Self-formed cavity quantum electrodynamics in coupled dipole cylindrical-waveguide systems

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Abstract: An ideal optical cavity operates by confining light in all three dimensions. We show that a cylindrical waveguide can provide the longitudinal confinement required to form a two dimensional cavity, described here as a self-formed cavity, by locating a dipole, directed along the waveguide, on the interface of the waveguide. The cavity resonance modes lead to peaks in the radiation of the dipole-waveguide system that have no contribution due to the skew rays that exist in longitudinally invariant waveguides and reduce their Q-factor. Using a theoretical model, we evaluate the Q-factor and modal volume of the cavity formed by a dipole-cylindrical-waveguide system and show that such a cavity allows access to both the strong and weak coupling regimes of cavity quantum electrodynamics.

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References and links
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1. Introduction

Cavity quantum electrodynamics (CQED) describes the interaction between quantum-emitters and electromagnetic modes of a cavity [1, 2] and has been explored for weak and strong coupling regimes for different cavities [3], such as microspheres [4–6], microtoroids [7, 8] and microdisks [9]. The modification of different aspects of the emission of quantum emitters, such as the existence of peaks, spontaneous emission linewidth, level shift, or radiation pattern, in the vicinity of dielectric interfaces and slabs [10–14], dielectric cylindrical waveguides [15–18], and resonators [19–22] have also been considered. Existence of peaks in the emission of quantum emitters in the vicinity of optical fibers have been observed experimentally [15] and studied theoretically and associated with the whispering gallery modes (WGMs) of fibers [14, 18].

The coupling of the emission from quantum emitters into guided and radiation modes of optical fibers has been studied [14–18, 23–29], and identified as a potential platform for developing devices for quantum information science [24, 27, 29]. The key point behind all these examples is that the radiative properties of an atom are not intrinsic but rather determined through the interaction between the atom and the available electromagnetic (EM) modes. These EM modes are determined by the physical structure.

Considering only the first few resonances, Fussell et al. [18] recognized that the radiation peaks of a dipole at the vicinity of an optical fiber have non-Lorentzian and asymmetric profiles and associated them with the presence of skew rays (radiation fields with non-zero longitudinal wave-vector components). For this reason, optical fibers, while providing transverse confinement due to cylindrical symmetry, have to date been believed to lack the confinement along the length of the fiber (due to translational symmetry) required to form a cavity and therefore exhibit only cavity-like behavior [18]. An approach to cavity formation that has been explored is to perturb the refractive index along the waveguide by some means, for example: truncating the structure, as in the case of microdisk cavities [9, 30, 31] and optical nanowires [32]; producing periodic modulation of the refractive index, as in the case of Bragg gratings in optical fibers [24, 25] or Bragg-stack mirrors in micropillars [33, 34]; or by producing a highly prolate shape in optical fibers (bottle microresonators) [35].

Here, we consider a dipole-fiber system, consisting of a z-oriented dipole located on the surface of a step index cylindrical glass-air fiber, extended along the z direction and show that such system leads to resonance radiation peaks. We reveal that the dipole-fiber system at resonance itself can provide the longitudinal confinement required to form a cavity, described here as a self-formed cavity. We examine and provide an accurate description of the contribution of different radiation modes, including skew rays, in the formation of resonance peaks of a dipole-fiber system for a wide range of resonance orders. In particular, we also show the relation between the radiation modes and WGM of the underlying 2D microdisk cross-section of the fiber. We discover that, for higher order radiation peaks, the contribution from skew rays shift outside the peak, resulting in resonance peaks that are purely associated with the radiation of the WGM of the underlying 2D microdisk. In this sense, we demonstrate that a dipole-fiber system can form a 2D cavity without any extra requirement for confinement in the longitudinal direction. The formation of this cavity results from locating a single dipole on the interface of the fiber. The physics behind this is similar to the case of a planar dielectric microcavity, in...
which spontaneous emission enhancement and the formation of localized modes are reported despite a lack of confinement in the lateral direction [20–22].

