Comparison of theoretical and experimental results for the directivity of panels and openings

John Laurence Davy (1), (2)

(1) School of Applied Sciences, RMIT University, GPO Box 2476V Melbourne, Victoria 3163, Australia
(2) CSIRO Materials Science and Engineering, PO Box 56 Highett Victoria 3190, Australia

ABSTRACT

A theoretical method has been developed for predicting the directivity of the sound that is radiated from one side of a panel, or an opening, which is excited by sound incident from the other side of the panel, or the opening, from a room or duct. This directivity needs to be known when one is predicting the sound level at an external position which is due to the radiation of sound from the roof, wall, ventilating duct or chimney flue of a factory. The theoretical method is essentially a two dimensional method, although it does include some three dimensional considerations. This paper compares this theoretical method with published experimental data. The theory presented in this paper agrees with the average trend of the experimental measurements. However the experimental results show significant variability about the theoretical predictions. This is believed to be due to both experimental and theoretical difficulties.

INTRODUCTION

This paper describes a theoretical method for predicting the directivity of the sound radiated from a panel or opening excited by sound incident on the other side. This directivity needs to be known when predicting the sound level at a particular position, such as that due to the sound radiation from a factory roof, wall, ventilating duct or chimney flue. The theory described in this paper has been compared (Davy 2008) with published experimental measurements of the directivity of the sound radiated from the ends of ducts in an anechoic room and out of doors and with experimental measurements of the directivity of a 6 mm thick window in the wall of a building.

In this paper, the theory is compared against the experimental measurements of David Oldham and his students. Oldham and Shen (1982) gave experimental measurements for an opening in the wall of a room. Shen and Oldham (1982), Oldham and Shen (1983) and Rowell and Oldham (1995a, 1995b, 1996) gave experimental measurements for single panels in the wall of a room.

THEORY

The effective impedance $Z_r(\phi)$ of a finite panel in an infinite baffle to a plane sound wave incident at an angle of $\phi$ to the normal to the panel is

$$Z_r(\phi) = Z_{wfi}(\phi) + Z_{wp}(\phi) + Z_{wft}(\phi)$$

(1)

where

$Z_{wfi}(\phi)$ is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of $\phi$ to the normal to the panel, on the side from which the plane sound wave is incident (this is the fluid loading on the incident side),

$Z_{wp}(\phi)$ is the wave impedance of the panel as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of $\phi$ to the normal to the panel, on the side from which the plane sound wave is incident (this is the fluid loading on the incident side),

$Z_{wft}(\phi)$ is the wave impedance of the fluid as experienced by the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of $\phi$ to the normal to the panel, on the side opposite to which the sound is incident (this is the fluid loading on the non-incident or transmitted side) and

$Z_{wfi}(\phi)$ is the wave impedance of the finite panel in an infinite baffle to a plane sound wave incident at an angle of $\phi$ to the normal to the panel, ignoring fluid loading.

It will be assumed that the fluid wave impedances on both sides are the same and the imaginary part of the fluid wave impedance will be ignored. That is

$$Z_{wfi}(\phi) = Z_{wp}(\phi) = \rho c \sigma(\phi)$$

(2)

where $\rho$ is the density of the fluid, $c$ is the speed of sound in the fluid and $\sigma(\phi)$ is the radiation efficiency into the fluid of one side of the finite panel in an infinite baffle, whose vibration is due to a plane sound wave incident at an angle of $\phi$ to the normal to the panel.

Reflections at the panel edges are ignored. The rms normal velocity $v_{rms}(\phi)$ of the panel due to a plane sound wave incident at an angle of $\phi$ to the normal to the panel which exerts an rms pressure $p_{rms}(\phi)$ is

$$v_{rms}(\phi) = \frac{p_{rms}(\phi)}{2 \rho c \sigma(\phi) + Z_{wfi}(\phi)}.$$  

(3)

The transmitted rms sound pressure $p_{rms}(\theta, \phi)$ which is radiated by the panel on the non-incident side to a receiving point which is at an angle of $\theta$ to the normal to the centre of the panel and a large distance from the panel (see Figure 1) is (Davy 2004)
\[ p_{\text{trms}}(\theta, \phi) \propto v_{\text{rms}}(\phi) \frac{\sin[ka(\sin \theta - \sin \phi)]}{ka(\sin \theta - \sin \phi)} \]  

