Language Difficulties In Mathematics Courses For Students From Non-English Speaking Backgrounds In The Transition From Secondary To Tertiary Education

THESIS
SUBMITTED BY
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IN FULFILLMENT OF THE REQUIREMENT FOR
THE DEGREE OF
DOCTOR OF PHILOSOPHY BY RESEARCH

IN

THE SCHOOL OF EDUCATION

RMIT UNIVERSITY
ABSTRACT

This research investigates the role of language in tertiary mathematics and the difficulties faced by second-language learners in grappling with the specialised vocabulary and discourse features of mathematics. The study began in response to observations while teaching Foundation Studies courses at RMIT University, which revealed some of the challenges faced by learners of mathematics due to lack of proficiency in English. A survey of literature revealed that very few studies have investigated language difficulties faced by learners of mathematics, from Non-English speaking backgrounds (NESB), at the tertiary level, despite growing number of NESB learners in universities of the developed nations. Placed in this setting, the goal of the research became an exploration into language-related difficulties faced by NESB learners of mathematics at the tertiary level and led to the framing of open ended research questions.

A mixed method, interpretive, case study methodology was used to probe two of the components of language use that are of importance in learning tertiary mathematics namely, reading and writing. The research was conducted in three parts that looked at reading and writing from different angles and enabled the use of multiple methods for corroborating results by triangulation. Although conducted as a case study of one group of students at one institution, the participation of students from three academic years and the nature of the data collected provided an opportunity to involve a larger number of students than is feasible in a conventional case study. Furthermore, the method resulted in quantifying most of the data which enabled the use of simple descriptive statistics measures to identify emerging patterns that assisted in drawing conclusions and answering the research questions. The use of effect size calculations displayed the magnitude of difference between unequal groups, allowing for comparison of naturally occurring groups in the sample.

Three instruments were developed for the three parts of the study. A Mathematics Language Comprehension Test was developed to assess comprehension of mathematical language, a Mensuration Task was designed to study student writing in
mathematics, and a Linguistic Complexity Rubric was modified to analyse the reading and comprehension demands of senior secondary and tertiary mathematics textbooks.

As a result of the collated findings from three parts, this study has identified several language difficulties experienced by tertiary NESB students of mathematics, and the effect of language background on the nature and level of difficulty experienced. These findings have widespread applications to comparable teaching and learning situations. Several implications call for action, and recommendations for practitioners and future researchers are provided in the light of the various findings. Two unique outcomes emerged from the deliberations of this study. One is a Revised Error Analysis Model to explain the possible errors made by tertiary NESB students in mathematics, and the other is a Text Analysis Framework which provides suggested guidelines for selection of textbooks that are suitable for given student cohorts.
DECLARATION

This thesis is the result of my own research and contains no material that has been submitted for the award of any other degree or diploma, to any University. To the best of my knowledge, it contains no material published by another person unless duly acknowledged by references in the thesis.

Nancy Ann Varughese

December, 2009
ACKNOWLEDGEMENTS

This research and its culmination in this thesis would not have been possible without the cooperation and encouragement of many people whose support is gratefully acknowledged.

First among them is my supervisor Professor Dianne Siemon. I am indebted to her for her untiring support, assistance and understanding. She was always there for me, readily available for consultation at the shortest notice even when she was out of the country. Despite her very busy schedule, Dianne was always ready to accommodate my schedule in completing this research alongside full time work. Her enthusiasm and energy were inspirational. She was always particular about quality and detail and yet she let me do my own thinking.

Secondly, I would like to thank my second supervisor Professor John Izard for introducing me to the concept of effect size, which became crucial to the methodology of my research. His expertise in the field, and readiness to offer advice have been of great assistance in developing my knowledge and confidence in using and interpreting effect size.

I would like to take this opportunity to thank two of my colleagues, who willingly participated as experts in one of the tasks of this research, for their time and effort. Thanks are also due to all my other colleagues who allowed me time to collect data from their mathematics classrooms and to all the students from my classes and theirs, who willingly participated in the various tasks.

Lastly, I would like to thank my family for their understanding and unconditional support throughout this endeavour. Thanks are due firstly, to my husband Varughese, for his encouragement and inspiration without which I would not have commenced such a daunting project, for his support throughout the period of my study, and for his attention to detail in proof-reading of my whole thesis in the very short time given. Secondly, I would like to thank my children for putting up with my ever busy schedule of work and study, their encouragement despite having to neglect their needs at times. Special thanks are due to my son Vineeth for saving my thesis from disaster with his software skills, and my daughter Veena for last minute editing of my power point presentation for my final review.
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LIST OF ABBREVIATIONS

RMIT    Royal Melbourne Institute of Technology (Melbourne, Australia)
FS      Foundation Studies
NESB    Non-English Speaking Background
L1      First Language
TAFE    Tertiary And Further Education
VCE     Victorian Certificate of Education
IELTS   International English Language Testing System
AEI     Australian Education International
IIE     Institute of International Education (U.S.)
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CHAPTER 1: INTRODUCTION

“Mathematics education begins in language, it advances and stumbles because of language, and its outcomes are often assessed in language” (Durkin, 1991, p 3).

The importance of language in mathematics education is evident in the above observation. My research is a quest to find out how mathematics education at the tertiary level advances or stumbles because of language especially for learners from Non-English Speaking Backgrounds (NESB).

Any research stems out of the need to find a solution to a problem, or curiosity about an unknown or unexplained phenomena or to delve into something that took place in the distant past. In any case it is the nature of the problem or curiosity and the context in which it occurs in conjunction with the values and beliefs of the researcher that determine the nature of the research (Green & Browne, 2005). Hence I feel it would be appropriate to give an account of the background and context of this research at the very outset.

1.1 Rationale

This research was motivated by my experiences as a teacher of international students in the multicultural teaching and learning environment at the Royal Melbourne Institute of Technology (RMIT) University. This section establishes the rationale for this study. Firstly, it looks at the trends in mathematics education and educational research that led to the emphasis on the role of language in learning. Secondly, it describes the background of this study and the incidents that led to a pilot study that paved the way for this research.
1.1.1 Progress and shifts in mathematics education

In a highly progressive world, teaching, learning, and educational outcomes have been constantly redefined to enable students to develop learning skills that are conducive to life-long learning and application of the knowledge acquired in appropriate situations. This helps them meet the demands of the ever-increasing body of knowledge that has to be attained by an individual in a lifetime. A teacher is no longer viewed as the source of all knowledge but a facilitator of the learning and inquiry process (Crawford, 2000; Knowles, 1975). As a consequence, it is felt that teachers should no longer focus on learning as the acquisition of a body of knowledge, but rather on ensuring that students develop the skills and confidence to access, process, and manage information as well as communicate effectively in a variety of ways.

Considerable progress has been made concerning our collective understanding about the nature of children’s developing mathematical knowledge and about the nature of effective teaching, learning, and problem solving in a wide range of topic areas (Schoenfeld, 2002). In my view, this emphasis on ‘information’ and ‘communication’ adds to the importance of language in education. Information and communication technology has brought in a whole new dimension to teaching and learning. Mathematics education is no exception. These trends have translated to tertiary mathematics education with the availability of modern technology ranging from sophisticated mathematical and graphing software to podcasting and ‘remote’ classrooms through video conferencing. Facilities such as the ‘Access Grid Room’ (AGR) available in the Mathematics department at RMIT University enable group-to-group interactive mathematical communication between participating institutions and sharing of expertise across Australia. The AGR network enables member institutions to share seminars, lectures, courseware and multimedia resources remotely and interactively and also provides an avenue for collaborative research with peers within Australia and internationally (RMIT, 2008). As a consequence of these advances, greater demand is placed on communication skills and the ability to comprehend and process information.
Moreover, access to tertiary education is no longer restricted to the elite few. Nor is it limited by state or national boundaries. Expansion of the higher education sector coupled with reduced government funding has placed universities under increased pressure and as a result students are increasingly seen as ‘clients’ and education as a product for sale. Commencing with the *Colombo Plan* for cooperation with South and Southeast Asia at the end of World War II, sponsorship of international students to study in Australian universities paved the way for progressively imposed levies for private international students, which escalated to 55% of the fees by the 1980s (Atweh & Clarkson, 2001). Although there was a complete decline of subsidies by the 1990s, this did not deter the increasing number of international students in Australian universities.

In 2007 there were 455,185 full fee paying international students in Australia representing a growth of 19% over the previous year (Australian Government, 2007). Asia continued to be the largest source of international students with China and India accounting for 23.5% and 14% of the total enrolments respectively. Strong growth was recorded for a number of nationalities between 2006 and 2007 such as India (64%), Vietnam (44%), Middle East (43%), Sri Lanka (35%), Brazil (23%) and Nepal (241%). Furthermore, the higher education (39%) and VET (27%) sectors together accounted for 66% of the total international student enrolment for 2007. Only 6% of enrolments were in the school sector.

On the other hand, technological progress has improved international travel and communication, and globalisation has seen the rise of multinational companies with international workers and outsourcing work to other countries. For instance, I am aware of an American company set up in Australia, having its payrolls processed by accountants and Human Resources personnel in India, and this requires individuals from all these countries communicating and understanding each other’s accounting systems and principles. Consequently, international standards and compatibility of curricula across the globe are becoming increasingly crucial.
These trends have created increasingly multicultural classrooms in Australian universities with growing numbers of mathematics learners from Non-English Speaking Background (NESB). The emphasis on information and communication places greater demands on language and communication skills. There is an increasing number of students in our tertiary classrooms who are faced with the challenge of communicating for academic purposes in a language with which they have little facility. The increase in numbers of students from diverse backgrounds in tertiary classrooms, has intensified the need for culturally responsive pedagogical approaches to teach effectively in a multicultural and multilingual classroom (Gollnick & Chinn, 2002).

As a consequence it is imperative that all practitioners in general, and mathematics educators in particular, become aware of the nature and extent of the likely difficulties faced by this growing cohort of students in our tertiary education system. This can only be achieved by including the role of language in mathematics teaching as a priority area in the mathematics education research agenda.

1.1.2 Background of this research

Foundation Studies (FS) is an accredited, tertiary-level program at RMIT University, which is offered exclusively to international students who have completed all or most of their schooling in another country and wish to pursue higher education in Australia. The successful completion of this program earns students a Certificate, recognised for entry to Degree and Diploma programs by Australian universities and Tertiary and Further Education (TAFE) colleges as well as international higher education institutions. This program is run by various schools of RMIT University in preparation for various higher education pathways such as Engineering, Health, Business, Art & Design, Media & Communication or Architecture.

The FS students have the option of choosing four subjects in addition to English, which are relevant to the Higher Education pathways of their choice. Three
different mathematics courses are included in the options available. The School of Life and Physical Sciences at RMIT University is responsible for offering these Mathematics courses to students of the Applied Science, Information Technology, and Business streams of FS. They serve as bridging courses between school and university mathematics for these international students. They are planned at a level that is at least equivalent to the Victorian Certificate of Education (VCE) Year 12 mathematics courses, although some extra topics are incorporated into each stream. The extra topics are selected on the basis of their relevance to their chosen pathways in higher education in Australia and prepare students to take up the required university mathematics courses. Students are provided with a wide range of subject combinations, which could include one or more of the three mathematics courses available.

Over a 100 students enrol in FS mathematics courses each year and these include students from Asian, African, European, South American, and the Middle Eastern countries. Until a few years ago, the majority of students came from Indonesia, Malaysia, Hong Kong, China and Singapore, with a few students each from other countries such as Taiwan, Japan, Macau, Korea, Thailand, Vietnam, Mauritius, Russia, Turkey, India, Pakistan, United Arab Emirates, Colombia, Kenya, Botswana and Angola. However, over the last three years there has been a notable increase in students from Middle Eastern countries and the Indian subcontinent including India, Pakistan, Bangladesh and Sri Lanka. Coming from such vastly different backgrounds and generally from countries where the language of instruction is not English, most FS students have very little knowledge of English.

Furthermore, some students complete Year 12 or A Level mathematics prior to joining FS, while a few start at the end of Year 11 or equivalent, replacing their final year of schooling by FS. This produces a mix of mathematical ability levels in every FS classroom. Also, differences between educational programs of the various countries in subject content as well as approaches to teaching and learning add to the variation among students.
A FS mathematics classroom is therefore a multilingual, multicultural environment, where mathematics is taught in English by teachers who do not speak the first language of the students. During this year of transition from school to university and to a new culture in a new country, not only do these students have to deal with the concepts and terminology of mathematics in English, they also have to deal with the unfamiliar language and culture of the host country. As a consequence, these mathematics classrooms are ideal for the study of language-related difficulties faced by NESB students learning mathematics at the tertiary level.

My interest in this research was prompted by my experiences and observations in the classrooms and examination halls of FS at RMIT University. While most FS students were high achievers in mathematics in L1 and their process skills could be transferred across into mathematics in English, there were a number of situations in the day to day teaching where student responses or questions alerted me to the level of difficulty that NESB learners experience with the linguistic features of spoken and written mathematics that are taken for granted by English speakers. However, two incidents during examinations that were particularly instrumental in motivating this research study are recounted below.

**Incident 1**

I was supervising the final examination of a FS mathematics course at the end of the first Semester, when one of my students put up her hand and pointed to the following question on projectile motion.

\[
\text{A shot is fired vertically upwards from the ground with a speed of 98 ms}^{-1}. \\
\text{Determine:} \\
\text{a) The maximum height it will attain.} \\
\text{b) The time taken to reach the maximum height.} \\
\text{c) How long will it remain more than 249.9 m above the ground?}
\]

She wanted to know whether the question was written correctly. I checked the question and assured her that it was fine. Five minutes later I realised that she was still struggling
with the question as she raised her hand again and asked, “Do I have to find how long it was above 249.9 m?” I affirmed this, although I was puzzled as she did not appear to experience any difficulty with the other parts of the question, which involved mathematical concepts and calculations related to projectile motion. To my surprise, a few minutes later her hand went up again and she asked me, “Do I have to find how much time it was above 249.9m?” I nodded again and saw the enlightened expression on her face as she quickly settled down and proceeded to answer the question. I was quite perplexed by all this until, back at my table, I read the question again and it dawned on me that to this student, ‘long’ meant distance and she had great difficulty interpreting it as ‘time’. I could now see why she could not make sense of the question having calculated the maximum height in part (a).

Incident 2

In the second semester of the same year, I carefully constructed another question on projectile motion for the Applied Mathematics examination. It was the cricket season and to me the following question seemed very relevant and interesting at the time.

During a cricket match, the batsman hit the ball at an angle of 30° and it was caught by a fielder 65 m from the point of projection just when the ball reached ground level.

a. Find the velocity at which the ball was projected.
b. What was the maximum height reached by the ball?
c. Given that the boundary line was 8 m further, what should have been the angle of projection for the ball to have reached the boundary to score a six?

During the examination however, a number of hands went up and it became apparent that the Asian students could not relate to the notion of ‘scoring a six’ in cricket. Once more I realised the extent to which unfamiliar ‘cultural references’ could hinder the understanding of otherwise mathematically able students.
These two incidents sensitised me to the special needs of the growing number of international students in our tertiary classrooms. A study among first year mathematics students in a South African university had shown me that NESB students had difficulties with a number of mathematical terms such as ‘integers’, ‘perimeter’ and ‘multiple’ (Varughese & Glenncross, 1996). A survey of research literature in the field showed that very little research had been conducted on the difficulties experienced by tertiary NESB students of mathematics. Apart from my interest in this field, it now became apparent that research into the relationship between language and mathematics was essential for effective teaching and learning to take place in a multicultural classroom. This motivated my research with the main aim of probing the extent and nature of these language-related difficulties encountered by NESB learners of tertiary mathematics in Australia.

1.2 A Preliminary Investigation

As a first step, I talked individually to students from different language backgrounds about the language difficulties they faced. However, it was not easy to converse with students about language difficulties when they are not fluent in English and I could not speak their first language. So my strategy was to talk to a few students from the same language background as a group. This was more successful as they would discuss among themselves in their own language and between them come up with a response in English. As a result of these informal conversations I was able to get a glimpse into some of the differences between languages and the potential difficulties these could cause in learning mathematics and performing well in assessments. A few examples of the language-related difficulties that emerged from these conversations are reported below.

Example 1

A word with two different meanings in English may be translated into two distinct words in other languages. One example was chosen and conversations with various groups revealed that ‘round’ (to two decimal places) and ‘round’ (circular) translate respectively to ‘gembu’ and ‘enkei’ in Japanese, ‘pembulatan’
and ‘bulat’ in Indonesian, and ‘tqurab’ and ‘daira’ in Arabic. That is, one word in English with two distinct meanings was represented by two different words in these languages.

Example 2

Differences in word order in two languages can cause loss of meaning in translation and contribute to misunderstanding, for example, ‘5 is greater than 3’ translated to Hindi would be ‘5, 3 se bada hai’ (5, 3 than greater is), or alternatively, ‘3 se 5 bada hai’ (3 than 5 greater is).

Example 3

A Chinese student, who performed very well in FS mathematics courses with nearly perfect scores on every examination, said that he had no difficulties with the concepts or skills required but usually spent an hour of the two hour examinations trying to comprehend the questions. In fact, I had observed him consistently referring to a bilingual dictionary both in classes and in examinations.

Example 4

Another story that emerged was recounted by a Middle Eastern student, who had to skip a question in a test because he could not comprehend the ‘intersection’ of two curves. He looked it up in a bilingual dictionary and the translation he obtained was a ‘traffic intersection’. He gestured to show that as it had been described as a circle with traffic lights in the dictionary, and the question did not make sense.

It became clear however, that such interviews would be very time consuming and could only provide very limited information, as the students who needed to be targeted did not possess the proficiency in English to be able to compare linguistic features of two languages or identify the pitfalls of miscomprehension.

In exploring literature, I found that a range of methods has been used by studies investigating the difficulties experienced by children learning mathematics in a
second language. For example, Lean, Clements, & Del Campo (1990) observed that while the one-to-one clinical approach is most suited for investigations involving early grades of primary school, it can be time consuming and expensive to implement. However, they advocated the use of pencil-and-paper tests with upper primary school students as this generates larger data sets than is feasible by the clinical approach. Jones (1982) developed the strategy of using tests which controlled for computational skills by using small numbers and varying the complexity of semantic structure. This strategy was used to investigate the linguistic and pedagogical factors affecting children’s understanding of arithmetic word problems. Eliminating the need for computational skills in these tests ensured that the difference in performance was caused by linguistic and pedagogical factors rather than differences in mathematical ability. It was felt that these strategies would be more suitable in the current situation and would enable me to obtain much more information from a larger number of students in a much shorter time.

In my position at RMIT University, I have been teaching FS mathematics courses for over seven years. In 2005 I also taught Mathematical Methods, which is a VCE (Yr 12) course offered to mature-age Australian students through the TAFE sector of RMIT University. This group included students from various backgrounds including migrants who were born or brought up in Australia. All students in this group were fluent in English. However, VCE students in TAFE are often returning to school after a break in education for various personal or health reasons, or because they wish to upgrade their qualifications after having worked for some time. Straber and Zevenbergen (1996) identify post-compulsory further mathematics education as a largely neglected and under researched area of mathematics education. They state that many of the students in this sector are adults returning to study and that problems faced by many learners are further exacerbated when gender, language, and cultural differences are considered. As I had access to the FS and VCE classes as part of my teaching schedule, I found myself in an ideal position to study the language difficulties faced by FS students. In particular, this situation allowed me to compare the performance of students who are proficient in English albeit with other setbacks in education (the
Prior to commencing formal study, I undertook a pilot study. Approximately 20 VCE students and 20 FS students were asked to complete a short six-item mathematics language test. In this test, students were asked to record relatively straightforward mathematics statements with potentially confusing language in symbols. For example, the expressions ‘the product of $x$ and $y$ is 24’ and ‘the sum of $a$ and $b$ is not less than 24’ were in two of the test items. Interestingly, both groups experienced similar difficulties with the language, although FS students seemed to fare slightly worse.

The observations derived from the pilot study are summarised in Table 1.1 below. As indicated above, the mathematical content of these test items was restricted to very basic arithmetic or algebraic operations on the grounds that any incorrect responses could be attributed to difficulty in comprehending the language used in the item (Jones, 1982).

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<tr>
<td></td>
<td></td>
<td>FS (N = 21) VCE (N = 20)</td>
</tr>
<tr>
<td>1</td>
<td>Product</td>
<td>48 85</td>
</tr>
<tr>
<td>2</td>
<td>Twice as many</td>
<td>29 40</td>
</tr>
<tr>
<td>3</td>
<td>Difference between Three times the</td>
<td>57 65</td>
</tr>
<tr>
<td>4</td>
<td>Sum Not less than</td>
<td>38 50</td>
</tr>
<tr>
<td>5</td>
<td>More than Half of</td>
<td>62 65</td>
</tr>
<tr>
<td>6</td>
<td>Sum At least More than Difference of</td>
<td>19 25</td>
</tr>
</tbody>
</table>
Results showed that the international FS students experienced greater difficulty than VCE students on all test items. However, both groups had difficulties comprehending Items 2, 4, and 6. The negative form of inequality, ‘not less than’, seemed to cause difficulties in Item 4 and the combination of four terms and expressions in Item 6 seemed to challenge both groups considerably with only 19% of the FS students answering it correctly. It was apparent that the linguistic features of the test items did have a bearing on performance on this simple mathematics test. This convinced me that a more in-depth investigation of the nature and extent of the difficulties experienced by these students was warranted. The findings and recommendations of such a study would provide valuable and much needed knowledge about the role of language in tertiary mathematics teaching and learning. The two aspects of language use that I was interested in, were reading and writing as these were the most relevant to undertaking tertiary study. I sought various ways to investigate the difficulties experienced by NESB students in these two aspects, leading to the subsequent design of the study. The resulting research study and its findings and recommendations are presented in this thesis.

1.3 Structure of the Thesis

The thesis is presented in eight chapters. Chapter 1 provides an introduction to the main study and details the rationale and background for the study in the context of the progress and shifts in mathematics education practice. It introduces the setting and relates some incidents that raised my curiosity and prompted me to carry out the informal interviews and the pilot study, the results of which are briefly illustrated here.

Chapter 2 is a survey of relevant literature related to the two aspects of this research namely, reading and writing in mathematics. Firstly, it traces the shifts in research paradigms over time and the current trends and priorities of mathematics education research in general with a view to establishing the increasing importance of information and communication technology, and
examines the role of language in learning in particular. It then reviews recent research linking language and mathematics in the context of multilingual classrooms, the language of texts and assessments that students are required to comprehend, and student writing in mathematics. This is examined in terms of mathematical genre, mathematics specific vocabulary, readability and linguistic complexity, culminating in a list of language-related difficulties in this field. This is followed by a discussion of theories of second language acquisition and error analysis in mathematics problem solving as this would identify the hurdles in acquiring a second language and the possible errors in using this language in the learning of mathematics. Lastly, the research questions that motivated the design and conduct of this study are presented at the conclusion of this chapter.

Chapter 3 begins with methodological considerations and a documentation of the theoretical frameworks that informed the selection of an appropriate research design and the procedure for the study. With a focus on the reading and writing aspects of language relevant to tertiary mathematics learning, it was decided to address the research questions in three parts. Firstly, by investigating language use in tertiary mathematics textbooks (Part I), secondly by examining student responses to a test of mathematical language (Part II), and thirdly by a detailed examination of student responses to a mathematical writing and interpretation task (Part III). The methods employed for each part of the study are explained in this chapter including a description of the participants of the study. It introduces the various instruments used for data collection and details the methods of data analysis employed for each. Ethical considerations and issues of trustworthiness are discussed in detail.

The findings of the three parts of the study are presented in the next three chapters, with one chapter devoted to each part.

Chapter 4 presents the results of Part I of the study which investigated the language used in mathematics textbooks and the challenges posed for tertiary mathematics students in the use of these textbooks. Three VCE Mathematical Methods textbooks and three first year university Mathematics textbooks were analysed for this purpose. The analysis involved an examination of the
readability levels and linguistic complexity of each textbook. This resulted in a comparison of readability tests and the development of a framework for the analysis of texts, which is presented in Chapter 7.

Chapter 5 presents the results of Part II which involved the administration of a Mathematics Language Comprehension Test with a view to studying the impact of the language of test items on student achievement. Two versions of the test were administered to FS students at the beginning and end of the year-long academic program and also to a reference group of VCE students. Various comparisons are made between these groups, as well as between gender and language groups.

Chapter 6 presents the results of Part III which explores the challenges faced by NESB students in writing and interpreting written texts. This was done by analysing written descriptions of a compound two dimensional geometric figure and evaluating the sketches produced from these descriptions. Student writing was systematically analysed and emerging patterns were categorised in terms of characteristics observed in the responses. The vocabulary used by each participant was also analysed. Associations between the various characteristics of the written descriptions and the corresponding sketches were examined and comparisons were made between gender and language groups.

Chapter 7 brings the collated findings together in a discussion of results and conclusions from all the three parts and inferences are drawn with reference to the literature. Two important outcomes of the research are presented. The first is a revised error analysis model based on the work of Newman (1977), to explain the language-related difficulties faced by tertiary NESB students. The second is a framework for evaluating textbook suitability. Responses to the individual research questions are provided from the collated findings and the models developed in this study.

Lastly, Chapter 8 presents an overview of the research. The aims of the research are reviewed and the adequacy of the methodology in addressing the research questions is discussed. This is followed by implications for practitioners and
future researchers. The implications for policy makers and teachers are discussed in terms of policy decisions and course design. Implications for authors are also examined in the light of the findings of the study. Avenues for future research that have been opened up by this study are considered and several suggestions put forward. The significance of this study and its limitations are discussed.
CHAPTER 2: LITERATURE REVIEW

The survey of literature presented in this chapter is a reflection of the exploratory nature of my research. Commencing with a search of literature for studies on language difficulties in learning mathematics, my quest spread out in various directions as the study progressed. I looked back at how interest developed in research in this field, investigated research paradigms for methodological considerations, sought out recommendations of experienced researchers in the development of instruments and appropriate methods of data analysis, and finally, looked for theoretical perspectives that could help explain and develop grounded theory from the findings. All aspects of this survey of literature are reviewed and organised into five main sections in this chapter, culminating in the research questions which guided this study.

The first section (2.1) looks into the changes over time in the field of mathematics education research corresponding to the trends and shifts in mathematics education examined in Section 1.1. This historical perspective is discussed under three subsections looking at shifts in research paradigms over time, factors that caused shifts in research priorities, and growing interest in the role of language in learning during the last two decades of the twentieth century. Having identified this area as a priority, the subsequent sections focus on examining the link between language and mathematics.

The second section (2.2) reviews more recent research on language and mathematics. Increase in numbers of multilingual and multicultural classrooms around the world appears to have prompted interest in the link between language and mathematics as shown by some of the relatively recent studies. Various social and political factors have contributed to the growth of multicultural environments resulting in bilingual or multilingual classrooms and a number of studies have investigated mathematics learning in these classrooms. These studies have focused on various aspects of language use in the teaching and learning of mathematics. However, the reading and writing components of language were felt
to be more relevant to the tertiary level of mathematics learning which involves reading and comprehending texts and assessments, and writing mathematically. The studies that are relevant to this research are briefly reviewed under three subheadings namely, teaching mathematics in bi/multilingual classrooms, the language of texts and assessments, and student writing in mathematics.

While the first two sections trace the development and nature of research in mathematics education in general and the link between language and mathematics in particular, Section 2.3 focuses on the findings that have emerged over the years about language use in mathematics. The discourse features of mathematics are discussed in terms of the ‘language of mathematics’ as well as ‘language in mathematics’. This is followed by a subsection that discusses the genres and register of the context specific language used in mathematics. This leads on to one of the foci of this study, namely, the difficulty level of mathematical texts. Difficulties can be experienced in reading or comprehending and can be caused by readability features such as word or sentence length as well as by linguistic features such as syntactic or semantic structure of a sentence. Hence, two subsections are devoted to literature on readability and linguistic complexity of written text.

The fourth section (2.4) considers the language-related difficulties in mathematics that have been identified in the literature and reflects on the bearing these difficulties could have on students from Non-English Speaking Backgrounds (NESB). In a bid to gain better insight into such difficulties faced by second language learners of mathematics, literature relating to theories of second language acquisition and error analysis are surveyed.

Section 2.5 summarises the literature review and establishes a rationale for the study to be reported here. The chapter concludes with the research questions framed in the light of these observations.

As discussed earlier, my search for studies linking language and mathematics took various directions as the review progressed. One of them was a historical
perspective as I sought to find how interest developed in this area of research. This is presented in the first section that follows.

2.1 Mathematics Education Research: Historical Perspectives

In the United States, Schoenfeld (2002) observes that “mathematics education began to coalesce as a discipline only a few decades ago, with its first professional meetings and journals appearing in the late 1960s and early 1970s. Its growth since then has been nothing short of phenomenal” (p. 483). As society has moved from an industrial age, through an age of electronic technologies towards an age of biotechnologies, research in mathematics education has found it necessary to move beyond simple cause and effect models to theories that view all of students, teachers, classrooms, courses, curricula, learning tools and minds as complex systems (Lesh, 2002). In other words, educational research has changed with the times and it has been recognised that clinical experiments and purely quantitative methods do not necessarily account for interactions within this complex system consisting of individuals who respond differently to teaching materials and learning environments.

2.1.1 Shifts in research paradigms

According to English (2002) it was only towards the end of the 20th century that emphasis shifted from purely quantitative and experimental designs to qualitative, phenomenographic, ethnographic paradigms in the US. Bauersfeld (1980) identified Erlwanger’s dissertation (Erlwanger, 1975) as a turning point for the emergence of the discussion of human interaction in the mathematics classroom (Bauersfeld, 1980). Carver (1978) made a case against using inferential statistical procedures to investigate best practice in mathematics classrooms as this could not reflect the individual responses and human interactions. This recognition saw the rise of alternative approaches to research such as individual interviews that could provide rich and in-depth information into learning processes. For instance, the Newman (1977, 1983) strategy of post-test interviews to analyse the errors in mathematical problem solving was acclaimed and used by other researchers in
Australia and other countries (Clements, 1980; Ellerton & Clarkson, 1996; Watson, 1980). Such trends also saw researchers increasingly become participants in the research settings rather than outside observers. Bishop (1991) called attention to the actual teaching process in classrooms during this crucial stage in the evolution of mathematics education. With this shift in thinking among educational researchers around the world, the last two decades of the 20th century saw mathematics educators acknowledge that all forms of mathematics education are surrounded by individual and cultural constraints that need to be taken into account.

Resultant shifts in paradigms saw an increasing use of qualitative as well as mixed methods which were combinations of both quantitative and qualitative methods. For example, Hart, Smith, Swars and Smith (2009) found that of 710 articles on research into mathematics education published in six journals from 1995 to 2005, only 21% were purely quantitative in nature, while 50% were qualitative and 29% used mixed methods. Although policy makers preferred experimental studies and empirical evidence, researchers and teachers welcomed the flexibility of alternative paradigms (English, 2002) as they offered better insight into the human interactions involved in individual situations.

2.1.2 Emergence of language in learning as a research priority

An important goal of research is to look beyond the immediate and find new ways of thinking about problems and potential solutions rather than quick fix solutions or answers to specific questions (Lesh & Lovitts, 2000). English (2002) identifies four catalysts for the shift in research priorities namely, national and international mathematics testing, influences from social, cultural, economic, and political factors, increased sophistication and availability of technology, and increased globalisation of mathematics education and research. Shifts in emphases in mathematics teaching from teacher-centred, formalistic approaches to student-centred, heuristic approaches (Schoenfeld, Kaput, & Dubinsky, 1998), advancement in technologies (Niss, 1999), and the emergence of
ethnomathematics linking mathematics and culture (Gerdes, 1996), were all factors that contributed to this change in priorities.

One of the priorities that emerged was the recognition of the importance of language in mathematics learning. This has been brought about by two major factors: the emphasis on problem solving, heuristic approaches, and the enormous increase in the numbers of second language learners in most developed nations.

Research into the first factor has investigated numerous aspects of mathematical problem solving by children, and the role of problem solving in mathematics education. Studies have looked into problem solving by the very young from as early as kindergarten stages (e.g., Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993), the importance of problem solving in the curriculum (Schoenfeld, 1985, 1992), the role of metacognition in children’s problem solving (Siemon, 1993), and the role of comprehension in problem solving (Cummins, Kintsch, Reusser, & Weimer, 1988). It has emerged from all these studies that problem solving is important for learning mathematics with understanding, and metacognition and comprehension are important in problem solving. In my view, both metacognition and comprehension are reliant on language proficiency, and consequently point to the importance of language in the learning of mathematics.

The second factor of increased numbers of second language learners in classrooms all over the world, can be attributed to several reasons. Political or social changes have brought students from minority groups into mainstream classrooms. For example, political changes in South Africa saw the end of segregated ‘black’ and ‘white’ education, and social reforms in other countries provided educational opportunities for language minorities, resulting in multilingual classrooms. Globalisation resulted in an increase in new migrants and expatriate workers in many countries and the current state of unrest in many parts of the world led to an increase in refugees which has contributed to the number of second language learners in host countries. Another recent trend is the influx of international students to tertiary institutions in developed countries. In my view, educational opportunities which were initially provided to foreign students with varying intentions such as specialisation in a discipline, assistance
to politically or economically unstable countries, or enhancement of diplomatic ties, have now become a multi-billion dollar ‘service export’ marketed and promoted by educational institutions and governments alike.

Data released by the U.S. Institute of International Education (IIE) in its annual *Open Doors* survey shows a consistent increase in international student enrolment in the U.S. since the 1960s. In the 2007/8 academic year, international student enrolment in U.S. colleges and universities increased by 7% to a record high exceeding 600,000 (Institute of International Education, 2008). The annual figures released by the Australian Education International (AEI) show that over 500,000 international students on a student visa were enrolled in Australia in 2008, of whom 79% were enrolled in the Higher Education, Vocational Education and Training (VET), or English Language Intensive Courses for Overseas Students (ELICOS) sectors, while other categories including the school sector, accounted for the remaining 21% (Australian Government, 2008). Data for Australia and U.S. provided by these websites are recorded in Table 2.1.

**Table 2.1**  International tertiary student enrolments in Australia and the U.S.

<table>
<thead>
<tr>
<th>Year</th>
<th>Australia</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>48,486</td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>134,959</td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>8,777</td>
<td>286,343</td>
</tr>
<tr>
<td>1990</td>
<td>386,851</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>188,277</td>
<td>514,723</td>
</tr>
<tr>
<td>2001</td>
<td>233,408</td>
<td>547,867</td>
</tr>
<tr>
<td>2002</td>
<td>274,877</td>
<td>582,996</td>
</tr>
<tr>
<td>2003</td>
<td>307,960</td>
<td>586,323</td>
</tr>
<tr>
<td>2004</td>
<td>325,356</td>
<td>572,509</td>
</tr>
<tr>
<td>2005</td>
<td>346,079</td>
<td>565,039</td>
</tr>
<tr>
<td>2006</td>
<td>383,818</td>
<td>564,766</td>
</tr>
<tr>
<td>2007</td>
<td>455,185</td>
<td>582,984</td>
</tr>
<tr>
<td>2008</td>
<td>543,898</td>
<td>623,805</td>
</tr>
</tbody>
</table>

*Source: Data released by IIE and AEI, 2008*
Australia and U.S. alone currently account for more than a million international students each year. With other countries such as the UK, Canada, and New Zealand with large cohorts of international students, and countries such as South Africa, and India with multilingual populations, potentially millions of students are learning mathematics in a language that is not their first language. This makes the need to understand the link between language, cognition, and academic performance all the more important.

2.1.3 Interest in the role of language in learning

The link between language and cognition has been recognised for decades. Traditional theories have helped our understanding of the conceptual development of children, and the role of language in shaping and producing thoughts, and influenced numerous educators over time (Ing, 1978; Lawton, 1978). One such study conducted three decades ago, suggested that the syntactic complexity of written mathematical problems influences the student’s ability to solve the problem (Larson, Parker, & Trenholme, 1978). However, the authors described it as a preliminary study using a small sample that helped raise interesting and thought provoking questions about the standardized tests of mathematical performance and arithmetic texts that are commonly utilized at the upper elementary and junior high school level. Austin and Howson (1979) looked at reading, writing, vocabulary and symbolism of mathematics and provided a bibliography of studies relating language and mathematics.

With changes in societies and the increase of language minorities in classrooms came the realisation that there was very little research on the link between language and the learning of mathematics, as well as on the role language plays in the assessment of mathematical concepts and skills. “The dearth becomes almost a void when one restricts one’s attention to students from a minority language group” ( Cuevas, 1984, p 140). This realisation brought about the shift in research priorities discussed in the previous section and raised interest in the role of language in mathematics learning. A study by Mestre (1986) identified error
patterns and areas of difficulty in problem-solving tasks for a group of Hispanic technical college students, and advocated the integration of teaching language skills with the teaching of problem-solving skills.

Over the last few decades of the twentieth century, there was growing awareness of the importance of language in the mathematics curriculum and a number of researchers were involved in identifying language factors that influence mathematics teaching and learning (Clarkson & Galbraith, 1992; Cummins, et al., 1988; Ellerton & Clarkson, 1996; Ellerton & Clements, 1991; Lean, et al., 1990). As a result, there was an emerging consensus that the linguistic features of word problems, textbooks, and tests affect understanding of word problems and consequently, performance on tests. Mousley and Marks (1991) encouraged more language-sensitive approaches in mathematics classrooms by teachers who model and actively teach the language of mathematics. They addressed various levels of discourse that occur in mathematics classrooms such as reading, talking, and writing of text that may be written, spoken, or in computer or calculator software. In their view, texts, tests and computers are all dependent on language. The role of student discussion and writing was emphasised in effective learning of the language of mathematics (Moschkovich, 1999; Shield & Galbraith, 1998).

During the 1990s mathematics educators were also becoming aware of the needs of second language learners (Lean, et al., 1990; MacGregor & Moore, 1991). However, most of this research was limited to young children and school mathematics. While there has been a substantial amount of research in mathematics education at the school level (Grouws, 1992), the amount at the tertiary level was still modest (Seldon & Seldon, 1999) and very few of them link language and mathematics learning. One such study by Varughese and Glencross (1996) conducted in a South African university found that first year students had difficulty with mathematical terms such as ‘multiple’, ‘integer’, and ‘perimeter’. This study involved students, whose first language was Xhosa, learning mathematics in English and suggested that second language learners at the tertiary level could have difficulties with the language of mathematics.
2.2 Language and Mathematics: Recent Research

The trend has continued in recent years and researchers have been looking into the relationship between language proficiency and performance on mathematics courses also at the secondary and tertiary level. This section focuses on research on the link between language and mathematics since 2000, which has grown as a consequence of the change in priorities discussed earlier. Linguistic diversity and lack of proficiency in English is not restricted to young children, but is fairly prevalent in tertiary classrooms as well. Consequently, “there is a growing interest in language requirements for tertiary study and in the provision of programmes that will assist students in their studies” (Barton & Neville-Barton, 2004, p.1). However this trend is not restricted to the tertiary level. For example, since this study commenced, a leading journal has devoted an entire issue to multilingual issues in mathematics education (Educational Studies in Mathematics, 2007, Volume 64, Issue 2). Three reasons that justify this focus are, increasing movement of people across international borders for work, education, opportunity, or peace, the rise of indigenous and minority movements bringing minority language groups into mainstream classrooms, and the growing interest and emphasis on culture-specific contexts such as code-switching or ethnomathematics (Barwell, Barton, & Setati, 2007).

My survey of literature showed that research into the link between language and mathematics has been conducted from different standpoints. Some researchers have investigated various aspects of teaching and learning mathematics in multilingual contexts, while others have looked at the components of language use in teaching and learning of mathematics. Of particular interest to me, were the findings from multilingual settings, and the reading and writing components of language. The findings from the relevant studies are summarised in the next three subsections.
2.2.1 Multilingual classrooms

Two types of multilingual classrooms were observed in the literature surveyed. One occurs in countries such as South Africa, India, Papua New Guinea or New Zealand which have at least one official language other than English, where students come from different linguistic backgrounds, the language of instruction is not always English, and teachers in most cases can speak the first language of the students. The second type of classroom occurs in countries like Australia and the US where students from different linguistic backgrounds are taught in English by teachers who are unlikely to speak the students’ first language. These two types of classrooms operate on very different classroom dynamics and require different teaching and learning strategies. Studies about both types of classrooms have yielded interesting results.

A number of studies have supported the notion that bilingualism is beneficial to learning mathematics and code-switching is a process that facilitates learning (e.g., Baker, 1993; Clarkson, 1991; Dawe, 1983; Secada, 1992; Setati & Adler, 2001). Clarkson (2007) studied the case of Year 4, Vietnamese students in Australia and suggests that some bilingual students have an advantage as they have greater metalinguistics skills and are more confident in solving difficult problems. Their ability to switch to their first language gives them the added confidence of familiar associations. A study investigating the language issues for senior Pasifika mathematics students (Year 12) in New Zealand found that complex mathematical sentences provide extra challenges and impede the learning of these students. The students performed better on instructional questions than on “word problems which required them to read a question or statement, think, analyze, and carry out appropriate computations” (Latu, 2005, p 489). The study further found that code-switching is a common practice in classrooms and students who use their first language while learning in English perform better than those who don’t.

On the other hand, translating to another language can at times result in change of meaning owing to differences in linguistic features of the two languages (Kern, 1994). A typical example is the study (Kazima, 2007) in Malawi that investigated
the responses of bilingual students in relation to the vocabulary used in teaching probability. The following excerpt from the discussion shows some of the difficulties that can arise as a result of code-switching.

Students in this study operate in two languages, Chichewa and English, relying heavily on the former. It is possible that when the students encounter English probability vocabulary, they interpret the words into Chichewa, do the thinking in Chichewa, and then translate their responses into English. This is different from English monolingual students who operate in one language only which is also the language of instruction. One specific example of the effect of Chichewa on students’ understanding of English probability words was observed on students’ responses to the words likely and unlikely. Since in Chichewa likely is understood as not unlikely, students might have difficulties with phrases such as not very likely and equally likely because in Chichewa they sound like not very not unlikely and equally not unlikely respectively. This kind of confusion does not exist for English monolingual students, although other kinds of confusion might arise (Kazima, 2007, p. 187).

This example highlights a situation that is typical of many multicultural classrooms today. NESB students face language difficulties that their teachers could not begin to fathom if they did not speak the students’ language. The confusion felt by the students in the above study was evident to the teacher/researcher only because of their own knowledge of the Chichewa language.

A study in South Africa compared the performance of two groups of first year calculus students, one group taught in their first language, and the other group taught in English, which was their second or even third language (Gerber, Engelbrecht, Harding, & Rogan, 2005). The group taught in English was composed of students from many different language backgrounds including Afrikaans and other African languages. The group taught in their first language was a group of Afrikaans students. While there was no significant difference between the performances of the two groups of second language learners, there was a significant difference between the achievement of Afrikaans students attending Afrikaans lectures and Afrikaans students attending English lectures.
Although Afrikaans is closer to English, than the other African language of South Africa, and Afrikaans students are colloquially bilingual, the study concluded that the Afrikaans students were not necessarily cognitively bilingual. This elicits an important distinction between conversational and academic proficiency in a language. A person may be able to speak a language for day-to-day purposes without acquiring sufficient association of ideas and concepts to develop a ‘cognitive framework’. This is supported by Barton and Neville-Barton (2003) who regard proficiency in mathematical English as an important factor. In fact, the learning of mathematics has been considered as a “matter of constructing mathematical meaning” which “requires a language for its internalization within the learner’s cognitive framework” (Clarke, Stephens, & Waywood, 1992, p. 185). Cognitive bilingualism would require the ability to construct mathematical meaning in both languages in question and a cognitive framework of familiar constructs and concepts to internalize this meaning. This distinction between colloquial and cognitive bilingualism relates to the work of Cummins (1979) who recognised a similar distinction between two forms of language proficiency referring to them as basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALPS). Although, in his later work (Cummins, 2000) refers to these terms as conversational and academic proficiency. These two terms will be the used for the rest of this study.

A series of studies were conducted in New Zealand (Barton, Chan, King, Neville-Barton, & Sneddon, 2005) to investigate issues surrounding the learning of mathematics by students who have English as an additional language (EAL). The first, smaller study found that there was at least a 10% disadvantage in mathematics achievement for EAL students on a test involving questions of different forms such as symbolic, and graphical. The second, and larger study involved a course test asking students to self-report on their understanding of the questions. The third study involved four parts in secondary schools and one part at undergraduate level, with all five parts confirming the findings above (Barton, et al., 2005). It was found that the level of disadvantage at third-year level was greater than that experienced in first-year. The conclusion of the researchers was that the language requirements and logical complexity of third-year mathematics in English are much greater than that of first-year mathematics and consequently
beyond the capabilities of many EAL students. This again confirms the need for academic proficiency in order to cope with the demands of rigorous academic language. Students in third year university are likely to have acquired colloquial bilingualism having spent more than two years in an English speaking environment and yet this cohort of EAL students were unable to cope with the cognitive complexities associated with the higher level of mathematics.

The other aspect of the link between language and mathematics relates to the ways in which language is used in the learning of mathematics. Literature relating to reading and writing that are relevant to the tertiary level are surveyed in the next two sections.

2.2.2 Language of texts and assessments

Reading in the context of mathematics learning involves reading text and assessment items. Most resources used in a mathematics classroom are texts, and most assessments involve reading and comprehending test items before responding to them. This section focuses on studies that have investigated this aspect of language use, although most of these are at the school level.