By evaluating the Purcell factor, Q-factor, effective modal volume, and coupling and decay rates, we show that the cavity formed by a dipole-fiber system can in either the weak or strong coupling regimes of quantum electrodynamics. In addition, we also show that a dipole-fiber system can, in principle, be used as an ultra-sensitive platform for sensing applications. Hence, we demonstrate that the cavity formed by a dipole-fiber system can provide an accessible platform for CQED and sensing applications.

2. Theory

Previously [26], we had considered photon collection from a dipole into the guided modes of an optical fibre. In that case, we studied the role of orientation on the collection efficiency. Here, we consider the emission of a \( z \)-oriented dipole located on the surface of a step index cylindrical waveguide, extended along the \( z \) direction, into the radiation modes of the fiber. For such geometry we show that while the emission captured into the guided modes approaches an asymptotic value, the radiation modes show strong resonance peaks which are associated with the whispering gallery modes (WGMs) that form in the cross section of the fiber. These resonances exhibit strong Purcell enhancement, which is a function of the core diameter of the fiber, and lead to the formation of a two dimensional cavity.

Following Ref. [26], we expand the total electric field of the dipole-waveguide system using \( z \)-propagating plane waves as:

\[
E(x,y,z) = \sum_j a_j e_j(x,y) e^{i\beta_j z} + \sum_{\nu} \int_{0}^{Q_{\text{max}}} a_{\nu}(Q) e_{\nu}(x,y,Q) e^{i\beta_{\nu}(Q)z} dQ + BK, \tag{1}
\]

in which \( a_j, a_{\nu}(Q) \) and \( e_j(x,y), e_{\nu}(x,y,Q) \) are coupling coefficients and modal vector fields for forward guided and radiation modes, respectively, \( \beta_j \)'s are the corresponding propagation constants, \( Q = (D/2)(k^2 n_j^2 - \beta^2)^{1/2} \), \( D \) is the core diameter, \( k = 2\pi/\lambda \), and \( BK \) represents the contribution of backward guided and radiation modes. In general, guided and radiation modes are orthogonal to each other in the sense that

\[
\int_{A_{\nu}^-} \left( e_i \times h_j^* \right) \cdot \hat{z} dA = n_{\text{co}} \delta_{ij} \quad \text{and the discrete and continuous values of the propagation constants } \beta \text{ of these modes, respectively, are bounded by [36]:} \quad \\beta_j \leq \beta_{j_{\text{co}}} \text{ for guided modes and } 0 \leq \beta_{j_{\text{cl}}} \leq kn_{\text{cl}} \text{ for radiation modes, where } n_{\text{co}} \text{ and } n_{\text{cl}} \text{ are the refractive indices of the core and cladding, respectively. The coupling coefficients } a_j \text{ are:}
\]

\[
|a_j|^2 = \frac{\omega^2}{16N_j^2} |\left( e_j^* \cdot p_0 \right)|^2,
\]

following [26], we also consider \( p(r_0, t) = p_0(r_0) e^{-i\omega t} \) for the dipole function, where \( \omega \) is the transition frequency of the dipole, \( p_0 \) is the dipole moment with the strength of \( p_0 \) and \( r_0 \) is the position of the dipole. Choosing such a function implies that the dipole is always emitting at the frequency of \( \omega \), as it is continuously excited by a CW source. Such a choice is appropriate since it allows the study of the spectral behavior of fiber resonances (as shown here) independent of the dipole emission spectrum. It would be possible to consider a dipole that can change state or exist in a superposition of the ground and excited states, in which case the overall emission spectrum would be a combination of the dipole and fiber resonance emission spectra.

The power captured in each mode is

\[
P_j = |a_j|^2 N_j
\]

and hence the total emitted power of the dipole-waveguide system is the sum of the power captured into all guided and radiation modes.
Fig. 1. Radiation (blue) and guided (green) power of a $z$-oriented dipole, located at the interface of a step-index tellurite-air fiber. TE-WGMs resonances $(m, p)$ are shown. Insets show transverse distribution of $|E_z(x, y)|^2$, respectively from left, for a peak and off-peak using Eq. (1) and a peak using the FDTD package Meep [39]. The core-clad interface is shown by white circles and the position of the dipoles is shown by black dots. The position of the dipole in the far right inset is at the top.

\[
P_{\text{total}} = \sum_j P_j + \sum_{\nu} \int_0^{Q_{\text{max}}} P_{\nu}(Q) dQ. \tag{3}
\]

The total power calculating using Eq. (3) results in similar values to those obtained through the FDTD method [26].

