A Novel Air Traffic Management Decision Support System

*Multi-Objective 4-Dimensional Trajectory Optimisation for Intent-Based Operations in Dynamic Airspace*

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A Novel Air Traffic Management Decision Support System

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DECLARATION

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13/10/2016
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Summary

This research project aimed to develop innovative Multi-Objective Trajectory Optimisation (MOTO) and Dynamic Airspace Management (DAM) models and tools to enhance the performance of Air Traffic Management (ATM) and Air Traffic Flow Management (ATFM) systems, simultaneously targeting safety, efficiency and environmental sustainability requirements in line with SESAR and NextGen operational concepts. The novel MOTO and DAM functionalities were implemented and numerically evaluated in the form of automated 4DT Planning, Negotiation and Validation (4-PNV) algorithms within an innovative ATM Decision Support System (DSS) prototype. The 4-PNV system exchanges optimal 4DT intents with Next Generation Flight Management Systems (NG-FMS) on board equipped traffic via suitable data-links. Two different MOTO algorithms were developed to selectively address the en-route context and Terminal Manoeuvring Area (TMA) sectors. A variable set of models and optimality criteria can be selected based on the flight phase, aircraft and airspace states. An operational 4DT smoothing algorithm translates the mathematical Continuous and Piece-Wise Smooth (CPWS) optimal trajectory in a flyable and concisely described 4DT intent, while retaining most of the optimality performances offered by the mathematical solution. The developed MOTO-4D and DAM algorithms support an automation-assisted planning of 4DT intents in strategic and tactical online timeframes, as well as traffic flow and airspace reconfigurations. The models adopted to introduce the environmental performance within the 4DT optimisation framework also enable an estimation of the environmental impacts.
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A Novel Air Traffic Management Decision Support System

Multi-objective 4-dimensional trajectory optimisation for intent-based operations in dynamic airspace

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In loving memory of
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# Table of Contents

**Chapter 1. Introduction**

1.1. Background

1.2. ATM and avionics modernisation

1.3. Environmental sustainability of aviation

1.4. Problem statement

1.5. Motivation and significance

1.6. Research aim and objectives

1.7. References

**Chapter 2. Overview of air traffic flow management and optimal scheduling principles**

2.1. Background

2.2. Demand-capacity balancing

2.3. Airport throughput modelling

2.4. ATC sector capacity modelling

2.5. Demand and operational capacity modelling

2.5.1. Demand prediction and macroscopic traffic flow models

2.6. Traffic Sequencing

2.6.1. Mathematical formulation and solution methods

2.6.2. Airport flow and capacity management

2.7. Departure-arrival problems

2.7.1. Mathematical formulation and solution methods

2.8. Gate-to-gate problems

2.9. Route complexity and conflict detection

2.10. Advanced ATFM concepts

2.10.1. Decentralised and distributed ATFM paradigms

2.10.2. Dynamic airspace management

2.11. Integration of trajectory optimisation concepts

2.12. Conclusions

2.13. References
Chapter 3. Aircraft trajectory optimisation in the ATM and avionics context ________ 3-1

3.1. Background ____________________________________________ 3-1
3.2. Optimal control problem __________________________________ 3-3
3.2.1. Dynamic constraints ___________________________________ 3-3
3.2.2. Path constraints ______________________________________ 3-4
3.2.3. Boundary conditions ___________________________________ 3-5
3.2.4. Cost functions and performance indexes __________________ 3-6
3.2.5. Resulting mathematical formulation ______________________ 3-6
3.3. Numerical solution techniques ______________________________ 3-7
3.3.1. Lagrangian relaxation and first order optimality conditions _____________ 3-8
3.3.2. Boundary-value problems _________________________________ 3-9
3.3.3. Iterative solution of unconstrained nonlinear programming problems ______ 3-10
3.3.4. Indirect methods ________________________________________ 3-12
3.3.4.1. Indirect shooting ______________________________________ 3-15
3.3.4.2. Indirect multiple shooting ______________________________ 3-15
3.3.4.3. Indirect collocation _________________________________ 3-16
3.3.4.4. Limitations _________________________________________ 3-16
3.3.5. Direct methods __________________________________________ 3-16
3.3.5.1. Direct shooting ______________________________________ 3-17
3.3.5.2. Multiple direct shooting ________________________________ 3-17
3.3.5.3. Local collocation methods _______________________________ 3-18
3.3.5.4. Global collocation methods ______________________________ 3-18
3.3.6. Heuristic methods ______________________________________ 3-21
3.4. Trajectory optimisation in the presence of wind ________________ 3-22
3.5. Multi-objective optimality ___________________________________ 3-24
3.5.1. Pareto optimality ________________________________________ 3-25
3.5.2. A priori articulation of preferences __________________________ 3-26
3.5.2.1. Weighted global criterion method _________________________ 3-27
3.5.2.2. Weighted min-max _____________________________________ 3-27
3.5.2.3. Weighted product ______________________________________ 3-28
3.5.2.4. Exponential weighted criterion ____________________________ 3-28
3.5.2.5. Lexicographic and sequential goal programming methods ____________ 3-29
5.7. Trajectory optimisation algorithm implementation 5-11
5.8. Trajectory optimisation for TMA operations 5-14
5.8.1. Transposition of the mathematical optimum in a flyable trajectory 5-18
5.8.2. Control input smoothing process 5-18
5.8.3. Operational trajectory realization process 5-22
5.9. Trajectory optimisation for the en-route context 5-26
5.10. Optimal airspace reconfiguration algorithm 5-28
5.10.1. Mathematical formulation 5-29
5.10.2. Implementation case study 5-31
5.11. Interactions between MOTO, DAS and AFP 5-33
5.11.1. TMA context 5-35
5.11.2. Large-scale rerouting due to unforeseen perturbations 5-37
5.12. Conclusions 5-38
5.13. References 5-39

Chapter 6. Numerical simulations 6-1
6.1. Terminal sequencing and spacing of arrival traffic 6-1
6.1.1. 4D-MOTO simulation case studies 6-5
6.1.2. Discussion of TMA 4D-MOTO results 6-15
6.2. En-route trajectory optimisation algorithm 6-16
6.2.1. Discussion of enroute 2D+T-MOTO results 6-18
6.3. Further enroute 2D+T-MOTO analysis 6-19
6.3.1. Flight 6 6-20
6.3.2. Discussion of further enroute 2D+T-MOTO results 6-22
6.4. Trajectory planning for the emergency timeframe 6-23
6.5. Dynamic airspace reconfiguration case study 6-25
6.6. Conclusions 6-28
6.7. References 6-29

Chapter 7. Conclusions 7-1
7.1. Recommendations for future research 7-5
Appendix A  Results of the MOTO-4D for en-route applications ...................... A-1
Appendix B  List of relevant publications .......................................................... B-1
Appendix C  Full list of references ........................................................................ C-1
Chapter 1

Introduction

1.1. Background

The continuous growth of civil air transport and the increasing adoption of manned and unmanned aerial vehicles for new and more traditional roles is posing significant challenges to aviation regulators, service providers and operators, as the current paradigms will not ensure the desired levels of safety, efficiency and environmental sustainability in the future unless substantial enhancements are implemented. Consequently, several large-scale renovation initiatives were launched in the last two decades. These research and development activities are investigating, in particular, the most promising technological and operational improvements to enhance the levels of safety, capacity, efficiency and environmental sustainability associated with current and likely future aviation business models in a holistic manner, hence by specifically improving the design, manufacturing, operation and lifecycle management of aircraft. In the operational domain, significant improvements are envisioned from the implementation of novel concepts and technologies in the Air Traffic Management (ATM) and avionics domain, in line with the evolutions originally envisioned within the Advisory Group for Aerospace Research and Development (AGARD) of the North Atlantic Treaty Organization (NATO), the Group for Aeronautical Research and Technology in Europe (GARTEUR) and the Future Air Navigation Systems (FANS) special committee of the International Civil Aviation Organization (ICAO) in the 1980s [1]. The ATM relies on a large set of operational measures to fulfil its mission of preventing collisions and promoting an ordered and expedite flow of air traffic [2, 3]. In the current centralised command and control-oriented ATM paradigm, these measures are typically based on amending the lateral, vertical and longitudinal navigation of aircraft as necessary, and can be performed at various operational timeframes. As depicted in Figure 1.1, these measures can only be ensured if suitable means are available to identify the
conflicts, negotiate a resolution and follow an alternative flight trajectory. For instrument flights in controlled airspace these essential functions are provided by Communication, Navigation, Surveillance, ATM (CNS/ATM) and Avionics (CNS+A) technologies. Human operators are still responsible for the majority of duties involved in the current ATM paradigm, which is largely procedural and hence more tactical than strategic in nature [4].

The Civil Air Navigation Services Organisation (CANSO) estimated that global ATM is currently between 92-94% fuel-efficient and that unrecoverable factors limit the attainable efficiency improvements to between 95-98% [6]. The International Air Transport Association (IATA) quotes figures of 12% fuel inefficiency in relation to ATM [7]. Efficiency improvements of individual ATM procedures implemented locally are thus deemed largely ineffective, especially in view of the foreseen steady traffic growth trends. In order to achieve the ambitious medium-long-term goals set for safety, efficiency and environmental sustainability, substantial improvements in the exploitation of the available airspace and airport resources are required.

Operational enhancements also have recognised potential benefits in mitigating environmental and health impacts of air transport, and effectively a number of
research activities in the last decades have explored these potential benefits. Some operational measures are potentially more effective than aircraft design enhancements, such as in the case of perceived noise and contrails. While some study led to the successful implementation of procedural improvements such as enhanced instrument departure and arrival routings, the effectiveness of these measures in the bigger picture is limited, especially due to their offline nature that rigidly constrains the flexibility of operations and does not cope efficiently with unforeseen airspace blockages. Therefore, research on CNS+A system implementations of optimised arrival and departure routing algorithms, which is still under way, will be crucial in delivering the required flexibility and versatility to facilitate online reconfigurations.

1.2. ATM and avionics modernisation

CNS+A technologies and operational concepts are expected to deliver some of the most significant contributions to enhancing the operational performances of air transport system. Although substantial progress was achieved since then, it is useful to mention some of the seminal works in the field of Air Traffic Management. Janic and Tosic undertook the modelling of terminal airspace capacity in the early 1980s [8]. Their analysis focussed on the basic case of a single runway in a relatively simple Terminal Manoeuvring Area (TMA) airspace. The study identified and modelled a number of presently well-known issues associated with air traffic. In particular, the throughput of the runway was identified as the key bottleneck of terminal operations. Effective arrival traffic sequencing solutions were then developed and became the basis of the presently enacted time-based operations. In 1992 Visser reviewed the state-of-the-art in the field of TMA traffic management [9], and identified the need for new automated systems to improve Communication, Navigation and Surveillance (CNS) for the by then forthcoming time-based operations. A further work [4] analysed the potential benefits offered by real-time trajectory optimisation algorithms in ground-based and airborne systems. The extensive adoption of Information Technology (IT) in support of ATM operations and the substantial progress in the optimal control theory for nonlinear applications prompted various researchers to investigate the opportunities and challenges
associated with the extensive adoption of automation in the forthcoming Trajectory Based Operations (TBO). Eurocontrol’s Programme for Harmonised ATM Research in Europe (PHARE) was undertaken and following its important results [10-12], a number of researchers have pursued more in-depth studies on automated ATM systems aimed at assisting the decision-making of human Air Traffic Controllers (ATCo), and even substituting him for a predefined set of tasks. In particular, Robinson [13], Erzberger [14] and Mueller [15-17] investigated ground-based ATM systems for automated online 4DT operations.

The growing political and industrial support around the evolutions initially envisioned by AGARD, GARTEUR, FANS and subsequent proposals ultimately led to the launch of major programmes such as the Single European Sky ATM Research (SESAR) in Europe and of the Next Generation Air Transportation System (NextGen) in the United States, as well as a number of other related initiatives around the globe. NextGen is a major collaborative research initiative pursuing ATM modernisation, operational improvements, technology maturation, scientific knowledge and integrated modelling, alternative jet fuels and targeted policy measures [18]. SESAR and the Clean Sky Joint Technological Initiative (JTI) for Aeronautics and Air Transport (Clean Sky) are the leading programmes outlining the future of air transportation in Europe by addressing both operational improvements and environmental issues [19-21]. These initiatives were established with the particular aim of governing and financially sustain the aviation modernisation in a harmonised framework. The Collaborative Actions for Renovation of Air Traffic Systems (CARATS) is the main ATM modernisation programme in Japan, which has entered its implementation phase after the long-term vision and roadmap were defined in 2010 [22]. Comprehensively, these major international programmes support the evolution of ATM into a highly automated, integrated and more collaborative system, allowing a more flexible and efficient management of airspace and airport resources through higher levels of automation and more accurate navigation to improve the capacity and maximise its exploitation. The ICAO’s recently established Aviation System Block Upgrades (ASBU) framework builds upon these major air navigation improvement programmes, aiming at the harmonised evolution of the CNS+A systems capabilities. An ASBU block consists of several modules, each relating targeted operational improvements with the governing
standards, procedures, technology and equipage required to implement them. The initiatives led by the National Aeronautics and Space Administration (NASA) in the United States resulted in creation of the Environmentally Responsible Aviation (ERA) project to explore and document the feasibility, benefits and technical risks of advanced vehicle concepts and technologies that will reduce the impacts of aviation on the environment. The outcomes of such initiatives have already resulted in improved fuel efficiencies, reduced noxious gas emissions and noise levels [23, 24].

In the Asia-Pacific region, a number of Air Navigation Service Providers (ANSP) formed the Asia and Pacific Initiative to Reduce Emissions (ASPIRE) in 2008, with the aim of trialling enhanced operational procedures. ASPIRE implements a pragmatic approach of targeting city-pairs where several “best practice” green procedures can be applied in concert to realise cumulative benefits. The ASPIRE Strategic Plan relates each ASPIRE “best practice” green procedure to an ASBU module. In Australia, optimised ATM procedures such as tailored arrivals [25] and the green Required Navigation Performance (RNP) project [26] have been implemented. These national initiatives are aligned with those of the Asia-Pacific region with Australia’s involvement in the ASPIRE programme [27]. Optimised Flextracks have been also established, allowing long-haul traffic to maximise the benefit offered by favourable winds in the relative low traffic densities encountered in the southern hemisphere. Soliciting aircraft equipage evolutions is one of the key ASBU objectives, but the relative youth of fleets in the fastest growing markets such as Asia-Pacific and South-America ensures that advanced CNS+A equipage is more commonly available where it is mostly needed in the future [27]. A significant number of concepts were proposed for implementation as part of the major ATM modernisation programmes. Figure 1.2 depicts the concepts selected for implementation as part of the SESAR ATM master plan [28]. The measures are classified based on their relevance for en-route operations, Terminal Manoeuvring Area (TMA) operations, airport operations and on the network. Green boxes represent Pilot Common Project (PCP) essential operational changes and blue boxes represent new essential operational changes.
Increasingly higher amounts of information will be collected, analysed and shared among ground-based systems and airborne avionics to more effectively deal with unpredicted events and mitigate disruptions. In order to optimally exploit these quickly growing amounts of information made available, an increase in automation support and a move away from the centralised command and control-oriented ATM paradigm towards more distributed/collaborative planning are deemed necessary. This will involve a redistribution of current ATM functions and services to other key players such as Airline Operation Centres (AOC) and flight crews, to improve the efficiency of the system as a whole. In summary, the key CNS+A advances identified by the major aviation modernisation programmes around the globe include [27, 29, 30]:

- Four Dimensional (4D) trajectory-based operations;
- Higher levels of Collaborative Decision Making (CDM) to allow all involved parties to participate in the enhancement of system performance by sharing and accessing more accurate and updated information;
• Role shifting of ATM from command and control oriented units to a highly automated and collaborative decision-maker in an interoperable network-centric environment;

• Dynamic Airspace Management (DAM) for an optimised exploitation of airspace capacity;

• Improved avionics and ATM systems Human Machine Interface and Interaction (HMI$^2$) design, interoperability and higher levels of automation;

• Performance-Based Communication, Navigation, Surveillance (PBC/PBN/PBS), enabling Performance-Based Operations (PBO).

In order to introduce these innovative concepts and ultimately progress along the planned evolutionary pathways, a number of new CNS+A technologies were considered essential, including:

• Avionics and ATM Decision Support Systems (DSS) featuring automation-assisted 4D Trajectory (4DT) planning and negotiation/validation functionalities;

• Enhanced ground-based and satellite-based aeronautical communications systems, involving a substantial exploitation of data-links;

• Enhanced navigation accuracy and integrity by means of Ground-, Avionics- and Satellite-Based Augmentation Systems (GBAS/ABAS/SBAS), promoting Global Navigation Satellite Systems (GNSS) as primary means of navigation;

• Enhanced ground-based and satellite-based surveillance, including Automated Dependent Surveillance Broadcast (ADS-B) and self-separation;

• A System Wide Information Management (SWIM) network.

The novel automated avionics and ATM DSS shall facilitate aircraft in flying user-preferred optimal flight paths (intents), limiting ATCo intervention to strategic high-level management and emergency assistance, so to decrease the overall workload without compromising the situational awareness. In order to do so, these DSS are expected to dependably produce feasible 4DT solutions that resolve the impending conflicts, while complying with the timeframe allocated for the complete ATM transaction. The increase in flexibility resulting from exploiting these real-time 4DT
planning and negotiation processes will enable greener and more efficient flight profiles even in presence of unforeseen perturbations.

In order to be economically and operationally viable, novel CNS+A technologies and operational concepts must be developed and deployed in a phased manner. As depicted in figure 1.3, the stages for such evolution within SESAR were defined based on the capability and consist of [28]:

- **Time-based Operations**: for which strategic and tactical ATM and Air Traffic Flow Management (ATFM) actions are aimed at an optimal traffic synchronisation. The time of arrival of traffic at specific points is the fundamental metric being estimated, managed and monitored by all the involved entities both on ground and airborne.

- **Trajectory Based Operations** (TBO): focusing on a further-evolved predictability, flexibility and environmental sustainability of air traffic, unleashing additional capacity. This stage involves the evolution of the legacy flight plans into dynamically managed 4DT, which become the continuously updated and negotiated reference plan for the aircraft mission.

- **Performance-Based Operations** (PBO): for which all the available CNS performance is exploited to establish a high-performance, network-centric, collaborative, integrated and seamless ATM system, supporting high-density operations. In this stage, ATM services are customised depending on the highest level of CNS performance provided by the involved traffic, enabling a further enhanced exploitation of airspace capacity.

![Figure 1.3: SESAR capability-based deployment stages [28.]](image-url)
TBO are based on the adoption of 4DT defining the aircraft’s position in three spatial dimensions (i.e., latitude, longitude and altitude) and in time from origin to destination, and on the associated precise estimation and correction of current and predicted traffic states [31]. Each aircraft is assigned a 4DT contract, which is determined by means of a CDM process involving novel ground-based and airborne DSS, evolving from the original reference business trajectory. Increased efficiencies and higher throughputs are obtained in a CNS+A context by actively managing 4DT. In the PBO context, the ATM services will be matched to the performance capability of aircraft. Airline deploying higher-grade PBO equipment will benefit easier access to congested areas and time slots. The regulations will impose requirements in terms of system performance rather than in terms of specific technology or equipment. Some of these CNS+A technologies are already approaching the market, while early stage advancements in the juridical framework are accommodating enhanced operational capabilities. Since most of the innovations currently being implemented were conceived from the operational point of view, the concurrent development of an adequate theoretical framework and the execution of extensive modelling and simulation activities are crucial.

1.3. Environmental sustainability of aviation

The aviation sector is directly responsible of a number of adverse effects on the environment and on the living beings. Although ground support vehicles and infrastructure participate to the overall total impacts, the mitigation of impacts directly associated with the aircraft flight mission sees a considerable involvement by researchers and policy-makers worldwide. Figure 1.4 depicts the main environmental impacts associated with the various flight phases of typical air transport missions. While some pollutants such as carbon dioxide (CO₂), unburnt hydrocarbons (HC), soot and sulphur oxides (SOₓ) are understood to adversely affect the environment along the entire flight, others either do not always eventuate, as in the case of condensation trails (contrails), or their adverse effects are reputed significant mostly in proximity of the ground, as in the case of aircraft noise and carbon monoxide (CO). Nitrogen oxides (NOₓ) in addition to their toxicity to living beings when emitted at higher concentrations in proximity of the ground
(tropospheric NO\textsubscript{X}) are also known to trigger a family of chemical processes ultimately causing the depletion of the ozone layer, and are thereby associated with a positive radiative forcing (stratospheric NO\textsubscript{X}).

The Advisory Council for Aviation Research and Innovation in Europe (ACARE) has set specific target goals to improve the environmental sustainability of aviation in its Strategic Research Agenda (SRA) and Strategic Research and Innovation Agenda (SRIA) [32, 33]. In Europe, air transportation causes approximately 3% of the total greenhouse gas emissions [34]. The Clean Sky programme implements the ACARE SRIA FlightPath 2050 objectives in multiple phases. Similarly to Clean Sky, other aviation modernisation programmes worldwide have set ambitious environmental objectives to reverse the adverse effects of air transport on the environment. Most programmes are attempting the achievement of the environmental targets by taking a holistic approach, hence encompassing the design, manufacturing, operation and lifecycle management of aircraft. In the operational domain, although the paradigm “less fuel consumed equates to less emissions” still meets the widest consent and is aligned with the financial interests of aircraft operators, it is evident that a number of
operational measures could be very effective in further mitigating environmental and health impacts, potentially more than design enhancements, such as in the case of perceived noise and contrails. On the global scale, the environment branch of the Air Transport Bureau (ATB) is in charge of the initiatives of ICAO in the area of environmental protection. The technical support for the implementation of these initiatives in terms of Standards and Recommended Practices (SARPS) is provided by the ICAO’s Committee on Aviation Environmental Protection (CAEP). The ICAO environmental branch periodically reviews the progresses achieved as well as the solutions proposed and adopted at national and international levels, leading to the publication of a comprehensive periodic environmental report [35]. Current emphasis is on improving the accuracy of aviation-related pollution figures by enhancing the measurement and estimation methods. Figures 1.5 and 1.6 represent the current state-of-the-art characterisations of Local Air Quality (LAQ). The values are yearly averages.

Figure 1.5: Predicted annual NO\textsubscript{X} and CO\textsubscript{2} concentrations (\(\mu g/m^3\)) in a medium-size airport, based on Landing Take-Off (LTO) cycle counting [35].
With respect to the modelling and assessment of the environmental impacts of aviation, researchers are now extending the investigation into the complex interdependencies of thermochemical processes associated with pollutant emissions and in the development of new measurement and estimation techniques to more accurately characterise aircraft emissions. In particular, new measurement systems are required to characterise aircraft emissions at higher spatial and time resolutions, thus allowing research in trajectory optimisation and more generally on aviation sustainability to more accurately formulate the mathematical problem and deliver more effective solutions [36-38]. Currently, in non-laboratory contexts, pollutant emissions are most commonly measured by means of air quality sampling/atmospheric sounding or model-based estimations. Air quality sampling stations in the vicinity of larger airports are the main source of pollution figures, but the wide range of neighbouring polluters and the cumulative aerodynamic advection and diffusion of pollutants affect the accuracy of the measurements and thus prevent discerning individual aircraft and specific phases of their flight. Pollutant emissions can also be empirically estimated based on commonly equipped thermodynamic and mechanical sensors that measure Fuel Flow (FF), Turbine Entry Temperature (TET), Exhaust Gas Temperature (EGT), Engine Pressure Ratio (EPR), engine rotation speeds (N₁, N₂, ...), acoustic vibrations and other thermomechanical parameters. These estimations rely upon empirical and semi-empirical models based on nominal engine and fuel conditions. Although typically adequate for Multi-Objective Trajectory Optimisation (MOTO) studies, these models show limited accuracy in
estimating emissions, as anomalies in engine or fuel conditions can void key assumptions and compromise the validity of these model-based estimations. More details on the assessment of environmental impacts of aviation are given in [39-45].

1.4. Problem statement

On-board algorithms for generating optimum trajectories were considered computationally realisable as early as the 1970s [46], and effectively the following two decades have seen the implementation of a number of new automatic flight and flight director modes for most phases of flight, targeting in particular the vertical guidance and optimised standard turns. Although the savings achieved implementing these solutions were very significant with respect to previous operational paradigms, these algorithms did not consider the lateral rerouting nor real-time information updates, therefore it has been proposed that more substantial gains can be achieved by adopting online optimisation functionalities for strategic and tactical replanning of 4D flight trajectories in an intent-based network-centric ATM scenario. As previously mentioned, 4DT represent a substantial evolution when compared to conventional flight plans and their associated limitations. From an operational perspective, since flight plans are submitted offline as a substantially static entity, unforeseen weather and traffic perturbations have the effect of progressively compromising their validity and optimality. Optimal 4DT planning and negotiation/validation capabilities are key features of novel avionics and ATM DSS designs, and introduce new variables and equations [4]. In addition to consistently and reliably fulfilling the transaction time requirements for strategic and tactical ATM operations, the 4DT optimisation functionalities will have to integrate all constraints and restrictions associated with air navigation rules, air traffic services and real traffic conditions in order to be ultimately implemented in airborne avionics and ground-based ATM DSS. The optimisation performed by current online DSS, such as FMS, is effectively still targeting the vertical flight profile only, as direct great circle arcs and constant-bank turns are implemented for lateral path planning. The generated route may be significantly suboptimal on a larger scale when considering wind field and airspace blockage due to traffic congestions, convective weather, volcanic ash and other localised phenomena. Furthermore, current DSS
address the needs arising from operating in dense traffic and complex airspace by allowing the addition of various constraints in terms of airspeed, altitude and flight path. Constraints and route amendments are mostly introduced through manual intervention of human operators (i.e. flight crews and ATCo), therefore their total number and update frequency are necessarily limited, and this is known to affect the routing optimality when traffic or weather conditions are rapidly evolving. Additionally, the unconstrained optimal flight profile is not regenerated nor considered between various route amendments, and this lead to progressive deviations from optimality. It is known that the occurrence of unfeasible constraints requires the implementation of MOTO techniques such as sequential goal programming. Another limitation of current systems requiring the implementation of MOTO techniques is due to the very narrow set of optimality criteria and their limited modelling. In particular, current DSS only consider direct operating costs related to fuel consumption and flight time and do not take into account the wind field and airspace blockage due to traffic and atmospheric phenomena. The limited optimality set does not allow the mitigation of actual environmental impacts, since their dependencies on geographic location (e.g. perceived noise, contrails), altitude and throttle settings (e.g. soot, unburned hydrocarbons, carbon monoxide, nitrogen oxides) are not captured by current algorithms. These limitations are further exacerbated by the growth of traffic. Increasing R&D initiatives are therefore addressing more effective and versatile mission and trajectory planning, optimisation and management functionalities incorporating MOTO algorithms to be implemented in novel airborne and ground-based CNS+A DSS. For instance, CNS+A implementations of MOTO are being explored as part of international research programmes such as the Systems for Green Operations Integrated Technology Demosrtator (SGO-ITD) of Clean Sky [47-59]. To implement the envisioned improvements in terms of safety, capacity, efficiency and environmental sustainability of air traffic operations, it is essential to correlate the trajectory design parameters to the targeted benefits. This can be done by introducing suitable models, objectives and constraints in MOTO algorithms to plan the optimal 4D intents and at the same time estimate the environmental impacts and operational costs [27]. Figure 1.7 represents the concept of MOTO to tackle multiple operational, economic and environmental criteria in the aviation context. In order to obtain optimal 4D intents
with respect to the set optimality objectives, the MOTO suite comprises a number of essential aircraft, environmental and operational models. These models include local/global weather, operational costs, pollutant emissions, airspace structure, contrails and aircraft noise. As the emphasis is on the noise perceived by the population on the ground the aircraft noise model must be complemented by suitable demographic distribution and digital terrain elevation databases.

![Multi-objective trajectory optimisation concept](image)

Figure 1.7: Multi-objective trajectory optimisation concept [27].

### 1.5. Motivation and significance

New airborne avionics and ground-based ATM DSS are being developed in order to enable 4D-TBO operational concepts and capabilities. A number of challenges are known to affect the application of MOTO functionalities in the online contexts, and particularly include:

1. computationally fast and efficient MOTO algorithm implementations, meeting the strict solution time requirements;

2. availability of secure, high-integrity and high-throughput air-to-ground data links for negotiation and validation of the generated 4DT intents;
3. availability of efficient and effective Human Machine Interface and Interactions (HMI²) formats and functions to enable the recalculation, review and customisation of optimised 4DT solutions by the human operators;

4. caching, progressive refinement and adaptation of optimal 4DT solutions within the algorithms to ensure the required reliability levels for dependable adoption of MOTO in automated 4DT planning functionalities;

5. synchronisation of 4DT predictions, constraints and performances between the ground-based systems and the aircraft for mathematical consistency;

6. capability to automatically generate and handle in real-time a considerable number of constraints arising from operation in dense traffic conditions, of which some can be unattainable and shall prompt recomputation of 4DT intents for a number of aircraft.

Research is currently addressing these limitations by enhancing the numerical implementations while at the same time outlining the operational and equipage requirements in view of the strict certification requirements.

1.6. Research aim and objectives

The aim of this research is to develop novel multi-objective trajectory optimisation and dynamic airspace functionalities suitable for integration in avionics and ATM systems to enhance the efficiency and environmental sustainability of air traffic. The following specific objectives were set:

1. **Review the state of the art in ATM/ATFM and avionics modernisation, particularly assessing the evolving requirements for online trajectory planning and the 4D-TBO context.**

2. **Perform a critical review of the literature on trajectory optimisation in the aviation domain, addressing both the theoretical framework and the proposed solution approaches.**

3. **Develop Multi-Objective 4D Trajectory Optimisation (MOTO-4D) algorithms suitable for integration in the next generation of ATM and avionics systems.**
4. Identify suitable models to integrate targeted operational, environmental and economic criteria in the formulated trajectory optimisation problem.

5. Develop suitable sequencing and spacing algorithms for deconflicting optimal 4DT intents when optimising the trajectories of multiple aircraft.

6. Develop suitable algorithms to translate the optimal trajectories generated by the MOTO-4D algorithms into 4DT intents complying with current standard to be exchanged on 4DT data-links, while retaining most of the optimality qualities of the original solution.

7. Define the functional architecture of a novel ATM DSS integrating the developed MOTO and DAM algorithms.

8. Perform numerical simulation activities in representative conditions to assess the viability of the developed algorithms.

A variable set of models and optimality criteria are to be considered based on flight phase, aircraft and airspace states. Additionally, different trajectory optimisation methods can be considered to better address the specificities associated with the different timeframes and/or different operational contexts. Due to the mostly varying altitude and constant level flight conditions respectively prevailing in the two scenarios, the TMA and en-route contexts are notably different. Consequently, two different multi-objective trajectory optimisation algorithms are developed: one algorithm for the TMA context and one for the en-route context.

In order to implement and numerically evaluate the newly developed functionalities, the project involved the conceptual development of an innovative ATM system software prototype featuring automated 4DT Planning, Negotiation and Validation (4-PNV) algorithms. The models adopted to introduce the environmental performance within the 4DT optimisation framework also support an estimation of the environmental impacts.

The work presented in this PhD Thesis is extensive in scope and addresses multiple topics. This is deemed beneficial to develop ATM system functionalities that successfully meet the contemporary issues of aviation and ATM research and development. A list of relevant publications developed based on the work carried out during this PhD research is given in Appendix B.
1.7. References


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Chapter 2

Overview of air traffic flow management and optimal scheduling principles

This chapter reviews the fundamental problems, the key mathematical models and the typical solution approaches that are investigated as part of Air Traffic Flow Management (ATFM) research and development programs. An introduction to the underlying Demand-Capacity Balancing (DCB) problem is given and the key models for capacity and demand are summarised. Three fundamental DCB problem formulations are described in particular: the sequencing problem, the arrival-departure problem and the gate-to-gate problem. Furthermore, brief mentions to conflict detection techniques and to more advanced ATFM concepts are included. The ATFM body of knowledge presented in this chapter is exploited for the development of optimal scheduling and conflict detection algorithms that are involved in the arrival sequencing and spacing algorithms implemented as part of this research project for the TMA environment.

2.1. Background

ATFM involves a number of measures to accomplish the overarching ATM mission of supporting a safe, efficient and expedite flow of air traffic. ATFM specifically aims at continuously matching the air traffic demand with the available capacity of airspace and airport resources. Both long-term (strategic) and short-term (pre-tactical) measures can be adopted to resolve perturbations arising due to unpredicted weather and capacity disruptions. The effectiveness of these measures largely depends on the amount, accuracy and timeliness of information exchanged. As a result, human operators crucially depend on forecast models, technology enablers and ATM DSS for taking better-informed and more effective decisions. The novel
functionalities that are being developed for 4D-TBO shall address the multiple superimposed problems that characterise the operation of airports and airspace sectors in a highly interconnected and interdependent air route network. Perturbations or disruptions occurring locally due to a variety of reasons can quickly affect a growing number of airports, airspace sectors and flights, unless effective mitigation measures are put in place. Strategic and tactical ATFM problems are all formulated as mathematical optimisation problems, in which either an individual solution or a set thereof is to be identified. Global solution approaches are explored to promote optimised flows across large number of airports and wider airspace regions. Optimal solutions are the ones for which the real scenario resembles the model implemented as part of the optimisation problem and all assumptions introduced remain valid. The uncertainties in input data and estimation models necessarily void some key assumptions, therefore pseudo-optimal solutions are the ones most typically pursued [60]. Uncertainties in input data depend on the limited amount and timeliness of information shared between flight crews, ATM operators, Air Navigation Service Providers (ANSP) and Airline Operations Centres (AOC).

2.2. Demand-capacity balancing

Air traffic demand is highly variable in space and time. Significant oscillations are evident throughout a single day, along the week and among the seasons. The highly variable demand faces a substantially constant capacity of ground infrastructures and airspace configuration, and this underlying mismatch contributes to major disruptions and delays if not properly addressed. Demand-Capacity Balancing (DCB) is the key objective of ATFM across all its implementations and timeframes. DCB aims to regulate the flow of traffic and the airspace configuration so that demand matches available capacity at all times. DCB is most useful when imbalances are predicted early enough to avoid tactical ATC intervention, and when this prerequisite is satisfied, Traffic Flow Management Initiatives (TFMI) performed in the en-route context yield the highest effectiveness. Two fundamental approaches are considered, typically in synergy, to pursue DCB in the en-route context: Airspace Flow Programs (AFP) and Airspace Management (ASM). A number of progressive improvements were proposed for both approaches and are currently being developed as part of all
major ATM modernisation initiatives. Challenges to the effectiveness of ATFM measures in the enroute context arise from uncertainties in trajectory prediction and weather forecasts over the longer ranges involved. Fig. 2.1 conceptually represents the relation between the predicted traffic demand and the planned airspace capacity for an Area Control Centre (ACC) facility. As depicted by the step shifts of the blue line in the figure, the capacity incurs discrete changes as airspace sectors are consolidated (i.e. merged) and de-consolidated (i.e. split). An optimised timing of these consolidations and de-consolidations can help minimising the under-capacity instances, by optimally fitting the predicted demand. This approach is referred to as Airspace Management (ASM) and is conceptually depicted in Fig. 2.2.

![Graph](image)

**Figure 2.1.** Conceptual representation of the relationship between original demand and capacity in an ACC.

An overall maximum ACC capacity associated with the available workforce, CNS/ATM infrastructure and airspace geography poses a hard limit for which sector de-consolidations may not be sufficient on their own to prevent under-capacity situations. This is exemplified by a morning traffic peak in Fig. 2.1 and 2.2. In these circumstances TFMI either in terms of GDP or AFP are required, re-distributing the demand on a more uniform profile, as depicted in Fig. 2.3.
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Figure 2.2. Representation of ASM initiatives in terms of sector consolidations and de-consolidations (green arrows).

Figure 2.3. Simultaneous changes to demand and capacity due to the concurrent implementation of AFP and ASM initiatives.
The importance of safeguarding some residual capacity to allow further tactical and opportunistic ATFM actions is also conventionally recognised. At present, Decision Support Tools (DST) for state-of-the-art ATFM and DAM are based on either analytic estimation or numerical simulation [61]. A very commonly adopted computing feature in ATFM decision making is the Fast-Time Simulation (FTS), and the associated critical evaluation of the resulting information. A major aspect for current and future DST development is the identification of an effective visualisation methodology for the complex underlying 4D reality. For example, simplified network visualisation models commonly fail to express the proximity of two closely spaced route segments. Studies of HMI\textsuperscript{2} factors are thus particularly crucial in the design of an effective ATFM DST. A recurring issue impeding simplified ATFM formulations is associated with the complex interdependencies in arrival and departure flows of closely spaced airports, that is, the ones sharing the same Terminal Manoeuvring Area (TMA). Major areas of research for ATFM DST include HMI\textsuperscript{2} factors to visualise complexity and dynamic density concepts, merging metering with sequencing (i.e. the integration with Arrival and Departure Management (AMAN/DMAN) operations) and the development of regional ATFM concepts that support multiple ANSP.

International initiatives seek to harmonise the methodologies by which different authorities determine and publish deemed capacities for resources [27]. For example, sector capacity can be specified by occupancy (the maximum number of aircraft that are simultaneously under the jurisdiction of the controller managing the ATC sector at any time) or by flow rate (the maximum number of aircraft that can enter that particular ATC sector in a specified period). Most authorities identified the need to adopt capacity limits correlated with ATCO workload. This yields recognised safety advantages, but there is no single standard for such workload models. Workload is dependent on a complex interaction of several factors, many of which are difficult to predict in advance. Traffic complexity permits a simplified estimation of ATCO workload. It seeks to characterise the cognitive effort associated with a particular configuration of airspace and air traffic. A number of unpredictable factors can manifest during operations, including [27]:

- Delays on departure.
• Queuing for take-off.
• Unscheduled, *pop-up* traffic.
• Winds aloft differing significantly from the forecasts.
• Weather deviations and avoidance of turbulence.
• Tactical ATC intervention for separation and sequencing.