(4)

where \( k \) is the wave number of the sound and \( 2a \) is the average length across the panel or opening in the plane containing the receiver and the normal to the panel or opening. Thus

\[ p_{\text{trms}}(\theta, \phi) \propto \frac{p_{\text{rms}}(\phi)}{2\rho c \sigma(\phi) + Z_{\text{rms}}(\phi)} \frac{\sin[ka(\sin \theta - \sin \phi)]}{ka(\sin \theta - \sin \phi)} \]  

(5)

Figure 1. Sound incident at an angle of \( \phi \) to the normal to a panel or opening and radiated at an angle of \( \theta \) to the normal.

The case where the incident sound is generated by a sound source in a room or duct is now considered. We assume that the sound pressure waves are incident at different angles \( \phi \) with random phases and mean squared sound pressures which are proportional to a weighting function \( w(\phi) \).

\[ |p_{\text{rms}}(\phi)|^2 \propto w(\phi). \]  

(6)

The weighting function is to account for the fact that sound waves at grazing angles of incidence will have had to suffer more wall collisions and therefore be more attenuated before reaching the panel. The total mean square sound pressure \( |p_{\text{trms}}(\theta)|^2 \) at the receiving point is

\[ |p_{\text{trms}}(\theta)|^2 \propto \int_{\pi/2}^{\pi/2} \frac{w(\phi)}{2\rho c \sigma(\phi) + Z_{\text{rms}}(\phi)} \left( \frac{\sin[ka(\sin \theta - \sin \phi)]}{ka(\sin \theta - \sin \phi)} \right)^2 d\phi \]  

(7)

The case when sound is incident from a source in a free field at an angle \( \theta \) to the normal to the panel and the panel radiates at all angles \( \phi \) into a room or duct is also of interest. In this case the weighting function \( w(\phi) \) is to account for the fact that sound waves radiated at grazing angles will have had more wall collisions and therefore be more attenuated before reaching the receiving position which is assumed to be a reasonable distance from the panel or opening which is radiating the sound. In this second case, we have to integrate over all angles of radiation \( \phi \) because of the reverberant nature of the sound. For this case, the impedance terms in the integral are functions of \( \theta \) rather than \( \phi \) and can be taken outside the integral. However in this study both cases are calculated using the formula for the first case which is shown above. This is because both cases should be the same by the principle of reciprocity and it is not clear which form of the formula is more appropriate.

For large values of \( ka \), the two cases of the formula will be similar. If \( ka \) is much greater than 1, the function

\[ \left( \frac{\sin[ka(\sin \theta - \sin \phi)]}{ka(\sin \theta - \sin \phi)} \right)^2 \]  

(8)

has a sharp maximum at \( \phi = \theta \) and is symmetrical in both \( \phi \) and \( \theta \) about the point \( \phi = \theta \). We can exploit these facts by evaluating the impedance terms for the first case at \( \phi = \theta \) and taking them out side the integral. This gives the formula for the second case.

To derive the angular weighting function, we assume that the sound source is distance \( b \) from the surface of the room containing the panel or opening and that the room width is \( g \) in the plane containing the incident sound ray (see Figure 2). If the sound ray is incident at an angle of \( \phi \) to the normal to the panel or opening, it travels a minimum distance of \( b \tan \phi \) parallel to the wall containing the panel or opening before hitting the wall. The sound which travels this minimum distance hits the wall approximately

\[ n = \frac{b}{g} \tan \phi \]  

(9)

times before reaching the panel or opening, where \( n \) is allowed to be a real number rather than an integer in order to give a smooth weighting function. If the sound absorption coefficient of the walls of the room is \( \alpha \), the sound intensity incident at an angle of \( \phi \) to the normal is proportional to

\[ w(\phi) = (1 - \alpha)^n = (1 - \alpha) \frac{Z_{\text{rms}}(\phi)}{2}. \]  

(10)

Equation (10) gives us the weighting function \( w(\phi) \). If \( \alpha \) is zero, a uniform diffuse field will be obtained. For rigid walled ducts, a value of \( \alpha \) equals 0.05 is recommended because this gives the best agreement between theory and experiment.