We use Eqs. (2) and (3) to find the guided and radiation power of a $z$-oriented dipole located at the interface of a step-index tellurite-air optical fiber, as shown in Fig. 1. Both guided and radiation powers are normalized to the dipole radiation power in bulk tellurite glass, $P_{\text{Te}} = \mu_0 p_0^2 \omega^4 n / 12 \pi c$. Duo to the normalization to $P_{\text{Te}}$, the results shown in Figs. 1 and 2 are independent of the dipole strength $p_0$. Note that the ratio of the radiation power to $P_{\text{Te}}$ is equal to the Purcell factor. Tellurite is a soft glass with a high refractive index $n = 2.025$. Incorporating single emitters within tellurite step-index fibers has recently been reported as a platform for hybrid quantum-photonic devices [27]. The main results presented here are generally true for any choice of core refractive index. However, it is generally known that higher refractive index contrast leads to smaller mode volume. It has also been shown that higher core-clad refractive index contrast leads to enhanced emission of a dipole at the vicinity of an optical fiber [26]. We have chosen tellurite glass as it has a wide transparency window from 500 nm to 4 $\mu$m, is amenable to fibre processing and has a high refractive index [27].

In Fig. 1 as the core diameter increases; the guided power (green line) approaches an asymptotic value ($\approx 0.3$), whereas the radiation power shows strong resonance peaks which become narrower and higher. The guided power includes the contribution from all guided modes as they appear and does not show any correlation to the resonance radiation peaks due to the orthogonality of the guided and the radiation modes.

We investigate the physics behind the radiation peaks by using an independent method that explains the formation of WGMs in a 2D circular disk, which is a representation of a thin cross section of the fibre, [37, 38]. We solve Maxwell’s equations for electric and magnetic fields of the form $F_i(r)e^{i(\omega t - \mathbf{k} \mathbf{r})}$, where $i = r, \phi, z$. For such a disk, TE (with non-zero components;
$E_{x}$, $H_{y}$, and $H_{\theta}$) and TM (with non-zero components; $E_{y}$, $E_{\theta}$, and $E_{\theta}$) solutions of Maxwell’s equations, for electromagnetic fields propagating around the disk, have the dispersion relations;

$$J'_m(n_{cl}x)H_m(n_{cl}x) - q_sH'_m(n_{cl}x)J_m(n_{cl}x) = 0,$$

(4)

where $x = kD/2$, $J_m$ and $H_m$ are the Bessel and Hankel functions, and $q_s = (n_{cl}/n_{co})$ for TE and $q_s = (n_{co}/n_{cl})$ for TM modes. Solutions of Eq. (4) determine the location of WGM resonances ($kD/2$ values) that can be labeled by $(m, p)$, where $m$ and $p$ are the azimuthal and radial mode numbers, respectively.

Equation (2) implies that the emission of a $z$-oriented dipole only couples into TE-WGMs, since $E_{y}$ is zero for TM-WGMs. It is possible to excite the TM-WGMs by using $r$ or $\theta$-oriented dipoles, since both $E_{r}$ and $E_{\theta}$ are non-zero for TM-WGMs. However, the coupling of the emission of a $z$-oriented dipole into TE-WGM is the strongest, since the electric field has only one non-zero component, $E_{z}$, that aligns along the dipole direction. Here, we use a $z$-oriented dipole, which only excites TE-WGMs.

We observe that the fundamental $(m, 0)$ TE-WGM resonances at $\lambda = 700\text{nm}$ are located within the radiation peaks of the dipole-fiber system, see Fig. 1, which have been obtained independently through Eqs. (2) and (3). This indicates that the radiation peaks are related to the WGMs of the 2D microdisk with diameter equal to that of the fiber. Although the connection between the peaks and WGMs has been commented on before [14, 18], there has been no complete study on the details of this connection. Here we show that every radiation peak, associated with one resonance $(m, p)$, in fact consists of a class of modes with different skew ray radiation components and find the condition for which the contribution of skew rays diminishes. This allows us to explain the relation between Q-factor of the radiation modes, which can be measured experimentally, and those of the WGMs.