These factors introduce significant uncertainties into the predicted traffic demand, with some (like winds aloft) having the potential to impact not just individual flights but clustered flight flows due to their geographic extent and relatively long duration.

Following the ICAO manual on collaborative ATFM, it can be noted that DCB can be implemented for resources such as airports, airways, fixes or sectors by employing specific ATFM measures across all phases of operation. Strategic DCB can be addressed by measures such as IATA airline schedule planning, airport slot co-ordination, ATM planning, airspace and procedure design, capacity analysis and performance prediction. Pre-tactical DCB measures include weekly and daily CDM meetings and Flexible Use of Airspace planning while tactical DCB measures include:

• Exit Separation Management and miles/minutes-in-trail techniques;
• Fix balancing and movable feeder fixes;
• Ad-hoc and pre-defined reroutes (conditional, preferential, alternate, “playbooks”);
• Level capping and tunnelling;
• Minimum departure intervals;
• Ground delay and ground stops;
• Airborne delay absorption and holding.

Traditional ATM automation systems provide some level of support for implementing ATFM measures, particularly during the tactical phase of operations. Emerging ATFM DSS provide additional capabilities, including [27]:

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- Demand prediction based on scheduled, filed or live traffic data and/or analytical estimation/numerical simulations;
- Fast-time simulation and “what-if” modelling of ATFM measures;
- Graphical visualisation of traffic (air situation display) and load (bar charts or similar);
- Common situational awareness (display of weather, aeronautical messages, constraints, etc.);
- CDM communication mechanisms (increasingly web-based);
- Compliance reporting and post-event analysis.

2.3. Airport throughput modelling

The hourly capacity of an aerodrome depends both on the number and layout of runways and their related navigation facilities as well as on a multitude of other factors associated with other ground infrastructures as well as local geographic and meteorological characteristics. Consequently, airport capacity frequently behaves as a random variable in relation to shifts in weather conditions, traffic composition, infrastructure conditions and human factors. Meteorological variables that significantly affect capacity include wind direction and intensity, precipitation, visibility, hazardous clouds and more severe hazards. The effects are exacerbated as different aircraft have very diverse weather capabilities, with some being capable of almost all-weather operations and others completely inhibited from operating. The resulting capacity of airports in terms of departure throughput and arrival acceptance rates can vary considerably and abruptly between zero and the maximum certified values. A maximum certified capacity can be calculated based only on the runway characteristics and layout. The Annex 11 of the Chicago Convention on Civil Aviation refers to the Procedures for Air Navigation Services-ATM (PANS-ATM) and to the Regional Supplementary Procedures (Doc. 7030) for the specific provision of separation minima based on the prevailing circumstances [2]. Minimum separation distances and intervals both on approach and on departure are dictated by the most restrictive requirements in terms of longitudinal separation. Whenever
surveillance information is available to the ATCo, the minimum lateral (horizontal) separation distance is prescribed in relation to the surveillance systems performance. Currently, the standard minimum lateral separation distance as recommended by ICAO based on either Primary Surveillance Radar (PSR), Secondary Surveillance Radar (SSR), ADS-B and/or Multi-Lateration (MLAT) systems is $r = 5.0 \text{ NM}$ [3]. According to ICAO, the minimum lateral separation distance can be reduced to $r = 3.0 \text{ NM}$ whenever the surveillance performance available so permits, and a further reduction to $r = 2.5 \text{ NM}$ is finally granted for aircraft established on the same final approach track within 10.0 NM from the runway threshold, provided that approach speeds, runway occupancy, braking action and wake turbulence factors are known and continuously monitored by the ATCo [3]. This further reduced minimum separation can be achieved only when a runway occupancy time consistently below 50 s is statistically demonstrated and when PSR and SSR surveillance with an update rate of at least once every 5 s is available with coverage reaching up to and including the short final [3]. This surveillance performance is normally provided only by approach-grade PSR and SSR located within or in close proximity to the airport. En-route grade PSR and SSR are typically not sufficient, as the update rate is reduced to achieve longer detection ranges and coverage at low altitudes is limited to minimise ground clutter and/or due to the radar being located farther away from the airport.

Wake turbulence separation standards are conventionally defined in terms of nautical miles (NM) on approach and in minutes on departure, as these are the metrics being most easily monitored by ATCo respectively in approach control and in the aerodrome control tower duties. In particular, wake turbulence separation criteria were defined by introducing 4 distinct categories based on the reference Maximum Take-Off Weight (MTOW) of aircraft: light (up to 7,000 kg), medium, heavy (more than 136,000 kg but not including AIRBUS A380) and super-heavy (currently only the AIRBUS A380). The minimum longitudinal separation distances on approach as recommended by ICAO are shown in table 2.1, whereas the minimum departure separation intervals are shown in table 2.2 [62].
Table 2.1. Minimum longitudinal separation distance on approach due to wake turbulence.

<table>
<thead>
<tr>
<th>Leading</th>
<th>Super-Heavy</th>
<th>Heavy</th>
<th>Medium</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>Super-Heavy</td>
<td>(not required)</td>
<td>(not required)</td>
<td>(not required)</td>
<td>(not required)</td>
</tr>
<tr>
<td>Heavy</td>
<td>6 NM</td>
<td>4 NM</td>
<td>(not required)</td>
<td>(not required)</td>
</tr>
<tr>
<td>Medium</td>
<td>7 NM</td>
<td>5 NM</td>
<td>3 NM</td>
<td>(not required)</td>
</tr>
<tr>
<td>Light</td>
<td>8 NM</td>
<td>6 NM</td>
<td>5 NM</td>
<td>(not required)</td>
</tr>
</tbody>
</table>

Table 2.2. Minimum departure separation interval due to wake turbulence

<table>
<thead>
<tr>
<th>Leading</th>
<th>Super-Heavy</th>
<th>Heavy</th>
<th>Medium</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>Super-Heavy</td>
<td>(not required)</td>
<td>(not required)</td>
<td>(not required)</td>
<td>(not required)</td>
</tr>
<tr>
<td>Heavy</td>
<td>2 min (not required)</td>
<td>(not required)</td>
<td>(not required)</td>
<td>(not required)</td>
</tr>
<tr>
<td>Medium</td>
<td>3 min 2 min</td>
<td>(not required)</td>
<td>(not required)</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>3 min 2 min</td>
<td>2 min</td>
<td>(not required)</td>
<td></td>
</tr>
</tbody>
</table>

Whenever a wake turbulence separation is not required, the minimum longitudinal separation distance equates to the minimum lateral (horizontal) separation distance. The minimum longitudinal separation distances on approach are translated into separation intervals by considering the ground speed of the leading aircraft, $v_i$, the ground speed of the trailing aircraft, $v_j$, the occupancy time of the leading aircraft on landing, $o_i$, the prescribed minimum longitudinal separation distance between the leading and the trailing aircraft due to wake turbulence, $s_{i,j}$ and the minimum lateral (radar) separation, $r$. Two alternative cases are obtained, depending on the relationship between the ground speed of the leading aircraft and the one of the trailing aircraft, and in particular [63]:

$$T_{i,j} = \max \left[ \frac{r + s_{i,j}}{v_j} - \frac{r}{v_i}, o_i \right] \text{ when } v_i > v_j \quad (2.1)$$

$$T_{i,j} = \max \left[ \frac{s_{i,j}}{v_j}, o_i \right] \text{ when } v_i \leq v_j \quad (2.2)$$

The minimum separation criteria on approach and on departure can lead to an inefficient runway usage, especially when the mix and sequence of traffic is suboptimal and/or highly inhomogeneous. In order to eliminate these sequence-related inefficiencies, airports traditionally resort to mixing arrival and departure
traffic, as this alternation allows neglecting the wake turbulence-related longitudinal separation requirements. Consequently, when the departure routings do not intersect approach routing, the total throughput in the case of mixed arrivals and departures is higher than the cases of arrivals-only and departures-only, and this is shown in Figure 2.4a (single runway operations), and Figure 2.4b (multiple runways operations).

Airports where surveillance systems and/or precision approach navigation facilities are limited or not available can implement analogous runway throughput estimation methods, though the more restrictive procedural minimum separation criteria will be predominant.

As mentioned before, further factors affecting arrival and departure throughputs at a particular airport include ground-based infrastructure as well as the geographic and meteorological characteristics of the airport region. In particular, the close proximity of mountain ranges or prominent peaks, as well as the frequent occurrence of low visibility, precipitation and/or strong winds not oriented along the runways are some of the factors that can considerably reduce an airport’s maximum hourly movements. In terms of ground-based infrastructure, the movements can be limited in relation to taxiway and apron layouts, number of parking stands and capacity of airport services including turnaround and refuelling vehicles and personnel, as well as security and fire-fighting services due to safety.

Figure 2.4. Total arrival and departure throughputs in case of single runway operations (a) and multiple runways (b).
2.4. ATC sector capacity modelling

According to ICAO, the capacity of an ATS unit depends on [3]:

- the level and type of ATS provided;
- the complexity of the sector/area/aerodrome and of the associated route structure;
- ATCO workload, including control and coordination tasks to be performed.
- navigation accuracy of aircraft and CNS system performance available;
- availability of support and alert systems;
- weather;

The ATS authority is responsible of assessing and declaring the capacity of control areas, control sectors and aerodromes [3]. Capacity should be expressed as the maximum number of aircraft which can be accepted over a given period of time in the concerned area [3], and this most often translates into hourly traffic flow limits. Some recommendations for the assessment and definition of ATS capacities were provided in Doc. 9426 Air Traffic Services Planning Manual [64]. The method developed by the United Kingdom Directorate of Operation Research and Analysis (DORATASK) and method developed by the German Messerschmitt Bolkow und Blohm (MBB) have been extensively used to estimate the effort required for management of individual traffic and to determine safe total hourly capacities for each sector [64]. Both methods can be applied both to TMA and to enroute sectors.

Adopting workload as the defining factor for ATC sector capacity is beneficial from the safety perspective, but unfortunately the workload is dependent on a complex interaction between a number of factors [65]. A measurable of growing scientific relevance for estimating the human ATCo workload is the ATC complexity, which better captures the distribution of tasks associated with cognitive work levels characterising a particular configuration of airspace and air traffic. The DORATASK method discerns the time allocated to observable tasks such as deconfliction, the time spent on non-observable tasks such as planning and periods of recuperation. Time spent on observable tasks is recorded and used to develop simulations for accurate estimation. Time spent on non-observable tasks is empirically postulated based on some measurable parameter such as the number of flight strip updates. The total ATCo workload is calculated as a sum of time spent on
observable and non-observable tasks, plus a percentage reserved to recuperation. This percentage was initially modelled to increase along the work shift to account for the cumulative effect of uncertainties and to compensate for ATCo’s fatigue. Upon model validation, it was ultimately found that the average workload should have been limited to 80% on a continuous basis and that peak loads of up to 90% should not have occurred more than 2.5% of the total time [64]. The MBB method involves the identification of working units associated with working actions and the identification of an airspace-related factor defining the conflict probability [64].

Welch et al. [66, 67] investigated the current operational capacity model used by the FAA in the National Airspace System (NAS) and based on the identified limitations developed a promising improvement. The current model, named Monitor Alert Parameter (MAP) is used by FAA to trigger operational traffic alerts, mainly by monitoring the hand-over traffic amounts. This simplified approach accurately estimates the inter-sector coordination workload and its linear growth with traffic count. The higher traffic densities typical of smaller sectors, leading to higher conflict rates, are effectively considered as the theoretical capacity is linearly proportional to the transit time. As a result, within the NAS, sectors with transit times lower than 4 minutes were found to be rare, as their maximum capacities would be up to 7. On the other hand, recurring tasks grow linearly with traffic amount. Consequently, in the MAP model the capacity limit for large sectors is no longer dominated by the linear dependence on transit time but by a hard limit count (18 aircraft). The current model can be written as [67]:

$$N_{MAP} = \min\left\{ G \frac{T}{\tau_t} ; 18 \right\}$$

Where $N_{MAP}$ is the MAP capacity, $G$ is the workload intensity, $T$ is the transit time of the sector, $\tau_t$ is the mean hand-over service time. A workload intensity of 1 equates to the capacity limit in nominal conditions. The mean hand-over service time was found to be currently set to $\tau_t = 36$ s. The transit time for a particular sector can be either computed from statistical data or with an empirical estimation such as:

$$T = \frac{\sqrt{A}}{v_{TAS}}$$
Where \( A \) is the horizontal surface area of the sector, and \( \bar{\nu}_{\text{TAS}} \) is the average True Air Speed (TAS) of the traffic in the sector. Both estimates are fairly accurate for sectors with limited vertical extent, but may prove unfit for sectors with considerable vertical extension, as transit times may be significantly different at varying altitudes.

An allowance of plus or minus three aircraft was also identified. Consistently with the assumptions of the MAP model, a maximum safe throughput of one flight every 36 s, which equates to 100 aircraft per hour, was determined for sectors with transit times up to approximately 11 minutes. Above that figure the throughput decreases according to the limitation of 18-aircraft within the sector. The authors of the study identified that peak flow capacities of up to 120 aircraft per hour were statistically occurring, but only for sectors with transit times of \( 300 \leq T \leq 600 \). As a consequence, sectors with volumes between 5000 and 30000 nmi\(^3\) were identified having the highest flow capacities, but only if a higher density of the air route network was planned for increasingly smaller sectors, since the maximum flow along individual routes is constrained by the longitudinal separation requirements.

Considering these factors, the authors of the study determined that the MAP model consistently over-estimates the safe capacity of smaller sectors, and did not account for the flow directionality.

An enhanced model was therefore proposed, for which the total workload intensity consists of four aggregate components, including a “background” monitoring component \( G_b \), a recurring task component, a hand-over component and a deconfliction component as \( G \):

\[
G = G_b + \frac{\tau_r N}{P} + \frac{\tau_c N}{T} + \frac{B N (N + 1)}{Q}
\]  

(2.5)

Where \( B = 2 s_h s_v \nu_{12} \tau_c \), \( \tau_r \) is the mean service time associated with recurring tasks, \( P \) is the associated recurrence period, \( \tau_c \) is the mean service time associated with deconfliction tasks, \( s_h \) and \( s_v \) are the horizontal and vertical distances associated with separation violations, \( \nu_{12} \) is the mean closing speed of aircraft that incur separation violation and \( Q \) is the airspace sector volume. The dependency on the fraction of aircraft that are either climbing or descending more than 2000 ft (\( F_{ca} \)) within the sector can also be integrated. A further modification of this model to
account for weather impacts was proposed. In particular, the following expression is proposed [67]:

$$G = G_b + (\tau_r + \tau_{wr}F_w) \frac{N}{P} + (\tau_t + \tau_{wtr}F_w) \frac{N}{T} + \frac{B N (N + 1)}{Q (1 - F_w)}$$ (2.6)

This formulation includes terms associated to the weather-induced mean service times for both recurring ($\tau_{wr}$) and hand-over coordination tasks ($\tau_{wtr}$) multiplied by the sector’s weather blockage factor $F_w$.

### 2.5. Demand and operational capacity modelling

In parallel with the modelling of human workload characterising studies that built upon the DORATASK and similar approaches, other works pursued the capacity modelling of airspace resources based on operational and geographic considerations [8, 68, 69]. These models outline the capacity limits of particular resources including air route segments, navigation waypoints and arrival/departure procedures and are typically used in conjunction with workload-based capacity models. Some early works on airspace capacity modelling were published in 1980s [8] and ‘90s [68, 69] specifically addressing the American National Airspace System (NAS). Although these were based on a number of considerations that are largely still valid, later works evolved substantially from them. Conventional models consider sectors as entities with several entry and exit points associated with traffic entering, exiting and flowing through. This principle has been adopted by many large scale models [70, 71]. Figure 2.5 depicts two airports O and D where flight $f$ is planned to depart from and arrive at, respectively. In order to arrive to airport D, flight $f$ must pass through either sector $V_1$, $V_2$ or $V_3$, which are characterised by various number of entry and exit points. This permits flight $f$ to access alternative routes in the case one of the original planned sectors becomes unavailable due to severe weather, which is an example of a pre-tactical or tactical ATFM initiative.
Each sector is represented as a volume of space, which can accommodate a number of aircraft depending on the throughput measured at the sector entry/exit points. The traffic either enters from adjacent sectors or originates from an airport within the sector itself. In order to define the network, each sector is connected to other sectors or airports. The traffic flow can be expressed in terms of aircraft entering and leaving the sector \( v \) during each time interval \( t \) as in:

\[
N_{v,t} = N_{v,t-1} + \sum_{f \in F} \sum_{e \in E_v^f} (w_{e,t}^f - w_{e,t-1}^f) - \sum_{f \in F} \sum_{x \in X_v^f} (w_{x,t}^f - w_{x,t-1}^f)
\]  

(2.7)

The number of aircraft in the sector, \( N_{v,t} \), is equivalent to the number of aircraft previously in the sector \( N_{v,t-1} \), and to the number of aircraft entering and leaving the sector at each entry point \( e \in E \) and exit point \( x \in X \). The variable \( k \) here represents any capacity metering element such as airports or sector’s entry/exit points. An overall capacity constraint can be defined on the total number of aircraft simultaneously in the sector: \( N_{v,t} \leq N_{\text{MAX}} \). When a sector is affected by severe weather, a capacity reduction is dynamically introduced as the weather cell progressively relinquishes portions of the sector volume in terms of altitude and horizontal extents. Capacity constraints force flights to reroute around most congested sectors.
2.5.1. Demand prediction and macroscopic traffic flow models

Traditional demand prediction methods rely on trajectory estimation in open-loop. This is done by retrieving or estimating the entry and exit times of each sector along the path. A normalised weather index is typically adopted for each sector, representing the portion affected by severe weather. Future Automation Concepts Evaluation Tool (FACET) evaluated advanced ATFM simulation tools to forecast both future traffic states and their related confidence levels. Both traditional methods and FACET methods are open-loop and thus do not take into account combined ATFM measures [72]. Chen and Sridhar developed a prediction model that takes into account ATFM actions as well as tactical ATM and airline actions, and is suitable for midterm predictions. Similarly to other works, a power law distribution formula was adopted to predict the sector-demand reduction rates [72]:

\[ h = 1 - \left( \frac{w_k - w_{low}}{w_{high} - w_{low}} \right)^\lambda \]  

(2.8)

where \( w_{low} \) is the lower weather index threshold and \( w_{high} \) is the weather index causing the entire sector capacity to be sequestered. This model showed good accuracies when assessed against real data, though the authors did not identify a general fitting exponent. The improvement of traffic estimation models to capture both predictable and unpredictable factors is an area of considerable active research.

As the implications arising from the discreteness of the traffic diminish with the increasing size of the volume considered, continuity can be at some point assumed when pursuing a macroscopic modelling of the traffic demand across very large areas [73]. The main advantage of this approach is that continuity is a physical principle accurately captured in a wide variety of natural processes (e.g., fluid dynamics, thermodynamics, electromagnetics, mechanics). In this respect, the Lighthill-Whitham model, modified by Richards to account for first-order continuity, so called LWR model, is the first macroscopic traffic flow model proposed for transport networks. The model is based both on the traffic densities, \( \rho \), defined as number of aircraft per unit length along route segments, and on traffic fluxes, \( q \), defined as the throughput in number of aircraft per unit of time on either route segments or at nodes. The following one-dimensional (1D) continuity equation can be therefore adopted in strictly macroscopic:
\[ \frac{\partial q}{\partial x} + \frac{\partial \rho}{\partial t} = s \]  

(2.9)

where \( s \) is a source term that accounts for aircraft either entering or exiting the sector not from neighbouring sectors. Although the continuity assumption is strongly restrictive when studying air traffic, especially at ATC sector scales, various accurate discrete models can be developed from this formulation, which retain the important property of convergence to the physical fluid dynamic principle. Additionally, a number of results of the LWR theory achieved for ground transport networks are applicable to air traffic as well.

### 2.6. Traffic Sequencing

Sequencing problems arise whenever a resource is subject to an air traffic demand greater than the available capacity. Solution methods normally attempt to determine the resource allocation schedule that maximises some figure related to the efficiency in exploiting the available capacity, or conversely the one that minimises a total weighted delay. This problem is equivalent to the single-machine scheduling with ready times of operations research. In the airport context, congestion problems essentially arise due to either:

- the mismatch between the nominal capacity figures adopted when granting slots to airlines and aircraft operators and the actual capacity available when accounting for weather conditions and other factors restricting the nominal capacity.

- the unexpected advance or delay of inbound traffic due to factors shortening or extending the en-route time, such as winds aloft, hazardous weather cells or clear air turbulence, which can coalesce in peaks of over-demand;

- other factors prompting inbound traffic to arrive earlier or later than planned, also including cascaded phenomena.

The sequencing problem for arrival traffic is conceptually represented in Figure 2.6, where two different arrivals route are depicted across the Terminal Manoeuvring Area (TMA), the Control Area (CTA) and the Aerodrome Traffic Zone (ATZ). Four different aircraft models are approaching the airport, characterised by an individual
runway. The segment in white represents the conventional shared final approach path, where a minimum safe longitudinal separation must be ensured at all times.

The problem can be stated as: for each inbound traffic determine the Allocated Time (AT) that optimises a predefined performance index considering the geometry of the approach, the Preferential Time (PT) of each traffic, the estimated runway occupancy, and the minimum longitudinal safe separation constraints. Higher degrees of complexity are introduced in relation to the differences in approach speed and in turbulence category of various aircraft, as well as to weather conditions, which as mentioned can vary the capacity available very significantly.

The following sections summarise the determination of longitudinal separation and runway throughput for arrival, and present the problem of scheduling jobs on a single machine with ready times minimising either total weighted completion time or total weighted delay.

Figure 2.6. Conceptual representation of the arrival sequencing scenario.
2.6.1. **Mathematical formulation and solution methods**

The scheduling cost functional can be written in a generic form that includes fictitious members both at the machine start time and between every job as [60]:

\[
z(S) = w_s \Delta T_s + \sum_{k=1}^{n} \left( w_{i_k} \Delta T_{i_k} + w_{i_k}^f \Delta T_{i_k}^f \right)
\]

(2.10)

where \( S \) is the desired schedule, \( z \) is the performance index to be minimised, \( w \) are weightings, \( \Delta T \) are delays. Fictitious jobs (\( J_{i_k}^f \)) are defined to run between the completion time of the previous job (\( C_{i_{k-1}} \)) and the start time of the following (\( t_{i_k} \)). This generalised form allows tracking and, if desirable, assigning a cost to both natural and inserted machine idle times (modelled as the fictitious jobs) between different jobs.

The first possible solution approach consists in prioritising the ready job for which the ratio between the job duration and the penalty weighting (\( b_i / w_t \)) is minimal. In this approach, which can be classified as a Simple Cheapest Insertion Heuristic (SCIH) [60], no idle time is inserted intentionally, as the job to be inserted must be selected from the ready queue. This scheduling method, called Heuristic H and documented in [74] is shown to require \( \log n \) steps. A more sophisticated method proposed by Belouadah and belonging to the family of known lower bounds methods consider the interruption of a running job if the duration time-to-weighting ratio (\( b_i / w_t \)) of a newly ready job compares favourably with the one of the already ongoing job [74]. The ongoing job is split between the already completed portion and the remaining portion.

2.6.2. **Airport flow and capacity management**

In order to operate at or close to the maximum hourly capacities, a precise synchronisation of all ground and runway operations is necessary and, as previously described, the unit of measure is the minute. Consequently, perturbations to any terminal and/or ground operations, which easily involve multiple minutes, can rapidly propagate not only to the entire airport but also to all arrival traffic within the TMA and progressively beyond. In order to mitigate these unavoidable perturbations and prevent or resolve major disruptions, the historical strategy consisted in building
over-capacity at airports. In addition to being costly, this approach is often not viable as airports are gradually engulfed by urbanisation that prevents successive expansions. Furthermore, some bottlenecks cannot be removed and this can void any benefit offered by infrastructure expansion. The progressive introduction of CNS equipment and DSS with prediction functionalities is supporting an evolutionary change to airport capacity and flow management.

2.7. Departure-arrival problems

Departure-arrival problems involve more complex scheduling problems arising when the resources are shared by different categories of users, that is, departures and arrivals, therefore an optimal alternated allocation must be determined. The departure/arrival problems can be formulated as multi-machine scheduling problems on parallel unrelated machines [60]. These, in turn can be approximated as assignment problems in order to be solved in polynomial time.

2.7.1. Mathematical formulation and solution methods

As mentioned, the departure-arrival problem can be formulated as a multi-machine scheduling problem on parallel unrelated machines. The multi-machine scheduling model at the basis of this formulation must assume that [60]:

- Each machine represents a single airspace/airport resource;
- Each traffic involves either a single job or no job at every machine;
- The job ready time represents the PT of a particular traffic at a particular airspace/airport resource in its route;
- The job starting time represents the AT of a particular traffic at a particular airspace/airport resource in its route;
- The job processing time represents the time required by an individual traffic to clear/vacate the corresponding resource.

The scheduling cost functional can be written as [60]:

\[ z(S) = \sum_{1 \leq s \leq m} \left( \sum_{1 \leq l \leq k_s} (w_{i,j,l} \Delta T_{i,j,l} + c_{i,j,l}) \right) \]  \hfill (2.11)
Where $c_{i,j,l}$ represents the cost of processing the $j^{th}$ job on the $i^{th}$ machine using $l^{th}$ slot. Assuming a constant processing time $b_{i,j}$ for any job on any machine, the multi-machine scheduling on parallel unrelated machines can be approximated as an assignment problem [75]. This assignment problem can be represented by the bipartite graph of figure 2.7. Each job concerns a different subset of machines and each machine has a different number of slots.

The solution of this assignment problem corresponds to the solution of the original problem subject to the condition that all job durations are all equal. A solution to the assignment problem is achievable in $O\left(m C \sqrt{n \log n}\right)$ steps.
2.8. Gate-to-gate problems

Gate-to-gate problems involve a global formulation in which departure scheduling, arrival scheduling and flight routing are considered together to attempt maximising the overall system efficiency. This problem can be translated in a multi-commodity flow problem [60]. The generalised formulation of this problem involves a set of airports \( A \) interconnected to any other in the set through an air route network \( A \), consisting of nodes and arcs. Each node is characterised by a distinctive 3-dimensional position in space. Each arc can only connect two nodes and is characterised by a specific geographical distance. Airports are also modelled by their associated ground route networks, which include nodes associated with gates, runways and ground intersections, as well as taxiway arcs. Within a time period \( T \), a set of aircraft \( K \) has to leave a gate of a departure airport and reach a gate of the destination airport. The ready time of a specific job represents the PT of that particular aircraft at a particular node. The start time, conversely, represent the AT of a particular aircraft at a particular node. Each aircraft can fly each segment at predetermined minimum and maximum speeds, respectively \( v_{\text{MIN}} \) and \( v_{\text{MAX}} \). Consequently, an upper limit is introduced to the delay accumulated en-route.

The gate-to-gate problem was formulated as a multi-machine scheduling problem in stages by Guercio [60]. By defining \( d_{i,k,j} \) the length of arc \( a_{i,k,j} \) connecting machine \( i \) at stage \( j \) with machine \( k \) at stage \( j+1 \), \( v_{l(i)}^{(l)} \) the speed of job \( j_l \) on the arc \( a_{i,k,j} \) connecting machine \( i \) at stage \( j \) with machine \( k \) at stage \( j+1 \), we can determine the maximum arc delay as [60]:

\[
AD_{\text{MAX},l,k,j}^{(l)} = d_{i,k,j} \left( \frac{1}{v_{\text{MIN},l,k,j}^{(l)}} - \frac{1}{v_{\text{MAX},l,k,j}^{(l)}} \right) \quad (2.12)
\]

By defining \( T_{R_{i,j}} \) the transit time of any job on machine \( i \) at stage \( j \), and with \( T_{R_{i,k,j}}^{(l)} \) the time spent by job \( l \) on arc \( a_{i,k,j} \) we have the following relationships [60]:

\[
T_{R_{i,k,j}}^{(l)} = T_{R_{i,k,j+1}}^{(l)} - AT_{T_{i,j}}^{(l)} \quad (2.13)
\]

\[
T_{R_{i,k,j}}^{(l)} = \frac{d_{i,k,j}}{v_{l,k,j}^{(l)}} + T_{T_{i,j}} \quad (2.14)
\]
The problem can be solved in terms of $AT_{i,j}^{(l)} \forall j \forall l$ by introducing suitable constraints to ensure the continuity of the path of each job, that minimum and maximum speeds (and associated arc delays) are met, that job processing on any machine starts at the beginning of an available slot, that any two jobs do not share the same slot on the same machine, that the processing of one job on a specific machine does not start before the ready time and finally that the start and finish of each job coincide with the desired start/finish machines. The gate-to-gate problem presented above can be translated in a multi-commodity flow problem in a particular directed graph [60]. In this formulation, a set of commodities share common arc capacities that restrict the total flow of commodities on that arc. A viable solution algorithm for this implementation involves the following steps [60]:

1. Prune the initial set of nodes from the ones not involved in any aircraft path
2. For each aircraft find the minimum and maximum time required to reach each vertex from the departure vertex;
3. Based on the determined minimum and maximum times, find vertex time intervals usable by all aircraft;
4. For each vertex, based on the transit time, find identify the admissible slots included in the time intervals. For each slot, the capacity of the vertex-slot is 1;
5. For each admissible couple of vertex-slots, establish in the network the associated arc;
6. For each aircraft, assign for each arc of the network a pseudo-infinite cost if the arc may not be used, or a cost proportional to the distance plus a cost proportional to the delay;
7. Insert fictitious vertexes in relation to the departure and destination vertexes to model the supply and demand.

A number of heuristic solution strategies were proposed. A heuristic algorithm following the First-Come First-Served (FCFS) principle at the destination involves the following steps:

1. Generate an ordered list of aircraft by increasing RT at a particular destination (if airborne) or by their PT if still on the ground;
2. Assign the first available and attainable slot to the first aircraft in the list, by solving the shortest path problem;
3. Delete the first aircraft and the assigned slot from the list and repeat steps 2 and 3.

An improved version that attempts the solution on en-route conflicts is:

1. For each aircraft ordered by increasing RT at a particular destination, assign the first available slot (mimicking the conventional FCFS heuristic algorithm above);
2. For each aircraft which transits a particular machine within the same slot already used by another aircraft, assign the first successive slot in that machine;
3. Repeat step 2 for all aircraft and all machines until all slots are deconflicted.

### 2.9. Route complexity and conflict detection

Strategic and tactical replanning of 4D flight trajectories due to unpredicted events may lead to new potential conflicts with other traffic. The possibilities of conflicts and close encounters in the 4D space-time domain are shown to be very large even in the case of a reduced set of flights [76-78]. The high complexity of the multiple-traffic de-confliction problem is already acknowledged by offline flight planning systems, which are largely focussing on constraining how many traffic will probabilistically be in a particular airspace sector or airway leg at the same flight level simultaneously. An optimised airway network design is currently adopted to minimise the probability of arising conflicts, and at the same time to limit the airspace/traffic complexity. In particular, most congested bidirectional air routes are
split in two unidirectional RNAV routes spaced by an optimal number of nautical miles. A third parallel RNAV route is sometimes introduced to allow segregating shorter-range traffic, which undergoes more frequent changes in altitude that would potentially disturb longer-range levelled traffic. This split introduces significantly higher flexibility for tactical de-confliction by exploiting the free semicircular flight levels of the unidirectional RNAV routes. The intersection of two route legs in the same semicircular flight level set is carefully designed to minimise the conflict probability. Fig 2.8 and 2.9 show a fraction of the Welsh airspace before and after the redesign. It can be noticed that the newly designed eastbound Upper Papa 2 (UP2) and Upper Lima 607 (UL607) are clustered together in proximity of the crossing with the north-south Upper Papa 16 (UP16), Upper November 862 and 864 (UN862 and UN864) cluster, so that northbound and southbound traffic can be deconflicted from the entire eastbound traffic flow in a single stage.

Figure 2.8. Route structure around the Brecon VOR (BCN) before the route redesign.
Although air route network redesign is the current state-of-the-art approach to conflict mitigation, significant research efforts are allocated to identify, implement, test, evaluate and certify efficient and reliable conflict detection algorithms in modern ATM systems. The simplest conceivable technique of determining conflicting trajectories is by calculating the horizontal and vertical separation of all discrete trajectory points in a pairwise manner. Although feasible, this strategy is known to lead to a combinatorial explosion phenomenon even for as low as two aircraft. Consequently, several alternatives have been investigated to identify a computationally efficient and effective conflict detection algorithm [76-108], and they may be summarily categorised between:

- Closest Point of Approach (CPA) geometrical analysis;
- Node/intersection slotting;
- Octant analysis;
- Traffic/trajectory clustering;
- Domain decomposition (also exploiting hyperplane separation methods).
Other techniques were also proposed and investigated to various extents [76-108]. The selection of a particular technique to be implemented depends on the problem formulation, and in particular on:

- the adopted trajectory data formats;
- the level of detail and accuracy granted by the employed trajectory prediction and monitoring algorithms;
- the computational performance that can be allocated to conflict detection;
- the 4-Dimensional extent of the search domain for possible conflicts;
- the overall level of system-wide information sharing.

Despite the substantial number of dependencies, multiple techniques will still be technically viable after the pruning, therefore a comparative evaluation will be performed as part of the research activities. Once a suitable conflict detection algorithm is identified, three possible design options will be available:

- automated conflict detection only (no automated conflict resolution functionality);
- automated conflict detection and automation-assisted resolution (e.g., providing multiple suggestions to the human ATCo);
- automated conflict detection and resolution.

When employed, the conflict resolution algorithm are loop-integrated with the trajectory optimisation algorithm and the conflict avoidance is accomplished by introducing either a mix of additional path and/or dynamic constraints, or heavy penalties in the cost functions (adopting the so-called “relaxation” described later). Both strategies were successfully undertaken in previous research activities [109-111].

### 2.10. Advanced ATFM concepts

The remarkable advances in CNS+A technologies is progressively allowing enhanced DCB strategies. New CNS technologies and ATFM strategies greatly increase the flexibility of routes and airspace configurations, which are both of great
use for DCB. Nevertheless, since at present ATM is still largely based on instructions dispatched verbally by air traffic controllers in individual radio transmissions, workload is still identified as being the overarching constraint for airspace capacity [61]. Recent works such as [71, 112] introduced more complex and accurate models and airspace constructs. Some preliminary Human-In-The-Loop (HITL) evaluations of the novel flexible airspace concepts led to positive acceptance by ATM operators [113], but a crucial aspect identified was the adoption of automated transfer-of-communication and more generally of TBO data-com equipment.

2.10.1. Decentralised and distributed ATFM paradigms

The importance of assessing the balance between centralised and distributed paradigms for different topics has been acknowledged by various researchers in the ATFM domain. Table 2.3, adapted from seminal works in the domain, outlines five progressive levels of ATFM decentralisation that were conceived in relation to the various ATM/ATFM duties [60]. Based on the table, it can be inferred that the best solution for a particular airport or airspace region largely depends on local operational and economic specificities. In particular, most studies highlight that no “one size fits all” strategy yields global optimality, as the various approaches show very different performances when applied to distinct cases, with some being better suited for very large airports serving as hub to multiple carriers, others being better fit to single-carrier hubs and others to smaller point-to-point airports.
Table 2.3: Progressive levels of decentralisation applied to various ATM/ATFM duties. Adapted from [60].

<table>
<thead>
<tr>
<th>Level of Decentralisation</th>
<th>Definition of Arrival Slots</th>
<th>Assignment of Arrival Slots to Individual Flights</th>
<th>Assignment of Departure Slots to Individual Flights</th>
<th>En-route Planning and Control</th>
<th>TMA, Ground Movement Planning and Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralised</td>
<td>ATFM service allocates arrival slots to individual flights</td>
<td>ATFM service assigns slots, each airline may cancel and substitute flights</td>
<td>ATFM service assigns departure slots to individual aircraft</td>
<td>ATFM service defines the authorized routings.</td>
<td>ATFM service prescribes and monitors the routing as part of the clearances.</td>
</tr>
<tr>
<td>Partially Centralised</td>
<td>ATFM service allocates sets of arrival slots to individual airlines</td>
<td>Individual airlines allocated their own sets of slots among their own flights</td>
<td>Airlines assign departure slots to individual aircraft, subject to ATFM service approval.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partially Decentralised (basic)</td>
<td>ATFM service informs airlines of the legal capacities at potentially congested airports</td>
<td>Airlines may bargain for the legal capacities. They may confirm, cancel or delay flights within their purchased slots.</td>
<td>Airlines assign departure slots to individual aircraft at their own discretion.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partially Decentralised (advanced)</td>
<td>ATFM service informs airlines about anticipated availability at congested airports.</td>
<td>Airlines have full freedom in managing the legal capacity for arrivals.</td>
<td>Airlines plan and control their routings. ATFM monitors for unavailability and conflicts.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decentralised (basic)</td>
<td>ATFM service allocates sets of arrival slots to individual airlines</td>
<td>Individual airlines allocated their own sets of slots among their own flights</td>
<td>Airlines assign departure slots to individual aircraft, subject to ATFM service approval.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decentralised (advanced)</td>
<td>ATFM service informs airlines of the legal capacities at potentially congested airports</td>
<td>Airlines may bargain for the legal capacities. They may confirm, cancel or delay flights within their purchased slots.</td>
<td>Airlines assign departure slots to individual aircraft at their own discretion.</td>
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</table>
2.10.2. Dynamic airspace management

In parallel with the implementation of 4D-TBO, the ATM research community is tackling the concurrent evolution of the airspace into a dynamically-optimised resource, taking into account traffic, airport and weather updates and forecasts and introducing enhanced strategies to vary capacity as needed. Although differences are present among the various national regulatory frameworks, it is possible to generally delineate three independent and coexisting regulatory aspects characterising the airspace configuration:

- Air Traffic Service (ATS) classification: it introduces a partitioning of the airspace that is essentially based on the Level of Service (LoS) offered by the Air Navigation Service Provider (ANSP), but has further substantial implications, so that, for example, distinct rules of the air are applicable for each class. ATS sectors are classified from Alpha (A) to Golf (G) in decreasing LoS levels. Different requirements for the airborne equipment are associated with each ATS class.