Figure 2. Calculating the number of wall reflections before sound hits the panel or opening at an angle of \( \phi \) to the normal.

In this study we use the radiation efficiency of a panel or opening of length \( 2a \) and width \( 2d \), which we approximate with the following equation (Davy 2004).

24-26 November 2008, Geelong, Australia

Proceedings of ACOUSTICS 2008

Acoustics 2008
\[
\sigma(\phi) = \begin{cases} 
\frac{1}{2\kappa ad} + \cos \phi & \text{if } |\phi| \leq \phi_1 \\
\frac{1}{2\kappa ad} + \frac{3\cos \phi - \cos \phi_1}{2} & \text{if } \phi_1 < |\phi| \leq \frac{\pi}{2}
\end{cases}
\] (11)

where

\[
\phi_1 = \begin{cases} 
0 & \text{if } ka \leq \frac{\pi}{2} \\
\arccos \left( \frac{\pi}{2ka} \right) & \text{if } ka > \frac{\pi}{2}
\end{cases}
\] (12)

and \(k\) is the wave number of the sound and \(2\alpha\) is the length of the panel in the direction of the receiver. \(\phi_1\) is the angle of incidence of the exciting sound pressure wave above which the radiation efficiency of a finite size panel differs from that of an infinite size panel. \(\phi_1\) is zero if the length of the finite size panel in the direction of radiation is less than half a wavelength.

For an opening with no panel in an infinite baffle we put \(Z_{\text{wp}}(\phi) = 0\). For a finite panel in an infinite baffle we use the infinite panel result for \(Z_{\text{wp}}(\phi)\). This result is expected to be the correct result when averaged over frequency, because this approach gives the correct result for point impedances when averaged over frequency and position on a finite panel.

\[
Z_{\text{wp}}(\phi) = m\omega \left[ i \frac{1}{2}(\omega/c) \sin^4(\phi) + \eta \frac{1}{\omega} \cos^4(\phi) \sin^4(\phi) \right]
\] (13)

where \(m\) is the surface density (mass per unit area) of the panel, \(\eta\) is the damping loss factor of the panel, \(\omega_c\) is the angular critical frequency of the panel and \(\omega\) is the angular frequency of the sound.

The directivity of the sound source is also included when it is in the duct or room. The sound source is modelled as a line source of length \(2r\) where \(r\) is the radius of the sound source. The directivity of the sound source is proportional to

\[
\sin^2 \left( kr \sin \phi \right) \sin \phi
\] (14)

where \(k\) is the wave number.

For angles of radiation close to 90° to the normal to the panel or opening, the effect of the diffraction by the panel or opening, or by the finite baffle in which the panel or opening is mounted, needs to be included (Davy 2007). \(p(\theta)\) is the ratio of the increased sound pressure to the sound pressure without the baffle for an angle of incidence or radiation of \(\theta\).

The baffle is of length \(2L\) in the plane containing the receiver (or source) and the normal to the baffle and of width \(2W\) in the direction at right angles to the above mentioned plane. Note that in (Davy 2007), the length and width of the baffle were assumed to be equal. The increase in sound pressure due to radiation (or incidence) of sound pressure of wave number \(k\) normally from (or onto) a panel or opening in a baffle is

\[
p(0) = 1 + p_w p_c
\] (15)

\[
p_w = \begin{cases} 
\sin(kW) & \text{if } kW \leq \frac{\pi}{2} \\
1 & \text{if } kW > \frac{\pi}{2}
\end{cases}
\] (16)

and

\[
p_c = \begin{cases} 
\sin(kL) & \text{if } kL \leq \frac{\pi}{2} \\
1 & \text{if } kL > \frac{\pi}{2}
\end{cases}
\] (17)

The limiting angle below which the sound pressure does not vary with angle of radiation (or incidence) is \(\theta_{\text{ad}}\). Notice that if \(L = a\), \(\theta_{\text{ad}} = \phi_1\). This means that both the radiation efficiency and the diffraction caused by a finite size panel differ from those of an infinite size panel for angles which are greater than the same limiting angle.

\[
\theta_{\text{ad}} = \begin{cases} 
0 & \text{if } kL \leq \frac{\pi}{2} \\
\arccos \left( \frac{\pi}{2kL} \right) & \text{if } kL > \frac{\pi}{2}
\end{cases}
\] (18)

