Optimisation of network systems for gas and water allocation and spot price dynamic modelling

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

Jonathan Plummer
Bachelor of Applied Science(Hons.)

School of Mathematical and Geospatial Sciences
College of Science, Engineering and Health
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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

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Summary

Figure 1 shows the contribution to the field provided in this thesis. The study of network allocation systems for gas and water is conducted with the original research (shown in red in Figure 1) placed in context as a logical extension of the existing research.

Figure 1: Thesis in the general context

This thesis studies the network of high pressure gas transmission pipelines in eastern Australia and the network of irrigation channels in the Goulburn-Murray irrigation district. The objective is to advance the study of network allocation systems. The research questions examined focus on three problems, optimising the delivery of commodities through networks, the optimisation of the cost of supply, and the modelling of prices of commodities that rely on network structures for delivery. A significant part of this research involves the collation of modelling data for the eastern Australian gas network. Three research questions will be addressed.
The first research problem is to estimate the price of irrigation water at the start of the irrigation season. Water allocated to holders of water entitlements in the Goulburn-Murray irrigation district are traded on a market exchange. This market has matured to the point where new products based on the water allocations can be introduced which is one of the objectives stated in The Intergovernmental Agreement on a National Water Initiative citation. A pricing model for European style option contracts with water allocations as the underlying commodity is available in [38], however options could only be considered after the season had commenced since the price of water exhibits large price jumps between seasons. In order to determine appropriate strike prices for the options an initial period of trading would have to take place to allow price discovery. The strike price is a crucial component of the specification of an option as the payoff is determined by the difference of the market price and the strike price. In this thesis the initial price of water allocations at the beginning of the season is estimated using the volume of water stored in Lake Eildon and the winter rainfall at the town of Jamieson upstream of the lake as predictor variables in a regression model. By providing a reliable estimate of the seasons opening water price a series of options on the price of water allocations can be introduced for trading. The benefit of allowing options on water to trade at the beginning of the season is that water uses can lock in a supply of water at an agreed price thus providing some certainty over one of the key inputs into their production.

The second research question addressed is the absence of a model for simulating the spot price of wholesale natural gas sold in the Victorian declared wholesale gas market. In this thesis a model to simulate this price is presented. With this model holders of a portfolio of energy contracts will now have the ability to ascertain the potential losses they may be exposed to in the spot market.

In the third research problem addressed in this thesis, the optimal routing of natural gas to the major demand centres in eastern Australia is addressed. Two formulations are presented with the first focussing on minimising shortfalls on days of simultaneous peak demand across all demand centres. Annual growth factors are applied to current peak day estimates and the model is run for each of the next 20 years and the size of any shortfalls are quantified. The network components that are causing the shortfalls are identified. In the second model the cost of supplying current peak day demand is minimised and the sensitivity of the solution to changes in production costs at supply nodes is discussed. The production costs are the most variable cost in the supply of natural gas. Gas resources are unique and finite and as different gas reservoirs are discovered and depleted the
production costs of different gas basins changes.

The structure of this thesis is arranged as follows. In Section two, the mathematics used in the literature regarding the networks studied is reviewed before the literature itself is discussed in Section three. The networks are discussed in detail in Section four. This section outlines the geographical reach and the physical characteristics of the network, including both the infrastructure that comprises the networks, and the commodities that are transported through them. A particular focus is given to the gas supply chain since gas reservoirs are unique and the gas in one reservoir can be quite different from another. Therefore the process of locating a gas field and processing its raw gas into the natural gas that is traded is worthy of mention as it has a wide variation in costs. In Section five mathematical models are presented and analysed to provide insight into the research problems identified previously, and in Section six the results of the analysis are discussed.

1 An introduction to networks

Networks appear in our lives in both the natural world and the environment we have constructed. A network is quite simply a group of connected points. The points are known as nodes or vertices and the connecting pathways as edges. Networks can be simple, consisting of a few nodes and edges or amazingly complex with millions of nodes and edges. In the following sections some notable types of networks are identified.

Electricity grids are networks consisting of generators, substations and transformers, which are generally represented as nodes, and high voltage transmission wires which form the edges joining the nodes [1]. They have many interesting features to study with one unusual feature being the possibility of cascading failures. One way a cascading failure might occur is if the electricity generated and the load demanded are not in balance causing the mains frequency to speed up or slow down. This instability in the system can cause infrastructure to fail which weakens the system and increases the probability other infrastructure in the system will also fail [42]. However, while network theory can be useful in understanding the structure of the network, it has not been used successfully to explain the complex nature of power grid failures, with most failures usually due to factors such as software bugs or operator error [73].
Transportation networks include railway lines, airline routes and roads. Commonly the nodes in these networks are geographic locations and the edges are the rail tracks, roads or air routes [73]. This is not always the case however and Sen et al. [95] model the Indian railway network identifying stations with nodes and a train which stops at any two stations as an edge between them, regardless of how many other stations the train may stop at on the way. The logic of this representation is that if a passenger can travel from station A to station B then it is a direct link, if the passenger needs to change trains this is not a direct link and their journey would be along two edges of the network.

River networks can be represented using graphs with each waterway being represented by an edge and the junction of two or more as a node. River networks do not have cycles and always form trees. Trees are connected, directed networks without cycles [73]. River networks are also directed networks as the water always flow in one direction. River networks are an important form of two-dimensional branching networks, furthermore they are natural examples of allometry in that the dimensions of different parts of the river network grow with respect with each other [43]. The vascular systems of plants from branching networks with similar scaling features as river systems, with resources necessary for the plant to grow distributed through a hierarchical network structure [116]. The root system of plants follow a similar structure as do many other biological systems such as the cardiovascular and respiratory networks [17].

Delivery and distribution networks include routes used by logistics companies, networks of sewerage and water pipelines, oil and gas pipeline networks, and networks of irrigation channels that form irrigation districts [73]. The postal service and logistics companies typically have large processing nodes where parcels and mail are sorted and routed to various destinations. The nodes in this type of network may be the local post office or warehouse, and the larger centralised sorting centres, the edges can represent the delivery routes. A vast amount of fresh water is used on irrigated farms and optimising irrigation networks is important as fresh water becomes a scarce commodity. Irrigation networks in their most basic form consist of reservoirs, channels and farms. The channels can be represented as edges of a graph and the reservoirs, farms, gates and other infrastructure that regulate flow, are represented as nodes. Oil and gas networks consist of drilling rigs, pipelines and refineries. The pipelines may be represented as edges and the drilling rigs and refineries as nodes. Construction costs are high and a typical problem would involve routing oil and gas through existing pipes to the refinery in an optimal manner.
2 Mathematics and physics of gas networks

The mathematics discussed in this section are used throughout the literature pertaining to gas network optimisation and price modelling. The physical properties of natural gas are discussed and the characteristics of the two main types of compressors used in gas networks presented. Compressors are often analysed in order to minimise the cost of gas transportation. There are many flow equations that have been developed to model gas flow through different types of networks and those featuring most prominently in the literature will be presented. Two types of incidence matrices commonly used to describe network topologies will be stated along with Kirchoff’s laws of flow balance. These topics are necessary for steady state analysis of gas flow networks. Analysing a gas network in its steady state is far more common in the literature than tackling transient analysis, however the basic equations that underpin transient analysis are discussed also.

Topics from stochastic calculus that have been applied to modelling financial assets are mentioned so that a spot price model for natural gas can be developed, in a similar vain the basics of optimisation theory needed for analysis of the gas transmission network are outlined. In this thesis a mathematical model for simulating the spot price of wholesale gas is developed. This model uses results from stochastic calculus which are presented in this chapter. In addition optimisation models are introduced in this thesis to analyse gas flow through the network. The models find the optimal path and optimal quantity of flow for various objective functions. The mathematics underlying these models is stated in this chapter.

2.1 Properties of natural gas

This section briefly outlines some of the fundamental properties of natural gas including temperature, pressure and density. The basic gas laws are then defined followed by the ideal gas law. Viscosity and the Reynolds number of flow are discussed as are the necessary concepts required to define equations for gas flow through pipes. This is followed by a derivation of Bernoulli’s equation and the general flow equation. Friction between the flowing gas and the pipeline is a critical factor in gas network analysis. Therefore a discussion on the friction factor needed for gas flow equations and a way to calculate this
factor are necessary. The Weymouth equation is one of several equations used to model the gas flow through high transmission gas pipelines. It provides a conservative estimate of the amount of gas able to be transported and is often chosen by researchers for this reason.

### 2.1.1 Temperature, pressure and density of a gas

The most commonly used parameters to describe the physical conditions under which natural gas is analysed in network analysis are temperature, pressure and density. Collectively this is known as the gas state.

The temperature of gas will be represented by $T$ and is measured on the Kelvin (K) temperature scale. In relation to temperature measured in degrees Celsius (C)

$$T = 273.15 + t^{\circ}C$$  \hspace{1cm} (1)

Before providing a definition for pressure we explain the idea of a continuum. A continuum is when the gas molecules are close enough to each other to be considered continuously distributed throughout a region of interest. The mean free path is the average distance a molecule travels before colliding with another molecule, if it is small in comparison to the characteristic dimension of a device, such as a gas pipeline, then the assumption of a continuum is reasonable [83].

Compressive force acting on an area results in pressure. Mathematically, an infinitesimal force $\Delta F$, acting on an infinitesimal area $\Delta A$ results in pressure $p$. Pressure is defined as:

$$p = \lim_{\Delta A \to 0} \frac{dF}{dA}$$  \hspace{1cm} (2)

Pressure is measured in pascals or kilopascals [kPa]. The pressure of the atmosphere at sea level is 101.3 kPa [83]. Atmospheric pressure is often used as a datum to measure the gauge pressure. Gauge pressure is the pressure measured above the atmospheric pressure [44].
Mass $G$ is a scalar quantity that measures the amount of matter in a substance. It is measured in kilograms (kg). The mass of a gas is constant and only changes if gas is added or removed, this is known as the principle of conservation of mass. The literature on gas network optimisation frequently refers to the mass flow rate through a pipeline.

Gas volume $V$ varies with temperature and pressure. As gas is compressible it expands to fill any area available to it. The specific volume of a gas is the volume occupied by its unit mass [96] and is represented mathematically by

$$v = \frac{V}{G}$$

The specific volume is measured in $[\text{m}^3\text{kg}^{-1}]$.

The density of a gas is the amount that can fit in a given volume. It is the inverse of the specific volume:

$$\rho = \frac{G}{V} = \frac{1}{v}$$

The specific weight $\gamma$ of a gas is its weight per unit volume:

$$\gamma = \rho \cdot g = \frac{g}{v}$$

where $g$ is the acceleration due to gravity measured in $[\text{ms}^{-2}]$. Specific weight is measured in $[\text{Nm}^{-3}]$.

### 2.1.2 Gas laws

An ideal gas is one in which the molecules are considered to be perfectly elastic and in which there is no attraction or repulsion between the gas molecules. An ideal gas obeys Boyle’s law, Charles’ law and the ideal gas equation.

Boyle’s law states that if the temperature is held constant, the volume of the gas
varies inversely with the absolute pressure:

\[
\frac{p_1}{p_2} = \frac{v_2}{v_1}
\]  

Charles’ law states that if the pressure on a gas is held constant, the gas volume is directly proportional to its temperature:

\[
\frac{v_1}{v_2} = \frac{T_1}{T_2}
\]

Also, if the volume is kept constant, then the gas pressure changes in proportion to its temperature

\[
\frac{p_1}{p_2} = \frac{T_1}{T_2}
\]

By combining Boyle’s law and Charles’ law we have

\[
\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}, \quad \text{or} \quad \frac{pv}{T} = \text{constant}
\]

The constant, known as the gas constant, is independent of the state of the gas but does depend on the properties of the gas, thus it is specific for each gas. The gas constant is denoted by \( R \) and is measured in units of \([\text{Jkg}^{-1}\text{K}^{-1}]\). Rewriting equation 9 as

\[
pv = RT
\]

we have the ideal gas equation which relates the pressure, volume and temperature of a gas. This is the equation of state for an ideal gas. The ideal gas equation approximately represents the behaviour of many gases at conditions close to normal atmospheric temperatures and pressures. It is possible to use equation 10 to calculate the following gas processes:

- the constant temperature of isothermal process;
- the constant pressure or isobaric process;
- the constant volume or isometric process;
- the zero heat transfer or adiabatic process.
The compression or expansion of an ideal gas can be described by the expression:

\[ p v^n = C \]  \hspace{1cm} (11)

where \( n \) is called the polytropic exponent and \( C \) is a constant. Such a process is called a polytropic process. Equation 11 is sometimes written in logarithmic form as

\[ \ln(p) = -n \ln(v) + c \]  \hspace{1cm} (12)

A real gas differs more from an ideal gas the more its density increases. The ideal gas equation can be modified in order to match it with the behaviour of gases other than an ideal gas. This is done by defining a factor \( Z \), called the compressibility factor, such that

\[ Z = \frac{p v}{RT} \]  \hspace{1cm} (13)

For an ideal gas \( Z = 1.0 \), and the various virial coefficients provide a series of corrections to the ideal-gas behaviour [75].

Boyle’s law and the equation of state are used in [31] to construct an equation to model the volume of gas stored in the pipes of the network during steady state flow. This volume of gas is known as the linepack.

2.1.3 Viscosity

Viscosity, \( \mu \), is the resistance to flow of a liquid. The lower the viscosity, the more easily a fluid will flow through a pipe and lower the pressure drop will be. Resistance to flow reveals itself as a shearing stress within a flowing gas and between the flowing gas and the container.

Related to viscosity is the kinematic viscosity. The kinematic viscosity is the absolute viscosity divided by the gas density:

\[ \nu = \frac{\mu}{\rho} \]  \hspace{1cm} (14)
The dimension of $\nu$ is $[m^2 s^{-1}]$. The viscosity of a gas depends on its temperature and pressure. The viscosity of a gas increases as its temperature increases. Similarly, the viscosity of a gas increases as the pressure increases.

2.1.4 The Reynolds number of flow

The Reynolds number is used to determine the type of flow in a pipe. At low velocities fluid particles move in parallel lines and retain the same relative position at successive cross-sections. This type of flow is known as viscous, streamline or laminar flow. As velocity is increased the particles no longer move in an orderly manner and cease to maintain their relative positions in successive cross-sections, this is known as turbulent flow. When the motion of a fluid particle is disturbed the force of inertia tends to carry it in the new direction while the viscous forces of the surrounding fluid tend to carry the particle in the direction of the stream. In laminar flow the viscous force is sufficient to overcome the force of inertia but in turbulent flow it is not. Therefore the criterion for determining the type of flow is the ratio of the viscous force to the inertial force [44].

$$Re = \frac{Dw\rho}{\mu}$$  \hspace{1cm} (15)

where:
$D$ is the inner diameter of the pipe
$w$ is the velocity of the gas
$\rho$ is the density of gas
$\mu$ is the viscosity.

Laminar flow occurs when the Reynolds number is below 2,000. Laminar flow is typical of low pressure gas distribution systems which are not considered in this thesis. A flow with a Reynolds number in the range $2,000 < Re \leq 4,000$ is said to be in the critical region between laminar and turbulent flow. The gas flow in high pressure transmission networks is typically in the fully turbulent region defined as a Reynolds number greater that 4,000. Fully turbulent flow can be further classified as turbulent flow in smooth pipes, turbulent flow in fully rough pipes and transition flow between smooth and fully rough pipes [96]. The flow through high pressure gas transmission pipes is considered to
be fully rough in [71]. This is a common assumption when modelling gas transmission networks.

### 2.1.5 Rate of flow

Figure 2: The mass flow rate of gas through a transverse reference plane in unit time

The mass flow rate is the mass of the gas which passes a transverse reference plane in unit time. The volume per unit time is known as the discharge rate or volume flow rate. Consider a gas flowing with a mean velocity \(w\) in a pipe with cross-sectional area \(A\) as depicted in Figure 2. In time \(t\) a gas particle will travel a distance \(x\), thus:

\[
w = \frac{x}{t}
\]

The volume flow rate \(Q\) is given by

\[
Q = \frac{Ax}{t} = Aw \quad [\text{m}^3\text{s}^{-1}]
\]

and the mass flow rate \(M\) is given by

\[
M = \frac{\rho Ax}{t} = \rho Aw \quad [\text{kgs}^{-1}]
\]
2.1.6 Continuity of flow

Figure 3: The flow of gas through a stream tube.

A stream line is a line or curve that is tangential to the velocity of flow at each instant. By definition there is no flow normal to a stream line and so a collection of stream lines are impermeable and form a tube known as a stream tube. If we consider the flow of gas in the stream tube in Figure 3 to be invariant over time, and define the velocity at sections one and two as \( w_1 \) and \( w_2 \) and the small cross-sectional areas at sections one and two as \( dA_1 \) and \( dA_2 \) we can derive the continuity equation for steady state flow. The mass flow rate is given by

\[
M = \frac{\rho A x}{t}
\]  

and therefore the masses of gas passing through sections one and two in unit time are

\[
dM_1 = \rho_1 w_1 dA_1 \quad \text{and} \quad dM_2 = \rho_2 w_2 dA_2
\]  

Furthermore, \( dM_1 = dM_2 \) as there is no gas flow out of the stream tube and the rate of flow is steady, thus we have that the mass of gas passing through any cross section of the stream tube in unit time is constant. We can sum across all the stream tubes in a pipe

\[
\int dM = \int \rho_1 w_1 dA_1 = \int \rho_2 w_2 dA_2
\]  

and assuming average velocities and constant densities across any cross-section we get

\[
M = \rho_1 w_1 A_1 = \rho_2 w_2 A_2
\]

which is the continuity equation for steady flow. For incompressible flow we have \( \rho_1 = \rho_2 \) and \( w_1 A_1 = w_2 A_2 = Q \) where \( Q \) is a constant [76].
2.1.7 Bernoulli’s equation

The total energy of gas flowing through a pipeline consists of various components. Bernoulli’s equation connects the components to form a conservation of energy equation [96]. For an ideal, frictionless, incompressible fluid Bernoulli’s equation can be written as

$$\frac{p}{\rho g} + \frac{w^2}{2g} + z = \text{constant}$$ (23)

Each component in the Bernoulli equation represents a component of the hydraulic head. The hydraulic head is the energy per unit weight of a fluid. The components are the pressure head, the velocity head and the elevation head. In equation 23 the pressure head is represented by $\frac{p}{\rho g}$, the velocity head by $\frac{w^2}{2g}$ and the head due to elevation, or potential head, by $z$.

We can derive Bernoulli’s equation with reference to Figure 4 which shows a fluid particle of cross-sectional area $A$ and length $dx$. The particle has velocity $w$ and acceleration $a$, and $p$ denotes pressure. If we assume that the particle is from an ideal frictionless fluid.
then forces acting on the particle are pressure and gravity.

The particle weight is given by

\[ W = Adx \rho g \]  

(24)

and, if we neglect the force of gravity and assume the direction of flow is positive, the pressure on the particle is given by

\[ pA - (p + dp)A = -dpA \]  

(25)

The force of gravity in the direction of motion is given by

\[ -W \sin \theta = -(\rho g Adx) \frac{dz}{dx} \]  

(26)

According to Newton’s second law the mass multiplied by the acceleration in the direction of motion is equal to the sum of the forces. Therefore we have

\[ M = \rho Adx \]  

(27)

and

\[ a = \frac{dw}{dt} = \frac{dw}{dx} \times \frac{dx}{dt} = \frac{dw}{dx} \times w \]  

(28)

Hence

\[ -dpA - (\rho g Adx) \times \frac{dz}{dx} = pAdx \times w \frac{dw}{dx} \]  

(29)

which when divided by

\[ \rho g Adx \]  

(30)

gives

\[ -\frac{1}{\rho g \frac{dx}{dx}} \frac{dp}{dx} - \frac{dz}{dx} = \frac{w dw}{g \frac{dx}{dx}} \]  

(31)

or

\[ -\frac{1}{\rho g \frac{dx}{dx}} + \frac{w dw}{g \frac{dx}{dx}} + \frac{dz}{dx} = 0 \]  

(32)

which are sometimes referred to as the Euler equations. Isothermal Euler equations are used to model the gas dynamics within pipes in [63]. For a fluid of constant density equation 32 can be integrated with respect to \( x \) to give Bernoulli’s equation [76].
2.1.8 Steady state flow

Steady state flow is a common assumption in gas network optimisation models. For example it is assumed in [60] when creating a strategic planning model for the construction of gas transmission networks.

As Bernoulli’s equation is a conservation of energy equation, for any two points along a streamline we have

\[
\frac{p_1}{\rho g} + \frac{w_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{w_2^2}{2g} + z_2
\]  

(33)

In reality energy is lost between points one and two due to various factors with friction being predominate in high pressure gas transmission networks. Equation 33 can be rewritten to account for the energy lost by adding an additional term:

\[
\frac{p_1}{\rho g} + \frac{w_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{w_2^2}{2g} + z_2 + h_f
\]  

(34)

Here \( h_f \) represents the total pressure lost due to friction between points one and two [76].

In this section the physical properties of natural gas have been outlined and the steady state flow equation stated. An understanding of the flow regime through a pipe is relevant to this work in order to gain an understanding of the common simplifying assumption most researchers employ when studying gas networks.

2.2 Compressors, regulators and valves in gas networks

Compressors, regulators and valves play an important role in gas transmission networks. As gas travels through the pipelines pressure is lost primarily due to friction, compressors restore the pressure difference necessary to keep gas flowing. When gas leaves the high pressure transmission network and enters the lower pressure pipes of the distribution network regulators are needed to reduce the gas pressure. Valves can be open or closed to allow or prevent gas flow through certain pipes in the network, they also provide
control over the rate of gas flow and to prevent gas from flowing in the wrong direction [76].

In this section the two main types of compressors will be discussed and their basic properties stated. A large part of the research literature focuses on the optimal use and placement of compressors in a gas network. The inclusion of valves and regulators is less common and their function will be covered only briefly. The significance of the compressor use is due to the fact that a portion of the gas being transported is consumed by the compressor to drive the compression process.

2.2.1 Compressors

The two types of compressors in high pressure gas transmission networks are centrifugal compressors and positive placement compressors. Centrifugal compressors are less efficient than positive displacement compressors but have the advantage of lower capital cost and lower maintenance costs. They are more commonly used in gas networks due to their operational flexibility. Centrifugal compressors create the pressure necessary for gas transport by using the centrifugal force created by spinning of the compressor wheel which translates the kinetic energy of the gas into pressure energy of the gas. Positive displacement compressors create pressure by allowing gas into a confined space and then reducing the volume of the space thus increasing the gas pressure. The gas is then released at high pressure into the pipeline. They are efficient and able to produce a wide range of pressures [96].

2.2.2 Compressors in series and parallel

Compressor stations generally configure the compressors in series or parallel depending on operation needs. A example of a series configuration is shown in Figure 5. Compressors in series raise the pressure in steps with each compressor compressing the same amount of gas. At the end of each stage of compression the gas temperature rises and it is often necessary to cool the gas before beginning the next stage. High gas temperatures reduce the volume able to flow through the pipeline [96].
Compressors are arranged in parallel to handle large volumes with each compressor handling part of the load and producing the same compression ratio [96]. Figure 5 shows an example of compressors operating in parallel. Positive displacement compressors are most often used in parallel as their efficiency falls at lower compression ratios [76].

**Figure 5: Schematic of compressors in series formation**

Centrifugal compressors compress gas to lower compression ratios than positive displacement compressors and this lends them to a series arrangement within the compressor [76].

**2.2.3 Centrifugal compressors**

Centrifugal compressors compress gas to lower compression ratios than positive displacement compressors and this lends them to a series arrangement within the compressor [76].
The steady flow work done by a centrifugal compressor is given by

\[ L_t = \int_{p_1}^{p_2} Vdp \]  

(35)

where:
- \( L_t \) = overall compressor work (J)
- \( V \) = volume (\( M^3 \))
- \( p \) = pressure (\( Pa \)).

In adiabatic compression there is no heat transfer between the gas and the surroundings and the relationship between pressure and volume is given by

\[ pV^\delta = C \]  

(36)

where
- \( \delta \) is the ration of specific heats of gas, \( \frac{C_p}{C_V} \)
- \( C_p \) is the specific heats of gas at constant pressure
- \( C_V \) is the specific heats of gas at constant volume
- \( C \) is a constant. The parameter \( \delta \) is known as the adiabatic exponent for the gas and ranges in value from 1.2 to 1.4 [96].

From equation 36 we can write

\[ pV^\delta = p_1V_1^\delta \]  

(37)

which can be rearranged to give

\[ V = V_1 \left( \frac{p_1}{p} \right)^{\frac{1}{\delta}} \]  

(38)

If we substitute equation 38 into equation 35 we have an expression for the total work of an adiabatic compressor:

\[ L_{adiabatic} = \int_{p_1}^{p_2} Vdp \]

\[ = V_1p_1^{\frac{1}{\delta}} \int_{p_1}^{p_2} p^{-\frac{1}{\delta}}dp \]

\[ = \frac{\delta}{\delta - 1} p_1V_1 \left( \frac{p_2}{p_1} \right)^{\frac{\delta - 1}{\delta}} - 1 \]  

(39)

where
$V_1$ is the initial volume
$p_1$ is the suction pressure
$p_2$ is the discharge pressure [76].

In polytropic compression there is no requirement of zero heat transfer between the gas and the surroundings. The relationship between pressure and volume in a polytropic process is given by:

$$pV^n = C \quad (40)$$

where $n$ is the polytropic exponent and $C$ is a constant [96]. By replacing the adiabatic exponent $\delta$ in equation 39 with the polytropic exponent $n$ we have an expression for the work done by a compressor during polytropic compression:

$$L_{polytropic} = \frac{n}{n-1} p_1 V_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad (41)$$

where

$V_1$ is the initial volume
$p_1$ is the suction pressure
$p_2$ is the discharge pressure [76]. A value for the polytropic exponent is needed before equation 41 is of use. The expression for polytropic efficiency, $\eta$ can be used:

$$\eta = \frac{n(\delta - 1)}{\delta(n - 1)} \quad (42)$$

The polytropic efficiency is determined by testing during the manufacturing process [76].

We can calculate the power required by the compressor using

$$N = M \cdot \Delta h \quad (43)$$

where:

$N = \text{power (kW)}$

$M = \text{mass flow (kgs}^{-1})$

$\Delta h = \text{change of enthalpy in the gas (kJkg}^{-1})[76]$. 
The polytropic efficiency can be written as

\[ \eta = \frac{\int_{p_1}^{p_2} vdp}{\Delta h} \]  

(44)

where:

\( v = \) specific volume \((m^3kg^{-1})\), thus the expression for the power required during gas compression can be written as

\[ N = M \int_{p_1}^{p_2} vdp \cdot \frac{1}{\eta} \]  

(45)

Under the assumption that compression is a polytropic process we can calculate the actual power required for gas compression as

\[ N = \frac{p_1 Q_1 n}{\eta(n - 1)} \left[ \left( \frac{p_1}{p_2} \right)^{\frac{n-1}{n}} - 1 \right] \]  

(46)

where:

\( p_1, p_2 = \) suction and discharge pressures respectively \((Pa)\)

\( Q_1 = \) the inlet flow \((m^3s^{-1})\) [76].

The overall efficiency of the compressor is function of the polytropic efficiency and the mechanical efficiency. If we represent this function as \( \phi \) we can write equation 46 as:

\[ N = \frac{p_1 Q_1 n}{\phi(n - 1)} \left[ \left( \frac{p_1}{p_2} \right)^{\frac{n-1}{n}} - 1 \right] \]  

(47)

where the power is measured in watts \(W\) [76].

Figure 7 shows the performance curve of a centrifugal compressor that can be driven at different speeds [96]. The flow rate is shown on the horizontal axis and the pressure ratio on the vertical. The dashed curves show the performance at the different operating speeds. The surge line indicated on the left of the operating envelop shows the minimum rate of flow at which the compressor output remains steady for each compression ratio. The limiting curve on the right is sometimes referred to as the stone wall limit. Increasing the flow beyond the stone wall limit does not correspond to an increase in pressure.
The performance of centrifugal compressors follow the affinity laws which state that the inlet flow and head vary as the as the speed and square of the speed of the compressor. The horsepower needed for compression changes as the cube of the speed [96]. The affinity laws can be summarised by the following equations.

\[
\frac{Q_2}{Q_1} = \left( \frac{N_2}{N_1} \right)
\]
\[
\frac{H_2}{H_1} = \left( \frac{N_2}{N_1} \right)^2
\]
\[
\frac{HP_2}{HP_1} = \left( \frac{N_2}{N_1} \right)^3
\]

where:

\( Q_1 = \) initial flow rate
\( Q_2 = \) final flow rate
\[ H_1 = \text{initial head} \]
\[ H_2 = \text{final head} \]
\[ N_1 = \text{initial compressor speed} \]
\[ N_2 = \text{final compressor speed} \]
\[ HP_1 = \text{initial horsepower} \]
\[ HP_2 = \text{final horsepower} \]

### 2.2.4 Positive displacement compressors

Positive displacement compressors increase the gas pressure by reducing the volume in which the gas is confined. They have higher efficiency than centrifugal compressors and can deliver compressed gas at a wide range of pressures [96]. However, positive displacement compressors have lower mechanical efficiencies due to the fact that they have more moving parts [76]. The two main types of positive displacement compressors are rotary screw compressors and reciprocating compressors. Rotary screw compressors use two helical shaped screw that mesh as they rotate, forcing the gas into a smaller volume while reciprocating processors use a piston to reduce the volume and increase the gas pressure. In the remainder of this section I will state the equations governing reciprocating compressor operations.

When the piston in a reciprocating compressor moves out (from point 4 to point 1 in Figure 2.2.4) gas is sucked into the cylinder though the suction manifold at constant pressure \( p_1 \). At point 1 the piston is fully extended the suction valve closes. At this point the delivery valve opens and the compressed gas is displaced by the piston at constant pressure \( p_2 \). The change in pressure as the piston moves in and out is shown by the curve from point 1 to point 2 [76].
The actual volume of gas displaced by the piston is given by

\[ V_p = \pi D_1^2 L n \]  

(51)

where:
- \( V_p \) is the piston displacement (\( m^{-3}s^{-1} \))
- \( D_1 \) is the piston diameter (\( m \))
- \( L \) is the length of stroke (\( m \))
- \( n \) is the revolutions per second (\( s^{-1} \))

In reality a clearance equal to

\[ c = 0.005L + 0.5 \text{ mm} \]  

(52)

is left so that the piston does not fill the cylinder completely at the bottom of the stroke. The clearance is left to allow for machining tolerances and thermal expansion. The clearance volume, \( V_0 \), is the total of the space between the closed valves and the piston and the extra clearance space. The ratio of the clearance volume and the piston displacement volume is

\[ C = \frac{V_0}{V_p} \]  

(53)
The actual cylinder capacity, $V_s$, is the volume of gas actually pumped. The ratio of the actual cylinder capacity to the piston displacement volume is called the volumetric efficiency and is given by

$$\lambda_V = \frac{V_s}{V_p} \tag{54}$$

To calculate the actual volumetric efficiency an allowance must be made for the clearance volume. The actual volumetric efficiency can be calculated using

$$\lambda_V = 1 - C \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n_1}} - 1 \right] \tag{55}$$

where $n_1$ is the polytropic exponent. We can calculate the flow through the compressor by using equations 54 and 55. The flow equation is

$$Q_1 = \lambda \left\{ 1 - C \left[ \left( \frac{p_2}{p_1} \right)^{\frac{1}{n_1}} - 1 \right] \right\} V_p \cdot s \tag{56}$$

where $s$ is the number of cylinders in the compressor. We can also calculate the theoretical power needed for the compression using

$$N = \frac{p_1 Q_1 n}{n - 1} \left[ \left( \frac{p_2}{p_1} \right)^{\frac{n - 1}{n}} - 1 \right] \tag{57}$$

The actual power required will be the theoretical power multiplied by a loss factor. The loss factor takes account of the pressure drop due to friction between the moving parts and other mechanical inefficiencies [76].

The procedure to construct the mainline system of a natural gas network is modelled in [91]. This paper concludes that the compression ratio directly affects the investment for compressor stations by determining the number of compressor stations required, the maintenance costs over the life of the network, and the energy consumed in their operation. The required compressor power is also considered a key factor in optimising the number of compressors. The compressor station investment is modelled as a function of the compression ratio and the compressor power. The lifetime cost of fuel consumed by compressor stations is modelled as a function of power and mechanical efficiency in [71]. This paper also notes the large effect the compression ratio has on the economics of the modelled network.
2.2.5 Valves and regulators

Regulators are used to reduce the pressure of the gas to a specified outlet pressure. They are used at intermediate delivery points on a high-pressure pipeline. Regulators are necessary at the junctions of high pressure transmission pipelines and lower pressure distribution pipelines. Valves in gas networks can be used to redirect flow from one pipeline to another and to shut off parts of a network. This is necessary for maintenance and in emergencies. Valves and regulators are often neglected in gas optimisation models. Two papers include valves and regulators are [54] and [66].

2.3 Steady-state analysis

When the flow of gas through a network does not change over time the network is said to be in steady state. A network in steady state can be described by a set of nonlinear equations

Various formulas have been derived to describe the steady state flow rate of gas in a pipe. These formulas relate the properties of the gas, such as gravity and compressibility factor, with the flow rate, the length and diameter of the pipes, and the distance and the pressure along the pipe [96].

2.3.1 General flow equation

In this section a derivation of the general flow equation

\[
Q_n = \sqrt{\frac{\pi^2 R_{air}}{64}} \times \frac{T_n}{p_n} \sqrt{\frac{\left( p_1^2 - p_2^2 \right) - \frac{2n^2_{air} S_{air} H}{ZR_{air} T}}{fS L T Z}} D^5
\]

is provided. This derivation references the more detailed derivation found in [76]. The general flow equation is derived from Bernoulli’s equation and makes the following as-
sumptions.
(1) Steady flow - for a steady flow rate the mass flow rate along the pipe is constant.
(2) The cross-sectional area of the pipe is constant.
(3) Isothermal flow - energy is lost due to friction with the pipe wall, this energy is dis-
sipated into the surroundings. The gas temperature remains constant.
(4) There is little change in kinetic energy along the pipe.
(5) There is constant compressibility of the gas along the pipe.
(6) The Darcy friction equation is valid for the pipe and the friction coefficient along the
pipe is constant.

We consider the pipe shown in Figure 9 and note that the gas pressure decreases as it
moves along the pipe because of the loss of pressure head needed to overcome the force of
friction [76]. The density of the gas also decreases and under the assumptions of steady
state flow and constant cross-sectional area we note from equation 22

\[ \rho_1 w_1 = \rho_2 w_2 \]  

(59)

that the velocity \( w \) must also change along the pipe. Therefore to calculate the frictional
resistance we consider a small element of the pipe in Figure 9 and integrate over the entire
length to obtain the total head lost. The element we consider is from \( x \) to \( x + dx \). We
have pressure \( p \) at distance \( x \) from the start of the pipe and pressure \( p + dp \) at distance
\( x + dx \), similarly the velocity is \( w \) at distance \( x \) and \( w + dw \) at distance \( x + dx \).

Figure 9: The gas pressure decreases as it moves along the pipe due the force of friction

If we assume the change in density across the element to be negligible we can write
Bernoulli’s equation across this element as

\[
\frac{p}{\rho g} + \frac{w^2}{2g} + z = \frac{p + dp}{\rho g} + \frac{(w + dw)^2}{2g} + (z + dz) + dh_f
\]

(60)

where \(h_f\) is the head loss due to friction and \(z + dz\) is the change in elevation. The changes in density and velocity change the kinetic energy by a small amount that can be neglected. We use Darcy’s equation

\[
dh_f = \frac{4f}{D} \frac{w^2}{2g} dx
\]

(61)

to calculate the head loss due to friction. The pipe diameter is represented by \(D\) and the friction factor by \(f\).

Rearranging equation 60 and including Darcy’s equation gives

\[
-dp = \frac{2f \rho w^2}{D} dx + \rho gz
\]

(62)

From equation 22 we see that

\[
w = \frac{\rho_1}{\rho} w_1
\]

(63)

where \(\rho = \rho_2\), and as we are considering isothermal flow we can write

\[
\frac{p}{\rho} = \frac{p_1}{\rho_1}
\]

(64)

Therefore the velocity \(w\) is given by

\[
w = \frac{p_1}{\rho} w_1
\]

(65)

and the density \(\rho\) by

\[
\rho = \frac{p_1}{\rho} \rho_1
\]

(66)

If the expressions for \(w\) and \(\rho\) are in included in equation 62 and the resulting equation is simplified it can be written as

\[
-dp = \frac{2f}{D} \rho_1 p_1 w_1^2 dx + \frac{p_1^2}{\rho_1} \rho_1 g dz
\]

(67)
The state equation 69 was defined earlier as

$$Z = \frac{pv}{RT} \quad (68)$$

from which we can see that

$$p_1 = \frac{ZRT}{v} \quad (69)$$

Substituting this expression for $p_1$ in equation 67 gives

$$-p\, dp = \frac{2f}{D} \rho_1^2 w_1^2 ZRT \, dx + \frac{p_2^2}{ZRT} g \, dz \quad (70)$$

The pipe in Figure 9 is elevated above the datum line. We can use the average pressure along the pipe, $p_{avg}$, in the term for the potential head due to the distance above the datum line, $\frac{p_2^2}{ZRT} g \, dz$. We also have from the volume flow rate, equation 17 and the continuity equation, equation 22 that

$$\rho_2^2 w_2^2 = \rho_2^2 Q_n^2 = \frac{\rho_n^2 Q_n^2}{\left(\frac{\pi D^2}{4}\right)^2} \quad (71)$$

where $D$ denotes the pipe diameter and the subscript $n$ denotes the standard conditions for pressure, $p_n \approx 0.1 \, MPa$ and temperature $T_n = 288 \, K$. Substituting this expression for $\rho_2^2 w_2^2$ into equation 70 gives us

$$-p\, dp = \frac{32f}{\pi^2} \rho_n^2 Q_n^2 ZRT \, dx + \frac{p_{avg}^2}{ZRT} g \, dz \quad (72)$$

We can express the density $\rho_n$ in terms of $p_n$ by considering the relationship of the gas constant for the gas to that of air $R_{\text{air}}$. At the same temperature and pressure the compressibility factor $Z$ is unity and the state equations for gas and air can be expressed as

$$p_n = \rho_n RT_n \quad \text{and} \quad p_n = (\rho_{\text{air}})_n R_{\text{air}} T_n \quad (73)$$

respectively. The specific gravity of a gas $S$ is the ratio of its density to that of air, thus

$$S = \frac{\rho_n}{\rho_{\text{air}}} = \frac{R_{\text{air}}}{R} \quad (74)$$

Therefore we can write the gas constant $R$ as

$$R = \frac{R_{\text{air}}}{S} \quad (75)$$
Then the density of the gas can be written as

\[ \rho_n = \frac{p_n}{RT_n} = \frac{SP_n}{R_{air} T_n} \] (76)

and substituting this equation for \( \rho_n \) along with the equation for \( R \) into equation 72 gives

\[ -p dp = \frac{32}{\pi^2} f \left( \frac{Sp_n}{R_{air} T_n} \right)^2 \frac{Q_n^2}{D^5} \frac{Z R_{air} T}{S} dx + \frac{p_{avg}^2 S}{ZR_{air} T} g dz \] (77)

which can be simplified slightly and written as

\[ -p dp = \frac{32}{\pi^2} f S Z T \frac{Q_n^2}{R_{air} D^5} \left( \frac{p_n}{T_n} \right)^2 dx + \frac{p_{avg}^2 S}{ZR_{air} T} g h dz \] (78)

We then need to integrate along the pipeline from \( x = 0 \) and \( p = p_1 \) to \( x = L \) and \( p = p_2 \) and solve for the flow rate \( Q_n \). The result of the integration is

\[ -\left( \frac{p_2^2 - p_1^2}{2} \right) = \frac{32}{\pi^2} f S Z T \frac{Q_n^2}{R_{air} D^5} \left( \frac{p_n}{T_n} \right)^2 L + \frac{p_{avg}^2 S}{ZR_{air} T} g h \] (79)

and solving for \( Q_n \) gives the general flow equation

\[ Q_n = \sqrt{\left( \frac{\pi^2 R_{air}}{64} \right) \times \frac{T_n}{p_n} \sqrt{\left( \frac{(p_1^2 - p_2^2) - 2p_{avg}^2 Sg H}{fS L Z T} \right) D^5}} } \] (80)

For a horizontal pipeline the general flow equation becomes

\[ Q_n = C \frac{T_n}{p_n} \sqrt{\left( \frac{(p_1^2 - p_2^2) D^5}{f S L Z T} \right) } \] (81)

where

\[ C = \sqrt{\frac{\pi^2 R_{air}}{64}} = \text{constant} \] (82)

There are several flow equations used to calculate the pressure lost during gas flow through a pipe, the main difference between them is the expression assumed for \( f \), the friction factor. [76].
2.3.2 The friction factor

The general flow equation for calculating the pressure drop as gas travels along the pipe requires a numerical value for the friction factor, $f$ \cite{69}. The friction factor is a dimensionless parameter that depends on the Reynolds number and the relative roughness of the pipe wall when the flow through the pipe is fully turbulent. The relative roughness is obtained by dividing the effective roughness of the pipe wall with the diameter of the pipe

$$\text{relative roughness} = \frac{e}{D} \quad (83)$$

The effective roughness $e$ is also commonly called the absolute roughness or the internal roughness \cite{96}. There are several empirical relationships for calculating the friction factor, in the next section one commonly used equation, the Colebrook-White equation is presented.

2.3.3 The Colebrook-White Equation

The Colebrook-White equation can be used to calculate the friction factor in gas pipelines when the flow is fully turbulent, that is, when the Reynolds number is greater than 4,000.

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{e}{3.7D} + \frac{2.51}{Re\sqrt{f}} \right) \quad (84)$$

where

- $f$ = friction factor
- $D$ = pipe inside diameter
- $e$ = absolute pipe roughness
- $Re$ = Reynolds number of the flow.

In smooth pipes the value of $e$ is small and may be neglected, thus the Colebrook-White equation reduces to

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{2.51}{Re\sqrt{f}} \right) \quad (85)$$

for turbulent flow in smooth pipes. For flow in fully rough pipes the large value for the Reynolds number means that the second term in equation 86 diminishes and the friction
factor can be calculated using

\[ \frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{e}{3.7D} \right) \]  \hspace{1cm} (86)

A table of typical friction factors is provided in [96] and reproduced in in Table 1.

