Analysis and Prediction of Tram Track Degradation

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed. I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

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Abstract

Transportation is the means to carry people and goods from one place to another, and has been very important in each stage of human civilization. Therefore, engineers have developed the transportation network day-by-day, aiming to provide for people’s comfort, needs and safety in the most sustainable way possible.

In the past, transport organisations generally concentrated on the construction and expansion of transport networks, but they have gradually moved from a focus on expansion to intelligently maintaining the existing assets in recent years. For this reason, degradation models have been developed in many transport management systems, with the aim of assisting track maintenance planning and reducing the costs of asset management.

Melbourne has the largest operating tram network in the world with 250 kilometers of double track (Yarra Trams, 2017a). Melbourne’s tram network is operated by the Yarra Trams organisation under franchise from the government of Victoria, Australia (Yarra Trams, 2017a). Yarra Trams organises the news, maps, timetables, service changes, real-time tram arrival information, and the construction and maintenance of the tram infrastructure.

Many variables are involved in ensuring that Melbourne tram system operates to safe and best practice standards. One of the main elements influencing the tram system is the track infrastructure. The condition of the track infrastructure affects network operations either directly or indirectly. In order to keep the track infrastructure in its best condition over the longest possible time period, a maintenance plan is required. This plan is essential for such a large network as it can help in recovering the serviceability of tram tracks from faults and damage and prevent further wear of the tracks.

Currently, manual inspections are still used to identify track maintenance activities across the network. These inspections identify the status of the tram tracks, whether the tracks need maintenance, the required level of maintenance and the time period needed to maintain the damaged tracks. Since the inspections are done by a number of maintenance teams, human errors are likely to occur. In addition, inaccurate prediction of the maintenance time frame and mistakes in the inspection and detection of track

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defects may occur. Therefore, prioritisation of the maintenance activities is a substantial challenge. High maintenance and operational costs may be the result of poorly planned maintenance schedules. In other words, very early or late maintenance of the tram tracks are very costly, as are unnecessary maintenance or replacement of tracks.

In order to solve this problem, this research investigates degradation models for tram tracks in Melbourne. The models are rigorously reviewed in order to determine the most appropriate model in terms of sustainability, safety, accuracy and long-term behaviour. A time-series stochastic model is developed using MATLAB software to predict the degradation of tram tracks. A regression model is also developed using SPSS software for comparison with the time-series model. The models are developed for straight and curves sections of the tram network.

The models were developed after analysing tram track variables over a period of time to find the relationship between the variables and track degradation. The variables include asset data variables (such as construction material, track surface, rail profile) and operational variables (such as annual rail usage, number of trips, route location). In this research, the annual rail usage (in million gross tons (MGTs)) is found to be the main variable affecting rail degradation using the gauge parameter of rails for curves and straight sections of the tram network.

Based on the developed prediction models, the maintenance activities of degraded rail tracks are identified within a specified time period. This will help to reduce the maintenance costs, save time and prevent occasional unnecessary maintenance activities. In addition, it will reduce interruptions to traffic and delays experienced by passengers.
List of Publications

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Chapter 1
Introduction

1.1 Background

In the past decades, railway infrastructure has been the main focus of transport organisations, representing the backbone of economic growth and huge financial investment. In addition to a focus on the construction and expansion of networks, transportation engineers have recently been working on maintaining the existing assets to meet the public and transport organisations demands with limited budgets. Therefore, the concept of asset management has become the main goal of transport organisations. It can be a device of communication between transport users, stakeholders, government officials and decision makers. This concept refers to the development, maintenance, and upgrading strategies of transport assets, which result in various benefits, such as long-term cost reduction, and safe, convenient and comfortable transport systems.

Some recent research areas have been the main interest of railway infrastructure managers, including maintenance planning, and the optimisation of decision-making and transport asset management systems. One of the main concerns is the degradation of urban rail tracks across transportation networks. Although the degradation process of rail tracks is usually very slow, it can lead to high-risk defects and failures with enormous financial costs. Therefore, the prediction of railway track geometry degradation is vital for the planning of maintenance and renewal actions, and is relevant in decision-support systems.

Maintenance and renewal activities are regularly applied on urban rail tracks of transportation networks to ensure the safety and continued operation of the rail system. Regular inspections of the track should be made before it degrades beyond acceptable limits. This will help to avoid unplanned maintenance activities, which are usually expensive and may cause low service quality.

Hence, the study of predictive degradation models is an important component of rail infrastructure. Degradation models can be applied on light and heavy rails. Light rails are different from heavy rails. Light rails are the lightweight passenger rail cars
operating singly (or in short, usually two-car, trains) on fixed rails in right-of-way that is not separated from other traffic for much of the way. Light rail vehicles are typically driven electrically with power being drawn from an overhead electric line via a trolley or a pantograph, also known as tramway or trolley car (American Public Transportation Association, 2014). However, heavy rails are high-speed, passenger rail cars operating singly or in trains of two or more cars on fixed rails in separate rights-of-way from which all other vehicular and foot traffic are excluded, also known as rapid rail or metropolitan railway (metro) (American Public Transportation Association, 2014). This research investigates the prediction of the degradation of light rails of Melbourne tram network. In turn, this research will enhance the optimisation of maintenance needs and improves track conditions for different degradation components, such as rail track loads, time, speed of vehicles and other variables. It will also help in scheduling rail tracks for maintenance operations before failure and within a specified timeframe. Therefore, this will decrease the impact on transportation services and the potential costs of maintenance activities and other factors.

1.2 Research Scope

This research reviews the current knowledge and contributes to research efforts of rail track degradation models. The main focus is to understand the behaviour of each model, taking into consideration the variables and factors. This research presents models to predict the tram track degradation of Melbourne network based on its influencing factors. It improves decision-making on degradation models in order to optimise and prioritise maintenance practices and minimise track defects.

1.3 Research Questions

This research addresses the following questions:

1. What are the variables/variables influencing rail track degradation?
2. How can rail degradation be modelled by incorporating these variables?
3. What is the most appropriate model for the Melbourne tram track degradation prediction?
4. What future rail track conditions are predicted from this study to be used for optimisation and prioritisation of rail maintenance?

1.4 Research Aim and Objectives

The broad aim of this research is to analyse and predict rail track degradation, particularly in Melbourne. In order to achieve this, a brief plan and a comparison of various degradation models are provided. The models are reviewed in order to make the most appropriate decision on the best model in terms of its accuracy, sustainability and long-term behaviour.

Consistent with this broad aim, the following objectives are established for this research and answer the addressed research questions (in Section 1.3):

- Critically assess and compare various rail degradation models proposed by researchers based on the variables influencing rail track degradation and the accuracy of the prediction results (Answers Question 3 in section 1.3) (see Chapter 2 for details).
- Analyse the factors influencing the tram track degradation of Melbourne network (Answers Question 1 in section 1.3) (see Chapter 4 for details).
- Develop two degradation models for Melbourne tram tracks based on existing studies as a function of the influencing variables and using a comprehensive field survey of the rail network (Answers Question 2 in section 1.3) (see Chapter 5 for details).
- Evaluate the accuracy and complexity of the developed degradation models in estimating the degradation of rail tracks (Answers Question 4 in section 1.3) (see Chapter 5 for details).
1.5 Thesis Structure

The thesis is structured to follow each stage of the research in the order in which it is undertaken. This thesis consists of six chapters, followed by the references and appendices, as summarised below:

![Flowchart of thesis structure](image)

**Figure 1.1: Research structure flowchart.**

Following this introduction, **Chapter 2** reviews in general the railway track system and provides some background knowledge on the structure of railway tracks and rail degradation. The chapter also reviews past studies on rail track degradation prediction models, discusses their major limitations, and concludes with a summary including the gaps in current knowledge to be filled by the research study.

**Chapter 3** presents an introduction to the study framework. This chapter describes the type and classification of datasets, addressing the sources of the data and how they were collected. The methodology of thesis is also discussed in this chapter. A comprehensive description of stochastic time-dependent model of the degradation of rail tracks follows.
Chapter 4 analyses the variables influencing railway tracks, using an appropriate analytical technique to evaluate the degradation of rail tracks. The most influential variables on tram track degradation are determined over time period using SPSS software.

Chapter 5 describes the detailed development of a stochastic time series model and a linear regression predictive degradation model using the variables affecting railroad tracks. A time series model is developed for the first time in rail degradation prediction. This chapter analyses the modelling of tracks for particular site sections of Melbourne railways. The degradation models are separately developed for curves and straight sections due to the variation of the track segments along the route. The chapter also discusses and explains the results of the proposed degradation models. The application of the degradation models can optimise and prioritise the maintenance activities of degraded rails and assist in minimising the costs, prevent unnecessary maintenance actions and save time.

Chapter 6 summarises the findings of this research. It also outlines some recommendations for future work to extend this research in order to benefit the railway industry.
Chapter 2
Review of Rail Track Degradation Models

2.1 Introduction

This chapter presents background information on the structure of rail track and its degradation. A general review of previous research on different degradation models in railways and other engineering fields is also presented for comparison. The aim of the chapter is to briefly determine the contributions of rail track degradation models and identify the gaps and the limitations of existing studies. This will help in making the decision on the most suitable and applicable degradation model for the present research study.

2.2 Overview of Railway Track

The core of the research of this thesis is the behaviour of rail tracks in terms of degradation modelling. To help understand this dynamic behaviour, it is essential first to provide an overview of the railway structure and rail degradation. In the next sections, rail track structure is briefly discussed, followed by a description of rail degradation and an overview of the degradation models developed to date and their classification, based on past research papers and studies.

2.2.1 Railway Track Structure

A railway is a track where the vehicle moves along two parallel rails. These rails support the wheels of the vehicles, which are usually trams or trains. The structure of a rail track is divided into two components:

5. Superstructure (top of the track),
6. Substructure (below the track).
The superstructure consists of the rails, the fastening system, rail pads and the sleepers, while the substructure consists of the ballast, sub-ballast and the subgrade (Hawari, 2007).

The most common types of rail track are traditional ballasted track and concrete slab (ballast-less) track (Esveld, 2001). On traditional ballasted tracks, the rail is fixed onto a wooden or concrete sleeper. The sleeper rests on a sheet of ballast that distributes the loading to the subgrade. The top ballast is positioned between the sleepers and on the shoulders to maintain longitudinal and lateral stability (Lyngby et al., 2008). Hence, routine maintenance of the rail track moving under loading forces is always required in order to restore the line and the level and clean or replace the ballast (Al-Douri et al., 2016). Figure 2.1 below shows the components of railway track structure.

![Cross-section of typical railway track](image)

**Figure 2.1: Cross-section of typical railway track (Hawari, 2007).**

### 2.2.2 Railway Track Degradation

Rail degradation is a failure process leading to a rail defect (fault). Various studies on rail degradation have been performed by a number of researchers (c.f. Zhang et al., 2000; Jovanovic, 2004; Zarembiski et al., 2005; Chang et al., 2010; Liu et al., 2010; He et al., 2015; Jovanovic et al., 2015; Soleimanmeigouni et al., 2016). There is a need to reduce rail degradation and predict rail failure in order to develop an effective rail maintenance strategy (Ferreira and Murray, 1997; Lovett et al., 2013; Ferreira and López-Pita, 2015).
A number of other factors contribute to the degradation of rail tracks. For instance, rail degrades due to wear and fatigue, which are greatly affected by the load applied on the rail track (Larsson, 2004). Larsson (2004) found that there is a strong relationship between rail track curvature and rail degradation. Narrow curves imply wear, while tangent track implies fatigue, as shown in Figure 2.2 (Lyngby et al., 2008).

![Figure 2.2: Wear and fatigue mechanisms as functions of curve radii (Lyngby et al., 2008).](image)

Other factors may also result in the degradation of rail tracks, such as the condition of assets (i.e. sleepers, fastenings and ballast) (Lichtberger, 2005; Lezin Seba et al., 2012), age of rails and axle load (Esveld, 2001; Fröhling, 2007), speed (Fröhling, 2007), traffic density (Larsson, 2004; Corshammar, 2005), traffic type and rail-wheel interaction (Knothe and Grassie, 1993; Fröhling, 2007), Million gross tons (MGTs) (Esveld, 2001), track curvature (Fröhling, 2007; Lyngby et al., 2008), rail size and rail profile (Kawaguchi et al., 2005), rail track construction (Esveld, 2001), rail track elevation and roughness (Hawari and Murray, 2008), rail track super-elevation and rail welding (Fröhling, 2007) and rail lubrication (Wilson, 2006).
2.3 A Classification Scheme for Track Degradation Models

Thus far, we have described the railway track structure and its degradation briefly. However, in the realm of modelling, it may be helpful to consider the railway track from the point of view of reliability. This section briefly discusses degradation models proposed in previous research studies.

Degradation models are mainly developed using track inspection data in order to predict future track conditions and provide information for the planning of maintenance and track behaviour. Various models of track degradation have established a variety of outcomes. There are four general approaches to rail track degradation modelling: mechanistic, statistical (empirical), mechanical-empirical and artificial intelligence models. Figure 2.3 shows a classification of rail track degradation models.