There is no increase of sound pressure at grazing angles of transmission (or incidence). This means that

\[p\left( \frac{\pi}{2} \right) = 1.\]
(19)

\(p(\theta)\) is obtained by linear interpolation in \(\cos(\theta)\). Note that this is different from (Davy 2007) where the linear interpolation was carried out in \(\theta\).

\[
p(\theta) = \begin{cases} 
\frac{p(0)}{\cos(\theta) \geq \cos(\theta_{\text{ad}})} \\
p(0) \cos(\theta) + \frac{p_c}{2} \cos(\theta_{\text{ad}} - \cos(\theta)) \text{ if } \cos(\theta) > \cos(\theta_{\text{ad}})
\end{cases}
\] (20)

The relative sound pressure level \(L(\theta)\) is the sound pressure level in the direction \(\theta\) to the normal, at the measurement distance from the centre of the panel or opening, minus the sound pressure level normal to the panel or opening, at the same measurement distance. Thus

\[
L(\theta) = 10\log_{10} \left[ p_{\text{rms}}(\theta) \right] - 10\log_{10} \left[ p_{\text{rms}}(0) \right] p^*(\theta)
\] (21)

If the transmission is into the shadow zone, that is \(\frac{\pi}{2} < |\theta| \leq \pi\), then the above calculations are carried out for \(\theta = \frac{\pi}{2}\) and the product \(p_{\text{rms}} \left( \frac{\pi}{2} \right) p^* \left( \frac{\pi}{2} \right)\) in equation (21) is multiplied by the following diffraction correction.

\[
D(\theta) = \frac{1}{1 - k_c \cos(\theta)}
\] (22)
where
\[ z = \frac{1}{\frac{1}{L} + \frac{1}{W}}. \] (23)

In practical situations, scattering from turbulence and other objects will place a lower limit on the relative sound pressure level. Let \( L_{\text{max}} \) be the maximum value of \( L(\theta) \). It is assumed that the scattered sound level is \( L_s = 20 \text{ dB} \) below \( L_{\text{max}} \). The predicted observed relative sound pressure level \( L_o(\theta) \) is
\[ L_o(\theta) = 10\log_{10} \left( 10^{L(\theta)/10} + 10^{L_s - L_{\text{max}}}/10 \right). \] (24)

\( L_o \) would usually be expected to be greater than 20 dB

**SOURCE AND RECEIVER DIRECTIVITY**

Most of the initial comparisons between the experimental results and the theory described in this paper were for the case of openings and panels mounted in the wall of a room where the sound source was external to the room and the microphone was inside the room. In these experiments the sound source was always rotated so that its direction of maximum sound radiation was directed towards the opening or the panel. In this situation, the directivity of the sound source had no effect on the results. The microphone in the room was relatively omnidirectional and thus its directivity also did not need to be considered. Thus there was no need to use equation (14).

However, when the directivity of the sound radiated from the opening at the end of a duct was considered, it became apparent that the directivity of the sound source at the other end of the duct needed to be included and equation (14) was introduced into theoretical model. It was initially thought that \( 2\pi \) in equation (14) should be set equal to the diameter of the sound source if this value was known. However it was observed that this did not always produce the best agreement between theory and experiment.

For each duct and each frequency, \( 2\pi \) was set equal to value that made the average value over angle of radiation of the difference between experiment and theory equal to zero or as close to zero as possible. These values of \( 2\pi \) varied over a wide range. However, somewhat surprisingly, it was observed that the values of \( 2\pi \) tended to decrease with increasing frequency. The average value of \( 2\pi \) over a large number of experimental results was approximately the wavelength \( \lambda \) of sound in air. This made average value of \( kr \) approximately equal to \( \pi \).

One of the reasons for this strange result is that smaller sound sources have to be used as the frequency is increased in order to achieve constant sound power output. Secondly as the frequency increases, loudspeakers only radiate efficiently from a decreasing area around the centre of their cones because of wave motion in their cones. Thus the physics tends to require the use of constant \( kr \) sound sources.

The range of the \( kr \) values was investigated further by setting \( kr \) for each duct equal to value that made the average value over all angles of radiation and all frequencies of the differences between experiment and theory equal to zero. The standard deviation over all angles of radiation and all frequencies of the differences between experiment and theory was determined as an estimate of the goodness of agreement between theory and experiment.