In the study conducted in Papua New Guinea with Grade 6 students, it was found that bilingual students competent in both languages performed better than monolingual students on mathematics assessments (Clarkson, 1992). Another large scale comparative study (Lean, et al., 1990) involving 2493 students, aged from 5 to 15 years, from Victoria and Papua New Guinea schools found that differences in performance could be attributed to degree of English language competence rather than numerical skills. Abedi and Lord (2001) conducted a large study involving 1174 Year 8 students from 11 schools in the Los Angeles area and found that students’ language background impacted on their performance and that modifying the linguistic structures in mathematics word problems is likely to affect student performance. This could be attributed to the requirement of a cognitive framework as discussed in the previous section, as it is possible that students from Papua New Guinea or other language backgrounds in
Los Angeles might not have had the opportunity to develop academic proficiency in English. Mousley and Marks (1991) have related the ability of computing correct solutions to mathematical problems, with the ability to read and interpret questions.

Cummins, Kintsch, Reusser, and Weimer (1988) found that correct solution to word problems were associated with accurate recall of problem structure, that solution errors were attributed to miscomprehension, and problems with abstract or ambiguous language were more likely to be miscomprehended. They support the ‘linguistic development view’, that it is the linguistic forms employed by certain word problems that make them difficult for children who have not yet acquired an interpretation for such verbal forms. In other words, these students have not developed the cognitive framework to which the linguistic forms of the problems can be linked. Ferrari (2004) in his study on U.S. freshman college students, points out that while it is customary to assume that at college level all learning problems that students experience in mathematics can be ascribed to deficiencies in their school curriculum, competence in ordinary language and the specialised language of mathematics are other sources of difficulty. He further cautions that these language factors need to be recognised, identified, and dealt with for mathematics to be taught with understanding.

Another American study investigated the impact of language characteristics in mathematics test items on the performance of a large sample of students including students with disabilities and English language learners of Years 4, 7 and 10 (Shaftel, Belton-Kocher, Glasnapp, & Poggio, 2006). This study analysed the linguistic complexity of test items on the Kansas General Mathematics assessments and found that language characteristics had greater effects on the Year 4 students than on Year 10 students. While the students with disabilities or the English language learners were not particularly affected, difficult mathematics vocabulary was found to have a consistent effect on performance of students of all grades. Multiple meaning words caused greater difficulties at the Year 4 level. This would suggest that the language used in mathematical assessments can have important consequences for students who are yet to acquire academic proficiency. I would surmise that this is true of NESB learners of mathematics at any level as
such students are likely to find it difficult to relate to an unfamiliar context of the problem especially when they are from a different cultural, or language background (Cooper & Dunne, 2000; Prins & Ulijn, 1998; Verschaffel, Greer, & De Corte, 2000). Such findings have important implications for teaching and assessment of mathematics and need to be conveyed to practitioners.

Campbell, Adams, and Davis (2007) contend that “life experiences, language, cognitive processes, and knowledge of and the ability to apply mathematical content, all interact in the solution process” (p. 6). According to this approach, several pieces of information need to be retained in working memory to solve a problem, and second language learners are likely to require extra working memory to process the demands of unfamiliar linguistic features and cultural contexts. Kern’s (1994) suggestion that this demand on working memory is eased by translation into first language fits in theoretically and explains why most NESB learners tend to switch codes while learning in English.

2.2.3 Student writing in mathematics

Writing is the other component of language that is important to mathematics learning at the tertiary level as most learning and assessment tasks at this level are written. In my view, mathematical writing requires cognitive proficiency and academic literacy, both of which are required in some form at all levels but particularly so at the tertiary level. This view is supported by literature on student writing in mathematics. Being proficient in academic literacy requires knowledge of the type of language that is predominantly used in classrooms and is related to learning (Kersaint, Thompson, & Petkova, 2009). While reading and comprehending this language can in itself present challenges to a number of students, expressing logical mathematical thought in writing requires a whole new level of skill and knowledge. Teacher expectations of the quality of students’ academic writing may vary with curriculum area, contexts, or tasks but students need to meet these expectations in order to be judged successful (Sheeran & Barnes, 1991). Studies have shown that writing in mathematics can be an integral part of the learning, teaching, evaluation, and assessment processes, and makes
students aware of their growth in understanding mathematical concepts as well as in their ability to explain them (Dougherty, 1996). Writing can be a valuable tool to enhance learning, as a written solution or logical reasoning requires academic proficiency, and written tasks are likely to help acquire this proficiency. Clarke, et al. (1992) support the inclusion of writing tasks such as journal writing that enable students to articulate their thinking and thereby empower them to have control of their own learning at a cognitive level. Chapman (1996) describes the way journal writing by her students proved to be extremely valuable in providing an insight into student thinking and diagnosing misconceptions. Writing in mathematics can thus promote thinking at the cognitive level, and use of academic language in students, and provide teachers with an insight into student capabilities and misconceptions.

Some examples of studies that have tried innovative ways to include writing into the mathematics curriculum are discussed here. Use of writing as a learning activity was tried at the Year 8 level in three schools in Queensland (Shield & Galbraith, 1998). Two types of written tasks were used for this study. One was to write a letter to a friend who had missed a class, to tell them about the mathematics that was learnt. The second was to write about how they would explain a mathematical idea to a friend who had expressed difficulty with a problem or concept. The study collected 290 expository writing samples which were used to develop a coding scheme and formulate a model of student writing. The researchers found “a consistency in style with a focus on doing a procedure as an algorithm” (p. 43). They concluded that the writing of students was constrained by the models of presentation that they were accustomed to, in teacher presentations and textbooks. This study suggests that the mode of discourse used by teachers and textbooks play an important part in the development of logical mathematical expression in students. In the case of NESB students, such activities could promote these skills with the added benefit of placing them in the context of everyday English such as in a letter to a friend.

Another example of a novel task of writing in mathematics was introduced in a calculus course for freshmen at Duke University (Gopen & Smith, 1990). The course included a standard first year calculus syllabus and a computer component
which required students to work collaboratively and then write individual weekly reports. These weekly lab reports included data, graphs, tables and one to three pages of expository writing, which helped students to grasp and learn concepts rather than just memorize them, and lecturers to gauge student understanding or misconceptions. The lecturers recount the success of this task as measured by student feedback. This study has shown that writing assignments in mathematics courses improves student comprehension and such written tasks can be successfully incorporated into the mathematics curriculum. While such a task may appear challenging for NESB students at first, this study suggests that these types of assignments may well help scaffold NESB students in developing academic proficiency in English.

Ferrari (2002) investigates spoken and written language use by two Year 7 classes in another novel task. This task required one class to study a floor plan for their school renovation, and as a group, come up with verbal description of the figure by means of a written text with no drawings. The other class was required to produce a sketch of the floor plan which they had not seen, from the verbal description of the first class. This investigation found that students adopted various means to describe the figure in written form, ranging from metaphoric references to labelling parts of the figure. During reading, reflecting, and discussing as a class group, they were seen to pay more attention to isolated words, and sometimes appeared to be inaccurate due to misconceptions but conveyed the required meaning. Use of verbal language to represent and communicate to peers not sharing the same context is recommended as a result of this study.

These examples in literature show that writing has an important place in the teaching and learning of mathematics and incorporation of more written tasks involving verbal descriptions enhances student understanding, scaffolds academic language development and provides teachers with an insight into student knowledge and understanding, as well as their misconceptions. While these studies have exemplified the importance of language in mathematics learning and suggested ways in which language use can be investigated, they also highlight the fact that the case of tertiary NESB mathematics learners in classroom settings
such as Australia, U.S., or UK, where all teaching is in English, has not been considered to date.

The next phase of my literature review was to investigate mathematical language itself. Having traced the history and development of interest in the link between language and mathematics, the next section looks the various aspects of language use in the teaching and learning of mathematics including the genre and register of mathematics as well as difficulties students have with these.

### 2.3 Language Use in Mathematics

Traditionally, mathematics was thought of as an area with minimal linguistic demands as it is not associated with elaborate writing tasks such as argumentative essays. Language and mathematics were treated as unrelated subjects and literacy and numeracy presented as separate and unrelated skills (Chapman & Lee, 1990). Mathematics was often seen as a collection of rules for manipulating unfamiliar symbols, something far removed from speech and writing. Probably this results from the fact that most elementary mathematics courses tend to be predominantly focussed on procedural techniques for working with numbers, symbols, and equations. However, advanced mathematics courses at university require understanding of interrelationships among a whole host of sophisticated concepts which can prove to be difficult for many students as evidenced by the study among New Zealand third year undergraduates (Barton, et al., 2005). Just as procedural mathematics courses tend to focus on formulae and systematic practice, with an emphasis on symbolic manipulation, so conceptual mathematics courses focus on proof and argument with an emphasis on correct, clear, and concise expression of ideas. This is a difficult but crucial leap for students making the transition from rudimentary to advanced mathematical thinking. It is at this stage that the link between language and mathematics becomes apparent. In fact, mathematics educationists “have moved beyond simplistic notions of mathematics being ‘language free’, or alternatively and conversely, of mathematics being a language” (Barwell, Leung, Morgan, & Street, 2005, p. 142). Instead, we have come to accept that mathematics is a discipline that uses
specialised vocabulary, which has to be spoken and written in conventional forms using specific rules and structure comprising the discourse features of mathematics. The relevant literature surveyed has been organised into four subsections. The first discusses the importance of mathematical discourse in learning, and the second, the genre and register of mathematics. This is followed by two subsections that examine two concepts that can help evaluate language use in mathematical text namely readability and linguistic complexity.

2.3.1 The importance of mathematical discourse in learning

Mathematical discourse consists of the spoken and written forms of language used to communicate mathematical reasoning, including the vocabulary and syntax used in such communications. It can be in receptive mode when hearing or reading, or in expressive mode when speaking or writing. There is growing interest in the classroom discourse of teachers and students and its place in learning (Huang, Normandia, & Greer, 2005; Lemke, 1989; MacGregor, 2002; O'Halloran, 2000; Pimm, 1991).

As with all school learning, a key challenge in mathematics teaching is to help students move from everyday, informal ways of construing knowledge into the technical and academic ways that are necessary for disciplinary learning in all subjects. Each subject area has its own ways of using language to construct knowledge, and students need to be able to use language effectively to participate in those ways of knowing (Schleppergrell, 2007, p. 140).

This is particularly true of mathematics and consequently, there is a growing recognition of the need for interdisciplinary cooperation with linguists (Barwell, et al., 2005). Huang, Normandia, and Greer (2005) concluded from their study of a secondary mathematics classroom, that instructional design plans would benefit by widening the range of discourse functions and incorporating mathematical thinking, talking, and writing into the curriculum. In fact, mathematics and language are intricately connected (Dale & Cuevas, 1992). The language of mathematics includes specialized vocabulary and discourse features and there are
universally accepted rules and structure to mathematical exposition. For instance perimeter of a polygon and circumference of a circle both refer to distance around the boundary but are not used interchangeably. As a consequence, it is essential that successful mathematics students understand the ‘language of mathematics’.

At the same time, with emphasis on problem solving approaches to teaching mathematics in schools and the application of mathematics in other disciplines, language plays an important part in deciphering what is required and/or meant by mathematical problems expressed in words. Thus it becomes important that students should understand the ‘language in mathematics’. Clarke (1993) in his discussion on the language of assessment expresses this very concern

The location of mathematical tasks in meaningful contexts for either instructional or assessment purposes ... acknowledging and utilising the situated nature of learning. We ... present our mathematical tasks in written form, and an emphasis on problem context requires that elaborate detail be provided to establish that context.... This language may require a vocabulary beyond the capability of many students, who in everyday life may interact with similar ‘meaningful contexts’ to those described in the problem situations but do so without the obligation to either encounter the situation in purely written form or to respond to the situation in written form (p. 215).

Most mathematics teachers must have encountered situations in their classrooms where mathematically able students have been confused by the language of the problem or been unable to produce high quality written responses. It would be natural to expect that such language issues pose greater challenges for NESB students as they are more likely to have difficulties with both the language and the cultural references used. In particular, international students who have had a medium of instruction other than English for their schooling and take up tertiary education in Australia are often faced with problems on two fronts. They have to understand the ‘foreign’ language used in the problems as well as follow the structure of the language of mathematics in this foreign language.

Unlike primary school students, tertiary students may have completed all or part of their schooling in a first language and may or may not have learnt English as a
second language. Consequently, we have students of widely varying English proficiency levels in our tertiary mathematics classrooms. This raises the question whether their proficiency in English affects their learning of mathematics and if so, what discourse features cause difficulties? Is it the technical language of mathematics, or the general English language used in mathematics problems that is unfamiliar to these students? Any study that focuses on these aspects of mathematics learning at the tertiary level must of necessity include a discussion of the language of mathematics as used in the main modes of mathematical discourse at this level, namely: oral or written language used by the student or teacher, and the language of mathematics texts and assessments.

As discussed earlier, the formal language of mathematics has a highly specialised vocabulary with strictly context-specific grammatical structure. “There is an increasing body of mathematics education research that indicates that one of the crucial roles of teachers of mathematics is to assist learners to acquire, in both receptive and expressive modes, the formal language of mathematics” (Ellerton & Clements, 1991, p. 12). Mathematics, when spoken, emerges in a natural language, when written it makes use of a complex rule-governed writing system (Pimm, 1991). This writing system with its specialised vocabulary, is the focus of the next section as this should guide the nature and focus of a study that aims to find the difficulties caused by this system for NESB students.

2.3.2 The genres and the register of mathematics

As in all expressions of language, the language of mathematics has accepted structural forms, which are used to make meaning. There are accepted conventions and patterns leading to the generation of context-specific ‘genres’ of mathematical language. According to Wallace and Ellerton (2004), language genres are forms in which discourse participants communicate within social contexts. A number of genres may be used within a mathematics classroom. Genres may differ according to the activity or the topic of the class. For example, the formal and technical genre of a mathematical proof will be different from that of an informal discussion or a teacher-student dialogue or a monologue
introducing a new concept. All of these may be different from the language used in examination or exercise questions. However, all of these genres of mathematics are bound by the grammatical rules of the root language (in this case English). “A language genre includes but is not limited to particular uses of terminology and grammatical structures found within the root language” (Wallace & Ellerton, 2004, p. 2). Gerofsky (1996) refers to genre as text type in terms of their linguistic and contextual features and examines the narrative, fictional and three-component structure of the genre of word problems.

On the other hand, to use any genre in mathematics, one has to be knowledgeable about the register of mathematics. A register is a kind of language associated with a particular situation. This is made up of words, phrases and structures of mathematics with specific mathematical meanings in particular contexts (Wallace & Ellerton, 2004). Halliday (1978) defines register as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings.... ‘mathematics register’, ...the sense of the meanings that belong to the language of mathematics” (p. 195). Ferrari (2004) differentiates ‘colloquial’ registers used in everyday life from ‘literate’ registers used for academic communications (in line with conversational and academic language defined in Section 2.2.1) and goes on to say that in mathematical language, the features of the literate register occur in extreme form. This is illustrated in the following passage which introduces the chapter on Normal Probability distributions in a textbook of engineering mathematics.

Turning from discrete to continuous distributions, in this section we discuss the normal distribution. This is the most important continuous distribution because in applications many random variables are normal random variables or they are approximately normal or can be transformed into normal random variables in a relatively simple fashion. Furthermore, the normal distribution is a useful approximation of more complicated distributions, and it also occurs in the proofs of various statistical tests.

The normal distribution or Gaussian distribution is defined as the distribution with the density
A number of words in this paragraph such as, continuous, distribution, normal, random, and variable have very precise meanings in this mathematical context, while having other familiar everyday meanings in other contexts. As this is the first paragraph of the chapter introducing the topic, knowledge of these words and concepts is clearly assumed. Furthermore, knowledge of several mathematical concepts and notations such as exponential functions (exp), mean ($\mu$), and standard deviation ($\sigma$) are also assumed. It is apparent that considerable amount of thought processing is required in reading and comprehending this short introductory passage in a mathematics text.

Chapman (1993) discusses some of the features of the register of school mathematics, which includes words from everyday language with different meanings in mathematics (mean, obtuse, improper) and words specific to mathematics (hypotenuse, integer), words borrowed from other languages (subtract, formula from Latin; domain, gradient from French; isosceles, pi from Greek) and words created from words or parts of words from other languages (histogram combining historia from Latin, gramme from French; hypotenuse from Greek hypo and teinein). Phrases or groups of words may become technical terms (degrees of freedom, highest common factor, ordered pairs). All of these may tend to create learning and understanding difficulties for students.

While the specialist language of mathematics may itself be a barrier to students’ understanding and concept development, this barrier becomes even greater when the specialist language is used in complicated sentence structures, such as occurs in many mathematics textbooks, or in language patterns which are unfamiliar to many students, for example, the passive form of words (Varughese & Glenncross, 1996).

As students progress through their mathematics learning beyond elementary level, they are increasingly reliant on written text. They are expected to read texts and notes, complete practise exercises, take written tests, write project reports, and

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right] \quad \sigma > 0$$

(Kreyszig, 2006, p. 1026).
complete other written tasks. Consequently, the language used in mathematics
textbooks, test items, lecture notes, and problem sets, are all of interest to this
study. For a written text to communicate the intended meaning, it has to be read
correctly and comprehended by the reader. This depends on the reading skill and
language proficiency of the reader as well as on the ‘readability’ and ‘linguistic’
complexity of the text. While readability is a measure of the ease with which the
text can be read, linguistic complexity tells us about the complexity of the
sentence structure and syntax used. The next sections discuss the literature on
readability and linguistic complexity of written texts.

2.3.3 Readability

The intention of an author whether of a textbook, a worksheet, or an examination
paper, is to transmit information to the reader. The author’s success will depend
on the readability of the text. Readability is concerned with the extent of the
match between reader and text. The term readability refers to all factors that
affect success in reading and understanding a text (Johnson & Johnson, 2002).
These range from motivation and interest and legibility of print to sentence
structure and word length. While some of these factors are highly subjective,
many of the widely used readability formulae quantify these features in relation to
the age of the reader.

A printed page of mathematical text may communicate comparatively
easily with children, or it may fail to communicate; it is important for
teachers of mathematics in both primary and secondary schools to
recognise whether children are likely to be able to read the page easily
(Shuard & Rothary, 1984, p. 1).

Readability formulae estimate how difficult the text is to read. It is literature from
the early stages of the development of these formulae that reveal the methods
used in arriving at readability levels for a given text. It was found that most
readability formulae involve putting word difficulty or sentence length into
mathematical formulae to obtain a readability score or a reading age level. “A
readability formula uses counts of language variables in a piece of writing in
order to provide an index of probable difficulty for readers. It is a predictive
device in the sense that no actual participation by readers is needed” (Klare, 1974, p 64). In this article, Klare (1974) presents more than 30 formulae for assessing readability and discusses the varied formulae developed and used for diverse purposes such as assessing the readability of government documents or brochures in a defence department. This presents the potential user of a readability formula with many formulae to choose from.

The Fog index, the Kincaid formula, the Flesch Reading Ease Formula, the Automated readability index, the McLaughlin SMOG formula and the Coleman-Liau formula are some of the popular methods of assessing the readability level of a text (Institute of Educational Sciences, 2003). With advances in technology many such tests have instant calculators available online which can compute these measures for any text that is submitted in typed format. However, online calculators do not always provide the method or reasoning behind these calculated measures. Four tests relevant to this study, which are easily available online, are discussed here.

The SMOG grading formula (McLaughlin, 1969) uses a count of the average number of words with three syllables or more (N) in samples of 30 consecutive sentences. The grade level and readability age of the text are calculated using the following formulae:

\[
\text{grade level} = (\text{square root of } N) + 3 \quad \text{and} \\
\text{reading age} = (\text{square root of } N) + 8 \text{ years.}
\]

The Flesch-Kincaid grade level of readability and the Flesch Reading Ease tests are used as standard tests by the US Government department of Defence (Institute for Simulation and Training, 2001) for assessing the readability of government documents. Average sentence length (L) and average word length (N) are used to determine readability as follows:

\[
\text{Grade level} = (L \times 0.39) + (N \times 11.8) - 15.59 \\
\text{Reading Age} = (L \times 0.39) + (N \times 11.8) - 10.59 \text{ years.} \\
\text{Reading Ease} = 206.835 - 0.846 N - 1.015 L
\]
Average word length is measured as number of syllables per word and average sentence length is calculated as number of words per sentence. Stokes (1978) and Harrison (1979) have shown that while there is a strong correlation between the various readability tests of groups of books, they may vary by several grades in their evaluation of the reading age or grade level of a single book. Furthermore, different passages from the same book may also vary by several grade levels. Thus the choice of formula as well as the section of the book may affect the results of readability tests. Another approach is the Cloze procedure which replaces certain words or symbols in passages by blanks. Participants then attempt to complete the passage and their score from correct responses is suggested as a valid and reliable measure of readability (Hater & Kane, 1975).

A relatively new method to evaluate readability level was developed as a result of a research project of the School Renaissance Institute and the Touchstone Applied Science Associates Inc. (TASA). This method is known as the Advantage TASA Open Standard (ATOS) readability test that minimises some of these limitations by adjusting variables used, taking into account book length and including the whole text in the analysis where possible. In most cases readability measures are calculated using formulae based on two variables: (1) semantic difficulty as measured by word length, word familiarity, or word frequency, and (2) syntactic difficulty as measured by sentence length (School Renaissance Institute, 2000). ATOS tests of readability levels eliminate sampling error by using high-speed scanners to analyse entire texts. As a result, the readability levels reflect an entire book, not just sampled passages as in other readability formulas. This could prove appropriate in measuring the readability of the English used in mathematics textbooks if whole books could be made available in text format or the whole book could be sent for analysis using scanners.

While a single test of a selected passage or passages may not be dependable, the whole book analysis or average of scores obtained on the various tests could provide information to help teachers make decisions regarding book selection. However, another limitation of readability tests is their inability to discriminate between language genres or technical registers. For instance a sentence with eight words using legal or scientific jargon and another sentence of the same length
from a children’s story book will be of very different readability levels. The order of words in a sentence, complex sentence structures or technical terms could make a significant difference to the difficulty level of the text which may not be picked up by a mathematical formula involving number of words or syllables. These issues are explored in the next section.

2.3.4 Linguistic complexity

Perera (1982) suggests that an alternative approach could be a more subjective analysis of the linguistic factors such as grammatical structure, or complexity of meaning that may contribute to the difficulty level of the text. This will have to be based on a more common sense approach relying on the teacher’s experience in identifying the needs of the students, the requirements of the subjects, as well as factors that could cause difficulties for a particular group of students. A written text of mathematics is likely to have technical words and phrases, sometimes in conjunction with symbols and is intended to convey precise, often complex mathematical meaning. In addition to accurate reading, such texts require decoding or interpreting for comprehension of the mathematical meaning intended. For instance, the phrase ‘rate of change’ has to be decoded to mean the derivative of one variable with respect to another, and involves specific process skills for computation. This is precisely the view expressed by Mousley and Marks (1991) that while word length, sentence length and sentence complexity are factors which affect the ease of decoding and comprehending text, mathematical words and symbolic statements often convey sophisticated messages and complex skills and ideas that require additional thought and effort for accurate comprehension.

Several studies have investigated the effect of linguistic structure of test items on performance (Abedi, Hofstetter, Baker, & Lord, 2001; Larson, et al., 1978; Shaftel, et al., 2006), the language use in word problems (Lean, et al., 1990; Verschaffel, et al., 2000), and particular linguistic features such as comparative terms have been found to cause difficulty (Cummins, et al., 1988; De Corte, Verschaffel, & De Win, 1985; Lean, et al., 1990). With the current trends in education of increased numbers of NESB learners at all levels, considerations of
reading and comprehension abilities are not restricted to age levels of readers. Mathematics textbooks at the high school and tertiary level may use the structured language of mathematics and tend to assume knowledge of certain terms and concepts. The language structure of a text can be very important to any reader but more so to an NESB learner.

Thus an analysis of the content presented by a text including its readability level as well as the register of mathematics and the genre used in various contexts can enhance the teachers’ ability to understand and address ‘language difficulties’ faced by their students. However, this calls for a clear understanding of the nature of language difficulties in the understanding of mathematics. The next section considers the language-related difficulties that have been identified in literature.

2.4 Language-Related Difficulties in Learning Mathematics

Students may be hindered in their understanding of mathematics by the very nature of the highly specialised language of mathematics. However, the causes for this may vary widely. For some it is the unfamiliar technical register of mathematics that causes difficulty, while for others it is the formal genre and rule bound structure of mathematical language. While the register and genre of the language of mathematics could pose problems for students in general, it is highly likely that this will prove even more difficult for NESB students, and there could be additional factors involved in the case of NESB students. The following subsections look at the difficulties that have been identified in the use of mathematical language, the challenges it poses for NESB learners, and the theories that could shed light on these challenges.

2.4.1 Categories of language difficulties in mathematics

Various attempts have been made to investigate the root causes of such difficulties in the mathematics register and systematically categorise them. Thompson and Rubenstein (2000) and Tapson (2000) have suggested some categories of such difficulties in the language of mathematics, and MacGregor and Moore (1991) have identified the use of propositions, passive voice, complex
sentences as well some difficult constructions in English, as hurdles to comprehension. A summary of the various categories of difficulties suggested by these researchers is compiled in Table 2.1 using examples presented in their work and my own experience as a mathematics teacher. This is information that all teachers of mathematics both at school and university levels must be equipped with, in the current climate of multicultural classrooms.

Table 2.2 Language difficulties in learning mathematics

<table>
<thead>
<tr>
<th>Difficulties</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguishing between pairs of similar words that refer to related concepts</td>
<td>Congruent &amp; similar, discrete and continuous, explicit &amp; implicit, bar chart &amp; histogram, convex &amp; concave, row &amp; column, hundreds &amp; hundredths.</td>
</tr>
<tr>
<td>Multiple usages or meanings of words</td>
<td>square, round, base, inverse, vertex, tangent</td>
</tr>
<tr>
<td>Words shared between English and mathematics with different meanings</td>
<td>power, volume, leg, event, right</td>
</tr>
<tr>
<td>Words shared by science and mathematics with different technical meanings</td>
<td>solution, radical, image, element, cell, tree</td>
</tr>
<tr>
<td>Ambiguity in use of closely related or interchangeable words</td>
<td>length, breadth, width, height, depth, thickness</td>
</tr>
<tr>
<td>Grasping mathematical phrases with precise meanings</td>
<td>at most, at least, not more than, if and only if</td>
</tr>
<tr>
<td>Words used only in mathematics</td>
<td>quotient, hypotenuse, isosceles, asymptote</td>
</tr>
<tr>
<td>Use of proposition affecting meaning</td>
<td>Water level rose by or rose to or rose from 5 cm.</td>
</tr>
<tr>
<td>Distinguishing between the use of articles and indefinite pronouns</td>
<td>a, an, the, some, all</td>
</tr>
</tbody>
</table>

These are but some of the difficulties associated with learning and understanding mathematics for both first language and NESB learners. With so many likely hurdles to comprehending mathematical language, it is only natural that students often misunderstand or confuse certain words and concepts used in mathematics. This problem is compounded in the case of NESB learners, as they have to grapple with some additional factors related to linguistic and cultural differences, which could cause difficulties in both understanding and expressing themselves in spoken or written responses. This is the focus of the next section.
2.4.2 What is involved for NESB students? A reflection

Several issues that came to light in the contexts of teaching mathematics to NESB students in Foundation Studies programs were discussed in Chapter 1. The above review of the literature has revealed features in the genre and register of mathematics that are known to cause difficulties for mathematics students. This section reflects on the language-related difficulties for NESB students. The reflections are derived from my experience with FS students and the language difficulties identified in literature have shown me that NESB students are likely to encounter hurdles to comprehension on account of the specialised vocabulary of the mathematics register, syntax, semantics and linguistic features of mathematical discourse, as well as cultural differences. I have listed these below with some examples:

- A word with two different meanings in English may be translated into two distinct words in other languages. For example, round (to two decimal places) and round (circular) translate respectively to ‘Gembu’ and ‘enkei’ in Japanese and ‘Pembulatan’ and ‘bulat’ in Indonesian.
- Conversely two different words in English may be represented by the same or similar words in another language. For example zero and empty can be translated to ‘shoonya’ in Sinhalese.
- Cultural differences may hinder understanding of mathematical problems despite having the necessary mathematical skills. For example Asian students may not relate to a situation on Australian-rules Football or Cricket and hence have difficulty understanding a problem in that context.
- Sentence structure could be vastly different in different languages creating difficulties.
- Certain phrases cannot be translated meaningfully into another language.

This is supported by Kersaint, et al. (2009), who in their recently published book, have categorised difficulties faced by NESB students at the school level, into five categories, namely, vocabulary, symbolic representation, syntax, semantics, and linguistic features of discourse. The next step was to locate relevant theories that could guide my research and help develop an understanding of these difficulties.
Two areas that I found to be relevant were theories on second language acquisition and theories on the link between language and errors in mathematical problem solving. These two areas are discussed in Sections 2.4.3 and 2.4.4 below.

2.4.3 Second language acquisition

Children seem to acquire language in a natural way attaining the ability to communicate around the same age, usually before they are two years old, irrespective of what language they speak. One language cannot therefore be considered to be more difficult than another. Yet, not everyone can achieve equal competency in a second or third language with the same ease. My purpose here was to examine theories about second language acquisition in order to gain a better understanding of ‘why’ NESB students face language difficulties.

Bialystock and Hakuta (1994) talk about complex relationships between various factors that influence second language acquisition. They argue that people learn second languages for different purposes and under different circumstances. The motivation and need to learn a new language, their own language background, and their ability and opportunity to practice the use of this new language, all affect the success and speed of acquisition. They also point out that children and adults are at different developmental stages of language learning. Children need to develop both a conceptual system for understanding the world around them as well as a linguistic system to communicate the concepts they have learnt. On the other hand, adults have already acquired a conceptual system in their first language and do not need to relearn it. Consequently, adults may learn a second language at a faster rate than children learning a second language, at the beginning, but this advantage is overtaken by younger learners over a longer period of time. Cummins (1981) suggests that development of academic proficiency in a student’s second language can take five to seven years. Another large scale study (Collier, 1987) found that 8 to 11 year old children acquired proficiency in English as a second language in the shortest time, requiring 2 to 5 years to reach the national norm, while 12 to 15 year olds required a longer period
of 6 to 8 years to achieve national norms for grade level academic proficiency when studying in a second language.

Older students, who are learning a second language, apply the knowledge and skills they developed in acquiring the first language to the process of learning the new language. “It is only with one foot placed squarely, securely within the known, the familiar, that the child can place the other foot in the beyond” (Lindfors, 1991, p. 282). However, “when we learn a new language, we’re not just learning new vocabulary and grammar, we’re also learning new ways of organizing concepts, new ways of thinking, and new ways of learning language.” (Bialystok & Hakuta, 1994, p. 122).

Barwell (2005) deliberates on two aspects about second language learning that have a bearing on this study. Firstly, he states that students will achieve conversational proficiency in English much earlier than proficiency in academic English, which could take a number of years to acquire. This is supported by the study conducted in South Africa, which found Afrikaans students display conversational bilingualism but not academic bilingualism (Gerber, et al., 2005). Secondly, Barwell (2005) examines the threshold hypothesis (Cummins, 2000) that high levels of proficiency in two languages leads to cognitive advantage just as low levels of proficiency in both languages leads to cognitive disadvantage, while high level proficiency in only one language offers no advantage or disadvantage. Barwell however, cautions that while this may partly explain underachievement in NESB students, linguistic proficiency may not be the only factor influencing performance.

There is evidence to suggest that second language learners tend to translate what they read into their first language, generally referred to as code-switching, before processing information (Clarkson, 2007; Kern, 1994; MacGregor & Moore, 1991; Setati, 1998). Kern (1994) studied the role of mental translation involved in reading in a second language. He found that translation decreases progressively with time as the students improve their proficiency in the second language. He discusses some of the functional benefits and disadvantages of code switching. He argues that translation to their first language facilitates semantic processing.
and eases memory constraints, owing to the familiar associations that are likely to exist in the learner’s first language. On the other hand, there are disadvantages caused by possibilities such as loss of accuracy in translation, loss of integrated meaning in word by word translation, a focus on first language representations rather than the original second language forms. This could lead to confusion or miscomprehension, and consequent errors in problem solving.

In summary, second language acquisition is affected by a number of factors such as age, length of exposure to second language, need and motivation to learn, academic proficiency in the first language, as well the existence of a cultural reference to facilitate translation to and from the first language. Whether or not these factors affect learning and performance in mathematics will become clearer if we look at the causes and types of errors made by mathematics learners. A relevant theory on error analysis is discussed in the next section.

2.4.4 Link between language and errors in problem solving

Pencil-and-paper tests have been an accepted mode of assessment in mathematics and continue to be so. In considering genuine assessment of student learning, mathematics educators had to give attention to the correct and incorrect answers given on these paper-and-pencil tests and look into the possible causes of errors made by students. “The most important body of evidence pertaining to issues associated with pencil-and-paper tests for school mathematics has been generated by what has become known as ‘Newman research’, which has been widely used in Australia, Oceania, and South East Asia” (Ellerton & Clarkson, 1996, p. 1000).

According to Newman (1977, 1983), any person carrying out a written mathematics task goes through a fixed sequence of five steps namely, reading, comprehension, transformation, process skills, and encoding. Newman (1983) also assigned errors due to other factors to a composite category, referred to as ‘Careless’.

It has been claimed that failure at any level in the sequence of steps above could result in an incorrect answer, and that this can be identified by a diagnostic
interview using a sequence of questions corresponding to the sequence of steps. Clements (1980) has compared data from three studies (Clements, 1980; Newman, 1977; Watson, 1980) which used Newman’s diagnostic interview method for error analysis and found that more errors were made by the students at the reading or comprehension stage than the process stages. Use of this procedure has drawn attention to the influence of language factors and “has generated a large amount of evidence pointing to the conclusion that far more children experience difficulty with the semantic structures, the vocabulary, and the symbolism of mathematics than with standard algorithms” (Ellerton & Clarkson, 1996, p. 1001).

2.5 Summary of Literature Review

Mathematics education research is relatively new, emerging as a discipline in the latter half of the 20th century. The last two decades have seen a growing recognition of the value of naturalistic, qualitative approaches to research in addition to the traditional positivist and quantitative approaches. There is an increasing awareness of the role of language in the learning of mathematics especially at the junior school levels.

A number of recent research studies have investigated the teaching and learning of mathematics in multilingual classrooms in countries such as South Africa, New Zealand, Australia and the United States of America. Code switching was found to be advantageous in coping with the challenges of learning mathematics in a second language. Various studies have reported that the language of assessments does affect performance of students. It was also found that the quality of student writing was an important factor in learning, and activities involving writing promotes mathematics learning.

Familiarity with the genre and register of mathematics has also been recognised as an important factor contributing to success in mathematics. Students are exposed to larger amounts of written text as they progress higher in their education. Readability and linguistic complexity of written text then become
factors than can hinder comprehension especially for students of NESB. A number of readability tests are available for measuring the readability of text based on word length, sentence length and syllable counts. Researchers have identified language-related difficulties that could hinder comprehension. However, there is very little research on these difficulties at the tertiary level for NESB students.

2.5.1 What is missing?

A second language learner of mathematics has to learn new concepts in an unfamiliar language, learn to make sense of unfamiliar cultural references, and master the very precise language of mathematics. However, the nature or effect of such difficulties and the likely pitfalls in understanding that these may lead to for NESB students have not been systematically examined. In the absence of such knowledge it is not realistic to expect tertiary mathematics teachers to understand, let alone support, students with language difficulties in the learning of mathematics. “Research is perhaps most appropriately carried out when there is uncertainty: when we recognize that we need to know more about a problem in order to solve it, or when we have identified a gap in our knowledge” (Green & Browne, 2005). Given the numbers of international students studying in English only classrooms, there is an urgent need for further research in this area in order to inform the work of tertiary mathematics educators.

While there have been theories about second language acquisition (Krashen, 1988), and studies about multilingual classrooms and bilingual code-switching (Barton & Neville-Barton, 2004; Clarkson, 2003; Qi, 1998; Setati & Adler, 2000, 2001), questions about learning mathematics in a second language remain unanswered and there is the need for research studies in this field especially at the tertiary level. Bilinguals who have already learnt most of the mathematics content in their first language and then revisit this mathematics in a second language may provide some answers (Galligan, 2004).

This is precisely the intention of this research study: to seek answers from students who are in the transition phase from school to university, have learnt
mathematics in other countries, mostly in a language other than English, and are now revisiting or learning mathematics in English. Foundation courses in a number of Victorian universities provide a bridging year for such students prior to entry into first year university courses. These students could provide us with a wealth of information about language difficulties in learning mathematics in English at the tertiary level.

Being involved in teaching mathematics to FS students at RMIT University seemed to provide an ideal opportunity to explore the language difficulties faced by NESB students at great depth. However, as this was felt to be “focused more on understanding the nature of the concepts under study and how they relate to each other” (Green & Browne, 2005, p. 25), it was thought best to explore and understand the various aspects of language difficulties faced by NESB students, rather than attempt to control or intervene in the research context. Green and Browne (2005) contend that “much qualitative research starts with open, exploratory questions, rather than formal hypotheses” (p. 25). The next stage was to formulate research questions and develop procedures best suited to find answers to these research questions. As a result, the research questions stated below were formulated to enable such an open ended exploration to take place.

2.5.2 Research questions

In view of my interests in language-related difficulties experienced by NESB students in tertiary mathematics classrooms and the literature reviewed in this chapter, this study is aimed at seeking answers to the following questions:

- What if any, language-related difficulties are experienced by senior secondary and tertiary students of mathematics?
- Do language-related difficulties have a bearing on the performance of NESB mathematics students at the tertiary level?
- Do NESB students from different language backgrounds differ in their comprehension and use of English language as it is used in the context of tertiary Mathematics?
• How do VCE and first year university Mathematics textbooks compare in their use of English language in relation to readability, linguistic complexity and use of the Mathematics Register?

The next step was to plan an appropriate method of research to produce satisfactory answers to these questions and this is done in Chapter 3.
CHAPTER 3: RESEARCH METHODOLOGY

The previous chapter discussed the research perspectives and literature related to the issues investigated by this study and formulated the research questions. This chapter describes the research design and methods chosen for the research. Chapters 1 and 2 examined the trends and shifts in research methodologies and the research agenda. A priority that emerged was the relationship between language and mathematics especially at the tertiary level and the dearth of research in this area strengthened the rationale for this research. The recognition of alternative approaches to the quantitative and scientific methods as more suitable to some social contexts, has resulted in the rise of mixed method researches. These developments are considered in the context of the aims and focus of this study.

The chapter is organised into six sections:
Section 3.1 discusses the methodological considerations that informed the research design. Section 3.2 discusses the research design that evolved from the nature of the research questions and the methodologies that were found suitable for this study. As an exploratory study, two aspects of language use in tertiary mathematics are investigated from three standpoints, forming three parts of the study. Section 3.3 describes the research settings and participants. Section 3.4 details the procedure for the study. This includes a description of the participants, the three parts of the study, and the methods of data collection used for each part. Section 3.5 describes the methods employed for the analysis of data. The three parts of the study involved entirely different methods of data collection and types of data, and consequently required different forms of analysis. Section 3.6 examines issues of trustworthiness of the research and addresses the validity and reliability aspects, as well as the ethical issues involved in conducting the research.
3.1 Methodological Considerations

Trends and shifts in priorities in the research agenda have been paralleled by shifts in research paradigms and choice of research designs. “Inquiry has passed through a number of paradigm eras” prepositivist, positivist, and postpositivist (Lincoln & Guba, 1985). While the positivist, scientific and empirical inquiry led to spectacular successes and discoveries in the field of science, the postpositivist era signifies a realisation that an alternative paradigm may be necessary for inquiries in the social sciences. This led to the advance of various research perspectives for example ethnographic, phenomenological, qualitative, humanistic or case study which are other names for naturalistic inquiry (Lincoln & Guba, 1985). Schwartz and James (1979) provided a broad analysis of a shift in paradigms and the new emerging paradigm was most comprehensively applied in the field of education by Lincoln and Guba (1985). Qualitative methods can be used to explore areas about which little is known, or to obtain intricate details about phenomena (Strauss & Corbin, 1998). Among various classifications and descriptions of research paradigms, Merriam (1998) advocates the typology developed by Carr and Kemmis (1986), which identifies three basic forms of educational research – positivist, interpretive, and critical.

While there are different paradigms and research designs available to the researcher today, and individual researchers may be predisposed to particular approaches, it is the nature of the research problem which guides the choice of methodology (Green & Browne, 2005). In the light of the discussions of the previous two chapters and the nature of the research problem, it would appear that this study is most suited to interpretive qualitative research in general, and naturalistic inquiry in particular as it is located in a natural setting with no interventions. Considerations that justify this view are discussed in this section.

Naturalistic inquiry is not intended to develop universal generalizations independent of time or context rather, “the aim is to develop shared constructions that illuminate a particular context and provide working hypotheses for the investigation of others” (Erlandson, Harris, Skipper, & Allen, 1993, p 45).
Rosnow and Rosenthal (2008) suggest that a descriptive orientation is often a necessary first step in the development of knowledge and theory in a field, and the aim of descriptive research is to describe what is happening rather than provide causal explanations. Given the emergent nature of the topic to be investigated here, this approach would seem to be the most appropriate for this study.

Strauss and Corbin (1998) describe grounded theory as theory ‘grounded’ in data that is systematically collected and analysed through research. Grounded theory is based on an iterative inductive and deductive cycle where theory emerges from data collected and not from testing deductively formulated hypothesis. Initially an avoidance of existing theories was advocated (Glaser & Strauss, 1968) so that theory could be developed in its purity without bias and preconceptions. However, Strauss and Corbin (1998) later conceded that existing research should be used as the starting point to guide design and data collection methods, taking care to avoid preconceptions. Charmaz (2000) critiques Glaser’s insistence on avoiding existing literature to prevent bias, refers to Strauss’s approach as “didactic and prescriptive rather than emergent and interactive” (p 524), and advocates a constructivist approach to the development of grounded theory. All of these considerations were taken into account in deciding how best to design a study that would provide answers to the research questions posed in the previous chapter. It was concluded that a study investigating language difficulties encountered by Non-English Speaking Background (NESB) learners of tertiary mathematics in a natural setting, with a view to developing grounded theory, would be best suited for the purpose.

Strauss and Corbin (1998) further clarify that some researchers may code qualitative data in a manner that can be analysed statistically and in effect quantify qualitative data. Such an approach enables the use of quasi-statistics to identify emerging patterns to assist in drawing conclusions (Becker, 1970). Since the nature and quantity of data lent itself to coding and quantifying, this process is utilized throughout this study and quasi-statistics put to use for interpreting results.
3.2 Research Design

As described in Chapter 1, the motivation for this study emerged from various day-to-day situations observed in the classrooms and examination halls of Foundation Studies (FS) at RMIT University in relation to the language difficulties faced by NESB learners of mathematics at the tertiary level. Furthermore, many of these students had learnt mathematics in a language other than English prior to coming to Australia. Such situations must be fairly common across universities in the developed countries rather than isolated cases. This was the motivation to investigate the issue in greater detail and determine the extent and nature of the linguistic challenges faced by otherwise mathematically capable students as they grapple with the specialised vocabulary and discourse features of mathematics in a language that is unfamiliar to them. Strauss and Corbin (1990) suggest that “choosing a research problem through the professional or personal experience” ... “may be more valuable an indicator for you of a potentially successful research endeavour” (pp. 35 - 36). It was hoped that my personal experience in teaching FS students would prove valuable in exploring the linguistic challenges faced by these NESB students.

As indicated above, it is the nature of the research that determines the research strategy. With the research issue identified, the next step was to find the appropriate research design which would help find answers to the research questions formulated in the previous chapter. “A research design is the logic that links the data to be collected (and the conclusions to be drawn) to the initial questions of a study” (Yin, 2003, p. 19). The question then was how best to identify the difficulties faced by NESB students.

Mestre (2000) argues that the “measures we use in research often drive the development of theories and that once a technique has been chosen to address research questions, the type of information that we hope to extract from the study is limited” (p. 151). However, it is not always possible to extract the desired information by a single measuring technique.
The selection of research techniques then, is of vital importance to the success of a research study. Unfortunately there are no simple, clear-cut criteria for matching methodology to research questions or theories. Statistical analyses and quantitative methods have proved their merits in the past and the richness of data obtained qualitatively has been recognised more recently (Schoenfeld, 2002). Both approaches have their merits and I strongly believe that an appropriate mix of techniques to suit the aim and purpose of the research is the key to success. In this way, larger amounts of rich data in naturalistic settings can be obtained and analysed using appropriate techniques of quasi-statistics to observe trends and draw conclusions. Accordingly, a mix of various methods as detailed in the following sections has been selected for this study, leading to a mixed mode research design.

The most suitable participants for the investigation of these issues are tertiary international students and the FS students at RMIT University are fairly representative of the international student community of any university with regard to the diversity of their cultural and educational backgrounds. As I was involved in teaching a number of mathematics courses for FS, they were an ideal and convenient sample for such a study. “Purposeful sampling is based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned” (Merriam, 1998, p. 61). However, the students I was involved in teaching were all from the Science, Engineering and Technology (SET) stream of one program of a particular institution and fall within an age range of 18 to 21 years. Hence it was felt that this research could be carried out as a case study of a group of students at one institution.

