LATE-TIME EFFECTS IN AIRBORNE EM:

INDUCED POLARISATION,

SUPERPARAMAGNETISM,

AND ROTATION

A thesis submitted in fulfilment of the requirements for the degree

of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Terence/Paul Kratzer

25 September 2013
Abstract

The aim of this thesis is to characterise and correct for three major late-time effects that are becoming increasingly important in airborne EM (electromagnetic) systems: induced polarisation, superparamagnetism, and rotation noise. The primary importance in characterising these effects is to improve the performance of AEM systems at late delay times. This aim is becoming increasingly important as near surface deposits are exhausted and the need for successful exploration to greater depths and in difficult environments mounts.

I identify the three late delay time effects from literature review. The approach to characterisation of each of these effects is similar: model the underlying phenomenon; fitting of the models to the data, thus obtaining deconvolved signals for each of the fitted models; subtraction of the unwanted signal(s) to leave pure AEM responses; and investigation of the coherence of the predicted SPM and AIP (Airborne IP) source parameters.

I model Inductive IP (IIP) using a combination of Warburg and exponential decay models as a basis for fitting electromagnetic data from ground TEM and airborne VTEM surveys. Observed decays are deconvolved into EM and IP constituents by constrained least squares fitting of basis functions, modified to account for transmitter waveforms. The method has been confirmed through synthetic modelling of 2D and 3D structures, and when applied to ground TEM or airborne TEM
data, obtains an estimate of apparent chargeability at each station or fiducial.

I use a $1/t$ decay, a time-dependance associated with magnetic viscosity, to model and fit SPM effects in AEM data. I identify the presence of SPM effects, as distinct from the decay of good conductors, by using this model as additional basis functions in constrained least-squares fitting.

I use the output of tri-axial rotation-rate sensors as a basis of a model for rotation noise, to predict and subtract the rotation noise from rigidly-coupled ARMIT magnetic field sensors.

In order to test out the usefulness of both the IP characterisation and rotation correction, I then use data from a low base frequency airborne IP and EM survey, which employed a low-noise B-field sensor coupled with rotation rate sensors. I then calculate chargeability over a synthetic IP source.

In the course of characterising these effects for better low-frequency performance, I make useful progress towards airborne induced polarisation. This includes characterisation of IP effects in an existing $\frac{dB}{dt}$ airborne EM system. In the case of a VTEM survey in Africa, the apparent chargeabilities mapped graphitic sediments and provided spatially consistent indications of clay concentrations.

I distinguish SPM effects in airborne electromagnetic survey data from the response of good conductors. Application of the method to airborne TEM (time-domain electromagnetic) surveys shows that the method allows correction of SPM and hence aids significantly in conductive target identification.

The approach towards rotation noise removal is successful in reducing rotation noise by one to two orders of magnitude at low frequencies. The survey over a synthetic IP source is easily able to determine location thanks to rotation correction, and the use of a B-field sensor is significantly more useful than a $\frac{dB}{dt}$ system, due to the low frequency response of such a sensor.
I also find that as these three effects (IP, SPM, and rotation noise) are all late-time, they often impact on each other. Specifically, I find that correction for rotation noise is important for accurate characterisation of induced polarisation, and is likely to be important for improved characterisation of superparamagnetism. Additionally, further investigation indicated that in some cases SPM and IP could be equal and opposite, either reducing the amplitude of the larger effect, or even resulting in a negligible net effect. This cancellation effect is also likely to be reduced with the improved late-time sensitivity that effective rotation correction brings.
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Chapter 1

Introduction

The airborne electromagnetic (AEM) method is a technique in common use in mineral exploration (Fountain, 1998). Ground-based EM methods, using low base frequencies and late times, in comparison to AEM have significantly more depth penetration, particularly in areas with a conductive overburden, it is not severely affected by rotation (other than wind noise), and there are data acquisition strategies to minimise the two other main late-time non-EM effects of superparamagnetism (SPM) and induced polarisation (IP) (Lee, 1984; Flis et al., 1989).

However there are many disadvantages to ground-based exploration: it is time-consuming and expensive, and it can be difficult or even impossible due to dense vegetation or extreme terrain. This leaves many areas unavailable to geophysical exploration.

Recently, helicopter-based Time-domain EM (TEM) has made significant improvements in internal signal-to-noise (S/N) ratios (Smith and Annan, 1997), which has had the dual effect of increasing the signals due to the fundamental inductive response, as well as the undesirable effect of increasing external noise. One way of viewing these effects is as a collection of insurmountable external sources of noise.
that form a noise floor, below which useful signal cannot be extracted, however noise from one point of view is signal from another.

Although there are many external sources of noise, including cultural noise and sferics, in this thesis I look specifically at the low-frequency (or equivalently, late-time) noise effects of rotation noise, SPM and IP effects (Kratzer and Macnae, 2012b, 2014; Kratzer et al., 2013b). I will characterise these noise sources, and in some cases view them no longer as noise but as signals from which useful information can be extracted. I will discuss this in more detail in later chapters.

In order to both better characterise these late-time effects (whether regarded as signal, unwanted signal, or noise), we require system sensitivity in lower frequencies. This is also necessary if we are to use AEM where only ground-based EM was previously useful.

I start by summarising the underlying physical principles of EM, AEM and SPM.

1.1 Classical Electrodynamics

The basis of classical electrodynamics is expressed conceptually as follows:

1. The electric flux through any closed surface is proportional to the enclosed electric charge;

2. Magnetic monopoles (i.e. “magnetic charges”) do not exist;

3. A magnetic field that changes with time produces an electric field; and

4. An electric current produces a magnetic field.
CHAPTER 1. INTRODUCTION

(a) Faraday’s Law - as per Equation 1.1.3.  (b) Ampere’s Law - as per Equation 1.1.4.

Figure 1.1.1: Graphical representation of two of Maxwell’s Equations (also known as Faraday’s and Ampere’s Laws respectively). From Grant and West (1965).

More precisely, these principles can be expressed mathematically, as Maxwell’s Equations:

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad (1.1.1) \]

\[ \nabla \cdot \mathbf{B} = 0 \quad (1.1.2) \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1.1.3) \]

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad (1.1.4) \]

where \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) is the magnetic field, \( \mathbf{J} \) is the current density, \( \rho \) is the charge density, \( \varepsilon_0 \) is the permittivity of free space, and \( t \) is time. Note also that \( \nabla \cdot \mathbf{A} \) represents the divergence of \( \mathbf{A} \), and \( \nabla \times \mathbf{A} \) the curl of \( \mathbf{A} \); later we will also use \( \nabla \mathbf{A} \) to denote the gradient of \( \mathbf{A} \), and \( \nabla \cdot \nabla \mathbf{A} = \nabla^2 \mathbf{A} \) to denote the Laplacian, or the divergence of the gradient of \( \mathbf{A} \).

Figure 1.1.1 shows the graphical representation of Equations 1.1.3 and 1.1.4.
1.1.1 Quasi-static approximation

For low frequencies, we can generally ignore wave propagation effects: the dis-
tance between transmitter and receiver circuits are generally small enough that delay
times due to propagation at the speed of light can be ignored. Using \( \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \) (here the magnetisation field \( \mathbf{M} = 0 \)), assuming no charges (\( \rho = 0 \)), and utilising the
vector identity

\[
\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla \cdot \nabla \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \tag{1.1.5}
\]

we can re-write the four Maxwell Equations as follows:

\[
\nabla^2 \mathbf{E} - \sigma \mu \frac{\partial \mathbf{E}}{\partial t} - \varepsilon \mu \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \tag{1.1.6}
\]

\[
\nabla^2 \mathbf{H} - \sigma \mu \frac{\partial \mathbf{H}}{\partial t} - \varepsilon \mu \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \tag{1.1.7}
\]

If we assume that \( \mathbf{E} \) and \( \mathbf{H} \) have a time dependance of the form

\[
\mathbf{H}(\mathbf{r}, t) = \text{Re}\mathbf{H}(\mathbf{r}, \omega)e^{i\omega t} \tag{1.1.8}
\]

\[
\nabla^2 \frac{\mathbf{E}}{\mathbf{H}} = i\sigma \mu \omega \frac{\mathbf{E}}{\mathbf{H}} - \varepsilon \mu \omega^2 \frac{\mathbf{E}}{\mathbf{H}}, \tag{1.1.9}
\]

where \( \omega = 2\pi f \) is angular frequency. In the air, which we can approximate as a vacuum (i.e. \( \mu = \mu_0 \), \( \varepsilon = \varepsilon_0 \), and \( \sigma = 0 \)), equation 1.1.9 simplifies to

\[
\nabla^2 \frac{\mathbf{E}}{\mathbf{H}} = 0 \tag{1.1.10}
\]
Typical values for rocks and minerals for permittivity are $\varepsilon \approx 9\varepsilon_0 \approx 8 \times 10^{-11} \text{ F/m}$, so that even for high values of $\omega$, such as 1 kHz and zones of low conductivity such as rocks of $\sigma = 10^{-3} \text{ S/m}$, the product $\varepsilon\mu\omega^2$ for the real term of equation 1.1.9 is much smaller than the product $\sigma\mu\omega$ for the imaginary term, we can use

$$\nabla^2 \frac{E}{H} = i\sigma\mu\omega \frac{E}{H} \quad (1.1.11)$$

which we can write as

$$\nabla^2 \frac{E}{H} = \sigma\mu \frac{\partial}{\partial t} \frac{E}{H} \quad (1.1.12)$$

We can gain a general understanding of the solutions to Equation 1.1.12 by assuming a magnetic field that propagates in the $x$-direction, is polarised in the $y$-direction, and varies sinusoidally with time. The solutions of Equation 1.1.12 then become (Grant and West, 1965):

$$H_y(x,t) = H_0 e^{i\omega x \pm x \frac{\sigma\mu\omega}{2} \sqrt{i\sigma\mu\omega}} \quad (1.1.13)$$

$$J_z(x,t) = \sigma E H_{\theta}(x,t) = J_0 e^{i\omega x \pm x \frac{\sigma\mu\omega}{2} \sqrt{i\sigma\mu\omega}}, \quad (1.1.14)$$

where $J_0 = H_0 \sqrt{i\sigma\mu\omega}$.

If we keep only the terms which remain finite as $x \to \infty$, and separate out the real and imaginary terms, we obtain:

$$H_y = H_0 e^{-x \sqrt{\frac{\sigma\mu\omega}{2}}} e^{i\left(\omega x - \frac{\sigma\mu\omega}{2}\right)}, \quad (1.1.15)$$

and

$$J_z = \sqrt{j\sigma\mu\omega} H_0 e^{-x \sqrt{\frac{\sigma\mu\omega}{2}}} e^{i\left(\omega x - \frac{\sigma\mu\omega}{2}\right)}. \quad (1.1.16)$$
Looking at the magnetic field, if we take the real part only we obtain:

$$H_y(x,t) = H_0 \cos\left(\omega t - x\sqrt{\frac{\sigma\mu\omega}{\rho}}\right)e^{-x\sqrt{\frac{\sigma\mu\omega}{2\rho}}}.$$  \hfill (1.1.17)

Figure 1.1.2 shows the diffusive behaviour of a magnetic field for various values of $\sigma\mu\omega$. We can see that for small values of $\sigma\mu\omega$, the magnetic field is barely attenuated by the conductor. In this case the currents are distributed throughout the conductor, while the current intensity is very small. When $\sigma\mu\omega$ is very large, the magnetic field will barely penetrate the conductor before significant attenuation occurs, and the induced currents (also known as ‘eddy currents’) responsible for the attenuation will be strong, and concentrated near the surface.

### 1.2 The EM method

The general idea behind the EM (electromagnetic) method is to generate a magnetic field (known as the primary field) near the earth’s surface, which, as described above, will induce eddy currents into any conductors nearby in the earth. These eddy currents in turn will produce a secondary magnetic field, which, being characteristic of the size, conductivity and distance of the conductor, can be recorded and
used to make an estimation of the conductive structure of the survey area.

There are two main methods of EM: Time Domain, or Transient, EM (TEM), and Frequency Domain EM (FEM). The FEM method will be discussed only briefly here, as this thesis is focused primarily on TEM.

In both FEM and TEM systems, the primary field is often generated using a wire loop or coil, producing a dipole magnetic field, and the secondary field is usually detected also using a wire coil, although more complex receiver systems, such as B-field receivers, are becoming more common, as will be discussed. A common ground-based EM configuration is the Slingram setup, which involves horizontal co-planar coils.

The FEM method generally uses a constant wave sinusoidal source current. The apparent resistivity of the surveyed area can be calculated using the quadrature component. The ratio of primary (or source) magnetic field to secondary magnetic field measured in the receiver can be used to calculate apparent resistivity. The correct method of calculation of apparent resistivity varies depending on the configuration of the system - for example, horizontal dipole; vertical dipole; line source; et cetera. The distance between the source and receiver determines the depth of investigation (Grant and West, 1965).

1.2.1 Time Domain EM

One disadvantage with the FEM method is that the secondary field, which is caused by the currents induced into any underground conductors, is several orders of magnitude smaller than the primary field, and so separation of the primary and secondary fields is difficult. TEM commonly (but not always) uses a primary field that has periods of ‘on-time’ and periods of ‘off-time’, and secondary field measurements are generally made during the off-time period, without the influence of the primary
field. A common waveform is an alternating square wave with 50% duty cycle (abbreviated “half-step”, see Figure 1.2.1). Base frequencies (or repetition frequencies - the number of full cycles per second) for individual airborne systems typically range from 25 Hz to 200 Hz, but may be as low as 0.125 Hz or as high as several kHz in ground systems.

For a 50% duty cycle system, after the primary current has been on a finite time, the primary current is switched off. This sudden change in the primary field will produce eddy currents in the underground conductor, in accordance with Faraday’s Law of Induction (Equation 1.1.3), and these eddy currents will act to oppose the change in magnetic flux in accordance with Lenz’s Law, producing currents on the surface of the conductor to do so.

Ohmic losses will cause the surface currents to dissipate as heat, and so the region beneath the surface will see a decreasing magnetic field. Eddy currents will also flow to oppose this change in magnetic field. As this is repeated the effective distribution of currents is an inward diffusion with time (although the currents themselves generally do not flow inwards).

**Smoke-rings** The large-scale nature of the currents was first described by Nabighian (1979), and are distributed as a ‘smoke-ring’ image of the transmitter loop that
Figure 1.2.2: EM coupling. $I_0$ is the current in the primary loop, $I_1$ the current induced in the conductor; $e_s$ and $e_p$ the EMF induced in the receiver loop by the primary and conductor respectively, $L$ and $R$ are the inductance and resistance of the conductor loop, and $M_{ij}$ is coupling coefficient between element $i$ and $j$. From Nabighian and Macnae (1991).

moves down from the surface and expands with time (see Figure 1.2.3). The smoke-ring of currents can be represented as a single loop of current, which moves downward with velocity

$$v = \frac{2}{\sqrt{\pi \sigma \mu t}}$$

(1.2.1)

and increases diameter as

$$\alpha = \sqrt{\frac{2t}{\sigma \mu}}$$

(1.2.2)

The equivalent current filament moves downward at an angle of $47^\circ$, whereas the actual volume of induced currents move downwards at $30^\circ$.

The vertical component of the magnetic field $b_z$ due to the smoke-ring in a halfspace (i.e. a 2-dimensional space, consisting of a vacuum in the upper half, and a conducting space in the lower), can be shown to asymptotically approach a time dependance of $b_z(t) \propto t^{-1.5}$, and for a conducting thin sheet, $b_z(t) \propto t^{-3}$ (Nabighian, 1979).
Conducting Loop  The EMF $\varepsilon$ in circuit $j$ due to current $I$ in circuit $i$ is given by Faraday’s Law (Equation 1.1.3):

$$
\varepsilon_j = -M_{ij} \frac{dI_i}{dt}
$$

(1.2.3)

where $M_{ij}$ is the coupling coefficient between circuits $i$ and $j$ (see Figure 1.2.2).