<table>
<thead>
<tr>
<th>Pipe material</th>
<th>Roughness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riveted steel</td>
<td>0.9 to 9.0</td>
</tr>
<tr>
<td>Welded steel</td>
<td>0.045</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.26</td>
</tr>
<tr>
<td>PVC, drawn tubing, glass</td>
<td>0.0015</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.3 to 3.0</td>
</tr>
</tbody>
</table>

The Australian Pacific LNG pipeline being constructed as part of an LNG export terminal located at Gladstone consists of welded steel pipe segments. The internal diameter of the pipe is 1,067 mm [107]. An estimate of the friction factor for this pipe would therefore be \( f = 0.0102 \).

The friction factor is an integral part of the general gas flow equation however, due to it being defined by a highly nonlinear function it must be solved iteratively or values must be read from a chart. Such charts are known as Moody diagrams. To allow direct calculation of the flow equation approximations to the friction factor are commonly used [69]. The Weymouth equation is one equation used that allow direct calculation of the gas flow through a pipe.

2.3.4 The efficiency factor

The efficiency factor allows for factors other than viscous forces that also reduce the amount of gas flowing through a pipe to be included in calculations. The internal surface of a pipe can have rust scaling or dirt deposits, it may also have welded joins and fittings etc. that increase drag losses as gas flows through the pipe. To account for these factors an effective roughness can be used when calculating the friction factor rather than the
absolute roughness. Furthermore, the Reynolds number is calculated using assumed values for gas density and viscosity, and the average velocity of gas flow, while the flow equations often use assumed values for specific gravity, compressibility and temperature. All of these variables vary with time and depend on the prevailing conditions during gas flow. Average values of these variables are used in calculations and the resulting errors can be incorporated into the efficiency factor [76].

The efficiency factor $E$ can be included in the expression for the friction factor as follows:

$$\sqrt{\frac{1}{f}} = E \sqrt{\frac{1}{f_t}} \quad (87)$$

where $f$ is the actual friction factor and $f_t$ is the theoretical friction factor. It may also be introduced via the general flow equation:

$$Q_n = C \frac{T_n}{p_n} E \sqrt{\frac{(p_1^2 - p_2^2)D^5}{f_tSLTZ}} \quad (88)$$

Equation 88 can be rearranged to give

$$p_1^2 - p_2^2 = \left( \frac{1}{C T_n} \right)^2 f_tSLTZ \frac{1}{D^5} \frac{1}{E^2} Q_n^2 \quad (89)$$

or

$$p_1^2 - p_2^2 = \alpha Q_n^2 \quad (90)$$

where

$$\alpha = \left( \frac{1}{C T_n} \right)^2 f_tSLTZ \frac{1}{D^5} \quad (91)$$

Equation 91 shows that the term $p_1^2 - p_2^2$ is inversely proportional to the square of the efficiency factor for a given flow equation $Q_n$ [76].

2.3.5 The Weymouth Equation

The Weymouth equation is used in [92] to model the flow rate through a pipeline. It is appropriate for use in high pressure, large diameter, high flow rate transmission pipelines. The friction factor used is dependent on the diameter only and is applicable in the fully
turbulent flow region \cite{96}

\[ Q = 3.7435 \times 10^{-3} E \left( \frac{T_b}{p_b} \right) \left( \frac{p_1^2 - p_2^2 e^4}{G T_f L_e Z} \right)^{0.5} D^{2.667} \]  \hspace{1cm} (92)

where

\( Q \) is the gas flow rate
\( T_b \) is the base temperature
\( p_b \) is the base pressure
\( T_f \) is the average gas flow temperature
\( p_1 \) is the upstream pressure
\( p_2 \) is the downstream pressure
\( L_e \) is the equivalent length of pipe segment
\( G \) is the specific gravity of the gas
\( Z \) is the compressibility factor of the gas
\( e \) is the internal pipe roughness
\( D \) is the diameter of the inside of the pipe
\( E \) is the pipeline efficiency factor

2.3.6 The Panhandle A equation

The Panhandle A equation was developed as a refinement of the Weymouth equation and is used in \cite{61} to model gas flow through high pressure pipelines. The equation may be written as:

\[ Q_n = 7.57 \times 10^{-4} \frac{T_n}{p_n} \sqrt{\frac{(p_1^2 - p_2^2)D^5}{f S L T Z}} \]  \hspace{1cm} (93)

The friction factor \( f \) is given by

\[ \sqrt{\frac{T}{f}} = 6.872(Re)^{0.073} E \]  \hspace{1cm} (94)

If we assume that the physical parameters of the gas are constant the Reynolds number can be defined using

\[ Re = C \frac{Q}{D} \]  \hspace{1cm} (95)
where $C$ is a constant. Using equation 95 the friction factor can also be obtained using

$$\sqrt{f} = \frac{14.94E(SQ_n)^{0.073}}{D}$$

which is applicable when considering natural gas. Under the further assumption that $Z = 0.95$, $T = 288 \text{ K}$ and $S = 0.589$, equation 93 can be rearranged and written as

$$p_1^2 - p_2^2 = KQ_n^{1.854}$$

where:

$$K = 18.43 \frac{L}{E^2D^{1.854}}$$

The Panhandle A equation is used to model gas flow through pipelines at pressures of over 700 kPa [76].

### 2.3.7 Pipe equations in general form

Commonly used pipe flow equations can be expressed in a general form as follows. Consider any pipe $k$, the equation of flow from node $i$ to node $j$ can be written as

$$\phi[(Q_n)_k] = K_k(Q_n^{m_1})_k = P_i - P_j = \Delta P_k$$

where

$\phi[(Q_n)_k]$ is the flow function for pipe $k$

$K_k$ is the pipe constant for pipe $k$

$(Q_n)_k$ is the flow in pipe $k$

$P_i = p_i^2$ and $P_j = p_j^2$

$m_1$ is the flow exponent. The flow exponent takes on different values depending on the pressure in the network. For a high pressure network $m_1 = 1.854$ [76].

For the remainder of this paper the terms $P$ and $\Delta P$ will be referred to as pressure and pressure-drop respectively, (as opposed to pressure squared and difference squared pressures).
Equation 99 can be rearranged to give

\[ \phi' (\Delta P_k) = (Q_n)_k = \left( \frac{\Delta P_k}{K_k} \right)^{\frac{1}{n_1}} \]  

(100)

To allow for gas to flow in either direction along the pipe we can make a further modification and rewrite equation 100 as

\[ (Q_n)_k = S_{ij} \left( \frac{S_{ij}(P_i - P_j)}{K_k} \right)^{\frac{1}{n_1}} \]  

(101)

where

\[ S_{ij} = \begin{cases} 
1, & \text{if } P_i > P_j \\
-1, & \text{if } P_i < P_j 
\end{cases} \]

Equation 101 is the general form of the flow equation for a high pressure gas transmission pipeline [76].

When referring to networks in steady state \( Q \) will be used in place of \( Q_n \) for the sake of simplicity.

### 2.4 Network topology

Consider the network shown in Figure 2.4. This network consists of four nodes, \( \{n_i : i = 1, \ldots, 4\} \), five pipes \( \{p_j : j = 1, \ldots, 5\} \) and three loads \( L_1, L_2 \) and \( L_3 \). Load \( L_1 \) is the supply into the network. By convention loads supplied to the network are identified by negative values and load demanded from the network by positive values. For network analysis it is necessary to have at least one reference node at which the pressure is known, typically the pressure is known at a source node and source nodes often serve as reference nodes. In Figure 10 node \( n_1 \) is the reference node. Reference nodes are independent, all other nodes and branches are dependent on the reference node. This fact is exploited in the reduced network techniques employed in [88] and [89]. The loads represent demands on the network, they may be positive, negative or zero. Positive loads represent gas supplied to the network, negative loads represent gas supplied from the network to consumers and zero loads represent nodes at which pipes join but gas is neither supplied to or demanded from the network. The total load demanded from
the network equals the total load supplied to it when the network is operating under steady-state conditions [76].

Figure 10: A directed network with four nodes, five pipes, and three loads

Each pipe in the network is assigned a direction arbitrarily, gas flow in that direction is indicated by a positive value for the flow. Gas flow in the opposite direction is indicated by a negative flow value. Finally, connected networks can form closed paths called loops. In Figure 2.4 the pipes $p_1$, $p_2$ and $p_3$ form loop $A$ and pipes $p_2$, $p_3$ and $p_5$ form loop $B$. Loops $A$ and $B$ are independent loops, the third loop consisting of pipes $p_1$, $p_3$, $p_4$ and $p_5$ can be derived from the two independent loops if the common pipe, $p_2$ is discarded [76].

2.4.1 The branch-nodal incidence matrix

The topology of a gas network is easily represented by a matrix. The branch-nodal incidence matrix $A = [a_{ij}]_{n \times m}$ has one row for each node and one column for each pipe. If pipe $j$ enters node $i$ then $a_{ij} = 1$; conversely, pipe $j$ leaves node $i$ then $a_{ij} = -1$; and
\( a_{ij} = 0 \) if pipe \( j \) is not directly connected to node \( i \). This can be written concisely as

\[
a_{ij} = \begin{cases} 
1, & \text{if pipe } j \text{ enters node } i \\
-1, & \text{if pipe } j \text{ leaves node } i \\
0, & \text{otherwise}
\end{cases}
\]  

(102)

### 2.4.2 The branch-loop incidence matrix

The loops in a gas network can be represented using a branch-loop incidence matrix, \( B \). Each row in the matrix represents an independent loop and each column a pipe. If pipe \( j \) has the same direction as loop \( i \) then \( b_{ij} = 1 \), conversely if pipe \( j \) has the opposite direction as loop \( i \) then \( b_{ij} = 1 \). If pipe \( j \) is not part of loop \( i \) then \( b_{ij} = 0 \). This can be written as:

\[
b_{ij} = \begin{cases} 
1, & \text{if pipe } j \text{ has the same direction as loop } i \\
-1, & \text{if pipe } j \text{ does not have the same direction as loop } i \\
0, & \text{otherwise}
\end{cases}
\]  

(103)

The reduced branch-node incidence matrix is defined in the same manner as the branch-nodal incidence matrix but with the rows corresponding to the reference nodes removed. It is written as \( A_1 = [a_{ij}]_{n_1 \times m} \), where \( n_1 \) is the number of nodes excluding the reference node [76].

### 2.4.3 Kirchoff’s laws

Kirchoff’s first law states the sum of flows into a node must be equal to the sum of flows out of the node [76]. Consider Figure 2.4.3 in which the flows through pipe \( j \) are denoted by \( Q_j \).
Figure 11: The sum of flows into a node must equal the sum of flows out of the node.

The nodal equations for this network are written as

\[
\begin{align*}
-Q_1 - Q_2 - Q_3 &= -L_1 \\
Q_1 + Q_4 &= L_2 \\
Q_2 - Q_4 - Q_5 &= L_3 \\
Q_3 + Q_5 &= L_4
\end{align*}
\]  

(104)

The equations in 104 can be written as

\[
L_i = \sum_{j=1}^{m} a_{ij} Q_j, \quad i = 1, 2, \ldots, n
\]  

(105)

or in matrix form as

\[
L_{n \times 1} = A_{n \times m} Q_{m \times 1}
\]  

(106)

where:

- \( \mathbf{L} \) is the vector of loads at the nodes
- \( \mathbf{Q} \) is the vector of flows in the branches
- \( \mathbf{A} \) is the branch-nodal incidence matrix [76].
In high pressure networks the drop in pressure in the branches can be related to the nodal pressures by the following set of equations

\[
\begin{align*}
\Delta P_1 &= P_1 - P_2 \\
\Delta P_2 &= P_1 - P_3 \\
\Delta P_3 &= P_1 - P_4 \\
\Delta P_4 &= -P_2 + P_3 \\
\Delta P_5 &= P_3 - P_4
\end{align*}
\] (107)

which can be written in matrix form as

\[
\Delta \mathbf{P} = -\mathbf{A}^T \mathbf{P}
\] (108)

where

- \(\Delta \mathbf{P}\) is the vector of pressure drops
- \(\mathbf{A}^T\) is the transpose of the branch-nodal matrix
- \(\mathbf{P}\) is the vector of nodal pressures [76].

If we substitute \(\Delta \mathbf{P}\) into equation 100 we have

\[
\mathbf{Q} = \phi'(-\mathbf{A}^T \mathbf{P})
\] (109)

We can then substitute this expression for \(\mathbf{Q}\) into equation 106 to get

\[
\mathbf{L} = \mathbf{A}_1[\phi'(-\mathbf{A}^T \mathbf{P})]
\] (110)

Equation 110 is the set of nodal equations that describe the gas network. It is solved to obtain values for nodal pressures [76].

Kirchoff’s second law says that there is zero pressure drop around a closed loop. If we denote the change in pressure as \(\Delta p\) and and adopt the convention that \(\Delta p\) is positive if the flow direction coincides with the branch direction the loop equations for Figure 2.4.3 are

\[
-\Delta p_1 + \Delta p_2 + \Delta p_4 = 0
\] (111)

for loop A and

\[
\Delta p_2 - \Delta p_3 + \Delta p_5 = 0
\] (112)
for loop $B$. The loop equations can be written in general form as

$$\sum_{j=1}^{m} b_{ij} \Delta p_j, \quad i = 1, 2, \ldots, k$$  \hspace{1cm} (113)

or in matrix form as

$$B \Delta P = 0$$  \hspace{1cm} (114)

where $B$ is the branch-loop incidence matrix, $\Delta P$ is the pressure in the branches [76].

### 2.5 Methods of steady state analysis

Most of the literature analysing gas transmission networks assumes the network is in steady state. For a network to be in steady state the flow of gas through the network must be independent of time. When in steady state the network can be described by a set of nonlinear equations that are also independent of time.

Generally steady state analysis seeks to use known values of the pressure at supply sources and of gas consumption in the nodes to calculate the values of gas flow through pipelines and the pressures at the nodes. The estimates must satisfy the flow rate equation and Kirchoff’s laws [76].

#### 2.5.1 The Newton-nodal method

Equation 110 gives a set of nodal equations that describe a gas network. We can rearrange the equation to get

$$A_1[\phi'(-A^T P)] - L = 0$$  \hspace{1cm} (115)

which says mathematically that the sum of flows into and out of the node is equal to zero, this is in accordance with Kirchoff’s first law. The Newton-nodal method starts with an initial estimate of the nodal pressures and seeks to improve it with each iteration. After each iteration the errors in the estimates leave imbalances in the flows through the node. These errors are a function of all the nodal pressures, except the reference node which
has a fixed pressure. The error at each node is denoted by \( f(P_1) \) where \( P \) is the vector of squared pressures. The set of all nodal errors is represented in a matrix

\[
F(P_1) = \begin{bmatrix}
  f_1(P_1, P_2, \ldots, P_n) \\
  f_2(P_1, P_2, \ldots, P_n) \\
  \vdots \\
  f_n(P_1, P_2, \ldots, P_n)
\end{bmatrix}
\] (116)

where \( F \) denotes a vector of functions [76].

Equation 115 can be rewritten to include the error functions as

\[
F(P_1) = A_1[\phi'(-A^T P)] - L
\] (117)

Equation 117 is solved iteratively and the approximations are improved after each iteration using

\[
P^{k+1} = P^k + (\delta P_1)^k
\] (118)

where \( k \) is the number of iterations. We can calculate \( \delta P_1 \) using the

\[
J^k(\delta P_1)^k = -[F(P_1)]^k
\] (119)

where

\[
J = \begin{bmatrix}
  \frac{\partial f_1}{\partial P_1} & \frac{\partial f_1}{\partial P_2} & \cdots & \frac{\partial f_1}{\partial P_n} \\
  \frac{\partial f_2}{\partial P_1} & \frac{\partial f_2}{\partial P_2} & \cdots & \frac{\partial f_2}{\partial P_n} \\
  \vdots & \vdots & \ddots & \vdots \\
  \frac{\partial f_n}{\partial P_1} & \frac{\partial f_n}{\partial P_2} & \cdots & \frac{\partial f_n}{\partial P_n}
\end{bmatrix}
\] (120)

is the Jacobian matrix [76].

The basic approach used in the Newton nodal method can be shown using the simple network shown in Figure 12

Figure 12: A network with three loads, five pipes, and no cycles
Using equation 121

\[
(Q_n)_k = S_{ij} \left( \frac{S_{ij}(P_i - P_j)}{K_k} \right)^{\frac{1}{m_1}}
\]

(121)

where

\[
S_{ij} = \begin{cases} 1, & \text{if } P_i > P_j \\ -1, & \text{if } P_i < P_j \end{cases}
\]

as the flow equation, the set of nodal equations can be written as

\[
f_2 = S_{12} \left[ \frac{S_{12}(P_1 - P_2)}{K_1} \right]^{\frac{1}{m_1}} + S_{32} \left[ \frac{S_{32}(P_3 - P_2)}{K_4} \right]^{\frac{1}{m_1}} - L_2
\]

\[
f_3 = S_{13} \left[ \frac{S_{13}(P_1 - P_3)}{K_2} \right]^{\frac{1}{m_1}} - S_{32} \left[ \frac{S_{32}(P_3 - P_2)}{K_4} \right]^{\frac{1}{m_1}} - S_{34} \left[ \frac{S_{34}(P_3 - P_4)}{K_5} \right]^{\frac{1}{m_1}} - L_3
\]

\[
f_4 = S_{14} \left[ \frac{S_{14}(P_1 - P_4)}{K_3} \right]^{\frac{1}{m_1}} + S_{34} \left[ \frac{S_{34}(P_3 - P_4)}{K_5} \right]^{\frac{1}{m_1}} - L_4
\]

(122)

and the Jacobian is

\[
J = \begin{bmatrix}
\frac{\partial f_1}{\partial P_1} & \frac{\partial f_1}{\partial P_2} & \cdots & \frac{\partial f_1}{\partial P_{n_1}} \\
\frac{\partial f_2}{\partial P_1} & \frac{\partial f_2}{\partial P_2} & \cdots & \frac{\partial f_2}{\partial P_{n_1}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_{n_1}}{\partial P_1} & \frac{\partial f_{n_1}}{\partial P_2} & \cdots & \frac{\partial f_{n_1}}{\partial P_{n_1}}
\end{bmatrix}
\]

(123)
Differentiating equation 122 with respect to $P_2$ gives

$$\frac{\partial f_2}{\partial P_2} = -\left(\frac{1}{m_1}\right) \cdot S_{12} \cdot \left[ \frac{S_{12}(P_1 - P_2)}{K_1} \right]^{\left(\frac{1}{m_1} - 1\right)} (P_1 - P_2)^{-1}$$

$$- \left(\frac{1}{m_1}\right) \cdot S_{12} \cdot \left[ \frac{S_{32}(P_1 - P_2)}{K_4} \right]^{\left(\frac{1}{m_1} - 1\right)} (P_1 - P_2)^{-1}$$

which can be written

$$\frac{\partial f_2}{\partial P_2} = -\frac{1}{m_1} \frac{Q_1}{\Delta P_i} -\frac{1}{m_1} \frac{Q_4}{\Delta P_4}$$

Proceeding in a similar manner gives the remaining values of the Jacobian matrix

$$J = \begin{bmatrix}
\left(\frac{Q_1}{\Delta P_i} + \frac{Q_4}{\Delta P_i}\right) & -\frac{Q_4}{\Delta P_4} & 0 \\
-\frac{Q_4}{\Delta P_4} & \left(\frac{Q_2}{\Delta P_2} + \frac{Q_4}{\Delta P_4} + \frac{Q_5}{\Delta P_5}\right) & -\frac{Q_4}{\Delta P_5} \\
0 & -\frac{Q_4}{\Delta P_4} & \left(\frac{Q_2}{\Delta P_2} + \frac{Q_5}{\Delta P_5}\right)
\end{bmatrix}$$

which can be written as $J = -A_1DA^T_1$, where $D = \text{diag}\left(\frac{1}{m_1} \frac{Q_i}{\Delta P_i}\right)$, for $i = 1, \ldots, m$.

Equation 119 is then used to make corrections to the estimates of the nodal pressures [76]. The Newton-nodal method is used in [77] to obtain estimates of the unknown pressure values.

### 2.5.2 The Hardy-Cross method

Equation 117 can also be solved using the Hardy-Cross method with the main difference being that this method solves for each node separately whereas the Newton method solves the system of equations as a whole. The Hardy-Cross method is used to find the nodal pressures in [29].

The first step is to find initial estimates to the pressures in equation 117. The nodal error for node $i$ is given by

$$f_i(P_1) = \sum_{j=1}^{m} a_{ij} [\phi'(-A^T P)] - L$$

The estimates are improved in iterations using

$$P_i^{k+1} = P_i^k + \delta P_i^k$$
where:

\[ \delta P_i^k = -(J_{ii}^k)^{-1} f_i(P_1)^k \]  
(129)

The term $J_{ii}$ is the diagonal term in the Jacobian used in the Newton method corresponding to node $i$. It is calculated using:

\[ J_{ii} = \frac{\partial f_i(P_1)}{\partial P_i} \]  
(130)

Both the Newton-nodal method and the Hardy-Cross method both rely on carefully chosen initial pressure estimates. If the initial estimates are too far away from the true values the process may diverge [76]. Nevertheless both methods have been used successfully to model gas networks.

2.6 Transient analysis

Steady state analysis of gas flows assumes constant flow rates, constant pressure and constant gas temperature over the period of analysis. Gas flow rates and the gas pressure usually are changing over time for various reasons. To model transient gas flow we can use partial differential equations. The equations may be linear but are generally nonlinear, they may be parabolic or hyperbolic of the first or second order. The basic equations describing the transient gas flow in a pipe are derived in the following section.

Transient gas flow through a pipeline is modelled using partial differential equations which makes solutions more difficult to find then under the assumption of steady state flow. Most researcher choose to assume steady state flow, however papers modelling transient flow include [18], [61] and [106]. Each of these papers assume isothermal flow and develop simplification techniques to find solutions to the transient flow equations. In this section the basic equations that underpin the models used in the literature are presented.
2.6.1 Basic equations for transient flow

It is not unreasonable to consider one dimensional flow of gas through a gas transmission network. One dimensional flow assumes that the rate of change of gas properties change negligibly in the direction normal to the stream line when compared to the rate of change along the stream line. For one dimensional flow the gas properties are functions of time and the distance along the axis of the pipe only.

2.6.2 Conservation of mass: Continuity equation

Since mass cannot be destroyed or created the mass of a system must remain constant. If we consider gas flowing through a control volume, by the law of conservation of mass the net mass flow rate into the control surface must equal the rate of increase of mass within the control surface. The amount of mass in the control volume \( V \) is given by

\[
m = \int \int \int_{V} \rho \, dV \tag{131}
\]

The mass entering and leaving the pipe in unit time is

\[
dm = dt \int \int_{V} \rho w \, dA \tag{132}
\]

The decrease of volume \( V \) in unit time is given by

\[
dm = -dt \int \int \int_{V} \frac{\partial \rho}{\partial t} \, dV \tag{133}
\]

we therefore can write

\[
\int \int_{V} \rho w \, dA = -\int \int \int_{V} \frac{\partial \rho}{\partial t} \, dV \tag{134}
\]

Applying the divergence theorem, which basically says the sum of outflow minus the sum inflows gives the net outflow, gives

\[
\int \int \int_{V} \left( \frac{\partial \rho w}{\partial x} + \frac{\partial \rho}{\partial t} \right) \, dV = 0 \tag{135}
\]
By definition equation 135 must be true for all values of $V$, therefore the condition

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho w}{\partial x}$$

must be satisfied. Equation 136 can be written in the form

$$\frac{\partial \rho A}{\partial t} = -\frac{\partial M}{\partial x}$$

which is known as the continuity equation. The continuity equation is the condition for the continuity of mass for one-dimensional gas flow in a rigid pipeline. Under the assumption of isothermal flow the equation of state can be written as

$$\rho c^2 = p$$

where $c$ is the speed of sound in the gas. Substituting this expression into equation 137 gives

$$\frac{\partial p}{\partial t} = A \frac{c}{c^2} = -\frac{\partial M}{\partial x}$$

Isothermal flow through pipelines is commonly assumed when modelling gas networks [76].

### 2.6.3 Newton’s second law: momentum equation

Newton’s second law, loosely stated, says that forces acting on a particle of fixed mass is equal to the rate of change of momentum of the particle. Therefore the sum of the forces acting on a particle moving in direction $x$ is equal to the rate of change of momentum of the particle

$$\sum F_x = \frac{d}{dt}(mw)$$

where $F_x$ is the component of the forces acting on the particle in direction $x$.

With reference to Figure 13 the gas in the control volume is denoted by $cv$ and the mass of gas in the region outside the control volume is denoted by $\sigma m_1$. At time $t + \Delta t$ the system is in state shown in Figure 14.
We can describe the dynamics of the system using

\[
\frac{d}{dt} (mw) = \lim_{\Delta t \to 0} \frac{[(mw)_{cv} + (mw)_{2}]_{t+\Delta t} - [(mw)_{cv} + (mw)_{1}]_t}{\Delta t}
= \frac{d}{dt} (mv)_{cv} + \lim_{\Delta t \to 0} \frac{[(mw)_{2}]_{t+\Delta t} - [(mw)_{1}]_t}{\Delta t}
\]

(141)

As \(\Delta t \to 0\)

\[
\lim_{\Delta t \to 0} \frac{(mw)_2}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta m_2 w_2}{\Delta t} = \rho_2 A w_2^2
\]

(142)

and

\[
\lim_{\Delta t \to 0} \frac{(mw)_1}{\Delta t} = \rho_1 A w_1^2
\]

(143)

therefore

\[
\frac{d}{dt} (mw) = \frac{d}{dt} (mv)_{cv} + \rho_2 A w_2^2 - \rho_1 A w_1^2
\]

(144)

Equation 144 says that when gas is flowing through the control volume the net force
acting on the gas is the sum of time rate of change of momentum within the control
volume, \( \frac{d}{dt}(mw)_v \), and the net change in the momentum flux, \( \rho_2 Aw_2^2 - \rho_1 Aw_1^2 \). The flux of momentum can be written as \( \frac{dm}{dt}w \), and as \( \Delta x \to 0 \)

\[
\frac{dm}{dt}w_2 - \frac{dm}{dt}w_1 = \frac{\partial}{\partial x}(\rho Aw^2)dx
\]

also

\[
\frac{d}{dt}(mw)_v = \frac{\partial}{\partial x}(\rho Aw dx)
\]

as \( \Delta x \to 0 \), thus

\[
\frac{d}{dt}(mw) = \frac{\partial}{\partial t}(\rho Aw dx) + \frac{\partial}{\partial x}(\rho Aw^2)dx
\]

The sum of the forces acting on the gas within the control surface are the net body force in the direction of flow, which is given by

\[
F \rho Adx \sin(\theta)
\]

where \( \theta \) is the angle between the horizon and the direction of flow \( x \); the force of pressure

\[
pA - \left( p + \frac{\partial p}{\partial x} A \right) = -A \frac{\partial p}{\partial x} dx
\]

and the shear force due to friction

\[
-\rho Aw^2 \frac{4f}{D} dx
\]

where \( f \) is the friction coefficient and \( D = \) the hydraulic mean diameter. In this paper \( D \) would be the diameter of the pipeline through which the gas flows.

The by substituting equations 147, 148, 149 and 150 into 140 we get

\[
-\frac{\partial p}{\partial x} - \rho w^2 \left( \frac{4f}{D} \right) = g \rho \sin(\theta) = \frac{\partial}{\partial x}(\rho w) + \frac{\partial}{\partial x}(\rho w^2)
\]

which is the general form of the momentum equation for one-dimensional flow [76].
2.6.4 Energy equation - the first law of thermodynamics

The first law of thermodynamics is a conservation of energy equation that can be written as

\[ \Omega - W = \Delta E \]  

(152)

where:
\( \Omega \) is the heat added to the system
\( W \) is the work done by the system
\( E \) is the change in energy of the system.

If we consider Figures 14 and 14 we can derive an expression for one-dimensional flow as follows. From the first law of thermodynamics we have

\[ (E_2 + E_{cv})_{t+\Delta t} - (E_1 + E_{cv})_t = \Delta \Omega - \Delta W \]  

(153)

Energy associated with the mass of the system can be further divided into the following components

\[ E = U + \frac{1}{2} mw^2 + mgz \]  

(154)

where:
\( U \) is the internal energy associated with molecular and atomic behaviour
\( \frac{1}{2} mw^2 \) is the kinetic energy
\( mgz \) is the potential energy associated with the location in the Earth’s gravitational field.

As \( \Delta t \to 0 \) equation 154 can be written as

\[ \frac{d\Omega}{dt} - \frac{dW}{dt} = \frac{dE_{cv}}{dt} + \frac{dm_{out}}{dt} \left( u_2 + \frac{w_2^2}{2} + gz_2 \right) - \frac{dm_{in}}{dt} \left( u_1 + \frac{w_1^2}{2} + gz_1 \right) \]  

(155)

where \( \frac{dm_{out}}{dt} \) and \( \frac{dm_{in}}{dt} \) are the mass flow rate out of and into the control volume respectively.

Work may be done at the boundaries of the system by the normal and tangential stresses. This is flow work. Excluding flow work, work done on the system at the upper boundary is denoted by \( \frac{dm_{in}}{dt}(p_1v_1) \). At the lower boundary work is done by the system. This is
denoted by \( \frac{dm_{\text{out}}}{dt} (p_2 v_2) \). We can therefore rewrite equation 155 as

\[
\frac{d\Omega}{dt} - \frac{dW_s}{dt} = \frac{dE_{cv}}{dt} + \frac{dm_{\text{out}}}{dt} \left( h_2 + \frac{w_2^2}{2} + g z_2 \right) - \frac{dm_{\text{in}}}{dt} \left( h_1 + \frac{w_1^2}{2} + g z_1 \right)
\]

(156)

where:

- \( \frac{dW_s}{dt} \) is the time rate of work (excluding flow work)
- \( h = u + pv \) is the specific enthalpy.

As \( \Delta x \to 0 \) equation 156 becomes

\[
\frac{d\Omega}{dt} - \frac{dW_s}{dt} = \frac{dE_{cv}}{dt} + \frac{dm_{\text{out}}}{dt} \left( h + \frac{\partial h}{\partial x} dx \right) + \frac{(w + \frac{\partial w}{\partial x} dx)^2}{2} + g \left( z + \frac{\partial z}{\partial x} dx \right) - \frac{dm_{\text{in}}}{dt} \left( h + \frac{w^2}{2} + gz \right)
\]

(157)

If we assume that \( \frac{dW_s}{dt} = 0 \) equation 158 can be written as

\[
\frac{d\Omega}{dt} = \frac{\partial}{\partial t} (\rho A dx) \left( u + \frac{w^2}{2} + dz \right) + \frac{\partial}{\partial x} (\rho w A) \left( u + \frac{w^2}{2} + dz \right) dx
\]

(158)

No heat is added or lost in adiabatic flow, thus \( d\Omega = 0 \). In isothermal flow the gas temperature \( T \) is constant while flowing through the pipe but \( d\Omega \neq 0 \). Isothermal flow is commonly assumed in the literature and is an important simplifying assumption in [18], [61] and [106]. For isothermal flow equations 139 and 151 are used. If we introduce the molar flow

\[
q_1 = \frac{\rho w A}{M_1}
\]

(159)

where \( M_1 \) is the molecular weight of the gas, and substitute the expression into equations 139 and 151 we have

\[
A \frac{\partial p}{c^2 \partial t} + M_1 \frac{\partial q_1}{\partial x} = 0
\]

(160)

and

\[
\frac{\partial p}{\partial x} + \frac{M_1 \partial q_1}{A} \frac{\partial t}{\partial t} + \frac{M_1^2}{A^2} \frac{\partial}{\partial x} \left( \frac{q_1^2}{p} \right) + \frac{M_1^2}{A^2} \frac{2 f q_1^2}{\rho D} + \rho g \sin(\theta) = 0
\]

(161)

It is possible to reduce equations 160 and 161 to one second order partial differential equation if required [76].

Simplified mathematical models based on the above equations for transient analysis have been constructed to study transient gas flow through pipelines although the assumption of steady-state flow is far more common when dealing with an already complex problem.
2.7 Topics from stochastic calculus for analysing spot prices and derivatives

Spot markets for natural gas exists in some markets around the world, including in Victoria. It is of interest to participants in this market to simulate the price evolution of wholesale gas traded in this market. The mathematics used in this endeavour are presented in the following sections.

2.7.1 Spot prices and derivatives

A spot price for a commodity is the price determined by a market for the commodity at the current time. Spot markets of different size and geographical scope exist for numerous commodities and financial assets. A derivative can be defined as a financial instrument that derives its value from another underlying financial variable. The underlying financial variable is often the prices of traded assets [52]. An option is a type of derivative that gives the holder of the option the right to sell or buy an asset at an agreed price called the strike price. All options have an expiry date. American style options allow the option holder to exercise their right to buy or sell at any time before the expiry date. European options allow the holder to exercise this right only on the expiry date. Spot market exist for temporary irrigation water in the Goulburn-Murray irrigation district in Murray-Darling basin as well as for wholesale natural gas traded within the Victorian declared transmission system. A stochastic differential equation has been developed by Cui & Schreider [38] to model the spot price temporary irrigation water in the Goulburn-Murray irrigation district and a pricing model for a series of European style options has been proposed. As yet not equivalent equations exist for the spot price of wholesale natural gas.

2.7.2 Brownian motion

Brownian motion is a core component of most models of financial assets. It was first used to describe the motion of a pollen particle suspended in a fluid, the particle is being
bombarded by the molecules of the fluid and moves in a random path. Klebaner \cite{62} describes Brownian motion, $B(t)$ as the basic model for the cumulative effect of pure noise and the displacement of the particle $B(t) - B(0)$ as a purely random process.

Brownian motion has the following defining properties. It has independent, normally distributed increments, that is $W(t) - W(s)$ for $t > s$ is independent of $W(u)$ where $0 \leq u \leq s$, and $W(t) - W(s)$ is distributed as a normal random variable with mean $\mu = 0$ and variance $\sigma^2 = t - s$. Brownian motion also has continuous paths so $W(t)$ $t \geq 0$ is a continuous function of $t$.

### 2.7.3 Stochastic processes

Brownian motion is not suitable as a model for asset prices as it allows for negative prices. In general asset prices are functions of one or more Brownian motions. For example, adding a term to allow for price inflation gives $X_t = W_t + \mu_t$, scaling the volatility is achieved using a constant $\sigma$, $X_t = \sigma W_t + \mu_t$ and taking the exponential of this process precludes the possibility of negative prices. The simple model of an asset price is then

$$X_t = e^{\sigma W_t + \mu}$$

In order to construct other asset price models we first define an Itô process. If we begin with a Brownian motion $W(t)$, and a family $X$ of random variables $X_t$ then this family of random variables satisfy the stochastic differential equation

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t$$

if for any $t$, we have that

$$X_{t+h} - X_t - h\mu(t, X_t) - \sigma(t, X_t)(W_{t+h} - W_t)$$

is a random variable with mean and variance which are $o(h)$ \cite{59}. 

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Itô’s lemma is an important result for understanding stochastic variables. Hull [52] provides an intuitive derivation of Itô’s lemma by extending known results from differential calculus. If we consider a continuous differentiable function \( G \) of two variables \( x \) and \( y \) it is well known that

\[
\Delta G \approx \frac{\partial G}{\partial x} \Delta x + \frac{\partial G}{\partial y} \Delta y \tag{165}
\]

The Taylor series expansion of \( \Delta G \) is

\[
\Delta G = \frac{\partial G}{\partial x} \Delta x + \frac{\partial G}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 G}{\partial x \partial y} \Delta x \Delta y + \frac{1}{2} \frac{\partial^2 G}{\partial y^2} \Delta y^2 + \ldots \tag{166}
\]

As \( \Delta x \to 0 \) and \( \Delta y \to 0 \) \( \Delta G \) becomes

\[
dG = \frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial y} dy \tag{167}
\]

We can extend equation 167 to cover functions of variables of processes by considering a variable \( x \) that follows an Itô process

\[
dx = a(x,t)dt + b(x,t)dz \tag{168}
\]

and \( G \) which is a function of time. We can expand equation 168 in a similar manner to equation 168. This gives

\[
\Delta G = \frac{\partial G}{\partial x} \Delta x + \frac{\partial G}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 G}{\partial x \partial t} \Delta x \Delta t + \frac{1}{2} \frac{\partial^2 G}{\partial t^2} \Delta t^2 + \ldots \tag{169}
\]

We can discretize equation 169 to get

\[
\Delta x = a(x,t)\Delta t + b(x,t)\epsilon \sqrt{\Delta t} \tag{170}
\]

and removing the arguments for the sake of clarity

\[
\Delta x = a\Delta t + b\epsilon \sqrt{\Delta t} \tag{171}
\]

shows that terms of order \( \Delta x^2 \) cannot be ignored since

\[
\Delta x^2 = b^2 \epsilon^2 \Delta t + \text{ terms of higher order in } \Delta t \tag{172}
\]
Using the variance of standard distribution and the fact that $E[\epsilon] = 0$ we have that $E[\epsilon^2] = 1$ and therefore $E[\epsilon^2 \Delta t] = \Delta t$. This means that we can treat $\epsilon^2 \Delta t$ as non-stochastic and equal to $\Delta t$ as $\Delta t \to 0$. It follows that $\Delta x^2 = b^2 dt$ as $\Delta t \to 0$. Using this result and taking limits as $\Delta t \to 0$ and $\Delta x \to 0$ gives

$$dG = \frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial t} dt + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 dt$$  \hspace{1cm} (173)$$

If we substitute for $dx$ from equation 168 we then have

$$dG = \left( \frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial G}{\partial x} b dz$$  \hspace{1cm} (174)$$

A more formal definition is constructed as follows. If we let $X^{(j)}_t$ be an Ito process for each $j$ satisfying

$$dX^{(j)}_t = \mu_j(t, X_t) dt + \sigma_j(t, X_t) dW_t$$  \hspace{1cm} (175)$$

and let $f(t, x_1, \ldots, x_n)$ be a twice differentiable function. Then $f \left( t, X^{(1)}_t, X^{(2)}_t, \ldots, X^{(n)}_t \right)$ is an Itô process and

$$d \left[ f \left( t, X^{(1)}_t, X^{(2)}_t, \ldots, X^{(n)}_t \right) \right] = \frac{\partial f}{\partial t} dt + \sum_{j=1}^n \frac{\partial f}{\partial X^j_t} \frac{\partial X^j_t}{\partial X^k_t} dX^j_t dX^k_t$$  \hspace{1cm} (176)$$

where $dX^j_t dX^k_t$ is defined by:

$$dt^2 = 0$$  \hspace{1cm} (177)$$

$$dt dW_t = 0$$  \hspace{1cm} (178)$$

$$dW_t^2 = dt$$  \hspace{1cm} (179)$$

In this definition all Itô processes are defined by the same Brownian motion however, in general this need not necessarily be the case [59].
2.7.5 Geometric Brownian motion

Geometric Brownian motion is a simple model of asset prices. It is defined by

\[ dS_t = \mu S_t dt + \sigma S_t dW_t \]  

where \( S_t \) is the asset price at time \( t \), \( \mu \) is the drift or trend of the asset prices and \( \sigma \) is the volatility or variability in the asset price. Using this model the log of the stock price follows a normal distribution. To see this let \( Y_t = \log(S_t) \) and apply Itô’s lemma

\[ dY_t = \left( \log[S_t] \right)^{\prime} \mu S_t dt + \left( \log[S_t] \right)^{\prime} \sigma S_t dW_t + \frac{1}{2} \left( \log[S_t] \right)^{\prime}\prime \sigma^2 S_t^2 dt \]

\[ = \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \]  

This is the equation defining a simple Brownian motion with drift \( \mu \) and volatility \( \sigma \). Therefore

\[ Y_t - Y_0 = \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma \sqrt{t} N(0,1) \]

and as \( S_t = e^{Y_t} \), we have that

\[ S_t = S_0 e^{\left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma \sqrt{t} N(0,1)} \]

This model is sometimes referred to as the log normal model of asset prices [59].

2.7.6 The Black-Scholes equation

The Black-Scholes equation enables the price of a European option to be priced under the following assumptions:

- The stock price follows a geometric Brownian motion with drift \( \mu \) and volatility \( \sigma \);
- It is possible to short sell securities without restrictions;
- There are no taxes or transaction costs and all securities are perfectly divisible;
- There are no dividends paid during the life of the option;
- There is no possibility of arbitrage;
- Securities can be traded continuously;
- The risk-free interest rate \( r \) is constant and the same for all maturities [52].
The price of a European call option depends on the asset price \( S \) and time \( t \), which we can write as \( C(S,t) \). Using Itô’s lemma the process followed by \( C(S,t) \) is

\[
dC = \left( \frac{\partial C}{\partial t}(S,t) + \mu S \frac{\partial C}{\partial S}(S,t) + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}(S,t) \right) dt + \sigma S \frac{\partial C}{\partial S}(S,t) dW_t \tag{184}
\]

We now consider a portfolio consisting of the option and \( \alpha = -\frac{\partial C}{\partial S}(S,t) \) of the stock. The idea is to create a portfolio without risk and conclude that it must therefore increase at the risk free rate of return. The amount of stock held is in the correct proportion so that changes in the stock price are offset by changes in the option price. The value of the portfolio is given by

\[
d(C + \alpha S) = \left( \frac{\partial C}{\partial t}(S,t) + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}(S,t) \right) dt \tag{185}
\]

which has no random component. Therefore the drift of \( C+\alpha S \) must be equal to \( r(C+\alpha S) \) and we conclude that

\[
\frac{\partial C}{\partial t}(S,t) + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}(S,t) = rC \tag{186}
\]

or

\[
\frac{\partial C}{\partial t}(S,t) + rS \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}(S,t) = rC \tag{187}
\]

Thus the value of the call option satisfies a second-order partial differential equation [59].