A case study is the preferred strategy “when the investigator has little control over events and when the focus is on contemporary phenomenon within some real-life context” (Yin, 2003, p. 1). As an exploratory study, the main aim of this research was to investigate language-related difficulties faced by NESB students in the learning of tertiary mathematics without any intervention or control groups. According to Yin’s (2003) typology of case study designs, this study could be classified as a single-case, embedded design where FS students of the SET stream
were treated as a single case and individuals as multiple embedded units of analysis. Of the several rationales put forward, the revelatory rationale is suggested when “an investigator has the opportunity to observe and analyse a phenomenon” not available to others for scientific investigation (Yin, 2003, p. 41). With this design it was envisaged that a large enough sample could be used to provide insight in the form of a revelatory case.

Case studies are not limited to “direct detailed observations as a source of evidence” rather “case studies can be based on any mix of quantitative and qualitative evidence” (Yin, 2003, p. 15). Becker (1970) advocated the explicit use of simple numerical results that could be derived from data and called it ‘quasi-statistics’. Maxwell (2005) suggests that the use of quasi-statistics as well as strategies for comparison will enhance the validity of qualitative case studies especially in multicase and multisite studies. It was decided to involve as many SET FS students as possible in the study, over two academic years as this would generate sufficient data to draw valid conclusions overall. As I am also interested to find if there are any differences between language groups, classifying information obtained and using quasi-statistics would make it feasible to handle the data systematically and provide strategies for comparison between groups or categories. Though the research design was primarily envisaged to be a qualitative case study, and descriptive statistics and effect size calculations were employed for comparisons, making it a mixed methods research.

An exploration into the language-related difficulties faced by tertiary NESB students in learning mathematics necessarily needs to consider possible sources of such difficulties in this context, that is, reading, comprehending, and writing. This suggested the use of various strategies to investigate these different aspects of the problem to obtain a comprehensive picture of the language-related difficulties experienced by NESB students at the tertiary level. The method of using different data sets, methods or approaches within a study to enhance validity or credibility is called triangulation (Green, 2005). The different methods or approaches within a study can complement each other to build up our knowledge. Yin (2003) advocates the process of triangulation using multiple sources of evidence from different lines of enquiry to corroborate and enhance the accuracy of findings in a
case study. “Most of the better case studies rely on a wide variety of sources...so that findings are based on the convergence of information from different sources, not quantitative or qualitative data alone” (Yin, 2003, p. 93). With this in mind, it was decided to investigate the problem from various perspectives using different methods in order to get a better insight into the situation and validate the findings by triangulation.

Thus the research design for this study can be classified as a mixed method case study within the theoretical framework of grounded theory qualitative research using the principle of triangulation to enhance the validity of its findings.

Language difficulties and understanding of the language of mathematics are abstract concepts that are not easily observable or measured. The first step in coming up with an effective research method is to identify the indicators of the construct or concept in question (Green & Browne, 2005). It was thus necessary to operationalize these constructs and identify indicators that could be observed or measured.

Language can be thought of in terms of reading, writing, listening, and speaking. However, given the nature of teaching and learning of tertiary mathematics, it was felt that the first two namely, reading and writing feature more prominently. While there is some level of listening involved in lectures, most mathematics lectures involve large quantities of written text on the board or on lecture slides and are likely to involve more reading and note-taking on the part of the students, than listening. Moreover, as described in Chapter 1 investigating listening or speaking was beyond the scope of this study as in-depth interviews were not feasible with students who had difficulty expressing themselves in English. Hence, it was decided to focus on the two aspects of reading and writing for the purposes of this study particularly on their common components: “vocabulary, syntax and understanding of text organisation alternatives” (Pressley & McCormick, 1995, p 393). Mathematical problem solving requires good reading comprehension which in turn calls for knowledge of vocabulary, decoding skills, and knowledge of social contexts in which the problems are set. It thus became
the goal of the research to probe all these aspects of reading, writing, vocabulary, and syntax in the context of tertiary mathematics learning.

As a consequence, it was decided to investigate language use by NESB learners from three perspectives dividing the study into three parts. The first part focuses on the nature and level of language use in popular textbooks of mathematics. The second part investigates the impact of specialised mathematical terms on student performance and the third part involves an analysis of the interpretation and use of the specialised vocabulary of mathematics by NESB learners in a task involving the construction and interpretation of written material. The three parts of the study are:

I. Language of texts
II. Language of tests
III. Language of student writing

This will be discussed in more detail in Sections 3.4.1, 3.4.2, and 3.4.3 respectively.

It was envisaged that the three parts of the study would complement each other and provide insights into the linguistic challenges experienced by NESB learners of tertiary mathematics. While this was a case study of students at one university, the use of tests, written tasks and text analysis provided the opportunity to produce quantifiable evidence of trends and allowed the use of a much larger group of participants than is generally feasible in more descriptive case studies. Furthermore, it was hoped that the various parts of the study would enhance the validity of the study by invoking the process of triangulation for corroborating the results obtained.

The details of the research plan and implementation together with the analysis of data are described in the next three sections.
3.3 Research Setting and Participants

The main participants of this research came from the mathematics classes of FS at RMIT University, during the academic years of 2005 and 2006. There were three intakes each year, in February, June, and August with three mathematics courses on offer for each intake. These were General Mathematics, Pure Mathematics, and Applied Mathematics in increasing order of mathematical complexity. I was involved in all three mathematics courses across four intakes during these two academic years. All the students from my classes as well as classes taught by other teachers were invited to participate in the study. Over the two academic years 90 FS students participated in this research.

While all the 90 students participated in Part II of the research, Part III was conducted in relation to the topic of mensuration in Applied Mathematics. As part of the research a task which involved the construction and interpretation of a written text, was administered to 23 Applied Mathematics students in my class in 2006. The student responses to this task yielded some very interesting results and it was decided to obtain some more data by repeating this task with the Applied Mathematics class of 2007. Though I was not personally involved with this group, the teacher concerned thought that the task was quite interesting and was keen to conduct the task in her class. The 12 students of this class were invited to complete the other tests of the research and thus included as participants of the research as well. In all there were now 35 participants for the Mensuration Task, and 102 FS participants over the three years all of whom had completed schooling in their respective countries and many of them had their prior education in languages other than English. Almost all of these participants were aged from 18 to 21 years.

In addition, during the first year of data collection in 2005, I was involved in teaching Year 12 Mathematical Methods for the Victorian Certificate of Education (VCE) offered to mature students of the Tertiary and Further Education (TAFE) sector of RMIT University. While many of these students were Australian Anglo Saxons, some of them were from other cultural and linguistic backgrounds, but were fluent in English, having completed some years
of education in Australia. However, the majority of these VCE students were mature age students who have had a break in their education due to individual reasons and may not have had much such success in school mathematics. This provided me with an opportunity to include some of these TAFE VCE students as a reference group for comparison purposes in order to investigate whether these students had any difficulties with the language of mathematics. All VCE Mathematical Methods students at RMIT were invited to participate in this study resulting in a group of 44 students over 2005 and 2006 participating in Part II of the research. All these students had completed at least Units 1 and 2 of VCE Mathematical Methods.

The next section describes the administration of the three parts of the study and the various instruments used in the process.

### 3.4 Procedure of the Study

A preliminary investigation in the form of informal interviews and a pilot study, with selected linguistic features embedded in a simple 6-item mathematics test, was conducted as described in Section 1.2. The results illustrated some of the language-related difficulties these students were facing. This prompted an extension of the investigation to other aspects of language use and a larger sample of students.

The first step in the research was to probe existing literature and identify technical vocabulary and syntactic structure known to be associated with difficulties in the understanding of mathematics (see Section 2.4). With this as the starting point, the next step was to investigate the level and impact of the language used in mathematics teaching and learning at the tertiary level and how NESB learners coped with this language use. As discussed earlier, learning mathematics at this level requires the use of language in many forms that predominantly involve reading and writing. Students are required to read and comprehend textbooks, class notes, word problems in textbook or tutorial exercises, test or examination items, and project or assignment tasks. They are
also expected to write mathematically when solving problems, or completing assessments or project reports. Both reading and writing involves mathematical and general English vocabulary as well as syntax and semantics of the language used. The aim of the research was to explore all these aspects of language use within the research design identified in the previous section. The three parts of the study are represented in the following diagram and described in detail in the next three subsections.
Figure 3.1 Overview of the research
3.4.1 Part I: The language of texts

The first part of the study was concerned with the mathematics reading material that students are exposed to at the tertiary level. Mathematics textbooks, notes, exercises, and examinations have to be interpreted successfully by mathematics students and the further they progress in their education at the tertiary level, the more they have to do this independently. The language used in some of the popular high school and university mathematics textbooks was the focus of this phase of the research. The reasons for the selection of the textbooks and the relevance of these books to this study are outlined below.

VCE is the Victorian equivalent of the Australian entry requirements for university programs. The FS program offers an alternative pathway to university programs for international students. While there are a number of textbooks for VCE mathematics courses, FS students are provided with RMIT University notes and exercises and are not required to buy any prescribed textbooks. However, they are encouraged to use Australian high school textbooks as references. Furthermore, on completion of the FS program, the NESB students are expected to use university textbooks as part of their university programs in the following year. Hence a selection of VCE and university Mathematics textbooks was felt to be appropriate for this part of the investigation.

Three VCE Mathematical methods texts and three university mathematics books were selected for analysis. The books selected are commonly prescribed textbooks or reference books in schools and universities and are readily available in libraries and most academic book stores. The VCE textbooks were *Maths Quest 12 Mathematical Methods* (Nolan, Phillips, Watson, Denney, & Stambulic, 2000), *Essential Mathematical Methods 3 & 4* (Evans, Lipson, Jones, & Avery, 2005) and *Macmillan Mathematical Methods* (Rehill & McAuliffe, 1999). These are among the most popular texts used in Victorian schools. These are also the VCE mathematics textbooks that are recommended as reference books for FS students, the main participants of this study. Two of these textbooks have been prescribed textbooks over the past few years for the TAFE VCE students of
RMIT University, who were also participants of this study albeit for reference purposes.

The university textbooks that were selected for the study were *Advanced Engineering Mathematics* (Kreyszig, 2006), *Probability and Statistics for Engineers and Scientists* (Hayter, 2007) and *Australian Business Statistics* (Selvanathan, Selvanathan, Keller, & Warrack, 2007). Table 3.1 lists these books with the relevant sections and abbreviations. These books are currently listed as recommended references in my teaching of mathematics for the Associate Degree in Engineering at RMIT University.

*Advanced Engineering Mathematics* is a popular book found on most university booklists for engineering students in Melbourne, it is available in bookstores and there are multiple copies available in university libraries. The textbook is in its ninth edition and has been in use for several years. *Australian Business Statistics* is the prescribed textbook for business students at Monash and RMIT universities. This book is in its fourth edition and I was impressed by its presentation and clear explanation of statistical concepts and recommended it to my students as a very useful reference book for the statistics component of their course even though they were engineering students. However, I needed a textbook that dealt with these concepts in an engineering context and this led to my selection of Hayter’s (2007) *Probability and Statistics for Engineers and Scientists*. This book is in its third edition, is very well presented, and all problems are presented in an engineering context with realistic data. As a large proportion of the participants of this study were aspiring to be engineers and some of them choose to do the Associate Degree in Engineering at RMIT University, it was felt that these books would form an appropriate sample for analysis.

Textbooks differ in their presentation of a topic as well as in layout of material. Most textbooks at this level present concepts and theory as sections of chapters and this is followed by a *Problem Set* of practice questions on the topic presented. Such problem sets could include numerical questions and/or word problems. My observation as a teacher has shown that students may or may not read all the theory presented especially in reference books however, the portion likely to be
used by most students would be the problem sets in a textbook. For a meaningful comparison of language use, it is necessary to choose comparable problem sets from all textbooks. Given that ‘word problems’ are likely to pose greater challenges in comprehension of language and use contextual words that may be unfamiliar to NESB learners, I decided to choose a topic that had word problems involving linguistic features identified in the literature such as comparative phrases, passive voice, or the mathematics register.

One such topic was identified as *Normal Probability Distributions* which had problem sets with a relatively high proportion of word problems. Wilson (2002) observes that a statistics course for engineers uses a considerable amount of subject specific language and concepts. This is particularly true of the topic of probability and is likely to present obstacles to NESB students. In each text, the topic of ‘normal probability distributions’ is presented either as a whole chapter or as a section of a chapter, and has problem sets or parts thereof, devoted to worded application problems. Hence it was decided to select a collection of word problems on ‘normal probability distributions’ from each of the textbooks as a sample for analysis. All six books could then be considered on comparable basis. While some books had a whole problem set devoted to normal probability distributions, others had this topic combined with other distributions. In the latter situation, word problems on normal probability distributions were selected from the problem set. Yet other books had several problem sets on different aspects of the topic with a few word problems in each. In this case the word problems were selected from each problem set. The details of the books and the selected materials together with the abbreviated reference which will be used from here on are given in the following table.
<table>
<thead>
<tr>
<th>Textbook</th>
<th>Abbreviation</th>
<th>Material selected</th>
<th>Number of problems</th>
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<tr>
<td>Advanced Engineering Mathematics (9\textsuperscript{th} Ed.) (Kreyszig, 2006)</td>
<td>UNI1</td>
<td>Problem Set 24.8 Problems 5 – 13</td>
<td>9</td>
</tr>
<tr>
<td>Probability and Statistics for Engineers and Scientists (Hayter, 2007)</td>
<td>UNI2</td>
<td>Problem Set 5.6: Problems 1 – 5, 11, 18 – 20, 22, 23, 25</td>
<td>12</td>
</tr>
<tr>
<td>Australian Business Statistics (Selvanathan, et al., 2007)</td>
<td>UNI3</td>
<td>Chapter 8 Problems Q20 – 24, 26, 31, 32, 35 – 41</td>
<td>16</td>
</tr>
<tr>
<td>Essential Mathematical Methods (Evans, et al., 2005)</td>
<td>VCE1</td>
<td>Chapter Review Problems</td>
<td>9</td>
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<tr>
<td>Macmillan Mathematical Methods (Rehill &amp; McAuliffe, 1999)</td>
<td>VCE2</td>
<td>Problem Set 12.02 Problems 7 – 22</td>
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</tbody>
</table>
Once the materials were selected, the aim was to compare the language in the six textbooks. For this, two aspects were considered. One was the readability of the text measured in terms of word length and sentence length, and the other was the linguistic complexity in terms of the syntactic and semantic features of the problems. The readability level of each problem set was calculated using the four techniques discussed in Section 2.3.3 of the literature review. As indicated earlier, any given readability formula may not be sufficient to determine an exact level of readability in a selected passage as the difficulty level of words or expressions may not be taken into consideration. However, it was felt that it was worth considering these tests to compare the texts using similar criteria. Also it was decided to explore methods for cross verification as the literature has shown that different readability tests may vary in their evaluation of readability levels. If all the readability tests concurred in their findings, then the six textbooks could be ranked in order of difficulty level for readability.

Although readability levels as calculated by the different formulae, are objective quantified measures calculated from sentence length and word length, they do not account for several other factors likely to affect comprehension of written text, such as inclusion of context specific vocabulary, syntactic structure of a sentence, abstractness of concepts presented, and multiple meanings of certain words. In addition, readability for NESB students could be affected by cultural references or unfamiliar contexts and their comprehension abilities in English may not necessarily correspond to their age. It was the goal of this study to investigate all these factors and for this purpose, all the problem sets were subjected to a linguistic complexity analysis. For this, each problem from the selected problem set was analysed using a rubric for linguistic complexity. This elicited the linguistic features including vocabulary use as well as syntactic structure. The structure and development of this rubric is described in detail in Section 3.5.1.

3.4.2 Part II: Language of tests

A number of characteristic difficulties associated with the context specific vocabulary used in mathematics were identified in Section 2.3.3. The second part of the study was
aimed at gauging the effect of linguistic features and specialised mathematical vocabulary on the difficulty level of test items for secondary and tertiary mathematics students and in particular for NESB learners. The participants involved in this part of the study and details concerning the design and administration of the Mathematics Language Comprehension Test which was used for this purpose are described below.

**Participants of Part II**

There were 102 FS and 44 Year 12 VCE students who participated in Part II of the study (See Section 3.3). The numbers of students by gender and language background in these groups are shown in the following tables and figures. The distribution of male and female students in each group as well as in the whole group is shown in Table 3.2. It was observed that the number of female students was very small in FS while there was more gender parity in the VCE cohort.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>88</td>
<td>14</td>
<td>102</td>
</tr>
<tr>
<td>% in FS</td>
<td>86.3</td>
<td>13.7</td>
<td>100</td>
</tr>
<tr>
<td>VCE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>24</td>
<td>20</td>
<td>44</td>
</tr>
<tr>
<td>% in VCE</td>
<td>54.5</td>
<td>45.5</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>112</td>
<td>34</td>
<td>146</td>
</tr>
<tr>
<td>% of Total</td>
<td>76.7</td>
<td>23.3</td>
<td>100</td>
</tr>
</tbody>
</table>

While the possibility of gender-related difference was of some interest, of greater interest to this study was the language backgrounds of the participants. The selection the FS students and adult VCE students from RMIT University for this study provided a rich data source as the students were from very diverse language backgrounds. All FS students were international students who had completed their prior education in other countries and many of them in languages other than English. A considerable number also had difficulty communicating in English. The VCE students on the other hand, were fluent in the English language. While many of them were mature age, Australian
students and some were from other cultural backgrounds, all of them had studied mathematics in Australia and had lived in Australia for some time.

Among the 102 FS students, 17 different languages were identified as first languages (L1). This included five students who had grown up in a foreign country as their parents worked there and had come to Australia for higher education. These students had English as their first language. The distribution of the first language of the FS students is represented by the pie chart in Figure 3.2.

![Figure 3.2](image)

**Figure 3.2**  Distribution of FS participants by language

Figure 3.2 shows that the majority of students came from three language backgrounds, Chinese (25 %), Indonesian (21 %) and Arabic (14 %). Of the remaining 14 languages identified, only Sinhalese (9 %) was identified by more than 5 % of the FS students as their first language. While some L1 like Chinese, Indonesian and Sinhalese formed relatively larger groups in their own right and exhibited some linguistic differences which merited independent analysis, some L1 with very small numbers would not provide conclusive answers to the questions being researched. Consequently, it was decided to form clusters of certain L1 based on their linguistic features and geographic location to enable meaningful comparison. Hence, a variable called *language group* was introduced for analysis purposes and L1 was broadly classified into eight language
groups as Chinese, Indonesian, Other Asian (referred to as ‘Asian’ in brief hereafter), Indian, Sinhalese, European, Middle Eastern, and English. The distribution of students according to assigned language group is shown in Figure 3.3.

![Distribution of FS students by Language Groups](image)

**Figure 3.3** Distribution of FS participants by language group

This distribution was used for comparison purposes whenever the influence of language background was investigated however, no measure of language proficiency was used.

**Instruments and administration**

Two versions of a Mathematics Language Comprehension Test were constructed for this part of the study. Each consisted of 14 items which included context specific mathematical vocabulary (eg. ‘product’, ‘reciprocal’, ‘perpendicular’) and potentially confusing linguistic features such as comparative phrases (eg. ‘no more than’, ‘at least’) rather than any great mathematical challenge. The construction of these items was guided by the language difficulties identified in the literature (Section 2.4.1). Some examples of the items used are given below.

- **Write using algebraic symbols:** The product of $x$ and $y$ is not less than 32
- **Sketch an isosceles triangle**
- **Find the square root of the reciprocal of 25**
- **Sketch a line through $M$ perpendicular to $AB$** (a line $AB$ and a point $M$ were given)
The two versions of the Mathematics Language Comprehension Test were referred to as Version 1 (Appendix 3) and Version 2 (Appendix 4). All the participants from FS were administered one version of Mathematics Language Comprehension Test at the beginning of their program and the other version towards the end of the year. Some of the groups were administered Version 1 as their first test and Version 2 as the second while the other groups were administered the tests in reverse order. This resulted in about half the sample taking Version 1 at the beginning of the year and the other half Version 2 as their first test.

The aim of testing the FS participants twice was to determine whether exposure to English in Australia for nearly a whole academic year made any difference to their comprehension of mathematical language. The use of two different versions reduced the risk of prior practise effect and allowed each participant to be tested on a total of 28 items. Reversing the order of administration eliminated effects of differences if any, in mathematical or linguistic complexity between the two versions. However, this was not intended as a pre-test and post-test design and there was no planned intervention in the interim period. The second test at the end of the year was intended to facilitate further exploration as to whether the language difficulties experienced by FS international students were still being experienced towards the end of their course as they prepared to enter university in Australia. Although it was not the main intent, a crude measure was included by making one item identical on both versions of the test.

The reference group of TAFE VCE students were also administered one of these versions for further comparisons as explained in Section 3.3. Student scores on each test were compared by version, by administration sequence, and between FS and TAFE VCE students. Comparisons between the FS and TAFE VCE students’ scores would show whether there was a difference between the performance of NESB learners and first language learners of mathematics on items with linguistic features known to be generally difficult for all students. It would also provide an insight into any language-related difficulties experienced by the TAFE VCE students, who differ substantially from secondary school VCE students and often have negative perceptions about themselves, mathematics, and their ability to achieve in mathematics (Straber & Zevenbergen, 1996). Student responses to individual test items were analysed in the
light of these findings to determine whether or not there was a relationship between the language used in test items and performance on the test.

3.4.3 Part III: The language of student writing

While comprehending technical vocabulary in items on a standard test of mathematics can pose difficulties for learners, it is far more difficult for them to produce a written text in mathematics. The third part of this research was focussed on an analysis of student writing and interpretation of written text. This part considered the extent to which the mathematical genre and register was used by NESB students in a written task and also the ability of NESB students to interpret a mathematical text written by another student. Ferrari (2002) recommends the use of verbal language to represent mathematical ideas and communicate them to peers who do not share the same context, as a useful means of studying language use by students. A similar strategy was used for this part of the study, though individual students were required to produce a written text to convey information about a composite geometric shape to another student who had not seen the figure.

Participants of Part III

Students from two Applied Mathematics classes of FS were the participants in this part of the study. They formed a subgroup of the whole group of participants and all except two of these students had also participated in Part II. Twenty three students from the Applied Mathematics class of 2006 and 12 more students from the Applied Mathematics class of 2007 were involved in this part of the study.

As this was a novel task, it was thought that a sample from an expert would provide a useful frame of reference. Niemi (1997) emphasises that performance differences that distinguish experts from novices in any subject area is the highly organised knowledge of experts in their field of expertise. Experts perceive more patterns among the information they know, relating it to the organised knowledge of concepts and principles of their area of expertise. In this case, the expertise required would be knowledge of Mathematics and English. Two lecturers from the school were invited to participate as ‘experts’ in this part of the study. These two colleagues are both
mathematics lecturers in the same department who have taught Applied Mathematics in FS. Both speak English only and were asked independently to comment on the task after completing it. They stated that it was not easy to describe these figures in words without the aid of sketches. However, with their mathematical knowledge and command of English language they produced very effective written descriptions enabling the other expert to sketch the figure. These descriptions of the two figures were used as models when comparing students’ descriptions.

The demography of this subgroup of participants was analysed by gender and linguistic background as was done for the whole group in Part II. In all, two experts and 35 FS students completed the Mensuration Task and the gender distribution of this group is shown in Table 3.3. The predominance of males in the sample population was reflected in this subgroup as well and there were only 5 females including one of the experts.

<table>
<thead>
<tr>
<th>Table 3.3</th>
<th>Distribution of Part III participants by gender</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
</tr>
<tr>
<td>Male</td>
<td>32</td>
</tr>
<tr>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
</tr>
</tbody>
</table>

The L1 background of this subgroup was analysed in terms of language groups as in Part II. This group was representative of 6 of the 8 language groups identified in Part II, as there were no students from European or English language background in this subgroup. Thus all the 35 students were classified as Chinese, Asian, Indonesian, Sri Lankan, Indian, or Arabic. The two experts, who were English speakers, have been treated as a separate language group called ‘expert’ but for purposes of analyses in order to differentiate them from the English L1 students. The distribution of participants by language groups is shown in Table 3.4.
Table 3.4  Distribution of Part III participants by language groups

<table>
<thead>
<tr>
<th>Language</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert</td>
<td>2</td>
<td>5.4</td>
</tr>
<tr>
<td>Asian</td>
<td>6</td>
<td>16.2</td>
</tr>
<tr>
<td>Chinese</td>
<td>7</td>
<td>18.9</td>
</tr>
<tr>
<td>Indonesian</td>
<td>3</td>
<td>8.1</td>
</tr>
<tr>
<td>Sri Lankan</td>
<td>7</td>
<td>18.9</td>
</tr>
<tr>
<td>Indian</td>
<td>5</td>
<td>13.5</td>
</tr>
<tr>
<td>Arabic</td>
<td>7</td>
<td>18.9</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
<td>100.0</td>
</tr>
</tbody>
</table>

As individual responses were analysed for this part of the study, participants were referred to by a code for purposes of analyses and presentation of results in Chapter 6. Firstly, the 35 names were placed in alphabetical order. The code was devised as a combination of letters and numbers consisting of the first letter M or F indicating their gender, followed by a four letter code indicating language background, and a number indicating their position in alphabetic order. This coding of participant identity enabled the presentation and discussion of specific samples of student writing without compromising confidentiality.

**Instruments and administration**

Two versions of a task on mensuration were created for this purpose (Appendices 6 & 7). This task was administered to the 35 FS participants in two Applied Mathematics classes of 2006 and 2007, and to the two experts as described in the previous section. Each version required one participant to write a description of a given geometric figure using words only that would allow another student to recreate the figure from the written text. Two compound two dimensional geometric figures (Figure A and Figure B) were constructed for this task each comprising multiple geometric shapes such as triangles, rectangles, circles and trapeziums. The two figures are shown in Figures 3.4 and 3.5 below.
Each class where the task was administered was divided into two groups. One group was given Version A of the task with Figure A to describe and the other group was given Version B with the different Figure B. They were allowed about 15 minutes to describe their respective figures as best as they could so that a student from the other group could sketch it from their written description without seeing the figure. At the end of this time the figures were collected back and the description sheet given to a student from the other group. The students were then asked to read the description, interpret it as best as they could, and attempt to reconstruct the original figure. The two experts were requested to complete the task in a similar fashion. One lecturer was given Figure A to describe and the other, Figure B. The descriptions were then exchanged and each lecturer attempted to reconstruct the figure from the given description. This provided an expert reference for each figure, which was used as a model for comparisons, and as a basis for the development of an evaluation rubric. Approximately half of the students and one of the experts wrote a description for Figure A, while the other half and the remaining expert wrote descriptions for Figure B.
The two versions of the task enabled all 35 participants to take on the roles of both describers and sketchers. In this way, all students could describe one figure and attempt to reconstruct the other. Although the two figures varied in their structure and location, both involved similar geometric shapes and required similar descriptions of structure and location for successful interpretation. Hence it was felt that combining the results of the two figures was justified in the analysis of descriptive language, and the quality of the resulting sketches. This task provided some interesting observations about language use and interpretation by NESB learners.

3.5 Data Analysis

As indicated earlier, a combination of qualitative and quantitative methods were employed in this study. “The process of observing, recording, analysing, reflecting, dialoguing and rethinking are all essential parts of the research process” (Erlandson, et al., 1993, p. 5). Though the research was predominantly qualitative and exploratory in nature, observable or measurable indicators as suggested by Green and Browne (2005) were identified for the abstract construct of ‘language difficulties’. Firstly, ‘language’ was approached from the two perspectives of ‘reading’ and ‘writing’ and each of these from the point of view of ‘vocabulary’ and ‘syntax’. Secondly, ‘difficulties’ were seen to arise in the contexts of ‘readability’ and ‘linguistic complexity’.

Where appropriate, data were quantified and empirical attributes such as traditional readability measures, number of mathematical vocabulary words, number of comparative phrases or cultural references and scores on simple tests were employed to measure these indicators. This allowed the use of a much larger sample of evidence than would have been feasible by purely qualitative methods. Analysis decisions “should inform, and be informed by, the rest of the design” (Maxwell, 2005, p. 95). Simple descriptive statistics and effect size calculations were then used to compare, contrast and evaluate student population effects, and to quantify observed results (Coe, 2006; Cohen, 1988). These were then used in the description of observed trends. A brief explanation of effect sizes is provided here, prior to the discussion of how the data were analysed in the next section.
As the research was not experimental, the comparisons made were often between existing program, gender, or language groups. Hence effect size was deemed to be the best measure in comparing groups that are not controlled for variables or equal in number. Effect size is a way of quantifying the difference between two groups using the standard deviation to contextualise the difference in means (Cohen, 1988). When looking at the difference between two groups, it is a measure of ‘how big the difference is’. Effect size \((d)\) is calculated using the means of the two groups and their pooled standard deviation, as

\[
d = (M_1 - M_2) / \sigma_{\text{pooled}}
\]

to take account of groups varying in size. Hedges's \(g\) is an inferential variation on Cohen's \(d\) that corrects for biases due to small sample sizes, computed by using the square root of the Mean Square Error from the analysis of variance testing for differences between the two groups (Glass, McGaw, & Smith, 1981; Hedges & Olkin, 1985). Effect size calculations with the Hedges’s \(g\) correction, were done for all parts of the research using Coe’s spreadsheet available online (Coe, 2006).

Effect size measures are adjusted for sample size. When comparing such groups, effect size is a measurement of how much the range of scores from the two groups overlap and can be interpreted in terms of the percent of non-overlap between the scores of the two groups. (Cohen, 1988) classified the size of the effect as small \((< 0.5)\), medium \((\text{between } 0.5 \text{ and } 0.8)\) or large \((> 0.8)\).

The table of ‘equivalents of \(d\)’ (Cohen, 1988) included in Appendix 10 provides an interpretation of the values obtained. For instance, a small effect size index of 0.1 indicates that there is a 7.7% non-overlap between two normally distributed populations of equal variance. In other words the combined area not shared by the two distributions is 7.7% of the total area covered by the two distributions. This also means that half of the scores of the distribution with the higher mean lies above 54% of the other distribution. An example of a small effect size cited by Cohen (1988) is the difference between the heights of 15- and 16- year olds.

A medium effect size is conceived by Cohen (1988) as one that is visible to the naked eye. A medium effect size of 0.5 indicates a 33.0% non-overlap between the two distributions. On the other hand, a large effect size indicates distributions that are well
separated with very little overlap as illustrated by the difference between mean heights of 14- and 18-year old girls (Cohen, 1988). A large effect size index of 3.2 for instance, indicates a 94.2% non-overlap between the two distributions, which means that half of the scores of the distribution with the higher mean lies above 99.9% of the scores of the other distribution.

Izard (2004) suggested the addition of another descriptor of effect size as *very small* (< 0.2) which could prove very useful in educational studies as distinguishing between differences of such small magnitude could be of importance to educators. Furthermore, Izard’s classification also took effect size values to the accuracy of two decimal places thereby refining the endpoints of the size descriptors within strictly assigned ranges. The descriptors for effect sizes and their assigned ranges are shown in Table 3.5 below.

<table>
<thead>
<tr>
<th>Effect size magnitude</th>
<th>Cohen’s descriptor with Izard’s modification</th>
<th>Assigned ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.2</td>
<td>Very small</td>
<td>0.00 to &lt; 0.15</td>
</tr>
<tr>
<td>0.2</td>
<td>Small</td>
<td>0.15 to &lt; 0.45</td>
</tr>
<tr>
<td>0.5</td>
<td>Medium</td>
<td>0.45 to &lt; 0.75</td>
</tr>
<tr>
<td>0.8</td>
<td>Large</td>
<td>0.75 or more</td>
</tr>
</tbody>
</table>

Thus effect size index and its interpretation provided a good measure of comparing various groups of unequal sizes as well as in determining the magnitude of difference between two mean values. For the purposes of this study, it was decided use effect size calculations whenever comparisons were made between two groups. It was also decided that *large* and *medium* effect sizes would be treated as indicative of significant differences between the two groups being compared. A *large* effect size would signify a substantial difference between groups with very little overlap in the distributions of the variable of interest. A *medium* effect size would also indicate a notable difference between the groups though less pronounced with some overlap between the distributions. A *small* effect size would indicate that while there was considerable
overlap between the two distributions under considerations, there were differences worth taking note of between the two groups. It was decided that a very small effect size (0.00 to < 0.15) would be considered negligible signifying that there were no notable differences between the two groups being compared. The overlap between distributions will be explained and illustrated the first time effect size calculations are used in Chapter 4.

Ensuring trustworthiness was an important priority at every stage of the research. Hence a number of data tables have been included in the results section of each of the three parts of the study as evidence of observed trends, and a ‘rich and thick description’ has been provided for the analysis of these results. This will be discussed in greater detail in Section 3.6.

Each part of the research involved a different mode of data entry and analysis. Part I analysed the language of the selected textbooks, Part II involved analysis of the results of the mathematics language tests, while Part III examined data from the Mensuration Task and analysed student writing as well as interpretation of written text. Each data set was then analysed using a mix of qualitative and quantitative methods. The methods employed for each part of the study are detailed in Sections 3.5.1 to 3.5.3.

### 3.5.1 Language use in mathematics textbooks

The main aim of this part of the research was to compare the language usage of the six textbooks selected. The selected problem sets with word problems on the topic of ‘normal probability distributions’ were subjected to the same analyses to determine firstly, the readability, and secondly, the linguistic complexity of the problems.

**Readability Analysis**

As discussed in Chapter 2, a number of formulae are available to determine the readability level of a text. The Flesch reading ease and the Flesch-Kincaid grade level are two measures of readability that are commonly used and readily available as a built-in facility in Microsoft Word 2007. Hence they were used for a preliminary evaluation of the readability levels of these selected problem sets from the six textbooks.
The results were then cross checked using two other methods, namely, the SMOG readability calculator available online and the ATOS readability test. Mathematical and scientific texts have particular characteristics such as the use of technical terms, formulae and symbols, which are taken into account when whole book analysis is carried out using the ATOS readability test. Although whole book analysis was not feasible for this study, the online ATOS readability test was used to provide an alternative evaluation. Readability levels for the problem sets of all six books were thus obtained by various methods and compared. This would enable the dependability of readability formulae for mathematics texts to be tested (see Section 2.3.3). The results are recorded and discussed in Chapter 4.

*Linguistic Complexity Analysis*

The second aspect was the analysis of the syntactic and semantic features in the selected materials. As indicated earlier, reading and comprehending a worded problem adds to the mental processing required in solving any mathematical problem. Often problems that use difficult linguistic features place greater demands on NESB learners in ways that do not affect fluent readers of English. For instance, the NESB students might focus on the language of the question in an effort to make sense of unfamiliar words, superfluous connecting phrases, or complex grammatical structures. The purpose of this part of the study was to identify such features of each problem from the selected problem sets, and determine the extent of their use in the selected textbooks.

A rubric used for the analysis of linguistic complexity of test items on a national mathematics test in the US was used as a basis for this part of the analysis (Shaftel, et al., 2006). This rubric was used for test items at the lower and middle school level and designed to cover all areas of school level mathematics.

The item rating rubric for scoring the linguistic features was reviewed by professionals including mathematics teachers, math assessment specialists, and a speech–language pathologist who specialized in second language learning. The features included the total number of words, sentences, and clauses in each item; syntactic features such as complex verbs, passive voice, and pronoun use; and vocabulary in terms of both mathematics vocabulary and ambiguous words (Shaftel, et al., 2006, p. 111).
My research required the analysis of tertiary textbooks and one selected topic on the aspects of language use identified in the literature review. A modified form of this rubric was developed and used for this purpose (Appendix 8). While retaining as much of the structure and content of Shaftel’s rubric as possible, some realignments and modifications were made to suit the purpose of this study.

In developing the rubric for this study, careful consideration was given to the main aim of this part namely, to identify features of word problems that were likely to cause difficulties for tertiary NESB learners despite being competent in the relevant mathematical concepts. Contextual references are to be expected in word problems as is the use of both technical vocabulary and syntactic structures appropriate to the particular mathematics topic. In fact it is a necessary part of professional preparation at this level that students of Engineering Mathematics for instance, be familiar with the contexts of engineering or students of Business Mathematics with the contexts likely to arise in that field.

Moreover topics involving more sophisticated mathematics often require complex sentence structures. This is particularly so for the topic of normal probability distributions that was selected for this analysis. Most problems on this topic are likely to require the use of comparative constructions, passive forms of verbs or propositional phrases (see examples below). In addition, there appeared to be another category of words referred to as *superfluous* words or phrases for the purposes of this study. Such superfluous words or phrases may appear to be non-essential fillers and the sentence may well be grammatically sound without them. However, they are sometimes necessary to convey the meaning of the sentence, to establish the context, to emphasise a fact, or in some cases, to add mathematical information. In any case they mean more information processing for the reader and consequently adding to the difficulty level of the text for NESB readers. The following excerpts from some of the selected problems illustrate these features.
Example 1

The weekly error (in seconds) of a brand of watch is known to be normally distributed. Only those watches with an error of less than 5 seconds are acceptable. ...

b) Determine the probability that fewer than two watches are rejected from a batch of 10 randomly selected watches (Evans, et al., 2005).

Example 2

The amount of sulphur dioxide escaping from the ground in a certain volcanic region in one day is normally distributed with a mean of... However, if a volcanic eruption is imminent, there are much larger sulphur dioxide emissions.

Under ordinary conditions, what is the probability of there being a daily sulphur dioxide emission larger than ...

If your instruments indicate ...have escaped from the ground on a particular day, would you advise that eruption is imminent? (Hayter, 2007).

As seen in these excerpts, the information provided is essential for dealing with the mathematics involved, but the text contains a number of passive verbs (e.g., rejected, is normally distributed), comparative constructions (e.g., fewer than, larger than) or superfluous phrases (e.g. under ordinary conditions, from the ground, in one day, on a particular day) that are potentially difficult for NESB learners trying to grapple with English words and phrases unfamiliar to them.

While textbook authors have to ensure that the mathematical content and standards required in professional practice have to be maintained in the problems presented in their texts, lecturers, teachers and tutors need to be aware of these potential linguistic pitfalls for the growing number of NESB learners in their classrooms. This was kept in mind as the primary goal of analysis using the rubric and guided the selection of the components of each section of the rubric and the specific criteria for counting each of them.

The information sought, was about the number of technical words, ambiguous words, potentially difficult syntactic structure like passive voice or comparative constructions and cultural references that could hinder understanding for NESB learners. Based on all
the considerations discussed earlier, the rubric developed for this study, was divided into four sections: readability related, syntactic, semantic, and cultural. Each section examined relevant components of language use counted on the basis of specific criteria. The *readability* section investigated problem length (measured by number of words in a problem), word length (measured by number of letters), and sentence length (measured by number of words per sentence). The rubric was used to count occurrences of these components in each problem from each selected problem set. The *syntactic* section scrutinised the grammatical constructions and recorded the average number of passive voice usages, pronouns, relative pronouns, and comparative constructions in each problem. The number of prepositional and other superfluous phrases was also examined. The *semantic* section of the rubric examined the words used in the problems for their technical and contextual meaning. The number of mathematical vocabulary words as well as mathematical words with other everyday meanings was counted. Words referring to potentially unfamiliar contexts were also counted. Also considered in this section of the rubric, were words or phrases that carried indirect or implied meaning relevant to the problem and essential for solving it. The last section of the rubric investigated *cultural references*. The problems were scanned for local colloquial usages that may be unfamiliar to international students or references specific to one culture that may be unfamiliar to students from other cultures. The components of the four sections of the rubric and the criteria used in counting each are summarised in Table 3.6., and the detailed rubric can be seen in Appendix 8. While there is some subjectivity in classifying the words in a problem, a uniform approach was maintained for all the selected problems.
Table 3.6  Components of Linguistic Complexity Rubric

<table>
<thead>
<tr>
<th>Section</th>
<th>Components</th>
<th>Criteria for counting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Readability related</strong></td>
<td>Total number of words</td>
<td>All words except abbreviated units, numbers and symbols counted literally rather than functionally (e.g. Standard deviation is counted as two words).</td>
</tr>
<tr>
<td></td>
<td>Number of long words</td>
<td>Words with 7 letters or more.</td>
</tr>
<tr>
<td><strong>Syntactic</strong></td>
<td>Number of sentences</td>
<td>Full sentences or numbered question parts.</td>
</tr>
<tr>
<td></td>
<td>Number of propositional/superfluous phrases</td>
<td>Phrases beginning with before, after, according to etc. or phrases that sentences could stand without. (e.g., on a given day, it is found that, is known to be, etc.)</td>
</tr>
<tr>
<td></td>
<td>Number of uses of passive voice</td>
<td>All uses of passive verbs (e.g., were sold, normally distributed, were rejected, were chosen etc.)</td>
</tr>
<tr>
<td></td>
<td>Number of complex verb forms</td>
<td>Verb forms of 3 words or more. (e.g., would have been, will be accepted)</td>
</tr>
<tr>
<td></td>
<td>Number of pronouns</td>
<td>All simple pronouns (e.g., she, their, them, it, etc)</td>
</tr>
<tr>
<td></td>
<td>Number of relative pronouns</td>
<td>Connecting a preceding noun to a dependent clause (e.g., that, who, whom, which)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Expletive ‘that’ were excluded. (e.g., claims that, finds that, probability that etc.)</td>
</tr>
<tr>
<td></td>
<td>Number of comparative constructions</td>
<td>All expressions that translated to mathematical statements with &lt;, &gt;, ≤ or ≥ or indicated minimum or maximum (e.g., more than, less than, at least, between, no greater than, faster, longer, smallest, etc.)</td>
</tr>
<tr>
<td></td>
<td>Number of complex negatives</td>
<td>Double negations, negatives combined with comparatives (e.g., no more than 5%)</td>
</tr>
<tr>
<td><strong>Semantic</strong></td>
<td>Number of mathematics vocabulary words</td>
<td>Any word with a specific meaning in mathematics (e.g., dimensions, perimeter, probability, mean, standard deviation, percentage, normal distribution etc.)</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Number of mathematics words with other meanings</td>
<td>Any word in the above category with another possible meaning in English (e.g., mean, normal, standard, distribution etc.)</td>
</tr>
<tr>
<td></td>
<td>Number of other unfamiliar contextual references</td>
<td>Any word that is not everyday use and a NESB learner may require a dictionary for. (e.g., surgical anaesthesia, component, fluorescent etc.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Cultural</strong></th>
<th>Number of local colloquial usages</th>
<th>Expressions unfamiliar outside Australia (e.g., Lotto)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of references to specific culture</td>
<td>(e.g., Australian rules football, cricket, Diwali, yum cha or words relevant to any other culture</td>
</tr>
</tbody>
</table>

Each problem from each of the selected problem sets was analysed using the rubric and the data was entered into an Excel spreadsheet. Mean values of these quantities were computed for each text book. For instance, mean word length, mean number of long words, technical words, comparative phrases, or cultural references gave an indication of the quality of language used in various textbooks in a problem set on the same topic. Once more the effect size of the differences in language use of the various texts was calculated from these data. Results are summarised in Chapter 4.

### 3.5.2 Part II: Language of test items

The data recorded for this part included student data (i.e., student name, FS group, gender, and language background) for all participants, their responses to each item on each version of the test and their total score on each test. All this data was entered into an Excel file, then coded and entered into an SPSS data file.
Firstly, the information provided by the participants was analysed to determine the proportions of the sample by gender and linguistic background. The next step was to analyse the results of the tests. The ‘comparison of means’ function on SPSS was used to compare students’ performance between the beginning of year test and the end of year test, as well as between Version 1 and Version 2. This was also used to determine whether there was any difference in performance between gender or language groups as well as between FS and TAFE VCE students.

The effect sizes of these differences were evaluated using Cohen’s $d$ with Hedges’ correction $g$ to determine the effect of the various differences among participants on performance on these tests. The results of all these analyses are discussed in detail in Chapter 5. Following this, the responses to individual questions on each test were analysed for error patterns. These were compared by language groups to determine whether there were any prevalent errors displayed by any of the language groups. The results of these analyses are summarised and discussed in Chapter 5.

3.5.3 Part III: Language use in student writing

The analysis of data related to the construction and interpretation of written text involved entirely different methods. The Mensuration Task administered for this part required two students to work independently as a pair as described in Section 3.4.3. Thus there was a ‘describer’ and a ‘sketcher’ involved in completing the task. Analysis involved a scrutiny of both the written text produced by the describer as well as the corresponding sketch drawn by the sketcher, and an exploration of possible relationships between features of the written descriptions and the quality of the resulting sketches. Evaluation of written text and the sketches were done by using rubrics developed on the basis of the responses of the experts.

Analysis of sketches

The first step was to evaluate the quality of the sketch produced by a student as a result of interpreting the written description of another student. This was done by giving a numeric score to each sketch produced based on a scoring rubric which was devised to ensure uniformity of the evaluation. The evaluation was based on the components of
the figure and a score out of 10 was allocated for structure and location of the five components of the figure. As each compound two dimensional figure used in the task consisted of five geometric shapes, an accurate sketch was expected to have these five shapes drawn correctly with reference to shape, relative size, position and orientation. Five points were allocated for structure and five for location. It was decided that structure would consist of shape and size of the five components. Five correct geometric shapes in a figure would fetch a score of 2.5 and the correct size ratio would account for the remaining 2.5 making up the total score of 5 for structure. Location was evaluated according to the position and orientation of the components of each figure. Components drawn in correct positions obtained a score of 2.5 and a further 2.5 was allocated for correct orientation of these five geometric shapes. This resulted in a numerical score for each sketch produced from the written description of a student. Examples to illustrate this method of allocating scores to sketches are included in Chapter 6. The quality of sketches produced as indicated by these numerical scores were then compared by gender and language groups in an attempt to identify factors likely to affect the interpretation of the written text and its translation into a drawn sketch.