- Airspace restrictions/Special Use Airspace (SUA): they are portions of the airspace characterised by a significant access restriction or natural/manmade hazard to aircraft operations. SUA are frequently associated with segregated operations such as by military, government, industrial and scientific research. SUA are likewise adopted to protect major cities as well as strategic and sensitive facilities such as prisons and power plants, from criminal threat.

- Air Traffic Control (ATC) sectors: they reflect the organisational needs of the ANSP to provide services to a large number of aircraft and over a vast geographic area, partitioning it in smaller sections that are under the jurisdiction of individual Air Traffic Controllers (ATCo).

The flexible/dynamic airspace concept was therefore introduced to overcome the rigidity of the conventional airspace structure, in order to actively manage the denominator of the DCB and consequently release further capacity. Under traditional Flexible Use of Airspace (FUA) concepts the geographic extent and activation times of SUA may be changed to minimise disruptions or maximise usage; however the ATC sector(s) containing the SUA remains unchanged. SUA therefore
cannot increase the capacity of an ATC sector, it can only decrease it and the aim of DCB in relation to SUA is to ensure that the timing of that decrease in capacity coincides with a decrease in traffic demand.

The long-term evolution of ATFM and airspace management is represented by the Dynamic Airspace Management (DAM) paradigm. While ATFM measures are largely dealing with the modulation of the demand, DAM is conceived to alter the capacity, by overcoming the rigidity of this conventional airspace structure. This can be achieved by modifying the geographic extent of the airspace sectors, morphing their boundaries to accommodate shifting traffic patterns, moving weather or other dynamic factors [27]. The capacity of the neighbouring ATC sectors may be temporarily decreased as a result, but this is a far more flexible concept that consolidating or de-consolidating sectors onto ATC positions. Regulatory considerations (ATCo ratings, traffic mix, etc.) may constrain the extent to which the sector boundaries may change but relatively moderate perturbations may still yield significant operational benefits for both ANSPs (maximise ATCo performance and manage staffing levels) and airline operators (minimise DCB measures such as reroutes and holding). Experimental studies on Flexible Airspace Management (FAM), an early Dynamic Airspace Sectorisation (DAS) strategy investigated in NextGen, highlighted the critical aspects in terms of the magnitude, frequency, timing and lead time preview of sector changes [113]. In particular, shifts involving large traffic volumes, major traffic flows or reverting upstream/downstream directions were found to have significant negative implications for both the workload and situational awareness of the ATCo. It was also recognised that the current practice involving Letters of Agreement (LOA) to regulate co-ordination at international boundaries posed a hard constraint for DAM and only allowed rigid and predefined rearrangements. It is nonetheless envisaged that more data sharing and interoperability amongst ATM systems will enable a progressive relaxation of cross-border arrangements. Significant research is required on the implications for inter-sector and inter-centre co-ordination. DAM techniques should not be restricted to ATC sectors and SUA. In the long term, ATS service classes, SUA and ATC sectors should be coherently modulated based on the inputs of suitable DAM DST, adopting integrally dynamic algorithms for estimation and decision logic, based on the available CNS performance level. In this perspective, DAM is considered an
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essential contribute to PBO. A number of different automated partitioning techniques have been evaluated for future DAM DSS implementations [114-116].

As the dynamics involved in 4D-TBO and DAM are mutually interdependent and both concepts are fundamentally based on real-time optimisation algorithms, significant research is required to address the loop dependence, stability and optimality metrics, promoting rapid convergence to global economic/environmental optimality. With respect to MOTO, in the short-term new airspace models will have to consider dynamically evolving lateral and vertical sector boundaries and continuously varying capacities. In the medium-long term, new combined traffic flow and airspace models integrating 4D-TBO and DAM will be required.

2.11. Integration of trajectory optimisation concepts

By introducing customised penalty terms associated with either the time or the distance travelled within congested airspace sectors, it is possible to numerically promote trajectory planning across less densely trafficked sectors both in current state-of-the-art flight planning tools and in novel trajectory optimisation algorithms. As an example, figure 2.10 and 2.11 depict 2D+T MOTO case studies around congested airspace sectors of the Swiss Area Control Centre (ACC).

Figure 2.10. 2D+T trajectory optimisation considering a congested sector.
2.12. Conclusions

To cope up with the steady growth of traffic and the increasing expectations in terms of safety, efficiency and sustainability performances, the ATM is progressively evolving towards more strategic paradigms, which crucially rely on DCB and are therefore in the domain of ATFM. This evolution is already well undergoing in some regions of the world where international cooperation allowed the implementation of effective large-scale initiatives, such as in Europe and in North America. This chapter described the fundamentals of DCB, the key problems addressed in ATFM and some implementation concepts. The models of capacity and demand were also reviewed. Some notion about the conflict detection algorithms was also given, in view of the implementation introduced later.

2.13. References


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Chapter 3

Aircraft trajectory optimisation in the ATM and avionics context

This chapter provides an in-depth review of the literature available on trajectory optimisation for aviation applications. The chapter introduces the theoretical framework, the main problem formulations and solutions approaches, including some detail on the numerical implementations aspects. The review identifies the most relevant approaches for the particular objectives set in this research. A dedicated review is performed with respect to multi-objective optimality.

3.1. Background

In general, an optimisation process involves the adoption of suitable algorithms, decision logics and heuristics to identify the best element from a finite or infinite set of available alternatives. Trajectory optimisation studies the methods to determine the best possible trajectory of a dynamical system in a finite-dimensional manifold, in terms of specific objectives and adhering to given constraints and boundary conditions. By this very definition, a substantial similarity between the statements of the Trajectory Optimisation Problem (TOP) and of the Optimal Control Problem (OCP) can be noted. This comes as no surprise, as the assumed origin of both is the Brachistochrone problem, posed by Johann Bernoulli in 1696 [117]. Consequently, the two designations are frequently interchanged in the literature. One distinction noted is about the mathematical nature of the unknown solutions, as in OCP the interest is about the identification of optimal input functions, whereas in TOP the search is typically restricted to a finite set of static input parameters [118]. This consideration partly justifies the rapid success of TOP solution methods based on
parameterisations. Furthermore, the formal distinction of control, state and output variables is not always implemented in TOP, possibly due to the investigation of solution strategies beyond the framework of the optimal control theory, which will be briefly mentioned here. Notwithstanding, the continued familiarity with the OCP formulation was very fruitful, as the theoretical and computational advances in optimal control were directly transposed in enhanced TOP solution strategies and more general in the aerospace vehicle Guidance, Navigation and Control (GNC) domain. After the early works on the solution of the Brachistochrone problem, further studies by Euler and Lagrange led to the establishment of the calculus of variations. Since, as mentioned, the sought optimal solutions are in the form of functions, the calculus of variations effectively refers to functional optimisation. Therefore, an early-developed approach to the solution of OCP that is still commonly considered at present is based on the calculus of variations. Conversely, the transcription of OCP in finite-dimensional Non-Linear Programming (NLP) problem is increasingly popular as a computationally fast OCP solution strategy. As expanded later, most OCP solution methods can be ascribed to either of these approaches, and therefore are conventionally categorised as either direct methods, if based on the transcription to a finite NLP problem, and indirect methods, if theoretical derivations based on the calculus of variation are implemented to formulate a Boundary-Value-Problem (BVP), and both approaches have led to very successful numerical implementations [4, 118-120]. The distinction between direct and indirect solution methods is less clear-cut than originally postulated, as some methods either fall in between or are intentionally hybridised. Additionally, more recent studies are addressing the theoretical analogies between the two philosophies, and the findings are progressively overruling the distinction [118]. A third class of OCP solution strategies is represented by heuristic methods, such as simulated annealing, evolutionary algorithms, tree/graph/pattern search and particle-swarm. Theoretical solution strategies for OCP – such as the ones developed exploiting parameterisations and the calculus of variation – were highly instrumental for the development of numerical solution techniques for TOP. Notwithstanding, in principle TOP can also encompass alternative non OCP-based formulations, including linear/nonlinear parametric optimisation problems on both continuous and discrete search spaces. This is particularly noteworthy in the aviation context, due to
the widespread reliance on a geographical organisation of airspace and air routes, upon which all flight trajectory descriptors have been based, as well as the rules of the air, piloting techniques, traffic separation criteria, and ATM lexicon. In particular, the current air navigation procedures, as well as the air route network, rely on discrete geographical descriptors for the horizontal flight path – known as Lateral NAVigation (LNAV) – and on altitude and airspeed constraints for the vertical flight profile – known as Vertical NAVigation (VNAV). Consequently, some studies in the operational aviation domain have approached the optimisation of flight trajectories within the current air navigation framework. This category of studies has been informally called *procedural optimisation*, as most frequently it involves the optimization of Standard Instrument Departure (SID) procedures and Standard Terminal Arrival Routes (STAR).

### 3.2. Optimal control problem

In view of the familiarity between the TOP and the OCP discussed previously, the most general and convenient way to formulate the TOP is based on the optimal control. Consequently, in line with the control theory, we introduce the vector of time-dependent state variables $\mathbf{x}(t) \in \mathbb{R}^n$, the vector of time-dependent control variables $\mathbf{u}(t) \in \mathbb{R}^m$, the vector of system parameters $\mathbf{p} \in \mathbb{R}^q$ and the time $t \in [t_0; t_f]$. In the following subsections we introduce the dynamic constraints, the path constraints, the boundary conditions and the cost functions. A consistent definition of all these components is fundamental to formulate a well-posed OCP and to perform an appropriate selection of the numerical solution method and of the multi-objective technique.

#### 3.2.1. Dynamic constraints

The specificity of the trajectory optimisation and optimal control with respect to other mathematical optimisation branches is the application to dynamical systems, i.e. in motion or transition along time. Therefore, a key component in the TOP formulation is the set of dynamic constraints, which are meant to reproduce the feasible motion of the system (i.e. the aircraft, in our case) within the TOP. A
system of Differential Algebraic Equations (DAE), consisting of the time derivatives of the state variables, is usually adopted to introduce the system dynamics, and the dynamic constraints are therefore written as:

$$\dot{x}(t) = f[x(t), u(t), t, p]$$  \hspace{1cm} (3.1)

Nonlinear dynamics are natively encompassed, while other cases such as discrete-time dynamics may also be accounted for by adopting adequately relaxed formulations. More details on optimal control of discrete-time systems can be found in [121-123].

### 3.2.2. Path constraints

In the generalised TOP formulation, all non-differential constraints insisting on the system between the initial and final conditions are classified as *path constraints*, as they restrict the path, i.e. the space of states and controls, of the dynamical system. In order to represent all possible non-differential restrictions on the vehicle motion, two types of algebraic path constraints are considered: *inequality constraints* and *equality constraints*. A generalised expression of an inequality constraint is:

$$g_i(x(t), u(t), t; p) \leq 0$$  \hspace{1cm} (3.2)

whereas an equality constraint can be written as:

$$h_i(x(t), u(t), t; p) = 0$$  \hspace{1cm} (3.3)

Equality constraints (eq. 3) can be considered a subset of inequality constraints, as they can be the result of two opposite inequalities such as in:

$$\begin{cases} g_{i,a}(x(t), u(t), t; p) \leq 0 \\ g_{i,b}(x(t), u(t), t; p) \leq 0 \end{cases}$$  \hspace{1cm} (3.4)

where $g_{i,a} = -g_{i,b}$, hence eq. (3.2) can account for both types. A compact representation of growing popularity due to a high similarity to the numerical implementation is [124]:

---

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\[ C_{\min} \leq C[x(t), u(t), t; p] \leq C_{\max} \] (2.5)

for which equality constraints are simply encompassed by imposing \((C_{\min})_i = (C_{\max})_i\). Each of the \(i\)-th compound inequalities of eq. \((3.5)\) can be split in two opposite inequality constraints similarly to eq. \((3.4)\). Despite the generality of the unified formulations (eq. \(3.2\) and \(3.5\)), it is typically preferable to treat equality constraints separately, as they introduce an additional relationship between some state and/or control variables, hence they usually allow the reduction of the number of states or controls in the TOP. Inequality constraints have a more ambiguous impact on the TOP, as sometimes the path of the dynamical system lies far from the constraint, and hence the latter could be safely ignored as it does not have any influence in the solution. Therefore, at each instant in the TOP time domain, an inequality constraint can either be active, if the system path intersects the constraint such that \(g_i(\cdot) = 0\) has to be enforced, or inactive, when \(g_i(\cdot) < 0\) is naturally verified, so that the constraint can be safely ignored. Consequently, for each inequality constraint the path of the dynamical system can be subdivided in constrained subarcs and unconstrained subarcs. Unfortunately, this is computationally nontrivial: usually, the occurrence and the exact number of constrained subarcs along the path is not known a priori, as also very frequently are unknown the locations of junction points between unconstrained and constrained subarcs, or between two diversely constrained subarcs.

### 3.2.3. Boundary conditions

Boundary conditions specify the values that state and control variables shall have at the initial and final times. Since boundary conditions are not always necessarily restricted to definite values, it is useful to adopt a generalised expression including relaxed conditions. Similarly to the already introduced expression of eq. \((5)\) for the path constraints, we then write the boundary conditions as:

\[ B_{\min} \leq B[x(t_0), x(t_f), u(t_0), u(t_f); p] \leq B_{\max} \] (3.6)

where equality conditions are still encompassed by imposing \((B_{\min})_i = (B_{\max})_i\).
3.2.4. Cost functions and performance indexes

In order to optimise a given performance, it is necessary to introduce a scalar value, the performance index, which by means of a suitably defined cost function quantifies the achievement of that particular objective. The optimisation process can then be translated into the mathematical minimisation (or maximisation) of such performance index. Generally speaking, a performance index may depend on a function of terminal state values and parameters, $\Phi[x(t_f), u(t_f), p]$, or on running costs along the system time, expressed as the integral function of one or multiple state variables, control variables, and parameters as $\int_{t_0}^{t_f} \psi [x(t), u(t), p] dt$. The generic TOP formulation involving a performance index $J_i$ that takes into account both components was introduced by Bolza [117, 125, 126], and is expressed as:

$$J_i = \Phi[x(t_f), u(t_f), p] + \int_{t_0}^{t_f} \psi [x(t), u(t), p] dt$$  \hspace{1cm} (3.7)

The optimisation is classified as single-objective when an individual performance index $J$ is introduced and multi-objective when two or multiple performance indexes $J_i$ are defined. Different objectives are typically conflicting, that is the attainment of a better $J_k$ would lead to a worse $J_h$, $\{h, k \in [1; n_f], h \neq k\}$. Hence, the optimisation in terms of two or more objectives generates a number of possible compromise choices. Therefore, a trade-off decision logic must be introduced to identify an individual solution, and this is the subject of multi-objective optimisation theory, which will be discussed in section 3.5.

3.2.5. Resulting mathematical formulation

In summary, having introduced the dynamics, the path constraints and the boundary conditions and the cost functions, the trajectory optimisation problem can be analytically stated as [124]:

“Determine the states $x(t) \in \mathbb{R}^n$, the controls $u(t) \in \mathbb{R}^m$, the parameters $p \in \mathbb{R}^q$, the initial time $t_0 \in \mathbb{R}$ and the final time $t_f \in \mathbb{R}$ $t_f > t_0$, that optimise the performance indexes
\[
J = \Phi[x(t_f), u(t_f), p] + \int_{t_0}^{t_f} \Psi[x(t), u(t), p]dt
\]

subject to the dynamic constraints

\[
\dot{x}(t) = f[x(t), u(t), t, p],
\]

to the path constraints

\[
C_{\min} \leq C[x(t), u(t), t; p] \leq C_{\max},
\]

and to the boundary constraints

\[
B_{\min} \leq B[x(t_0), x(t_f), u(t_0), u(t_f); p] \leq B_{\max}.
\]

### 3.3. Numerical solution techniques

Figure 3.1 outlines a comprehensive tree of techniques that were proposed for the solution of TOP, of which the most commonly adopted will be described in this section. Some of the techniques will not be discussed in detail due to their limited diffusion. Details on temporal finite element methods based on weak Hamiltonian formulation for the solution of OCP are available in [4, 127]. As briefly mentioned in section 3.1, for the solution of the TOP two mainstream strategies have been extensively adopted, namely direct methods and indirect methods [4, 118-120]. In the first class, also defined by the motto “discretise then optimise” the determination of the unknown control function is attempted directly, and this involves the discretisation of the infinite-dimensional TOP into a finite-dimensional Non-Linear Programming (NLP) problem. In the indirect methods, which historically emerged beforehand and are defined by the motto “optimize then discretise”, analytical manipulations based on the calculus of variations are exploited to transform the OCP into a nonlinear Boundary-Value Problem (BVP).
Figure 3.1. Outline of the techniques proposed for the solution of trajectory optimisation problems.

### 3.3.1. Lagrangian relaxation and first order optimality conditions

The **Lagrangian relaxation** consists in approximating a constrained optimization problem with an unconstrained one. The process involves constructing a new function $\mathcal{L}$, called **Lagrangian**, which incorporates the constraints $c$ multiplied by a vector of additional unknowns, the Lagrange multipliers, $\lambda_i$:

$$\mathcal{L}(x, \lambda) = F(x) - \lambda^T c(x)$$  \hspace{1cm} (3.8)

The optimisation process is then applied to the Lagrangian. Some additional conditions are introduced to ensure that the solution of the relaxed optimisation problem converges to the solution of the constrained optimisation. The augmented functional in the case of the problem of Bolza can be written as:

$$J = \int_{t_0}^{t_f} [\psi + \lambda^T (x - f) + \mu^T (x - g_x)] \, dt + [\phi + v^T (x - b_x)]_{t_f}$$  \hspace{1cm} (3.9)
where $\lambda$, $\mu$ and $\nu$ are the vectors of Lagrange multipliers respectively for dynamic constraints, path constraints and boundary conditions, with dimensions consistent with the related constraints. In the BVP solution process, some of these Lagrange multipliers are promoted to the rank of adjoints or “co-states” and treated separately. Convergence to the optimality of the augmented functional is ensured by first-order optimality conditions. As an example, the set of conditions for a path and boundary constrained problem is:

$$ f_u - \psi_u \lambda = 0 $$  \hspace{1cm} (3.10)

$$ \int_{t_0}^{t_f} (f_p - \psi_p \lambda) \, dt + (b_p + \phi_p \nu)_{t_f} = 0 $$  \hspace{1cm} (3.11)

$$ \dot{\lambda} - f_x + \psi_x \lambda = 0 $$  \hspace{1cm} (3.12)

$$ (\lambda + b_x + \phi_x \lambda)_{t_f} = 0 $$  \hspace{1cm} (3.13)

### 3.3.2. Boundary-value problems

BVP are differential problems for which the conditions on the value of the unknown solution are prescribed at the boundary(ies) of the domain. In TOP applications, BVP are usually multi-dimensional and involve Dirichlet boundary conditions (also called of the first type or fixed). As BVP in the aerospace domain are most commonly encountered when dealing with problems of fluid dynamics, thermodynamics and structural analysis, a large portion of the solution methods were developed for these applications. A general introduction to the BVP theory can be found in [128-131], whereas more details on state-of-the-art solution techniques, with particular emphasis on problems arising in the fluid-dynamics domain can be found in [132-134]. If the Lagrange and Mayer terms are quadratic in $\{x, u, p\}$ and the constraints are linear in $\{x, u, p\}$ the BVP is linear and some originally proposed techniques to attain a numerical solution of the TOP included the methods of complementary functions, the adjoint variables and the particular solutions [135]. Given that considerable nonlinearities are present in most of the models involved in aerospace TOP, the reduction of nontrivial cases into linear quadratic BVP is
typically precluded. Systems involving either Mayer or Lagrange functionals that are nonquadratic in \( \{x, u, p\} \) or constraints that are nonlinear in \( \{x, u, p\} \) can be solved iteratively or exploiting some kind of heuristics. Iterative solution techniques can be either based on gradient methods or on higher order methods [135]. Gradient methods use at most the first derivatives of the function under analysis, while \textit{quasilinearisation} methods involve at most the second derivatives of the function under analysis. Both categories require the solution of a linear BVP at each iteration.

A scalar numerical convergence performance has to be defined to assess the efficiency a specific iterative BVP solution algorithm. The following definitions of convergence performances were proposed [135]:

\[
P = \int_{t_0}^{t_f} N(x - \phi) \, dt + N(\psi)
\]

\[
Q = \int_{t_0}^{t_f} N(\lambda - f_x + \phi_x \lambda) \, dt + \int_{t_0}^{t_f} N(f_u + \phi_u \lambda) \, dt
\]

\[
+ N \left[ \int_{t_0}^{t_f} (f_p - \phi_p \lambda) \, dt + (g_p + \psi_p \mu)_{t_f} \right] + N (\lambda + g_x + \psi_x \mu)
\]

where \( N \) is the norm operator. The algorithm should stop when either \( P \) or \( Q \) attain a certain desired tolerance, i.e.:

\[
P \leq \epsilon_1 \lor Q \leq \epsilon_2
\]

with \( \epsilon_1 \ll \epsilon_2 \), since the compliance to the constraints (metric \( P \)), shall prevail over optimality (metric \( Q \)) in most practical implementations.

### 3.3.3. Iterative solution of unconstrained nonlinear programming problems

Similarly to the nonlinear BVP arising in indirect solution approaches, the NLP problems involved in the direct solution of TOP are typically solved by iterative algorithms, for which the basic theory is briefly mentioned here, or through some kind of heuristics. Extensive details on NLP theory and on the development of computationally efficient NLP solution algorithms can be found in [130, 136, 137].
Adopting the n-dimensional Taylor series expansion of $F(x)$ to the third term we may write:

$$F(x^{(k+1)}) \equiv F(x^{(k)}) + \nabla_x F(x^{(k)}) \cdot s^{(k)} + \frac{1}{2} s^{(k)^T} H(x^{(k)}) s^{(k)}$$ (3.17)

Where

$$s^{(k)} \triangleq x^{(k+1)} - x^{(k)}$$ (3.18)

$$\nabla_x F(x) = \left[ \frac{\partial F(x)}{\partial x_1}, \ldots, \frac{\partial F(x)}{\partial x_n} \right]^T$$ (3.19)

$$H(F(x)) = \begin{bmatrix}
\frac{\partial^2 F(x)}{\partial x_1^2} & \ldots & \frac{\partial^2 F(x)}{\partial x_1 \partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 F(x)}{\partial x_n \partial x_1} & \ldots & \frac{\partial^2 F(x)}{\partial x_n^2}
\end{bmatrix}$$ (3.20)

The necessary conditions for a local minimiser, $\bar{x}$, are:

$$\nabla_x F(\bar{x}) = 0$$ (3.21)

$$s^T H|_{\bar{x}} s \geq 0$$ (3.22)

Whilst the sufficient conditions for a strong local minimiser are:

$$\nabla_x F(\bar{x}) = 0$$ (3.23)

$$s^T H|_{\bar{x}} s > 0$$ (3.24)

An iterative NLP solution can thus be formulated so that the search direction at step $k$ based on the n-dimensional Newton method is written:

$$s^{(k)} = -[H^{-1}\nabla_x F(x)]_{x^{(k)}}$$ (3.25)

In a number of aerospace applications, the evaluation of derivatives of the objective function or of the constraints at every step may be computationally cumbersome;
therefore various heuristic strategies were studied and implemented in NLP solvers. A number of strategies are based on the recursive update of the Jacobian or Hessian matrixes, as [137]:

\[ D^{(k+1)} = D^{(k)} + U(\Delta F, \Delta x) \]  \hspace{1cm} (3.26)

For instance, an n-dimensional generalisation of the secant method providing a recursive update of the Jacobian that minimises the Frobenius norm, proposed by Broyden, is:

\[ D^{(k+1)} = D^{(k)} + \left( \frac{\Delta F - D^{(k)} \Delta x}{\Delta x^T \Delta x} \right) \]  \hspace{1cm} (3.27)

For the Hessian matrix, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update can be implemented, which is a rank two positive definite secant approximation:

\[ D^{(k+1)} = D^{(k)} + \frac{\Delta g \Delta g^T}{\Delta g^T \Delta x} - \frac{D^{(k)} \Delta x \Delta x^T D^{(k)}}{\Delta x^T D^{(k)} \Delta x} \]  \hspace{1cm} (3.28)

Heuristic strategies formulated to recursively determine the inverse of the matrixes were less successful. It is also important to observe in this context that the quasi-Newton methods based on recursive updates manifest a superlinear instead of quadratic convergence rate.

In addition to recursive updates of matrixes, other notable factors for the development of computationally efficient NLP solvers include the treatment of infeasible, redundant or rank-deficient constraints and of discontinuities, scaling, sparsity and mesh refinements, above others. Considerable factors also involve the integration schemes and the matrix calculus algorithms adopted. More details on fast and efficient implementations of iterative NLP solution algorithms are given in [137].

### 3.3.4. Indirect methods

As previously mentioned, indirect methods are a family of TOP solution methods in which the solution is attempted by applying the theory of the calculus of variations, a recipe which is sometimes summarised as “optimise then discretise”. Therefore, by
means of suitable analytical derivations, the problem of Bolza is transformed into an augmented BVP. The Hamiltonian function, augmented with the dynamic and path constraints by means of the Lagrangian relaxation, hence including the co-state $\lambda$ is:

$$\mathcal{H}(x, \lambda, \mu, u, t) = L + \lambda^T f[x, u, t, p] - \mu^T C[x, u, t; p]$$  \hspace{1cm} (3.29)

where $\mu$ are the Lagrangian multipliers associated with the path constraints. Boundary conditions are also augmented with the co-state:

$$\omega_{min} \leq \omega[x(t_0), x(t_f), \lambda(t_0), \lambda(t_f), t_0, t_f, p] \leq \omega_{max}$$  \hspace{1cm} (3.30)

The dynamic equations of the resulting Hamiltonian Boundary-Value Problem (HBVP) are:

$$\begin{align*}
    \dot{x} &= \frac{\partial \mathcal{H}}{\partial \lambda} \\
    \dot{\lambda} &= -\frac{\partial \mathcal{H}}{\partial x}
\end{align*}$$  \hspace{1cm} (3.31)

The TOP is therefore brought back to a two-point or multi-point BVP. The adoption of variational calculus for the approximate solution of OCP is described in detail in [138]. An example of the variational approach applied for the transcription of a TOP with no path constraints on the states, and a representative application to the minimization of the Direct Operating Costs (DOC), i.e. fuel costs and time costs, in the two-dimensional (2D) case, is described in [46]. The first step taken is to adjoin the dynamic constraints to the cost function as follows:

$$J = \phi(x(t_f), t_f) + \int_{t_0}^{t_f} \{\psi(x, u, t) + \lambda^T (f(x, u, t) - \dot{x})\} \, dt$$  \hspace{1cm} (3.32)

This leads to the definition of the following Hamiltonian function:

$$\mathcal{H}(x, u, t) = \psi(x, u, t) + \lambda^T f(x, u, t)$$  \hspace{1cm} (3.33)

so that:

$$J = \phi(x(t_f), t_f) + \int_{t_0}^{t_f} \{\mathcal{H}(x, u, t) - \lambda^T \dot{x}\} \, dt$$  \hspace{1cm} (3.34)
Integrating by parts yields:

\[
J = \phi(x(t_f), t_f) - \lambda^T(t_f)x(t_f) + \lambda^T(t_0)x(t_0) + \int_{t_0}^{t_f}\{\mathcal{H}(x, u, t) + \dot{\lambda}^T(t)x(t)\} \, dt \tag{3.35}
\]

The variation of the performance index with respect to the states and controls is:

\[
\delta J = [\nabla^T_x \phi - \lambda^T] \delta x]_{t_f} + [\lambda^T \delta x]_{t_0} + \int_{t_0}^{t_f}\{(\nabla^T_x \mathcal{H} + \dot{\lambda}^T) \delta x + \nabla^T_u \mathcal{H} \delta u\} \, dt \tag{3.36}
\]

The co-state equations are therefore written as:

\[
\nabla^T_x \mathcal{H} + \dot{\lambda}^T = 0 \rightarrow \dot{\lambda}^T = -\nabla^T_x \mathcal{L} - \lambda^T \cdot \nabla_x f \tag{3.37}
\]

\[
\lambda^T(t_f) = \nabla^T_x \phi \big|_{t_f} \tag{3.38}
\]

hence eq. 36 can be rewritten as:

\[
\delta J = [\lambda^T \delta x]_{t_0} + \int_{t_0}^{t_f}\nabla^T_u \mathcal{H} \delta u \, dt \tag{3.39}
\]

the necessary condition for \( J \) to be minimum is that:

\[
\nabla^T_u \mathcal{H} = 0 \quad t \in [t_0, t_f] \tag{3.40}
\]

Whenever the controls are constrained as

\[
C(u, t) \leq 0 \tag{3.41}
\]

then for all admissible \( \delta u \), i.e. simultaneously fulfilling all constraints \( C \), we shall have:

\[
\nabla^T_u \mathcal{H} \triangleq \delta \mathcal{H} \geq 0 \tag{3.42}
\]

which represents Pontryagin’s Maximum Principle [139]. Although indirect methods for the solution of TOP were overlooked with the rise of efficient direct
solution methods and of the computational capability to quickly solve very large NLP problem, recent research has identified some advantages that a combined approach exploiting both the calculus of variation and NLP may provide. Indirect collocation methods mentioned hereafter are an example of such combined approach, which is actively researched at present.

3.3.4.1. **Indirect shooting**

The indirect shooting, also known as simple shooting, is one of the most basic iterative methods to attempt the solution of the BVP. The system of Hamiltonian dynamics is integrated numerically together with an initial guess from $t_0$ to $t_f$. If the obtained terminal conditions differ from the desired boundary conditions by more than a set tolerance $\varepsilon$, an updated initial guess set is generated and another integration is performed. In practice, the indirect shooting is almost never viable for the solution of aerospace TOP, as the Hamiltonian dynamics have a very bad conditioning, which is further exacerbated as the integration intervals $t \in [t_0; t_f]$ increase. Furthermore, the integration on long time intervals may require a considerable computation time.

3.3.4.2. **Indirect multiple shooting**

As an attempt to resolve the limitations of the indirect shooting, in the indirect multiple shooting the time interval is divided into $n_i + 1$ subintervals. The indirect shooting method is then applied individually to each subinterval, that is, the HBVP are integrated together with the initial guesses on the state and co-state values. After each successful integration, the interface conditions

$$
\begin{align*}
    x(t_i^-) &= x(t_i^+) \\
    \lambda(t_i^-) &= \lambda(t_i^+)
\end{align*}
$$

are imposed and a root-finding iteration is introduced to minimise the objective function associated with the discrepancies $(x(t_i^-), \lambda(t_i^-)) - (x(t_i^+), \lambda(t_i^+))$ until a satisfactory threshold is attained. The size of the HBVP is effectively increased due to the inclusion of the interface values of the state and co-state at the boundaries of each interval. By partitioning the integration interval in a sufficiently high number
of subintervals, the issues associated with the hypersensitivity of the Hamiltonian dynamics can be mitigated and a rapid convergence may be eventually attained.

3.3.4.3. **Indirect collocation**

Collocation methods such as the ones introduced in the following section in the context of direct methods can be applied to parametrise the state and co-state of the HBVP associated with the OCP. Methods following this approach are defined indirect collocation methods. Indirect collocation methods were recently considered for the solution of aerospace TOP [118, 140]. These approaches have promising qualities when the application to TOP is considered, though the limitations in terms of flexibility intrinsic of indirect methods are still compounding.

3.3.4.4. **Limitations**

The most severe limitations encountered when adopting indirect solution methods are related to the initial guess and the overall flexibility of the approach. In particular, since the co-state variables are not representative of real physical entities, their initial guess is challenging. Additionally, the analytical derivation performed prior to establish the HBVP is significantly dependent on the original TOP formulation and this notably restricts the applicability and flexibility of the approach, effectively limiting the application of indirect methods to specific problems where their adoption is advantageous, such as in the case of trajectory optimisation in the presence of wind, discussed in section 3.4.

3.3.5. **Direct methods**

As previously mentioned, direct methods involve the transcription of the infinite-dimensional problem in a finite-dimensional NLP problem, hence following the approach summarised as “discretise then optimise”. The transcription into finite-dimensional NLP problem can be either performed by introducing a control parameterisation based on arbitrarily chosen analytical functions, as in transcription methods, or by adopting a generalised piecewise approximation of both control and state variables based on a polynomial sequence of arbitrary degree, as in collocation methods. In both cases the transcribed dynamical system is integrated along the time interval \([t_0; t_f]\). The search of the optimal set of discretisation parameters is
formulated as a NLP problem, which is solved computationally by exploiting the
most efficient numerical optimisation algorithms available. In direct transcription
methods, a basis of known linearly independent functions \( q_k(t) \) with unknown
coefficients \( a_k \) is adopted as the parameterisation in the general form:

\[
\begin{align*}
z(t) &= \sum_{k=1}^{N} a_k q_k(t) \\
&= \sum_{k=1}^{N} a_k \phi_k(t) \\
&= \sum_{k=1}^{N} a_k \phi_k(t)
\end{align*}
\] (3.44)

Considerable research on computationally efficient TOP solution algorithms based
on direct transcription was performed by Betts et al. [141-149]. Analogously,
remarkable work on computationally efficient TOP solution algorithms based on
direct collocation methods was performed by Rao et al. [150-157].

3.3.5.1. Direct shooting
In the direct shooting and multiple direct shooting, the parameterisation is performed
on the controls \( u(t) \) only, the dynamic constraints are integrated with traditional
numerical methods such as the Runge-Kutta family, and the Lagrange term in the
cost function is approximated by a quadrature approximation. A generalised
parameterisation of control variables can be written as:

\[
\begin{align*}
\mathbf{u}(t) &= \sum_{k=1}^{N} \mathbf{c}_k q_k(t) \\
&= \sum_{k=1}^{N} \mathbf{c}_k \phi_k(t)
\end{align*}
\] (3.45)

3.3.5.2. Multiple direct shooting
In the multiple shooting, the analysed time interval is partitioned into \( n_t + 1 \)
subintervals. A direct shooting method is then applied to each subinterval.
Continuity of the state is enforced at the interfaces similarly to multiple indirect
shooting, as in the following expression:

\[
\begin{align*}
\mathbf{x}(t^-_t) &= \mathbf{x}(t^+_t)
\end{align*}
\] (3.46)
3.3.5.3. **Local collocation methods**

By introducing distinct parameterisations on both the control variables and the state variables, it is possible to implicitly integrate the dynamics, that is approximate the integral of dynamics with a quadrature as:

\[
\int_{t_i}^{t_{i+1}} f[x(s), u(s), s, \mathbf{p}] ds \approx \sum_{j=1}^{Q} \beta_{ij} f[x(\tau_j), u(\tau_j), \tau_j, \mathbf{p}] \quad (3.47)
\]

where \(\tau_j\) are the nodes of the quadrature approximation. The Lagrange term of the cost function \(\psi\) is also approximated with a numerical quadrature. This methodology is at the basis of a family of direct OCP solution methods, named collocation methods.

3.3.5.4. **Global collocation methods**

In the global collocation methods, the direct solution of the OCP is attempted by enforcing the evaluation of the state and control vectors in discrete collocation points across the entire problem domain. Pseudospectral Methods (PSM) are considered one of the most computationally effective families of global collocation techniques available for the direct solution of large nonlinear OCP. They are based on a global collocation of orthogonal (spectral) interpolating functions. Due to their considerable success, we review their working principle in more detail. Further information is given in [118, 153]. As an initial step, their application involves the introduction of the non-dimensional scaled time \(\tau \in [-1, 1]\) so that:

\[
t = \frac{t_f - t_0}{2} \tau + \frac{t_f + t_0}{2}
\]

(3.48)

Such change of variable involves that the differential operator is transformed as follows:

\[
\frac{d}{dt} = \frac{2}{t_f - t_0} \frac{d}{d\tau}
\]

(3.49)

If the final time is not known (i.e. it is either unconstrained or inequality-constrained in the boundary conditions), \(t_f\) will be an additional unknown variable that will have to be determined by the NLP solver. The states and control variables of the OCP are
approximated by a set of polynomials of order $N$, and the problem is thereby discretised in $N + 1$ nodes. These interpolation polynomials must be an orthogonal basis in the discretised space. Hence, they have to satisfy the null scalar product property:

$$ P_i(x_j) \ast P_k(x_l) = 0 \quad \forall \; i \neq j, \forall \; k, l \in \{1, \ldots N + 1\} $$  \hspace{1cm} (3.50)

Although various families of interpolating polynomials can be successfully adopted and comprehensive dissertations may be found in [158, 159], the best implementations of PSM in terms of computational efficiency adopt simple interpolation polynomials in conjunction with a careful selection of the collocation nodes distribution. For such reasons, the basic Lagrange polynomials are frequently adopted for the interpolation of states and controls. A Gaussian quadrature rule guarantees the order of accuracy of the discretisation associated with the quadrature. Adopting the interpolation polynomials $P_k(\tau)$ on the $N + 1$ nodes $\tau_k$, the states are approximated as:

$$ \tilde{x}_i(\tau) = \sum_{k=1}^{N} \tilde{x}_i(\tau_k) \cdot P_{i,k}(\tau) $$  \hspace{1cm} (3.51)

and the controls are approximated as:

$$ \tilde{u}_j(\tau) = \sum_{k=1}^{N} \tilde{u}_j(\tau_k) \cdot P_{j,k}(\tau) $$  \hspace{1cm} (3.52)

The evaluation of the dynamic constraints (i.e. the state equations) is then performed in the nodes only, leading to a problem of finite dimensions. The dimension of the discrete problem is not the same in all cases, though. Lagrange polynomials of order $N$ are expressed as:

$$ P_k(\tau) = \prod_{j \neq k} \frac{\tau - \tau_j}{\tau_k - \tau_j}, \quad \forall \; j \in [0,N] $$  \hspace{1cm} (3.53)

As an example, a representation of Lagrange polynomials of the $4^{th}$ order for equally spaced nodes (basic case) is given in figure 3.2, whereas figure 3.3 depicts the same polynomials applied to Chebyshev nodes.
Figure 3.2. 4th order Lagrange interpolation polynomials for equally spaced nodes.