If the current in the transmitter is $I_0 u(t)$, where $u(t)$ is the step function, and the current in the conductor is $I(t)$, then the EMF induced in the conductor is given by:

$$
\varepsilon_1(t) = -M_{01} \frac{d}{dt} [I_0 u(t)]
$$

(1.2.4)

we must then add the EMF due to resistance and back-EMF in the loop:

$$
\varepsilon_1^\dagger(t) = -RI(t) - L \frac{d}{dt} [I(t)]
$$

(1.2.5)

These potentials must both sum to zero, giving us:

$$
- \left( L \frac{d}{dt} + R \right) I(t) = M_{01} I_0 \frac{d}{dt} [u(t)]
$$

(1.2.6)

Making the approximation that $I(\tau) \approx -\frac{M_{01} I_0}{L} \frac{d}{dt} [u(t)]$, and $\tau \ll \frac{L}{R}$, we can solve this
for $I(t)$ and obtain (Grant and West, 1965):

$$I(t) \approx -\frac{M_{01}I_0}{L}e^{-R/L} \quad (1.2.7)$$

We can also obtain the EMFs induced in the receiver coil by the transmitter and the buried circuit:

$$e_2^{(P)} = -M_{02} \frac{d}{dt} [I_0 u(t)] = -M_{02}I_0 \delta(t), \quad (1.2.8)$$

and

$$e_2^{(S)} = -M_{12} \frac{d}{dt} I(t) = \frac{M_{01}M_{12}}{L} I_0 \left[ \delta(t) - \frac{R}{L} e^{-R/L} \right] \quad (1.2.9)$$

where $\delta(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\tau} & 0 \leq t \leq \tau \\ 0 & t > \tau \end{cases}$

Figure 1.2.4 shows the form of the current, fields and EMF in the transmitter and receiver.

Equation 1.2.9 therefore shows us that for a particular wire loop, the secondary magnetic field will decay exponentially, with a time-constant that depends on the conductivity and the size of the loop, and with an amplitude that depends on the depth and the size. If we generalise the secondary magnetic field from any conductor to be that of the aggregate field of a large number of wire loops, with different sizes and conductivities, we have an effective way of characterising an isolated conductor at depth. We will use this technique in Chapter 2.
Figure 1.2.4: A visual description of the fields and currents involved in time-domain EM. From Grant and West (1965).
1.2.1.1 Airborne EM

As discussed above, the benefits of Airborne EM are that a larger area can be surveyed in much less time, areas that are inaccessible to ground based survey due to dense vegetation or steep terrain are surveyable, and difficult ground conditions are removed as a problem.

Airborne EM is similar in concept to ground based EM, however as signal falls off with $1/r^3$ (for $dB/dt$ systems), larger transmitter coils and more sensitive receiver coils are required due to the increased distance from the earth.

Airborne platforms can either be helicopter or fixed-wing aircraft. Figure 1.2.5 shows a photo of VTEM, a helicopter TEM system which is the system used to acquire data which we utilise in Chapters 2 and 3. Configuration varies significantly between fixed wing and various helicopter-towed systems: VTEM is a concentric loop setup, meaning the receiver is concentric to the transmitter. Other configurations include a trailing receiver with transmitter mounted on a fixed-wing platform; both the receiver and transmitter mounted on the aircraft; or even a two-aircraft system, in which one aircraft carries the transmitter and one the receiver. Passive systems are also in use - ZTEM uses sferics as a source and therefore carries only a receiver but requires a ground reference. In this thesis however I will use the term EM to refer only to controlled source EM systems.

1.3 Late-time effects in TEM: noise and signal

There are many unwanted signal sources that we regard as noise in EM systems, including cultural noise, such as power lines, VLF signals, and train lines; sferics; and system internal noise such as sensor noise, data acquisition noise, and power supply noise.
Additional to these noise sources are other non-EM effects, some of which could be considered to be signal depending on the context of the survey. In this thesis, I look at non-EM effects, specifically late-time effects, that can affect the signal received, and which have the potential to disrupt an EM survey. I will then identify and test strategies to characterise and hence reduce the undesirable effects caused by these late-time non-EM signals.

It is important to note that late-time (or equivalently, low-frequency) effects in EM systems are becoming increasingly significant. A recent (in geological terms) historical trend of increasing commodity prices has led to economic feasibility for mining deeper and more difficult mineral deposits, the discovery of which has driven demand for more sensitive and suitable geophysical instrumentation. Various external noise sources and limitations are dominant at higher frequencies, such as sferics and conductive overburden response (Buselli et al., 1998). In order to continue to increase sensitivity, it becomes necessary to work at lower frequencies, which are better able to penetrate conductive overburden, and which are less susceptible to interference. This work requires instrumentation better suited to lower frequencies, for example, B-field receivers such as the ARMIT sensor (Macnae, 2012a).

Other sources of interference exist at these lower frequencies, however. Argu-
ably the three most important low frequency (late-time) effects are rotation noise, induced polarisation, and superparamagnetism, which are the three effects I will discuss in this thesis. As we will see, all three are related, in that one can often affect another, and as such, in some cases, must be considered together.

1.3.1 Rotation Noise

There are many different types of magnetic field receiver (or magnetometer) that can be used in geophysics. Every magnetometer can be broadly classified as either scalar or vector: either they measure only the total strength of the field, or they measure the strength of the field in one direction only.

Generally in the EM method, vector magnetometers are used, because the additional information provided by separated magnetic field components allows more constraints to be placed when modelling the range of structures that could produce the secondary field.

When a vector magnetic field sensor rotates within a magnetic field, the coupling of the sensor to the magnetic field necessarily changes. This produces rotation noise: noise induced into receiver coils due to the rotation of the receiver coil within the earth’s magnetic field. It is often difficult to distinguish from a signal of interest, and therefore is not easily filtered out using signal processing techniques. There are various techniques to do so, which will be covered in more detail in Chapter 4. Prior to the research detailed in Chapter 4, rotation noise suppression was only significantly effective above 25 Hz. For this reason, the lowest base frequency usefully used in a commercial Airborne EM system was 25 Hz (Vrbancich et al., 2005a), limiting efficacy for deep surveys and beneath conductive cover.
1.3.1.1 Rotation noise in Airborne IP

Rotation noise is a major impediment towards Airborne IP. This is because in order to accurately characterise the IP signal, long off-times are required due to the slow rate of IP decay. As expected from ground IP experience (Sumner, 1976), the fundamental induction response (EM decay) dominates the signal in the early times, but drops off rapidly in the later times, leaving the IP signal. Therefore in order to accurately determine chargeability, low base frequencies are required. Ground-based IP systems generally use base periods of between 2 to 16 seconds, but can be as much as 40 seconds (Seigel et al., 2007).

1.3.2 Induced Polarisation

There are two main mechanisms that cause induced polarisation: electrode polarisation, and membrane polarisation. They generally occur in metallic minerals and clays respectively. Induced polarisation was first detected as an electrochemical effect, described by Conrad Schlumberger in 1920 in his article Étude sur la prospection électrique du sous-sol. It is a technique most useful for detection of very small amounts of metallic minerals, in some cases as low as 0.5% by volume Sumner (1976), where other techniques such as DC resistivity or EM would fail to show any contrast. In other cases such as disseminated mineralisation, poor electrical connectivity may result in insignificant conductivity contrast, but a good IP signature may be present. Another advantage of IP is the low background response - other techniques rely on the contrast of physical properties, however the presence alone of IP is sufficient to define an anomaly.

IP is therefore mainly utilised in mapping disseminated sulphides bodies, Carlin-type gold deposits, etcetera. It has also been used, through measuring membrane polarisation, with clay deposits; dirty sands; oil-bearing sands (Schmutz et al., 2010);
CHAPTER 1. INTRODUCTION

landfill, waste disposal monitoring and contamination detection (Aristodemou and Thomas-Betts, 2000); and groundwater (Vacquier et al., 1957).

1.3.2.1 Electrode polarisation

Electrode polarisation is caused when small electronically conducting particles are distributed in an electrolytically conducting host, such as occurs in disseminated mineralisation. When an electric field is present, the ions in the electrolyte will migrate and collect on the surface of the electronic conductors. The electrons within the metallic conductors will then mirror these surface charges, forming a double layer. If neither ions in the electrolyte nor electrons in the metal can cross the interface and pass into the other medium, direct current cannot flow and no charge transfer is possible, and the interface is “perfectly polarisable”, and appears electrically like a capacitor.

If, however, the electrolyte contains active ions, some will undergo electrochemical charge transfer reactions. At some location on the metal surface a cation accepts an electron from the metal, and at another location, a metallic atom loses an electron, oxidising to the ionic form and goes into the electrolyte. When the interface is subjected to perturbing electric fields, the net current density crossing this interface is no longer zero. In this case, the system will be imperfectly polarisable (Wong, 1979).

1.3.2.2 Membrane Polarisation

Almost all rock minerals absorb a net negative charge when in contact with normal pore fluids. Membrane polarisation occurs when this negative charge build-up occurs on the surface of clay particles, or the edges of layered or fibrous materials or
cleavage faces, attracting a diffuse cloud of mobile positive ions in the surrounding electrolytic conducting fluid or medium, and repelling negative ions (see Figure 1.3.1). If passageways between clay sheets (or pore paths) are small, the randomly distributed cations in the electrolyte can pass easily through this cationic cloud but the anions (which are generally physically larger than cations) will be blocked, and when an electric field is present, causing ions to migrate, this will cause a polarising effect (Madden and Cantwell, 1967; Sumner, 1976).
1.3.2.3 Galvanic IP

Undertaking a galvanic source induced polarisation survey is conducted using source electrodes are inserted into the ground carrying the primary excitation current, and additional measurement electrodes are used to measure the resulting potential difference. Two of the most common electrode configurations are the dipole-dipole array (generally used for profile surveys - see Figure 1.3.3a), and the Wenner array (for sounding surveys - see Figure 1.3.3b), but any number of configurations is possible. In time-domain IP surveys, the source current is often the half-step commonly used in EM surveys (Figure 1.2.1), however with a much lower base frequency: cycle times are generally from 2 to 16 seconds, as induced polarisation decays have a much slower rate of change than the fundamental induction response. This will be covered in more detail in Chapter 2.

1.3.3 Superparamagnetic Effects

1.3.3.1 Diamagnetism

A diamagnetic substance has a negative magnetic susceptibility $\chi$ (or equivalently, has a permeability $\mu$ less than that of free space ($\mu_0 = 4\pi \times 10^{-7} H/m$), because $\mu = \mu_0(1 + \chi)$). This means that an external magnetic field $H$ induces a magnetisation
CHAPTER 1. INTRODUCTION

Figure 1.3.4: Model for electrode Induced Polarisation mechanism

M which is in the opposite direction to H. This is caused by the orbital motion of the negatively charged electrons. Diamagnetism is a characteristic of all materials, however it will be the dominant form of magnetism only in those materials that have completely filled electron shells, because in such materials the electron spins are in up/down pairs, and thus have zero total dipole moment. The most common diamagnetic materials (those in which diamagnetism is dominant) in geophysics are graphite, gypsum, marble, quartz and salt (Telford et al., 1976).

1.3.3.2 Paramagnetism

In a material in which the sub-shells are not completely filled, an external magnetic field tends to cause the magnetic moments of the unpaired electrons to align in the same direction as the field, reinforcing it. Once the field is removed, the magnetic moments once again become randomly orientated due to thermal agitation, and the material loses magnetisation. This is known as paramagnetism, and these materials have a positive magnetic susceptibility: the induced magnetic field will be in the same direction as the external magnetic field.
1.3.3.3 Ferromagnetism, ferrimagnetism and antiferromagnetism

A ferromagnet also has unpaired electrons, however, in addition to the paramagnetic effect, there is a tendency for magnetic moments to align parallel with each other, and in fact this state is a lower energy state, so that once the external magnetic field has been removed, the magnetic moments stay parallel. The three ferromagnets; iron, cobalt and nickel, have susceptibilities $10^6$ times that of diamagnetic and paramagnetic materials (Telford et al., 1976), so it is a much greater effect. As temperature increases, thermal motion increases and therefore ferromagnetism decreases, until at the Curie temperature, where it disappears completely, and the material becomes paramagnetic.

Ferrimagnetism is similar to ferromagnetism, however ferrimagnetic materials have magnetic domains divided into regions which may be aligned in opposition to each other, and yet retain a net magnetic moment when the external field is zero. This is possible when either one set of sub-domains has a stronger magnetic alignment than the other, or when there are more of one sub-domain than the other. Examples are magnetite, titanomagnetite, ilmenite, iron oxides, iron and titanium oxides, and pyrrhotite.

If the antiparallel sub-domains do cancel out, so that the net moment is zero, the material is known as antiferromagnetic (Stacey, 1963); an example of this is hematite.

1.3.3.4 Superparamagnetism

When a ferromagnetic or ferrimagnetic particle becomes small enough, so that they become single-domain, the direction of magnetisation can flip randomly between two anti-parallel directions, below the materials Curie temperature. The mean time between flips is known as the Néel relaxation time.
When the magnetic field of these particles is measured over a time much longer than the Néel relaxation time, in the absence of an external magnetic field, their magnetisation averages zero. An applied external magnetic field will magnetise the particles similarly to a paramagnet, however with much larger susceptibility. Once the external field is removed, the bulk magnetisation will relax exponentially back to zero, with a time-constant dependant upon the shape, volume, type of material, and temperature.

This effect can be a problem in EM exploration as a large population of super-paramagnetic particles with a spectrum of relaxation appears as a decay signal with a time dependence $1/t$ - as will be discussed in Chapter 3, this signal can appear to be the magnetic field due to currents induced within a large buried conductor.
Chapter 2

Induced Polarisation

The majority of the content in this chapter has been published (Kratzer and Macnae, 2012b,a), and therefore the Society of Exploration Geophysicists (SEG) owns the copyright. It is reproduced here in compliance with the SEG license agreement. I have chosen to use the term “we” from the published paper to describe the work undertaken, reflecting the fact that I undertook the work with input from my supervisor.

2.1 Introduction

Induced polarisation (IP) is an important method of geophysical exploration for disseminated sulphide mineralisation for which no airborne methodology exists (Thomson et al., 2007). Conventional sources for IP signals are galvanic currents injected into the ground, with the secondary fields detected with grounded electrodes measuring voltage (conventional IP) (Oldenburg and Li, 1994) or magnetometers measuring the magnetic field of primary and secondary currents (MIP) (Chen and Oldenburg, 2006). As well as galvanic sources, IP effects have been detected using inductive sources. It has been observed (Lee, 1981; Flores and Peralta-Ortega, 33
2009) for example that in a time domain EM (electromagnetic) system with coincident transmitter and receiver loop, inductive induced polarisation (IIP) effects may be visible as negative transients. Weidelt (1982) has further shown that, for this coincident loop case, negative transients cannot come from induction in a purely conductive earth. However, the lack of negative transients does not preclude the presence of IP, as the IP effect takes finite time to build up and fall off; it therefore may not be evident in measured delay times. In addition, a longer fundamental inductive response, as would be found over conductive ground, may completely obscure any IIP response (Smith and Klein, 1996).

Nonetheless, the concept of Airborne IP is extremely attractive, as it would allow large areas to be surveyed more efficiently than is possible for ground methods. Some research has been undertaken using a grounded transmitter with an airborne magnetic receiver (airborne MIP), but this has not led to any commercial systems (Thomson et al., 2007). The effects of IP have been occasionally observed in Airborne TEM (time-domain EM) data, as reported by Smith and Klein (1996), in permafrost affected ground, but no commercial services are known to exist that extract IP parameters from AEM (airborne EM) data. We discuss methodology in this paper to achieve limited IP detection and characterisation in TEM and airborne TEM data.