Analysis of written text

Secondly, the written text produced by each describer was analysed for the type of the language used to convey meaning to the other student who was required to draw the sketch. The written text was firstly examined for commonalities and differences to facilitate analysis.

It was observed that most of the responses were either descriptive in nature, that is, they were aimed at creating a picture of the figure for the reader, or they were procedural, that is, giving directions to the reader for constructing the figure. These preferences are referred to as style for the purposes of this study. Responses were not always amenable to discrete classification. A few participants used a mix of both styles in their text. In fact, it could be observed that this classification is a continuum which is more probabilistic than deterministic.
Accordingly, all responses were classified as descriptive or procedural based on the predominant style adopted by the student. It was also observed that students approached the description of the figures in two different ways. Some students tended to look at the whole picture and describe parts thereof, while other students concentrated on individual components of the figure and tried to connect them into a whole. The method adopted by the students in tackling the written task was referred to as approach. The written texts were thus classified into holistic or componential approaches based on this difference. These two classifications resulted in four possible types of written text: descriptive holistic, descriptive componential, procedural holistic and procedural componential as shown in Table 3.7 below.

<table>
<thead>
<tr>
<th></th>
<th>Holistic Approach</th>
<th>Componential Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptive</td>
<td>Descriptive holistic</td>
<td>Descriptive componential</td>
</tr>
<tr>
<td>Procedural</td>
<td>Procedural holistic</td>
<td>Procedural componential</td>
</tr>
</tbody>
</table>

This matrix was used to analyse the written descriptions of all participants including the two experts, and all the descriptions were thus classified in terms of these four categories. Sketch scores were compared between these categories to determine whether the style and approach used in the writing had any impact on the quality of the resulting sketches. The impact of gender and language background on writing style and approach was also investigated using frequency analysis.

Next, the language used was analysed by scrutinising the words used in the description of a figure. All words that described mathematical shapes, concepts, dimensions, position or orientations were included in the count. Articles, conjunctions, or prepositions were ignored. Each descriptive text was then assessed for linguistic characteristics in terms of the vocabulary used. This analysis was guided by the requirements of a description that would lead to an accurate sketch of the figure provided. It was expected that more words and phrases from the mathematical register,
and those giving precise directions, dimensions, and orientation would result in more accurate sketches. With this in mind, the words and phrases used were classified as mathematical register, locators or qualifiers. A clear distinction was necessary between these categories for effective classification and elimination of any confusion regarding ambiguous cases. The mathematics register was defined as any word with a specific meaning in mathematics such as ‘rectangle’, ‘trapezium’, ‘annulus’, ‘diameter’, ‘height’, ‘vertices’ etc. Other words/phrases used were classified as locators or qualifiers depending on their function in a sentence. For instance, ‘above’, ‘between’, ‘left’, ‘inside’, ‘on each side’ etc. were locators while ‘big’, ‘smaller’, ‘outer’, ‘equal’, ‘this semicircle’, ‘other end’, ‘third rectangle’ etc. were classified as qualifiers. Certain ambiguous cases which could be used in different categories were listed clearly so that validity was maintained. For instance, it was decided that ‘draw a right-angled triangle on each side’ had ‘each side’ used as a locator while ‘each triangle was right angled’ used ‘each’ used as a qualifier. All this information for each written text produced by the participants and the two experts was collated using an Excel spreadsheet.

Analyses were carried out determine whether gender or language background impacted on the types of descriptions used by students. The number of mathematical register words, locators and qualifiers used in the description were also compared among gender and language groups. The effect sizes of the various differences were calculated using the mean and standard deviation of the sketch scores of the various subgroups. These were then used as indicators for further qualitative analyses to get an insight into language use and interpretation by NESB learners of mathematics.

Lastly, the association between the features of the written description and the quality of the corresponding sketches was investigated by determining the correlation between the numbers of types of words and the sketch scores.

The analyses used in the three parts of the research provided an insight into the language difficulties faced by NESB learners of mathematics at the tertiary level. The results of these analyses are given in detail in Chapters 4, 5 and 6 respectively.
3.6 Establishing Trustworthiness

In many qualitative, naturalistic studies, it may not be possible or meaningful to apply conventional methods to establish credibility. Furthermore naturalistic studies often involve the researcher as part of the research setting and deal with human beings in existing relationships such as students or colleagues. Nevertheless the “applied nature of educational inquiry makes it imperative that researchers and others have confidence in the conduct of the investigation and in the results of any particular study” (Merriam, 1998, p 199). Qualitative researchers need to address the concerns of others regarding both credibility and ethical issues. This is dealt with in the following two subsections.

3.6.1 Credibility considerations

In 1985 Lincoln and Guba (1985) identified four constructs that researchers operating within the positivist paradigm strive to achieve, namely truth value, applicability, consistency and neutrality, leading to the four criteria that are important in demonstrating the trustworthiness of a research study in this context, that is, internal validity, external validity, reliability, and objectivity. They then go on to suggest four criteria that are appropriate for a naturalistic paradigm namely, credibility, transferability, dependability, and confirmability as the naturalist’s equivalents of the four conventional terms above (Lincoln & Guba, 1985).

During the decade that followed after 1985, there were theoretical debates about the strategies to achieve the criteria that ensure the trustworthiness of qualitative research (Connelly & Clandinin, 1990; Lincoln, 1995; Merriam, 1998; Stake, 1994; Wolcott, 1994). These include triangulation involving multiple approaches of investigating the same issue, prolonged engagement or long term observation, member checks taking data or interpretations back to participants for endorsement (referred to as respondent validation) by Maxwell (2005), peer examination asking colleagues to comment on emerging findings, clarifying researcher’s position regarding assumptions, worldview and theoretical orientation, providing detailed ‘rich thick description’ enabling readers to determine how closely another situation resembles the settings and providing an ‘audit trail’ in the form of detailed documentation of procedures enabling another
researcher to replicate the study. All these views were considered in the design of this mixed method research and the constructs needed to establish trustworthiness, the corresponding derived criteria for both research paradigms as well as some of the suggested strategies are summarised in Table 3.8.

### Table 3.8 Criteria for establishing trustworthiness

<table>
<thead>
<tr>
<th>Construct</th>
<th>Criteria for Quantitative research</th>
<th>Alternative Criteria for Qualitative research</th>
<th>Some Suggested Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truth value</td>
<td>Internal validity</td>
<td>Credibility</td>
<td>• Triangulation</td>
</tr>
<tr>
<td>Applicability</td>
<td>External validity</td>
<td>Transferability</td>
<td>• Member check (respondent validation)</td>
</tr>
<tr>
<td>Consistency</td>
<td>Reliability</td>
<td>Dependability</td>
<td>• Long term observation</td>
</tr>
<tr>
<td>Neutrality</td>
<td>Objectivity</td>
<td>Confirmability</td>
<td>• Peer examination</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

More recently with the increased acceptance and use of qualitative research, there has been a great interest in developing formal standards to establish credibility of such research. There has been a call for public disclosure of methods as a criterion (Lincoln, 2001) and documentation tables providing “detailed explanations of how research questions are related to data sources, how themes or categories are developed, and how triangulation is accomplished” (Anfara, Brown, & Mangione, 2002, p 30). While the discussion continues, it has been contended that “it is neither desirable, nor possible to reach consensus about or prescribe standards of evidence” in the heterogenic field of qualitative research (Freeman, deMarrais, Preissle, Roulston, & St. Pierre, 2007, p. 25). Rather, it has been concluded that it is best that qualitative researchers take the steps to
address concerns of credibility ensuring that their claims are justified by evidence in the data and applying criteria that are relevant to the type of research being conducted.

This was the approach taken in conducting this research. From the research design, selection of participants, administration of instruments, through to data analysis, every effort was made to achieve results that would provide trustworthy answers to my research questions and prove useful to future practitioners. This is illustrated by the documentation table (Anfara, et al., 2002) shown in Table 3.9 which shows how multiple data sources corroborated findings by triangulation to address each research question.

Table 3.9 Documentation of method

<table>
<thead>
<tr>
<th>Focus of Research Question</th>
<th>Sources of data and basis for claims</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q1: Language difficulties faced by tertiary NESB students</strong></td>
<td>Literature review: difficulties identified in literature Part I: readability and linguistic complexity of textbooks Part II: responses to the mathematics language test Part III: written descriptions and sketches in the Mensuration Task</td>
</tr>
<tr>
<td><strong>Q2: Impact of language difficulties on performance</strong></td>
<td>Part I: difficult language features identified in textbooks Part II: impact of language of test items on test scores Part III: quality of student writing and interpretation of written text by students</td>
</tr>
<tr>
<td><strong>Q3: Impact of language background on use of English language</strong></td>
<td>Part II: performance on test compared by language groups Part III: student writing and interpretation compared by language groups</td>
</tr>
<tr>
<td><strong>Q4: How do textbooks compare in use of English language</strong></td>
<td>Part I: comparison of language use in three VCE and three university textbooks Part III: preferences of style and approach observed in the Mensuration Task</td>
</tr>
</tbody>
</table>
The exploratory approach of this study made it predominantly qualitative in nature, although the method incorporated the use of some empirical data and descriptive statistics. This mixed method approach enabled me to draw on the strengths of both paradigms. The nature of data eliminated concerns of subjectivity due to researcher or participant bias. Yet the use of my own classes in the research maintained the involvement and contact of researcher with participants facilitating long term observation, which is one of the strengths of qualitative research. What follows is an examination of the strategies and procedures adopted during this research with regard to the four constructs of trustworthiness discussed above.

Validity or credibility “deals with the notion that what you say you have observed is, in fact, what really happened. In the final analysis validity is always about truth” (Shank, 2002, p 92). It is “the trustworthiness of inferences drawn from data” (Eisenhart & Howe, 1992, p 644). The very purpose and exploratory nature of this research was geared towards gleaning the ‘truth’ regarding the language-related difficulties encountered by NESB students at the tertiary level. Data collection and analysis have been well documented for each of the three parts of the research. The use of descriptive statistics measures and effect size calculations has provided a means to observe trends and justify the conclusions reached. The observations provided during the informal interviews with a few students in the preliminary study, and with the teachers as experts in Part III serve as member checks. Moreover, the use of different methods to investigate different aspects of language use has enabled corroboration of results. Thus any concerns about the credibility of this study are taken care of by the procedure of the study.

Transferability or external validity of the findings is also ensured subject to certain limitations of a case study. The participants of the study have been shown to be of very diverse cultural and language backgrounds, and are likely to be fairly representative of other cohorts of international students in many parts of the world. They represent many different linguistic backgrounds and cultures suggesting that the conclusions and inferences are likely to be transferable to other similar multilingual classrooms. While they need to be applied with caution to other situations such as multilingual classrooms where teachers and students share a first language and teaching occurs in a second language, there are numerous commonalities in language-related difficulties faced by
mathematics students universally as seen by the various studies discussed in Chapter 2. A rich and thick description has been provided about the research setting and methods used for data collection and analysis sufficient to enable researchers elsewhere to replicate the study meaningfully or adapt it to suit their circumstances.

The results obtained from the three parts of the research, carried out from three different standpoints, complement each other and provide verification using triangulation so that all inferences drawn can be justified. Investigation of three aspects of language use in the three parts, provide multiple methods of data collection and analysis. For instance, likely difficulties in comprehending the specialised vocabulary of mathematics are tested by the mathematics language test in Part II and by the interpretation of written text in Part III, and the textbooks are analysed in Part I to gauge the level of mathematics vocabulary used. The results obtained from all three parts can be used to corroborate each other in determining whether tertiary NESB learners have difficulties with the specialised vocabulary of mathematics. It is hoped that the reliability of the study will be strengthened by triangulation, to increase the *dependability* of this study.

The *confirmability* criterion was addressed by the research design and data analysis. The nature of the data collected in each of the three parts of the study has drawn on the strengths of both paradigms of research. While the analysis and interpretation retained the richness and detail of qualitative and constructivist paradigm, the quantified data and observations were not subject to influence by researcher or participants. Furthermore, interpretation of results has been described and documented in detail. Thus it is hoped that the validity, reliability and objectivity concerns have all been addressed by the design of the study.

### 3.6.2 Ethical considerations

Action research and participant observation in a classroom situation involves careful planning to address potential ethical issues. As this research involved students, a formal application was made to the Human Research Ethics Committee of RMIT University, written permission was obtained from the Head of the FS unit, and all ethical norms
were carefully adhered to. Firstly participation was completely voluntary and students were informed that data collected would be used for research purposes. A plain language statement was issued at the outset to all potential participants and the purpose and method of study was explained to them. Every participant signed a consent form prior to taking part in the study.

Though these steps were taken as a precaution, there was no concern at any stage of this research breaching any ethical norms, as the instruments administered only identified language difficulties in general. Student identity was not revealed at any stage and all data was kept in the custody of the researcher and treated as confidential. The results of this research had no bearing on the performance of the students in their course. Any information gleaned was only to improve teaching strategies for the future. In fact the nature of the research was such that most students were keen to participate and determine how the language of mathematics might be impacting their performance.

This chapter has described the three part research design investigating the reading and writing aspects of language from three different standpoints. Each part used different instruments and techniques and hence called for varying methods of data analysis. Appropriate strategies applied in each case have also been described. The trustworthiness and ethical considerations have been discussed. The results of these analyses are presented in the next three chapters.
CHAPTER 4: RESULTS I
LANGUAGE USE IN MATHEMATICS TEXTBOOKS

This chapter presents the results of Part I of the study, which investigated language use in three Victorian Certificate of Education (VCE) and three university mathematics textbooks. The aim of this part of the study was to assess the level of language use and hence identify possible linguistic challenges that students in general and learners from Non-English Speaking Backgrounds (NESB) in particular, may face in using textbooks at the tertiary level. Larson et al. (1978) raised the question “do commonly employed arithmetic texts and workbooks at the upper elementary and junior high school level take into account syntactic complexity when attempting to teach and provide practise in basic and advanced mathematical concepts?” (p. 85). One of the research questions of this study raises a similar question about the textbooks at the tertiary and senior secondary level and this chapter is a response to that.

The results of this part are presented as observations from the analyses of readability and linguistic complexity of written texts. Apart from some inferences about the textbooks that were analysed, all conclusions and implications are considered in Chapters 7 and 8 in conjunction with results from the other parts of the study.

The six textbooks were selected from those available on the basis of several considerations as described in Chapter 3. All six textbooks, coded as UNI1, UNI2, UNI3, VCE1, VCE2, and VCE3, for the purposes of this study (see Section 3.4.1 and Table 3.1), are currently in use by students at the secondary school or tertiary level and each has its own strengths. Having used each of these textbooks at some stage in my teaching career, either as a textbook or as a reference book, I had intuitive evaluations about the general standard of each book. Some books were appreciated for their simple or clear presentation of topics, others were selected for the variety of worked examples or practice questions, while others were selected for the challenging and/or contextual problems they presented. However, an objective analysis of the actual language used and the breakdown into its components threw new light on the linguistic aspects of each book as well as the relative standing of each book in relation to the others investigated.
As detailed in Chapter 3, Problem Sets or parts thereof, with word problems on the topic of ‘normal probability distributions’ were chosen for analysis from each of the textbooks. Firstly, this was a topic found to be common to VCE, Foundation Studies (FS), and university mathematics courses of many programs such as engineering, science and business. Secondly, the topic typically involved word problems with several mathematical vocabulary words, linguistic features that have been identified as difficult, and a variety of social contexts.

A sample problem from a university textbook and one from a VCE text book are given below.

Sample 1
The breaking strength X [kg] of a certain type of plastic block is normally distributed with a mean of 1250 kg and a standard deviation of 55 kg. What is the maximum load such that we can expect no more than 5% of the blocks to break? (UNI 1)

Sample 2
The owner of a new van complained to the dealer that he was using, on average, 18 litres of petrol to drive 100 km. The dealer pointed out that the 15 litres/100 km referred to in an advertisement was ‘just a guide and that actual consumption will vary’. Suppose that the distribution of fuel consumption for this make of van is normal with a mean of 15 litres/100 km and standard deviation 0.75 litres/100 km.

a. How probable is a van that uses at least 18 litres/100 km?
b. What does your answer to (a.) suggest about the manufacturer’s claim?
c. Find C_1 and C_2 such that the van’s fuel consumption is more than C_1 but less than C_2 with a probability of 0.95. (VCE1)

Two forms of analyses were conducted: the readability of the text, and the linguistic complexity of the language used. The readability of each problem set was determined using four different techniques and the linguistic complexity was analysed using a rubric as described in Section 3.5.1. The results of these analyses are presented in the following sections.
4.1 **Readability Measures of Problem Sets**

The literature review in Section 2.3.3 identified several tests for evaluating the readability levels of text. These measures were seen as a first step in the analysis of the selected textbooks. Readability levels are indicated either as a grade level of the readers it is intended for, or as a score that rates the reading ease of the text. As discussed in the literature review, while these tests continue to be used, it is acknowledged that they do not always agree in their evaluation of readability as they use different criteria for calculating these measures. Studies in the past (e.g., Stokes, 1978, Harrison, 1979) found that scores obtained from the various tests varied, in some cases by several grade levels. Hence, four different tests were selected to assess the readability of these books. The rationale was that despite issues of variability of results, four tests would provide a more objective measure of readability of the six texts for comparison purposes. All four tests were readily available and could be carried out by submission of the required text online. This enabled easy evaluation of all six problem sets by each of the four methods. Initial observations suggested that the results of the four tests provided very different evaluations and the preliminary conclusion was to agree with the previous studies that readability tests could not provide reliable measures. I was curious about these results and this added another dimension to my research, which was to determine why the results were so varied. This prompted the linguistic complexity analysis of the six problem sets using a rubric developed for the purpose. A closer look at the results of the various readability tests in conjunction with the linguistic complexity analysis helped and contributed to a more detailed understanding of the value and relevance of such tests.

This part of the study, originally intended as just one aspect of and a brief preliminary investigation into language use in mathematics, turned out to be a long journey of exploration into ways of assessing language use in mathematics textbooks. The following sections present the findings at the various stages of this long journey.

All six problem sets were subjected to the Flesch Reading Ease test, as well as the Flesch-Kincaid, SMOG, and ATOS tests of readability grade levels (see Section 2.3.2) and the results were compared. The results obtained from all these tests are summarised
in the table below and the numerical values obtained are explained in the following paragraph.

Table 4.1  Readability levels of selected texts by score or grade level

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Flesch Reading Ease score</th>
<th>Grade level of readability Flesch-Kincaid</th>
<th>SMOG</th>
<th>ATOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNI1</td>
<td>39.6</td>
<td>14.5</td>
<td>13.7</td>
<td>6.9</td>
</tr>
<tr>
<td>UNI2</td>
<td>38</td>
<td>13.2</td>
<td>11.3</td>
<td>9.2</td>
</tr>
<tr>
<td>UNI3</td>
<td>50.1</td>
<td>10.5</td>
<td>10.5</td>
<td>7.6</td>
</tr>
<tr>
<td>VCE1</td>
<td>50.7</td>
<td>8.1</td>
<td>13.7</td>
<td>9.5</td>
</tr>
<tr>
<td>VCE2</td>
<td>48</td>
<td>11.8</td>
<td>14.7</td>
<td>8.6</td>
</tr>
<tr>
<td>VCE3</td>
<td>65.8</td>
<td>6.5</td>
<td>13.0</td>
<td>8</td>
</tr>
</tbody>
</table>

The Flesch Reading Ease score rates text on a 100 point scale and is calculated using the average sentence length and average number of syllables per word. A higher score on this scale indicates easier reading. The ideal score for most standard documents is suggested to be between 60 and 70. A score of 70 may be understood by a Grade 7 student while a legal document is more likely to have a score of 10. According to the results, VCE3 secured the highest score for reading ease and was the only textbook with a score above 60 suggesting that the language used in the problem set of VCE3 was the easiest to read. This was followed by VCE1 and UNI3 with scores above 50. VCE2, UNI1, and UNI2 were rated as more difficult reading with UNI2 rated most difficult to read.

The other tests of readability used three different formulae to compute a grade level indicator as a measure of readability. On these tests, a score of 9 indicated that the text was readable by a student of Grade 9. Hence a score above 12 can be assumed to indicate more difficult reading beyond that of school texts, for example, formal communications, technical jargon, and legal documents. As indicated earlier, the results
of various tests have been found to vary. This was observed to be the case in relation to these analyses as well, with the different tests producing quite different results.

The four tests used different criteria and formulae for assessing readability as discussed in Section 2.3.3, and consequently, the range of values used for presenting results was found to vary from test to test. For instance, the Flesch-Kincaid grade levels of the six texts ranged between 6 and 14, while the SMOG grade levels lay between 10 and 15 and the ATOS levels between 6 and 10. These were not easily comparable and could be the reason that these results appeared to differ. So for each test result, it was decided to rank the six textbooks in increasing order of difficulty. Although it is acknowledged that such a ranking would not take account of the magnitude of differences between the textbooks, the aim was to obtain a consensus of the relative difficulty level of these textbooks. It was hoped that despite the incomparable range of scores, ranking the results for each test would produce some uniformity in these readability ratings and provide a ranking of the books in order of reading difficulty. The rankings obtained by the six textbooks on each readability test are presented in Table 4.2 below. A rank of 1 indicates it is the easiest to read and a rank of 6 indicates the most difficult.

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Readability Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flesch</td>
</tr>
<tr>
<td>UNI1</td>
<td>5</td>
</tr>
<tr>
<td>UNI2</td>
<td>6</td>
</tr>
<tr>
<td>UNI3</td>
<td>3</td>
</tr>
<tr>
<td>VCE1</td>
<td>2</td>
</tr>
<tr>
<td>VCE2</td>
<td>4</td>
</tr>
<tr>
<td>VCE3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2  Readability rankings of selected texts
While it was known that the actual grade levels evaluated by the various tests could vary, it was an unexpected outcome that no two tests agreed in the relative ranking of the texts. For instance, the problem set from UNI2 was attributed with a relatively higher difficulty level by two of the tests while the SMOG test contradicted this. Among the six texts analysed, UNI1 was given the highest grade level of 14.5 by the Flesch-Kincaid test (ranked sixth, most difficult to read) and the lowest grade level of 6.9 by the ATOS test (ranked first the easiest to read). It could be seen that there was no simple and definitive way of stating that one book was more readable than another on the basis of these tests. While the immediate response was to conclude that readability tests were not reliable and would not serve the purpose of this study, it was decided that it would be more appropriate to reserve judgements until further investigations were made. With this in mind, the four rankings received by each book were scrutinized to get an overall consensus suggested by these results.

While the Flesch Reading Ease scores and the Flesch-Kincaid grade levels agreed on the first four ranks, they disagreed on the fifth and sixth ranks. However, the general impression gleaned from all four rankings suggests that UNI2 may be more difficult to read, while VCE3 and UNI3 are the easiest to read. Somewhere in-between this continuum lie VCE2, VCE1 and UNI1, possibly in decreasing order of difficulty. This suggested ranking of text readability was noted and will be discussed later in conjunction with other results in Section 4.3.

This was followed by a search for other important factors such as syntactic and semantic structure of the language used in each text (Mousley & Marks, 1991; Shaftel, et al., 2006), which could provide a measure of the linguistic complexity of a text. As the main goal of this part of the research was to determine the difficulties NESB students are likely to face in using these textbooks, it was decided to employ a more in-depth analysis of the language used in the problem sets. This led to the development of a modified version of the Linguistic Complexity Rubric developed by Shaftel, et al. (2006) which was used to analyse each worded problem, in every problem set (see Section 3.5.1).
4.2 Linguistic Complexity of Word Problems

It is recognised that word problems are more difficult to solve, as they require additional interpretation of the context of the problem before it can be related to the mathematical concepts involved and relevant process skills such as differentiation, integration, factorisation, or computational skills can be utilized. It has been suggested that “much of the difficulty children experience with word problems can be attributed to difficulty in comprehending abstract or ambiguous language” (Cummins, et al., 1988, p. 405). As the problem sets selected for this part of the study consisted of word problems on normal probability distribution, they were likely to use some common mathematical vocabulary and linguistic structures. For instance, comparative phrases such as ‘no more than’, ‘exceeding’, and ‘at least’, are common in problems involving normal probability distributions. Each worded problem of each problem set was analysed using the rubric described in Section 3.5.1. The results are summarised in the Table 4.3 in terms of the average instance per problem for each textbook.
Table 4.3  Linguistic complexity analysis of selected texts

<table>
<thead>
<tr>
<th>Textbook</th>
<th>VCE</th>
<th>VCE1</th>
<th>VCE2</th>
<th>VCE3</th>
<th>UNI1</th>
<th>UNI2</th>
<th>UNI3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average numbers per problem</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Readability related</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total words</td>
<td>90.00</td>
<td>68.75</td>
<td>55.60</td>
<td>39.25</td>
<td>86.42</td>
<td>76.38</td>
<td></td>
</tr>
<tr>
<td>Long words</td>
<td>22.44</td>
<td>17.44</td>
<td>14.00</td>
<td>9.88</td>
<td>24.00</td>
<td>17.94</td>
<td></td>
</tr>
<tr>
<td>% of long words</td>
<td>24.94</td>
<td>25.36</td>
<td>25.18</td>
<td>25.16</td>
<td>27.77</td>
<td>23.49</td>
<td></td>
</tr>
<tr>
<td>Sentences</td>
<td>4.89</td>
<td>3.94</td>
<td>4.07</td>
<td>1.50</td>
<td>4.75</td>
<td>4.94</td>
<td></td>
</tr>
<tr>
<td>Avg words per sentence</td>
<td>18.41</td>
<td>17.46</td>
<td>13.67</td>
<td>26.17</td>
<td>18.19</td>
<td>15.47</td>
<td></td>
</tr>
<tr>
<td><strong>Syntactical</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Superfluous phrases</td>
<td>2.22</td>
<td>1.75</td>
<td>1.40</td>
<td>1.75</td>
<td>2.33</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
<td>Passive voice</td>
<td>4.33</td>
<td>2.56</td>
<td>2.60</td>
<td>0.88</td>
<td>2.83</td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td>Complex verb forms</td>
<td>0.11</td>
<td>0.31</td>
<td>0.27</td>
<td>0.13</td>
<td>0.08</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Pronouns</td>
<td>0.78</td>
<td>0.44</td>
<td>1.00</td>
<td>0.50</td>
<td>1.42</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>Relative pronouns</td>
<td>0.44</td>
<td>0.56</td>
<td>0.47</td>
<td>0.13</td>
<td>1.33</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>Comparative constructions</td>
<td>2.78</td>
<td>2.19</td>
<td>2.40</td>
<td>1.13</td>
<td>2.58</td>
<td>2.13</td>
<td></td>
</tr>
<tr>
<td>complex negatives</td>
<td>0.00</td>
<td>0.13</td>
<td>0.07</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td><strong>Semantical</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maths vocabulary</td>
<td>5.33</td>
<td>4.50</td>
<td>4.67</td>
<td>3.75</td>
<td>4.67</td>
<td>4.38</td>
<td></td>
</tr>
<tr>
<td>Maths words with other meanings</td>
<td>4.44</td>
<td>3.88</td>
<td>3.60</td>
<td>3.13</td>
<td>3.83</td>
<td>3.81</td>
<td></td>
</tr>
<tr>
<td>Unfamiliar contexts</td>
<td>2.22</td>
<td>0.75</td>
<td>0.47</td>
<td>1.50</td>
<td>0.83</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>Indirect/implied meaning</td>
<td>0.67</td>
<td>0.31</td>
<td>0.13</td>
<td>0.00</td>
<td>0.25</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td><strong>Cultural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local colloquial usages</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Cultural references</td>
<td>0.00</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.13</td>
<td></td>
</tr>
</tbody>
</table>
Contrary to expectation, it was observed that the university textbooks were not necessarily more complex than the VCE textbooks in terms of the features analysed. This is probably because the VCE textbooks, which are intended mainly for Victorian students, tend to assume a good grasp of English while the university textbooks are expected to cater for a much more global readership as these books are marketed internationally in many countries.

The following sections discuss the observations derived from the four sections of the rubric in more detail. Effect size calculations have been used to enable pair-wise comparisons between the six books to determine the magnitudes of these differences. The method and interpretation of these analyses are presented in detail to provide a rich and thick description (Lincoln & Guba, 1985; Merriam, 1998) to facilitate replication by other researchers as part of ensuring trustworthiness.

4.2.1 Readability related features

The first section of the rubric analysed readability features of the selected texts and consisted of three components, problem length, word length, and sentence length.

Problem length

The first component that was considered under the readability related aspects was the problem length as measured by the number of words in a problem. Problem length varied considerably from problem to problem in each book and ranged from 23 to 168 words across the six books. As the number of problems in each book also varied greatly, effect size, which took into account the distributions of problem length, provided a better measure of the differences in mean problem lengths between the various textbooks.

Paired comparisons between the six textbooks resulted in effect size calculations for 15 pairs generated by considering all combinations of the six books taken two at a time. It was found that the effect size ranged from very small to large and there were a number of large effect sizes. To make the comparisons more visible it was decided to represent this information in a matrix form. The textbooks were arranged in descending order of
problem length from left to right and from top to bottom, and effect sizes of all 15 pair-wise comparisons were represented as shown in Table 4.4, and effect sizes were calculated for each pair-wise comparison. The descending order of means ensured that all effect sizes were positive. This representation provided a much clearer picture and trends became clearly visible with the application of a colour coding. The effect sizes of large, medium, small and very small were respectively colour coded as red, orange, green, and blue, as shown by the legends at the foot of each table. This matrix representation has been used for all other effect size calculations for the remaining analyses of linguistic complexity.

Effect sizes across the pair wise comparison for the six textbooks ranged from very small to large as shown by the colour coding of the magnitude of differences. The colours show a fairly regular pattern consistent with order of the decreasing means. The first effect size in the top left corner indicates the magnitude of difference between the highest and the second highest mean problem lengths, while the top right corner cell represents the magnitude of the difference between the largest and the smallest. That is, the effect sizes consistently increase from left to right and decrease from top to bottom along the table. Any exceptions to this trend are easily visible and indicate a difference in the distributions of the books concerned. This method of interpreting effect sizes in terms of the overlap of distribution and visible colour coding is explained in detail for the first component of one section of the rubric, and similar arguments are applied in other cases.
Table 4.4  Effect size of difference in mean problem lengths

<table>
<thead>
<tr>
<th>Textbook (Mean problem length)</th>
<th>UNI2 (86.42)</th>
<th>UNI3 (76.38)</th>
<th>VCE2 (68.80)</th>
<th>VCE3 (55.6)</th>
<th>UNI1 (39.25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCE1 (90)</td>
<td>0.09</td>
<td>0.41</td>
<td>0.60</td>
<td>1.17</td>
<td>1.88</td>
</tr>
<tr>
<td>UNI2 (86.42)</td>
<td>_</td>
<td>0.28</td>
<td>0.46</td>
<td>0.92</td>
<td>1.40</td>
</tr>
<tr>
<td>UNI3 (76.38)</td>
<td>_</td>
<td>_</td>
<td>0.23</td>
<td>0.74</td>
<td>1.41</td>
</tr>
<tr>
<td>VCE2 (68.80)</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>0.43</td>
<td>1.00</td>
</tr>
<tr>
<td>VCE3 (55.6)</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The largest effect size between the mean problem lengths was 1.88 for VCE1 and UNI1. This shows that there is a 79.4% non-overlap between the lengths of the problems in these two books. As explained in Section 3.5, from Cohen’s (1988) table for the equivalents of $d$ (Appendix 10), this indicates that half the problems in VCE1 were longer than 97.7% of the problems in UNI1. In other words half of the problems in a VCE textbook were longer than almost all the problems in a textbook for advanced engineering on the same topic.

On the other hand, VCE1 and UNI2 had the closest means in problem length with a very small effect size of 0.09. This translated to a very small non-overlap of 7.7% between the distributions of problem lengths in the two textbooks or equivalently, the upper half of the problems of VCE1 were longer than the shortest 54% of the problems of UNI2. This indicated that the VCE textbook had almost the same distribution of problem length as a university statistics textbook for engineering students.

A small effect size of 0.41 in the difference between VCE1 and UNI3 indicated a 27.4% non-overlap which meant that 50% of the problems in VCE1 were longer than
65.5% of the problems in UNI3. Similar pair-wise comparisons were made between all the six books to probe for any interesting trends displayed by the data.

For the pair wise comparisons involving the three university textbooks, UNI2 and UNI3 both showed large effect sizes in their differences from UNI1 in mean problem length. Both these effect sizes indicated a 68.1% non-overlap showing that approximately half of the problems in both texts were longer than 91.9% of the problems in UNI1. This indicated that the problems on ‘normal probability distributions’ in statistics textbooks for both engineering and business students were longer than problems in the advanced engineering textbook. It was interesting to note that although the mean length of a problem in UNI2 was about 86 words and that of UNI3 was about 76 words, the magnitude of difference with UNI1 was slightly more for UNI3 compared to UNI2. This case is an illustration of standard deviation and sample size being taken into account in the calculation of effect sizes. The standard deviation (indicating spread) for UNI2 (40.9) being larger than that of UNI3 (30.4), accounted for the larger overlap of UNI2 with UNI1 resulting in the smaller effect size as illustrated by the sketch below.

![Figure 4.1 An illustration of overlap of distributions](UNI1-UNI2-UNI3)

For the pair wise comparisons of the three VCE textbooks, it was seen that VCE1 showed a medium effect size with VCE2 and a large effect size with VCE3. This meant that half of the problems of VCE1 were longer than about 73% of the VCE2 problems and 89% of the problems of VCE3. Furthermore an effect size of 0.43 between the latter two books indicated that half of the problems in VCE2 were longer than 66% of the problems in VCE3. This demonstrates that problem length varied considerably between the three textbooks of VCE Mathematical Methods selected for this study.
Overall comparisons of problem lengths in the six textbooks showed that University mathematics textbooks do not necessarily have longer problems than VCE textbooks on the same topic in this case normal probability distributions. VCE1 had the longest word problems of all six books. However, two of the university statistics books had longer problems than the other two VCE textbooks. Both university statistics textbooks had longer problems than the advanced engineering textbook and the other two VCE textbooks. These results are discussed in conjunction with the readability levels in Section 4.3.

Word length

While it is evident that longer problems require more reading and comprehension, it cannot be assumed to be the sole factor affecting readability. The next component that was considered likely to impact readability was the number of long words in a problem. As in the study by Shaftel, et al. (2006) for the purposes of this study, a word containing seven letters or more was defined as long.

While the number of long words per problem varied greatly owing to the varying problem lengths, it was found that all the textbooks were similar in the percentage of long words in relation to the total number of words per problem. It was observed that approximately 25 percent of the words in a problem were composed of long words. UNI2 had the highest percentage of long words, followed by VCE2, UNI1 and VCE3 with VCE1 and UNI3 having a slightly smaller proportion of long words per problem. Once more effect sizes were calculated to determine the magnitude of the difference displayed between the textbooks with respect to the mean percentage of long words per problem and the results are shown in matrix form in Table 4.5 below.
The textbooks were arranged as before, in descending order of the percentage of long words. It was observed that the effect sizes of the differences in the mean percentage of long words used in a problem varied from very small to medium. However, the variation was systematic and consistent with the decrease in means. The largest effect size was displayed between the UNI2 and UNI3, the books with the maximum and minimum percentage respectively of long words per problem. No irregularities were observed in the transition through the colour codes indicating that the percentage of long words in problems, were in the order that was evident from the percentage counts and there were no unusual distributions. This suggested that most books contained a similar proportion of long words in problems on normal probability distributions and it was observed that mathematics vocabulary words, such as ‘normally’, ‘distributed’, ‘probability’, and ‘deviation’, were used in almost all of the problems. A few other contextual words such as ‘manufacturer’, ‘jeweller’, or ‘intelligence’ accounted for the small differences in percentage of long words used.

Of the VCE textbooks, VCE1 which had the longest problems had the lowest percentage of long words, indicating that the authors used more words but relatively fewer long words. VCE2 which had shorter problems seems to use more long words.
VCE3 had fewer words and a lower percentage of long words per problem. These results add weight to the readability measures which ranked VCE3 as the easiest to read.

Among the university texts, UNI2 which had the longest problems was also found to use the greatest percentage of long words in a problem. UNI1 found earlier to have the shortest problems had a greater percentage of long words in these problems. UNI3 had the least percentage of long words among all the books probably making it easier to read. This again was in agreement with the combined results of the four readability tests which ranked UNI3 as the easiest and UNI2 as relatively more difficult with UNI1 ranked somewhere in between.

While there were no great differences between the texts on word length, problems with 25% or more of the words classified as long, called for a fairly high level of reading skill, considering that every word in a problem was counted, including ‘the’, ‘a’, ‘an’, ‘in’, and ‘on’. The long words that were mathematical vocabulary words like ‘normal’, ‘distribution’, ‘standard deviation’, and ‘variance’, would have been taught and repeatedly used in class. However, long words that related to the situational context may be unfamiliar to NESB learners and might hinder reading.

**Sentence length**

The third component of readability to be considered was average sentence length. The mean sentence length was calculated for each problem, from the total number of words in a problem and the number of sentences used. These values were then used to calculate the mean sentence length of the whole problem set for each text. It was observed that UNI1 used the longest sentences, followed by VCE1. VCE3 and UNI3 had the shortest sentences on average. Table 4.6 shows the mean sentence lengths for each text and the effect sizes of the differences between pairs of textbooks.
It can be seen from these results that there were considerable differences in sentence length between the various textbooks. The problem set from UNI1 had 32 words on average in each sentence. This was much larger than the means displayed by all the other texts. The next highest mean of nearly 20 words per sentence was seen in the problem sets from VCE1 and VCE3 with an average use of around 14 words had the shortest sentences among these six textbooks.

This difference of the mean sentence length, of the problems of UNI1 relative to the mean sentence length of the problems in all the five other books, is reflected in the large effect sizes seen in Table 4.6. These effect sizes coded red in the table, ranged from 1.05 (VCE1) to 1.91 (VCE3). This indicates that about 97% of the problems in VCE3 had a mean sentence length shorter than the top half or median value of the sentence lengths in the UNI1 problem set. VCE3, which used relatively shorter sentences in its problem set, also displayed large effect sizes in differences in sentence length with VCE1, VCE2 and UNI2, and a small effect size with UNI3. As far as sentence length is concerned, UNI1 was thus seen to stand out as different to others for
its lengthy sentences while VCE3 stood out at the other extreme for its relatively shorter sentences in relation to the other books.

Comparison of the three VCE textbooks showed that VCE1 and VCE2 had very similar mean sentence lengths as evidenced by the very small effect size of 0.04. This indicates an almost 100% overlap of the two distributions showing that the two books have almost identical distributions of sentence length in their problems. However, the difference of mean sentence length of both these books from VCE3 was large to the extent that half of the problems in both VCE1 and VCE2 were longer than at least 80% of the problems in VCE3.

Among the three university textbooks, UNI1 had very little overlap with the other two textbooks in their distributions of mean sentence length. The difference in means between UNI2’s problem set and UNI3’s problem set also shows a medium effect size of 0.60. This indicates that all the three textbooks differ in their style of writing on the component of sentence length. These differences in sentence lengths are likely to affect the readability of the text. However this will be compared with the results of the readability tests in Section 4.3.

### 4.2.2 Syntactic aspects

The syntax of a sentence is another major factor that affects the understanding of a text (Mousley & Marks, 1991). Very often grammatical structures like passive voice and comparative phrases hinder sentence clarity. This is particularly the case for NESB learners. As discussed in Section 3.5.1 five specific usages were targeted by this section of the rubric and the results of the analyses are presented here.

**Superfluous phrases**

The first component to be considered was the use of superfluous phrases. The operational definition of *superfluous phrases* for this study was taken as phrases that made no difference to the structure of the sentence but were included either to present some detail of the context of the problem or some mathematical information such as units of measurement as seen in the examples in Section 3.5.1. For a person fluent in
English, phrases such as on a particular day, from the ground, and under ordinary conditions could help in comprehending the question by clarifying the context. However, for NESB learners, as I have observed in FS classes, superfluous phrases appear to require additional time to process and often lead to the use of a dictionary to grasp the meaning and relevance to the problem.

It was observed that all textbooks used such phrases in word problems on normal probability distributions. The average number of usages per problem and the effect sizes of differences between the various textbooks in mean number of superfluous phrases are presented in Table 4.7 below.

<table>
<thead>
<tr>
<th>Textbook (Mean no. of superfluous phrases)</th>
<th>VCE1 (2.22)</th>
<th>UNI1 (1.75)</th>
<th>VCE2 (1.75)</th>
<th>UNI3 (1.625)</th>
<th>VCE3 (1.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNI2 (2.33)</td>
<td>0.04</td>
<td>0.24</td>
<td>0.28</td>
<td>0.31</td>
<td>0.43</td>
</tr>
<tr>
<td>VCE1 (2.22)</td>
<td>__</td>
<td>0.33</td>
<td>0.36</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>UNI1 (1.75)</td>
<td>__</td>
<td>__</td>
<td>0.00</td>
<td>0.09</td>
<td>0.29</td>
</tr>
<tr>
<td>VCE2 (1.75)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.09</td>
<td>0.30</td>
</tr>
<tr>
<td>UNI3 (1.625)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Effect size analysis showed that the magnitudes of difference ranged from very small to medium and there were no large differences. UNI2 had the largest mean number of superfluous phrases, with UNI3 and VCE3 showing the least. It was observed that the greatest effect size was not between UNI2 and VCE3 with the largest and smallest means, but between VCE1 and VCE3 and this is visible as an irregularity in the transition of the colours. This was because the distribution of UNI2 had a greater spread.
(st. dev. of 2.84) compared to VCE1 (st. dev. of 1.56), resulting in a greater overlap with the distribution of VCE3. This suggests that there is considerable difference between the two VCE textbooks with the highest and lowest mean number of superfluous phrases. Once more, VCE3 among the VCE books, and UNI3 from the university books, used a smaller number of superfluous phrases in their problems which could result in easier reading for NESB students when considering this particular component. Superfluous phrases were found to consist of short or long words, and involve any of the syntactic or semantic features being considered. In every case, they added to the length of the sentence. Hence it is not surprising that these results support the results obtained on sentence length as well as the rankings of the Flesch Reading Ease and Flesch-Kincaid tests. Average sentence length was one of the factors in the calculation of these readability measures (see Section 2.3.2). This suggests that the number of superfluous phrases in a problem contributes to reading difficulty on two fronts, increased sentence length and the inclusion of additional components such as long words, pronouns, relative pronouns, or unfamiliar contexts.

**Passive voice**

Another grammatical structure that is likely to cause linguistic difficulties is the passive form of verbs (Mousley & Marks, 1991; Shaftel, et al., 2006). Some uses of passive voice such as ‘is normally distributed’ or ‘was randomly selected’ occur fairly frequently in word problems of normal probability distributions. However, other passive voice uses such as ‘sold’, ‘received’, ‘time taken’, ‘is exceeded by’, ‘is to be exceeded by’, or ‘it is known that’, varied between the textbooks. The number of uses of passive verbs in each problem was recorded and the mean number of usages per problem for each text was calculated. The means and the magnitude of difference between these means by pairwise comparison are shown in Table 4.8.
<table>
<thead>
<tr>
<th>Textbook (Mean no. of passive verbs)</th>
<th>UNI2 (2.83)</th>
<th>VCE3 (2.6)</th>
<th>VCE2 (2.56)</th>
<th>UNI3 (1.75)</th>
<th>UNI1 (0.88)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCE1 (4.33)</td>
<td>0.46</td>
<td>0.69</td>
<td>0.81</td>
<td>1.43</td>
<td>1.78</td>
</tr>
<tr>
<td>UNI2 (2.83)</td>
<td>__</td>
<td>__</td>
<td>0.10</td>
<td>0.42</td>
<td>0.67</td>
</tr>
<tr>
<td>VCE3 (2.6)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.43</td>
<td>0.82</td>
</tr>
<tr>
<td>VCE2 (2.56)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.48</td>
</tr>
<tr>
<td>UNI3 (1.75)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
</tr>
</tbody>
</table>

The textbook with the most number of passive verbs per problem was VCE1 with an average of over 4 usages per problem. This was followed by UNI2, VCE3 and VCE2 all with an average of between 2 and 3 passive verbs per problem. However, UNI3 and UNI1 used passive verbs to a lesser extent. Based on the criterion of passive voice usage it would appear that UNI1 might be simpler to read followed by UNI3. It was interesting to note that these observations were closely aligned with the ranking according to the ATOS readability test. This could be a reflection of the additional criteria such as word familiarity that are used in the ATOS readability score calculations.

Effect sizes of the differences between books varied from very small to large. VCE1 with an average of 4.33 passive usages per problem showed considerable difference from the mean of all the other textbooks. An effect size of 1.78 with UNI1 indicated that half of the problems in the VCE book VCE1 used more passive verbs than about 96% of the problems from UNI1, the book for advanced engineering. In fact, the upper half of the distribution of number of passive usages in VCE1 was above at least 69% of the distributions of all the other textbooks.
On the whole, results showed that the VCE textbooks tended to use more passive verbs compared to the university texts, with the exception of UNI2. Two of the popular university books, UNI3 for business students and UNI1 for advanced engineering students both had fewer uses of passive voice. This suggests a perception of audience on the part of the authors and could be a deliberate simplification of language which might be advantageous to NESB students. Further research is needed to determine whether this is a response to the needs of multicultural classrooms or the natural preference of writing style of these authors.

