRICING OF MERGERS & ACQUISITIONS USING REAL OPTIONS GAMES .................................................................................................................................................. 60
   5.1. INTRODUCTION ............................................................................................................ 60
   5.2. TWO-PERSON M&A MODEL USING REAL OPTIONS GAMES .................................. 61
   5.3. CONCLUSION ............................................................................................................... 67

6. PSYCHOLOGICAL PRICING OF MERGERS & ACQUISITIONS USING REAL OPTIONS SIGNALLING GAMES ............................................................................................................................................. 69
   6.1. INTRODUCTION ............................................................................................................ 69
   6.2. TWO-PERSON M&A MODEL USING REAL OPTIONS SIGNALLING GAMES .............. 71
   6.3. CONCLUSION ............................................................................................................... 78

7. SUMMARY, LIMITATIONS AND SUGGESTED EXTENSIONS ............................................. 80
   7.1. SUMMARY .................................................................................................................... 80
   7.2. CONTRIBUTION TO KNOWLEDGE ............................................................................. 81
   7.3. LIMITATIONS OF THIS RESEARCH ........................................................................... 81
   7.4. POSSIBLE EXTENSIONS OF THIS RESEARCH ......................................................... 82
   7.5. SUGGESTED EXTENSIONS ......................................................................................... 82

REFERENCES ............................................................................................................................. 83
Table of Figures

CHAPTER THREE
1. Strategic form of the two-person merger & acquisition model with complete information .......... 40
2. Normal form of the two-person merger & acquisition model with complete information .......... 41
3. Strategic form of the two-person merger & acquisition model with incomplete information .......... 45
4. Normal form of the two-person merger & acquisition model with incomplete information .......... 46

CHAPTER FOUR
1. Strategic form of the two-person merger & acquisition model with prospect theory .................. 52
2. Normal form of the two-person merger & acquisition model with prospect theory .................. 53
3. Simulation results for the Two-person Merger & Acquisition Model ........................................ 56
4. Acquirer’s payoff profile for the Two-person Merger & Acquisition Model ................................ 57
5. Target’s payoff profile for the Two-person Merger & Acquisition Model .................................. 57

CHAPTER FIVE
1. Binomial Option Pricing (CRR) Model ....................................................................................... 62
2. Offer Path in the Binomial Option Pricing (CRR) model ......................................................... 64
3. Player Strategies and Option Pricing ......................................................................................... 64
4. Probabilities based on Option Type ......................................................................................... 65
5. Pay-offs for each Player for playing Specific Strategies .......................................................... 66

CHAPTER SIX
1. Binomial Option Pricing (CRR) Model ....................................................................................... 71
2. Player types and probabilities: Risk-taking Acquirer and Optimistic Target ............................... 72
3. Player pay-offs: Risk-taking Acquirer and Optimistic Target .................................................. 73
4. Player types and probabilities: Risk-taking Acquirer and Pessimistic Target ............................ 74
5. Player pay-offs: Risk-taking Acquirer and Pessimistic Target ................................................ 75
Chapter One

Introduction

1.1 Introduction

Merger and acquisition pricing has traditionally been conducted using Corporate Finance theory that relates to valuation methods like discount cash flow analysis, price-earnings ratio, transaction multiples. However, researchers have started to find in the research area of Behavioural Finance that there are psychological factors that impact the valuation of financial assets, which includes pricing mergers and acquisitions. Baker, Pan and Wurgler (2009) have recently undertaken an empirical study of mergers and acquisitions in the US looking at 7498 mergers that took place during the period 1st January 1984 to 31st December 2007. They found that the majority of these mergers had a final agreed price of sale that was close to the 52-week high stock price of the target company (company that is being purchased by the acquirer). This is an interesting finding as the 52-week high stock price for a target company may be much higher than its current stock price. However, Baker, Pan and Wurgler (2009) explained that this 52-week high (target) stock price acted like a psychological anchor, which was suitable to the target company’s stockholders as they believed that they were obtaining a reasonable price for the sale of their company. While, also satisfying the stockholders of the acquiring company making them feel that they were not paying too much for purchasing the target company.

This thesis intends to develop a theoretical model called the two-person Merger & Acquisition model with incomplete information to understand how such transactions can be analysed in practice. It will then extend this theoretical model to incorporate prospect theory (Kahneman and Tversky 1979). It is interesting to include prospect theory rather than using the concept of Expected Utility theory, as prospect theory provides for more realistic risk perceptions that investors are using in valuing financial assets. This thesis will then try to extend the two-person Merger & Acquisition model by including Prospect theory. A further extension to this model is to incorporate the Cox-Ross-Rubenstein discrete-time American option pricing model to value M&A transactions.
1.2 Research questions and their significance

The research problem that this thesis intends to solve is to develop a theoretical model to analyse merger and acquisition pricing. It is seen that merger and acquisition pricing has psychological pricing factors and cannot be estimated accurately using traditional financial models.

This research problem raises the following research questions that will be answered by this thesis:

1. Can we develop a two-person Merger & Acquisition model with incomplete information to analyse real world merger & acquisition transactions?

2. Does prospect theory have any impact on the results when included in the two-person Merger & Acquisition model?

3. Can this model be extended to incorporate discrete-time American option pricing (real options games) to make it more applicable in practice?

4. Does signaling have any impact when included in two-person Merger & Acquisition using real options games?

1.3 An original contribution to knowledge

This thesis makes the following original contributions to knowledge that would help extend the theory in the area of psychological pricing of mergers and acquisitions:

1. Develops a two-person Merger & Acquisition model to analyse M&A pricing under the conditions of an incomplete information game and prospect theory (Kahneman and Tversky 1979). Previously, researchers have not considered analyzing M&A pricing using these methodologies.

2. This two-person Merger & Acquisition model is extended further to include the Cox-Ross-Rubenstein American Option discrete time option pricing model (Cox, Ross and Rubenstein 1979) and signaling behavior in such real option games. This is the first model that includes real options signaling games application to M&A pricing in such a manner that helps organization price M&A deals in everyday business environment.
1.4 Outline of the thesis’ argument

As we have noticed the area of mergers and acquisition pricing is impacted by psychological pricing factors, therefore it is hard to price such transactions with traditional finance models. The research area of behavioural finance has played a strong role in extending the theory related to the psychological deviations associated to the pricing of assets. For example, Baker, Pan and Wurgler (2009) have extended this research into the area of psychological pricing of mergers & acquisitions. This thesis considers the challenge of psychological pricing factors on merger & acquisition pricing and looks to develop a two-person Merger & Acquisition (M&A) model with incomplete information, prospect theory and discrete-time American option pricing. This model is a two-person game between the acquirer and target firms that participate in the M&A transaction. Game theory assists in analyzing the behavioural factors that impact such transactions.

The next chapter will review literature from the field of game theory and negotiation taking into account the game theoretic application to mergers and acquisitions. Chapter three will then develop the two-person Merger and Acquisition (M&A) model, that is, a two-person incomplete information game between the acquirer and target firms that form the merger or acquisition transaction. Chapter four will then extend this two-person Merger and Acquisition model to incorporate prospect theory (Kahneman and Tversky 1979). Prospect theory provides a more realistic reflection of how humans behave towards gains and losses, and in order to make the two-person M&A model more applicable to real world M&A transactions. It is important to find out if the application of prospect theory changes the equilibrium obtained in Chapter three.

Chapter five then extends the two-person M&A model that has been developed in chapters three and four, however it includes real options (Cox-Ross-Rubenstein discrete American option) pricing tree to obtain actual dollar values for the M&A transaction rather than using relative values used in previous chapters.Primarily, as relative values do not make much sense when valuing real world M&A transactions. Chapter six then adds the concept of signaling to the model developed in chapter five and chapter seven summarizes the discussion in this thesis and provides limitations and suggestions for the extension of the work completed in this thesis.
CHAPTER TWO

LITERATURE REVIEW & METHODOLOGY

2.1 Introduction

This chapter will review the literature related to game theory and negotiation as well as the application of this literature to the field of mergers and acquisitions. The literature in the area of negotiation relates to game theory and psychology. In this chapter we will initially discuss the literature broadly in the research area of negotiation. Later in this chapter we will then analysis the literature that discusses the application of game theory and negotiation techniques to the field of mergers and acquisitions. This discussion will help us identify the gap in knowledge and it will provide a basis for developing the two-person merger & acquisition model in the following chapter.

2.2 Game Theory and Negotiation

Negotiation theory is important as it helps us understand the complexities of human interaction where individuals need to mediate to meet their needs. This area of research incorporates game theory and psychology. Game theory provides a solution to the negotiation problem from a socio-economic outlook. On the other hand, psychological negotiation theory applies cognitive, behavioral and social psychological theories to negotiation. We also have models that have assimilated game theoretic and psychological concepts, for example, Neale and Northcroft (1991) and Hausken (1997).

Game theory is a field of study that analyses human interaction from a mathematical perspective. Game theory was initially introduced in the paper “Zur theorie der Gesellschaftsspiele” written by Von Neumann and Morgenstern (1928) and a subsequent book “Theory of Games and Economic Behaviour” written by Von Neumann and Morgenstern (1944) that discussed a concept called zero-sum games. Prior to going any further we need to state that games consist of players (two or n-players), who choose a set of strategies (actions) that are played in a sequence (i.e. each player get to implement an action), which result in a pay-off (outcome). Zero-sum games are those games where the positive pay-off (gain) for the winner is equal to the negative pay-off (loss) for the loser in the game.
Game theory research was later extended and now there are two forms of game theory, which are the strategic (non co-operative) and axiomatic (co-operative) forms. Non co-operative game theory has its beginnings from the work of John Nash (1951), while co-operative game theory has been introduced by Von Neumann and Morgenstern (1944). Other contributions in this field were from Shapley (1977), Shapley and Shubik (1954), Aumann and Drèze (1974), Myerson (1977) and others. The two approaches to game theory analyse problems in two different ways, as presumed the strategic form analyses game theoretic problems from a strategic representation (i.e. epitomized by a decision tree structure that shows the different paths that a player could follow on this decision tree).

The following summary and discussion of game theory concepts are contained in Zeephongsekul (2011). The strategic form game through the following representation:

$$\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$$ (1)

Where, $N = \{1, 2, 3 \ldots n\}$ signifies the set of players, $S_i$ is the set of strategies that Player $i$ can implement and $u_i(s)$ is a function defined on the Cartesian product set $S = \prod_{i \in N} S_i$, which denotes the pay-off to Player $i$ when a specific combination of strategies $s \in S$ was selected. Though, if this game is played against Nature, then the pay-off is defined as the expected value as per probability theory.

Definition 1. Let $A_i, i = 1, 2, 3 \ldots n$ be any family of sets. The Cartesian product of sets can be written as $A_1 \times A_2 \times \ldots A_n = \prod_{i = 1}^{n} A_i$ that is the set of tuples $(a_1, a_2, \ldots, a_n)$ where each $a_i \in A_i$.

Definition 2. A non-cooperative game $\Gamma$ is called a constant-sum game if there exists a constant $c$ such that $\Sigma_{i \in N} u_i(s) = c \ \forall \ s \in S$. There is a special case where $c = 0$ and $n = 2$ represents a two-person zero sum game.

Definition 3. Two games $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ and $\tilde{\Gamma} = (N, (S_i)_{i \in N}, (\tilde{u}_i)_{i \in N})$ are strategically equivalent if for $\forall i \in N$ there exists numbers $a_i > 0$ and $b_i$ such that

$$\tilde{u}_i(s) = a_i u_i(s) + b_i \ \forall \ s \in S$$ (2)

The same optimal strategies exist for games that are strategically equivalent and that all constant sum games are equivalent to zero-sum games. The main
advantage of setting up a game in strategic form is due to its mathematical
traceability, though additional commentary is required when representing strategic
form games otherwise it seems like players implement their strategies
simultaneously. However, the strategic form does not provide information like
timing of each move or information each player may have at each point in time.

The axiomatic approach works differently by providing a set of axioms that imply a
unique solution for the game theoretic problem. Chaterjee and Samuelson (1983),
Holmstrom and Myerson (1983), Cramton (1992), Satterwaite and Williams (1989)
and Gresik (1995) are some of the game theorists who have contributed to the
development of the strategic form, while, some researchers, for example Myerson
(1979; 1984; 1985), sought to amalgamate the axiomatic and strategic forms.
Harsyani (1966, 1967 and 1968) made a seminal contribution to the growth of the
strategic form. Chaterjee and Samuelson (1983) and Rubenstein (1985) then
extended this model. In the meantime, Selten (1975) has also provided a seminal
contribution to game theory by providing the sub-game perfect equilibrium theory.

The main objective of game theory is to determine optimal strategies (equilibrium
points) for each player within a game. Several concepts of equilibria exist however
the Nash Equilibrium Point (NEP) provided by Nash (1951) is the perhaps the most
useful concept. Prior to discussing the Nash Equilibrium Point, it will be better to
discuss the concept of Dominance Solvability and that will lead into the NEP
discussion. Before we discuss Dominance Solvability, we should review the
following Substitution Notation that will be used further:

For any player $i \in N$ let $S_{-i}$ denote the Cartesian product set $\prod_{j \neq i} S_j$. Here, $S_{-i}$
denotes the set of all possible combinations of strategies for all players except
Player $i$. A typical member of $S_{-i}$ will be denoted by $s_{-i}$. Therefore, a member $s$ of
$S = \prod_{i \in N} S_i$ can be represented by $s = (s_i, s_{-i})$ where $s_i$ is a strategy in $S_i$.

We can now discuss the dominant strategy solution that can be defined as:

Definition 4. For a Player $i$, his/her strategy $s_i^*$ is weakly dominated or simply
dominated by another strategy $s'_i$ if:

\begin{align}
    u_i(s'_i, s_{-i}) & \geq u_i(s^*_i, s_{-i}) & \forall s_{-i} & \quad (3) \\
    u_i(s'_i, s_{-i}) & \geq u_i(s^*_i, s_{-i}) & \text{for some } s_{-i} & \quad (4)
\end{align}
In other words, strategy $s'_i$ does as well as $s_i$ against every other strategy adopted by their opponents and against some it does strictly better. If $s'_i$ weakly dominates every other strategy then it is called a weakly dominated strategy or simply dominant strategy. Further, the strategy $s'_i$ strongly dominates $s_i$ if:

$$u_i(s'_i, s_{-i}) > u_i(s_i', s_{-i}) \quad \forall s_{-i}$$  \quad (5)

Definition 5. A game is dominance solvable if a unique combination of strategies is reached by a sequence of iterated elimination of strategies (IEDS). This concept is widely used in game theory as it is an easy way to find the Nash Equilibrium Point (NEP). Disadvantages of the IEDS method are that:

1. Certain strategies only become dominant after the elimination of other strategies and after several rounds of elimination, still a rational equilibrium point in not obtained; and
2. The order in which the strategies are eliminated is important.

Nash Equilibrium Point (NEP) was a seminal contribution to game theory provided by Nash (1951). John Von Neumann (1928; 1944) provided the first equilibrium point, also called saddle point solution concept of two-person zero sum games. Effectively, the NEP is a saddle point for games that are not zero sum; as a result Nash’s solution generalizes the solution provided by Von Neumann.

Definition 6. A combination of strategies $s^* = (s^*_1, s^*_2, ..., s^*_n) \in S$ is a NEP if it satisfies the following property:

$$u_i(s'_i, s^*_{-i}) \geq u_i(s_i, s^*_{-i}) \quad \forall s_{-i} \text{ and } \forall i$$  \quad (6)

The equation provided above can be illustrated, in a two-person game with two strategies – $a_1$ and $a_2$ for player 1 and $b_1$ and $b_2$ for player 2. In this case, if $(a_1, b_1)$ is an NEP then:

$$u_1(a_1, b_1) \geq u_1(a_2, b_2)$$  \quad (7)

$$u_2(a_1, b_1) \geq u_2(a_2, b_2)$$  \quad (8)

This definition states that if a combination of strategy $s^*$ is a NEP, if a player deviates from his/her Nash strategy then he will not gain, if his/her opponents adhere to their Nash strategies. Further, it does not imply that the Nash equilibrium
will be a dominant strategy. Whereas an IEDS will always be a Nash equilibrium, but the converse does not hold.

Definition 7. For each strategy $a_i$ selected by Player 1 in a two-person game, the reaction set $R_2(a_i)$ of Player 2 consists of the set of strategies available to Player 2 that will give him/her the best return to Player 1’s strategy $a_i$, i.e.

$$R_2(a_i) = \{ c_j \in S_2 : u_2(a_i, c_j) \geq u_2(a_i, b_j) \} \quad \forall b_j \in S_2$$  \hspace{1cm} (9)

Rule 1. The combination of strategies $(a^*, b^*)$ is a NEP if $a^*$ is the best reply to $b^*$ and $b^*$ is to $a^*$.

Further, we notice that a Nash Equilibrium Point (NEP) may not exist is some game where players have played pure strategies, i.e. a player will select a single action from within a set of actions. However, an NEP will always exist if we extend the space of available strategies that includes mixed strategies, i.e. a player can play a combination of pure strategies.

Definition 8. Suppose $S_i = \{s_{i1}, s_{i2}, ..., s_{in}\}$ is the set of pure strategies available to Player i. A mixed strategy for Player i is a discrete probability distribution over his/her mixed strategies, i.e. a vector of non-negative numbers $\bar{\rho} = (p_1, p_2, ..., p_M)$ such that $\sum_{j=1}^{M} p_j = 1$. The set of mixed strategies over $S_i$ will be denoted by $\mathcal{V}(S_i)$.

We playing a mixed strategy a player i will pick a strategy $s_j$ with a probability $p_j$. In other words, a pure strategy is simply a mixed strategy where $p_j = 1$ for some $j$.

As a result, when players apply mixed strategies, the resulting pay-off can be represented as an expected value as discussed in the next definition.

Definition 9. Suppose Player i has available to him/her a set of strategies $S_i = \{a_1, a_2, ..., a_M\}$ and Player j has available to him/her the set $S_j = \{b_1, b_2, ..., b_M\}$. Let $\bar{\rho} = (p_1, p_2, ..., p_M) \in \mathcal{V}(S_i)$ and $\bar{q} = (q_1, q_2, ..., q_M) \in \mathcal{V}(S_j)$, then the expected pay-offs for Players i and j are respectively:

$$u_1(\bar{\rho}, \bar{q}) = \sum_{i=1}^{M} \sum_{j=1}^{N} p_i q_j u_1(a_i, b_j)$$  \hspace{1cm} (10)

$$u_2(\bar{\rho}, \bar{q}) = \sum_{i=1}^{M} \sum_{j=1}^{N} q_i p_j u_2(a_i, b_j)$$  \hspace{1cm} (11)

Definition 10. A combination of mixed strategies $(\bar{\rho}^*, \bar{q}^*)$ is a NEP in mixed strategies if it satisfies the following property:
The following result is pivotal in game theory and was first proved by Nash (1951). Theorem 1. At least one NEP exists in mixed strategies in any finite co-operative game. Where, the number of players and strategies available to them are finite.