Figure 3.3. 4th order Lagrange polynomials for Chebyshev nodes.
Chebyshev Pseudospectral Methods (CPM) adopt Chebyshev polynomials of order $N$. An application of the Chebyshev pseudospectral method to aircraft dynamics is found in [160] and involves the evaluation of the Chebyshev trigonometric polynomials:

$$P_N(\tau) = \cos(N \cos^{-1} \tau)$$  \hspace{1cm} (3.54)

in the $N + 1$ nodes:

$$\tau_k = \cos \frac{k \pi}{N}, \quad k \in [0, N]. \hspace{1cm} (3.55)$$

Two recently adopted PSM variants are the Gauss PSM and the Legendre-Gauss-Lobatto (LGL) PSM [161]. Gauss PSM are based on the Gauss-Legendre quadrature, whereas the LGL PSM are based on the LGL quadrature, also simply known as Lobatto quadrature. Gauss PSM are specifically conceived to ensure that the Karush-Kuhn-Tucker (KKT) conditions are identical to the discretised first-order optimality conditions. Legendre polynomials may be calculated by using the Rodrigues formula:

$$P_N(\tau) = \frac{1}{2^N N!} \frac{d^{(N)}}{d\tau^{(N)}}[(\tau^2 - 1)^N]$$  \hspace{1cm} (3.56)

The Legendre-Gauss-Lobatto (LGL) nodes are the $N + 1$ zeros of the polynomial:

$$L_N(\tau) = (1 - \tau^2) \hat{P}_N(\tau)$$  \hspace{1cm} (3.57)

where $\hat{P}_N(\tau)$ is the first derivative of the Legendre polynomial of degree $N$ [162]. In Gauss PSM the dynamic constraints are not collocated at the boundary nodes, whereas in the LGL PSM the evaluation of states and controls is performed also at the boundary nodes, thereby the dimension of the NLP problem is increased by two additional nodes.

3.3.6. Heuristic methods

Although heuristics are involved to a certain extent in most numerical solution algorithms at various levels, in this section we briefly mention the solution strategies
for aerospace TOP that involve a substantial exploitation of heuristics and metaheuristics for the optimization of aircraft flight trajectories. A number of researchers proposed TOP solution strategies exploiting evolutionary algorithms, as they can natively feature nonlocal search scopes, hence detecting global optimality regions. Evolutionary Algorithms (EA) refer to a class of algorithms emulating the natural evolution processes, which are considered computationally intelligent since their pruning of unsuitable solutions emulates a machine learning process. For mathematical optimisation purposes, a particularly useful evolution process is the natural selection, for which a population of individuals (genotypes) evolve towards the fittest. Cross-overs and mutations are the genetic processes most commonly implemented to generate new off-springs to be added to the population, which may or may not prove fitter. Mutations and crossovers are of nondeterministic nature and allow EA to overcome the nonglobal convergence manifested by most iterative solution methods, causing convergence to local minimisers if the initial guess is not within the region of global optimality. The most desired quality of EA is therefore that they are natively suitable for the research of the global optima, since the method produces an arbitrary number of non-deterministic initial guesses and the algorithm itself is not restricted on the concept of the search scope. Among other research activities, EA were exploited as part of the research within the SGO-ITD of Clean Sky by the Green Systems for Aircraft Foundation (GSAF) academic cluster [47, 48, 50-56, 58, 59, 163].

3.4. Trajectory optimisation in the presence of wind

A trajectory optimisation problem of significant interest in the aviation domain consists in the determination of the optimal routing in the presence of winds, which was originally tackled by Zermelo as early as the 1930s [164, 165], and solicited several decades of active research. The potential environmental and economic benefits offered by an optimal routing in a given wind field are very substantial in most cases and furthermore in case of long-range air transport routes. Notwithstanding, the limited accuracy of medium-to-long term wind forecasts and the increasing air traffic densities that progressively restrict the freedom of lateral and longitudinal routing may compromise, and sometimes revert, the gains. The
simplest case consists in determining the optimal routing of an aircraft travelling horizontally at constant cruise airspeed between two known points in a known and constant wind field, ignoring the fuel consumption as well as variations in all other meteorological variables. In addition to loosening the constant altitude constraint, other generalisations involve the adoption of constant or variable non-uniform wind fields (i.e. 2D/3D/4D), more sophisticated modelling of the aircraft dynamics and of the atmosphere, multiply-connected search domains, and more complex cost functions addressing fuel consumption or other objectives in addition to flight time. As this research was largely performed before the recent widespread adoption of direct TOP solution methods, a considerable portion of the proposed numerical solution strategies are based on the calculus of variation. A very comprehensive treatment addressing the theory and solution of the trajectory optimisation problem under space-time varying meteorological conditions was produced by De Jong and the Royal Netherlands Meteorological Institute in 1974 [166]. The study led to the definition of a unified theoretical approach to the problem formulation taking into account both space non-uniformities and time variations of the meteorological conditions and flight performances. Theoretical manipulations led to viable alternative formulations involving gradient equations for the time of transfer and a phase velocity equation for the airspeed. Considerations of the real operational conditions restricting the available choices in flight routing highlighted the suitability of graph optimisation methods for global optimality. Some iterative solution algorithms were proposed, including a special graph algorithm, and their viability in flight planning was numerically demonstrated. Bijlsma discussed the case of optimal aircraft routing in general wind fields and the most promising strategies for the development of a computational solution algorithm [167]. The convergence issues and non-global convergence of iterative solution methods are raised and support the adoption of algorithm based on graph theory. One such method capitalising on the advantages of the calculus of variation and of the graph theory is outlined. The general case with variable cruise speed \( v \) and wind speed \( u \) is introduced by the following dynamic constraints, where the sole control variable, \( \theta \), is the ground track azimuth:
\[\dot{x}_1 = v(t, x_1, x_2, \theta) \cos \theta + u_1(t, x_1, x_2, \theta) \quad (3.58)\]
\[\dot{x}_2 = v(t, x_1, x_2, \theta) \sin \theta + u_2(t, x_1, x_2, \theta) \quad (3.59)\]

Leading to the following Euler-Lagrange equations:
\[\dot{\lambda}_1 = -\lambda_1 \left( \frac{\partial v}{\partial x_1} \cos \theta + \frac{\partial u_1}{\partial x_1} \right) - \lambda_2 \left( \frac{\partial v}{\partial x_1} \sin \theta + \frac{\partial u_2}{\partial x_1} \right) \quad (3.60)\]
\[\dot{\lambda}_2 = -\lambda_1 \left( \frac{\partial v}{\partial x_2} \cos \theta + \frac{\partial u_1}{\partial x_2} \right) - \lambda_2 \left( \frac{\partial v}{\partial x_2} \sin \theta + \frac{\partial u_2}{\partial x_2} \right) \quad (3.61)\]
\[-\lambda_1 v \sin \theta + \lambda_2 v \cos \theta = 0 \quad (3.62)\]

Due to the impracticality of developing trajectory optimisers strictly based on the solution of BVP with the required flexibility to be implemented in avionics and ATM systems, researchers have investigated alternative approaches and heuristics. A method based on mesh discretization that does not require iterative solution is presented in [168]. Another approach based on wind networking was also recently proposed [169, 170]. This approach is especially valuable to increase the trajectory predictability in oceanic airspace and other extents outside primary radar surveillance.

### 3.5. Multi-objective optimality

As mentioned in Section 3.1, the optimisation in terms of multiple conflicting objectives \(J_k = Q_k(p), k \in [1, n_f]\) leads to large set of solutions that can be considered optimal in some sense that will be expanded in the following section. Therefore, a trade-off selection strategy must be introduced in order to identify a single optimal solution from the large set of compromise solutions, and this is the purpose of multi-objective optimisation theory, discussed in this section. Conflicting objectives arise when introducing multiple environmental, economic and operational criteria. Furthermore, as previously mentioned in section 1.4, the implementation of constraints that are either unfeasible or contrasting the attainment of better optimality can hinder the optimisation process or introduce numerical ambiguity, which also has
to be addressed by adopting suitable multi-objective optimality techniques. This is particularly noteworthy if the operation in real traffic environment is considered, as the arbitrary number of constraints introduced in the online tactical timeframe to resolve traffic conflicts can hamper the optimisation process. In all these instances it is necessary to complement the optimisation algorithm with predefined heuristics, decision logics and Human Machine Interface and Interaction (HMI) formats for that increase the versatility and effectiveness of the trajectory planning algorithm.

A comprehensive and detailed review covering a large number of multi-objective optimisation strategies for engineering applications is given in [171]. This section resumes the techniques that are particularly suited for the optimisation of aircraft trajectory with respect to multiple operational, economic and environmental criteria, while complying with an arbitrary number of constraints. As we will describe in more detail, this decision making process is usually performed either by means of an \textit{a priori} articulation of preferences (i.e. beforehand), or of an \textit{a posteriori} articulation of preferences (i.e. afterwards). Some other strategies were proposed and adopted, including progressive articulation of preferences and no articulation of preferences [171]. In the aerospace domain, bi-objective trajectory optimisation, i.e. in terms of fuel costs and flight time costs, has been widely studied and has led to results that have been exploited for some time in the aviation domain. An important advantage is that by limiting the study to two objectives it is possible to introduce a single scalar value, such as the Cost Index (CI) that is implemented in most of the current generation FMS, to account for the trade-off between the two conflicting objectives. In the emerging environmentally sustainable aviation research, it is nonetheless valuable to account for several different objectives in a flexible multi-model / multi-objective optimisation framework. Pollutant emissions, fuel consumption, perceived noise, convective weather and turbulence avoidance, operative costs and contrail formation are examples of the most common objectives currently studied.

### 3.5.1. Pareto optimality

A point in the design space \( p^* \in P \) is defined \textit{Pareto optimal} if and only if there does not exist another point \( p \in P \) such that \( Q(p) \leq Q(p^*) \) and \( Q_i(p) < Q_i(p^*) \) for at least one \( i \). The \textit{Pareto front}, also called the \textit{Pareto frontier}, is the set of all the
Pareto optimal points $p^*$. Pareto optimal points are non-dominated, that is, there does not exist another solution that strictly dominates the Pareto optimal solution in terms of any objective. The Pareto front is the multi-objective and multi-dimensional equivalent of the single optimal solution of single objective optimisation. Due to the fact that in many applications a single solution is ultimately pursued even in large complex problems, multi-objective optimisation techniques shall lead to the identification of a single optimal solution that must be Pareto optimal, eventually at least in the weak sense, and thus must belong to the Pareto front.

3.5.2. A priori articulation of preferences

In the a priori articulation of preferences approach, the user adopts a formulation of the multi-objective optimality that involves either a quantitative or a qualitative combination or prioritisation of the various objectives $J_k, k \in [1, n_k]$ and hence leads to the definition of a single combined objective $\tilde{J}$. A single-objective TOP solution method is employed to optimise the combined objective, ultimately leading to an individual optimal solution. This approach is schematically represented in figure 3.4. Methods falling in the a priori category analyse ways to define the combined objective starting from the various possible user-defined preference articulations.

![Diagram](image-url)

Figure 3.4. Schematic representation of the a priori articulation of preferences.
3.5.2.1. Weighted global criterion method
A very common choice for the condensation of the various objectives in a single cost function is to assign a weight to each objective function $Q_i(p)$ or to a functional of the objective and sum them together. By defining $Q_0^0$, the utopia point, that is $Q_0^0 = \min_p \{Q_i(p) \mid p \in P\}$, we can introduce the following combined weighting:

$$J = \left\{ \sum_{i=1}^{n_f} w_i^T [Q_i(p) - Q_0^0]^s \right\}^{1/s} \tag{3.63}$$

where $w_i \neq 0 \forall i$ is the weight assigned by the user to each single objective. Generally $\sum_{i}^{n_f} w_i = 1$, but this is not strictly necessarily. An important subcase is when $w_i = 1 \forall i$, $r = 1$ and $s = 2$, so that the weighted global criterion is actually the geometric distance (modulus) of the optimal solutions from the utopia point. Another subcase, which is the simplest and perhaps the most common method to combine the various objectives in a single one is the weighted sum method, for which $Q_0^0 = 0 \forall i$, and $r = s = 1$, so that

$$J = \sum_{i=1}^{n_f} w_i Q_i(p) \tag{3.64}$$

For all the weighted global criterion methods, an adequate normalisation of the objective functions $Q$, even if not explicitly mandated, is fundamental to attain the intended balance of importance. This aspect is nontrivial, as the results will significantly depend on the choice of reasonable reference values for normalisation. The presence of a nonzero utopia point in the generalised formulation partially alleviates this criticality, by replacing absolute magnitudes with relative ones.

3.5.2.2. Weighted min-max
The weighted min-max, also called weighted Chebychev method, endeavours to optimise the worst performance among the various objectives at any given time, as in:

$$J = \max_k (w_k [Q_k - Q_0^0]) \tag{3.65}$$
The weighted min-max may frequently require more iterations than the weighted global criterion to reach the optimal solution, but at every step the structure of the evaluated $\bar{J} = Q(\mathbf{p})$ will be simpler than the weighted global criterion, potentially improving the computational speed when complex nonlinear objective functions are considered.

### 3.5.2.3. Weighted product

A different kind of weighting was proposed for which the combined performance index is defined as a product of all the objective functions, to the power of the assigned weight $w_i$, as in:

$$\bar{J} = \prod_{i=1}^{n_f} [Q_i(\mathbf{p})]^{w_i}$$  \hspace{1cm} (3.66)

It is important to note that in order to attain a non-trivial solution, it is strictly necessary that the condition

$$Q_i(\mathbf{p}) \neq 0, \forall \; i$$  \hspace{1cm} (3.67)

is enforced at all times. An advantage of this formulation compared to the weighted global criterion is the diminished dependence on the quality of the normalisation performed, so this approach can prove useful when the range of the objective function is unknown or unbounded. A strong limitation of the weighted product, on the other hand, is that when nonlinearities are present in one or more of the objective functions $Q(\mathbf{p})$, the computational complexity of the optimisation may increase considerably.

### 3.5.2.4. Exponential weighted criterion

To overcome the limitation to convex portions of the search domain imposed by weighted global criterion methods, an exponential weighted formulation can be adopted, as in:

$$\bar{J} = \sum_{i=1}^{n_f} \left(e^{pw_i} - 1\right)e^{pQ_i(\mathbf{p})}$$  \hspace{1cm} (3.68)
As in the weighted product, a significant advantage is the diminished dependence on the quality of the normalisation performed, while nonlinearities in the objective functions still affect the computational performances significantly.

3.5.2.5. Lexicographic and sequential goal programming methods

Another strategy for a priori articulation of preferences is the definition of an order of importance for the various objectives. A single objective optimisation step is then performed for each objective $J_k$ in the defined order, enforcing that the performance of the new step $Q(p^{(k)})$ is equal or better than the previous step $Q(p^{(k-1)})$ for all the objectives between 1 and $(k)$, as in:

$$Q_j(p^{(k)}) \leq Q_j(p^{(k-1)}), \quad \forall j \in [1; k] \quad (3.69)$$

Since the original version of the method appears heavily unbalanced towards the first objective, a relaxed version for which in each new step a limited freedom to worsen the performance of the previous step is introduced in order to attain a more balanced optimal solution

$$Q_j(p^{(k)}) \leq \left(1 + \frac{\delta_j}{100}\right) Q_j(p^{(k-1)}), \quad \forall j \in [1; k] \quad (3.70)$$

3.5.2.6. Physical programming

A notable physical programming approach was proposed, for which the user can introduce intuitive considerations and unstructured information that are used as design metrics to construct the combined objective function. In particular, the user may define quantitative ranges to particular parameters, based on qualitative considerations, such as desirable, tolerable, undesirable and unacceptable. The user-defined classifications and ranges are then translated in a number of structured numerical objective functions as:

$$Q_a(p) = \log \left(\frac{1}{dm} \sum_{i} \tilde{Q}_i[Q_i(p)]\right) \quad (3.71)$$
The most interesting advantage of the physical programming is that the user can directly intervene on the solution region and restrict the part of the Pareto front studied.

3.5.3. A posteriori articulation of preferences

In the *a posteriori articulation of preferences* approach, a single optimal solution, belonging to the Pareto front, is chosen after the whole set or a portion of it has been already determined. This approach is schematically represented in figure 3.5. Methods in this category aim essentially at *populating* the Pareto front with an even distribution of points, in order to reduce the computational requirements, increase the effectiveness of the selection and so that the user can perform the final choice from a set of sufficiently diversified solutions.

![Figure 3.5. Schematic representation of the a posteriori articulation of preferences.](image)

3.5.3.1. Physical programming

Similarly to the a priori implementation, Physical programming can be applied to translate unstructured information supplied by the user into decision criteria. As already discussed, the physical programming approach involves the definition of desirable, tolerable, undesirable and unacceptable ranges of objective functions and constraint values (ranges for the state and control variables are defined by inequality
constraints of TOP). An advantage of the a posteriori physical programming approach in a closed-loop implementation is that the method restricts the search domain in addition to the solution region, by discarding the combination of performance indexes that produced unacceptable solutions. This can prevent computational resources from being wasted in calculating Pareto optimal points lying outside of the acceptable portions of the Pareto front.

3.5.3.2. Normal boundary intersection

In the Normal Boundary Intersection (NBI) strategy, the user-supplied weightings \( w \) are modulated in order to obtain an even distribution of Pareto optimal points. The NBI strategy can be formulated as:

\[
\text{Minimise } \lambda, \text{ subject to: } \Phi w + \lambda n = Q(p) - Q^0
\]

where \( \Phi \) is the pay-off matrix made by the column vectors of the objective functions \( Q(p_i) - Q^0 \) at the minimum of the \( i \)-th objective function. The vector \( w \geq 0, \sum_i^n w_i = 1 \) are parameters to be systematically modified to obtain the complete Pareto front. It must be pointed out that the method does not provide sufficient condition for Pareto optimality, so some generated solutions are actually weakly optimal or suboptimal points.

3.5.3.3. Normal constraint method

The Normal Constraint (NC) method is a modification of the NBI method for which a tactic to filter suboptimal solution is encompassed. For this method, an arbitrary number of evenly distributed sample points are determined in the utopia hyperplane as a linear combination of the vertices with consistently varied weights. Each sample is then correlated to a Pareto optimal solution through a single objective optimisation process.

3.6. Conclusions

The theoretical results achieved and the very efficient solution methods proposed for the optimal control of nonlinear dynamical systems offer unique opportunities for exploitation in aircraft path planning. The greatest advantage lies in the fact that the
optimality of the solution is mathematically supported. Pseudospectral transcription methods are emerging as the most efficient solution techniques and yield a considerable flexibility in their implementation. Suitable multi-objective formulations are, on the other hand, essential to capture multiple conflicting objectives and unfeasible constraints. This chapter reviewed the fundamental solution techniques and multi-objective optimality strategies that can be adopted for aircraft trajectory optimisation applications. The knowledge gathered from this review is exploited when designing the Multi-Objective 4D Trajectory Optimisation (MOTO-4D) algorithms presented in chapter 5.

3.7. References


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Chapter 4

Aircraft trajectory optimisation models and optimality criteria

This chapter presents the various optimality criteria and constraints that were developed or extracted from the literature to support the MOTO-4D algorithms in addressing the operational, economic and environmental performances of an aircraft in flight and to reproduce realistic weather and airspace/traffic conditions. The aim of this modelling activity was to derive adequate models for integration in the 4-PNV system prototype, achieving an acceptable trade-off between accuracy and computational efficiency. In particular, this chapter introduces simplified formulations that were developed for the gaseous pollutant emissions of conventional turbofan/turboprop aircraft engines, which will be some of the main objectives tackled in the MOTO algorithms.

4.1. Background

Optimality criteria and constraints are introduced in the optimisation problem by means of suitable models. Figure 4.1 outlines the interdependencies introduced in principle by models adopted in MOTO for environmental sustainability studies. These models can either consist of one or more mathematical functions, which are either analytically derived from physical/geometrical principles or empirically formulated, or consist of raw numerical data, usually in tabular form. From the theoretical point of view, it is desirable to identify representative functions in analytical form, i.e. being accurate, simple, versatile and universal, though these qualities are frequently conflicting in nature. The identification of functions in analytical form has the advantage of leading to results of higher scientific significance and/or of more general applicability, which are the most relevant from the research perspective. Another advantage of adopting functions in analytical form is that the effectiveness of
numerical solution strategies can be accurately estimated. In practice, in a number of applications, it is necessary to deal with numerical data in tabular or other forms. Some detail is therefore given to techniques for the exploitation of raw data, and to the necessary pre-arrangements to ensure numerical stability and convergence.

4.2. Flight dynamics

As the geometric design of flight trajectories is deeply related to the dynamics of the body in aerial motion, flight dynamics are a core component, and are discussed beforehand. The
focus of this work is on fixed-wing transport aircraft; hence the models introduced are specifically tailored to this category of flying platforms. A detailed discussion on the derivation of flight dynamics equations from first principles is described in [172-175].

4.2.1. Rigid body models

Assuming the aircraft to be a rigid body with a static mass distribution, an accurate model of its flight dynamics can be introduced, which complement the equilibrium of forces along the coordinate axes of a suitable Cartesian reference frame located in the centre of mass of the aircraft, named body frame, with the equilibrium of their momentums. This model involves a high number of parameters to define the properties of inertia and of aerodynamic stability and control forces. Adequate experimental and numerical investigations are typically required in order to define the parameters with good precision. For the implementation in TOP, and for other applications including flight simulation and trajectory estimation, flight dynamics are typically transcribed in a set of Differential Algebraic Equations (DAE). An introductive description of such derivation may be found in [172]. The set of DAE and complementary kinematic relations defining the six Degree of Freedom (6DOF) rigid body dynamics of a fixed-wing aircraft are [176]:

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} = \frac{g}{W} \begin{bmatrix}
F_{x,b} \\
F_{y,b} \\
F_{z,b}
\end{bmatrix} + g \begin{bmatrix}
-s\theta \\
s\phi \ c\theta \\
\phi \ c\theta
\end{bmatrix} + \begin{bmatrix}
rv - qw \\
pw - ru \\
qu - pv
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
I_{xx,b} & 0 & -I_{xz,b} \\
0 & I_{yy,b} & 0 \\
-I_{xz,b} & 0 & I_{zz,b}
\end{bmatrix}^{-1} \cdot
\begin{bmatrix}
M_{x,b} + (I_{yy,b} - I_{zz,b})qr + I_{xz,b}pq \\
M_{y,b} + (I_{zz,b} - I_{xx,b})pr + I_{xz,b}(r^2 - p^2) \\
M_{z,b} + (I_{xx,b} - I_{yy,b})pq - I_{xz,b}qr
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_f \\
Y_f \\
Z_f
\end{bmatrix} = \begin{bmatrix}
c\theta \ c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\
c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\
-s\theta & s\phi c\theta & c\phi c\theta
\end{bmatrix} \begin{bmatrix}
u \\
v \\
w
\end{bmatrix} + \begin{bmatrix}
\frac{v_{wx,f}}{v} \\
\frac{v_{wy,f}}{v} \\
\frac{v_{wz,f}}{v}
\end{bmatrix}
\]
Multi-objective 4-dimensional trajectory optimisation for intent-based operations in dynamic airspace

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
1 & s\phi s\theta/c\theta & c\phi s\theta/c\theta \\
0 & c\phi & -s\phi \\
0 & s\phi/c\theta & c\phi/c\theta
\end{bmatrix} \cdot \begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (4.4)

where:

\(u, v, w\): translational velocity components along the three axes of the body reference frame [m s\(^{-1}\)];

\(p, q, r\): rotational velocity components around the three axes of the body reference frame; respectively representing rolling, pitching and yawing rates [rad s\(^{-1}\)];

\(\phi, \theta, \psi\): Euler angles, respectively representing bank, elevation and azimuth/heading Euler rotations [rad];

\(x_f, y_f, z_f\): components of the relative position vector between the Earth-fixed reference frame and the body centre of mass [m];

\(F_{x_b}, F_{y_b}, F_{z_b}\): resultants of the aerodynamic and propulsive forces acting along the three axes of the body reference frame [N];

\(M_{x_b}, M_{y_b}, M_{z_b}\): resultants of the aerodynamic and propulsive moments acting around the three axes of the body reference frame [N m];

\(I_{xx_b}, I_{yy_b}, I_{zz_b}, I_{xz_b}\): non-null components of the inertia tensor [kg m\(^2\)];

\(v_{Wx_f}, v_{Wy_f}, v_{Wz_f}\): components of the wind vector along the three axes of the Earth-fixed reference frame [m s\(^{-1}\)];

\(W\): aircraft weight [N], which may be either constant or subject to fuel consumption.

In particular, eq. 4.1 represent the translational dynamics, eq. 4.2 the rotational dynamics, eq. 4.3 the kinematics and eq. 4.4 the Euler rotations of the body frame with respect to the Earth-fixed reference frame. Rigid-body models are considered unsuitable for the calculation of trajectories on medium-long timeframes, and this is both due to the large dimensions and complexity of the resulting TOP/estimation problem, as well as the presence of short period modes that are keen to introduce numerical instabilities [177]. Rigid body models are nevertheless fundamental for the study of transition manoeuvres and more in general for the dynamic stability and control analysis and design of aircraft, and are successfully adopted in a
number of trajectory optimisation studies, in combination with a careful selection of path constraints.

4.2.2. Point-mass models

A commonly adopted approach to derive a simplified set of equations of motion for atmospheric flight is based on the approximation of the aircraft as a point-mass object thereby neglecting all aspects associated to its rotational dynamics. The resulting dynamics are characterised by only Three Degrees Of Freedom (3DOF) – i.e. the three spatial coordinates – which is the name adopted to define this family of models. These models are based on Newton’s second law expressed along the coordinate axes of the body frame, and on the expression of the motion of such frame with respect to an inertial reference frame of convenience. All aspects associated with the rotational state of the aircraft are neglected. The model can involve either a constant mass or a variable mass. Models belonging to the first category are adopted when the analysed timeframe is relatively limited, so that the fuel consumption may be neglected, or when no fuel is consumed, such as in the case of sailplanes or total engine failures. As an example, the following set of DAE is associated with a variable mass 3DOF model [172]:

\[
\begin{align*}
\dot{v} &= \frac{g}{W} (T \cos \epsilon - D - W \sin \gamma) \\
\dot{\gamma} &= \frac{g}{v W} \cdot [(T \sin \epsilon + L) \cos \mu - W \cos \gamma] \\
\dot{\chi} &= \frac{g}{v W} \cdot (T \sin \epsilon + L) \sin \mu \\
\dot{\phi} &= \frac{v \cos \gamma \sin \chi + v_{w,\phi}}{R_E + z} \\
\dot{\lambda} &= \frac{v \cos \gamma \cos \chi + v_{w,\lambda}}{(R_E + z) \cos \phi} \\
\dot{z} &= v \sin \gamma + v_{w,z} \\
\dot{m} &= -FF
\end{align*}
\]

where the state vector consists of the following variables:

\(v\): longitudinal velocity (scalar) \([\text{m s}^{-1}]\);

\(\gamma\): flight path angle (scalar) \([\text{rad}]\);

\(\chi\): track angle (scalar) \([\text{rad}]\);
\( \phi \): geographic latitude [rad];

\( \lambda \): geographic longitude [rad];

\( z \): flight altitude [m];

\( \epsilon \): thrust angle of attack [rad];

\( m \): aircraft mass [kg];

and the variables forming the control vector are:

\( T \): thrust force [N];

\( N \): load factor [ ];

\( \mu \): bank angle [rad];

Other variables and parameters include:

\( D \): aerodynamic drag [N];

\( v_w \): wind velocity, in its three scalar components [m s\(^{-1}\)];

\( g \): gravitational acceleration [m s\(^{-2}\)]

\( R_E \): Earth radius [m];

\( FF \): fuel flow [kg s\(^{-1}\)].

Frequently the modelling is restricted to the flight profile in the vertical plane or in the horizontal plane only.

### 4.2.3. Energy state models

Energetic approaches to the solution of minimum cost-to-climb were developed by Erzberger and colleagues, assuming only that aircraft energy is monotonically increasing during climbs and monotonically decreasing during descents.

### 4.3. Turbofan and turboprop engine models

This section will briefly outline useful models for fuel consumption and thrust with respect to current generation turbofan/turbojet and turboprop engines. Different propulsive technologies are potentially associated with vastly dissimilar energy sources and consumption rates, and
this is increasingly evident with the appearance of novel alternative propulsion systems. Notwithstanding, most of the currently operated aircraft rely on air-breathing internal combustion engines of the turbofan or turboprop type, employing hydrocarbon fuels, which are an established technology with efficient and cost-effective production and supply chains. Within the MOTO context, steady state empirical models are commonly employed to reproduce the dependencies of thrust and fuel flow on altitude ($z$), true airspeed ($v_{TAS}$), temperature and throttle ($\tau$), which are the only ones directly associated with the aircraft trajectory. The following empirical expressions were adopted in the development of Eurocontrol’s Base of Aircraft Data (BADA), to determine the climb thrust and the fuel flow $FF$ of a turbofan propelled aircraft, which operationally equates to the maximum thrust $T_{MAX}$ in all flight phases excluding take-off [178]:

$$T_{MAX} = C_{T1} \left(1 - \frac{H_p}{C_{T2}} + C_{T3}H_p^2\right) \left[1 - C_{T5}(\Delta T - C_{T4})\right]$$ (4.6)

$$FF = \max \left[ \tau T_{MAX} C_{f1} \left(1 + \frac{v_{TAS}}{C_{f2}}\right), \ C_{f3} \left(1 - \frac{H_p}{C_{f4}}\right) \right]$$ (4.7)

where $\tau$ is the throttle control, $H_p$ is the geopotential pressure altitude in feet, $\Delta T$ is the deviation from the standard atmosphere temperature in kelvin, $v_{TAS}$ is the true airspeed. $C_{T1} ... C_{T5}, C_{f1} ... C_{f4}$ are the empirical thrust and fuel flow coefficients, which are also supplied as part of BADA for a considerable number of currently operating aircraft [178]. Similarly, the following empirical expressions were adopted in BADA to determine the maximum thrust and fuel flow of a turboprop propelled aircraft:

$$T_{MAX} = \frac{C_{T1}}{v_{TAS}} \left(1 - \frac{H_p}{C_{T2}}\right) + C_{T3}$$ (4.8)

$$FF = \max \left[ \tau T_{MAX} C_{f1} \left(1 - \frac{v_{TAS}}{C_{f2}}\right) \cdot \left(\frac{v_{TAS}}{1000}\right), \ C_{f3} \left(1 - \frac{H_p}{C_{f4}}\right) \right]$$ (4.9)

where again $C_{T1} ... C_{T5}, C_{f1} ... C_{f4}$ are also supplied as part of BADA for a considerable number of currently operating aircraft [178]. The accuracy of the empirical models and of the coefficients supplied as part of Eurocontrol’s BADA were analysed in [179].
Due to the very competitive specific energies and energy densities, which are furthermore important in the weight/volume sensitive aerospace domain, no significant change in the market-share of hydrocarbon fuels is expected in the near-term future. These aspects, combined with the lengthy and costly development and certification processes to be undertaken by any innovation in the aviation domain, introduce a considerable technological inertia. Given the generational lifespan of aircraft models, it is expected that petroleum-based fuels will still be the largest source of chemical energy for aeronautical propulsion at least for the next two decades. For all these reasons, current generation jet fuels are widely targeted in trajectory optimisation studies for the assessment of aviation environmental impacts in the future.

### 4.4. Pollutant emissions

Emission Indexes ($EI$) specific to each Atmospheric Pollutant ($AP$) species were introduced in order to distinguish the dependencies of pollutant emissions from the Fuel Flow ($FF$), and are very frequently adopted in TOP studies. The general expression to calculate the total emission of the $AP$ from its associated Emission Index ($EI_{AP}$) expressed in [Kg$_{AP}$/Kg$_{Jet\text{-}A\text{-}1}$] is:

$$\int_{t_0}^{t_f} EI_{AP}(\tau) \cdot FF(\tau, v, z) \, dt \quad [\text{Kg}]$$

where $FF$ is the fuel flow [Kg s$^{-1}$].

The ICAO has established an extensive and constantly updated databank for engine emissions based on data collected independently by a number of entities, which proves being a valuable starting point for simplified empirical engine emission models [180]. The fuel-specific $EI$ are measured at the standard throttle settings defined in ICAO Annex 16 volume 2 [181]. In particular, for an exclusively subsonic engine, the reference throttle settings are take-off (100% of rated engine thrust), climb (85%), approach (30%), and idle (7%).

Eurocontrol’s BADA employs an empirical calculation method for the $FF$ as a function of the engine thrust ($\tau$), of the ambient pressure and of the true airspeed conditions [178]. For the implementation of Eq. 76 in a trajectory optimiser, it is convenient to refer to the differential expression:
The various aviation-related AP taken into consideration have different dependencies, which are discussed separately. The most relevant carbon-related exhaust products of fossil fuels are carbon dioxide ($\text{CO}_2$), carbon monoxide (CO) and unburned hydrocarbons (HC). CO and HC are significantly noxious for both the environment and the living beings, therefore are primarily targeted. Significant amounts of CO and HC are generated during the incomplete combustion incurring at low throttle settings [182]. An empirical model for CO and HC emissions ($E_{\text{CO/HC}}$) at mean sea level based on nonlinear fit of turbofan engines experimental data available in the ICAO emissions databank is:

\[
\frac{d \text{AP}}{dt} = EI_{\text{AP}}(\tau) \cdot FF(\tau, v, z) \left[ \frac{\text{Kg/s}}{} \right]
\]  

(4.11)

where, as a first estimate, the fitting parameters $c_{1,2,3}$ accounting for the CO emissions of 165 currently operated civil turbofan engines from the ICAO emissions database are $c = \{0.556, 10.208, 4.068\}$ for CO and $c = \{0.083, 13.202, 1.967\}$ for HC [183]. Figure 4.2 and 4.3 represent the experimental data and the empirical models.

![Figure 4.2. Empirical fit of CO emissions as a function of the throttle for a large number of currently operated turbofan engines [27].](image-url)
Figure 4.3. Empirical fit HC emissions as a function of the throttle for a large number of currently operated turbofan engines [27].

All the carbon contents of the fuel that are not transformed into CO or HC are transformed in CO₂, which has a positive radiative forcing impact and therefore is a major contributor to the greenhouse effect. The reference value is 3.16 tonCO₂/tonJet-A1 [35]. At rising combustion temperatures the atmospheric nitrogen increasingly reacts with oxygen, generating a family of nitrogen-based combustion products, the nitrogen oxides (NOX), which are associated with important impacts and shall therefore be mitigated. Based on the ICAO emission databank, an empirical curve fit model can be introduced for the NOX emission index at mean sea level based on the throttle setting. The following expression, plotted in Figure 4.4, is an example of such curve fitting comprehensively accounting for 177 currently operated civil aircraft engines [183]:

\[
EI_{NOX}(\tau) = 7.32 \tau^2 + 17.07 \tau + 3.53 \quad \left[ \frac{g}{Kg} \right]
\]  

(4.13)
In order to obtain an accurate estimate of pollutant emissions at height, a methodology commonly referred to as “Method 2” was developed by Boeing in 1995 [184, 185]. The method proposes an empirical correction to account for installation effects, and subsequently introduces $EI$ corrections based on ambient temperature, pressure and relative humidity. The turbofan and turboprop engine models adopted in BADA and presented in section 4.3 implement the altitude dependency as part of the empirical fuel flow models following this philosophy. Further information on the modelling of aircraft emissions is available in [45, 56].

### 4.5. Operational costs

Operational costs of aircraft are a fundamental aspect of trajectory optimisation. It is necessary in particular to capture the economic dependencies on flight time, which is typically a conflicting criterion with respect to fuel consumption and gaseous emissions. Moreover, some sorts of taxation scheme have dependencies with the flown trajectory, and should also be modelled. By taking into consideration Maintenance Costs (MC), flight Crew Costs (CC), Schedule Costs (SC), Cabin Services (CS) and Fees/Taxes (FT), the Total Costs (TC) of a commercial transport mission can be expressed as:
\[ TC = MC(t) + CC(t) + SC(t) + CS(t) + FT(t, x) \]  

(4.14)

Maintenance, crew and ownership costs generally are characterised by a linear dependence on flight time, but crew rotations and shifts introduce step increases when the section is long enough. Schedule costs are a highly nonlinear component, capturing all implications of a suboptimal timetable and fleet exploitation, including unnecessary stopovers in the rotation. Cabin services are also typically nonlinear, for example involving steps when additional on board meals are introduced. Fees and taxes encompass a wide range of different dependencies such as landing/parking fees, carbon taxes etc.

### 4.6. Atmosphere and weather

Diversions around unpredicted hazardous weather cells and extensive flight periods in regions of headwind or crosswind conditions have substantial negative effects on all environmental and economic performances of a flight, in addition to potentially causing delays that perturb operations and negatively impact passenger satisfaction. Furthermore, many of the models introduced in the trajectory optimization framework require local atmospheric data as input. For all these reasons accurate and updated weather data are essential for advanced CNS+A DSS such as the 4-PNV system. Meteorological data handled by ground-based ATM/ATFM systems shall correspond to data handled by airborne avionics systems as far as practical. This ensures consistency in the 4DT planning and negotiation/validation processes and supports full interoperability, which is an essential aspect for the functional air/ground integration being implemented as part of the CNS+A roadmap. It is therefore convenient to briefly review the most recent standards and planned evolutions of weather data services for aviation and propose their implementation in the ground-based ATM DSS development [186].