**Pronouns**

The next component to be considered for syntactic analysis was the use of pronouns. Table 4.9 presents the mean number of pronouns per problem for each problem set of each text, and the magnitude of the differences between means. While pronouns are helpful in clarifying the person or object that is being referred to in the text, it can create hurdles in comprehension (Campbell, et al., 2007). According to Shaftel et. al (2006), “pronouns might be expected to cause confusion for less skilled linguists because they introduce a (possibly ambiguous) reference to another sentence element” (p. 121). Below a certain threshold of English proficiency, as is the case for many NESB learners, additional processing is often required to comprehend a text.
Table 4.9  Effect size of difference in use of pronouns

<table>
<thead>
<tr>
<th>Textbook (Mean no. of pronouns)</th>
<th>UNI3 (1.06)</th>
<th>VCE3 (1.0)</th>
<th>VCE1 (0.78)</th>
<th>UNI1 (0.5)</th>
<th>VCE2 (0.44)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNI2 (1.42)</td>
<td>0.19</td>
<td>0.24</td>
<td>0.35</td>
<td>0.52</td>
<td>0.62</td>
</tr>
<tr>
<td>UNI3 (1.06)</td>
<td>—</td>
<td>0.04</td>
<td>0.20</td>
<td>0.42</td>
<td>0.49</td>
</tr>
<tr>
<td>VCE3 (1.0)</td>
<td>—</td>
<td>—</td>
<td>0.18</td>
<td>0.45</td>
<td>0.51</td>
</tr>
<tr>
<td>VCE1 (0.78)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td>UNI1 (0.5)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.08</td>
</tr>
</tbody>
</table>

UNI2 was the book with the highest number of pronouns on average. Unlike the rankings on other criteria so far, UNI3 and VCE3 used more pronouns than VCE1, UNI1, and VCE2. These results do not appear to align with the results of any of the readability tests. This could possibly be explained by the inclusion of common pronouns (such as ‘he’, ‘she’, ‘it’, ‘they’, or ‘them’) being short and familiar words. Some readability tests use number of syllables and others use reading list of familiar words as criteria in their computations. Hence, more pronouns in a text could have contributed to the reduction of readability levels. In fact the presence of pronouns could prove useful as they occur more often with the active form of verbs hence ensuring a reduction of the more difficult passive form.

Relative pronouns

The next component considered was the average number of relative pronouns used in the word problems. Use of relative pronouns such as ‘that’, ‘which’, or ‘whose’, have also been found to cause difficulties for readers (Shaftel, et al., 2006), and the effects are likely to be more pronounced in the case of NESB learners. Problems from UNI2 had the largest number of relative pronouns per problem, while UNI1 had the least. These results and effect size calculations for the magnitude of these differences are
shown in Table 4.10 below. The difference between UNI2 with the largest mean and UNI1 with the least mean displayed the largest effect size and the other differences followed the expected pattern according to the increasing or decreasing order of the means as can be observed by the colour coding.

These results showed that two of the university texts used more relative pronouns than all the three VCE texts while the third, UNI1, used the least. Furthermore UNI1 showed a medium to large effect size with all the other texts indicating that UNI1’s use of relative pronouns was considerably less compared to all the other books. This could probably be attributed to the author’s relatively cryptic writing style as UNI1 also recorded the shortest problems. The three VCE texts were not very different in their use of relative pronouns borne out by the very small or small magnitude of difference between them.

Table 4.10  Effect size of difference in use of relative pronouns

<table>
<thead>
<tr>
<th>Textbook (Mean no. of relative pronouns)</th>
<th>UNI3 (0.63)</th>
<th>VCE2 (0.56)</th>
<th>VCE3 (0.47)</th>
<th>VCE1 (0.44)</th>
<th>UNI1 (0.13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNI2 (1.33)</td>
<td>0.60</td>
<td>0.65</td>
<td>0.74</td>
<td>0.72</td>
<td>0.97</td>
</tr>
<tr>
<td>UNI3 (0.63)</td>
<td>__</td>
<td>0.08</td>
<td>0.20</td>
<td>0.24</td>
<td>0.69</td>
</tr>
<tr>
<td>VCE2 (0.56)</td>
<td>__</td>
<td>__</td>
<td>0.12</td>
<td>0.16</td>
<td>0.60</td>
</tr>
<tr>
<td>VCE3 (0.47)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.03</td>
<td>0.51</td>
</tr>
<tr>
<td>VCE1 (0.444)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Large  Medium  Small  Very small
Comparative constructions

Any expression that translates to greater than (>\), or less than (<\, \leq) was taken as a comparative construction and these occur in most problems on probability distributions. Several studies over the years (e.g., Cummins, et al., 1988; De Corte, et al., 1985; Lean, et al., 1990; Shaftel, et al., 2006) have shown that comparative terms and comparison problems are difficult for students. Expressions such as ‘no more than’, ‘not exceeding’, ‘at least’, or ‘at the most’, are often confusing and necessitate considerable thought before accurate comprehension and application to the context of the problem. The average number of comparative constructions in each problem and the magnitude of the differences between texts are shown in Table 4.11 below.

Table 4.11 Effect size of difference in use of comparative constructions

<table>
<thead>
<tr>
<th>Textbook (Mean no. of comparative constructions)</th>
<th>UNI2 (2.58)</th>
<th>VCE3 (2.4)</th>
<th>VCE2 (2.19)</th>
<th>UNI3 (2.13)</th>
<th>UNI1 (1.13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCE1 (2.78)</td>
<td>0.09</td>
<td>0.22</td>
<td>0.39</td>
<td>0.43</td>
<td>1.07</td>
</tr>
<tr>
<td>UNI2 (2.58)</td>
<td>—</td>
<td>0.10</td>
<td>0.23</td>
<td>0.26</td>
<td>0.78</td>
</tr>
<tr>
<td>VCE3 (2.4)</td>
<td>—</td>
<td>—</td>
<td>0.16</td>
<td>0.21</td>
<td>0.99</td>
</tr>
<tr>
<td>VCE2 (2.19)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>UNI3 (2.13)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Large  Medium  Small  Very small

It can be seen that the magnitude of difference between most of the books were either small or very small. However, UNI1 had the least number of comparative constructions on average and showed a large effect size with all the other five textbooks. This suggested that UNI1 used significantly less comparative constructions than other books. This could again be attributed to writing style as UNI1 was the book with the shortest problems and least number of relative pronouns as seen earlier. While UNI2 had one of the highest averages in use of comparative constructions, it demonstrated a greater overlap with books with lower means. This is owing to a wider spread (highest standard
deviation of 2.24) in the distribution of comparative constructions, with some problems having fewer and others a larger number of comparative constructions. A number of problems with relatively fewer comparative constructions resulted in greater overlap with the texts with lower mean number of comparative constructions, while a number of problems with relatively higher number of comparative constructions accounted for the higher average value.

In general it was observed that the VCE textbooks seemed to use more comparative constructions on average compared to the university textbooks with the exception of UNI2. This suggests that authors of university textbooks appear to be more mindful of the fact that they cater for a multicultural audience while the VCE textbooks are mainly targeted at Victorian secondary school students.

4.2.3 Semantic aspects

The next section of the rubric dealt with the semantic aspect of the language used in the problem sets. As indicated earlier, word problems at this level of mathematics involve both the technical vocabulary of mathematics as well as the vocabulary required to establish the context of the problem. Mathematical vocabulary could affect readability for any student, and mathematical words with other meanings in everyday English could exacerbate the matter. However, local colloquial usages or cultural reference could be an additional hurdle that hinders comprehension for NESB learners who are often international students or recent migrants (Shaftel, et al., 2006). Hence this section of the rubric took mathematical vocabulary, mathematical words with other meanings, and other context specific usages into consideration.

Mathematical terms

Firstly, the number of mathematics vocabulary words in each problem was counted and averages obtained for each textbook. Most of the problems in all textbooks included the terms ‘normally distributed’, ‘mean’ and either ‘variance’ or ‘standard deviation’. This accounted for an average of about three mathematical terms per problem. The results of analysis of the six books showed that the number of mathematical terms ranged from 2 to 8 per problem. The averages for the various texts were compared, and
the magnitudes of difference between the texts in terms of their distributions were computed using effect sizes. These results are presented in Table 4.12 below.

Once more VCE1 topped the list with an average of over five mathematics vocabulary words per problem followed by UNI2 and VCE3, while UNI1 emerged as the textbook with the lowest number of mathematical terms per problem. UNI1 displayed large effect sizes with all but one textbook indicating that the distribution of the number of mathematical words of UNI1 had little overlap with the distributions of the other textbooks. In other words most of the problems in UNI1 used fewer mathematical vocabulary words than the other textbooks. On the other hand, VCE1 with a medium to large effect size with all the other textbooks. This indicates that half of the problems of VCE1 used more mathematical words than most of the other textbooks. This could be partly because VCE1 was also the book with the longest problems on average.

Table 4.12 Effect size of difference in use of mathematical terms

<table>
<thead>
<tr>
<th>Textbook (Mean no. of Mathematical terms)</th>
<th>UNI2</th>
<th>VCE3</th>
<th>VCE2</th>
<th>UNI3</th>
<th>UNI1</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCE1 (5.33)</td>
<td>0.52</td>
<td>0.49</td>
<td>0.66</td>
<td>0.66</td>
<td>1.15</td>
</tr>
<tr>
<td>UNI2 (4.67)</td>
<td></td>
<td>0.00</td>
<td>0.19</td>
<td>0.26</td>
<td>1.17</td>
</tr>
<tr>
<td>VCE3 (4.67)</td>
<td></td>
<td></td>
<td>0.17</td>
<td>0.24</td>
<td>0.93</td>
</tr>
<tr>
<td>VCE2 (4.50)</td>
<td></td>
<td></td>
<td></td>
<td>0.11</td>
<td>0.86</td>
</tr>
<tr>
<td>UNI3 (4.38)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.54</td>
</tr>
</tbody>
</table>

Large | Medium | Small | Very small

It can be seen that students at this level are likely to encounter four to five mathematical terms on average in a word problem. Familiarity with such technical words will impact on students’ ability to comprehend and solve problems and teachers need to be aware
that NESB students in particular are likely to face difficulties if they are not familiar with the relevant terminology in English. It was interesting to note that university books do not use any more mathematical terms than the VCE books. This suggests that the necessary terminology for undergraduate mathematics, at least for the topic of probability distributions, is introduced at the VCE level. However, this might be a problem for students in undergraduate courses who have completed school mathematics in another language and may not be familiar with these terms in English, and they could be at a disadvantage compared to the students who have completed VCE.

**Mathematical words with other meanings**

Having looked at the number of mathematical terms used in a problem, it was decided to determine how many of these words have other meanings in English. While mathematical terms have to be used in problems, they may interfere with the readability and comprehension when these terms have other meanings in another context. It was notable that the four mathematical terms identified as common to most problems namely, ‘normal distribution’, ‘mean’, ‘variance’ and ‘standard deviation’, involved six words with other everyday meanings and connotations in English. The number of different mathematical words that had other meanings was counted for each problem and the means per problem for each textbook as well as the effect sizes of the differences are recorded in Table 4.13 below.
Table 4.13  Effect size of difference in use of Maths words with other meanings

<table>
<thead>
<tr>
<th>Textbook (Mean no. of Maths words with other meanings)</th>
<th>VCE2</th>
<th>UNI2</th>
<th>UNI3</th>
<th>VCE3</th>
<th>UNI1</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCE1 (4.44)</td>
<td>1.08</td>
<td>0.91</td>
<td>0.44</td>
<td>0.78</td>
<td>2.15</td>
</tr>
<tr>
<td>VCE2 (3.88)</td>
<td></td>
<td>0.07</td>
<td>0.08</td>
<td>0.29</td>
<td>1.32</td>
</tr>
<tr>
<td>UNI2 (3.83)</td>
<td></td>
<td></td>
<td>0.01</td>
<td>0.22</td>
<td>0.98</td>
</tr>
<tr>
<td>UNI3 (3.81)</td>
<td></td>
<td></td>
<td></td>
<td>0.14</td>
<td>0.46</td>
</tr>
<tr>
<td>VCE3 (3.60)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.42</td>
</tr>
</tbody>
</table>

The focus of investigation in the next component of the rubric was the number of possibly unfamiliar contexts in the problems. Very often word problems are set in
contexts that may be unfamiliar to the students. While it is important to acquaint students with contexts where they are likely to use mathematics in real life, they may present hurdles in the processing of information and reading comprehension. It can be expected then that NESB learners with limited proficiency in English may find some of these words totally new and need a dictionary to make any sense of the problem, a scenario often witnessed in my FS classrooms.

The classification of words as ‘unfamiliar’ was based on my personal experience of teaching NESB students as well as my knowledge of three other languages. While some subjectivity is acknowledged in this regard, the classification was for comparative purposes, and every attempt was made to maintain consistency by applying the same criteria for classification each time. As discussed in Section 3.5.1, words for which I expected at least some students to refer a bilingual dictionary, were classified as unfamiliar. For instance, ‘a manufacturer of resistors’ appearing in a problem would cause some students to refer to a dictionary and ‘resistors’ was classified as an unfamiliar word. However, ‘a manufacturer of watches’ is unlikely to cause much difficulty as ‘watch’ is an adopted word in many languages and was not counted as unfamiliar. The mean number of unfamiliar references per problem and the magnitude of differences between the texts are presented in Table 4.14 below.
VCE1 had the highest number of unfamiliar context words per problem on average, followed by UNI1. Both these books showed a large magnitude of difference with most of the other books indicating that there was very little overlap between the distributions of these books and the others. The magnitude of difference between the three books with lower means was either small or very small. VCE3 recorded the lowest mean number of unfamiliar context words. This was owing to the familiar contexts such as ‘height’, ‘milk cartons’, ‘fish’, ‘coin’, and ‘die’, used in most of the problems. This could be because of the increased attention to real life mathematics in Australian schools as opposed to the more narrow contexts of Engineering and Business. There were no irregularities in the transition through the colours representing the magnitudes of difference which were consistent with the order of the means, that is, increasing from left to right and decreasing from top to bottom. This suggests that these distributions had similar spreads about the mean and the largest effect size was between the books with the highest and lowest mean number of unfamiliar contexts.
Another aspect of mathematics problems especially at this level is some of the implied meaning in a sentence which was to be used in solving the problem. A student with lack of proficiency in language is likely to miss the point of the question. For instance, a problem that says ‘the length of certain items is normally distributed with a mean of 7 cm and standard deviation of 1.2 cm. Calculate the probability that a randomly selected item has a length less than 5 cm’, poses a direct question. Instead a problem in UNI 2 that reads ‘the length of certain items is normally distributed with a mean of 7 cm and standard deviation of 1.2 cm. What is the probability that, if two items are placed side by side, the difference in their lengths is less than 1 cm’, was classified as indirect as it involves implied meaning and consequently requires more mental processing prior to applying the concept of normal probability. The next section of the rubric recorded the number of instances for each problem from each textbook. Mean values and effect sizes showing the magnitudes of the differences between the various books are shown in Table 4.15.

Table 4.15  Effect size of difference in use of indirect / implied meanings

<table>
<thead>
<tr>
<th>Textbook</th>
<th>VCE1 (0.67)</th>
<th>UNI3 (0.38)</th>
<th>VCE2 (0.31)</th>
<th>UNI2 (0.25)</th>
<th>VCE3 (0.13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNI1 (1.50)</td>
<td>0.97</td>
<td>1.63</td>
<td>1.75</td>
<td>2.03</td>
<td>2.52</td>
</tr>
<tr>
<td>VCE1 (0.67)</td>
<td>—</td>
<td>0.39</td>
<td>0.49</td>
<td>0.61</td>
<td>0.87</td>
</tr>
<tr>
<td>UNI3 (0.38)</td>
<td>—</td>
<td>—</td>
<td>0.10</td>
<td>0.22</td>
<td>0.46</td>
</tr>
<tr>
<td>VCE2 (0.31)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.11</td>
<td>0.35</td>
</tr>
<tr>
<td>UNI2 (0.25)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Large  Medium  Small  Very small

UNI1 and VCE1 that showed the highest mean number of indirect references per problem indicating that these books generally contained more challenging problems,
while VCE3 had the least suggesting it contained more straightforward problems. It was observed that UNI1 displayed a large magnitude of difference with all the other textbooks and VCE1 had a large effect size in its difference from VCE3 and a medium effect size with VCE2 and UNI2. UNI3 which was ranked third highest in the use of indirect meanings also had a medium effect size with VCE3. With an effect size of 2.52, it became apparent that the distributions of UNI1 and VCE3 had very little overlap.

Overall, there was no evidence that either the VCE or university books used more indirect questions. Of the three VCE textbooks, VCE1 used more indirect questions followed by VCE2 and VCE3 had very few indirect questions. Among the university books, contrary to observations on many of the other criteria, UNI2 was ranked as having the least number of indirect questions. This suggests that the author of this book for engineering students chose to use direct questions while introducing a number of unfamiliar contexts mainly in the engineering field.

4.2.4 Cultural issues

The last section of the rubric investigated the nature and frequency of cultural references, if any, in the VCE and tertiary mathematics textbooks. In general it was observed that there were very few cultural references (as defined in Section 3.5.1) in the textbooks at this level. The contexts and language used were found to be more professional (such as engineering or business related) than social (e.g., related to a local festival or food). Hence the occurrence of colloquial usages or words specific to particular cultures was less likely compared to a primary school mathematics text, which may refer to ‘treats for Halloween’ or ‘lamington fingers’ both of which are likely to be unfamiliar to some cultures such as Asians. While all the textbooks introduced numerous contexts from real life situations in the problem settings of normal probability distributions, they were all found to be ‘culturally neutral’ such as ‘emissions during a volcanic eruption’, ‘diameter of components’, ‘lifetime of streetlight bulbs’, or ‘annual return of shares’. There were no references to specific cultures such as local or religious festivals or holidays that students from other cultures
would not understand. The words and phrases that did get noted under this criterion were too few to merit quantifying or any calculations.

For instance, one problem mentioned the scores required for admission to a ‘grammar school’. It was felt that not all countries had these schools or used the term grammar schools. Hence it was possible that students from such countries might not make sense of the context and spend extra reading time in processing the information. Another such reference was to a ‘talk back show’ on radio which may also be unfamiliar to students from certain cultural backgrounds with censored media. Certain usages that might be taken for granted in the case of English speakers, may be unfamiliar to NESB readers were for instance, ‘jigsaw puzzle’ or ‘eye fillet steak’. An expression such as ‘sold as seconds’ may not be in use in all countries and could be potentially confusing to NESB readers.

4.3  In Summary

The use of textbooks is an important part of tertiary education and the ability to do so independently and comprehend the material presented is essential for success at this level. This chapter sought to determine the challenges that accessing information from textbooks could present to NESB students by investigating the level of language use in mathematics textbooks.

Six mathematics textbooks, three VCE texts and three university mathematics texts, were analysed for their readability and linguistic complexity. Results of the four readability tests applied to these texts appeared to vary considerably in their evaluations and did not provide a consistent ranking of the difficulty level of the texts as they used different criteria to calculate readability. However, the results taken together appear to provide a general consensus that, among these six books, UNI2 might prove relatively more linguistically challenging and VCE3 and UNI3 might be the easiest to read. The readability levels of the other three books namely VCE2, VCE1, and UNI1, would be somewhere in-between in increasing order of reading difficulty. These results were kept aside until further investigations revealed possible reasons for this disparity in the results of these tests.
A more detailed analysis using a Linguistic Complexity Rubric gave some insight into
the language use in these selected problem sets. The rubric examined four aspects of
text known to impact ease of reading or comprehension, readability, syntax, semantics
and cultural references. Once more the six textbooks were found to vary greatly in their
complexity level as measured by the different criteria used in the rubric. The results of
the comparison, including the magnitude of the differences between books on each
criterion, were presented individually in the previous section. While this provides rich
and detailed information regarding these books, the variability of results on the various
criteria, make it difficult to arrive at any consensus regarding the linguistic complexity
of the books overall. Hence, as in the case of the readability tests, the six books were
ranked in increasing order of difficulty according to the results obtained on each
criterion. This gave a rank from 1 to 6 on each criterion with 1 indicating the lowest
mean score and 6 the highest mean score. The ranks of all the six books by each criteria
are summarised in Table 4.16.

This rank list on each criterion of linguistic complexity gave a better picture of the
linguistic features of each textbook. For instance, it could be seen that while UNI1 had
the shortest problems on average, it used the longest sentences and contained the
greatest number of indirect questions. It was also attributed with the least numbers of
passive verbs, relative pronouns, comparative constructions, and mathematics
vocabulary words. Thus a closer look at the table provides an insight into the individual
features of each textbook and enables educators to determine suitability of each text for
a particular group of students.

On the whole, a general consensus gleaned from the ranks in the table indicate that
VCE1 and UNI2 are likely to be more linguistically challenging with ranks of 6 or 5 on
a number of criteria, followed by VCE2 and UNI3, and UNI1 and VCE3 that appear to
be simpler in language use. This seems to agree to a certain extent, with the general
consensus obtained from the rankings on the readability tests shown in Table 4.2
although the order is not identical. Tables 4.2 and 4.16 were scrutinised in detail and
some more points of interest became evident.
Table 4.16   Linguistic complexity ranking of selected texts

<table>
<thead>
<tr>
<th>Readability Related</th>
<th>Syntactic</th>
<th>Semantic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem length</td>
<td>% of long words</td>
<td>Sentence length</td>
</tr>
<tr>
<td>UNI1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>UNI2</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>UNI3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>VCE1</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>VCE2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>VCE3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
The formulae used for computing both, the Flesh Reading Ease score and the Flesch-Kincaid grade level are both functions of the average number of syllables per word and average sentence length (see Section 2.3.3). These relate to the ‘percentage of long words’ and ‘sentence length’ criteria of the Linguistic Complexity Rubric. Some similarities in the ranking of these criteria and the Flesch tests can now be discerned. However, the formulae involve both these criteria while the rubric just provides a count and hence cannot be expected to be identical. Similarly, the SMOG grade level is a function of the average number of words with three or more syllables which relates to the criteria of ‘long words’. The rankings for the SMOG grade level and the percentage of long words did show similar trends. However, the most interesting fact to emerge was the nearly identical ranking on the ATOS test with three of the rubric criteria namely average number of ‘passive verbs’, ‘comparative constructions’, and ‘mathematics vocabulary words’, as well as a close similarity with a fourth criteria, which is ‘problem length’. This indicates that the ATOS test matches more of the criteria that are likely to affect readability than the other readability tests considered. The results of this study seem to justify the claim of the publishers of the ATOS test (Renaissance Learning, 2006) that substantial amount of latest reading practice and achievement data was incorporated into the design of the ATOS readability formula making it more reliable on a number of counts.

On the basis of all the results, it can be concluded that readability and linguistic complexity can be assessed for any text using a combination of readability tests and the Linguistic Complexity Rubric developed for this study, to support informed choices by educators. Each readability test may have its own merit provided the user knows the criteria used in calculation and uses the appropriate test to suit the needs of the students in question. The ATOS readability test appears to be a better measure of readability and linguistic complexity and although insufficient on its own, it is well complemented by a Linguistic Complexity Rubric such as the one used in this study.

It was noted that the samples from university texts were not necessarily more linguistically challenging than those from VCE textbooks. On the contrary it was
felt that while the VCE texts were primarily tailored for Australian students, university texts were probably meant to cater for a wider section of students including foreign students and international editions.

This part of the study has exposed the possible language-related challenges (namely, readability-related, syntactic or semantic) that NESB students are likely to face in the use of tertiary mathematics textbooks. Furthermore, it provides a model for determining the linguistic complexity and readability of a text which could be used by interested parties such as teachers, authors, publishers, or researchers to determine the suitability of texts/written material for a wider range of audience. The derivation of this model will be presented and discussed in Chapter 7. The implications of these results in conjunction with findings from the other parts of the research more generally are discussed in Chapters 7 and 8.
CHAPTER 5: RESULTS II
LANGUAGE OF TEST ITEMS

Written assessments form an important part of tertiary mathematics courses and comprehending test items is crucial to achieving success. This was the focus of Part II of this study. As discussed in Section 2.2.2, a number of studies over the years (e.g., Abedi & Lord, 2001; Shaftel, et al., 2006) have investigated and established the link between the language of test items and students’ mathematical performance. However, they concentrated on school mathematics up to the tenth grade level. The aim of Part II of the study was to investigate the impact of the language of test items on the performance of tertiary students from Non-English Speaking Backgrounds (NESB), the results of which are presented in this chapter.

As described in Section 3.4.2, the procedure involved administering a Mathematics Language Comprehension Test to Foundation Studies (FS) students as well as to a reference group of Victorian Certificate of Education (VCE) students from the Tertiary and Further Education (TAFE) sector of RMIT University. Student responses were analysed to observe error patterns and to look for possible patterns in the students’ responses that may be associated with program, gender, and/or language background. As this often resulted in comparisons between small groups of unequal sizes, effect size was employed to determine the magnitude of difference between the mean scores of various groups on these tests. Furthermore, FS students were tested at the beginning and end of their one year program to determine whether exposure to an English speaking environment had made any significant difference to performance on a mathematics language test. Two versions of the test were developed for this purpose to eliminate practice effects (see Section 3.4.2).

While the number of female students participating in this study was very small, comparisons between gender groups were used to look for any trends that might merit further investigation rather than general conclusions. In keeping with the
The main aim of this research was on the comparison between language groups and identification of common language difficulties prevalent among NESB students.

5.1 Results of the Mathematics Language Comprehension Test

The aim of this part of the research was to determine the effect of linguistic features and specialised mathematics vocabulary on the difficulty level of test items for NESB learners of mathematics at the tertiary level. This was achieved by analysing student responses to test items that involved selected linguistic features generally known to cause difficulties in mathematics (see Section 2.4). As described in Section 3.4.2, the test items were specifically designed to include context specific mathematical vocabulary but posed no mathematical challenges. Performance on this test could therefore be mostly attributed to the mathematical vocabulary used in the test items.

Performance on the Mathematics Language Comprehension Test was analysed on two fronts. Firstly, performance was compared by gender and language groups of students in order to investigate the influence of student background on performance as explained in Section 3.5.2. This was followed by an analysis of errors made by students on these test items to look for patterns. Errors were also compared by language groups of participants. As described in Section 3.4.2, the procedure of the research involved the administration of two versions of the test and next section (5.1.1) establishes the parity of the two versions and the validity of the results obtained from these comparisons. Section 5.1.2 then examines the relationship between student background and performance on the Mathematics Language Comprehension Test. Section 5.2 analyses student responses to each item on the test and the results are presented in two subsections. Section 5.2.1 examines the error patterns displayed in the student responses and Section 5.2.2 compares these errors by language backgrounds.
5.1.1 Comparison of results by program, version and administration sequence

The effect of mathematics vocabulary on test scores was investigated by comparing performance on the two versions of the test, the scores of FS students at the beginning and at the end of the year, and the scores of FS and TAFE VCE students. Mean scores were compared for this purpose and the magnitude of the difference between two means was determined using effect size.

All FS students were tested at the beginning and the end of the year. Two equivalent versions of the test were made for this purpose to eliminate practice effect. Approximately half of the students did Version 1 (Appendix 3) at the beginning and Version 2 (Appendix 4) at the end of the year. The other half completed the tests in reverse order. One item (Item 9) was identical on both versions and a comparison of performance on this item for all FS students at the beginning and end of the year provided a crude measure of the change in success rate from beginning of year test to the end of year test. Comparison of results of Item 9 on the two tests showed that 84% of the FS students responded correctly to this item on their first test and 89% of the FS students on the second test. This suggests that there was very little difference in performance between the beginning and end of the year on the same item.

Although the TAFE VCE students were only tested once for referencing purposes (Section 3.4.2), again approximately half did Version 1 and half did Version 2. Student performance on the two versions was compared to verify that the two tests were equivalent in the difficulty level of mathematics and language used in test items. The results shown in Table 5.1 demonstrate that the two versions displayed similar mean scores for the FS group as well as for all participants including FS and TAFE VCE students taken as a group.
Table 5.1  Student performance on two versions of the Mathematics Language Comprehension Test

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>N</th>
<th>Std. Dev</th>
<th>Effect size</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FS students</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Version 1</td>
<td>7.82</td>
<td>102</td>
<td>2.72</td>
<td>0.12</td>
</tr>
<tr>
<td>Version 2</td>
<td>7.49</td>
<td>102</td>
<td>2.72</td>
<td></td>
</tr>
<tr>
<td><strong>All students</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Version 1</td>
<td>7.91</td>
<td>121</td>
<td>2.65</td>
<td></td>
</tr>
<tr>
<td>Version 2</td>
<td>7.58</td>
<td>132</td>
<td>2.81</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 5.1 above, the effect size of the difference between the means of Version 1 and Version 2 for both FS students as well as the whole group was found to be very small. These results, in conjunction with the fact that the order of testing was reversed for half of the sample to eliminate any practice effect, show that the two versions of the test are of comparable difficulty.

The next step was to compare the mathematics language comprehension skills of the NESB students at the start of the FS program and towards the end of the year long program. The results shown in Table 5.2 suggest very little difference which confirms the result indicated by the crude measure of comparing performance on Item 9. The magnitude of this difference was investigated using effect size calculations and it was found that the effect size of the difference between performance at the beginning and the end of the year was small (see Table 5.2).

While there was no planned intervention during the course of the year, participation in the FS mathematics courses as well as general exposure to English in Australia, could have contributed to the small improvement in scores. On the other hand, the fact that there the difference in performance between the beginning and end tests was not larger, goes to show that linguistic features can pose difficulties for FS students despite exposure to English and an English speaking environment for a year. These students displayed a high level of mathematical skills and, by the end of the FS courses, demonstrated the ability to solve mathematical problems in algebra, statistics, probability, and calculus, including word problems. However, they seemed to have considerable difficulty
with the mathematics vocabulary words in the simple Mathematics Language Comprehension Test. For example, most FS students could easily find the equation of a straight line perpendicular to a given line. This involves computing the gradient of the required line as the ‘negative reciprocal’ of the gradient of the given line, and they are taught this in precisely those words. However, the majority of students failed to give the correct response when required to write the ‘reciprocal’ in the Mathematics Language Comprehension test, both at the beginning and at the end of the year. This suggests that NESB students are likely to continue experiencing language difficulties despite FS courses when they pursue university mathematics courses the following year, unless something is done to remedy this in the future.

The next part of the analysis involved comparison of the results of the NESB students with those of the reference group, the TAFE VCE students. This was done to determine whether there was any observable difference in performance between the groups. Firstly, the scores of the FS students on the first test they were given at the beginning of the year was compared to those of the VCE students and it was found that despite fluency in English, the VCE group did not fare much better than the FS group as shown in Table 5.2.

Table 5.2 Performance on Mathematics Language Comprehension Test by program

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean score (out of 14)</th>
<th>N</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS Year Beginning</td>
<td>7.38</td>
<td>102</td>
<td>2.69</td>
</tr>
<tr>
<td>FS Year End</td>
<td>7.93</td>
<td>102</td>
<td>2.74</td>
</tr>
<tr>
<td>VCE</td>
<td>7.91</td>
<td>44</td>
<td>2.81</td>
</tr>
</tbody>
</table>

The difference in means between the various groups in Table 5.2 and the resulting effect size of these differences are shown in Table 5.3. Comparison of the FS
beginning of year results with the VCE results showed that the magnitude of difference was very small. Comparison of end of year results for the FS students with VCE results showed that not only was the difference in scores reduced but it had switched in favour of the FS students although the effect size was still very small. This shift in relative position is indicated by the positive and negative signs of the effect size with negative sign implying the difference is in favour of the second mean and vice versa. This suggests that the year-long FS courses might have put these students at par with the TAFE VCE students in comprehension of the language of mathematics. However, it has to be kept in mind that TAFE VCE students are not from mainstream secondary schools but are students who are often completing VCE after a break in education. While fluent in English, these students were likely to be out of touch with mathematics and the results suggest that this could have affected their comprehension of the mathematical test items.

Table 5.3  Effect size of differences between results of program groups

<table>
<thead>
<tr>
<th>Results compared</th>
<th>Difference in Means</th>
<th>Effect Size of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS Year Begin Vs FS Year End</td>
<td>-0.55</td>
<td>-0.20</td>
</tr>
<tr>
<td>FS Year Begin Vs VCE</td>
<td>-0.53</td>
<td>-0.19</td>
</tr>
<tr>
<td>FS Year End Vs VCE</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The mean scores of all groups were between 7 and 8 out of a maximum of 14 indicating that these students had responded incorrectly to approximately half of the items. This suggests that technical mathematical vocabulary and some linguistic features of test items are likely to affect the performance for NESB learners as well as TAFE VCE students owing to lack of familiarity with the technical terms. The implications of these results are discussed in Chapter 7.
5.1.2 Relationship between student background and performance

The next stage was to explore the distribution of the scores and determine whether any particular group of NESB students were affected more than others by language difficulties. This analysis used the results of the FS students only. Having established the parity of the two versions of the Mathematics Language Comprehension Test, and the insignificant difference between performance on the tests at the beginning and end of the year, it was felt that looking at all of the 28 items on the two versions of the test was justified for the purposes of identifying trends in the performance of NESB students overall. As all FS students had taken both versions of the test, the scores of both tests were added together to form a total score out of a possible maximum of 28.

Firstly, differences by gender were investigated and it was found that the mean scores of male and female students were not very different. Effect size calculations showed that the effect of gender on performance on the language tests was very small, (see Table 5.4 below). This result is to be expected given that language-related difficulties are probably more related to the students’ exposure to learning mathematics in English, and is unlikely to be differentiated by gender.

Table 5.4 Performance on Mathematics Language Test by gender

<table>
<thead>
<tr>
<th>Gender</th>
<th>Mean Total Score</th>
<th>N</th>
<th>Std. Dev</th>
<th>Effect Size of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>15.36</td>
<td>88</td>
<td>5.16</td>
<td>0.07</td>
</tr>
<tr>
<td>Female</td>
<td>15.00</td>
<td>14</td>
<td>5.02</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15.31</td>
<td>102</td>
<td>5.11</td>
<td></td>
</tr>
</tbody>
</table>

Following this, the results were analysed to determine whether there was any notable degree of difference in performance between the participants from different language groups. The categorization of the language groups of the participants was described in Section 3.4.2.
The mean of the combined scores for each group is shown in Table 5.5. Some variations and similarities were observed between language groups in these mean scores. The magnitudes of such differences were investigated using pair wise calculations of effect size (see Table 5.6). While no general claims can be made given the relatively small sample size of each language group, some trends were clearly apparent. This was followed up with a closer look at the performance of each group and the individual participants of these groups where possible.

<table>
<thead>
<tr>
<th>L1 Group</th>
<th>Mean</th>
<th>N</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian</td>
<td>12.57</td>
<td>14</td>
<td>5.68</td>
</tr>
<tr>
<td>Chinese</td>
<td>16.00</td>
<td>26</td>
<td>4.55</td>
</tr>
<tr>
<td>Indonesian</td>
<td>15.33</td>
<td>21</td>
<td>4.28</td>
</tr>
<tr>
<td>Middle Eastern</td>
<td>14.50</td>
<td>18</td>
<td>6.28</td>
</tr>
<tr>
<td>Indian</td>
<td>17.29</td>
<td>7</td>
<td>4.27</td>
</tr>
<tr>
<td>European</td>
<td>20.50</td>
<td>2</td>
<td>0.71</td>
</tr>
<tr>
<td>Sri Lankan</td>
<td>14.33</td>
<td>9</td>
<td>4.12</td>
</tr>
<tr>
<td>English</td>
<td>19.20</td>
<td>5</td>
<td>5.63</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>15.31</strong></td>
<td><strong>102</strong></td>
<td><strong>5.11</strong></td>
</tr>
</tbody>
</table>

The Chinese (mean 16) and Indonesian students (mean 15.3) appeared to perform better than the remaining Asian groups including Vietnamese, Korean, Thai and Malay students (mean 12.6). A closer look showed that the four Korean and three Vietnamese students in the sample had mean scores of 10.5 and 9.3 respectively. While these individual groups were too small to make generalised conclusions, it became apparent that the lowest mean score was recorded for these students from other ‘Asian’ language backgrounds. This was followed by the Sri Lankan and Middle Eastern groups. The individual means of each independent first language group is reported in Appendix 9.

All the nine students in the Sri Lankan group were Sinhalese speaking and the scores seemed to indicate that they faced considerable difficulty on a Mathematics Language Comprehension Test. The Middle Eastern group also
displayed language difficulties with a mean score of 14.5. A closer look revealed that of the two language backgrounds comprising the group, it was the Arabic speakers who seemed to have greater difficulties with a mean score of 13.3 compared to the Persian speaking group with a mean score of 18.8. The students in the Indian languages group displayed a relatively higher score than the Asian, Chinese, Indonesian and Middle Eastern groups, possibly because many Indian students are exposed to some English in their multilingual classrooms. The European students (who were all NESB) scored the highest (mean 20.5) followed by students who had indicated English as their first language (mean 19.2).

Effect size calculations were carried out to investigate the effect of language background on performance on these tests and the results are reported in Table 5.6. As there were eight language groups involved, pair wise calculations were made to examine all possible combinations. It was seen that while some language groups did not differ much, some others exhibited a large effect size. Once more a matrix representation was used to present the results of these pair-wise calculations. This is reflected in Table 5.6 using the same colour coding that was indicated in earlier tables that is, red, orange, green, and blue, for large, medium, small, and very small effect sizes respectively. The results of these effect size calculations were interpreted in terms of the percentage of non-overlap between distributions as given in the table of Cohen’s ‘Equivalents of d’ in Appendix 10.

It was seen that the only two groups showing a very small effect size were the Middle Eastern group and the Sri Lankan group. This indicated that the distribution of the scores of these two groups was almost identical. Although the difference between the means was greater between the European and the Asian groups, the largest effect size was between the European and Sri Lankan groups. This was due to the larger variation (standard deviation of 5.7) in the scores of the Asian group when compared with the Sri Lankan group (standard deviation of 4.1). This indicates that there were some scores in the Asian group higher than those of the Sri Lankan group, resulting in greater overlap between the distribution of scores of Asian students and the European students.
Table 5.6  Effect size of difference between language groups in performance

<table>
<thead>
<tr>
<th>Lang Groups</th>
<th>English (19.2)</th>
<th>Ind (17.29)</th>
<th>Chin (16)</th>
<th>Indones (15.33)</th>
<th>M East (14.5)</th>
<th>S. Lankan (14.33)</th>
<th>Asian (12.57)</th>
</tr>
</thead>
<tbody>
<tr>
<td>European (20.50)</td>
<td>0.22</td>
<td>0.72</td>
<td>0.98</td>
<td>1.19</td>
<td>0.94</td>
<td>1.45</td>
<td>1.37</td>
</tr>
<tr>
<td>English (19.2)</td>
<td>—</td>
<td>0.36</td>
<td>0.66</td>
<td>0.83</td>
<td>0.74</td>
<td>0.97</td>
<td>1.12</td>
</tr>
<tr>
<td>Indian (17.29)</td>
<td>—</td>
<td>—</td>
<td>0.28</td>
<td>0.44</td>
<td>0.46</td>
<td>0.67</td>
<td>0.86</td>
</tr>
<tr>
<td>Chinese (16)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.15</td>
<td>0.28</td>
<td>0.37</td>
<td>0.68</td>
</tr>
<tr>
<td>Indones (15.33)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.15</td>
<td>0.23</td>
<td>0.55</td>
</tr>
<tr>
<td>M East (14.5)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.03</td>
<td>0.31</td>
</tr>
<tr>
<td>S Lankan (14.33)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The scores of the European group showed a _large_ effect size with all the groups except the English and Indian groups. This suggests that the European NESB students performed considerably better than the Asian, Sri Lankan, Middle Eastern and Chinese language groups on the Mathematics Language Comprehension Test. The other _large_ effect sizes noted were in the difference of the English group with the Indonesian, Sri Lankan and Asian groups. On the other hand, the Asian group with the lowest mean score showed a _medium_ or _large_ effect size with all groups except the Middle Eastern and Sri Lankan groups. Cohen’s definition and classification of effect sizes indicate that _medium_ or _large_ effect sizes represent an observable difference between the groups being compared.

Thus it could be seen that there were notable differences between a number of language groups on their performance on the Mathematics Language
Comprehension Test. The exact magnitude of difference between each of the groups can be seen in Table 5.6. This suggests that language background of students does affect interpretation of the language of test items and hence their academic performance.

Several factors could be responsible for differences between students hailing from different language backgrounds. Firstly, students from China, Vietnam and some Arabic countries may have completed their schooling in a language other than English. On the other hand, students from a multilingual country like India may have been exposed to English, even though many schools in India do have local languages as the main language of instruction. Another factor that could have had a role in language difficulties could be the written script. All the language groups except the European group had a different script for their native languages. It is possible that some students were still not sufficiently familiar with the English and Roman script. This could possibly explain why European students (comprising Spanish and Portuguese) scored higher than other groups on the tests despite having problems communicating in English, though it needs further research.

These trends observed were noted and the differences probed further by the next section which analyses the responses to the test items, and error patterns displayed by language groups.

5.2 Student Responses to Test Items

Having looked at the general trends of student performance on the Mathematics Language Comprehension Test, I turned my attention to the items themselves with the primary aim of analysing the errors made by NESB students. As all of the 102 FS students had undertaken both versions of the test, it was possible to record the proportion of students correctly responding to each item on each version (see Figure 5.1). The percentage of correct responses on the 14 items on each version of the test is represented by the graph in Figure 5.1 below.
Correct responses to Mathematics Language Comprehension Test items

Of the 28 items on the two versions of the test, there were only 6 items that 80% or more of FS students could do correctly while, 11 items had less than 50 percent of correct responses. Incorrect responses were analysed for indicators of error patterns. Responses that indicated some form of misinterpretation by the student were categorised as error patterns while calculation errors were all grouped together as incorrect responses. The language features of these items, the error patterns in responses, and the relationship between language background and these errors are discussed in the next two subsections. Section 5.2.1 examines the errors observed in student responses to each item and Section 5.2.2 compares these error patterns by language groups to determine whether language background had any bearing on student errors.
5.2.1 Error analysis of the Mathematics Language Comprehension Test items

All 14 items on each test were analysed for error patterns. In a bid to understand the possible reasons for the difficulties experienced by the FS students, responses to each item were analysed by error type. Some examples of the items and types of errors are discussed in this section. These and other prevalent errors that were observed in the responses to the items are summarised in Table 5.7 at the end of this section.

Question 1 of Version 1 required the students to write the product of $x$ and $y$ is 15 using mathematical symbols. While 59 percent of the responses were correct, it was apparent that about a third of the responses were incorrect because ‘product’ was interpreted as ‘sum’ and the response was $x + y = 15$. A few students interpreted the item as a ‘solve for $x$ and $y$’ problem and tried to find values that satisfied the condition. The distribution of student responses to this question is represented in Fig 5.2. It was clear that while 10% of the incorrect answers are due to conceptual or calculation errors, a majority of students who responded incorrectly, were misled by the context specific meaning of the word ‘product’. It has been demonstrated by their work in class that FS students are capable of multiplying numbers in fractional, decimal, or scientific notation, as well as simplifying complex algebraic expressions involving multiplication. Hence this result highlights the influence of the language of the item on performance.

Question 2 of Version 1 confirmed the finding that the phrase there are twice as many students as desks was often expressed as $D = 2S$ rather than $S = 2D$ (Cocking & Chipman, 1988). Less than a third of the students responded correctly and 57% interpreted it as the number of desks being twice the number of students as shown in Figure 5.2. Although this conceptual error occurs whether or not the problem was given in the student’s first language, it represents an increased level of difficulty for NESB students because of the additional translation required. Cocking and Chipman (1988) suggested that the order and syntax of the English sentence can mislead students into literally translating words to numbers and symbols and prevent them from correctly interpreting the conceptual meaning.
Another error pattern was observed in relation to Question 5 of Version 1. This item required students to write an expression for the sum of $m$ and $n$ is at least 3 more than the difference of $m$ and $n$ using mathematical symbols. There were a number of linguistic features that caused difficulty in this case. About 17% of the students responded correctly and a number of patterns were observed among the remaining responses. The greatest proportion of students appeared to be confused by the usage ‘at least 3 more than’ and responses indicated that they interpreted the statement as ‘equal’, ‘greater’ or ‘less than’ ($=, >, <$) instead of the ‘greater than or equal’ ($\geq$) notation required. Some others had difficulty translating ‘3 more than’ into an inequality. Examples of student responses to this item are given below.

Correct responses: $(m + n) \geq (m - n) + 3$, or $(m + n) - (m - n) \geq 3$

Incorrect responses: $(m + n) = (m - n) + 3$, or $> (m - n) + 3$, or $< (m - n) + 3$,

$(m + n) + 3 = (m - n)$, or $> (m - n)$, or $< (m - n)$,

$3(m + n) > (m - n)$

The rest of the responses were not identifiable as specific language errors but it was apparent the respondents had difficulty interpreting the statement mathematically, and the comparative construction in the item contributed significantly to this difficulty. Figure 5.2 includes the distribution of errors for this item.

Similar misinterpretations were apparent in other items that used comparative phrases in the text, such as not less than 24 (Question 3, Version 1), 15 more than $p$ (Question 4, Version 1), not less than 32 (Question 1, Version 2), at least 5 more than 2a (Question 3, Version 2), and 4 more than the numerator (Question 4, Version 2).