2.7.7 The Black-Scholes pricing formulas

To solve the Black-Scholes equation first rewrite equation 187 as

\[
\frac{\partial C}{\partial t} + \left( r - \frac{1}{2} \sigma^2 \right) S \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 \left( S \frac{\partial}{\partial S} \right)^2 C = rC \tag{188}
\]

Then by letting \( S = e^Z \) equation 188 we have

\[
\frac{\partial C}{\partial t} + \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial C}{\partial Z} + \frac{1}{2} \sigma^2 \frac{\partial^2 C}{\partial Z^2} = rC \tag{189}
\]

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Then if we consider the time remaining until the option expires, \( \tau = T - t \), \( \ref{eq:189} \) becomes

\[
\frac{\partial C}{\partial \tau} - \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial C}{\partial Z} - \frac{1}{2} \sigma^2 \frac{\partial^2 C}{\partial Z^2} = -rC
\]  

\( \text{(190)} \)

The price of the option is the current value of a possible future cash flow \( D \), where \( C = e^{-r\tau} D \). We then have

\[
\frac{\partial D}{\partial \tau} - \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial D}{\partial Z} - \frac{1}{2} \sigma^2 \frac{\partial^2 D}{\partial Z^2} = 0
\]

\( \text{(191)} \)

At time \( t \) the mean value of \( Z \) is \( Z(0) + (r - \frac{1}{2} \sigma^2) t \), therefore if we let

\[
y = Z + \left( r - \frac{1}{2} \sigma^2 \right) \tau
\]

\( \text{(192)} \)

the first order term can be eliminated and the equation becomes

\[
\frac{\partial D}{\partial \tau} = \frac{1}{2} \sigma^2 \frac{\partial^2 D}{\partial y^2} = 0
\]

\( \text{(193)} \)

Equation \( \ref{eq:193} \) is the one-dimensional heat equation which can be solved to give

\[
C(S, t) = SN(d_1) - Ke^{-(T-t)}N(d_2)
\]

\( \text{(194)} \)

where \( N(x) \) is the cumulative normal distribution and

\[
d_1 = \frac{\log \left( \frac{S}{K} \right) + \left( r + \frac{1}{2} \sigma^2 \right) (T - t)}{\sigma \sqrt{T - t}}
\]

\( \text{(195)} \)

\[
d_2 = \frac{\log \left( \frac{S}{K} \right) + \left( r - \frac{1}{2} \sigma^2 \right) (T - t)}{\sigma \sqrt{T - t}}
\]

\( \text{(196)} \)

The Black-Scholes equation has the important property that the value of the derivative is always a smooth function of \((S, t)\) which means it can be used for a variety of complicated derivatives [59].

### 2.7.8 Jump-diffusion processes

The Black-Scholes formula is dependent on replicating the payoff structure of the option by investing in the stock. To make this possible it must be assumed that the underlying
stock returns can be described by a continuous stochastic process [67]. It is well known that prices of stocks and other financial assets can crash at which time the assumption of continuity is invalid. Merton proposed a continuous path stochastic processes with jumps to simulate the possibility of discontinuous jumps in the price. The continuous part of the model can be modelled using a geometric Brownian motion while a Poisson process can be used to model the price jumps [67].

A Poisson process can be used to model a phenomenon in which we are waiting for an event to occur. A Poisson process can be defined as follows. For each $t \geq 0$, let $N_t$ be an integer-valued random variable. If $N_t$ satisfies the following properties:

(i) $N_0 = 0$
(ii) $s < t \Rightarrow N_s$ and $N_t - N_s$ are independent
(iii) $N_s$ and $N_{t+s} - N_t$ are identically distributed
(iv) $\lim_{t \to 0} \frac{P(N_t=1)}{t} = \lambda$
(v) $\lim_{t \to 0} \frac{P(N_t>1)}{t} = 0$

then

$$P(N_t = n) = e^{-\lambda t} \left(\frac{\lambda t}{n!}\right)^n$$

that is, $N_t \sim Poisson(\lambda t)$ [30].

Merton’s jump-diffusion model can be written as

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma dW_t + dN(t)$$

where $\mu$ is the instantaneous expected return, $\sigma$ is the instantaneous variance of return, $dW_t$ is a standard Brownian motion, $dN(t)$ is a Poisson process, $\lambda$ is the mean number of arrivals per unit time and $k = E[Y - 1]$ where $Y - 1$ is the percentage in stock and $E$ the expectation operator over the random variable $Y$ [67]. The random variable governing the size of the price jumps can be defined as appropriate and without restriction, Cui & Schreider [38] use five constants to simulate the price jumps seen in empirical data of the temporary water price in the Goulburn-Murray irrigation area.

It can be seen from equation 198 that when $\lambda = 0$ the equation is the geometric Brownian
motion, which can be solved using the Black-Scholes pricing formula 194. However, when \( \lambda = 1 \) the correct proportions of stock and option that make the portfolio risk free cannot be maintained and the Black-Scholes formula cannot price the option. Joshi \[59\] writes Merton’s jump diffusion model as

\[
\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + (J - 1)dN(t)
\]  

(199)

and under the assumption that jumps are all of the same size provides

\[
\sum_{j=0}^{\infty} e^{-\lambda T} \frac{(\lambda T)^j}{j!} \text{BS}(S_0e^{(\mu-r)T}, J, r, T, K)
\]  

(200)

where \( \text{BS}(S, \sigma, r, T, K) \) is the Black-Scholes option pricing formula for an option with spot price \( S \), volatility \( \sigma \), interest rate \( r \), expiry \( T \) and strike price \( K \). In practice the infinite series would be cut off after a finite number of terms. Other more complicated formulas can be derived for multiple jump sizes \[59\].

### 2.7.9 Mean-reverting processes

Mean-reverting processes tend to oscillate around a long term average or equilibrium state. A mean-reverting process first proposed by Ornstein and Uhlenbeck \[111\] can be written as

\[
dX_t = (\theta_1 - \theta_2 X_t)dt + \theta_3 dW_t, \quad X_0 = x_0
\]  

(201)

where \( \theta_1, \theta_2 \in \mathbb{R} \) and \( \theta_3 \in \mathbb{R}^+ \). For \( \theta_2 > 0 \) mean reverts to an equilibrium state. For \( t \geq 0 \) the Ornstein-Uhlenbeck process has finite variance. Equation 201 can be solved by using Ito’s lemma. In differential form Ito’s lemma may be written as

\[
f(t, X_t) = f_t(t, X_t)dt + f_x(t, X_t)dX_t + \frac{1}{2} f_{xx}(t, X_t)(dX_t)^2
\]  

(202)

If \( X_t \) is a Brownian motion this can be simplified to

\[
df(t, W_t) = \left( f_t(t, W_t) + \frac{1}{2} f_{xx}(t, W_t) \right) dt + f_x(t, W_t)
\]  

(203)
If we choose \( f(t, x) = xe^{\theta_2 t} \) we have

\[
f_t(t, x) = \theta_2 f(t, x), \quad f_x(t, x)e^{\theta_2 t}, \quad f_{xx}(t, x) = 0
\]  

(204)

Therefore we have

\[
X_t e^{\theta_2 t} = f(t, X_t)
\]

\[
= f(0, X_0) + \int_0^t \theta_2 X_u e^{\theta_2 u} du + \int_0^t e^{\theta_2 u} dX_u
\]

\[
= x_0 + \int_0^t \theta_2 X_u e^{\theta_2 u} du + \int_0^t e^{\theta_2 u}[(\theta_1 - \theta_2 X_u) du + \theta_3 dW_u]
\]

\[
= x_0 + \theta_1 e^{\theta_2 t} - 1 + \theta_3 \int_0^t e^{\theta_2 u} dW_u
\]

(205)

then dividing through by \( e^{\theta_2 t} \) gives

\[
X_t = \frac{\theta_1}{\theta_2} + \left( x_0 - \frac{\theta_1}{\theta_2} \right) e^{-\theta_2 t} + \theta_3 \int_0^t e^{-\theta_2 (t-u)} dW_u
\]

(206)

Furthermore, for \( t > 0 \) the transitional density \( P(t, X_t|X_0 = x_0) \) is Normal with expected value and variance given by

\[
E[X_t|X_0 = x_0] = \frac{\theta_1}{\theta_2} + \left( x_0 - \frac{\theta_1}{\theta_2} \right) e^{-\theta_2 t}
\]

(207)

and

\[
\text{Var}[X_t|X_0 = x_0] = \frac{\theta_2^2 \left( 1 - e^{-2\theta_2 t} \right)}{2\theta_2}
\]

(208)

respectively and for \( \theta_2 > 0 \) the Ornstein-Uhlenbeck process is ergodic [53].

2.8 Optimisation

In this thesis optimal solutions to commodity allocation problems are solved using the optimisation techniques discussed in this section. Optimisation entails finding the ‘best’ solution to a problem from an ‘acceptable’ set of solutions. The way this solution set is formed and the way the ‘best’ solution is defined can be used to divide optimisation problems into different segments which can then be solved using appropriate techniques.
In general an optimisation problem may be represented as

\[
\min_X f(x) \quad (209)
\]

where \( x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n \). The range of values that the optimal solution can take in the above formulation is not bounded and can take on any value in \( \mathbb{R} \), such problems are called unconstrained optimisation problems. A constrained optimisation problem may be written as

\[
\min_X f(x) \\
\text{subject to } g_j(x) \leq b_j, \quad j = 1, 2, \ldots, m \\
r_j(x) = c_j \quad j = 1, 2, \ldots, m \quad (210)
\]

In this case the set of possible solutions, known as the feasible set, is a subset of \( \mathbb{R}^n \).

The technique for solving an optimisation problem depends on the type of problem being optimised.

2.9 Selected topics from convex analysis

This section will present the necessary theory from convex analysis needed for the development of optimisation theory.

2.9.1 Convex sets

A line segment between \( x \) and \( y \) in \( \mathbb{R}^n \) is the set of points on the straight line between \( x \) and \( y \). If the point \( z \) lies on the line we can write \( z = \alpha x + (1 - \alpha) y \) where \( \alpha \in [0, 1] \) and define the line segment as

\[
\{ \alpha x + (1 - \alpha) y : \alpha \in [0, 1] \} \quad (211)
\]
A point \( z = \alpha x + (1 - \alpha)y \) is called a convex combination of \( x \) and \( y \). A set \( S \subseteq \mathbb{R}^n \) is a convex set if for all \( x, y \in S \) the line segment joining \( x \) and \( y \) is in \( S \).

### 2.9.2 Convex functions

A function \( f(x) \) over a convex set \( S \) is a convex function if

\[
f(\alpha x + (1 - \alpha)y) \leq \alpha f(x) + (1 - \alpha)f(y)
\]

(212)

The function is strictly convex if the strict inequality holds. A function \( f \) is concave if \( -f \) is convex and strictly concave if \( -f \) is strictly convex.

Optimising a convex function over a convex set has the desirable property that any local minimum is also a global minimum, furthermore if the set is strictly convex then the minimum is unique.

### 2.10 The gradient vector

For any function \( f(x) \in C^1 \), where \( C^1 \) denotes the set of continuous, differentiable functions, the vector of first order partial derivatives at point \( x \) is

\[
\nabla f(x) = \begin{bmatrix}
\frac{\partial f}{\partial x_1}(x) \\
\frac{\partial f}{\partial x_2}(x) \\
\vdots \\
\frac{\partial f}{\partial x_n}(x)
\end{bmatrix}
\]

(213)

which may be referred to as the gradient vector. For smooth functions the gradient vector is perpendicular to the contour at \( x \) and is in the direction of maximum increase of \( f(x) \).
2.11 The Jacobian and Hessian matrices

The Jacobian matrix is the matrix of first-order partial differential equations of the function \( f(x) \in C^1 \)
\[
\nabla f(x) = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1}(x) & \frac{\partial f_1}{\partial x_2}(x) & \cdots & \frac{\partial f_1}{\partial x_n}(x) \\
\frac{\partial f_2}{\partial x_1}(x) & \frac{\partial f_2}{\partial x_2}(x) & \cdots & \frac{\partial f_2}{\partial x_n}(x) \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_m}{\partial x_1}(x) & \frac{\partial f_m}{\partial x_2}(x) & \cdots & \frac{\partial f_m}{\partial x_n}(x)
\end{bmatrix}
\] (214)

The matrix of second order partial derivatives of the function \( f(x) \in C^2 \) where \( C^2 \) denotes the set of continuous, twice differentiable functions
\[
\nabla^2 f(x) = \begin{bmatrix}
\frac{\partial^2 f}{\partial x_1^2}(x) & \frac{\partial^2 f}{\partial x_1 \partial x_2}(x) & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n}(x) \\
\frac{\partial^2 f}{\partial x_2 \partial x_1}(x) & \frac{\partial^2 f}{\partial x_2^2}(x) & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n}(x) \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2 f}{\partial x_n \partial x_1}(x) & \frac{\partial^2 f}{\partial x_n \partial x_2}(x) & \cdots & \frac{\partial^2 f}{\partial x_n^2}(x)
\end{bmatrix}
\] (215)

is known as the Hessian.

2.12 Quadratic forms

A quadratic function of \( n \) variables can be written as
\[
f(x) = x^T M x
\] (216)

where \( M \) is an \( n \times n \) real symmetric matrix. For non-zero vectors of \( x \) and \( x^T M x > 0 \) the quadratic form is said to positive definite, it is negative definite if \( x^T M x > 0 \). For \( x^T M x \geq 0 \) the quadratic form is positive semi-definite for all \( x \) and negative definite if \( x^T M x \leq 0 \).
2.13 Testing for convexity

The Hessian matrix can be used to test if a function is convex. A function $f(x) \in C^2$ defined over an open convex set $S$ is convex if and only if the Hessian is positive semi-definite for all $x \in S$. If the Hessian is positive-definite then the function is strictly convex.

2.14 Unconstrained minimisation

An unconstrained minimisation problem seeks to find minimum value of a function $f(x)$ without restriction on the range of the function. The problem may be written as

$$\min_{X} f(x)$$  \hspace{1cm} (217)

where $x = (x_1, x_2, \ldots, x_n)^T \in X \subseteq \mathbb{R}^n$. In this section the necessary requirements for a minimum to exist will be stated, as will the theory to determine whether the minimum is a local or global minimum and whether it is a unique.

2.14.1 Local and global minima

A point $x^* \in X$ is a local minimiser of $f(x)$ over the set $X$ if $f(x) \geq f(x^*)$ for all $x \in X$ and $\|x - x^*\| < \varepsilon$, where $\varepsilon$ is some positive number and $\| \cdot \|$ denotes the Euclidean norm. The point $x^*$ is a global minimiser of $f(x)$ over $X$ if $f(x) \geq f(x^*)$ for all $x \in X$. Replacing the weak inequalities with strong ones give the conditions for strong local minimisers and strong global minimisers respectively. Local and global maxima can be identified by considering $-f(x)$.
2.14.2 Necessary and sufficient conditioners for local minimisers

When considering unconstrained optimisation problems a necessary condition for a strong local minimum of \( f(x) \) over the interior of the set \( X \) at the point \( x^* \) is that

\[
\nabla f(x^*) = 0
\]

A sufficient condition is that \( H(x^*) \) be positive-definite. For boundary points we first define a directional derivative and a feasible direction. The directional derivative of \( f(x) \) in the direction of \( d \) is given by \( \nabla^T f(x)d \) and a direction is feasible if there exists an \( \alpha > 0 \) such that \( x + \alpha d \in X \). Then the first order necessary condition is

\[
\nabla^T f(x^*d) \geq 0
\]

for feasible directions of \( d \).

2.15 Classical methods for constrained optimisation

A constrained optimisation problem of the form

\[
\min_x f(x)
\]

subject to \( h_j(x) = 0 \quad j = 1, 2, \ldots, m < n \)

(220)

can be transformed into unconstrained optimisation problems by adding Lagrange multipliers. The new problem is written as

\[
L(x, \lambda) = f(x) + \sum_{j=1}^{m} \lambda_j h_j(x) = f(x) + \lambda^T h(x)
\]

(221)

The idea of adding the Lagrange multipliers to the problem is to change the constrained optimisation problem into a form that allows the first-order conditions of the unconstrained problem to be applied.

The necessary first order conditions for a local minimiser \( X^* \) can be presented in terms of
the Lagrangian function [99]. First a definition of a regular point is provided. A point \( \mathbf{x}^* \) can be defined as regular point of the constraints if the gradient vectors of the constraints are linearly independent, or equivalently if the Jacobian matrix of \( \mathbf{h}(\mathbf{x}) \)

\[
\nabla \mathbf{h}(\mathbf{x}) = \begin{bmatrix}
\frac{\partial L}{\partial x_1}(\mathbf{x}) \\
\frac{\partial L}{\partial x_2}(\mathbf{x}) \\
\vdots \\
\frac{\partial L}{\partial x_n}(\mathbf{x})
\end{bmatrix}
\]

(222)

is of full rank. Then if \( f(\cdot), h_j(\cdot) \in C^1 \), the necessary condition for \( \mathbf{X}^* \) to be a local minimiser is that the stationary point \((\mathbf{x}^*, \lambda^*)\) of the Lagrangian function coincide with \( \mathbf{x}^* \). Stated another way, there exists \( \lambda^* \in \mathbb{R}^m \) such that

\[
\frac{\partial L}{\partial x_i}(\mathbf{x}^*, \lambda^*) = 0, \quad i = 1, 2, \ldots, n
\]

and

\[
\frac{\partial L}{\partial \lambda_j}(\mathbf{x}^*, \lambda^*) = 0, \quad j = 1, 2, \ldots, m.
\]

(223)

In the form of a mathematical theorem: Let \( \mathbf{x}^* \) be a local minima of \( f(\mathbf{x}) \) where \( f : \mathbb{R}^n \to \mathbb{R} \) subject to the constraints \( h(\mathbf{x}) = 0 \), \( \mathbf{h} : \mathbb{R}^n \to \mathbb{R}^m \), and assume that \( \mathbf{x}^* \) is regular. Then there exists \( \lambda^* \in \mathbb{R}^m \) such that

\[
Df(\mathbf{x}^*) + \lambda^T Dh(\mathbf{x}^*) = 0^T
\]

(224)

where \( D \) is the derivative operation [32].

2.15.1 Second order necessary conditions for constrained optimisation

The second-order necessary condition for constrained optimisation first requires a definition of the tangent space. The constraints in equation 220 describe a surface \( S = \{ \mathbf{x} \in \mathbb{R}^n : h_1(\mathbf{x}) = 0, \ldots, h_m(\mathbf{x}) = 0 \} \). The tangent space on the surface \( S \) at point \( \mathbf{x}^* \) can then be defined as the set \( T(\mathbf{x}^*) = \{ \mathbf{y} : Dh(\mathbf{x}^*)\mathbf{y} = 0 \} \). Then if we let \( \mathbf{x}^* \) be a local minima of \( f(\mathbf{x}) \) where \( f : \mathbb{R}^n \to \mathbb{R} \) subject to the constraints \( h(\mathbf{x}) = 0 \), \( \mathbf{h} : \mathbb{R}^n \to \mathbb{R}^m \), \( m \leq n \), and assume that \( f, \mathbf{h} \in C^2 \) and that \( \mathbf{x}^* \) is regular. Then there
exists $\lambda^* \in \mathbb{R}^m$ such that

$$Df(x^*) + \lambda^T Dh(x^*) = 0^T$$

and

For all $y \in T(x^*)$, we have $y^T L(x^*, \lambda^*) y \geq 0$ (225)

which is a necessary second-order condition for a local minima. If there exists a point $x^* \in \mathbb{R}^n$ and $\lambda^* \in \mathbb{R}^m$ such that

$$Df(x^*) + \lambda^T Dh(x^*) = 0^T$$

and

For all $y \in T(x^*), y \neq 0$, we have $y^T L(x^*, \lambda^*) y > 0$ (226)

we have sufficient second-order conditions for a local minima.

In words, if $x^*$ satisfies the first-order condition, and $L(x^*, \lambda^*)$ is positive-definite on $T(x^*)$, then $x^*$ is a strict local minima. Similarly, if $x^*$ satisfies the first-order condition, and $L(x^*, \lambda^*)$ is negative-definite on $T(x^*)$, then $x^*$ is a strict local minima [32].

2.15.2 Inequality constraints and Karush-Kuhn-tucker conditions

Optimisation problems with inequality constraints take the form

$$\min_x f(x)$$

subject to

$$h_i(x) = 0 \quad i = 1, 2, \ldots, m < n$$

$$g_j(x) \leq 0 \quad j = 1, 2, \ldots, p$$

(227)

Before stating the first-order necessary condition for a local minima we first define active inequality constraints as those where $g(x^*) = 0$ at $x^*$ and inactive inequality constraints as those where $g(x^*) < 0$ at $x^*$. Then, if we let $J(x^*)$ denote the set of active inequality constraints at $x^*$ we can define $x^*$ as a regular point if $\nabla h_i(x^*)$, for $1 \leq i \leq m$ and
\( \nabla g_j(\mathbf{x}^*) \) where \( j \in J(\mathbf{x}^*) \) are linearly independent. The first-order necessary conditions can then be stated as follows. Assuming that \( f, \mathbf{h}g \in C^1 \) and that \( \mathbf{x}^* \) is a regular point and a local minimiser of \( f \) and satisfies the constraints \( \mathbf{h}(\mathbf{x}^*) = 0 \) and \( g(\mathbf{x}^*) \leq 0 \). Then there exists \( \lambda^* \in \mathbb{R}^m \) and \( \mu^* \in \mathbb{R}^p \) such that:

\[
\mu^* \geq 0 \\
Df(\mathbf{x}^*) + \lambda^{*T}D\mathbf{h}(\mathbf{x}^*) + \mu^{*T}Dg(\mathbf{x}^*) = 0^T \\
\mu^{*T}g(\mathbf{x}^*) = 0
\]

These first-order necessary conditions can be used to identify potential minima which can then be investigated further [32].

Before stating the second-order necessary conditions we first define the matrix

\[
L(\mathbf{x}, \lambda, \mu) = F(\mathbf{x}) + [\lambda \mathbf{H}(\mathbf{x})] + [\mu \mathbf{G}(\mathbf{x})]
\]

where

\[
[\lambda \mathbf{H}(\mathbf{x})] = \lambda_1 \mathbf{H}_1(\mathbf{x}^*) + \ldots + \lambda_m \mathbf{H}_m(\mathbf{x}^*)
\]

and

\[
[\mu \mathbf{G}(\mathbf{x})] = \mu_1 \mathbf{G}_1(\mathbf{x}^*) + \ldots + \mu_m \mathbf{G}_m(\mathbf{x}^*)
\]

where \( \mathbf{H}_k(\mathbf{x}) \) is the Hessian of \( h_k \) at \( \mathbf{x}^* \) and \( \mathbf{G}_k(\mathbf{x}) \) is the Hessian of \( g_k \) at \( \mathbf{x}^* \).

The second-order necessary conditions for a local minimum can then be stated as follows. Let \( \mathbf{x}^* \) be a local minimum of equation 227 and assume that is regular. Then there exists \( \lambda^* \in \mathbb{R}^m \) and \( \mu^* \in \mathbb{R}^p \) such that:

\[
\mu^* \geq 0 \\
Df(\mathbf{x}^*) + \lambda^{*T}D\mathbf{h}(\mathbf{x}^*) + \mu^{*T}Dg(\mathbf{x}^*) = 0^T, \mu^{*T}g(\mathbf{x}^*) = 0 \\
\text{and for all } \mathbf{y} \in T(\mathbf{x}^*) \text{ we have } \mathbf{y}^T L(\mathbf{x}^*, \lambda^*, \mu^*) \mathbf{y} \geq 0
\]

(229)

where \( T(\mathbf{x}^*) = \{ \mathbf{y} \in \mathbb{R}^n : D\mathbf{h}(\mathbf{x}^*) \mathbf{y} = 0, Dg_j(\mathbf{x}^*) \mathbf{y} = 0, j \in J(\mathbf{x}^*) \} \) which is the tangent space defined by the active constraints [32].

Before stating the second order sufficient condition for \( \mathbf{x}^* \) to be a strict local minimiser
we first define the set $\tilde{T}(x^*, \mu^*) = \{y : Dh(x^*)y = 0, Dg_j(x^*)y = 0, j \in \tilde{J}(x^*, \mu^*)\}$ and $\tilde{J}(x^*, \mu^*) = \{i : g_i(x^*) = 0, \mu^*_i > 0\}$. Then, if $f, h, g \in C^2$ and there exists a feasible point $x^*$ and vectors $\lambda^* \in \mathbb{R}^m$ and $\mu^* \in \mathbb{R}^p$ such that

$$
\begin{align*}
\mu^* &\geq 0 \\
Df(x^*) + \lambda^* Dh(x^*) + \mu^* Dg(x^*) &= 0, \mu^* T g(x^*) = 0 \\
\text{and for all } y \in \tilde{T}(x^*, \mu^*), y \neq 0, \text{ we have } y^T L(x^*, \lambda^*, \mu^*) y > 0
\end{align*}
$$

then $x^*$ is a strict local minimiser of equation 227. For $x^*$ to be a strict local maximiser we have a similar definition with the difference being the $\mu^* < 0$ and $L(x^*, \lambda^*)$ is negative definite on $\tilde{T}(x^*, \mu^*)$ [32].

2.16 Mathematical programming

Mathematical programming techniques have been developed to overcome some of the short comings of optimisation using calculus. In particular, the ability to incorporate inequality constraints into the problem is a valuable option that mathematical programming allows. We are also able to include more constraints than decision variables by using mathematical programming, the classical techniques discussed earlier preclude these possibilities.

2.17 Linear programming

Linear programs contain an objective function and a set of constraints that are all linear. The set of constraints includes non-negativity constraints and may include equality or inequality constraints as well.

The general formulation of a linear programming problem with $n$ decision variables and
The decision variables are denoted by the $x_i$, and the coefficients of the decision variables by $c_i$ where $i = 1, 2, \ldots, n$. The coefficients of the decision variables in the constraint function are denoted by $a_{ij}$, for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$ and the constants $r_j$ bound the constraint functions.

We can make use of the summation operator to write the problem more compactly.

$$
\begin{align*}
\min_{x_1, x_2, \ldots, x_n} & \quad z = c_1 x_1 + \ldots + c_n x_n \\
\text{subject to} & \quad a_{11} x_1 + \ldots + a_{1n} x_n \geq r_1 \\
& \quad \vdots \\
& \quad a_{m1} x_1 + \ldots + a_{mn} x_n \geq r_m \\
& \quad x_i \geq 0 \quad (i = 1, 2, \ldots, n)
\end{align*}
$$

(231)

We can also write the problem using matrix notation. The decision variables are represented by the column vector $\mathbf{x}$, and their coefficients in the objective function are represented by the column vector $\mathbf{c}$. The coefficients of the decision variables in the constraint function are represented by matrix $\mathbf{A}$, and the constraint bounds by vector $\mathbf{r}$.

$$
\begin{bmatrix}
 x_1 \\
 x_2 \\
 \vdots \\
 x_n
\end{bmatrix}, \quad
\begin{bmatrix}
 c_1 \\
 c_2 \\
 \vdots \\
 c_n
\end{bmatrix}, \quad
\begin{bmatrix}
 a_{11} & a_{12} & \cdots & a_{1n} \\
 a_{21} & a_{22} & \cdots & a_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}, \quad
\begin{bmatrix}
 r_1 \\
 r_2 \\
 \vdots \\
 r_m
\end{bmatrix}
$$
The problem is then written as

\[
\begin{align*}
\min_{x} & \quad z = c'x \\
\text{subject to} & \quad Ax \geq r \\
\text{and} & \quad x \geq 0
\end{align*}
\]  

(233)

For the case of \( n = 2 \) decision variables we can solve a linear programming problem using a graphical solution method. The non-negativity constraints restrict our solution to the non-negative quadrant. We then treat each constraint as an equation and plot the lines, before reducing the solution space further by noting the type of inequality constraint and considering only those points that satisfy all the constraints. The set of points that satisfy all the constraints is known as the feasible region and each point as a feasible solution. To find the optimal solution from this set we consider the intersections of the constraint functions or the intersection of a constraint function with an axis. These are known as the extreme points of the feasible region. By graphing the objective function as a family of parallel lines, with the value of \( z \) either increasing or decreasing depending on whether we are maximising or minimising the function, we find the optimal solution at the point where the objective function meets one of the extreme points. We can then find the value of the optimal solution by solving the equations of the two intersecting lines for the values of the two decision variables. For problems involving more than two decision variables we need another technique to find the optimal value of the objective function.

When the objective function contains \( n \) decision variables we working in an \( n \)-dimensional space, however the principals that we discussed for problems with two decision variables can be extended to the case of \( n \) decision variables. The non-negativity constraints reduce the feasible solution space to the non-negative orthant. The constraint functions reduce the feasible solution space further to a subset of the non-negative orthant, and once again, the optimal solution occurs at one of the extreme points of the feasible solution space and can be identified using the objective function.

As the optimal solution always occurs at an extreme point of the feasible solution space in linear programming problems, we only need to identify the set of extreme points and then find the optimal one. This is because the set of feasible solutions is always a closed
2.17.1 Convex sets and linear programming

In linear programming problems an objective function with \( n \) decision variables

\[
z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n
\]

defines a hyperplane. The set of points in the hyperplane defined by equation 234, which we will denote by \( H \), can be shown to be a convex set. It also divides an \( n \)-dimensional space into two half-spaces, which may be called open or closed half-spaces if the hyperplane is excluded or included respectively. Similarly, all the constraints in a linear programming problem form half spaces with those utilising strict inequalities being closed and those with weak inequalities being open half-spaces, and as with the objective function, each of these half-spaces is also a convex set. As any feasible solution must lie in the intersection of each of the \( m \) constraints and \( n \) non-negativity restrictions we can make use of the fact that the intersection of a finite number of closed convex sets forms another closed convex set. Thus the set of feasible solutions must also be a convex set. We will denote the set of feasible solutions as \( F \).

**Extreme points and the optimal solution:** We now discuss the interior, boundary and extreme points of the set \( F \). Informally we may define a boundary point \( b \) as a point such that in any neighbourhood of \( b \) we must have a point that is not in \( F \). Continuing informally, an interior point can be defined as a point \( i \) such that in any neighbourhood of \( i \) all points are in the set \( F \), and an extreme point of \( F \) is a point that cannot be derived from the convex combination of any other two points in \( F \). In linear programming problems the hyperplane defined by the objective function is increased or decreased until it contains no interior points of set \( F \), but at least one boundary point. A hyperplane which satisfies this requirement is called a supporting hyperplane. Supporting hyperplanes have the following useful properties. Firstly, if a point \( u \) is a boundary point of a closed convex set, then there is at least one supporting hyperplane at \( u \), and secondly, for a closed convex set bounded from below, there is at least one extreme point in every supporting hyperplane. Thus, we know that the optimal solution occurs at a boundary point and that it is also an extreme point. These two facts give rise to the simplex method of
solving linear programs which provides a systematic method for checking extreme points until the optimal solution is found.

Furthermore, if the set of feasible solutions $F$ is a closed convex set and the objective function is a continuous concave function over the set then the local maximum is also the global maximum. Should the objective function be strictly concave then the maximum is unique. Unlike classical optimisation, in linear programming these sufficient conditions for the global optimum are satisfied as the feasible set is always a closed convex set and the objective function is continuous and linear.

2.17.2 Duality

Thus far in our discussion we have referred to linear programming problems in terms of maximisation or minimisation, however for every maximisation problem we have a corresponding minimisation problem, and for every minimisation problem we have a corresponding maximisation problem. The original problem is known as the primal problem and the corresponding program is known as the dual problem. If the primal program seeks to minimise $Z$ then its dual will seek to maximise a new variable $Z^*$, and similarly if the primal problem seeks to maximise $Z$, then its dual will seek to minimise $Z^*$. The primal and dual problems have the same optimal value and it is always possible to translate the solution for one into the solution for the other. Therefore by solving one we have the solution for the other.

**The dual program:** The relationship between the primal and dual programs for a maximisation problem are shown in equation 235.

\[
\begin{align*}
\text{Primal} & : & \max_x & z = c'x & \rightarrow & \min_x & z^* = r'y \\
\text{subject to} & & A x & \leq r & & A' y & \geq c \\
\text{and} & & x & \geq 0 & & y & \geq 0
\end{align*}
\]

(235)

and similarly, the relationship between the primal and dual problems for a minimisation
problem are shown in equation 236

\[
\begin{align*}
\text{Primal} & \quad \min_x z = c'x \\
\text{subject to} & \quad Ax \leq r \\
\text{and} & \quad x \geq 0 \\
\end{align*}
\]  \quad \rightarrow \quad
\begin{align*}
\text{Dual} & \quad \max_x z^* = r'y \\
\text{subject to} & \quad A'y \geq c \\
\text{and} & \quad y \geq 0 \\
\end{align*}
\]  (236)

To change the primal problem into the dual problem we must first change the objective from maximising to minimising or vice versa. In the notation above an asterisk is attached to the dual problem to indicate that it is the dual. The inequalities in the constraints of the primal problem are reversed in the dual however the non-negativity restrictions are left unchanged. The coefficient matrix for the constraints in the dual problem is simply the transpose of the coefficient matrix of the constraints in the primal problem. The constraints in the dual problem are bound by a column vector of constants that is the transpose of the coefficients on the decision variables in the objective function. The coefficients of the decision variables in the dual problem is the transpose of the column vector of constraint bounds in the primal problem.

The primal problem has \( n \) decision variables and \( m \) constraints while the dual problem has \( m \) decision variables and \( n \) constraints. The difference in the number of decision variables and constraints may lead to the dual problem being easier to solve than the primal problem. The problem with the fewest constraints will have a solution space of a smaller dimension and will therefore fewer dummy variables making the computation process more efficient. It may also be desirable to solve a maximisation problem since we have an initial basic feasible solution and therefore have no need for artificial variables.

However, even if the dual problem is easier to solve it is the primary problem that we are endeavouring to solve. Two results assists in this case, firstly provided optimal feasible solutions exist, the optimal value of the primal and dual objective functions are the same. Also there is a relationship between the choice variable in the primal program and the dummy variable in the dual. If the optimal value of the choice variable is non-zero in the primal then the corresponding dummy variable in the dual is zero and if the dummy variable in the primal program is non-zero in the the optimal value of the corresponding choice variable in the dual is zero.
The dual of a linear program can serve as a tool to facilitate a solution for the primal program, however it also has a significant economic meaning. The dual variables represent the opportunity cost (or shadow price) of using some resource. A shadow price represents the change in the objective function realised when a constraint is relaxed. Maximising the primal problem is the same as minimising the opportunity cost. Dixon & Parmenter [41] describe the use of shadow prices in an economic/pollution problem. Reducing pollution comes at the cost of resources absorbed and the pollution generated through the reduction activities. Constrained linear programming models of economic output have been used to examine the structure of an economy, net of pollution abatement costs and subject to exogenous constraints on the levels of net pollution. The shadow prices of these constraints are then interpreted as the opportunity costs of reducing pollution.

2.18 Concluding remarks

The mathematics discussed in this section is used throughout the literature pertaining to gas network optimisation and price modelling. The physical properties of natural gas were discussed and the characteristics of the two main types of compressors in gas networks presented. There are many flow equations that have been developed to model gas flow through different types of networks and those featuring most prominently in the literature were discussed. Two types of incidence matrices commonly used to describe network topologies were stated along with Kirchoff’s laws of flow balance. These topics are necessary for steady state analysis of gas flow networks. Analysing a gas network in its steady state is far more common in the literature than tackling transient analysis, however the basic equations that underpin transient analysis were discussed also. Topics from stochastic calculus that have been applied to modelling financial assets are mentioned so that a spot price model for natural gas can be developed, in a similar vain the basics of optimisation theory needed for analysis of the gas transmission network are outlined.
3 Literature review and background

In this section of the state of the research concerning the gas and water delivery networks is discussed. Much of the literature on natural gas networks is focused on minimising transportation costs. The cost of transporting natural gas is largely due to the cost of compressing the gas to create the required pressure difference between the origin and destination nodes. For networks in the planning stage optimisation techniques have been applied to find the best network configuration to minimise both the cost of construction and the cost of operating and maintaining the network. Natural gas is increasingly being used as a fuel source for electricity generators. Research on the interaction between gas and electricity networks is discussed in this section. Markets for natural gas are expanding as regional networks are connected, both by pipeline and by shipping liquefied natural gas (LNG). There is also diminishing government involvement in many markets with competition between market participants being permitted and encouraged. The research on modelling markets for natural gas is reviewed as is the literature on modelling commodity spot prices, including those that have been applied to natural gas. The state of research on water networks is reviewed, beginning with a discussion concerning water allocations and entitlements, and followed by research in the used of water markets and trade as an allocation tool.

3.1 Minimising compressor consumption

The literature on optimising gas networks is growing as the demand for natural gas and the size and complexity of gas networks grows. Much of the literature concentrates on minimising the cost of transportation through the network. Pressure difference in the pipes drive gas flow and gas compressors are used to create the pressure difference. The compressors consume gas as their fuel source and attempts to minimise transport costs typically focus on minimising the gas consumed by the compressors while satisfying demand and maintaining system pressure restrictions.

Gas transmission pipelines can run for hundreds of kilometres and energy and pressure are lost primarily because of friction between the inner surface of the pipe and the gas although energy is also lost through heat exchange between the gas and the environment.
As a result it is necessary in most cases to restore the pressure differential periodically along the route by installing compressor stations. The compressor stations can consume as much as 5% of the gas transported and as such maximising flow while minimising gas consumed by compressors is an important consideration for pipeline operators. Mathematical models of the compressor operation and the fuel cost problem are presented in [119]. The complexities of the problem which include pressure drops at the network nodes, the mass-flow rate through each pipeline, the fact that compressors can be bought on or taken off-line and valves can be open or closed are all noted. These factors result in a complex problem which includes nonlinear constraints and decision variables and a non-convex, discontinuous feasible solution set.

There are stochastic variables to contend with as well, demand is uncertain for example, and gas flow through the network is transient and thus time-dependent. These factors form a problem that can describe as a transient technical optimisation problem. In [66] a model that is intended to be an aid for a gas network operator is proposed with the intention of simplifying the problem to allow usable results to be obtained within a 15 minute time frame. To achieve this the focus is placed on the time-dependent and discrete aspects of the problem while the stochastic variables are neglected. The result is a mixed-integer linear programming problem with the nonlinearities approximated using sum of squares constraints. The proposed model divides the set of edges into pipe, compressor and valve subsets. The pipes have connections as a subset. The connections are connecting pipes of short lengths in which there is no pressure loss. The set of valves has control valves as a subset. The model includes binary variables for compressors and valves which depend on whether they are operating or not, and additional binary variables to indicate the time step at which a compressor begins operating or is shut down. The nodes in the network represent suppliers, consumers and intersections of pipelines. In each time period the state of each valve and compressor is considered, as is the flow and pressure of the connections. The fuel consumption by the compressors is modelled by a nonlinear function that is neither convex nor concave. This function is approximated by a piece-wise linear function. The gas dynamics in the pipes are described by three partial differential equations with the assumption of a constant temperature throughout the network and horizontal pipeline segments. The system of equations is made discrete in time and space by considering it across hourly time steps and at the beginning and end of pipeline segments only. The resulting set of nonlinear equations are approximated by linear functions with errors deemed acceptable for practical applications. Further transient conditions are considered regarding the minimum runtime and downtime for compressors while the cost of switching of compressors is also included. The objective
function is divided into two parts, one for the gas consumed running the compressors and the second for the switching costs associated with their operation. The model was tested on two artificial networks and on one actual network located in western Germany. The model was not able to provide solutions in all of the scenarios envisaged, however the results for the simulation on the one real network suggest the approach is worthy of further investigation.

Steinbach [103] approaches the same problem of minimising gas transportation costs by minimising gas consumed by compressors while providing usable solutions that can be used as a tool to aid the network operator in real time. The proposed solution is a model that treats the dynamic equations for the network elements as discrete in a similar manner to [66]. They too neglect the stochastic component assuming reliable weather forecasts will ensure predictable demand and focus on developing a solution method for the partial differential equations governing the flow of gas through the pipes while assuming the discrete aspects of the problem are solved independently. They were able to implement an algorithm that can solve the system of discretised partial differential equations with an improvement on the speed and amount of memory needed by the public domain algorithms currently being used. Kolb, Lang and Bales [63] approach the fuel minimisation problem as a nonlinear mixed-integer problem and use techniques from discrete optimisation. This necessitates discretisation of the nonlinear equations governing gas flow and gas consumed by compressors, and the approximation of all other constraints by piece-wise linear equations. They model a network as directed finite graph, this fixes the direction of gas flow. The edges of the graph correspond to the components of the network. The gas flow is explained by the isothermal Euler equations, with a friction term, which are discretised using a simple finite-difference method so that high dimensional nonlinear terms are avoided. The focus of their work is to introduce an adaptive linearisation method that can maintain accuracy while handling the complexity of the model. Numerical results on two simplified network suggest accurate results can be obtained using this technique.

Rios-Mercado, Wu, Scott, and Boyd [88] propose a reduction technique that exploits the properties of the gas pipelines. The problem as formulated is to find the optimal values for the pressures at the nodes and the flow rate through the pipes and compressors that minimises the fuel used by the compressors. Constraints are placed on the pressure at each node, the operating limits of the compressors, the gas flow through each pipe and the mass flow balance at each node. Under the formulation of the general problem the feasible
domain for the variables governing compressor operation, namely the mass flow rate and the suction and discharge pressures, are typically non-convex, the pipe flow equations form a non-convex set and the fuel minimisation functions in the objective function are typically non-convex, nonlinear and often discontinuous. These characteristics make the fuel minimisation problem difficult to solve. By using the proposed reduction technique the problem can be simplified so that the fuel minimisation functions in the objective function depend on the suction pressure and the discharge pressure at each compressor station.

The reduction technique proposed in [88] is explained as follows. Firstly the network is divided into a set of disjoint sub-networks by removing the compressor station arcs and leaving only the nodes and pipes. Then a new network is constructed by considering each sub-network as a single node and replacing the compressor arcs. The new network is called a reduced network. Each network has a unique reduced network which may have a graph represented by a tree or be a graph with cycles, in either case it will have a less complicated structure that the original network graph. In the case where the reduced network is a tree it can be shown that the mass flow rates through the compressor stations can be fixed if the supplying flow rates, the sources, at all nodes are given. If the reduced network is a tree the source value at the nodes is the sum of the source values in the sub-network represented by each node. Thought of in this way the sources at the nodes in the reduced network are fixed, and as the reduced graph is a tree, the flow rates through the edges of the reduced network are uniquely determined. Furthermore, as each edge in the reduced network represents a compressor station in the original network, it means that the flow rates through all the compressor stations are known. The sources at the nodes in the sub-network, including those connected to compressor stations, are also known, and it can be shown that the flow rates through all the pipes in the sub-network can be uniquely determined. The pressures at all the nodes in the sub-network are uniquely determined by the pressure at a reference node and these pressures will rise and fall as the pressure at the reference node rises and falls. The general problem is now simplified as the flow rates through all the compressor stations is known, thus fuel minimisation functions depend only on input and output pressure at the compressors. Therefore the objective function depends only on the input and output pressures also.