2.1.1 Background

The IP phenomena for mineral exploration has its basis in electrochemistry - more specifically, the energy storage mechanisms that occur at any metallic-electrolytic conductor interface. The effect on electrical impedance is dispersive, and in the simplest cases involves proportionality to the inverse square root of frequency. This is known as a Warburg impedance, and it has been discussed by a number of authors;

Wong (1979) combined electrochemical and electrical potential theory to derive a model for the IP effect in disseminated mineralisation. He found that his model behaved very much like the Warburg model when it was dominated by electrochemical diffusive processes (e.g. sulphide source IP), and like the Debye (or exponential decay - the fundamental electromagnetic inductive response) when there were no electrochemical reactions. In the time domain, following turn-off of a current that has been flowing for a long time, these models are:

\[
\frac{V_S(t)}{V_P} = \frac{\rho_L - \rho_H}{\rho_L} e^{-t/\tau} u(t),
\]

(2.1.1)

for the Debye decay (also incidentally the fundamental inductive response), and

\[
\frac{V_S(t)}{V_P} = \frac{\rho_L - \rho_H}{\rho_L} e^{j/\tau} \text{erfc} \left( \sqrt{\frac{1}{\tau}} \right) u(t),
\]

(2.1.2)

for the Warburg (or IP) decay, where \( V_S \) and \( V_P \) are the secondary and primary voltages, \( \rho_L \) and \( \rho_H \) are the low- and high-frequency asymptotes of resistivity, \( t \) is time, \( \tau \) is the time constant, \( \text{erfc} \) is the complementary error function, and \( u(t) \) is the step function. Wong (1979) showed that where mineralisation had a distribution of grain sizes, a distribution of Warburg time constants would result and be required to fit observed decays.

A common empirical formulation for IP in the literature is the Cole-Cole equation for impedance \( Z \), expressed in the frequency \( (f) \) domain (Cole and Cole, 1941):

\[
Z(\omega) = R_0 \left[ 1 - m \left( \frac{1}{1 + (i\omega\tau)^\epsilon} \right) \right],
\]

(2.1.3)

where \( m \) is the chargeability, \( \tau \) is the time constant, \( \omega = 2\pi f \), and \( \epsilon \) is the fre-
quency dependence. When $c = 1$, equation 2.1.3 can be represented in the time
domain as Equation 2.1.1 (the Debye decay), and when $c = 0.5$, equation 2.1.3 can
be represented in the time domain as equation 2.1.2 (the Warburg decay).

Distributed time constants have been used extensively for electromagnetic wave-
form decomposition. Stolz and Macnae (1998) derived an expression for the re-
response of an isolated conductor to an alternating repetitive square wave sampled
by finite window widths, for the fundamental inductive response, and used this ex-
pression as a basis for decomposition of the response into the time constant, or $\tau$,
domain. We will extend this approach to transform data containing both fundament
inductive EM response and IP effects into the $\tau$-domain.

2.2 Method

2.2.1 Basis Functions

The current in an isolated body, after excitation, decays with the nature of the decay
characterised by a distribution of time constants (West and Macnae, 1991). Associ-
ated EM responses exhibit this same set of decays, occasionally with complications
from changing geometrical coupling if currents migrate. The response $A_{EM}(t, T)$
of any extended or local conductor to a primary field waveform consisting of a
repeated, alternating square-wave with 50% duty cycle (abbreviated 'half-step’) of
period $T$, can be represented as a finite sum using the approach of Stolz and Macnae
(1998) to be:

$$A_{EM}(t, T) = \sum_{m} A_{m} \left( e^{-t/\tau_{m}} \left( 1 - e^{-T/2\tau_{m}} + e^{-T/\tau_{m}} + e^{-3T/2\tau_{m}} - \cdots \right) \right), \quad (2.2.1)$$
where there is an amplitude $A_m$ associated with each $\tau_m$ and which, after analytical infinite summing using the Taylor series, becomes

$$A_{EM}(t, T) = \sum_m A_m \left( e^{-t/\tau_m} \frac{1 - e^{-T/2\tau_m}}{1 + e^{-T/\tau_m}} \right). \quad (2.2.2)$$

From there we can continue as per Stolz and Macnae (1998) to obtain the off-time response in finite sampling windows $t_k$ to $t_{k+1}$:

$$A_{EM}(t_k, t_{k+1}, T) = \sum_m \frac{A_m \tau_m}{t_{k+1} - t_k} \left( 1 - e^{-T/2\tau_m} \right) \left( e^{-t_k/\tau_m} - e^{-t_{k+1}/\tau_m} \right). \quad (2.2.3)$$

The basis functions for the off-time EM part of our decay with half-step excitation are constructed from equation 2.2.3, and these are shown in Figure 2.2.1 for the case where $T = 40$ ms and each $A_m = 1 \mu$V.

The IP basis functions can be constructed in a similar manner. The effects of a half-step waveform when convolved with equation 2.1.2 produces the off-time IP response:

$$B(t, T) = \sum_p \left( e^{x_0,p} \text{erfc}(x_{0,p}) - e^{x_1,p} \text{erfc}(x_{1,p}) + e^{x_2,p} \text{erfc}(x_{2,p}) + e^{x_3,p} \text{erfc}(x_{3,p}) - e^{x_4,p} \text{erfc}(x_{4,p}) - \cdots \right), \quad (2.2.4)$$

which can be written as:

$$B(t, T) = \sum_p \left[ e^{x_{0,p}} \text{erfc}(x_{0,p}) + \sum_{i=1}^{\infty} \left( (-1)^i e^{x_{(2i-1),p}} \text{erfc}(x_{(2i-1),p}) + (-1)^{i+1} e^{x_{(2i),p}} \text{erfc}(x_{(2i),p}) \right) \right], \quad (2.2.5)$$

where $x_{i,p} = \sqrt{\frac{t+iT/4}{\tau_p}}$. 
Figure 2.2.1: EM basis functions for a 25 Hz half-step. Time constants are $\tau = 10^{-6}$ to $10^{-1}$ seconds in 21 steps. The sensitivity cutoff at 10% is shown as the dashed line. Only time constants between $10^{-5}$ s and $10^{-3}$ s are above the 10% cutoff, with peak sensitivity at approximately $\tau = 5.6 \times 10^{-5}$ s.

We assert without proof that, as first implied by Wong (1979) and exactly analogous to the fact that (Stolz and Macnae, 1998) any time-domain EM response can be fit by a weighted sum of exponential decays, any simple time-domain IP response can be fit by a weighted sum of Warburg decays. Equation 2.2.5 is thus an electrochemically and physically consistent expression that Wong showed in his paper can fit observed sulphide IP responses, and to our view should fundamentally be preferred to the common empirical single or dual Cole-Cole parameter description introduced by Pelton et al. (1978). Wong (1979) also described how the Warburg spectrum could be approximated using a sum of several exponential decays. In order not to destabilise our solution by allowing opposite exponential decay basis functions with positive (EM) and negative (IP) coefficients, we ensured that EM data was fitted using a carefully selected range of exponential decay time-constants, and negative IP responses were fitted using Warburg decays. This will be discussed in more detail in the following sections.
CHAPTER 2.  INDUCED POLARISATION

The time-domain IP response for finite width sampling windows extending from $t_k$ to $t_{k+1}$ is given by:

$$C(t_k, t_{k+1}, T) = \frac{1}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} B(t, T) \, dt.$$  \hfill (2.2.6)

Because of the form of $B(t, T)$, this is not easily integrated, but since $B(t, T)$ is a slowly varying function we can safely approximate it linearly, to get the IP response to a half-step function for finite width sampling windows:

$$C(t_k, t_{k+1}, T) = \frac{1}{2} [B(t_k, T) + B(t_{k+1}, T)].$$  \hfill (2.2.7)

The normalised IP basis functions are shown in Figure 2.2.2. However these basis functions represent the IP or MIP response to half-step function primary current (from say galvanic injection), not the IP response caused by the induced secondary currents in the ground. We will first analyse the case where the ground EM decay can be approximated with a decay of single time-constant, such as the response of a small, isolated target.

If $I_0(t) = I_0 u(t)$ is the primary current, and $I_1(t)$ is the induced current with exponential decay, its approximate amplitude is given by (Fitterman and Labson, 2005):

$$I_1(t) = \frac{M_{01} I_0}{L_1} e^{-\gamma/2} u(t),$$  \hfill (2.2.8)

where $M_{01}$ is the mutual inductance between the transmitter and the target, and $L_1$ is the self-inductance of the target. This exponentially decaying current $I_1(t)$ rather than $I_0(t)$ becomes the primary current for the inductive IP (or IIP) effect, in an isolated target. Furthermore, an inductive coil receiver measures not the mag-
netic field $B$ of currents in the ground, but rather their time-derivative $\frac{dB}{dt}$.

The IP response to a half-step function is given by equation 2.2.7, but for a coil receiver we require the time-derivative of the response to $I_1(t)$. To get this, we will convolve $I_1(t)$ with the IP response to an impulse excitation - equivalent to the time-derivative of the IP response to step function excitation.

When performing convolution in linear systems where a time-derivative is involved, it does not matter which function we differentiate, or

$$A_{IP}(t) = I_1(t) \ast \frac{d}{dt}C(t) = \frac{d}{dt}I_1(t) \ast C(t).$$

(2.2.9)

In this case, $M_{12} \frac{d}{dt}I_1(t)$ is the signal we would receive in our detection coil (Fitterman and Labson, 2005):

$$M_{12} \frac{d}{dt}I_1(t) = e_s(t) = -\frac{d\Phi_{12}}{dt} = M_{12} \left( \frac{1}{\tau}I_1(t) - \frac{M_{01}I_0}{L_1}\delta(t) \right) = \frac{M_{01}M_{12}}{L_1}I_0 \left[ \frac{e^{-t/\tau}u(t)}{\tau} - \delta(t) \right].$$

(2.2.10)

Therefore, we can then convolve equation 2.2.10 with equation 2.1.2 to obtain the inductive IP response of an isolated target.

Although $e_s(t)$ is the signal received in the receiver coil, when using equation 2.2.5 to fit real data, we have had to numerically reconstruct $e_s(t)$, as there are important characteristics, such as the large negative impulse, that are not received in data measured only in the off-time.

For our EM basis functions, we will set:
Figure 2.2.2: Warburg response to a 25 Hz half-step function excitation. Time constants are $\tau = 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ and $10^{-5}$ seconds. Shown in the dashed line is the sensitivity cutoff of 10%, at the time of 1 ms delay, when negative values from the IP response often become evident; $\tau = 1$ s and $\tau = 10^{-5}$ s (the fastest and slowest time constants) are both below this cutoff. Peak sensitivity is at approximately $\tau = 10$ ms, at delay time $t = 1$ ms.

The negative impulse function $\delta(t)$ in $e_\tau^s(t)$ therefore has area

$$\delta(t)dt = \frac{-M_0 I_0}{L_1},$$

and the initial value at $t = 0$ of $e_\tau^s(t)$ is

$$e_\tau^s(0) = \frac{M_0 M_{12} I_0}{L_1 \tau}.$$

As measurement of this initial value during the on-time to off-time transition is difficult, it can instead be estimated by backwards extrapolation (Macnae and Baron-Hay, 2010).

For an isolated target, we can define the chargeability $m$ as the ratio of the IP to EM starting amplitudes. This definition is consistent with the usual frequency
Figure 2.2.3: Example of the convolution of an observed and extrapolated EM decay with synthetic IP responses. IP time constants are the same as those used in Figure 2.2.2. In this case, the peak sensitivity is at $\tau = 1$ ms (at delay time $t = 1$ ms).

Our targets are likely to be complex and located in a conductive host, or under cover. We therefore only obtain an apparent chargeability $m_a$ from the ratio of the inductive limit EM response to the starting IP response, and therefore we need to ensure that the IP basis functions also hold equation 2.2.11 to be true. Our received response $e_s^2(t)$ includes factor $M_{12}$, the coupling coefficient between the target and the receiver loop – and as $M_{12}$ is unknown when we reconstruct $\frac{d}{dt}I_1(t)$, we need to remove $M_{12}$ as a variable by normalising $\frac{d}{dt}I_1(t)$ to have unit area.

To illustrate the results of this process on real data, we took a typical airborne electromagnetic decay from a VTEM survey to be described later in the paper, from which we backwards extrapolated (Macnae and Baron-Hay, 2010) to predict the response at zero delay time under a half-step assumption, and then convolved the resulting reconstructed response with a range of synthetic IP decays (as described in equation 2.1.2). The set of IIP basis functions that would be excited by the sample VTEM decay are shown in Figure 2.2.3. Different EM decays would lead to different IP basis functions.


2.2.2 Time constant sensitivity

The signal magnitude from either an EM or IP decay is dependent on the time constant of the decay. This is evident in Figure 2.2.1; the decays with very slow time constants have low initial amplitudes because our primary current is reversed every half-period, and therefore the average effect for a slow decay is very small. The decays with very fast time constants also have small initial amplitudes, because by the time we start sampling the decay is almost over. This means that we have a range and peak sensitivity, outside of which any fitting will be of lower quality. We have chosen for illustration a cutoff of 10% of the peak sensitivity - any decays with time constants below this cutoff will not be as well resolved as those of larger amplitude, and when removed from our basis functions help to increase process stability. This also brings us back to the findings of Wong (1979); that a Warburg decay could be fitted using the sum of several exponential decays. We ultimately use constraints to ensure that positive EM decays are not used to fit negative IP decays, hence giving “false negatives” (indicating a lack of chargeability where there should be a chargeable response).

Note that for the EM decays, amplitudes are largest for the earliest samples, but for the IP decays we are most interested in fitting data for delay times at which negatives begin to appear. This for example is approximately 1 ms for the results combining an observed EM decay with synthetic IP responses in Figure 2.2.3.

Our IP basis functions are constructed in part from the EM decay observed at any particular point - every set of IP basis functions is different. This means that our sensitivity range may vary from one point to the next. For example, the Warburg decays in Figure 2.2.2 have peak sensitivity at (IP time constant) $\tau_{IP} = 10$ ms, but
when we look at the actual basis functions for point 1 of the VTEM survey, the
peak sensitivity is at $\tau = 1$ ms. While not quantitatively modelled here, we can
state that it is necessary for EM effects to have largely decayed before IP effects
can be detected. This implies that in conductive ground, a low base frequency for
the AEM system is required in order to allow EM effects to decay before the later
time-channels are sampled. At present, AEM systems are limited to about 25 Hz
base frequencies, as motion noise (Buselli et al., 1998) overwhelms any signal in
later delay time channels at lower base frequencies.

### 2.2.3 Constrained Least-squares decomposition

We employed the approach of Stolz and Macnae (1998) to quantitatively fit the EM
components of an observed response, with the addition of IP basis functions to fit
the IP components of the observed response. This approach is outlined here - for
full details of EM fitting the reader should refer to Stolz and Macnae (1998).

We construct a basis function matrix (equation 2.2.14), containing EM (Debye)
decays with $m$ time constants $\tau_m$, IP (Warburg) decays with $p$ time constants $\tau_p$,
including a set of smoothing constraints:

$$
\begin{pmatrix}
R_1 \\
R_2 \\
\vdots \\
R_n
\end{pmatrix}
= 
\begin{pmatrix}
A_{n_i}^{\text{CAP}} & A_{n_1\tau_1}^{\text{EM}} & \cdots & A_{n_{1m}}^{\text{EM}} & A_{n_1\tau_1}^{\text{IP}} & \cdots & A_{n_{1p}}^{\text{IP}} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -\lambda & \lambda & \cdots & 0
\end{pmatrix}
\begin{pmatrix}
ad_{\tau_1}^{\text{CAP}} \\
ad_{\tau_1}^{\text{EM}} \\
\vdots \\
ad_{\tau_p}^{\text{IP}}
\end{pmatrix}
$$

(2.2.14)
The smoothing factor $\lambda$ for the EM part of the basis function is needed for stabilisation as discussed by (Macnae and Baron-Hay, 2010) and found numerically by the Prediction Error Sum of Squares (PRESS) method, which involves fitting an average decay to the EM basis functions, and sequentially removing each data point to determine how well the removed point is predicted by the model - this is sequentially solved for a range of $\lambda$ values. In the case of VTEM, a negative EM decay $A^{CAP}(t)$ with time constant $\tau_{cap} = 25 \mu s$ (Macnae and Baron-Hay, 2010) is required to fit parasitic capacitance effects.

As evident from Figure 2.2.3, at later delay times (when the Warburg decay starts to dominate the response), variation of time constant does not significantly affect the shape of the decay - hence resolution in the IP time-constant domain is poor. For this reason, we found it to be unnecessary to include more than one or two Warburg basis functions, or to include any regularisation between them.