Table 1 shows the values of \( kr \) and the standard deviations for 11 outdoor measurements on circular ducts with diameters ranging from 0.305 to 1.22 m (Neish 1997, Potente et al. 2006). The values of \( kr \) range from 1.55 to 3.3. The standard deviations range from 1.7 to 4.6 dB.

**Table 1.** Values of \( kr \) and standard deviations over all angles of radiation and all frequencies of the differences between experiment and theory for 11 outdoor measurements on circular ducts.

<table>
<thead>
<tr>
<th>Diameter of Duct (m)</th>
<th>Length of Duct (m)</th>
<th>Measurement distance (m)</th>
<th>( kr )</th>
<th>Standard deviation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.305</td>
<td>3</td>
<td>1</td>
<td>3.3</td>
<td>2.3</td>
</tr>
<tr>
<td>0.305</td>
<td>3</td>
<td>3</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>0.4</td>
<td>8</td>
<td>2</td>
<td>1.55</td>
<td>1.7</td>
</tr>
<tr>
<td>0.61</td>
<td>3</td>
<td>2</td>
<td>3.25</td>
<td>2.5</td>
</tr>
<tr>
<td>0.61</td>
<td>6</td>
<td>4</td>
<td>2.6</td>
<td>3.6</td>
</tr>
<tr>
<td>0.914</td>
<td>4.8</td>
<td>3</td>
<td>2.3</td>
<td>4.5</td>
</tr>
<tr>
<td>0.914</td>
<td>7.8</td>
<td>3</td>
<td>1.8</td>
<td>3.6</td>
</tr>
<tr>
<td>0.914</td>
<td>7.8</td>
<td>6</td>
<td>2.65</td>
<td>3.3</td>
</tr>
<tr>
<td>1.22</td>
<td>12</td>
<td>3</td>
<td>1.55</td>
<td>2.8</td>
</tr>
<tr>
<td>1.22</td>
<td>12</td>
<td>6</td>
<td>2.2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

**Table 2.** Values of \( kr \) and standard deviations over all angles of radiation and all frequencies of the differences between experiment and theory for 18 anechoic room measurements on ducts.

<table>
<thead>
<tr>
<th>Duct cross section</th>
<th>( kr )</th>
<th>Standard deviation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>85 mm diameter</td>
<td>4.1</td>
<td>2.9</td>
</tr>
<tr>
<td>85 mm diameter</td>
<td>5</td>
<td>3.1</td>
</tr>
<tr>
<td>80 x 80 mm</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>120 x 40 mm</td>
<td>2.75</td>
<td>2.7</td>
</tr>
<tr>
<td>40 x 120 mm</td>
<td>4.6</td>
<td>1.7</td>
</tr>
<tr>
<td>80 x 40 mm</td>
<td>4.1</td>
<td>2.9</td>
</tr>
<tr>
<td>40 x 80 mm</td>
<td>6</td>
<td>3.0</td>
</tr>
<tr>
<td>85 mm diameter</td>
<td>30</td>
<td>4.3</td>
</tr>
<tr>
<td>Pure tone excitation</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>112 mm diameter</td>
<td>3.37</td>
<td>1.1</td>
</tr>
<tr>
<td>120 x 120 mm</td>
<td>8</td>
<td>2.9</td>
</tr>
<tr>
<td>80 x 160 mm</td>
<td>8</td>
<td>2.5</td>
</tr>
<tr>
<td>160 x 80 mm</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>80 x 240 mm</td>
<td>16</td>
<td>3.2</td>
</tr>
<tr>
<td>240 x 80 mm</td>
<td>3.05</td>
<td>3.9</td>
</tr>
<tr>
<td>130 mm diameter</td>
<td>20</td>
<td>8.0</td>
</tr>
<tr>
<td>260 mm diameter flange</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>130 mm diameter</td>
<td>13</td>
<td>6.5</td>
</tr>
<tr>
<td>80 x 160 mm</td>
<td>6</td>
<td>5.1</td>
</tr>
<tr>
<td>160 x 80 mm</td>
<td>2.8</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 2 shows the values of \( kr \) and the standard deviations for 18 anechoic measurements on ducts with cross sectional dimensions ranging from 40 to 240 mm (Croft 1979, Sutton 1990, Dewhirst 2002, Li 2005). The values of \( kr \) range from 2.75 to 13 for ducts driven with third octave bands of random noise which are unflanged except for the thickness of their wall sound insulation. The flanged duct had a \( kr \) of 20 and the duct driven with a pure tone had a \( kr \) of 30. The standard deviations range from 1.1 to 8.0 dB. The three biggest standard deviations were for three of Li’s four measurements. Li measured at angles of one degree increments. It maybe that
his measurements picked up more deviations than the other coarser angular measurements.