![Classification of rail track degradation models](image)

**Figure 2.3:** Classification of rail track degradation models.
2.3.1 Mechanistic Models

Mechanistic models are based on fundamental theories of modelling behaviour. In other words, they are based, by theory or testing, on the mechanical properties of track components. Mechanistic models cover the calculation of forces and stresses in order to assess the degradation of the rail (Zhang et al., 2000).

Various studies have proposed mechanistic models based on records and data measured on rail tracks to explain the degradation process. In this section, two sub-models are presented:

1. Models based on Japanese studies (Satoh, 1959; Satoh et al., 1961; Sato, 1995; Yousefikia et al., 2014).
2. Models based on Austrian studies analysing the development of track quality from a passenger’s point of view (Hummitzsch, 2005).

The Japanese railway companies established a relationship of the settlement of railway ballast according to cyclic loading (Sato, 1995). The following equation was applied for assessing the track deformation ($y$) of the heavy-haul narrow gauge and the high standard gauge:

$$ y = \gamma \left( 1 - e^{-\alpha x} \right) + \beta x \quad (2.1) $$

where:

- $x$ represents the repeated number of loadings or tonnage carried by the track,
- $\alpha$ is the vertical acceleration required to initiate slip and can be measured using spring-loaded plates of the ballast material on a vibrating table,
- $\beta$ is a coefficient proportional to the sleeper pressure and peak acceleration experienced by ballast characteristics and presence of water,
- $\gamma$ is a constant dependent on the initial packing of the ballast material.

Sato (1995) found that traffic, time, track condition and humidity are the most important variables in the mechanism of rail track degradation.
In another study, TU Graz studied settlement developments in Austria based on a quality index, which represents accelerations in the vehicle caused by track irregularities (Lyngby et al., 2008). This index comprises both horizontal and vertical deviations in rails together with a lack of super-elevation and speed. An exponential development of track quality index over time was found, giving the following expression for track quality:

\[
Q = Q_0 \times e^{-b \times t}
\]  

(2.2)

where:

- \(Q\) is the track quality index,
- \(Q_0\) is the initial track quality,
- \(b\) is a constant.

The exponential structure of the Austrian model shows that the increase of the roughness of the rail tracks leads to more dynamic forces while trains pass. These forces cause deformation of the track geometry, which increases the variations of the train/track interaction forces and speeds up the track degradation process.

### 2.3.2 Statistical (Empirical) Models

Statistical models have been widely used in degradation prediction studies. They are based on observations of rail track performance and the variables influencing it, including traffic, track components, and maintenance variables. These models provide a method of simulating real-life situations with mathematical equations to forecast the future behaviour of rail track and its degradation. They can be explained under three different types: deterministic, probabilistic and stochastic. Probabilistic models are categorized into three sub-models: continuous probability distributions, Bayesian models and Markov models. Figure 2.4 shows a classification of statistical degradation models.
2.3.2.1 Deterministic Models

Deterministic models are usually developed through experimentation where the performance of the rail track is related to the factors influencing its degradation. This approach requires a relatively large number of variables, including train speed, geometry and operations of the rail (i.e. axle weight, line speed and traffic volume) (Audley and Andrews, 2013) and accumulated tonnage (in MGTs) (Esveld, 2001; Zwanenburg, 2009; Guler et al., 2011). Linear and exponential forms of deterministic models were the first attempt in rail degradation modelling, due to their simplicity of mathematical expression and their ability to show a direct relationship between the input and output variables (Hasan, 2015). Therefore, these models predict the condition of rail and its degradation deterministically by ignoring random errors in prediction.

The Office for Research and Experiments (ORE) of the International Union of Railways (UIC) investigated the fundamentals of the degradation mechanism of railroad track (Dahlberg, 2001). A deterministic ORE model was proposed to estimate rail degradation according to various studies (Corbin and Kaufman, 1975; Subramanian and Kumar, 1978; ORE Question, 1998; Shafahi and Hakhamaneshi, 2009). Accordingly, traffic
volume, dynamic axles and speed were identified as the most important variables in influencing rail track degradation. The structure of the ORE deterministic model is as follows:

\[ e = e_0 + h T^\alpha (2Q)^\beta v^\gamma \]  \hspace{1cm} (2.3)

where:
- \( e_0 \) is the degradation directly after tamping,
- \( h \) is a constant,
- \( T \) is the traffic volume,
- \( Q \) is the dynamic axle,
- \( v \) is the speed,
- \( \alpha, \beta, \) and \( \gamma \) are the variables estimated from experimental data.

The ORE model has also been analysed based on data obtained from American and Indian railways by Corbin and Kaufman (1975) and Subramanian and Kumar (1978), respectively. These studies explain why the power spectral density form of representation of track irregularities is necessary and illustrate how this valuable tool can be applied to railway track geometry data to assist in the understanding and management of the permanent way.

From the deterministic viewpoint, various studies have shown a linear relationship between track faults and accumulated tonnage (Esveld, 2001; Zwanenburg, 2009; Guler et al., 2011). Accumulated tonnage (in million gross tons (MGTs)) is found based on the operating data provided by adding all the axle loads (in metric tons) of all trains that have run through the analysed section. Therefore, the linear relationship is as follows:

\[ \sigma = c_1 + c_0 T \]  \hspace{1cm} (2.4)

where:
- \( \sigma \) is the standard deviation of longitudinal-levelling faults (mm),
- \( c_1 \) is the initial standard deviation measured after renewal (mm),
- \( c_0 \) is the degradation rate (mm/MGT).
T is the accumulated tonnage between maintenance operations (MGT).

Although some researchers have found a non-linear relationship, such as the polynomial (Jovanovic, 2004), exponential (Veit, 2007) and multi-stage linear (Chang et al., 2010) models, the linear relationship is still commonly used based on many other studies, including those of Liu et al. (2010) and Andrade and Teixeira (2011).

2.3.2.2 Probabilistic Models

Although it is difficult to analyse the probability of rail track degradation due to various factors such as the environment, the structural materials and the construction quality, three probabilistic models are mainly discussed: continuous probability distributions (state-based), Bayesian models and Markov models (time-based). Figure 2.4 shows the classification of probabilistic degradation models.

- Continuous Probability Distributions (State-Based)

It is recommended for a continuous probability distribution model to be applied in a certain state within a known elapsed time since the last maintenance activity. A probability distribution model was carried out in a study of the Norwegian National Rail Administration (called Jernbaneverket (JBV)) (Podofillini et al., 2006). This model was developed to calculate the risks and costs following an inspection strategy; it also covers issues of the rail failure process using the actual inspection and maintenance procedures followed by the railway company (Podofillini et al., 2006). Accordingly, time, speed and rail route location are found to be variables most influencing rail degradation using a continuous probability distribution model. The model structure is shown in the following equation:

\[
E [D (\tau, \tau', t_w)] = f_I Q (\tau, \tau', t_w) \times Pr [\text{rail breakage } | \text{crack undetected}] \times Pr [\text{undetected breakage } | \text{rail breakage}] \times Pr [\text{derailment } | \text{undetected breakage}]
\]  

(2.5)
where:
\[ E[D(\tau, \tau', t_w)] \] is the expected number of derailments per year,
\( f_I \) is the yearly frequency of crack initiations,
\( Q(\tau, \tau', t_w) \) is the number of cracks missed by per year,
\( \tau, \tau', t_w \) are the variables directly related to rail derailment, including time, speed and route of rail.

Another track degradation model was proposed by Zio et al. (2007), describing the progression of defects in relation to the JBV (Podofillini et al., 2006; Zio et al., 2007). The railway network is modelled within a multi-state perspective in which each rail track section is analysed in different discrete states depending on the track degradation and its condition. The degradation model presents a state diagram of the defects (Figure 2.5). There are six sections in this diagram; the track condition \( h_j \) in each section \( j \) (\( j = 1, 2, \ldots, n \)) is discretised in \( \delta + 1 = 6 \) levels. Section level 5 corresponds to a section with zero defects (i.e. in perfect condition). The degradation levels \( h_j \) (\( h_j = 4, 3, 2, 1 \)) correspond to gradually critical track conditions. The section of level \( h_j = 0 \) corresponds to rail breakage (i.e. complete failure). The downward and upward arrows in the diagram shown in Figure 2.5 indicate the stochastic transitions of defect growth and repair, respectively (Zio et al., 2007). A study using this probability distribution model showed that the growth of defects (i.e. sorted as ‘high risk’ or ‘low risk’) depends on the expected times of failure. It also depends on the speed at which trains pass over the rail track.
Hierarchical Bayesian Models

Hierarchical Bayesian models (HBMs) are flexible statistical models that provide a general prediction of railway track geometry degradation (Andrade and Teixeira, 2015). HBMs allow the assessment of the relationship between different components of consecutive rail track sections, including the deterioration rates and the initial quality variables (Bernardo, 2003; Andrade and Teixeira, 2015). A HBM was developed for the main Portuguese railway line Lisbon-Oporto. It assesses two main quality indicators related to rail track geometry degradation: the standard deviation of longitudinal level defects (SDLL) and the standard deviation of horizontal alignment defects (SDHA) (Andrade and Teixeira, 2015). HBMs assume variables to be random variables, the uncertainty of which can be quantified by a prior distribution (Andrade and Teixeira, 2012; Andrade and Teixeira, 2013; Andrade and Teixeira, 2015).

This prior distribution $p(\theta)$ is combined with the traditional likelihood $p(y \mid \theta)$ to obtain the posterior distribution of the variables of interest (Andrade and Teixeira, 2015). The posterior distribution $p(\theta \mid y)$ of the parameter $\theta$ given the observed data $y$ can be computed according to Bayes' rule as:
\[ p(y) = \frac{p(y|\theta).p(\theta)}{\int p(y|\theta').p(\theta') d\theta' \propto p(y|\theta).p(\theta)} \]  \hspace{1cm} (2.6)

where:
- \( \theta \) is a random variable whose value to be estimated,
- \( y \) is a random variable the value or probability distribution of which is known,
- \( P(\theta | y) \) is posterior distribution of \( \theta \) given \( y \) which relates to \( \theta \) via a model,
- \( P(y | \theta) \) is the likelihood to observe \( y \) given unknown \( \theta \) or the sampling distribution of \( D \) given known \( \theta \),
- \( P(\theta) \) is prior probability of \( \theta \).

It was found that the calculation of the prior distribution is a very important step in every Bayesian model application. However, every case applying this model finds that the joint posterior distribution \( p(\theta | y) \) has a reasonably high dimension, and integration through numerical methods must rely on Markov Chain Monte Carlo (MCMC) methods, which are built in such a way that their stationary distribution is the desired posterior distribution (Bernardo, 2003; Turkman et al., 2003; Andrade and Teixeira, 2015; Ntzoufras, 2009).

Overall, the application of the HBM model to a sample of operational and maintenance data showed that the HBM exhibited a worse fit of the quality indicator \( SD_{HA} \) compared to the quality indicator \( SD_{LL} \), suggesting that horizontal alignment defects are less predictable (Andrade and Teixeira, 2015).

- **Markov Models (State-Based)**

Markov models are statistical models that analyse the infrastructure of the rail tracks at different condition levels over time. They also consider the hazard deterioration rate to assess the uncertainty of track degradation (Bai et al., 2015). Shafahi (2009) developed a Markov model calculating the Track Quality Index (TQI) in a range between 0 and 100 (mapped onto 5 states) in relation to track unevenness, twist, alignment and gauge.
measurements. The model structure consists of a transition matrix showing the probabilities of various states of rail tracks at any time, n, as follows:

\[
p(n) = (p(X(n) = 1), p(X(n) = 2), \& p(X(n) = 5))
\]

(2.7)

where:

X(n) is the track state at time n,
p(X(n) = j) is the probability that a track is in state j at time n.

Shafahi and Hakhamaneshi (2009) found that the Markov model appears to be superior to conventional regression models, such as the ORE model (Refer to Section 2.3.2.1). Although more studies and enhancement of the model are needed, the application of the Markov model for Iranian railways proved to be a reasonable method for the allocation of maintenance funds (Shafahi, 2009).

Lyngby (2008) proposed a 50-state Markov model for Norwegian railway tracks, analysing the twist on each section of track up to 50mm in order to estimate the failure rates for the railway line. Deterioration rates were also analysed in this model, depending on the geometry of the track section, whether it is straight, curve or transitional. The model also studied frequency optimisation between track geometrical sections. The Markov model covers a phase type distribution used for time to failure. The model consists of two degraded failure states, two critical failure states and an acceptable state, as shown below in Figure 2.6.

![Diagram of General Markov failure model](image.png)

Figure 2.6: General Markov failure model (Lyngby et al., 2008).