Before the least squares fitting process, we made a number of pre-processing steps to equation 2.2.14 in order to improve stability. Firstly, we can estimate channel noise levels from EM data using a number of methods, we used the n’th difference method (Macnae, 2011) for airborne data. In numerical fitting, we then inversely weight the basis functions $A(t, \tau)$ at each point with the predicted or measured noise level for that channel, as well as the corresponding channel in the observed data $R(t)$. The weighting accentuates any IP activity within the decay, because the later channels have less noise (because they average over a longer duration) and therefore have higher weighting. Secondly, we normalise the amplitudes of the basis functions $A(t, \tau)$ so that the first channel of every basis function has unit amplitude for further improvement of stability in the least squares process. De-normalisation of $a(\tau)$ is therefore necessary following least-squares fitting to allow calculation of the chargeability. The general procedures of weighting and normalisation are described by Macnae et al. (2010).
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We then use QR decomposition (Lawson and Hanson, 1974), as implemented in the MATLAB optimisation toolbox, to obtain a least-squares solution to the problem, with non-negative constraints to the solution. We found it important to check the resulting amplitudes, as inclusion of unresolved faster decays (e.g. unrealistically small time constants) can destabilise the results, leading to unrealistically large amplitudes (up to 20 orders of magnitude higher than the other amplitudes).

2.3 Results

2.3.1 Synthetic Modelling of Airborne IP responses

We first successfully tested our proposed methodology using two different airborne EM forward modelling programs that allowed for IP properties to be specified, specifically ArjunAir and AMGPlate. We then tested the method on two data sets where we believed that IIP effects were present: A ground TEM survey from Mexico and an airborne VTEM survey from Africa. This paper reports on the results of these first two tests of our apparent chargeability calculation method.

2.3.1.1 Polarisable Quarterspace

We constructed a number of models and ran numerical simulations for an airborne half-step system using program ArjunAir, to verify the method described above. The basic model consisted of two quarterspaces discretised over a volume 1 km deep and 3 km wide. The western half of the model is non-chargeable, with a resistivity of 100 $\Omega\cdot$m, and the eastern half is chargeable. The simulation was run with differing IP parameters for the chargeable half.

Normally distributed noise is proportional to the square root of the sample window duration, however we found that the ideal weighting (based on ensuring suf-
ficient amplitude for the later delay times) for this model was instead to use the square of the sample window duration. This has the effect of emphasising the importance of the IP results to the least squares solution, at the expense of fitting the EM solution. This observation may be important in other datasets, depending on the magnitude of the negatives observed.

The system modelled was an airborne transmitter with an in-loop receiver, base frequency for the simulated survey was 4 Hz, and it was 'flown' at 30 m. Instead of a distribution of Warburg impedances with differing $\tau$ as we have used, ArjunAir describes IP in terms of equivalent Cole-Cole parameters $m$, a time constant $\tau$ and a frequency dependence $c$. If $c = 0.5$, then the Cole-Cole formulation reduces to a Warburg model, the case $c = 0.2$ to 0.3 corresponds to a range of $\tau$’s spread over several orders of magnitude. Figure 2.3.1a presents a synthetic model profile as calculated by ArjunAir for the case $m = 0.6$, $\tau = 1$ ms, $c = 0.5$. Each synthetic decay was fitted with equation 2.2.5 to predict the IP parameters. The results of the simulations, converted back to apparent chargeability using the method described above, is shown in Figure 2.3.1b. Cases with $m = 0.6$, $c = 0.3$ or 0.5 (which are typical values for real world minerals (Pelton et al., 1978)) and $\tau$’s between 0.3 ms and 3 ms produced reasonable estimates of chargeability $m$ over the polarisable quarterspace.

For the model with chargeability $m = 0.2$, the IP effect produced was insufficient to allow the least squares to fit the IP basis functions to the data. The long time constant model ($\tau = 3$ ms) had predicted chargeability approximately half the correct chargeability. Additionally, the discontinuity between the two quarterspaces cause significant disturbance due to the AEM system having non-linear lateral sensitivity to structures within a few hundred metres. Overall no model has false positives for
(a) Response for the ArjunAir polarisable quarterspace basic model, with $m = 0.6$, $\tau = 1$ ms, $c = 0.5$. Curves show the response at increasing delay times from the top, with the earliest channel (20 $\mu$s) at the top (largest response signal), and the last channel (52.9 ms) at the bottom.

(b) Results of ArjunAir modelling. The results for the basic model with $m = 0.6$, $\tau = 1$ ms, $c = 0.5$ is in blue, with variations and the corresponding varied parameter shown in red, green, teal and magenta.

*Figure 2.3.1: ArjunAir modelling.*
the presence of IP (except near the interface of the two quarterspaces), and three of the five models predicted chargeability reasonably accurately. This model suggests that a conventional airborne TEM system is sensitive to IP effects when strongly polarisable material is located near-surface, and when the host is resistive enough.

2.3.1.2 Polarisable Plate in host and under cover

We obtained a number of model responses for a polarisable plate in a half-space, simulated by program AMGplate (Hanneson, 1992). The AEM system simulated was a coincident-loop helicopter TEM system with transmitter and receiver located 40 m above the ground. Models consisted of a horizontal thin (1 m thick) plate, of strike length 400 m and along-line extent 200 m at a depth 100 m below the surface, located in a 0.005 S/m host. Three series of models were used: polarisable plate in a non-polarisable halfspace (models a – d), polarisable plate in a slightly polarisable halfspace (models e – h), and polarisable plate in host with a thin 5 S surface conductor (models i – l). Table 2.1 details the variable parameters for each model. In all cases, IP was approximated in the forward models with Cole-Cole parameters with $c = 0.3$. Two example calculated response profiles are presented in Figures 2.3.3 and 2.3.2a.

A summary set of profiles of the apparent chargeability needed to fit the data is presented in Figure 2.3.2b, and show realistic apparent chargeability predictions for models a – h. Apparent chargeabilities are lower than the intrinsic chargeabilities due to the attenuating effect of the conductive material above the plate(s). A significant observation is that IP time constants $\tau = 0.1$ ms lead to apparent chargeabilities smaller than estimated when the IP time constant is around $\tau = 1$ ms. This
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(a) Response for the polarsable plate synthetic model c.

(b) Summary profiles of apparent chargeability.

Figure 2.3.2: AMGplate modelling.
Table 2.1: AMGplate models. Host conductivity is $\sigma_H = 0.005 \text{ S/m}$. All chargeable volumes have frequency constant $c = 0.3$. The plate has conductance $\sigma_P = 10 \text{ S}$. The overburden cover model (i – l) has a conductive layer of thickness 5 m, with $\sigma_L = 1 \text{ S/m}$, and $m = 0.1, \tau = 1 \text{ ms}$.

<table>
<thead>
<tr>
<th>Model</th>
<th>Plate $m$</th>
<th>Plate $\tau$ (ms)</th>
<th>Host $m$</th>
<th>Host $\tau$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.05</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>0.05</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>0.5</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>0.5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>f</td>
<td>0.05</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>g</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>h</td>
<td>0.5</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>i</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>j</td>
<td>0.05</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>k</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>l</td>
<td>0.5</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

The conductive cover model has significantly reduced our predicted apparent chargeability, which is expected, again due to attenuation from the conductive cover. Although the algorithm is useful for this scenario, in that IP effects are apparent, a much lower base frequency of the EM system would be needed to get sufficient penetration of the 5 S of conductive cover simulated here, which lowering of base frequency is not currently possible in commercial AEM systems as previously discussed.

Also worth mentioning is that only the models c, d, g and h have negatives in the data; interestingly models a, b, e and f show apparent chargeability with reasonable accuracy (the apparent value half the true value) without the presence of any negat-
Figure 2.3.3: Response for the AMGplate polarisable plate synthetic model a. Note the lack of any negative response over the polarisable plate, which does not appear to hinder least squares fitting (see Figure 2.3.2b).

...ive data samples in the profiles (e.g. Figure 2.3.3). We thus have confidence that the basis function method will successfully detect polarisable material in many cases even when negatives are not present in the data, but where observed decays become “too fast” to be consistent with the EM effects observed in earlier time channels.

2.3.2 Ground data: the El Arco porphyry copper deposit

Flores and Peralta-Ortega (2009) have reported mineral discrimination using IP extracted from ground-based TEM soundings over the El Arco deposit in Mexico, inverting the data to derive the parameters of the Cole-Cole model using an iterative least-squares approach. The authors kindly supplied us with the TEM data for this survey. We ran this data through our fitting process, and the fits and apparent chargeabilities are shown in Figure 2.3.4. Generally, we were able to obtain a almost perfect fit to the data decays as can be seen on the decay plots. Our fitted chargeabilities are compared to other geophysical results plotted on top of a schematic drill-section of the El Arco deposit from a study by Garcia (1978) (Figure 2.3.5). Our highest apparent chargeability by far occurs at the location where the copper...
Figure 2.3.4: Decay (Voltage, dots) and fits (lines) as a function of delay time for the El Arco deposit (30 Hz), with associated chargeabilities m. Data locations are points 1 to 14, top to bottom, left to right.
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Figure 2.3.5: Copper ore section and associated IP, resistivity, magnetic and gravity profiles for the El Arco orebody, with chargeability calculated as described in text, using TEM data from Flores and Peralta-Ortega (2009)) (original geophysical data and figure from Garcia (1978)). Calculated chargeabilities are shown as ◊’s.

We found that to obtain reasonable consistency in the fitted results, it was important to use the same smoothing constant (λ) for every point. For El Arco, we initially estimated λ for each point, using the PRESS method, then used the median of all λ’s to include as the smoothing constraint in equation 2.2.14. The apparent chargeabilities we calculated are spatially consistent with mineralisation and the amount of cover, but significantly smaller (approximately an order of magnitude) than the chargeabilities predicted from inversion by Flores and Peralta-Ortega (2009), which are generally around 0.2 to 0.4. This we attribute to attenuation of the apparent IP response by fairly thick conductive cover. At this stage of our research, we have not developed any methodology to predict intrinsic chargeabilities through for example inversion: rather we are investigating methodology for stable extraction of an apparent chargeability from TEM data.
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2.3.3 Airborne VTEM Survey Example

2.3.3.1 Survey Area

We used data from a VTEM survey over a shallowly-weathered area in Africa with superficial clays (mostly kaolinite) located in the valley floors. Many clays are known to have a significant IP response (Vacquier et al., 1957). The VTEM data was processed using program EMFLow (Macnae et al., 1998) to produce conductivity-depth-sections, which all showed that conductivity was concentrated in the near surface in this area. Figure 2.3.6 presents a map of the cumulative conductance in the top 10 m of the survey area. As well as weakly conductive zones within the lower-lying ground, the most significant conductor in the survey area is a unit with a south-east strike that is located to the east. This conductor consists of graphitic sediments, which rocks contain mixed electronic and ionic conductors and hence would be likely to exhibit polarisation.

In conductivity-depth sections derived from the AEM survey, all conductors were near-surface. Before IIP detection is attempted, data are first corrected for
altitude variation of the AEM system, using the inverse-cube relationship of Green (1998). The airborne data for this survey has had DC offset (levelling) applied by the contractor based on forcing the last channel to approximately zero - a significant problem for our method of fitting where the last channel would be negative if IP were present. To get around this we added a small negative DC offset to the entire dataset, and appended a negative DC ’constant’ in the basis function matrix. This approach effectively unconstrains the DC level (contaminated by levelling procedures) and produced good results and good fits to much of the data - Figure 2.3.7 shows two examples of data and the corresponding fits - one at location A (a location of negligible chargeability) and one at location B (a location of substantial apparent chargeability - \( m = 0.67 \)) (locations A and B are marked in Figures 2.3.6 and 2.3.8). Figure 2.3.8 shows the apparent chargeability map for the VTEM survey predicted after altitude correction and fitting. Of note is the lack of chargeable ground in vicinity of the high ground to the north-east and south-west, which is consistent with the expectation that transported clays would be in lower ground.

The spatial consistency of mapped apparent chargeability is good, and the magnitude of chargeability is generally consistent with that expected of graphitic deposits (in the conductor to the east), and for clays (the smaller responses elsewhere). There are some isolated predictions of apparent chargeability along the graphitic zone that are greater than one, which is probably due to problems in the survey data at longer delay times due to DC offset issues or motion noise.

2.3.3.2 Multispectral Data

We made attempts towards verification of the mapped apparent chargeability distribution as being spatially consistent with clays through the use of data from the
Figure 2.3.7: Decay and corresponding fits as a function of delay time for two sample decays obtained during the VTEM survey. Location A is in a region of low chargeability, while location B is in a region of substantial chargeability. Locations A and B are marked in both Figures 2.3.6 and 2.3.8.

Figure 2.3.8: Apparent chargeability map for the VTEM survey. The graphites appear chargeable as expected. Lower chargeabilities show spatial consistence with thicker clays towards the west.
Japanese Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) multispectral sensor. The technique described by Crosta et al. (2003), involving the use of Principal Components Analysis (PCA) of ASTER multispectral data, was utilised, to produce spatial distribution patterns optimised for Kaolinite mapping, which is the main clay mineral present at the survey area. The mineral electromagnetic spectrum of Kaolinite (see Figure 2.3.9) has two characteristic features, one each at band 6 (reflective) and band 7 (absorptive) of the ASTER sensor (Yamaguchi et al., 1998).

A PCA using Bands 1 to 9 of the ASTER multispectral data of the survey area shows that Principal Component 8 (PC8) has the highest contrast between Band 6 (which was strongly positive) and Band 7 (strongly negative). This indicates that spatial distribution of PC8 will show areas of Kaolinite as low values (see Figure 2.3.10).

Although there is some correlation with apparent chargeability (Figure 2.3.8), overall it is low. ASTER only sees surface substances however, whereas IIP is only
likely to detect clays with substantial thickness. At this point, we would suggest that the methodology we have presented is encouraging, and warrants further study and testing on additional data sets.

2.4 Discussion

In the decomposition of observed transients with N samples into m constituent EM and p constituent IP basis functions several choices may be made. Simplistically, we could in theory set the solution up as an over-determined problem provided m+p < N. In practice, at some stations fewer than N data are above the noise level, and we may make poor choices of basis function time constants, so that in general we need to allow for the solution to an under-determined problem, with the number of unknowns (m+p basis function amplitudes) exceeding the number of useful data.

The use of MATLAB lsqnonneg algorithm based on the simplex method enables an “over-determined” solution to an under-determined problem without the need for regularisation, through successive selection of only an optimum few basis functions needed to fit the data to an acceptable level. The imposition of smoothness con-
straints between adjacent time-constants is an alternative approach to the solution of underdetermined problems, with the tradeoff between data fit and smoothness constraints controlled by a “Tikhonov” regularisation parameter (Zhdanov, 2002).

The question arises into choice of IP basis functions: do we allow for an extensive set of generic Cole-Cole models with non-linear variables $m$, $\tau$, $c$, or should we choose a limited set. Effersø (2000) suggests that at least 20 data (over 3 decades in frequency) are required to predict $m$ and $c$ of a Cole-Cole model within 25% with typical noise in conventional IP, and that with these 20 data, $\tau$ is only predictable within say an 80% error. With the few data points above noise available after the airborne EM decays in our case, we have chosen to fix $c$ and $\tau$ at typical values, and solve for $m$. We see no reason to assume that a range of $\tau$’s need to be fitted to data given their limited resolvability, and have chosen to use the $c$ value of 0.5, the Warburg decay, indicative of metallic IP targets with uniform grain size. Typical values for clay are $c = 0.75$ (Leroy and Revil, 2009), while graphite was quoted by Pelton et al. (1978) as having frequency dependence $c$ in the 0.3 range. With the few late time data of small amplitude responding to IP effects, we do not believe that the VTEM data here have enough data above the noise level to hope to discriminate $c$ or $\tau$. We have therefore focussed on the “detection” of IP, rather than attempting its spectral characterisation.