In addition, we have also constructed $|E_{z}(x, y)|^2$ of the radiation mode of the dipole-waveguide system for $D = 0.8\mu\text{m}$ (peak (5, 0)) and $0.87\mu\text{m}$ (off-peak), using Eq. (1), and for $D = 1.7\mu\text{m}$ (peak (12, 0)) using the FDTD package Meep [39]. The $|E_{z}(x, y)|^2$ distributions show 10 and 24 nodes for the peaks at $D = 0.8$ and $1.7\mu\text{m}$, respectively, which matches with the expected $2m$ nodes of the corresponding $m = 5$ and $m = 12$ WGM resonances. In contrast, the $|E_{z}(x, y)|^2$ distribution at $D = 0.87\mu\text{m}$, off-peak, does not show any such node pattern.

Similar behavior is observed when the core diameter is fixed and the radiation peaks are examined as a function of the dipole transition wavelength, indicating that different combination of $(D, \lambda)$ can be associated with one resonance $(m, p)$. This can be inferred from Eq. (4) if the material dispersion is negligible over the wavelength range of interest. Note that the Q-factor of a radiation peak is calculated by $Q = \lambda_{\text{res}}/\Delta\lambda$, where $\lambda_{\text{res}}$ and $\Delta\lambda$ are the resonance wavelength and linewidth of the radiation peak (for constant $D$), respectively.

The radiation power has been calculated by integrating the radiation power associated with each $\beta$ within the range of $\beta = 0$ to $\beta = \beta_{\text{max}} = kn_{cl}$, see Eq. (3). As $\beta$ is the propagation constant along the $z$-direction, the radiation power associated with values of $\beta \approx 0$ are completely contained within the transverse plane, while those associated with $0 < \beta < kn_{cl}$ consist of non-transverse radiation mode, i.e., skew modes. For these modes within the core region $k_t^2 + \beta^2 = (kn_{co})^2 = (2\pi n_{co}/\lambda)^2$, where $k_t$ is the transverse component of the propagation constant in the core region. Hence, the angle between the propagating direction of the radiation field and the $z$ axis can be defined as $\theta = \cos^{-1}(\beta/k_{n_{co}})$ implying that the condition $0 < \beta < kn_{cl}$ corresponds to $90^\circ > \theta > \cos^{-1}(n_{cl}/n_{co}) \approx 60^\circ$ (for tellurite glass with $n = 2.025$).

To explain the relation between the WGM of a 2D disk and the radiation modes, we compare the radiation modes associated with WGM (12, 0) and (29, 0) in Figs. 2(a) and 2(b), with Q-factors of 895 and $1.3\times10^5$, respectively. Both modes show asymmetric profiles with steeper leading edge compared to their trailing edges. Such asymmetry has been observed and associ-
Fig. 2. Radiation power of a \( z \)-oriented dipole located at the interface of a tellurite-air step-index fiber and due to the integral of all \( 0 < \beta < kn_{cl} \), (blue curves). Other peaks correspond to different ranges of \( \frac{\beta}{kn_{cl}} \) bins, i.e., \( 0 < \frac{\beta}{kn_{cl}} < 1 \) as labeled, and the summation of these peaks is shown (red circles). (a) and (b) correspond to TE-WGM resonances \((12,0)\) and \((29,0)\), as labeled by black circles.