This can be further analysed by the following two corollaries:

Corollary 1. For a pair of strategies \((\bar{p}^*, \bar{q}^*)\) to be a NEP in a two-person game, it is necessary and sufficient that:

\[
\begin{align*}
u_1(a_i, \bar{q}^*) &\leq u_1(\bar{p}^*, \bar{q}^*) &\forall \text{ pure strategies } a_i \in S_i \\
u_2(\bar{p}^*, b_j) &\leq u_2(\bar{p}^*, \bar{q}^*) &\forall \text{ pure strategies } b_j \in S_2
\end{align*}
\]

In the next corollary, we define the support of a mixed strategy \(\bar{p}\) to be all those pure strategies that have a positive probability of being selected, i.e. strategy \(a_i\) belong to the support of \(\bar{p}\) if \(p_i > 0\).

Corollary 2. Let \((\bar{p}^*, \bar{q}^*)\) be a NEP in a two-person game. If a pure strategy \(a_i\) belongs to the support of \(\bar{p}^*\), then

\[
u_1(a_i, \bar{q}^*) = u_1(\bar{p}^*, \bar{q}^*)
\]

Similarly, if a pure strategy \(b_j\) belongs to the support of \(\bar{q}^*\), then

\[
u_2(\bar{p}^*, b_j) = u_2(\bar{p}^*, \bar{q}^*)
\]

To provide the first part of this result, the second part follows similarly, assume that \(q^*(a_i) > 0\) but

\[
u_1(a_i, \bar{q}^*) < u_1(\bar{p}^*, \bar{q}^*)
\]

From corollary 1 it follows that

\[
u_1(a_j, \bar{q}^*) \leq u_1(\bar{p}^*, \bar{q}^*) &\forall \text{ pure strategies } a_j \neq a_i
\]
Multiplying both sides of equation (18) by $\tilde{q}^*(a_i)$ gives

$$u_i(a_i, \tilde{q}^*) < u_i(\tilde{p}^*, \tilde{q}^*) \quad (20)$$

and multiplying both sides of (19) by $\tilde{q}^*(a_j)$ gives

$$u_i(a_j, \tilde{q}^*) < u_i(\tilde{p}^*, \tilde{q}^*) \quad \forall \text{ pure strategies } a_j \neq a_i \quad (21)$$

Therefore, adding (20) and (21) results in

$$u_i(\tilde{p}^*, \tilde{q}^*) < u_i(\tilde{p}^*, \tilde{q}^*) \quad (22)$$

which is a contradiction. This contradiction proves that

$$u_i(a_i, \tilde{q}^*) = u_i(\tilde{p}^*, \tilde{q}^*) \quad (23)$$

This result states that all pure strategies that could be used, i.e. have positive probability of being selected, will provide the same optimal payoff when used against an opponent’s optimal mixed strategy.

Further, we have seen that there could be more than one NEP for each game. Selten (1975) has attempted to refine the concept of a NEP where games are dynamic in nature and have several consecutive moves or stages.

Definition 11. A sub-game is part of a game in extensive form which has the following properties:

- Starts at a single decision node $x$,
- Contains all successive nodes including the branches that lead from node $x$,
- If a sub-game contains an information set then it must contain all nodes of that information set.

Definition 12. A strategy combination is a sub-game perfect Nash equilibrium of a game if:

- It is a NEP for the entire game, and
The strategy combination restricted to every sub-game in which it is played is also a NEP for that sub-game.

Theorem 2. In a game of perfect information, i.e. each node belongs to an information set consists solely of that node, the sub-game perfect NEPs coincide with the solution obtained by the method of iterated elimination of dominated strategies (IEDS).

When the total number of pure strategies in a moderately game can be huge, for example, in games like chess or bridge and as mixed strategies is a discrete probability distribution on the pure strategy space. It can be prohibitively time consuming to find all the mixed strategies for such games. On the other hand, behavioural strategies specify a probability distribution on a set of alternatives at each of the player’s information sets. If a player opts to use a behavioural strategy then he/she can choose strategies at each move (with these subsequent decisions being uncorrelated) rather than trying to find these strategies in the beginning of the game.

Definition 13. Suppose a player has \( I \) information sets. His/her behavioural strategy is a collection of \( I \) probability distributions, one each over the set of alternatives in an information set. Behavioural strategies are useful as they simplify the calculation of probabilities associated with the selection of strategies as they reduce the dimensions of the space of distributions. Further, several mixed strategies can induce the same behavioural strategy and these strategies are said to be behaviorally equivalent. While, not all mixed strategies are induced by behavioural strategies, but given a behavioural strategy there always exists a mixed strategy (not necessarily unique) that induces a behavioural strategy. Also, if a game has perfect recall, i.e. all players can recall every decision at every past node, then all behaviorally equivalent mixed strategies have the same value in the game. An important consequence in such games is that the behavioural strategies will generate the equilibrium point for the game, so a player can restrict themselves to the class of behavioural strategies.

When we look at two-person zero sum games (TPZSG) we notice that not all TPZSG have equilibrium points in pure strategies, i.e. saddle points. Though, if the set of strategies are extended to include mixed strategies, then all TPZSG will have an equilibrium point in mixed strategies. Von Neumann and Morgenstern (1944) provided the fundamental theorem for TPZSG known as the Minimax theorem.
Mixed strategies are a probability distribution on a set of available strategies. Let \( \mathcal{V}(S_i) \) be the set of mixed strategies available to Player \( i \), where \( i = 1,2 \). Then, the expected pay-off to Player 1, if he selects a mixed strategy \( \tilde{\rho} = (p_1, p_2, \ldots, p_m) \in \mathcal{V}(S_1) \) and Player 2 selects \( \tilde{q} = (q_1, q_2, \ldots, q_m) \in \mathcal{V}(S_2) \) is:

\[
\tilde{\rho} A \tilde{q}' = \sum_{i=1}^{m} \sum_{j=1}^{n} p_i q_j a_{ij}
\] (24)

Where in (24), \( \tilde{q}' \) refers to the transpose of \( \tilde{q} \). Note that because the game is zero-sum the expected pay-off to Player 2 is \( -\tilde{\rho} A \tilde{q}' \).

Theorem 3. (Minimax theorem). All TPZSGs have solutions in mixed strategies, i.e. there exist \( \tilde{\rho}^* \) and \( \tilde{q}^* \) such that

\[
\max_{\tilde{\rho} \in \mathcal{V}(S_1)} \min_{\tilde{q} \in \mathcal{V}(S_2)} \tilde{\rho} A \tilde{q}' = \min_{\tilde{q} \in \mathcal{V}(S_2)} \max_{\tilde{\rho} \in \mathcal{V}(S_1)} \tilde{\rho} A \tilde{q}' = \tilde{\rho}^* A \tilde{q}^{**}
\] (25)

\[
= \tilde{\rho}^* A \tilde{q}^{***}
\] (26)

The common value of the game is denoted by \( v \).

Another way of viewing (25) and (26) is if Player 1 and Player 2 adopt their optimal mixed strategies then

\[
\tilde{\rho} A \tilde{q}^{**} \leq \tilde{\rho}^* A \tilde{q}^{***} = v \leq \tilde{\rho}^* A \tilde{q}' \forall \tilde{\rho}, \forall \tilde{q}
\] (27)

In other words, no player will gain if he deviates from his optimal strategies while the other player sticks to his optimal strategies. For example, if Player 1 selects any other strategies except his optimal strategies and Player 2 sticks to an optimal strategy, then Player 1 will get at most the value of the game and could do worse.

While, the Nash equilibrium point proved a stronger result since it covers all non-co-operative games and not just two-person zero-sum games. However, solutions to non-zero sum games do not often have the same properties that are available in saddle points.

We can use Linear Programming (LP) to solve two-person zero-sum games. In fact, a solution to any TPZSG can be obtained from solving a standard LP problem. Many problems occurring in industry can be formulated as LP problems, e.g. transportation, scheduling, resource allocation and financial services. LP problems are usually solved (if a solution exists) by the well-known Simplex method. An LP is
the problem of finding the vector \( \tilde{x} = (x_1, x_2, ..., x_n) \in \mathbb{R}^n \), the n-dimensional Euclidean space, which maximizes the linear function:

\[
z = \tilde{c} \tilde{x}' = c_1x_1 + c_2x_2 + \cdots + c_nx_n
\]  

(28)

Subject to the constraints

1. \( A \tilde{x}' \leq \tilde{b}' \) where \( A \) is an \( m \times n \) matrix of columns, \( \tilde{b} = (b_1, b_2, ..., b_m) \in \mathbb{R}^m \), and

2. \( \tilde{x} \geq \tilde{0} \)

The region in \( \mathbb{R}^n \) that satisfies the above constraints are called the feasible region of the LP problem. An LP problem can also be succinctly summarized by the notation:

\[
\max \{ \tilde{c} \tilde{x}' : A \tilde{x}' \geq \tilde{b}', \tilde{x} \geq 0 \}
\]  

(29)

As such (29), is often referred to as the primal problem. Associated with each LP problem is a dual problem which is defined by:

\[
\min \{ \tilde{b} \tilde{y}' : A \tilde{y}' \geq \tilde{c}', \tilde{y} \geq 0 \}
\]  

(30)

Notice that the dual of the dual problem is the primal problem. The next result is the famous Duality theorem:

Theorem 4. If the primal problem has an optimal solution \( \tilde{x}_0 \), then the dual problem also has an optimal solution \( \tilde{y}_0 \). Moreover the following identities hold:

\[
\tilde{c} \tilde{x}_0' = \tilde{b} \tilde{y}_0', \tilde{y}_0 (\tilde{b} - A \tilde{x}_0') = 0 \text{ and } \tilde{x}_0 (A \tilde{y}_0' - \tilde{c}') = 0
\]  

(31)

Conversely, if the last two equations in equation (31) [called the complementary slackness conditions] holds for some feasible \( \tilde{x} \) and \( \tilde{y} \), then they are optimal solutions to the primal and the dual problem respectively.

Consider a game with a \( m \times n \) pay-off matrix \( A \). Assume all elements of the matrix are positive, otherwise add a large number to each element to get a new matrix which has the same solutions but a different value as the original game. For each \( j = 1, 2, ..., n \), the quantity:
\[ x_1 a_{1j} + x_2 a_{2j} + \cdots + x_m a_{mj} \]  

Represents the expected pay-off to Player 1 if he adopts strategy \( \bar{x} \) while Player 2 selects pure strategy \( j \). Let \( \bar{v}_x \) equals the minimum of these pay-offs over all \( j = 1, 2, \ldots, n \). Note that this value is the pay-off corresponding to the strategy that Player 2 will adopt since he wishes to minimize Player 1’s gain. It follows that:

\[
\begin{align*}
x_1 a_{11} + x_2 a_{21} + \cdots + x_m a_{m1} & \geq \bar{v}_x \\
x_1 a_{12} + x_2 a_{22} + \cdots + x_m a_{m2} & \geq \bar{v}_x \\
\vdots & \\
x_1 a_{1n} + x_2 a_{2n} + \cdots + x_m a_{mn} & \geq \bar{v}_x
\end{align*}
\]

Dividing both sides of the inequalities (33) by \( \bar{v}_x \) and calling \( \bar{u} = \frac{u_i}{\bar{v}_x} > 0 \), \( i = 1, 2, \ldots, m \) will result in the set of inequalities

\[ A' \bar{u}' \geq \bar{f}' \]  

Where \( f = (1, 1, \ldots, 1) \) is a vector of ones.

Note that the value of the game is given by \( v = \max_{\bar{x}} \bar{v}_x \) (i.e. Player 1 will maximize this value to obtain the Maxmin value) and the corresponding mixed strategy that achieves this is obviously his optimal mixed strategy. Since \( f \bar{u}' = 1/\bar{v}_x \), the ratio will be a minimum at the value of the game. Therefore, the procedures for obtaining the optimal strategies for Player 1 and corresponding value of a game involving solving the LP problem:

\[
\min \{ f \bar{u}' : A' \bar{u}' \geq \bar{f}', \bar{u} \geq 0 \}
\]

Analogously, let \( \bar{y} \) be an arbitrary strategy for Player 2 and denote the maximum of

\[
y_1 a_{i1} + y_2 a_{i2} + \cdots + y_n a_{in}, \quad i = 1, 2, \ldots, m
\]

by \( v_{ij} \). It therefore follows that

\[
\begin{align*}
y_1 a_{11} + y_2 a_{12} + \cdots + y_n a_{1n} & \leq \bar{v}_y \\
y_1 a_{21} + y_2 a_{22} + \cdots + y_n a_{2n} & \leq \bar{v}_y
\end{align*}
\]
Dividing both sides of the inequalities (37) by $\bar{v}_y$ and calling $w_i = y_i/\bar{v}_y > 0, i = 1, 2, ..., n$ will result in the set of inequalities

$$A\bar{w}' \leq \bar{j}'$$

(38)

Since the value of the game is given by $v = \min_{\bar{v}_y} \bar{v}_y$ (i.e. Minmax value) and $\bar{j}'\bar{w}' = 1/\bar{v}_y$, the ratio will be a maximum at the value of the game. Therefore, the procedures for obtaining the optimal strategies for Player 2 and corresponding value of a game involves solving the LP problem:

$$\max\{\bar{j}'\bar{w}' : A\bar{w}' \leq \bar{j}', \bar{w}' \geq 0\}$$

(39)

The conclusion is therefore that a solution of a two-person zero-sum game (TPZSG) can be reduced to a solution of a pair of dual linear programming problems. Incidentally, we have also solved theorem 3.

Moving forward to discuss incomplete (asymmetric information), we notice that incomplete information games are different to imperfect information games, as imperfect information refers to the uncertainty regarding the position of a player in the decision tree, i.e. games of imperfect information contain information sets with multiple nodes. Though, incomplete information games go further, where players may not even know some relevant information, for example, opponent’s strategies, pay-offs or beliefs. Harsanyi (1966, 1967, 1968a and 1968b) developed a structure that allows incomplete information games to be analysed as imperfect information games.

We can formalize games with incomplete information, as a set of players $N = \{0, 1, ..., n\} \forall i \in N$ and we must specific the set of action $A_i$ available to Player $i$ and the pay-off function as $u_i(\cdot)$. Further, we introduce two additional factors into this game, which are a set of possible types $T_i$ and a probability function (density) $p_i$ over $T_i$. Define $A = \prod_{i \in N} A_i$ and $T = \prod_{i \in N} T_i$, i.e. the set of combinations of actions and types (type and behaviour are used interchangeably in this thesis) on the n players respectively, for example, a typical member of $A$ is $\bar{a} = (a_1, a_2, ..., a_n)$ and of $T$ is $\bar{t} = (t_1, t_2, ..., t_n)$. Using the notation $T_{-i}$ to represent the set of all possible combinations of types for all players except Player $i$, the probability $p_i(t_{-i}|t_i), t_{-i} \in T_{-i}$ is the conditional probability representing what Player $i$ believes to be the
opponent’s type if his/her own type was \( t_i \). The pay-off function \( u_i(\cdot) \) is defined over the set \( A \times T \), i.e. \( u_i(\tilde{a}; \tilde{t}) \) is the pay-off to Player \( i \) if players choose their actions as specified in \( \tilde{a} \) and their types are specified by \( \tilde{t} \). We can represent an incomplete game as:

\[
\Gamma^i = (N, (A_i)_{i \in N}, (T_i)_{i \in N}, (p_i)_{i \in N}, (u_i)_{i \in N})
\]  

(40)

Such a game is called a \textit{Bayesian game} due to the fact that if nature draws a type vector \( \tilde{t} \) with probability distribution \( P \) (in the finite case), then \( p_i(t_{-i}|t_i) \) can be obtained using Bayes’ Rule:

\[
p_i(t_{-i}|t_i) = \frac{p(t_{-i}|t_i)}{\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i)}
\]  

(41)

Similar formula exists for the infinite case but involves a density function and integral rather than probability mass functions and sum respectively. A game of incomplete information involves:

1. nature draws a type vector \( \tilde{t} = (t_1, t_2, ..., t_n) \in T \)
2. nature reveals \( t_i \) to Player \( i \) but not to other players
3. players simultaneously choose strategies \( s(t_i) = a_i \) that is a function from \( T_i \) to \( A_i \)
4. finally, Player \( i, i = 1, 2, ..., n \), receives the pay-off \( u_i(a_1, a_2, ..., a_n) \).

In this case, as we have allowed nature to make the first move, this incomplete information game has now transformed into an imperfect information game. The normalized form of this game \( \Gamma^b \) is referred to as the Bayesian equivalent to the game of incomplete information \( \Gamma^i \). The following definition explains how to calculate the equilibrium point in such games.

Definition 14. In the game \( \Gamma^i \) given by equation (41), the strategies \( (s^*_1, s^*_2, ..., s^*_n) \) are a pure strategy \textit{Bayes-Nash equilibrium} if for each Player \( i \) and his types \( t_i \), \( s^*_i(t_i) \) solves,

\[
\max_{a_i \in A_i} \sum_{t_{-i} \in T_{-i}} u_i( s^*_1(t_1), s^*_2(t_2), ..., s^*_{i-1}(t_{i-1}), a_i, s^*_{i+1}(t_{i+1}) \ldots, s^*_n(t_n); t) \ p_i(t_{-i}|t_i)
\]  

(42)
This equation essentially states that no player would deviate from their optimal strategy. Otherwise, they will obtain a lower pay-off if their opponents play their optimal strategies. Also, the weighting of the pay-off $u_i$ with the distribution $p_i(.)$ arises players are uncertain about their opponent’s type and need to optimize an expected value based on that uncertainty. For example, where $N = \{1, 2\}$, this definition can be read as:

$$\sum_{t_2 \in T_2} u_1 (s^*_{1}(t_1), s^*_{2}(t_2)); t_1, t_2) \geq \sum_{t_2 \in T_2} u_1 (a_1(t_1), s^*_{2}(t_2)); t_1, t_2) \quad (43)$$

and

$$\sum_{t_2 \in T_2} u_1 (s^*_{1}(t_1), s^*_{2}(t_2)); t_1, t_2) \geq \sum_{t_2 \in T_2} u_1 (s^*_{1}(t_1), a_2(t_2)); t_1, t_2) \quad (44)$$

Theorem 5. Let $\Gamma^{i}$ be a game of incomplete information and $\Gamma^{b}$ be the Bayesian game equivalent to $\Gamma^{i}$. In order that the n-tuple $s^* = (s^*_{1}, s^*_{2}, ..., s^*_{n})$ be a Bayes-Nash equilibrium point of the game $\Gamma^{i}$, it is necessary and sufficient that this n-tuple $s^*$ be a Nash Equilibrium Point (NEP) in the game $\Gamma^{b}$.