In addition to the terms ‘product’ and ‘sum’ discussed earlier, a number of other mathematical vocabulary words such as ‘difference’, ‘numerator’, ‘denominator’, ‘hundredth’, ‘prime factor’, ‘even number’ ‘reciprocal’, ‘isosceles’, ‘perpendicular’, ‘quadrilateral’, and ‘perimeter’, were observed to be misunderstood and this was reflected in the responses to the relevant items. In both versions the item with the term ‘reciprocal’ seemed to challenge most students with only 15% and 22% responding correctly in Version 1 and Version 2.
respectively. In Version 1, Question 13 required students to write down “the square root of the reciprocal of 25”. Response analysis showed that 77% made errors in interpreting the ‘reciprocal’ while 3% made errors in determining the square root. In Question 13 of Version 2 the students were required to write down the reciprocal of the square of 5. In this case, 78% of the students were unable to respond correctly though only 49 responses could be clearly classified as a misinterpretation of ‘reciprocal’. It was apparent that students had difficulty in combining two concepts involving mathematical terms ‘reciprocal’ and ‘square’ in the same item. Student responses to both these items are also represented in Figure 5.2.

These results suggest that words that were unique to the mathematics vocabulary affected performance of NESB students on the respective items to varying levels. The linguistic features that appeared to cause problems in an item, the proportion of correct responses, and the most prevalent errors for some of the notable items are summarised in Table 5.7.
Figure 5.2 Distribution of student responses to selected items
Table 5.7  Analysis of selected item responses

<table>
<thead>
<tr>
<th>Linguistic feature / mathematical vocabulary in item</th>
<th>Correct (%)</th>
<th>Error prevalence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product is 15</td>
<td>59</td>
<td>sum (31%)</td>
</tr>
<tr>
<td>Product not less than 32</td>
<td>36</td>
<td>Sum (11%), difficulty with not less than (33%)</td>
</tr>
<tr>
<td>Students twice the number of desks</td>
<td>26</td>
<td>2s = d (57%)</td>
</tr>
<tr>
<td>Not less than</td>
<td>57</td>
<td>greater (28%)</td>
</tr>
<tr>
<td>15 more than p is half of q</td>
<td>54</td>
<td>Combining more than and half of</td>
</tr>
<tr>
<td>Sum, at least 3 more than, difference</td>
<td>17</td>
<td>Difficulty with at least 3 more than (46%)</td>
</tr>
<tr>
<td>Sum at least 5 more than 2a</td>
<td>21</td>
<td>Difficulty with at least more than (42%)</td>
</tr>
<tr>
<td>Prime factor</td>
<td>45</td>
<td>prime, misinterpreted 1 as prime</td>
</tr>
<tr>
<td>Isosceles</td>
<td>44</td>
<td>Equilateral (24%), right (4%)</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>43</td>
<td>Wrong shape (57%), interpretation as parallelogram, square or rectangle (20% among correct respondents)</td>
</tr>
<tr>
<td>perimeter</td>
<td>71</td>
<td>Area (9%)</td>
</tr>
<tr>
<td>square root of reciprocal</td>
<td>15</td>
<td>Reciprocal (79%), square root (3%)</td>
</tr>
<tr>
<td>hundredth</td>
<td>54</td>
<td>Hundred times (18%)</td>
</tr>
<tr>
<td>Denominator is 4 more than numerator</td>
<td>25</td>
<td>Difficulties with denominator/ numerator (5%), more than (13%),</td>
</tr>
</tbody>
</table>
5.2.2 Comparison of errors on test items by language groups

The next step was to investigate if there was any association between language background and particular error patterns displayed. The responses provided by students of the eight language groups to each test item on the Mathematics Language Comprehension Test were scrutinised for this purpose and the notable observations are summarised here.

The NESB European students appeared to perform well in mathematical comprehension and did not display many of the prevalent errors identified in the previous section. However, they too found the items involving the word ‘reciprocal’ and some items involving comparative phrases difficult. While these students were not fluent in English, the use of the same script as English and similar words in their first language, may explain their relative ability to comprehend mathematical vocabulary. For instance ‘product’, ‘sum’, and ‘parallel’ are translated to ‘producto’, ‘suma’, and ‘paralelo’ in Spanish, and ‘soma’, ‘produto’, and ‘paralela’ in Portuguese. On the other hand, they responded to the items involving ‘reciprocal’ incorrectly though this translates to ‘reciprococ’ in both languages. This suggests that the mathematical term ‘reciprocal’ is not well known to students in any language. However the small sample size of European students calls for further research before conclusions can be drawn.

Another language group that performed very well on the Mathematics Language Comprehension Test consisted of participants who had indicated English as their first language. These students were born and brought up in countries other than their country of origin such as Indian students whose parents live in America or Kenya or a Zimbabwean student who had lived and completed schooling in India. Although these students performed well on most items, they did display a few of the error patterns exhibited by the sample in general. For instance, items involving comparative phrases and the items involving ‘prime’, ‘hundredth’, and ‘parallelogram’ prompted error responses from some of these students.
While the Indian group also displayed relatively fewer of the prevalent errors noted in the previous section, this group did have difficulty with ‘prime’, ‘isosceles triangle’, ‘quadrilateral’, and three of the items with comparative constructions. This group also had considerable difficulty with ‘reciprocal’ giving a range of incorrect responses.

Students from the remaining language groups accounted for most of the error patterns identified in the previous section. Of these, the Chinese and Indonesian groups displayed the largest proportion of errors followed by the Asian and Middle Eastern groups. All the items with less than 40% of correct responses appeared to cause difficulties for these language groups. Furthermore the prevalent errors noted in Table 5.7 in the previous section were all exhibited by these groups. This was particularly so for the Chinese students despite displaying a higher mean score than other Asian language groups.

Students who are less exposed to a multicultural environment are likely to have completed all of their schooling in their first language. Moreover many of these languages involve entirely different vocabulary and written scripts. It is possible that these factors, taken together or alone, may generate a much greater level of cognitive demand before a mathematical problem can be solved. It appears that students from such L1 backgrounds were more likely to experience difficulty in comprehending mathematical language as demonstrated by the results of the Mathematics Language Comprehension Test. This suggests that a different script of native languages such as Chinese, Sinhalese, or Arabic, and prior schooling in their first language does have a bearing on the level of comprehension of mathematical language and consequently on the performance of NESB students in mathematics assessments.

5.3 Summary of Findings

The results of this part of the research indicate that the performance of tertiary NESB mathematics students was affected by the language of test items. Several prevalent errors due to lack of comprehension were apparent from student
responses as shown in Table 5.7. These were largely concerned with the use of linguistic features such as comparative constructions and technical mathematical vocabulary. It was observed that items involving comparative phrases such as ‘at least’, ‘no more than’, and mathematical terms such as ‘parallel/perpendicular’, ‘sum/product’, ‘isosceles’, ‘reciprocal’, and ‘hundredth’, hindered comprehension and hence affected performance on these items.

TAFE VCE students returning to school after a break although fluent in English also had difficulties with the specialised vocabulary of mathematics. Furthermore, FS students seemed to have similar difficulties both at the beginning and the end of the academic year indicating that exposure to an English speaking environment for a whole year had not eliminated these particular language-related difficulties. While these students did demonstrate very good mathematical skills and manage to solve complex symbolic mathematical problems in particular topics, they had difficulties with a simple test on mathematical vocabulary. This has important implications for educators at two different levels. Firstly, university mathematics lecturers need to be aware of these likely sources of errors for students in their classrooms. Secondly, programs such as FS need to consider including activities in their mathematics curriculum that directly address these issues. This is supported by Moschkovich (1999) who emphasised the need for second language learners to experience the building of vocabulary by participating in mathematical discussions that encourage justifying thinking and interpreting meaning.

The investigation into the influence of gender showed that there was very little difference in performance between the male and female students suggesting in this instance at least, that gender did not have any bearing the level of difficulty faced by these students. However, considerable difference was observed between the performance of different language groups, indicating that language background did have an influence on the level of language difficulties. Students from multilingual countries like India seemed to experience less difficulty, and students from European language backgrounds that used the English scripts also seemed to fare better, despite having problems communicating in English. Chinese and Indonesian students demonstrated a slightly better grasp of the language of test items than students from other Asian language backgrounds such
as Vietnamese and Korean. Middle Eastern and Sri Lankan students also seemed to experience relatively greater difficulty in comprehending the language of test items. This suggests that further research into these languages and education systems is needed to understand the reasons for this.

Overall, there was evidence that the language used in test items did have a bearing on performance of tertiary students suggesting that all teachers of mathematics at this level need to be conscious of the challenges faced by the growing number of NESB students in our classrooms. The implications of these results in conjunction with findings from other parts of the research are discussed in Chapters 7 and 8.
CHAPTER 6: RESULTS III

LANGUAGE USE IN STUDENT WRITING

The first part of the research explored the linguistic features that are prevalent in the mathematics textbooks used by secondary and tertiary students. The second part demonstrated how certain linguistic features of test items impacted NESB student performance on a test of mathematical comprehension. The third part of the research had a different focus and was concerned with the mathematical writing of NESB students. Most assessments in tertiary mathematics courses are likely to involve written tasks be it tests, examinations, assignments or project reports.

Performance tasks provide information about high levels of knowledge and performance that is difficult to obtain by other means...certain performance abilities, such as the ability to explain one's ideas, are increasingly viewed as essential components or as aspects of subject area knowledge in themselves (Niemi, 1997, p. 243).

This part of the research explored the difficulties faced and the strategies adopted by various individuals and groups when confronted with an impromptu writing task. The participants of this part of the study were students of Applied Mathematics which is the most advanced of the three mathematics courses offered to FS students. Students who take up Applied Mathematics are usually mathematically able students who also study Pure Mathematics as a compulsory subject. Their situation is comparable to the Specialist Mathematics students in VCE who study Mathematical Methods as a co-requisite for VCE and are generally very capable mathematically. However, these FS participants came from very diverse language backgrounds and had widely varying language skills in English as discussed in Chapter 3. Many of them had completed all their schooling in their first language. The language background of students was used for classification purposes and no measure of language proficiency was employed in this predominantly qualitative study.
This part of the study involved the administration of a Mensuration Task. The nature of the task required each student to describe a compound two dimensional figure which was made up of multiple geometric shapes. Two figures referred to as Figure A and Figure B were created for the purpose (see Appendices 5 & 6). As described in Section 3.4.3, each student (the describer) was shown one of the figures and was required to produce a description of the figure that would enable another student (the sketcher) who had not seen the figure to sketch it from the written description. This would not be an easy task for anyone without the aid of any form of drawing. It has been shown in Part II of the study that these NESB students had difficulty in comprehending mathematical vocabulary and linguistic features. Hence it was expected that it would be a challenge for these students to convey ideas in written form effectively enough to enable a reader to reproduce the figure described. The aim of this part of the study was to investigate the nature and extent of these challenges by analysing the written text produced by each student and evaluating the quality of the sketch produced from this description. In addition, two experts were invited to participate in this task and their responses were taken as models when comparing student descriptions as discussed in Section 3.5.3.

The analyses involved in this part of the research required an entirely different approach to those adopted for Parts I and II (see Section 3.5.3). The main aim of this task was to test the writing skills of these NESB students. The success of the task was dependent on the describer’s success in depicting the compound figure in words to enable another person to sketch it. It was felt that a good indicator of the success of a student in conveying precise geometrical information in written form would be the quality and accuracy of the sketch resulting from this description. While many other factors such as the handwriting or spelling skill of the writer, or the reading and comprehension skill of the reader, could also affect the quality of the sketch, an accurate sketch by a person who had not seen the original figure could only result from a clear and precise description. Hence a correct sketch would be an indicator of appropriate language use in the description although an inaccurate sketch could be attributed to a number of factors. It was also felt that investigating the relationship between the written descriptions and the resulting sketches would provide an insight into the writing,
and interpretation skills of NESB students. The next section presents the results of the analysis of the sketches produced. The participants have been referred to using a code as described in Section 3.4.3 consisting of M or F indicating their gender, followed by a four letter code indicating language background, and a number indicating their relative position on an alphabetic list of names.

### 6.1 Analysis of Sketches

The sketches produced by the 35 students varied greatly in quality. While some students managed to produce a perfect reproduction of the original figure, others were unable to interpret any of the components correctly. Some others sketched all or most of the shapes correctly but appeared to misinterpret the position, orientation, or relative size. The aim of this section was to determine the factors that impacted on the quality of the diagrams produced by the sketchers. All the sketches were scrutinised in detail and comparisons were made between gender and language groups. However, many of the groups were small and no patterns or trends could be discerned by mere observation of the sketches. It was felt that the quality of the sketches needed to be quantified to enable the use of simple measures of descriptive statistics and calculations of effect sizes of the differences observed. If present, an effect size > 0.15 would show trends in differences between gender or language groups despite small numbers and unequal group sizes.

As detailed in Section 3.5.3 the quality of sketches was quantified by assessing each sketch in terms of its *structure* and *location* using a scoring rubric. Each compound figure consisted of five geometric shapes. The *structure* of these compound figures was scored for correct shape and size out of a total of 5 points, and the *location* of the shapes in the figure was scored in terms of their positions and orientation out of a further 5 points. Thus each sketch had a numerical score out of 10. These *sketch scores*, made up of *structure* and *location* scores, became indicators of the quality of the sketch and hence a measure of the quality of written text describing the compound figure. Some sample sketches are shown below along with an explanation of their scoring based on *structure* and *location*.
as illustrations of the scoring rubric. Figure 6.1 shows two sketches of Figure A and the scores they obtained on each component while Figure 6.2 shows two sketches of Figure B with a similar breakdown of scores.

Figure 6.1 Illustration of scoring rubric for Figure A
This sketch of Figure B has all five component shapes in correct position and orientation. The shape of the trapezium and size of the semi-annulus are incorrect and the figure was scored as follows:

**Structure**
- Shape: 2.0
- Size: 2.0

**Location**
- Position: 2.5
- Orientation: 2.5

Total Sketch Score: 9.0

This sketch shows three of the component shapes in correct size ratio. The semicircle drawn was not accepted as the required shape was a half annulus. The two longer rectangles are in correct position and orientation but the smaller rectangle while correctly oriented, is in the wrong position. A total score of 5.5 was obtained as follows:

**Structure**
- Shape: 1.5
- Size: 1.5

**Location**
- Position: 1.0
- Orientation: 1.5

Total Sketch Score: 5.5

**Figure 6.2** Illustration of scoring rubric for Figure B

The sketches drawn by all the participants were scored in this manner and the summary statistics of these scores are shown below in Table 6.1. The decision to combine the results of the two comparable figures was discussed in Section 3.5.3. The wide range of scores was an indication of the great variation in the quality of sketches produced by the participants.
Table 6.1  Summary statistics of sketch scores of all participants

<table>
<thead>
<tr>
<th></th>
<th>Structure</th>
<th>Location</th>
<th>Sketch score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.29</td>
<td>2.88</td>
<td>6.18</td>
</tr>
<tr>
<td>Min</td>
<td>0.5</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Max</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.17</td>
<td>1.50</td>
<td>2.50</td>
</tr>
</tbody>
</table>

The next step was to investigate possible factors that might explain the variation in sketch scores. In line with the overall aim of the research, the sketch scores were firstly compared by gender and language groups of the sketchers.

6.1.1  Impact of gender of sketchers on sketch scores

The sketch scores were compared in gender groups and the results are summarised in Table 6.2. The female participants scored slightly higher on both the structure and location components and hence on the total sketch score. With the unequal group sizes, effect size calculations were required to determine the magnitude of these differences before any inferences could be made. The effect sizes of these differences are also shown in Table 6.2.

Table 6.2  Comparison of sketch scores by gender groups of sketchers

<table>
<thead>
<tr>
<th>Gender</th>
<th>Structure</th>
<th>Location</th>
<th>Sketch score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Mean 3.22</td>
<td>2.81</td>
<td>6.03</td>
</tr>
<tr>
<td></td>
<td>Min 0.5</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Max 5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>1.13</td>
<td>1.41</td>
</tr>
<tr>
<td>Female</td>
<td>Mean 3.80</td>
<td>3.30</td>
<td>7.10</td>
</tr>
<tr>
<td></td>
<td>Min 1.5</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Max 5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>1.44</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Effect Size of difference -0.49 -0.32 -0.42
It was seen that the effect size of the difference in means between the male and female participants on the *structure* score was *medium* and on the *location* score was *small*. The resulting total sketch score also showed a *small* effect size. This suggests that the female participants were more likely to interpret the written description of the figure than the male participants in general and this difference was more pronounced on the *structure* of the figure. While the number of female participants was too small to generalise, these results were noted for further discussion of observed trends in conjunction with other results.

### 6.1.2 Impact of language background of sketchers on sketch scores

The next comparison was made between language groups of the sketchers to determine whether language background had any impact on the quality of students’ sketches. On visual observation of the sketches, it was difficult to identify the variation of quality between language groups. Comparison of sketch scores with their component structure and location scores showed that while there was no characteristic pattern in the sketches or errors that were peculiar to any language group, there were some trends in the variations between the groups.

This suggested that certain language groups may have had more students who were successful in interpreting the descriptions and producing higher quality sketches. It was felt that this merited further scrutiny. The mean and standard deviation of component and total sketch scores for each language group, as well as the minimum and maximum scores for each group, are given in Table 6.3. Given the size of each language group, effect sizes were computed to determine the magnitude of difference between the various language groups.

The two experts were treated as a separate language group (Expert) and their average score of 9.75 with a relatively small standard deviation reflected their nearly perfect scores of 10 and 9.5. The student scores on the other hand presented a different scenario. It was observed that the Chinese students produced the highest quality sketches followed by the Sri Lankan, Indonesian and Middle Eastern groups. Indian students and Asian students produced the poorest sketches.
Minimum and maximum scores varied considerably within a language group and between groups.

Table 6.3  Comparison of sketch scores by language groups of sketchers

<table>
<thead>
<tr>
<th>Language group</th>
<th>Structure</th>
<th>Location</th>
<th>Sketch Score</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expert</strong></td>
<td>Mean 5.0</td>
<td>4.8</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>Min 5.0</td>
<td>4.5</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>Max 5.0</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev 0.0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Asian</strong></td>
<td>Mean 2.6</td>
<td>1.6</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>Min 0.5</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Max 4.0</td>
<td>3.0</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>Std. dev 1.5</td>
<td>1.2</td>
<td>2.3</td>
</tr>
<tr>
<td><strong>Chinese</strong></td>
<td>Mean 3.9</td>
<td>4.0</td>
<td>7.9</td>
</tr>
<tr>
<td></td>
<td>Min 2.5</td>
<td>1.5</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>Max 5.0</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev 0.8</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td><strong>Indian</strong></td>
<td>Mean 2.6</td>
<td>2.3</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>Min 1.5</td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Max 3.5</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev 0.7</td>
<td>0.7</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>Indonesian</strong></td>
<td>Mean 3.2</td>
<td>2.7</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td>Min 1.5</td>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>Max 5.0</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev 1.8</td>
<td>2.3</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Middle Eastern</strong></td>
<td>Mean 3.3</td>
<td>2.2</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>Min 2.0</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>Max 4.5</td>
<td>3.5</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev 0.8</td>
<td>1.0</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>Sri Lankan</strong></td>
<td>Mean 3.4</td>
<td>3.4</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>Min 1.5</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>Max 5.0</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev 1.3</td>
<td>1.6</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Effect sizes for these differences were computed, and the results are shown in Table 6.4 below. It can be seen that there are a number of large and medium effect sizes between various language groups showing that there were notable differences between the quality of sketches produced by students from these language backgrounds.
Table 6.4  Effect size of difference in sketch scores by sketcher language groups

<table>
<thead>
<tr>
<th></th>
<th>S. Lankan (6.79)</th>
<th>Indonesian (5.83)</th>
<th>Mid East (5.5)</th>
<th>Indian (4.83)</th>
<th>Asian (4.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese (7.93)</td>
<td>0.45</td>
<td>0.74</td>
<td>1.33</td>
<td>1.76</td>
<td>1.69</td>
</tr>
<tr>
<td>S. Lankan (6.79)</td>
<td>__</td>
<td>0.28</td>
<td>0.53</td>
<td>0.79</td>
<td>0.94</td>
</tr>
<tr>
<td>Indonesian (5.83)</td>
<td>__</td>
<td>__</td>
<td>0.12</td>
<td>0.34</td>
<td>0.50</td>
</tr>
<tr>
<td>Mid East (5.5)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.18</td>
<td>0.63</td>
</tr>
<tr>
<td>Indian (4.83)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.15</td>
</tr>
</tbody>
</table>

For instance, it can be seen that the Chinese students had a mean sketch score that was considerably higher than that of other language groups resulting in large or medium effect sizes with every other group. This suggests that the Chinese students were able to interpret the written descriptions and produce more accurate sketches compared to the other language groups. This was followed by the Sri Lankan and Indonesian groups which also scored considerably higher than the Middle Eastern, Indian and other Asian language groups. These results indicate that language background of students had considerable impact on the quality of sketches produced. It was felt that these results merited further investigations to find explanations for, or further confirmation of, these differences between language groups.

For this purpose attention was turned to the other aspect of the task, namely the written descriptions by the students. Each sketch that was assessed above was produced from the written description of the compound figure by another participant. In fact it has been argued earlier that the accuracy of a sketch was
considered as a measure of the success of the written description of the original compound figure by the describer. The next step then was to analyse these written descriptions to ascertain the writing and comprehension skills of NESB students.

For instance, for the two sketches of Figure A illustrated in Figure 6.1, one secured the maximum possible score of 10 points [M – INDN17], while the other obtained a score of 5 points [M – INDN9]. The descriptions of two students that resulted in these sketches were scrutinised for possible features that might help explain the difference in the quality of the sketches. The two written texts are transcribed below:

(Read the whole thing before you start drawing)!

Start drawing a rectangle from the bottom with horizontal length 3 cm and vertical length of 1 cm. Make sure the rectangle is in the middle bottom.

Just above the rectangle, draw a circle of diameter 3 cm. The circumference of the circle and the top horizontal line of the rectangle should meet together. Then draw a second circle of diameter 2 cm within the first (3 cm diameter) circle. Just above the first circle draw a trapezium of base length 5 cm, height 1 cm and top length 3 cm. Then draw a right angle triangle on the right side of the circle. The hypotenuse of the triangle starts from the top right corner of the rectangle and ends at the bottom right corner of the trapezium. The base of the triangle is of 1 cm which starts where the top horizontal line of the rectangle ends. The height of the triangle is 3 cm. Now this triangle is symmetrical so draw the same triangle as how it would reflect on the left side of the circle. Now finally, shade the rectangle, shade the trapezium, shade both of the triangles and shade the outer circle (donut). (M – INDN17)

This description resulted in the sketch that scored 10 points with all components drawn correctly. The description of the same figure by another student that resulted in a sketch score of 5 points is transcribed below.
A trapezium on the top of concentric circle with the longer parallel side touching the circle. The length of shorter parallel side is 3 units and the height of trapezium is 1 unit which is equal to the difference of the radius of concentric circles. Two congruent triangles, one on left and other on right side of the circles having height of 3 units & base of 1 unit are placed such that their top vertices touches the bottom vertices of trapezium. And the bottom non right angled vertices are joined by a rectangle touching the circle of dimensions 3 units X 1 unit. (M – INDN9).

While the linguistic and mathematical comprehension skills of the sketcher must have played a role in the accuracy of the sketches, the features of the written text could also have been partly responsible for the level of accuracy obtained by the sketcher. The first description is more elaborate, and also addresses the reader directly. A few direct instructions (“read the whole thing before you start drawing”, “make sure the rectangle is in the middle bottom”), some clear location guides (“horizontal”, “hypotenuse starts at the top right corner of the rectangle and ends at the bottom right corner of the trapezium”), and precise mathematical terms (right angled triangle, hypotenuse, trapezium) that characterise the first description might have contributed to the better interpretation.

The second describer on the other hand does not give instructions but describes the figure he perceives in fewer words. Some of the information is implied and not explicitly stated. For instance, the radii of the concentric circles are not directly given but implied in the statement that the difference of the radius of the concentric circles equals the height of the trapezium which is 1 unit. Similarly there is no mention that the triangles are right angled, but is implied by the “non right angled vertices” which indirectly state that the third vertex is right angled. This indicates that a describer’s style of writing and the choice of vocabulary could have contributed to the accuracy of the sketches produced.
The next step then was to investigate this possibility. For this purpose, the language used in the written description of the compound two dimensional figures by the two experts and the 35 students was systematically examined. The responses displayed some interesting characteristics, which were analysed for emerging patterns.

Firstly, the written text produced by the students in describing a geometric figure was analysed for the type of writing they employed to convey information. As defined in Section 3.5.3, the responses were classified into categories of style and approach. Writing style and approach were then compared on the basis of gender and language groups to determine the impact of these factors on language use. Secondly, the vocabulary used by the participants in their descriptions were analysed and compared between gender and language groups. The following sections present the results of these analyses.

### 6.2 Categories of Student Writing

On reading the student descriptions certain recurrent patterns were observed. Most participants resorted to one of two styles in their writing. They either described the figure by trying to produce a picture for the reader, or they set out a procedure for the reader to follow which would result in a reconstruction of the figure. Thus two predominant styles of writing were identified in the student responses, which I will refer to as descriptive or procedural, as indicated earlier in Section 3.5.3. While there were a few students who exhibited a combination of both styles in their writing, they did tend to have one of them as the dominant style.

Apart from their style of writing, the participants also adopted two different approaches to their descriptions. While some tended to start with the whole figure and describe the parts thereof, others started with the components of the figure and tried to put these together to form the whole picture. I will refer to the two approaches taken as holistic or componential.
Thus all the participants’ responses were categorised under *style* and *approach* as detailed in Section 3.5.3. These resulted in four possible categories of student writing for this task, namely, *descriptive holistic, descriptive componential, procedural holistic* and *procedural componential*. However, analysis revealed that all the writing samples fell into three categories which will be explained later in the section. The following passages are excerpts from student descriptions that illustrate these categories of *style* and *approach*. Any grammatical or mathematical vocabulary errors in the student writing have been retained in these excerpts.

*The figure is divided by five parts. There are big rectangle with 5 cm as its horizontal length and 4 cm as its vertical length. This big rectangle divided by four area. The first part is rectangle with 5 cm as its horizontal length and 1 cm as its vertical length. It is placed at the bottom of the big rectangle. The second part is rectangle with....... It is placed at the top of the big rectangle. The third and fourth parts are placed between the first and second parts. The third part is a trapezium with.....the semicircle is exactly at the right end of the big rectangle. ......there is a small semicircle inside the big semicircle with diameter......exactly beside the vertical side of the fourth part.* (F – IDSN21)

This student has described a figure that has five parts and then provided a description of each part with size and location specified. Hence, this text was categorised as *descriptive holistic*. On the other hand, the two descriptions shown earlier (by M – INDN17, and M – INDN9) demonstrate two other categories of *procedural componential* and *descriptive componential*.

The following excerpt from description by M – INDN17 provides instructions for drawing a sketch commencing with a part and building up to the complete diagram. This is an example of a *procedural componential* description.
Start drawing a rectangle from the bottom with the horizontal length 3 cm and vertical length of 2 cm. .......Just above the rectangle draw a circle of diameter 3 cm. .....Draw a second circle of diameter...... within the first....Just above the first circle draw a trapezium....... then draw a right angled triangle to the right side of......Now finally shade the rectangle, shade the trapezium.......and shade the outer circle (donut). (M – INDN17)

A descriptive componential description of the same figure is demonstrated in the written text provided by the other student below. It can be seen that the student is describing the figure, starting with individual components and building up the figure in parts.

A trapezium is on the top of concentric circle ......The length of the shorter parallel side is 3 units and the height of the trapezium is 1 unit which is equal to the difference of the radius of concentric circles. Two congruent circles are on......bottom non right angled vertices are joined by a rectangle touching the circle....of dimension 3 units x 1 unit. (M – INDN9)

Each writing sample was classified in this manner in terms of style and approach. The distribution of participants in these categories was investigated using the cross-tabulation facility of SPSS. The results reveal an interesting pattern (see Table 6.5).

<table>
<thead>
<tr>
<th></th>
<th>Holistic</th>
<th>Componential</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptive</td>
<td>9</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>Procedural</td>
<td>0</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>9</strong></td>
<td><strong>28</strong></td>
<td><strong>37</strong></td>
</tr>
</tbody>
</table>

Table 6.5      Distribution of participants by *Style* and *Approach*
Of the 37 participants including the two experts who undertook this task, over three quarters used a *descriptive style*. Similarly, close to 25% of the 37 participants adopted a *holistic approach* and 75% used a *componential approach*. Of the four possible categories, the greatest proportion fell into the category of *descriptive componential* comprising half the population. The remaining half were equally divided between the *descriptive holistic* and the *procedural componential* groups. It was observed that no one adopted a *procedural holistic* form of writing. On reflection this is not surprising as a *procedural* style would generally consist of instructions to draw the figure and would naturally tend to start with individual shapes (components) leading up to the whole picture. Even if a student were to start with a holistic statement such as “You have to draw a figure made by joining 5 shapes...” a procedural style would then lend itself to giving instructions to draw each component, making the approach predominantly *componential*. This may explain the absence of a *procedural holistic* type of writing among the responses.

Following these analyses it was accepted that there may not be four types of writing as originally presumed, but essentially only three types namely, *descriptive holistic, descriptive componential,* and *procedural componential* at least in this group of participants.

An excerpt from an example of an extremely *procedural componential* approach is given below. In this instance, the student gave directions for plotting a graphical representation of Figure A using coordinate references.

```
Begin with the origin point O (0, 0), draw x- and y-axis. Take O as centre to draw two circles one with d = 1 unit and the other is 2 units. From O (0, 0), point out 4 points (2, 1), (2, -1) (-2, -1) (-2, 1). From those 4 points you can draw a rectangle with two circles inside it. Then draw a something (dunno how to call it) for 4 point (-2, 1), (-1, 2).... And another rectangle in the bottom from 4 points (-1, -1),....And the last two triangles (-2, 1).... (M – ASN24)
```
While this enabled the describer to avoid referring to the trapezium by name, the sketcher (M – CHNS13) appeared to be confused with the detail, and sketched it incorrectly scoring 5.5 as shown in Figure 6.3.

![Figure 6.3 Sketch by M – CHNS13](image)

Since this was the only sample of writing that used this method of description, I was curious about how the reader had interpreted it. I completed the sketch independently following the student’s directions. The directions were very precise and coordinates accurately specified to produce the exact figure, though there was one flaw in the student’s description namely, the use of ‘d’ to represent radius. The student sketching the figure had interpreted ‘d’ to be the diameter by convention. This is a good example of how a mathematically able student could use mathematical symbols, equations or graphs to support the use of language and written words.

The natural question that followed was whether writing style or approach affected the sketch scores to any extent. It was found that the descriptions in the procedural componential category had the highest mean sketch score of 7.6, followed by the descriptive componential category which had a mean sketch score of 6.1. The mean sketch score of the descriptive holistic category was found to be 5.0. Effect size calculations showed that the magnitude of difference between these means also varied. It was found that there was a large magnitude of difference between the mean sketch scores of descriptive holistic and procedural componential descriptions. The magnitude of difference between each
of these categories and the descriptive componential category was medium. This suggests that a procedural componential description was easier to interpret and follow directions and resulted in better quality sketches. Among the descriptive styles of writing, the componential approach appeared to be more successful compared to the holistic approach in achieving more accurate sketches. This seems to suggest that procedural and componential forms of writing were easier for NESB students to interpret and translate into a sketch. This could be because students were more familiar with procedural mathematics or that the language used in procedural and componential forms is simpler and more direct.

The next step was to look for factors that could have influenced this preference for writing style and approach. Further analyses were carried out to determine whether gender or language background had an impact on these preferences.

### 6.2.1 The impact of gender on writing style and approach

The next step in the analysis investigated whether gender impacted on the type of writing adopted by the participants. The purpose of this analysis was to look for any possible trends in the distribution of style and approach among male and female participants. The results are shown in Table 6.6, and are presented in a matrix form by style and approach in each gender group. This provides four cells for each gender group representing the four possible combinations of style and approach. The cells for procedural holistic types of writing have been shaded to indicate that these were not displayed in the data. The zeros on the other hand show that the particular category was not utilized by that gender group.

<table>
<thead>
<tr>
<th>Language</th>
<th>Style</th>
<th>Holistic</th>
<th>Componential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Descriptive</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Procedural</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>Female</td>
<td>Descriptive</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Procedural</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
It was interesting to note that majority of the male and all the female participants adopted a descriptive style of writing. This suggested that the preferred style was descriptive especially among the female participants. It was also noted that both experts employed a descriptive style.

The data on approach shows a much more pronounced difference between the genders. It had been observed in the frequency analysis earlier that a componential approach was the preferred type of writing for the majority (nearly 75%) of participants. However, the breakdown by gender shows that while about 84% of males used the componential approach, 80% of females employed the holistic approach in their writing. This was reinforced by the fact that the male expert used the componential approach while the female expert used the holistic approach.

Thus from the data available it appeared that gender did have an impact on the writing style and approach used by participants and this was more evident in their preferred approach than in their preferred style.

6.2.2 The impact of language background on writing style and approach

The next point of curiosity was whether language background had any impact on the style and approach used in writing. The distribution of style and approach by each language group is shown in Table 6.7. Several trends appear to be worth noting. As in the previous table, the procedural holistic category is shaded to indicate that this type of writing was not observed at all. However, the zeros are significant in that they indicate that no one in that language group exhibited that particular type of writing. As before, ‘Expert’ was considered as an independent language group for purposes of analysis.
Table 6.7 Writing Style and Approach by language groups

<table>
<thead>
<tr>
<th>Language (Group size)</th>
<th>Style</th>
<th>Approach</th>
<th>Holistic</th>
<th>Componential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert (2)</td>
<td>Descriptive 1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Asian (6)</td>
<td>Descriptive 0</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Chinese (7)</td>
<td>Descriptive 3</td>
<td>4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Indian (5)</td>
<td>Descriptive 1</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Indonesian (3)</td>
<td>Descriptive 1</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Middle Eastern (7)</td>
<td>Descriptive 4</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Sri Lankan (7)</td>
<td>Descriptive 0</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

As discussed earlier, both experts used a descriptive style but one used a holistic approach while the other a componential approach. Of the 35 student participants, the Chinese, Indonesian and Middle Eastern students all used a descriptive style with some adopting holistic and others a componential approach. None of these participants used the procedural style. On the other hand, Indian, Sri Lankan and Asian participants used both descriptive and procedural styles. However, it was interesting to note that all these participants adopted a componential approach. No student in these three groups took a holistic approach in writing. Although it is beyond the scope of this study, future investigations could determine whether the students’ first language or educational background influenced these preferences in any way. As observed earlier, the majority of participants of this part of the study demonstrated a descriptive componential writing type. It now became apparent that language background could have been partly responsible for this preference.
6.3 Vocabulary of Written Descriptions

When required to produce a written description of a geometric figure without the aid of a diagram, one is forced to depend on words to communicate the individual shapes and their location and orientation to the reader. This calls for either the use of precise vocabulary such as ‘rectangle’, ‘trapezium’, or ‘annulus’, or more general descriptions in terms of ‘equal sides’, ‘parallel lines’, or ‘circle inside another circle’. Secondly, the relative positions of the geometric shapes in the compound figure had to be clearly defined. ‘A circle inside the rectangle’ or ‘a semicircle on the right end of the big rectangle’ would indicate the relative position of the shapes. Thus several features of the written text were seen to be important for achieving the desired end namely, an accurate sketch of the compound figure by the reader.

The considerable variation of the linguistic skills of the participants was reflected in the written descriptions produced. While some were very brief, others tended to be lengthy. Some used precise mathematics vocabulary such as ‘trapezium’, ‘concentric’, ‘hypotenuse’ or ‘vertical’. Others tried to convey the same ideas with limited use of mathematical terms and resorted to describing the various components of the compound figure in terms of ‘rectangles’, ‘triangles’, ‘circles’, and ‘parallel lines’. In addition the compound figure comprised multiple geometric shapes placed in specific positions and orientations. This needed to be conveyed clearly in the descriptions for accurate sketches to be produced. Hence the use of words giving directions for location and orientation were also important in this task.

The over-arching goal of this part of the research was to investigate the writing ability of NESB students. This was achieved by scrutinising the language features of the written text and the accuracy of the resulting sketch. Thus an in-depth analysis of the vocabulary used in the written texts became the next focus.

Firstly, a total word count was made for each written response. Secondly, the nature of these words was then analysed. In counting the number of written words
it was decided to ignore articles, conjunctions and propositions such as ‘a’, ‘the’, ‘and’, ‘of’ etc. All mathematical terms such as ‘rectangle’, ‘radius’, or ‘length’ and all words that indicated location or relative position such as ‘above’, ‘inside’, ‘beside’, or ‘left’ were included in the count. All the words that were counted were classified as mathematical register, locators or qualifiers as explained in Section 3.5.3 with operational definitions, examples and distinctions between these categories.

The count of mathematical register words was intended to find out how many different mathematical terms were used in the description of the figure. This was to determine whether use of appropriate mathematical vocabulary impacted the accuracy of the sketches produced by NESB readers. One term such as ‘rectangle’ was counted only once even if it was used repeatedly. In other words it was a measure of the extent of the student’s vocabulary. On the other hand, all locators and qualifiers were counted to determine how clearly the position and orientation of the shapes were specified by the writer. Hence if ‘inside’ was used 3 times and ‘above’ was used twice in one description, this would count as 5 locators.

All data were entered into a spreadsheet and detailed analyses were carried out to investigate the use of vocabulary in the written description of a figure. It was observed that participants varied greatly in their use of vocabulary. For instance, the number of words used to describe each figure varied from 20 to 110 and while some participants used as many as 30 qualifiers, there were others who used none at all. The descriptive statistics obtained from preliminary analysis of the word counts are shown in Table 6.8. Data from the experts’ writing are also included in this table.
Table 6.8  Descriptive statistics of word counts

<table>
<thead>
<tr>
<th></th>
<th>Students</th>
<th></th>
<th></th>
<th></th>
<th>Experts</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Exp 1</td>
<td>Exp 2</td>
</tr>
<tr>
<td>Words</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maths</td>
<td>20</td>
<td>110</td>
<td>48.77</td>
<td>19.51</td>
<td>102</td>
<td>53</td>
</tr>
<tr>
<td>Register</td>
<td>4</td>
<td>15</td>
<td>8.09</td>
<td>2.42</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>Locators</td>
<td>2</td>
<td>24</td>
<td>8.29</td>
<td>4.03</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Qualifiers</td>
<td>0</td>
<td>30</td>
<td>9.80</td>
<td>5.86</td>
<td>21</td>
<td>5</td>
</tr>
</tbody>
</table>

These data indicate the variation in students’ use of vocabulary in a writing task. Just as they used different styles and approaches in their writing, it can be seen that students differed greatly in their choice of words. While some students used only 2 locators in their text, others used as many as 24. The question then was whether there was any relationship between the vocabulary use in the written task and the accuracy of the sketch produced by the reader. Interestingly, the two experts who had demonstrated success in conveying the compound figure in writing but in different ways, also showed considerable variation in their choice of words. From Table 6.8 it is apparent that there is a considerable difference between the students and experts in their use of vocabulary especially in the domain of the mathematics register. Further analyses were carried out to determine the specific nature of these differences.

It was observed that the experts had the highest average for total number of words used as well as for the mathematics register though their use of locators and qualifiers was not very different from that of the students. However, it was apparent that the experts used precise names of geometric shapes and more of the mathematically correct expressions which probably eliminated the need for more qualifiers or locators. This is illustrated by excerpts from the description of Figure A by an expert and two different students.

Expert: Inside the trapezium is a shaded annulus of outer diameter 3 so that the outer edge of the annulus touches the parallel sides of the trapezium. The inner diameter of the annulus is 1 unit.
**M – SLKN30:** There are two circles in the middle of the rectangle, where the small circle is situated inside the big circle. The radius of the big circle is 1.5 while the radius of the small circle is 0.5. The big circle has been shaded but not the small one.

**M – CHNS32:** There are two triangle on the side of a big rectangular and a big circle in the middle. The middle of the big circle there is another small circle. The trapezium, small rectangular, 2 triangles and the big circle are shaded.

In the first excerpt, the expert used six words from the mathematics register (trapezium, annulus, diameter, edge, parallel, side), one locator (inside) and four qualifiers (shaded, outer \( \times 2 \), inner). On the other hand, the student M – SLKN30 used three words from the mathematics register (circles, rectangle, radius), two locators (middle, inside) and eight qualifiers (two, big \( \times 3 \), small \( \times 3 \), shaded) while M – CHNS32 has used four mathematical terms (rectangle, circle, triangle, trapezium), three locators (side, middle \( \times 2 \)) and ten qualifiers (two \( \times 2 \), big \( \times 4 \), another, small \( \times 2 \), shaded).

Another point of difference evident in the excerpts was that the expert described an unshaded trapezium with a shaded annulus in the centre with two additional triangles on the side, whereas M – CHNS32 saw it as a large rectangle with two shaded triangles and a big circle in the middle with another unshaded smaller circle inside it. Some other students described the latter part as a smaller circle cut off from the bigger one. So it was possible that while lack of proficiency was a constraint for most students, some of the differences in the description and vocabulary use came from differences in perception of the compound figure. As illustrated by the excerpts above, components of Figure A were perceived either as a rectangle with two triangles inside on either side, or as a trapezium with two triangles outside on either side.

In some cases the difference in descriptions could be attributed to difference in mathematical expertise while in some cases the description was restricted to ‘rectangle’, ‘circle’ and ‘triangle’ because the student may not have been familiar
with the words ‘trapezium’ or ‘annulus’ in English. In the case of young children, such differences have been considered in terms of developmental and learning progression in composing shapes (Clements, Wilson, & Sarama, 2004). Young children for instance, would not have been exposed to terms such as ‘trapezium’, or ‘annulus’ and could only have used limited vocabulary. However, the NESB students at this level present a very different situation in that they are mature, relatively mathematically competent, and yet their written descriptions appear to be at very elemental stages due to lack of competence in the English language.

The next step in the analysis was to examine these observed variations and determine whether gender or language background had any impact on the vocabulary used in a writing task.

6.3.1 Impact of gender on vocabulary usage

As there were only four female students in the group and the remaining 31 were males, effect size calculations, which take sample size and standard deviation from the mean into account, were used to explore differences if any, between gender groups in this sample. As comparisons between groups would involve mean and standard deviation, the two experts were excluded from the calculations involving gender groups to avoid influencing the mean.

Mean and standard deviation calculations for gender groups showed considerable difference as shown in Table 6.9. It was observed that female students used more words on average in their descriptions compared to the male participants. It was also seen that this difference was largely due to the greater number of locators and qualifiers used by the female participants, rather than mathematics register words. The effect sizes of all these differences are also shown in Table 6.9.
**Table 6.9**  
Vocabulary use by gender groups

<table>
<thead>
<tr>
<th>Gender</th>
<th>Words</th>
<th>Maths Register</th>
<th>Locators</th>
<th>Qualifiers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Mean</td>
<td>47</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>20</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>88</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>17</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Female</td>
<td>Mean</td>
<td>68</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>43</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>110</td>
<td>21</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>30</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Effect Size of difference</td>
<td>1.11</td>
<td>0.04</td>
<td>1.31</td>
<td>2.00</td>
</tr>
</tbody>
</table>

It was seen that the effect size of the difference in means was *very small* for the mathematics register. All the other effect sizes were *large*. This signals that both male and female participants appeared to have similar mathematical knowledge but the female students used considerably more words that specified location and other qualities of the geometrical shapes than the male students. This difference could be the reason for the difference in the sketch scores between the genders observed in Section 6.1.1. Considering the fact that four out of the five female participants sketched from the descriptions of male participants, it would appear that female students related to the mathematical terms in the descriptions and sketched the ‘structures’ correctly but were unable to ‘locate’ these shapes in the correct position as there were fewer *locators* and *qualifiers* in the descriptions provided by the male describers.

### 6.3.2 Impact of language background on vocabulary

The next aim was to investigate whether language background of students impacted on their use of vocabulary in a written task. A preliminary analysis of the simple descriptive statistics for each language group showed that the
variations in vocabulary use were not the same for all language groups. The results are summarised in Table 6.10.