For the scale model anechoic room measurement considered in the next section, the procedure described at the start of this section was repeated. The average value of $kr$ was 1.78 and this value was used for the theoretical calculations used in the next section.

**COMPARISON WITH EXPERIMENT**

Oldham and Shen (1982) conducted a scale model investigation of the sound radiation from a large aperture in a building. They used a box with external dimensions of 0.5 x 0.5 x 0.5 m which they rotated in an anechoic room. The internal dimensions of their box were approximately 0.3 x 0.3 x 0.3 m, but two of the inner walls were inclined in order to improve the diffusion of the sound field. Four piezoelectric tweeters having a frequency range extending to 40 kHz were placed in the four lower corners of the model room with their axes inclined to the walls. 1/3 octave band filtered white or pink noise was supplied to the loudspeakers to produce a reverberant field inside the model room. Spherical diffusers of diameter about 4 cm were hung from the ceiling in order to improve the diffusion of the sound field.

No reverberation times were given, so the absorption coefficient of the internal walls of the box was assumed to be 0.05 for the theoretical calculations in this section.

In their paper Oldham and Shen gave results for three of their different apertures sizes (0.1 x 0.05, 0.1 x 0.1 and 0.05 x 0.1 m) for the octave band frequencies from 2.5 to 40 kHz at angles in 10° increments from 0° to 90°. The first length of the two aperture dimensions is the length of the aperture in the plane of measurement. The differences between their experimental and the theoretical sound pressure levels relative to the sound pressure levels normal to the apertures, when averaged over all angles and all frequencies were -0.1, 0.7 and -1.6 dB respectively. The standard deviations of these differences were 1.6, 2.2 and 2.3 dB respectively.

![Figure 3](image1.png)

**Figure 3.** Comparison of theory with Oldham and Shen’s average results for an aperture in the wall of a room when $ka = 9.2$ as a function of angle of radiation relative to the normal.

Oldham and Shen (1982) observed that their results depended mainly on the value of the product of the frequency with the length of the aperture in the plane of measurement. They averaged all of their results which had the same values of this product. The difference between their average results and the theoretical results when averaged over all angles and all the different products of frequency with aperture length in the plane of measurement was 0.8 dB. The standard deviation of the differences was 1.4 dB. Figure 3 shows the comparison as a function of angle of radiation relative to the normal when $ka = 9.2$. Figure 4 shows the comparison as a function of $ka$ at an angle of radiation relative to the normal of 90°.

![Figure 4](image2.png)

**Figure 4.** Comparison of theory with Oldham and Shen’s average results for an aperture in the wall of a room at an angle of radiation relative to the normal of 90° as a function of $ka$.

Shen and Oldham (1982) measured the directivity of the sound insulation of a 5 mm thick concrete panel measuring 0.2 x 0.15 m and a 0.05 mm thick aluminium panel measuring 0.1 by 0.05 m using the box described above in an anechoic room. Their paper gives continuous level recorder plots from -90° to +90°. The author of this paper read off the values of on these plots at 15° intervals and averaged the values for the negative and positive angles with the same magni-
tudes. The differences between their experimental and the theoretical sound pressure levels relative to the sound pressure levels normal to the panels, when averaged over angles and frequencies were 1.0 and 1.5 dB respectively. The standard deviations of the differences were 6.2 and 2.0 dB respectively. Figures 5 and 6 show the comparisons as a function of $ka$ at an angle of radiation relative to the normal of $60^\circ$ for the 5 mm concrete panel and of $75^\circ$ for the 0.5 mm aluminium panel.

Figure 6. Comparison of theory with Shen and Oldham’s results for a 0.5 mm thick aluminium panel in the wall of a room at an angle of radiation relative to the normal of $75^\circ$ as a function of $ka$.