ated with skew rays; rays with non-zero \( z \)-component of the wave vector [18]. The TE-WGM resonances, shown by vertical black lines and black circles, are located within the leading edge of the radiation peaks and to the left of the peak maxima. The separation between TE-WGM resonance and the radiation peak maximum is much larger for mode \((12,0)\), 98 pm, than \((29,0)\), 0.26 fm. For both resonances, we have evaluated the radiation integral in Eq. (3) for different bins of \( \beta \) values, e.g. \((0 - 0.02), (0.02 - 0.04), ..., (0.1 - 1.0) \times kn_{cl} \) for \((12,0)\) in Fig. 2(a), and presented them as a function of core diameter. The key significant conclusions are; I) overall radiation peaks \((12,0)\) and \((29,0)\) are the envelopes of underlying radiation peaks that are associated with different \( \beta \) bins (i.e., radiation modes propagating with different \( z \)-component wavevector), II) the \((12,0)\) and \((29,0)\) radiation peaks are mainly due to the contribution of radiation power associated with the first five bins of \( \beta \), III) curves associated with higher \( \beta \) bins are shifted to larger core diameters and they are much broader, IV) the curve associated with the lowest \( \beta \) bin matches the corresponding TE-WGM resonance, shown by black circle in Figs.
2(a) and 2(b). Hence the leading edge of the overall radiation peak is steeper than the trailing edge and curves associated with larger values of $\beta$ contribute to the trailing edge, determining the linewidth and the asymmetric profile of the overall peak.

Examination of the resonance peaks also reveals the main result of this study; the peak associated with TE-WGM resonance $(29, 0)$ is mainly due to contributions from a very small range of $\beta$, i.e. $0 < \beta < 1 \times 10^{-4} \times k_{\text{ncr}}$, compared to that of $(12, 0)$, i.e. $0 < \beta < 0.1 \times k_{\text{ncr}}$. These ranges of $\beta$s are equivalent to $90^\circ > \theta \gtrsim 89.997^\circ$ and $90^\circ > \theta \gtrsim 87.169^\circ$ respectively. This means that the peak associated with the $(29, 0)$ resonance has a very small contribution from non-transverse propagating modes (skew modes) and is almost purely radiating in the transverse plane, which explains the extremely small linewidth (of the order of fm) and ultrahigh quality factor (as shown later) of this peak. This clarifies the concept of self-formed cavity: a 2D cavity forms at high resonance orders, since only radiation modes with $\beta \approx 0$ contribute to the radiation peak.

At such a resonance, the radiation is localized within a transverse plane normal to the fiber axis and at the position of the dipole. This is similar to the case of a dielectric planar microcavity in which a localized mode forms in spite of no lateral confinement. For such a mode, similar to the above discussion, the divergence angle of the emitted radiation narrows as the spatial mode number (or the mode Q-factor) increases [22].

To qualitatively explain the above conclusion, we consider the radiation within the core region, for which $k^2 = (2\pi n_{\text{ncr}} / \lambda)^2 = k_r^2 + k_\phi^2 + \beta^2$, in which $k = (k_r, k_\phi, \beta)$ is the wave vector in cylindrical coordinates. We impose a resonance condition similar to one that is used in multilayer optical waveguides [40]: the phase accumulation associated with $k_\phi$, over a round trip of the circumference of the fiber cross section, should be $2l\pi$, i.e. $k_\phi \times \pi D = 2l\pi$, where $l$ is an integer. This results in the following relation between resonance diameter $D$ (at fixed wavelength) or resonance wavelength (at fixed core diameter);

$$D_{\text{res}} = 2l / \sqrt{(2\pi n_{\text{ncr}} / \lambda_{\text{res}})^2 - k_r^2 - \beta^2} \quad (5)$$