The reduction technique proposed reduced the number of variables from the sum of the nodes, pipes and compressors to, at most, two pressure variables for each compressor station, it simplifies and linearises some of the nonlinear pipe flow constraints and reduces
the number of nonlinear equality constraints remaining. The trade off for simplifying the problem using this technique is that the network flow equations for each sub-network need to be solved. In the case where the reduced network contains cycles, the flow rates cannot be uniquely determined. In this case the mass flow rate though the compressor stations satisfies a system of linear equations and the number of independent variables in the system is equal to the number of fundamental cycles in the associated reduced network. The mass flow rate must be bounded and this bound forms a feasible set of values for the mass flow rate. The fuel minimisation problem then becomes one of minimising a function of the mass flow rate over this feasible set. Therefore the reduction technique can still be used. This reduction technique can reduce the size of a complex problem by more than an order of magnitude without disrupting the mathematical structure of the problem. Rios-Mercado, Kim, and Boyd [89] note that there are few effective algorithms for optimising networks with cyclic structures. A cyclic network is a network with at least one cycle containing two or more compressor station arcs. They suggest a heuristic solution procedure for the fuel minimisation problem on a network with cycles. The problem is modelled as a nonlinear non-convex network flow problem and a two stage iterative solution technique is proposed. To simplify the model it is assumed that the network is composed only of pipes, nodes and compressors. The compressors are the only control variables over the network and are considered to be open, bypassed or closed depending on the suction and discharge pressure differential and flow rate through the compressor. If the discharge pressure is greater than the suction pressure it is open, if they are equal it is bypassed and if there is no flow it is closed. It is further assumed that there are no self-loops in the system and bypass over any compressor station is not allowed. A compressor is bypassed if there is a directed sequence of pipes connecting two nodes for which there is an alternative route containing at least one compressor station. Lastly, at each node the net inflow is known and at each delivery node the minimum pressure limit is known. The first step is to decompose the network into a set of disconnected subgraphs by removing the compressor stations from the network. The decomposition of the network allows for the flow variables to be calculated for each subgraph if the compressor station flow rates are fixed. The focus is therefore on analysing the compressors participating in the network structure. The concept of the reduced graph is introduced in a similar manner to [88] to facilitate this analyses. In the reduced graph the subgraphs of the larger network are compressed into a single node and therefore the structure of the subgraphs does not affect the structure of the reduced graph. If the reduced network has no cycles, then the flow rates through the arcs, which correspond to the flow rates of each compressor station in the larger network, are uniquely determined. If the reduced graph does have cycles the flow rates through the compressors must be determined in the context of the
underlying optimisation procedure.

The solution procedure proposed is to begin with a set of feasible values for the flow rates and calculate the flow variables for each subgraph using a set of linear equations derived during the initial partitioning of the larger graph. Having values for the flow variables for the network leaves the values of the pressure variables to be determined. As each subgraph is connected, knowing the pressure at any node in the subgraph allows the pressure at all other nodes in that subgraph to be calculated. In addition, if there exists a direct path between two nodes in the subgraph then the pressure at the first node can be written as a function of the second node. Then the problem at each subgraph can be simplified by including the nonlinear functions of the one reference node and removing all other nonlinear equations from the problem. This simplification leaves the reduced problem containing only the pressure variables of the reference nodes in each subgraph. The reduced problem is formulated so that it can be converted into a sequential decision structure which allows dynamic programming to be efficiently applied. The degree of the sequential decision structure varies depending on the structure of the subgraph. If the subgraph has a single input node and single output node then the reduced problem has a sequential decision structure. However, if the subgraph has multiple input or multiple output nodes, then the problem has a non-sequential structure. A further assumption is made to exclude structures of this type and allow either multiple inputs and a single output of multiple outputs and a single input.

The second stage of the solution process is to find the set of flow variables that improve the objective function. With given flow variables, and considering a cyclic network structure, an attempt to find a better value for the objective function can be made by adjusting the given flow variables, and using the residual network and negative cycles concepts from the field of graph theory. Computational experimentation and was found to be effective in both cyclic and non-cyclic networks with the average cost reduction across all experiments being 27%.

Sanaye and Mahmoudimehr [92] note that non-sequential dynamic programming will find the global optimal solution to the compressor fuel minimisation problem. The advantage of using dynamic programming is that it is insensitive to the non-linearity, non-convexity and discontinuities that exist in this problem. However this approach cannot be used to analyse cyclic networks without the flow rates being known in advance. The problems
arise with cyclical networks because the rate of flow through each pipe and compres-
stor cannot be uniquely specified in advance and must be included among the decision variables. This means that the non-sequential dynamic programming method is multi-
dimensional and will require impractical computation times. The solution proposed in this paper is to use the genetic algorithm method as an alternative. This method reduces the probability of converging to a local optima by beginning the search for the global optimum at different points in the solution space and provides the further advantage of relying only on the objective function. The results of a comparison between using non-sequential dynamic programming and the genetic algorithm method on straight, branched and cyclic structures showed the genetic algorithm was a viable alternative for both cyclic and non-cyclic networks.

Jamshidifar [54] uses a slightly different objective function when approaching the fuel minimisation problem. The problem formulation includes valves, in addition to compressors, as network components used to control the flow through the network, and proposes a weighted sum of two functions with one related to the number of compressor stations operating and the other related to the total amount of their fuel consumption. The rationale for the unique objective function is that the cost of a compressor overhaul is substantial, and therefore running the network with the fewest number of compressors operating reduces this cost by reducing the frequency of compressor overhauls necessary. The operating compressors are constrained to their feasible operating domains. The graph reduction technique described in [88] is employed and dynamic programming is then used to calculate the pressure variables at fixed flow rates as in [89]. A genetic algorithm approach is then used to find the flow variables. In the case of a straight or tree structure, the flow variables of the reduced graph can be found uniquely. This is not the case if the reduced graph has cycles. A set of estimated flow variables that satisfy the constraints is used as a starting point and improvements to the objective function are sought by modifying the estimates using a genetic algorithm approach. The search space of the problem is reduced by noting that there is only one independent flow variable for each independent cycle in the network and searching only for the value of these variables. After a value is found at each iteration the dependent flow variables in the cycle can then be calculated as can the flow variables of the valves and compressors that are not part of a cycle. Simulation scenarios on sample networks with up to 16 compressor stations found savings in power consumption across all simulations with a maximum saving of 22% when compared to the optimisation system currently used by the system operator. In addition to the seven simulated configurations only one required the use of more operational compressors then the solution provided by the current optimisation method.
3.2 Design and operation of gas transmission networks

Gas transmission networks are costly to construct and operate. As such optimising the network design can save substantial amounts of money over the life of the network. Historically the network design has been seen as a conceptual design case and not as an optimisation problem [60], however optimisation techniques are now being bought to bear in the hope of realising some of the cost savings available. Babonneau, Nesterov, and Vial [15] note the difficulties in solving a nonlinear non-convex optimisation problem, namely problems with convergence to local minima and the numerical complexities and instabilities. Their approach is to avoid non-convexity by focusing on what they describe as the minimum energy principle, according to which stationary flows minimise total energy dissipated by the system. This problem is convex with linear constraints on the network flows. Pressures can be adjusted by compressors or regulators and they use this feature to look for a system of pressures that satisfy the network constraints while remaining compatible with the stationary flows. Failure to solve this problem suggests that reinforcement of the network is desirable and they proceed to develop a model to minimise the cost of reinforcing the network using pipe diameters on the reinforcing arcs as decision variables and placing constraints on the total investment cost. By finding a suitable approximation to the cost function this new problem is also convex and tractable. The framework is further extended to deal with the procurement problem. This problem is treated in a similar manner with a linear cost function and linear constraints on flows and thus a convex solution space.

Babonneau, Nesterov, and Vial formulate the transportation problem as follows. The network is represented as an oriented graph with the set of nodes partitioned into supply nodes, demand nodes and transit nodes. The set of arcs is divided into passive arcs, active arcs with a compressor, and active arcs with a regulator. An active arc has zero length and no gas is transported through it, instead pressure is increased via a compressor or decreased via a regulator. Gas is transported on passive arcs. This formulation is unusual in that it effectively breaks an arc into two adjacent components, one for transport and the other for pressure. Constraints are applied to the supply and demand nodes and to ensure that the mass balance equation is satisfied at transit nodes. Gas flow is determined by the pressure difference between the originating node and the destination node with the effects of friction incorporated, and the direction of flow is determined by the sign. Constraints are placed on the active arcs to ensure the flows are not negative and to ensure compressors increase the pressure and regulators decrease the pressure, and a
multiplying factor can be set to zero to take the compressor out of the formulation. The objective is to solve the problem for a compatible pair of flows and pressures without putting constraints on the pressures. Any compatible pair of flows and pressures can be interpreted as an optimal primal dual solution of a strictly convex problem. However, constraints on pressures are usually required due to limitations of the physical system, this leads to nonlinear constraints and the more complex problem discussed previously. To find a feasible and compatible pair of flows and pressures to the linear problem a two stage process is proposed. This is necessary as the solution to the strictly convex problem is unique for the flow variables but not necessarily for the dual pressure variables. Unfortunately the two step procedure provides no guarantee a solution will be found, nor does it provide proof that no solution exists. However, by changing the initial parameters that govern the pressures for active arcs and the pressures at delivery and supply nodes the problem can be re-calculated in an attempt to find a solution. This approach has some similarities to non-convex optimisation where there is no guarantee that the algorithm used will find a solution, even if one exists, however in the event of failure, a new starting point can be used in a second attempt.

Najibi and Taghavi [71] seek to optimise the design of the gas transmission network by minimising the life-cycle cost of the network. The main components of a gas transmission network are the pipes themselves, and the compressor stations. The construction, maintenance and operation of these two components comprise the life-cycle cost with a factor applied to reflect the inflation rate and interest rate (also called the discount rate) over the life of the network. The initial investment for the pipeline includes the cost of design, procurement and installation, while the maintenance cost is set at 5% of the initial investment cost with inflation and discount factors applied. The initial compressor cost includes similarly relevant factors relating to the technical specifications of compressors and 5% of the initial investment costs, with the same discount and inflation factor applied, is used for the maintenance costs. The cost of running the compressors is in gas consumed. The estimate of gas consumed is set at 0.5% of the gas transmitted multiplied by the gas price, and discounted in the same manner as the other costs. Constraints are placed on the flow regime through the pipes. The flow regime through the pipes is considered fully rough and the Reynolds number of the flow must be above a critical value. A flow equation between two compressor stations must be satisfied and a maximum allowable pressure constraint is applied for each pipe. Each pipe must also satisfy a stability constraint based on the pipe diameter and thickness. The compression ratio must be within a stipulated range for the compressors to work efficiently. It is noted that the overall economics of the network are sensitive to the compression ratio at
which the compressors operate. The velocity of gas flow is limited to minimise erosion corrosion and the temperature at which gas leaves a compressor must be maintained below a maximum value to prevent damage to equipment. Finally, the size, thickness and construction material are limited to a set of discrete values that reflect commercial pipes available in the market place. A rank-optimisation methodology is used to solve this problem. Initially, this methodology involves finding all combinations satisfying the technical and economic constraints. Once the feasible set of solutions is found the life cycle cost is calculated for each of them with the minimum considered optimal. Their results show that increasing working pressure in the pipelines increases the optimum flow rate and lowers the minimum life-cycle cost of the network.

Ruan et. al [91] also apply a rank-optimisation model to minimise the life-cycle cost of a gas transmission network. Constraints are placed on many of the same variables as in [71] although they include a constraint on the distance from one compressor to the next while neglecting to constrain the velocity of gas flow and the output temperature from the compressor stations. Their analysis identified the network design pressure, pipeline diameter and compressor ratio as the variables most affecting the life-cycle cost. Increasing design pressure increases the initial investment in pipes as the thickness of the pipe wall must be increased to contain the extra pressure. However, increasing the pressure reduces the velocity of the gas thus reducing corrosion of the inner surface of the pipe. A further benefit of increasing the pressure is that the network requires fewer compressor stations. By doubling the diameter of the pipe they find that the amount of gas that can be transported increases by as much 5.6 times, however the pipeline diameter is constrained in the model to maintain stability and a larger diameter pipe again increases the initial investment cost. Compression ratio also influences the number of compressors needed with higher compression ratios requiring fewer compressors but those required must be more powerful and will therefore consume more gas over the life of the network. A case study of two possible designs for a network to be constructed, and using data provided by the operator, found that higher pressures allowed the increased compressor costs to be offset by the decrease in pipeline maintenance costs resulting in the total cost of the optimal design being 94% of the alternative.

Kabirian and Hemmati [60] focus on optimising the expansion of an existing network over a long time horizon under the assumption that the current network satisfies all demand. The decision variables are the type and location of additional pipelines and compressors. The objective function is the discounted cost of construction and operation
of the network, as in [91] and [71]. Their approach is to break the long time horizon into shorter time segments, consisting of years, during which the available usable structure cannot be changed. That is, the structure at the beginning of each time segment must satisfy consumer demand until the beginning of the next time segment. While the usable network cannot be changed the network itself can be upgraded by adding additional compressor stations and pipelines which will be become operational at the start of the next time segment. They chose to construct their network graph with pipelines and compressors as edges, in a similar manner to [15], and five types of nodes: demand nodes, supply nodes, transshipment nodes and nodes for the compressor station entrance and the compressor station exit. Demand and supply nodes are forecast by the modeller. Once added to the network they may not be removed but may be modified at the end of each time segment. Transshipment nodes are potential locations for pipes to join the existing network and are set by the modeller at the start of each time segment. The model finds the optimal transshipment node to join and that node is considered active. Transshipment nodes where no joint occurs are inactive. Furthermore, this model allows for pipes and compressors of various specification, unlike [91] and [71] where only one type of each is selected in the optimal solution. Capital costs depend on the type of compressors installed and the type and length of pipelines installed. Operating costs are limited to compressor operating costs and the cost of the gas itself which is dependent on the source of supply over the time segment. The operating costs are a function of a steady-state mass flow rate through the network and pressures in the nodes.

As the network model invented by Kabirian and Hemmati is expanded over each time segment a sub-model that minimises the operating costs for the new time segment is run to find the optimal mass flow rate for that period of time. As is the case for many steady state simulation models of pipeline networks, the sub-model has flow variables for each edge and variables for the pressure at each node. The objective function is constrained by mass balance constraints, a nonlinear equality constraint on each pipe that represents the pressure drop and the flow, and a nonlinear nonconvex set which represents the feasible operating limits for pressure and flow for each compressor station. The objective function is given by a nonlinear function of flow rates and pressures. The sub-model minimises the operating costs, which like the larger model are the compressor operating costs and the price of gas. The compressor operating costs are a function of the type of compressor in question, the incoming mass flow rate, and the pressures at the entrance node and the exit node. The gas price for each supplier is a function of the mass flow rate and pressure required, and must be known in advance and supplied to the model. Kabirian and Hemmati’s sub-model is integrated into the larger model to be optimised.
In contrast to models discussed earlier that seek to provide an operational tool to network managers where providing solutions quickly, see [103] and [66], the model they propose is intended for planning over a long time frame and therefore focuses on completeness. This focus on completeness introduces interdependencies between the decision variables, the optimisation in the current time step depends on the values from all previous time steps, and complex nonlinear constraints and a nonlinear objective function. The solution method they propose is heuristic random search method which sequentially and randomly assigns values to decision variables and evaluates the objective function. The optimum development plan is the one with the lowest cost. While the case study conducted was on a highly simplified network and used a discount rate of zero, this type of model is worthy of exploration and extension due to the large amounts of money that must be committed to extending natural gas transmission networks if, as expected, the use of natural gas increases.

Woldeyohannes and Majid [117] provide a simulation model for various configurations of transmission pipeline network which includes mathematical formulations for all of the key components of the network. The model creates parameter estimates for pressure and flow using a method based on the Newton-Rapshon numerical technique. A network of gas transmission pipelines includes the pipelines and several non-pipe components. Simulating the network by considering the pipelines alone is the simplest configuration to tackle. Adding the non-pipe components, of which the compressor stations are perhaps the most important, increases the complexity of the task. The variables of the compressor operation might include the speed of the compressor, the suction pressure and temperature and the flow through the compressor. The problem may be further complicated by considering two-phase flow or the transportation of gas and gas liquids through the pipe. Transporting gas and gas liquids changes the amount of friction through the pipe. Older pipelines will also tend to have higher amounts of friction due to corrosion and accumulation of detritus.

### 3.3 Gas and electricity networks

Security of supply of natural gas is an increasing concern in the UK and Europe. In the UK around 70% of homes use natural gas and in the services and industrial sector about 40% of businesses also use gas. The electricity sector is also a heavy user with gas
powered generators producing about 45% of electricity output [98]. The threat to future supply in the UK is likely to be through the decreasing ability to meet peak demand and the possibility of a failure at a key piece of infrastructure [104]. This is a serious threat to Australian gas supplies as well. Historical disruptions to gas supply in Australia give credence to the assertion of interruptions because of infrastructure failure. The fire and explosion at the Longford gas processing plant in Victoria in 1998 killed two people and left the state without gas for 19 days [51] and the Varanus Island fire in Western Australia in 2008 resulted in a significant decrease in gas supplied to the state and a large economic loss. The Western Australian government’s enquiry found the failure was as result of corrosion to a high pressure pipeline that subsequently ruptured [20]. Prices in the Victorian wholesale market for gas reached the ceiling price on November 22nd, 2008 as a result of coincident planned and unplanned outages of gas infrastructure [9]. While no load shedding occurred on the day participants in the wholesale market were exposed to significant financial losses as a result of the failure. The future inability to meet peak demand in the Eastern Australian gas market is also a plausible hypothesis with the large LNG export projects that are currently being built, the legislation to price carbon emissions and the environmental impacts of coal seam gas production all factors in the debate [40].

There is an increasing dependency between electricity networks and natural gas networks as the use of gas to fuel electricity generators increases. When compared to conventional coal plants, gas-fired and combined-cycle power plants have high efficiency, lower capital investment costs, lower environmental impact, and increased operational flexibility [77]. With the increased interdependency comes a risk that disruption of the gas supply will have a flow on affect to the electricity supply. A real-time balance between generation and load is required by an electric power system under normal and fault conditions. Gas fired generators can respond quickly to contingencies should the need arise. If an imbalance between generation and load is not eliminated the mains frequency may increase or decrease and lead to a loss of stability of the system through electromechanical and electromagnetic transients. Voltage collapses and more generators will lose synchronisation leading to the likelihood of a blackout. Therefore the supply of natural gas to gas fired power plants is critical to prevent the failure of the electrical power system [64]. Conversely, gas fired generators are heavy users of natural gas and can deplete linepack quickly. Linepack is the amount of gas stored in pipelines. If the linepack is depleted rapidly it may lead to the pressure in gas transmission pipelines falling to their minimum pressure limits. Therefore the modelling of the integrated gas and electricity supply systems is an important area of research.
Chaudry, Jenkins and Strbac [31] model the United Kingdom gas network as an integrated part of the electricity supply system. The electricity and gas networks are linked through gas powered electricity generators that are connected to both networks. Growth in European gas usage is expected to be driven largely by an increased use of gas powered electricity generators and as such there is going to be an increased interdependence between the supply of natural gas and the supply of electricity. Their rationale in modelling the gas network as an integral part of the electricity supply system is to answer questions about optimal transmission investment of gas and electricity infrastructure, security of supply and the ramifications of the loss of a major gas terminal on the availability of electricity, the environmental benefits of using gas to generate electricity as opposed to using coal, and the optimal usage of gas and electricity infrastructure to match supply and demand under the physical constraints of the infrastructure. The gas network component of the system is modelled using daily time-steps that take into account the gas flow, linepack volume and important infrastructure such as compressors and storages. Their simulations showed significant gas load shedding (failure to meet demand) of customers was necessary when a key piece of infrastructure, the Bacton terminal, was taken off line. The Bacton terminal is one of three gas terminals in the UK that process gas from the North Sea gas fields and is the single largest piece of infrastructure in the UK gas network [98]. When a second piece of infrastructure, the Rough storage facility, was also taken off-line load shedding increased. The rough storage facility can supply approximately 10% of UK peak day gas demand [123]. The simulations modelled demonstrate the importance of alternative sources of gas supply when key infrastructure fails.

Damavandi, Kiaei, Sheikh-El-Eslami and Seifi [120] also model gas as part of the electricity supply system but emphasis the difference in the dynamics of electricity and gas networks. The electricity network moves at high speeds and variations in load can be compensated for quickly so the network reaches a steady state condition rapidly. The movement of gas through a network is far slower and rapid increases in demand can affect the dynamics of the gas network for hours after the demand spike has passed. This is important as gas powered generators are heavy users of gas and can cause demand spikes when a generation unit is bought online. Their modelling shows the importance of gas velocity and the distance between network nodes, particularly supply and demand nodes, when planning for the addition of gas powered generators to a network.

Pantoš [77] notes that the increasing use of gas to fuel electricity generators increases the complexity of managing congestion on electricity transmission infrastructure. A market
based congestion management model that includes the natural gas network is proffered. The model uses Benders decomposition with a sub-problem that ensures the feasibility of scheduling electricity supplied by gas fired power generators. The natural gas sub-problem models the gas network in a steady-state with constraints on pressure, compressor usage and the mass-flow balance at each node. The objective of the sub-problem is to minimise the sum of the slack variables at the natural gas nodes. The slack variables represent gas load shedding variables and are added to make sure the problem is always feasible. In the physical sense, the slack variables represent virtual gas shedding at each delivery point to eliminate mismatches. Storages are included in the model as both a load and a supply on the natural gas system. Privately owned storages located at gas fired electricity generators do not directly affect the gas supply network and are not directly included in the model, however they are implicitly included as they affect gas consumption. Privately owned storages can be interpreted as an energy buffer. In the event the electricity network system becomes congested the system operator executes the model and accepts bids from generators, including those using gas as fuel, and a feasible solution is found. Next the feasibility of the gas network to supply the gas fuelled generators that had their bids accepted is assessed. If the gas sub-problem proves infeasible Benders cuts are used to form constraints on the gas usage of gas-fired generators and the sub-problem is added to the master problem. This iterative process continues until a feasible solution for both the electricity grid and the natural gas network is found. The nonlinear mass-flow balance equations are solved using the Newton-Raphson method. The constraints on pressure limits at each node, and the power limits and compression ratio ranges of the compressor are all nonlinear as stated previously. Successive linear programming is applied to solve theses equations iteratively.

In the United States the linkages between the electricity and gas networks are complex and the security of gas supply is also a focus of attention among researchers. Liu, Shahidehpour, Fu and Li [65] use the same method to model security-constrained unit commitment. The model minimises power system operating costs by taking into consideration constraints on electricity transmission, gas transmission and natural gas contracts. The supply and transportation of gas is typically negotiated under separate contracts. The contract for transportation capacity on transmission pipelines is generally categorised as firm or interruptible. In the United States gas fired generators commonly prefer interruptible transportation contracts because they are cheaper. However gas fired generators are heavy users and require high pressures and their gas supply is often curtailed first in emergency situations. Liu, Shahidehpour, Fu and Li model the gas contracts as a bundle of supply and firm transportation capacity, however the inclusion of the contracts is
novel and reaffirms the system security issues arising from integrated gas and electricity systems.

Liu, Shahidehpour, and Wang [64] note that much of the work on modelling integrated gas and electricity networks focused on steady-state flows through the gas transmission system. They develop a bi-level programming formulation to coordinate the scheduling of electricity while taking into account the time dependent aspects of gas supplied to gas fired generators. They propose modelling the gas flow through gas pipelines using a set of partial differential equations as opposed to steady state flows. The scheduling problem for the gas system minimises the compressor costs while satisfying the transient transmission constraints, gas transportation contracts and system pressure requirements. The gas system is linked to the electricity system via the gas fired generating units and the consumption rates of the gas fired units in the electricity scheduling problem are set equal to the gas loads in the gas scheduling problem.

Woo, Olson, Horowitz, and Luk [118] found a causal relationship, in both directions, between the price of wholesale electricity and natural gas in California. To understand the link in markets they discuss the hypothetical ramifications of unseasonal weather. In the advent of an increase in electricity demand driven by hot weather the price of electricity should rise, all other factors being equal. The increase in price results in an increase in the so called ‘spark spread’, the the margin between the electricity price and the cost of the gas used as fuel in generators, and encourages gas-fired generators to buy gas to produce electricity, thus raising the demand for gas and resulting in higher gas prices. Alternatively cold winter temperatures increase gas demanded by residential and commercial users for heating, driving up the gas price and raising the operating costs, and therefore supply bids offered by gas fired electricity generators shifting the supply curve and ultimately market price up. Gas fired generators accounted for about 25% of electricity supply in the United States in 2005 while the share of generating capacity located in California was about 50%. Gas powered generators are the marginal supplier of electricity in the summer months. The study of the California market found a causal link between the wholesale prices of electricity and natural gas and the authors postulate this link exists in markets with substantial gas fired generation.
3.4 Models of spot price dynamics

Natural gas is often traded under long term contracts with specified delivery volumes and agreed prices. Futures markets also exists where gas is bought and sold at an agreed price for delivery at a future date. Markets for immediate delivery, known as spot markets operate in some areas as well. In this section the literature relevant to modelling the dynamics of spot prices is discussed. In subsequent sections a model of the spot price of natural gas in the declared wholesale gas market, a market for natural gas that operates in the state of Victoria, Australia, is developed. To the best of my knowledge no model of the spot price in this market has been published previously.

Commodities are traded on financial markets in much the same way as other financial assets and many of the models from financial mathematics have been adapted to model the price dynamics of commodities. However, there is a fundamental difference between commodities and other financial assets, namely commodities have a physical presence that financial assets, such as shares in a public company or options on future interest rates, don’t have. In this section the literature pertaining to the price dynamics of commodity prices is discussed and models within the context of the arbitrage pricing theory framework are considered. The value in having better models and better estimation methodologies is that commodity producers, users and financial intermediaries can be better informed and better manage their financial risks [35].

The famous Black-Scholes paper on the pricing of options and corporate liabilities [25] puts forward the idea that if options are correctly priced then there is no opportunity to make certain profits by creating combinations of long and short positions in options and stocks. That is, arbitrage opportunities don’t exist if options are correctly priced. The Black-Scholes valuation model assumes that the logarithm of the asset price follows a geometric Brownian motion, with a constant variance over time and has continuous sample paths, which if true implies that the return on the asset is normally distributed. This implication is questioned in [67] where it is noted that the return on financial assets does not follow a continuous sample path but in fact is discontinuous with jumps in price being observed. The solution proposed is to incorporate a Poisson process into the valuation model to simulate the random arrival of new information that has more than a marginal effect on price. Models of this type are often referred to as jump-diffusion models and several such models are developed in [57] and [37] with the view to
incorporating positive and negative price jumps in the underlying asset price. A jump diffusion model with the jumps following a Poisson process is used by Cui & Schreider [38] to model the price dynamics of irrigation water traded in the Goulburn-Murray irrigation district in southern Australia. This pricing model for irrigation water has allowed options with irrigation water as the underlying asset to be proposed with the view of allowing participants in this market to better manage their financial risks.

A further limitation to the geometric Brownian motion model is the fact that it also has a constant volatility term structure. The volatility term structure is the amount of volatility, or variance, in the returns on futures over increasing time horizons. This is contradicted by the empirical evidence of commodities futures prices which show the volatility term structure follows a decreasing function as the maturity increases. That is futures with longer to maturity are less volatile. This can be explained by mean-reversion in commodity prices [35]. The term structure of futures prices is analysed in [19] to test for investor expectations of mean reversion in agricultural commodities and crude oil. The magnitude of mean reversion was found to be high in these markets whereas little evidence exists to suggest futures prices on financial assets mean revert.

The price series for oil, coal and natural gas is examined in [81] with a view to understanding the stochastic dynamics of price evolution and how to model it. The series for natural gas begins in 1919 and includes 75 years of data for the annual average of producers prices in the United States. For all price series, mean reversion to a stochastically fluctuating trend line is advocated with the argument that models of this nature capture in a non-structural way what theory suggests should be driving prices, namely the total long-run marginal cost. Structural models incorporating changes in supply and demand are considered to be less useful for long-run forecasting due to difficulty in forecasting the explanatory variables needed for such models. In the model advocated the total marginal cost includes user costs associated with reserve accumulation and resource depletion. However, the caveat to the conclusion that mean reversion to a stochastic trend line is appropriate is that the rate of mean reversion is slow and may take up to a decade to occur.

The physical presence of commodities adds the cost of storage, and the dynamic of inventory to the other factors influencing the price. The theory of storage suggests that the cost of storing commodities, the interest foregone while the commodity is stored and
the convenience yield of the stored commodity explains the difference between contemporaneous spot and futures prices [46] often observed in futures markets for commodities. The convenience yield is the benefit of optionality that accrues to the owner of the commodity being stored but not to the owner of the futures contract. The owner of the commodity can choose to satisfy unexpected demand in the market for the commodity, or unexpected demand for another product for which the commodity is a raw input. In [82] the interrelationship of inventory level, rates of production and the spot price are discussed. It is argued that the levels of these factors are determined via the equilibrium of the spot market and the market for storage and that the equilibrium in these markets is affected by, and has an affect on, changes in the level of price volatility. The relationship between spot prices, futures prices and the level of inventory is also discussed. This paper also puts forward the idea of considering the production facilities, such as oil wells, refineries and pipelines, as call options on the commodity itself. Call options increase in value as the spot price for the underlying asset increases, thus the option to produce and supply more of the commodity has a value to the owner of these production assets. Inventories for storable commodities allow fluctuations in demand to be accommodated to an extent without the costs involved in changing the level of production. Producers set levels of inventory and levels of production using the spot price for immediate sale and the price of storage. The price of storage is equal to the benefit to inventory holders from a marginal unit of inventory held. This adds further evidence to the suggestion that commodity prices mean revert.

Three models of the stochastic behaviour of commodity prices are compared by Schwartz in [93]. Each model takes into account mean reversion and each incremental change in the model increases in complexity. The first model is a one-factor model that considers the logarithm of the price to be mean-reverting. In this model the logarithm of the price follows an Ornstein-Uhlenbeck process. The Ornstein-Uhlenbeck process solves differential equations of the form \(dX_t = (\theta_1 - \theta_2 X_t)dt + \theta_3 dW_t\), with \(X_0 = x_0\), \(\theta_3 \in \mathbb{R}^+\) and \(\theta_1, \theta_2 \in \mathbb{R}\). The second model adds the convenience yield as a second stochastic factor which is also assumed to follow a mean-reverting Ornstein-Uhlenbeck process. The convenience yield is not directly observable and is generally calculated by computing the difference between two futures prices with different maturities. This model effectively treats the commodity as an asset that pays a stochastic dividend yield in the form of the convenience yield. The third model adds a stochastic interest rate using the mean-reverting Ornstein-Uhlenbeck process developed in [112] to model interest rates. The three models are compared in terms of their ability to price existing futures contracts and the relationship of these contracts to the valuation of other financial and real assets. The
valuation of natural resource investment, such as a gas or oil field, relies on the stochastic process assumed to govern the price of the commodity. The evidence presented in this paper argues for mean reversion in prices as an important consideration when evaluating potential natural resource projects. Further extensions to include uncertainty in the equilibrium to which prices revert were introduced by Schwartz & Smith in [94] and an $n$-factor model is proposed in [35] where the specific commodity and level of complexity that is tolerable by the modeller determines the number of factors included.

The value of the convenience yield of natural gas is analysed in [114] and it is determined that air temperature as well as storage levels influences its value. A large fraction of the demand for natural gas in the European markets analysed is used for heating and is thus sensitive to changes in temperature. Temperature is also used as an exogenous factor on gas spot prices in [105] and applications to the valuation of derivatives and risk premiums are demonstrated. Sørenson [100] proposes a stochastic differential equation as in [93] and [94] with a seasonal component modelled by a parametrised linear combination of trigonometric functions with seasonal frequencies and applies this model to agricultural commodities futures. To model the dynamics of the futures curve for seasonal commodities Borovkova & Geman [26] replace the spot price with the average forward price as the first factor in a multi-factor model. They argue that for seasonal commodities the shape of the forward curve is largely determined by seasonal factors and cite UK electricity and natural gas prices futures as examples. Futures prices of these two energy commodities have historically exhibited a price premium when the expiration date is in the winter months.

The study of seasonality as a deterministic factor is extended to that of a stochastic factor in [68]. They argue that limited storage of natural gas and the absence of low cost transportation makes supply unresponsive to changes in demand and therefore natural gas prices are highly seasonal. The model presented is described by the authors as an $(n + 2m)$-factor model with the assumption that the logarithm of the spot price is the sum of $n$ non-seasonal factors and $m$ seasonal stochastic factors. The non-seasonal factors are as described in [93], [94] and [35], namely the convenience yield, interest rate and the rate of mean reversion. The seasonal components are modelled through a complex trigonometric component which is expressed by two stochastic differential equations. When applied to the Henry Hub gas futures contracts they conclude that the model allowing for stochastic seasonality outperforms standard models in which seasonality is deterministic. Furthermore they find that stochastic seasonality implies the volatility of
future returns is also seasonal.

The cointegration of spot prices in both gas and electricity are considered by de Jong & Schneider in [39]. Two time-series are said to be cointegrated if they have a common stochastic trend, in this case stationarity can be achieved by finding a suitable linear combination of the two series [78]. Literature on the co-movements of two commodities is rare with the focus generally on co-movements between futures of different maturity but for the same commodity. For example the co-movements of the daily returns of natural gas forwards are considered in [101] with the intention of better understanding the way volatility is transmitted across futures of varying maturities, and the co-movements of crude oil futures on the NYMEX and International Petroleum exchanges are analysed in [47]. However, many energy portfolios consist of multiple commodities, for example oil-indexed gas contracts and gas fired electricity generators[74]. With electricity generators being large and growing consumers of natural gas the study of co-movements between electricity and natural gas is of interest. While the individual price series may be non-stationary the theory of co-integration suggests one or more weighted combinations of the series may be stationary. In some cases a stationary series can be achieved by first-order differencing or seasonal adjustments. Co-integration implies that there are economic factors that make variables move together stochastically over time. Co-integration was found by de Jong & Schneider between gas spot prices in three European markets with the link attributed to the physical connection of the markets through gas pipelines. Co-integration was also found between the gas spot markets and the price of electricity on the Amsterdam Power Exchange on what on a forward time scale. The reason proposed to explain the looser connection between the gas and electricity spot prices was the existence of the longer-term gas supply contracts gas fired electricity generators typically hold.

The concept of cointegration of forward curves is extended in [74] where a framework to simulate the evolution of two commodity forward curves, heating oil and natural gas futures, is provided. The model captures the local dependence structure, which are the volatilities, marginal densities and the correlations of the movements of the daily forward curves, as well as the global dependence structure which describes the long-term relationships existing between commodity prices. The forward curve is decomposed into short-term shocks and long-term shocks with volatility being stochastic and possible seasonal. The short-term shocks only affect the short-term futures prices, they are factors such as changes in temperature, transportation disruptions or transitory supply shortages. The long-term shocks are factors that have the potential to impact long-term energy
prices. The analysis highlights two long-term relationships, one between the convenience yields and the second between the long-term forward prices. As well as evidence of a causal relationship between short-term shocks in both markets and large swings between the the co-movements of oil and gas forward curves. It is envisaged that the model can help in estimating the risks in different hedging strategies for multi-commodity portfolios.

Gemen and Ohana [49] seek to validate using the slope of the forward curves for oil and natural gas, seasonally adjusted, as a proxy for inventory and to analyse the relationship between inventory and price volatility for these two commodities. The slope of the futures curve has the advantage of being visible on commodity exchanges while data on inventories is not as readily accessible. The research confirms that this proxy for inventory works well. The study of spot price volatility, for example in [46], shows that the variance of commodity spot prices for agricultural commodities and metals decreases with inventory levels. The research by Gemen and Ohana confirms this correlation exists for crude oil and to a lesser extent natural gas. They find a global negative correlation between inventory and volatility for crude oil but for natural gas this negative correlation is only seen when the inventory is below the historical average and that occurrences of this increase dramatically during winter periods.

### 3.5 Water entitlements and allocations

In many parts of the world there is competition for use of available water supplies between industry, agriculture, human needs and the needs of the environment. All must be balanced to maintain a safe and sustainable water supply. The literature on the allocation, delivery and price of water is reviewed in this section. Traditionally water catchments were modelled using bespoke tools applicable to one specific geography however, over the last two decades generalised models that can be calibrated to any system have been developed. These generalised models can be grouped into two families of models, one using some form of heuristic method, such as the integrated quantity and quality model or IQQM, and the other using mathematical programming techniques such as the resource and allocation model known as REALM [80]. The IQQM models river systems using nodes and connecting links that can be configured to simulate any river system. It has a modular design that allows for the addition of new components as they are developed. Originally the model incorporated modules for water quality, water quantity
and rainfall-runoff. The initial purpose of the IQQM was as a replacement for monthly simulation studies conducted by the New South Wales Department of Land and Water Conservation. The monthly simulations were suitable for long term planning when the objective was to maintain a secure water supply for consumers, however with the transition from being a water provider to a manager of water resources a new modelling tool was needed to incorporate factors such as environmental flows and water quality [97]. The IQQM model has been used to assess the sensitivity of runoff in Australia to the effects of climate change. Measuring runoff is particularly useful as a measure of climate change because it lies upstream of hydrological use and human use but integrates rainfall, evapotranspiration and the characteristics of the landscape [85]. The IQQM can be used to model both unregulated and regulated rivers but has the disadvantage of complexity making parametrisation difficult. It was used in comparison to a simple modelling technique known as the integrated flood and flow modelling (IFFM) using the Macquarie River in the Murray-Darling basin. The effects of river regulation was tested at three gauges by comparing actual data to modelled data both before regulation and after regulation. It was found that the IQQM underestimated the impact of river regulation at one of the gauges while both models provided similar results at the other two, thus the simpler statistical models could potentially provide more reliable estimates of the affects of river regulation [87] with a less complex model. The IQQM was used in [58] to model the affects of climate change on the water resources of the Macquarie river catchment. Climate change scenarios were selected from nine climate models and used to perturb the long term daily rainfall and potential evaporation in the catchment. The impact on water storages, environment flows and allocation of water for irrigation were then analysed by using Monte Carlo simulation to generate a probability distribution function. Thresholds for irrigation allocations and environmental flows were then determined. The analysis found that variations in local rainfall due to climate change most significantly impacts the range of uncertainty of the change in water resources. The over use of water for irrigation in the Murray-Darling basin led the council of Australia governments to institute a set of reforms aimed at securing environmental flows and optimising the allocation of water to holders of water entitlements. An important feature of the reforms was the introduction of markets where water can be reallocated through trade during times of shortages. Water users were given legal entitlements to an allocation of water and an independent body was established to determine if sufficient water was available, and when it would be allocated to entitlement holders. The IQQM is able to simulate water allocations on the catchment scale but the advent of water markets in which water entitlements are reallocated through trade is beyond the capabilities of the model [121], therefore new models were needed. The REALM system is a generalised model that can be configured
to any water system in both urban and rural settings. It can generate stochastic data to
test security of supply, environmental flows and water planning scenarios [80] and can be
modified to test the impact of new system modifications or operating procedures [113]. It
can also model the trade in water allocations. All water systems in Victoria have REALM
simulation models [80] as do many other water systems in other Australian states. The
structure and operation of the REALM system is outlined below with reference to [113]
and [80]. The REALM system takes input of a streamflow file, a system demand file and
a system file. The streamflow file contains data on climatic conditions and unregulated
river flow that is available for harvesting, the demand files contain unrestricted demand
and the system files contain information on the water system, such as configuration and
operating procedures and rules. The elements of the system are converted into a set of
demand and supply nodes and connecting arcs, the connecting arcs are subject to con-
straints and the mass-flow balance constraints are applied to the nodes. The network
flows are then solved using a linear program and output files are produced. The REALM
system allows detailed input of the key features of a system such as reservoir capacity, the
type of capacity inflows and evaporation. Penalties on flow paths can be created to direct
flow, the connecting arcs can be modelled as minimum flow, or maximum flow and trans-
mision losses can be factored in as fixed quantities or percentage of flow. REALM uses a
hierarchy to allocate water within the system during each time step. Firstly evaporation
losses in reservoirs are satisfied, then transmission losses in carriers. The third level of
precedence is to satisfy demands, which may be subject to restriction, then spillage from
the system is minimised. The fourth level of precedence in each time step is to satisfy
minimum flow requirements before the final priority of meeting the end of season reservoir
volumes. This hierarchy is built into the code using a system of penalties that cannot be
altered by the user [80]. The logic is that demand can only be satisfied if evaporation and
transmission losses are first accounted for, similarly the volume in a reservoir at the end
of the season must first allow for all water consumed and lost within the system during
the season. A model of the Goulburn water system, which is comprised of the Goulburn,
Broken, Campaspe and Lodden valleys in northern Victoria was created using REALM.
This region is a particularly productive agricultural region with most of the water being
used for irrigation. The Goulburn system is a complex system with twenty storages and
58 demand centres. In 2005-06 more than 50% of water used for irrigation in Victoria was
consumed in the Goulburn system [50]. The Goulburn river is the largest in the system
and has the most reliable flow. Farmers in the Campaspe and Loddon valleys can be sup-
plied from the Goulburn river via an interconnecting channel. The interconnected nature
of the system gives rise to complex operating procedures based on water availability in
storages and the level of river flow. The actual operating procedures in the system were
developed using the experience over time of the water authority managing the area. In REALM a system of penalties is used to model these procedures [80].

Hydro-climatic uncertainty and the effect on water allocations is examined in [80] where REALM is used to provide a probability distribution of water allocations. The REALM system is used as a forecasting tool to provide information to water entitlement holders on the likelihood of increases in water allocations. The current volume of water in storage is known and estimates of the timing and amount of inflows is made using 110 years of historical data. Using the current storage levels as a starting point, and creating 109 replicates of two years duration from the historical data a series of two-year simulations can be run. The different climatic conditions for each replicate give a probability distribution for the amount of water allocations over the next two years. The effects of hydro-climatic uncertainty on irrigator decision making in the Goulburn system is analysed in [50]. The estimated costs associated with hydro-climatic uncertainty ranged as high as 5-7% for the dairy industry but was negligible for the horticultural industry. The sensitivity of water allocations and several other output functions of REALM were tested in [27]. In this paper the sensitivity of the REALM model is examined by generating sample paths for the systems parameters and analysing the model outputs. Commonly parameter values are not known with accuracy and/or are difficult to measure therefore sensitivity analysis is useful in identifying the most significant parameters on the model outputs. Sensitivity analysis can also be used to identify second-order interactions and non-linear relationships in the model [27].