We have discussed game theoretic literature above and this information will be used as a basis to develop the two-person Merger and Acquisition (M&A) model in the following chapters. For ease of explanation of the two-person M&A model, this thesis will use real numbers (to represent gain or loss instead of variables, e.g. +1 or -1 instead of +a or -a) in the decision trees that are built using the Gambit software package (version 0.2007.12.04; further information can be found at http://www.gambit-project.org/doc/index.html).

While, we have reviewed game theoretic concepts, it is now important to see what game theoretic research has been undertaken that relates to mergers and acquisitions. This is important as the two-person Merger and Acquisition (M&A) model that will be developed in this thesis has a basis in game theory. Therefore, prior to developing this model, we need to review research in this area.

Fridolfsson and Stennek (2010) provide an endogenous merger model that explains how rival companies analyse mergers pre-empting that their rivals will undertake mergers themselves. This model helps increase the share prices of the merged
company, but ends up reducing profits. Croson et al. (2004) explain that mergers improve market efficiency and increase synergies, while takeovers create positive and negative synergies. They provide a game theoretic method to predict takeovers and explain outcomes of such transactions. Damme and Pinkse (2005) discuss the benefits of merger simulation as such simulations can help with deeper understanding of such transactions.

Hansen (1987) on the other hand provides a theory of choice that uses a framework of bargaining under asymmetric information to understand if the acquirer should pay cash or offer its stock when purchasing the target firm. Asymmetric information refers to the situation where one player has more information compared to their opponent. In this case, the target firm knows more about itself than the acquirer knows about the target firm. Therefore, cash provides more certainty than stock for the target, while the acquirer does not know exactly the value of the target. Jensen (1987) also adds to this discussion by saying that takeovers add more value than mergers for acquirers. As, acquirers obtain on average a 4-8 percent return on takeovers compared to zero percent return on mergers. However, in a hostile takeover, it is seen that target firm shareholders benefit getting anywhere between a 30-50 percent premium compare to takeover transactions that are non-hostile. Bradley et al. (1988) agree with Jensen (1987) stating that acquiring firms usually obtain a 7.4 percent return in a successful tender for the target firm. However, when the number of bidders increases the return for the target increases and that of the acquirer decreases with the supply of target’s stock being positively sloped. They also noticed that the legal or regulatory environment had minimal impact on these results.

While, game theory research provides the basis on the discussion in this thesis, we also need to note that mergers and acquisitions transactions have a significant portion of negotiation associated with them. So, it is important to discuss negotiation theory to obtain a holistic background.

Negotiation utilizes game theory research to analyse conflicts from a socio-economic perspective (Bazerman, Lewicki and Sheppard 1991; Brett, Goldberg and Ury 1990; Kochan and Katz 1988). The five types of negotiation approaches that have been recognized are: normative/perspective approach, individual difference approach, sociological structural approach, cognitive information processing approach and the social contextual approach (Carroll and Payne 1990; Kramer,
Raiffa (1982) has identified an additional approach that uses negotiation analysis.

Extending the discussion from game theory into bargaining theory, we realize that Edgeworth (1881) presented the first bargaining theory. Since then there have been subsequent bargaining models developed by Nash (1950), Osborne and Rubinstein (1990), Roth (1983), Roth and Schoumaker (1983), Roth (1985) Roth, Prasnikar, Okuno-Fujiwara and Zamir (1991), Camerer (1997) and Carraro, Marchiori and Sgobbi (2005).

Brams (1994) has also provided an interesting concept called the Theory of Moves, which intends to provide another solution using dynamic games. Brams and Taylor (1996) extend this concept to provide a solution for the fair division of cutting a cake. They believe that if the first player cuts the cake then her opponent can choose the slice of the cake. Using this approach, it will make the first player more honest in the division of the cake. Otherwise, if the first player cuts the cake disproportionately, then the opponent can easily chose the bigger slice of the cake first. However, this may not work as well when we have heterogeneous products, where each player may have different preferences and may or may not like the product being divided sufficiently enough for the division to be fair.

Further, Hopmann (1995) analyses the similarity between bargaining and problem solving and states that negotiation is a value creating activity compared to problem solving. As he states that while the players in a negotiation will try to maximize their share of the gain, they will also end up increasing the overall pay-off in the negotiation. This extends the learning from Brams and Taylor’s (1996) discussion on dividing a fixed sized pie. Hopmann (1995) believes that the competitive approach increases the overall gain within the negotiation. Problem solving on the contrary is a co-operative process that requires the players to work constructively towards a mutual resolution. As a result, he states that problem solving results in an equitable, durable and efficient negotiated solution and is useful in resolving complex negotiations.

Brams (1990), Hoffmann et al (1991) and Roth (1991) reinforce that selfishness is critical for players to succeed in a non co-operative game in order to achieve a superior outcome. However, Pruitt and Rubin (1986) and Loewenstein, Thompson and Bazerman (1989) explain that players concentrate sometimes on reducing the final pay-offs of their opponents instead of finding strategies to increase their pay-
off within the game. Studies have also shown that consideration regarding fairness in games can also undesirably affect the outcome in a game (Guth and Tietz 1990; Hoffmann et al 1991; Kahneman, Knetsch and Thaler 1986, 1987; Ochs and Roth 1989; Roth 1991). As fairness in games is important, it has also been noticed that players frequently distribute the pay-offs symmetrically when no external influences exist in a game (Allison, McQueen and Schaarfl 1992; Messick 1992; Messick and Schell 1992; Bazerman and Neale 1983; Northcroft and Neale 1986, 1987).

Schelling (1960) showed that if a player undertakes an irrevocable commitment with inefficient delays, then there is a strong possibility that this player will be set to win the game, as it will indicate to the opponent that this player will do anything to win. Cunyat (2001) explains inefficient delays as the delay that occurs when a committed player chooses an incorrect level of commitment that may cause a delay in obtaining a suitable pay-off in the game. He gives an example of some complete information game settings like concession games, where a strong commitment by a player may end up setting back rather than assisting the player win the game.

Schelling (1960) provides an example to explain his reasoning stating that if an invading army burnt their ships once they touch land, it would indicate to the enemy the irrevocable commitment of this army, as the only way for this invading army to survive will be to win this war. Such a strong commitment can sometimes make the opponent reconsider their actions, which would not occur if such irrevocable commitment was not made by the invading army in the first place. On the contrary, Fershtman and Seidmann (1993) explain that negotiations between the players may decrease the pay-offs for each player due to inefficient delays. Cunyat (2001) has stated that the inefficient delays can be eliminated by getting the opponent to agree to terms early due to the irrevocable commitment.

Pruitt and Kimmel (1977) and Thibaut and Kelley (1959) explain that long term cooperation is a considerable factor in negotiations. However, it has been noticed that noisy social dilemmas can cause misperception and result in reciprocity from opponents (Bendor 1987, 1993; Bendor et al 1991; Fundenberg and Maskin 1990; Molander 1985; Mueller 1988; Wu and Axelrod 1995; Young and Foster 1991). Researchers have also identified that inefficient outcomes occur where players are matched in toughness in bargaining. When viewed on the basis of Thibaut-Kelley (1959) degree of reciprocity, if players show less toughness, then the opponent may exploit that behavior (Komorita and Bremmer 1968). Further, Komorita and
Esser (1975) and Smith et al (1982) explain that opponents will show reciprocity; if they have been exploited when they showed less toughness in a bargaining situation, for example, provide less concession to subsequent actions.

Players increase their toughness in bargaining to counteract for behavior to circumvent their other players from reducing their toughness and causing a downward spiral (Druckman and Harris 1990). In complex negotiations players may give up less important issues, but may act tougher when dealing with more important issues (Pruitt 1981; Weingart 1993). Other researchers encourage issues to be split into smaller issues in order to enable each issue to be negotiated with the same significance as the less important issues (Lax and Sebenius 1986; Pruitt 1981).

Heifetz and Segev (2001) analyse evolutionary toughness in games concluding that asymmetric information between players results in a moderate degree of toughness. Intimidation and allowances by players are seen to impact the demand and communication patterns between players in a game. Bacharach and Lawler (1981), Jervis (1976) and Schelling (1960) have shown that conflict spiral and bilateral deterrence models can explain the demand and communication patterns among players in a game. Moreover, it is noted that the conflict spiral model does not justify the demand and communication patterns; instead the power relationship between players describes the coercive and concessional behavior providing for quicker compromises (De Dreu 1995; Carnevale and Pruitt 1992; Thompson 1990). Though, research has shown that the bilateral deterrence model can justify the demand and communication patterns in a game (De Dreu 1995; Bacharach and Lawler 1981; Lawler, Ford and Blegen 1988; Lawler and Bacharach 1987; Lawler 1992).

It is also seen that when a player makes a threat, either the opponent will cooperate, otherwise it will repudiate by undertaking threatening actions as well making it hard to negotiate a successful outcome (Axelrod 1980; Leng 1993). While, players do need to explicitly make the threat to the opponent, it seems that it is better for weaker players to make concessions rather than threats. However, this does not help either as each player’s power does not increase through the remaining part of the negotiation (Bacharach and Lawler 1981; Komorita, Sheposh and Braver 1968; De Dreu 1995).
Mogy and Pruitt (1974) believe that the credibility of a threat is inversely proportional to the cost of implementing this threat. Supporting this ideology, it is also seen that an increased cost to apply force or to repudiate will reduce the threat of this attack (Bacharach and Lawler 1981; Lawler, Ford and Blegen 1988). It is often believed that balancing the power of each player in the game may be suitable to obtain a fair outcome, however this may adversely impact the outcome as well because inequality in power could result in a more efficient outcome (Ippolito and Pruitt 1991; Laskewitz, Van de Vliert and De Dreu 1994; De Dreu 1995).

Raiffa, Richardson and Metcalfe (2002) believe that game theoretic solution to negotiation problems can be normative and may not be practical in application. On the other hand, it is seen that psychology related solutions to negotiation problems can be descriptive, providing insight into individual decision making. Luce and Raiffa (1967) asserts that game theoretic and psychology are complimentary to providing insight into negotiation issues. However, it is seen that the following shortcomings exist for game theoretic solutions:

1. Probability of obtaining multiple equilibria with no method to find a single optimal outcome,

2. Game theory assumes that players are completely rational and this is inconsistent with reality. The concept of bounded awareness provided by Bazerman and Chugh (2004) is more suitable to most games in reality,

3. As factors related to players and the environment is not completely understood and the occurrence of psychological factors, limits the prescriptive power of the game theoretic framework, and

4. In reality, conflict situations could include infinite relationships between the actions of each player, their opponents and the environment, which is hard to replicate in game theory as it requires specific limits (Luce and Raiffa 1957).

2.3 Cognitive, Social and Behavioural Psychology in Negotiation

Psychology is an important research area in negotiation as it provides input from the cognitive, behavioural and social psychology aspects. This is critical as it
explains the individual decision making of each player and also its social interaction with their opponents within a game.

Social identity is important to players in a game and they will try to be fair in a game, displaying accountability where social identity is at risk, as social identity represents the interpersonal relationship within the players in a game (Kramer, Pommerenke and Newton 1993; Hogg and Abrams 1988; Tajfel and Turner 1987; Turner 1987). Dawes and Thaler (1988) and Kramer and Brewer (1984) reinforce this feedback regarding social identity and fairness of outcomes. We also notice that accountability can be a significant concern as players want to be accepted by others in the game (Baumeister and Hutton 1987; Tetlock 1985). Players also want to look competent, co-operative and fair as this will impact the final pay-off in the game (Bond 1982; Ginzel, Kramer and Sutton 1992; Greenberg 1990; Reis and Gruzen 1976). While, it is still important to exhibit toughness and competitiveness as this selfish behaviour improves their pay-off as well (Carnevale, Pruitt and Button 1979; Sutton and Kramer 1990).

While, low accountability from players in a game will make them more self-interested, it is likely that players will never be completely co-operative even when there is higher accountability as each player wants to improve their pay-off from the game (Diener et al 1976; Prentice-Dunn and Rogers 1982). Existing research supports that social judgement can have an impact on individual decision making within games and this occurs as biases from social judgement can result in selective attention to information, under or over weighting of specific information and polarised evaluation of social stimuli that will widen the difference between the issues being negotiated (Fiske and Taylor 1991; Messick and Mackie 1989). On the other hand, if the importance of social identity is reduced then it could have a positive impact of reducing the differences between players and more likely to result in a better outcome (Fiske and Taylor 1991). This is aided by the fact that players in a game co-operate with their opponents when there is a greater likelihood to interacting with each other in subsequent games (Ben-Yoav and Pruitt 1984; Axelrod 1984; Kreps and Wilson 1982).

When players are co-operatively working with each other, they follow some decision heuristics that help them to manage the distribution of resources within the game (Allison and Messick 1990). Such heuristics assist players in maximizing their interests within a group (Kramer, Pommerenke and Newton 1993). In addition, Keenan and Wilson (1990) explain that bargaining theory emphasizes that
we understand the motives of players and how it will affect it will impact their decision making, recommending that social contextual analysis can be used for such analysis.

Korobkin and Guthrie (2004) discuss some heuristics that help players in making their decisions. Anchoring is one of these heuristics, where players can provide a specific price in their initial offer (offer and bid are used interchangeably in this thesis and have the same meaning) and this seems to restrict the range of prices that we would otherwise achieve. Availability is another such heuristic, where players provide offers in a way that it will make their opponent likely chose that offer. Framing on the contrary is similar to anchoring, where players can display their offer in comparison to the worst case scenario. This will make the offer attractive and there is some possibility that the opponent may accept this offer. Finally, another heuristic called contrast effects, where players can offer multiple offers with their preferred offer looking superior to other potential offers. This may result in the opponent positively considering the superior offer.

Continuing the discussion, Korobkin and Guthrie (2004) state that it can often become hard for players to objectively consider offers made to them. So, players can use approaches to assist them, starting with anchoring and adjustment – where a player will chose an anchor and then try to move sequentially (adjustment) towards their expected value. This can often be sub-optimal when another player is choosing the anchor as it could completely bias the final outcome. We have recently also discussed the concept of availability – this approach works well in case we have historic data on what offers were accepted by opponents, in which case similar offers can be provided. Though, this may not work that well, when the preferences of a player’s opponents changes over time and when these historically accepted offers are no longer suitable. The last approach that can be used is called self-serving evaluations – where players try to analyse the preferences that their opponents would hold and then make offers based on those preferences (Lord et al 1979; Weinstein 1980). Nonetheless, there is a likelihood that players will not be unbiased (have ego-centric bias) in their judgment and this may result in the offer being rejected by their opponent.

Bendersky and Curhan (2003) examine player’s preferences and state that preferences can change even within a single game, which makes it hard to judge an opponent’s preference accurately. On the contrary, it is seen that players prefer making offers rather than receiving offers which is in line with dissonance theory.
(Festinger 1957) and reactance theory (Brehm 1972). Dissonance theory explains that players will undertake actions to reduce the cognitive dissonance between two conflicting states (Festinger 1957). While, the reactance theory states that players may overvalue equally weighted options due to potential likelihood of losing the attractive features of the rejected option (Brehm 1972; Brehm and Brehm 1981; Wicklund 1972; Flonta 2002; Ross 1995; Stillinger, Epelbaum, Keltner and Ross 1991; Bazerman and Neale 1983; Neale and Bazerman 1991). In addition to dissonance theory, the self-perception theory is based on the fact that players are unsure of their preferences and base their preferences on the previous choices they have made (Bem 1967; 1972).

Players analyse the behaviours of their opponents, for example, strategies and tactics. Similarly, their opponents analyse their behaviours (Fortgang, Lax and Sebenenius 2003; Morris, Larrick and Su 1999; Tinsley, O’Connor and Sullivan 2002). However, at a higher level, it is important to look at the social interaction between players, analyzing factors like ethics, fairness and trust (Lewicki, McAllister and Bies 1998; Naquin and Paulson 2003; Pruitt and Rubin 1986). While, understanding behaviour is important, we can also face significant inefficiencies when asymmetrical information existing in games as inefficient delays occurs in reaching a mutually acceptable outcome (Myerson and Satterwaite 1983). In addition to asymmetrical information effects, empirical experiments have shown that players can display an endowment effect (toughness) that negatively impacts the possibility of concluding successful negotiations (Kahneman et al 1990).

It has also been seen that players often accept inefficient pay-offs were they are reluctant to improve their pay-offs (Aleimi, Fos et al 1990; Prasnikar and Roth 1992; Roth 1995; Weingart 1996). Theoretical and experimental studies have been conducted to analyse this phenomenon (McClenen 1990; Varoufakis 1991; Kersten and Noronha 1998; Bazerman and Neale 1991; Neale and Bazerman 1991; Adler and Graham 1989; Hofstede 1989; Faure and Rubin 1993). However, it has been noticed that players do not use Pareto optimal pay-offs that includes distrust, fairness and other psychological factors (Korhonen, Phillips et al 1998; Bartos, Tietz et al 1983; Tietz and Bartos 1983; Thompson and Loewenstein 1992). Korhonen et al (1998) argue that providing inducements to players may progress the negotiation towards more efficient outcomes. Rapoport et al (1995) have also added that time and external influences can also have a substantial impact on the final pay-offs received by players.
2.4 Prospect theory

Von Neumann and Morgenstern’s (1944) work was important in the field of game theory as we discussed earlier. However, they also set the framework for understand utility for a psychological perspective by providing the concept of Expected Utility theory. This work was later extended by Kahneman and Tversky (1979), where they explained that humans react differently to positive and negative events. Prospect theory and cumulative prospect theory provided by Kahneman and Tversky (1979; 1992) highlight the following factors that impact human behaviour:

1. Certainty Effect: humans would prefer sure gains but will be willing to gamble to reduce losses.
2. Isolation Effect: humans have varying preferences resulting in different choices when the same preferences are provided in another form.
3. Value Function: humans have a steeper convex function for losses compared to a concave function for gains.
4. Non-linear transformation: humans overweight small probability events and underweight high probability events.