### 2.5 Conclusion

We have used Warburg basis functions, modified by adaptive convolution with secondary field EM waveforms, to fit both simulated and real IP decays in ground and airborne geometries. Good fits to modelled and observed have been obtained in all cases. Resulting chargeabilities are mostly accurate (for the simulated data), and plausible (for ground and airborne field data), and spatial distribution of chargeab-
ility for the airborne field data is good. The method requires further optimisation in terms of accounting for conductive cover, which attenuates the IP response and therefore produces a lower apparent chargeability. Additionally, noise in the later delay times is significant for airborne data (later than approximately 2 ms), and this probably contributes significantly to reducing the accuracy of extracting the IP response.
Chapter 3

Superparamagnetism

The majority of the content in this chapter has been published (Kratzer et al., 2013b,a). As in the previous chapter, I have maintained the usage of “we” as published, rather than claiming all the work as my own.

3.1 Introduction and Background

SPM effects in AEM surveys can be a source of needless expense in exploration if not identified, due to unnecessary drilling or further exploration. AEM surveys are commonly used for conductor mapping. Figure 3.1.1 shows for example the EMFlow apparent conductance (surface to 600 m depth) for a VTEM survey in the Mwese area in Africa. There are many high conductance anomalies on the map that would appear at first glance to be suitable drilling candidates in this dataset. With recent reductions in AEM noise levels, SPM effects (explained in the next section) have become an issue in low-amplitude, late-delay time data, and many anomalies have been drilled based on mistaken interpretation (Mutton, 2012). The challenge lies in determining which of these anomalies indicate basement conductor responses, and which indicate SPM responses.
3.1.1 SPM Theory

Superparamagnetism (SPM) is an effect exhibited by certain fine-grained magnetic minerals, that on application of an external magnetic field, gradually align atomic spins to develop a net magnetic field. The acquired magnetic field then decays when the external magnetic field is removed. Time constants of interest to paleomagnetists vary from microseconds to billions of years (Dunlop and Özdemir, 2001). SPM particles with millisecond range time constants affect TEM systems and develop through weathering of ferrous rocks in the normal regolith development process (Barsukov and Fainberg, 2001), and when these particles are present at surface they are easily redistributed though physical transport processes and may be widespread or at times be concentrated in paleochannels. When the paramagnetic materials are at surface, the external field from the transmitter is large and hence the SPM effect will be prominent.

SPM is evident in single-domain ferromagnetic particles. Depending on the
material, ferromagnetic particles become single-domain at room-temperature when their radius is $10^{-9}$ to $10^{-7}$ m (Barsukov and Fainberg, 2001). The SPM particle usually has two stable directions of magnetic moment which are antiparallel. The direction of the magnetic field can spontaneously flip under thermal agitation, with the mean time between changes given by the Néel relaxation time $\tau_N$:

$$\tau_N = \frac{1}{f_0} e^\frac{K_u v}{kT},$$

(3.1.1)

where $K_u$ is a factor dependent on the anisotropy (or shape) of the particle, $v$ is the volume of the particle, $k$ is the Boltzmann constant, $T$ is the temperature, and $f_0$ is the frequency factor, which is characteristic of the material, and is typically $10^9$ Hz (Barsukov and Fainberg, 2001). The product $K_u v$ is thus the energy barrier associated with switching between the two stable directions of magnetic moment.

Under zero external magnetic field in a shielded chamber, thermal agitation can raise the energy of the particle above the energy barrier, and in a large ensemble of SPM particles, the wide distribution of orientations of the SPM particles leads to a net magnetic field of zero (Stacey, 1963). In the Earth’s field, some SPM particles acquire a parallel magnetic field leading to a net magnetic moment, however this signal will generally be too small to detect in AEM systems. Magnetic fields from larger maghemite particles are often present as magnetic noise (Stanley et al., 1992).

In soil containing SPM particles, there is a wide range of domain shapes and sizes, leading to a spectrum of relaxation times $\tau_N$. Billings et al. (2003) show that if the external primary field $H$ is a step function, then for a distribution of particles, where each specific $\tau_N$ has equal probability over a wide range, the resulting magnetisation (assuming zero equilibrium magnetisation) of the SPM particle $M$ is given by
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\[ M(t) = H\chi_0 F(t), \quad (3.1.2) \]

where \( \chi_0 \) is the DC susceptibility, and the after-effect function is given by \( F(t) = e^{-t/\tau} \) for a single relaxation time, and for a spectrum of relaxation times \( f(\tau) \), for \( \tau \) between zero and infinity.

\[ F(t) = \int_0^\infty f(\tau)e^{-t/\tau}d\tau. \quad (3.1.3) \]

The magnitude of \( F(t) \) is dependent upon the concentration and distribution of particles that we choose, however the time dependence is proportional to \( \log(e^t) \) and therefore for a \( \partial B/\partial t \) receiver commonly used in AEM systems, the SPM signal is proportional to \( t^{-1} \).

SPM is often a problem in mineral exploration because unrecognised it can result in incorrect apparent resistivities (Buselli, 1982). These anomalous apparent resistivities often appear on conductivity depth images (CDI) (Macnae and Lamontagne, 1987) to indicate deep strong conductors. This is because although one exponential decay term will provide a poor fit to SPM decay (see Figure 3.1.2), a range of exponential decay terms used to fit the response of an isolated body (West and Macnae, 1991; Stolz and Macnae, 1998) can also be used to fit the \( t^{-1} \) decay of an ensemble of SPM particles (see Figure 3.1.3). Typically the range of time constants required to fit this pseudo-conductor consist of very long time constants, which are normally characteristic of good conductors.

One issue that has prevented simple identification of SPM effects is that \( t^{-1} \) decays may numerically provide a good fit to a conductor under regolith that has a decay that is large in amplitude in the early to middle time data (Figure 3.1.4(b)). Further, inductive IP effects which produce negatives at late time distort isolated
fitting of a $t^{-1}$ response. Additionally, the late time signal decay from conductors with long time constants is significantly affected and poorly characterised if the energising field frequency is not low enough (Watts, 1997). With AEM systems limited to 20-25 Hz, decays from slow time constant conductors and SPM may be poorly characterised.

Our solution to this is to provide the fitting algorithm with an SPM basis function that can be used to fit an SPM decay if it is present. The aim of this research is to reduce the occurrence of conductor false positives, as well as improving target prioritisation.

### 3.2 Method

Previous work has described basis function decomposition of AEM data (Macnae et al., 1998) for EM response characterisation. This approach has recently been extended to airborne IP effect detection through least-squares fitting of both EM and IP basis functions as described in Kratzer and Macnae (2012b). This last paper provides a comprehensive description of basis function fitting methodology, which we will briefly summarise below (see equation 3.2.2). To fit for SPM effects we add a single basis function representing an inverse delay-time:

$$A^{SPM}(t_k, t_{k+1}) = \frac{1}{2(t_k + t_{k+1})}, \quad (3.2.1)$$

where $t_k$ and $t_{k+1}$ are the start and end of the sample windows with respect to the primary field turn-off. We then construct our least squares problem:
Figure 3.1.2: Single exponential fit to example VTEM field decays. Both data have regolith conductor present.
Figure 3.1.3: Range of exponentials fit to VTEM field decays. Data is the same as that used in Figure 3.1.2.
Figure 3.1.4: $t^{-1}$ fit to VTEM field decays. Data is the same as that used in Figures 3.1.2 and 3.1.3.
\[ \mathbf{R} = \begin{bmatrix} A^{EM} & A^{IP} & A^{SPM} \\ \Lambda & 0 & 0 \end{bmatrix} \begin{bmatrix} a^{EM} \\ a^{IP} \\ a^{SPM} \end{bmatrix} \] (3.2.2)

where \( R \) is our time-series data, \( A^{EM} \) are our exponential EM basis functions, \( A^{IP} \) our IP basis functions, \( A^{SPM} \) is our SPM basis function from equation 3.2.1, \( \Lambda \) is the smoothing parameter bidiagonal matrix of \(-\lambda\) and \( \lambda\), and \( a^{EM} \), \( a^{IP} \) and \( a^{SPM} \) are the EM, IP and SPM amplitudes.

AIP effects in AEM were accurately modelled with AIP as well as AEM basis functions as described by Kratzer and Macnae (2012b). In the current work, we found that a number of modifications were necessary to the AIP fitting process, in order to reliably detect SPM. For example, we found that if any significant weight was given to minimising errors in fitting the early channels (< 0.5 ms delay) in VTEM, the SPM basis function was not used, and so these early channels were not used in the processing.

We have found that normalisation of the EM decay basis functions is necessary to prevent the large dynamic range of data leading to unstable fitting. This process may have the effect of allowing unstable fits of the very long and very short decays by the fitting algorithm. In order to reduce the incentive of the least-squares fitting algorithm to fit very long EM time constants to SPM signals, we also normalise the EM smoothing matrix \( \Lambda \) (defined above) (along with the EM basis functions).

The most inconvenient but necessary modification was that we found that the smoothing parameter \( \lambda \) (of the smoothing matrix \( \Lambda \)) could not be determined analytically. This has been found previously by a number of authors in many different fields of research, and it is generally accepted that this must be done experimentally. It should be noted that unlike our IP decay fitting (Kratzer and Macnae, 2012b), the
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SPM substances that respond most strongly to a magnetic field are on the surface. We can therefore assume excitation by the primary field only.

3.2.1 Finding the smoothing parameter

We used a synthetic AMGplate model (Hanneson, 1992) with a polarisable plate target. Model d from Kratzer and Macnae (2012b) was employed, which consisted of a horizontal thin (1-m thick) chargeable plate, of strike length 400 m and along-line extent 200 m at a depth of 100 m below the surface, with chargeability \( m = 0.5 \), IP time constant \( \tau = 1 \), Cole-Cole frequency constant \( c = 0.3 \) (Cole and Cole, 1941), and DC conductance 10 S, located in a host with conductance 0.005 S/m. We added artificial SPM decay to the last five stations in the model, in the form of Equation 3.2.1 (and with SPM amplitude varying spatially in an approximate cosine manner to emulate a basement conductor response). We then fitted the model using the set up described in equation 3.2.2, using 200 different values of \( \lambda \) spread logarithmically between 0.01 and 10. Twenty-four EM time constants, from 0.04 ms to 200 ms were used. Figure 3.2.1 shows an example of the data, corresponding fit, and the residual of the fit, for point +300m of the model (where synthetic SPM has been added) for \( \lambda = 0.02 \). Figure 3.2.2 is a ’stacked events’ plot of the results, where more intense colours indicate that where more of the 200 fits have determined a particular value of either chargeability (red) or SPM (blue). It shows how there is a strong likelihood of a chargeable body (red) in the centre of the model, with possible chargeabilities varying from 0.2 to 0.8 in the centre. The black bar indicates the actual location of the plate with true chargeability of 0.5.

The fitting also correctly identifies the synthetic SPM data added to the model (blue), varying in dimensionless amplitude from 0 to 0.5. No false positive SPM indications are present in the fitting, however some fits indicate false negatives.
Thus Figure 3.2.2 gives us a good indication of what to expect when IP or SPM effects are present in synthetic data.

In order to conduct a sensible and efficient fitting of field data, we need to pick one value for \( \lambda \). Figure 3.2.3 shows the model response in \( \lambda \)-space, for both IP and SPM. From these figures, we can pick an appropriate compromise for \( \lambda \) as will be discussed.

We know from Figure 3.2.2 that it is very likely there is a chargeable region in the centre of the model, and an SPM region to the right, and unlikely that there are SPM or IP regions elsewhere. Figure 3.2.3 shows that a larger value for \( \lambda \) indicates SPM to the right of the model which we know to be likely, however this value of \( \lambda \) also indicates chargeability of almost 100\% for areas which we also know are unlikely to contain any chargeable material. Therefore the most appropriate value for \( \lambda \) seems to be approximately 0.02, where there is a chargeable section in the middle of approximately 0.5, and an SPM section to the right of approximately 0.5. The results for \( \lambda = 0.02 \) is shown on Figure 3.2.2 as black circles (IP) and black triangles (SPM). It is also worth noting that based on this selection criteria,
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Figure 3.2.2: Predicted chargeability and SPM amplitude of an AMGplate model with SPM (blue) and IP (red). A total of 200 fits are shown, each for a different smoothing constant, (logarithmically from 0.01 to 10), and then stacked. Colour intensity indicates the number of fits that showed anomalous IP or SPM in that location. One additional fit is shown in black points (circles for chargeability, triangles for SPM), for the best case of $\lambda = 0.055$. SPM stacked 'events' range from $n = 1$ (light blue) to 7 (dark blue), and IP stacked events range from $n = 3$ (light red) to 5 (dark red).

subsequent experience has found that the appropriate value for $\lambda$ seems to be larger for AEM field data than it is for noise-free and simple synthetic data.

3.3 Results

3.3.1 AMGplate

Figure 3.3.1 show the results for AMGplate model d with synthetic SPM added (i.e. the same data used to generate figure 3.2.2). Figure 3.3.1(a) is the raw model CDI from data presented in Figure 3.3.1(b) the profile. The synthetic SPM appears as a very strong basement conductor in the CDI.

Data was then fitted with AEM, AIP and SPM basis functions. The SPM component was then stripped (by re-producing the decays from the least-squares solution, with the SPM amplitude set to zero) EMFlow rerun to produce the CDI shown
Figure 3.2.3: Predicted chargeability and SPM amplitude for AMGplate model d, over a smoothing constant range of 0.01 to 0.15. Ignoring the information we have on the model, and based on the sweep shown in the cumulative figure above, we can determine that the most suitable value for lambda is 0.02, as this captures both the strong IP indication in the centre, the weak (or zero) IP elsewhere, and the SPM to the east.
in Figure 3.3.1(c), from the data (Figure 3.3.1(d)). Although the false deep conductor is still present, significant attenuation of the synthetic SPM artefact is obvious. Also notable is that the real (plate) conductor image has not been affected by the removal of the SPM component.

3.3.2 Kapalagulu Test Lines

In order to quantify the fitting efficacy on real data, we used a line of VTEM data over an area that we also had Magnetic Viscosity Meter (MVM) data (Mutton, 2012). The two lines we will look at here are known as Kapalagulu Test Lines 1 and 2. They are of particular interest because as well as the MVM data, they both have known real conductors - Test Line 1 has a basement conductor under non-SPM soil, with adjacent SPM, and Test Line 2 has a basement conductor under SPM. Profiles of the test lines are shown in Figures 3.3.2(a) and 3.3.3(a).

After undergoing the preliminary process described in the previous section for finding an appropriate $\lambda$ (for Test Line 1 $\lambda = 7.8$, and for Test Line 2, $\lambda = 5.5$), we fitted the lines. We then re-synthesised the decay curves, stripping out the SPM component (SPM-stripped profiles are shown in Figures 3.3.2(c) and 3.3.3(b)), and used EMFlow to produce CDIs of the two test-lines. Also shown for Test Line 1 is the profile of an alternative method of SPM stripping: instead of setting the SPM amplitude to zero and then re-synthesising the decays based only on the fitted EM exponentials, we have reconstructed the SPM decay and then subtracted it from the original data, to produce the profile shown in Figure 3.3.2 (b).

Figures 3.3.4 and 3.3.5 show the results of this fitting. The figures show MVM data, as well as raw CDI (still containing SPM), and the CDI with the fitted SPM component removed.

Although it is apparent that the fitting process is not perfect, overall the results
Figure 3.3.1: AMGplate (a) CDI and (b) profile from data with synthetic SPM present, and (c) CDI and (d) profile of stripped data. The central conductor is at a true depth of 100 m.
Figure 3.3.2: Test Line 1 profile: (a) raw profile; (b) raw with fitted SPM data subtracted; (c) SPM-stripped profile. $\lambda = 7.8$ and $\tau_{EM}$ ranges from 20µs to 100ms.
Figure 3.3.3: Test Line 2 profile: (a) raw profile; (b) SPM-stripped profile. $\lambda = 5.5$ and $\tau_{EM}$ ranges from 20µs to 100ms.