### Table 6.10 Vocabulary use by language groups

<table>
<thead>
<tr>
<th>Language group</th>
<th>Words</th>
<th>Maths Register</th>
<th>Locators</th>
<th>Qualifiers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expert</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>78</td>
<td>16</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>Min</td>
<td>53</td>
<td>11</td>
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<td>5</td>
</tr>
<tr>
<td>Max</td>
<td>102</td>
<td>21</td>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>Std. dev</td>
<td>35</td>
<td>7</td>
<td>3</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>45</td>
<td>7</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Min</td>
<td>31</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Max</td>
<td>57</td>
<td>11</td>
<td>10</td>
<td>14</td>
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<tr>
<td>Std. dev</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Chinese</strong></td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>52</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Min</td>
<td>31</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Max</td>
<td>73</td>
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<td>16</td>
</tr>
<tr>
<td>Std. dev</td>
<td>15</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Indian</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
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<td>Mean</td>
<td>54</td>
<td>11</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Min</td>
<td>34</td>
<td>8</td>
<td>6</td>
<td>6</td>
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<tr>
<td>Max</td>
<td>88</td>
<td>15</td>
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<td>15</td>
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<tr>
<td>Std. dev</td>
<td>21</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Indonesian</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>73</td>
<td>8</td>
<td>13</td>
<td>19</td>
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<tr>
<td>Min</td>
<td>44</td>
<td>7</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Max</td>
<td>110</td>
<td>9</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>Std. dev</td>
<td>34</td>
<td>1</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td><strong>Middle Eastern</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Mean</td>
<td>37</td>
<td>7</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Min</td>
<td>20</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Max</td>
<td>72</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>Std. dev</td>
<td>19</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td><strong>Sri Lankan</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>47</td>
<td>8</td>
<td>9</td>
<td>8</td>
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<tr>
<td>Min</td>
<td>24</td>
<td>6</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>67</td>
<td>9</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>Std. dev</td>
<td>18</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

A comparison between the language groups showed that the Indonesian group used the largest number of words on average, accounted for by the largest number of locators and qualifiers, while their mathematics register usage was no more than other groups. The Indian group with the next highest mean number of words, used the highest number of mathematics register words while their use of
**locators** and **qualifiers** was no more than other groups. The Middle Eastern group had the lowest count on average in all categories of words used. Such differences were noted to be used in conjunction with other results in discussions in the last section.

These results suggest that there are differences between the language groups in their use of vocabulary in a written task. Once again, with unequal group sizes and varying standard deviations from the mean, effect size calculations were employed to determine whether any of the differences between the means of the various language groups in their use of the **mathematics register**, **locators**, and **qualifiers**, were noteworthy. The results of various pair-wise effect size calculations are presented in Tables 6.11 to 6.13. The representation here follows the same conventions that were introduced and discussed in Chapters 4 and 5.

The first comparison was made on the use of the **mathematics register** (Table 6.11). It was noted that the Indian group used more mathematical terms than other language groups as shown by the *large* effect sizes. There was no difference between the Indonesian, Chinese and Sri Lankan groups and all three groups showed a *small* effect size with the Asian group and a *medium* effect size with the Middle Eastern group. Similarly it was observed that there was no difference between the Asian and Middle Eastern groups in their use of the mathematics register.
Table 6.11  Effect size of difference in use of mathematics register by language groups

<table>
<thead>
<tr>
<th></th>
<th>Indonesian (8)</th>
<th>Chinese (8)</th>
<th>S. Lankan (8)</th>
<th>Asian (7)</th>
<th>Mid East (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian (11)</td>
<td>1.04</td>
<td>1.13</td>
<td>1.35</td>
<td>1.22</td>
<td>1.51</td>
</tr>
<tr>
<td>Indonesian (8)</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.34</td>
<td>0.50</td>
</tr>
<tr>
<td>Chinese (8)</td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.37</td>
<td>0.47</td>
</tr>
<tr>
<td>Sri Lankan (8)</td>
<td></td>
<td></td>
<td></td>
<td>0.43</td>
<td>0.59</td>
</tr>
<tr>
<td>Asian (7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
</tr>
</tbody>
</table>

This was followed by an analysis of the use of *locators* by the six language groups. The effect sizes of the differences between the language groups in the use of locators are shown in Table 6.12. The results demonstrate that there are considerable differences between the various language groups. The Indonesian group tended to use more *locators* compared to the other language groups and the difference was evident from the *medium* or *large* effect sizes displayed with all other groups. At the lower end, the Middle Eastern group used considerably fewer *locators* in their descriptions as seen by *medium* to *large* effect sizes with all other groups except the Asian group. The Asian group also used relatively fewer *locators* and showed *medium* or *large* effect sizes with three of the groups. These differences are discussed in the summary at the end of this chapter.
Table 6.12  Effect size of difference in use of Locators

<table>
<thead>
<tr>
<th></th>
<th>Chinese (9)</th>
<th>Sri Lankan (9)</th>
<th>Indian (8)</th>
<th>Asian (7)</th>
<th>Mid East (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indonesian (13)</td>
<td>0.64</td>
<td>0.64</td>
<td>0.65</td>
<td>0.95</td>
<td>1.12</td>
</tr>
<tr>
<td>Chinese (9)</td>
<td>__</td>
<td>0.00</td>
<td>0.27</td>
<td>0.72</td>
<td>0.93</td>
</tr>
<tr>
<td>Sri Lankan (9)</td>
<td>__</td>
<td>__</td>
<td>0.27</td>
<td>0.72</td>
<td>0.93</td>
</tr>
<tr>
<td>Indian (8)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.30</td>
<td>0.54</td>
</tr>
<tr>
<td>Asian (7)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Next was a comparison of the use of qualifiers and the magnitude of differences between the language groups are shown in Table 6.13.

Table 6.13  Effect size of difference in use of Qualifiers

<table>
<thead>
<tr>
<th></th>
<th>Indian (10)</th>
<th>Chinese (10)</th>
<th>Asian (10)</th>
<th>Mid East (8)</th>
<th>S. Lankan (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indonesian (19)</td>
<td>1.02</td>
<td>1.17</td>
<td>1.10</td>
<td>1.25</td>
<td>1.34</td>
</tr>
<tr>
<td>Indian (10)</td>
<td>__</td>
<td>0.00</td>
<td>0.00</td>
<td>0.35</td>
<td>0.40</td>
</tr>
<tr>
<td>Chinese (10)</td>
<td>__</td>
<td>__</td>
<td>0.00</td>
<td>0.36</td>
<td>0.41</td>
</tr>
<tr>
<td>Asian (10)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.36</td>
<td>0.41</td>
</tr>
<tr>
<td>Mid East (8)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Legend:
- Large
- Medium
- Small
- Very small
As in the case of locators, the Indonesian students who used the most qualifiers displayed a large effect size in comparison to all the other groups, indicating that they used a significantly larger number of qualifiers in their description. On the other hand, the Sri Lankan students who used relatively fewer qualifiers in their descriptions also showed a medium effect size with most groups. This signals that there is a significant difference in the use of qualifiers by the Sri Lankan groups from the other language groups. The Middle Eastern group also used fewer qualifiers and showed a small difference with all other groups except the Sri Lankan group. Differences between the other groups were very small and consequently not noteworthy.

6.3.1 Impact of vocabulary on sketch scores

The question then was, whether the number of mathematics register words, locators, or qualifiers had any association with the scores obtained on the corresponding sketches. It was interesting to note that Expert 1 used 102 words and several mathematical vocabulary words as well as measurement specifications and Expert 2 was able to produce a perfect reproduction of Figure A from the description. Expert 2 on the other hand used 53 words including mathematical vocabulary words but omitted specifying some measurements resulting in Expert 1 producing a sketch with the correct shapes but an error in the position of one of the shapes in the figure. Investigations were extended to the whole group of participants to look for any associations between vocabulary used in writing and the scores obtained on the sketches.

As a first step, scatter diagrams were constructed using Excel to determine the association between the vocabulary used in the description and the score obtained on the sketch. The sketch score was plotted against the total number of words used, words from the mathematics register, locators and qualifiers respectively to investigate whether any of these vocabulary components had a noteworthy association with the sketch score. While none of these components seemed to have a marked impact on the score, it was found that all of them had a medium
level positive association. This indicates that sketch quality was directly related to the use of these components of vocabulary.

A representation of the association between the total number of words and the sketch score is shown in Figure 6.4. This sketch shows a clear positive trend indicating that, in general, the sketch score increased with the number of words used in the description. As indicated in Section 3.5.3, a coefficient of determination ($r^2$) of 0.24 indicates that the number of words in the description was approximately 24% responsible for the score obtained on the sketch. In this case, the correlation coefficient ($r$) between number of words and sketch score was 0.49. This was evidence of a medium level of correlation between the length of the description and the score obtained on the sketch. This suggests that to some extent sketch quality increased with the length of the written description of the figure. This seems highly likely as descriptions that are very brief may not have conveyed sufficient information to draw an accurate sketch.

![Figure 6.4](image_url)  
**Figure 6.4** Association between number of words and sketch score

While it was evident that a longer description was insufficient on its own to produce a high quality sketch, the positive association between number of words and the sketch score, shows that a more detailed description helped comprehension and consequently contributed to a more accurate sketch. This was investigated further by constructing similar scatter diagrams to study the
associations between the categories of words used and the sketch scores. It was found that while locators and qualifiers also showed positive trends in their association with the sketch score, as well as with the scores obtained for structure and location separately, they did not have as much impact on the scores as the number of words. However, the number of words used from the mathematics register appeared to have relatively more impact on the sketch score. The scatterplot between mathematics register and the sketch score showed a positive association as shown by the trend line in Figure 6.5. The coefficient of determination of 0.16 indicated that the sketch score was about 16% dependent on the mathematics register used and the correlation coefficient between them was 0.4. This suggests that the quality of the sketch improved by a moderate extent with the increase in the number of mathematics register words used in the description.

![Mathematics Register and Sketch score](image)

**Figure 6.5** Association between mathematics register and sketch score

It was found that lengthier or more detailed descriptions using more words resulted in slightly higher scores on the sketches and more use of mathematical words also helped in scoring higher on the sketches. I was then interested to see if there was any relationship between language groups or gender and performance on this task. This was investigated by calculating the average score obtained on the sketches produced from the descriptions of the various student groups,
according to gender and language background. The results of these analyses are presented in the next two tables.

**Table 6.14  Sketch scores of describers by gender groups**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Structure</th>
<th>Location</th>
<th>Sketch score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Mean 3.33</td>
<td>2.80</td>
<td>6.13</td>
</tr>
<tr>
<td></td>
<td>Min 0.5</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Max 5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>1.15</td>
<td>1.50</td>
</tr>
<tr>
<td>Female</td>
<td>Mean 3.1</td>
<td>3.4</td>
<td>6.50</td>
</tr>
<tr>
<td></td>
<td>Min 1.5</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Max 5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>1.43</td>
<td>1.56</td>
</tr>
</tbody>
</table>

**Effect Size of difference**

| Effect Size of difference | 0.19 | -0.39 | -0.14 |

The formula for the calculation of effect size involves the difference of means of the two groups. Hence a positive effect size indicates that the first group has a larger mean and a negative effect size indicates the opposite. It can be seen that the male participants scored higher on structure while the female participants scored higher on location with a greater margin in the effect size. This resulted in a very small effect size of the difference on the overall sketch score in favour of the female participants. This supports the findings of the earlier Section 6.2.1 that while there was very little difference in the mathematics register words used in their descriptions of the male and female students, the female students used considerably larger number of locators and qualifiers. These results suggest that female students tend to be more verbose and descriptive in communicating the same ideas than their male counterparts. This possibly helped the sketchers’ using descriptions produced by females to position the shapes more accurately resulting in a higher location score.

Next, sketch scores were compared by the language groups of the describers. This time the aim was to determine whether there was any difference between the scores obtained on the descriptions provided by the students of various language
groups. The means, standard deviations, and range of the structure, location, and sketch scores for each language group are shown in Table 6.15.

Table 6.15  Sketch scores of describers by language groups

<table>
<thead>
<tr>
<th>Language group</th>
<th>Structure</th>
<th>Location</th>
<th>Sketch Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert</td>
<td>Mean</td>
<td>5.0</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>5.0</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>Asian</td>
<td>Mean</td>
<td>3.6</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
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<td>Chinese</td>
<td>Mean</td>
<td>2.6</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>4.0</td>
<td>4.0</td>
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<tr>
<td></td>
<td>Std. dev</td>
<td>0.8</td>
<td>1.1</td>
</tr>
<tr>
<td>Indian</td>
<td>Mean</td>
<td>4.3</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>3.5</td>
<td>1.5</td>
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<tr>
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<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>0.6</td>
<td>1.5</td>
</tr>
<tr>
<td>Indonesian</td>
<td>Mean</td>
<td>3.3</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>0.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Middle Eastern</td>
<td>Mean</td>
<td>2.6</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>0.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Sri Lankan</td>
<td>Mean</td>
<td>3.1</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Std. dev</td>
<td>1.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

It was observed that there was considerable variation in the mean scores of the various groups. Hence effect size calculations were employed to determine the magnitude of these differences. Pair-wise calculations were made between the six language groups on sketch scores as well as the structure and location scores. These results are shown by the matrix representation in Tables 6.16, 6.17, and 6.18. The number of large and medium effect sizes suggests that there is a notable difference in the scores obtained by the various language groups.
Table 6.16  Effect size of difference in sketch scores of describers by language groups

<table>
<thead>
<tr>
<th></th>
<th>Indonesian (6.5)</th>
<th>Asian (6.33)</th>
<th>Sri Lankan (5.93)</th>
<th>Chinese (5.07)</th>
<th>Mid East (4.57)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian</td>
<td>0.82</td>
<td>0.93</td>
<td>0.86</td>
<td>1.82</td>
<td>1.88</td>
</tr>
<tr>
<td>(8.5)</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Indonesian</td>
<td>__</td>
<td>0.07</td>
<td>0.17</td>
<td>0.74</td>
<td>0.88</td>
</tr>
<tr>
<td>(6.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asian</td>
<td>__</td>
<td>__</td>
<td>0.13</td>
<td>0.63</td>
<td>0.81</td>
</tr>
<tr>
<td>(6.33)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sri Lankan</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.32</td>
<td>0.49</td>
</tr>
<tr>
<td>(5.93)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chinese</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.27</td>
</tr>
<tr>
<td>(5.07)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Large  Medium  Small  Very small

There was a large effect size between the sketch scores of Indian students and the sketch scores of all the other groups suggesting that the descriptions provided by the Indian students resulted in better quality sketches. This could be related to the findings of the earlier section that Indian students used the largest number of mathematics register and relatively high number of qualifiers in their descriptions. They were also found to employ only the componential approach though they used both descriptive and procedural styles suggesting that descriptions of components with more mathematical register words resulted in better interpretation and higher quality sketches. The access and exposure to English in the multilingual setting in India might have contributed to their ability to communicate their ideas in written form.

Indonesian students with the next highest sketch scores for their descriptions were the group found to use the second highest number of Mathematics Register words and the maximum number of locators in their descriptions. The Middle Eastern students, who used the least number of words of all categories in their
description, appear to experience greater difficulty expressing their ideas in written English. It can be seen from the effect sizes above, that the sketches produced from the descriptions of these participants had the least scores on average and demonstrated considerable differences from the other groups. Moreover, the sketches produced from the descriptions provided by Middle Eastern and Chinese students, who tended use the *descriptive* style, generally attracted the lowest sketch scores.

Similar trends were observed in the differences between the *structure* and *location* scores of the various language groups as demonstrated by the effect sizes shown in the Tables 6.17 and 6.18.

**Table 6.17** Effect size of difference in structure scores by language groups of describers

<table>
<thead>
<tr>
<th></th>
<th>Asian (3.58)</th>
<th>Indonesian (3.33)</th>
<th>S.Lankan (3.14)</th>
<th>Chinese (2.64)</th>
<th>Mid East (2.64)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian (4.3)</td>
<td>0.74</td>
<td>1.47</td>
<td>0.83</td>
<td>2.14</td>
<td>1.95</td>
</tr>
<tr>
<td>Asian (3.58)</td>
<td>__</td>
<td>0.23</td>
<td>0.30</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>Indonesian (3.33)</td>
<td>__</td>
<td>__</td>
<td>0.12</td>
<td>0.83</td>
<td>0.75</td>
</tr>
<tr>
<td>S.Lankan (3.14)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.37</td>
<td>0.36</td>
</tr>
<tr>
<td>Chinese (2.64)</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>__</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Large* | *Medium* | *Small* | *Very small* | *Very small* |
Table 6.18  Effect size of difference in location scores by language groups of describer

<table>
<thead>
<tr>
<th></th>
<th>Indonesian (3.17)</th>
<th>S.Lankan (2.79)</th>
<th>Asian (2.75)</th>
<th>Chinese (2.43)</th>
<th>Mid East (1.93)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian</td>
<td>0.56</td>
<td>0.83</td>
<td>0.90</td>
<td>1.27</td>
<td>1.59</td>
</tr>
<tr>
<td>Indonesian (3.17)</td>
<td></td>
<td></td>
<td>0.24</td>
<td>0.52</td>
<td>0.83</td>
</tr>
<tr>
<td>S.Lankan (2.79)</td>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
<td>0.57</td>
</tr>
<tr>
<td>Asian</td>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
<td>0.59</td>
</tr>
<tr>
<td>Chinese (2.43)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.41</td>
</tr>
</tbody>
</table>

These results confirm the trends observed in the scatter diagrams that the vocabulary used in the descriptions has a positive association with the scores obtained on the sketches and that language background of the writer impacts on the style, approach, and vocabulary use in their descriptions and consequently, on the sketches produced by the readers of their descriptions.

6.4 Summary and Discussion

This part of the research provided an insight into language use by NESB students in writing and interpreting written text. The impact of gender and language background was also looked at in detail. Though the number of female participants and certain language group participants was too small to warrant conclusive evidence, some clear patterns emerged from these results.

Preliminary analysis of the written text of all 37 participants suggested that two styles and two approaches were adopted to describe each of the two figures, resulting in four categories of writing types. However, on detailed analysis and
classification of each individual text only three writing types were evident: descriptive holistic, descriptive componential, and procedural componential found. It was concluded that the procedural holistic category may not exist as originally presumed, at least for this group of participants.

A comparison of the sketches resulting from each of the three writing types exhibited showed that the procedural style and componential approach were more likely to be associated with higher sketch scores.

The descriptive style appeared to be more prevalent especially among the female participants. However, there was a difference between the male and female participants in their choice of writing approach. The majority of male participants employed the componential approach while most female participants appeared to take the holistic approach. On this basis it would appear that gender did impact the type of writing adopted by the participants, but further research is needed to explore this trend in more detail.

Language background appeared to have an impact on the type of writing employed by students in completing this task. The trends displayed by this group of students indicate that students from the Chinese, Indonesian and Middle Eastern backgrounds tended to use descriptive styles of writing though they used both approaches. On the other hand, the Indian, Sri Lankan and Asian background tended to use only a componential approach though they used both styles. These preferences could have been partly responsible for the differences in sketch scores obtained by describer language groups.

Vocabulary used in descriptions was also analysed in terms of the number of words, the mathematics register, locator words, and qualifier words. On average, experts used more words from the mathematics register than the students. It was found that there was a wide variation among the students in their use of these words. Female participants tended to use more qualifiers and locators than the male participants while using the same range of mathematical terms. This was reflected in the scores obtained on the corresponding sketches produced from their descriptions. While the sketches drawn from the description of the male
students scored higher on *structure*, the descriptions by the female students fared better on *location* and the overall *sketch score*.

On the other hand comparison of sketch scores by gender of sketchers showed that female sketchers are more likely to interpret the sketches correctly and the difference was more pronounced in relation to the structural aspects of the figures. This could be partly due to the difference in the types of writing employed by the male and female participants. It was seen in Table 6.9 that there was no difference in the *mathematics register* used by both gender groups though they differed considerably in their use of *locators* and *qualifiers*. These results taken in conjunction, suggest that the female participants were more likely to relate to the mathematical terms used by the male students and draw correct structures but were not able to locate these structures correctly possibly as a consequence of the fewer *locators* and *qualifiers* used by the male students in their descriptions.

Language background was also found to have an impact on vocabulary. Indonesian students used the largest number of words but the difference was in the number of *locators* and *qualifiers*. Indian students used more mathematical terms and the Middle Eastern students used the least number of words from all categories. These differences will be discussed in more detail in Chapter 7.

Investigation of association between vocabulary and the sketches showed that the total number of words used as well as the number of words from the *mathematics register* had a positive association with sketch score. The number of *locators* or *qualifiers* used in a description did not affect the sketch scores as much though they also had positive associations with sketch score.

There was an interesting difference between the quality of sketch scores and the written descriptions produced by students from the same language group. For instance, the Indian sketch scores were among the lowest suggesting that they were not able to interpret the descriptions of students with less developed linguistic skills. However, the quality of their descriptions was generally the highest due to their greater facility with language especially with respect to their
use of the *mathematical register*. Conversely, the sketch scores of the Chinese, Sri Lankan and Indonesian students tended to be higher in comparison to the quality of their descriptions.

In summary, a number of differences were found between gender groups, language groups, and between experts and novices that warrant further investigation. This study suggests that these observed differences are worthy of further investigation. All these findings in conjunction with the findings of the other parts of the study are discussed in Chapter 7.
CHAPTER 7: PULLING IT ALL TOGETHER

This research investigated language difficulties faced by tertiary students from Non-English Speaking Backgrounds (NESB) in learning mathematics. The study was conducted in three parts focusing on three different aspects involving the reading and writing components of language use in mathematics learning. The preceding chapters reported the results of the three parts of the research. This chapter brings these results together and the discussion compares and collates the findings of the individual parts with a view to answering the research questions that guided this study. The first section (7.1) collates the results from the three parts of the study and examines these in the light of relevant literature. The findings are summarised and discussed in terms of existing and emerging theories.

The two major outcomes of this exploratory study are presented in the two sections that follow. Section 7.2 describes a model to help explain the nature of the language difficulties and errors faced by tertiary NESB students in mathematical problem solving. Section 7.3 provides a framework for selection of textbooks, which provides suggested guidelines for selection of textbooks that are suitable for given student cohorts.

Section 7.4 considers each research question in the light of the findings of the study, and reviews how the outcomes of this research support the findings of previous studies as well as pave the way for future research.

7.1 Findings from Collated Results

The Foundation Studies (FS) program at RMIT University is designed to enable international students, who lack proficiency in English, become more proficient in using English for academic purposes, and articulate into our higher education programs. It entails a compulsory core English course and four other courses in content areas relevant to the chosen academic and career pathways of individual
students. The FS program appears to be successful in enhancing the language skills of these students as there is a perceivable difference in their communication skills in English by the end of the year which is demonstrated by their progress in the English course, their presentations and project reports in various subjects, and their participation in debates during second semester as part of their English curriculum. In addition, students are required to complete the mathematics and other elective courses with stipulated grades before they qualify for the FS certificate. It has been shown that the metalinguistic skills of bilingual students could be an advantage in learning mathematics (see Section 2.2.1). Despite this, many FS students appear to have on-going difficulties coping with the technical language and genre of academic subjects which may impact their capacity to complete subsequent university mathematics courses. This study set out to answer the ‘what’ and ‘to what extent’ questions about these language-related difficulties and to find possible explanations for these difficulties. The first step in gaining an understanding of this was to explore the language backgrounds of the international students involved and their relative proficiency with the English language. The next section summarises the observations related to this aspect of the study.

7.1.1 English Language proficiency of the participants

FS students of RMIT University vary considerably in their proficiency in English. Not only do they come from very diverse language backgrounds, but also from very different educational backgrounds. The 102 participants of this study came from 17 different first language (L1) backgrounds as described in Sections 3.4.2 and 3.5.2. For purposes of data analysis, they were grouped into the language groups of Indonesian, Chinese, Asian, Indian, Sri Lankan, Middle Eastern students, European and English. The students from Europe spoke languages such as Spanish and Portuguese as L1, and were not proficient in English. The students who had indicated English as L1 were students born or brought up as expatriates in a country which was not their own. For example, there were Indian students from Zimbabwe and America, and an African student from India, whose parents lived and worked as expatriates in those countries. These students were fluent in
English and were enrolled in FS because they had only completed the equivalent of Year 11 in their respective countries and needed an Australian Year 12 equivalent for University entrance. With the exception of these few students none of the other participants were very proficient in English.

Students who came from multilingual countries such as India might have been taught in or at least exposed to English, although it might be their second or third language. Such students were generally more proficient in English compared to students who were taught in local languages such as Chinese, or came from monolingual countries where English was hardly used in day to day or official communications. A FS mathematics classroom is thus composed of a wide range of proficiency levels in English. Teaching in FS classrooms over the years has enabled me to observe a number of behaviour patterns as students learn mathematics in English for the first time. Two of these are of particular interest to this study. The first is that students tend to sit in language groups whenever possible and very often resort to L1 for discussions among themselves when solving problems. The second is that many students frequently refer to bilingual dictionaries. The paperback versions are increasingly being replaced by electronic versions in the classroom, although the students are not allowed electronic dictionaries in FS examinations. Both these situations, switching to L1 and the use of bilingual dictionaries, indicate that students constantly translate the problems into L1 and sometimes translate back and forth between two languages as they progress through a problem. It is natural under these conditions that students draw on their knowledge and associations in L1, and “we can find signs of first language influence in immersion bilingual programs where input is often primarily from the teacher and not from peers” (Krashen, 1988, p. 66).

7.1.2 Summary of findings

The findings from the three parts of the study that were detailed in the three preceding chapters are collated together in this section. This study focused on two aspects of language use that feature predominantly in teaching and learning tertiary mathematics namely, reading and writing. The common components of
vocabulary, syntax and text organisation alternatives were examined in relation to both reading and writing from three different standpoints forming the three parts of the study. Part I of the study involved the analysis of the linguistic features of six selected textbooks of tertiary mathematics. Part II entailed a Mathematics Language Comprehension Test to examine the effect of linguistic features on performance. Part III involved a task where one student (describer) was asked to describe a composite geometric shape in words, without the aid of sketches, and another student (sketcher) was asked to sketch the figure from the description of the first student. Differences between language groups were also investigated for both aspects of reading and writing.

Findings on NESB students’ mathematical reading

Results from Part II demonstrated the impact of mathematical vocabulary and linguistic features of test items on performance. The Mathematics Language Comprehension Test showed that many of the difficulties identified in the literature concerning school-aged students (Section 2.4) were experienced by these NESB students at the tertiary level. Mathematical vocabulary words such as sum, product, isosceles, quadrilateral, numerator or denominator, parallel or perpendicular, and reciprocal clearly posed difficulties for many students. This was true of both the FS students as well as the VCE students in TAFE. It was seen that both FS and VCE students had a mean score that ranged between 7 and 8 out of a maximum of 14 on the Mathematics Language Comprehension Test (see Table 5.2) indicating that these students responded incorrectly to half of the items on the test. Classroom observations as a teacher of both these groups and their academic performance have shown that these VCE students were fluent in English, although not very proficient in mathematics or mathematical English, and most FS students although lacking proficiency in English, were mathematically capable. It is likely that many Australian students who take up mathematics in university may fall into one of these two categories and consequently, have difficulties with these linguistic features. This however, needs further research.
Part III of the study took a different approach and investigated the interpretation of written text by NESB students. This involved reading and comprehending mathematical descriptions written by other students who were not very proficient in English themselves, and reconstructing a geometrical figure from these descriptions. Analysis of resulting sketches revealed that the quality of the sketches was affected by features of the written text such as, style and approach. It was also found that the vocabulary features including total number of words as well as number of words from the mathematics register used in the description of the geometric sketch affected comprehension and, as a consequence, the quality of sketch produced by the sketcher. The findings suggest that the procedural style and componential approach are likely to be relatively easier to comprehend, and NESB students require detailed explanations using more mathematics register words, locators and qualifiers as seen in Section 6.3.1.

The results from Parts II and III of the research have implications for the use of textbooks as well. Most textbooks that were analysed in Part I were found to exhibit many of these linguistic features that were identified as difficult by the Mathematics Language Comprehension Test in Part II. Features such as word length, sentence length, or mathematics vocabulary words that were analysed by the Linguistic Complexity Rubric in Part I, can be interpreted with new meaning in the light of the findings of Part III. In particular, it can be seen that lengthy problems need not be a negative but can be a positive if it helps clarify meaning for the reader. Use of more mathematics register words in problems would also appear to be beneficial provided they have been introduced previously in the text and clearly defined.

Syntactic features such as comparative constructions (e.g. at least or no more than) were also causes of misinterpretation of questions by a number of students in Part II. This is supported by Lean, Clements, and Del Campo (1990) who demonstrated the difficulty that young children have with comparative constructions. Such features were also identified in the textbooks analysed in Part I. Consequently, many NESB students are likely to face difficulty in reading and interpreting textbooks, lecture notes, problem sets, or test items.
Findings on NESB students’ writing in mathematics

Both Parts II and III of the study involved paper-and-pencil tasks which required written responses from students. While the Part II test was designed with an emphasis on the comprehension of linguistic features and only simple written responses were required, Part III involved a fairly challenging descriptive writing task intended to convey very precise information. The analysis of student writing revealed certain characteristics. Two styles and two approaches to writing were identified resulting in three combinations adopted by the students namely, descriptive holistic, descriptive componential, and procedural componential. The findings of Part III reinforced the importance of mathematical vocabulary in writing and the difficulties faced by NESB students in constructing and interpreting mathematical writing. It was seen that FS students had immense difficulty in producing a written description of a composite geometric figure. This indicates that they are likely to experience difficulties in expressing mathematical ideas where a non-formulaic explanation is required. This has implications for students in terms of the quality of project reports, assignments, and written examinations, but it also suggests they may have ongoing difficulties in the workplaces for which they are being prepared. For instance, a degree program in Engineering or Business might include two or three courses respectively of Engineering Mathematics or Business Mathematics/Statistics. It is the responsibility of the mathematics educators to ensure that all students who successfully complete their programs become cognitively proficient in the mathematical and contextual language of their respective fields.

Differences in performance across language groups

There were 102 FS participants involved in this study. They came from 17 different L1 backgrounds and were classified into 8 language groups for meaningful analysis. A number of trends were observed in the differences between these language groups in both Part II and Part III. Various factors could account for these observed results. As discussed earlier, the FS students with English as L1 had attended school as expatriates in a foreign country and hence were fairly fluent in English. Indian students, coming from a multilingual society may also have completed some or all of their schooling in English. This could
partly explain the higher scores of these two groups on the Mathematics Language Comprehension Test. European students on the other hand had as much difficulty communicating in English as the Asian and other language groups. However, they scored considerably higher on the language comprehension written test. This could be because of the difference in scripts used in these languages. Of all the language groups involved in this study, only the European group used the English script for their first languages. All the others had different scripts such as Chinese, or Arabic. It is likely that the European group found it easier to read the written items in the test as they were familiar with the English and Roman scripts. Moreover a number of mathematical terms sound very similar in their first languages. For instance, ‘sum’, ‘product’, and ‘reciprocal’, that appeared to cause difficulty on the language test, translate to ‘suma’, ‘producto’, and ‘reciproco’ in Spanish, and ‘soma’, ‘produto’, and ‘reciproco’ in Portuguese. As discussed in Section 5.2.2, while this could explain some of the results observed in this part of the study, some difficulties with mathematical vocabulary words transcend language barriers. For instance, while familiarity with ‘sum’ and ‘product’ benefited the European group, the same did not apply to ‘reciprocal’. It was evident that some terms are not familiar to students and could cause difficulties in any language. In any case, the errors in responses of students who had such difficulties can be classified as comprehension errors whether or not translation was involved.

Written responses from Part III of the study also demonstrated differences between language groups. The preferences displayed in writing style and approach showed some clear trends (Table 6.7). For instance, all the Chinese, Indonesian, and Middle Eastern students adopted descriptive styles without exception while the Indian, Sri Lankan, and Asian groups used both descriptive and procedural styles with roughly half of the students in each group adopting one style. On the other hand, in the case of approaches to writing, the situation was reversed. The Chinese, Indonesian, and Middle Eastern groups showed no particular preference with about half of each group adopting a holistic approach while the other half used a componential approach. However, the Indian, Sri Lankan, and Asian groups seemed to have a preference for the componential approach with every student adopting this approach without exception. This could
be a reflection of one or more of several factors such as: students’ educational backgrounds (e.g., whether the emphasis was on procedural computation skills or analytical problem solving skills), personal learning styles and/or personality traits, differences in exposure to English, or it could be that language structure of L1 influenced the organisation of written text due to code switching. The sample size of language groups in this part of the study was too small to make generalisations about style or approach preferences within language groups. However, the trends observed are interesting and merit further research with larger groups in the future.

Vocabulary use in the written description also appeared to vary with language groups. Indonesian students used the largest number of words in their descriptive text. However, the difference came from the use of a larger number of locators and qualifiers. Indian students used the next highest number of words in their descriptions but their difference came from the use of larger number of mathematics register words. These were followed by the Chinese, Sri Lankan, and Asian groups with the Middle Eastern group using the least number of words. It was found that the Middle Eastern group used the least number of words in all categories of mathematics register, locators and qualifiers. This could again be a reflection of several factors. Indian students could be more familiar with the mathematics register as they had some exposure to English in schools, while other language groups could have been less proficient in using these words. It could also have been the nature and extent of code switching prior to a written task that affected the number and type of words used in the descriptions. Although beyond the scope of this study, it would be interesting to investigate whether the vocabulary used in a written task in languages such as Indonesian or Arabic was very different, and emerged differently when translated into English.

Language background also seemed to have a bearing on sketch scores. Sketches were drawn by interpreting the written descriptions required by the Mensuration Task of Part III of the research. It was found that Chinese students scored the highest on their sketches followed by the Sri Lankan students. The Indonesian and Middle Eastern students came next followed by the Indian and Asian groups who scored the least. It was interesting to note that the Indian group which fared
well in their use of vocabulary had difficulties interpreting other students’ descriptions. Conversely, the Indonesian group students who demonstrated less proficiency in constructing written text in English, had fewer difficulties with interpreting the descriptions of other students. This suggests that students who were more proficient in the use of English and mathematical vocabulary in their descriptions, had difficulty interpreting the descriptions that lacked suitable vocabulary whereas students who were not very adept in the use of appropriate vocabulary were able to make better sense of sparsely worded or imprecise descriptions.

Findings from the three parts suggest that tertiary NESB students do face language difficulties in both reading and writing mathematical texts, which is likely to affect their performance in mathematics. The next section revisits the literature to help interpret these findings.

7.1.3 Revisiting the literature

Second language acquisition theories discussed in the literature review (Section 2.4.3), inform us that the process of acquiring proficiency in a second language can take several years ranging from two years to eight years (Collier, 1987; Cummins, 1981). Immersion in English for a year is therefore no guarantee that a mature student has attained mastery of the language at an advanced cognitive level. Observations in FS classrooms suggest that by the end of the year, many students are still at a stage where they revert to code-switching to solve difficult problems despite the improvement in communication skills in English. This supports the study in South Africa where Afrikaans students were found to be colloquially bilingual but not necessarily cognitively bilingual (Gerber, et al., 2005), and the observation that second language learners achieve conversational proficiency long before proficiency in academic English (Barwell, 2005). This is further corroborated by results from both Parts II and III of this study. Results on the Mathematics Language Comprehension Test at the beginning and end of the year show that, language difficulties at the cognitive level persist at the end of the FS program despite demonstrated process skills in mathematics, and improved
communication skills in English. It was evident from the written descriptions in Part III that these students experienced considerable difficulty writing mathematically and their use of the mathematical register and genre differed considerably from that of an expert writer. This indicates that it is likely that many FS students who commence higher education programs may not have achieved proficiency in English at the academic cognitive level in the limited time available and may need specifically targeted support to enable them to complete their studies.

Results from Part II detailed in Chapter 5 indicate that the language of test items influenced performance on the Mathematics Language Comprehension Test. The paper-and-pencil test generated a larger data set than was possible by clinical interviews (Lean, et al., 1990), and enabled investigation of linguistic factors by controlling for mathematical skills required (Jones, 1982). Several prevalent errors were observed in student responses (see Table 5.7) which could be attributed to lack of comprehension. Furthermore, considerable differences were observed in the performance of different language groups (see Table 5.6), indicating that L1 did influence language comprehension. This was corroborated by the results of Part III presented in Chapter 6. Clear differences were observed in vocabulary use between language groups as shown in Table 6.10, and the magnitudes of these differences are shown in Tables 6.11 – 6.13. Similarly, considerable differences were observed between the sketch scores of language groups (see Tables 6.15 – 6.18). Sketch scores were indicators of comprehension of the written descriptions by the sketcher as well as the quality of writing of the describer. Once more it was evident that L1 had considerable bearing on students’ mathematical writing and comprehension of written text.

Results from both these parts of the research point to comprehension difficulties and several linguistic features were identified as problematic in this respect. Most of these identified features such as comparative phrases, or specialised mathematical vocabulary were found in the textbooks that were analysed in Part I of the study (see Chapter 4). This suggests that NESB students are likely to face difficulties comprehending these texts and solving problems presented in them.
This further corroboration of the likelihood of difficulties from a third angle lends credibility to the results by triangulation, as multiple approaches were used to investigate the same issue (Lincoln & Guba, 1985; Merriam, 1998).

Taken together these results have convinced me that tertiary NESB students do face a number of difficulties in learning mathematics, that these difficulties were predominantly related to language comprehension and writing skills rather than mathematical process skills, and that a student’s L1 had a significant bearing on the difficulties experienced. My next step was to interpret and explain the hurdles to competent reading, comprehending, and writing, in terms of the relevant literature. This led me to the literature on student errors in mathematics.

The error classification scheme proposed by Newman (1977) serves as a useful frame of reference for the steps involved in the process of solving a mathematical problem and the associated likelihood of making errors. Although this was designed and tested for younger learners of mathematics, it seemed to me that such a model could provide an insight into the difficulties faced by NESB learners of tertiary mathematics, and the likelihood of possible errors as a consequence of lack of language proficiency. So I set out to think through the steps involved in Newman’s error analysis model and how it could relate to a tertiary NESB student and what emerged is a revised model that could explain the results observed in my study.

### 7.2 A Revised Error Analysis Model

Newman (1977, 1983) proposed that in solving a one-step, written mathematics problem, a person needs to follow a fixed sequence of five steps, reading, comprehension, transformation, process skills, and encoding (see Section 2.4.4). This model has been used in other studies in Australia (e.g., Casey, 1978; Clarkson, 1980; Watson, 1980), and in other countries such as Brunei, India, Papua New Guinea, Singapore, Philippines and Thailand, to help explain the influence of language factors on mathematics learning (Ellerton & Clarkson,
The ‘Newman research’ method provides a structured format for a diagnostic interview after a paper-and-pencil test. There is a sequence of suggested questions intended to identify the precise stage at which an error was made. Although such studies were conducted at the primary school level, it was felt that with some modification, this model could be used to study and explain the errors made by tertiary NESB students. It was not the interview structure that was of interest to me for this purpose, rather the five steps in the sequence that have come to be known as Newman’s hierarchy. One other composite factor combining carelessness and motivation were also proposed by Newman in this analysis of errors. Clements (1980) modified this model to include ‘question form’ and presented the model in the form of a diagram shown in Figure 7.1. This diagram is notionally divided into two sections namely, ‘characteristics of the question’, and ‘interaction between the question and the person attempting it’. The five steps of Newman’s hierarchy, together with motivation and carelessness that impact across these steps are shown under the second section on the right side of the partition.
The components of this model are considered one by one and modified to form a revised model suitable for tertiary NESB students. While the Newman research and the resulting hierarchy model is a guideline for diagnostic interviews, my revised model offers a tool to understand and explain the stages and processes that have potential for errors in mathematical problem solving by tertiary NESB students. Firstly, it needs to be argued that most of the components of this model would seem to be applicable at any level of mathematics. Errors can be caused either by characteristics of the question or as a result of interaction between the question and the person attempting it. Hence these two sections are retained in my revised model and the modifications made in each section are elaborated below.

**Characteristics of the question**

The original model had ‘question form’ representing the characteristics of the question that could lead to errors. At the tertiary level most questions are presented in language and need comprehension. Secondly, the characteristic of
the question that was the focus of this study is the language of the question. Hence the characteristic on the left hand side of the partition has been revised to ‘question language’. Part II of this study has shown that the language used in the question can affect comprehension and identified linguistic features associated with errors. Part I demonstrated that many of these same language features were embedded in the textbooks at this level. As a consequence, the question form used in texts is highly likely to be a cause of error for NESB students. Four linguistic features ‘technical terms’, ‘comparative phrases’, ‘passive verbs’, and ‘contextual words’ that were identified by this study as contributing to errors made by NESB students, have been listed as the components of question language. These features have the potential to impede comprehension and cause errors in mathematical problem solving even at the post secondary level, especially for NESB students and students who are not proficient in the technical vocabulary of mathematics.

Interaction between the question and the person attempting it

The components in the right hand side of the partition in Clements’ representation and the modifications in my revised model are discussed in this section.

The five steps in Newman’s hierarchy shown on the right side of the partition in Figure 7.1 can be meaningfully extended to problem solving at the post-secondary level. If a student is proficient in English, reading errors at tertiary level could be attributed to carelessness, such as missing a negative sign, or neglecting the ‘not’ in a multiple choice question that reads ‘which of the following is not a function?’. However, in the case of NESB students, these reading errors can be caused by lack of reading skills in English, a comparable situation to that experienced by young children in the early primary years. Comprehension errors are likely to be relevant to all tertiary mathematics students as well due to the complexity of language used in higher levels of mathematics and this is particularly so for NESB students. The remaining three steps in the hierarchy namely ‘transformation’, ‘process skills’ and ‘encoding’ are also relevant and are potential causes of error at the highest level of mathematics. For instance, a student attempting to find the area under the curve of a complicated
function by integrating could make transformation, processing or encoding errors. Furthermore, carelessness can cause errors in any of these steps along the way, as shown in the diagram above. Hence it can be argued that all these aspects of the model are valid at any level of mathematics and these have been retained in my revised model.

The factor from Clements’ diagram that is likely to operate differently in post-secondary and post-compulsory phases of education is motivation. FS students are fee paying, international students who are keen to complete the program successfully and proceed with their higher education programs the following year. The VCE students in TAFE are mature students returning to school after a break and are usually there to upgrade their qualifications for some specific purpose. While individual situations can affect the level of motivation in mature students as well, it can be argued that students at this level are there because they want to, or need to achieve a particular qualification. In either case, these students can be expected to be motivated and exceptions to this assumption are unlikely to be in class as attendance is not compulsory. In any case, this assumption is particularly true about the participants of this study. Hence, it was felt that for these particular tertiary students, lack of motivation could be disregarded as a cause of error in solving a problem.

On the other hand, an important additional factor that is more likely to have a bearing on the errors made by tertiary NESB students, is ‘translation’ to L1 or ‘code switching’ back and forth between English and L1. On reading a question, NESB students very often tend to translate it into L1 for better comprehension and then switch back into English for subsequent stages. Others might continue the thought processes in L1 till the end of the problem, or switch back and forth at each stage. Translation to L1 plays an important role in comprehension and successful completion of subsequent stages. While translation facilitates semantic processing there is risk of loss of accuracy, loss of integrated meaning as well as shift of focus to L1 forms rather than the intended second language form (Kern, 1994).
On translation, students need to draw on a familiar ‘knowledge and culture base’ (the mathematical, linguistic and social context) to be able to relate to the problem. “Interpretation of a text is hardly a plain translation (based on vocabulary and grammar), but involves the context the text is produced within (including participants and goals)” (Ferrari, 2004, p. 386). The familiar knowledge and culture base in a mature student’s L1 is a key element that is missing from the model in relation to tertiary NESB students. As a consequence, I have modified the Newman model and Clements diagram to incorporate the key elements of ‘first language knowledge and culture base’, and ‘translation’ to L1, and ‘code switching’ (translation back and forth) between languages as a source of possible error for NESB students solving mathematical problems.

The ‘first language knowledge and culture base’ component in the diagram represents the familiar associations between words, concepts and cultural contexts that exist in L1 for the student. Observed behaviour of NESB learners shows that on reading a question that involves written text, they translate it to L1, sometimes referring to a bilingual dictionary, or by seeking assistance from peers or the teacher. Once translated, they draw on their knowledge base in L1 as well as the cultural base they are familiar with, to make sense of the problem and comprehend it. Thereafter they could continue with the transformation and process skills in either English or L1 depending on the complexity of the problem. If intermediate steps using language are required, they may need to switch back to English in order to write down the steps. If on the other hand, the steps are symbolic such as differentiation or integration steps, it is likely that they may continue to process in L1 as long as the same symbols apply. This is depicted by the two-way arrows indicating possible code switching in any direction between English and L1 at any stage. Double headed arrows are used to represent this code switching together with associations in the first language and culture base. While translation from English to L1 occurs between reading the question and comprehending it, code switching between the two languages may occur at any or all subsequent stages for various parts of a question. Each code switching compounds the risk of making errors owing to possible flaws. For instance, miscomprehension, lack of equivalent words in two languages, and different ways of encoding mathematics in two languages, or a difference in the
cultural base, could all lead to an error or multiple errors in solving a problem. As mentioned earlier, carelessness is a factor that can occur at any of the stages across the board, as indicated the vertical arrow spanning all the other components in the diagram.

This revised model for the analysis of possible errors, incorporating these additional factors is represented by Figure 7.2.

![Figure 7.2: Revised model for analysis of errors by tertiary NESB students](image)

This modification of a tried and tested model helped in understanding the findings of this study, and the results obtained in the various parts of the research took on a new meaning in the light of this new model for the analysis of errors.
7.2.1 Testing the explanatory power of the revised error analysis model

Going back to the beginning of my research, the two incidents mentioned in Section 1.1.2 can now be explained within this framework. A mathematically capable NESB student who was able to read the question and use process and encoding skills to answer some parts of a question involving complex mathematical concepts, was hindered by her inability to comprehend ‘how long will it remain 249.9 m above the ground’ and was unable to proceed with a relatively simpler part of the question. To this student, ‘long’ indicated a distance and she could not relate it to the context of time. It can be seen that between the steps of reading and comprehension, translation into L1 and failure to find a meaningful L1 equivalent in her knowledge and culture base, were responsible for her inability to proceed.