RTCA DO-340 introduces an advanced concept of use for Meteorological (MET) data link services [187]. These are specified in terms of service category, method of delivery and of the weather information involved. The MET services are classified into 2 categories. Category 1 services are the primary means of delivering MET information and may be exclusively relied on to support decisions without questioning their validity, while Category 2 services are useful for making non-critical decisions but should not be relied upon as the sole source of information. Category 1 services comprise both MET and Aeronautical Information Service (AIS) data links. MET data links are used on-board aircraft to provide weather
information for supporting flight crew decisions. There are three types of pilot decision support services which reflect the different planning and execution needs of the flight crew: a Weather Planning Decision Service (WPDS), a Weather Near-term Decision Service (WNDS) and a Weather Immediate Decision Service (WIDS). WPDS provides weather information for strategic planning such as in the case of changes in routing or cruise altitude due to ATFM initiatives or destination airport closures. In such scenarios, it is assumed that the flight crew has an advance time of 20 minutes or more to comprehensively evaluate the situation and plan/validate diversions or route amendments. WNDS provides weather information for tactical decision-making such as avoiding hazardous weather cells (including cumulonimbus, icing, turbulence, etc.) especially in terminal arrival/departure operations. In these cases, the flight crew has limited time for replanning and co-ordination with ATM operators, typically between 3 and 20 minutes. WIDS is conceived to provide weather information for freshly detected or quickly evolving weather hazards in order to allow the flight crew to initiate an emergency avoidance or abort take-off/landing. These decisions are assumed to require immediate action from within a few seconds to 3 minutes.

All the services are supported by three delivery modes: Broadcast, Demand and Contract. Broadcast data link delivery service issues continuous regular transmissions of MET information to all aircraft within range. Demand and Contract data links delivery services require two-way communication between the aircraft and ground station, where the flight crew initiates a request for specific MET information which the ground station then responds to. For the Demand service, the ground station only needs to respond to an initiated request, while for the Contract service, the information request is usually pre-coordinated and this requires the ground station to monitor the aircraft and provide the MET information at predefined time intervals or position.

RTCA DO-308 specifies four different categories for MET data formats [188]. These are: point data, area data, vector graphics and gridded data. Point and area data are given in alphanumeric strings and, as their name suggest, provide weather information on either a single geospatially located point or over an area delimited by a polygonal line. Examples of point data include the conventional Meteorological Terminal Air Report (METAR) and Terminal Aerodrome Forecast (TAF), whereas examples of area data include Significant Meteorological Information (SIGMET). Vector graphic data are images, represented by vectors, points, lines or other geometric entities and can be used to mark out volumes of
interest. Gridded data, typically in the form of General Regularly-distributed Information in Binary format (GRIB), consist in a 4-dimensional structured grid (latitude, longitude, altitude, time) of weather data with a forecast time dimension. The amount of MET information exchanged is also dependent on the delivery method. Broadcast services always transmit the full set of weather information, while demand and contract will usually transmit a subset of information based on the initiated request and thus have shorter transmission times. The nature of the decision service also affects the type of information provided. For example, Table 4.1 shows the information provided by the three pilot decision support services.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time horizon</td>
<td>Greater than 20 mins</td>
<td>3 mins to 20 mins</td>
<td>Less than 3 mins</td>
</tr>
<tr>
<td>Profile grid-point data</td>
<td>Offline and strategic online operations</td>
<td>Tactical online operations</td>
<td>Emergency operations</td>
</tr>
<tr>
<td>Airport Equivalent</td>
<td>METAR (incl. RVR), TAF</td>
<td>RVR, gusts and wind shear</td>
<td></td>
</tr>
</tbody>
</table>

As WPDS is used for route planning and optimisation, the information transmitted has a significantly longer time horizon than WNDS and WIDS. The geographic extent of WPDS MET data is also significantly larger as WPDS data is conceived for strategic large-scale replanning and diversions, whereas WNDS information is used for local tactical rerouting or altitude changes and might involve higher fidelity and resolution, while these tasks are not supported by WIDS. The profile grid-point data is mainly used for arrival and departure planning and involves MET information along the planned flight profile only. The information provided in WIDS is used for more tactical operations such as Airborne Separation Assurance Systems (ASAS). Hazardous weather information is similar across all the three services, while for airport weather, WPDS and WNDS provide more comprehensive information in the form of METAR, TAF and Runway Visual Range (RVR), while WIDS provides only visibility and wind shear weather data. RTCA DO-324 specifies the Required Communication Performance (RCP) for the service delivery [189]. These are determined in terms of Transaction Time (TT) and can be either the RCP Transaction Time ($TT_{RCP}$) or the Normal Transaction Time ($TT_{95}$), both expressed in seconds. A transaction is defined as the
basic unit of an interaction between peer parties which includes one or more operational messages that are transmitted from one party to the other. $T_{T_{RCP}}$ is the maximum time for completion of a transaction, while $T_{T_{95}}$ is the time before which 95% of all transactions should be completed. Different TT requirements are defined for airport, terminal and en-route domains. These are tabulated in Tables D7 to D15 of RTCA DO-324 [189].

The recommended quality of MET information is defined in ICAO Annex 3 [190], which lists the desirable accuracies of measurements forecasted data for a range of weather data. A partial list is provided in Table 4.2, which shows the desired quality of measured data and also the TAF data. The rest of the document also provides recommendations regarding forecast data for general trends, take-off and en-route cases. However, these are not operational requirements but rather desirable accuracy figures to fulfil typical operational needs. RTCA DO-308 also identifies a list of candidate MET products [188]. In particular, the Global Forecast System (GFS) used for planning and offline/strategic online optimisation purposes falls under the World Area Forecast Centre group shown in Table 4.3.

<table>
<thead>
<tr>
<th>Weather data</th>
<th>Accuracy of measurement</th>
<th>Accuracy of forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind direction</td>
<td>± 10°</td>
<td>± 20°</td>
</tr>
<tr>
<td>Mean surface wind</td>
<td>± 1 knot up to 10 knots</td>
<td>± 10 5 knots</td>
</tr>
<tr>
<td>Visibility</td>
<td>± 50 m up to 600 m</td>
<td>± 200 m up to 800 m</td>
</tr>
<tr>
<td>Cloud amount</td>
<td>± 1 okta</td>
<td>One okta below 1500 ft</td>
</tr>
<tr>
<td>Cloud height</td>
<td>± 33 ft up to 330 ft</td>
<td>± 100 ft up to 1000 ft</td>
</tr>
<tr>
<td>Air temperature</td>
<td>± 1°C</td>
<td>± 1°C</td>
</tr>
</tbody>
</table>
Table 4.3. Reference global forecast MET data for WPDS [188].

<table>
<thead>
<tr>
<th>World Area Forecast Centre Information</th>
<th>Data Format</th>
<th>Refresh Rate</th>
<th>Validity (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind (Latitudinal and Longitudinal)</td>
<td>Gridded</td>
<td>6 hr</td>
<td>36</td>
</tr>
<tr>
<td>Temperature</td>
<td>Gridded</td>
<td>6 hr</td>
<td>36</td>
</tr>
<tr>
<td>Humidity</td>
<td>Gridded</td>
<td>6 hr</td>
<td>36</td>
</tr>
<tr>
<td>Tropopause – height, temperature, direction</td>
<td>Gridded</td>
<td>6 hr</td>
<td>36</td>
</tr>
<tr>
<td>Maximum wind – speed, direction, height</td>
<td>Gridded</td>
<td>6 hr</td>
<td>36</td>
</tr>
<tr>
<td>Significant weather charts</td>
<td>Vector</td>
<td>6 hr</td>
<td>13</td>
</tr>
</tbody>
</table>

The WPDS data described above have been sufficient for flight planning and offline/strategic online trajectory optimisation applications within airborne avionics and ground-based ATM DSS, as the main factors driving the lateral/vertical planning are the 4D wind and temperature fields. Nevertheless, a significant gap is represented by the lack of information regarding weather cells and other adverse phenomena. Consequently, in terms of future CNS+A evolutions, ICAO’s ASBU roadmap acknowledges that further improved meteorological services are required to implement advanced functionalities such as the ones involved in 4D-TBO. Long-range weather forecasts (such as GRIB and METAR) are already being supplemented by nowcasting techniques for the provision of ATM and ATFM services, particularly in the terminal area. Some of these advanced weather services are already acknowledged by RTCA in DO-308, as shown in Table 4.4. In particular, the US National Centre for Atmospheric Research’s Thunderstorm Identification, Tracking, Analysis and Nowcasting (TITAN) system is an example of a nowcasting system belonging to the NCWF class and used by ANSPs in the US, Australia and South Africa.

Nowcasting provides detailed current weather information extrapolated up to 6 hours into the future, typically with sub-kilometre spatial resolution and a temporal resolution in the order of minutes. Nowcasting systems use sophisticated algorithms to track and extrapolate individual storm cells from weather radar information. Aviation weather service providers are increasingly supplementing this information with satellite imagery and ground-based sensors such as lightning detectors. The ability to downlink weather data turns datalink-equipped aircraft into mobile weather sensors able to report on the weather in their vicinity. Current-generation ATM systems are capable of receiving this data and exporting it to aviation weather service providers. Such data can contribute to a “4D weather cube”, supplementing
weather data along major air routes and allowing those sections of the cube that are of the most interest to aviation users to be updated more frequently and more accurately. However, the availability of a near real-time, high-resolution full 4D Weather Cube is not far off. In 2015 SESAR successfully demonstrated a web-services-enabled SWIM implementation of the FAA NEXTGEN 4D Weather Cube concept in their “Optimising trajectories over the 4DWeatherCube” SWIM Masterclass. Further research and development is currently being carried out as part of SESAR’s TOPMET project.

Table 4.4. Advanced METLINK products for flight planning in the USA and Europe [188].

<table>
<thead>
<tr>
<th></th>
<th>Data Format</th>
<th>Refresh Rate</th>
<th>Validity (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>National Weather Service (NOAA)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>National Convective Weather Forecast (NCWF)</td>
<td>Gridded/Vector</td>
<td>5 min</td>
<td>1</td>
</tr>
<tr>
<td>Graphical Turbulence Guidance (GTG)</td>
<td>Gridded</td>
<td>15 min</td>
<td>0.25</td>
</tr>
<tr>
<td>Current Icing Product (CIP)</td>
<td>Gridded</td>
<td>1 hr</td>
<td>N/A</td>
</tr>
<tr>
<td>Forecast Icing Potential (FIP)</td>
<td>Gridded</td>
<td>1 hr</td>
<td>3</td>
</tr>
<tr>
<td><strong>WIMS (FLYSAFE)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WIMS thunderstorm</td>
<td>Gridded/Vector</td>
<td>5 min to 6 hr</td>
<td>0.2-1</td>
</tr>
<tr>
<td>WIMS turbulence</td>
<td>Gridded/Vector</td>
<td>6 hr</td>
<td>36</td>
</tr>
<tr>
<td>WIMS icing</td>
<td>Gridded/Vector</td>
<td>15 min to 12 hr</td>
<td>0.24-24</td>
</tr>
<tr>
<td>WIMS wake vortex</td>
<td>Gridded/Vector</td>
<td>1 to 6 hr</td>
<td>2-12</td>
</tr>
</tbody>
</table>

The weather model currently implemented in the 4-PNV system processes the global weather data available from the National Climatic Data Center (NCDC) of the National Oceanic and Atmospheric Administration (NOAA) and selectively extrapolates the required information on a structured 4D grid. The data currently employed is extracted from the GFS, collected on a 0.25° latitude and longitude resolution grid, updated every 6 hours (4 times daily) and including projection of up to 180-hours in 3-hours intervals. Figure 4.5 depicts the 3-dimensional wind and temperature fields at some of the typical cruise altitudes of a jetliner for a 15-hour advance forecast sampled at 0600 UTC of April 3rd 2015. Figure 4.6 depicts the 3-dimensional relative humidity field in the same conditions.
Whenever an interpolation is performed to translate data tabulated at pressure altitudes into Cartesian altitudes and to extrapolate data at higher resolution grids, the following expression (known as barometric formula) should be adopted to determine intermediate values instead of linear interpolation:

\[
\frac{P}{P_0} = \left[ \frac{T_0}{T_0 + L (h - h_0)} \right]^{\frac{\gamma}{R^*}}
\]

where \( P \) is the extrapolated pressure, \( P_0 \) and \( T_0 \) are reference pressure and temperature, \( h \) and \( h_0 \) are the altitude of extrapolation and reference points, \( L \) is the locally evaluated lapse rate and \( R^* \) is the specific gas constant for air.
Figure 4.6. Relative Humidity 3D field at typical jetliner cruise altitudes [183].

The calculation and exploitation of weather conditions and forecasts are actively researched topics, as the influence in the operational efficiency and in the accuracy of TOP results is recognised. Further details are given in [191].

4.7. Noise

The noise generated by various mechanical, thermochemical and fluid dynamic processes within the engine as well as around the aircraft structure is propagated through the atmosphere, which acts as a low pass filter due to thermo-fluid-dynamics and molecular processes [192]. When the receiver is at a sufficient distance, most contributes can be merged into a single point source with non-uniform distribution [192]. By means of a suitable propagation model, and thanks to the adoption of apposite Demographic Distribution Databases (D3) and Digital Terrain Elevation Databases (DTED), noise footprint on the
ground can be determined. Notwithstanding, the high physical complexity associated with the emission and propagation of aircraft noise limits the adoption of accurate models to offline optimisation studies, typically performed to optimise the departure and approach paths of aircraft, which are the portions of the flight in close proximity to the ground. The specific noise problem formulation also depends on the governing regulations, which mandate different output metrics [193-195]. More specifically, due to the interactions between the sound waves emitted by various locations at different frequencies and the aircraft structure at various attitudes and airspeeds, noise emissions are characterised by highly uneven and nonlinear 4D profiles. The vagaries of the atmosphere in terms of unsteady and non-uniform composition and thermodynamic state will further affect the propagation of sound waves and thus alter the noise footprint on the ground. Notwithstanding, some assumptions may enable the integration of simplified noise models in real-time avionics and ATM system implementations for evaluation of an estimated noise footprint of generated trajectories. For instance, the propagation of noise generated by a point source with non-uniform distribution can be formulated adopting a Noise-Power-Distance (NPD) model. The integration of this model with a simplified worst-case emission and with limited resolution and computationally optimised DTED and D3 can enable a quasi-real-time noise model implementation. A number of optimisation case studies specifically targeted the minimisation of perceived noise emissions in the design and redesign of departure and arrival procedures [196-202]. More details on the modelling of noise emission and propagation are given in [192, 194, 195, 202, 203].

4.7.1. Optimisation of arrival and departure trajectories

Figure 4.7 and 4.8 present the results of the optimisation of departure trajectories of a medium-range narrow-body airline transport aircraft carried out by Quaglia and Madani implementing the Integrated Noise Model (INM) [195, 204]. The optimisation objectives consisted in minimum noise exposure and fuel consumption. The 70 dB SEL area is one of the key measurable of noise exposure, and was thereby adopted for noise mitigation [194]. Nondominated Sorting Genetic Algorithms (NSGA) were adopted for these offline studies, enabling the identification of global optimality. Both case studies implemented the same engine thrust restrictions. In comparison to the original departure trajectory, the noise-optimised one consists of a less steep initial climb to attain higher airspeed, followed by a
steep zoomed segment and subsequently a gradual transition to attain the desired altitude and airspeed.

Figure 4.7. Noise footprint of a typical flight departure from London Heathrow, highlighting the 70 dB Sound Exposure Level (SEL) area [204].
Figure 4.8. Noise footprint of an optimised departure trajectory, reducing the 70 dB SEL area by 47% [204].

Although these case studies were performed offline with dedicated computing facilities, an implementation of suitably approximated models in CNS+A DSS for real-time online applications is planned.

4.8. Condensation trails

Contrails are increasingly addressed in aviation sustainability research due to their recognised radiative forcing potential [40]. Contrails are formed when the combination of local atmospheric temperature and relative humidity are suitable, so that the hot water vapour in the engine exhausts leads to locally attain or exceed liquid saturation conditions, generating droplets that subsequently freeze. The discriminant for such phenomenon is referred to Schmidt-Appleman criterion, and is based on the evaluation of the slope of the exhaust mixing curve (MS):
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\[ MS = \frac{p c_p EI_{H_2O}}{\epsilon (Q - \frac{v}{\kappa})} \text{ [Pa/K]} \]  

where \( p \) is the local atmospheric pressure [Pa], \( c_p \) is the specific heat capacity of air [J Kg\(^{-1}\) K\(^{-1}\)], \( EI_{H_2O} \) is the water vapour emission index, \( \epsilon \) is the ratio of molar masses of water vapour and air, \( Q \) is the specific fuel combustion heat [J Kg\(^{-1}\)], \( v \) is the true airspeed [m s\(^{-1}\)] and \( \kappa \) is the thrust specific fuel consumption [Kg N\(^{-1}\) s\(^{-1}\)]. The formed contrails are susceptible of convective and thermodynamic processes, which may lead to a rapid dissipation of the formed contrail or to a belated formation. In particular, the theory distinguishes three phases: jet, vortex and dispersion phases, which are governed by different physical processes and are hence modelled individually. More details are given in [205-209]. In relation to the growing environmental and social concerns, during the last decade the research in aircraft trajectory optimisation has increasingly addressed contrail avoidance, and a number of approaches were proposed [30, 210-215]. Among other solutions, a 4D mapping of the lifetime and radiative forcing associated with contrails was proposed by the authors for real-time implementations of contrail models [30, 215]. An altitude-constrained 2-Dimensional plus Time (2D+T) trajectory optimisation case study addressing the conflicting objectives of long-lasting contrails and of fuel consumption is depicted in Figure 4.9.

\[ \text{Figure 4.9. 2D+T MOTO with respect to contrail lifetime and fuel consumption [215].} \]
4.9. Airspace and air traffic models

In addition to reproducing real operational conditions, the trajectory optimisation algorithms shall implement the constraints and operational characteristics deriving from ATM route, altitude and airspeed clearances, and translate them in entities that are meaningful for the numerical optimisers. As section 3 discussed, these entities are either in the form of boundary conditions, path or dynamics constraints or cost functions. Therefore, airspace and air traffic modelling both in nominal conditions and in presence of considerable disruptions were studied as part of the trajectory optimisation research. Figure 4.10 shows the partition of the upper European airspace into sectors at FL310.

Figure 4.10. Partition of the upper European airspace into sectors at FL310.

4.10. Conclusions

This chapter introduced and briefly described the various models that are typically adopted in trajectory optimisation studies. While flight dynamics, engine thrust and to a lesser extent weather models are applicable across the entire trajectory optimisation spectrum, the models for pollutants, operational costs, noise, condensation trails and airspace / ATM constraints are less common but are crucially important for environmental sustainability assessments. As
documented in the subsequent sections, the trajectory optimisation algorithms implemented in this project integrate the models presented in this chapter in suitable arrangements.

4.11. References


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4-27
Chapter 5

ATM system prototype
design and implementation

This chapter describes the design and implementation of a 4-PNV ATM system prototype that integrates the developed MOTO and DAS algorithms. In particular, the chapter presents the CNS+A concept of operations and system architectures that were developed as part of this doctoral research to capture the concepts introduced in chapters 1-2 and lay the foundation for the design of the 4-PNV system prototype featuring the implemented MOTO and DAS techniques. The chapter subsequently presents the negotiation loops that were developed considering the use of MOTO algorithms both on-board and on the ground. Furthermore, the timeframe definitions and the requirements for the design and verification of the proposed MOTO and DAS algorithms are introduced. Subsequently, the detailed structures of the implemented MOTO and DAS algorithms are presented and the important issue of the interaction between MOTO, DAS and traffic flow optimisation processes is finally addressed. Some preliminary simulation case studies are also presented to demonstrate the algorithm capabilities.

5.1. Avionics and ATM DSS in the TBO context

The automated ATM DSS shall allow the equipped aircraft to fly user-preferred optimal flight paths without increasing the workload or compromising the situational awareness of flight crews and Air Traffic Control Operators (ATCO) [183]. This shall be attained by relinquishing the repetitive low-level tasks and restricting their responsibilities to high-level and emergency decisions. In order to cope up with the unpredictable disruptions intrinsically affecting the air traffic flows, novel automated
ATM DSS include functionalities for online air traffic flow optimisation considering high-density airspace, re-routing and re-scheduling in real-time. This is accomplished by sharing constraints and restrictions, as well as other relevant aircraft and airport performance and status, and optimisation criteria through enhanced data links in real time.

Modern FMS are the primary airborne DSS providing automated navigation and guidance services. Figure 5.1 depicts the architecture of NG-FMS, which acts as the airborne DSS and has been designed to integrate 4D-TBO. The key components of a typical FMS architecture are the Flight Management Computer (FMC), Multi Control Display Unit (MCDU), Flight Control Unit (FCU), Electronic Flight Instrument System (EFIS), Multi-Function Display (MFD) and Navigation Display (ND) [183]. Current day FMS provide area navigation (RNAV) services adhering to the Required Navigation Performance (RNP) levels in all flight profiles ensuring accuracy, availability, continuity and integrity by constantly monitoring the performance [216]. A number of integrated navigation and guidance functionalities are adopted in the FMS architecture for meeting the required performance levels [217]. FMS incorporating autopilots have the capability of ensuring the Required Time of Arrival (RTA) and do not have any database size or processing issues [218]. A number of concepts including an air-ground negotiation prototype for generating safe, fuel-efficient, very accurate, and air-ground synchronized 4D-trajectories were proposed exploiting flight segment groundspeed profiles and linking GNSS data to the aircraft FMS with feedback control [218].

NG-FMS were conceived as the key avionics enablers for generating globally optimal trajectories that fulfil the safety, operational, and environmental requirements in the 4D-TBO context. The key TBO functionalities of the NG-FMS design presented here are MOTO for both flight planning and real-time operations, 4DT monitoring, negotiation/validation with the ATM DSS and real-time rerouting and information updating. The integration of specific functions including Sense-and-Avoid (SAA) and integrity augmentation functions is also studied [219, 220]. Integrity management modules are used to generate caution (predictive) and warning (reactive) flags based on inputs from different sensors/systems and predefined decision logics. A loss of data leads to re-initialising of the 4DT planning and
subsequently the 4DT optimisation process. For instance, the main causes of GNSS signal outage and degradation in flight are identified and modelled to implement integrity thresholds and guidance algorithms in the Avionics-Based Integrity Augmentation (ABIA) system [221, 222]. The 4DT planning algorithms implemented in novel CNS+A DSS shall exploit the latest advances in MOTO, incorporating emissions and operational cost models. Issues such as trajectory and information synchronisation are addressed in order to ensure consistency of the computed intents between the aircraft and the ground. The airborne and the ground-based DSS shall therefore implement compatible trajectory prediction/estimation models, so that a consistent mathematical solution can be determined and implemented. This is accomplished by standard airspace, trajectory and aircraft descriptors and constraints to be shared between the aircraft and the ground.

Figure 5.1. NG-FMS architecture for remotely piloted aircraft.
5.2. CNS+A concept of operations

The CNS+A concept of operations in the online 4DT-IBO context is schematically represented in Fig. 5.2. This scenario is conceptually similar to the “democratic” 4D-TBO paradigm envisaged by Visser [4]. The 4-PNV system features 4DT planning, data exchange and negotiation/validation with the Next Generation of Flight Management Systems (NG-FMS) on board manned aircraft and RPAS [223-225]. The optimised 4DT intents generated by the NG-FMS are downlinked to the ground-based 4-PNV system via a Next Generation Aeronautical Data-Link (NG-ADL) (flow of information 1 in Fig. 5.2).

Several 4DT intents are generated by each NG-FMS, providing multiple options to identify a conflict free solution for each traffic in real-time [226], hence decreasing the length of negotiation time and reducing the need for ATM to calculate optimal 4DT remotely. Once an optimal conflict-free 4DT has been identified, reviewed and approved by the human Air Traffic Control Operators (ATCO), the 4DT validation is accomplished and a clearance is uplinked to the NG-FMS (flow of information 2).

Figure 5.2. Concept of operations of the online 4DT intent negotiation and validation in the CNS+A context.
When valid 4DT cannot be identified among the NG-FMS intents, the 4-PNV based on the performance weightings defined as part of an automation-assisted Collaborative Decision-Making (CDM) process calculates a new family of optimal 4DT and uplinks them to the aircraft. The NG-FMS then analyses the received 4DT, assessing the compliance with aircraft performance and operational envelope, and presents them to the flight crew/remote pilots in enhanced Human-Machine Interface and Interactions (HMI²) formats. After reviewing and selecting the preferred solution, the flight crew/remote pilots issue a confirmation to the 4-PNV (flow of information 3) [183]. The increased automation allows more aircraft to fly optimal flight paths, limiting the intervention of human operators to higher-level and emergency decisions.

5.3. Assumed operational timeframes

A large number of aspects in ATM are affected by uncertainties whose effects are integrated over time. Since the involved dynamics are often non-deterministic in nature, it is convenient to adopt a reference timeframe convention and formulate different optimisation problems, specifically tailored for each timeframe. The timeframe definition should be governed both by the numerical considerations and by operational motivations. From the numerical perspective, similarly governing dynamics and uncertainties of comparable magnitudes are requirements for a well-posed mathematical problem and hence good numerical performances. From the operational perspective, timeframe conventions that are already familiar or easily assimilated by the human operators are essential for the rapid and faultless deployment of new technologies. Based on these general principles, the first key distinction is between the online phase, when the air transportation mission is active, and the offline phase, when the mission is inactive. In the most recent terminology, ground operations form part of the online phase as well, as far as including payload boarding and disembarkation phases. In fact, variations in the ground timings are significantly propagated to the airborne phase. This heavy interdependence promote the formulation of a seamless gate-to-gate optimisation problem [34]. A subsequent timeframe distinction is based on the timespan before the occurrence of an unpredicted event affecting air traffic operations. Such unpredicted events would
include any occurrence perturbing the successful outcome of the mission, such as potential conflicts, weather cells, airspace or airport congestions, and system failures. Barnier and Allignol’s temporal classification of ATM operations provides a practical reference for this study, hence the adopted timespans are strategic, tactical and emergency [100].

The definition of the operational timeframes adopted in our research is presented in Figure 5.3 [227, 228], where the stages specifically targeted in the design of novel CNS+A DSS are encircled. A considerable level of coordination and management is performed before the flight is initiated, i.e. in the offline contexts. These stages are typically associated with less stringent requirements in terms of computational time; therefore a variety of models and optimisation strategies were investigated and substantial research activities are continuing in the subject. Although trajectory optimisation is actively investigated for offline contexts, the specific implementations are beyond the scope of this research and further details may be found in [48, 70, 229-237]. On the other hand, the online contexts impose increasingly restrictive requirements as the predicted conflict/hazard approaches.

![Figure 5.3. Assumed timeframes for the offline and online air traffic operations.](image)

In the strategic online context, the effect of uncertainties on a longer integration timespan induces a suboptimal estimation, but on the other hand, the longer time available for calculation and decision-making enables the adoption of more complex
models and algorithms in more extensive search domains. In the tactical online timeframe, uncertainties have limited effects, but a safe solution to the problem shall be attained in real-time, thus the involved models, decision logics and the TOP formulation are necessarily simpler and typically based on approximations and linearisations. The emergency phase traditionally involves robust and faultless decision logics for safe avoidance. The delineation between strategic, tactical and emergency differ in the literature and are usually set as part of the problem formulation and a distinction based on the type of action to be taken is recently prevailing [27, 238]. As most of the unpredicted disruptions incur while the flight mission is already active, i.e. in the online contexts, substantial expectations are put on the novel automated avionics and ATM DSS that will specifically target these stages. While systems conceived for the emergency context, also called “safety-nets”, will always be based to a large extent on safe avoidance logics, MOTO will play an increasingly higher role in the DSS addressing online strategic and tactical contexts, where significant ATM and ATFM duties are exerted.

Strategic and tactical level ATM problems have to be analysed in order to establish the top-level operational and technical requirements for the ATM systems. The novel CNS+A systems shall accommodate dense airspace, rerouting and rescheduling in real-time or quasi-real-time. All models shall be validated by simulation in realistic test scenarios. Simulation cases will cover all phases of flight and airspace configurations. Particular emphasis shall be devoted to the Terminal Manoeuvring Area (TMA). In the TMA, in fact, ATM online tasks are often critical since trajectories of climbing and descending aircraft intersect, requiring frequent tactical interventions, and more generally the limited runway capacity causes the arrival traffic to queue up, even when disruptions are not present. While concepts such as 4DT operations and Continuous Curved Descent Approaches (CCDA) are predicted to reduce TMA congestion, practical implementation aspects have to be evaluated, especially in the IBO context.

5.4. Top-level 4-PNV system design requirements

The 4-PNV model for “single attempt” negotiation of 4DT is based on the following requirements [226]:
1. The system shall identify a conflict-free 4DT for each traffic;
2. The system shall have a high computational efficiency, compatible with the online tactical operational timeframe defined in the previous section;
3. The system shall be able to negotiate and validate the 4DT intents of multiple aircraft, with multiple equipage in dense air traffic conditions;
4. The system shall reduce human operator’s workload and improve his situational awareness;
5. The system shall improve the overall optimality of air traffic in terms of environmental and economic sustainability objectives.

5.5. 4-PNV system architecture

Figure 5.4 depicts a schematic architecture of the 4-PNV system prototype. As described in sections 5.1 and 5.2, the online processes are distributed across the NG-FMS/NG-MMS, the Airline Operations Centres (AOC), the Air Navigation Service Providers (ANSP), the regional Air Traffic Flow Management (ATFM) centre and ATM units.
The 4-PNV analyses and selects the globally optimal combination of intents to best achieve global objectives, whilst ensuring that the validated 4DT are free from traffic conflicts, known or likely weather hazards and congested sectors.

5.6. Intent negotiation and validation loops

The online tactical rerouting capability requirements prescribe that 4-PNV validation algorithms are to be designed to achieve single-attempt negotiations. A 4DT negotiation protocol was developed in EUROCONTROL’s DOC 97-70-14 [12] and can be adopted to iteratively generate an optimal flight path, taking all constraints into consideration. The customized negotiation loops developed for the 4-PNV DSS were derived from previous research [226] and are depicted in Fig. 5.5 and Fig. 5.6, which respectively represent the 4-PNV initiated and the NG-FMS initiated loops. The shared trajectory intents include the aircraft’s unique identification and model, the wake-turbulence category, and the vector of 4DT waypoints in the prescribed format.

Figure 5.5. 4-PNV initiated 4DT intent negotiation/validation loop.
The ground-based 4-PNV system is the protagonist of the strategic online scenario as it retains a continuously updated global situational awareness, thanks to the surveillance infrastructure, to the aeronautical fixed services network and to in-built trajectory predictions algorithms. Uncertainties, such as long-term forecast weather phenomena, airport closures or sectors saturations, prompt the 4-PNV to initiate a strategic replanning and negotiation by up-linking new constraints to the NG-FMS, which are then incorporated as part of the on-board 4DT optimisation. Alternatively, the 4-PNV may compute optimal 4DT and uplink them for validation by the aircraft. If, after on-board evaluations performances/constraint violations exist (e.g. turn radius, climb rate), the aircraft downlinks a rejection message together with a new intent to the 4-PNV for validation. Multiple negotiation loops are allowed in the strategic online scenario but minimised thanks to the generation of multiple optimal 4DT intents for each aircraft. If for particular reasons the newly introduced constraints cannot be satisfied without violating aircraft/crew limitations (i.e., manoeuvrability, fuel limits, qualifications, crew hours, etc.), then the systems will display a suitable error message to the human operators for alternative actions. This situation is normally very unlikely and in current paradigms usually leads to diversion to alternate destination airports. Further research is recommended to assess which unusual situations may prompt the trajectory negotiation to fail, what heuristics could be implemented to address these cases and when, conversely, the process must necessarily be handed over to human operators. In the tactical online scenario, either the NG-FMS or the 4-PNV may initiate 4DT intent negotiations. The 4-PNV will act mainly as a centralised decision maker. The NG-FMS may initiate the trajectory negotiation due to locally detected weather changes, aircraft performance degradation, equipment failures or on-board emergency situations. Other manoeuvre-related factors such as inefficient heading changes, and unachievable climb/descent rates and altitudes due to the actual aircraft weight may also be causes of NG-FMS initiated negotiation loops. In the tactical online scenario, a “single-loop” negotiation is ultimately sought due to the reduced time and stringent traffic management commitments. Similarly to the strategic online, if a solution cannot be obtained through trajectory negotiation then ATCO’s and pilots’ direct intervention will be considered. In order to be adopted for both strategic online and tactical online contexts, with a reference time horizon of 5 minutes, it is assumed that
the total duration of optimization, negotiation and validation processes must remain under 180 seconds. Trajectories are checked for traffic conflicts and separation from hazardous phenomena. The validation algorithm assesses the lateral and vertical separation criteria and includes a simplified wake vortex modelling to assess the longitudinal separation. The Collaborative Decision Making (CDM) is substantially enhanced through the exploitation of semi-automated negotiation schemes.

![Diagram](image)

**Figure 5.6. NG-FMS initiated 4DT intent negotiation/validation loop.**

### 5.7. Trajectory optimisation algorithm implementation

This section outlines the overall approach and the specific techniques adopted to implement the selected MOTO and dynamic airspace reconfiguration techniques into software algorithms. The two MOTO algorithms as well as the Dynamic Airspace Sectorisation (DAS) algorithm are all based on an optimisation problem formulation. The top-level layout of an optimisation algorithm is represented in Fig. 5.7. A set of models representing real-world physical processes receives a vector of control inputs $\mathbf{u}$ and external conditions $\mathbf{k}$ to generate vectors of state variables $\mathbf{x}$ and output variables $\mathbf{y}$. These two vectors are then processed by suitably defined cost functions, which based on user-defined logics determine individual or arrays of performance indexes $\mathbf{J}$. The numerical optimiser module processes the performance indexes and
the state variables (as applicable) to identify a new vector of control inputs and external conditions. The process is repeated iteratively until some condition on the optimality of \( J \) is met, and at that stage the control inputs and the output variables associated with the identified optimal solution are stored. While this top-level layout can also describe the in-workings of population-based optimisation algorithms such as the one briefly mentioned in section 3.3.6, these were excluded from implementation as their preliminary assessment did not yield acceptable computation speed performances, but further research to develop suitably customised population-based algorithms for real-time aircraft trajectory optimisation applications is recommended.

A number of challenges are known to affect the application of MOTO functionalities in the online contexts, and particularly include:

1. computationally fast and efficient MOTO algorithm implementations, meeting the strict solution time requirements;
2. availability of secure, high-integrity and high-throughput air-to-ground data links for negotiation and validation of the generated 4DT intents;
3. availability of efficient and effective Human Machine Interface and Interactions (HMI²) formats and functions to enable the recalculation, review and customisation of optimised 4DT solutions by the human operators;
4. caching, progressive refinement and adaptation of optimal 4DT solutions within the algorithms to ensure the required reliability levels for dependable adoption of MOTO in automated 4DT planning functionalities;
5. synchronisation of 4DT predictions, constraints and performances between the ground-based systems and the aircraft for mathematical consistency;

6. capability to automatically generate and handle in real-time a considerable number of constraints arising from operation in dense traffic conditions, of which some can be unattainable and shall prompt recomputation of 4DT intents for a number of aircraft.

7. capability to generate a limited set of sufficiently diverse solutions which are globally-optimal and belonging to the Pareto front, especially when using a single non-population-based optimisation algorithm as in our case.

Research is currently addressing these limitations by enhancing the numerical implementations while at the same time outlining the operational and equipage requirements in view of the strict certification requirements. In this research project, two different MOTO algorithms have been implemented and evaluated, targeting respectively the enroute context and the Terminal Manoeuvring Area (TMA) context.

The fundamental distinction between these two settings lies in the fact that in the enroute scenario the altitude is constrained along the entire trajectory, whereas in the TMA context altitude constraints are only introduced in boundary conditions (initial and final altitudes). This is required in order for the optimiser to identify a feasible 4DT intent that attains a predefined approach fix, merge point or TMA exit gate at a certain desired altitude. The altitude constraint enforced along the entire enroute flight path typically consists in a constant altitude, but can also involve predefined Flight Level (FL) changes, and in both cases it reduces the original 4D-MOTO in an actual 2-Dimensional plus Time (2D+T) optimisation (2D+T-MOTO). The preference for this approach emerged during the research activities, as it was identified as more appropriate for near-term implementation in current generation system. This consideration also included a higher level of situational awareness due to the fact that current Human Machine Interfaces (HMI) are more effective in representing the lateral trajectory than the vertical profile. In addition to these motivations derived from current best practices, due to the potentially very long enroute phase of airline transport aircraft (i.e. up to ~7,500 NM) and according to the literature, it was identified that a fully 4D MOTO process for this entire phase would have often resulted in an exceedingly vast search space, preventing quasi-real-time
solution algorithms to be implemented, unless under severe restrictions or assumptions. Additionally, an optimal 4DT might be exceedingly complex in terms of number of manoeuvres and associated waypoints for datalink trajectory exchange, potentially voiding the requirement of this work in terms of feasible implementation in next generation ATM and avionics systems. Consequently, noting that as mentioned in section 1.4 many strategies are available to compute an optimised vertical profile, our study focussed on the more challenging problem associated with the lateral routing, effectively leading to a simplified multi objective Zermelo problem, which will become progressively more relevant when free routing will be extensively implemented in most airspace regions.

The selection of the specific problem formulation and of the numerical solution method for the planning of optimal 4DT intents in the TMA with respect to one or multiple user-defined criteria is based on recent research findings both within the research team and in the literature. In particular, while geometric path-planning algorithms similar to the enroute 2D+T MOTO implementation are considered more mature and resilient to route inconsistencies, the adoption of an optimal control-based formulation integrating a full set of three degrees of freedom (3DOF) flight dynamics introduces a level of accuracy which is considerably higher than the one attainable with geometric path-planning methods. Furthermore, the adaptation of geometric path-planning algorithms to a truly 4-Dimensional problem necessitates an increase in algorithm complexity that partly jeopardizes the benefits associated with such implementation. Such adaptation may for instance involve the integration of separate interdependent horizontal and vertical iterative sub-optimisation routines.

A TMA MOTO implementation distinct from the enroute MOTO implementation is desirable, as it fulfills the role of a technology demonstrator to assess the feasibility of state-of-the-art optimal control-based solution techniques, increasingly investigated in the scientific literature, to the application domain, where only non-optimal control-based implementations are currently adopted.