The degraded failure states are denoted as D₁ and D₂. D₁ represents minor degraded
cracks where observations are made more regularly so that critical degradation failure does not occur. However, D2 stands for larger cracks so that the failure is fixed directly. The critical failure states are denoted as $F_1$ and $F_2$. $F_1$ represents failures due to degradation, which can be fixed using preventive repair if they are found at an early inspection stage. However, $F_2$ implies shock failures, which can happen due to the application of large external forces on the rail. Moreover, Figure 2.6 shows that there is a constant rate $\lambda$ to experience shock failure $F_2$, if critical failures do not exist. Reaching critical failure $F_1$ requires the rail to reach degraded states $D_1$ and $D_2$. Therefore, the rail is divided into small partitions 1 m in length in order to have one partition falling in one of the states OK, $D_1$, $D_2$, $F_1$ or $F_2$.

Another Markov model was proposed by Prescott and Andrews (2013) for the degradation, inspection and maintenance of a single one eighth of a mile section of UK railway track. The model studies the changing deterioration rate and maintenance of the track section. It is also used to examine the effects of changing the level of track geometry degradation starting from the good condition of the rail until it reaches a critical value at which maintenance is needed.

Figure 2.7 shows 10 states of the Markov model of track section degradation (solid arrows) and inspection before the first tamp (dashed arrows). The shaded states indicate that the track condition is revealed, which usually happens at inspection every 28 days ($\theta$). However, in the period before inspection, the state of the track geometry may not reveal the true extent of the degradation, which may cause disorder in the level and condition of the track state (Prescott and Andrews, 2013). For instance, condition A of the track may be worse than condition K. In addition, $\lambda_{ij}$ indicates the rate of degradation, where $i$ shows the level of degradation (1: good to crit, 2: crit to spd, 3: spd to cls) and $j$ shows the maintenance history of the track section. ‘spd’ is the speed limit, ‘cls’ is the line closure and ‘crit’ is the critical degradation rate.
2.3.2.3 Stochastic Models

Stochastic models are usually statistical theories based on historical records and data (Andrews, 2012). They propose the distribution of time to degradation events and predict their performance. Such models are based on what has actually happened and account for variability through the use of probability, although by their nature, they do not provide insight into the underlying physics (Andrews, 2012).

Various research studies have developed stochastic models for rail degradation prediction. Rail characteristics (i.e. rail type, sleeper type), time and rail geometry (including tamping activities) are the main variables used in stochastic modelling. Yousefikia et al. (2014) proposed a review of stochastic models based on their data analysis of rail tracks along tram routes in Melbourne, Australia. From this study’s viewpoint, the rail track is considered to be regular when it carries out its function under
operating conditions for a certain period of time; if this is not the case, the rail track fails (Yousefikia et al., 2014). Hence, failure progress can be identified in several ways. The gamma process is the most common model used for failure progression and continuous time stochastic processes. Refer to Meier-Hirmer et al. (2009) for further details on systems with gamma deterioration activity.

Lyngby et al. (2008) analysed Markov and stochastic degradation models. They stated that rail track geometry could be displayed better as a stochastic model due to the observed variability (Lyngby et al., 2008). Other rail degradation prediction and maintenance planning researchers have developed stochastic modelling in their studies, including Mishalani and Madanat (2002), Ahmad and Kamaruddin (2012), Zakeri and Shahriari (2012), Vale and Ribeiro (2014), Andrews et al. (2014), and Ye and Xie (2015).

Another geometry stochastic deterioration model was developed by Guler et al. (2011), which analyses the effects that various track characteristics, environmental conditions, maintenance and renewal policies have on the deterioration of each of the track variables measured by recording vehicles. This study concluded that natural disasters such as flooding and falling rocks augment the rate of geometry deterioration, whilst snow and landslides have no influence (Guler et al., 2011). This study also found that the increase of curvature or gradient or line speed raises the rate of geometry deterioration. Therefore, this study showed that sleeper type and rail type (continuously welded rail (CWR) or jointed rail) have an effect on geometry deterioration, with CWRs deteriorating at a slower rate (Guler et al., 2011). However, the results of the study also showed that increasing the annual tonnage decreased the rates of deterioration. This invalidates the model, as it is widely known that increasing annual tonnage increases deterioration rates.

A stochastic model was also proposed by Quiroga and Schnieder (2011b), to investigate a heuristic-based method for tamping intervention scheduling, and they then developed it into a Monte Carlo simulation for the processes of track ageing and restoration. The model uses 20 years of track measurement train data obtained from the French railway operator SNCF (Quiroga and Schnieder, 2011b). This model does not consider measurements taken in the first three months after an intervention because it assumes
that all tracks experience a bedding-in process (Quiroga and Schnieder, 2011b). Therefore, only datasets for which the time between tamps is at least one full year are used to increase the accuracy of the model. This assumption reduces the applicability of the model to the UK network and a number of geometry deterioration processes. Therefore, the model hypothesis of this study includes two assumptions, as follows:

1. The degradation value $\text{NL}_{\text{init}}$ is achieved after the nth tamping intervention. It is defined as a log-normally distributed stochastic variable;

$$\text{N}\text{L}_{\text{init}} \sim \text{LN} \left( \mu_{\text{NL}_{\text{init}}} (n), \sigma^2_{\text{NL}_{\text{init}}} (n) \right)$$  \hspace{1cm} (2.8)

where:
- $\mu$ is the mean value,
- $\sigma^2$ is the variance.

2. The degradation value evolves between two tamping activities. It is defined by an exponential function of the form:

$$\text{NL}_{\text{init}} e^{b_n (t - t_n)} + \epsilon (t)$$  \hspace{1cm} (2.9)

where:
- $t$ is the time,
- $t_n$ is the time at which the last tamping activity took place,
- $b_n$ is a log-normally distributed stochastic variable,

$$b_n \sim \text{LN} \left( \mu_b (n), \sigma^2_b (n) \right)$$  \hspace{1cm} (2.10)

- $\epsilon_n(t)$ is a normally distributed variable with mean value 0,

$$\epsilon_n(t) \sim \text{N}(0, \sigma^2_\epsilon)$$  \hspace{1cm} (2.11)

Although it is assumed in this model that the rail track undergoes exponential deterioration, there is no evidence to substantiate the claim of an exponential deterioration pattern. Hence, plots of the SNCF rail network sections included in the paper do not show that the track geometry follows an exponential deterioration pattern.
In addition, Andrews (2012) developed a Petri net stochastic model for the degradation, inspection, maintenance and renewal of track sections. This model investigates the efficiency of the asset management process employed and predicts the state of the track geometry (Shang, 2014). Statistical distributions of times to given levels of geometry deterioration are included, considering the effects of maintenance on the rate of deterioration. However, a better understanding of the degradation process needs to be established to support the development of accurate models based on historical data, such as the effects of maintenance (Audley and Andrews, 2013).

Vale and Lurdes (2013) proposed a stochastic model of geometrical track degradation, adopting the Portuguese Railway Northern Line as a case study. Statistical and probabilistic analyses were performed for different vehicle speed groups, showing that the rate of degradation of the standard deviation of the longitudinal level is similar for both rails. Therefore, the rate of degradation of the longitudinal level has an asymmetric distribution with heavy tailedness, and can be described as follows:

\[ \gamma = \frac{\mu_3}{\sigma^3} \]  

(2.12)

where:

\(\gamma\) is the skewness of a random variable \(X\),
\(\mu\) is the third moment about the mean,
\(\sigma\) is the standard deviation of the variable.

It was found that the Dagum distribution, usually adopted to represent income distribution, fits very well the geometrical track degradation process for the Portuguese Railway Northern Line in terms of the longitudinal level (Vale and Lurdes, 2013). The Dagum distribution represents the model in the analysis of three variables of function \(F(x)\), which is defined by:

\[ F(x) = \left[ 1 + \left( \frac{x}{\beta} \right)^{-\alpha} \right]^{-k} , \quad x > 0 \]  

(2.13)
where:

\( \alpha, \beta \) and \( k \) are positive variables.

Parameter \( \beta \) is a scale parameter, while \( \alpha \) and \( k \) are shape variables.

- **Time Series Models**

The time series approach can be classified under stochastic models (Machiwal and Jha, 2012). A time series is a sequence of data points, typically consisting of consecutive measurements or observations of quantifiable variable(s), made over a time interval. The observations are usually consecutive and taken at regular intervals (days, months, years). Typical examples of time series are seen in many application areas such as economics (e.g. monthly data on unemployment), finance (e.g. daily exchange rates), and the environment (e.g. daily rainfall, air quality readings).

Stochastic models are used to model a time series without considering the physical nature of the time series (Box and Jenkins, 2015). Common stochastic models using time series approaches include pure random (or white noise) models, autoregressive (AR) models, moving average (MA) models, autoregressive moving average (ARMA) models, and autoregressive integrated moving average (ARIMA) models (Machiwal and Jha, 2012).

Time series models have been used in different studies and in different areas of engineering in order to predict future conditions based on existing time-dependent variables (see Zhang, 2003; Khashei and Bijari, 2011; Wijeweera et al., 2014).

A study in systems engineering of a degradation path modelling method based on time series analysis was proposed by Gao et al. (2012). The study was applied on a life test electric circuit in accordance with the sensitive variables. The sensitive variables were identified using a time series stationary test, and were the input voltage and the minimum cathode current for regulation. A time series ARIMA model was established for the degradation of sensitive variables, denoted by ARIMA \((p, d, q)\), and can be expressed as:
\[ \nabla^d X_{it} = Y_{it} = (\sum_{k=1}^{p} \phi_{ik} Y_{i(t-k)} + \sum_{l=1}^{q} \varphi_{il} R_{i(t-l)}) , \quad t=1,2,3,\ldots \]  \hfill (2.14)

where:

- \( i \) is the number of samples, \( \nabla^d \) expressed as a d-order difference,
- After d-order difference, sequence \( Y_{it} \) is a stationary series coming from sequence \( X_{it} \),
- \( p \) and \( q \) is the autoregressive order and the moving average order, respectively,
- \( \phi_1, \phi_2, \ldots, \phi_p \) are autoregressive coefficients,
- \( \varphi_1, \varphi_2, \ldots, \varphi_q \) are moving average coefficients,
- \( R_t \) is the irregular component.

The application of an ARIMA time series model fits well with the measured data, the prediction is accurate, and the relative error is small. In addition, the model shows a significant correlation between the sensitive variables.

Time series models have also been used in rail degradation prediction. A study was conducted by Grossoni et al. (2015) to assess the role of the longitudinal variability of vertical track stiffness in long-term degradation. A time series approach is presented to correlate track stiffness properties with track degradation.

The main track variables used in the model are:

- Rail section: 60E1;
- Rail pad vertical dynamic stiffness
- Vertical support stiffness
- Sleeper mass
- Sleeper spacing

From a mathematical point of view, a time series \( \{Y_t\} \) is said to follow an ARIMA model if the \( d^{\text{th}} \) difference \( W_t = \nabla^d Y_t \) is a stationary ARMA process (Grossoni et al.,
For instance, for a stationary ARIMA \((p, d, q)\) model with \(d=0\) and with a mean equal to:

\[
W_t = \mu + \varphi_1 W_{t-1} + \varphi_2 W_{t-2} + \cdots + \varphi_p W_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}
\]

(2.15)

where:

- \(\mu\) is a mean component,
- \(e_t\) is the weighted sum of neighbouring error values,
- \(\varphi_p\) and \(\theta_q\) are the estimated coefficients values of the variables.

This study concluded that not only the mean value of track stiffness impacts significantly the degradation rate of ballast, but also its longitudinal variability. However, this study suggests further research, including a better understanding of track stiffness characteristics based on a larger and more representative set of measurements.

Other studies have also been published using time series modelling on railway tracks. Some examples are (Hipel, 1994; Quiroga and Schnieder, 2010; Czop and Mendrok, 2011; Quiroga and Schnieder, 2011a).

### 2.3.3 Mechanical-Empirical Models

Mechanical-empirical models represent a combination of both mechanistic and statistical models in order to cover the track condition of railways. These models are based on an understanding of the behaviour of a system’s components, coupled with direct observations, measurements and extensive data records. These models have been used around the globe in order to develop degradation models for railway tracks.

Sadeghi and Askarinejad (2008) used both mechanical and empirical models to improve the track condition of railways. Three different variables were shown to influence the rate of track degradation: Track Quality Indices (TQIs), traffic variables, and
maintenance variables. In addition, the total equivalent million gross tons (EMGTs) is taken into account as a major maintenance parameter. TQIs are divided into two sections: the track geometry index (TGI), representing the statistical analysis of the track geometry conditions (i.e. profile, twist and alignment), and the track structure index (TSI), representing the mechanical analysis of the track (i.e. the condition of the rails, sleepers, and fastening systems). For more details, refer to (Sadeghi and Askarinejad, 2007).

In the degradation model, a mathematical expression indicating the future condition of the track was established, taking into consideration the roles of the track’s main influencing variables. A correlation was obtained between the track degradation coefficient and time, incorporating the degradation coefficient (DC) and the main track variables (Sadeghi and Askarinejad, 2007).