3.6 Water markets and water prices

The scarcity of water has created considerable interest in reforms aimed at optimising its use. At the council of Australian governments meeting in 1994 the state governments in eastern Australia agreed to a set of water reforms to allow water to be traded. This marked a shift in policy away from meeting increasing demand with new sources of supply to a process of optimising the use of currently available water [22]. The objective was to be achieved by allowing a market mechanism to reallocate water to its most efficient use [115] without the need for government intervention. There are two types of water markets in operation in the Murray-Darling basin, one for water entitlements and the other for water allocations. A water entitlement is a right to a share of a water
resource, an allocation is the actual amount of water allocated to each entitlement in each irrigation season. These are often referred to as markets for permanent water and for temporary water respectively. In the various countries and regions where markets exist for similar types of entitlements and allocations it was found in [23] that the market for allocations is more active than for entitlements. This is the case in the Murray-Darling where restrictions on the trade of permanent water out of a region are still in place [28] which is an obvious impediment to trading volume. The price for water allocations in the Goulburn-Murray irrigation district has also appreciated far more than the price for entitlements between 1993-2003 [23], however water trading was still a new phenomenon over this period. The newness of the market, the restrictions on trading entitlements out of a district and the drought conditions that prevailed during the end of the study period may have impacted on the price of water allocations to a greater extent than for entitlements. Water markets are designed to mitigate to some extent the considerable uncertainty irrigators face when planning capital investment and when deciding on how much to plant each season. These decisions must be made with the added uncertainty of water availability. The opportunity to trade water allows some of the uncertainty to be removed as water is available if the irrigator is willing to pay any price. The uncertainty of a variable hydro-climate creates seasonal opportunity costs for individual entitlement holders. In [28] it is argued that water is reallocated through trading in the temporary market according to these opportunity costs. The major determinant of the opportunity cost was found to be capital investment decisions which have been made based on the reliability of water allocations made to entitlements. Interestingly, the price of most commodities was found to have little impact on the price of water entitlements [23] with only the price of wine grapes showing a positive influence on price. Some farmers of high value crops have accepted higher prices as water allocations have been reduced during years of drought. There is evidence to suggest that the significant investments made in growing perennial crops and in dairy farming are influencing farmers to accept higher prices for water as necessary to protect their investment. During the period from 1993-2003 dairy prices and the price of temporary water showed a negative correlation [23] which supports this assertion.

There is limited literature on the factors that drive volume and price in the market for temporary water allocations. Regression and correlation analysis have been applied to a hypothesised set of possible factors affecting the price of water allocations [22]. These factors are the price of commodities produced in the region, the amount of seasonal allocation and rainfall, the price of substitute inputs such as feed for irrigated pasture, and the potential loss of capital investment through lack of water. The price for water in
the temporary market was found to be driven by the size of current seasonal allocations, the amount of rainfall and the amount of evaporation. When prices are high there were more transactions but lower volumes [22] which could reasonably be expected since any rainfall would make the water purchased at a high price unnecessary. Infrastructure upgrades in the Goulburn-Murray irrigation district are designed to allow farmers to have greater control over when and how to use their allocated water, particularly with respect to the timing and speed of application to farms. This should allow efficiency gains through targeted application of small amounts of water in anticipation of future rainfall events, which the market data suggests is the pattern of behaviour of farmers when their water supply is curtailed. Farmers of high value crops could shore up their water supply by buying more entitlements and therefore receive a greater share of the total water allocated. This would give them a greater control over their water supply, however in years when water is abundant they would have an over supply and there is likely to be few buyers in the market. One option currently available to irrigators in the Goulburn system is to carryover water into the next season. This is a new innovation which allows water to be stored in the unused portion of reservoirs. There is a cost to this strategy in that volumes drop by 5% when carried over to the next season. There is also a risk that the storage will reach 100% capacity and spill. Should this occur water carried over from the previous season is the first water lost. This is water that could have been sold in the market. As the markets for both permanent and temporary water mature the opportunity to introduce new products arises. The National Water Initiative [36] sets out the goal of allowing products based on water entitlements that can be traded. The clause states that the entitlements can be leased, traded in part or as a whole, or in any other trading option that might arise in the future. Several types of products are potentially available with a derivative that has water as the underlying commodity being one. Currently derivatives are not available in any water market in Australia despite there widespread use in markets for other commodities. Price spikes, where the price of temporary water allocations reached more than $1,000 per megalitre show that there is considerable financial risk [84] and no easily applied strategy to reduce this risk. Changes in the way water allocations are made have also shifted the burden of risk management to the farmer. Water is allocated base on storage levels and estimates of future inflows. Allocations are then increased if conditions allow [24]. Thus the farmer must make investment decisions at the start of the season without control over the volume or cost of available water. In order to create an effective hedge against a risk there needs to be market participants with an opposing risk. One potential barrier to the introduction of derivatives is that irrigators are likely to have similar risk profiles. Food processors, who benefit from low prices and producers, who benefit from high prices have suitable
opposing risks [24] but the size of food processors, the fact that there are relative few of them and their position as the buyers of the food produced present significant problems. In many markets traders and speculators provide the opposing positions however there is a reluctance to allow access to market participants that wish to engage in these types of activities. Alternatives to allowing greater access to the market is to allow a market maker to operate. A market maker will take the opposing risk of a market participant. If there are sufficient transactions the positions held be the market maker will largely cancel each other leaving little risk. The remaining risk of financial loss is recouped by buying and selling the products at a spread sufficiently large to cover potential losses. While the need and intention to introduce new products is clear, the research on what types of products to introduce is sparse. A pricing model for temporary water allocations is provided in [38]. A pricing model with which to simulate prices is a vital component and an important step forward to reach the goals set out in the National Water Initiative as it allows options contracts on water to be valued. However, one impediment to the introduction of options on water allocations was the large jumps seen in the price of water between irrigation seasons. In order to set a strike price for an option it is necessary to frame a price range. For example, if the market price of one megalitre of water is $100 on October 1st, a series of options with an expiration date of November 1st could be introduced with strike prices ranging from $70 to $130 and increasing in $10 increments. However, while interseason price jumps were in the order of 100% or more it was impossible to frame a suitable range of strike prices. An approach to estimating the price of temporary water allocations is needed to accurately frame a series of options that can be traded at the beginning of the irrigation season. This research question is addressed in section 4.1.

3.7 Summary of the literature on the analysis of gas and water networks

A review of the important papers analysing the various features of gas and water networks was presented in the preceding sections. A large amount of literature relating to the optimisation of gas networks seeks to minimise the fuel used by gas compressors in transporting natural gas and several techniques were discussed. Methods of optimisation are increasingly being applied to the design of gas networks in an effort to minimise the substantial capital costs involved in the construction of the necessary infrastructure. Several papers were reviewed outlining the techniques proposed. The increasing use of
natural gas as a less carbon intensive method to generate electricity has seen the links between gas and electricity networks become stronger. System security is an issue for both networks, and the considerable physical and economic damage that can occur due a failure in either network has seen an increase in the literature devoted to this topic. This literature is reviewed and a model to minimise shortfalls in supply is presented in this thesis. A second model used to simulate the spot price of wholesale gas is stated and the relevant literature used to develop this model was reviewed. The research published relating to the modelling of irrigation basins in eastern Australia was also discussed. This thesis presents regression model that can be used to estimate the price of irrigation water at the beginning of the irrigation season in the Goulburn-Murray irrigation district and the published literature focusing on this region was reviewed.

In this work the security of supply problem is addressed in the context of the gas network in eastern Australia. Furthermore, a sub-network of this gas network is studied through which gas traded on a spot market is transported. This spot market has many similarities to the spot market for irrigation water in the Goulburn-Murray irrigation district in south-eastern Australia which has been the subject of previous research. Both of the commodities traded in these spot markets are transported through a delivery network and both exhibit considerable price volatility. A model of the price of irrigation water exists however the inability to predict the price of water at the beginning of the season means it is not useful until a period of trade has passed that allows for prices to be discovered. In this work a model that can be used to predict the price at the beginning of the season is developed. With regard to the spot of natural gas, no model exists. In this work a model is developed to address this gap in knowledge.

4 The nature of the networks studied

Two networks are analysed in this thesis, the network of irrigation channels in the Goulburn-Murray irrigation district, and the gas transmission pipelines forming the eastern Australian gas network. In this section the salient features of these networks are outlined.
4.1 Water trading in the Goulburn-Murray irrigation district

The Goulburn-Murray Irrigation District (GMID) is Australia’s largest irrigation district. It covers 1.6 million hectares in central Victoria and stretches from the Great Dividing Range in the east, down the Murray River to Swan Hill in the West and as far south as the Goulburn weir near Nagambie. The volume of water entitlements in the GMID is approximately 1,000GL. The district has a large dairying industry and produces fruits and nuts and other produce that rely on irrigation to ensure a reliable water supply. Several large food processing factories are also located in the region and they provide the majority of off-farm employment. The GMID includes four discrete irrigation areas that are supplied from Lake Eildon: Central Goulburn, Shepparton, Rochester and Pyramid Boort. Lake Eildon has a storage capacity of 3,250GL. Two discrete irrigation areas nearby, Murray Valley and Torrumbarry, are supplied from the Murray River, with diversions primarily at Yarrawonga and Torrumbarry Weirs. The major storages that supply the Murray irrigation districts are the Hume and Dartmouth dams which have capacity of about 3,000GL and 4,000GL respectively, although water stored in these dams is shared between New South Wales, Victoria and South Australia and is not all available for use in the GMID.

4.1.1 The 1A Greater Goulburn Trading Zone

The water trading zones for Victorian regulated water systems is shown in Figure 16 and the possibilities for trade to and from each trading zone.

All offers to trade are subject to an environmental impact assessment and are contingent on system capacity. The 1A Greater Goulburn trading zone includes Lake Eildon in the south, the Goulburn river from Lake Eildon to the Goulburn Weir, the east Goulburn main channel and the irrigation areas to the east of Shepparton, the Waranga basin and the central Goulburn irrigation area to the west of the lower Goulburn river, the Waranga west main channel which leads to the Rochester irrigation area, which is just west of the lower Campaspe river, and the Pyramid-Boort irrigation areas east of the Loddon river.
It is possible to sell temporary water within the zone and to the following trading zones with minimal restrictions: 1B Boort, 3 Lower Goulburn, 4C Lower Campaspe, 6 Vic Murray Dartmouth to Barmah, 6B Lower Broken Creek, 7 Vic Murray Barmah to South Australia, 10A NSW Murray above Barmah Choke. Backtrade sales are possible to the following zones: 4A Campaspe, 5A Loddon. Backtrade is selling temporary water to an irrigation area upstream. Water entitlements sold from the 4A Campaspe and 5A Loddon zones to irrigators in 1A Greater Goulburn create opportunities for back trade. Water allocations to these entitlements are based on allocations made to the irrigation areas where the entitlements originated and where the water is stored, in this case the Campaspe and Loddon systems. When an allocation is made to these entitlements water flows from the Campaspe and Loddon systems to the Goulburn system which creates the opportunity for an equivalent amount of water to be traded back.

It is possible to buy temporary water from within 1A Greater Goulburn and from the following trading zones with minimal restrictions: 4A Campaspe, 4C Lower Campaspe, 5A Loddon. Back trades are possible from the following zones into 1A Greater Goulburn: 1B Boort (limits may apply to the net trade out of this zone), 3 Lower Goulburn, 6 Vic Murray Dartmouth to Barmah, 6B Lower Broken Creek, 7 Vic Murray Barmah to South Australia, 10A NSW Murray above Barmah Choke, 10B Murray irrigation areas. The NSW irrigation districts, 10A NSW Murray above Barmah Choke and 10B Murray irrigation areas, are not shown on this map. Backtrade sales from the Murrumbidgee and Darling systems, both located in NSW, may also be possible at times. The opportunities for buying water as a result of back trade are far larger than the opportunities to sell. The Victorian water register calculates the opportunities for back trade, on July 26th 2010 the opportunity existed to trade 98,379ML to the Goulburn Valley via back trades, the Goulburn Valley system encompasses 1A Greater Goulburn, 1B Boort and 3 Lower Goulburn. An example of how a back trade from the Murray system to the Goulburn system works is provided in [70].

The ability for the relative free trade in temporary water makes 1A Greater Goulburn a good zone from which to source data.
Figure 15: The Goulburn-Murray irrigation district.

Figure 16: Water Trading Zones for Victorian Regulated Water Systems.

Water trading zones for Victorian regulated water systems

as at February 2009

Legend
- Shaded areas: irrigation areas / districts
- Unshaded areas: other entitlements
- Trading zones numbers

<table>
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<tr>
<th>Zone</th>
<th>Trading Capability</th>
<th>Trading Zone Date</th>
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</tr>
</tbody>
</table>

Note: Trading zones 1, 2, and 3 are subject to trade restrictions.
4.1.2 Infrastructure upgrades in the Goulburn-Murray irrigation district

A large investment in rationalising and upgrading irrigation infrastructure is being made in the GMID. The upgrade will be implemented in two stages. The first stage is due for completion in 2013 and it is estimated it will account for 225 GL of water currently lost in the system due to faulty metering, evaporation, seepage or by other means. This water will be shared between irrigators, the environment and urban users. It is designed to consolidate the channel system from around 6,300km to approximately 2,300km and to modernise it. Water meters will be upgraded to meet a new national standard and channel gates will be automated. It is expected that water will be available in a time frame that is near to “on-demand” and that there will also be opportunities for farm automation using soil moisture monitoring and irrigation scheduling. Stage 2 of the project is estimated to account for a further 200 gigalitres of water and is projected to be completed by 2018.

The efficiency with which market participants can execute a strategy for water use should increase with the automation of the irrigation system. Operating channel gates remotely and taking delivery of water for use almost immediately can offer opportunities for productivity gains. Buyers of temporary water often wait until just before they want to use the water and then enter the market [21]. A logical course of action since it might rain making the purchased water superfluous. If the system upgrade proceeds as planned it will allow for a higher level of sophistication in farm planning since buyers and sellers can take action at what they consider the optimal time. Furthermore, accurately accounting for water use and the opportunity to make better use of technology should increase the amount of water available for trading. The depth of a market, i.e. the volume traded, and the liquidity of a market, i.e. the frequency of transactions, are important factors to consider when considering the effectiveness of a market. A deep, liquid market will produce the most accurate price signals.

4.1.3 The Victorian water grid

The Victorian water grid is a network of rivers, channels and pipes linking Victoria’s major water systems. The state government is currently expanding the grid to allow
water to be moved around Victoria more easily. Two key pipelines built as part of the expansion will directly effect the water market in the GMID. The Goldfields super pipe was completed in May 2008. It has the capacity to deliver up to 18 GL a year to Ballarat and up to 20 GL a year to Bendigo. The sugarloaf interconnector will link the Goulburn river to Melbourne and it is expected to be able to deliver as much as 75GL to Melbourne annually. At maximum capacity these two pipes can move 113 GL from the GMID to supply the urban areas around Melbourne, Bendigo and Ballarat. The expectation is that this water will be sourced from the expected savings made from the infrastructure upgrades.

There is also a desalination plant under construction that will have the capacity to supply up to 150GL of water to Melbourne, currently about one-third of annual requirements. The desalination plant is a water source that is independent of rainfall so it can supply water to Melbourne at the same rate during droughts. So, while the expanded water grid sees water flowing out of the GMID, it is envisaged [72] that the stable supply of water to Melbourne from the desalination plant when combined with the projected extra water from the infrastructure upgrades could increase the supply reliability of water entitlements and allow an increase in trading of temporary water in the water markets. The water grid expansion may also increase the number of market participants in the future.

4.1.4 Policy changes

Perhaps the most significant policy change affecting the temporary water market in Victoria in recent years was the introduction of carryover. Initially introduced in March 2007 as an emergency drought response carryover was adopted permanently in December 2007. At the end of the 2007-08 seasons up to 30% of the volume of a water entitlement could be kept for use in a subsequent season. This was increased to a maximum of 50% of entitlement volume at the end of the 2008-09 season and 50% was again allowed to be carried over at the end of the 2009-10 season. After a review of the carryover rules several policy changes were implemented for the 2010-11 season. Entitlement holders now have an allocation bank account (ABA), water allocations made throughout the season are made to this account and can be used by the irrigator. In the 2010-11 season entitlement holders were allowed to carryover all of the water in their allocation bank account which was allowed to be up to 100% of entitlement volume. Any water in excess of the entitle-
ment volume could be carried over and stored in a spillable water account (SWA). Water in the SWA cannot be used or traded until the water resource manager makes a very low spill risk declaration, once the declaration is made any water in the SWA can be used or sold. Allocations are made to the water share in the current season until the current seasons’ allocations and the carried over volume reach 100% of entitlement volume. Any subsequent allocations are stored in the SWA until the very low spill risk declaration is made, after which they can be used or traded. If the storage spills, water in the SWA will be lost proportionately across all SWAs. All carried over water is subject to a 5% deduction in volume to account for evaporation losses. The carryover policy allows water to retain its value as the end of the season approaches. If unused water has to be returned to the consumptive pool at the end of the season, as was previously the case, it is likely that water will be used inefficiently.

A further change in the policies governing the use of water GMID is the introduction of a new reserve policy. Under the reserve policy in operation until the end of the 2009-10 season, water was reserved for system operations after April 1st or once the allocation volume to high reliability water shares reached 100%. In the drought conditions experienced over recent years allocations haven’t reached 100% and insufficient water has been held in reserve to meet all operating requirements by the beginning of the next irrigation season. The result was uncertainty over whether there would be any water to make an initial allocation in August when the irrigation season begins. A new reserve policy was introduced in the 2010-11 season. It’s intent is to accumulate sufficient system reserves to allow the distribution system to be run even under the most extreme drought conditions and to ensure enough water is available by August 15th in each year for initial allocations to be made and carryover water to be delivered. Under the new policy, to build the reserve for the following season water will be allocated to high reliability water shares until 30% of their entitlement volume is met. After that water will be allocated equally to high reliability water shares and to the system reserve until allocations to the water shares reaches 50% of entitlement volume. Once allocations reach 50% all water available until April 1st will be allocated to fulfilling current season allocations.

Modelling done as part of the Northern Region Sustainable Water Strategy [109] estimates that allocations will be reduced by about 1% over the long term, however on a year to year basis this could be as high 20%. While the modelling suggests only a marginal reduction in allocations over the long term, trading patterns may be effected since more water will be set aside earlier in the season which reduces the volume available for trading. However
once the new reserve system is in place it will allow trading to begin as soon as the season begins. In fact trading can be continuous since the buyer will know with certainty that they can take delivery of water at the start of the season whereas previously they would have had to wait until the distribution system was operational. This means that the price of temporary water allocations will better reflect changes in supply and demand over the winter months.

4.2 The eastern Australian gas network

Australia has two distinct gas networks, one on the west coast and another on the east. Discoveries of large gas fields in the Cooper and Gippsland basins in the late 1960’s marked the beginning of the east coast network while gas fields discovered in the Carnavon basin formed the foundations of the natural gas industry in Western Australia. The east coast network can be broken down further into demand centres which are based around the states and their largest east coast cities and industrial centres. The connected network is a relatively new phenomenon, historically each state had its own unique gas market and transmission network. Typically a large field was discovered, a production facility was built and a high pressure transmission pipeline constructed from the field to a demand centre. Government involvement in the industry was high with distribution and transmission networks often owned by state governments. Long term contracts between the companies developing the fields and government utilities provided relatively stable cashflows which allowed for investment in the expansion of the gas networks. In the decades since, new fields have been found, new pipelines have been built, government ownership of assets has decreased and government regulation has diminished. The increased use of gas, the new fields discovered and the new pipelines built have formed the interconnected network of pipelines that can be called the Eastern Australian Gas Network, a series of high pressure transmission pipelines linking all the major producing gas fields to all the major demand centres in eastern Australia. Figure 17 was published in the 2011 Gas Statement of Opportunities [6]. It shows the key components of the Eastern Australian gas network.
Figure 17: The Eastern Australian Gas Network.
4.2.1 Energy resources in Australia

Australia has abundant energy resources with large amounts of coal, natural gas and uranium. It is a large exporter of coal for energy and steel making and a large exporter of LNG. It has crude oil resources but is a net importer of oil. Uranium resources are developed as an export commodity but are not used in any significant quantity domestically with use restricted to medical and research applications. Uranium is not used to produce electricity in Australia. All states and territories in Australia have access to natural gas as an energy source and it is used by both heavy and light industry, in the residential sector, and for generating electricity. Although the use of natural gas is widespread, the interest both domestically and internationally in using it as a substitute for other fuels has increased in recent years due to its favourable environmental characteristics. Each joule of energy released from burning natural gas emits less carbon dioxide, as well as less sulphur dioxide, nitrogen oxides and fewer particles when compared to coal and oil [90]. There is also a reluctance to use, or increase the use of, nuclear power in some countries with gas being considered as a viable alternative. Natural gas can be consumed at or near the supply source, for example to generate electricity for mines and associated infrastructure or for large industrial use such as the production of fertiliser or minerals processing. Alternatively the gas fields can be connected to high pressure transmission pipelines that transport the gas over large distances to major demand centres. An example is the gas from the Gippsland basin off the coast of southern Australia that is transported via the Longford-Melbourne Pipeline to the city of Melbourne and on the Eastern Gas Pipeline to the city of Sydney. A third use of natural gas is as an export commodity in the form of LNG. Natural gas chilled to minus 161°Celsius at sea level transforms into a liquid. When in liquid form the gas occupies a tiny fraction of the volume it does as a gas, (about $\frac{1}{600}$th), which makes it possible to transport via ship. Australian exports of LNG in the September quarter of 2011 were worth $3$ billion [2] with gas supplied from the Carnavon basin off the western coast of Australia and from the Bonaparte basin in the Timor Sea off the coast of north western Australia [6].

4.2.2 Types of hydrocarbons

Methane is a hydrocarbon occurring naturally in gaseous form at normal pressures and temperatures [110], it is commonly referred to as natural gas or sometimes as ‘sales gas’. 
Hydrocarbons are organic compounds consisting of hydrogen and carbon and although they consist of only two elements they exist in a variety of compounds, the smallest are gaseous and the largest are solids. Hydrocarbons are fossil fuels formed by the degradation of organic matter over millions of years. The material that covers the organic matter forms rock reservoirs in the Earth’s crust and the pressure and heat change the matter into coal, oil or gas depending on the composition of the matter and the conditions specific to each reservoir [102]. Natural gas can be broadly categorised as unconventional or conventional gas. Conventional gas is further classified as associated and non-associated gas depending on the presence of oil. While associated gas occurs with oil it is desirable, where possible, to extract the oil while leaving the gas in place as the pressure of gas in the reservoir helps with the oil extraction. Once extracted the oil is separated from the gas that is present. Associated gas is often high in natural gas liquids such as ethane, propane, butane and pentane which are removed and can be sold separately. The gas remaining in depleted oil fields can be harvested later if desired. Preparations are being made to extract gas from the Tuna field in the Gippsland basin; oil has been produced from the field for sometime. Non-associated gas is found in reservoirs with little or no crude oil or gas liquids which makes its extraction easier than associated gas. Once the reservoir is tapped the gas flows under its own energy to the surface where the temperature and pressure are lowered so that any liquids present condense and can be separated from the mixture. Other sources of gas are generally referred to as unconventional gas. Two types of unconventional gas that currently support large scale production are shale or “tight” gas and coal bed methane, which is known as coal seam gas in Australia. While conventional gas is found in rock reservoirs where it is trapped in small empty spaces between the rock and can be extracted by the drilling of wells into the reservoir, shale gas is found in low permeability reservoirs that require some form of stimulation before the gas can be extracted. This can be done by creating fractures in the rock to create a channel for the gas to flow through. Shale gas is produced on a commercial scale in North America. Coal seam gas is methane that forms in coal seams. During the coalification process gas is produced and absorbed into the micropores of the coal surface where it is held in place by water pressure. The gas is released by drilling a well into the seam and reducing the water pressure by pumping the water out. Because coal has a very large internal surface area, coal seam gas deposits have the potential to hold more gas per unit volume than conventional reservoirs. Generally coal seam gas production requires more wells to be drilled than conventional reservoirs however the gas produced is generally pure methane and requires little processing [102].
4.2.3 How gas networks evolve in an Australian context

The Eastern Australian Gas Network is comprised of several smaller networks that have evolved separately around state based markets. The government of the state of Queensland has published several reviews of the states’ gas market. In the 2011 review [108] a short history of gas use in the state was provided. It is instructional as an example of how gas markets developed in the eastern Australian states. The first commercial gas discoveries in Queensland were around the town of Roma which is situated about 475 kilometres west of Brisbane. The volume of gas discovered in the fields was sufficient to supply the Roma power station and in 1961 it was converted to use a gas-fired generator. As the gas production capacity increased a project began to link Wallumbilla, which is 30 kilometres east of Roma, to Brisbane the largest city in Queensland. The Roma-Brisbane Pipeline started operating in 1969 and is still in use. As gas fields around Roma began to be depleted a new field was being developed at Ballera in south west Queensland. The Moomba gas plant in central Australia is the closest production facility to the Ballera fields and a 190km pipeline was constructed to connect the two. Raw gas was sent from the Ballera fields to be processed into sales gas at Moomba and then sent to customers in Adelaide to the south and Sydney to the south east, as well as regional customers along the two routes. To connect the Ballera fields to the Brisbane market the South West Queensland Gas Project was undertaken. Three foundation customers agreed to buy gas in sufficient volumes to provide a level of confidence that the cost of construction was likely to be recovered and the South West Queensland Pipeline was built to connect the Ballera fields to the Roma-Brisbane pipeline at Wallumbilla [108]. Foundation customers with long term contracts are the typically method of developing new gas infrastructure in Eastern Australia. State governments have also been involved in the early life of gas networks and regulation of access to infrastructure and pricing of that access was common. As assets have matured and diversity of supply has improved government regulation has reduced.

The history of gas use in Queensland is a typical example of how gas resources were exploited in Eastern Australia. Firstly a gas field was discovered and its size determined to ensure it was large enough for production. The time frame over which production could be maintained was also considered, this is important as all fields will be depleted in time. If production could be maintained for long enough to justify the large initial capital expenditure required the field was developed. Once the size and duration of production was established potential buyers are found and supply contracts are agreed. The
supply contracts, sometimes called gas sales agreements, were typically long term contracts that ran for as long as 15 years or more. The long term contracts were deemed necessary to limit the risk involved and are still typical for ‘green field projects’, that is projects that begin with little or no existing infrastructure. Similar negotiations took place to determine pipeline access and tariffs for transporting gas on the new pipeline. When the supply and transmission contracts were in place a high pressure transmission pipeline was built that connected the production facility at or near the field either directly to the customers or to a ‘city gate’, where the gas could enter the low pressure distribution network before connecting to the end users. The progressive development of resources and infrastructure, often backed by long term supply contracts and sometimes with government involvement was typical of gas market development in Australia.

4.2.4 Exploration, extraction and production of methane

The natural gas industry begins with exploration, and when exploration is successful, extraction and production. The exploration process is dependent on the expected value of the next discovery exceeding the exploration cost, including drilling unsuccessful wells, and the cost of developing and producing from the field discovered [48]. The raw gas extracted from the Earth must be processed to meet quality specifications before it can be transported and consumed. Exploration, extraction and processing can be considered the ‘upstream’ part of the gas supply chain. The upstream part of the process is usually centred around a processing facility that provides a node or hub. Small lateral pipelines connect wells to the facility and supply it with raw gas for processing. The processed gas is then transferred to the high pressure transmission network for transportation. The large fields in Australia generally have a key node where a major production facility is located, three such facilities are Moomba, Longford and Roma which are processing nodes for the Cooper, Gippsland and Bowen/Surat basins respectively. After processing the next stage in the supply chain is transportation. Gas is transported via the transmission network. The transmission network is a set of pipelines where gas is transported under high pressure over large distances. Once the destination is reached the pressure in the transmission pipelines is reduced and the gas enters lower pressure pipelines called the distribution network.

The final stage of the domestic gas supply chain connects the transmission network to the
end user. The pressure of the gas is reduced at ‘city gates’ and it enters the distribution network, a network of lower pressure pipelines that connect to the consumer. Large consumers such as power stations and big manufacturing facilities may connect directly to the transmission pipeline network and reduce the gas pressure at their own facilities before use [13]. When gas is destined for export the supply chain will differ and depend on particular circumstances however gas must still be discovered, extracted, processed and transported with the major difference being the transportation method. The gas will be liquefied at LNG export terminals and then loaded on specially designed ships which can maintain the gas in its liquid form until reaching the destination. The gas is then unloaded at ports built to accommodate LNG tankers before being re-gasified.

4.2.5 Competition in the upstream supply chain

Competition concerns are worth considering when looking at the upstream portion of the gas supply chain, particularly in the basins where development began in the early years of natural gas use in Australia. Where large companies dominate ownership of the production facilities they may be in a position to engage in anti-competitive practices. The Cooper and Gippsland basin are two basins that began supplying major cities in the 1970’s and are still crucial sources of supply today. The industry in eastern Australia has developed around these two gas basins and their gas processing facilities. Both facilities have an owner/operator with at least two thirds ownership. Where smaller companies are involved their stake in the projects is far smaller than that of the dominate company which, as a consequence, can be expected to dominate decisions regarding the production process. Historically, gas was also sold on common terms and conditions although this is now less common. The marketing of wholesale gas is also subject to legislation that prohibits anti-competitive behaviour and the Australian Competition and Consumer Commission can investigate marketing arrangements if it deems it necessary to ensure adequate competition exists [13].

Transmission pipelines are natural monopolies and there is little benefit in constructing parallel pipelines between two destinations if both have significant spare capacity. Transmission pipelines operate between safe maximum and minimum pressure limits and provided the maximum pressure is not exceeded the amount of gas transported can be increased by adding a compressor station along the route. Gas under higher pressure
will travel faster and more gas can flow through the pipe thus increasing the capacity. A pipeline segment that loops around a choke point can also be constructed to increase capacity making it unnecessary and impractical to build two independent competing pipelines from a production node to a demand centre. This makes pipelines natural monopolies and access to transmission pipelines in Australia has been regulated in the past however the amount and level of regulation has been declining as more producing fields and connecting pipelines have been added to the pipeline network.

4.2.6 Estimating the size of hydrocarbon reserves

Oil and gas exploration begins with identifying prospective locations and the drilling of wells to test the prospect. When a resource is found more wells are drilled to begin determining the extent of the resource. As the precise amount of oil or gas enclosed in a reservoir in the Earth’s crust is impossible to determine estimates of the volume are made. As a way of comparing various resources an industry standard has been developed for this purpose. The Petroleum Resource Management System (PRMS) is used to provide estimates of the size of oil and natural gas resources. The 2010 Gas Statement of Opportunities [10] gives a background to the PRMS. The PRMS was developed by the American Association of Petroleum Geologists, the Society of Petroleum Engineers, the Society of Petroleum Evaluation Engineers and the World Petroleum Council and is commonly used within the petroleum industry to evaluate oil and gas reserves. All quantities of oil or gas are included in an identified resource, this includes amounts that are not yet discovered but thought to exist, or amounts that may prove to be unrecoverable. The volumes are then categorised based on the likelihood that they will one day be able to sustain production. A summary of the categories used are as follows. The quantity of the resource that has already been recovered is recorded as current production, the reserves are the part of the resource which are considered commercially recoverable, contingent resources are resources that are less certain than reserves to be recoverable. Prospective resources are the estimated volumes thought to exist and that may have the potential to be recovered. Prospective resources are likely to be targets for exploration wells to confirm the presence of oil or gas. The estimated quantities of the resource that can not be recovered are referred to as unrecoverable. There may be several reasons why parts of a resource are unrecoverable, there may also be technological improvements that make unrecoverable resources recoverable in the future.
Reserves are volumes of oil and gas that are expected to be recovered from identified resources. Due to the uncertainty in determining reserves, estimates are made and classified based on the degree of certainty that they are accurate. They are defined as ‘Proved’, ‘Probable’ and ‘Possible’ estimates of reserve size and are commonly denoted as ‘1P’, ‘2P’, and ‘3P’. Proved reserves can be estimated with reasonable certainty to be the amount of oil or gas that is commercially recoverable, ‘reasonable certainty’ is taken to be at least a 90% chance. Probable reserves are less certain estimates. Probable reserves are taken to have a 50% chance that more than the estimate will be recovered. Possible reserves provide the least certainty of resource size. When considering possible reserves it can be assumed that there is a 10% or better chance that the amount of oil or gas recovered will exceed the estimate. Proven plus probable reserve estimates, ‘2P’ estimates, are considered a fair estimate or recoverable reserves and are used in the Gas Statement of Opportunities 2010 [10], as well as in reports by publicly listed companies, which are required by law to provide price sensitive information to the public via the stock exchange on which they are listed. The size of a company’s oil and gas reserves is price sensitive information.

Reserve estimates can be improved by drilling more wells and conducting more analysis, technological advancements and production data can also be used. This can in turn increase the size of proven or probable reserves. Attempts were made to find coal seam gas reserves in Australia by companies with experience in extracting gas from coal deposits in the United States, these attempts proved largely fruitless and by the mid-1990’s most of the attempts had been abandoned. The increase in the coal seam gas reserves that have subsequently occurred have been the result of improvements in technology and increased understanding of the geology of the coal fields where the exploration has taken place [16]. In addition, reserves estimates from the Gippsland and Cooper-Eromanga basins have been updated over the four decades since they first begin operating. Production data and increased understanding of the resource together with the drilling of more exploration wells make the updating of reserve estimates possible.

4.2.7 Eastern Australian gas basins and reserves

Eastern Australia has six basins where oil and natural gas are produced, these are the Bowen-Surat basin, the Cooper-Eromanga basin, the Gippsland basin, the Otway basin,
and the Bass basin. The Bowen-Surat basin is located parallel to the Queensland coast in Figure 17, the Cooper-Eromanga basin straddles the borders of Queensland, New South Wales and South Wales in central Australia, the Otway basin is located south west of Melbourne in Figure 17 while the Gippsland is located south east of Melbourne.

The conventional gas basins in eastern Australia are the Gippsland, Bass, and Otway basins and the Cooper-Eromanga basin. The Gippsland, Bass, and Otway basins are located off the southern coast of mainland Australia in Bass Strait, the narrow stretch of water that separates the mainland from the island of Tasmania. The Cooper-Eromanga basin is located in central Australia and covers a large area around the border of the state of Queensland and the state of South Australia. The Bowen-Surat basin, which covers large parts of Queensland is a major source of coal seam gas and a small amount of coal seam gas is also produced from the Sydney basin in New South Wales [6]. Production comes primarily from the Gippsland, Bass and Otway basins, the Cooper basin and the Bowen-Surat basin. Only small amounts of gas are produced from the Sydney basin.

The Gippsland basin has been supplying gas to the city of Melbourne since 1969. It is predominately an off-shore basin with several gas platforms in Bass Strait connected by pipeline to an onshore processing facility at Longford, which is located about 174 km east of Melbourne. Since the construction of the Eastern Australian Gas Pipeline in the year 2000, gas from the Gippsland basin has supplied the Sydney market in New South Wales and Canberra in the Australian Capital Territory, as well as some regional centres along the pipeline route. The Tasmanian Gas Pipeline was built in 2002 to connect the Longford processing plant to the state of Tasmania to allow gas from the Gippsland fields to supply the Tasmanian demand for gas. Remaining 2P reserves in the Gippsland basin are currently estimated at 5,455 PJ in the 2011 Gas Statement of Opportunities [6].

The Otway basin is located in the Otway region of Victoria, approximately 150 km west of Melbourne. The field covers the coastal region of western Victoria and the south east corner of South Australia. The producing fields in the basin are off-shore and supply the Melbourne region via the South West Pipeline and Adelaide via the South East Australia Gas Pipeline (SEAGas pipeline). The basin has only been developed over the last decade and has more producers than the older fields where one dominant producer/operator largely controls the resource development. The Otway basin is currently estimated to have 991 PJ of 2P reserves remaining [6].
The Bass basin lies in Bass Strait south of Melbourne, between the Gippsland basin and the Otway basin. It is the most recently developed of the three and supplies gas through lateral pipeline to a processing facility at the town Lang Lang which is located approximately 85 km south west of Melbourne. The gas is then used to satisfy a portion of the Melbourne demand. Remaining 2P reserves in the Bass basin are currently estimated to be 266 PJ [6].

The Eromanga basin covers a large area of central Australia predominately in Queensland and South Australia, but also extending as far south as New South Wales and as far west as the Northern Territory. The Cooper basin is a smaller region of the Eromanga basin located around the Queensland and South Australia border. The Cooper-Eromanga basin has been an important energy producing region since gas was first discovered in the 1960’s. Gas is processed at the Moomba processing plant and has been transported using the Moomba-Adelaide Pipeline to supply Adelaide since 1969. The Moomba-Sydney Pipeline was constructed in 1976 and gas from the Cooper basin began supplying Sydney. The Ballera processing facility is located in the Cooper Basin and processes gas from surrounding fields. The gas has been transported using the South West Queensland Pipeline to supply Brisbane since 1997 and via the Carpentaria Pipeline to supply Mt. Isa since 1998. There are 1,396 PJ of 2P reserves remaining in the Cooper-Eromanga basin based on current estimates.

The Bowen-Surat basin is a large region located parallel with the east coast of Queensland and extending south into New South Wales. It is a well established coal producing region and supplies coal in large quantities for export and domestic use. Conventional gas was found around the Roma region and the Roma-Brisbane Pipeline was completed in 1969 to supply gas to Brisbane. Current estimates of conventional gas reserves remaining in the basin are reasonably large at 205 PJ however when considered alongside the reserves of coal seam gas they are insignificant. Exploration conducted over the last fifteen years in the Bowen-Surat basin has seen coal seam gas gas reserves increase rapidly to the current estimate of 33,719 PJ. Gas from the producing fields can be shipped to Brisbane on the Roma-Brisbane Pipeline or to Gladstone on the Queensland Gas Pipeline. There is existing demand from industry at Gladstone and four confirmed or likely to be confirmed projects to produce LNG for export. The different production methods for coal seam gas, which requires many wells to be drilled and so allows incremental development, allowed smaller companies to become gas producers however larger firms have now consolidated ownership. The basin has more than a dozen production facilities, reflecting the different
methods used to produce coal seam gas as compared to conventional gas.

4.2.8 Reserve estimates

Table 2 condenses data from Table 3-1 and Table 3-2 which appear in Chapter 3 on pages 9 & 10 of the 2011 Gas Statement of Opportunities [6]. The estimates were taken as at December 31st 2010 and show the initial 2P estimate of gas reserves and the remaining 2P and 3P estimates of reserves in producing basins in eastern Australia. There are other basins with significant gas reserves in eastern Australia and there are plans for some of these basins to come into production in the future however none are currently producing on a significant scale. For a detailed look at the rise and fall of reserves in eastern Australia see Table 2-1 of [16] which has periodic estimates of reserves beginning in 1975 and ending in 2009.

<table>
<thead>
<tr>
<th>Name</th>
<th>Initial 2P Reserves</th>
<th>Remaining 2P Reserves</th>
<th>Remaining 3P Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bowen-Surat</td>
<td>35,761</td>
<td>33,924</td>
<td>52,686</td>
</tr>
<tr>
<td>Gippsland</td>
<td>14,020</td>
<td>5,455</td>
<td>7,163</td>
</tr>
<tr>
<td>Cooper-Eromanga</td>
<td>8,023</td>
<td>1,396</td>
<td>1,612</td>
</tr>
<tr>
<td>Otway</td>
<td>1,572</td>
<td>991</td>
<td>1,518</td>
</tr>
<tr>
<td>Bass</td>
<td>347</td>
<td>266</td>
<td>337</td>
</tr>
<tr>
<td>Total</td>
<td>60,048</td>
<td>42,325</td>
<td>63,785</td>
</tr>
</tbody>
</table>

4.2.9 Reserve depletion and augmentation

Natural gas is a finite resource and discovered reserves deplete as the gas is extracted and consumed. Exploration around existing and prospective fields can augment reserves however in time they too will be consumed. Consideration must be given to the likely size of reserves in the future and the way gas is consumed in the present.

Gas was discovered in the Gippsland basin in the 1960’s and bought into production. The city of Melbourne is Australia’s second largest city and provides an obvious market, and
its temperate climate provided an obvious use. Most homes in Melbourne are connected to the gas distribution network and can use gas for home heating, among other things. The widespread use over a long period of time has seen a depletion of the resource to less than half of its original 2P estimate. It should be noted that it is still considered a highly prospective basin and new production is expected to commence in the near future.

The Cooper-Eromanga basin was being developed over a similar time frame and provided gas to Sydney, Australia’s largest city, and Adelaide the state capital of South Australia. The reserve depletion from the initial 2P estimates of 8,023 PJ to the current estimate of 1,396 PJ demonstrates its historic importance as major source of energy. While conventional gas has been produced in the Bowen-Surat basin over the same period of time as the Gippsland and Cooper-Eromanga, and shows the same depletion of 2P reserves, it is the huge increase in coal seam gas reserves over the last 15 years that are changing the dynamic of the gas network in eastern Australia. The remaining 2P reserves of coal seam gas are more than four times larger than those of conventional gas. It is the size of these reserves that is underpinning the LNG export facilities around the Queensland city of Gladstone, and potentially changing the volumes of gas and the direction of flow around the pipeline network. Coal seam gas reserves and LNG exports are also expected to impact the domestic price for gas with some suggesting an increase to a local equivalent of the LNG export price.

4.2.10 Demand

Historically the states in eastern Australia have developed their gas infrastructure independently based on their unique needs and opportunities. As demand grew new sources of supply were discovered and the state based gas networks expanded. Over the first decade of the 21st century pipelines linking the networks were constructed and the producing basins in eastern, southern and central Australia are linked to all the major demand centres in Australia other than those in Western Australia and the Northern Territory.

The largest consumers of gas on a state by state comparison are Victoria and Queensland. The profile of gas use in these two states is very different with a large proportion of gas used in the residential sector in Victoria and relatively little for gas powered electricity
generation. While in Queensland the converse is true. Demand is seasonal in Victoria, cold weather increases demand, while in Queensland demand is not seasonal. The remaining states in eastern Australia share similar characteristics although not to the same extremes. The diversity of demand profile reinforces the notion of separate entities maturing independently before merging into one network.