5.8. Trajectory optimisation for TMA operations

For the TMA implementation of the MOTO-4D algorithms, we focus on the terminal area, which involves relatively short arrival and departure trajectory legs, of typically
less than 60 nautical miles (nmi). For this scenario the MOTO-4D algorithm proposed here is based on direct methods of the pseudospectral family, which are currently the most computationally efficient techniques available for the solution of large nonlinear OCP [118]. As previously mentioned in section 3.3.5, in direct transcription methods the OCP is discretised so that the state and control trajectories are represented by vectors of values at distinct nodes in time. The computation of a solution for the discretised optimal control problem involves a number of mathematical computations, the most computationally intensive of which are those approximating the derivatives of the state trajectories at the discretisation nodes and integrating the cost functionals. Over the last few years, PSM discretisation techniques have emerged as the most suitable computational methods for solving optimal control problems owing to their accuracy and speed, with an impressive convergence rate known as spectral accuracy [49]. In fact, for smooth problems, spectral accuracy implies an exponential convergence rate. For such reasons, a PSM solution method is adopted here for the TMA implementation of MOTO, which is analogous to the one presented in [49].

The MOTO-4D algorithm for terminal area operations is represented in Fig. 5.8. The trajectory optimisation algorithm receives a combined objective as input from an a priori multi-objective articulation of preferences process. A mathematically optimal 4DT is generated by PSM. This 4DT is characterised by discretised Continuous/Piecewise Smooth (CPWS) state profiles, while control variables are mostly unconstrained. This CPWS trajectory in general is not flyable by human pilots nor by conventional Automatic Flight Control Systems (AFCS), as it includes transition manoeuvres involving multiple simultaneous variations in the control inputs. Moreover, the discretised CPWS consists of a very high number of overfly 4D waypoints, which would have unacceptable impacts on the NG-ADL bandwidth usage and pose high processing power requirements. Therefore, two post-processing stages are introduced: a Control Input Smoothing (CIS) algorithm and an Operational Trajectory Realisation (OTR) procedure.

Solving a multi-objective TOP results in a Pareto set of optimal solutions. Therefore, a multi-objective optimality formulation and decision strategy such as the ones reviewed in section 3.5 need to be implemented. As described in section 3.5, a priori
articulation of preferences strategies feature a number of scalarisation techniques to determine a combined objective and efficiently reduce the multi-objective TOP to a series of single-objective TOP to populate the Pareto frontier. As described in section 3.4, the three most applied scalarisation methods are the linear weighted sum (LWS), the normal boundary intersection (NBI) and the normal constraint (NC) methods. The multi-objective articulation of preference adopted in our simulation case studies to incorporate multiple economic and environmental criteria is the weighted product, described in section 3.5.2.3. In particular, the weightings listed in Table 5.1 were adopted when addressing the conflicting objectives of CO/HC emissions and NO\textsubscript{X} emissions in the TMA MOTO-4D numerical verification case studies. The selection of this particular a priori scheme draws from the theoretical foundation presented in section 3.5.2 and is based on the fact that the weighted product offers significant resilience to the actual range of the individual cost/performance values, therefore preventing the need for any customization or adaptiveness of the weightings across the various simulation test cases. The main drawback of the weighted product is associated to the very large magnitude range of the resulting combined objective, frequently exceeding the double precision limits of the computation platform. This limitation was considered and the values of individual costs/performances were scaled to remain within 2 orders of magnitude of a baseline value.

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Fuel weighting ($w_{\text{Fuel}}$)</th>
<th>CO weighting ($w_{\text{CO}}$)</th>
<th>HC weighting ($w_{\text{HC}}$)</th>
<th>NO\textsubscript{X} weighting ($w_{\text{NO}_x}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trajectory 1</td>
<td>1</td>
<td>4.000</td>
<td>4.000</td>
<td>0</td>
</tr>
<tr>
<td>Trajectory 2</td>
<td>1</td>
<td>3.333</td>
<td>3.333</td>
<td>0.667</td>
</tr>
<tr>
<td>Trajectory 3</td>
<td>1</td>
<td>2.667</td>
<td>2.667</td>
<td>1.333</td>
</tr>
<tr>
<td>Trajectory 4</td>
<td>1</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>Trajectory 5</td>
<td>1</td>
<td>1.333</td>
<td>1.333</td>
<td>2.667</td>
</tr>
<tr>
<td>Trajectory 6</td>
<td>1</td>
<td>0.667</td>
<td>0.667</td>
<td>3.333</td>
</tr>
<tr>
<td>Trajectory 7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4.000</td>
</tr>
</tbody>
</table>

The comprehensive MOTO-4D algorithm architecture for TMA operations is depicted in Fig. 5.8. In addition to the aforementioned aspects, this algorithm structure captures all the elements previously discussed in chapters 1-4.
A number of standard NLP solvers can be adopted. The predominant ones are *SNOPT*, *IPOPT* and *fmincon*. *SNOPT* is a software package for solving large-scale optimisation problems (linear and nonlinear programs). It is implemented in Fortran 77 and distributed as source code against a license fee. *IPOPT* is an interior-point optimiser which was written to solve large-scale nonlinear optimisation problem. The algorithm is written in C++ and distributed open-source. Finally, *fmincon* is a function included in MATLAB’s optimisation toolbox which seeks the minimiser of a scalar function of multiple variables, with a region specified by linear constraints and bounds. The TMA MOTO algorithm can be configured to use any NLP solver, including *fmincon*, *SNOPT* or *IPOPT*.
5.8.1. Transposition of the mathematical optimum in a flyable trajectory

Two sequential processes have been implemented to transpose the mathematically optimal trajectory into an operationally flyable 4DT. The first of the two steps consists in a Control Input Smoothing (CIS). The second step is termed Operational Trajectory Realisation (OTR). The aim of these two processes is to develop a 4DT that is characterised by:

- a sequence of standardised trajectory descriptors, of limited length, to facilitate system processing and data-link exchange for air-ground negotiation and validation;
- involving manoeuvres which can be flown by currently equipped autopilots and human pilots (in terms of control input histories);
- retains a consistent portion of the optimality characteristics of the original mathematical optimum in terms

5.8.2. Control input smoothing process

By adopting 3DOF flight dynamics, the output calculated upon numerical convergence of the 4D-MOTO algorithm for TMA sequencing and spacing consists in time-history vectors of discretised Continuous and Piece-Wise Smooth (CPWS) state variables. This is due to the fact that by definitions all dynamics and path constraints are satisfied when convergence is successfully attained, and these constraints prevent spikes or jumps in the state vector histories. Additionally, the three spatial position variables \( \{\phi, \lambda, z\} \) in the state vectors are also characterised by a first-order derivative that is continuous along the entire domain, that is, they are \( C^1(t) \), because their dynamics is governed uniquely by the other states \( \{v, \gamma, \chi, m\} \). Unfortunately, no smoothness constraint is imposed on the three control inputs, which as described in section 4.1.2 involve:

- \( T \): thrust force [N];
- \( \mu \): bank angle [rad];
- \( N \): load factor [ ];
These control inputs are obtained by actuating respectively the throttle, the ailerons and the elevators, and may only achieve the desired control input value after a finite transient time. Additionally, the actuators themselves also cannot undergo step or impulse variations in the practical world. Unfortunately, the output of optimal control based MOTO formulation can involve a consistent amount of step or impulse changes to the control inputs, as they are allowed by the mathematical formulation and no constraint on the rate of change can be put in the conventional OCP when adopting a conventional 3DOF\(^1\). To resolve these occurrences, the CIS process features a customised version of the manoeuvre identification algorithm originally developed by Sabatini, R. [239], which is executed along the entire output vectors to determine the flight manoeuvre being pursued in every segment based both on the segment’s data and on previous segment’s data. The manoeuvre modes implemented and their identification criteria are depicted in Table 5.2, whereas the transition probabilities also considered in the manoeuvre identification algorithm are depicted in Fig. 5.9.

<table>
<thead>
<tr>
<th>Manoeuvre Modes</th>
<th>Identification Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>GROUND OPERATIONS (GO)</td>
<td>Weight on wheels</td>
</tr>
<tr>
<td>STRAIGHT AND LEVEL (SL)</td>
<td>No Pitch/Bank, Constant Altitude/Heading</td>
</tr>
<tr>
<td>HORIZONTAL TURN (HT)</td>
<td>No Pitch/Bank, Constant Altitude, Heading Variation</td>
</tr>
<tr>
<td>TURNING CLIMB (TC)</td>
<td>Pitch up, Altitude increase, Nonzero Bank, Heading Variation</td>
</tr>
<tr>
<td>STRAIGHT CLIMB (SC)</td>
<td>TAKE OFF Positive FPA, No Bank, Flaps and/or Gear Down</td>
</tr>
<tr>
<td></td>
<td>STRAIGHT CLIMB No Bank, Positive FPA</td>
</tr>
<tr>
<td>TURNING DESCENT (TD)</td>
<td>CURVED APPROACH Negative FPA, Nonzero Bank, Heading Variation, Flaps and/or Gear Down</td>
</tr>
<tr>
<td></td>
<td>TURNING DESCENT Negative FPA, Nonzero Bank, Heading Variation</td>
</tr>
<tr>
<td>STRAIGHT DESCENT (SD)</td>
<td>STRAIGHT APPROACH No Bank, Constant Negative FPA, Constant Heading, Flaps and/or Landing Gear Down</td>
</tr>
</tbody>
</table>

\(^1\) Adopting 6DOF dynamics models, the smoothing of attitude rates and thrust can be implemented directly in the OCP introducing appropriate constraints on the actuator commands, which are the control inputs. Additionally, a higher level of user control is available adopting 6DOF-based OCP formulations, as the number of state and control variables are increased. Nevertheless, 6DOF based formulations are characterised by numerical instabilities associated with longitudinal, lateral and coupled rigid-body stability characteristics ([177] Imado, F., Heike, Y., and Kinoshita, T., "Research on a New Aircraft Point-Mass Model", Journal of Aircraft, vol. 48, no. 4, pp. 1121-1130, Jul-Aug 2011.)
The CIS process exploits the manoeuvre characteristics extracted by the manoeuvre identification algorithm and recomputes the control inputs required to fly the optimal CPWS trajectory while meeting user-defined constraints on the rate (or frequency) of change in the control inputs between one segment and the following. In particular, the following restrictions respectively on bank angle rate, load factor rate and throttle rate, representing conventional medium-large transport aircraft, are introduced in our implementation:

\[ |\dot{\phi}| \leq \dot{\phi}_{\text{MAX}} = 12.5^\circ/s \]  
\[ |\dot{N}| \leq \dot{N}_{\text{MAX}} = 0.25 \text{ s}^{-1} \]  
\[ |\dot{T}| \leq \dot{T}_{\text{MAX}} = 0.25 \text{ s}^{-1} \]

These smoothing constraints are added to the control input constraints specified in the 4D-MOTO pseudospectral implementation, which for a medium-large transport aircraft are:

\[ -25.0^\circ \leq \mu \leq 25.0^\circ \]
Combining eq. 5.1 with eq. 5.4, it is possible to calculate the maximum frequency of change in bank angle:

\[ f_{\delta\mu} \leq \frac{\dot{\mu}_{\text{MAX}}}{\Delta\mu} = 0.5 \text{ Hz} \]  

(5.7)

where the maximum bank angle change used is \( \Delta\mu = 25^\circ \), corresponding to the difference between the maximum bank angle and the horizontal level attitude. The choice of not implementing the maximum total bank range is opportune as the transport aircraft is not expected to pass from turning in one direction to the opposite direction in a single manoeuvre. Similarly, combining eq. 5.2 with eq. 5.5, it is possible to calculate the maximum frequency of change in load factor:

\[ f_{\delta N} \leq \frac{\dot{N}_{\text{MAX}}}{\Delta N} = 1 \text{ Hz} \]  

(5.8)

where the maximum load factor change used is \( \Delta N = 0.25 \), corresponding to the maximum difference from the horizontal level condition. The choice of not implementing the maximum total load factor range is opportune as the transport aircraft is not expected to execute positive and negative \( g \) transitions in a single manoeuvre. Analogously, combining eq. 5.3 with eq. 5.6, it is possible to calculate the maximum frequency of change in throttle:

\[ f_{\delta\tau} \leq \frac{\dot{\tau}_{\text{MAX}}}{\Delta\tau} = 0.25 \text{ Hz} \]  

(5.9)

where the maximum change in throttle setting adopted is \( \Delta\tau = 1 \).

Computationally efficient low-pass filters implementing eq. 5.7-5.9 as cut-off frequencies are implemented in the 4D-MOTO CIS process, whose results are detailed later.
5.8.3. **Operational trajectory realization process**

The output of the CIS process consists in discretised vectors of control and state variables that do not involve control spikes. These trajectories still cannot be implemented in real flight operations for two reasons:

1. the combination and frequency of control input changes included in the 4DT is still beyond the capabilities of human pilots and current commercially-available Automatic Flight Control Systems (AFCS) in a very high number of segments.

2. the overall number of trajectory segments is very high and unconstrained, and this poses both a technical problem for guidance systems that may not efficiently process such amounts of guidance data, as well as a system ergonomics issue due to the fact that this trajectory would be too complex to be amended by human operators when necessary.

It is noted, in particular, that the output of optimal control formulation based MOTO algorithms involves a considerable portion of flight modes that can be accurately approximated by simple conventional aircraft piloting modes, as those are characteristic solutions of the same 3DOF equations implemented in the MOTO routine. Furthermore, due to the impracticality of describing lateral flight paths only in terms of fly-over points, fly-by as well as more sophisticated IFR routing procedures were adopted in aviation. It is therefore desirable for flight crews and ATM operators to be presented with an optimal 4DT consisting of both fly-over and fly-by waypoints.

The purpose of the OTR process is to transpose the operationally smooth trajectory delivered by the CIS process into a concisely described RNAV 4DT that can be efficiently processed by ground-based and airborne systems, and that can be easily amended by the human operator if necessary.

To achieve the stated purpose, all output data from CIS algorithm are reprocessed and information is extracted, in particular, in terms of turn radius and flight path angle for each and every segment. Multiple segments in the original trajectory are then clustered together in either level/climbing/descending fly-by or fly-over legs. The clustering is performed when the variation of turn radius or flight path angle are...
limited in a 6.4% band. Whenever this threshold is exceeded, the cluster is split and a new leg is introduced. Both fly-by and overfly turns are extrapolated from the CIS output.

Particular conditions are introduced to ensure that the extrapolated fly-by turns meet the specifications set for Required Navigation Performance (RNP) trajectory descriptors. In particular, the following conditions are implemented to generate a 4DT that complies with RTCA DO-236C and DO-229D [240, 241]:

1. The fly-by transition is defined by the following equations [240]:

$$ R = \frac{GS^2}{g \tan \mu} $$

$$ Y = R \tan(0.5 \delta \chi) $$

$$ GS = |\bar{V}_{TAS} + \bar{V}_{\text{wind}}| = \begin{cases} 500 \text{ kts (low altitude)} \\ 750 \text{ kts (high altitude)} \end{cases} $$

$$ \mu = \begin{cases} \min[0.5 \delta \chi, 23.0^\circ] \text{ (low altitude)} \\ 5.0^\circ \text{ (high altitude)} \end{cases} $$

$$ Y > 20.0 \text{ nmi} \Rightarrow \begin{cases} Y = 20.0 \text{ nmi} \\ R = \frac{20.0}{\tan(0.5 \delta \chi)} \end{cases} $$

2. The geometry of the Fixed Radius Transition (FRT) is defined by the track change $\delta \chi$ and the radius $R$. The Lead Distance $Y$ from the turn initiation to the waypoint and the Abeam Distance $X$ between the waypoint and the point of the circular arc abeam the waypoint are defined based on the radius and the track change as per the following equations:

$$ X = R \cdot \left( \frac{1}{\cos \delta \chi/2} - 1 \right) $$

$$ Y = R \cdot \tan(\delta \chi/2) $$

3. When transitioning from one airway to another, if both require a FRT at the common waypoint, the smaller of the two radii applicable shall be selected. In the case one of the two airways does not involve a FRT, the FRT of the other shall be implemented.
The criteria for implementing the definitions above are given in Table 5.3, adapted from RTCA DO-229D [242]. As the TMA implementation of 4D-MOTO developed within this research project mainly addresses terminal sequencing and spacing, the criteria for the “Feeder, Departure and Missed Approach” context were implemented.
Table 5.3. Geometric characteristics of the fly-by theoretical transition area. From [241].

<table>
<thead>
<tr>
<th>Context</th>
<th>Track change ($\delta \chi$)</th>
<th>Maximum Radius ($R$)</th>
<th>Maximum Turn Initiation Distance ($Y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High altitude ($h \geq 19,500$ ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$&lt; 24.1^\circ$</td>
<td>93.7 nmi</td>
<td>$R \cdot \tan \frac{\delta \chi}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\geq 24.1^\circ$</td>
<td>$\frac{Y}{\tan \frac{\delta \chi}{2}}$</td>
<td>20.0 nmi</td>
<td></td>
</tr>
<tr>
<td>Low altitude ($h \geq 19,500$ ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\leq 46.0^\circ$</td>
<td>$\frac{Y}{\tan \frac{\delta \chi}{2}}$</td>
<td>3.65 nmi</td>
<td></td>
</tr>
<tr>
<td>$&gt; 46.0^\circ$</td>
<td>8.59 nmi</td>
<td>$R \cdot \tan \frac{\delta \chi}{2}$</td>
<td></td>
</tr>
<tr>
<td>Feeder, Departure and Missed Approach</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\leq 46.0^\circ$</td>
<td>$\frac{Y}{\tan \frac{\delta \chi}{2}}$</td>
<td>1.78 nmi</td>
<td></td>
</tr>
<tr>
<td>$&gt; 46.0^\circ$</td>
<td>4.18 nmi</td>
<td>$R \cdot \tan \frac{\delta \chi}{2}$</td>
<td></td>
</tr>
<tr>
<td>Initial Approach</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\leq 46.0^\circ$</td>
<td>$\frac{Y}{\tan \frac{\delta \chi}{2}}$</td>
<td>1.45 nmi</td>
<td></td>
</tr>
<tr>
<td>$&gt; 46.0^\circ$</td>
<td>3.41 nmi</td>
<td>$R \cdot \tan \frac{\delta \chi}{2}$</td>
<td></td>
</tr>
<tr>
<td>Intermediate Approach</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\leq 46.0^\circ$</td>
<td>$\frac{Y}{\tan \frac{\delta \chi}{2}}$</td>
<td>1.00 nmi</td>
<td></td>
</tr>
<tr>
<td>$&gt; 46.0^\circ$</td>
<td>2.38 nmi</td>
<td>$R \cdot \tan \frac{\delta \chi}{2}$</td>
<td></td>
</tr>
</tbody>
</table>
5.9. Trajectory optimisation for the en-route context

As described in chapter 3, alternative formulations of TOP that are not based on the optimal control theory have been investigated in the literature and successfully applied to the aircraft path planning problem. By definition, all these formulations necessarily relinquish system dynamics, therefore are valid only when assuming quasi-steady state (i.e. that accuracy impacts due to neglecting transients are small).

In the enroute phase of transport aircraft, transients occur only in relation to turns and cruise altitude changes, or due to control manoeuvres performed in reaction to external disturbances, such as clear air turbulence. Consequently, enroute operations typically involve a fairly small number of manoeuvres of sufficiently limited individual duration, occurring most typically in the proximity of a waypoint along the route. In particular, based on current Area Navigation (RNAV) trajectory descriptors, the number of turns and altitude variations is always necessarily smaller than the number of route waypoints $n_{WP}$, which is also limited, therefore:

$$n_M \leq n_{WP} \quad (5.17)$$

As a result, non-optimal control-based formulations offer satisfactory accuracy when applied to the enroute phase of transport aircraft. This approach features continuously in the literature and is used to develop efficient path-planning algorithms, though only one or at most two objectives were considered.

Compared to optimal control-based implementations, a considerable simplification of the problem formulation is obtained by adopting steady-state-based approximations. These formulations involve a suitably defined trajectory model, typically of geometric nature, allowing a direct exploitation of NLP or heuristic-based algorithms to determine the optimal path. By adopting one of the possible multi-objective optimality formulations presented in section 3.4, it is possible to extend this approach to an arbitrary number of objectives and constraints, including conflicting/unfeasible constraints. If necessary, empirical relationships can be developed to statistically capture the dependency of time discrepancies on the number of waypoints.

The geometric 4D trajectory model implemented in the enroute MOTO algorithm is directly derived from the assumption of constrained cruise altitude (i.e. either constant or with predefined changes). The 4D trajectory is parametrised by the
latitude, longitude, altitude and crossing time variables at a discrete number of waypoints. Compared to the 4D-MOTO for TMA sequencing and spacing, which involved both fly-by and fly-over waypoints, only fly-over waypoints are adopted in the enroute MOTO. As this limitation is also reflected in the optimised trajectory that is processed by the avionics guidance and control systems, this assumption does not impact the model accuracy.

In the implemented enroute MOTO algorithm, the impacts of wind 4D field variations are captured adopting the dynamic programming paradigm, hence by recursively re-optimising the remaining path to the target waypoint considering the most up-to-date conditions. In essence, the environmental conditions are dynamic and are taken into consideration when a route optimisation is performed. The optimised trajectory is adaptive to dynamic weather changes. This of course requires non-standard atmosphere and weather model data which need to be supplied by an online service provider. Flown trajectories accounting for weather can be optimised in 2 ways:

1. Offline using forecasted weather and traffic data supplied to the MOTO optimiser
2. Initially generated offline and recalculated using an updated weather model by either a next generation FMS or the ATCo.

The enroute MOTO algorithm processes input information associated with original flight plan route, navigation database, airspace sector boundaries, weather, avoidance regions. The algorithm defines the upper and lower boundaries of the search region and uses the original trajectory as input guess. The optimisation parameters include a throttle setting and the cross-track deviation, effectively introducing two scalar degrees-of-freedom for each waypoint of the optimal trajectory. The optimisation algorithm is executed recursively on the still-to-be-flown portion of the trajectory, to consider updated weather forecasts. The number of output fly-over waypoints is at least equivalent to the number of input waypoints, but is augmented as necessary by including an additional waypoint in order to have at least one waypoint every 250 nmi. This is necessary as it is assumed that updated weather forecasts will be made available every 30 minutes, prompting a further optimisation loop.
The computational cost of the 2D+T-MOTO algorithm relies on the amount of discretised waypoints, and this number can be defined by the user. More points increase the accuracy at the cost of computational complexity however figure below indicates that a minimum amount of points for an optimal solution exists. This can be determined by analyzing the optimality of the solutions with different number of route points, for instance in terms of total flight time. Initial increases in the number of points enable a more accurate avoidance trajectory that improves the optimality achievable, but beyond a certain number further increases in the amount of points only increases computational time without benefits in solution optimality and accuracy, as depicted in Figure 5.10.

![Figure 5.10. Relationship between flight time and the amount of waypoints describing the trajectory based on a Melbourne to Dubai enroute mission implementing 2D+T-MOTO using minimum time, 4D wind field and contrail avoidance areas [243].](image)

### 5.10. Optimal airspace reconfiguration algorithm

Automated partitioning techniques were numerically evaluated in [113-116, 244-258]. An algorithm for Dynamic Airspace Sectorisation (DAS) is proposed here, as outlined in Figure 5.11.
At each iteration the conditions of weather and airspace segregations are processed to identify the relinquished airspace portions. Subsequently, the current configurations of airspace sectors and traffic flows within the ACC jurisdiction are processed. A check on the balance between the estimated demand and the updated capacity is therefore performed. In case an under-capacity or over-capacity is identified, load balancing and DAS resolutions are attempted in terms of sector consolidations and de-consolidations, and traffic re-routing. If the demand-to-capacity balance is still suboptimal, further attempts are undertaken.

5.10.1. Mathematical formulation

By introducing dynamic expressions of capacity $C_i(t)$ and demand $D_i(t)$ for the $i$-th airspace sector, DCB is governed by the following expression:
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\[
\Delta_i(t) = \frac{d C_i(t)}{dt} - \frac{d D_i(t)}{dt} \tag{5.18}
\]

where \( \Delta_i(t) \) is the sector occupancy trend. In order to model the demand, we may adapt the Reynold's transport theorem:

\[
\frac{d D_i(t)}{dt} = \sum \left( \dot{D}_i(t) \right)_{in} - \sum \left( \dot{D}_i(t) \right)_{out} + \sigma_i(t) \tag{5.19}
\]

where \( \dot{D}_i \) represent the flow of demand (hence of traffic) from/to the adjacent airspace sectors, and \( \sigma_i(t) \) is a source/sink term accounting for airports within the sector. In order to comply with an arbitrarily limited sector capacity, it is possible to introduce thresholds for the predicted demand, which will trigger a local airspace reconfiguration. In particular, neglecting dynamically morphing sectors, Flexible Airspace Management (FAM) strategies involve sensor splitting and merging. Research is currently underway to assess the viability of a variable sector capacity and 4-Dimensionally morphing sectors. The key aspect lies in the introduction of capacity models that entail 4-Dimensional dynamics. We present an approach that is based on the forthcoming CNS/ATM and Avionics (CNS+A) operational scenario. The novel CNS+A systems being developed for Performance Based Operations (PBO) are conceived to meet the high CNS performance requirements that will enable an increase in the traffic density without compromising flight safety. The capacity can be therefore expressed as a function of the CNS performance as follows. The capacity factor of airspace sector \( i \) due to Communication Performance (CP) alone can be expressed as a factor of the minimum CP guaranteed by all aircraft \( j \) concurrently in the sector, as an example in term of Communication Integrity (CI):

\[
(k_e)_i \propto \min_j CI_{i,j} \tag{5.20}
\]

Similar expressions can be written for Navigation Integrity (NI) and Surveillance Integrity (SI):

\[
(k_N)_i \propto \min_j NI_{i,j} \tag{5.21}
\]

\[
(k_S)_i \propto \min_j SI_{i,j} \tag{5.22}
\]

The dependence upon time is introduced by taking into account the aircraft predicted to enter and exit the airspace sector. The total sector capacity is also correlated with the minimum guaranteed level of integrity provided by all the CNS systems, and therefore:
The dependence on the minimum CNS performance levels guaranteed by the entire sector traffic and not on the average is a result of assuming homogeneous traffic separation models in the entire sector, which is a requirement a significant increase in the ATCo workload.

5.10.2. Implementation case study

The algorithm was implemented and preliminarily assessed in representative traffic conditions for central Europe. Figure 5.12 depicts a medium resolution discretisation of the approximately 500 nmi x 250 nmi ACC into 34 triangular partitions by means of a 2D meshing algorithm.

Figure 5.12. Discretisation of the ACC en-route airspace with a medium density mesh.

Figure 5.13 depicts a higher resolution discretisation into approximately 150 triangular partitions. As described in the previous sections, the DAS/ATFM algorithm analyses the traffic flow and attempts a consolidation/de-consolidation of the airspace to optimise the demand-to-capacity balance over a certain time horizon.
The objective of 6 sectors is introduced to minimise the required ATC workforce and attain high demand-to-capacity optimality. An example of the algorithm result is represented in Figure 5.14, where the traffic flows are simplified by straight vectors, depicted in blue.

In this case, the originally introduced objective of 6 sectors was not attained, as the algorithm identified that 8 sectors were required to maintain a sufficient capacity along the defined time horizon of 3 hours.

As the dynamics involved in DAS, load balancing and aircraft trajectory planning are mutually interdependent and all are fundamentally based on real-time optimisation algorithms, significant research is required to address the loop dependence, stability and optimality metrics, promoting rapid convergence to global economic/environmental optimality. With respect to trajectory planning, in the short-term new airspace models will have to consider dynamically evolving lateral and vertical sector boundaries and continuously varying capacities. In the medium-long term, new combined traffic flow and airspace models integrating DAS will be required.
5.11. Interactions between MOTO, DAS and AFP

As conceptually represented in Fig. 5.15, MOTO, DAS and TFMI including AFP have extensive interdependencies in the strategic online context. In particular:

- the replanning of optimal 4DT performed by MOTO depends on traffic flow and airspace constraints also accounting for Traffic Flow Management Initiatives (TFMI), which are transposed in either boundary conditions or path constraints;

- the identification of AFP to be enacted to optimise the DCB depends on the airspace structure and on active 4DT intents, which define the input data used to estimate downstream demand levels;

- the optimal reconfiguration of airspace sectors performed by DAS depends on sector demand levels and on active 4DT intents.
Figure 5.15. Conceptual representation of the interactions between MOTO, DAS and AFP.

Ideally, the introduction of novel optimisation processes such as MOTO and DAS should not take place at the cost of relinquishing the existing optimisation processes, such as the DCB optimisation currently pursued by enacting AFP/TFMI. This consideration is acknowledged in our research project, as we impose that these three interrelated ATM processes shall synergically achieve their own individual aims, unleashing a higher level of global optimality.

A robust strategy to resolve the potentially hindering loop dependencies identified in the mutual reconfigurations of trajectories, sectors and traffic flows consists in defining a Hierarchical Prioritisation (HP) of the three optimisation processes. This approach effectively interrupts the loop dependencies but the solution determined with this methodology can be theoretically suboptimal. This is due to the fact that the HP reduces the reachable set of solutions, while the theoretical optimality can only be reached by a particular sequence of actions. This and other similar problems are not uncommon when attempting the optimisation of complex systems, and the scientific community is currently exploring the full implications and opportunities as part of the research in the “Viability Theory” [259], which have important implications for the ATFM domain.

As a consequence, the HP must be carefully selected in order not to excessively compromise the optimality nor revert the theoretical benefits of integrating MOTO and DAS in ATFM systems. Two different HP were therefore identified to address
distinct circumstances, starting from the relatively simple sequencing and spacing in the TMA context.

5.11.1. TMA context

As discussed in section 2.4, the throughput of either departing traffic or arriving traffic crucially depends on the minimum spacing required for wake turbulence separation. Consequently, a suboptimal sequencing (e.g. involving alternated wake turbulence category aircraft) can have considerable impacts, potentially leading to temporary saturation when the nominal demand levels of arrival and departure traffic at a particular airport exceed approximately 66–75% of the available capacity.

In this context it is therefore recommended that the optimal sequencing of traffic performed by ATFM systems gets the highest priority and is performed considering all traffic within an extended AMAN (XMAN) horizon, in line with the XMAN implementation formulated within the SESAR programme, also leading to the determination of the optimal arrival-departure runway usage ratio.

The 4D-MOTO should then be performed based on the arrival slots allocated to each traffic to compute the 4DT that offers the best economic and environmental performances while complying with the operational constraint consisting of the allocated slot and eventual precluded airspace (e.g. convective weather or military areas). A DAS process should lastly be executed to amend sector shapes and consolidations/deconsolidation times as opportune, using the optimised 4DT as input and only considering operator workload. This process is schematically represented in Fig. 5.16.
Figure 5.16. Hierarchical prioritisation of MOTO, DAS and DCB in the TMA context.
5.11.2. Large-scale rerouting due to unforeseen perturbations

Unpredicted events prompting large-scale rerouting of multiple aircraft, or significant deviations from the original Estimated Times Enroute (ETE) can void the validity of the active Network Operations Plan (NOP) that outlined the original traffic flow demand and ATM capacities. Unpredicted wind shifts causing highly inefficient or even unsafe flight conations are one typical instance for this to occur, and may affect multiple aircraft.

In these severe circumstances, the online replanning of 4DT for all involved aircraft is to be pursued. This must consider updated environmental conditions which must be supplied to the optimiser. The effectiveness of both DCB and DAS optimisations crucially depend on the accuracy of demand estimates, which are also void when the original trajectories are no longer viable.

As a consequence, it is recommended that 2D+T MOTO is preliminarily carried out for all involved aircraft. Subsequently, DAS optimisation should take place for all enroute sectors, attempting to accommodate the new demand without introducing additional constraints. This also reduces the complexity of the optimisation process and subsequently reduces the computation time required for mass optimisation protocols. Finally, TFMI should be enacted as necessary only if demand peaks that could not be addressed by DAS (e.g. at the arrival airport) still exceed the maximum capacity. This process is schematically represented in Fig. 5.17.
Figure 5.17. Hierarchical prioritisation of MOTO, DAS and DCB in the large-scale rerouting context.

5.12. Conclusions

In this chapter, the state-of-the-art of CNS+A implementations and the requirements assumed for the design of the proposed MOTO-4D and DAS algorithms were outlined. The chapter subsequently described the design of the 4-PNV system prototype as well as the implementation of MOTO-4D and DAS algorithms for the intended implementations. The important issues emerging when considering the interactions between MOTO, DAS and TFMI were considered and a possible solution approach based on hierarchical prioritisation was presented.
5.13. References


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Chapter 6
Numerical simulations

This chapter conveys a detailed description of the numerical simulation and evaluation activities performed addressing MOTO-4D algorithms as part of the ATM system prototype. The research carried out in this phase consisted of the preparation of an offline simulation environment for the 4-PNV system prototype models implementation and verification. Simulations were performed in a limited but representative set of conditions in an industry-grade ATM development scenario (THALESLAND). MATLAB was the selected Integrated Development and Testing Environment (IDTE). The simulation activities were aimed at verifying the viability of the MOTO-4D and DAS algorithms developed for the 4-PNV DSS. Results highlighted that the implemented algorithms consistently achieve the set 4-PNV DSS design requirements.

6.1. Terminal sequencing and spacing of arrival traffic

All MOTO algorithms conceived for CNS+A integration require extensive validation by simulation in realistic test scenarios. Simulation cases shall cover all phases of flight and airspace configurations. Particular emphasis is given to the TMA, as trajectories of climbing and descending aircraft usually intersect, requiring frequent tactical ATM interventions, and more generally the constrained airport capacity can lead the arrival traffic to stack, even when disruptions are not present. While concepts such as 4DT point-merge and Continuous Curved Descent Approaches (CCDA) are currently experimented to reduce TMA congestion [260-263], further implementation aspects are being investigated, especially in the 4DT-TBO context. The Arrival Manager (AMAN) is responsible of the optimised sequencing and spacing of arrival traffic towards a single final approach segment. The AMAN scenario is the most representative case study of online tactical TMA operations for ATM DSS implementations of MOTO-4D. The ground-based ATM system implementing MOTO-4D algorithms identifies the best arrival sequence among the
available options in terms of multiple and conflicting objectives. Longitudinal separation is enforced at the merge-point in the form of path constraints and boundary conditions to ensure sufficient separation upon landing, and to prevent separation infringements in the approach phase itself. Notwithstanding, the ATM DSS implementing MOTO-4D shall be capable to perform point-merge at any metering point. After the initial set of optimal intents has been stored in the ATM DSS, the point-merge sequencing algorithm allocates the available time slots according to suitable optimal scheduling decision-logics. As a reference, assuming a minimum longitudinal separation of 4 nautical miles on the approach path for medium category aircraft approaching at 140 knots, the allocated time slots are characterized by a time spacing ranging between 90 and 160 seconds depending on the wake-turbulence categories of two consecutive aircraft. The MOTO-4D implemented for the TMA scenario is designed to plan optimal sequenced and spaced Continuous Descent Arrival (CDA) trajectories of aircraft inbound from different TMA entry points and characterised by conflicting Estimated Times of Arrival (ETA) at the final approach fix or exit point. This situation is frequently found in the TMA of congested hubs as well as farther out at upstream feeder sectors, as traffic flows from different arrival routings need to be sequenced and spaced at the metering point.

Although the THALES scenario involves a densely trafficked airport (TSKH) with 13 different arrivals as well as two departures, the density of this traffic (in terms of time spacing) is insufficient to assess the MOTO-4D algorithm effectively. Additionally, the vast majority of the inbound traffic (n. 1, 3, 5, 12, 13, 14, 15, 16, 18, 19, 20) share the same inbound route (CIVET SLP KHV) and while this setup accurately represents airports typically neighbouring FIRs with limited air route structures, it restricts the generality of the simulation-based algorithm. Consequently, the simulation case studies performed retain basic data from the THALESLAND scenario in terms of geographic location and final approach characteristics, but the conflicting traffic is randomly generated at arbitrary TMA entry points.

At the beginning of the MOTO-4D algorithm execution, two initial 4DT guesses are generated exploiting the MOTO-4D for each traffic consisting in minimum time and
minimum fuel intents. These initial guesses serve only as reference for the algorithm to compute the optimal sequencing and spacing, as such they are not treated by the CIS and OTR processes. Based on these initial 4DT guesses, the optimal sequencing is determined through processes described in section 2.7 by defining the arrival slots to be assigned to each traffic for required spacing. Subsequently, new optimal 4DT intents are generated to fulfil the defined slot. These intents are treated by the CIS and OTR processes described in sections 5.8.2 and 5.8.3 to determine operationally smooth, concisely described and flyable 4DT. For the purpose of clarity, Fig. 6.1 and 6.2 visualise the AMAN simulation results implementing MOTO-4D algorithms implementing the time constraints from the terminal sequencing and spacing algorithm but adopting only minimum fuel as objective. Waypoints and lines depicted in magenta in Fig 6.1 represent the flyable and concisely-described 4DT consisting of a limited number of fly-by and overfly 4D waypoints, obtained through the CIS and OTR processes.

Figure 6.1. Results of the 4-PNV in the AMAN scenario.
Fig. 6.2 depicts the computed 4DT in an AMAN time-distance plot. AMAN implementations of MOTO-4D such as the one presented above are actively researched and evolving to address inaccurate trajectory predictions, air-ground trajectory synchronisation issues, unforeseen perturbations and the dynamic handling of multiple altitude/airspeed/path constraints in real-time.

![Figure 6.2. AMAN schedule plot of the optimised 4DT intents.](image)
6.1.1. 4D-MOTO simulation case studies

The initial conditions and the numerical results relative to the MOTO-4D simulation case study n. 1 are detailed in Table 6.1. Random realistic weather conditions were initially simulated, but the very minimal spatial variations of the associated atmospheric parameters in the limited extent of the TMA prompted to assume constant weather fields instead to improve computational performances (with negligible impacts to accuracy). The constant conditions adopted in this simulation case study were wind from 160 at 6 knots, surface pressure of 1006 mbar, temperature of 27 degrees Celsius. An overview of the validated and operationally viable 4DT is given in Fig. 6.3, and an AMAN time-distance plot of the validated results is provided in Fig. 6.4. Fig. 6.5 to 6.9 depict the time histories of selected control and state variables, highlighting the effectiveness of the CIS process.

Table 6.1. Initial conditions and results of the 4D-MOTO.