In this equation, both $\beta$ and $k_r$ are continuous and bounded by $0 < \beta(k_r) < (2\pi n_{\text{ncr}} / \lambda)$. The above equation leads to an important conclusion; for every mode $l$, there is a continuum class of WGM resonances that are associated with different values of $\beta$, skew rays, and $k_r$, spiral rays. For resonances where $\beta \approx 0$ and $k_r \approx 0$, the equation reduces to a ideal WGM resonance condition for a circular disk, which represents an electromagnetic (EM) field that is propagating around the circumference of the disk and evanescently decaying outside (due to the imaginary part of $k_\phi$) the disk, but without any power radiating in $r$ or $z$ directions. However, for resonances with non-zero $\beta$ or $k_r$, the EM field propagates around the circumference of the disk, but at the same time has power radiating in $r$ and $z$ directions. For these resonances as $\beta$ and $k_r$ increase, the resonance core diameter $D_{\text{res}}$ shifts to higher values for constant mode order and wavelength, which is consistent with the behavior observed in Figs. 2(a) and 2(b). A striking observation of Eq. (5) is that even for $\beta \approx 0$, i.e., no skew rays, there still exist a class of resonances, 2D spiral rays, for which the EM field is confined within the two-dimensional transverse plane, but radiating in the $r$ direction with different values of $k_r$. These values of $k_r$ correspond to different phase velocities along the $r$ direction, which determine how quickly the spiral arm of each wave opens as the wave propagates to infinity. Based on the behavior observed in Fig. 2 and Eq. (5), one can expect that the resonance core diameter associated with $k_r \neq 0$ shifts to larger values as $k_r$ increases, resulting in an asymmetric profile. This indicates that even for $\beta \approx 0$, one can not expect to achieve ideal WG resonances with Lorentzian shape and linewidths of Im($k_\phi$)/2Re($k_\phi$) [37, 38].

In addition to radiation modes for which $k_\phi$ is quantized (i.e. locked to the fiber circumference), there is another class of radiation modes for which all three wave-vector components
Fig. 3. (a) Purcell factor, (b) Q-factor, (c) effective modal volume, and (d) coupling coefficient and decay rate as functions of fiber core diameter at $\lambda = 700\text{nm}$. Calculations in (d) are for a quantum dot with a spontaneous emission lifetime of 1$n$s inside a tellurite glass with $n = 2.025$. The dashed line represents a decay rate of 1$GHz$ for the quantum dot used for this calculation [31].

$\langle k_z, k_q, \beta \rangle$ are continuous. This class of radiation modes are not locked to the structure and are orthogonal to radiation modes with locked $k_q$, and hence do not show any resonances and Purcell enhancement. They contribute to a uniform background radiation power as it is evident in Fig. 1.

The above discussion shows that higher order radiation peaks are mainly due to the formation of a 2D cavity and hence one should be able to use these peaks to study cavity quantum electrodynamic (CQED) effects. To investigate this, we have calculated the Purcell factor, $F_p$, Q-factor, modal volume $V_{\text{eff}}$, coupling coefficient, $g/2\pi$, and the decay rate, $\kappa/2\pi$, of the cavity formed by the dipole-fiber system, as a function of core diameter of the fiber as shown in Figs. 3(a)–3(d). As the core diameter increases so as the resonance orders. The data points in Figs. 3(a)–3(d) correspond to resonances; (12,0), (14,0), (16,0), … (28,0) and (29,0).

In studies where the cavity modes have been directly evaluated through analytical or numerical methods (e.g., [31]), the effective modal volume has been calculated using $V_{\text{eff}} = \int \epsilon(r)|E(r)|^2d^3r/\max[\epsilon(r)|E(r)|^2]$. Here, we have calculated $V_{\text{eff}}$ using $F_p =$
The modal volume of the radiation peaks which includes overall contribution from all skew and spiral WGMs. While both Q-factor and modal volume increase exponentially as functions of core diameter, the Q-factor increases faster, which results in an exponential increase of the Purcell factor. Note that the increase of the effective mode area and Q-factor for higher order resonance modes has also been observed in two dimensional planar microcavities [21].

Considering a quantum dot (QD) with a spontaneous emission lifetime of $\tau = 1$ ns, we calculate the coupling rate between the cavity and the QD, $g/2\pi$ [31] and the decay rate, $\kappa/2\pi$ as:

$$g/2\pi = (1/2\tau_{sp})\sqrt{(3\lambda_0^2/\tau_{sp}c)/(2\pi n^3 V_{eff})}$$

and

$$\kappa/2\pi = c/(\lambda_{cav}Q),$$

respectively, in which $\lambda_{cav}$ is the resonance wavelength of the cavity. Figure 3(d) shows that for fibers with core diameters up to 2.23 $\mu m$ the decay rate of the cavity is higher than the coupling rate, indicating that the cavity is in the weak coupling regime. However, for larger core diameters the cavity operates in the strong coupling regime. For comparison, we have also considered a decay rate of 1 $GHz$ for the quantum dot used in this study [31], which has been shown as a dashed line in Fig. 3(d). Such a decay rate corresponds to a linewidth of order of 1.6 $pm$ at the resonance wavelength of 700 nm, which is much higher than the linewidths of radiation resonance peaks (22.0)-(29.0) (last five data points in Fig. 3(d)). This indicates that for these peaks the overall decay rate of the quantum dot, which is a combination of the free space decay rate and the cavity decay rate, is entirely determined by the cavity rate since it is much narrower than that of the free space. This shows the suitability of a dipole-fiber system to study both strong and weak coupling regimes of CQED.

While the above discussion provides a phenomenological evidence that dipole-fiber system can operate in the strong coupling regime of CQED, it is possible to provide a direct evidence of the strong coupling by showing the vacuum Rabi splitting in the emission spectrum. In order to model this, we need to extend the model presented here to include dipoles that can change state or exist in a superposition of the ground and excited states. Such a model will naturally include the dipole decay rate and resonance frequency, which when combined with the decay rates and resonance frequencies of fiber WGMs will lead to Rabi splitting of the emission spectrum.

High Q-factors of the radiation peaks of the dipole-fiber system indicates that this configuration can also be used for refractive index sensing applications. We define the sensitivity of radiation peaks as the amount of shift per unit refractive index (RIU) change of the cladding, normalized to the peak’s linewidths (LW), and plot it as a function of core diameter (resonance modes (12, 0) to (29, 0)) as shown in Fig. 4. The sensitivity increases exponentially for higher order resonance modes. Considering that it is possible to locate the position of a resonance to 1/100 of its linewidth [41], the minimum shift that can be measured using the resonance peak (29, 0) in Fig. 4 (with the linewidth of order of 0.54fm) is of the order of 0.0054fm, which leads to a theoretical minimum detectable $\delta_\eta \approx 5 \times 10^{-10}$ for the dipole-fiber system considered here. These results suggest the potential of the dipole-fiber system as a platform for ultra-sensitive refractive index sensing. The Q-factor of the resonance modes considered here only includes radiation losses. In practice there are other sources of losses such as scattering or material losses that reduce the Q-factor of the resonances and hence the sensitivity of these resonances to refractive index changes.

Refractive index sensing based on the WGMs of cylindrical waveguides such as microcapillaries has been studied both experimentally and theoretically [42–46]. In these studies the WGMs of the cylindrical waveguide are excited either by extended laser beams of tapered fibers [42, 43] or by fluorescent emission of an ensemble of quantum dots [45] or dye-doped polymer, which have been coated on the inner walls of the capillary (see [46] and references therein for an overview of different methods). However, the key point that leads to the ultrasensitivity of the dipole-fiber system and distinguishes this work from others is the selective
Fig. 4. Sensitivity; shift per refractive index unit (RIU) per linewidth (LW) as a function of core diameter.

coupling of a dipole emission to those WGMs that have almost zero skew components ($\beta = 0$).

3. Discussion and conclusion

We have discovered that a 2D cavity forms for higher order resonances of the radiation power of a $z$-oriented dipole located at the interface of a longitudinally invariant cylindrical waveguide. For these resonances, the peak of radiation modes with non-zero longitudinal wavevector, skew modes, shift outside the resonance formed by the WGM of the microdisk cross-section of the waveguide. As a result, skew radiation modes, which exist due to the longitudinally unbounded nature of the cylindrical waveguide, do not contribute into high order WGM resonances. Based on a model that accounts for the total radiation and guided power of the dipole-waveguide system, we have evaluated the Purcell factor, Q-factor, effective modal volume, and coupling and decay rates and demonstrated that dipole-waveguide cavity can operate in both weak and strong coupling regimes of CQED. In addition, we have also shown that the dipole-waveguide cavity can be used as an ultra-sensitive architecture for sensing applications. Considering that atomic-level surface roughness quality can be achieved in fabricating and tapering glass fibers with micro cross-sections, we believe that the dipole-waveguide cavity provides a simple and practical platform for CQED and sensing applications.

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