Figure 3.3.4: Kapalagulu test line 1 (flight 5) results. This line covers a NiS orebody in the west, and SPM to the east. The area has been ground-truthed for SPM using a MVM (magnetic viscosity meter). A raw data CDI was produced using EMFlow. Processing using $\lambda = 7.8$, and subsequent SPM stripping and a re-calculated SPM removed CDI is shown. Note that the true depth of this conductor at 176150 m E is approximately 80 to 100 m.
Figure 3.3.5: Kapalagulu test line 2 (flight 10) results. This line covers a NiS mineralisation (around 178000 m E) under surficial SPM. The area has been ground-truthed for SPM using a MVM (magnetic viscosity meter), and a raw data CDI was produced using EMFlow. Processing using $\lambda = 5.5$, and subsequent SPM stripping and a re-calculated SPM removed CDI is shown. Note that the true depth of this conductor is approximately 80 to 100 m.

are good, and correlate with known ground-truthed data. The basement conductors remain, and the SPM effect is fairly effectively reduced. The conductor underneath the SPM - presumably the more challenging scenario - in Test Line 2 is successfully retained. It should be noted that the colour scales for conductivity are different for the raw and SPM-removed CDIs. The tendency of finite conductors to be imaged at too great a depth on a CDI is a known artefact of using a 1D approximation to fit the small amplitude data from a 2D or 3D source (Macnae et al., 1998).

3.3.3 Mwese

The Mwese survey area (Figure 3.1.1) contains large amounts of SPM, as well as a few identified basement conductors. Figure 3.3.6 shows an image of conductance after fitting and stripping of the SPM component, using EMFlow conductivity map-
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Figure 3.3.6: EMFlow conductance map (0m to 1000m depth) for Mwese dataset, with areas of detected SPM effects shaded in dark hues.

ping. Good conductors are yellow to red (in both Figures 3.1.1 and 3.3.6), and SPM in dark hues (Figure 3.3.6 only).

Figure 3.3.7 shows detailed area 2 from Figure 3.3.6 and outlines a number of conductors, both confirmed and unconfirmed, as well as SPM. Both of the unconfirmed conductors appear to be real, and additionally SPM appears to partially cover another of the confirmed conductors. The SPM indicated is unconfirmed. This area appears to have offset SPM and conductive source anomalies.

Figure 3.3.8 shows detail of a confirmed (drilled) conductor in the Mwese dataset (detail 1). Processing suggests that SPM is present over the top of the conductor, however this has not been confirmed. A second indication of SPM in the north-west of the map demonstrates the limitations of the method - this is actually a confirmed conductor, but our algorithm shows it as SPM. This is the only occurrence of a misclassification of a conductor as SPM we have seen using this fitting process in
limited testing to date. Figure 3.3.9 shows an area (detail 3 of Figure 3.3.6) of both confirmed and unconfirmed SPM, as well as one unconfirmed conductor.

### 3.4 Discussion and Conclusion

We fitted for SPM effects using a $t^{-1}$ basis function, along with other (EM and IP) basis functions in order to provide improved target prioritisation, by indicating where SPM effects are contributing a significant signal. This provides a tool that allows exploration resources to be focused on targets that show the greatest potential to be economic. A major challenge is the basis function matrix stabilisation, and once this has been attained, correctly determining the appropriate value for the smoothing constant $\lambda$ is difficult and time consuming, but completely necessary for a stable solution. An inappropriate value for $\lambda$ often leads to instability such that small changes to input data leads to disproportionately large changes in the output.

Some limitations include the fact that we implicitly assume a uniform SPM time constant distribution when we assume a $t^{-1}$ SPM decay. Barsukov and Fainberg
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Figure 3.3.8: Detail 1 in the Mwese dataset. SPM is shown as shading and contours. The processing indicates SPM over a good conductor (in purple), but also misclassifies a conductor as SPM to the top left.

Figure 3.3.9: Detail 3 in the Mwese dataset. These SPM responses appear as conductors in uncorrected data (Figure 3.1.1).
(2001) showed that some physical SPM processes can vary by as much as $t^{-1\pm 0.2}$, however we have not allowed for this variation. We believe that although allowing for this would further generalise the process, it could also potentially destabilise the fit and produce “false positives” (i.e. indications of SPM effects where there are no SPM effects). Additionally, we have not modified the SPM basis function for primary field shape variation, and we have not corrected SPM signals for height.

The process we have implemented seems to fit SPM when it is present, but it is not infallible. Our opinion is that it appears that it may become a useful tool for target prioritisation.
Chapter 4

Rotation Noise

The majority of the content in this chapter is to be published (Kratzer and Macnae, 2014).

4.1 Introduction and Background

Rotation noise, or unwanted signal due to rotations of a receiver within the Earth’s magnetic field, is a perhaps the main impediment to low base-frequency AEM surveys (Buselli et al., 1998; Vrbancich et al., 2000). Rotation noise is generally the most significant AEM noise source below 1kHz, followed by sferics spikes caused by lightning (see Figure 4.1.1), and at times cultural noise. Low base frequencies are needed to detect very conductive targets, particularly under highly conductive cover. Many ground EM systems collect data with base frequencies less than 10 Hz for optimum detection of such targets (King, 2007).

Low base-frequency systems are also of interest in airborne induced polarisation (AIP), as the IP signal only begins to dominate over the fundamental induction response in the later time channels (Kratzer and Macnae, 2012b). Additionally, lower
base frequency AEM systems would allow greater penetration through conductive cover than in conventional AEM surveys, as well as better characterisation and/or identification of other late-time effects, such as superparamagnetism (Kratzer et al., 2013b).

There have been several methods, such as stacking, published to reduce noise uncorrelated with desired signals in EM systems (McCracken et al., 1986; Macnae et al., 1984). More recently, Munkholm (1997) suppressed motion noise by estimating the direction of the Earth’s field from three-component magnetic field data, re-projecting the data towards the direction of the field (thence obtaining a signal that is minimally coupled to rotation), and by summing this re-projected field obtained a rotation noise suppressed signal. A reliable and widely used airborne method is to suspend the receiver coils in a dampening and/or suspension system (Lee et al., 2001; Barringer, 1963). In normal operation, rotation noise is reduced through suspension systems that decouple bird rotations from the sensor (Turner et al., 2002), and through the use of elastic suspension systems (Morrison et al., 2007; Kuzmin and Morrison, 2009). With housing / bird size and sensor mass constraints, elastic suspension systems typically have resonances at a few Hertz, and are only effective in greatly reducing vibrations and rotations at frequencies higher than about 20 Hertz. Residual rotation noise has therefore restricted the useful base frequencies of time domain AEM systems to 25 Hz or greater (Macnae, 2007; Vrbancich et al., 2005a). Both fixed-wing and helicopter-borne AEM systems using elastically suspended sensors have encountered a rotation noise limit at 25 Hz. One exception to the suspended rigid sensor approach is that of Hoistem, which uses a large loop of wire directly attached to a frame as a sensor. This paper does not address the issues of such a sensor.

Outside mathematically repressing or physically reducing rotation noise, attempts have been made to characterise it through observation of the rotations and

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comparison with magnetic sensor data. Kratzer and Vrbancich (2007) used GPS and inertial navigation instrumentation to track the motion of a test bird, in order to correct anomalous EM responses due to pendulum motion. Davis et al. (2006) used data from video recordings and GPS to track the motion of an AEM receiver and design a filter to remove the effects of bird swing.

Our method bears similarities with that of Smiarowski et al. (2010), who used data from flight instruments (other than attached rotation-rate sensors) as a set of basis functions to calculate and subsequently remove the effect of coupling changes between the transmitter and receiver, in order to obtain better on-time data in time-domain AEM systems. Smiarowski et al. (2010) achieved optimum prediction of transmitter-receiver geometry rather than the prediction of 3 component receiver rotations I am discussing here. Here we use a sensitive 3-axis rotation rate sensor, rigidly attached to a sensor to generate a transfer function between rotation rates and rotation noise, in order to predict rotation noise and subsequently remove it from AEM data. The success of this research should result in the commercial availability of useful 12.5 Hz AEM systems in the near future.

4.2 Method

4.2.1 Theory

Take a single-component B-field sensor rigidly attached to a rotation rate sensor, in an external magnetic field of magnitude $B_E$. If the directional magnetic field sensor is moving sinusoidally, in one axis, with angular frequency $\omega$ over an angular range $\phi_0$, the angle of the sensor is given by:

$$\phi(t) = \phi_0 \cos(\omega t), \quad (4.2.1)$$
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Figure 4.1.1: AEM noise source spectral amplitudes. After Buselli et al. (1998).
and the field seen by the sensor will be:

\[ BS = B_E \cos(\theta + \phi) \]

\[ = B_E \left( \cos\theta \cos\phi + \sin\theta \sin\phi \right) \]

\[ = B_E \left[ \cos(\theta) \cos(\phi_0 \cos(\omega t)) - \sin(\theta) \sin(\phi_0 \cos(\omega t)) \right], \tag{4.2.2} \]

where \( \theta \) is the angle between the magnetometer and the Earth’s field (see Figure 4.2.1).

When the magnetometer is perpendicular to the earth’s field, i.e. \( \theta \approx 90^\circ \):

\[ BS \approx B_E \sin(\phi_0 \cos(\omega t)). \tag{4.2.3} \]

When the magnetometer is aligned (or close to aligned) with the earth’s field, i.e. \( \theta \approx 0 \):

\[ BS \approx B_E \cos(\phi_0 \cos(\omega t)). \tag{4.2.4} \]

As can be seen, \( BS \) is dependant on both the coupling angle and the rotation rate.
Equation 4.2.4 is a function of the form \( \cos(\cos(x)) \). As \( \cos(x) \) is an even function (i.e. \( \cos(x) = \cos(-x) \)), regardless of whether the inner term \( \phi_0 \cos(\omega t) \) is positive or negative, \( B_S \) is always positive, and has maxima at extrema of \( \phi_0 \cos(\omega t) \), and minima where \( \phi_0 \cos(\omega t) = 0 \): these events happen twice per full cycle; it therefore has a fundamental frequency \( 2\omega \). This is not the case with Equation 4.2.3, because \( \sin(x) \) is an odd function.

Note that I have taken a very simple model in order to illustrate the frequency-doubling effect; in a real system, there will be a wide range of rotational frequencies present, and therefore combinations of various trigonometric and other functions will be required. This will be discussed in more detail in the Basis Functions section.

### 4.2.2 Sensor Frequency Response

Manufacturer specifications for the rotation rate sensor document a flat frequency response in the 1 Hz - 50 Hz passband. As Equation 4.2.2 shows, the induced rotation noise is proportional to trigonometric functions of the attitude of a B-field sensor, whereas the rotation rate sensor measures the angular rate of change. In order to integrate the rotation rate sensor data for attitude information, I can either sum numerically, or alternatively, in the frequency domain, divide by \( j\omega \):

\[
\Theta = \frac{\tilde{g}}{j\omega} + k, \quad (4.2.5)
\]

where \( \Theta \) is attitude in the Fourier domain, \( \tilde{g} \) is the angular rate of change data from the rotation rate sensor, \( j \) is the imaginary unit, and \( k \) is an (unknown) constant, representing the attitude frame of reference offset.

The problem with numerical integration is that as \( \omega \) approaches zero, \( \Theta \) in Equa-
tion 4.2.5 approaches infinity, hence instabilities arise. This is essentially the same as the unknown constant of an indefinite integral.

The ARMIT B field sensor behaves as a single-pole filter (Macnæ, 2012a), with a corner frequency at 5.4 Hz: above the corner frequency the ARMIT B-field response is flat, and below this frequency response decreases with \(\frac{1}{f}\); hence it is a \( \frac{dB}{dt} \) sensor below 5.4 Hz. In order to correct for this I construct a frequency-domain single-pole filter:

\[
\tilde{B}^* = \tilde{B} \cdot \frac{\omega_c + j\omega}{j\omega},
\]

(4.2.6)

where \( \tilde{B} \) indicates the B-field in the Fourier domain, \( \tilde{B}^* \) is the corrected B-field data, \( \omega_c \) is the corner frequency, and \( j \) is the imaginary unit. I can then apply the inverse Fourier transform to to obtain \( \tilde{B}^* \), the corrected time-domain B-field data.

To properly generate a transfer matrix between the rotation rate sensor and the ARMIT sensor, I must therefore apply both the “filters” (Equation 4.2.5 to the rotation rate sensor data, and Equation 4.2.6 to the ARMIT data). Alternatively, I can combine both filters, which, when applied to the rotation rate sensor data, both integrates it and gives it the same (non-flat) frequency response as the ARMIT sensor. This has the advantage of “rolling off” the rotation rate sensor data at the corner frequency of the ARMIT sensor with \( \frac{1}{f} \), hence eliminating the instability problems with numerical integration at low frequency:

\[
a = \left( \frac{\omega_c + j\omega}{j\omega} \right)^{-1} \cdot \frac{1}{j\omega}
= \frac{1}{\omega_c + j\omega}
\]

(4.2.7)
4.2.3 Basis Functions

As can be seen from Equation 4.2.2 that the induced rotation noise has components proportional to both $\sin(\theta)$ and $\cos(\theta)$. I must therefore construct a set of basis functions that contain these terms. I will also add Hilbert transformed data to allow for any phase shift, as phase shift was observed in simulation (I will discuss this later). Equation 4.2.8 shows the set of 13 basis functions.

$$G = \begin{bmatrix} \sin(g_a) & \sin(H(g_a)) & \cos(g_a) & \cos(H(g_a)) & k \end{bmatrix}$$

(4.2.8)

where $g_a$ are the filtered (with the filter in Equation 4.2.7) angular rate of change time-series data from the three rotation rate sensor axes $(x_a, y_a, z_a)$, $H$ indicates the Hilbert transform, and $k$ is the DC function, a constant needed to account for small electronic DC offset levels in the B field data.

I then construct a least-squares problem using the ARMIT B-field data and the basis functions $G$, in order to solve for the transfer matrix $T$:

$$B = GT$$

(4.2.9)

4.2.4 Simulation

I simulated the 1D motion of a single-component B-field sensor rigidly attached to a rotation rate sensor, in an external magnetic field. I simulated the motion for multiple attitudes relative to the magnetic field. The motion was sinusoidal, with a frequency of 5 Hz, over an angular range of $10^\circ$. I observed that for high-angle attitudes (i.e. those angles that represent the sensor closer to perpendicular to the field), the resulting signal from the simulated magnetometer is close to sinusoidal,
with a frequency of 5 Hz, and with a large amplitude (Figure 4.2.3). A relatively
small component of the field appears at double the frequency of motion (10 Hz: ap-
parent in the frequency domain - see Figure 4.2.2). As previously discussed, this is
to be expected from the cosine-coupled case. As the attitude decreases towards the
direction of the field (i.e. the sensor becomes more cosine-coupled), amplitude de-
creases, DC offset increases, and the second frequency at 10 Hz becomes relatively
more prominent (Figure 4.2.2).

Using this simulated data I then attempt to remove the rotation noise, using
rotation rate “measurements”. To do this I produce a number of basis functions that
I use for least squares fitting. The least-squares fitting will select the appropriate
combination of basis functions that allow the best reproduction of the “measured”
magnetic field.

Figure 4.2.2 shows simulated motion noise for 5 different attitudes, with an
added sinusoidal “signal” of 15 Hz, together with the results of least squares fitting
using the basis function of Equation 4.2.8. The peaks due to motion noise, seen at
5 Hz and 10 Hz, have been completely removed, leaving only the 15 Hz “signal”
peak.

Note that in this chapter I assume that the motion noise due to coupling changes
to the received signal are small enough (relative to the earth’s field) to be ignored.
This is a reasonable assumption, given that the earth’s field is approximately 50,000
nT, while the signals of interest in airborne EM are often less then 10 nT (see
Chapter 5).

It also appears that the magnetic field is phased shifted relative to the rate of
rotation (see Figure 4.2.3). I initially included Hilbert transformed data, as men-
tioned in the previous section to account for this phase shift. The Hilbert transform
provides a finite time series in quadrature to the original data. For a single sinusoid,
a cosine at the same frequency is in quadrature. Equations 4.2.5 and 4.2.7 theoretically account for the phase differences between the rotation-rate and B field data if sensors are perfectly linear and the assumed ARMIT corner frequency is exact. Nonetheless, I kept the Hilbert transformed results in our fitting process as I found that including these stabilised the result, and allowed fitting of dB/dt (not shown here) as well as B field data.