Kahneman and Tversky (1992) also discuss the following factors in their cumulative prospect theory framework:

1. Framing effects: humans should come up with a consistent sequence of choice when analyzing their preferences (Arrow 1982). However, Kahneman and Tversky (1986) explain that people vary their choices when the same problem is structured differently.
2. Nonlinear Preferences: Expected utility theory (Von Neumann and Morgenstern 1944) states that utility is linear for risky prospects. Though, Allias (1953) advises that humans have non-linear preferences for their different choices.
4. Risk Seeking: Humans are willing to gamble on a large win compared to a small but certain pay-off. On the other hand, they will accept the small certain loss and will not gamble by taking on the possibility of a large loss.
5. Loss Aversion: Humans have a skewed perception where losses impact them more than gains (Kahneman and Tversky 1984; Tversky and Kahneman 1991).
Barberis, Huang and Santos (2001) and Levy and Levy (2002) have shown that prospect theory impacts asset prices. Levy (1997) and McDermott (1998) have shown that prospect theory also influences international relations. Fiegenbaum (1990) undertook an experimental study across 85 industries and explained successful firms took three times less risk than less successful firms.

Prospect theory explains the way humans perceive economic risk and this has a substantial impact on the way negotiations are undertaken. Khanemann and Tversky (1979) have stated that this pattern of risk perception can be generalised to other fields of human decision making. Curhan, Elfenbein and Xu (2006) have used a psychological framework called the subjective value inventory to analyse risk in negotiations.

2.5 Conclusion

This chapter has discussed the concepts of game theory and psychological theory that relates to negotiation theory research. In this chapter, we start by discussing the strategic and axiomatic forms of game theory. Then, the discussion extends to understanding dominant solvability and Nash Equilibrium Point (NEP) concepts. This discussion is then extended to consider the literature on sub-game perfect Nash equilibrium point and behavioural strategies. The concept of Bayes-Nash equilibrium has then been reviewed to understand games with incomplete information. While, we discuss game theory in the initial parts of this chapter, the literature review moves on to discuss psychological negotiation theory and prospect theory.

The following chapter will develop on this literature to develop the Two-person Merger & Acquisition (M&A) model with complete information to understand how the acquirer and target firms will behave in a complete information game. Chapter 4 will then extend this model to incorporate incomplete information and prospect theory. Chapters 5 and 6 will then extend this model to include real options games.
3.1 Introduction

Mergers and Acquisitions (M&A) is a significant research area in the field of Corporate Finance. There is substantial finance literature on how merger and acquisition transactions can be priced (valued). Currently, the investment banking industry uses the concepts of Balance Sheet valuation models (Book value, Liquidity value and Replacement cost), Dividend Discount model, Price Earnings ratio, Discount Cash Flow analysis, Transaction multiples and Economic Profit model to price such transactions (see Marren 1993 for more information). Merger and acquisition pricing has been a challenging task, primarily as different valuation (pricing) methods can provide different results. A pertinent question is the offer price that should be used in any M&A transaction?

In most cases, the acquiring company (buyer) and the target company (seller) may not always agree on the valuation because the acquirer primarily wants to purchase the target company for the lowest valuation possible. While, the target company’s stockholders will want the highest valuation for which they can sell their company. This leads to a series of negotiations between the acquirer and the target company to either come up with an agreed price or to decide not to go through with this transaction.

Therefore, interestingly none of these models provide accurate estimates of offer prices for the purposes of mergers and acquisitions. These corporate finance models provide a theoretical offer price that should be offered. However, due to a biased system of beliefs that humans have where good is preferred to bad, these models fall short in providing suitable estimates of offer prices that will be acceptable for merger and acquisition activities.

To support this argument, Becher (2000) has shown that on average target stocks have gained 22% with the combined firm gaining 3% when he analysed US bank mergers through the period 1980-97. This shows that these mergers could have been underpriced, otherwise there would not have been such a large increase in
the target and merged firm stock prices if the mergers were priced accurately. Rhodes-Kropf and Viswanathan (2004, p.2685) add to this discussion stating that “...potential market value deviations from fundamental values on both sides of the transaction can rationally lead to a correlation between stock merger activity and market valuation. Merger waves and waves of cash and stock purchases can be rationally driven by periods of over- and undervaluation of the stock market. Thus, valuation fundamentally impacts mergers.” This shows that market valuations of mergers are not being accurately calculated that cause under/over-valuation of M&A transactions. Rhodes-Kropf et al. (2005) review the breakdown of the market-to-book value (MV) ratio and state that the MV ratio is driven by short-term and long-term factors. They state that misvaluations can occur depending on if a low or high MV ratio firm mergers with a low or high MV ratio firm. They believe that market-to-book value ratio, method of payment and neoclassical explanations for merger misvaluation are factors that can provide an understanding of when misvaluation may occur in mergers.

Moeller et al. (2005) undertake a study of acquiring firms from 1998 to 2001 and find that their values fell by around 12 percent primarily as acquirers took on a large number of small mergers with negative synergies and with extremely high valuations. As this period coincides with the internet boom, it may be possible that the high valuations were seen as acceptable at that moment in time. However, with the subsequent bust of the internet bubble, such mergers were seen as overpriced and resulted in the acquiring firms losing value. Rad and Beek (1999) also found in an empirical study of European bank mergers that target firms usually obtained positive returns compared to acquirer. Where, results showed that returns were higher for bigger banks as they were more efficient in integrating the target firms.

Gupta and Gerchak (2002) also add to this discussion by stating that higher operational synergies between the target and the acquirer increase the merger valuation by the acquirer – this would mean that an increase in synergies should increase potential future revenues. If these future potential earnings are not included in these corporate finance valuations of M&A transactions then there is a possibility for underpricing of this valuation to occur? Further, supporting this discussion, Mukherjee, Kiymaz et al. (2004) interviewed Chief Financial Officers of leading companies reviewing mergers, acquisitions and divestures that occurred during the period from 1990 to 2001. They found that mergers occur mainly to increase operating efficiencies, while acquisitions are undertaken to increase diversification or efficiencies and divestures were done to focus on profit making enterprises. They found that the Discounted Cash Flow model was utilised more
than the Market Multiples method for both listed and unlisted companies. They also
found that acquirers have most often used their own weighted average cost of
capital to value the target’s future cash flows that can cause errors in M&A pricing
and found that this was one of the prime reasons for persistent overvaluation of
targets.

While, valuation methods may be the reason for under/over-valuation of M&A
pricing, there also seems to be a behavioural finance bias to these valuations.
Baker, Pan and Wurgler (2009) and Baker, Ruback and Wurgler (2004) do indicate
that behavioural factors can impact the pricing of targets in mergers and
acquisitions. Baker, Pan and Wurgler (2009) through their empirical research of US
companies through the duration of 1st January 1984 to 31st December 2007, have
found that deals for listed companies usually settled around the 52-week high price
of a stock for a target company. This occurs as the 52-week high stock price acts
as a psychological anchor for the acquirer and target companies, where the
acquirer’s stockholder do not think that they are paying too much and the target’s
stockholder perceive this amount as sufficient payment for selling their company.
As a result, they have suggested that it may be worthwhile that acquirers provide
an offer price close to the 52-week high price to purchase the target company.

Merger and acquisition pricing can therefore been seen as a negotiation between
the acquirer and the target to agree on a price at which the transaction can be
finalised. As a result, we need to use game theory to review this concept further.
Therefore, before we move any further with this discussion it will be worth
analyzing some past research in the area of game theory and to understand what
relevant research has been done where game theory has been applied to merger
and acquisition pricing. Previous research shows that game theory provides an
insight into strategic human interaction in such financial negotiations.
Contemporary game theory has two forms: non co-operative game theory (Nash
1951) and co-operative game theory (Von Neumann and Morgenstern 1944;
Shapley 1953; Shapley 1977; Shapley and Shubik 1954; Luce and Raiffa 1957;
Aumann and Drèze 1974; Myerson 1977). Both these types of game theories are
the essential basis for research in the area of game theoretic negotiation theory
(including bargaining and auction theory).

Von Neumann and Morgenstern (1944) and Nash (1950; 1951; 1953) have
suggested two game theory approaches to resolve bargaining problems: axiomatic
or strategic. The axiomatic approach (sometimes called co-operative theory)
assists by providing a set of valuable axioms. On the other hand, the strategic
approach models outcomes in a non co-operative game. Mergers and acquisitions would come under the premise of non co-operative games, as the acquirer and target firms can either gain or lose value depending on the valuation at which the target firm is purchased and the synergies obtained through post-merger integration. Initially, the axiomatic approach led the way due to the findings of Von Neumann and Morgenstern (1944). But, Rubenstein’s (1982) solution of alternating players with discounting pay-off shifted the research to move towards the strategic form approach.

In this chapter, we will model this M&A pricing problem as a two-person zero sum game with incomplete information. Games with complete information are games where all players know the strategies and pay-offs of other players in the game. Rasmusen (2006, p.53) differentiates by saying that “In a game of complete information, all players know the rules of the game. Otherwise, the game is one of incomplete information”. Effectively, the two-person Merger & Acquisition game can initially be defined as a two-person game with complete information as both the acquirer and target know the potential strategies and pay-offs that each player can implement. However, it is not a game of perfect information that includes information on what strategies each player will choose. It is important to make this differentiation as the equilibrium attained in both these games may be different. The two-person Merger & Acquisition model with complete information is then extended to include incomplete information, where the target or the acquirer or both do not know the type of their opponent.

Though, previous research in two-person games has been extensively undertaken, for example, Axelrod’s (1984) tit-for-tat game that is a special type of two-person repeated game, where the second player follows the strategy that the first player followed in the previous round. Two-person games have been found to be simplistic and game theorists have often developed N-person games as they believe this provides a better representation of real world situations (Rasmusen 2006). Nonetheless, two-person games are also useful as in this circumstance where a merger or an acquisition usually only occurs in a two-person scenario. Other potential suitors may exist but these games can often be reviewed as multiple two person games or can be extended to include additional players (in which case it will be an N-person game with three or four players). In this thesis, we will only consider the situation where mergers or acquisitions occur in the two-person game scenario.
The objective of this chapter is to develop a two-person merger and acquisition game theoretic model to understand if the Nash equilibrium point (NEP) for this game equals the mid-point between the initial offer price provided by an acquirer and sale price provided by the target firm. In order to answer this question, the structure of this chapter has been set out as follows: the introduction section has reviewed the literature related to merger and acquisition pricing and game theory, and identified that a gap exists where a game theoretic model could be developed to analyse merger and acquisition pricing. The next section will develop the two-person merger and acquisition game theoretic model with complete information and extend it to include incomplete information. This section will also review the strategic and normal forms of each game as well as provide the relevant results. Finally, the third section will provide a conclusion to this chapter and explain if the mid-point was identified as the stable equilibrium state for this game.

3.2 Two-person Merger & Acquisition model with complete information

The two-person merger & acquisition model with complete information is a two-person game between the Acquirer (Buyer) and Target (Seller). In this game, the acquirer provides a price to purchase the target (which is known to both the acquirer and target in the complete information game) and this price can be negotiated (i.e. can be increased or reduced) by the acquirer and (increased or kept stable by the) target till a final price is attained. In this model, only three iterations of negotiations have been considered: the acquirer will initially start at a price from where he will either increase or decrease the offer, the target can then ask for an increase (as the target would like to get paid more to sell) or keep the same price (where the target does not believe he can obtain a higher price to sell) and finally the acquirer can increase (if the acquirer believes that he needs to pay more to purchase the target) or decrease the price (if the acquirer believes that he can decrease the price due to an adverse change in the target’s situation), as would occur in business practice in a merger or acquisition scenario. However, the results at the end of this chapter will identify if the stable equilibrium state for this game is equal to the mid-point price (between the acquirer's offer price and target's offer price)?

The intention of this two-person merger & acquisition model with complete/incomplete information is to find the saddle point of this game that will explain what price would be suitable to the acquirer and target to complete this transaction. The saddle point is a set of strategies that align with the stable equilibrium state which players would play in order to best their optimal outcome.
The saddle point can be explained as follows: both the acquirer and target can select from a set of strategies $S$. A combination of strategies $s^* = (s^*_1, s^*_2, ..., s^*_n) \in S$ is a saddle point if it satisfies the property:

\[ u_1(s^*_1, s^*_2) \geq u_1(s_1, s^*_2) \quad \forall s_1 \in S_1 \]  
\[ u_2(s^*_1, s^*_2) \geq u_2(s^*_1, s_2) \quad \forall s_2 \in S_2 \]

Where, $s^*_1$ is an optimal strategy that is played by player 1 (acquirer) and $s^*_2$ is an optimal strategy played by player 2 (target) in this game. $s_1$ and $s_2$ is any other strategy that is not necessary an optimal strategy. While, $u_i$ is the pay-off that is obtained by each player for playing their strategy.

This property states that if $s^*$ is a saddle point then a player who deviates from this optimal strategy will not gain if all other players adhere to their optimal strategies. In other words, if a player does not follow this optimal strategy, then it is possible they could end up with a worse pay-off than their opponents.

The strategic form (diagrammatic form of the game providing the sequences of strategies played by each player) and normal form (tabular form of the game providing the final outcomes of each strategy pair) of the two-person merger and acquisition game theoretic model with complete information have been provided below to discuss the potential strategies that the acquirer and target could follow in this game. The extensive and normal form have been developed using the Gambit software package (version 0.2007.12.04; further information can be found at [http://www.gambit-project.org/doc/index.html](http://www.gambit-project.org/doc/index.html)).

Figure 1 provides the strategic form of the two-person M&A model, where the acquirer will make the initial offer and the target will decide on the same node, if he wants to accept that offer. If the offer is rejected, then the acquirer can either increase or reduce his bid in the future that is represented by the next node. An increase in bid occurs when the acquirer wants to purchase the target and is will to pay more. However, it could be possible that between the time when the initial offer was provided and when the subsequent offer is provided, the target’s value could have decreased due to any number of reasons. As a result, the acquirer can either increase or reduce the offer based on the circumstance at that moment in time. Similarly, the game will continue till the time where either the target accepts the offer or if the acquirer does not want to provide any further offers as it has
reached his limit where he does not believe he would like to pay a higher amount to purchase the target firm.

**Figure 1. Strategic form of the two-person merger & acquisition model (2-P M&A model) with complete information**

The strategic form of this game can be translated into the normal form (see figure 2 below). Strategies in the normal form can be directly translated back to the strategic form for both the acquirer and target. For example, the strategy “III” for acquirer and “IS” for target would mean that the acquirer will initially choose the “Increase Bid” strategy in the first play. In response to this action the target will choose the “Increase Bid” strategy as well because it follows the strategy “IS” which means it will “Increase Bid” if the acquirer chooses the “Increase Bid” and it will opt for the “Stable Bid” strategy if the acquirer chooses “Reduce Bid”. Similarly, the acquirer will choose the “Increase Bid” strategy as it follow the “III” strategy. This occurs as the acquirer will choose the “Increase Bid” strategy if the target had chosen the “Increase Bid” strategy or the “Stable Bid” strategy.
However, before we move on to develop the normal form of the game, it is important to explain how the pay-offs have been calculated in the strategic form. For each time, the bid has been increased (strategy: “I” Increase Bid) by the acquirer he receives a pay-off of +1 and every time the acquirer reduces the bid (strategy: “R” Reduce Bid) he receives a pay-off of -1. Therefore, the strategies “RII” and “IRR” will receive a pay-off of 0 for the acquirer because in the first round he will increase the bid and then reduce the bid in the second round. Similarly, the target will receive the opposite of these pay-offs, i.e. +1 for Increase Bid “I” and 0 for keeping the bid Stable - Stable Bid “S”. Therefore, the pay-off for “SI” and “IS” will be similar for the Target with the pay-off being +1.

*Figure 2. Normal form layout of the two-person merger & acquisition model (2-P M&A model) with complete information*

<table>
<thead>
<tr>
<th>Player 1’s Strategies</th>
<th>Player 2’s Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>II: -3</td>
</tr>
<tr>
<td>III</td>
<td>III: -2</td>
</tr>
<tr>
<td>IIR</td>
<td>IIR: -2</td>
</tr>
<tr>
<td>IRI</td>
<td>IRI: -1</td>
</tr>
<tr>
<td>IRR</td>
<td>IRR: 0</td>
</tr>
<tr>
<td>RII</td>
<td>RII: 0</td>
</tr>
<tr>
<td>RIR</td>
<td>RIR: 2</td>
</tr>
<tr>
<td>RRI</td>
<td>RRI: 1</td>
</tr>
<tr>
<td>RRR</td>
<td>RRR: 2</td>
</tr>
</tbody>
</table>

Based on the strategies and pay-offs provided in the strategic and normal forms of this game, you will notice that the acquirer is at a greater disadvantage than the target as he can lose 3 units if the price is increased by the acquirer and target (upper most point of the strategic form game) compared to when the price is reduced (lower most point of the strategic form game). This occurs due to the strategies followed by the target. The target company will never reduce the price, but will opt to increase the price where possible. This causes an imbalance in the pay-off structure and it may result in a situation where equilibrium state is more suitable to the target than the acquirer.