<table>
<thead>
<tr>
<th>ID</th>
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<th>405</th>
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<td>B753</td>
<td>A320</td>
<td>A320</td>
<td>A333</td>
</tr>
</tbody>
</table>
| Initial Latitude | 22° 55' 53" | 22° 50' 19" | 22° 47' 6" | 23° 1' 42" | 22° 42' 35"
| Initial Longitude | 119° 39' 27" | 120° 28' 13" | 119° 36' 31" | 119° 57' 40" | 119° 34' 3" |
| Initial Altitude | 16,452 ft | 9,109 ft | 5938 ft | 11,480 ft | 9,272 ft |
| Min. Time ETA | 497 s | 370 s | 466 s | 420 s | 478 s |
| Min. Fuel ETA | 507 s | 377 s | 498 s | 468 s | 546 s |
| Assigned order in sequence | 4 | 1 | 3 | 2 | 5 |
| Assigned slot | 636 s | 375 s | 549 s | 462 s | 723 s |
| Points in original CPWS trajectory | 42 | 80 | 75 | 83 | 89 |
| Points in flyable 4DT | 19 | 18 | 11 | 23 | 18 |
Figure 6.3. Overview of 4D-MOTO results.
Figure 6.4. AMAN scheduling plot of 4D-MOTO results.
Figure 6.5. Aircraft #402 – Original intents and smooth deconflicted 4DT result.
Figure 6.6. Aircraft #404 – Original intents and smooth deconflicted 4DT result.
Figure 6.7. Aircraft #403 – Original intents and smooth deconflicted 4DT result.
Figure 6.8. Aircraft #401 – Original intents and smooth deconflicted 4DT result.
Figure 6.9. Aircraft #405 – Original intents and smooth deconflicted 4DT result.
The following Fig. 6.10 to 6.12 depict the Pareto fronts of the 4DT generated by the MOTO-4D algorithm for some of the simulated flights when considering a trade-off between minimum carbon monoxide (CO) and minimum nitrogen oxides (NO\textsubscript{X}) emissions. The Pareto front addressing the two conflicting objectives is depicted in the bottom right graphic, whereas the individual exhaust emissions assessed against fuel consumption are visible in the remaining three graphics.

Figure 6.10. Pareto front of flight #402.
Figure 6.11. Pareto front of flight #403.

Figure 6.12. Pareto front of flight #404.
6.1.2. Discussion of TMA 4D-MOTO results

All initial 4DT intents are generated in less than 10 s on a recurrent basis, however this computational time is based on a MATLAB simulation environment (running on a standard PC with Intel Core i7 processor and 8 GByte RAM). An implementation into compiled languages such as C, C++, Java or Fortran is expected to yield even better results. Although some of the minimum fuel initial guesses suggest numerical instability or incomplete convergence, this is due to the fact that neither the CIS nor the OTR processes are applied and the 4D-MOTO algorithm consistently succeeds in determining the optimal operational 4DT that meets the assigned slot requirements and that can be successfully implemented in current-generation avionics/ATM systems. In particular, the sequenced and spaced 4DT intents are treated by CIS and OTR processes, removing all control input discontinuities and reducing the considerable number of input points by an average factor of four. The effectiveness of the CIS process can be noticed by observing the time histories of key states and controls of the validated trajectories (blue lines) in Fig. 6.5 to Fig. 6.9, especially when compared to the minimum fuel/time initial guesses (red and green lines). The targeted optimisation objectives were successfully achieved by the trajectories n. 1 and n. 7 which respectively represented minimum CO/HC and minimum NOX. The remaining trajectories 2 to 6 offered compromise performance between these two extreme cases. While for flight #402 (Fig 6.10) intent 4 was characterised by outstanding performances with respect to most of the other solutions, this is an atypical case possibly due to inaccuracies in the CRJ-900 dynamics, as most Pareto fronts generated with the MOTO-4D featured smooth performance ranges. Faster computational performances were recorded with heavier aircraft models, and conversely slower convergence was observed with lighter aircraft. This can be explained by the fact that a more limited manoeuvrability restricts the search space, improving the computation speed, and that the higher inertia associated with aircraft mass reduces the characteristic frequency of instability modes, reducing the requirements in terms of time resolution/intervals in the trajectory optimisation algorithms.
6.2. En-route trajectory optimisation algorithm

A suitable test scenario was made available from the THALES Centre for Advanced Studies in ATM CASIA) simulation environment. This simulation scenario, named THALESLAND, is recurrently used by THALES for ATM software development and testing and is therefore well suited for research and development activities. All the required information on the data location and structure was also delivered by CASIA staff. As part of the THALESLAND dataset, exercise n. 272 (EXE0272) was provided, featuring an ideal distribution of traffic for ATM software testing and evaluation. The assessment of 2D+T-MOTO was performed by developing a customised scenario based on THALESLAND airspace data and EXE0272 traffic information. The THALESLAND simulation scenario including the exercise n. 272 relieves researchers from the need for data of real air traffic, which is subject to national security and confidentiality restrictions. A dedicated toolset was developed for THALESLAND input data parsing. Additional utilities for integrating EUROCONTROL Base of Aircraft Data (BADA) aircraft dynamics parameters as well as Global Forecast System (GFS) and a dedicated radial wind velocity profile model for tropical revolving storms were also developed. Fig. 6.13 represents the THALESLAND airspace with the original EXE0272 scenario traffic in Lambert conic conformal projection. In order to maximise the number of aircraft necessitating rerouting, the scenario involved the passage of a tropical revolving storm, whose evolution was not previously accurately estimated. The modelled tropical revolving storm also defines the 4D wind field at all points in the THALESLAND airspace. In particular, the radial wind velocity profile was calculated adopting the revised model from [264]. The initial weather and airspace conditions at the beginning of the simulation and the original flight trajectories are depicted in Fig. 6.14, while an overview of the results calculated using 2D+T-MOTO on all the affected aircraft is given in Fig. 6.15. The original flight data and the performance metrics are detailed for every rerouted traffic in the following subsections. Three distinct intents have been generated for each rerouted traffic: a minimum fuel, a minimum time, and a minimum emissions, defined as having an equal weighting for CO, HC and NOX emissions. A detailed specification of the
simulation case studies implementation activities required to perform the verification campaign is included in appendix.

Figure 6.13. Lambert conic conformal representation of THALESLAND scenario.

Figure 6.14. Original flight trajectories and initial wind field conditions.
6.2.1. Discussion of enroute 2D+T-MOTO results

For all the 34 aircraft that were subject to rerouting, the 2D+T-MOTO algorithm successfully identified three different 4DT intents simultaneously fulfilling the typhoon avoidance constraint and the airspace boundaries.

With respect to flights n. 1, 2, 3, 4, 12, 13, 16, 17, 18, 20, 24, 30, 34 and 35, all the three intents calculated by the 2D+T-MOTO algorithm successfully achieved a higher optimality compared with the original trajectory, regardless of its validity. In several of these cases the minimum fuel and minimum emission results are to a certain extent similar in nature. This can be expected because according to eq. 4.10-4.11, the emissions are also a function of the fuel burn. Nevertheless, important differences, especially in terms of flight time, can be noted.

In a number of cases, the optimality of one or multiple avoidance trajectories was deemed not satisfactory, either with respect to the key performance indicators, or to the geometric layout of the avoidance trajectory. An additional test campaign, described in the following section, was therefore performed for aircraft 6, 7, 9, 11, 23, 32 and 37.
6.3. Further enroute 2D+T-MOTO analysis

Flights n. 6, 7, 9, 11, 23, 32 and 37 were selected for an additional investigation campaign, as one or multiple of their calculated avoidance trajectories either did not feature satisfactory levels of optimality in the key performance indicators, or manifested considerable path distortions. In particular, macroscopic path distortions can be noticed in the minimum emission trajectory of flights 6, 7, 9, 11 and in all optimal avoidance trajectories of flights 32 and 37. Whereas the optimality of the results was deemed unsatisfactory in case of flight 23. Although they represented a reduced number of occurrences that could be managed manually by the ATM operator, a new 2D+T-MOTO process was therefore executed for flights 6, 7, 9 and 11 to assess whether changes in the resolution of the optimal trajectories and in the cost functions could improve the smoothness and optimality of the numerical solution. Consequently, this time the three different objectives of minimum CO, minimum HC and minimum NO\textsubscript{X} emissions were selected. Furthermore, the reference distance and time for the segmentation of the trajectories were respectively 100 nmi and 15 minutes.
6.3.1. Flight 6

The minimum emissions avoidance trajectory of flight n. 6 was affected by macroscopic distortions. The original and the updated optimal trajectories are plotted in Figure 6.16, and the associated performance indicators are detailed in Table 6.2. The fuel-time Pareto diagram of the updated results is given in Figure 6.17.

Figure 6.16. Results of the updated enroute 2D+T-MOTO optimisation performed for flight n. 6.
Table 6.2. Updated enroute 2D+T-MOTO results for flight n. 6.

<table>
<thead>
<tr>
<th>Original (unfeasible)</th>
<th>Min. CO</th>
<th>Min. HC</th>
<th>Min. NOX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>239.6</td>
<td>253.9</td>
<td>264</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.83</td>
<td>37.4</td>
<td>36.69</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>106.89</td>
<td>118.18</td>
<td>115.94</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>19.74</td>
<td>21.64</td>
<td>21.49</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.82</td>
<td>3.11</td>
<td>3.06</td>
</tr>
<tr>
<td>NOX [kg]</td>
<td>691.75</td>
<td>784.97</td>
<td>743.55</td>
</tr>
</tbody>
</table>

Figure 6.17. Updated fuel-time Pareto diagram of original and optimised trajectories of flight n. 6.


6.3.2. Discussion of further enroute 2D+T-MOTO results

For flights n. 5, 6, 7, 8, 9, 10, 11, 21, 23, 27, 29, 32 and 37 the original trajectory was conflicting with the typhoon avoidance region constraint, therefore the higher optimality achieved by the original trajectory was only theoretical. In practice, only the 2D+T-MOTO algorithm produces optimal and valid trajectories. In a more comprehensive validation scenario involving human-in-the-loop test cases, initial non-optimal weather-avoidance paths would be manually planned by the flight crew and/or by the duty controller in an offline/online strategic timeframe, as it is currently done on a regular basis in these situations. This manually amended trajectory would then be compared with the 2D+T-MOTO intent, allowing a more thorough experimental evaluation. This activity is beyond the scope of this research but it is recommended for the future industrial validation of the newly developed MOTO functionalities.

The macroscopic distortions present in the trajectories of flights 6, 7, 9 and 11 were reduced in magnitude but not completely eliminated in the updated results. This can be explained observing the wind field characteristics, which promote a significant southward shift of eastbound traffic. It is concluded that the implementation of custom hybrid-heuristic optimisation techniques is required to attain global optimality in these cases.

These further results highlight an improvement in the solution accuracy with the increase in the number of waypoints. The important findings of section 5.9 (Fig. 5.10) nonetheless highlight that there is a clear point beyond which no substantial differences in the solutions are obtained.
6.4. Trajectory planning for the emergency timeframe

Safety-critical applications of trajectory optimisation algorithms were investigated for airborne emergency DSS, also known as safety-nets. Research on real-time implementations of trajectory optimisation has been pursued in connection with the development of SAA systems. As an example, robust parallelised direct shooting solution methods with a posteriori decision logics were implemented for the generation of safe obstacle avoidance trajectories as part of the research on Laser Obstacle Avoidance Systems (LOAS) for manned and unmanned aircraft [109-111, 265]. Figure 6.18 depicts a number of feasible and unfeasible avoidance trajectories generated by LOAS in real-time in the presence of multiple obstacles. Adopting a multi-phase trajectory optimisation formulation, the selection of the optimal trajectory along the first phase (safe steering) is typically based on minimising a cost function of the form [266]:

\[
J = w_t \cdot t_f + w_f \cdot m(t_f) - w_d \cdot D(t_f) - w_{id} \cdot \int D(t) dt
\]  

(6.1)

where \(D(t)\) is the slant distance of the host platform along the avoidance trajectory from the avoidance volume associated with the obstacle, \(t_f = t|\rho > 0\) is the time at which the safe avoidance condition is successfully attained (also defined “time-to-safety”), \(m(t)\) is the host platform’s mass and \(\{w_t, w_d, w_{id}, w_f\}\) are the positive weightings attributed to time, distance, integral distance and fuel respectively. In time-critical avoidance applications (i.e., closing-up obstacles with high relative velocities) appropriate higher weightings are used for the time and distance cost elements. The knowledge in the area of SAA systems and safety-nets for Remotely Piloted Aircraft Systems (RPAS) is evolving rapidly in view of the strict certification requirements. Current research is addressing, in particular, the fusion of cooperative and non-cooperative surveillance data for accurate detection, tracking and safe steering.
Figure 6.18. Example of trajectory optimisation applied to the generation of safe obstacle avoidance trajectories in emergency DSS.
6.5. Dynamic airspace reconfiguration case study

The DAS algorithm was developed considering the airspace and traffic characteristics of some of the enroute sectors characterised by the highest density in the world. Some characteristics of the THALESLAND airspace and of EXE0272 traffic data made them not well suited for the assessment of the implemented DAS algorithm. In particular:

1. The DAS implementation assumed traffic prevailingly entering and exiting the FIR from the external borders, whereas significant portions of EXE0272 flights originate and/or terminate in the local airports.

2. The DAS implementation assumed a FIR with spatially homogeneous characteristics in terms of route density and ATM Level of Service, where a large portion of the airspace was experiencing demand levels close to the capacity limit set due to human operator workload. Conversely, the DAS algorithm does not address capacity restrictions associated with throughput limitations at specific elements such as RNAV intersections or route segments. These limitations are not mitigated by deconsolidating sectors or morphing their borders, and can only be addressed successfully by TFMI.

3. The traffic levels required for DAS to show its benefit are necessarily high, as the capacity limit imposed by human operator workload is in the order of 18 traffic simultaneously under his jurisdiction [67]. This consideration elicits the adoption of traffic density higher than the one available in EXE0272.

4. The external boundary of the THALESLAND airspace includes a convex vertex, which could not be processed by the available 2D meshing algorithm.

Consequently, the updated DAS algorithm was assessed introducing representative airspace and traffic data pseudo-randomly generated by means of dedicated functions. Input data involved the array of FIR vertexes, the sector capacity limit, the number of FIR entry/exit fixes, the number of flights whose routes consisted in a combination of the generated FIR entry/exit fixes, a reference cruise airspeed and the number of desired sectors. Table 6.3 details the input parameters used. The 2D triangular mesh generated to partition the FIR is shown in 6.16. The consolidations
determined by the DAS algorithm are shown in Fig. 6.17, and the final sectorisation is depicted in Fig. 6.18.

Table 6.3. DAS algorithm input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airspace vertexes longitudinal coordinates [nmi]</td>
<td>[0, 550, 420, -10]</td>
</tr>
<tr>
<td>Latitudinal coordinates of the vertexes [nmi]</td>
<td>[0, 70, 310, 100]</td>
</tr>
<tr>
<td>Maximum capacity</td>
<td>18 aircraft</td>
</tr>
<tr>
<td>Number of FIR entry/exit fixes</td>
<td>9</td>
</tr>
<tr>
<td>Number of flights</td>
<td>46 aircraft</td>
</tr>
<tr>
<td>Desired number of ATC sectors</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 6.19. Detail of the triangular 2D mesh generated by the meshing algorithm.
Figure 6.20. Results of the DAS algorithms, detailing the calculated consolidations (in green).

Figure 6.21. Final FIR sectorisation layout after implementing the calculated consolidations.
The DAS algorithm calculated that 5 sectors were required at the starting instant to maintain the demand levels below capacity at all times in the analysed timeframe. It can be seen that while a significant portion of the routes run along the FIR long side, the overall orientation of all final sectors is perpendicular to the FIR. This configuration is sometimes considered suboptimal according to the current body of knowledge, but it is directly driven by the meshing performed. Additionally, two air routes are returning to the former control sector after a brief transit across the corner of another sector, and this is also undesirable [267-269]. A notable feature of this updated version of the DAS algorithm are that no “orphan” sectors including only one air route or none at all are generated. When comparing these results with the ones presented in section 5.10.2, it shall be noted that the air route network structure, the number of flights along each route and the original objective in terms of desired number of sectors are different.

6.6. Conclusions

The numerical simulation case studies that were performed supported the verification of the developed algorithms for their intended applications. Representative traffic conditions were introduced and the computational performances were assessed against the set requirements for strategic and tactical online timeframes. The simulations involved cases both in standard atmospheric conditions and in real weather from the models introduced in section 4.5. The en-route implementation of MOTO was mostly assessed in terms of optimality with respect to the set environmental and economic performances. Conversely, the MOTO-4D was mainly assessed in terms of capability to meet the overflight time calculated by the terminal sequencing and spacing algorithm. The TMA MOTO-4D case study featured dense arrival traffic conditions with respect to current separation standards, whereas considerably higher traffic densities could be considered for the en-route case study, therefore future research should assess in detail the scalability of the en-route MOTO algorithms.
6.7. References


Chapter 7

Conclusions

Various research initiatives have undertaken the theoretical developments and possible practical implementations of trajectory optimisation techniques to improve the economic, operational and environmental performances of air traffic. A number of studies specifically targeted the significant gains that could be attained by optimising individual manoeuvres, procedures, flight phases or entire flight routes. Yet, a number of challenges were identified when considering the operational deployment of these promising solutions in the civil aviation domain, as the conventional and largely procedural Air Traffic Management (ATM) operational paradigm would have posed significant restrictions. More recently, the research and development of novel avionics and ATM Decision Support Systems (DSS) has been pursued, introducing 4D Trajectory-Based Operations (TBO) paradigms. These systems are designed to deploy advanced air-to-ground trajectory planning, negotiation and validation functionalities to enhance the safety, efficiency and environmental sustainability of air traffic operations. The adoption of continuously updated 4-Dimensional Trajectory (4DT) descriptors with high navigational accuracy and predictability, in conjunction with data-link based 4DT negotiation and validation functionalities are providing an opportunity for the full exploitation of trajectory optimisation methods into the next generation avionics and ATM systems. Trajectory optimisation algorithms capable of addressing multiple and often conflicting objectives have a clear potential to enable real-time planning and re-planning of more environmentally efficient and economically viable flight routes by simultaneously addressing the dynamic nature of both weather and air traffic conditions. The progresses in optimal control and nonlinear programming have recently resulted in very efficient numerical solution methods, enabling real-time applications even when complex nonlinear models and multiple constraints are introduced.
In view of the recognised potential, this research project investigated the development and integration of innovative Multi-Objective 4D Trajectory Optimisation (MOTO-4D) algorithms and dynamic airspace functionalities in a next-generation ATM system prototype. Both optimal control-based and geometric formulations were successfully implemented and verified in realistic conditions. Optimal control-based formulations were identified as the most suitable for Terminal Manoeuvring Areas (TMA), as they can best match the complexities introduced by combined climb/descend and turn profiles by implementing three degrees of freedom (3DOF) point-mass aircraft dynamics models. On the other hand, geometric formulations based on steady-state aircraft dynamics were regarded as most suitable for the replanning of cruise trajectories in the enroute scenario, as transients are limited in number and duration, and the cruise altitude is most frequently constrained a priori. These MOTO-4D functionalities can effectively address the need for replanning flight trajectories in the online strategic and tactical contexts when considerable changes of contingent conditions occur. In particular, the activities highlighted a promising pathway for the implementation of MOTO functionalities in real systems. The performance improvements that can be achieved compared to manual replanning performed by the operator are significant, though uncertainties in long-term weather forecasts can potentially reduce some of the benefits. Additionally, the quality of the results is significantly affected by the accuracy of the adopted aircraft data, highlighting the potential benefits of using more accurate propulsive and aerodynamic parameters in future developments.

In the broader ATM research context, automation-assisted replanning functionalities for strategic and tactical online applications are bound to become an essential part of the 4D-TBO implementation plan. This supports the opportunity of implementing MOTO and dynamic airspace functionalities, which would also offer significant improvements to the flexibility of ATFM operations. It is therefore anticipated that a combination of MOTO and Dynamic Airspace Management (DAM) techniques will effectively enable the next step in the CNS+A driven evolutions of the global ATM network.
The conclusions of this research project are here summarised in terms of achieved research objectives:

1. **Review the state of the art in ATM/ATFM and avionics modernisation, particularly assessing the evolving requirements for online trajectory planning and the 4D-TBO context.**

   A comprehensive review of the most recent achievements and research endeavours in ATM/Air Traffic Flow Management (ATFM) and avionics relevant to online trajectory planning, 4D-TBO and dynamic airspace was accomplished. Some important considerations for the design of 4DT planning algorithms based on trajectory optimisation were identified and implemented in the subsequent algorithm development stages.

2. **Perform a critical review of the literature on trajectory optimisation in the aviation domain, addressing both the theoretical framework and the proposed solution approaches.**

   An extensive review of approximately 200 publications was performed, including major works in the domain as well as specific implementations that were deemed very relevant for ATM and avionics implementations. In addition to leading to a major publication, this critical review supported the identification of the most suitable trajectory optimisation techniques for ATM system implementation in the considered operational conditions.

3. **Develop Multi-Objective 4D Trajectory Optimisation (MOTO-4D) algorithms suitable for the next generation of ATM and avionics systems.**

   Two alternative implementations of MOTO were successfully developed and verified in representative conditions as part of this research project, addressing the operational specificities of the scenarios for which they were conceived, respectively Terminal Manoeuvring Area (TMA) and en-route contexts, while at the same time fulfilling the stringent time requirements for possible integration in strategic and tactical online ATM implementations.

4. **Identify suitable models to integrate targeted operational, environmental and economic criteria in the formulated trajectory optimisation problem.**
Key models were identified for flight dynamics, engine thrust, exhaust emissions and weather. Additionally, the knowledge gathered with respect to airspace structure and route descriptors supported the design of the required 4DT realisation models. All these models were implemented and numerically assessed in representative conditions.

5. **Develop suitable sequencing and spacing algorithms for deconflicting optimal 4DT intents when optimising the trajectories of multiple aircraft.**

A set of analytical models supporting the development of automated sequencing and spacing algorithm based on DCB was developed during the research project. More specifically, the three fundamental mathematical models governing ATFM DCB were introduced and described in detail current and proposed models for ATCo workload-based sector capacities. Additionally, the delicate aspect of integrating the interdependent optimisation processes involved in MOTO, dynamic airspace and Traffic Flow Management Initiatives (TFMI) were addressed, and two robust prioritisation strategies specifically for the TMA and for the enroute scenarios were developed.

6. **Develop suitable algorithms to translate the optimal trajectories generated by the MOTO-4D algorithms into 4DT intents complying with current standard to be exchanged on 4DT data-links, while retaining most of the optimality qualities of the original solution.**

The necessary steps to translate the mathematically optimal trajectory generated by optimal control-based solution methods into a flyable and conventionally described 4DT intent were identified and implemented. In particular, the mathematically optimal trajectory generated by pseudospectral optimal control algorithm are processed by a Control Input Smoothing (CIS) algorithm to filter out the unfeasible control input variations. Subsequently, the smooth mathematical trajectory is processed by a suitably developed Operational Trajectory Realisation (OTR) algorithm to be translated in a series of conventional 4DT descriptors, including fly-by and overfly waypoints with suitable altitude/time constraints.
7. **Define the functional architecture of a novel ATM DSS integrating the developed MOTO and DAM algorithms.**

Based on the implemented MOTO and DAM algorithms, a novel ATM DSS prototype was conceptually designed. The functional architecture developed for this new ATM DSS integrates automated 4DT Planning, Negotiation and Validation (4-PNV) functionalities and captures the interactions with Next Generation Flight Management Systems (NG-FMS) on-board manned and unmanned aircraft through suitable data-links, meeting the requirements set for the 4D-TBO context. The functional architecture was designed considering the existing ATM structure, hence including regional, national and local ATM/ATFM units.

8. **Perform numerical simulation activities in representative conditions to assess the viability of the developed algorithms.**

The developed MOTO and DAM algorithms were verified by means of numerical simulations in representative conditions. In particular, an offline validation campaign was carried out implementing representative airspace, traffic and weather data and adopting different objectives including emissions, fuel and time. The representative airspace and traffic data from THALESLAND were adopted to test the enroute implementation of MOTO. Representative pseudo-random data was used for assessing the TMA implementation of MOTO and the automated sectorisation algorithm for DAM. The results demonstrate that the newly developed MOTO techniques consistently generate solutions meeting the requirements for employment in future ATM DSS for 4D-TBO.

### 7.1. Recommendations for future research

Further research activities are needed to address the following areas:

- **Human Machine Interface and Interactions (HMI²) evolutions for 4D-TBO** accommodating MOTO and DAM functionalities. Aspects not yet fully addressed by the research community include the ergonomic and human-factors impacts of ATM system evolutions and how, in particular, new
technologies such as MOTO and Dynamic Airspace Sectorisation (DAS) can be fully exploited to enhance airspace capacity and human operators’ productivity. Suitable forms of human-machine teaming are therefore an essential asset to fully enable these new concepts, including adaptive forms of HMI\(^2\) based on measurable cognitive and workload states.

- **Impact of Unmanned Aircraft Systems (UAS).** In the CNS+A context, the general case is that of multiple manned/unmanned aircraft performing either cooperative or non-cooperative surveillance. In terms of granting the required levels of operational safety when considering the integration of unmanned traffic in airspace characterised with dense air traffic and high levels of Air Traffic Services (ATS), the emphasis is on CNS+A equipment that can meet strict performance requirements while also supporting enhanced ATM functionalities. These systems will enable the UAS Traffic Management (UTM) paradigm. Research is on-going on a regional as well as global scale for demonstrating the feasibility of UTM concepts. In the United States, the National Aeronautics and Space Administration (NASA) along with UAS operational and regulatory stakeholders is researching on prototype technologies for a UTM system that could develop airspace integration requirements for enabling safe and efficient UAS operations (especially targeting low-altitude tasks performed by unmanned aircraft). The UTM system provides to remote pilots information needed to maintain separation from other aircraft by reserving areas for specific routes, with consideration of restricted airspace and adverse weather conditions.

Strategic separation coordination and some tactical separation management are considered as part of UTM in addition to contingency management, surface operation management and capacity-demand balancing. In the long term, UTM research is envisaged to integrate in particular, the functionalities for trajectory prediction and negotiation, conflict detection and resolution, separation/spacing monitoring and optimisation for mixed manned/unmanned aircraft traffic both within individual controlled airspace sectors and across multiple sectors belonging to a Flight Information Region (FIR), also considering the inbound/outbound flows from adjacent FIRs. Additionally,
UTM supports functions associated with airspace design and management, geo-fencing, congestion management, authenticated operations and weather predictions. As a significant portion of the UAS fleet is specifically employed on an opportunity-basis with little to no advance planning, one of the key challenges that need to be addressed is the access of this opportunity UAS traffic to controlled airspaces and the subsequent implications on design and development of future ATFM systems.

- **CNS+A System Certification.** As part of the progressive transition to an integrated and more strategic CNS+A scenario, contemporary certification considerations for CNS/ATM systems will become applicable to ATFM systems as well. In the CNS+A context, ATM and avionics systems are integrated in various aspects with the airborne systems, with a functional integration between air, ground and space systems. Integrated systems demand greater safety verification efforts, considering the complex nature of the interactions taking place across these systems. The verification, validation and certification of the individual components as well as the “integrated” CNS+A systems fully supporting TBO and Performance-Based Operations (PBO) are crucial to ensure that the required levels of network-wide safety are met. At present, traditional standards and regulations are in place through ICAO Annexes 8 and Doc 9760 for the individual components of the system, with more emphasis on the airborne systems than for the ground-based systems. These standards will need to evolve to account for the functional integration of CNS+A systems in the 4D-TBO and PBO contexts.
Appendix A  Results of the MOTO-4D for en-route applications

This appendix includes all the scenario data and results of the en-route MOTO algorithm verification.

Flight 1

Flight n. 1 is a westbound traffic whose original route is obstructed by the passage of the tropical revolving storm. The details of the flight are given in Table 1. The original and optimal trajectories are plotted in Figure 1, and the associated performance indicators are detailed in Table 2. The fuel-time Pareto diagram of the results is given in Figure 2.

Table 1.  Details of flight n. 1.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>FDX104</th>
<th>Type</th>
<th>MD11 (Jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XCMO</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>340</td>
<td>Activation</td>
<td>00:45</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MO CS CAS GEOFF L66 CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time</td>
<td>13.3s</td>
<td>11.6s</td>
<td>20.3s</td>
</tr>
</tbody>
</table>

Figure 1.  Results of the enroute 2D+T-MOTO optimisation performed for flight n. 1.
Table 2. Enroute 2D+T-MOTO results for flight n. 1.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>165.9</td>
<td>153.6</td>
<td>153.6</td>
<td>157.5</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>72.97</td>
<td>68.02</td>
<td>68.02</td>
<td>68.49</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>13.4</td>
<td>12.45</td>
<td>12.45</td>
<td>12.58</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.9</td>
<td>1.79</td>
<td>1.79</td>
<td>1.81</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>483</td>
<td>452.31</td>
<td>452.31</td>
<td>450.14</td>
</tr>
</tbody>
</table>

Figure 2. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 1.
Flight 2
Flight n. 2 is a westbound traffic whose original route is partly affected by the tropical revolving storm. The details of the flight are given in Table 3. The original and optimal trajectories are plotted in Figure 3, and the associated performance indicators are detailed in Table 4. The fuel-time Pareto diagram of the results is given in Figure 4.

Table 3. Details of flight n. 2.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>EVA666</th>
<th>Type</th>
<th>A310 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XNEB</td>
<td>Destination Airport</td>
<td>TSTP</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>360</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>LAVIL LASER LILLY L94 BESTO BULAN APU</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>22.6s</td>
<td>18.9s</td>
<td>22.4s</td>
</tr>
</tbody>
</table>

Figure 3. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 2.
Table 4. Enroute 2D+T-MOTO results for flight n. 2.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>214.8</td>
<td>220.6</td>
<td>202.9</td>
<td>237.3</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>15.56</td>
<td>15.18</td>
<td>15.41</td>
<td>15.45</td>
</tr>
<tr>
<td>CO$_2$ [t]</td>
<td>49.17</td>
<td>47.96</td>
<td>48.71</td>
<td>48.82</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.31</td>
<td>1.29</td>
<td>1.3</td>
<td>1.32</td>
</tr>
<tr>
<td>NO$_x$ [kg]</td>
<td>274.3</td>
<td>258.2</td>
<td>281.3</td>
<td>254.2</td>
</tr>
</tbody>
</table>

Figure 4. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 2.
Flight 3

Flight n. 3 is a westbound traffic whose original route is obstructed by the passage of the tropical revolving storm. The details of the flight are given in Table 5. The original and optimal trajectories are plotted in Figure 5, and the associated performance indicators are detailed in Table 6. The fuel-time Pareto diagram of the results is given in Figure 6.

Table 5. Details of flight n. 3.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>ARV825</th>
<th>Type</th>
<th>B788 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure</td>
<td>XECT</td>
<td>Destination</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>360</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>TAXON TASIA L66 CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time</td>
<td>54.2s</td>
<td>53.7s</td>
<td>61.1s</td>
</tr>
</tbody>
</table>

Figure 5. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 3.
Table 6. Enroute 2D+T-MOTO results for flight n. 3.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>315</td>
<td>299.8</td>
<td>301.8</td>
<td>320.7</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>27.55</td>
<td>26.49</td>
<td>27.28</td>
<td>26.49</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>87.06</td>
<td>83.7</td>
<td>86.2</td>
<td>83.72</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>16.08</td>
<td>15.41</td>
<td>15.78</td>
<td>15.73</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.3</td>
<td>2.21</td>
<td>2.27</td>
<td>2.21</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>563.4</td>
<td>546.22</td>
<td>573.23</td>
<td>518.9</td>
</tr>
</tbody>
</table>

Figure 6. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 3.
Flight 4

Flight n. 4 is an eastbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 7. The original and optimal trajectories are plotted in Figure 7, and the associated performance indicators are detailed in Table 8. The fuel-time Pareto diagram of the results is given in Figure 8.

Table 7. Details of flight n. 4.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>PYT727</th>
<th>Type</th>
<th>B744 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XSEW</td>
<td>Destination Airport</td>
<td>XCMO</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>380</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>WEEA WANKA GASPA KENOJ CAS MO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>4.8s</td>
<td>3.1s</td>
<td>3.5s</td>
</tr>
</tbody>
</table>

Figure 7. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 4.
Table 8. Enroute 2D+T-MOTO results for flight n. 4.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>73</td>
<td>64.8</td>
<td>62.8</td>
<td>83.7</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>12.64</td>
<td>11.1</td>
<td>11.19</td>
<td>13.65</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>39.93</td>
<td>35.09</td>
<td>35.36</td>
<td>43.15</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>7.35</td>
<td>6.48</td>
<td>6.47</td>
<td>8.04</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.05</td>
<td>0.93</td>
<td>0.93</td>
<td>1.14</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>260.78</td>
<td>227.69</td>
<td>235.13</td>
<td>272.76</td>
</tr>
</tbody>
</table>

Figure 8. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 4.
Flight 5

Flight n. 5 is a westbound traffic whose original route is partly affected by the tropical revolving storm. The details of the flight are given in Table 9. The original and optimal trajectories are plotted in Figure 9, and the associated performance indicators are detailed in Table 10. The fuel-time Pareto diagram of the results is given in Figure 10.

Table 9. Details of flight n. 5.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>EVA748</th>
<th>Type</th>
<th>B744 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XNEB</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>360</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>LAVIL LASER L94 BESTO W2 CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>34.3s</td>
<td>23s</td>
<td>29.9s</td>
</tr>
</tbody>
</table>

Figure 9. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 5.
Table 10. Enroute 2D+T-MOTO results for flight n. 5.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>189</td>
<td>215.1</td>
<td>198.7</td>
<td>227.6</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.21</td>
<td>36.46</td>
<td>36.17</td>
<td>36.81</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>104.96</td>
<td>115.12</td>
<td>114.3</td>
<td>116.31</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>19.9</td>
<td>22.18</td>
<td>21.47</td>
<td>22.77</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.78</td>
<td>3.06</td>
<td>3.02</td>
<td>3.09</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>636.31</td>
<td>682.68</td>
<td>709.47</td>
<td>671.86</td>
</tr>
</tbody>
</table>

Figure 10. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 5.
Flight 6
Flight n. 6 is an eastbound traffic whose original route is obstructed by the passage of the tropical revolving storm. The details of the flight are given in Table 11. The original and optimal trajectories are plotted in Figure 11, and the associated performance indicators are detailed in Table 12. The fuel-time Pareto diagram of the results is given in Figure 12.

Table 11. Details of flight n. 6.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>VRG07</th>
<th>Type</th>
<th>B773</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSTP</td>
<td>Destination Airport</td>
<td>XNEB</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>330</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>APU BESTO BULAN NANCY N88 BISON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>26.8s</td>
<td>19.7s</td>
<td>33.4s</td>
</tr>
</tbody>
</table>

Figure 11. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 6.
Table 12. Enroute 2D+T-MOTO results for flight n. 6.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>239.6</td>
<td>264.9</td>
<td>252.2</td>
<td>294.2</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.83</td>
<td>36.87</td>
<td>37.22</td>
<td>37.89</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>106.89</td>
<td>116.52</td>
<td>117.62</td>
<td>119.74</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.82</td>
<td>3.07</td>
<td>3.1</td>
<td>3.17</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>691.75</td>
<td>747.89</td>
<td>782.14</td>
<td>738.6</td>
</tr>
</tbody>
</table>

Figure 12. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 6.
Flight 7
Flight n. 7 is an eastbound traffic whose original route is obstructed by the passage of the tropical revolving storm. The details of the flight are given in Table 13. The original and optimal trajectories are plotted in Figure 13, and the associated performance indicators are detailed in Table 14. The fuel-time Pareto diagram of the results is given in Figure 14.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>DHL200</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSTP</td>
<td>Destination Airport</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>350</td>
<td>Activation</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>PN APU MANNX M77 MAXIM</td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>84s</td>
<td>81s</td>
</tr>
</tbody>
</table>

Figure 13. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 7.
Table 14. Enroute 2D+T-MOTO results for flight n. 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>284</td>
<td>322</td>
<td>298.1</td>
<td>365.9</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>51.06</td>
<td>57.91</td>
<td>56.24</td>
<td>60.03</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>161.33</td>
<td>182.99</td>
<td>177.72</td>
<td>189.7</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>31.08</td>
<td>35.24</td>
<td>33.61</td>
<td>38.06</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>4.28</td>
<td>4.85</td>
<td>4.7</td>
<td>5.07</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>954.91</td>
<td>1083.34</td>
<td>1087.65</td>
<td>1060.9</td>
</tr>
</tbody>
</table>

Figure 14. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 7.
Flight 8

Flight n. 8 is an eastbound traffic whose original route is obstructed by the passage of the tropical revolving storm. The details of the flight are given in Table 15. The original and optimal trajectories are plotted in Figure 15, and the associated performance indicators are detailed in Table 16. The fuel-time Pareto diagram of the results is given in Figure 16.

Table 15. Details of flight n. 8.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>PAL712</th>
<th>Type</th>
<th>B744 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSTP</td>
<td>Destination Airport</td>
<td>XESG</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>370</td>
<td>Activation</td>
<td>00:00.5</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>ANLOT YU GI GANDA M24 GOTIM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>89.9s</td>
<td>94.6s</td>
<td>92.5s</td>
</tr>
</tbody>
</table>

Figure 15. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 8.
Table 16. Enroute 2D+T-MOTO results for flight n. 8.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>269.3</td>
<td>273.1</td>
<td>267.6</td>
<td>289</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>46.78</td>
<td>47.43</td>
<td>46.48</td>
<td>50.2</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>147.82</td>
<td>149.89</td>
<td>146.87</td>
<td>158.63</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>27.65</td>
<td>28.03</td>
<td>27.47</td>
<td>29.67</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>3.91</td>
<td>3.96</td>
<td>3.88</td>
<td>4.19</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>926.18</td>
<td>939.15</td>
<td>920.24</td>
<td>993.92</td>
</tr>
</tbody>
</table>

Figure 16. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 8.
Flight 9

Flight n. 9 is an eastbound traffic whose original route is obstructed by the passage of the tropical revolving storm. The details of the flight are given in Table 17. The original and optimal trajectories are plotted in Figure 17, and the associated performance indicators are detailed in Table 18. The fuel-time Pareto diagram of the results is given in Figure 18.