The model is expressed in two forms: one based on the relationship between the track geometry conditions and time (Equation 2.16), and the second based on structural visual inspections of the mechanistic conditions of the track components over time (Equation 2.17).

\[
\frac{TGI_2}{TGI_1} = \alpha_4 \exp(\beta_1 V + \beta_2 \text{EMGT} + \beta_3 TGI_1) \times [\lambda_1 T^4 + \lambda_2 T^3 + \lambda_3 T^2 + \lambda_4 T + 1]
\]

\[
(2.16)
\]

\[
\frac{TSI_2}{TSI_1} = \alpha'_4 \exp(\beta'_1 V + \beta'_2 \text{EMGT} + \beta'_3 TSI_1) \times [\lambda'_1 T^4 + \lambda'_2 T^3 + \lambda'_3 T^2 + \lambda'_4 T + 1]
\]

\[
(2.17)
\]

where:

TGI_2 is the future track geometry index,
TGI_1 is the present track geometry index,
TSI_2 is the future track structure index,
TSI_1 is the present track structure index,
T is the time (in seconds).

Furthermore, a model was formulated providing the correlation between TGI_2 and TSI_2
to limit the study to fewer inspections, as follows:

\[
TSI_2 = \eta_1 \eta_2 \eta_3 \eta_4 TGI_2
\]  
(2.18)

where:
\(\eta_1\) to \(\eta_4\) are factors representing the influence of train speed (V), equivalent million gross tons (EMGTs), and time (T).

Obtaining linear correlations between the ratio of \(TGI_2/TSI_2\) and the influencing variables, the following expressions are obtained for \(\eta_1\) to \(\eta_4\):

\[
\begin{align*}
\eta_1 &= \kappa_1 V + \kappa_3 \quad (2.19) \\
\eta_2 &= \kappa_3 EMGT + \kappa_4 \quad (2.20) \\
\eta_3 &= \kappa_5 TGI_1 + \kappa_6 \quad (2.21) \\
\eta_4 &= \kappa_7 T + \kappa_8 \quad (2.22)
\end{align*}
\]

where:
\(\kappa_1\) to \(\kappa_8\) are the constant coefficients.

In general, the behaviour of the rail track varies at different segments; therefore, this degradation model is separately developed for each segment, such as curves, turnouts and straight lines.

Another study was conducted by Ahac and Lakusic (2015) at the University of Zagreb in Croatia. The study was based on tram track maintenance planning using gauge degradation modelling. They followed a mechanical-empirical model to define the rate of degradation using statistical regression analysis. This regression determines the speed of degradation of the dependent and independent variables, which are the track quality and the period of track exploitation, respectively. Two types of tram tracks were observed during the study: the indirect elastic rail fastening system and the stiffer direct elastic rail fastening system (Ahac and Lakušić, 2015).
The findings of this study indicated that the correlation between the rate of tram track gauge degradation during exploitation and the stiffness of its fastening system can be defined by dividing the results into three groups as follows: values of tram track exploitation intensity to approximately 35 million gross tons (MGTs), after an increase in exploitation intensity above 35 MGT, and values of tram track exploitation intensity above 45 MGT.

For values close to 35 MGT for tram track exploitation intensity, the degradation modelling for both observed systems found equal regression coefficients with very high determination coefficients (0.95 ≤ R² ≤ 0.98). Based on these results, it was concluded that the effects of fastening system stiffness on the gauge degradation rate are negligible in the initial stages of tram track exploitation (Ahac and Lakušić, 2015). After an increase in exploitation intensity above 35 MGT, the gauge degradation speed significantly decreases on tracks with direct elastic fastening systems. Lastly, for values of rail track exploitation intensity above 45 MGT, the proposed models do not specify a precise prediction of gauge degradation behaviour.

Briefly, the conclusions of this study were that the period of significant gauge degradation during tram track exploitation was shorter than in the case of the indirect elastic fastening system with lower stiffness. Therefore, it would be preferable to adjust the track geometry quality control and maintenance cycles according to track stiffness in order to optimise the track maintenance procedures and extend the life cycle of the tracks. It would also be better if preference could be given to indirect elastic rail fastenings when selecting structural elements for new rail tracks.

According to these researchers, the study was limited by the availability and form of the input data on rail tracks. This was needed for the creation of the database on which the modelling was based. This may have caused the lack of accuracy of the prediction models. Therefore, increasing the accuracy of models requires further monitoring of rail tracks (Refer to Table 2.1).
2.3.4 Artificial Intelligence Models

Artificial intelligence (AI) models are built and tested for the prediction of track quality. They are a type of machine learning model used to predict degradation in infrastructure, especially rail tracks. In this research, AI models are divided into two sub-types: artificial neural network models (ANNs) and neuro-fuzzy models, as shown in Figure 2.8.

![Figure 2.8: Categories of artificial intelligence degradation models.](image)

2.3.4.1 Artificial Neural Networks

ANNs are also known as connectionist models. These models use known or expected principles based on human brains and contain simple processors called neurons, which are connected to each other through weighted connections called synaptic weights (Guler, 2014). Therefore, various variables can be identified, such as number of layers, nodes, type of network and functions. In addition, the weight of these variables is modified and the data of the network are presented appropriately (Shafahi et al., 2008). Lastly, the network data should be examined using some known data, so that probable errors can be adjusted. Shafahi et al. (2008) presented a study testing an optimal network of 3 layers and 5 neurons in each internal layer. The data of this network were divided in two sets randomly: the training set (82% of the data) and the test set (18% of the data). The results of the neural network modelling of these data sets showed that the combined
track record index (CTR) predictions are at the level of the previous year or one year below that. The results of this study concluded that there is 33% accuracy of the neural network within one level wrong and 67% using the correct level.

### 2.3.4.2 Neuro-Fuzzy Models

A combination of artificial neural networks (ANNs) with a fuzzy inference system (FIS) is called neural fuzzy or neuro-fuzzy systems. Briefly, FISs show a proposition that may be inaccurate (Dell'Orco et al., 2008; Shafahi et al., 2008). The proposition restricts the possible values of a variable (x) and is represented by means of a membership function ($\mu_p(x)$). In this model, a fuzzy set is introduced by its membership function that falls in the interval [0,1].

Hybrid neuro-fuzzy models are considered one of the modern neuro-fuzzy approaches. The neural network and the fuzzy system are joined in a homogenous manner. Hence, this may be analysed as a special neural network with fuzzy variables or as a fuzzy system in a parallel distributed form (Shafahi et al., 2008). Jang et al.’s ANFIS model was one of the first hybrid neuro-fuzzy models proposed. This model is based on human knowledge and input-output data pairs (Jang et al. 1993). Hence, in this model, rule exertion and output variables are calculated by train data. The training algorithm is usually hybrid or back propagation (Shafahi et al., 2008). Based on all these conditions, the neuro-fuzzy model is produced. The findings of the model predictions and observed data for a sample set show that there is 27% accuracy with one level wrong and 73% accuracy with the correct level. Comparing this model with an ANN model, the use of the neuro-fuzzy model improved the results by 6%.

### 2.4 Limitations of Existing Studies

Based on the preceding review of the literature, the major limitations of the existing rail track degradation models become apparent. The literature review has outlined a number of areas where further research could overcome gaps in existing knowledge, which are indicated below:
• The network of the mechanistic models may be inconvenient and ineffective because it cannot be applied on track sections as they vary along the rail (i.e. turnouts, straight lines, curves). This explains why the studies found using mechanistic degradation models are quite old and recent research papers are very rare.

• The review shows that there is great variation in each type of statistical degradation model. Although deterministic models work well for large datasets and appear more attractive in degradation modelling, the rate of degradation of deterministic models varies between track sections and the models do not account for uncertainty (i.e. input variables and model geometry are not well known). Deterministic models also suffer from the potential of missing important factors in the causality of degradation, which in turn invalidates the models.

• In addition, probabilistic models of rail track degradation are not common due to the lack of historical data related to the geometrical quality of tracks for research purposes. Based on the literature review, Markov models restrict the detail of the analysis. In other words, this approach is limited to small track models. Furthermore, the transitions between asset states must occur at a constant rate. Bayesian models, which are limited in the research, rely on Markov models when high numerical dimensions occur. Continuous probability distributions were found to offer more realistic results. However, this model type is recommended for use in certain states, depending on the data provided for the case study.

• Stochastic models have been widely used in recent research studies. However, this type of model may need more explanation of its application and procedures for future research.

• The mechanical-empirical approach provides model development for different track segments, such as curves, turnouts, straight lines, tunnel lines and bridge lines. The degradation of lines on bridges, curve-bridges and turnouts shows a higher rate in comparison with other types; therefore, this model requires more attention, especially in maintenance and inspection scheduling.

• AI models, including ANNs and neuro-fuzzy models depend on different variables, such as the number of layers and nodes, the type of network and fuzzy variables. These models are the latest models in degradation studies. However, few research papers were found on them, as they are very new. In addition, the variables
influencing track degradation in these model types are not clearly explained and poorly defined.

All of these models have strengths and weaknesses. Table 2.1 shows a basic comparison of various degradation models discussed in the literature review. The table presents the different variables of each model type as well as their strengths and weaknesses. Based on this comparison, we found that stochastic models under the category of statistical approaches are the most appropriate models for degradation studies of rail tracks in Melbourne, because the strengths of these models outweigh their weaknesses.
Table 2.1: Comparison of different track degradation models.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Variables</th>
<th>Strengths</th>
<th>Weaknesses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanistic</td>
<td>• Track settlement,</td>
<td>• Based on laboratory experimental data sources,</td>
<td>• Challenging, intensive, time-consuming.</td>
</tr>
<tr>
<td></td>
<td>• Track deformation,</td>
<td>• Clearly address track settlement and degradation,</td>
<td>• Measurement of the variables influencing rail structure may be difficult or poorly understood.</td>
</tr>
<tr>
<td></td>
<td>• Track geometry (e.g. gauge),</td>
<td>• Suitable for maintenance of a particular section of rail track.</td>
<td>• Materials of rail structure are not homogenous.</td>
</tr>
<tr>
<td></td>
<td>• Track Quality Index (TQI).</td>
<td></td>
<td>• Difficulties in applying the model for different sections of rail track.</td>
</tr>
<tr>
<td>Deterministic</td>
<td>• Traffic volume,</td>
<td>• Work well for large data sets.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Dynamic axle,</td>
<td></td>
<td>• Potential to miss important degradation factors during application,</td>
</tr>
<tr>
<td></td>
<td>• Speed,</td>
<td></td>
<td>• Do not account for uncertainty (i.e. input variables and model geometry are not well known).</td>
</tr>
<tr>
<td></td>
<td>• Accumulated tonnage (MGT),</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Axle loads.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistical (Empirical)</td>
<td>• Speed restrictions or line closure,</td>
<td>• Reasonable procedure and realistic findings,</td>
<td>• Not common due to lack of historical data,</td>
</tr>
<tr>
<td>Probabilistic</td>
<td>• Track Quality Index (TQI),</td>
<td>• Ability to deal with large numbers of datasets to achieve more accurate results.</td>
<td>• Difficulties in predicting probability of track degradation,</td>
</tr>
<tr>
<td></td>
<td>• Standard deviation of longitudinal level defects (SDL),</td>
<td></td>
<td>• Bayesian models rely on Markov models, especially when high numerical dimensions occur.</td>
</tr>
<tr>
<td></td>
<td>• Horizontal alignment defects (SDHA),</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Number of cracks missed per year,</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Rail breakage.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stochastic</td>
<td>• Time,</td>
<td>• Ability to deal with large numbers of datasets</td>
<td>• No evidence to validate the claim of an exponential degradation pattern.</td>
</tr>
<tr>
<td></td>
<td>• Degradation rate of longitudinal level.</td>
<td>• Achieve more accurate results.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Widely used in rail prediction studies</td>
<td></td>
</tr>
<tr>
<td>Mechanical-empirical</td>
<td>• Track Quality Index (TQI),</td>
<td>• Applicable to different track segments (e.g. curves, turnouts, straight lines),</td>
<td>• Show a higher rate of degradation of lines in bridges, curve-bridges and turnovers in comparison with other model types.</td>
</tr>
<tr>
<td></td>
<td>• Traffic variables,</td>
<td>• Applicable to more accurate and less costly future maintenance procedures.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Maintenance variables (EMGT),</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Degradation Coefficient (DC),</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Time.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Artificial Neural Networks</td>
<td>• Number of layers,</td>
<td>• Calibrating model with an optimization algorithm,</td>
<td>• Presence of many effective factors resulting in more errors,</td>
</tr>
<tr>
<td>(ANNs)</td>
<td>• Nodes,</td>
<td>• Optimising variables of model.</td>
<td>• Validation of membership functions.</td>
</tr>
<tr>
<td>Artificial Intelligence</td>
<td>• Type of network and functions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neuro-Fuzzy</td>
<td>• Fuzzy sets,</td>
<td>• Finding fuzzy rules from numerical data,</td>
<td>• Complexity in abstracting fuzzy rules,</td>
</tr>
<tr>
<td></td>
<td>• Fuzzy membership functions.</td>
<td>• Considering human imprecise perception,</td>
<td>• Connections of a proposition may be imprecise,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Categorising variables into different categories</td>
<td>• Difficulty in calibrating model variables.</td>
</tr>
</tbody>
</table>
2.5 Summary

This chapter has presented a review of existing models of rail track degradation and explained their strengths, weaknesses and variables. On the basis of previous studies, stochastic models, which fall under the category of statistical approaches, are most suitable for the Melbourne CBD rail track network. They can deal with large datasets and achieve more accurate results than other models, and are widely used in degradation prediction studies, although they may require more understanding and clarification of their application. Also, the input variables of stochastic models are available in the dataset of Melbourne case study. Therefore, this study applies this model to a case study approach and proposes strategies to minimise the degradation of rail tracks and determine their maintenance needs.
Chapter 3
Research Framework

3.1 Introduction

The review of the literature identified the current state of practice and knowledge in the prediction of the degradation of rail tracks. This research project aims to develop a model to predict the degradation of tram tracks based on historical data and measurements of rail track geometry collected over the past years. It also aims to predict the future conditions of the rail tracks and estimate the maintenance planning activities needed for deteriorated rail tracks. Based on a comprehensive review of previous research of rail degradation models, this chapter presents an overview of the location of the study, the data collected and the methodology developed in this research.