Let’s review this game further and try to solve it using the method of linear programming. As this model is a zero sum game, we will be able to analyse all the mixed strategies that each player can use and the value of the game using linear programming (c.f. Section 2.2).
The following linear equations had to be solved in order to find the mixed strategies for the acquirer (player 1):

\[
\begin{align*}
\text{min} & \quad u_1 + u_2 + u_3 + u_4 \\
\text{Subject to} & \quad -3u_1 - 3u_2 - 2u_3 - 2u_4 \geq 1 \\
& \quad -3u_1 - 3u_2 \geq 1 \\
& \quad -u_1 - u_2 - 2u_3 - 2u_4 \geq 1 \\
& \quad -u_1 - u_2 \geq 1 \\
& \quad -2u_1 + 2u_2 - 2u_3 \geq 1 \\
& \quad 2u_1 + u_2 \geq 1 \\
& \quad 2u_1 + 2u_2 + u_3 + 2u_4 \geq 1 \\
& \quad u_1 \geq 0, \quad u_2 \geq 0, \quad u_3 \geq 0, \quad u_4 \geq 0
\end{align*}
\]

The two-person merger and acquisition game theoretic model with complete information has been solved using a linear program developed in Microsoft Excel (see chapter 2 for information on zero-sum games and linear programming). Results for these linear equations for Player 1 are provided below:

<table>
<thead>
<tr>
<th>Target Cell (Max)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell</td>
<td>Name</td>
<td>Original Value</td>
<td>Final Value</td>
</tr>
<tr>
<td>$H:$28</td>
<td>Value of Objective Function Total</td>
<td>$4$</td>
<td>$0.2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adjustable Cells</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell</td>
<td>Name</td>
<td>Original Value</td>
<td>Final Value</td>
</tr>
<tr>
<td>$D:$17</td>
<td>Values to be determined $u_1$</td>
<td>$1$</td>
<td>$0.104512797$</td>
</tr>
<tr>
<td>$E:$17</td>
<td>Values to be determined $u_2$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$F:$17</td>
<td>Values to be determined $u_3$</td>
<td>$1$</td>
<td>$0.095487203$</td>
</tr>
<tr>
<td>$G:$17</td>
<td>Values to be determined $u_4$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraints</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell</td>
<td>Name</td>
<td>Cell Value</td>
<td>Formula</td>
</tr>
<tr>
<td>$H:$19</td>
<td>Constraints Total</td>
<td>$0.295487203$</td>
<td>$H:$19&lt;=$I:$19$</td>
</tr>
<tr>
<td>$H:$20</td>
<td>Total</td>
<td>$0.486461609$</td>
<td>$H:$20&lt;=$I:$20$</td>
</tr>
<tr>
<td>$H:$21</td>
<td>Total</td>
<td>$0.504512797$</td>
<td>$H:$21&lt;=$I:$21$</td>
</tr>
<tr>
<td>$H:$22</td>
<td>Total</td>
<td>$0.695487203$</td>
<td>$H:$22&lt;=$I:$22$</td>
</tr>
<tr>
<td>$H:$23</td>
<td>Total</td>
<td>$0.6$</td>
<td>$H:$23&lt;=$I:$23$</td>
</tr>
<tr>
<td>$H:$24</td>
<td>Total</td>
<td>$0.6$</td>
<td>$H:$24&lt;=$I:$24$</td>
</tr>
<tr>
<td>$H:$25</td>
<td>Total</td>
<td>$1$</td>
<td>$H:$25&lt;=$I:$25$</td>
</tr>
<tr>
<td>$H:$26</td>
<td>Total</td>
<td>$1$</td>
<td>$H:$26&lt;=$I:$26$</td>
</tr>
<tr>
<td>$D:$17</td>
<td>Values to be determined $u_1$</td>
<td>$0.104512797$</td>
<td>$D:$17&lt;=$E:$17$</td>
</tr>
<tr>
<td>$E:$17</td>
<td>Values to be determined $u_2$</td>
<td>$0$</td>
<td>$E:$17&lt;=$E:$17$</td>
</tr>
<tr>
<td>$F:$17</td>
<td>Values to be determined $u_3$</td>
<td>$0.095487203$</td>
<td>$F:$17&lt;=$E:$17$</td>
</tr>
<tr>
<td>$G:$17</td>
<td>Values to be determined $u_4$</td>
<td>$0$</td>
<td>$G:$17&lt;=$E:$17$</td>
</tr>
</tbody>
</table>
The following linear equations had to be solved in order to find the mixed strategies for the target (player 2):

\[
\begin{align*}
\text{max} & \quad w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 \\
\text{Subject to} & \quad -3w_1 - 3w_2 - 1w_3 - 1w_4 - 1w_5 - 1w_6 + 1w_7 + 1w_8 \leq 1 \\
& \quad -3w_1 - 3w_2 - 1w_3 - 1w_4 + 2w_6 + 2w_8 \leq 1 \\
& \quad -2w_1 - 2w_3 - 1w_5 - 1w_6 + 1w_7 + 1w_8 \leq 1 \\
& \quad -2w_1 - 2w_3 + 2w_6 + 2w_8 \leq 1
\end{align*}
\]

\[w_1 \geq 0, w_2 \geq 0, w_3 \geq 0, w_4, w_5 \geq 0, w_6 \geq 0, w_7 \geq 0, w_8 \geq 0\]

Results for these linear equations for Player 2 are provided below:

### Target Cell (Min)

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
<th>Original Value</th>
<th>Final Value</th>
<th>Vx</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$43</td>
<td>Value of Obective Function Total</td>
<td>8</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

### Adjustable Cells

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
<th>Original Value</th>
<th>Final Value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$36</td>
<td>Values to be determined w1</td>
<td>1</td>
<td>0</td>
<td>w1</td>
</tr>
<tr>
<td>$E$36</td>
<td>Values to be determined w2</td>
<td>1</td>
<td>0</td>
<td>w2</td>
</tr>
<tr>
<td>$F$36</td>
<td>Values to be determined w3</td>
<td>1</td>
<td>0</td>
<td>w3</td>
</tr>
<tr>
<td>$G$36</td>
<td>Values to be determined w4</td>
<td>1</td>
<td>0</td>
<td>w4</td>
</tr>
<tr>
<td>$H$36</td>
<td>Values to be determined w5</td>
<td>1</td>
<td>0</td>
<td>w5</td>
</tr>
<tr>
<td>$I$36</td>
<td>Values to be determined w6</td>
<td>1</td>
<td>0</td>
<td>w6</td>
</tr>
<tr>
<td>$J$36</td>
<td>Values to be determined w7</td>
<td>1</td>
<td>0.1</td>
<td>w7</td>
</tr>
<tr>
<td>$K$36</td>
<td>Values to be determined w8</td>
<td>1</td>
<td>0.1</td>
<td>w8</td>
</tr>
</tbody>
</table>

### Constraints

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
<th>Cell Value</th>
<th>Formula</th>
<th>Status</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$38</td>
<td>Constraints Total</td>
<td>$L$38&lt;=$M$38</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$L$39</td>
<td>Total</td>
<td>1 $L$39&lt;=$M$39</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$L$40</td>
<td>Total</td>
<td>1 $L$40&lt;=$M$40</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$L$41</td>
<td>Total</td>
<td>1 $L$41&lt;=$M$41</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$D$36</td>
<td>Values to be determined w1</td>
<td>0 $D$36=0</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$E$36</td>
<td>Values to be determined w2</td>
<td>0 $E$36=0</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$F$36</td>
<td>Values to be determined w3</td>
<td>0 $F$36=0</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$G$36</td>
<td>Values to be determined w4</td>
<td>0 $G$36=0</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$H$36</td>
<td>Values to be determined w5</td>
<td>0 $H$36=0</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$I$36</td>
<td>Values to be determined w6</td>
<td>0 $I$36=0</td>
<td>Binding</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$J$36</td>
<td>Values to be determined w7</td>
<td>0.1 $J$36=0</td>
<td>Not Binding</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$K$36</td>
<td>Values to be determined w8</td>
<td>0.1 $K$36=0</td>
<td>Not Binding</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Results showed that the value of the game is +1 for the acquirer and -1 for the target. This means that the acquirer can offer a price slightly below the average price and would likely yet be able to purchase the target company.
The mixed strategies followed by the acquirer were RRI and RRR (with equal probability of 0.5) and the target followed the mixed strategies II and SI (with the probability 0.523 and 0.477 respectively). This outcome seems plausible in everyday business practice as the acquirer would like to reduce the price they pay and the target would like to increase the price they get for selling their company. This result seems a little counter-intuitive as initially we felt that the pay-off structure would support the target. However, the acquirer and target would adopt strategies that provide them the best outcome, in this case these strategies relate to an outcome of (1, -1). If either the acquirer or the target follow any other strategy then it is unlikely that they will do as well as following this strategy to obtain the (1, -1) equilibrium point.

3.3 Two-person Merger & Acquisition model with incomplete information

If we now consider a two-person merger and acquisition game with incomplete information with the conditions that both target and acquirer, do not know the type of the other party (i.e. probability with which the acquirer will choose “Increase Bid” or “Reduce Bid” or the target will choose “Increase Bid” or “Stable Bid”). In everyday business practice, acquirer and target companies do not disclose their expected offer and bid prices because they do not want the opponent to know how much they are ready to pay or how much they want before they will accept the offer. This can occur due to many reasons; one of them is that the target does not want to provide a low price because it may be able to obtain a higher price if the acquirer is ready to pay more. Similarly, the acquirer does not want to disclose his expected offer price because he does not want to pay too much for purchasing the target company. Both the acquirer and target company will try to find out approximately what the expected offer and bid price their opponent has in mind and then they will try to provide a price that suits them (i.e. acquirer will start off by providing a lower offer price) and then they will try to negotiate their way through to an equilibrium price.

To understand this game further we develop the strategic form of this incomplete information game that has been developed using the strategic form game for the complete information game (initially discussed in figure 1).
Figure 3. Strategic form layout of the two-person merger & acquisition model (2-P M&A model) with incomplete information.
Figure 4. Normal form layout of the two-person merger & acquisition model (2-P M&A model) with incomplete information

<table>
<thead>
<tr>
<th>Player 1's Strategies</th>
<th>Player 2's Strategies</th>
<th>II</th>
<th>IS</th>
<th>SI</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>III**I</td>
<td>-3.00</td>
<td>-3.00</td>
<td>-2.00</td>
<td>-2.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.67</td>
<td>-1.00</td>
<td>-1.33</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.67</td>
<td>0.33</td>
<td>-1.33</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-0.33</td>
<td>-1.00</td>
<td>0.00</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-0.33</td>
<td>0.33</td>
<td>0.00</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-3.00</td>
<td>-3.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.67</td>
<td>-1.00</td>
<td>-0.67</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.67</td>
<td>0.33</td>
<td>-0.67</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-0.33</td>
<td>-1.00</td>
<td>0.67</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-0.33</td>
<td>0.33</td>
<td>0.67</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-2.00</td>
<td>-2.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.00</td>
<td>-0.33</td>
<td>-1.33</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.00</td>
<td>1.00</td>
<td>-1.33</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>0.33</td>
<td>-0.33</td>
<td>0.00</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>0.33</td>
<td>1.00</td>
<td>0.00</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.00</td>
<td>-1.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.00</td>
<td>-0.33</td>
<td>-0.67</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>-1.00</td>
<td>1.00</td>
<td>-0.67</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>0.33</td>
<td>-0.33</td>
<td>0.67</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>III**I</td>
<td>0.33</td>
<td>1.00</td>
<td>0.67</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-2.33</td>
<td>-2.00</td>
<td>-1.67</td>
<td>-1.33</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-2.33</td>
<td>-2.00</td>
<td>-0.33</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-1.00</td>
<td>-0.67</td>
<td>-1.67</td>
<td>-1.33</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-1.00</td>
<td>-0.67</td>
<td>-0.33</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-1.00</td>
<td>0.00</td>
<td>-1.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-2.33</td>
<td>-1.33</td>
<td>-1.67</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-2.33</td>
<td>-1.33</td>
<td>-0.33</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-1.00</td>
<td>0.00</td>
<td>-1.67</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-1.00</td>
<td>0.00</td>
<td>-0.33</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>RIII**I</td>
<td>-1.00</td>
<td>2.00</td>
<td>-1.00</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-1.67</td>
<td>-2.00</td>
<td>-1.00</td>
<td>-1.33</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-1.67</td>
<td>-2.00</td>
<td>0.33</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-0.33</td>
<td>-0.67</td>
<td>-1.00</td>
<td>-1.33</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-0.33</td>
<td>-0.67</td>
<td>0.33</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-1.67</td>
<td>-1.33</td>
<td>-1.00</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-1.67</td>
<td>-1.33</td>
<td>0.33</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-0.33</td>
<td>0.00</td>
<td>-1.00</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td>R**IRR</td>
<td>-0.33</td>
<td>0.00</td>
<td>0.33</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>R**RRR</td>
<td>1.00</td>
<td>2.00</td>
<td>1.00</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>
The strategic form of the two-person Merger & Acquisition (M&A) model has been shown in figure 3. This incomplete information game is an extension of the complete information game discussed previously in this chapter. In an incomplete information game, there is some information, for example, either the strategies, types or other factors that are not completely known to a player’s opponent. Information about incomplete information games was previously explained in chapter 2 of this thesis. In figure 3, we notice that information sets exist for certain strategies that a player could action and it becomes hard for the opponent to understand the node at which the game is at a certain point in time due to the existence of these information sets. Otherwise, the two-person M&A model with incomplete information has the same strategies followed by the acquirer (increase or reduce bid) and by the target (increase or stable bid), as were followed in the complete information game. Further, due to the existence of incomplete information each player has a type that is associated with the probability of that type.

In this game, we have added a probability that the seller could be optimistic (33.3% of the time) or pessimistic (66.6% of the time) and the target could be risk taking (33.3% of the time) and risk averse (66.6% of the time). These probabilities can be changed, but have been set to show that it is possible to find the stable equilibrium state even when the type (behavior) of the opponent is not known completely. As discussed earlier, this occurs in reality where players guess the type (behavior) of their opponents, which helps them assess which strategies to choose.

When we assess this incomplete information game, the value for the acquirer is +1 and for the target is -1. The acquirer (player 1) in this case will choose the strategy “R*RRI” and “R*RRR” (with the probability of 0.499 respectively) and target (player 2) will choose the strategy “II” (with the probability of 0.489) and “SI” (with the probability of 0.511). This is similar to the strategies that the acquirer and target selected in the complete information. This shows that the incomplete information does not change the optimal strategies of either the acquirer or the target. This could occur due to the fact that the acquirer’s primary motive is to reduce the price (use strategy “R*RRR”) and the target’s motive is to increase the price (use strategy “II”). However, both do not want to push the opponent too harshly to avoid the deal being broken, therefore they also use the strategies “R*RRI” and “SI”. This usually is seen to occur in reality as acquirers and target companies use optimal mixed strategies similar to what is proposed in order to obtain the best possible outcome.
3.4 Conclusion

This chapter explains that in a two-person merger and acquisition game when the acquirer and target negotiate they can often find the equilibrium (saddle) point in the game that lies at the mid-point of the acquirer’s offer price and the target’s bid price. It is also seen that the acquirer uses an optimal mixed strategy of “RRR” and “RRI”, while the target uses the optimal mixed strategy of “II” and “SI”. This seems viable in everyday business practice as acquirer’s will try to negotiate down the price, while the target will try to negotiate a price increase. This chapter further support the proposition that merger and acquisition pricing has a strong psychological basis that can be reviewed further using game theory.
CHAPTER FOUR

PSYCHOLOGICAL PRICING IN MERGERS & ACQUISITIONS USING PROSPECT THEORY

4.1 Introduction

We have developed the two-person Merger & Acquisition (M&A) model with incomplete information in chapter 3. Further, we realize that psychological pricing is a significant factor in M&A valuations. Due to a biased system of beliefs and psychological anchoring in M&A negotiation the Corporate Finance models fall short in providing suitable estimates of offer prices that will be acceptable for M&A transactions and usually under or over-value such transactions. Baker, Pan and Wurgler (2009) and Baker, Ruback and Wurgler (2004) indicate that behavioural factors can impact the pricing of targets in M&A transactions. In addition to psychological factors like fairness, optimism and risk-aversion, humans also view gains and losses on financial transactions differently.

Khaneman and Tversky (1979) have shown that humans are risk-averse and prefer deterministic gains, while they are willing to gamble on reducing their losses. This means that both the acquirer and target firms in a merger or acquisition transaction will have biased view of losses compared to gains. Also, Prospect theory explains that humans prefer positive events over negative events. It will be important to include the implications of Prospect theory in the two-person M&A model developed in the previous chapter, in order to make the results more realistic and to bring it closer to real world decision making. It will also be interesting to understand if the strategies followed by the acquirer and target as well as their pay-offs change in this game, if we include Prospect theory. This can be analysed further by applying Prospect theory (Khaneman and Tversky 1979) to the two-person Merger & Acquisition (M&A) model with incomplete information.

The objective of this chapter is to extend the two-person M&A model with incomplete information to include prospect theory and to understand if the equilibrium obtained in this model is different to that obtained in the incomplete information model developed in chapter 3 that did not include prospect theory. The reason we are including prospect theory in the two-person M&A model is due to the fact that humans do not view financial gains and losses in the same vain. Humans usually prefer certain gains and will be happy to speculate in order to reduce their
losses. Therefore, it is likely that acquirers would prefer a reduction in the offer price more than an increase and the target firm would more likely be interested in an increase in the offer price rather than a decrease as per prospect theory. Therefore, the inclusion of prospect theory in the two-person M&A model should provide a more realistic outcome that can be applied in practice. To analyse this further the next section of this chapter will extend the two-person M&A model to include Prospect theory. While, the third section will discuss the results of a simulation that explains what behaviour an acquirer (risk taking or risk averse) and target (optimistic or pessimistic) should show in such a negotiation and there pay-offs. It will also compare this outcome (strategies and pay-offs) with the outcome obtained in chapter 3 to understand if Prospect theory has had any effect? Finally, the last section of this chapter will summarise the discussion.

4.2 Two-person Merger & Acquisition model with prospect theory preferences

The two-person M&A model is an incomplete information game between the acquirer and the target, where both players need to agree to a price that will be suitable to them for the sale of the target company to the acquirer. The extensive form for this game can be seen in figure 1 that depicts a three stage game between the acquirer (buyer) and target (seller). In this game, the acquirer can choose either the “Increase Bid” or “Reduce Bid” strategy and the target can choose the “Increase Bid” or “Stable Bid” strategy. This game also has two chance nodes which are associated with probabilities related to the expected behaviour of the acquirer if it is either “risk taking” or “risk averse” and if the target is “optimistic” or “pessimistic”. These types have been defined as follows:

1. risk-averse: a person can be called risk-averse, when he is opposed to taking risks or only willing to take small risks (Macmillan dictionary 2012)

2. risk-taking: a person is seen to be risk-taking when he takes on risk to achieve a benefit (Macmillan dictionary 2012)

3. optimistic: means when a person is hopeful and confident about the future, for example, he was optimistic about the deal (Oxford dictionary 2012)

4. pessimistic: means when a person tends to see the worst aspects of things or believe the worst will happen, for example, he was pessimistic about his future prospects (Oxford dictionary 2012)
Here, probabilities are assigned based on the expected behaviour (type) that the acquirer or target may show. If these probabilities are changed then the equilibrium point in this game may change as well.