Queensland has significant demand for natural gas in the capital city of Brisbane and in the regional industrial city of Gladstone as well as in and around the mining town of Mt. Isa. Annual demand amounted to 201 PJ in 2010 [6]. In Queensland there is limited demand from homes for heating and most of the gas is used by industry and for electricity generation, about 50% and 40% respectively [10]. Natural gas is mostly supplied from the Bowen-Surat and Cooper-Eromanga basins. The largest pipelines in Queensland are the Roma-Brisbane Pipeline, the South West Queensland Pipeline, the Queensland Gas Pipeline and the Carpentaria Gas pipeline.

The Roma-Brisbane Pipeline is 440 kilometres long and has a reported capacity of 233 TJ per day with plans announced by the pipeline owner to increase the capacity by approximately 10%. The South West Queensland Pipeline begins at Roma and runs west to Ballera and then on to the Moomba gas plant in the Cooper-Eromanga basin. The final leg from Ballera to Moomba is known as the QSN link. The addition of the QSN link brings the length of the pipeline to 935 kilometres and provides a pipeline link to demand centres in south eastern Australia. The QSN link allows for sales gas to be directly sent to southern markets, previously only raw gas was transported from Ballera to Moomba where it was processed into sales gas before being sold to customers. The reported capacity of the South West Pipeline is 384 TJ per day. The Carpentaria Gas Pipeline begins at Ballera in south western Queensland and runs north to Mt. Isa. The pipeline is 840 kilometres long and has a capacity of 119 TJ per day. Gas is generally supplied from the Cooper-Eromanga basin or from the Bowen-Surat basins. The Queensland Gas Pipeline runs from Roma to Gladstone on the coast north of Brisbane. It has a length of 630 kilometres and a maximum daily capacity of 145 TJ. This pipeline is a key piece of infrastructure in the developing LNG projects which will export gas from Gladstone.

The state of Victoria in southern Australia has the most sophisticated network of natural gas transmission and distribution pipelines in the country. Gas can be injected into the Victorian Declared Transmission System (DTS) at five locations and withdrawn at over
Gas consumed in Victoria in 2010 amounted to 213 PJ [6]. Victoria has a higher proportion of gas consumption by the residential sector than other states with average residential use of about 65 PJ per year [13]. Gas is used for cooking, hot water heaters and most importantly home heating. As a consequence demand is heavily influenced by the weather with large amounts of gas consumed during the coldest months of the year, for example, the peak gas demand day in 2010 occurred on the 29th of June when 1,207 TJ of gas was consumed [8].

The discovery of commercial quantities of gas in the Gippsland basin off the south east coast of the state in the 1960’s has seen significant amounts of infrastructure develop and the Gippsland basin supplies a large proportion of gas to the Victorian market. The Otway gas basin has since been developed off the south west coast of Victoria and a third smaller gas basin, the Bass basin has recently been developed to add to the larger two. These gas basins have historically supplied the Victorian market however there are now pipelines running to South Australia, Tasmania, New South Wales and the Australian Capital Territory and gas can also be delivered directly to these markets.

The Longford gas processing plant currently has a capacity 1,145 TJ per day. It is the largest processing plant in eastern Australia. A smaller gas processing plant at Orbost in Gippsland has a production capacity of 90 TJ per day. The newest and smallest producing basin is the Bass Basin. Gas is processed at the Lang Lang gas plant and supplied to the DTS at a maximum rate of 70 TJ per day. There is also a storage facility for LNG at Dandenong south-east of Melbourne. If these three facilities are grouped, 1,463 TJ/day of gas can be supplied. Gas from the Otway basin is processed at the Minerva and Otway gas plants, which have a capacity of 81TJ/day and 205 T.J/day respectively. There is also an underground storage facility with a capacity of 500 TJ. As a collective these facilities can supply 786 TJ/day to the market [122].

The largest transmission pipeline in the DTS is the Longford-Melbourne pipeline. It has a length of 797 kilometres and a maximum capacity of 1,030 TJ per day [6]. Gas is sourced from the Gippsland fields, processed at Longford and is used to supply the Melbourne demand centre via the Longford-Melbourne Pipeline. The Longford plant and the Longford-Melbourne Pipeline are the largest pieces of infrastructure for the supply, processing and delivery of natural gas in eastern Australia. The South West Pipeline transports gas from the Otway processing plants to Melbourne. The South West Pipeline
has a length of 144 kilometres and a maximum capacity of 353 TJ per day [6].

Major pipelines connected to the DTS include the Eastern Gas Pipeline, which runs from Longford to Sydney, the SEAGas Pipeline from Otway to Adelaide, the Tasmanian Gas Pipeline from Longford to Hobart, and the New South Wales to Victoria Interconnect (NSW-Vic Interconnect).

In 2010 about 130,000 TJ of natural gas were consumed in New South Wales and the Australian Capital Territory (ACT) [6]. The majority of the Gas used is imported from the Cooper Basin via the Moomba-Sydney pipeline and the Gippsland basin via the Eastern Gas Pipeline.

Historically gas consumed in South Australia was processed at the Moomba gas plant in the Cooper-Eromanga basin and transported along the Moomba-Adelaide Pipeline to Adelaide, the largest city in South Australia. The development of the fields in the Otway basin lead to the construction of the SEAGas Pipeline which runs from the onshore processing plants of the Otway basin to Adelaide, allowing a competing source of supply. The construction of the QSNLink, which extended the South West Queensland Pipeline from Ballera to the Moomba gas plant now also allows gas from the Bowen/Surat basin fields to supply South Australia. The closure of some industrial manufacturing plants that used natural gas as well as imports of electricity, resulting in a decline in gas powered generation in the state, has seen demand for gas in South Australia decline in the years leading up to 2008 [13]. The state of South Australia used approximately 102,000 TJ of gas in 2010 [6].

Construction of the Tasmanian Gas Pipeline was completed in 2002 and the state of Tasmania has access to natural gas as a fuel source for the first time. Approximately 10,000 TJ of gas was consumed in Tasmania in 2010 [6] with all of it being transported on the Tasmanian Gas Pipeline from the Longford plant which remains the only source of supply. A new gas fired power plant that is under construction is likely to increase the use of natural gas in the state when construction is complete.
4.2.11 Processing of natural gas

Natural gas consists mainly of methane and small amounts of other light hydrocarbons. Mokhatab, Poe, and Speight provide an example of a ‘typical’ composition of natural gas in Table 1-1 on page 3 of the “Handbook of Natural Gas Transmission and Processing” [69]. The table is reproduced in Table 3 and although they note that the composition can vary widely most gas fields can be expected to contain a similar mix of hydrocarbons. The Australian energy market operator describes natural gas as 70%-90% methane with the balance ethane, propane and butane, as well as carbon dioxide, carbon monoxide, nitrogen, water with traces of nitrogen oxides, mercury and other heavy metals as well as hydrogen sulphide [5].

Table 3: Typical Composition of Natural Gas

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Volume (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>&gt; 85</td>
</tr>
<tr>
<td>Ethane</td>
<td>C₂H₆</td>
<td>3 – 8</td>
</tr>
<tr>
<td>Propane</td>
<td>C₃H₈</td>
<td>1 – 2</td>
</tr>
<tr>
<td>Butane</td>
<td>C₄H₁₀</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Pentane</td>
<td>C₅H₁₂</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>1 – 2</td>
</tr>
<tr>
<td>Hydrogen sulphide</td>
<td>H₂</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>N₂</td>
<td>1 – 5</td>
</tr>
<tr>
<td>Helium</td>
<td>He</td>
<td>&lt; 0.5</td>
</tr>
</tbody>
</table>

All pipeline owners and operators will have specifications for the composition of gas that can be transported through their pipelines to avoid damage and limit maintenance requirements. The owner of the Eastern Gas Pipeline, for example, provide the specification of gas quality in section 3.2.1 Natural Gas Specification of the Eastern Gas Pipeline Measurement Manual [56]. Among other things the gas must be free of material such as dust, waxes or gums, liquids or other matter that may damage or interfere with the operation of the pipeline. A minimum and a maximum temperature of 2°C and 50°C are specified, as is a maximum oxygen content of 0.2% by volume. Total inert gases, of which carbon dioxide and nitrogen are two, are capped at a maximum of 7% by volume and maximum contents are specified for hydrogen sulphide (5.7 mg/m³) and sulphur (25 mg/m³). The total for sulphur excludes the amount added to gas to give it an odour, which is a necessary safety precaution. In Section 2.1 of the EGP Operations Manual [55] the pipeline owner stipulates that the gas must be free of substances deemed objectionable which are
stated to include various trace metals and other compounds and radioactive substances. Different pipeline owners may have specifications that differ slightly from those above but the general purpose is to ensure a regular quality of gas is supplied safely and with minimal damage to the transmission pipelines.

Raw gas must be processed before it can enter the transmission pipeline network for transportation to demand centres. Gas processing plants separate the components of the raw gas flowing from the gas field for sale or disposal and regulate delivery pressure into the transmission pipeline [69]. The raw gas must be processed to remove the heavier hydrocarbons and other miscellaneous compounds from the methane. The heavier compound hydrocarbons such as ethane, propane, butane and pentane, known as natural gas liquids, have a variety of uses and are a valuable by-product of natural gas production [102].

4.2.12 Processing facilities in eastern Australia

The 2011 Gas Statement of Opportunities [6] identifies and provides brief descriptions of the production facilities in the Eastern Australian Gas Network, a summary of which is provided in the following section and then utilised in modelling the network.

The Bowen and Surat basins in Queensland produce gas predominately from coal seam deposits and the number and size of the processing facilities reflect the different nature of the production process when compared to the conventional off-shore gas fields near the Victorian coast. The production process for coal seam gas requires the drilling of more wells than conventional gas fields which means that fields can be developed in incremental stages. Incremental development, as well as the history of development of coal seam gas in Queensland, which has been used in the past to power electricity turbines, sometimes exclusively, explains the large number of processing plants in the state. There are four gas processing plants that process conventional gas, Kincora, Rolleston, Silver Springs, and Yellowbank. The conventional gas processing facilities tend to be located further apart than the facilities processing coal seam gas and as a result each has its own lateral pipeline. The Rolleston and Yellowbank facilities connect to the Queensland Gas Pipeline and the Kincora and Silver Springs facilities connect to the Wallumbilla processing plant.
The Wallumbilla processing plant is located at the Roma supply node and connects to South West Queensland Pipeline, the Queensland Gas Pipeline and the Roma-Brisbane Pipeline. The maximum capacity of the Yellowbank, Rolleston, Kincora, and Silver Springs plants is, in descending, order 30 TJ/day, 30 TJ/day, 25 TJ/day, and 8.8 TJ/day respectively [6].

The Kenya processing plant is the largest in the Bowen-Surat basin with a maximum capacity of 160 TJ/day. The Kenya plant has a connection to the Roma-Brisbane Pipeline. The Berwyndale South and Talinga plants are located in close proximity to the Kenya plant. Berwyndale South has a stated maximum capacity of 140 TJ/day and has a connection to the Roma-Brisbane pipeline and to another smaller pipeline, the Braemar pipeline. The Braemar pipeline connects to the Braemar power station which is a gas powered generator that provides electricity during times of peak demand. The Talinga plant can produce 120 TJ/day of processed gas at maximum capacity. The Scotia and Peat plants are located north of the cluster on the same lateral pipeline as Berwyndale South and can process 30 TJ/day and 15 TJ/day respectively. Another large processing facility in the Bowen/Surat basin is the Fairview plant which has a maximum daily production capacity of 136 TJ/day. The Spring Gully, Taloona and Strathblane processing plant are all part of the one development. The Taloona plant has a lateral pipeline linking it to the Roma supply node. The remaining processing plants in the Bowen and Surat basins are Kogan North which feeds the Roma-Brisbane Pipeline and has a maximum daily capacity of 12 TJ/day, Dawson Valley the first coal seam gas plant, which can produce 16 TJ/day, and Wungoona which has a maximum daily production capacity of 20 TJ/day.

The Ballera and Moomba plants process conventional gas from the Cooper-Eromanga basin. The Moomba gas plant is one of the longest working plants in Australia with gas processed at Moomba being shipped to Adelaide in 1969. It now processes raw gas from over 100 fields in the basin. The maximum capacity at Moomba is 390 TJ/day. In the winter season the Moomba plant creates its own demand of 54 TJ/day, if this gas is sourced from its own supply the production capacity in the winter season would be reduced to 336 TJ/day. The Ballera plant processes gas from about 45 gas fields in the surrounding region. It has a maximum daily capacity of 100 TJ/day and until recently supplied gas to Mt.Isa and Brisbane, which is 1,200km east of Ballera. With the completion of the QSN Link pipeline segment of the South West Queensland Pipeline to Moomba, gas processed at Ballera can now supply demand in the southern states [6].
The majority of gas produced from the Gippsland basin is processed at the Longford plant which is by far the largest gas processing plant in the Eastern Australian Gas Network. The Longford plant has a maximum daily capacity of 1,145 TJ/day [122]. Gas processed at Longford supplies demand in Victoria, Tasmania, New South Wales and the Australian Capital Territory directly via connecting transmission pipelines. A second plant, the Orbost gas plant, originally processed gas from the Patricia-Baleen field, and currently processes gas from the Longtom field now that Patricia-Baleen has been depleted. Gas processed at Orbost, which has a maximum daily capacity of 90 TJ/day [122], feeds into the Eastern Gas Pipeline [122]. The newest producing fields off the Victorian coast are located in the Bass basin. The Bass basin is off the south-east coast of Victoria in Bass Strait and gas is processed at the Lang Lang gas plant which is located south east of Melbourne. Lang Lang can process gas at up 70 TJ/day [122]. The Dandenong LNG storage facility is located near the Dandenong city gate where the Longford-Melbourne pipeline and gas sourced from the plant at Lang Lang converge to supply Melbourne gas demand. The capacity of the Dandenong LNG storage is 158 TJ [122].

The Otway basin has two processing plants, Otway and Minerva. The Otway plant can process 205 TJ/day and the Minerva plant 205 TJ/day [122]. The Iona underground storage and processing plant is also located in the Otway basin. The Iona plant has a total capacity of 500 TJ/day [122] and processes gas from the Casino, Henry and Netherby fields which are located off the Otway coast. The Iona processing plant also injects gas withdrawn from the Iona underground storage facility, which is built in a depleted onshore field, into the the South West Pipeline. The South West Pipeline is a major component of the Victorian Declared Transmission System and the gas is transported from the Otway region to the city gate just west of Melbourne. Gas processed at Iona also supplies Adelaide via the SEAGas Pipeline. The nearby Minerva and Otway plants process gas from the Minerva and Thylacine fields respectively.

4.2.13 Transportation of natural gas

The transportation of natural gas is an important component of the model developed in the current work. The optimal routing of natural gas on days where all demand centres are at peak demand is modelled to minimise supply shortfalls and identify network
components that are likely to need capacity expansions as demand grows.

The options for transporting natural gas currently in use in Australia are by using pipelines, as is done for most domestically consumed gas, or as LNG when the gas is to be exported. Other methods of transportation are as a transformed manufactured products, such as fertilizer say, which could then be transported by conventional methods such as road or rail, or by transforming the gas into electricity and transporting it through the electricity grid [102]. A transportation method not currently in use on a large scale is as compressed natural gas which is gas transported in containers under pressure. Using specifically designed ships compressed natural gas can be transported in a similar manner to LNG and could be a useful complement to transportation by LNG tanker. In certain circumstances compressed natural gas may be a more cost efficient method of transportation than LNG, particularly over shorter distances and for smaller demand volumes [45].

Gas transported over land is generally done using gas pipelines. Pipelines are not always the most effective transportation solution as they are inflexible and can only deliver gas along the route of the pipeline, in addition when a country must import gas the pipelines must cross international borders which brings politics and diplomacy into the equation. An alternative for transportation between nations is as LNG which is simply the liquid form of natural gas. The liquefaction and re-gasification needed for LNG transport is complex and expensive and may not be viable on anything other than a very large scale operation however LNG has been used effectively for decades and the trade is increasing with new import and export terminals being built and extended around the world. Australia is among the largest exporters of LNG in the world and the volumes exported will rise in coming years as construction is completed on existing and new export facilities.

In Australia gas for domestic consumption is transported by pipelines. High pressure transmission pipelines are used to transport gas from the fields and processing centres as gas under pressure has a reduced volume and can be transported over long distances more efficiently. The high transmission pipelines connect to ‘city gates’ at large demand centres where the pressure is reduced before entering lower pressure distribution networks. It is the distribution pipelines that connect to the end user in most cases, although heavy users of natural gas may connect directly to a transmission pipeline. The main advantage
of using pipelines for transport are the low cost when compared to other transportation methods, however should the pipeline be shut down for any reason the facilities receiving the gas often need to be shut down as well. Pipelines must maintain a minimum and not exceed a maximum pressure and gas can be stored in pipelines by allowing the pressure in the pipeline to increase up to its maximum level. However, compressor stations and regulators are needed along the route to maintain the gas pressure at safe levels. This increases costs as the distance travelled increases.

After the processing of raw gas into a purer concentration of methane it is ready for transportation and sale. Pipelines are the main transportation method used in Australia when gas is consumed domestically. The network consists of transmission pipelines that typically have large diameters and operate under high pressure with the purpose of linking producing fields with demand centres. Since the year 2000 several pipelines have been built to link the regional networks. Theses pipelines are the Eastern Gas Pipeline which runs from the Longford plant in the Gippsland basin to Sydney in Figure 17, the Tasmanian Gas Pipeline which supplies gas to Tasmania from the Longford plant, the SEAGas Pipeline which connects production facilities in the Otway basin to Adelaide [12]. The other major connecting pipeline constructed in recent years is the QSN Link in Queensland which extends the South West Queensland Pipeline from Ballera in southwest Queensland to the Moomba processing plant and the southern portion of the Eastern Australian Gas Network.

Queensland has four high transmission pipelines linking the production nodes at Roma and Ballera to the demand centres in Brisbane, Gladstone and Mt.Isa directly, and the demand centres to the south indirectly via the Moomba production node. The Roma-Brisbane Pipeline links processing facilities in and around Roma to the city of Brisbane to the east, the Queensland Gas Pipeline also originates at Roma and supplies gas to Gladstone. The third pipeline beginning at Roma is the South West Queensland Pipeline which runs west to Ballera and then on to Moomba where gas is shipped to supply demand from the cities in the southern part of the network. The fourth high transmission pipeline located in Queensland is the Carpentaria Gas Pipeline which begins at Ballera and runs north to Mt.Isa.

The Roma-Brisbane Pipeline is the oldest major pipeline for transporting natural gas on a large scale in Australia having begun life in 1969 [108]. The capacity has been expanded
over its lifetime to its current maximum daily capacity of 233 TJ/day and lateral pipelines have been added to allow the injection of coal seam gas from new production facilities. Roma was the initial source of gas when the pipeline was constructed and the pipeline begins at Roma which is 30 km west of Wallumbilla. The precise location of transmission and lateral pipelines is at Wallumbilla, however when considering the Eastern Australian Gas Network the pipeline can be said to begin at the Roma node.

The Queensland Gas Pipeline runs from the Roma node north west to the coastal town of Gladstone where it supplies industrial customers and retail customers through a local distribution network. There are also two lateral pipelines running along the coast to Townsville in the north and Bundaberg and Maryborough in the south. Gladstone is the site for the LNG plants currently under construction and the capacity of the Queensland Gas Pipeline may need to be expanded to meet the demand of the new plants. The maximum daily capacity was increased in 2010 and the amount of gas flowing increased once able to do so [6]. The current reported capacity of the Queensland gas pipeline is 145 TJ/day [122].

The South West Queensland Pipeline runs from Roma to Ballera and then on to Moomba via the QSN link. At Roma it connects with the Roma-Brisbane Pipeline and the Queensland Gas Pipeline and at Ballera it connects with the Carpentaria Gas Pipeline. The QSN link, which is considered part of the South West Queensland Pipeline, ends at the Moomba processing plant and allows gas from the Bowen and Surat basins in Queensland to supply the southern demand centres via the Moomba-Adelaide Pipeline and the Moomba-Sydney Pipeline. The Roma and Barcaldine power stations are supplied by the South West Queensland Pipeline and have a capacity of 74 MW and 55 MW respectively. The maximum capacity of the South West Queensland Pipeline is 384 TJ/day [122].

The mining town of Mt.Isa is the destination of the Carpentaria Gas Pipeline which originates at Ballera. The gas is mainly used in industry for mineral processing and to generate electricity. The maximum capacity of the Carpentaria Gas Pipeline is 119 TJ/day with the average transported daily in the 2009 and 2010 calendar years being 88 TJ/day. In 2010 on May 5th, 106 TJ of gas flowed north from Ballera to Mt.Isa, this is the maximum amount transported in a single day in the 2009 and 2010 calendar years [6].
South Australia is the origin of two key pipelines in the Eastern Australian Gas Network, both beginning at the Moomba processing facility. The Moomba-Adelaide Pipeline runs south to Adelaide, the capital of South Australia and the Moomba-Sydney Pipeline runs south-east to Sydney in New South Wales.

The Moomba processing plant is one of the oldest in Australia and has been processing gas from the Cooper/Eromanga basin since 1969 for supply to Adelaide via the Moomba-Adelaide Pipeline. The Moomba-Pipeline has a maximum capacity of 241 TJ/d. There are two lateral pipelines that are also supplied via the Moomba-Adelaide Pipeline. The western lateral supplies industry in the towns of Port Pirie and Whyalla which are both located in South Australia to the north-west of Adelaide, and the eastern lateral supplies the town of Mildura in Victoria which is located east of Adelaide.

The Moomba-Sydney Pipeline has a maximum capacity of 439 TJ/d and supplies Sydney and Canberra as well as regional towns along the pipeline route [122]. A high transmission pipeline connects the Moomba-Sydney Pipeline with the Victorian declared transmission system at the regional New South Wales town of Young. Gas can flow into or out of the Moomba-Sydney pipeline at this point but there is no gas production or processing at Young.

The state of Victoria is the largest consumer of natural gas in the network and has the most complex gas transmission and distribution system. The Longford production node is by far the largest in the network and is the origin for the Longford-Melbourne pipeline, the Eastern Gas Pipeline and the Tasmanian Gas Pipeline. The other major processing region is at the Otway node. The South West Pipeline begins at the Otway node and runs east to Melbourne while the SEAGas Pipeline, runs north west to Adelaide.

The Longford-Melbourne Pipeline runs from the Longford processing plant to the city of Melbourne which is approximately 174 km west of the plant. It was initially constructed in 1969 to supply gas extracted from the Gippsland basin to Melbourne and has a current capacity of 1,030 TJ/d [122]. The Longford-Melbourne Pipeline is part of the Victorian declared transmission system.

The Eastern Gas Pipeline was constructed in the year 2000 and has a current maximum
capacity of 289 TJ/d in the summer season and 291 TJ/day in the winter season [122]. The pipeline supplies gas from the Gippsland basin to Sydney and Canberra and to regional towns and businesses along the pipeline route which begins at the Longford processing plant and ends at Sydney. As well as sourcing gas from the Longford processing plant gas is injected from the Orbost processing plant and the VicHub interconnect facility. The VicHub interconnect allows gas to be exported from or imported to the Victorian declared transmission system with export limited to 135 TJ/d and imports limited to 150 TJ/d.

The Tasmanian Gas Pipeline provides the only supply of gas to the island state of Tasmania. It was constructed in 2002 and runs from the Longford processing plant to Hobart via Bell Bay on the Tasmanian coast with a lateral pipeline supplying the regional town of Port Latta. The maximum capacity of the Tasmanian Gas Pipeline is 129 TJ/d [122].

The South West Pipeline as another key pipeline in the Victorian declared transmission system. Constructed in 1999 it has a maximum capacity of 353 TJ/d [122] and supplies gas from the Otway basin to Melbourne.

The South East Australian Gas Pipeline, or SEAGas pipeline, supplies gas from the Otway basin fields to Adelaide. The pipeline has been in operation since 2004 and the maximum capacity is now 314 TJ/d [122]. The pipeline currently transports more than half of Adelaide’s gas requirements [6]. According to the pipeline owner the firm capacity of the pipeline is fully contracted with three foundation customers on long term agreements but interruptible gas supply contracts and firm capacity expansion could still be negotiated at the time of writing.

The New South Wales-Victoria Interconnect was constructed in 1998 to provide a link between the Victorian declared transmission system and the Moomba-Sydney Pipeline. Gas can flow into the Victorian declared transmission system at a maximum rate of 90 TJ/d [122]. The is the possibility to reverse the flow of gas in this pipeline and export gas from the Victorian DTS under certain operational conditions specified by the market regulator AEMO.
4.2.14 Regulation of transmission pipelines

Transmission pipelines are natural monopolies as it is easier to add capacity to an existing pipeline than to build a competing pipeline. Bottlenecks can be by-passed by adding an additional pipeline around the constraint, this known as looping and if the specification of the pipeline allows, additional compressors can be added to increase capacity also. A competing pipeline must be be built entirely from producer to customer [13]. Given this and the fact that the pipelines are privately owned some are subject to government regulation. Regulated pipelines are said to be covered pipelines and may be subject to full regulation or light regulation. Full covered pipelines submit an access arrangement to the Australian Energy Regulator (AER) for approval. The access agreement must give a price and a service that is commonly sought by most customers wishing to transport gas through the pipeline [12]. Light regulation only requires the publishing of a tariff and the terms and conditions on the owners website. Disputes arising between those seeking access and the owners of lightly regulated pipelines can be taken to the AER for arbitration [12]. Covered pipelines can have the regulation reviewed and removed and the number pipelines subject to regulation is declining.

4.2.15 Gas transportation tariffs for transmission pipelines

The price of wholesale gas and the transportation costs are both necessary to supply gas to consumers. The markets for gas and pipeline access therefore share the characteristics of being largely bi-lateral long-term contracts that are mostly confidential. Pipeline tariffs will often be a combination of an access cost and a cost for the volume transported. Interruptible or non-firm access may also be negotiated in some cases. The major demand centres of Melbourne, Adelaide and Sydney/Canberra have direct pipeline access to two gas basins. Retailers supplying customers in these cities therefore have increased choices of supply. When considering alternative sources of wholesale gas the buyer must compare the price of the gas and the cost of transportation in combination [12]. Differences in transportation charges may result from the distance the gas must be transported, the geography that the pipeline must traverse, or for economic reasons such as the capital costs involved in construction and the amount of depreciation the pipeline costs have accrued over time. The cost of transporting gas through the transmission network is a relative small part of the cost retail customers pay for gas, with estimates for capital city
customers in the range of 2%-7%. It is nevertheless significant enough to warrant study in an attempt to lower it.

4.3 Summary of the key features of the networks analysed

This section began by outlining the key features of the Goulburn-Murray Irrigation District with a particular focus on the Greater Trading Zone, the trading zone where the price data and volumes traded was collected. Infrastructure upgrades and policy changes were outlined and there potential influence on the water market was discussed. The second part of this section outlined the nature of the network of gas transmission pipelines in eastern Australia and gas a summary of how gas networks have evolved in Australia. A brief background on energy resources in Australia was given and an overview of how gas is discovered and how the size of gas fields is determined. The producing gas basins in Australia were mentioned with reserve estimates and a discussion on the finite nature of gas reserves. The major demand centres were outlined and the different demand profiles discussed. The size and seasonal nature of Melbourne demand was noted, satisfying Melbourne demand in the winter requires a large amount of production capacity. This results in excess capacity in the summer when Melbourne gas demand is approximately half as large. The production facilities and transmission pipelines were discussed, with the size and importance of the Longford production plant mentioned. Pipelines are natural monopolies and some are subject to regulation, the nature of this regulation was outlined and the difference between fully covered pipelines and those only subject to light regulation was discussed before pipeline tariffs were discussed.

5 Network models and analysis

In this section three research questions will be addressed. Firstly a solution to the problem of estimating the price of temporary allocations of irrigation water at the beginning of the season will be discussed. The solution to this problem will allow a market in options to be established which will in turn give farmers greater control over the input costs of production. Secondly, a pricing model to simulate the spot price of wholesale natural
gas sold in the Victorian declared wholesale gas market is presented. This model will give holders of a portfolio of energy contracts the ability to ascertain the potential losses they may be exposed to in the spot market. The third model optimises the delivery of gas to major demand centres. In the first formulation of the model supply shortages are minimised under simultaneous peak day demand conditions. The size of any shortfalls are quantified as demand for gas grows according to forecast annual growth factors. In the second formulation of the model the cheapest way to supply existing peak day demand is stated, and the sensitivity of the solution to changes in production costs at supply nodes is discussed. Production costs are the largest cost in the supply of natural gas, they are also unique and finite so that changes in costs are highly likely to occur as existing production is depleted and new gas reservoirs are developed.

5.1 Estimating the price of temporary water at the start of the irrigation season

The beginning of the irrigation season is a period of high uncertainty for irrigators. It is precisely this time when a series of options on the price of temporary water allocations is most useful. In buying a portfolio of options contracts an irrigator can gain some certainty over the volume of water they will have available to them in the season and the price they will have to pay. A hindrance to the introduction of a series of options over the price of temporary water at the beginning of the season is the jump in prices seen between the closing price at the end of the previous season and the starting price at the beginning of the next. In order to trade an option a strike price must be set, the payout of the option is determined by the difference between the market price and the strike price. Therefore an estimate of the market price at the start of the season is necessary to set appropriate strike prices. In this section a regression model using the volume of water in Lake Eildon and the amount of winter rainfall at Jamieson, upstream of Lake Eildon as predictor variables is proposed.
5.1.1 Seasonal allocations

Water entitlements are divided into different classes in different jurisdictions in the Murray-Darling basin. In the Goulburn-Murray irrigation district water entitlements are called water shares and are classed as high reliability or low reliability. High reliability water shares are expected to receive 100% of entitlement volume in 95 years out of 100. Table 4 shows the opening and final allocation of temporary water to high reliability water shares in the 1A Greater Goulburn trading zone from the 2002-03 season to the 2009-10 season. It can be seen that the maximum allocation has been less than 100% in five years out of eight. No allocations have been made to low reliability water shares in over ten years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Opening Allocation</th>
<th>Maximum Allocation</th>
<th>Date 100% Reached</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-03</td>
<td>35%</td>
<td>57%</td>
<td>-</td>
</tr>
<tr>
<td>2003-04</td>
<td>48%</td>
<td>100%</td>
<td>3 Nov. 2003</td>
</tr>
<tr>
<td>2004-05</td>
<td>55%</td>
<td>100%</td>
<td>15 Nov. 2004</td>
</tr>
<tr>
<td>2005-06</td>
<td>34%</td>
<td>100%</td>
<td>1 Nov. 2005</td>
</tr>
<tr>
<td>2006-07</td>
<td>0%</td>
<td>29%</td>
<td>-</td>
</tr>
<tr>
<td>2007-08</td>
<td>0%</td>
<td>57%</td>
<td>-</td>
</tr>
<tr>
<td>2008-09</td>
<td>0%</td>
<td>33%</td>
<td>-</td>
</tr>
<tr>
<td>2009-10</td>
<td>0%</td>
<td>71%</td>
<td>-</td>
</tr>
</tbody>
</table>

Low seasonal allocations force irrigators to curtail production, cease production temporarily, or enter the market and buy enough water to maintain production. Irrigators with permanent plantings have no choice but to buy water to keep their trees alive. Prices for water spiked as high as $1000 ML in the spring of 2007 after a particularly harsh period of drought and the risks associated with low seasonal allocations cannot be hedged effectively at present.
5.2 Results

In this section a statistical analysis of the historical data on water price dynamics and storage volume dynamics in the Lake Eildon is presented. Following this a model to estimate the jump in prices between seasons is formulated and the results discussed.

5.2.1 Analysis of historical water price dynamics and storage volume dynamics

Table 5 shows summary statistics for the price of temporary allocations since the 2002-03 season. An analysis of price data from 1998-99 to 2002-03 is provided in [21]. It is clear that water users are exposed to a large variation in prices. Farmers of perennial crops must have water each season and currently face an unhegable price risk. An option on the price of temporary water allocations can help manage some of this risk.

<table>
<thead>
<tr>
<th>Year</th>
<th>Median</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-03</td>
<td>$369.25</td>
<td>$353.18</td>
<td>$500.00</td>
<td>$105.00</td>
<td>$107.96</td>
</tr>
<tr>
<td>2003-04</td>
<td>$69.99</td>
<td>$80.51</td>
<td>$125.00</td>
<td>$12.00</td>
<td>$45.19</td>
</tr>
<tr>
<td>2004-05</td>
<td>$68.53</td>
<td>$63.71</td>
<td>$102.50</td>
<td>$25.60</td>
<td>$19.12</td>
</tr>
<tr>
<td>2005-06</td>
<td>$55.00</td>
<td>$53.91</td>
<td>$102.50</td>
<td>$12.50</td>
<td>$16.30</td>
</tr>
<tr>
<td>2006-07</td>
<td>$500.00</td>
<td>$518.76</td>
<td>$950.00</td>
<td>$296.00</td>
<td>$152.56</td>
</tr>
<tr>
<td>2007-08</td>
<td>$500.00</td>
<td>$545.62</td>
<td>$1,054.00</td>
<td>$200.00</td>
<td>$281.55</td>
</tr>
<tr>
<td>2008-09</td>
<td>$350.00</td>
<td>$399.89</td>
<td>$650.00</td>
<td>$271.00</td>
<td>$117.85</td>
</tr>
<tr>
<td>2009-10</td>
<td>$177.25</td>
<td>$218.55</td>
<td>$427.50</td>
<td>$114.28</td>
<td>$60.00</td>
</tr>
</tbody>
</table>

One impediment to the introduction of options on water allocations was the large jumps seen in the price of water between irrigation seasons. In order to set a strike price for an option it is necessary to frame a price range. For example, if the market price of one megalitre of water is $100 on October 1st, a series of options with an expiration date of November 1st could be introduced with strike prices ranging from $70 to $130 and increasing in $10 increments. However, while interseason price jumps were in the order of 100% or more it was impossible to frame a suitable range of strike prices.
Cui & Schreider [38] described water price dynamics as a Brownian motion with jumps. This process was obtained as a solution of a jump-diffusion stochastic partial differential equation. However, the solution was obtained only for intra-seasonal trading. They proposed a solution to the problem of the price jumps across seasons by limiting options trading to individual seasons and waiting until the market determined each season’s price for temporary water before creating a series of options for trade. However, to get the full economic benefits available from introducing options contracts on temporary water, the options need to be available at the time of maximum uncertainty for water users. This is before the beginning of the irrigation season when decisions need to be made on the size and type of the crops to produce. To allow option contracts to be traded at the beginning of the season, the pricing formula must be extended to add an interseasonal jump.

A further complicating factor was the uncertainty in the timing and size of initial water allocations. However, the introduction of the carryover provisions have meant that prices have been more consistent across seasons in recent years and with the new reserve system water carried over can be accessed at the start of the season. Therefore, two significant barriers to the introduction of options on temporary water have been mollified.

Figure 18: Water Allocation Trading Price and Volume.

Figure 18 shows the volume of trade (bars - right axis) and the pool price paid (line graph - left axis) in the 1A Greater Goulburn trading zone. The vertical lines show the change
in financial year (June 30th) which corresponds loosely to the winter break between
irrigation seasons. The jump in prices between the last trade made in the financial year
and the first trade in the next season can be clearly seen up until the 2007-08 season.

There was a clear pattern in trading activity over the years from the start of the 2002-03
season to the end of the 2005-06. The last trade of the season is lower than the first
trade of the next season in all years and the volumes traded also tail off as the end
of the season approached. Allocations made late in the season were of little value as
there wasn’t sufficient time for many irrigators to put the water to productive use. This
left little option but to sell it at any price in the market. The time to the end of the
irrigation season clearly influenced the price paid for water. This distortion in trade
shows the effects of confining the use of water allocations to the season in which they
were allocated and it exacerbated the natural price jumps caused by the uncertainty of
events over the period between seasons. In particular, uncertainty over the amount of
winter rainfall and subsequent inflows into storages which was significant in determining
the size of early season allocations.

With the introduction of the carryover rules in March 2007 you can see that this caused
a change in the pattern of trade. The price of temporary water increased late in the
season. Water retained its value since it could be bought in the autumn and used in the
spring. The volume traded was still low and a large jump in prices was seen between
the 2006-07 and 2007-08 seasons. Two contributing factors to the size of the jump were
the extremely low water allocation in that season, water entitlements in the Goulburn
system were allocated only 29% of their entitlement volume, and the proximity of the
announcement that carryover was possible to the end of the season. It is likely that many
entitlement holders had used the small amount of water they had been allocated and
the only way to make use of carryover was to buy water in the market. Given the small
volumes traded this was not an option taken by many (see Figure 18). The price jump
in 2007 was 70%, with the last trade recorded on the 7th of June 2007 being $505.75
and the first trade of the new season recorded on the 30th of August being for $862.00.
The sizes of the inter-season price jumps between the 2008-09 and 2009-10 seasons was
much smaller, inter-season price jumps and the volumes traded late in the season have
also been higher then in previous years. Irrigators have adjusted there behaviour and
made greater use of the carryover provisions. In 2008 a maximum of 50% of entitlement
volume could be kept for use in the next year. The last recorded trade of the season was
on the 12th of June and the price was $550. The first trade of the next season was on
the 31st of July at $580 per megalitre. A price jump of 5.5%. The 50% of entitlement volume limit was kept in place in 2009. The final trade of the season was at $400 per megalitre on June 4th and the first trade of the subsequent season was at $402.50 on the 6th of August. A price change of less than 1%.

Figure 19: Water Price and Storage Volume.

The volume of water stored in Lake Eildon and the price paid for water on the same date are shown in Figure 19. The winter rainfall in 2006 was particularly poor and the subsequent winter rains were also below average. The volume of water in storage shifted down and the price paid for water shifted up. There is clearly a negative correlation between the price of water and the amount of water in storage, however in years 2003, 2004 and 2005 the declining price paid for water as the season progressed was persistent even as the amount of water in storage declined. Before the 2006-07 season all water allocations had to be used in the season they were made. Water allocated late in the season had less value than water allocated early in the season since for many farmers the time to plant and harvest a mature crop had passed before the water was made available. This water entered the market and put downward pressure on the price. As the end of the season approached unused water fell in value and the volumes traded declined.
The town of Jamieson is located upstream from Lake Eildon where the Goulburn and Jamieson rivers meet. The rainfall between irrigation seasons at Jamieson, along with the change in price and the volume stored in Lake Eildon at the beginning of the season are shown in Table 6. Lake Eildon is subject to storage management rules. The manager of the resource considers the volume in the lake when deciding on whether water is released to irrigators.

Table 6: Winter Rainfall, Lake Eildon Volume and Price Jump

<table>
<thead>
<tr>
<th>Year</th>
<th>Price Jump</th>
<th>Winter Rainfall (mm)</th>
<th>Lake Eildon Volume (GL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>$160.00</td>
<td>477</td>
<td>745</td>
</tr>
<tr>
<td>2004</td>
<td>$72.45</td>
<td>472</td>
<td>1,085</td>
</tr>
<tr>
<td>2005</td>
<td>$54.40</td>
<td>430</td>
<td>1,177</td>
</tr>
<tr>
<td>2006</td>
<td>$287.50</td>
<td>175</td>
<td>772</td>
</tr>
<tr>
<td>2007</td>
<td>$332.00</td>
<td>375</td>
<td>739</td>
</tr>
<tr>
<td>2008</td>
<td>$156.65</td>
<td>327</td>
<td>679</td>
</tr>
<tr>
<td>2009</td>
<td>$85.98</td>
<td>323</td>
<td>664</td>
</tr>
<tr>
<td>2010</td>
<td>-$12.00</td>
<td>281</td>
<td>1,100</td>
</tr>
<tr>
<td>2011</td>
<td>$18.00</td>
<td>366</td>
<td>2,950</td>
</tr>
<tr>
<td>2012</td>
<td>$17.00</td>
<td>338</td>
<td>3,000</td>
</tr>
</tbody>
</table>

5.2.2 Formulation of model

In formulating the model the seasonal rainfall and the volume in storage are assumed to be key factors affecting the price paid for water. Lake Eildon is dammed and the volume is assumed to be predominantly influenced by controlled outflows. The seasonal rainfall is assumed to be the key determinate of inflow into the lake. Low winter rainfalls in 2012-2012 led to higher volumes due to the decision by the water management authority to restriction of outflows. A plot of the price paid for water against the amount of water stored in Lake Eildon shows that the price falls as the volume increases. However, when the volume of Lake Eildon moves above a critical value the relationship between price paid and volume in storage breaks down. In formulating the model a volume of 7,400ML was estimated to be this critical value. The critical value selected is approximately 45% of the maximum storage volume of the lake. A multiple regression model using an indicator variable was used to estimate the effect of winter rainfall and volume of water in storage on the price jump between irrigation seasons. When the volume of Lake Eildon was above 7,400ML the indicator variable was set to 1, otherwise it is equal to zero. The indicator in this model is of a physical nature. In this work while other models were explored the
results were deemed unsatisfactory. The regression analysis is shown in Table ??.

If we let $x_1$ be the volume of water stored in Lake Eildon, $x_2$ be the amount of winter rainfall recorded at the Jamieson weather observation station, and $x_3$ be the indicator variable where

$$x_3 = \begin{cases} 
1 & \text{if } x_1 < 7.4 \\
0 & \text{otherwise}
\end{cases}$$

then the regression equation can be written as

$$E[P] = 583 - 0.284x_1 - 0.422x_2 - 1.788x_3 + 5.11x_2x_3$$

where $P$ is the price jump. The analysis was carried out using Matlab software and the results are shown in Table 7. The variables analysed in the model are shown in Table 7 to be significant and are justifiably included in the model.