3.2 Location of Study

Melbourne has the largest operating tram network in the world with 250 kilometres of double track. Yarra Trams is responsible for the day-to-day operation of Melbourne's extensive tram network under a franchise agreement with the Victorian Government (Yarra Trams, 2017a). Yarra Trams operates more than 450 trams with over 1700 tram stops across 24 tram routes and a free City Circle tourist tram. Seventy-five per cent of Melbourne's tram network operates on roads shared with other vehicles. The average speed of a tram is 16 km/h and within the CBD this drops to 11 km/h. Yarra Trams travels more than 24.6 million kilometres each year on timetabled services. It operates 31,500 scheduled tram services each week, which results in trams operating for approximately 20 hours per day and a team of 24-hour operations staff completing network maintenance and cleaning.

This research covers the entire network of Yarra Trams consisting of 24 tram routes and the city circle (Yarra Trams, 2017b). Figure 3.1 shows Melbourne tram network.
3.3 Research Methodology

The aim of this research is to analyse and predict rail track degradation, particularly in Melbourne. To achieve this aim, a research methodology is followed using available historic and measurement data as input for our research model (Figure 3.2). The main input data of the research are the automated inspection data, asset data and operation data (Refer to section 3.4 for details). An analysis of the dataset will be applied to find the main variables affecting tram track degradation of Melbourne network (Refer to Chapter 4 for details). Accordingly, two degradation models are developed, a linear regression and a time series model, for curve and straight sections in order to predict the
degradation of tram tracks in the future (Refer to Chapter 5 for details). Hence, the outputs of the research model are the variables affecting tram track degradation, the degradation rate of rail tracks and rail maintenance strategies are concluded (Refer to Chapter 5 for details).

![Research Model Diagram](image)

**Figure 3.2: Research model.**

### 3.3.1 Degradation Data Analysis Using SPSS

Different variables of the datasets are analysed using SPSS software to evaluate their impact on tram track degradation over particular sections (i.e. curves and straights) and in relation to the gauge parameter. These variables include rail profile, rail type, track surface, curve radius, number of trips and annual rail usage (in million gross tons, MGTs). Samples of data on curves and straights are used in this research. The data do not include any maintenance work over the period from 2010 to 2015. This helps predict the degradation of rail tracks more accurately and precisely with fewer errors and outliers. Based on the data, an analysis of curves and straights is carried out in order to identify the factors that affect the degradation of rail tracks. Therefore, the most influencing variables on tram track degradation are identified to develop predictive degradation models over time.

A correlation analysis and analysis of variance (ANOVA) are presented using SPSS software for continuous variables and categorical variables, respectively. Continuous
variables are the annual rail usage (in MGTs), number of trips and curve radii. Categorical variables are rail type, rail profile and track surface. In this research, the analysis of these variables shows their relationship with the changes in gauge value and track changes. Based on the significance and correlation values, the most influencing variable of rail degradation is identified according to the changes in gauge value of rail tracks from 2010 to 2015.

### 3.3.2 Linear Regression Model

A predictive degradation model named a regression model is developed in order to predict the degradation status of rail tracks in future. It also helps to identify the condition of the track and whether it needs to be repaired or not. The purpose of developing such a simple model is to compare it with another more complex model. This will help determine the most suitable model for the research in terms of accuracy and efficiency. The development of a simple regression model is important to check whether a simple model will be able to accurately predict the rail degradation. The model complexity and model accuracy will be compared to another degradation model, named time series model. Details of the time series model are provided in section 3.3.3.

In this research, the identified variables, gauge parameter and annual rail usage (MGT) values, are used to develop a linear regression model over time using SPSS software. First, the relationship between the changes in gauge value and MGT values is identified. The output results of this model show the estimated variables and coefficients of the model using training data samples. Accordingly, the accuracy of the linear regression model is determined and analysed for curves and straight sections. The observed data versus the estimated data of curves and straights are also plotted to graphically interpret the trend of rail degradation prediction.

### 3.3.3 Predictive Time Series Model

A time series stochastic model is developed using MATLAB software to predict the degradation of rail tracks over time. The time series model is developed using gauge
defects and MGT variables from 2010 to 2015. Based on the variation and high noise of the gauge defect values, a time series model called AutoRegressive Moving Average with eXogenous input (ARMAX) best fitted our data. It is a linear dynamic system model and a type of time series model. This model is developed for the first time in the degradation prediction of light rails. The application of the time series model shows in details its structure, accuracy and complexity. The model is applied for straight and curve sections using training data.

The statistical variables and coefficients of ARMAX time series are estimated for curves and straight segments. The observed data is plotted versus the estimated data of the model in order to show the trend of rail degradation prediction. This will also help identify the future condition of Melbourne tram tracks and predict the maintenance needed for damaged rail tracks.

3.3.4 Comparison of Applied Models

The linear regression model and time series model for straight and curve sections are compared. The performance of both models is evaluated to identify the influence of model complexity on the accuracy of the models in rail degradation prediction. The accuracy of each model is compared in order to detect the most accurate and suitable predictive degradation model for Melbourne tram tracks. The development of a suitable degradation predictive model is important to evaluate the condition of degraded rail tracks. This will help plan the maintenance strategy for damaged rail tracks. In this research, an evaluation of the maintenance activity was applied for degraded tram tracks. This will identify whether the degraded tram track needs to be maintained. It classifies whether the tram track needs to be repaired or replaced. This evaluation will help optimise and prioritise maintenance planning activities as well as save time and costs of unnecessary maintenance applications. Figure 3.2 shows a summary of the research model.
3.4 Dataset

Transport organisations collect large amounts of data when carrying out inspection and maintenance procedures. Datasets for this research were collected by Yarra Trams using an automated track inspection vehicle. This vehicle is designed for the Melbourne tram network. It runs along the entire tram track system and collects geometry and ride measurements through GPS and a wheel encoder that collect data to the computer. The automated inspection vehicle captures rail track geometry faults and rail surface deviations, including gauge defects, alignment (vertical and horizontal), twists (short and long), corrugations and high impacts. These measurements are then converted to a condition index for each track segment along each rail route. These measurements can also be used as a guide for annual renewal and maintenance planning.

The datasets of this research were measured along the tram rails according to the identity number of the tram, a series of segments (measured in metres) and specified line sections (measured in kilometres). The datasets include four types of data as follows:

- **Automated Inspection Data**

The automated inspection data are collected every six months from 2009 to 2015. The dataset contains measurements of rail track geometry for every track segment of 0.25m including:

- Track gauge: This is the distance between rail tracks and is measured at a right angle between the inner faces of the load-bearing rails. The standard railway gauge of trams is 1435 mm (4 ft 8.5 in).
- Alignment (also defined as horizontal alignment): This is the change in the curvature of rail over a certain cord length. Our dataset covers two alignment measurements: short alignment (over 5m chord length) and long alignment (over 10 m chord length).
- Twist (also known as crosslevel or cant gradient): This is the change in crosslevels of curve rail tracks over a specified cord length (expressed in mm/m). Our dataset includes measurements of three twist lengths: short twist (1.8 m long), medium twist (3.5 m long) and long twist (10 m long).
Crosslevel (also known as super-elevation or cant): This is the difference in height between the top surfaces of two rails of the railroad track. The crosslevel is zero when there is no difference in height between the surfaces of rails at the measured point. The crosslevel is negative (also known as reverse crosslevel) when the outer rail of curve track has a lower height than the inner rail and is positive when otherwise. Cant is super-elevation expressed in the unit of angle. The standard cant of straight rail tracks is designed to be 7°.

The dataset also includes measurements of acceleration, coordinates and speed of the tram at the time of data collection.

- **Asset Data**

Rail asset data include information such as track categories (i.e. straights, curves, crossovers, H-crossings, terminus and sidings), track location (i.e. route area, route number, tram ID number) and construction material (i.e. rail profile, rail type, track surface material, sleeper type, curve radius and installation dates).

- **Operational Data**

The operational data relate to the usage of the network, and include the annual million gross tons (MGTs), which is the load passing over each rail track segment without passengers. Gross tons is the product of total weight including the weight of locomotives as well as the weight of the average annual daily traffic volume passing the tram rails. The operational data also include the number of trips, which is the number of trams passing over each rail track segment.

Other factors may affect the degradation of rail tracks, including environmental factors, type of vehicle (leading to different axle loads and wheel-rail interfaces), ballast degradation (i.e. foundation instability) and other traffic loads. However, this research ignores the influence of these factors due to the limitation of information and to avoid complicating the model development. Table 3.1 shows a summary of the collected datasets from 2009 to 2015.
Table 3.1: Summary of datasets.

<table>
<thead>
<tr>
<th>Dataset variables</th>
<th>Tram track identification</th>
<th>Route area</th>
<th>Route number</th>
<th>Tram ID number</th>
<th>Tram Track segments (in m)</th>
<th>Line sections (in km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail geometric variables</td>
<td></td>
<td>Gauge</td>
<td>Twist (1.8m, 3.5m, 10m long)</td>
<td>Crosslevel</td>
<td>Alignment (5m, 10m long)</td>
<td></td>
</tr>
<tr>
<td>Tram track categories</td>
<td></td>
<td>Straights</td>
<td>Curves</td>
<td>Crossovers</td>
<td>H-Crossings</td>
<td>Terminus</td>
</tr>
<tr>
<td>Construction information</td>
<td></td>
<td>Rail profile (i.e. 41 kg, 42 kg, 43 kg, 57 kg, 60 kg)</td>
<td>Rail type</td>
<td>Track surface (i.e. Asphalt, Concrete, Open)</td>
<td>Curve radius (in m) (ranging between 0 and 190 m)</td>
<td>Installation date (in years)</td>
</tr>
<tr>
<td>Million Gross Tons (MGT)</td>
<td></td>
<td>MGT (in Mpa)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of trips</td>
<td></td>
<td>Tram trips</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tram speed</td>
<td></td>
<td>(in km/hr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 3.5 Summary

This chapter has presented the research framework of the thesis. It has discussed the research methodology and presented details of the study location and the datasets collected for model development on rail track degradation. The datasets were used to identify different input variables for degradation modelling. Therefore, this chapter has provided information about the different types of input variables used in the degradation modelling process. A brief outline of the outputs has been highlighted for further discussion in the following chapters.
Chapter 4

Analysis of Variables Influencing Rail Track Degradation

4.1 Introduction

Chapter 3 discussed the data sets for corridor modelling and the methodology of the present research. This chapter presents an analysis of the impact of different data variables on tram track degradation. The chapter shows the relationship between these variables and the changes in gauge value of tram tracks over time using SPSS software. A summary of the variables which affect tram track degradation is also provided for research model development. The analysis of the variables which affect track degradation is used to develop rail degradation prediction models.

4.2 Factors Affecting Tram Track Degradation

The aim of this section is to examine the relationship between different variables in the provided data and track degradation. The variables can be divided into two different categories: continuous and categorical variables. For the purpose of the degradation analysis, the changes in gauge value at every two consecutive years is calculated. Hence, the relationship between the different variables and the changes in gauge value can be analysed. Using SPSS software, a correlation analysis is adopted for continuous variables while ANalysis Of VAriance (ANOVA) is adopted for categorical variables. The analysis shows whether the changes in gauge value are statistically correlated with the data variables. Based on this, the main variables influencing rail degradation are detected for both curves and straight sections.

4.2.1 Continuous Variables

A continuous variable is a variable that can take any numerical value and is measured. It is a variable that has an infinite number of possible values. The dataset considered in this
research includes the following continuous variables:

- **Annual Million Gross Tons (MGTs):** Average MGT value for the track where the inspection point is located.
- **Trips:** Average number of trips for the track where the inspection point is located.
- **Curve Radius:** Curve radius for the track on which the inspection point is located.