Before we move further, we should note that the pay-offs for each strategy is relative to any other strategy in this game. So, the “Increase Bid” strategy has a pay-off of +1 (for the target as the target receives a greater return from the acquirer) and -1 (for the acquirer as he will have to pay more to purchase the target firm). Further, the strategic form of the game provided in figure 1 below is the same as the strategic form game shown in figure 3 of chapter 3 (two-person M&A model with incomplete information), except that the positive outcomes for the acquirer and target are only 0.45 times of the equivalent negative outcomes. This occurs are prospect theory’s loss aversion co-efficient states that humans prefer positive events only 0.45 times as much as an equivalent negative event (Khaneman and Tversky 1979).

However, we use the concept called Prospect theory (Khaneman and Tversky 1979), which states that humans prefer positive events at least twice as much as negative events. Therefore, positive events are divided by 2.25 (loss aversion co-efficient, see Khaneman and Tversky 1992) to equate them to negative events (see Metzger and Reiger 2010 for application of prospect theory to game theory). As a result all positive events would have a pay-off equal to +0.44 instead of +1. On the other hand, the “Reduce Bid” strategy has a pay-off of +1 (for the acquirer or +0.44 after considering prospect theory) and -1 (for the target). The “Stable Bid” strategy will be associated with a pay-off equal to 0.

The extensive form (diagrammatic form of the game providing the sequences of strategies played by each player) and normal form (tabular form of the game providing the final outcomes of each strategy pair assuming strategies are played simultaneously) of the two-person M&A model are provided in figure 1 and table 1 respectively. The extensive and normal form have been developed using the Gambit software package (version 0.2007.12.04; further information can be found at http://www.gambit-project.org/doc/index.html).
Figure 1. Strategic form of the two-person merger & acquisition model (2-P M&A model) with prospect theory preferences
In the extensive form of the two-person M&A model (figure 1), the base case is when the acquirer is risk taking and the target is pessimistic. Further, the pay-offs are decreased by 50 per cent from the base case, if the acquirer is risk averse and the target is pessimistic. On the contrary, the pay-offs are increased by 50 per cent, if the acquirer is risk averse, but the target is optimistic. The pay-offs are increased 100 per cent from the base case when the acquirer is risk taking and the target is optimistic. The intent of this concept is that a risk taking acquirer will be willing to pay more compared to a risk averse acquirer. While, an optimistic target will ask for more in order to sell his company compared to a pessimistic target.

It is important to recall that we are dealing with an incomplete game, where the acquirer does not know if the target is optimistic or pessimistic and the target does not know if the acquirer is risk taking or risk averse. Due to this fact, we have information sets around similar node types. For example, if we need to calculate the pay-off for the acquirer when he uses the “IIII” strategy against the “II” strategy used by the target. We will need to add the pay-offs for each instance where the acquirer will use the “IIII” strategy multiplied by the acquirer and target’s probabilities. So, this would be (1.0) x (0.1) x (-6) + (1.0) x (0.9) x (-4.5) + (0.0) x (0.1) x (-3) + (0.0) x (0.9) x (-1.5), which would equal -4.65. Similarly, the pay-off for the target will be calculated being +2.07.

<table>
<thead>
<tr>
<th>Player 1’s Strategies</th>
<th>Player 2’s Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>IS</td>
</tr>
<tr>
<td>III</td>
<td>(-4.65,2.07)</td>
</tr>
<tr>
<td>IIIR</td>
<td>(-1.95,0.86)</td>
</tr>
<tr>
<td>IIR</td>
<td>(-4.65,2.07)</td>
</tr>
<tr>
<td>IIIRR</td>
<td>(-1.95,0.86)</td>
</tr>
<tr>
<td>RIIR</td>
<td>(-1.55,0.68)</td>
</tr>
<tr>
<td>IRRR</td>
<td>(0.39,-1.26)</td>
</tr>
<tr>
<td>RIRR</td>
<td>(-1.55,0.68)</td>
</tr>
<tr>
<td>RIR</td>
<td>(0.39,-1.26)</td>
</tr>
<tr>
<td>RII</td>
<td>(-4.25,1.89)</td>
</tr>
<tr>
<td>RIIR</td>
<td>(-1.55,0.68)</td>
</tr>
<tr>
<td>RIRR</td>
<td>(-4.25,1.89)</td>
</tr>
<tr>
<td>RIRR</td>
<td>(-1.55,0.68)</td>
</tr>
<tr>
<td>RRRR</td>
<td>(-1.26,0.39)</td>
</tr>
<tr>
<td>RRAIR</td>
<td>(0.68,-1.55)</td>
</tr>
<tr>
<td>RRIR</td>
<td>(-1.28,0.39)</td>
</tr>
<tr>
<td>RARR</td>
<td>(0.68,-1.55)</td>
</tr>
</tbody>
</table>
Nash (1951) states that players who play strategies that coincide with the Nash Equilibrium Point (NEP) will succeed against any opponent who differs from their strategy that coincides with an NEP. Therefore, both the Acquirer and Target can select from a set of strategies $S$, where a combination of strategies $s^* = (s_1^*, s_2^*) \in S$ is an NEP if it satisfies the property:

$$u_1(p^*, q^*) \geq u_1(p^+, q^-) \quad \forall p^+ \in V(S_1)$$

$$u_2(p^*, q^*) \geq u_2(p^+, q^-) \quad \forall q^- \in V(S_2)$$

Where, $u_1$ is the utility for Player 1 and $u_2$ is the utility for Player 2 for playing their strategy.

The Nash Equilibrium Point (NEP) for this game will be the strategy “RRRR” or “RRIR” played by the acquirer and the strategy “II” played by the target, which will result in a pay-off of $(0.68, -1.55)$. Neither that acquirer nor the target will choose any other strategy; otherwise they run the risk of ending up with a lower pay-off. This shows that the NEP (equilibrium state) for this game is closer to the acquirer’s offer than the target’s bid price, where the acquirer is risk averse and the target is optimistic. It would be interesting to analyse the behaviour of the acquirer and target further to understand what behaviour will maximize the pay-off for both the players.

4.3 Optimal behaviour of the Acquirer and Target

If we change the probabilities for the acquirer’s risk taking – risk averse behaviour and the target’s optimistic – pessimistic behaviour in the two-person M&A model. Then, we could possibly understand what behaviour would be best for these players to adopt in such a game? Four possible combinations can occur in this model, which are:

1. Target: Optimistic; Acquirer: Risk Taking
2. Target: Pessimistic; Acquirer: Risk Taking
3. Target: Optimistic; Acquirer: Risk Averse
4. Target: Pessimistic; Acquirer: Risk Averse

In the real world, M&A transactions are conducted in similar circumstances as depicted by the two-person M&A model – as both incomplete information and prospect theory preferences exist. Due to these factors, valuations can be higher or
lower than what Corporate Finance valuation models would recommend - some relevant examples are the Centro Properties Fund (US assets) that has been recently bought by the Blackstone group: Centro’s US assets were sold at a larger discount due to indebtedness (Smith 2011), MySpace was bought by News Corporation: for a premium at $580 million, but News Corp sold MySpace for a meagre sum of $35 million (Stelter 2011) and Bank of America purchased Merrill Lynch at a substantial discount (Moore 2009). Other more recent examples are the merger negotiation between SAB Miller and Foster’s group (Greenblat 2011), AMP and AXA Asia Pacific (Murdoch 2011) and others have psychological pricing variances.

The two-person M&A model has identified the behaviours (types) of the acquirer (risk taking – risk averse) and target (optimistic – pessimistic) that will change between players depending on the M&A transaction. It is also often hard to find the behaviours of opponents and the associate probabilities to these behaviours in order to calculate potential NEPs for such games. However, this model does provide us some significant insight on how such M&A transactions can be analysed more accurately, which will help the acquirer or target, develop and simulate strategies that it can follow against its opponent. An example of such a simulation of the behavioural probabilities of the acquirer and target is provided in Table 2.

A simulation of the two-person M&A model will help us understand if the acquirer should choose to be risk taking or risk averse and the target will be able to gauge if he should act optimistically or pessimistically. The data from the simulation of the two-person M&A model in table 2 has shown that the Nash Equilibrium Point for such a series of games lies at (2, -2), which relates to the acquirer being a risk taker and the target acting pessimistically. In an everyday scenario such a result makes sense because if the acquirer is a risk taker, he will be ready to offer more for the sale. While, on the other hand if the target is pessimistic then he will be more likely to negotiate with the acquirer on a reasonable price.
**Figure 3. Simulation results for the Two-person Merger & Acquisition model**

<table>
<thead>
<tr>
<th>Acquirer (Buyer)</th>
<th>Target (Seller)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimistic Seller</td>
</tr>
<tr>
<td>1.0</td>
<td>(1.76, 4)</td>
</tr>
<tr>
<td>0.9</td>
<td>(1.716, 3.9)</td>
</tr>
<tr>
<td>0.8</td>
<td>(1.672, 3.8)</td>
</tr>
<tr>
<td>0.7</td>
<td>(1.628, 3.7)</td>
</tr>
<tr>
<td>0.6</td>
<td>(1.584, 3.6)</td>
</tr>
<tr>
<td>0.5</td>
<td>(1.54, 3.5)</td>
</tr>
<tr>
<td>0.4</td>
<td>(1.496, 3.4)</td>
</tr>
<tr>
<td>0.3</td>
<td>(1.452, 3.3)</td>
</tr>
<tr>
<td>0.2</td>
<td>(1.408, 3.2)</td>
</tr>
<tr>
<td>0.1</td>
<td>(1.364, 3.1)</td>
</tr>
<tr>
<td>0.0</td>
<td>(1.322, 3.0)</td>
</tr>
</tbody>
</table>

Note: The values in the table represent the results of the simulation for different risk levels of the acquirer and target. The values are in a range format (minimum, maximum).
Figure 4. Acquirer’s pay-off profile for the Two-person Merger & Acquisition model

Figure 5. Target’s pay-off profile for the Two-person Merger & Acquisition model
However, if we consider the opposite situation, where the acquirer is risk averse and the target is optimistic, then it is less likely that a successful negotiation will result. As, the acquirer will be unwilling to negotiate a higher price for the merger and the target will be unlikely to accept a lower price for this transaction. Such a situation will most likely result in the merger to break down. The situation will not be much different when the acquirer is risk averse and the target is pessimistic – in this case, the M&A valuation will likely favour the acquirer compared to the target, which may result in the target discontinuing the transaction. However, if the acquirer was risk taking and the target was optimistic, there still is a risk that the target may ask for more than the acquirer is willing to offer, which may result in the merger talks being stalled.

In everyday business practice, we find all different varieties of people, some who are risk taking, while others are risk averse. Also, some are optimistic and others are pessimistic. It is possible that not only these but other behavioural traits can be used and even combined to develop more realistic M&A valuation games. The pay-offs will change based on the behavioural characteristics and strategies that are chosen to represent the M&A valuation game. It is inevitable that game theory will be used more often for M&A valuation as it helps support the psychological pricing variances that are not available through traditional Corporate Finance valuation techniques. Those techniques do provide some theoretical basis to the M&A valuation decision, however due to behavioural biases, it is not possible to obtain an understanding of what would be considered as the most optimal offer (Nash Equilibrium Point) that will provide the best result to the acquirer and the target. Further, the concept of prospect theory that has such a significant impact on pricing, cannot be considered by the traditional valuation model. Therefore, the two-person M&A model has been developed to help provide a tool that could be used to simulate M&A pricing that can provide a platform for valuing potential M&A transaction that relate to real world situations.

Finally, we notice that the strategies followed by the acquirer and target remain the same in the two-person M&A model with Prospect theory as compared to the standard two-person M&A model (without Prospect theory) that we developed in previous chapter. On the contrary, however the pay-off for the acquirer decreased to +0.68 (instead of +1.55; as a positive event is only 0.45 times as good as a negative event) compared to the pay-off for the target remaining the same as that obtained in the previous chapter being -1.55. In essence, we notice that there is marginal difference in applying Prospect theory to the two-person M&A model and the standard M&A model provides a good proxy to the model that incorporates
Prospect theory. This also means that the results provided by the standard two-person M&A model are close to decisions that people will make in the real world. As a result, in the next chapter, we can extend the standard two-person M&A model to include real options games with no requirement to include Prospect theory equivalent pay-offs. The intent in the next chapter is to remove the relative pay-off structure in the model and to use an option pricing structure to provide for realistic pricing of strategies that will be followed by the acquirer and target. By doing this we obtain a specific figure (dollar value) that the acquirer can pay to successfully purchase the target firm, which will be represented by the Nash equilibrium in that game.

4.4 Conclusion

In this chapter, prospect theory (Khaneman and Tversky 1979) is included in the two-person Merger & Acquisition (M&A) model with incomplete information to understand if prospect theory has any impact on the outcomes of this model. Prospect theory (Kahneman and Tversky 1979) states that humans react differently to financial gains and losses. In order to make the outcome of the two-person M&A model more realistic, it is important to include prospect theory in this model. Results of the two-person M&A model show that the strategies used by the acquirer are “RRRR” or “RRIR” against the target’s strategies “II”. As a result, the pay-off to the acquirer is +0.68 and the target is -1.55. When this model is used to simulate the behaviour of the acquirer and target, it is seen that it would be better for the acquirer to be risk taking and the target to be pessimistic. This would help as the acquirer would be willing to pay more, while the target will be willing to accept less for the sale of the target company.

Additionally, this chapter has also confirmed that results obtained from the standard two-person M&A model are close to those obtained from the two-person M&A model incorporating Prospect theory. As, Prospect theory provides for more realistic financial biases made by humans between gains and losses, it was important to incorporate it in the two-person M&A model to test if the equilibrium that was obtained in the previous chapter was still valid when such a bias between gains and losses is incorporated into this model. By validating this we have also opened the road for the standard two-person M&A model to be extended (without having to incorporate Prospect theory) and to include real options games, which will allow the two-person M&A model to provide a useable dollar figure rather than a relative pay-off for the equilibrium price in this game that the acquirer will pay to purchase the target firm.
CHAPTER FIVE

PSYCHOLOGICAL PRICING IN MERGERS & ACQUISITIONS USING REAL OPTIONS GAMES

5.1 Introduction

Mergers and acquisition (M&A) transactions are difficult to accurately value in practice, primarily due to the psychological factors involved in such transactions. Traditional finance models like the net present value and discount cash flow based approaches, for example, Rappaport (1998) or option pricing models, like Black and Scholes (1993) and Cox, Ross and Rubenstein (1979), do not incorporate subjective factors. Therefore, they could be considered less accurate in pricing M&A transactions.

Further, researchers have used game theory in order to evaluate M&A transactions, for example, Farrell and Shapiro (1990) and Hirshleifer and Ping (1990). However, even these papers have not been able to consider psychological factors in M&A pricing sufficiently. Other concepts applied to M&A pricing include valuing M&A synergies using real options (Kinnunen 2010) and using real options games (Yu and Xu 2011). Both these studies have been quite interesting in understanding M&A pricing. However, they lack the concept of incomplete information and behavioral types of the two players playing the game. Baker, Pan and Wurgler (2009) have shown that psychological factors do impact M&A pricing, and they state that the 52-week high stock price of the target firm can be a psychological anchor for the target’s shareholders. As a result, the acquirer may consider paying an amount greater than the 52-week high target stock price for the target’s shareholders to accept this offer.

In this chapter, we develop a two-person M&A model using real options games, primarily extending the standard two-person M&A model developed in chapter 3. However, the inclusion of real options games helps by replacing the relative pay-off structure used in the standard model with a realistic dollar value of the equilibrium price. As a result, the two-person M&A model incorporating real options games can provide a real (dollar value) equilibrium price that the acquirer can pay to purchase the target firm. Such a model makes more sense to an M&A practitioner compared to using relative values, which cannot be directly applied to real world M&A transactions. Moving forward we acknowledge the steps that will be required by a
user of the model to build the two-person M&A model incorporating real options games (these steps are an extension of the steps described in the previous chapter):

1. Develop a binomial tree (using the Cox, Ross and Rubinstein model) to analyse the bid/ask prices and strike prices (based on the acquirer and target’s type).

2. Develop a two-person incomplete information game that M&A model to analyse how the synergies are distributed between the acquirer and target based on the behavioural types of each player.

3. For each stage of this game, the acquirer will offer the high and low price provided by the binomial tree and this offer will be accepted or rejected depending on how close it is to the strike price. The Nash Equilibrium point of the game will be selected as the set of strategies followed by the acquirer and target. Otherwise, the game will move to the next level where the acquirer will increase the offer and if the offer cannot be increased any further then the game will end.

The following section will review the structure of the binomial option pricing tree and the incomplete information game that represents the two-person M&A model. It will also show how the model is used to analyse the Nash equilibrium point for the game. Finally, the last section of this chapter will conclude by summarizing the discussion.

5.2 Two-person Merger & Acquisition model with real options games

The aim of the two-person M&A model (incorporating real options games) is to find the optimal strategy that the acquirer and target should play given the incomplete information. This chapter extends the two-person M&A model by using real options games. Real options games as applied to this model through a combination of a binomial option pricing model that provides the pay-offs for an incomplete information game. In order to develop this real options game, we need to first develop the binomial option pricing framework that will be used in this game. A representation of a binomial tree is provided in figure 1 with the acquirer type of risk-averse or risk-taking and target type of optimistic-pessimistic. The spot price (spot price is defined as the current market price of the stock) in this game is $25 billion and strike prices (strike price is the price at which the stock option will have no value; i.e. a put option will have a value when the stock price is below this level
and call price will have a value when the stock price is above this level) are $25, $27.5, $30 and $37.5 billion.

In figure 1, the binomial tree is based on the fact that the spot price is $25 billion (i.e. example of a firm’s price on the stock market) and the price will increase or decrease with an underlying volatility of 10 per cent (volatility is defined as the variance in the underlying price of the stock) and a risk free rate of 5 per cent (risk free rate is defined as the interest rate that is provided by the central bank, e.g. the US federal reserve or the Reserve Bank of Australia, on overnight accounts for local banks held with them). Using this information the stock prices are calculated in figure 1 for an increase and decrease in the stock price across four nodes (levels), for example, the stock price increases from $25 billion (in node 1) to $33.75 billion (in node 4) on the top part of the binomial tree, while it reduces to $18.52 (in node 4) in the bottom part of this tree.

**Figure 1. Binomial Option Pricing (CRR) Model**
Option prices are calculated using the Cox, Ross, Rubenstein (1979) model, as this binomial tree calculates an American option (an option that can be exercised at any time prior to its expiry date) price, the price of a call is calculated as max [0, spot – strike] and for a put it is max [0, strike – spot]. Option price for each node in a European option is calculated differently, where the option price would be calculated first at the end node and then option prices would be calculated backward at each node through the process of backward induction.