In the second incident, it can be seen that the Asian students were hampered in their attempts to comprehend a question because they were unfamiliar with the game of cricket. The question required them to calculate the angle of projection required for the ball to reach the boundary to score a six. Despite the clue that ‘reach the boundary’ provided in the question, ‘scoring a six’ could not be sensibly translated into L1 as there was nothing equivalent in their knowledge and culture base, resulting in lack of comprehension. A similar situation has been reported in a recent study in the US, where an NESB pre-service elementary school teacher who failed a mathematics course was interviewed. It was found that although she knew the mathematics to solve a problem involving positions of players in baseball, she was not able to create a mental model because she knew nothing about the game of baseball (Campbell, et al., 2007).

In my pilot study it was evident that both FS and VCE students had difficulties interpreting the language of the six sample questions. This was clearly reinforced by the results of the Mathematics Language Comprehension Test administered in Part II of the research. International students who lacked proficiency in English (FS students) and mature students, who had had some setbacks or breaks in education and were returning to school (VCE students in TAFE), had difficulties comprehending the language of the test items. The Mathematics Language
Comprehension Test was designed to test *comprehension* of mathematical language and did not involve any advanced mathematical *process skills*. The results indicate that many students in this representative sample were impeded between the *read* and *comprehend* steps of the hierarchy. This suggests that both NESB students as well as students returning to school after a break are likely to experience difficulty *comprehending* the technical language of mathematics thereby not proceed beyond the second step in the problem solving sequence, despite having the *process skills* needed to solve the problem. As these students comprise a significant proportion of university mathematics classrooms, these difficulties could mean that a sizeable proportion of students might be at risk of failing or at least underachieving in their subsequent tertiary mathematics courses.

The analysis of item responses to the tests of Part II discussed in Section 5.2 and summarised in Table 5.7 indicated that a number of mathematics vocabulary words and linguistic features such as comparative phrases hindered *comprehension*. This can now be explained in terms of the model. For example, it was seen that ‘product’ was interpreted as ‘sum’ by 31% of the students. This was clearly a *comprehension* error. Since this was a language comprehension test it must be noted that students were not allowed dictionaries, or clarification of any linguistic features, and encouraged to answer each question to the best of their knowledge. This could have blocked the *translation* pathway to L1 *knowledge base* for some students which might account for 10% of students making other errors. Other students might have *translated* product as ‘result’ in L1 and interpreted it as sum, although this cannot be confirmed without further research. However, it was noted that the European students although not proficient in English, had no difficulty with this item. The fact that ‘product’ translates to ‘producto’ in their L1 could account for their ability to draw on their *knowledge base* to *comprehend* the item and respond correctly.

I have observed over the years that most FS students have good process skills in mathematics and cope well with the mathematics courses which were taught in English. These courses deal with high levels of mathematics such as differential equations or complex numbers, but focus on skills rather than applications.
However, when it came to topics with word problems such as mensuration, statistics and probability, the FS students generally experienced more difficulty and required considerable support from teachers. These were also difficulties between the steps of reading and comprehension, due to the question language. The Mathematics Language Comprehension Test administered to FS students both at the beginning and end of the year showed very similar results. This suggests that exposure to English speaking environment for a whole year had very little effect on their performance on the Mathematics Language Comprehension Test and that these students are likely to face the same difficulties with reading, comprehending or writing in their subsequent university mathematics classes.

The results of Part III of the study showed that FS students experienced considerable difficulty in producing a written description of a composite geometric figure. This suggests that they are likely to experience difficulties in expressing mathematical ideas in words (possibly involving code switching back to English and encoding). Although the Mensuration Task is substantially different to the type of problems used to develop the original model, NESB student responses to this task can be understood in terms of the revised error analysis model. For example, faced with the task of describing a composite geometric shape, a NESB student would have to draw on his or her L1 knowledge and culture base to visualise the component shapes, think about how to describe these in terms of their position and location, then translate that into an effective description in English. The use of less technical descriptions, such as describing all four-sided shapes as rectangles or an annulus as ‘two circles’, could indicate a lack of familiarity with the relevant mathematical terms in English, a translation error, or the lack of equivalent terms in L1. Further evidence of the explanatory power of the revised model is provided by an analysis of the trends observed in relation to writing styles and approaches in Section 6.2. It can now be seen that the differences in the writing styles and approaches of different language groups could be partly due to the linguistic features of their L1 or differences in emphasis on process skills versus analysis in their respective education systems. This is also an indication of the interaction between a student’s knowledge and culture base and the translation and transformation processes, in this case from L1 to
English and from a diagram to written text, that serves to compound and confound NESB student attempts to make sense of mathematical tasks.

Other linguistic features peculiar to English or a particular L1 could also have a bearing on students’ ability to comprehend, transform, and solve a mathematical problem. For instance, Middle Eastern languages are read from right to left, and for a student who has completed all previous schooling in a language such as Arabic, this could pose considerable difficulties. Reading from left to right could require conscious effort and might lead to comprehension or careless errors. For example, direction is very important in subtraction and an Arabic student tempted to read 5 – 4 from right to left might end up making an error. Apply this to the mathematics at the tertiary level with its own conceptual complexities, and the risk of errors is compounded as illustrated by the following test item on the concept of area under a curve evaluated using definite integrals. A multiple choice question given as part of a short quiz conducted in a FS Pure Mathematics is given below. There are numerous complexities involved in this question as a result of mathematical concepts that can be confusing to a student who is used to reading right to left. Area between two curves is computed as the integral of the difference of the upper and lower curve functions \( f(x) - g(x) \) in this case) from a lower limit to an upper limit along the \( x \)-axis, making D the correct choice.

The area bounded by the curves \( f(x) \), \( g(x) \) and the lines \( x = -3 \) and \( x = -1 \) is equal to:

\[
\int_{-3}^{-1} [f(x) - g(x)] dx
\]

A) \( \int_{-3}^{-1} [f(x) - g(x)] dx \)

B) \( \int_{-3}^{-1} [f(x) + g(x)] dx \)

C) \( \int_{-3}^{-1} [g(x) - f(x)] dx \)

D) \( \int_{-3}^{-1} [f(x) - g(x)] dx \)

E) \( \int_{-1}^{-3} [f(x) + g(x)] dx \)
However, there are several pitfalls or students who might revert to reading from right to left, even momentarily. The order of numbers on the negative side of the \(x\)-axis could cause them to choose the option with limits from -1 to -3, or the order of subtraction of the two functions could cause them to choose \(g(x) - f(x)\) and these errors could lead them to choose A or C as the correct option. These could be classified as reading errors possibly derived from the culture base of the model. However very often Roman numerals or Western mathematical notations are used in Arabic writing, in which case the risk of such errors may be reduced.

In this way, the revised error analysis model provides an explanation for the incidents that prompted this research as well as the results of the research. This model sheds light on many of the errors made by tertiary NESB students and their preferences for writing styles, and provides an insight into the difficulties experienced by lack of proficiency in English.

The next section describes the second outcome of this research, derived from the analysis of textbooks in Part I.

### 7.3 A Framework for Selecting Suitable Texts

Although the four readability tests used in Part I of the study did not provide consistent results in their rankings of the readability levels of the six textbooks, taken together, they seem to indicate that some textbooks could provide more linguistic challenges than others and some might be easier to read. The results of the linguistic complexity analysis seemed to agree with this general consensus of how the textbooks varied in their use of language (see Section 4.2). For instance, it was found that there were considerable variations in problem length, sentence length, and the number of long words per problem in the mathematics textbooks analysed. Findings from Part III have shown that longer descriptions with more precise mathematics register words, locators, and qualifiers produced higher quality sketches. This suggests that textbooks that have very brief or cryptic descriptions and used the linguistic features identified as 'difficult', could present considerable challenges for comprehension especially for NESB students.
Lengthy problems in themselves are unlikely to be a source of language difficulties if they provide a clearer and more detailed description of the context and the requirements of the problem. Consequently, it is important to choose the appropriate method to judge the suitability of a textbook depending on the needs of the students.

It was argued in Chapter 4 that any single readability test might not provide an accurate measure of the difficulty level of a book especially at the tertiary level with academic and technical language involved. Analysis of a representative selection from the book using the Linguistic Complexity Rubric in conjunction with a suitable readability test appears to be more appropriate in deciding what is best for a particular group of students. However, these findings are not confined to tertiary students, and could be applied to determine the suitability of any book for intended readers. It is important to match the requirements of the intended readers and the criteria used in the readability test for the choice of books to be effective and appropriate.

The four tests of readability levels that were analysed in Part I of this study were found to focus on different aspects of readability (Section 2.3.2), which used singly or in combination may be of benefit in particular situations. The SMOG grade level is computed as a function of the average number of ‘long words’. It is computed as the average number of words with three or more syllables in 30 consecutive sentences. Both the Flesch Kincaid grade level and the Flesch Reading Ease score are calculated using ‘average sentence length’ and ‘average word length’. In this case, average word length is measured by number of syllables per hundred words. The ATOS test uses semantic and syntactic difficulty to evaluate readability levels. Syntactic difficulty is measured by sentence length and semantic difficulty by word length, word familiarity and word frequency, obtained from reading practice and achievement data. The six mathematics textbooks analysed in this study were ranked differently by the four readability tests demonstrating the difference in emphases of the various tests. These evaluations were found to agree with evaluations of different aspects of the Linguistic Complexity Rubric. This suggests that the choice of readability test should be decided on the basis of its focus and used in conjunction with the
Linguistic Complexity Rubric or parts thereof when making decisions about the suitability of a book for a particular group of readers. These considerations led to the development of a framework to assist teachers make informed decisions about textbook selection for particular student cohorts.

Ideally, a teacher who wishes to select a textbook will have the needs of the students in mind. For instance, if the students concerned are young children from an English speaking background, the teacher may only be interested in their capacity to read unfamiliar long words. In this situation it would be sufficient to apply the SMOG grade level and the readability section of the Linguistic Complexity Rubric. On the other hand, if the intended readers are older NESB students, then readability and semantic and syntactic aspects might be of more concern and the ATOS test in conjunction with the Linguistic Complexity Rubric is more likely to provide a better evaluation of possible texts. The flowchart in Figure 7.3 illustrates the proposed framework for the analysis of textbooks depending on the specific needs of the intended readers.
Figure 7.3 Framework for selection of textbooks
While this framework is based on four selected readability tests, further research is needed to examine the extent to which other readability tests might work with this framework. The next section answers the research questions in the light of the collated results of the three parts of the study.

### 7.4 Responses to Research Questions

The aims of this research were addressed by the four research questions stated in Section 2.6. The responses to these questions draw on all three parts of the research as appropriate and provide a holistic picture of the difficulties faced by NESB students undertaking tertiary mathematics courses. The following table shows how the findings from the three parts of the study have informed the claims made in relation to each research question.

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<tr>
<th>Research Question</th>
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#### 7.4.1 Research Question 1:

**What if any, are the language-related difficulties experienced by senior secondary and tertiary mathematics students?**

This question effectively consisted of two parts, one asking ‘are there language-related difficulties’, and a second asking ‘what’ these are. The results observed in Chapters 4, 5 and 6 have provided answers to both parts.

The response to the first part of the question is that tertiary mathematics students experience many language-related difficulties. This is evidenced by the findings
from Parts II and III of the research as well as Part I which found that many of the identified sources of linguistic difficulty are present in the mathematics textbooks used at this level. In response to the second part of the question, several features listed in an earlier section on summary of findings (Section 7.1.2) and detailed in the results presented in Chapters 4, 5 and 6 were found to pose difficulties for students. In addition, the revised error analysis model provided explanations for these language-related difficulties, and the errors NESB students are likely to make as a consequence. As indicated by the Pilot study and confirmed by Part II of the research, vocabulary and linguistic features of test items appeared to have a bearing on performance on mathematics tests. Senior secondary and tertiary students are heavily reliant on written text in the learning of mathematics. They have to read lecture notes and textbooks, comprehend the language of problems before they can solve them, present written solutions to problems and assessment tasks and possibly write up project reports. The revised error analysis model has also highlighted the likely pitfalls for NESB students in all these processes. Part I of the research showed that many of the linguistic features that caused these difficulties are found to a large extent in the tertiary textbooks that were analysed. Hence these students are likely to face difficulties when directed to use textbooks for reading and self-study. Thirdly, the written task in Part III of the study also demonstrated difficulties encountered by FS students in a written task. This indicated that lack of proficiency in English was likely to impact the quality of written tasks such as project reports or analytic proofs. The findings from all three parts of the research concurred in showing that NESB students are likely to experience language-related difficulties in their attempts to learn mathematics at the tertiary level and the revised error analysis model has provided a framework for explaining these difficulties.

7.4.2 Research question 2

What influence do language-related difficulties have on NESB students’ learning of mathematics at the tertiary level?

This question was aimed at determining the extent to which language-related difficulties influenced the learning of NESB students. The response to this question is informed by the results of all three parts of this research which clearly
established that language-related difficulties impacted the performance of NESB students and by implication, their capacity to learn mathematics at this level. It became evident from Part II that linguistic features of test items had a bearing on test scores. This indicates that NESB students are more likely to experience difficulties comprehending written text, which is a significant component in learning mathematics at this level. These difficulties that hinder students in the reading and comprehension stages of the revised error analysis model are likely to cause errors in the subsequent stages and prevent students from demonstrating their learning despite possessing process skills in mathematics.

It was also found that students differed in their preferred style and approach to writing. This preference also affected their ease of comprehension as did the extent to which they used the mathematical register, locators or qualifiers. Detailed and longer descriptions using more qualifiers and familiar and/or standard mathematical terms were more easily comprehended. This suggests that textbooks or lecture notes that are cryptic, brief or abstract may limit NESB students’ access to learning materials. As the procedural style appeared to be preferred, abstract conceptual descriptions in texts could be relatively more difficult for many students. It was evident that most NESB students had considerable difficulty writing a mathematical text. All these factors are likely to have a bearing on academic performance in examinations, project reports and any other assessment tasks due to students’ inability to demonstrate their learning.

An important aspect of learning at the tertiary level is self study using recommended textbooks and reference books. Part I of the research found that many of the features discussed above are evident in the tertiary mathematics textbooks that were analysed. Hence these students are likely to face challenges in effectively comprehending concepts and procedures which in turn can affect their capacity to learn. Furthermore, the style and approach preferences of students observed in Part III impacted on their comprehension of written text. Therefore these preferences might also have a bearing on how they relate to the writing style of authors of textbooks.
7.4.3 Research question 3

Is there a difference between NESB students from different language backgrounds in their comprehension and use of the language of Mathematics?

The aim of this question was to investigate whether there is a relationship between language background and comprehension and use of English language by tertiary students of mathematics from different NESB backgrounds. This was answered by both Parts II and III of the research. The results of both parts demonstrated that there was a difference between the various language groups in performance on the Mathematics Language Comprehension Test as well as the Mensuration Task. These differences were explained using the revised error analysis model in terms of possible errors during translation and code switching or lack of equivalents in students’ first language knowledge and cultural base.

Thus it could be seen that language background did have a bearing on academic performance in the learning of tertiary mathematics. Middle Eastern and Asian groups appeared to experience more language-related difficulties in all the aspects studied, namely, performance on the language test, descriptive mathematical writing, and interpreting written text to produce a sketch. Chinese, Indonesian and Indian students demonstrated greater ability in the same tasks, while students from European-language backgrounds scored relatively higher on the mathematics language tests despite having problems communicating in English. All these results imply that language background does impact on the relative capacity to comprehend and use the language of mathematics.

It was also found from the results of Part III that writing style and approach differed with language background. Features of written text such as number of words and the use of mathematics register, locators, and qualifiers all showed differences with language background. Differences were also observed in interpretation of written texts as demonstrated by sketch scores. It was found that the groups which demonstrated greater ability in language tests or written descriptions did not necessarily demonstrate better interpretation skills especially if the text was not in clear and precise language. All these observed trends have been discussed in the earlier sections.
7.4.4 Research question 4

How do VCE and first year university Mathematics textbooks rate in their readability levels, linguistic complexity of English and use of the Mathematics Register?

This question sought to determine how different textbooks compared in their use of English language. This was the focus of Part I where comparison of three VCE Mathematical Methods textbooks and three university mathematics textbooks, showed the extent of difficult language features that were identified in each. Readability tests showed that textbooks varied in their level of language use (see Sections 4.2 and 7.1.2) and the use of linguistic features such as *mathematics register/ technical terms, comparative phrases, passive verbs* and *contextual words* (see Sections 4.3, 7.1.2 and Figure 7.2). As the six representative textbooks selected for this study, were found to use all these features identified as challenging for NESB students, these students are likely to experience difficulties in reading and comprehending textbooks at this level, and proceed with other stages of problem solving identified in the revised error analysis model.

The findings showed that the university mathematics textbooks did not necessarily use more difficult language features than the school texts. Each book displayed individual features and no single book could be ranked as most difficult or the easiest on all of the criteria. This has implications for the selection of textbooks, in that the requirements and expected outcomes of the course as well as the demography of its users must be taken into consideration.

7.5 Conclusion

Over the years teachers and researchers have sensed, and investigated the relationship between language and mathematics, and the relationship between language proficiency and performance in mathematics, although predominantly at the primary and junior school level. The few studies that have investigated these issues at the senior secondary or tertiary level, have done so in multilingual classrooms rather than in English speaking environments where NESB students
learn mathematics in English. Barton et al. (2005) conducted a series of three studies which led to the conclusion that NESB students struggle to learn mathematics in English and the level of difficulty increases with progression of students to higher levels of mathematics at university. Although conducted for NESB students, their study involved local second language learners who had completed schooling in New Zealand and it is possible that at least some of the lecturers and tutors spoke the students’ L1. My study on the other hand, is the first in my knowledge, to carry out such research in a tertiary setting where international NESB students are taught in English by teachers who do not speak the L1 of their students.

The collated results from the three parts of this study have established that NESB students do face a number of language-related difficulties that appear to affect learning of tertiary mathematics. Language-related difficulties are clearly impacting students’ capacity to comprehend and construct written text – this means that these students do not necessarily have the means to engage with the learning materials despite having the opportunity to do so. This signals several implications for educators, teachers and authors, which are discussed in the next chapter.
CHAPTER 8: IMPLICATIONS AND RECOMMENDATIONS

The previous chapter drew on the findings of this study to support a revised error analysis model, develop a framework for analysing the suitability of texts for particular audiences, and address the research questions posed at the outset of this study. This final chapter looks back at the research journey, its outcomes and implications, including suggestions for further research, and considers the significance and limitations of the study. The first section (8.1) examines the goals of the research, the appropriateness of the research questions, and the adequacy of the methodology in achieving these goals. Section 8.2 is a call for action in the light of the findings and discusses the implications of these for practitioners, policy makers, teachers and lecturers, as well as authors and publishers. Section 8.3 puts forward several suggestions for further research. The significance of this study is outlined in Section 8.4 and its limitations in Section 8.5. The chapter ends with a concluding statement for the thesis in Section 8.6.

8.1 The Research Journey

This study was conducted in the context of the increasing number of multicultural classrooms in developed nations and the consequent rise in numbers of NESB mathematics students. It was established in Section 2.1.2 that this count of NESB students includes a large number of students in Australian tertiary classrooms who have arrived to pursue higher education in this country. It was found that of the 500,000 international students in Australia in October 2008, 79% were enrolled in the tertiary or higher education sector (Australian Government, 2008). Prior to enrolment in any tertiary educational program, all international students are required to meet minimum stipulated scores on the standardised test of English proficiency, the International English Language Testing System (IELTS). While some may just meet the minimal requirement of a score of 6 on IELTS to secure a place in the higher education programs here, they may not be as proficient in their use of English language as students from English speaking
backgrounds. Others who fall short of the IELTS requirements, take a year before commencing higher education programs to complete bridging courses such as Foundation Studies (FS) at RMIT, which accepts students with a minimum overall score of 5.5 on IELTS.

My research began in response to my curiosity to know more about the difficulties experienced by learners of mathematics in Foundation Studies (FS) due to lack of proficiency in English. It all began with two separate incidents in FS examinations that alerted me to the fact that some students were experiencing difficulties comprehending particular examination questions solely due to their language or cultural backgrounds (see Section 1.1.2). I decided to follow this up by talking to students from various language backgrounds. However, having a conversation about language was not an easy task when the students were not fluent in English and I could not speak their first language. I resorted to having a few informal conversations with small groups of students who spoke the same first language, so that with some discussions and deliberations in their own language amongst themselves, they were able to respond to my questions. The information gleaned, including some specific difficulties that emerged during these informal interviews, was recorded and these instances convinced me that language-related difficulties in learning tertiary mathematics merited further research.

A first step was to access existing knowledge and a survey of literature revealed that numerous researchers (e.g., Abedi & Lord, 2001; Barwell, et al., 2005; Chapman, 1993; Chapman & Lee, 1990; Cuevas, 1984; Cummins, et al., 1988; Ellerton & Clements, 1991; MacGregor & Moore, 1991; Mousley & Marks, 1991) identified language as a factor affecting learning of mathematics at the primary or junior school level. Others (e.g., Barwell, 2005; Barwell, et al., 2007; Clarkson, 1991, 1992, 2003; Galligan, 2004; Moschkovich, 1999; Secada, 1992; Setati, 1998; Setati & Adler, 2001) established that language proficiency impacts the performance of NESB learners of mathematics, also at the junior school level. However, there are relatively fewer studies of this nature at the senior secondary and tertiary level involving NESB students. Those that do exist are fairly recent and tend to be limited to the classrooms of multilingual societies such as South
Africa (e.g., Gerber, et al., 2005; Varughese & Glenncross, 1996) or New Zealand (e.g., Barton & Neville-Barton, 2003; Barton & Neville-Barton, 2004). In multilingual countries such as these, NESB students tend to come from minority groups or native majority groups and teachers and lecturers might speak the L1 of these students (e.g. Afrikaans, Xhosa, Zulu in South Africa, or Maori in New Zealand). In these situations, teachers are likely to be familiar with students’ cultural backgrounds or the students themselves have lived and schooled in these countries for several years as was the case with the New Zealand study. The situation is not the same in the case of the international NESB students in universities in developed countries, who are learning mathematics in English taught by lecturers who do not speak the students’ first language. The growing number of such NESB students fall into a unique category. There was no evidence that I could find, of any research into the language difficulties faced by international NESB students in the learning of tertiary mathematics in developed countries with monolingual teachers in English speaking environments.

As a consequence, the goal of the research became an exploration into the language-related difficulties faced by NESB learners of mathematics undertaking a bridging course in a tertiary environment. The exploratory nature of the study led to open ended questions rather than hypotheses testing (Green & Browne, 2005). Four research questions were framed to explore

(i) what, if any, are the language difficulties faced by NESB students in learning tertiary mathematics,
(ii) whether these difficulties have any relationship with the performance of these NESB students in mathematics,
(iii) whether the language background of students affect their use of English language in the context of tertiary mathematics, and
(iv) how VCE and university mathematics textbooks compare in the use of linguistic features likely to cause difficulties (see Section 2.6).

The number of students undertaking FS at RMIT has nearly doubled in the past five years. This year, approximately 200 students in total (five intakes per year) are currently enrolled in this program. This is an educational situation where students from vastly different cultural and linguistic background and with varying
levels of proficiency in English are taught by teachers who do not speak their first
languages: a situation of ‘immersion’ in English, as they are taught in English for
the whole year. English is a compulsory subject and students choose four other
elective subjects including three mathematics courses that are comparable to, but
progress a little beyond, the three Year 12 VCE mathematics courses (see Section
1.1.2). The FS students of RMIT University formed a convenient sample that was
representative of the diverse community of international students in the tertiary
and higher education sector in Australia.

It was felt that of the four components of language use, reading, writing,
speaking, and listening, the first two were the most relevant to learning tertiary
mathematics, particularly as students’ facility with oral language tended to be
somewhat in advance of their reading and writing skills. Hence the study focused
on different aspect of the reading and writing components of mathematics
learning. It was observed that FS students, who were capable of advanced
mathematical processes such as integration, often had difficulties with topics such
as mensuration and probability involving technical terms and word problems. The
teaching of statistics and probability, which uses many terms with very specific
mathematical meaning and unfamiliar concepts, has been recognised as a likely
source of student difficulty (Wilson, 2002). As the focus of the study was the
language-related difficulties experienced by NESB learners, the areas selected for
investigation were normal probability distribution and mensuration. The study
was conducted in three parts with the aim of investigating three aspects of
language-related difficulties faced by NESB students. Part I examined language
use in Mathematics textbooks in the word problems of normal probability
distributions, Part II looked at how the language of test items affected
performance in mathematics, and Part III analysed student’s capacity to construct
and interpret written text on the topic of mensuration. It was felt that this
multipronged approach would offer the best means of providing a rich insight into
the language difficulties faced by tertiary NESB students of mathematics.

Although conducted as a case study of one group of students at one institution,
the participation of students from three academic years and the nature of the data
collected provided an opportunity to involve a larger number of students than is
feasible in most case studies. While caution needs to be exercised in generalising the findings of a case study, the sample was sufficiently large to provide evidence of clear trends. Furthermore, the methods used resulted in quantifying most of the data, which enabled the use of effect size to determine the magnitude of differences between groups and to identify trends. The use of a mixed method interpretive case study methodology supported the development of grounded theory which proved useful in addressing the research questions. The three parts of the study provided a means of examining different aspects of reading and writing, and the results from the three parts used for corroboration offered a form of triangulation.

8.2 Findings That Call for Action

The findings of this research have shown that NESB students studying tertiary mathematics in English face a number of language-related difficulties. Much of the mathematics learning at the tertiary level requires reading and writing. A task which is exacerbated by the fact that mathematics has its own specialised vocabulary in addition to the contextual vocabulary related to the field in which the mathematics is being applied, in this case, Engineering and Business. This presents several difficulties for NESB students both in comprehending written text and writing mathematically.

This study found that many of the linguistic features identified as difficult for learners of school mathematics cause difficulties for tertiary NESB students and these features are prevalent in the mathematics textbooks that were analysed. The language of test items was found to have a bearing on performance and language background appeared to influence comprehension and use of English language. Writing in mathematics was found to be the most challenging task, which suggests that these students are likely to face difficulties in presenting logical arguments or written reports. The findings of the study conducted in New Zealand (Barton, et al., 2005), detailed in the literature review, indicated that NESB students had greater difficulty coping with the logical writing required in third-year mathematics compared to the more computational first-year
mathematics. This suggests that the FS students are likely to face even greater difficulties in higher levels of mathematics requiring logical arguments and proofs despite the year long FS program in an English speaking environment. Proficiency in English has been found to be essential for cognitively demanding academic tasks (Spanos, Rhodes, Dale, & Crandall, 1988), and studies such as Cummins (1981) and Collier (1987) suggest that development of academic proficiency in a student’s second language could take several years. Parts II and III of this study have shown that many international students who undertake FS do not achieve the academic proficiency in English at the end of the one year program.

Analysis of student writing showed that there were preferred styles and approaches to writing. Language background also had an influence on student choice of style and approach. The procedural style appeared to be easier to comprehend. Vocabulary use was also found to vary with language background and some of the differences between language groups were quite pronounced suggesting that student cohorts cannot be treated as homogenous groups with similar needs.

The results of this study have several implications for the teaching and learning of tertiary mathematics. Educators need to be aware of the needs of NESB students and make efforts to accommodate these students without compromising the standard of the mathematics courses. This calls for action at all levels as discussed in the following sections.

### 8.2.1 Implications for policy makers

First and foremost is a need for consultation and communication between the policy makers and the implementers. Policy makers are persons in Government Departments of Education and Immigration, or administrators in university chancellery or academic faculties, who make decisions about the qualifying requirements for student visas and entry into academic programs in universities. The findings of this research suggest that policy makers should determine the
minimal levels of language proficiency required for higher education programs through wider consultation with academics in the classroom. This should be well communicated to all prospective students and training packages should be provided for those who lack the required proficiency. Lack of language proficiency should be recognised as a form of disadvantage and remedied as it has become apparent that such students are not able to gain full benefit of participating in university mathematics courses. One remedy could be more intervention programmes for language skills such as a course on academic English, incorporated into programs at all levels. Another could be to include multilingual tutors in academic support units such as the learning skills unit at RMIT University, who are able to assist students by communicating in their first languages. Alternatively, there should be an option of a longer period for bridging courses depending on the needs of the students. For instance, two year FS programs could be introduced for students at the lower end of the acceptable IELTS scale of 5.5. At present such students who fail FS, either repeat the program, or drop out of their higher education pathways and opt for diplomas, or in extreme cases, return to their countries. It would be beneficial to introduce a two year FS program with students with severe language disadvantages taking both years and others going directly to the second year.

Secondly, academics need to become aware of relevant educational theories such as second language acquisition theories. While academics are experts in their fields, they cannot be expected to have the ability to understand or cope with individual learning difficulties. While some form of professional teaching qualification would be ideal for all academics, at least ESL training, or professional development sessions on educational principles such as second language learning or the language difficulties experienced by these learners could be made mandatory.

Thirdly, students enrolled in a program should be provided all support to benefit from their education. They should be encouraged to use facilities that might be available such as learning skills units and other student support programs available in most universities. They should be encouraged to have a self-evaluation of language difficulties experienced, and inform lecturers, authorities,
or academic support staff. All of this can be achieved by providing international students with a sound orientation, induction and mentoring upon arrival.

8.2.2 Implications for teachers

This study investigated language-related difficulties from different perspectives and the findings have many implications for tertiary mathematics teachers in their day to day professional activities. Kersaint, et al. (2009), referring to school mathematics classrooms, said that “it is our professional obligation to find ways to address the needs of all students in our classrooms...we do not select our students and we cannot change them” (p. 75). On the contrary, in the tertiary sector, we select our students and they are fee paying students at our universities. University Departments of Mathematics need to acknowledge and address the needs of the growing number of NESB students enrolled in their courses. This needs to be done in realistic ways that do not detract from the quality of mathematics or professional preparation offered to students in courses such as Engineering Mathematics or Business Statistics. In other words, the solution does not lie in avoiding technical language or contextual words but in scaffolding NESB students in developing academic and professional language skills as they progress through the mathematics courses. Ferrari (2004) recommends the exploitation of verbal language as a tool to describe and justify procedures and suggests that discussions between students may help develop linguistic skills. Siemon (1997) also recommends an emphasis on the construction of meaning and a classroom culture where discussion and justification of procedures with peers and teachers is encouraged and nourished. However, teachers need to be sensitive to cultural differences and provide a supportive learning environment.

Lecturers of tertiary mathematics need to be aware of the needs of different groups in planning their teaching and preparing notes. Tertiary students who lack proficiency in English or mathematical vocabulary are likely to face difficulties following a fast paced lecture where lecturers are likely to assume knowledge of simple mathematical words or linguistic features. The steps in the Revised Error Analysis Model presented earlier could apply to this listening aspect of language
as well. The lecturer’s spoken words, or some terms or concepts on a lecture slide/whiteboard could trigger code switching for an NESB student. Any error in translation could cause the student to lose track of the argument or concepts being presented.

These implications with regard to specific teacher actions are discussed in more detail below.

**Selecting textbooks**

Mathematics teaching at the high school and university level, is heavily reliant on textbooks. As students progress through higher education, they are increasingly required to read and understand the content of texts and problem sets for success in mathematics courses. It is inevitable that certain levels of reading competence will be required to grasp the ideas presented in textbooks and to interpret and solve problems successfully. The technical vocabulary of mathematics adds to this requirement of reading competence. The contextual references and vocabulary used in application problems place further demands on the reader especially in the case of NESB learners.

In selecting textbooks, teachers or lecturers are likely to base their assessment of textbooks on the difficulty level of the mathematical concepts and problems presented rather than the complexity of language use. For instance, a teacher is likely to look for examples to illustrate various concepts and mathematical techniques and the variety and range of applications involved. If they do consider clarity of explanation it is not generally from the perspective of NESB learners but from the lecturer’s understanding of the mathematical concepts or techniques involved. The criteria will differ according to the level of mathematics involved. However, the level of English used and the syntactic structure of the sentences used in a text are less likely to feature in the selection criteria for textbooks. It must be recognised that while a textbook might be technically correct and meet all the long term requirements of the mathematics course, its usefulness could be diminished if it cannot be used effectively by its intended users including NESB learners. Hence the framework developed as a result of this study for the analysis
of textbooks, is a means for assisting lecturers make more informed decisions about text selection.

The results of this study show that, teachers would be well advised to consider linguistic features as well in selection of prescribed texts, recommended reading, or additional references. The NESB students would require books with simple but ample explanations, repeated and graduated use of mathematical vocabulary. The results of this study suggest that a long problem does not necessarily mean it would be more difficult for NESB students. The content and language use is of equal importance. The results show that if a problem is lengthy due to the presence of superfluous information and contains linguistic features such as passive verbs, comparative phrases, or complex verb forms, it is likely to hinder readability and comprehension. On the other hand a problem that is lengthy due to more precise information being given, or the explanation of mathematical terms, or the explicit statement about what is required, will be better comprehended.

While most teachers would have criteria for the textbooks they select, this study identifies some additional factors that teachers should be aware of. A selective use of an appropriate readability test together with relevant aspects of the Linguistic Complexity Rubric can assist teachers in making informed decisions about the choice of textbooks. The flowchart presented in Section 7.2.3, provides a suggested framework for such an analysis.

Course planning

The findings of this research also have implications for course design. Lecturers will be well advised to consciously plan to accommodate students who might have language difficulties. This is by no means a call to reduce quality or avoid technical language use. Rather, it is recommended that while content need not be compromised or changed, an intentional effort will be required to scaffold the improvement of language skills of the less proficient students. MacGregor (1993) warns against trying to help weaker students by reducing the reading and writing components of a lesson which will result in lack of cognitive growth in the long
run. Instead, careful introduction of mathematical vocabulary and repeated, clear use of linguistic features such as comparative phrases and passive voice in class discourses will familiarise students to these aspects prior to facing assessments. Encouraging students to develop a glossary of the required mathematical and contextual terms, or providing them with one, could go a long way towards scaffolding students in the development of their mathematics-specific vocabularies. A conscious effort to include both descriptive and procedural styles of writing in lecture notes and other written instructions could accommodate all students and help them relate to the material. Inclusion of more written tasks and oral presentations in class could help students develop confidence in communicating mathematically, and enhance the skills necessary for assessment tasks. Gopen and Smith (1990) have strongly advocated the use of writing in mathematics based on their successful experience of introducing a major report writing exercise in their college calculus course at Duke University, although their study was not designed with NESB students in mind. While a major focus on writing may further disadvantage NESB learners, more exposure to relatively minor written assignments and report writing components in courses such as FS could enhance students’ thought processes as well as their written expression in mathematics.

Writing assessment tasks

It is evident from the findings of this study that the language of test items does affect NESB student performance in mathematics. It is important to familiarise students with mathematical vocabulary and prepare them for the contexts in which mathematics will be used in the workplace. However, teachers must make an effort to restrict the use of unfamiliar contexts in test items. In other words, care must be taken to introduce as many mathematics register words and contextual words in examples and problems done in class to provide students with an opportunity to familiarise themselves with these words and phrases. Passive voice, comparative phrases and other complicated linguistic features must also be introduced progressively so that students do not feel overwhelmed by language hurdles in a mathematics examination. While providing maximum exposure to a variety of linguistic features in class and tutorial time, the language used in
assessment tasks must be simple, clear, and unambiguous. The findings of Part III showed that there are clear preferences between the descriptive and procedural style of writing and the latter was more easily comprehended by these students. Hence a mix of both styles in writing the items on an assessment task could assist in comprehension.

It is recognised that “the experience of gathering evidence, of writing a convincing report, and making an oral presentation are considered valuable components” of assessments and can enhance learning (Izard & Haines, 1993, p. 237). It was seen in Part III of the study that NESB students have considerable difficulty writing mathematically. This has implications for project reports, assignments, and written examinations. Detailed guidelines should be provided where written submissions such as project reports are required for assessment purposes as many NESB students have considerable difficulty expressing their ideas in English and might tend to skip essential components such as an introduction, details and explanations, or conclusion. A degree program in Engineering or Business might include two or three courses respectively of Engineering Mathematics or Business Mathematics/Statistics. It is the responsibility of the mathematics educators then, to ensure that all students who take up their programs become cognitively proficient in the mathematical and contextual language relevant to their particular field of study and future workplace. An instrument such as the Mathematics Language Comprehension Test used in Part II could prove useful in judging progress in achieving such language proficiency.

8.2.3 Implications for Textbook Authors and Publishers

A number of issues raised in this study have direct implications for authors and publishers of textbooks. The literature review and the findings of this study have identified some of the features that are likely to cause difficulties for NESB students. Many of these features were found to varying extents in the six textbooks that were analysed. As these texts are fairly typical of VCE and university textbooks, these findings are likely to apply to similar textbooks at this
level. This indicates some issues that authors should be conscious of when they write mathematics textbooks namely, maintaining good quality language, reducing unnecessarily complex sentences with passive voice and minimising other difficult linguistic features. Problem contexts can be clearly and explicitly stated. While long and unfamiliar words or long sentences might hinder readability, very cryptically word problems might be difficult to comprehend. Thus, the use of more detailed descriptions using simpler words, and clearly stated problems would help the students who are less proficient in English. At the same time care must be taken to introduce as many technical terms and contextual words in the descriptive sections, worked examples, and problem sets to help prepare the students to tackle these with ease in examinations and assessments.

An author’s style and approach to writing can also help or hinder students in comprehension of the text. For instance, it was found that procedural componential descriptions resulted in higher quality of sketches. A few procedural worked examples and a few simple problems using a procedural style at the beginning of problem sets can scaffold less proficient students with a preference for that style in solving more linguistically challenging problems later on in the problem set. However, quality and standard of mathematics teaching cannot be compromised at this level of education, and authors do have a responsibility to prepare the users of their textbooks for workplaces at the end of their chosen pathways. Thus, problem sets in any textbook should be a suitable mix inclusive of style and approach preferences, simple and challenging problems, and procedural as well as contextual application problems, all of these gradually scaffolding students to build up skills in both mathematical and English language use.

It has been argued that “teachers, schools and policy makers need to value such inter-disciplinary dialogue and, crucially, provide time for it to happen and space for it to flourish” (Barwell, et al., 2005, p. 146). The following functional flow chart summarises the suggestions for action at the various levels in the above discussion and illustrates how such inter-disciplinary dialogue might be achieved in a systematic manner.
Figure 8.1   Call for action flowchart
8.3 Recommendations for Further Research

One of the goals of this study was to extend what is known about the nature of mathematics education at the tertiary level, particularly in relation to NESB learners. While it has achieved this purpose in many respects, it has raised a number of other questions that merit further research. Furthermore, the methodology and findings of this research open up several avenues for further research. There is need for research to investigate current practices, and improve teaching and learning strategies at the tertiary level.

- This study was conducted solely at RMIT University and could be replicated at any other university to determine whether NESB students face the same difficulties elsewhere. Secondly, the sample size was limited by the number of FS students in the mathematics classes over the two academic years during which data was gathered. A study that could be extended to several universities over a longer period of time would provide more conclusive evidence of the trends observed in this study.

- Findings from this study indicate that, NESB students as well as the TAFE VCE students who are returning to school after a break, had difficulty comprehending mathematical vocabulary and certain linguistic features of test items. This suggests that other Australian students in university classrooms might be having similar difficulties. This could be researched further by investigating the language of assessments used in university mathematics courses and the effect of modifying the language of assessment questions. However, this can be done on a research instrument not used for assessment purposes as it may not be appropriate or ethical to modify actual assessment items. The Mathematics Language Comprehension Test and the Mensuration Task could be administered to a large number of students from both NESB and English speaking backgrounds to identify students’ specific language needs and inform teaching at this level.
• The findings of this study point to several difficulties experienced by NESB students at the tertiary level. Further research could be focused on addressing the issues raised and on evaluation of action taken.

• The trends observed in language and gender groups of this sample could be further investigated if the study was extended to a larger sample of students. For instance, trends in writing style and approach, or patterns of vocabulary use, shown by the small groups of this study could be verified by such projects.

• It would be interesting to determine how NESB students think or write mathematically in their first languages and how this affects their language use in English. However, this requires experts who know these languages as well as English and this might only be possible through large scale funded projects.

• A longitudinal study using the instruments developed for this study could follow the progress of FS students through the mathematics courses they take in their higher education pathways to monitor the extent to which the language difficulties identified initially remained a problem or not.

• This study has put forward methods of analysing mathematics textbooks and has shown how readability tests used in conjunction with the Linguistic Complexity Rubric could be used to analyse any textbook for teaching or research purposes. Alternatively, the framework in Figure 7.3 could be used as a basis for developing an instrument for tertiary students that incorporates these issues and suggests actions to increase the probability of successful communication.

• Newman’s error analysis approach is being applied to many schools in New South Wales (White, 2009). The Revised Error Analysis Model for tertiary NESB students, developed as a result of this study could form the basis of a future Australian Research Council funded study. A study
comprising mathematics educators from various language backgrounds that could include in-depth interviews of NESB students to gain a better understanding of the advantages as well as possible errors and pitfalls in code-switching between English and L1.

- This study provides several methodological approaches that future researchers can emulate or modify for use in other contexts. It has provided an example of a mixed method approach where a number of qualitative and abstract concepts were quantified enabling the effective use of simple statistical measures to look for trends followed by the use of effect sizes to determine the magnitudes of difference between groups.

### 8.4 Significance of the Study

This study has made a significant contribution to what is known about the language-related difficulties experienced by NESB students at the tertiary level. It provides methodological and practical implications for researchers and tertiary mathematics teachers.

- This research has called attention to the gap in knowledge regarding mathematics education at the tertiary level particularly regarding the relationship between mathematics teaching, and a student’s language background and English proficiency.

- This study is particularly relevant at a time when the number of international students in Australia and most other developed countries has increased dramatically. Growing numbers of NESB students in our tertiary classrooms call for an understanding of the language difficulties likely to be faced by these students in coping with the discourse features of university mathematics courses. This study is a response to this need and provides important information to educators.

- Although conducted as a case study, the sample was a very diverse cohort of students representative of international NESB students at any tertiary
institutions and the findings can be meaningfully interpreted or applied in those contexts as relevant.

- This study provides several methodological aspects that can be adopted by other researchers and reaffirms the strength of a mixed method approach. Despite the exploratory nature of the study, the method allowed for a relatively large sample size than would be feasible in a typical case study. This enabled quantifying of most data, which provided a means for identifying trends in the observed results, and abstract concepts like 'language difficulty' were made measurable through the use of defined indicators that were amenable to simple statistical analyses. This supported the use of effect size to determine the magnitudes of the differences observed between groups. These methods provided objective ways of analysing data and making inferences, while retaining the strengths of qualitative research such as long term observation from within the research setting.

- The multi-pronged method adopted by this research provided a means of corroborating the findings from the three parts of the study using the triangulation principle. These findings have provided useful information for lecturers and teachers of university mathematics courses, FS and other bridging courses, policy makers, as well as authors and publishers of tertiary mathematics textbooks.

- The three instruments developed or modified for this study namely, the Mathematics Language Comprehension Test, the Mensuration Task, and the modified Linguistic Complexity Rubric would appear to offer valuable tools for further research in this area.

- Two frameworks developed as a result of this study could also prove valuable for future researchers and practitioners. One is the Revised Error Analysis Model for NESB learners of mathematics and the other is the Text Analysis Framework.
• This study has provided numerous avenues for future research as discussed in the previous section, and mathematics education could benefit greatly by other researchers replicating, extending or modifying this research to increase our collective knowledge in this important domain.

8.5  Limitations of the Study

Any study that involves human participants in real life situations as opposed to artificially created laboratory conditions has unavoidable limitations and this study is no exception.

• This is a case study conducted with students of one program at RMIT University in Melbourne, Australia. Although they appear to be representative of the wider community of international students, this is an assumption that needs to be verified before transfer of findings to other situations.

• Although larger than most qualitative case studies, it has to be acknowledged that the sample is not sufficient to make conclusive generalisations or infer causality, and hence the inferences had to be limited to observed trends.

• As an exploratory study, while the multi-pronged, quantified approach provided wider scope for data collection, it limited the amount of detail that was available. In depth interviews and individual case studies, which might have provided richer data, were simply not feasible in this situation owing to language constraints - I did not speak the first language of the participants and they were not fluent enough to communicate language issues in English.
8.6  In Conclusion

In the relatively young field of mathematics education, interest in the link between language and mathematics learning is fairly new. Recent studies have investigated multilingual classrooms and the impact of language of assessment items on performance at the primary and junior school levels, but there is a very limited amount of research in this field at the tertiary level. Consequently, educators have little or no access to research-based knowledge or advice about the difficulties faced by the growing number of NESB students in tertiary classrooms. This thesis has been an attempt to explore an under researched area, gather knowledge subject to the constraints of a case study, and provide motivation for further research in the field. The exploratory study has revealed several aspects of language-related difficulties experienced by NESB learners of tertiary mathematics in an English speaking environment. The findings have implications at all levels of mathematics teaching and learning and recommendations have been made for policy makers, lecturers and teachers, authors and publishers of textbooks, and for the students themselves. Furthermore, this study has produced useful instruments for assessing language difficulties and linguistic complexity of texts. Above all, this study has put forward a Revised Error Analysis Model for tertiary NESB students and a Text Analysis Framework which are likely to prove useful in further research and practice. With numerous avenues suggested for further research, it is hoped that this study will pave the way for many more studies in the field of language and mathematics education, and development of much needed knowledge in this field especially at the tertiary level.


Wilson, R. J. (2002). "What does this have to do with us?" *Teaching Statistics to Engineers.* Paper presented at the Sixth International Conference On Teaching Statistics, Cape Town, South Africa.


APPENDICES