To examine the relationship between the continuous variables and the changes in gauge value, a correlation analysis is adopted for curves and straight sections using SPSS software. Correlations measure how variables are related. The values of the correlation coefficient range between -1 and 1. The sign of the correlation value shows the direction of the relationship (positive or negative). The absolute value of the correlation coefficient indicates the strength of the relationship between the two variables. Larger absolute values signify stronger relationships. In other words, higher correlation values indicate higher association between the variable and the changes in gauge value. Positive and negative correlation values show that the variable and the changes in gauge value are positively and negatively correlated, respectively.

A summary of the correlation values and their associated significance levels (P-values) is shown below in Tables 4.1 and 4.2. The significance level (or P-value) is the probability of obtaining results as extreme as that observed. The P-value indicates whether the associated correlation value is statistically significant and whether the two variables are linearly related. If the P-value is very small (less than 0.05), the correlation is statistically significant and the two variables are linearly related. If the P-value is greater than 0.05, the correlation value is not statistically significant and the two variables are not linearly related.

Tables 4.1 and 4.2 show a summary of the correlation analysis for curves and straights for the most recent years available in the datasets (2014-2015).
Table 4.1: Correlation between changes in gauge value and continuous variables of curve segments: 2014-2015.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGT</td>
<td>0.295</td>
<td>0</td>
</tr>
<tr>
<td>Trips</td>
<td>0.203</td>
<td>0.054</td>
</tr>
<tr>
<td>Curve Radius</td>
<td>-0.12</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 4.2: Correlation between changes in gauge value and continuous variables of straight segments: 2014-2015.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGT</td>
<td>0.207</td>
<td>0</td>
</tr>
<tr>
<td>Trips</td>
<td>0.003</td>
<td>0.92</td>
</tr>
<tr>
<td>Curve Radius</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

As Tables 4.1 and 4.2 show, ‘MGT’ is positively correlated with changes in gauge value for curves and straights, and the P-value is 0, which means that it is statistically correlated. The positive value indicates that as ‘MGT’ increases, the changes in gauge value increases. ‘Trips’ has a P-value greater than 0.05 for curves and straights (Refer to Tables 4.1 and 4.2). This explains that ‘Trips’ is not significantly correlated with the changes in gauge value. ‘Curve Radius’ is only available for curve segments. The analysis shows that ‘Curve Radius’ is not significantly correlated with the changes in gauge value for curves (Refer to Table 4.1).

4.2.2 Categorical Variables

A categorical variable is a variable that can take on one of a limited, and usually fixed, number of possible values, assigning each individual to a particular group or category. A categorical variable is one that simply allows you to assign categories, but you cannot clearly order the variables. In the dataset considered, the categorical variables are the
following:

- Rail Profile: Rail profile associated with the inspection point (i.e. 41 kg, 42 kg, 43 kg, 57 kg, 60 kg, 96 lb, 102 lb)
- Rail Type: Rail type associated with the inspection point
- Track Surface: A categorical variable to indicate the track surface (i.e. Concrete, Asphalt, Open).

To examine the relationship between the categorical variables and the changes in gauge value, ANOVA was adopted for curves and straight sections using SPSS software. One-way analysis of variance (ANOVA) is used to determine whether there are any statistically significant differences between the means of two or more independent groups. This analysis statistically compares the average of changes in gauge value in different categories of each variable. It shows whether the relationship between each of the categorical variables and the changes in gauge value is statistically significant.

Tables 4.3 and 4.4 show a summary of the ANOVAs for curves and straights for 2014 and 2015, including F-values and significance values. The higher F-values indicate greater differences between the changes in gauge value in different categories of the variable. The significance value, if less than 0.05, suggests the differences are statistically significant. If the significance value is equal to or greater than 0.05, this means that the relationship between the changes in gauge value and the categorical variable is not statistically significant.

Table 4.3: ANOVA for changes in gauge value and categorical variables of curve segments: 2014-2015.

<table>
<thead>
<tr>
<th>Variable</th>
<th>F value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail Profile</td>
<td>1.68</td>
<td>0.16</td>
</tr>
<tr>
<td>Rail Type</td>
<td>0.19</td>
<td>0.67</td>
</tr>
<tr>
<td>Track Surface</td>
<td>9.7</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Table 4.4: ANOVA for changes in gauge value and categorical variables of straight segments: 2014-2015.

<table>
<thead>
<tr>
<th>Variable</th>
<th>F value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail Profile</td>
<td>5.18</td>
<td>0</td>
</tr>
<tr>
<td>Rail Type</td>
<td>4.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Track Surface</td>
<td>6.92</td>
<td>0.001</td>
</tr>
</tbody>
</table>

According to Table 4.3, the ‘Track Surface’ of curve segments is statistically significantly associated with the ‘Changes in gauge value’ (Significance value = 0.002 < 0.05). However, ‘Rail Profile’ and ‘Rail Type’ for curve segments are not statistically significantly associated with ‘Changes in gauge value’ as their significance values are greater than 0.05 which means that there is no significant correlation between ‘changes in gauge value’ and those variables.

For straight sections, Table 4.4 shows that ‘Rail Profile’, ‘Rail Type’ and ‘Track Surface’ are important categorical variables for modelling the changes in gauge value. The significance values of these variables are less than 0.05, which makes them statistically correlated with the changes in gauge value. However, based on the F-values, ‘Track Surface’ has the greatest impact (F-value = 6.92), and ‘Rail Type’ has the lowest, with the smallest F-value (F-value = 4.04).

A summary of the correlation analysis of other years (From 2010 to 2014) is displayed in Tables 4.5 and 4.6 for curves and straight sections. For curves, the analysis shows that the ‘MGT’ factor of the P-value is less than 0.05 is positively correlated with the changes in gauge value for all years (From 2010 to 2014). However, ‘Trips’ and ‘Curve Radius’ are not statistically correlated with the changes in gauge value for all years between 2010 and 2014 because they have P-values greater than 0.05 (Refer to Table 4.5). For straight segments, the correlation analysis shows that the ‘MGT’ variable is significantly correlated with the changes in gauge value’ and the P-values for all years from 2010 to 2014 are less than 0.05. However, the ‘Trips’ variable is not statistically correlated with the changes in gauge value for all years between 2010 and 2014 because they have P-values greater than 0.05 (Refer to Table 4.6).
Table 4.5: Correlations between changes in gauge value and continuous variables of curve segments: 2010-2014.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
<td>MGT</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>Trips</td>
<td>-0.19</td>
<td>-0.14</td>
</tr>
<tr>
<td>Curve Radius</td>
<td>-0.19</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

Table 4.6: Correlations between changes in gauge value and continuous variables of straight segments: 2010-2014.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>MGT</td>
<td>0.001</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

A summary of the ANOVAs for 2010 to 2014 is displayed in Tables 4.7 and 4.8 below for straight and curve sections. For curve segments, the analysis shows that the variables of ‘Rail Profile’, ‘Rail Type’ and ‘Track Surface’ are not statistically significantly associated with the changes in gauge value (Refer to Table 4.7). For straight segments, the analysis shows that ‘Rail Profile’ is a common significant factor for all years from 2010 to 2014. However, ‘Rail Type’ and ‘Track Surface’ are not statistically significant for all years.

Table 4.7: ANOVA for changes in gauge value and categorical variables of curve segments: 2010-2014.

<table>
<thead>
<tr>
<th>Variable</th>
<th>F value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>0.73</td>
<td>5.23</td>
</tr>
<tr>
<td>Rail Profile</td>
<td>1.41</td>
<td>0.04</td>
</tr>
<tr>
<td>Track Surface</td>
<td>0.07</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Table 4.8: ANOVA for changes in gauge value and categorical variables of straight segments: 2010-2014.

<table>
<thead>
<tr>
<th>Variable</th>
<th>F value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010-2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011-2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012-2013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013-2014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail Profile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.78</td>
<td>9.82</td>
<td>4.93</td>
</tr>
<tr>
<td>Rail Type</td>
<td>0.67</td>
<td>6.25</td>
</tr>
<tr>
<td>Track Surface</td>
<td>0.01</td>
<td>3.87</td>
</tr>
</tbody>
</table>

Based on the above tables, the variables influencing the changes in gauge value can be determined for straight and curve segments. Table 4.9 shows a summary of the variables impacting the gauge degradation for all years from 2010 to 2015. Based on this table, it can be concluded that ‘MGT’ is the common influencing variable for all years from 2010 to 2015. Hence, this research will focus on the degradation of the changes in gauge value in relation to the ‘MGT’ factor over the period between 2010 and 2015.

Table 4.9: Summary of variables influencing gauge degradation for curves and straights: 2010-2015.

<table>
<thead>
<tr>
<th>Variables Influencing Gauge Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>2010-2011</td>
</tr>
<tr>
<td>2011-2012</td>
</tr>
<tr>
<td>2012-2013</td>
</tr>
<tr>
<td>2013-2014</td>
</tr>
<tr>
<td>2014-2015</td>
</tr>
</tbody>
</table>
4.3 Summary

This chapter has discussed the variables influencing the degradation of rail tracks. This chapter categorised the datasets provided in the research into categorical and continuous variables. These variables were analysed using SPSS software to determine the significant variables influencing the degradation of rail tracks. According to this analysis, MGT was the common variable influencing the degradation of rails based on the changes in gauge value. Therefore, this research will focus on the MGT factor as it is the variable which most affects tram track degradation.
Chapter 5

Rail Degradation Prediction Models

5.1 Introduction

Chapter 4 presented an analysis of different rail track variables and showed the variables which most affect the degradation of Melbourne rail tracks among the considered factors. This chapter presents two rail degradation prediction models: a time-series degradation model and a linear regression model. This chapter explains the step-by-step procedures in modelling using the variables affecting tram tracks over time. Time-series and linear regression models are developed for straight and curve sections using MATLAB and SPSS software, respectively. The results of the proposed models are discussed and the accuracy of each model is identified. In addition, a comparison of the models is presented, followed by a summary of the models’ results.

5.2 Relationship between Variables

In order to find the most applicable degradation prediction models for this research, it is important to find the relationship between the different variables and input variables. Rail geometric degradation is usually quantified with many different defects.

Railway degradation is affected by time and different variables (i.e. longitudinal levelling defects, horizontal alignment defects, cant defects, gauge deviations and track twist). However, these variables are generally not provided in the datasets. In this research, the most important defects leading to rail degradation are gauge deviation defects and tonnage per year (in MGTs).

The entire data used for this research include gauge and MGT values from 2010 to 2015, respectively. Gauge is plotted with respect to MGT in Figure 5.1. This clearly shows that there is a meaningful relationship between gauge and MGT.
Therefore, the gauge model $gauge(t+1)$ is considered to be a function of $gauge(t-i)$ for $i = 0$ to $n$ and the $MGT(t-i)$ for $i = 0$ to $n$ the function can be described by Equation 5.1:

$$
Gauge(t+1) = f(gauge(t-i), MGT(t-i)) \quad \text{for } i=0,1,...n
$$

Therefore, the time series and regression predictive models of this research follow a linear pattern based on the linear relationship between gauge defects and MGT values.
5.3 Time Series Model Development

5.3.1 Time Series Models

Time series modelling has been a dynamic research area over last few decades. It aims to carefully collect and rigorously study the past observations of a time series to develop an appropriate model which describes the structure of the series. A time series, classified under time-dependent models, is a signal that varies over time, such as the currency exchange rate, the unemployment rate in a country, the world’s population, the amount of rainfall in a particular area. Time series modelling is used for many engineering applications such as water resources and environmental systems (Hipel and McLeod, 1994; Milly et al., 2008), traffic and road engineering (Jilani et al., 2007; Min and Wynter, 2011; Lv et al., 2015).

A time series is a sequential set of data points, measured typically over successive times. A characteristic of time series models is that the current value is usually dependent on previous values (Adhikari and Agrawal, 2013). It is mathematically defined as a set of vectors \{x(t), t = 0,1,2,\ldots\}, where t denotes the time elapsed (Hipel and McLeod, 1994; Raicharoen et al., 2003; Cochrane, 2005; Brockwell and Davis, 2016), and the variable \(x(t)\) is treated as a random variable. The mathematical expression describing the structure of a time series is termed a stochastic process (Hipel and McLeod, 1994; Fuller, 2009; Box and Jenkins, 2015). A time series is usually generated by a white noise signal, which drives a dynamic system (Ljung, 1999; Wei, 2006; Chatfield, 2016). The dynamic system is then identified with the time series data. A time-series model can be linear or non-linear, depending on whether the current value of the series is a linear or non-linear function of past observations (Brockwell and Davis, 2016). In the present study, the dynamic system is assumed to be linear because there is a linear relationship between gauge and MGT values (Refer to Figure 5.1).