The call price and put price that have been calculated in figure 1 and 2 align with the spot price in the fourth node (level) of this game. You will notice that these options can be out of the money in some cases, where their value is zero. The intent of developing this option pricing model is to find out which option will be viable and then to use these option prices as pay-offs in an incomplete game to find the Nash equilibrium of such a game. This will explain the optimal strategy for the acquirer and target in such a real options game as well as provide the specific price (i.e. provided by Nash equilibrium point) that the acquirer can pay to purchase the target firm.

Based on the binomial tree, the price of the option will either increase or decrease, which will result in the target either accepting or rejecting this offer in view of the strike price of this option. If the offer is accepted then the game will conclude. Otherwise, the acquirer will increase the offer to the next higher node in the binomial tree. The potential path of the acquirer’s bid is shown by the blue, brown and red triangles in figure 1 and by the yellow coloured boxes in figure 2. Further, however we note that, the offer will not increase if the next higher node has a greater value than the strike price for that acquirer/target type. This can be seen in figure 2, where the offer price is increased by the acquirer in the instance where the strike price is $35 billion and the maximum increase in spot price by the fourth node is $33.75 billion.

If the target rejects this offer of $33.75 billion at the fourth node, then the value of the spot should be greater than $35 billion. However, this is not viable for the acquirer – the game will terminate at the fourth node (if the acquirer is risk-averse and the target is optimistic). Nonetheless, if the acquirer and target have any other type that relate to the other strike prices of $25, $27.5 or $30 billion, then you will notice the game will terminate at the first node because the strike prices are much lower. The yellow squares in figure 2 provide the path of the offers that the
acquirer will provide in this two-person M&A model using real options games. This path is a rule that we have created in this model, however it can be changed to suit the acquirer because not all acquirers will simply increase the offer at every subsequent node. Some acquirers may reduce the offer price where they notice that the target’s financial condition has deteriorated or for any other reason the acquirer believe it will not be willing to pay the same or greater amount to purchase the target.
Figure 2. Offer Path in the Binomial Option Pricing (CRR) Model

Based on the binomial option pricing tree that has been developed, we can associate them to the two strategies that can be played by the acquirer and target. These strategies are identified in figure 3.

**Figure 3. Player Strategies and Option Pricing**

<table>
<thead>
<tr>
<th>Acquirer’s Strategies</th>
<th>Target's Strategies</th>
<th>Option Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1 = Increase Bid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a2 = Decrease Bid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b1 = Accept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2 = Reject (if Reject, then moves to next Game - Acquirer Increases Bid)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Target** - Holds (Short Put) a Call Option - As he will gain from the price increase (Obtain a higher price for the sale)

**Acquirer** - Holds (Short Call) a Put Option - As he will gain from the price decrease (Pay less to buy the Target)

**Strike/Exercise Price** - Amount the Acquirer is expected to Offer & Price the Target will likely Accept - based on each Type

---

**Cox, Ross & Rubinstein (CRR) Model**

<table>
<thead>
<tr>
<th>Firm Value</th>
<th>$25 Billion</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.05</td>
</tr>
<tr>
<td>sigma</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**X** stands for - Exercise Price

**Call Price**

<table>
<thead>
<tr>
<th>Target</th>
<th>Acquirer</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.75</td>
<td>1.25</td>
</tr>
<tr>
<td>30.54</td>
<td>7.37</td>
</tr>
<tr>
<td>27.63</td>
<td>12.38</td>
</tr>
<tr>
<td>25.00</td>
<td>16.48</td>
</tr>
</tbody>
</table>

**Put Price**

<table>
<thead>
<tr>
<th>Target</th>
<th>Acquirer</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.62</td>
<td>20.47</td>
</tr>
<tr>
<td>22.62</td>
<td>18.52</td>
</tr>
<tr>
<td>22.62</td>
<td>16.48</td>
</tr>
<tr>
<td>22.62</td>
<td>14.58</td>
</tr>
</tbody>
</table>

---

- Risk Taking (t11)
- Optimistic (t21)
- Risk Averse (t12)
- Pessimistic (t22)
These strategies are associated with the call and put option prices that are provided in figures 1 and 2. In order to find the pay-offs for these strategies for the acquirer and target, we first need to state the probabilities associated with these strategies in this incomplete information game - these probabilities were taken as an example and can be modified based on the probabilities related to each player's type, shown in figure 4.

**Figure 4. Probabilities based on player type**

<table>
<thead>
<tr>
<th>Probability</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(t_{11},t_{21})$</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>$P(t_{12},t_{21})$</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$P_{1}(t_{21}</td>
<td>t_{11})$</td>
<td>0.80</td>
</tr>
<tr>
<td>$P_{1}(t_{22}</td>
<td>t_{11})$</td>
<td>0.40</td>
</tr>
<tr>
<td>$P_{1}(t_{21}</td>
<td>t_{12})$</td>
<td>0.67</td>
</tr>
<tr>
<td>$P_{1}(t_{22}</td>
<td>t_{12})$</td>
<td>0.25</td>
</tr>
<tr>
<td>$P_{2}(t_{11}</td>
<td>t_{21})$</td>
<td>0.67</td>
</tr>
<tr>
<td>$P_{2}(t_{12}</td>
<td>t_{21})$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Once we consider the probabilities associated to the type and strategy of each player, we are able to calculate the pay-offs that each player will receive for playing these strategies. These pay-offs are provided in figure 5.

In figure 5, we can see that the optimal strategy that the acquirer can play will be a2a2 and the target will play the strategy b1b1. This will result in the acquirer gaining more than the target in this game. The reason that the acquirer will decrease is that the acquirer does not want to pay any more than required to purchase the target and the target may accept the offer because there is a one in four chance that the acquirer is risk-taking while the target is optimistic. The payoff obtained by the acquirer is +$7.3791 billion and that payoff for the target is zero. The price that the acquirer will offer based on the Nash equilibrium will be $22.62 billion. This happens as the acquirer will reduce the offer price and the target will accept, not knowing if the price will increase or decrease in the future.
Figure 5. Pay-offs for each player for playing specific strategies

<table>
<thead>
<tr>
<th>Acquirer's Pay-off Profile</th>
<th>Target's Pay-off Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1 b2</td>
<td>b1 b2</td>
</tr>
<tr>
<td>a1 7.37 Type = (t11,t21)</td>
<td>a1 2.63 Type = (t11,t21)</td>
</tr>
<tr>
<td>a2 12.38</td>
<td>a2</td>
</tr>
<tr>
<td>b1 b2</td>
<td>b1 b2</td>
</tr>
<tr>
<td>a1 2.37 Type = (t11,t22)</td>
<td>a1 2.63 Type = (t11,t22)</td>
</tr>
<tr>
<td>a2 7.38</td>
<td>a2</td>
</tr>
<tr>
<td>b1 b2</td>
<td>b1 b2</td>
</tr>
<tr>
<td>a1 4.88</td>
<td>a1 2.63 Type = (t12,t21)</td>
</tr>
<tr>
<td>a2</td>
<td>a2</td>
</tr>
</tbody>
</table>

Nash Equilibrium - Acquirer/Target

<table>
<thead>
<tr>
<th>Acquirer's Optimal Strategy</th>
<th>Target's Optimal Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1b1 b1b2 b2b1 b2b2</td>
<td>b1b1 b1b2 b2b1 b2b2</td>
</tr>
<tr>
<td>a1a1 3.1854 3.1854</td>
<td>a1a1 2.6293 1.3146 1.3146</td>
</tr>
<tr>
<td>a1a2 4.8749 3.1854 1.6895</td>
<td>a1a2 1.3146 1.3146 -</td>
</tr>
<tr>
<td>a2a1 5.6895 5.6895</td>
<td>a2a1 1.3146 -</td>
</tr>
<tr>
<td>a2a2 7.3791 5.6895 1.6895</td>
<td>a2a2 - - -</td>
</tr>
</tbody>
</table>

Nash Equilibrium - Acquirer/Target

<table>
<thead>
<tr>
<th>Acquirer</th>
<th>Target</th>
<th>Acquirer</th>
<th>Target</th>
<th>Acquirer</th>
<th>Target</th>
<th>Acquirer</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1b1</td>
<td>b1b2</td>
<td>b2b1</td>
<td>b2b2</td>
<td>b1b1</td>
<td>b1b2</td>
<td>b2b1</td>
<td>b2b2</td>
</tr>
<tr>
<td>a1a1</td>
<td>3.1854</td>
<td>2.6293</td>
<td>3.1854</td>
<td>1.3146</td>
<td>-</td>
<td>1.3146</td>
<td>-</td>
</tr>
<tr>
<td>a1a2</td>
<td>4.8749</td>
<td>1.3146</td>
<td>3.1854</td>
<td>1.3146</td>
<td>1.6895</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a2a1</td>
<td>5.6895</td>
<td>1.3146</td>
<td>5.6895</td>
<td>-</td>
<td>-</td>
<td>1.3146</td>
<td>-</td>
</tr>
<tr>
<td>a2a2</td>
<td>7.3791</td>
<td>-</td>
<td>5.6895</td>
<td>1.6895</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5.3 Conclusion

In conclusion, the intention of this chapter was to provide the two-person M&A model as a potential framework that could be used to incorporate psychological pricing in M&A transactions using real options games. In most cases, M&A transactions have a lot of uncertainty surrounding the pricing of the target and it becomes perplexing for the acquirer if the offer should be increased and for the target as to what offer would be sufficient without rejecting any potential M&A transaction. This chapter intends to capture the incomplete information and explain how the acquirer and target can understand what optimal strategy they can play in such a game.
As a result, this chapter answers these key questions, where the option pricing model explains the highest offer price that an acquirer would offer in based on the different types of the acquirer and target. This should explain to the acquirer and target, as to the boundaries of the potential sale price that may eventuate. However, more interestingly, it is the type of the acquirer and target that really determines the outcome. In an incomplete information game, we do not know the type of the opponent. However, based on the probability of each type we can make some assessment on the likelihood of the strategies and offers that the opponent will provide, which should provide us some understanding of the strategies that we should follow to obtain the best outcome. For example, if the acquirer has a higher probability of being risk-taking, it makes sense for the target to act optimistically and demand a higher price. On the contrary, if the acquirer finds the target to be pessimistic, then he should act as if he is risk-averse by pushing down the price.

Therefore, we realise that each player’s type can be a strong indicator on what strategies are followed and as a result the final outcome of the M&A transaction. It is often seen that acquirers pay more than they should have paid to take over the target. So, it may useful to understand that it is not about how much more you can pay to take over a target firm, but to understand the dynamics of the game and the behaviour of the players in the game. This will help both the acquirer and the target optimise their outcome for an M&A transaction.

Finally, it is also important to state that the acquirer will gain +$7.3791 billion and the target will gain zero value in this game based on the Nash equilibrium point in this game. Therefore, based on the strategies that are played by the acquirer being a2a2 and the target being b1b1, we notice that the price that the acquirer can offer the target would be $22.62 billion that should be accepted by the target. The reason that this offer will be accepted is that the target does not know the type of the acquirer and he may feel that it could be possible that the acquirer may reduce the offer further. Moreover, these results are in line with these results stated in chapters 3 and 4. In previous chapters, as in this chapter, the acquirer would attempt to reduce the offer price in order to purchase the target at a lower price. Though, in the model represented in chapters 3 and 4, the target could opt to either increase offer or keep the offer stable (at the same level). In that case, the target preferred to increase offer, as it wanted to increase the price it would obtain for the sale. In contrast, the model in this chapter only allows the target to accept or reject the offer and the target’s optimal strategy is to accept this offer because it does not know the type of the acquirer and if future offers may be lower than the existing offer.
CHAPTER SIX

PSYCHOLOGICAL PRICING IN MERGERS & ACQUISITIONS USING REAL OPTIONS SIGNALLING GAMES

6.1 Introduction

In the previous chapters we have developed the two-person Mergers and Acquisition (M&A) model to include incomplete information and real options games. We did this as we realized that existing finance models like net present value and discount cash flow models, for example, Rappaport (1998) and option pricing models using real options pricing to value mergers and acquisition transactions, for example, Black and Scholes (1993) and Cox, Ross and Rubenstein (1979), are not sufficient in valuing M&A transactions. This occurs are psychological factors like risk-aversion and optimism impact the valuation of such transactions. As a result, chapter 5 discussed the concept of the two-person M&A model using real options games. The issue with real options games however is that the actions of each opponent can only be judged with the probability of each action occurring. However, when we added signaling to this type of game, it helps us get a better indication of which action is likely to occur and we do not simply depend on the probability of each event.

In this chapter, we will extend that two-person M&A model with real options games (that we developed in the previous chapter) to include signaling in games. Some examples of signaling behavior are for example when the target firm repurchases (share buyback) from the market or if it has an announcement stating that it will have above expected profit in the coming 6 months. Such signals make the intentions of the target clear. Similarly, the acquirer can provide signals by slowly purchasing more shares of the target firm or by purchasing one of the suppliers that supply products to the target firm. Such actions by the acquirer provide a signal of the intentions of the acquirer of taking over the target. It is important to understand that signals help indicate the direction that the acquirer or target will likely take going forward in the incomplete information game and provide insight into the likely action.

In the previous chapter, we discussed that researchers have used game theory to evaluate M&A transactions, for example, Farrell and Shapiro (1990) and Hirshleifer and Ping (1990). We also reviewed the real options have been used for M&A pricing (Kinnunen 2010) as well as real options games (Yu and Xu 2011). Though, it was identified that previous research chapters have not used real options games to analyse incomplete
information and behavioral types of players in an M&A transaction. However, it has been seen that psychological pricing does exist in M&A transaction. With some empirical evidence provided by Baker, Pan and Wurgler (2009), as they show that psychological factors do impact M&A pricing, and they state that the 52-week high stock price of the target firm can be a psychological anchor for the target’s shareholders. As a result, the acquirer may consider paying an amount greater than the 52-week high target stock price for the target’s shareholders to accept this offer.

In this chapter, we extend the two-person M&A model using real options games to include signalling behaviours undertaken by the acquirer and target, as a result, extending this model to be called the two-person M&A model using real options signalling games. This model has the same conditions as the model provided in chapter 5, except when players exercise an option they effectively provide a signal to their opponent of their intentions. The reason that the two-person M&A model with real options signalling game will be developed to understand the price the acquirer should offer at each step of the game to purchase the target. It is critical to explain how signalling will occur in this game – in practice, acquirers and targets can signal by making moves that can either show their intention (for example, the acquirer buys a substantial amount of shares of the target firm) or intend to deceive the opponent by making them feel that they will take an action when they actual will do something different (for example, the target will get another company to purchase a major stake in the target to stop the acquirer from taking over the target. These signals will impact the underlying stock price that will eventually change the price of the options (represented by the binomial options pricing tree). Also, signalling can occur through the acceptance/reject of the target to the potential offers provided by the acquirer (for example, the target will reject an offer if it feels that it is worth more than the acquirer is offering) and the acquirer’s decision to increase/decrease the offer price (for example, a decrease in the offer price will indicate that the acquirer feels the target will be worth less in the future).

Therefore, the approach to analyse this real options signalling game can be analysed by undertaking the following steps these are similar to the steps listed in the previous chapter except it including signalling behaviour (at each step the signal will indicate to the acquirer if he should increase or decrease the offer):

1. Developing a binomial tree (using the Cox, Ross and Rubinstein model) to analyse the bid/ask prices and strike prices (based on the acquirer and target’s type).
2. Developing a two-person incomplete information game that M&A model to analyse how the synergies are distributed between the acquirer and target based on the behavioural types of each player.

3. For each stage of this game, the acquirer will offer the high and low price provided by the binomial tree and this offer will be accepted or rejected depending on how close it is to the strike price.

4. Based on the exercise of the option that is held by the acquirer (option to buy) and target (option to sell), the opponent will get an idea of the intention of their opponent. The opponent can then take the next action to protect its interest in the game. In an incomplete information game, the provision of a signal of a player’s actions makes it much easier for the opponent to improve his pay-off due to the increasing certainty of the first player’s action (i.e. exercise of the option).

The following section will explain how this model in this chapter is different from the model developed in chapter five.

6.2 Two-person Merger & Acquisition model with real options signaling games

The aim of the two-person M&A model using real options games is to find the optimal strategy that the acquirer and target should play given the incomplete information. However, when we add the signalling behaviour in this model (signalling behaviour in this thesis is defined as any action that can be seen by the opponent that the acquirer or target would take to achieve their target price), it changes the model, as the acquirer can change the offer once they see the target exercising the option. For example, if the acquirer sees the target announcing a higher profit margin or looking for a white knight (getting another company to protect itself from being forcefully being taken over by the acquirer – this is done by the other company purchasing a large portion of the target’s outstanding shares, which does not leave enough shares on the market for the potential acquirer to take over the target) – this signals to the acquirer that it needs to act quickly to purchase the target firm quickly. Similarly, the target firm may see the acquirer purchasing the shares of the target in the secondary market - this provides a signal to the target that it needs to protect itself from a potential takeover.

At every node, the acquirer can either increase or decrease the offer price depending on the signal that he receives from the target. In the model discussed in the previous chapter the acquirer would only increase the offer price (until the exercise price of the option is not reached), if the previous offer was rejected by the target. This is not the case in this model which includes signalling as the acquirer and target get new information from their
opponent. As a result, the offer price can be increased or decreased based on the option exercise. Also, what is important is the perception of the option exercise. For example, the target purchases its own shares from the secondary market, showing that it is optimistic about its future conditions. However, the target’s management may be pessimistic about the future outcomes and the target may be just taking this action in order to send a wrong signal to the acquirer. So, that the acquirer believes this signal and increases its offer for the target.

Moving forward with discuss how we develop this game – in order to develop this real options game, we need to first develop the binomial option pricing framework that will be used in this game. A representation of a binomial tree is provided in figure 1 with the acquirer type of risk-averse or risk-taking and target type of optimistic-pessimistic. The spot price in this game is $25 billion and strike prices are $25, $27.5, $30 and $37.5 billion.