If the signal is periodic with an angular frequency $\omega$, I can represent it as:

$$ u = e^{i\omega t}. \quad (4.2.10) $$

The Hilbert transform $H$ for a signal $u$ has the property:

$$ \mathcal{F}(H(u))(\omega) = (-i\text{sgn}(\omega)) \cdot \mathcal{F}(u)(\omega), \quad (4.2.11) $$

where sgn is the signum function, and $\mathcal{F}$ is the Fourier transform. I therefore apply the Hilbert transform to obtain a $90^\circ$ phase shift. This combined with the raw data (i.e. with no phase shift), allows the least squares fit to select the appropriate amount of phase-shift, if any is required, through linear combination. For example, a $45^\circ$ phase shift can be obtained by using equal parts of the raw and Hilbert transformed data. This allowance for arbitrary phase shift also allows for imperfections in our physical system later on, such as sensor misalignment and ARMIT sensor corner frequency ($\omega_c$) calculations.

4.2.5 Test Rig

I constructed a test-rig (the ’cube’), consisting of a rigid cubic structure, containing a 3-axis rotation rate sensor at the centre of the cube, and coils for the 3-axis ARMIT
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Figure 4.2.2: Extraction of 15 Hz signal from synthetic data containing 5 Hz rotation frequency, with different orientations of rotation axis compared to Earth’s magnetic field.

Figure 4.2.3: Simulation results - time domain, for various attitudes relative to the external magnetic field. The black dashed line represents the rate of rotation. Note the phase shift between the rotation rate data (dashed) and the magnetic field data.
sensor (Macnae, 2012a) along the edges. In order to characterise the cube, I first collected data from the cube in a remote area known to be magnetically quiet. With the cube suspended in such a way as to allow small oscillatory motion in all axes (see Figures 4.2.4 and 4.2.5), and with motion stimulus, I recorded data from both ARMIT sensors and rotation rate sensor. Although the ARMIT sensor can deliver both $B$ and $dB/dt$ signals simultaneously (Macnae and Kratzer, 2013), I collected only $B$ field data as I am primarily interested in low (<30 Hz) frequencies.

I recorded cube data for a number of different orientations of the cube (relative to the earth’s field). I found that the transfer matrix changes depending on this orientation; this will be discussed in more detail in section 4.2.5.1.
4.2.5.1 Transfer Matrix Generation

There are two possible approaches to removing rotation noise. I either generate a transfer matrix using data with an external TEM signal present, and apply it to the same dataset used to generate the matrix, or I generate a transfer matrix using “quiet” data (with no external signal), and apply this quiet transfer matrix to data with a TEM signal present, in which the cube is orientated in the same direction. The advantage of the former approach is that I do not require a quiet data line in order to generate the transfer matrix, and the advantage of the latter approach is that there is no chance that the least-squares process used to generate the transfer matrix will fit for the TEM signal, preventing the possibility of another noise source.

Another advantage of the former approach is that I can break the dataset up into segments, producing a sequence of transfer functions, thence allowing for gradual changes in orientation. The challenge then lies in choosing an appropriate duration for each segment - too short and the transfer function is likely to contain the TEM
signal, too long and the noise reduction result is poor.

Using the basis function described in Equation 4.2.8, I produce a transfer matrix, relating the 3-axis rotation rate sensor signals to resulting motion noise in the 3-axis ARMIT sensor. Note that I also normalised the basis function matrix to improve stability.

Figure 4.2.6 shows the power spectrum for rotation noise prediction using a 0.5 second segment. Here the rotation signal has been attenuated by approximately 1 order of magnitude. It should also be noted that the rotation noise correction below
2 Hz is basically a DC offset, as I am using a 0.5s segment size. It can also be seen from the detail of 3 to 30 Hz (Figure 4.2.8) that although from 0.1 to 5 Hz the noise correction is good, in the range 5 to 12 Hz, my attempt to reduce noise has failed, and in fact I have added noise. This is also evident in the time domain (Figure 4.2.7). Using a 3 second segment, although I have minimal signal attenuation, the noise fitting capability is significantly reduced. Through empirical observation, I found that a segment size of 1 second tends to yield the best results, however this value is likely to depend on characteristics of the dataset, such as the base frequency of the transmitter, and the frequency composition of the rotation noise.

For the “quiet” transfer function approach, I set up a ground transmitter loop nearby (approximately 10m away), producing a signal typical of an airborne EM system, with a base frequency of 5 Hz. Using the transfer matrix found previously in the “no signal” environment, I predicted the rotation noise using the rotation rate sensor signal and subtracted it from the B-field signal. The results are shown in the frequency domain in Figure 4.2.10. At its best (~2 Hz - the apparent resonance of the setup), the prediction has removed almost 2 orders of magnitude of motion noise, whilst the TEM signal is retained with minimal attenuation.
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Figure 4.2.9: Power spectra for segmented fitting (segment duration of 3 sec).

Figure 4.2.10: Power spectra for the “quiet” transfer function approach. Again, $B_u$ is the uncorrected $u$-component of the B-field, and $B'_u$ is the corrected signal. The “static noise” data shows the characteristics of the cube when hanging with very little movement, and no transmitted signal: the rotation correction has managed to correct rotation noise to significantly below that “static” level.
4.2.5.2 Transfer Matrix Analysis

In order to observe the behaviour of the transfer matrix with variation of attitude, I conducted tests with a similar setup to that described above, adjusting attitude horizontally (around the vertical axis) and collecting 60 second data blocks in 10 degree increments for the full 360 degrees. In this area, at the time of survey, the magnetic inclination was $-67.8^\circ$ (sourced from www.ga.gov.au).

The transfer matrix plotted in figure 4.2.11 was produced by splitting each 60 second block into 10 second intervals, and least-squares fitting (Equation 4.2.9), as previously described. The median value for the 6 resulting transfer function values (one for each 10 second block) are then plotted, for each $10^\circ$ increment, producing a plot of transfer function versus attitude relative to the Earth’s magnetic field. After calculation, the basis functions were cropped by 1 second at the start and end to eliminate end-effects caused by the FFT (Fast Fourier Transform) process on a finite time series.

Immediately obvious is that the attitude of the cube directly determines the magnitude of the effect of motion, as simulation suggests. It can be seen that for the u- and v-components (top and middle in Figure 4.2.11), the axis of rotation that affects coupling is the z-axis, with a small amount of phase shift (as the Hilbert-transformed data is used). The x- and y-axis data is required, but coupling does not vary with attitude. For the w-component (bottom in Figure 4.2.11), the x- and y-axis are the important axes of rotation, and in fact the z-axis does not contribute any relevant data. Reviewing figure 4.2.4, it is apparent that this should be expected, as the z-axis and w-component are aligned, and so rotation around the z-axis does not change incident magnetic flux to the w-component.

At both 60 degrees and 270 degrees there are large spikes in the transfer matrix. This appears to be due to different frequency composition: Figure 4.2.12 shows
Figure 4.2.11: Transfer matrix behaviour, for the full range of horizontal attitudes. Only the 'sine' components are shown here for clarity. \( x_a, y_a, \) and \( z_a (= g_a) \) are the three rotation sensor axes; and \( H(x_a), H(y_a), \) and \( H(z_a) (= H(g_a)) \) are the Hilbert-transformed rotation sensor axes (see Equation 4.2.8). 60 second data blocks were taken every 10 degrees, and processed in 10 second segments. The median values for the 10 second segments are plotted. The axes of rotation that change with attitude are displayed in solid lines while the non-changing axes are dotted for clarity. Note that the displayed transfer matrix has been de-normalised for display purposes (after being normalised for the least-squares fitting process).
the power spectra for two “consistent” orientations (i.e. where the transfer matrix is continuous to its neighbours), and the two inconsistent orientations (where there are spikes in the transfer matrix). It is clear that the two orientations that have large spikes in the transfer matrix have a higher proportion of 0.3 Hz power than the other orientations.

One aim of conducting transfer matrix analysis was to characterise the cube, allowing the transfer matrix to be generated semi-analytically; so that with knowledge of the direction of a survey line and the magnetic inclination, a pre-calculated transfer matrix could be applied. As can be seen, the frequency composition of the rotational data is extremely important, and due to the inadequate signal to noise ratio present in this particular set of transfer matrices (in particular, at 60° and 270°), it appears that this approach may not yield good results. Ideally I would produce the transfer matrix by sweeping the cube through an appropriate range of rotational frequencies (say 0.1 to 30 Hz) for each axis, producing a much more generalised transfer matrix, possibly allowing a transfer matrix to be generated semi-analytically.
4.3 Discussion and Conclusion

I have constructed a 3 component magnetic field sensor with integrated 3 component rotation rate sensor for rotation noise prediction. It appears that the best approach to noise prediction and removal is to determine the transfer matrix in location under “quiet” (no TEM signal) conditions for each of the orientations that EM survey lines will be conducted along, before applying this transfer matrix to the working dataset to remove rotation noise. This approach appears to have minimal effect on the TEM signal whilst still performing good rotation noise prediction. In the case that this approach is not possible, good results are still obtainable applying rotation corrections to “live” data, with use of an appropriate length of processing segment to allow for attitude drift through a survey line. These results extend previous work on system geometry prediction to predict and correct for elastically suspended sensor rotations. Analysis of the transfer matrix for a range of attitudes of our test-rig demonstrates that the transfer matrix is stable, and even predictable. It therefore should be possible, with further improvement, to generate a transfer matrix based on line direction alone, removing the requirement for a “quiet” (or passive) line, therefore reducing survey time and removing the risk of transfer matrix corruption from external noise sources. The best approach for this, a study for future work, seems to be a rotational frequency sweep for each B-component, the axis of rotation being the B-component under investigation. This may address the inconsistencies seen in the two datasets (60° and 270°) that have different frequency composition and therefore produce inconsistent transfer matrices. Another improvement that could be made which would potentially increase the efficacy of rotation noise reduction is the addition of data from a 3-component flux-gate magnetometer, which would supply the missing constant $k$ from Equation 4.2.5.

Practically, if an order of magnitude reduction in rotation noise can be achieved
in flight at frequencies below 20 Hz, this should allow for adequate correction of 12.5 Hz base frequency AEM data to make this data useful in exploration through conductive cover.
Chapter 5

Airborne Induced Polarisation

5.1 Introduction

The Induced Polarisation method is important for disseminated sulfide mineralisation exploration, however although the effects of IP are sometimes seen in Airborne electromagnetic (AEM) data (Smith and Klein, 1996), no airborne methodology currently exists (Thomson et al., 2007).

Here, I extend on the work of Kratzer and Macnae (2012b), where progress was made towards quantification of the IP signals often seen in AEM data, based on the work of Cole and Cole (1941) and Wong (1979). It was concluded in Kratzer and Macnae (2012b) that for useful quantification, significantly lower system base frequency was required, as the IP effect begins to dominate over the EM induction effect at later delay times, due to a slower rate of decay. Implicit in this conclusion is that the IP effect dominates at lower frequencies than EM. This effect can be seen in Figure 5.1.1, which shows one (i.e. $\frac{dB}{dt}$) decay from a VTEM field survey over a chargeable area, which has been deconvolved into $\dot{B}_{EM}$ and $\dot{B}_{IP}$ decays (here I will use $\dot{B}$ to indicate the signal received by a $\frac{dB}{dt}$ system such as VTEM). Although
the $\dot{B}_{EM}$ response dominates early, the much slower rate of decay of $\dot{B}_{IP}$ makes it dominant in the later channels. There are two requirements that can be drawn from this observation, in order to progress towards a commercial AIP system:

Firstly, direct measurement of the B field during an AIP survey will result in a higher signal-to-noise ratio, as the B field necessarily contains more energy in the lower frequency bands than does $\dot{B}$. Figure 5.1.2 shows field data as seen by an ARMIT B-field sensor (Macnae, 2012b) over a synthetic IP source, again having been deconvolved into $B_{EM}$ and $B_{IP}$ components. This demonstrates the significant advantage of using a B-field sensor for inductive IP; the IP effect dominates much earlier, leaving more channels containing significant IP signal. Additionally, due to the significantly lower dynamic range of the B-field in general (noting the linear scale in Figure 5.1.2), the later channels may have a higher signal-to-noise ratio than the later channels in a $\dot{B}$ system in which the EM signal plunges towards the noise floor.

Secondly, as rotation noise is the dominate noise source at lower base frequencies (Buselli et al., 1998; Vrbancich et al., 2000), an effective method of removing rotation noise is required. Kratzer and Macnae (2014) describe a method using a
Figure 5.1.2: $B_{EM}$ and $B_{IP}$ decay comparison. Note here that the y-scale is linear: in contrast to the high dynamic range of the $\dot{B}$ decays of Figure 5.1.1, the $B$ decays have a much lower dynamic range, and the IP effect begins to dominate over the EM response earlier than the $\dot{B}$ decay. Again, solid lines are positive, dashed lines are negative.

triad of rotation rate sensors to predict rotation noise in an oscillating ground-based sensor, and subsequently remove it from the $B$ signal.

Here I employ both of these developments to predict and remove rotation noise in a low base-frequency (8.33 Hz) ARMIT B-field sensor airborne IP survey, in the vicinity of a synthetic IP source.

## 5.2 Method

### 5.2.1 IP Basis Functions

#### 5.2.1.1 Warburg

Here I will derive the B-field IP response ($B_{IP}$) for $c = 0.5$, the Warburg decay, which is the model that Wong (1979) found closely fitted his electrochemically-based model for disseminated sulfides. To obtain the IP response to any arbitrary excitation, I require the impulse response; i.e. the response of the IP function to an Dirac delta excitation $\delta(t)$ (Kratzer and Macnae, 2012b). I can then convolve the
impulse response with any excitation to produce the expected IP response to that excitation.

Although Kratzer and Macnae (2012b) derived the IP step response, I was not able to find an analytic time derivative to obtain the impulse response. To get around this I use linearity of the following convolution $a = b * c$:

$$
\frac{d}{dt} a = \frac{d}{dt} b * c = b * \frac{d}{dt} c = \frac{d}{dt} (b * c),
$$

(5.2.1)

This allows me to take a time derivative of either argument, and as I have already the scaled time-derivative of $I_1(t)$, being the current in the receiver coil $I_2(t)$ given by:

$$
I_2(t) = M_{12} \frac{d}{dt} I_1(t),
$$

(5.2.2)

I can simply normalise $I_2(t)$ and convolve it with the IP step response $C(t)$ (after some additional modification of $I_2(t)$ to account for system sampling limitations - see details in Kratzer and Macnae (2012b)). I can then determine that the IP response $\dot{B}_{IP}$ for a $B$ system such as VTEM:

$$
\dot{B}_{IP} = \frac{d}{dt} I_1(t) * C(t).
$$

(5.2.3)

For the ARMIT B-field sensors however, $B_{IP}$, rather than $\dot{B}_{IP}$, is received. Integrating both sides, Equation 5.2.1 therefore allows me to say that:
Figure 5.2.1: Example $B_{IP}$ basis functions, for frequency dependence $c = 0.5$ (i.e. Warburg decay). A total of 14 different decays are shown, with time constants $\tau_{IP}$ spaced logarithmically from $10^{-6}$ to 3 seconds; dotted lines indicate intermediate times of $3 \times 10^{-6}$ s, $3 \times 10^{-5}$ s, etc.

\[
B_{IP} = \int \left( \frac{d}{dt} I_1(t) * C(t) \right) dt \\
= I_1(t) * C(t).
\]

So I can simply take the received ARMIT signal, normalise to remove the attenuation effect of $M_{12}$, and convolve it without modification with the IP step function, to obtain the IP response to the induced current excitation. This function then becomes the basis function, unique for each point. Figure 5.2.1 shows some examples of the B field IP decay for various time constants.

5.2.1.2 Debye

In this research, due to logistical constraints, the IP source was synthetic, consisting of a wire loop closed by a capacitor. The response of such a source will be a Debye decay (Wong, 1979), i.e. $c = 1$. 
For \( c = 0.5 \), Wong (1979) showed that the time domain step response is given by:

\[
\frac{V_S(t)}{V_P} = \frac{\rho_L - \rho_H}{\rho_L} e^{x^2} \text{erfc}(x(t)), \tag{5.2.4}
\]

and for \( c = 1 \):

\[
\frac{V_S(t)}{V_P} = \frac{\rho_L - \rho_H}{\rho_L} e^{-x^2} u(t), \tag{5.2.5}
\]

where \( x = \sqrt{t/\tau} \).