In general, time series models can have many forms and represent different stochastic processes. Two time series models are widely used in the research literature: Auto Regressive (AR) and Moving Average (MA) (Hipel and McLeod, 1994; Box and Jenkins, 2015; Chatfield, 2016). Combining these two, the Auto Regressive Moving
Average (ARMA) and Auto Regressive Integrated Moving Average (ARIMA) models have been proposed in the literature. These models have been developed by researchers in different disciplines, such as economics, engineering and science (Hipel and McLeod, 1994; Jilani et al., 2007; Box and Jenkins, 2015). In this chapter, a suitable time series model is developed based on the analysed data and input variables of the present research. This model type is used for the first time in rail degradation prediction which makes it an effective contribution to rail engineering studies. In this research, the development a time series model will predict the degradation status of Melbourne tram tracks in order to optimise and prioritise rail maintenance practices in the future.

5.3.2 Model Performance

A time series is usually generated by a white noise signal, which drives a dynamic system. In the present research, the dynamic system is assumed to be linear. To model the system, an ARMAX model is suggested focusing on the changes in gauge value and MGT factor to predict the degradation of tram tracks in Melbourne over the following years. As a result, this will help predict the future maintenance applications needed for tram tracks, resulting in lower costs, less effort and time saving.

The AR time-series model is a very common type of system representation with few linear variables (Nelles, 2013). In AR models, the output of the system is derived in an autoregressive manner to the previous values of outputs by filtering the white noise \( \nu(k) \), as shown in Equation 5.2:

\[
y(k) = \frac{1}{D(q)} \nu(k)
\]

where:

- \( y(k) \) is the output in time \( k \) and \( \nu(k) \) is white noise.

Since it is clear that time series models are not sufficiently accurate without considering the input, the Auto Regressive with eXogenous input (ARX) model is the extended form of the AR model, which can be written as Equation 5.3:
\[ y(k) = \frac{B(q)}{A(q)} u(k) + \frac{1}{D(q)} v(k) \]  \hspace{1cm} (5.3)

As a result of high noise in the gauge values, different dynamic system models have been evaluated based on the data and Auto Regressive Moving Average with eXogenous input model (ARMAX) fitted best with the lowest mean absolute error. Therefore, this method has been selected for system modelling. The ARMAX model is in fact the extended noise model of the ARX with more flexibility. The ARMAX model is one of the most useful models in linear dynamic system modelling, although the model is non-linear in variables. ARMAX can be described as shown in Equation 5.4:

\[ A(q) y(k) = B(q) u(k) + C(q) v(k) \]  \hspace{1cm} (5.4)

The predictor of ARMAX can be written as Equation 5.5:

\[
\hat{y}(k|k-1) = \frac{B(q)}{C(q)} u(k) + \left(1 - \frac{A(q)}{C(q)}\right) y(k)
\]  \hspace{1cm} (5.5)

The predictor is stable if \( C(q) \) is stable. The prediction error of the ARMAX model can be written as follows:

\[ e(k) = \frac{A(q)}{C(q)} y(k) - \frac{B(q)}{C(q)} u(k) \]  \hspace{1cm} (5.6)

The estimation of the ARMAX model can be achieved by the following procedure. First, an ARX model estimation for the data is calculated, as shown in Equation 5.7:

\[
\hat{\phi}_{\text{ARX}} = (X^T X)^{-1} X^T y
\]  \hspace{1cm} (5.7)

Next, the ARMAX model variables are calculated following a non-linear procedure. By using non-linear least square methods, the model variables can be identified. For the non-linear least square models the computation of the gradients is necessary.
As the squared error is 
\[ e^2(k) = (y(k) - \hat{y}(k))^2, \quad \frac{\partial e^2}{\partial \varphi} = -2e(k) \frac{\partial \hat{y}}{\partial \varphi}. \]
Therefore, the gradient of the estimated model must be calculated.

By multiplying both sides of Equation 5.5 by \( C(q) \), the equation can be rewritten as shown below:

\[
C(q) \hat{y}(k|k-1) = B(q)u(k) + (C(q) - A(q))y(k)
\]

(5.8)

The differentiation \( \frac{\partial e^2}{\partial \varphi} \) yields the differentiation Equation 5.4 with respect to \( a_i, b_i, c_i \).

\[
C(q) \frac{\partial \hat{y}(k|k-1)}{\partial a_i} = -y(k - i)
\]

(5.9)

Therefore,

\[
\frac{\partial \hat{y}(k|k-1)}{\partial a_i} = \frac{1}{C(q)} y(k - i)
\]

(5.10)

Equation 5.10 should be calculated with respect to \( b_i, c_i \) which yields:

\[
C(q) \frac{\partial \hat{y}(k|k-1)}{\partial b_i} = u(k - i)
\]

(5.11)

and,

\[
\frac{\partial \hat{y}(k|k-1)}{\partial c_i} = \frac{1}{C(q)} y(k - i) - \hat{y}(k - i|k - i - 1)
\]

(5.12)

Therefore, the gradient can be calculated by the above equations. Experience has shown that the above equations converge to global optimal variables (Zhang, 2003).
5.3.3 Model Validation

Validation is the task of demonstrating that the time series model is reasonably replicate the actual system. Model validation is an important step in judging how good a model is with respect to the system. It ascertains the goodness of the model and whether the assumptions proposed to model the system are reasonable with respect to the real system. In the presented model, a number of assumptions have been implemented to develop the time series model. The assumptions are as follows:

- The model system is assumed to be a linear dynamic system and the relations are exclusively assumed to be linear.
- The time-series involved are weakly stationary or integrated in some order (which implies restrictions on the values of the unknown coefficients, as well as their constancy).
- All observed time series are combinations of white noises only, and perhaps a constant.
- The ARMAX model predictor shown in Equation 5.5 in Section 5.3.2 is stable even if the A(q) polynomial is unstable. However, the C(q) polynomial is required to be stable.
- The ARMAX model variables and gradient follow a non-linear procedure using non-linear least square methods.

More details of the assumptions are provided in Section 5.3.2 showing the performance of the time series model.

Different input variables are used in the development of the time series model. The changes in gauge value of tram tracks and the MGT factor are the main input variables used in the time series ARMAX model. 270 samples of data have been used for each of curve and straight segments to develop the models. The data has been divided into two subsets. The first subset is the training data, or the actual data used to train the model computing the gradient, output variables and biases. The second subset is the testing data. It is used to test the developed model with real world data, and is not used to develop the model. In the present model, 70% of the data has been used for training and
30% for testing. The developed model predicts the gauge degradation of tram tracks in Melbourne in future using the changes in gauge value of the previous years and their MGT values. When modelling the system, the R-squared criterion is used to find the best training set in modelling. This criterion shows the goodness of the model and its accuracy. A sample x is used as training data to find the R-squared values of the test data. For greater model accuracy, sample k of the highest R-squared value is used to model the gauge values. Accordingly, the output variables of the developed time series model are identified. The estimated variables of time series model are proposed when sample k is used for training purposes using the following Equation 5.13:

\[ y(k) + a_1 y(k - 1) + \ldots + a_m y(k - n) = b_1 u(k - n_m) + \ldots + b_n u(k - n - n_m) + e(k) + c_1 e(k - 1) + \ldots + c_m e(k - m) \]  \hspace{1cm} (5.13)

Various forecast measures have also been performed in order to determine the goodness of fit of the proposed time series model. The coefficient of determination, also called R-square, represents the strength of the relationship of common variation in two time series or variables. It is used to analyse how well the gauge degradation values can be predicted using the input variables of the time series model.

Another forecast measure is the Mean Absolute Error (MAE), which is defined as:

\[ \text{MAE} = \frac{\sum |\text{Real value} - \text{Estimated value}| \text{ Number of data}}{\text{Number of data}} \]  \hspace{1cm} (5.14)

MAE measures the average absolute deviation of predicted gauge values from the original values. It shows the magnitude of the overall error due to prediction. For a good prediction, the obtained MAE should be as small as possible. It also depends on the scale of measurement and data transformations. Our research model shows the MAE values for curve and straight segments using sample k as training data and the rest of the data as testing data.

Another forecast measure is the Mean Squared Error (MSE). It is a measure of the average squared deviation of predicted values. MSE gives an overall idea of the error in prediction. It is sensitive to the change of scale and data transformations. Although MSE
is a good measure of overall prediction error; it is not as easily interpretable as the other measures discussed previously. Therefore, the focus of the interpretation will be mainly on MAE and R-squared prediction measures. Details of the prediction measures, equations and output results are provided in the following section.

5.3.4 Results

To find the best sample for use in modelling the gauge values, the R-squared criterion needs to be calculated. This research includes 270 samples of data for curve and straight sections. Figures 5.2 and 5.3 demonstrate the R-squared value for the test data if sample \( x \) is used to model the gauge values. Sample \( x \) is used as training data to find R-squared values for curves and straights sections.

![R-squared values for test data with sample \( x \) used as training data for curves.](image)

Figure 5.2: R-squared value for test data with sample \( x \) used as training data for curves.
Figures 5.2 and 5.3 show the changes in R-squared values of the model when it is trained using different samples. In some samples, the R-squared value is relatively higher. In other samples, R-squared is ‘zero’ lying on the x-axis. This represents a model that does not explain any of the variation in the response variable around its mean. Therefore, a sample k of the highest R-squared is chosen to develop the model. Sample k is the training data that has the highest R-squared value and using this sample for the training, the result of MAE for each test data in the whole data set is summarised in Figures 5.4 and 5.5.
Figure 5.4: MAE on test data with sample k used as training data for curves.

Figure 5.5: MAE on test data with sample k used as training data for straights.
Figures 5.4 and 5.5 show that if the model is trained by sample k and is tested on the other data, the MAE is just below 0.1. The MAE is calculated using Equation 5.14 provided in Section 5.3.3.

The estimated variables if sample k is used for the training purpose are presented in Equation 5.13 in Section 5.3.3 and Table 5.1 below.

Table 5.1: Estimated variables of time series model.

<table>
<thead>
<tr>
<th>Curve data</th>
<th>Straight data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(z) = 1 - 1.504 z^{-1} - 0.4602 z^{-2}$</td>
<td>$A(z) = 1 - 0.6795 z^{-1} - 0.6168 z^{-2}$</td>
</tr>
<tr>
<td>$B(z) = 6.227e - 0.9$</td>
<td>$B(z) = -1.368e - 0.7$</td>
</tr>
<tr>
<td>$C(z) = 1 - 0.9948 z^{-1}$</td>
<td>$C(z) = 1 + z^{-1}$</td>
</tr>
</tbody>
</table>

According to Table 5.1, the values for the input data are very low, due to the higher values of MGT in respect to the gauge values. The observed data versus the estimated data of gauge values are plotted in Figures 5.6 and 5.7, showing the accuracy of the model.
Figure 5.6: Observed versus estimated gauge values (in mm) on test data for curves.

Figure 5.7: Observed versus estimated gauge values (in mm) on test data for straights.
To summarise the findings of this study, the most important variables influencing tram track degradation are summarised in Table 5.2.

Table 5.2: Statistical variables of time series model.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Curve test data</th>
<th>Straight test data</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.890</td>
<td>0.979</td>
</tr>
<tr>
<td>MSE</td>
<td>0.503</td>
<td>0.320</td>
</tr>
<tr>
<td>MAE (in mm)</td>
<td>0.079</td>
<td>0.055</td>
</tr>
</tbody>
</table>

R-squared is a statistical measure of how close the data are to the fitted regression line. The higher R-squared value, the better the model fits the data. According to Figures 5.6 and 5.7 and Table 5.2, the developed time series model shows that R-squared values are 89% and 97.9% for curves and straights gauge values, respectively. The R-squared values are high, which indicates that the model explains most of the variability of the response data near its mean and is sufficiently accurate in predicting tram track degradation.

MSE stands for the mean squared error. It is a measurement of the variation between predicted (or estimated) and observed gauge values. Smaller values for MSE indicate closer agreement between predicted and observed results. In this research, MSE values are equal and less than 0.5 (Refer to Table 5.2), which signifies small and acceptable data variations between observed and predicted gauge values.

Mean absolute error (MAE) is another measurement showing the variation between predicted and observed gauge values (in mm). MAE has the same unit as the original data and it can only be compared between models whose errors are measured in the same units. MAE values are 0.079 and 0.055 for curves and straights data, respectively (Refer to Table 5.2). These values are very low, which indicates small data variations and closer agreement between predicted and observed gauge values.
5.4 Linear Regression Model Development

5.4.1 Linear Regression Models

Linear regression is the most commonly used techniques for investigating the relationship between one dependent variable and one or more independent variables. The simplest form of the regression equation with one dependent and one independent variable is defined as \( y = c + b \times x \), where \( y \) = estimated dependent variable, \( c \) = constant, \( b \) = regression coefficient, and \( x \) = independent variable. Linear regression can be used to identify the strength of the effect that the independent variable(s) has on a dependent variable. It can also be used to predict the effects or impacts of changes of the dependent variable based on the independent variable(s). That is, regression analysis helps us to understand the extent to which the dependent variable changes with a change in one or more independent variables. Moreover, regression analysis predicts trends and future values. In the present research, a linear regression model is developed to predict trends and future estimates of the degradation of tram data over time.