**Table 7: Regression Analysis**

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coefficient</th>
<th>Standard Deviation</th>
<th>T-Statistic</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>583.3</td>
<td>74.64</td>
<td>7.82</td>
<td>0.016</td>
</tr>
<tr>
<td>Lake Eildon Volume</td>
<td>-0.2839</td>
<td>0.0839</td>
<td>3.38</td>
<td>0.077</td>
</tr>
<tr>
<td>Winter Rainfall</td>
<td>-0.4416</td>
<td>0.1274</td>
<td>3.47</td>
<td>0.074</td>
</tr>
<tr>
<td>Indicator</td>
<td>-1.788</td>
<td>247.6</td>
<td>7.22</td>
<td>0.019</td>
</tr>
<tr>
<td>Interaction</td>
<td>5.106</td>
<td>0.7126</td>
<td>7.17</td>
<td>0.019</td>
</tr>
</tbody>
</table>

**Table 8: Actual Price Jump and Predicted Price Jump**

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual Price Jump</th>
<th>Predicted Price Jump</th>
<th>Absolute ($) Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>$160.00</td>
<td>$160.59</td>
<td>$0.59</td>
</tr>
<tr>
<td>2004</td>
<td>$72.45</td>
<td>$66.52</td>
<td>$5.93</td>
</tr>
<tr>
<td>2005</td>
<td>$54.40</td>
<td>$58.97</td>
<td>$4.27</td>
</tr>
<tr>
<td>2006</td>
<td>$287.50</td>
<td>$286.40</td>
<td>$1.10</td>
</tr>
<tr>
<td>2007</td>
<td>$332.00</td>
<td>$335.62</td>
<td>$3.62</td>
</tr>
<tr>
<td>2008</td>
<td>$156.65</td>
<td>$128.60</td>
<td>$28.05</td>
</tr>
<tr>
<td>2009</td>
<td>$85.98</td>
<td>$114.19</td>
<td>$28.21</td>
</tr>
<tr>
<td>2010</td>
<td>-$12.00</td>
<td>$146.47</td>
<td>$158.47</td>
</tr>
<tr>
<td>2011</td>
<td>$18.00</td>
<td>-$416.48</td>
<td>$434.48</td>
</tr>
<tr>
<td>2012</td>
<td>$17.00</td>
<td>-$418.31</td>
<td>$435.31</td>
</tr>
</tbody>
</table>
5.2.3 Discussion of modelling results

Changes to policy, particularly the introduction of the carryover option, make finding the critical value difficult however with a stable policy environment it should be possible to provide a more robust estimate. An understanding of this critical value will be important when market participants consider managing the risk of a low seasonal water allocation.

Table 8 shows the performance of the regression equation against the price jumps observed for the ten years from 2003 to 2012. It can be seen in Table 8 that the regression model produces good results initially, however the relationship breaks down in the later years of the period analysed. There have been significant policy changes in how the market functions since 2009. The advent of the carryover provisions has allowed water to be stored through the winter months. This is a significant change in the way the market operates. Furthermore, the delivery system for the stored water is now operational at the beginning of the season so the water carried over can be delivered and utilised as soon as required.

Another major factor in the breakdown is the fact that the drought experienced in the early part of the study period has broken and the volume of water in Lake Eildon has risen to levels approaching capacity. The fact that the storage has been at or near capacity in recent years is a structural change in the price series. Irrigation water has become abundant and the risk of low seasonal allocations due to below average winter rainfall has not been as significant as seen previously. The volume of water available in the Goulburn-Murray irrigation district has been sufficient for the water authority to make allocations to low reliability water shares in the Campaspe and Loddon valleys for the first time in over a decade. While the jumps in price between irrigation seasons have been diminished since the introduction of the carryover provisions there has also been abundant rainfall. The dynamics of the volume stored and price paid for water during a prolonged drought, such as the one experienced from 2003 to 2009 are still to be discovered.
5.2.4 Closing Remarks

The market for temporary water allocations in the northern Victoria regulated river systems has matured over the last decade. Most irrigators have traded water, either by buying water or selling it. Policy changes have been considerable but have reached a point where unnecessary distortions to the price due to policy have become minimal, the price of water is now determined by appropriate factors such as the climatic conditions and the value of the commodities produced with it rather than the time to the end of the irrigation season, as was the case in the early part of the decade. New products based on water entitlements were envisaged as part of the National Water Initiative, see Clause 58 part (iii) [36], and several water brokers have attempted to begin a market in futures on temporary water allocations. The advantage options have over futures is that they allow the option holder to benefit from positive price movements while protecting against negative movements. Futures are a contract to trade at an agreed price, thus the protection from negative price movements is inherent in the contract but the potential to benefit from positive price movements is removed.

5.3 Spot price model for simulating the weighted average daily price of wholesale natural gas in the declared wholesale gas market

Natural gas sold wholesale in the Eastern Australian Gas Network is generally done under bilateral arrangements that are both long term and confidential. The contracts include price reviews at agreed times and increases of around 80%-90% of the consumer price index (CPI) each year are typical [14]. As the contract terms are confidential there is no price transparency for the bulk of wholesale gas traded in eastern Australia, however in 2006 wholesale gas was sold around the range of $3.50/GJ to $3.80/GJ to large buyers [90]. Using the year end percentage change in the CPI, measured in the December quarter, and taking 90% of this value as the yearly increase, a rough approximation of the wholesale gas price at the end of 2011 might be in the range of $4.10/GJ to $4.45/GJ. The wholesale price in Eastern Australia is largely driven by domestic supply and demand and is not linked to the international price of oil as there is currently no export market. As a result gas in Eastern Australia is relatively cheap when compared to prices in other parts of the
The construction of an LNG export industry on the east coast of Australia, with prices linked to the price of oil, is expected to see domestic wholesale prices rise [108].

Two spot markets for wholesale gas exist and the amount gas traded on them is increasing. The longest running and most liquid is the Declared Wholesale Gas Market which has operated in Victoria since 1999. The second spot market is the Short Term Trading Market (STTM) which is a market for wholesale gas delivered to designated trading hubs. The STTM began operating with two trading hubs, one in Sydney and one in Adelaide. A third trading hub in Brisbane has since been added.

The spot market for wholesale gas in Victoria is known as The Declared Wholesale Gas Market (DWGM). It is the longest running market for wholesale natural gas in eastern Australia and is essentially a market for trading imbalances between supply and demand. Natural gas is widely used in Victoria and there are over 2 million customers with an annual demand of about 200 PJ, excluding demand from gas powered electricity generators [11]. There is a high proportion of residential gas use in the total demand profile for the state with many homes using gas for heating. As a result demand is highly seasonal with increased consumption in winter. The predominance of long term, bilateral gas supply agreements makes responding to large fluctuations in demand problematic. If the temperature is significantly lower than forecast on a particular day then demand for gas can be significantly higher than expected. Gas supply contracts generally include a daily amount with the flexibility to supply more or less within an agreed range. However if customer demand exceeds the available supply under the contract the retailer will not be able to satisfy their customers’ requests for gas. Conversely, supply contracts are generally take-or-pay contracts and as such there is little to be gained by refusing supply when demand is lower than expected. The DWGM acts to complement long term contracts by providing a spot market where imbalances in daily supply and demand can be traded. In 2009 about 10-20% of the wholesale gas used in Victoria was traded in the DWGM [12].
5.3.1 Victorian Declared Transmission System (DTS)

The DWGM applies to the Victorian Declared Transmission System (DTS), the network of pipelines that connect production facilities to major demand centres in the state. The largest demand comes from the Melbourne metropolitan area where the bulk of the population live however there is also significant demand from regional towns. The wholesale gas price determined in the DWGM applies equally to all demand centres in the DTS regardless of their location. The major transmission pipelines in the DTS are the Longford-Melbourne Pipeline, the South West Pipeline and the NSW-Vic Interconnect. The Longford-Melbourne Pipeline connects the production facilities at Longford to the Melbourne city gate at the south eastern fringes of the metropolitan area. The South West Pipeline transports gas from the Otway production facilities to the city gate near Melbourne on its western edge. At the city gate gas enters the low pressure distribution network that connects to retail customers. The DTS was initially isolated from the Eastern Australian Gas Network however this is no longer the case. Links are now provided by the NSW-Vic interconnect, which connects to the Moomba-Sydney Pipeline, the Eastern Gas Pipeline, which runs along the eastern seaboard from Longford to Sydney, the SEAGas pipeline which supplies gas from the Otway basin to Adelaide and the Tasmanian Gas Pipeline which supplies Tasmania with gas from the Longford plant.

Figure 20 shows the pipelines in the DTS.
There are several locations where gas is injected into the DTS, with the three largest indicated in Figure 20 as red rectangles. Gas can be injected from the producing fields in the Gippsland, Bass and Otway basins, from stored gas in a depleted gas reservoir at Iona in the Otway basin, and from interconnecting pipelines to the wider Eastern Australian Gas Network. The capacity of the storage facility at Iona is 12,000 TJ and gas can be injected into the DTS at a rate of 340 TJ/day [5]. The Victorian DTS also has an LNG storage facility where gas can be stored and injected into the system at short notice to maintain supply and to keep the system pipelines within the bounds of safe operating pressures. The capacity of the LNG storage is 665 TJ and the liquefied gas can be returned to a gaseous state and injected into the system at a rate of 10 TJ per hour [5].

The Gippsland basin fields, the Longford processing plant and the Longford-Melbourne pipeline were the first, and are still by far the largest source of supply to the DTS. An
additional piece of infrastructure called the VicHub has been built at Longford. The VicHub allows gas from the Gippsland fields to flow through the Eastern Gas Pipeline and supply Sydney, Canberra and some markets in regional New South Wales as well as allowing gas from the Eastern Gas Pipeline to be injected into the DTS to supply the Victorian market. The Longford supply node is a key point in both the DTS and wider Eastern Australian Gas Network. Gas can also be injected at the Otway supply node which is located west of Melbourne from the Otway and Minerva processing plants and from the Iona underground storage reservoir. Gas from the Yolla field in the Bass basin is injected into the DTS via a pipeline that connects to a processing plant at the town of Lang Lang which is located approximately 85 kilometres south east of Melbourne. The New South Wales-Victoria interconnect allows gas to be injected into or withdrawn from the DTS. A connection facility at the town of Culcairn, which is located north of Melbourne and across the border into southern New South Wales, connects with a lateral of the Moomba-Sydney pipeline. The large, and sometimes quite sudden, changes in demand can be smoothed to some extent by vaporising gas stored as LNG and injecting it into the DTS. The Dandenong LNG plant is located on the edge of the Melbourne metropolitan area and can be used to maintain the pipeline within safe operating pressures and to satisfy demand when necessary [5]. There are approximately 120 other points in the DTS where gas is withdrawn to satisfy customer demands [3].

5.3.2 The Gas Day and Scheduling

The following information described in this section is provided by the Technical Guide to the Victorian Gas Wholesale Market [11].

The DWGM operates on a 24 hour schedule known as the Gas Day. The Gas Day contains five schedules at which the market operator prepares an operating schedule and a pricing schedule based on information provided by participants in the DWGM. Market participants are those entities that are eligible to inject or withdraw gas from the DTS. The schedules are prepared at 6am, 10am, 2pm, 6pm and 10pm and apply for the remainder of the gas day. The operating schedule provides individual market participants with an injection schedule and a schedule for controllable withdrawals while the pricing schedule sets the price per gigajoule of wholesale gas traded at that schedule.
5.3.3 Controllable and uncontrollable withdraws

Gas withdrawals from the DTS are classified as controlled or uncontrolled withdrawals. Controllable withdrawals may be gas withdrawn into the storage facility or exported from the DTS to supply other markets. Most gas withdrawn from the DTS is uncontrollable and is determined by retail customers who consume gas as required. If a market participant supplies gas that can be withdrawn by their customers as the customers requires they must submit hourly demand forecasts to the market operator. For specific sites that use large amounts of gas quickly, such as gas powered electricity generators or large industrial customers, hourly demand forecasts for the site must be provided. The site specific demand forecasts must include information on the volatility of gas consumption at the site as large customers can consume gas quickly enough to put the system under strain. For smaller customers of the gas provider an aggregate system wide demand forecast must be provided.

The pricing schedule determines the price at which gas is traded at each schedule. Market participants intending to inject gas into or make controllable withdrawals from the DTS submit bids that include the quantity they are intending to trade and the price they are willing to pay. Bids can be staggered in up to ten steps so that a market participant can include bids to inject a small quantity of gas at a low price and increase the injection quantity for higher prices, conversely bids for controllable withdrawals will have larger quantities at low prices and smaller quantities at higher prices. The bids range in price from $0/GJ to $800/GJ. The price for each schedule is when the incremental bids for injections satisfy the forecast for uncontrollable withdrawals and the decremental bids for controllable withdrawals.

The initial schedule at 6 am is known as the beginning of the day schedule and market participants submit details of how much gas they intend to inject into and withdraw from the DTS over the next 24 hours. Gas injected into the DTS is generally within the control of the market participants as they have contracts with gas producers to supply gas up to an agreed amount each day. Therefore an imbalance in each individual participants supply and demand profiles will generally exist.

At each schedule market participants submit bids at which they are willing to inject gas
into and withdraw gas from the DTS. Bids are limited to a range of between $0/GJ and $800/GJ. Each schedule has a market clearing price at which all gas is traded. There are opportunities to revise daily supply and demand forecasts at 10 am, 2 pm, 6 pm and 10 pm. At the four schedules following the 6 am schedule market participants can resubmit their injection and withdrawal schedules for the remainder of the gas day.

5.3.4 Linepack

As well as allowing gas to be transported from place to place the network of pipelines act as a storage tank for natural gas. The gas contained in the pipelines at any time is called linepack. The amount of linepack gas can be increased up to a limit, as natural gas is compressible, and gas stored as linepack can provide a means to meet an increase in demand [5]. During the course of the gas day the amount of linepack varies since the rate on injection of gas into the system is relatively constant while demands fluctuate.

The market operator sets a target for the amount of linepack in the system at the end of the gas day. The end of day linepack target maintains the minimum pressure requirements needed to maintain the pipeline network within safe operating pressures. The end of day linepack target in 2011 was 340 TJ [5].

5.3.5 Types of payments in the declared wholesale gas market

There are two main types of payments in the declared wholesale gas market, imbalance payments and deviation payments. Both types of payments will be explained in this section.

Providers of gas to retail customers try to forecast their daily demand, and thereby avoid exposure to the fluctuations of the spot market. At each schedule they submit bids for injections and any controllable withdrawals they might have as well as a forecast for their uncontrollable withdrawals for the remainder of the gas day. An imbalance will exist if the amount they are scheduled to inject differs from the amount they or their customers
withdraw. This imbalance is settled using imbalance payments. Imbalance payments are daily payments and can be positive or negative. If a market participant injects more gas than they withdraw they are selling gas to the market and if the reverse is true they are buying gas from the market [11]. At each schedule a market participants imbalance is multiplied by the market price at that schedule, the daily imbalance payment is then calculated as the sum of the imbalance payments calculated at each schedule [11].

Deviation payments are calculated based on the difference between a market participants scheduled level of injections and withdrawals and the actual amount they do inject and withdraw. Should they deviate from their scheduled injection and withdrawal quantities they will create a deficit or surplus in the amount of gas in the system. A deficit will need to be made up at the next schedule by requiring more gas to be injected which can potentially increase the price of gas traded at that schedule. Similarly, a surplus will add to the amount of gas in the system which may depress the price at the next schedule. As a result the price for deviation payments uses the market price calculated at the schedule following the deviation. As with imbalance payments deviation payments can be positive or negative. They are calculated by multiplying the deviation amount by the price at the next schedule and summing these payments over the gas day.

5.3.6 The weighted average daily imbalance price

The market operator provides data on weighted average daily prices which are calculated from the imbalance and deviation payments made by market participants. The formula for deriving the price is as follows:

$$\frac{\sum_{S,MP} |imb_{S,MP}| + \sum_{SI,MP} |dev_{SI,MP}|}{\sum_{S,MP} |imb_{S,MP}| + \sum_{SI,MP} |dev_{SI,MP}|}$$

where:

$\$imb_{S,MP} = $ of imbalance payments for market participant MP in schedule S

$\$dev_{SI,MP} = $ of deviation payments for market participant MP in schedule SI
The weighted average daily imbalance price basically sums the absolute values of imbalance and deviation payments made by the market participants and divides this by the amount of gas traded. The weighted average daily imbalance price provides a useful value for analysis of the daily price of gas in the DWGM. Figure 21 shows the weighted average daily price in $/GJ from March 1st, 2009 to September 30th, 2011.

Figure 21: The weighted average daily price of wholesale gas in the declared wholesale gas market.

A model of the form

\[ dX_t = (\theta_1 - \theta_2 X_t)dt + \theta_3 dW_t, \quad X_0 = x_0 \]  

(239)

where \( \theta_1, \theta_2 \in \mathbb{R}, \) and \( \theta_3 \in \mathbb{R}_+ \), is proposed to simulate the price series shown in Figure
21. In this model $X_t$ represents the price at time $t$, $W_t$ is a Brownian motion, $\theta_1$ and $\theta_2$ ensure the model is mean reverting and $\theta_3$ magnifies or diminishes the volatility. This model was chosen in preference to the well known Black-Scholes model to incorporate mean-reversion, a feature this is known to be present in commodities markets and was implemented using the R software package.

The estimates of the coefficients using the weighted average daily price in $$/GJ from March 1st, 2009 to September 30th, 2011 are shown in Table 9 and the simulated price series of the weighted average daily price using equation 239 is shown in Figure 22.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>1.058</td>
<td>0.107</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.509</td>
<td>0.048</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>1.005</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Figure 22: A simulation of the weighted average daily price.
On inspection it can be seen from Figure 22 that the salient features of the market have been captured. While no quantitative testing was conducted to assess the goodness of fit in future work the Akaike Information Criteria could be used to test the fit of the proposed model against that of the Black-Scholes model.

5.3.7 Discussion in regard to the model of the wholesale gas price

The conjunction of electricity and gas networks has been mentioned previously in this thesis in relation to the physical infrastructure that comprise the networks. It must be noted that not only are the networks increasingly intertwined but the markets for each are also. For example, most energy companies in Australia provide both electricity and gas to retail customers. The value of a spot price model for natural gas is in providing a simulation tool that can be used for scenario testing of a portfolio of energy contracts. To date no model of the spot price of wholesale natural gas traded in the Victorian market has been published. Therefore, while energy companies can model their long term gas supply contracts they have no tool for modelling the impact of the spot price on the portfolio.

5.4 Optimisation model for gas supply in Eastern Australia

Australia is a large consumer of natural gas and demand is growing rapidly. The security of supply is crucial in maintaining this growth and encouraging the use of natural gas as a substitute for other fuel sources that have a greater impact on the environment. South-eastern Australia has several producing gas fields and a sophisticated network of transmission pipelines connecting them to the major demand centres in the regions’ largest cities and major industrial centres.

There are several optimisation problems of interest when modelling the Eastern Australian gas network, however little has been published. With the increasing demand for natural gas the capacity of the network to meet this demand is worthy of analysis. Research has been conducted on minimising gas transportation costs through transmission
networks using gas consumed as fuel in the compressors. Recall that gas compressors create the pressure difference that causes gas to move through a pipe. No data is currently published on compressor fuel use in the eastern Australian network, however a proposal to include compressor fuel use by gas day was approved for future publication [4]. The compressor fuel use in the Victorian DTS sub-network is far less than the 3-5% suggested in [119] as an average estimate of gas consumed internally by a network for gas transportation. In fact on a peak demand day of 1,200 TJ with all compressors running, the fuel used by the compressors in the DTS is only about 0.5% of total gas demand [79]. It should be noted that the distances involved in gas transportation in other parts of the eastern Australian network are considerably larger and when compressor data is available a least cost transportation analysis using compressor fuel consumed as the cost would be useful. In this section a model to minimise the cost of supply is presented using estimates of production and transportation costs.

There a surprisingly few research works devoted to the optimisation of gas supply in Australia, possibly because this problem primarily attracts the attention of commercial structures rather academic researchers. Liner programming (LP) based optimisation techniques are being used for the practical operation of the Victorian subnetwork. The LP model for clearing of the natural gas market in Victoria is described in [86]. The gas model presented in these works uses a quite complicated flow dynamic approach for approximating the gas transportation over the pipeline system. This model is then linearised and solved using LP techniques, where decision variables are the physical parameters of the model and constraints are defined by the linearised fluid dynamics equations as well as the limits for upper and lower allowable pressures in the pipes. The technical details of the paper are described in [79] which serves as a companion paper to [86].

In the following chapter two models will be presented. The first will determine the adequacy of the current network to meet simultaneous peak day demand across all demand centres in the network. The pipelines that are fully utilised under such a scenario are candidates for capacity expansion. The second model will focus on the cost of supply. Estimates of production costs at each production node are provided, as are estimates of pipeline transportation tariffs. These estimates can be used to formulate a model to analyse the dynamics of the network under different production profiles. The physical processes of the system are considered exogenously, via capacity constraints on the pipe segments, gas flow directions, the mass balance at pipe junctions and production capacity.
The infrastructure data used in the network model is taken from the Int 911 Standing Capacity Report which is available for download from the gas market bulletin board [122]. Operators of network facilities, such as pipelines, production plants and gas storages must provide capacity information annually or when there is a material change to the capacity of the plant that is expected to remain in force for 12 months or longer. The Figures cover both winter and summer seasons. The estimates of pipeline tariffs are taken from analysis provided to the AEMO by the Core Energy Group [33] which was included in the 2012 Gas Statement of Opportunities [7].

5.5 Pipeline capacity

Table 10 provides data on the pipeline subscripts and maximum daily quantity (MDQ) that can be transported by the pipe [6]. Only the Eastern Australia gas pipeline which runs from Longford to Sydney provides different capacity estimates for the summer and winter seasons.
<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Maximum daily quantity summer</th>
<th>Maximum daily quantity winter</th>
<th>cost $/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Longford - Melbourne</td>
<td>1030</td>
<td>1030</td>
<td>$0.24</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Otway - Melbourne</td>
<td>353</td>
<td>353</td>
<td>$0.27</td>
</tr>
<tr>
<td>$x_3$</td>
<td>NSW - VIC. Int.</td>
<td>90</td>
<td>90</td>
<td>$0.41</td>
</tr>
<tr>
<td>$x_4$</td>
<td>Otway - Adelaide</td>
<td>314</td>
<td>314</td>
<td>$0.73</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Moomba - Adelaide</td>
<td>241</td>
<td>241</td>
<td>$0.65</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>439</td>
<td>439</td>
<td>$0.88</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Young - Sydney</td>
<td>439</td>
<td>439</td>
<td>$0.88</td>
</tr>
<tr>
<td>$x_8$</td>
<td>Longford - Sydney</td>
<td>289</td>
<td>291</td>
<td>$1.16</td>
</tr>
<tr>
<td>$x_9$</td>
<td>Longford - Hobart</td>
<td>129</td>
<td>129</td>
<td>$2.00</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>384</td>
<td>384</td>
<td>$0.96</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>Ballera - Mt.Isa</td>
<td>119</td>
<td>119</td>
<td>$1.44</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>384</td>
<td>384</td>
<td>$0.96</td>
</tr>
<tr>
<td>$x_{13}$</td>
<td>Roma - Brisbane</td>
<td>233</td>
<td>233</td>
<td>$0.51</td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>Roma - Gladstone</td>
<td>145</td>
<td>145</td>
<td>$0.90</td>
</tr>
</tbody>
</table>
5.6 Production capacity

The production nodes in the network are the sum of production capacities for the facilities in the region. The Longford node includes the Longford, Lang Lang and Orbost gas plants as well as the Dandenong LNG storage. The Otway node includes the Minerva and Otway gas plants and the Iona underground storage. The Moomba node includes the Moomba gas plant, and the Ballera node includes the Ballera gas plant. The Roma node includes the Dawson Valley, Fairview, Keny, Kincora, Kogan North, Berwyndale South, Peat, Rolleston, Scotia, Silver Springs, Spring Gully, Strathblane, Talinga, Talonga, Wumgoona and Yellowbank gas plants.

Estimates of production costs are taken from analysis provided to the AEMO by the Core Energy Group [34]. All production costs necessary to produce gas meeting the specifications for delivery and sale through the transmission network are included in the estimates. In this section the estimates used in this thesis to model the cost of gas supplied to eastern Australian demand centres are presented.

Oil and gas are often found together in conventional fields. Recall that conventional hydrocarbon fields are underground reservoirs where oil and gas are trapped in large amounts. While unconventional gas is produced largely from coal seams or from hydraulic fracturing of rock formations, and other less common methods. The composition of the hydrocarbons in conventional fields may include a large percentage of liquids which significantly increases the value of production from the field. The presence of liquid hydrocarbons makes the cost of gas production at some fields negligible and gas was routinely flared or burnt off during the extraction of more valuable liquid hydrocarbons such as crude oil. The basins with fields containing gas and liquids in eastern Australia are the Cooper basin which is represented in the models in this thesis by the Moomba production node, and the Bass and Gippsland basins, which are combined and represented by the Longford production node. In order to make a comparison between sources of supply the cost of gas production exclusive of liquid hydrocarbons is used in this thesis.

Table 11 shows the current daily production capacity of the producing nodes connected to the eastern Australian gas network. This data is available from the National Gas Market Bulletin Board website [122].
Table 11: Daily capacity at production nodes

<table>
<thead>
<tr>
<th>Production node</th>
<th>Name</th>
<th>Summer (TJ/day)</th>
<th>Winter (TJ/day)</th>
<th>cost $/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{15} )</td>
<td>Longford</td>
<td>1,463</td>
<td>1,463</td>
<td>$2.03</td>
</tr>
<tr>
<td>( x_{16} )</td>
<td>Otway</td>
<td>786</td>
<td>786</td>
<td>$4.06</td>
</tr>
<tr>
<td>( x_{17} )</td>
<td>Moomba</td>
<td>390</td>
<td>336</td>
<td>$3.79</td>
</tr>
<tr>
<td>( x_{18} )</td>
<td>Ballera</td>
<td>100</td>
<td>100</td>
<td>$3.79</td>
</tr>
<tr>
<td>( x_{19} )</td>
<td>Roma</td>
<td>922.8</td>
<td>922.8</td>
<td>$3.28</td>
</tr>
</tbody>
</table>

5.7 Peak day demand forecasts

The peak day demand forecasts used in the network model are taken from the Int 912 report which is available for download from the gas market bulletin board [122]. Peak day demand for Adelaide is reported explicitly. The Figure for the Victorian Principal Transmission System is used for the Melbourne demand node, the Sydney and A.C.T forecasts are combined in the Sydney demand node in the network model. Peak day demand forecasts are not provided for demand centres in Queensland or Tasmania. Each of these demand centres are supplied by a single pipeline and the maximum daily demand forecast for these pipelines is used as a proxy for the peak day demand in those demand centres. The Moomba gas plant has a winter demand of 54 TJ/day. This amount is subtracted from the maximum daily production available from Moomba in the winter season. Table 12 shows the current peak day demand forecasts.

Demand growth for each demand node uses growth factors published in [6]. Forecasts are provided by the market operator AEMO for peak day demands that are estimated to have a 5% chance of occurring. Estimates are provided for three economic scenarios in [6], in this analysis the largest growth factor is chosen for each demand node. No data is published explicitly for Mt.Isa, Gladstone and Brisbane. These demand centres use the growth factor published for the state of Queensland.
Table 12: Forecast maximum daily demand (TJ)

<table>
<thead>
<tr>
<th>Demand node</th>
<th>Name</th>
<th>Summer (TJ/day)</th>
<th>Winter (TJ/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>Melbourne</td>
<td>650</td>
<td>1,135</td>
</tr>
<tr>
<td>$d_2$</td>
<td>Adelaide</td>
<td>410</td>
<td>345</td>
</tr>
<tr>
<td>$d_3$</td>
<td>Mt. Isa</td>
<td>120</td>
<td>107</td>
</tr>
<tr>
<td>$d_4$</td>
<td>Gladstone</td>
<td>145</td>
<td>168</td>
</tr>
<tr>
<td>$d_5$</td>
<td>Brisbane</td>
<td>180</td>
<td>219</td>
</tr>
<tr>
<td>$d_6$</td>
<td>Sydney</td>
<td>310</td>
<td>389</td>
</tr>
<tr>
<td>$d_7$</td>
<td>Hobart</td>
<td>50</td>
<td>61</td>
</tr>
</tbody>
</table>

5.7.1 Summer demand growth

Table 13 lists the current peak day demand and the estimated annual growth in the 1 in 20 summer peak day demand forecasts.

Table 13: Forecast growth for summer maximum daily demand (TJ)

<table>
<thead>
<tr>
<th>Demand node</th>
<th>Name</th>
<th>Summer (TJ/day)</th>
<th>Growth factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>Melbourne</td>
<td>650</td>
<td>5.4%</td>
</tr>
<tr>
<td>$d_2$</td>
<td>Adelaide</td>
<td>410</td>
<td>1.4%</td>
</tr>
<tr>
<td>$d_3$</td>
<td>Mt. Isa</td>
<td>120</td>
<td>7.4%</td>
</tr>
<tr>
<td>$d_4$</td>
<td>Gladstone</td>
<td>145</td>
<td>7.4%</td>
</tr>
<tr>
<td>$d_5$</td>
<td>Brisbane</td>
<td>180</td>
<td>7.4%</td>
</tr>
<tr>
<td>$d_6$</td>
<td>Sydney</td>
<td>310</td>
<td>5.1%</td>
</tr>
<tr>
<td>$d_7$</td>
<td>Hobart</td>
<td>50</td>
<td>7.3%</td>
</tr>
</tbody>
</table>

Table 14 shows the forecast peak day demand for each demand centre over the next 20 years using the growth factors in Table 13.
Table 14: Forecast growth for summer maximum daily demand (TJ)

<table>
<thead>
<tr>
<th>Year</th>
<th>Melbourne</th>
<th>Adelaide</th>
<th>Mt.Isa</th>
<th>Gladstone</th>
<th>Brisbane</th>
<th>Sydney</th>
<th>Tas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>650</td>
<td>410</td>
<td>120</td>
<td>145</td>
<td>180</td>
<td>310</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>685</td>
<td>416</td>
<td>129</td>
<td>156</td>
<td>193</td>
<td>326</td>
<td>54</td>
</tr>
<tr>
<td>3</td>
<td>722</td>
<td>422</td>
<td>138</td>
<td>167</td>
<td>208</td>
<td>342</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>761</td>
<td>427</td>
<td>149</td>
<td>180</td>
<td>223</td>
<td>360</td>
<td>62</td>
</tr>
<tr>
<td>5</td>
<td>802</td>
<td>433</td>
<td>160</td>
<td>193</td>
<td>239</td>
<td>378</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>846</td>
<td>440</td>
<td>171</td>
<td>207</td>
<td>257</td>
<td>398</td>
<td>71</td>
</tr>
<tr>
<td>7</td>
<td>891</td>
<td>446</td>
<td>184</td>
<td>223</td>
<td>276</td>
<td>418</td>
<td>76</td>
</tr>
<tr>
<td>8</td>
<td>939</td>
<td>452</td>
<td>198</td>
<td>239</td>
<td>297</td>
<td>439</td>
<td>82</td>
</tr>
<tr>
<td>9</td>
<td>990</td>
<td>458</td>
<td>212</td>
<td>257</td>
<td>319</td>
<td>462</td>
<td>88</td>
</tr>
<tr>
<td>10</td>
<td>1,043</td>
<td>465</td>
<td>228</td>
<td>276</td>
<td>342</td>
<td>485</td>
<td>94</td>
</tr>
<tr>
<td>11</td>
<td>1,100</td>
<td>471</td>
<td>245</td>
<td>296</td>
<td>368</td>
<td>510</td>
<td>101</td>
</tr>
<tr>
<td>12</td>
<td>1,159</td>
<td>478</td>
<td>263</td>
<td>318</td>
<td>395</td>
<td>536</td>
<td>109</td>
</tr>
<tr>
<td>13</td>
<td>1,222</td>
<td>484</td>
<td>283</td>
<td>342</td>
<td>424</td>
<td>563</td>
<td>116</td>
</tr>
<tr>
<td>14</td>
<td>1,288</td>
<td>491</td>
<td>304</td>
<td>367</td>
<td>455</td>
<td>592</td>
<td>125</td>
</tr>
<tr>
<td>15</td>
<td>1,357</td>
<td>498</td>
<td>326</td>
<td>394</td>
<td>489</td>
<td>622</td>
<td>134</td>
</tr>
<tr>
<td>16</td>
<td>1,431</td>
<td>505</td>
<td>350</td>
<td>423</td>
<td>525</td>
<td>654</td>
<td>144</td>
</tr>
<tr>
<td>17</td>
<td>1,508</td>
<td>512</td>
<td>376</td>
<td>454</td>
<td>564</td>
<td>687</td>
<td>154</td>
</tr>
<tr>
<td>18</td>
<td>1,589</td>
<td>519</td>
<td>404</td>
<td>488</td>
<td>606</td>
<td>722</td>
<td>166</td>
</tr>
<tr>
<td>19</td>
<td>1,675</td>
<td>527</td>
<td>434</td>
<td>524</td>
<td>651</td>
<td>759</td>
<td>178</td>
</tr>
<tr>
<td>20</td>
<td>1,766</td>
<td>534</td>
<td>466</td>
<td>563</td>
<td>699</td>
<td>798</td>
<td>191</td>
</tr>
</tbody>
</table>
5.7.2 Winter demand growth

Table 15 lists the current peak day demand and the estimated annual growth in the 1 in 20 winter peak day demand forecasts.

<table>
<thead>
<tr>
<th>Demand node</th>
<th>Name</th>
<th>Winter (TJ/day)</th>
<th>Growth factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>Melbourne</td>
<td>1,135</td>
<td>4.6%</td>
</tr>
<tr>
<td>$d_2$</td>
<td>Adelaide</td>
<td>345</td>
<td>0.2%</td>
</tr>
<tr>
<td>$d_3$</td>
<td>Mt. Isa</td>
<td>107</td>
<td>6.3%</td>
</tr>
<tr>
<td>$d_4$</td>
<td>Gladstone</td>
<td>168</td>
<td>6.3%</td>
</tr>
<tr>
<td>$d_5$</td>
<td>Brisbane</td>
<td>219</td>
<td>6.3%</td>
</tr>
<tr>
<td>$d_6$</td>
<td>Sydney</td>
<td>389</td>
<td>5.1%</td>
</tr>
<tr>
<td>$d_7$</td>
<td>Hobart</td>
<td>61</td>
<td>7.9%</td>
</tr>
</tbody>
</table>

Table 16 shows the forecast peak day demand for each demand centre over the next 20 years using the growth factors in Table 15.
Table 16: Forecast growth for winter maximum daily demand (TJ)

<table>
<thead>
<tr>
<th>Year</th>
<th>Melbourne</th>
<th>Adelaide</th>
<th>Mt.Isa</th>
<th>Gladstone</th>
<th>Brisbane</th>
<th>Sydney</th>
<th>Tas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,135</td>
<td>345</td>
<td>107</td>
<td>168</td>
<td>219</td>
<td>389</td>
<td>61</td>
</tr>
<tr>
<td>2</td>
<td>1,187</td>
<td>346</td>
<td>114</td>
<td>179</td>
<td>233</td>
<td>409</td>
<td>66</td>
</tr>
<tr>
<td>3</td>
<td>1,242</td>
<td>346</td>
<td>121</td>
<td>190</td>
<td>247</td>
<td>430</td>
<td>71</td>
</tr>
<tr>
<td>4</td>
<td>1,299</td>
<td>347</td>
<td>129</td>
<td>202</td>
<td>263</td>
<td>452</td>
<td>77</td>
</tr>
<tr>
<td>5</td>
<td>1,359</td>
<td>348</td>
<td>137</td>
<td>215</td>
<td>280</td>
<td>475</td>
<td>83</td>
</tr>
<tr>
<td>6</td>
<td>1,421</td>
<td>348</td>
<td>145</td>
<td>228</td>
<td>297</td>
<td>499</td>
<td>89</td>
</tr>
<tr>
<td>7</td>
<td>1,487</td>
<td>349</td>
<td>154</td>
<td>242</td>
<td>316</td>
<td>524</td>
<td>96</td>
</tr>
<tr>
<td>8</td>
<td>1,555</td>
<td>350</td>
<td>164</td>
<td>258</td>
<td>336</td>
<td>551</td>
<td>104</td>
</tr>
<tr>
<td>9</td>
<td>1,626</td>
<td>351</td>
<td>174</td>
<td>274</td>
<td>357</td>
<td>579</td>
<td>112</td>
</tr>
<tr>
<td>10</td>
<td>1,701</td>
<td>351</td>
<td>185</td>
<td>291</td>
<td>380</td>
<td>609</td>
<td>121</td>
</tr>
<tr>
<td>11</td>
<td>1,780</td>
<td>352</td>
<td>197</td>
<td>309</td>
<td>403</td>
<td>640</td>
<td>130</td>
</tr>
<tr>
<td>12</td>
<td>1,861</td>
<td>353</td>
<td>210</td>
<td>329</td>
<td>429</td>
<td>672</td>
<td>141</td>
</tr>
<tr>
<td>13</td>
<td>1,947</td>
<td>353</td>
<td>223</td>
<td>350</td>
<td>456</td>
<td>707</td>
<td>152</td>
</tr>
<tr>
<td>14</td>
<td>2,037</td>
<td>354</td>
<td>237</td>
<td>372</td>
<td>485</td>
<td>743</td>
<td>164</td>
</tr>
<tr>
<td>15</td>
<td>2,130</td>
<td>355</td>
<td>252</td>
<td>395</td>
<td>515</td>
<td>781</td>
<td>177</td>
</tr>
<tr>
<td>16</td>
<td>2,228</td>
<td>355</td>
<td>268</td>
<td>420</td>
<td>548</td>
<td>820</td>
<td>191</td>
</tr>
<tr>
<td>17</td>
<td>2,331</td>
<td>356</td>
<td>284</td>
<td>447</td>
<td>582</td>
<td>862</td>
<td>206</td>
</tr>
<tr>
<td>18</td>
<td>2,438</td>
<td>357</td>
<td>302</td>
<td>475</td>
<td>619</td>
<td>906</td>
<td>222</td>
</tr>
<tr>
<td>19</td>
<td>2,550</td>
<td>358</td>
<td>321</td>
<td>505</td>
<td>658</td>
<td>952</td>
<td>240</td>
</tr>
<tr>
<td>20</td>
<td>2,667</td>
<td>358</td>
<td>342</td>
<td>536</td>
<td>699</td>
<td>1,001</td>
<td>259</td>
</tr>
</tbody>
</table>
5.8 Formulation of optimisation problem to minimise supply shortfalls

This section outlines the formulation of the optimisation problems analysed. The first seeks to minimise the supply shortfalls on a day of maximum demand subject to constraints on production capacity and pipeline capacity. The configuration of the network for this analysis is shown in Figure 23.

Figure 23: Network to be optimised in the minimum shortfall analysis
5.9 Objective function to minimise supply shortfalls

The objective function to minimise any shortfall in demand is shown in equation 240.

\[
S(x) = \min_x (d_1 - x_1 - x_2 - x_3)^2 + (d_2 - x_4 - x_5)^2 + (d_3 - x_{11})^2 + (d_4 - x_{14})^2 \\
+ (d_5 - x_{13})^2 + (d_6 - x_7 - x_8)^2 + (d_7 - x_9)^2
\]  

(240)

The demand is represented by the \(d_j\), where \(j = 1, 2, \ldots, 7\). The \(x_i\), \(i = 1, 2, \ldots, 14\) represent the transmission pipes in the network. The amount of gas to flow through each pipe to satisfy demand is the solution sought.

5.10 Constraints

The constraints are represented as

\[
x_1 + x_2 + x_3 \leq d_1 \\
x_4 + x_5 \leq d_2 \\
x_{11} \leq d_3 \\
x_{14} \leq d_4 \\
x_{13} \leq d_5 \\
x_7 + x_8 \leq d_6 \\
x_9 \leq d_7
\]

The constraints on production are represented as

\[
x_1 + x_8 + x_9 \leq y_1 \\
x_2 + x_4 \leq y_2 \\
x_5 + x_6 - x_{10} \leq y_3 \\
x_{10} + x_{11} + x_{12} \leq y_4 \\
-x_{12} + x_{13} + x_{14} \leq y_5
\]
5.11 Results of optimisation to minimise supply shortfalls

The model to minimise shortfalls is run for both summer and winter seasons. The results show that demand centres with a single pipeline are more vulnerable to supply shortages than those with two or more sources of supply. It is also clear that the system is more vulnerable to supply shortfalls in the winter season, primarily because of the large variation in seasonal demand in Melbourne. To ensure supply in the winter season there is necessarily an excess of production capacity in the summer season.

5.11.1 Modelling results for the summer season

Table 17 shows the shortfalls emerging over the time period considered for the summer season. It can be concluded that in the sense of satisfying the summer demand all Eastern Australia capital cities are quite secure except Brisbane. Shortfalls in gas supply were predicted to begin in Brisbane in five years in the absence of capacity upgrades. The next region experiencing shortfalls is Tasmania where shortfalls are predicted in 15 years. The security of gas supply to the regional industrial towns of Mt.Isa and Gladstone is less certain. Mt. Isa has the potential for gas curtailment under present demand forecasts, whereas Gladstone is predicted to be in the same position next year.

Tables 18 and 19 show the pipeline utilisation for each segment in the network over the next 20 years under the scenarios described previously. Analysis of these tables shows that the summer shortfalls in Mt Isa, Gladstone and Brisbane are related to the limited capacities of single carriers connecting these demand nodes with the rest of the systems. Other carriers which could potentially be a bottle neck in the supply system are NSW-Victoria interconnect, which connects the Young Junction in New South Wales with Melbourne, and the Eastern Gas pipeline which connects the Longford production site with the Sydney/Canberra demand node. The capacity of the Eastern Gas pipeline is a scarce resource even at present. The Eastern Gas pipeline reaches its maximal capacity in year six in the future for summer peaks.
Table 17: Projected summer peak day shortfalls (TJ)

<table>
<thead>
<tr>
<th>Year</th>
<th>Melb.</th>
<th>Adelaide</th>
<th>Mt.Isa</th>
<th>Gladstone</th>
<th>Brisbane</th>
<th>Sydney</th>
<th>Tas.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>11</td>
<td>0</td>
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<td>0</td>
<td>20</td>
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<td>19</td>
<td>22</td>
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<td>0</td>
<td>0</td>
<td>41</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>30</td>
<td>35</td>
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<td>0</td>
<td>0</td>
<td>64</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>41</td>
<td>48</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>96</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>52</td>
<td>62</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>138</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>65</td>
<td>78</td>
<td>43</td>
<td>0</td>
<td>0</td>
<td>186</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>79</td>
<td>94</td>
<td>64</td>
<td>0</td>
<td>0</td>
<td>237</td>
</tr>
<tr>
<td>9</td>
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<td>0</td>
<td>93</td>
<td>112</td>
<td>86</td>
<td>0</td>
<td>0</td>
<td>290</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>109</td>
<td>131</td>
<td>109</td>
<td>0</td>
<td>0</td>
<td>349</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>126</td>
<td>151</td>
<td>135</td>
<td>0</td>
<td>0</td>
<td>412</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
<td>144</td>
<td>173</td>
<td>162</td>
<td>0</td>
<td>0</td>
<td>478</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>164</td>
<td>197</td>
<td>191</td>
<td>0</td>
<td>0</td>
<td>552</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>185</td>
<td>222</td>
<td>222</td>
<td>0</td>
<td>0</td>
<td>628</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>207</td>
<td>249</td>
<td>256</td>
<td>0</td>
<td>5</td>
<td>717</td>
</tr>
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5.11.2 Modelling results for the winter season

The security of the system is much more vulnerable during the winter than in summer. As mentioned previously, demand in Melbourne is highly seasonal and is expected to grow at a moderate pace from a high base. The shortfalls in supply for the winter season are shown in Table 20. Gladstone currently has a very real possibility of supply curtailment even under current peak day demand forecasts, which is one year earlier than for summer. Shortfalls for Mt Isa are predicted to start in year three, which is two years later than for summer. Mt Isa currently has a slightly larger forecast for peak day demand in the summer than in the winter which explains the earlier shortfalls predicted by the model. Gas is used for electricity generation in Mt Isa and the increased demand in summer may be because of increased electricity demand. Temperatures in Mt Isa are extremely hot in the summer months so it is likely that air conditioners are used extensively which increases the demand for electricity.