Table 17. Details of flight n. 9.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>VRG719</th>
<th>Type</th>
<th>B773 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSTP</td>
<td>Destination Airport</td>
<td>XNEB</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>330</td>
<td>Activation</td>
<td>00:24</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>APU BESTO BULAN N88 BISON</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 2D+T MOTO Calculation Time (Min Fuel / Time / Emis.) | 25.5s | 19.9s | 30.2s |

Figure 17. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 9.
Table 18. Enroute 2D+T-MOTO results for flight n. 9.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>240.3</td>
<td>264.9</td>
<td>252.3</td>
<td>294.3</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.92</td>
<td>36.88</td>
<td>37.23</td>
<td>37.9</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>107.2</td>
<td>116.54</td>
<td>117.63</td>
<td>119.75</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>19.8</td>
<td>21.59</td>
<td>21.54</td>
<td>22.56</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.83</td>
<td>3.08</td>
<td>3.1</td>
<td>3.17</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>693.76</td>
<td>748.01</td>
<td>782.27</td>
<td>738.68</td>
</tr>
</tbody>
</table>

Figure 18. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 9.
Flight 10

Flight n. 10 is a westbound traffic whose original route is partly affected by the tropical revolving storm. The details of the flight are given in Table 19. The original and optimal trajectories are plotted in Figure 19, and the associated performance indicators are detailed in Table 20. The fuel-time Pareto diagram of the results is given in Figure 20.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>KAL922</th>
<th>Type</th>
<th>B773 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XNEL</td>
<td>Destination Airport</td>
<td>TSTP</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>310</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>LAVIL LASER L94 LILLY NEWBY LAGER L94 BESTO APU</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>24.4s</td>
<td>20.1s</td>
<td>25.5s</td>
</tr>
</tbody>
</table>

Figure 19. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 10.
Table 20. Enroute 2D+T-MOTO results for flight n. 10.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>191.8</td>
<td>219.3</td>
<td>186.3</td>
<td>226.9</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>28.04</td>
<td>28.18</td>
<td>29.04</td>
<td>28.15</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>88.61</td>
<td>89.06</td>
<td>91.76</td>
<td>88.94</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>16.64</td>
<td>17.44</td>
<td>16.95</td>
<td>17.6</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.34</td>
<td>2.37</td>
<td>2.42</td>
<td>2.37</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>550.03</td>
<td>514.24</td>
<td>593.82</td>
<td>506.2</td>
</tr>
</tbody>
</table>

Figure 20. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 10.
Flight 11

Flight n. 11 is an eastbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 21. The original and optimal trajectories are plotted in Figure 21, and the associated performance indicators are detailed in Table 22. The fuel-time Pareto diagram of the results is given in Figure 22.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>BAW022</th>
<th>Type</th>
<th>B773 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSTP</td>
<td>Destination Airport</td>
<td>XNEB</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>330</td>
<td>Activation</td>
<td>00:31</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>APU BESTO BULAN N88 BISON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>26.5s</td>
<td>20s</td>
<td>30.4s</td>
</tr>
</tbody>
</table>

Table 21. Details of flight n. 11.

Figure 21. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 11.
**Table 22.** Enroute 2D+T-MOTO results for flight n. 11.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>240.3</td>
<td>264.9</td>
<td>252.3</td>
<td>294.3</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.92</td>
<td>36.88</td>
<td>37.23</td>
<td>37.9</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>107.2</td>
<td>116.54</td>
<td>117.63</td>
<td>119.75</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>19.8</td>
<td>21.59</td>
<td>21.54</td>
<td>22.56</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.83</td>
<td>3.08</td>
<td>3.1</td>
<td>3.17</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>693.76</td>
<td>748.01</td>
<td>782.27</td>
<td>738.68</td>
</tr>
</tbody>
</table>

**Figure 22.** Fuel-time Pareto diagram of original and optimised trajectories of flight n. 11.
Flight 12
Flight n. 12 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 23. The original and optimal trajectories are plotted in Figure 23, and the associated performance indicators are detailed in Table 24. The fuel-time Pareto diagram of the results is given in Figure 24.

| Table 23. Details of flight n. 12. |
|------------------|----------------|----------------|----------------|
| Callsign         | Type           | A320 (jet)     |
| Departure Airport| XCCS           | Destination Airport | TSKH          |
| Cruise FL        | 320            | Activation     | 00:30          |
| Flight Plan      | CS CAS L66 CIVET SLP KHV |               |
| 2D+T MOTO Calculation Time (Min Fuel / Time / Emis.) | 13s | 11.5s | 16.9s |

Figure 23. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 12.

A-23
Table 24. Enroute 2D+T-MOTO results for flight n. 12.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>185.7</td>
<td>190.3</td>
<td>161.5</td>
<td>210.8</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>7.82</td>
<td>7.09</td>
<td>7.37</td>
<td>7.39</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>24.71</td>
<td>22.39</td>
<td>23.3</td>
<td>23.35</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>4.68</td>
<td>4.58</td>
<td>4.28</td>
<td>5.02</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>0.65</td>
<td>0.6</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>150.5</td>
<td>122.61</td>
<td>152.85</td>
<td>121.93</td>
</tr>
</tbody>
</table>

Figure 24. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 12.
Flight 13
Flight n. 13 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 25. The original and optimal trajectories are plotted in Figure 25, and the associated performance indicators are detailed in Table 26. The fuel-time Pareto diagram of the results is given in Figure 26.

Table 25. Details of flight n. 13.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>NBT019</th>
<th>Type</th>
<th>B753 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XCMW</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>340</td>
<td>Activation</td>
<td>00:30</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>CAS GEOFF L66 CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>16.2s</td>
<td>11.1s</td>
<td>16.5s</td>
</tr>
</tbody>
</table>

Figure 25. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 13.
Table 26. Enroute 2D+T-MOTO results for flight n. 13.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>161.6</td>
<td>159.5</td>
<td>148</td>
<td>171.7</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>10.71</td>
<td>10.12</td>
<td>10.24</td>
<td>10.34</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>33.83</td>
<td>31.97</td>
<td>32.36</td>
<td>32.68</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>6.41</td>
<td>6.17</td>
<td>6.04</td>
<td>6.45</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>0.9</td>
<td>0.85</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>206.1</td>
<td>188.72</td>
<td>203.68</td>
<td>186.65</td>
</tr>
</tbody>
</table>

Figure 26. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 13.
Flight 14

Flight n. 14 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 27. The original and optimal trajectories are plotted in Figure 27, and the associated performance indicators are detailed in Table 28. The fuel-time Pareto diagram of the results is given in Figure 28.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>KAL001</th>
<th>Type</th>
<th>A388 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XENM</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>400</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MISTY MARLA MAMUT M77 YU CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>39.5s</td>
<td>36.8s</td>
<td>43.2s</td>
</tr>
</tbody>
</table>

Figure 27. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 14.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>263.2</td>
<td>239.7</td>
<td>239.7</td>
<td>247.4</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>60.71</td>
<td>55.7</td>
<td>55.7</td>
<td>56.46</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>191.84</td>
<td>176.02</td>
<td>176.02</td>
<td>178.4</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>34.13</td>
<td>31.3</td>
<td>31.3</td>
<td>31.75</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>5.04</td>
<td>4.63</td>
<td>4.63</td>
<td>4.69</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>1500.8</td>
<td>1382.66</td>
<td>1382.66</td>
<td>1388.47</td>
</tr>
</tbody>
</table>

Figure 28. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 14.
Flight 15

Flight n. 15 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 29. The original and optimal trajectories are plotted in Figure 29, and the associated performance indicators are detailed in Table 30. The fuel-time Pareto diagram of the results is given in Figure 30.

Table 29. Details of flight n. 15.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>QFA803</th>
<th>Type</th>
<th>A388</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XECT</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>400</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>TAXON TASIA TROUT KWANG L66 CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 2D+T MOTO Calculation Time (Min Fuel / Time / Emis.) | 52.6s | 50.8s | 71.3s |

Figure 29. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 15.
Table 30. Enroute 2D+T-MOTO results for flight n. 15.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>300.5</td>
<td>282</td>
<td>282</td>
<td>292.9</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>69.3</td>
<td>65.53</td>
<td>65.53</td>
<td>66.83</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>219</td>
<td>207.08</td>
<td>207.08</td>
<td>211.17</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>38.96</td>
<td>36.83</td>
<td>36.83</td>
<td>37.59</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>5.76</td>
<td>5.44</td>
<td>5.44</td>
<td>5.55</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>1713.28</td>
<td>1626.62</td>
<td>1626.65</td>
<td>1643.34</td>
</tr>
</tbody>
</table>

Figure 30. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 15.
Flight 16

Flight n. 16 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 31. The original and optimal trajectories are plotted in Figure 31, and the associated performance indicators are detailed in Table 32. The fuel-time Pareto diagram of the results is given in Figure 32.

Table 31. Details of flight n. 16.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>AFR349</th>
<th>Type</th>
<th>B744 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XENM</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>340</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MAXIM MISTY M77 YU CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time</td>
<td>68.9s</td>
<td>59.7s</td>
<td>73.6s</td>
</tr>
</tbody>
</table>

Figure 31. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 16.
Table 32. Enroute 2D+T-MOTO results for flight n. 16.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>290.3</td>
<td>294.3</td>
<td>269.5</td>
<td>311.2</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>52.39</td>
<td>49.34</td>
<td>51.43</td>
<td>49.59</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>165.54</td>
<td>155.92</td>
<td>162.53</td>
<td>156.69</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>32.63</td>
<td>31.98</td>
<td>31.22</td>
<td>33.04</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>4.41</td>
<td>4.18</td>
<td>4.31</td>
<td>4.22</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>946.87</td>
<td>850.75</td>
<td>966.63</td>
<td>831.97</td>
</tr>
</tbody>
</table>

Figure 32. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 16.
Flight 17
Flight n. 17 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 33. The original and optimal trajectories are plotted in Figure 33, and the associated performance indicators are detailed in Table 34. The fuel-time Pareto diagram of the results is given in Figure 34.

Table 33. Details of flight n. 17.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>PYT389</th>
<th>Type</th>
<th>B772 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XESG</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>380</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>GOTIM GUEST M24 GI SLP KHV</td>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>65.9s / 60.9s / 62s</td>
</tr>
</tbody>
</table>

Figure 33. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 17.
Table 34. Enroute 2D+T-MOTO results for flight n. 17.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>305</td>
<td>301.7</td>
<td>296.1</td>
<td>313.1</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>32.75</td>
<td>31.82</td>
<td>32.77</td>
<td>31.39</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>103.49</td>
<td>100.56</td>
<td>103.54</td>
<td>99.2</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>19.93</td>
<td>19.50</td>
<td>19.73</td>
<td>19.60</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.75</td>
<td>2.67</td>
<td>2.74</td>
<td>2.64</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>612.51</td>
<td>588.75</td>
<td>624.73</td>
<td>565.45</td>
</tr>
</tbody>
</table>

Figure 34. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 17.
Flight 18

Flight n. 18 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 35. The original and optimal trajectories are plotted in Figure 35, and the associated performance indicators are detailed in Table 36. The fuel-time Pareto diagram of the results is given in Figure 36.

Table 35. Details of flight n. 18.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>Type</th>
<th>Departure Airport</th>
<th>Destination Airport</th>
<th>Cruise FL</th>
<th>Activation</th>
<th>Flight Plan</th>
<th>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PYT313</td>
<td>B772 (jet)</td>
<td>XENM</td>
<td>TSKH</td>
<td>360</td>
<td>00:00</td>
<td>MAXIM MISTY M77 YU CIVET SLP KHV</td>
<td>65.3s 53.6s 64s</td>
</tr>
</tbody>
</table>

Figure 35. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 18.
Table 36. Enroute 2D+T-MOTO results for flight n. 18.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>306.5</td>
<td>306.2</td>
<td>285</td>
<td>314.7</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.73</td>
<td>31.8</td>
<td>33</td>
<td>31.09</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>106.6</td>
<td>100.49</td>
<td>104.28</td>
<td>98.25</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.84</td>
<td>2.69</td>
<td>2.77</td>
<td>2.65</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>603.7</td>
<td>548.84</td>
<td>611.24</td>
<td>522.74</td>
</tr>
</tbody>
</table>

Figure 36. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 18.
Flight 19

Flight n. 19 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 37. The original and optimal trajectories are plotted in Figure 37, and the associated performance indicators are detailed in Table 38. The fuel-time Pareto diagram of the results is given in Figure 38.

Table 37. Details of flight n. 19.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>SAA121</th>
<th>Type</th>
<th>A388 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XENM</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>400</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MIS60</td>
<td>MISTY M77 YU CIVET SLP KHV</td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time</td>
<td>40.2s</td>
<td>34.4s</td>
<td>43.4s</td>
</tr>
</tbody>
</table>

Figure 37. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 19.
Table 38. Enroute 2D+T-MOTO results for flight n. 19.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>243.9</td>
<td>238.6</td>
<td>238.6</td>
<td>246.2</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>56.25</td>
<td>55.45</td>
<td>55.45</td>
<td>56.17</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>177.75</td>
<td>175.23</td>
<td>175.23</td>
<td>177.5</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>31.62</td>
<td>31.16</td>
<td>31.16</td>
<td>31.59</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>4.67</td>
<td>4.61</td>
<td>4.61</td>
<td>4.67</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>1390.6</td>
<td>1376.44</td>
<td>1376.44</td>
<td>1381.43</td>
</tr>
</tbody>
</table>

Figure 38. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 19.
Flight 20

Flight n. 20 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 39. The original and optimal trajectories are plotted in Figure 39, and the associated performance indicators are detailed in Table 40. The fuel-time Pareto diagram of the results is given in Figure 40.

Table 39. Details of flight n. 20.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>SIA013</th>
<th>Type</th>
<th>A388 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XECT</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>380</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>TAXON TASIA TROUT L66 CIVET SLP KHV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 40. Details of flight n. 20.

| 2D+T MOTO Calculation Time (Min Fuel / Time / Emis.) | 57.3s | 52.2s | 65.3s |

Figure 39. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 20.
Table 40. Enroute 2D+T-MOTO results for flight n. 20.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>298.6</td>
<td>288.8</td>
<td>288.8</td>
<td>299.2</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>68.6</td>
<td>67.36</td>
<td>67.35</td>
<td>67.19</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>216.8</td>
<td>212.86</td>
<td>212.83</td>
<td>212.33</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>39.13</td>
<td>38.36</td>
<td>38.36</td>
<td>38.42</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>5.7</td>
<td>5.6</td>
<td>5.6</td>
<td>5.59</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>1526.81</td>
<td>1512.23</td>
<td>1511.99</td>
<td>1478.05</td>
</tr>
</tbody>
</table>

Figure 40. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 20.
Flight 21

Flight n. 21 is an eastbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 41. The original and optimal trajectories are plotted in Figure 41, and the associated performance indicators are detailed in Table 42. The fuel-time Pareto diagram of the results is given in Figure 42.

Table 41. Details of flight n. 21.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>ARV682</th>
<th>Type</th>
<th>A330 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XSEW</td>
<td>Destination Airport</td>
<td>XCCS</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>380</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>WEEWA WANKA GASPA KENOJ CAS CS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)

<table>
<thead>
<tr>
<th>Min Fuel</th>
<th>Min Time</th>
<th>Min Emis</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3s</td>
<td>3.2s</td>
<td>3.8s</td>
</tr>
</tbody>
</table>

Figure 41. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 21.
Table 42. Enroute 2D+T-MOTO results for flight n. 21.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>75.8</td>
<td>87.5</td>
<td>76.6</td>
<td>92.5</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>6.73</td>
<td>7.02</td>
<td>7.22</td>
<td>7.11</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>21.27</td>
<td>22.19</td>
<td>22.82</td>
<td>22.47</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>3.95</td>
<td>4.27</td>
<td>4.18</td>
<td>4.4</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>0.56</td>
<td>0.59</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>135.12</td>
<td>131.39</td>
<td>151.77</td>
<td>129.79</td>
</tr>
</tbody>
</table>

Figure 42. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 21.
Flight 22
Flight n. 22 is a westbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 43. The original and optimal trajectories are plotted in Figure 43, and the associated performance indicators are detailed in Table 44. The fuel-time Pareto diagram of the results is given in Figure 44.

Table 43. Details of flight n. 22.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>MARNR38</th>
<th>Type</th>
<th>P8 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XNEW</td>
<td>Destination Airport</td>
<td>XCMW</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>340</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>WISKY WAL30 WALTA WELLAL DONMU CS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>8s</td>
<td>6.4s</td>
<td>9.6s</td>
</tr>
</tbody>
</table>

Figure 43. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 22.
A Novel Air Traffic Management Decision Support System
Multi-objective 4-dimensional trajectory optimisation for intent-based operations in dynamic airspace
© Alessandro G. M. Gardi

Table 44. Enroute 2D+T-MOTO results for flight n. 22.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>133.6</td>
<td>134.1</td>
<td>129.4</td>
<td>150.2</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>12.09</td>
<td>12.12</td>
<td>12.25</td>
<td>12.45</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>38.21</td>
<td>38.31</td>
<td>38.71</td>
<td>39.35</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>7.21</td>
<td>7.23</td>
<td>7.19</td>
<td>7.73</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.01</td>
<td>1.01</td>
<td>1.02</td>
<td>1.05</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>235</td>
<td>235.25</td>
<td>247.09</td>
<td>226.12</td>
</tr>
</tbody>
</table>

Figure 44. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 22.
Flight 23

Flight n. 23 is an eastbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 45. The original and optimal trajectories are plotted in Figure 45, and the associated performance indicators are detailed in Table 46. The fuel-time Pareto diagram of the results is given in Figure 46.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>UAL94</th>
<th>Type</th>
<th>B744 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSKH</td>
<td>Destination Airport</td>
<td>XESG</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>370</td>
<td>Activation</td>
<td></td>
</tr>
<tr>
<td>Flight Plan</td>
<td>GRUMP Gi GANDA M24 GOTIM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>98.1s</td>
<td>96s</td>
<td>105s</td>
</tr>
</tbody>
</table>

Figure 45. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 23.
Table 46. Enroute 2D+T-MOTO results for flight n. 23.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>269.2</td>
<td>276.8</td>
<td>271.3</td>
<td>292.4</td>
</tr>
<tr>
<td>Fuel t</td>
<td>46.75</td>
<td>48.08</td>
<td>47.13</td>
<td>50.79</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>147.74</td>
<td>151.93</td>
<td>148.92</td>
<td>160.49</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>27.63</td>
<td>28.42</td>
<td>27.85</td>
<td>30.02</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>3.9</td>
<td>4.01</td>
<td>3.93</td>
<td>4.24</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>925.74</td>
<td>951.94</td>
<td>933.07</td>
<td>1005.61</td>
</tr>
</tbody>
</table>

Figure 46. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 23.
Flight 24

Flight n. 24 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 47. The original and optimal trajectories are plotted in Figure 47, and the associated performance indicators are detailed in Table 48. The fuel-time Pareto diagram of the results is given in Figure 48.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>QFA57</th>
<th>Type</th>
<th>A388 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XNMN</td>
<td>Destination Airport</td>
<td>XSMN</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>360</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>NANNA NEV60 NEVER L398 SAMAD</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 47.** Details of flight n. 24.

| 2D+T MOTO Calculation Time (Min Fuel / Time / Emis.) | 10.3s | 8s | 9s |

Figure 47. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 24.
Table 48. Enroute 2D+T-MOTO results for flight n. 24.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>104.9</td>
<td>104</td>
<td>104.3</td>
<td>109.7</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>24.64</td>
<td>24.25</td>
<td>25.2</td>
<td>24.28</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>77.86</td>
<td>76.63</td>
<td>79.62</td>
<td>76.71</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.05</td>
<td>2.02</td>
<td>2.1</td>
<td>2.03</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>508.5</td>
<td>498.21</td>
<td>529.46</td>
<td>484.39</td>
</tr>
</tbody>
</table>

Figure 48. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 24.
Flight 25

Flight n. 25 is an eastbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 49. The original and optimal trajectories are plotted in Figure 49, and the associated performance indicators are detailed in Table 50. The fuel-time Pareto diagram of the results is given in Figure 50.

Table 49. Details of flight n. 25.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>NBT116</th>
<th>Type</th>
<th>B753 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XCCS</td>
<td>Destination Airport</td>
<td>XESG</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>370</td>
<td>Activation</td>
<td></td>
</tr>
<tr>
<td>Flight Plan</td>
<td>CS MWV ROWLY RASTA GUEST GOTIM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time</td>
<td>20s</td>
<td>25.3s</td>
<td>23.7s</td>
</tr>
</tbody>
</table>

Figure 49. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 25.
Table 50. Enroute 2D+T-MOTO results for flight n. 25.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>151.1</td>
<td>147.7</td>
<td>147.7</td>
<td>157.5</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>30.47</td>
<td>30.55</td>
<td>30.55</td>
<td>31.07</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>5.59</td>
<td>5.57</td>
<td>5.57</td>
<td>5.72</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.82</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>201.73</td>
<td>205.92</td>
<td>205.89</td>
<td>202.59</td>
</tr>
</tbody>
</table>

Figure 50. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 25.
Flight 26

Flight n. 26 is an eastbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 51. The original and optimal trajectories are plotted in Figure 51, and the associated performance indicators are detailed in Table 52. The fuel-time Pareto diagram of the results is given in Figure 52.

Table 51. Details of flight n. 26.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>GREY19</th>
<th>Type</th>
<th>C17 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XCCS</td>
<td>Destination Airport</td>
<td>XESG</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>330</td>
<td>Activation</td>
<td>00:10</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>CS MWV ROWLY RASTA GUEST GOTIM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>21.4s</td>
<td>20.6s</td>
<td>26.6s</td>
</tr>
</tbody>
</table>

Figure 51. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 26.
Table 52. Enroute 2D+T-MOTO results for flight n. 26.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>152.5</td>
<td>154</td>
<td>146.4</td>
<td>173.8</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>44.31</td>
<td>45.44</td>
<td>44.98</td>
<td>45.95</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>8.46</td>
<td>8.62</td>
<td>8.4</td>
<td>9.24</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.17</td>
<td>1.20</td>
<td>1.19</td>
<td>1.23</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>266.07</td>
<td>276.33</td>
<td>283.13</td>
<td>256.19</td>
</tr>
</tbody>
</table>

Figure 52. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 26.
Flight 27

Flight n. 27 is an eastbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 53. The original and optimal trajectories are plotted in Figure 53, and the associated performance indicators are detailed in Table 54. The fuel-time Pareto diagram of the results is given in Figure 54.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>JAL083</th>
<th>Type</th>
<th>B773 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSTP</td>
<td>Destination Airport</td>
<td>XNEB</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>340</td>
<td>Activation</td>
<td>00:01</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>APU BESTO BULAN N88 BISON</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)

|                | 31s   | 20.5s  | 28.4s  |

Figure 53. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 27.
Table 54. Enroute 2D+T-MOTO results for flight n. 27.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>238.5</td>
<td>260.7</td>
<td>251.6</td>
<td>276.9</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.55</td>
<td>36.55</td>
<td>36.72</td>
<td>35.94</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>106.02</td>
<td>115.51</td>
<td>116.04</td>
<td>113.57</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.79</td>
<td>3.04</td>
<td>3.06</td>
<td>3</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>708.21</td>
<td>770.3</td>
<td>792.59</td>
<td>727.93</td>
</tr>
</tbody>
</table>

Figure 54. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 27.
Flight 29

Flight n. 29 is an eastbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 55. The original and optimal trajectories are plotted in Figure 55, and the associated performance indicators are detailed in Table 56. The fuel-time Pareto diagram of the results is given in Figure 56.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>QFA166</th>
<th>Type</th>
<th>A388 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XNMU</td>
<td>Destination Airport</td>
<td>XSMU</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>370</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>NANNA NEV30 NEVER M521 NIVEN</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 55. Details of flight n. 29.

2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)

|                     | 23.2s | 22.8s | 22.8s |

Figure 55. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 29.
Table 56. Enroute 2D+T-MOTO results for flight n. 29.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>126.1</td>
<td>135.1</td>
<td>133.2</td>
<td>136.8</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>29.51</td>
<td>31.6</td>
<td>31.17</td>
<td>32.02</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>93.25</td>
<td>99.87</td>
<td>98.51</td>
<td>101.18</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>16.97</td>
<td>18.17</td>
<td>17.92</td>
<td>18.41</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.46</td>
<td>2.63</td>
<td>2.59</td>
<td>2.66</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>634.13</td>
<td>679.12</td>
<td>669.86</td>
<td>688.05</td>
</tr>
</tbody>
</table>

Figure 56. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 29.
Flight 30

Flight n. 30 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 57. The original and optimal trajectories are plotted in Figure 57, and the associated performance indicators are detailed in Table 58. The fuel-time Pareto diagram of the results is given in Figure 58.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>PYT501</th>
<th>Type</th>
<th>A388 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XENM</td>
<td>Destination Airport</td>
<td>TSKH</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>380</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MAXIM 3220N15500E 2950N15000E 2540N14000E GROTY GANDA SKP KHV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>58.5s</td>
<td>53.6s</td>
<td>67s</td>
</tr>
</tbody>
</table>

Figure 57. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 30.
Table 58. Enroute 2D+T-MOTO results for flight n. 30.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>314.3</td>
<td>284.6</td>
<td>284.6</td>
<td>290</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>72.21</td>
<td>66.36</td>
<td>66.36</td>
<td>65.13</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>228.18</td>
<td>209.71</td>
<td>209.71</td>
<td>205.82</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>41.19</td>
<td>37.8</td>
<td>37.8</td>
<td>37.25</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>6</td>
<td>5.52</td>
<td>5.52</td>
<td>5.42</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>1607.11</td>
<td>1489.84</td>
<td>1489.84</td>
<td>1432.94</td>
</tr>
</tbody>
</table>

Figure 58. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 30.
Flight 32

Flight n. 32 is an eastbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 59. The original and optimal trajectories are plotted in Figure 59, and the associated performance indicators are detailed in Table 60. The fuel-time Pareto diagram of the results is given in Figure 60.

Table 59. Details of flight n. 32.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>PYT932</th>
<th>Type</th>
<th>B744 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSKH</td>
<td>Destination Airport</td>
<td>XNEB</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>370</td>
<td>Activation</td>
<td>00:00</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>ARLEN</td>
<td>CIVET TIGER</td>
<td>2613N13000E 2940N14000E 3250N15000E 4500N15500E</td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>113.9s</td>
<td>112.8s</td>
<td>87.5s</td>
</tr>
</tbody>
</table>
Table 60. Enroute 2D+T-MOTO results for flight n. 32.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>292.2</td>
<td>313</td>
<td>310.4</td>
<td>341.8</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>50.74</td>
<td>55.92</td>
<td>55.46</td>
<td>56.73</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>160.34</td>
<td>176.69</td>
<td>175.25</td>
<td>179.28</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>30</td>
<td>32.8</td>
<td>32.53</td>
<td>33.96</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>4.24</td>
<td>4.66</td>
<td>4.63</td>
<td>4.74</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>1004.68</td>
<td>1127.83</td>
<td>1118.63</td>
<td>1093.58</td>
</tr>
</tbody>
</table>

Figure 60. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 32.
Flight 34

Flight n. 34 is a westbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 61. The original and optimal trajectories are plotted in Figure 61, and the associated performance indicators are detailed in Table 62. The fuel-time Pareto diagram of the results is given in Figure 62.

Table 61. Details of flight n. 34.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>THJLG</th>
<th>Type</th>
<th>AT72 (turboprop)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XCMW</td>
<td>Destination Airport</td>
<td>TSTP</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>220</td>
<td>Activation</td>
<td>00:50</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MWV CAS CRAIG L601 BULAN APU</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>13.3s</td>
<td>14.4s</td>
<td>12.5s</td>
</tr>
</tbody>
</table>

Figure 61. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 34.
Table 62. Enroute 2D+T-MOTO results for flight n. 34.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>323.9</td>
<td>377.6</td>
<td>259.6</td>
<td>377.6</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>5.15</td>
<td>3.27</td>
<td>5.01</td>
<td>3.27</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>16.26</td>
<td>10.32</td>
<td>15.84</td>
<td>10.32</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>2.91</td>
<td>2.02</td>
<td>2.8</td>
<td>2.02</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>0.43</td>
<td>0.27</td>
<td>0.42</td>
<td>0.27</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>122.11</td>
<td>59.61</td>
<td>133.67</td>
<td>59.6</td>
</tr>
</tbody>
</table>

Figure 62. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 34.
Flight 35
Flight n. 35 is a westbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 63. The original and optimal trajectories are plotted in Figure 63, and the associated performance indicators are detailed in Table 64. The fuel-time Pareto diagram of the results is given in Figure 64.

Table 63. Details of flight n. 35.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>FDX076</th>
<th>Type</th>
<th>MD11 (Jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>XCMO</td>
<td>Destination Airport</td>
<td>XNMN</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>360</td>
<td>Activation</td>
<td>01:15</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>MO RIVER R925 NEVER</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>4.3s</td>
<td>4.1s</td>
<td>4.3s</td>
</tr>
</tbody>
</table>

Figure 63. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 35.
A Novel Air Traffic Management Decision Support System

Multi-objective 4-dimensional trajectory optimisation for intent-based operations in dynamic airspace

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Table 64. Enroute 2D+T-MOTO results for flight n. 35.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>110.8</td>
<td>110.2</td>
<td>110.2</td>
<td>110.2</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>15.01</td>
<td>14.91</td>
<td>14.91</td>
<td>14.91</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>47.42</td>
<td>47.12</td>
<td>47.12</td>
<td>47.12</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>8.57</td>
<td>8.51</td>
<td>8.51</td>
<td>8.51</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.25</td>
<td>1.24</td>
<td>1.24</td>
<td>1.24</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>332.51</td>
<td>330.46</td>
<td>330.46</td>
<td>330.46</td>
</tr>
</tbody>
</table>

Figure 64. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 35.
Flight 36
Flight n. 36 is an eastbound traffic whose original route is only marginally affected by the wind field variations associated with the tropical revolving storm. The details of the flight are given in Table 65. The original and optimal trajectories are plotted in Figure 65, and the associated performance indicators are detailed in Table 66. The fuel-time Pareto diagram of the results is given in Figure 66.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>GREY23</th>
<th>Type</th>
<th>C17 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure</td>
<td>XCCS</td>
<td>Destination Airport</td>
<td>XESG</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>330</td>
<td>Activation</td>
<td>00:20</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>CS MWV ROWLY RASTA GUEST GOTIM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time</td>
<td>22.9s</td>
<td>21.9s</td>
<td>27.2s</td>
</tr>
</tbody>
</table>

Figure 65. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 36.
Table 66. Enroute 2D+T-MOTO results for flight n. 36.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>152.5</td>
<td>154</td>
<td>146.4</td>
<td>173.8</td>
</tr>
<tr>
<td>CO$_2$ [t]</td>
<td>44.31</td>
<td>45.44</td>
<td>44.98</td>
<td>45.95</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>8.46</td>
<td>8.62</td>
<td>8.4</td>
<td>9.24</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.17</td>
<td>1.2</td>
<td>1.19</td>
<td>1.23</td>
</tr>
<tr>
<td>NO$_x$ [kg]</td>
<td>266.07</td>
<td>276.33</td>
<td>283.13</td>
<td>256.19</td>
</tr>
</tbody>
</table>

Figure 66. Fuel-time Pareto diagram of original and optimised trajectories of flight n. 36.
Flight 37

Flight n. 37 is an eastbound traffic whose original route is obstructed by the tropical revolving storm passage. The details of the flight are given in Table 67. The original and optimal trajectories are plotted in Figure 67, and the associated performance indicators are detailed in Table 68. The fuel-time Pareto diagram of the results is given in Figure 68.

Table 67. Details of flight n. 37.

<table>
<thead>
<tr>
<th>Callsign</th>
<th>GREY57</th>
<th>Type</th>
<th>C17 (jet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Airport</td>
<td>TSBB</td>
<td>Destination Airport</td>
<td>XCMW</td>
</tr>
<tr>
<td>Cruise FL</td>
<td>330</td>
<td>Activation</td>
<td>00:05</td>
</tr>
<tr>
<td>Flight Plan</td>
<td>BB 3227N12400E NANCY 2932N13457E CRAIG CAS MWV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D+T MOTO Calculation Time (Min Fuel / Time / Emis.)</td>
<td>13.6s</td>
<td>19.7s</td>
<td>18.2s</td>
</tr>
</tbody>
</table>

Figure 67. Results of the enroute 2D+T-MOTO optimisation performed for flight n. 37.
**Table 68.** Enroute 2D+T-MOTO results for flight n. 37.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. Fuel</th>
<th>Min. Time</th>
<th>Min. Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>179.3</td>
<td>189.2</td>
<td>190</td>
<td>213.3</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>52.09</td>
<td>57.29</td>
<td>58.39</td>
<td>56.85</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>9.95</td>
<td>10.75</td>
<td>10.9</td>
<td>11.38</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>1.38</td>
<td>1.51</td>
<td>1.54</td>
<td>1.52</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>312.8</td>
<td>356.08</td>
<td>367.55</td>
<td>318.86</td>
</tr>
</tbody>
</table>

**Figure 68.** Fuel-time Pareto diagram of original and optimised trajectories of flight n. 37.
Further en-route case studies

The further simulation analyses presented in this section were carried out to study the errors introduced by the transit of the tropical revolving storm across the trajectory. In particular, the behaviour of the optimisation algorithm was assessed with respect to an increased number of route points. As it can be inferred by the results, the shifts are diminished in magnitude but do not disappear entirely.

**Flight 7**

The minimum emissions avoidance trajectory of flight n. 7 was affected by macroscopic distortions. The original and the updated optimal trajectories are plotted in Figure 69, and the associated performance indicators are detailed in Table 69. The fuel-time Pareto diagram of the updated results is given in Figure 70.

---

![Figure 69. Results of the updated enroute 2D+T-MOTO optimisation performed for flight n. 7.](image-url)
Table 69. Updated enroute 2D+T-MOTO results for flight n. 7.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. CO</th>
<th>Min. HC</th>
<th>Min. NOx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>284</td>
<td>297.6</td>
<td>307.2</td>
<td>362</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>51.06</td>
<td>56.14</td>
<td>55.35</td>
<td>59.0</td>
</tr>
<tr>
<td>CO$_2$ [t]</td>
<td>161.33</td>
<td>177.4</td>
<td>174.91</td>
<td>186.44</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>31.08</td>
<td>33.55</td>
<td>33.66</td>
<td>37.51</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>4.28</td>
<td>4.69</td>
<td>4.64</td>
<td>4.98</td>
</tr>
<tr>
<td>NOx [kg]</td>
<td>954.91</td>
<td>1085.72</td>
<td>1037.15</td>
<td>1039</td>
</tr>
</tbody>
</table>

Figure 70. Fuel-time Pareto diagram of original and updated optimised trajectories of flight n. 7.
Flight 9

The minimum emissions avoidance trajectory of flight n. 9 was affected by macroscopic distortions. The original and updated optimal trajectories are plotted in Figure 71, and the associated performance indicators are detailed in Table 70. The fuel-time Pareto diagram of the updated results is given in Figure 72.

Figure 71. Results of the updated enroute 2D+T-MOTO optimisation performed for flight n. 9.
Table 70. Updated enroute 2D+T-MOTO results for flight n. 9.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. CO</th>
<th>Min. HC</th>
<th>Min. NOₓ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>240.3</td>
<td>251.9</td>
<td>262.3</td>
<td>285.5</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.92</td>
<td>37.17</td>
<td>36.77</td>
<td>36.69</td>
</tr>
<tr>
<td>CO₂ [t]</td>
<td>107.2</td>
<td>117.44</td>
<td>116.19</td>
<td>115.93</td>
</tr>
<tr>
<td>CO [kg]</td>
<td>19.8</td>
<td>21.5</td>
<td>21.49</td>
<td>21.85</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.83</td>
<td>3.1</td>
<td>3.07</td>
<td>3.07</td>
</tr>
<tr>
<td>NOₓ [kg]</td>
<td>693.76</td>
<td>780.99</td>
<td>748.76</td>
<td>714.39</td>
</tr>
</tbody>
</table>

Figure 72. Fuel-time Pareto diagram of original and updated optimised trajectories of flight n. 9.
Flight 11

The minimum emissions avoidance trajectory of flight n. 11 was affected by macroscopic distortions. The original and updated optimal trajectories are plotted in Figure 73, and the associated performance indicators are detailed in Table 71. The fuel-time Pareto diagram of the results is given in Figure 74.

Figure 73. Results of the updated enroute 2D+T-MOTO optimisation performed for flight n. 11.
Table 71. Updated enroute 2D+T-MOTO results for flight n. 11.

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Min. CO</th>
<th>Min. HC</th>
<th>Min. NO\textsubscript{X}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel Time [min]</td>
<td>240.3</td>
<td>252.1</td>
<td>262.6</td>
<td>285.9</td>
</tr>
<tr>
<td>Fuel [t]</td>
<td>33.92</td>
<td>37.2</td>
<td>36.83</td>
<td>36.74</td>
</tr>
<tr>
<td>CO\textsubscript{2} [t]</td>
<td>107.2</td>
<td>117.55</td>
<td>116.39</td>
<td>116.08</td>
</tr>
<tr>
<td>HC [kg]</td>
<td>2.83</td>
<td>3.1</td>
<td>3.07</td>
<td>3.07</td>
</tr>
<tr>
<td>NO\textsubscript{X} [kg]</td>
<td>693.76</td>
<td>781.73</td>
<td>750.25</td>
<td>715.41</td>
</tr>
</tbody>
</table>

Figure 74. Fuel-time Pareto diagram of original and updated optimised trajectories of flight n. 11.
Appendix B  List of relevant publications

Book chapters


Journal articles


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Refereed conference proceedings


**Technical R&D Reports (refereed)**


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Appendix C   Full list of references


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