5.4.2 Model Performance

In this research, gauge defects and MGT are the dependent and independent variables respectively of the regression model. The R-squared value (\( R^2 \)) denotes the goodness-of-fit of the model. It shows how close the regression line is to the observed values. \( R^2 \) is expressed in Equation 5.15 as follows.

\[
R^2 = \frac{\sum (\hat{y} - \bar{y})^2}{\sum (y - \bar{y})^2}
\]

(5.15)

where:

\( \hat{y} \) is the regression line,
\( \bar{y} \) is the mean of observed values,
\( y \) is the observed values.
In this research, $y$ represents the observed values of gauge defects. The $R^2$ value shows the level of accuracy of the prediction regression model and how much the improvement of prediction is. If the $y'$ value is close to 0, this means that the regression model is not highly accurate. If the $R^2$ value is 1, this means that the prediction model is highly accurate and the regression line fits the data perfectly. The adjusted $R^2$ is also taken into account in the regression analysis. The adjusted $R^2$ is a model accuracy measure that tends to estimate the fit of the linear regression. The adjusted R-squared compares the descriptive power of regression models that include diverse numbers of predictors. The adjusted R-squared is a modified version of R-squared for the number of predictors in a model. The value of the adjusted $R^2$ is always equal to or less than $R^2$. Similar to $R^2$, an adjusted $R^2$ value of 1 indicates that the regression perfectly predicts the degradation of the gauge values of rail tracks.

The standard error of the estimate $s_y$ is also used as a measurement of the model goodness. $s_y$ is the standard deviation of the error variable. It shows the difference between the observed values, $y$, and the predicted values, $y'$, on the regression line. $s_y$ is expressed in Equation 5.16:

$$s_y = \sqrt{\frac{\sum(y - y')^2}{n - 2}}$$

(5.16)

where:

- $y$ is the observed value,
- $y'$ is the predicted value,
- $n$ is the number of samples.

See Gelman and Hill, (2007) for further details on regression analysis.
5.4.3 Model Application

Linear regression is an analysis that assesses whether one or more predictor variables explain the dependent variable. This research proposes a linear regression to predict the gauge degradation of rail tracks in the future. For the present research, the regression analysis representing the degradation of rail tracks was applied to curve and straight sections, separately. The model was developed using 270 data samples, taking into consideration the MGT factor and changes in gauge value. The gauge model \( gauge(t + 1) \) is considered to be a function of \( gauge(t - i) \) for \( i = 0 \) to \( n \) and \( MGT(t - i) \) for \( i = 0 \) to \( n \) the function can be expressed as:

\[
Gauge(t + 1) = f(gauge(t - i), MGT(t - i)) \quad \text{for } i=0,1,\ldots n \quad \text{and } t \text{ is the time} \quad (5.17)
\]

In modelling the tram data using regression analysis, several assumptions were made as follows:

- Linear regression assumes the relationship between the independent and dependent variables to be linear. It is also important to check for outliers, since linear regression is sensitive to outlier effects.
- Linear regression assumes that there is little or no multicollinearity in the data. Multicollinearity occurs when the independent variables are too highly correlated with each other. This refers to a situation in which two or more explanatory variables in a regression model are highly linearly related. A perfect multicollinearity occurs if the correlation between two independent variables is equal to 1 or −1.
- Linear regression analysis requires that there is little or no autocorrelation in the data. Autocorrelation is a characteristic of data in which the correlation between the values of the same variables is based on related objects. Autocorrelation occurs when the residuals are not independent of each other. In other words, the value of gauge \((t+1)\) is not independent of the value of gauge \((t-i)\).

For the current regression modelling, 70% of our data are used for training and 30% for testing. Training data are used to develop the model and testing data are used to validate the model built. 270 samples of data are used for regression modelling for each of the
curve and straight sections. The main input variables are the changes in gauge value and MGT values. Based on the regression analysis, the following measures about the output variables are drawn:

- The estimated variables of the linear regression model are determined without any constant value input. Accordingly, linear regression equations are produced for curves and straights. In addition, outliers are checked and removed to obtain a good linear regression fit. To show the results visually, observed values versus estimated values are plotted for curves and straights.

- The coefficient of determination (denoted by $R^2$) is a key output of regression analysis. It measures how well the regression line approximates the real data points. The $R^2$ can be calculated using Equation 5.15 in Section 5.4.2. Adjusted $R^2$ values are also identified to perfectly predict the goodness of fit of regression modeling.

- The standard error (denoted by $s_y$) is also an output of regression modeling. It provides an overall measure of how well the model fits the data. The standard error represents the average distance that the observed values fall from the regression line. It determines the error of the regression model on average using the units of the response variable. Smaller values are better because it indicates that the observations are closer to the fitted line.

Details of the output measures and results are provided in the next section.

**5.4.4 Results**

By running the regression analysis with the gauge value, Gauge (t), as the dependent variable and Gauge (t-1) and MGT (t-1) as independent variables, the regression coefficients if sample k is used for training purpose are presented below:
Table 5.3: Estimated variables of linear regression model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Curve data</th>
<th>Straight data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter value</td>
<td>Parameter value</td>
</tr>
<tr>
<td>α</td>
<td>1.075</td>
<td>1.186</td>
</tr>
<tr>
<td>β</td>
<td>1.462e-8</td>
<td>2.53e-8</td>
</tr>
<tr>
<td>Number of observations</td>
<td>270</td>
<td>270</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.997</td>
<td>0.987</td>
</tr>
</tbody>
</table>

Therefore, we retrieved the following expression for gauge (t+1):

For curves: \( \text{Gauge (t)} = 1.075 \text{ Gauge (t-1)} + 1.462e-8 \text{ MGT (t-1)} \) \hspace{1cm} (5.18)

For Straights: \( \text{Gauge (t)} = 1.186 \text{ Gauge (t-1)} + 2.53e-8 \text{ MGT (t-1)} \) \hspace{1cm} (5.19)

As shown in Equations 5.18 and 5.19, the relationship between MGT and gauge defects follows a linear fit. The observed data versus the estimated data of curves and straights are plotted in Figures 5.8 and 5.9 and show the accuracy of the model. Figures 5.8 and 5.9 also show the regression line of the test data. It describes the behavior of a set of data. In other words, it a line that best fits the trend of given data using regression model and results in estimating the R-squared value.
Figure 5.8: Observed versus estimated gauge values on test data for curves.

Figure 5.9: Observed versus estimated gauge values on test data for straights.
Table 5.4 shows a summary of the statistical variables for both curve and straight sections.

Table 5.4: Statistical variables of regression model.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Curve test data</th>
<th>Straight test data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.997</td>
<td>0.987</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.997</td>
<td>0.987</td>
</tr>
<tr>
<td>$s_y$</td>
<td>0.269</td>
<td>0.815</td>
</tr>
</tbody>
</table>

As Figures 5.8 and 5.9 and Table 5.4 show, the R-squared value indicates that the model explains most of the variability of the response data near its mean and is sufficiently accurate to predict tram track degradation.

5.5 Discussion

5.5.1 Comparison of Developed Models

As the main objective of the present study is to compare the performance of two different prediction models, time series and linear regression, a comparison of the results of both models was carried out to evaluate their performance and clarify the distinction between both approaches.

As discussed in previous sections, time series and linear regression models are developed for curves and straights sample data. The R-squared value is important to determine the accuracy of each prediction model. Therefore, it is found that the R-squared values of the linear regression are 0.997 and 0.987 for curves and straights, respectively (Refer to Table 5.5). In addition, an R-squared value is obtained for the time series approach of 0.890 for curves and 0.979 for straights (Refer to Table 5.5). Therefore, the results show that both models are highly accurate in predicting tram track degradation. This result clearly indicates that both linear regression and time series models work well for rail degradation prediction.
Table 5.5: Comparison of time series and linear regression models.

<table>
<thead>
<tr>
<th>Model</th>
<th>R² value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Curve data</td>
</tr>
<tr>
<td>Time series</td>
<td>0.890</td>
</tr>
<tr>
<td>Linear regression</td>
<td>0.997</td>
</tr>
<tr>
<td>Number of observations</td>
<td>270</td>
</tr>
</tbody>
</table>

The application of linear regression is more suitable for this research, as it is simpler and less complex than the application of the time series model. However, both models are applicable and show high accuracy of model prediction.

5.5.2 Maintenance Planning

It is important to predict how rail tracks degrade over time to help predict the maintenance activities needed in the future. The development of degradation prediction models can be used to identify the maintenance activities needed for degraded rails. It simulates the intervention of the maintenance applications based on the track condition and on the main model variables. In this research, the main model variables are the annual rail usage (MGT) and the gauge values of rails. Based on these variables, the degradation of rails is predicted every 6 months using two degradation models: a time series model and linear regression. The developed models can identify the level of degradation of rail tracks. Using the results from degradation prediction models, it can be identified whether the degraded rails need to be repaired or not for the next 6 months up to a year. If the predicted degradation of rail tracks is high, the rails will need to be repaired. If this is not the case, the rail tracks may not need any maintenance for the next 6 months up to a year. Therefore, the developed models can help predict the maintenance activities of degraded rails. This will result in the optimisation of maintenance activities, minimisation of maintenance costs, prevention of unnecessary maintenance actions and time saving.
5.6 Summary

This chapter has provided an overview of the modelling of rail degradation using MATLAB and SPSS software. A time series model, called ARMAX, and a linear regression were developed to predict the degradation status of tram tracks over the Melbourne network. Both models were applied for straight and curve sections on different data samples. The input and output variables and the modelling procedures have been explained. Details of the model development have been provided and the rail degradation prediction results from the models have been presented. A comparison of regression and time series models has been applied. The comparison shows that both models are applicable and highly accurate. However, the application of the regression model is simpler and more accurate in predicting the rail track degradation of the present research.
Chapter 6

Conclusions and Future Research

6.1 Conclusions

It is important for maintenance authorities to understand how light rail tracks deteriorate over time based on various factors. It is also important for them to know how the rail tracks behave in the long run in order to predict maintenance activities as accurately as possible. In turn, this will help avoid unnecessary maintenance activities, reduce expenses and save time.

This thesis has reported the results of research on the degradation prediction of light rail tracks in Melbourne. The study identified degradation prediction for curve and straight railroad segments based on their influencing variables. An analysis of the variables affecting rail degradation was conducted using SPSS software. The analysis of the collected data showed that rail usage (in MGT) and gauge defects are the main and most common variables in rail degradation. In addition, two models were proposed to predict rail degradation according to these variables: a time-dependent model and a linear regression model. The time series model was developed using MATLAB software, while the linear regression model was developed using SPSS software. The models were trained by different trained data and tested/validated using the rest of the data.

A comparison of the two models showed that both models followed the gauge value with a very low error percentage. Therefore, both models can be applied to predict tram track degradation. However, linear regression was more accurate and less complex than time series in model application.

In this research, other variables were not considered such as twist and alignment and we only focus on curve and straight segments. In future studies, the proposed models can be extended to these variables and can be analysed on other segments such as terminus and crossovers.
6.2 Research Contributions

The study focuses on modelling the degradation of rail tracks on Melbourne tram network. This research contributes specific knowledge of predictive degradation models applied in the rail sector. A review of these models was provided based on past studies in order to decide on the preferable model type for this case study. Therefore, the contributions of the present research can be summarised as follows:

- The thesis analysed and modelled the degradation of rail tracks using a stochastic time-series model. The first contribution is that the stochastic model was developed for light rail degradation prediction. The advantage of this model is that it can deal with large datasets and achieve more accurate degradation prediction results than other models based on the review of past research studies. It was also widely used in prediction studies, although it may require more understanding and clarification of its application. Therefore, this research proposed a stochastic degradation model, namely time series, using the variables affecting rail degradation.

- Another contribution is that the time series model was used for the first time in rail degradation prediction. This research covered a new model type which has not been used in rail degradation prediction before. The variables in this research were the gauge and annual rail usage (MGT) values of rail tracks. The application of this model was provided for straight and curve sections, due to the limited data on other sections. The thesis also proposed a linear regression model to evaluate the rail degradation prediction for straights and curves. Gauge and MGT values were also used in the model application.

- A comparison of both models, time series and linear regression, was applied. This comparison was important to identify the accuracy and how well each model works for Melbourne tram data. It showed that both models were highly accurate and worked well for tram data. However, the application of linear regression tended to be simpler, easier and more accurate than the time series model.
6.3 Future Research Directions

This research has focused on the degradation of tram tracks using gauge variables of curves and straight segments. In future, research can apply other model types to different rail sections. Possible research topics are as follows:

- Time series and regression models can be applied using other variables than gauge, such as twist and alignment.
- Time series and regression models can be applied on other rail segments such as termini, crossovers and sidings.
- Other degradation models can be developed, such as artificial neural networks, to compare the results with those of the proposed time series and regression models.
- Maintenance planning can be expanded in more detail and cover more topics, such as the development of maintenance optimisation models in order to identify the time when maintenance is needed for degraded rails.
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