Figure 7. Comparison of theory with Oldham and Shen’s results for a 1 mm thick plexiglass panel in the wall of a room when $ka = 87$ as a function of angle of radiation relative to the normal.

The average differences appear to show reasonable agreement, but the 6.2 dB standard deviation shows that this is not the case for the thick concrete panel. The problem appears to be the inadequacy of equation (13) to properly model the wave impedance of a finite panel in the vicinity of and above the critical frequency of the panel. There has been some suggestion of this in previous comparisons made by the author on thin panels whose critical frequencies were near the high frequency end of the frequency range measured. It is much more obvious for a thick panel whose critical frequency is near the low frequency end of the frequency range measured.

Equation (13) is only strictly valid for the forced wave in an infinite panel. One of the problems with equation (13) when it is applied to a finite panel which Ljunggren (1991) has pointed out, is the spatial rise length. It can take a considerable distance from the edge of the panel for the forced bending wave to reach the velocity implied by equation (13). It is possible for this distance to be much greater than the dimensions of the panel. A second problem is that the waves reflected from the edges of the panel propagate with the free bending wavelength rather than the trace wavelength forced by the airborne sound. An attempt was made to model the vibration of the 5 mm thick concrete panel as the forced wave in an infinite limp panel and the rest of the vibration as free bending waves. Unfortunately this approach made the disagreement worse. It produced an average difference of -2.6 dB and a standard deviation of 8.3 dB.

The value of the damping loss factor used in the theoretical model for the concrete panel was 0.01 which, although it is at the upper end of the expected range, is acceptable for an in situ concrete panel. However, although the agreement for the aluminium panel is reasonable, this was only obtained by setting the damping loss factor to 0.1 which far too high for an aluminium panel. Again the inadequacy of equation (13) is believed to be the problem.

Oldham and Shen (1983) measured the directivity of the sound insulation of a 1 mm thick plexiglass panel measuring $0.3 \times 0.1$ m, a 2 mm thick plexiglass panel measuring $0.2 \times 0.2$ m, a 0.05 mm thick aluminium panel measuring 0.1 by 0.05 m, a 6 mm thick plexiglass panel measuring $0.1 \times 0.3$ m, a 6 mm thick plexiglass panel measuring $0.3 \times 0.1$ m and a 8 mm thick plexiglass panel measuring $0.3 \times 0.2$ m. Their measurements were made using the same box as described above and their paper gives results for the octave band frequencies from 1.25 to 40 kHz. The differences between their experimental and the theoretical sound pressure levels relative to the sound pressure levels normal to the panels, when averaged over angles and frequencies were respectively 0.4, 0.4, 1.3, 3.0, 4.1 and 5.3 dB respectively. The standard deviations of the differences were 1.6, 4.0, 2.6, 6.0, 7.3 and 7.4 dB respectively. The general trend is for both the magnitudes of
the averages and the standard deviations to become greater as the panel thickness increases and decreases the critical frequency. To obtain theoretical values at the coincidence peaks which were in reasonable agreement with the experimental peaks, damping loss factors of 0.23 and 0.1 were assumed for plexiglass and aluminium. These values are about one hundred times greater than the typical values of 0.002 and 0.001. Again these problems are believed to be due to the inadequacy of equation (13) in the vicinity of and above the critical frequency of the panel. Figures 7 to 10 show comparisons of theory with experiment for plexiglass panels of thickness 1, 2, 6 and 8 mm. Figure 10 shows the problems of modelling the behaviour of panels at and above their coincidence frequencies.

Rowell and Oldham (1995a, 1995b and 1996) used near field acoustical holography to measure the directivity of the sound insulation of homogeneous, profiled and composite panels for the octave band frequencies from 125 Hz to 4 kHz. The homogeneous panels that they measured were 6 mm thick aluminium panels measuring 0.92 x 1.2 m and 1.2 x 0.92 m. They also gave the results of far field measurements on a 6 mm thick aluminium panel measuring 2.4 x 1.2 m in one third octave frequency bands from 2 to 6.3 kHz. The differences between their experimental and the theoretical sound pressure levels relative to the sound pressure levels normal to the panels, when averaged over angles and frequencies were respectively -1.0, -0.6 and -0.1 dB respectively. The standard deviations of the differences were 2.6, 3.6 and 6.1 dB respectively. A damping loss factor of 0.23 which is equal to that used for plexiglass was used to obtain these results. The use of a large loss factor is required because the experimental coincidence peaks are not as large as expected and tend to disappear altogether at higher frequencies. This is discussed by Rowell and Oldham (1996).