You will also notice a column stating the call and put option prices in figure 1. As this binomial tree calculates an American option, the price of a call is calculated as max [0, spot – strike] and for a put it is max [0, strike – spot]. The call and put option prices that have been calculated in figure 1 and 2 align with the spot price in round four of this game. You will notice that these options can be out of the money in some cases, where their value is zero. The intent of developing this option pricing model is to find out which option will be viable and then to use these option prices as pay-offs in an incomplete game to find the Nash Equilibrium of such a game. This will explain the optimal strategy for the acquirer and target in such a real options signalling game.
You will notice that figure 1 (above) is the same as figure 1 in the previous chapter. The reason that this figure has been provided here is that it is the base case with which we build figure 2 (provided below). Based on the binomial tree, the price of the option will either increase or decrease, which will result in the target either accepting or rejecting this offer in view of the strike price of this option. If the offer is accepted then the game will conclude. Otherwise, the acquirer will increase or decrease based on the signal that is received from the target - offer will not increase if the next higher node has a higher value than the strike price for that acquirer/target type.

Where the offer price is increased by the acquirer in the instance where the strike price is $35 billion and the maximum increase in spot price by the fourth round is $33.75 billion. If the target rejects this offer of $33.75 billion in the fourth round, then the value of the spot should be higher than $35 billion. As, this is not viable for the acquirer the game will terminate in round four if the acquirer is risk-averse and the target is optimistic. However,
if the acquirer and target have any other type that relate to the other strike prices of $25, $27.5 or $30 billion, then you will notice the game will terminate in round one. In addition however the offer can also be reduced in this game, if the acquirer believes from the signal provided by the target that target may be pessimistic and may be willing to accept a lower offer. This world mean that offer prices could be anywhere below the exercise price as listed in figure 1 for each option and the path of the offer price will be defined at each step in the game as additional information becomes available with the exercise of options by the acquirer or target.

We realise that signals in a game make it easier for the opponent to find out the type of the player, if these signals are picked up correctly. So, let’s look at an instance where the acquirer is risk taking and the target is optimistic (results are the same if the acquirer is risk averse and the target is pessimistic). Provided below are the probabilities of each player type and the pay-offs for the game.

**Figure 2. Player types and probabilities: Risk-taking Acquirer and Optimistic Target**

| P(t11,t21) | 0.9 | P(t11,t22) | 0.2 |
| P(t12,t21) | 0.1 | P(t12,t22) | 0.2 |
| P1(t11|t11) | 0.82 | P1(t12|t11) | 0.18 |
| P1(t21|t12) | 0.33 | P1(t22|t12) | 0.67 |
| P2(t11|t21) | 0.90 | P2(t21|t21) | 0.10 |
| P2(t11|t22) | 0.50 | P2(t21|t22) | 0.50 |

Based on these probabilities we notice that the probability is highest when the acquirer is risk taking and the target is optimistic. Figure 3 provides the pay-off for this game and the Nash equilibrium is at (0.2500, -0.2500). Also, it is noticed that the acquirer will choose the strategy **a2a2**, which means that the acquirer will try to reduce the offer.
Figure 3. Player pay-offs: Risk-taking Acquirer and Optimistic Target

### Acquirer's Pay-off Profile

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>4.46</td>
<td>10.00</td>
</tr>
<tr>
<td>b2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Type = (t11,t21)**

### Target's Pay-off Profile

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>b2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Type = (t11,t22)**

### Acquirer's Pay-off Profile

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>b2</td>
<td>2.50</td>
<td>-</td>
</tr>
</tbody>
</table>

**Type = (t12,t21)**

### Target's Pay-off Profile

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>b2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Type = (t12,t22)**

### Acquirer's Optimal Strategy

#### Legend:
- **a1** = Increase Bid
- **a2** = Decrease Bid
- **b1** = Accept
- **b2** = Reject (if Reject, then moves to next Game - Acquirer Increases Bid)

### Target's Optimal Strategy

#### Target's Strategies
- Holds (Short Put) a Call Option - As he will gain from the price increase (Obtain a higher price for the sale)
- Holds (Short Call) a Put Option - As he will gain from the price decrease (Pay less to buy the Target)

#### Strike/Exercise Price
- Amount the Acquirer is expected to Offer & Price the Target will likely Accept - based on each Type

### Nash Equilibrium - Acquirer/Target

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>b2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Option Type**

- **Target** - Holds (Short Put) a Call Option - As he will gain from the price increase (Obtain a higher price for the sale)
- **Acquirer** - Holds (Short Call) a Put Option - As he will gain from the price decrease (Pay less to buy the Target)
It is in the acquirer’s best interest to reduce the offer, so that it can purchase the target for a lower price. Obviously, an optimistic target will reject this offer because the target is confident that he will be able to achieve a higher offer to sell his company. As, is the result shown in figure 3, with the target choosing the strategies \( b_2b_1 \), which means that he will reject the offer in the instance when the target is optimistic and will accept the offer when the target’s type is pessimistic. This result seems logical as an optimistic target will not agree to a reduced offer, while a pessimistic target will accept a lower offer worrying that he may not be able to get a better offer elsewhere and that there could be a possibility that the acquirer may reduce the offer again.

Let’s compare this result to the condition where the player types are opposing each other, for example, the acquirer is risk taking but the target is pessimistic or the acquirer is risk averse and the target is optimistic. You will notice significant difference between these results with the player play-offs shows in figure 5 below. To start off we see that the probability of the acquirer being risk taking and the target being pessimistic is the highest in the probabilities shown for each player type in figure 4 below.

**Figure 4. Player types and probabilities: Risk-taking Acquirer and Pessimistic Target**

<table>
<thead>
<tr>
<th></th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(t_{11}, t_{21}) )</td>
<td>0.1</td>
</tr>
<tr>
<td>( P(t_{12}, t_{21}) )</td>
<td>0.9</td>
</tr>
<tr>
<td>( P(t_{11}, t_{22}) )</td>
<td>0.3</td>
</tr>
<tr>
<td>( P(t_{12}, t_{22}) )</td>
<td>0.4</td>
</tr>
<tr>
<td>( P_1(t_{21}</td>
<td>t_{11}) )</td>
</tr>
<tr>
<td>( P_1(t_{22}</td>
<td>t_{11}) )</td>
</tr>
<tr>
<td>( P_1(t_{21}</td>
<td>t_{12}) )</td>
</tr>
<tr>
<td>( P_1(t_{22}</td>
<td>t_{12}) )</td>
</tr>
<tr>
<td>( P_2(t_{11}</td>
<td>t_{21}) )</td>
</tr>
<tr>
<td>( P_2(t_{12}</td>
<td>t_{21}) )</td>
</tr>
<tr>
<td>( P_2(t_{11}</td>
<td>t_{22}) )</td>
</tr>
<tr>
<td>( P_2(t_{12}</td>
<td>t_{22}) )</td>
</tr>
</tbody>
</table>

Further, the pay-offs for this game is provided in figure 5 below. This pay-off matrix shows that the Nash equilibrium for this game lies at \((2.500, -2.500)\). You will notice that the acquirer has managed to reduce the offer by a greater amount in this game compared to the previous game (see pay-off matrix in figure 3). In this instance, the acquirer has reduced the offer by $2.5 billion rather than $0.25 billion as in figure 3. Therefore, the acquirer will keep its strategy as \( a_2a_2 \) and the target adopts the strategy \( b_2b_1 \). This occurs due to a simple reason, that the acquirer can take advantage of the target if he finds out that the target is pessimistic when the acquirer is risk taking. In which case, the acquirer will aggressively reduce the offer and this will send a signal to the target that the acquirer is risk taking and if the target happens to be pessimistic, then it will accept the offer.
Figure 5. Player pay-offs: Risk-taking Acquirer and Pessimistic Target

<table>
<thead>
<tr>
<th>Acquirer's Pay-off Profile</th>
<th>Target's Pay-off Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b1</strong></td>
<td><strong>b2</strong></td>
</tr>
<tr>
<td>a1</td>
<td>4.46</td>
</tr>
<tr>
<td>a2</td>
<td>10.00</td>
</tr>
</tbody>
</table>

| **b1** | **b2** | **b1** | **b2** |
|---------------------------|--------------------------|
| **b1** | **b2** | **b1** | **b2** |
| a1 | - | - | Type = (t11,t22) | a1 | - | - | Type = (t11,t22) |
| a2 | 5.00 | - | | a2 | 5.00 | - |

| **b1** | **b2** | **b1** | **b2** |
|---------------------------|--------------------------|
| **b1** | **b2** | **b1** | **b2** |
| a1 | - | - | Type = (t12,t21) | a1 | - | - | Type = (t12,t21) |
| a2 | 2.50 | - | | a2 | 2.50 | - |

| **b1** | **b2** | **b1** | **b2** |
|---------------------------|--------------------------|
| **b1** | **b2** | **b1** | **b2** |
| a1 | - | - | Type = (t12,t22) | a1 | - | - | Type = (t12,t22) |
| a2 | - | - | | a2 | - |

<table>
<thead>
<tr>
<th>Acquirer's Optimal Strategy</th>
<th>Target's Optimal Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b1</strong></td>
<td><strong>b1</strong></td>
</tr>
<tr>
<td>a1a1</td>
<td>0.4465</td>
</tr>
<tr>
<td>a1a2</td>
<td>2.6965</td>
</tr>
<tr>
<td>a2a1</td>
<td>2.5000</td>
</tr>
<tr>
<td>a2a2</td>
<td>4.7500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nash Equilibrium - Acquirer/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b1b1</strong></td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td><strong>b1b1</strong></td>
</tr>
<tr>
<td>a1a1</td>
</tr>
<tr>
<td>a1a2</td>
</tr>
<tr>
<td>a2a1</td>
</tr>
<tr>
<td>a2a2</td>
</tr>
</tbody>
</table>

Legend:

**Acquirer’s Strategies**
- a1 = Increase Bid
- a2 = Decrease Bid

**Target’s Strategies**
- b1 = Accept
- b2 = Reject (if Reject, then moves to next Game - Acquirer Increases Bid)

**Option Type**
- Target - Holds (Short Put) a Call Option - As he will gain from the price increase (Obtain a higher price for the sale)
- Acquirer - Holds (Short Call) a Put Option - As he will gain from the price decrease (Pay less to buy the Target)
- Strike/Exercise Price - Amount the Acquirer is expected to Offer & Price the Target will likely Accept - based on each Type
However, if the target happens to be optimistic, then the target will reject the offer as the target believes that he can obtain a higher offer. As the acquirer is risk taking, he will not increase the offer and will act aggressively, in which case the deal will not go any further. Results are the same if the acquirer is risk averse and the target is optimistic.

In this chapter, we have discussed two instances, initially that the players have similar types (acquirer is risk taking and target is optimistic) and opposing types (acquirer is risk taking and target is pessimistic). Also, we have noticed that it is important for the acquirer and target to correctly guess the type of their opponent because if they misjudge their opponent's type then the deal can either not go any further or the target may end up accepting a lower offer than he could have obtained otherwise.

This chapter has extended the learning in the previous chapter by showing that signalling can have an impact on the outcome of the M&A transaction, as it provides clues for the player to find out the type of their opponent. While, this chapter tried to explain the signalling concept using two instances, however it is harder to provide this explanation as the solution for this game is more dynamic as players will learn more about their opponent through these signals as the game proceeds. As a result, the probabilities of the player types will keep changing till the end of the game.

6.3 Conclusion

In conclusion, the intention of this chapter was to provide the two-person M&A model as a potential framework that could be used to incorporate psychological pricing in M&A transactions using real options signalling games. In most cases, M&A transactions have a lot of uncertainty surrounding the pricing of the target and it becomes perplexing for the acquirer if the offer should be increased and for the target as to what offer would be sufficient without rejecting any potential M&A transaction. This chapter intends to capture the incomplete information and explain how the acquirer and target can understand what optimal strategy they can play in such a game.

As a result, this chapter answers these key questions, where the option pricing model explains the highest offer price that an acquirer would offer in based on the different types of the acquirer and target. This should explain to the acquirer and target, as to the boundaries of the potential sale price that may eventuate. However, more interestingly, it is the type of the acquirer and target that really determines the outcome. In an incomplete information game, we do not know the type of the opponent. However, based on the probability of each type we can make some assessment on the likelihood of the strategies and offers that the opponent will provide, which should provide us some understanding of
the strategies that we should follow to obtain the best outcome. For example, if the acquirer has a higher probability of being risk-taking, it makes sense for the target to act optimistically and demand a higher price. On the contrary, if the acquirer finds the target to be pessimistic, then he should act as if he is risk-averse by pushing down the price.

Therefore, we realise that behaviour of each player can be a strong indicator on what strategies are followed and as a result the final outcome of the M&A transaction. It is often seen that acquirers pay more than they should have paid to take over the target. So, it may useful to understand that it is not about how much more you can pay to take over a target firm, but to understand the dynamics of the game and the behaviour of the players in the game. This will help both the acquirer and the target optimise their outcome for an M&A transaction.

This chapter intends to explain that the signals that they receive from their opponent helps them change their decision as it provides more information in the game. Therefore, the use of real options signalling games will assist improve decision making by the acquirer and target to optimise their pay-offs in the M&A transaction. It is seen that acquirers could either increase or decrease their offer in the game with the offer always staying below the exercise price of each option. Though, additional information becomes available through the exercise of options by the acquirer and target throughout the game. This leads to the change in offer price and the offer price will depend on the signals that are communicated as well as the correct interpretation of these signals by the opponent.
7.1 Summary

Merger and acquisition pricing can be complicated due to the impact of psychological pricing factors that cannot be incorporated into traditional financial models. This thesis develops a two-person Merger & Acquisition model that incorporates: incomplete information games, prospect theory and discrete-time American option pricing. This thesis has been structured in the following manner:

1. Chapter one starts by providing an introduction to this thesis and sets the scene for the remain part of the this thesis,

2. Chapter two reviews the literature in the research areas of game theory and negotiation that provides the basis for the discussion on merger and acquisition pricing in chapters 3-5,

3. Chapter three develops the two-person Merger & Acquisition model with incomplete information games, which explains how an M&A transaction can been seen as a two-person incomplete information game and it is solved to find an optimal solution,

4. Chapter four extends the model that is developed in chapter three to include Prospect theory, which helps make the results of this model to be more accurate to real world problems,

5. Chapter five develops this model by applying real options games that incorporates the Cox-Ross-Rubenstein model for discrete time American option pricing to the model with an incomplete information game, and

6. Chapter six extends the model to include real options signaling games.

7. Chapter seven (this chapter) summarizes the discussion in this thesis and its potential applications and further extensions.


7.2 Contribution to knowledge

This thesis intends to analyse how to price a merger & acquisition and it is found that psychological factors affect this pricing. Baker, Pan and Wurgler (2009) and other researchers have shown that psychological pricing can have significant impact on pricing and this is not taken into consideration when using traditional financial models like discount cash flow analysis, price earnings multiples etc. This thesis therefore contributes to knowledge in the following manner:

1. Develops a two-person Merger & Acquisition model to analyse M&A pricing under the conditions of an incomplete information game. Previously, researchers have not considered analyzing M&A pricing in such a model.

2. This two-person Merger & Acquisition model is developed further to include Prospect theory (Kahneman & Tversky 1979), which makes this model more realistic in its application as humans show behavioural biases in financial decision making. No other model exists for M&A pricing that includes Prospect theory.

3. This two-person Merger & Acquisition model is developed further to include the Cox-Ross-Rubenstein American Option discrete time option pricing model with incomplete information to understand how M&A pricing can be undertaken in practice. This is the first model that includes real options games application to M&A pricing in such a manner that helps organization price M&A deals in everyday business environment.

7.3 Limitations of this research

The following limitations exist in relation to the research undertaken in this thesis:

1. *Psychological pricing theory limitations:* The two-person Merger & Acquisition model developed in thesis is a theoretical model. It is intended to consider only psychological pricing issues related to mergers & acquisitions in relation to the chapters: Baker, Pan and Wurgler (2009) and Kahneman and Tversky (1979). Other psychological deviations could exist in merger and acquisition pricing that has not been considered.

2. *Corporate Finance limitations:* The research area of corporate finance, behavioural finance and quantitative finance is extensive. This thesis did not undertake an extensive literature review of all these fields as it would be
prohibitive to do such an extensive search. The author undertook specific research and reviewed literature in the areas of game theory, negotiation, bargaining, corporate finance, behavioural finance (related to prospect theory) and mergers & acquisitions. This provided the author and readers of this thesis a sound understanding of the research that would form the basis of this research problem.

7.4 Possible applications of this research

Merger and acquisition pricing is difficult to undertake as discussed throughout this thesis. The two-person Merger & Acquisition model developed in this thesis can be used to price potential targets for both listed and unlisted companies. It can also be used to price companies to undertake venture capital and private equity investments. This can be achieved as the stock price can be used for listed companies and the total assets/liabilities can be used for unlisted companies as a reference to price a merger or acquisition transaction. As a result, we can see that this two-person Merger & Acquisition model can have a wide range of application in pricing listed and unlisted companies.

Management implications of such a two-person M&A model is simply that acquirer and target firms should not only weigh M&A transactions based on theoretical valuation obtained from traditional financial models. In addition to these models there are subjective factors that can significantly impact the pricing of an M&A transaction, which could have a significant impact on both the valuation and success of such transactions. As a result, it is important to incorporate such subjective factors into such valuations, in order to make sure that the best possible strategy is chosen when undertaking M&A deals.

7.5 Suggested Extensions

The possible extensions of this model are as follows:

1. The two-person Merger & Acquisition model developed in this thesis can be extended to include other psychological factors that would impact M&A transactions,

2. This model can be extended to an N-person model to incorporate Portfolio Selection, in which case a market portfolio of existing M&A and non-M&A stocks that can be priced, and
3. It could be possible to extend this model to price intellectual property like patents, copyrights, trademarks and other non-tangible property in both M&A and non-M&A situations.

4. This model can be extended to differentiate between mergers (where both acquirer and target will share the new entity) and acquisitions (where acquirer takes over the target) – this is important as the management of the acquirer and target will treat these transactions differently as they fear losing control on the new entity and more importantly fear losing their jobs.

5. This model can also be extended to include different payment options for the acquisition, i.e. cash or shares. It could be possible that the type of payment may change the behaviour of the acquirer and target firm.

6. This model can also be modified to further understand how the information asymmetry affects the negotiation process between the acquirer and target firms and how these behavioural factors (and any other behavioural factors) will impact such information asymmetry in such games.

7. The decision tree in this model can also be modified where the target has the option of accepting the acquirer’s initial offer. In reality, the target could accept the acquirer’s initial offer and that would end the game. Also, the acquirer doesn’t have an option to keep the price stable after the target’s offer. A discussion on M&A regulation and legal issues related to such offers could make the settings of this game more realistic and this game could be modified to add other relevant factors that make the application of this model more realistic.
REFERENCES