The winter shortfalls for the remaining demand nodes start earlier in winter. Brisbane will be at risk of supply shortfalls earlier than other cities, with demand outstripping supply in year three. Melbourne will experience difficulties with gas supply in the 7th year, Tasmania in the 11th, and Sydney in year 12. The size of the shortfalls in the southern demand centres raise the possibility that, in the absence of significant gas discoveries in the Gippsland and Otway basins, gas may have to be sourced from the coal seam gas fields in Queensland. Much of the gas reserves in Queensland are earmarked for export as LNG which will put domestic customers in competition with those in Asia where currently gas prices are significantly higher.

Tables 21 and 22 contain modelling results for pipeline utilisation in the winter season. Several pipelines are utilised at full capacity immediately under present demand conditions, namely the NSW-Victoria interconnect, the South-West pipeline, the Eastern Gas pipeline and the Queensland Gas pipeline. In the summer season the South-West pipeline had excess capacity until year five, after which it was fully utilised and the Eastern gas pipeline has excess capacity until year six after which it is fully utilised.

The fact that the NSW-Victoria interconnect, the South-West pipeline and the Eastern Gas pipeline are constrained reinforce the large seasonal variation in Melbourne’s demand
Table 20: Projected winter peak day shortfalls (TJ)

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profile and the importance of the Longford supply node. It is the large winter demand in Melbourne that causes the South-West pipeline to be fully constrained in year one of the period considered, whereas in the summer season there is excess capacity until year five. While the Longford production node can supply the Longford-Melbourne pipeline, the Eastern gas pipeline and the Tasmanian gas pipeline when operating at full capacity.

The western section of the Moomba-Sydney pipeline, which runs from the Moomba production node to the pipe junction at Young is constrained six years earlier in the winter season, year 11 as opposed to year 17. Again we can conclude that this is largely as a result of the large Melbourne demand as, although demand is increasing in the Sydney node there is still excess capacity in the eastern section of the Moomba-Sydney pipeline.

Demand in Tasmania is the forecast to grow more rapidly than anywhere else in the eastern Australian gas network, however present demand is small relative to other demand nodes and there is significant excess capacity in the Tasmanian gas pipeline to accommodate the rapid growth rate. Therefore, even though demand is supplied through a single connection, this pipe is not forecast to be constrained until year eleven. In contrast the other demand centres with single connections, namely Mt.Isa, Brisbane and Gladstone face the possibility of supply shortages immediately in the case of Gladstone, year two in the case of Brisbane and in year three in Mt.Isa.
Table 21: Winter season pipeline utilisation in years 1 to 10

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Table 22: Winter season pipeline utilisation in years 11 to 20

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<td>161</td>
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5.12 Formulation of optimisation problem to minimise the cost of supply

This section outlines the formulation of the optimisation problem to minimise the cost of supply. In this formulation production levels are included as decision variables along with the amount to ship through each pipe. Shortfalls are not permitted and the model is run for both the summer and winter seasons. As shortfalls are not permitted there is no optima solution for 100% of peak day demand in the summer season as demand cannot be satisfied under the present operating constraints of the network. This is because the current published peak day demand forecast for Mt.Isa in the summer season is 120 TJ/day while the current maximum daily quantity able to be transported through the Carpentaria gas pipeline which supplies Mt.Isa is 119 TJ/day. For this analysis the capacity of this pipeline is increased to 120 TJ/day in the summer season to allow feasible solutions to be found. A similar situation exists in the winter season where Gladstone has a forecast peak day demand of 168 TJ/day. The published capacity of the Queensland gas pipeline that supplies Gladstone is increased from 145 TJ/day to 168 TJ/day to allow feasible solutions to be found. As this model seeks to minimise costs the primary interest is where gas should be sourced from when a demand centre has multiple supply options.

The configuration of the network for this analysis is shown in Figure 24.
5.13 Objective function to minimise the cost of supply

The objective function to minimise the cost of supply is shown in equation 241.

$$S(x) = \min_x \sum_{i=1}^{19} c_i x_i$$  \hspace{1cm} (241)$$

where $c_i$ denote the cost of using asset $i$ and the $x_i$ denote asset $i$. For example, the cost of transporting gas on the Longford to Melbourne pipeline, represented in the model by $x_1$, is $c_1$. As the costs are fixed by minimising the $x_i$ the optimal solution will achieve the minimum cost of supply.
5.14 Constraints

The constraints are represented as

\[
\begin{align*}
    x_1 + x_2 + x_3 &= d_1 \\
    x_4 + x_5 &= d_2 \\
    x_{11} &= d_3 \\
    x_{14} &= d_4 \\
    x_{13} &= d_5 \\
    x_7 + x_8 &= d_6 \\
    x_9 &= d_7
\end{align*}
\]

These constraints ensure the demand is met at each demand node, that is no shortfalls are permitted to reduce costs.

The constraints on production are represented as

\[
\begin{align*}
    x_1 + x_8 + x_9 &= x_{15} \\
    x_2 + x_4 &= x_{16} \\
    x_5 + x_6 + x_{10} &= x_{17} \\
    x_{10} + x_{11} - x_{12} &= x_{18} \\
    x_{12} + x_{13} + x_{14} &= x_{19}
\end{align*}
\]

As the amount of production at each production node is now subject to change in the search for the optimal solution, the flow out of each production node must equal the amount of gas to be shipped along each pipeline.

The formulation of this problem allows for changes in demand to be studied in the context of the changes in production amounts and transportation routes. By reducing maximum daily demand, production nodes or transportation arcs that were previously constrained will have excess capacity and may replace higher costing sources of supply.
5.15 Results of optimisation to minimise the cost of supply

In this section the results of minimising the cost of supply in the summer and winter seasons are discussed. The configuration of the network, with Mt.Isa, Gladstone, Roma and Tasmania supplied through a single connection, and the large fluctuation in seasonal demand in Melbourne makes the competition to supply Melbourne, Adelaide and Sydney of primary interest for this model.

5.15.1 Minimum cost of supply for the summer season

Tables 23 and 24 show the modelling results for minimising the cost of supply in the summer season. The results show the amount of gas transported through each pipe segment and the amount produced at each production node for decreasing percentages of maximum daily demand.

It can be seen from Tables 23 and 24 that the large production capacity and low cost of the Longford plant mean that it is the cheapest way to satisfy gas demand in Melbourne. It is also the cheapest way to supply Sydney even though transportation tariffs on the Eastern gas pipeline form Longford to Sydney are among the highest in the network. Higher production costs at the Otway supply node and the excess in production and pipe capacity emanating from Longford in the summer season mean that the South-West pipeline from Otway to Melbourne is idle in the summer. Adelaide demand is first supplied from Moomba via the Moomba-Adelaide pipeline and then from Otway through the SEAGas pipeline. At 50% of peak day demand all of Adelaide's gas can be supplied through the Moomba-Adelaide pipeline and the Otway supply node and its connecting pipelines are unused. The low demand from Melbourne in the summer relative to the winter means the New South Wales -Victorian Interconnect, which runs from the Young junction to Melbourne is unused. A similar situation exists in Queensland where the large production capacity at Roma means that Brisbane and Gladstone demand can be satisfied and gas sent west along the South-West Queensland pipeline to Ballera, and then on to Mt.Isa. This leaves the QSNLink from Ballera to Moomba unused. Both the New South Wales -Victorian Interconnect and the QSNLink have value in that they increase the diversity of supply options.
Table 23: Amount transported to minimise the cost of supply in the summer season

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_{11}$</td>
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<td>83</td>
<td>71</td>
<td>60</td>
<td>48</td>
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<td>72</td>
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<td>102</td>
<td>87</td>
<td>73</td>
<td>58</td>
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Table 24: Amount produced to minimise the cost of supply in the summer season

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<th>Segment</th>
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<th>90%</th>
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<th>70%</th>
<th>60%</th>
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<th>40%</th>
<th>30%</th>
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<td>128</td>
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<td>100</td>
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<td>260</td>
<td>228</td>
<td>195</td>
<td>163</td>
<td>130</td>
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Using the results in Table 23 as a baseline it is interesting to change the costs of production to analyse their impact on where gas is sourced. The large capacity and low cost of production at Longford make it a key piece of infrastructure. It is interesting to explore the results of different production costs at Longford and how they would affect flow around the network. If all other decision variables are held steady there is no change in the baseline profile shown in Table 23 until production costs at Longford rise to $4.09/GJ. At this price production from Longford falls from 989 TJ/day to 719 TJ/day and at $4.10/GJ production falls to 636 TJ/day before stabilising. The reduced production at Longford sees flow on the Longford-Melbourne pipeline fall to 380 TJ/day and 297 TJ/day at production costs of $4.09/GJ and $4.10/GJ respectively. The reduced production at Longford and reduced supply to Melbourne on the Longford-Melbourne pipeline is made up by increased production at Otway and flow to Melbourne on the South-West pipeline.

Table 25 shows the changes in the network variables at this critical price point in Longford production costs.

<table>
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<th>Segment</th>
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<th>$4.10/GJ</th>
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<td>270</td>
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<td>989</td>
<td>719</td>
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<td>Otway</td>
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<td>169</td>
<td>439</td>
<td>522</td>
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</table>

The next critical point for production costs at Longford is at $4.39/GJ which causes production to fall from 636 TJ/day to 548 TJ/day, and to 508 TJ/day at production costs of $4.40/GJ. Production at Otway is unchanged as there is no longer any capacity to supply Melbourne via the South-West pipeline. The fall in production at Longford is replaced with increased production at Moomba which increases from 262 TJ/day in the baseline scenario to its maximum daily capacity 390 TJ/day when Longford production costs are at or above $4.40/GJ. The reduced daily production at Longford sees an equivalent reduction in daily gas flow on the Eastern gas pipeline to Sydney. When Longford produces at a cost of $4.39/GJ flow on the Eastern gas pipeline falls to 201 TJ/day, and when Longford produces at a cost of $4.40/GJ flow on the Eastern gas pipeline falls to 161 TJ/day. To satisfy Sydney demand Moomba gas is transported on the Moomba-Sydney pipeline with both segments of this pipeline increasing flow in steps equal to the declines on the Eastern gas pipeline at the same Longford production costs. Table 26 summarises the changes in...
the decision variables.

Table 26: Affect of rise in production costs at Longford to $4.40/GJ

<table>
<thead>
<tr>
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<th>$4.39/GJ</th>
<th>$4.40/GJ</th>
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<td>109</td>
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<td>289</td>
<td>201</td>
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</tr>
<tr>
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<td>989</td>
<td>636</td>
<td>548</td>
<td>508</td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>Moomba</td>
<td>262</td>
<td>262</td>
<td>350</td>
<td>390</td>
</tr>
</tbody>
</table>

The next step down in Longford production comes when production costs hit $4.74/GJ when production falls to 470 TJ/day and at $4.75 when production falls to 363 TJ/day. The production is made up at Otway in equivalent steps. As Longford production falls so too does the flow to Sydney on the Eastern Australia gas pipeline, the reduced flow to Sydney is replaced with an increase in flow on the Moomba-Sydney pipeline segments which run from Moomba to the Young junction and from there to Sydney. As production at Moomba is already at its maximum daily quantity, gas is redirected from Adelaide to Sydney and flow from Moomba to Adelaide on the Moomba-Adelaide pipeline is reduced to 96 TJ/day. The balance of Adelaide demand is made up with gas produced at Otway and shipped from there to Adelaide on the SEAGas pipeline. The flow on the SEAGas pipeline is maximised at these production costs and 314 TJ/day is transported. Table 27 summarises the changes in the decision variables when Longford produces at these cost levels. To replace Longford supply to Melbourne Otway production costs would have to fall to $2.00/GJ when 241 TJ/day would be shipped from Otway to Melbourne on the South-West pipeline. At production costs of $1.99/GJ the maximum capacity of the South-West pipeline, 353 TJ/day, would be used to supply Melbourne with gas produced at Otway. The volume produced at Longford would fall by the same amount, as would the volume shipped to Melbourne on the Longford-Melbourne pipeline.

The next critical cost point for production at Longford is at $5.80. When costs reach this level production falls to 356 TJ/day and at $5.81 Longford production is 347 TJ/day. Since Ballera and Moomba production levels are already at the maximum daily quantity for those facilities, and the excess production capacity at Otway is stranded by capacity constraints on the SEAGas and South-West pipelines the decline in production at
Longford is replaced with gas produced at the Roma supply node which increases daily production to 361 TJ/day. This extra production is sent west to Ballera on the South-West Queensland pipeline, which increases the flow on this pipeline to 36 TJ/day, then from Ballera to Moomba on the QSNLink which has a daily flow rate of 16 TJ. The QSNLink is the final pipeline constructed in the network and under the assumptions in this analysis is underutilised when production costs in the southern part of the network are low, however these fields are generally considered to be decline while there are abundant reserves around the Roma supply node and it is reasonable to expect to QSNLink to be used more as production costs rise. From Moomba the extra gas produced at Roma is transported to Sydney on the Moomba-Sydney pipeline, via the Young junction. Flow on these two pipe segments increases from 294 TJ/day to 310 TJ/day. The Eastern gas pipeline is not used when production costs at Longford reach $5.81 TJ. Table 28 shows how each decision variable is affected by the increase in production costs at Longford.

Holding all other variables equal, Longford production costs would have to rise to $6.25 before it is viable to replace it with more production from Roma. Were production costs at Longford to rise to this level Roma production would increase to 405 TJ/day, and to 451 TJ/day should Longford production costs reach $6.26/GJ. Flow on the South-West Queensland pipeline would increase to 126 TJ/day, and flow on the QSNLink to 126 TJ/day. The large capacity on the Moomba-Sydney pipeline would be further utilised with the eastern segment from Moomba to the Young junction increasing its daily flow rate to 400 TJ/day. At the Young junction 310 TJ/day would continue on the eastern
Table 28: Affect of rise in production costs at Longford to $5.81/GJ

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$5.79/GJ</th>
<th>$5.80/GJ</th>
<th>$5.81/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>21</td>
<td>294</td>
<td>301</td>
<td>310</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Young - Sydney</td>
<td>21</td>
<td>294</td>
<td>301</td>
<td>310</td>
</tr>
<tr>
<td>$x_8$</td>
<td>Longford - Sydney</td>
<td>289</td>
<td>16</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>0</td>
<td>16</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>20</td>
<td>27</td>
<td>20</td>
<td>36</td>
</tr>
</tbody>
</table>

$\times$

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$6.24/GJ$</th>
<th>$6.25/GJ$</th>
<th>$6.26/GJ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{15}$</td>
<td>Longford</td>
<td>989</td>
<td>347</td>
<td>352</td>
<td>361</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>Roma</td>
<td>345</td>
<td>361</td>
<td>405</td>
<td>451</td>
</tr>
</tbody>
</table>

Under the assumptions of this scenario, at least 207 TJ of production is needed at Longford, regardless of the cost of production, in order to satisfy forecasts summer peak day demand in Sydney, and 90 TJ/day would be sent down the New South Wales-Victoria interconnect to supply Melbourne. This 90 TJ/day of supply from Roma to Melbourne would be cheaper than producing gas at Longford and shipping it to Melbourne on the Longford-Melbourne pipeline. The daily flow rate on the Longford-Melbourne pipeline would fall to 207 TJ/day. The changes in decision variables that occur at the critical cost of point of Longford production are shown in Table 29.

Table 29: Affect of rise in production costs at Longford to $6.26/GJ

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$6.24/GJ$</th>
<th>$6.25/GJ$</th>
<th>$6.26/GJ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Longford - Melbourne</td>
<td>650</td>
<td>297</td>
<td>253</td>
<td>207</td>
</tr>
<tr>
<td>$x_3$</td>
<td>NSW - VIC. Int.</td>
<td>0</td>
<td>0</td>
<td>44</td>
<td>99</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>21</td>
<td>310</td>
<td>354</td>
<td>400</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>0</td>
<td>16</td>
<td>60</td>
<td>106</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>20</td>
<td>36</td>
<td>80</td>
<td>126</td>
</tr>
</tbody>
</table>

$\times$

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$6.24/GJ$</th>
<th>$6.25/GJ$</th>
<th>$6.26/GJ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{15}$</td>
<td>Longford</td>
<td>989</td>
<td>347</td>
<td>303</td>
<td>257</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>Roma</td>
<td>345</td>
<td>361</td>
<td>405</td>
<td>451</td>
</tr>
</tbody>
</table>
demand at all demand centres in the network.

If we shift the focus to the Moomba production node and again keep all other variables fixed, we can explore the impact on the network of changes in the production cost structure at Moomba. In the baseline scenario the production costs at Moomba are estimated to be $3.79/GJ and the amount produced is 262 TJ/day. The first reduction in production comes as production costs rise to $4.14 GJ/day when production falls to 211 TJ/day, at $4.15 GJ/day production falls to 117 TJ/day. The reduction in Moomba production sees the daily flow on the Moomba-Adelaide pipeline fall from 241 TJ/day in the baseline scenario to 190 TJ/day when production costs are at $4.14/GJ and to 96 TJ/day when production costs hit $4.15/GJ. The reduction in Moomba production and supply to Adelaide are replaced by an equivalent increase in production at Otway and flow on the SEAGas pipeline from Otway to Melbourne. Otway production steps from 169 TJ/day in the baseline scenario to 220 TJ/day and on to 314 TJ/day. All of this production is transported through the SEAGas pipeline. Table 30 shows the changes in decision variables when Moomba production costs reach $4.15/GJ.

Table 30: Affect of rise in production costs at Moomba to $4.15/GJ

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$4.13/GJ</th>
<th>$4.14/GJ</th>
<th>$4.15/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_4$</td>
<td>Otway - Adelaide</td>
<td>169</td>
<td>169</td>
<td>220</td>
<td>314</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Moomba - Adelaide</td>
<td>241</td>
<td>241</td>
<td>190</td>
<td>96</td>
</tr>
<tr>
<td>$x_{16}$</td>
<td>Otway</td>
<td>169</td>
<td>169</td>
<td>220</td>
<td>314</td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>Moomba</td>
<td>262</td>
<td>262</td>
<td>211</td>
<td>117</td>
</tr>
</tbody>
</table>

Moomba gas produced in the same volume until production costs reach $5.20/GJ when production falls to 84 TJ/day, and at $5.21/GJ production falls to zero. At these production costs 314 TJ/day of gas is shipped to Adelaide from the Otway production node on the SEAGas pipeline and 96 TJ of gas produced at Roma is transported to Adelaide via the South-West Queensland pipeline, the QSNLink and the Moomba-Adelaide pipeline in order to meet the peak day summer demand of Adelaide.

With estimated production costs of $4.06/GJ the Otway production node is the highest cost gas producer in the network. In the summer season Melbourne demand is easily supplied from Longford and in the baseline scenario no gas from Otway is needed to
satisfy demand. The smaller capacity of the Moomba-Adelaide pipeline means that the lower cost gas from the Moomba production node cannot be shipped in the necessary quantity to satisfy summer peak day demand in Adelaide and 169 TJ/day of production from Otway is shipped to Adelaide through the SEAGas pipeline to make up the shortfall. There is excess capacity on the SEAGas pipeline which can be used to displace production from Moomba if Otway production costs fell $3.71/GJ. The affect network is identical to Moomba production costs rising to $4.15/GJ as shown in Table 30.

5.15.2 Minimum cost of supply for the winter season

Tables 31 and 32 show the modelling results for minimising the cost of supply in the winter season. As for summer, the results show the amount of gas transported through each pipe segment and the amount produced at each production node for decreasing percentages of maximum daily demand.

Once again the low cost production available from Longford is used to supply Melbourne through the Longford-Melbourne pipeline and Sydney through the Eastern gas pipeline. However, the maximum daily quantity that can be transported through the Longford-Melbourne pipeline is insufficient to satisfy demand and higher cost Otway gas is needed on peak demand days. At 90% of peak demand Melbourne can be supplied with gas from Longford and the South-West pipeline from Otway to Melbourne is unused. The production profile at Otway declines more rapidly in the winter than in the summer season. Production at Otway is not needed at 60% of peak day demand, as opposed to 50% in the summer. A large part of Adelaide demand is for electricity generation which peaks in the summer, the earlier cessation of production at Otway is a function of the lower demand at Adelaide in the winter.

If we conduct a similar analysis of the affects on the network of changes in production costs, using Table 32 as a baseline, the large capacity and low cost of Longford must be considered the most important source of production. The estimate of production costs at Longford is $2.03/GJ. Holding all other variables equal, there is no change in the baseline production and flow volumes until production costs rise to $4.09/GJ at Longford. When production costs rise to this level, production at Longford falls to 1,178 TJ/day and at
Table 31: Minimising the cost of supply in the winter season

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Longford - Melbourne</td>
<td>1030</td>
<td>1022</td>
<td>908</td>
<td>795</td>
<td>681</td>
<td>568</td>
<td>454</td>
<td>341</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Otway - Melbourne</td>
<td>105</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_3$</td>
<td>NSW - VIC. Int.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_4$</td>
<td>Otway - Adelaide</td>
<td>107</td>
<td>70</td>
<td>35</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Moomba - Adelaide</td>
<td>238</td>
<td>241</td>
<td>241</td>
<td>241</td>
<td>207</td>
<td>173</td>
<td>138</td>
<td>104</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>98</td>
<td>59</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Young - Sydney</td>
<td>98</td>
<td>59</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_8$</td>
<td>Longford - Sydney</td>
<td>291</td>
<td>291</td>
<td>291</td>
<td>272</td>
<td>233</td>
<td>195</td>
<td>156</td>
<td>117</td>
</tr>
<tr>
<td>$x_9$</td>
<td>Longford - Hobart</td>
<td>61</td>
<td>55</td>
<td>49</td>
<td>43</td>
<td>37</td>
<td>31</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>Ballera - Mt.Isa</td>
<td>107</td>
<td>96</td>
<td>86</td>
<td>75</td>
<td>64</td>
<td>54</td>
<td>43</td>
<td>32</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_{13}$</td>
<td>Roma - Brisbane</td>
<td>219</td>
<td>197</td>
<td>175</td>
<td>153</td>
<td>131</td>
<td>110</td>
<td>88</td>
<td>66</td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>Roma - Gladstone</td>
<td>168</td>
<td>151</td>
<td>134</td>
<td>118</td>
<td>101</td>
<td>84</td>
<td>67</td>
<td>50</td>
</tr>
</tbody>
</table>
Table 32: Minimising the cost of supply in the winter season

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Amount produced TJ/d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x15</td>
<td>Longford</td>
<td>1382</td>
<td>1367</td>
<td>1248</td>
<td>1110</td>
<td>951</td>
<td>793</td>
<td>634</td>
<td>476</td>
</tr>
<tr>
<td>x16</td>
<td>Otway</td>
<td>212</td>
<td>70</td>
<td>35</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x17</td>
<td>Moomba</td>
<td>336</td>
<td>300</td>
<td>261</td>
<td>241</td>
<td>207</td>
<td>173</td>
<td>138</td>
<td>104</td>
</tr>
<tr>
<td>x18</td>
<td>Ballera</td>
<td>100</td>
<td>96</td>
<td>86</td>
<td>75</td>
<td>64</td>
<td>54</td>
<td>43</td>
<td>32</td>
</tr>
<tr>
<td>x19</td>
<td>Roma</td>
<td>394</td>
<td>348</td>
<td>310</td>
<td>271</td>
<td>232</td>
<td>194</td>
<td>155</td>
<td>116</td>
</tr>
</tbody>
</table>

a cost of $4.10/GJ production is 1,134 TJ/day. The fall in production at Longford is matched by falls of the same volume through the Longford-Melbourne pipeline, where the daily amount transported to supply Melbourne falling to 826 TJ/day at production costs of $4.09/GJ, and to 782 TJ/day at production costs of 782 TJ/day. Production at Otway replaces Longford production and the South-West pipeline from Otway to Melbourne replaces the volume shipped on the Longford-Melbourne. The additional production at Otway and transportation on the South-West pipeline are equal in volume to the reduction at Longford and shipped on the Longford-Melbourne pipeline. Table 33 summarises the changes in the decision variables of a rise in production costs at Longford to $4.10/GJ.

Table 33: Affect of a rise in production costs at Longford to $4.10/GJ

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$4.08/GJ</th>
<th>$4.09/GJ</th>
<th>$4.10/GJ</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x1</td>
<td>Longford - Melbourne</td>
<td>1030</td>
<td>1030</td>
<td>826</td>
<td>782</td>
</tr>
<tr>
<td>x2</td>
<td>Otway - Melbourne</td>
<td>105</td>
<td>105</td>
<td>309</td>
<td>353</td>
</tr>
<tr>
<td>x15</td>
<td>Longford</td>
<td>1382</td>
<td>1382</td>
<td>1178</td>
<td>1134</td>
</tr>
<tr>
<td>x16</td>
<td>Otway</td>
<td>212</td>
<td>212</td>
<td>416</td>
<td>460</td>
</tr>
</tbody>
</table>

The volume of production at Longford is unchanged until prices hit $4.74/GJ when production declines to 1,022 TJ/day and on to 927 TJ/day when production costs are $4.75. The decline in Longford production is again replaced with production at Otway which rises to 572 TJ/day, and then to 667 TJ/day. The flow on the Longford-Melbourne pipeline is unchanged as the flow on the South West pipeline reached its maximum daily
quantity when Longford production costs rose to $4.10/GJ. The reduced production at Longford sees a matching reduction in daily flow on the Eastern Australian pipeline which runs from Longford to supply the Sydney demand node. The daily flow on the Eastern Australian gas pipeline when Longford production costs are $4.75/GJ is 84 TJ/day. The major part of Sydney peak day demand is supplied from the Moomba production node, however Moomba production operates at its maximum daily capacity even in the baseline case. Therefore the majority of gas produced at Moomba gas is diverted from Adelaide and sent to supply Sydney while Otway gas is used to satisfy Adelaide demand. The result is that the flow of gas from Moomba to Sydney on the east and west segments of the Moomba-Sydney pipeline increases to 305 TJ/day while the flow from Moomba to Adelaide on the Moomba-Adelaide pipeline falls to 31 TJ/day. Otway production reaches 667 TJ/day and the SEAGas pipeline from Otway to Adelaide reaches its maximum capacity of 314 TJ/day. The changes in the values of the decision variables when Longford production costs reach $4.75/GJ are shown in Table 34.

Table 34: Affect of rise in production costs at Longford to $4.75/GJ in the winter season

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
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<th>$4.74/GJ</th>
<th>$4.75/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_4$</td>
<td>Otway - Adelaide</td>
<td>107</td>
<td>107</td>
<td>219</td>
<td>314</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Moomba - Adelaide</td>
<td>238</td>
<td>238</td>
<td>126</td>
<td>31</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>98</td>
<td>98</td>
<td>210</td>
<td>305</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Young - Sydney</td>
<td>98</td>
<td>98</td>
<td>210</td>
<td>305</td>
</tr>
<tr>
<td>$x_8$</td>
<td>Longford - Sydney</td>
<td>291</td>
<td>291</td>
<td>179</td>
<td>84</td>
</tr>
<tr>
<td>$x_{15}$</td>
<td>Longford</td>
<td>1382</td>
<td>1134</td>
<td>1022</td>
<td>927</td>
</tr>
<tr>
<td>$x_{16}$</td>
<td>Otway</td>
<td>212</td>
<td>460</td>
<td>572</td>
<td>667</td>
</tr>
</tbody>
</table>

The next critical cost point for Longford production comes at $5.80/GJ where production falls to 889 TJ/day and on to 843 TJ/day at $5.81/GJ. With Moomba already operating at its maximum daily capacity and Otway constrained by the lack of capacity on the SEAGas and South-West pipelines, through which Otway production connects to Adelaide and Melbourne respectively, the fall in Longford production is matched by an increase in production at Roma. Roma production rises from 394 TJ/day in the baseline case to 478 TJ/day when Longford production costs reach $5.81/GJ. The increase in Roma production sees a commensurate increase in flow on the South-West Queensland pipeline from Roma to Ballera, and the same increase in flow on the QSNLink from Ballera to
Moomba. From Moomba the increase in Roma production is shipped to supply Sydney which sees the daily flow on the two segments of the Moomba-Sydney pipeline increase to 389 TJ/day. The reduced production at Longford sees the flow on the Eastern Australian gas pipeline fall to zero. Table 35 shows how the decision variables change as Longford production costs rise to $5.81/GJ.

Table 35: Affect of rise in production costs at Longford to $5.81/GJ in the winter season

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$5.79/GJ</th>
<th>$5.80/GJ</th>
<th>$5.81/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>98</td>
<td>305</td>
<td>343</td>
<td>389</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Young - Sydney</td>
<td>98</td>
<td>305</td>
<td>343</td>
<td>389</td>
</tr>
<tr>
<td>$x_8$</td>
<td>Longford - Sydney</td>
<td>291</td>
<td>84</td>
<td>46</td>
<td>0</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>0</td>
<td>0</td>
<td>38</td>
<td>84</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>7</td>
<td>7</td>
<td>45</td>
<td>91</td>
</tr>
<tr>
<td>$x_{15}$</td>
<td>Longford</td>
<td>1382</td>
<td>927</td>
<td>889</td>
<td>843</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>Roma</td>
<td>394</td>
<td>394</td>
<td>432</td>
<td>478</td>
</tr>
</tbody>
</table>

As production costs at Longford reach $6.25/GJ production drops to 820 TJ/day and at costs of $6.26/GJ production falls to 793 TJ/day. The flow of gas to Melbourne on the Longford-Melbourne pipeline falls from 782 TJ/day to 732 TJ/day. Production at Roma steps up to 501 TJ/day and on to 528 TJ/day. The increase in Roma production is again shipped on the South-West Queensland pipeline to Ballera and then on the QSNLink from Ballera to Moomba. The amount transported on these pipelines increases to 141 TJ/day and 134 TJ/day respectively. The extra gas from Roma is then transported on the west segment of the Moomba-Sydney pipeline which runs from Moomba to the Young junction. At the Young junction gas flows into the New South Wales-Victoria interconnect to supply Melbourne demand. The daily flow on the Moomba-Sydney pipeline west is now at its maximum of 439 TJ/day while the flow on the New South Wales-Victoria interconnect is at 50 TJ/day. These changes in the values of the decision variables are shown in Table 36.

In the winter season the baseline scenario sees Moomba producing at is maximum daily capacity of 336 TJ/day. This production rate is maintained until production costs reach $4.14/GJ when production falls to 228 TJ/day, and at $4.15/GJ production at Moomba
Table 36: Affect of rise in production costs at Longford to $6.26/GJ in the winter season

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$6.24/GJ</th>
<th>$6.25/GJ</th>
<th>$6.26/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Longford - Melbourne</td>
<td>1030</td>
<td>782</td>
<td>759</td>
<td>732</td>
</tr>
<tr>
<td>$x_3$</td>
<td>NSW - VIC. Int.</td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>50</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Moomba - Young</td>
<td>98</td>
<td>389</td>
<td>412</td>
<td>439</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>0</td>
<td>84</td>
<td>107</td>
<td>134</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>7</td>
<td>91</td>
<td>114</td>
<td>141</td>
</tr>
<tr>
<td>$x_{15}$</td>
<td>Longford</td>
<td>1382</td>
<td>843</td>
<td>820</td>
<td>793</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>Roma</td>
<td>394</td>
<td>478</td>
<td>501</td>
<td>528</td>
</tr>
</tbody>
</table>

falls to 129 TJ/day. Flow on the Moomba-Adelaide pipeline falls from 238 TJ/day in the baseline scenario to 130 TJ/day at production costs of $4.14/GJ and to 31 TJ/day at production costs of $4.15/GJ. Supply to Adelaide is secured by increasing production at Otway from 212 TJ/day to 320 TJ/day and on to 419 TJ/day when Moomba production costs reach $4.15/GJ. The extra supply from Otway increases the amount of gas flow through the SEAGas pipeline from 107 TJ/day to 215 TJ/day before reaching its maximum capacity of 314 TJ/day.

Table 37: Affect of rise in production costs at Moomba to $4.15/GJ

<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Baseline</th>
<th>$4.13/GJ</th>
<th>$4.14/GJ</th>
<th>$4.15/GJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_5$</td>
<td>Moomba - Adelaide</td>
<td>238</td>
<td>31</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Ballera - Moomba</td>
<td>0</td>
<td>0</td>
<td>44</td>
<td>129</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>Roma - Ballera</td>
<td>7</td>
<td>91</td>
<td>114</td>
<td>141</td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>Moomba</td>
<td>336</td>
<td>336</td>
<td>228</td>
<td>129</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>Roma</td>
<td>394</td>
<td>478</td>
<td>501</td>
<td>528</td>
</tr>
</tbody>
</table>

Production at Moomba is steady until costs reach $5.20/GJ when production falls to 85 TJ/day. At production costs of $5.21/GJ production at Moomba is zero. The production needed to supply Adelaide is found at Roma. When production at Moomba is zero the forecast peak day winter demand for Adelaide of 345 TJ is supplied by 31 TJ of production
at Roma which is transported via the South-West Queensland pipeline, the QSNLink and the Moomba-Adelaide pipeline, and 314 TJ of gas produced at Otway which reaches Adelaide through the SEAGas pipeline.

The larger winter demand in Melbourne sees Otway production utilised more with 212 TJ/day being produced in the baseline scenario. Of this production 105 TJ/day is sent to supply Melbourne on the South-West pipeline and 107 TJ/day is sent to supply Adelaide via the SEAGas pipeline. These volumes are maintained until Otway production reaches $5.12/GJ when Otway production falls to 210 TJ/day and then to 209 TJ/day at a production cost of $5.13/GJ. The small reduction in volumes is a function of the lack of excess capacity on the Moomba-Adelaide pipeline. In the baseline scenario this pipeline transports 238 TJ/day from Moomba to supply Adelaide. The maximum daily quantity that can be shipped using this pipeline is 241 TJ/day. The 3 TJ/day drop in production at Otway is replaced with an equivalent amount produced at Roma and shipped to Adelaide through the South-West Queensland pipeline, the QSNLink, and then the Moomba-Adelaide pipeline.

6 Concluding statements

In this thesis three research questions relating to the eastern Australian gas network and the Goulburn-Murray irrigation district have been analysed and solutions presented. The problem of estimating the price of irrigation water at the beginning of the season has been addressed. The value of this research is that it allows new risk management tools to be utilised by irrigators in the region. For example, derivatives that have the price of water as the underlying commodity could be developed. The introduction of a derivatives market has the potential to give farmers greater certainty over their production costs by removing uncertainty over the price of one key input to production, the price of water.

The second research question addressed in this thesis involves the lack of a model with which to simulate the price of wholesale natural gas sold at spot prices. Energy companies in Australia sell both gas and electricity to retail customers. Both commodities are bought in the wholesale market using long and short term contracts with various generic and unique features, and both are subject to regulation with regard to the price that can
be charged to retail customers. The market for electricity is well established and includes a spot market and a variety of hedging instruments that participants can use to manage risk. However, while long term gas sales agreements can be included when modelling a portfolio of energy contracts held by a company, the daily price fluctuations of natural gas have thus far been neglected.

As stated, most gas consumed in eastern Australia is subject to long term gas sales agreements however a spot market for wholesale natural gas has existed in Victoria since 1999. A model of the spot price for wholesale natural gas sold in Victoria is of interest to an industry partner who see it as the final component needed to model a portfolio of energy contracts held by their customers. The spot price of gas is volatile and as such market participants are exposed to a risk that previously has been unable to quantify. The spot price of gas shows enough volatility to make the use of long term averages insufficient to reflect the risks that market participants are exposed to by fluctuations in the spot price. In this thesis a stochastic differential equation is presented as a suitable model for the spot price of wholesale natural gas sold in Victoria. The parameters of the model have been estimated and the equation can be used to simulate a price series with which energy contracts can be valued. The model captures the volatility seen in the historical price series and can give insight into the daily fluctuations in price. This model fills the gap in knowledge and allows energy companies to gain insight into the financial risks or their portfolio of energy contracts.

A new spot market has been operating since 2011 with gas traded at the Adelaide, Sydney and Brisbane demand nodes. The price history available from these markets and the volumes traded are not yet sufficient to create a simulation model. However, pricing models with which to simulate the spot price of gas at these nodes is likely to be useful and should be a topic for future research. The addition of pricing models for the new trading nodes would complete the picture for energy companies. Another logical extension of the model presented in this thesis is the creation of hedging instruments, similar to those used in the electricity market and developed for use in the market for irrigation water in the Goulburn-Murray irrigation district. This would remove unnecessary risks for energy companies and allow for longer term planning based on future price certainty.

Furthermore this thesis has contributed to the existing research in the field of network allocation systems by modelling the gas supply system in eastern Australia using network
optimisation methods. The network system is a complex and dynamic system, in this thesis the network is stylised and the key variables identified. An important contribution to this field of research is that the data which must underpin any model of the network is collated and presented in one document. This is the third research question outlined in the introduction to this thesis for which a solution has been presented.

Two models of the eastern Australian gas network were formulated. The first minimises the shortfalls in simultaneous peak day demand across all demand centres in eastern Australia. Annual growth factors are applied to each demand centre to model the network over the next two decades. The growth factors are estimates based on the make up of the market and expectations of the growth likely for each market segment. The value of this model is in the fact that key pieces of infrastructure that quickly become constrained can be identified for capacity upgrades. A stable and secure gas supply is a key component underpinning economic growth. The importance of a secure gas supply is increasing as the interconnections between electricity and gas networks increases. Gas fired electricity generators are increasing in number in eastern Australia with many generators being engineered to provide baseload electricity as opposed to the historically more common peak demand generators. The increase in gas demand from electricity generators makes the secure supply of adequate gas vitally important for the safety and prosperity of the community. The model presented in this thesis is the first step to developing a dynamic model of the network system.

Each demand centre in the gas network has its own unique demand profile and the model was run for both the summer and winter seasons. The highly seasonal demand at the Melbourne demand node was found to be the most influential on the direction and volume of gas flow through the system. In the summer season capacity constraints in three of the four pipelines in Queensland are evident. The transmission pipes supplying Mt.Isa and Gladstone reach their capacity in years one and two in the model, and the Roma-Brisbane pipeline which supplies gas to Brisbane was shown to become constrained in year five. The model clearly identified that expansion of these pipelines should be a priority. In the winter season the increase in demand from Melbourne sees the system at risk of supply shortages earlier in the study, in addition the shortages are more widespread. The fact that each demand centre in the network, other than those in Queensland, has two or more connecting pipelines allows the large amount of excess capacity available from the Longford supply node in the summer to be utilised effectively to minimise supply shortfalls. However in the winter the greater demand from Melbourne brings
forward the existence of supply shortages by a full decade. It is clear from this study that the excess production capacity from Longford is not utilised optimally. This excess production capacity could be used to generate electricity in the summer when Melbourne experiences spikes in electricity demand. A study of the nature of gas demand from electricity generators and the effects on the supply network would be of interest and could be considered as a logical extension of this model. This would require a dynamic approach to system constraints as electricity generators consume large amounts of gas in short time periods which can deplete the amount of gas in the pipeline and lower the pressure to critical levels.

The second model formulated minimises the cost of supplying gas to consumers. The model includes both the cost of production and the cost of transportation, which together constitute the cost of supply. Much of the research on gas transportation focuses solely on transportation costs and neglects the cost of production. However, transportation costs are largely a factor of the distance covered and are a small part of the price paid by consumers. In contrast, the cost of production is a larger component in the price paid by consumers and is highly variable across gas basins, and even across fields within the same basin. Gas from conventional basins are typically larger and have longer production profiles than gas sourced from coal seam gas fields. Conventional gas fields off-shore present different costs to those located on-shore. Natural gas is often found with heavier hydrocarbons such as propane or butane, and with liquid hydrocarbons like crude oil, which may increase the value of exploiting certain fields at the expense of others. Therefore the variability of production costs is of interest to owners of transmission pipelines, who may see the volume transported through their pipes decline if production costs rise and large gas users find alternative sources of supply. It is also of interest to industry that may be appraising different regions for the construction of new plants, factories, or gas powered electricity generators. This thesis provides a sensitivity analysis for the cost of production in eastern Australia and highlights the changes in system dynamics bought about by the changes in production costs.

This model further emphasises the importance of the Longford production node. Longford has the largest production capacity and lowest production costs. The cost of production at Longford would have to rise significantly before it becomes cheaper to source gas from other sources in both the summer and winter seasons. In particular, the large demand for gas in the winter season sees Longford production costs having to rise higher in winter than in summer before Longford gas is displaced as the lowest supplier. The model also
confirms that the production plant at Moomba is a key piece of infrastructure. Moomba sits at the junction of the Queensland network and the network in the southern states. There are large reserves of gas in Queensland and large investment in developing LNG export capacity. However, it is a point of debate as to whether there are sufficient reserves to supply all of the exports contract entered into. It is also a point of debate as to the timing of exploitation of the gas reserves in Queensland, which typically have shorter production lives than those in the Gippsland basin around Longford and in the Cooper basin around Moomba. The fact that Moomba is a low cost supplier, that there are significant reserves of gas nearby, and the geographical location of the plant makes Moomba an important source of supply. A study of the nature of gas reserves around Moomba, with a focus on production costs, and the effects of increases of the capacity of connecting infrastructure would be a logical extension of the models presented in this thesis. Modelling the effect of gas exports through Gladstone on the eastern Australian gas network is a topic for future research with regard to the direction and volume of gas flow and the price paid for natural gas by consumers.

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