The profiled panels measured were a quasi-sinusoidally corrugated 0.5 mm thick steel panel and three trapezoidally corrugated 0.7 mm thick steel panels. The directivity of the sound insulation was measured in both a plane at right angles to the corrugations and in a plane parallel to the corrugations. The differences between their experimental and the theoretical sound pressure levels relative to the sound pressure levels normal to the panel, when averaged over angles and frequencies were respectively, 0.3, 0.3, 0.8, -2.2, -0.5, -0.8, 0.2 and 2.3 dB respectively. The standard deviations of the differences were 3.5, 4.7, 4.1, 7.1, 4.7, 6.3, 3.1 and 8.6 dB respectively. The damping loss factor used for all the theoretical calculations on the profiled panels was 0.0004 which is in the normal range for steel. The magnitudes of the averages and the standard deviations in a plane parallel to the corrugations were equal to or larger than those in a plane at right angles to the corrugations. This is believed to be due to the much lower critical frequency parallel to the corrugations and the problems with equation (13) at and above the critical frequency. Figures 11 and 12 show comparisons of theory with Rowell and Oldham’s results for a quasi-sinusoidally corrugated 0.5 mm thick steel panel measured perpendicular and parallel to the corrugations respectively.

Rowell and Oldham (1996) used near field acoustical holography to measure the directivity of the sound insulation of thermally insulating material sandwiched between and bonded to two metal panels. The directivity of the sound insulation was measured in two planes at right angles to each other. The differences
between their experimental and the theoretical sound pressure levels relative to the sound pressure levels normal to the panels, when averaged over angles and frequencies were -0.9, 0.3, -1.8 and -2.5 dB respectively. The standard deviations of the differences were 5.7, 5.2, 4.8 and 4.5 dB respectively. The damping lost factor used for the theoretical calculations on the composite panels was 0.008. The critical frequencies used for the two panels were 500 and 700 Hz respectively.

**CONCLUSION**

On average, the theory presented in this paper agrees with the experimental calculations of Oldham, Shen and Rowell. On average, the theory presented in this paper agrees with the experimental measurements of Oldham, Shen and Rowell. On average, the theory presented in this paper agrees with the experimental data about the theoretical data is large. It must be remembered that the experimental measurements are difficult to make accurately because of breakout noise through the walls of the box and residual reflections in the anechoic room. Never the less, it is believed that equation (13) does not adequately model the impedance of a finite panel in the vicinity of and above the critical frequency.

**REFERENCES**

Croft, G. J. 1979, Noise Directivity of Exhaust Stacks, Final year thesis for the Honours Degree of Bachelor of Engineering, University of Adelaide, Adelaide, Australia. This thesis has been lost and Croft’s results only exist as a graph in Dewhirst (2002).


Dewhirst, M. 2002, Exhaust Stack Directivity, Final report, Final year design project for the Degree of Bachelor of Engineering, University of Adelaide, Adelaide, Australia.

Li, X. 2005, Milestone report for determination of the duct directivity in an anechoic chamber, University of Adelaide, Adelaide, Australia


Neish, M. J. 1997, Predicting sound directivity at a ventilation duct termination, Final year project, School of Mechanical Engineering, The University of Technology, Sydney, Australia.


Figure 12. Comparison of theory with Rowell and Oldham’s results for a quasi-sinusoidally corrugated 0.5 mm thick steel panel measured parallel to the corrugations at an angle of radiation relative to the normal of 90° as a function of ka .

The average across of all 27 average differences is 0.4 dB. The root mean square of the 27 standard deviations is 4.8 dB. Thus the theory does a reasonable job of predicting the average trend of the experimental data, but the variation of the experimental data about the theoretical data is large. It must be remembered that the experimental measurements are difficult to make accurately because of breakout noise through the walls of the box and residual reflections in the anechoic room. Never the less, it is believed that equation (13) does not adequately model the impedance of a finite panel in the vicinity of and above the critical frequency.