Kratzer and Macnae (2012b) showed that the IP step response for a half-step excitation of base-period \( T \) is given by:

\[
C_{\text{Warburg}}(t, T) = \sum_p \left[ e^{x_0^2} \text{erfc}(x_{0,p}) + \sum_{i=1}^{\infty} \left( (-1)^i e^{x_{(2i-1),p}^2} \text{erfc}(x_{(2i-1),p}) + (-1)^i e^{x_{(2i-1),p}^2} \text{erfc}(x_{(2i),p}) \right) \right], \tag{5.2.6}
\]

where \( x_{i,p} = \sqrt{t + t/\tau_p} \). It can be seen therefore that to obtain the half-step response for \( c = 1 \) (the Debye decay), I can simply replace the \( e^{x^2} \text{erfc}(x) \) term with \( e^{-x^2} \), to obtain the half-step IP response:

\[
C_{\text{Debye}}(t, T) = \sum_p \left[ e^{-x_0^2} + \sum_{i=1}^{\infty} \left( (-1)^i e^{-x_{(2i-1),p}^2} + (-1)^i e^{-x_{(2i),p}^2} \right) \right]. \tag{5.2.7}
\]

Equation 5.2.7 is the step response I will use here only to fit for the synthetic chargeable source of \( c = 1 \). Surveys of real IP sources will require IP step responses of the Warburg form (Equation 5.2.6). It should also be noted that in taking this
final step, Equation 5.2.7 is the numerical version of the analytically derived Debye decay calculated for the EM basis functions (Equation 5.2.8).

### 5.2.2 EM Basis Functions

Stolz and Macnae (1998) showed how the EM response to a half-step excitation of period $T$ is given by:

$$B_{EM}(t_k, t_{k+1}, T) = \sum_{m} \frac{A_{m} \tau_{m}}{t_{k+1} - t_{k}} \frac{1 - e^{-T/2\tau_{m}}}{1 + e^{-T/\tau_{m}}} \left( e^{-t_{k}/\tau_{m}} - e^{-t_{k+1}/\tau_{m}} \right). \tag{5.2.8}$$

For a $\dot{B}$ system, the basis functions would be the time derivative of Equation 5.2.8, which is similar to Equation 5.2.8 but without the $\tau_{m}$ factor. Here, however, I directly measure $B_{EM}$, and so the EM basis functions are composed of Equation 5.2.8. Figure 5.2.2 shows the EM basis functions for a range of $\tau_{EM}$.

**Figure 5.2.2: $B_{EM}$ basis functions. A total of 25 decays are shown, with time constants $\tau_{EM}$ logarithmically spaced from 30\(\mu\)s to 1s.**
5.2.3 AIP survey

I used data from an airborne survey, consisting of a 100\(m^2\) square ground-base transmitter loop carrying a 40 Amp current (for a dipole moment of 400,000 A \(\cdot\) m\(^2\)) of 50\% duty cycle square-wave, with period 8.33 Hz. Situated 300m to the north was placed a synthetic IP target, consisting of a wire loop closed with a capacitor, designed such that the synthetic IP loop response amplitude was comparable to that of a large porphyry target at a depth of 150 m below surface.

5.3 Results

5.3.1 Noise removal and preprocessing

I will take a small section of 40m of the survey, recorded in the vicinity of the synthetic IP. Rotation noise was removed as previously achieved as specified in Section
Figure 5.2.4: Hilbert-transformed basis functions, illustrating the edge-effects and the sampling technique I used to minimise them. Dashed lines indicate the sampled segments, and solid lines indicate the centre 50% of the sample used to avoid edge-effects.

Figure 5.3.1: Detail of 440m to 480m from Figure 5.2.3

5.1 above. I used one second windows with 50% overlap to remove edge effects from the FFT (Figure 5.2.4), a technique which although removed problems caused by edge effects, introduces small discontinuities between segments, and probably contributes to noise.

The results of the rotation noise removal are good (and fairly typical for this survey), with approximately one to two orders of magnitude of noise amplitude removed in the apparent main band (~0.5 Hz) - see Figure 5.3.1. A detailed view of the received field for one full cycle at position 467m is shown in 5.3.2.

Due to a technical synchronisation problem with the equipment during this survey (intermittent GPS data), removal of the primary field from the received field in the data was not straightforward, and a peak detection algorithm was used for syn-
Figure 5.3.2: Detail of noise removal at position 467m.

Figure 5.3.3: Primary field removal.

chronisation together with data from a transmitter flyover to subtract the primary field from the received field in order to produce the secondary field. An example of this for one decay is shown in 5.3.3. The peak finding technique was not perfect, and in some areas an imperfect removal of the transmitted field appears to contribute to noise. For example, the decays in 5.3.4 had systematic misfits in the 2 ms region, which I attribute to imperfect transmitted field removal. In order to reduce this effect, I found that it was vital to stack decays - here I stack 4 decays for each point.
5.3.2 Debye IP source fitting

I fitted for the resulting secondary field in the off-time using 100 EM decays, stepping logarithmically from $\tau_{EM} = 10 \mu s$ to $\tau_{EM} = 1.6s$; as well as 20 (Debye) IP decays stepping logarithmically from $\tau_{IP} = 1.0s$ to $\tau_{IP} = 10^3s$ I used smoothing constant $\lambda = 0.05$. I did not use a DC basis function as the data has not been DC shifted, and neither did I include SPM or parasitic capacitance basis functions.

I found that in order to obtain a good fit to the decays, it was necessary for me to include unrealistically long $\tau_{EM}$ basis functions, and it is obvious from Figure 5.3.5 that the least squares fitting process is attempting to match up the short $\tau_{IP}$ basis functions against the long $\tau_{EM}$ basis functions; this can be observed in the near continuous line of short $\tau_{IP}$ and long $\tau_{EM}$ in Figure 5.3.5.

This effect is likely due to the problems of fitting data which contains both positive and negative Debye decays, as Kratzer and Macnae (2012b) did not observe this in fitting Warburg decays (albeit for $\dot{B}_{IP}$ data).

When I calculate the apparent chargeability $m$ for this line, using
Figure 5.3.5: Graphical representation of the tau space for the EM/IP fitting. Note the almost continuous band of short $\tau_{IP}$ and long $\tau_{EM}$, due to coefficient cancellation. The synthetic IP source is clearly visible at 550m as a strong band of $\tau_{IP} = 100s$.

\[ m = \frac{\Sigma A_{IP}}{\Sigma A_{EM}}, \]  

(5.3.1)

where $A_{IP}$ and $A_{EM}$ are EM and IP amplitudes respectively. I find that the location of the synthetic IP source is very obvious at 550m (see Figure 5.3.6). However, the apparent chargeability is significantly lower than expected, at 20 ppt. Under the assumption that the Debye decays are cancelling, I can recalculate chargeability ignoring the very long $\tau_{EM}$’s and very short $\tau_{IP}$’s. The results of both calculations are shown in Figure 5.3.6, and it is clear that the recalculated chargeability gives a better representation: apparent chargeability is 0.2 at the peak (a more plausible result for the synthetic source), and is zero elsewhere (excluding two points showing chargeability of 0.5 to 0.9 ppt - probably due to low signal to noise ratio - these points are 400m from the transmitter, and signal amplitude is very small here).

Another method I looked at was to allow only the dominant $\tau_{IP}$ in the basis func-
Figure 5.3.6: Apparent chargeability section for the survey, for chargeability calculated for both all $\tau$’s (●’s) and only relevant $\tau$’s (×’s). The synthetic IP source is located at 550m. Gaps are due to transmission breaks, in the event that ‘quiet’ (Kratzer and Macnae, 2014) transfer functions were required for QC checking (I did not utilise this method here).

It should be noted here that the dominant IP basis function is one of $\tau_{IP} = 100\,s$. This is a far slower decay than would be expected from the synthetic source used in this experiment, however when a more realistic basis function of $\tau_{IP} = 1\,ms$ was employed, it was obvious that a much slower decay was required. I believe that this is due to imperfectly removing the primary field from the received field, and I do not expect properly synchronised data to require as large $\tau_{IP}$. 

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Figure 5.3.7: Apparent chargeability fitting using only the dominant $\tau_{IP} = 100s$ (see in Figure 5.3.5). Note that the scale here is linear: the range of apparent chargeability is much smaller than in Figure 5.3.6 as the procedure is fitting even the very small chargeability detected when not directly over the target.

5.4 Discussion and conclusion

I have successfully determined apparent chargeability from an airborne platform for a synthetic IP target that behaves (electrically) as a geologically realistic target, using a low base frequency EM system made possible through the use of low-noise B field sensors and rotation noise prediction. Significant room for improvement exists in a number of areas, including a properly synchronised transmitter and receiver to accurately extract only the secondary field, an airborne co-located transmitter, and potentially a flux-gate magnetometer to remove the ambiguity of an integration constant offset. Additionally, a better technique to remove edge-effects which does not introduce discontinuities would also decrease noise.

A survey over a real disseminated sulfide source could also potentially yield improved results as the IP decay from disseminated sulfides, being of the Warburg form, is dissimilar to the fundamental inductive response and therefore will not cause least-squares fitting ambiguity.

Results from this research could be applied to other areas in which late-time
data are important, such as superparamagnetism, where low-noise late-time data are important to distinguish SPM from large conductors (Kratzer et al., 2013b); AEM bathymetry (Vrbancich et al., 2005b), and development of AEM systems for deep EM survey or areas with highly-conductive cover.
Chapter 6

Discussion and Conclusion

6.1 Discussion

The focus of this thesis was on three major late-time effects in airborne TEM systems; i.e. effects which occur at later delay times after current switch-off than the fundamental induction response, being IP, SPM, and rotation. These effects are often considered as noise in EM systems, and indeed, the primary aim of this thesis was to improve airborne EM data by characterisation and removal of these noise sources, but as I have shown, with improving sensors and effective characterisation, value can be added to the exploration process by viewing two of them as signal: airborne IP has obvious advantages in disseminated sulfide exploration, and SPM in various other deposits (Barsukov and Fainberg, 2001). Therefore utilisation of these noise sources can be considered a secondary aim of this thesis.

As EM sensors improve and internal system noise decreases, it will become increasingly important to take the effects studied here into account, not only because they provide useful information during geophysical exploration, but also to prevent these effects becoming part of a noise floor and limiting the performance of data
CHAPTER 6. DISCUSSION AND CONCLUSION

from the fundamental inductive response.

6.2 Conclusion

I refined the model for disseminated mineralisation developed by Wong (1979) to produce the IP decays expected when the source and receiver are both inductive. Using both simulated and real IP decays for a \( \frac{dB}{dt} \) system, in ground and airborne geometries, I was able to produce good fits to modelled and observed data in all cases. Resulting chargeabilities are mostly accurate (for the simulated data), and plausible (for ground and airborne field data), and spatial distribution of chargeability for the airborne field data is good.

SPM effects can be fitted in a \( \frac{dB}{dt} \) system, using a \( t^{-1} \) basis function, along with other (EM and IP) basis functions, by indicating where SPM effects are contributing a significant signal. This provides a tool that allows exploration resources to be focused on targets that show the greatest potential to be economic. A major challenge is the basis function matrix stabilisation, and once this has been attained, correctly determining the appropriate value for the smoothing constant \( \lambda \) is difficult and time consuming, but completely necessary for a stable solution. An inappropriate value for \( \lambda \) often leads to instability such that small changes to input data leads to disproportionately large changes in the output. Some limitations include the fact that I implicitly assume a uniform SPM time constant distribution when I assume a \( t^{-1} \) SPM decay. Barsukov and Fainberg (2001) showed that some physical SPM processes can vary by as much as \( t^{-1\pm0.2} \), however I have not allowed for this variation. I believe that although allowing for this would further generalise the process, it could also potentially destabilise the fit and produce “false positives” (i.e. indications of SPM effects where there are no SPM effects). Additionally, I have not modified the SPM basis function for primary field shape variation, and I have not
corrected SPM signals for height. The process I have implemented appears to fit SPM when it is present, but it is not infallible. My opinion is that it appears that it may become a useful tool for target prioritisation.

A three component magnetic field sensor with integrated three component rotation rate sensor can be used for rotation noise prediction. I found that good results could be obtained by using a set of basis functions that allowed phase shift between rotation and resulting rotation noise, to produce a transfer matrix. It was important to determine an appropriate processing window - one short enough to perform good noise removal, but long enough that the transmitted signal was not affected. Analysis of the transfer matrix for a range of attitudes of our test-rig demonstrates that the transfer matrix is stable, and even predictable (although noisy).

Using B-field basis functions and B-field output, together with rotation-rate sensors, I can calculate chargeability for an airborne survey over a synthetic IP source. Noise was introduced into this particular survey from poor transmitter/receiver synchronisation, and difficulties were encountered due to the source having a $c = 1$ frequency constant and therefore being a Debye response and cancelling with the fundamental induction response, but the source was easily located regardless.

### 6.3 Implications

There are a number of areas for further research that would build on the work documented in this thesis. I will summarise them here, before providing a more detailed explanation below:

- Accounting for conductive cover attenuation;

- Rotation noise in SPM;
• Using a B-field sensor for SPM affected areas;

• The combined effects of SPM and IP;

• Rotation noise segment size;

• Filtering for rotation noise;

• Broad spectrum transfer matrix for the cube; and

• Integration of a flux-gate magnetometer.

To expand on this list: quantification of IP signals requires further optimisation in terms of accounting for conductive cover, which attenuates the IP response and therefore produces a lower apparent chargeability, which will become an important area for further research as airborne IP emerges as a viable technology.

A further refinement for calculating the IP response involves a double convolution step: after convolving the inductive response with the Warburg step response we obtain the IP response underground. Therefore, in order to obtain the IP response above ground, we can assume that as the magnetic field from the IP response moves up through to the surface, due to linearity it will undergo the same transformation as the fundamental inductive response did as it moved down from the surface. Thus I refine the solution by convolving the underground IP response with the fundamental inductive response once again. This method was not employed in this thesis but it is a refinement that has been tested subsequently, and should be employed in future use.

I did not look at removal of rotation noise with SPM, however, being a late time effect it is likely that a lower base-frequency AEM survey with rotation noise removal would make identification and removal of SPM even more effective. Additionally, a B-field sensor such as ARMIT would potentially also contribute towards
the problem of SPM, due to the lower frequency response of a B-field system. The
combination of IP and SPM was briefly looked at subsequent to this research, and it
was found that in synthetic models, the combination of IP and SPM were equal and
opposite to a close enough degree that they cancelled - meaning that with dominant
SPM, an addition of IP simply decreased indication of SPM without indicating the
presence of IP, and with dominant IP, the addition of SPM decreased chargeability,
and indicated no SPM present. It is possible that extending delay times, or use of a
B-field sensors, would prevent this happening. The cancellation of these effects was
not investigated further as no relevant field data was available, but it could become
an important area for further research.

In terms of fitting for rotation noise, fitting segment size has a big effect on the
quality of the solution. A rigorous method for determining optimal segment size
would be an area for further research.

It should also be noted that should the rotation noise be confined to very low
frequencies (<2 Hz), there is evidence that careful filter design can remove just
as much rotation noise as can be removed through prediction and subsequent sub-
traction (Macnae personal communication, 2013). This of course is only relevant
in systems that are able to confine rotations to that band; currently it appears that
this is achievable only through increasing the moment of inertia of the receiver, al-
though it is conceivable that better mechanical damping could change this. It should
also be noted that this technique is not additive with the effect of prediction with
rotation-rate sensors. Research into filtering for rotation noise could be an important
alternative area for removal of this effect from AEM data.

It may also be possible, with further improvement, to generate a transfer matrix
based on line direction alone, removing the requirement for fitting for each segment,
therefore reducing survey time and removing the risk of transfer matrix corruption.
from external noise sources. The best approach for this, a study for future work, seems to be a rotational frequency sweep for each B-component, the axis of rotation being the B-component under investigation. Another improvement and area for further research which would potentially increase the efficacy of rotation noise reduction is the addition of data from a three component flux-gate magnetometer, which would supply the missing constants that are lost with a rate sensor.
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