CONSTRUCTING PATHS TO MULTIPLICATIVE THINKING: BREAKING DOWN THE BARRIERS

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SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE SCHOOL OF EDUCATION COLLEGE OF DESIGN & SOCIAL CONTEXT RMIT UNIVERSITY

June, 2011
In loving memory of my husband
Marty Breed
30.04.1957 – 06.06.2008
DECLARATION

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third part is acknowledged; and ethics procedures and guidelines have been followed.

Margarita H. Breed
February, 2011
I wish to express gratitude to my supervisor Dianne Siemon for her guidance, support, and scholarship through the duration of my candidature. Without her unwavering professional and personal care, the love I have for this work would not be as deep nor as rich. She has shared so much of her knowledge and experience, while at the same time guiding my voice.

Thank you to Claudia Johnstone for supporting the editorial process by reading, questioning, and attending to the minute details that I would have missed, and to my son Julian Wallbridge for creating the images of the butterflies.

Thank you to my children, Ben, Jasmin, Phoebe, Julian, and Gracie, and other family members, Catherine, Christian and Jo for their interest and understanding throughout. Without their love and support it would have been hard to continue.

I would also like to formally acknowledge all the students who participated in this research, especially the students with whom I worked so closely. This is for them.
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ABSTRACT

Large scale research on numeracy in Years 5 to 9 identified an 'eight-year' range in student achievement at each year level which was almost entirely due to difficulties associated with multiplication, division, fractions, decimals and proportional reasoning (Siemon, Virgona & Corneille, 2001). This study was supported by an Australian Post-graduate Award Industry (APAI) scholarship as part of a larger, follow up research project on the development of multiplicative thinking in Years 4 to 8, the Scaffolding Numeracy in the Middle Years Project (SNMY), that was funded by the Australian Research Council and the Victorian and Tasmanian Departments of Education in 2003-2006. The SNMY Project used Rasch modelling (Rasch, 1980) to develop a Learning and Assessment Framework for Multiplicative Thinking (LAF) on the basis of 3200 student responses to a range of rich tasks designed to evaluate student thinking in relation to the key aspects of number referred to above (Siemon, 2004; Siemon, Breed, Dole, Izard, & Virgona, 2006).

The APAI study was designed to explore what it would take to address the learning needs of students identified to be at-risk in terms of the LAF in a timely and effective manner. At-risk in this context, refers to students performing well below the level expected in relation to multiplicative thinking who relied on modelling and count-all strategies to solve relatively simple problems involving small whole numbers. The research design was premised on the view that learning is socially and culturally situated and that research conducted from this perspective should be responsive to and informed by events and interactions as they emerged. As a consequence, a design experiment methodology was adopted as it accommodated an evolving innovative learning environment and simultaneous evaluation of the innovations introduced in that environment (Brown, 1992). The context for the design experiment was the mathematics classroom, with the researcher working directly with a small group of at-risk Students in Year 6 to support a sustainable shift from additive to multiplicative thinking. Beginning and end data from the SNMY project was used to obtain data for selection and monitoring purposes.

The sample comprised ‘performance outliers’ identified by the SNMY Project assessment data, that is, students performing well outside what is generally expected for their year level in school mathematics. Three groups were constructed for the purposes of the study: at-risk intervention students (ARI), at-risk non-intervention students (ARN), and successful (SC) students. These groups were chosen from like schools in the metropolitan cluster of schools involved in the SNMY project to evaluate the extent to which the intervention made a difference. Comparison across and between the two at-risk groups
(ARI and ARN) and the SC group determined the degree of similarity and difference between the students, as well as the impact of the intervention program on student learning outcomes. Data was collected over three distinct periods: a Pre-Intervention (Pre-I) Phase (December 2004 – May 2005), an Intervention Phase (May 2005 – November 2005), and a Post-intervention (Post-I) Phase (November 2005- December 2005). During the Pre-I and Post-I Phases, interviews were administered to all students to determine their beliefs about learning mathematics as well as the strategies they used to solve multiplicative tasks. The Intervention Phase took place over a period of 18 weeks, from May 2005 to November 2005. Three hour-long sessions were held each week with all ARI students. In addition to developing conceptual understanding with a view to supporting students to make the transition from additive to multiplicative thinking; the sessions were focussed on building student confidence, developing procedural fluency, working mathematically, and improving communication skills.

Beginning and end data from the SNMY project revealed that all nine ARI students, who were assessed at the lowest level of the LAF (Zone 1) in the Pre-I Phase, were assessed at Zones 4 or 5 in the Post-I Phase. That is, they had access to an extended range of more efficient strategies to solve problems involving multiplication and division and were beginning to work with simple fractions and decimals. In contrast, the five ARN students and eight SC students made little, if any shift, in terms of the LAF over this period, suggesting that the shift in performance of the ARI students was due to their participation in the intervention program. However, the analyses of the Pre-I and Post-I interviews with all three groups of students suggest that something more than access to valid and reliable diagnostic tools and targeted teaching advice in the form of the SNMY materials was needed to make a difference to student learning outcomes at this level.

In addition to confirming the critical role of beliefs and attitudes and the importance of experiencing success, the interview data suggests that successful intervention at this level is much more heavily dependent on the depth of teacher knowledge for teaching mathematics and the extent to which he or she is able to respect and engage with the emerging identities of students, than it is in the early years of schooling. In particular, the capacity to create mutually endorsing, productive communities of learners appears to be strongly related to the teacher’s own sense of identity and agency and the extent to which they actually care about their student’s welfare and being beyond the mathematics classroom.
The results of this study have important implications for improving student learning outcomes in mathematics at this level of schooling. The overall results of the SNMY project suggest that up to 35% of students in Years 7 and 8 do not have the foundation knowledge, skills and understandings to successfully participate in further school mathematics (Siemon et al., 2006). While the intervention program adopted in this instance was specific to the learning needs of the children involved, much of the content and many of the pedagogical practices are likely to be relevant to students in this 35%.

To tease out the issues involved, the study was necessarily limited in its scope and design to a relatively small sample of students with intervention provided by the researcher. Further research is needed to determine what would be needed to support classroom teachers to achieve the same result. However, there are key characteristics that are directly transferable in relation to teacher knowledge, pedagogy, and relationships. Given the critical importance of meeting the learning needs of at-risk students, the results of this study suggest that, at the very least, two immediate considerations are important. First, the provision of opportunities for extended professional development on the ideas and strategies involved in scaffolding the shift from additive to multiplicative thinking. Second, the appointment of mathematics specialists to primary schools to support diagnosis of learning needs, and support for targeted teaching.
CHAPTER 1
INTRODUCTION

The process that led me undertake this research began the first day I attended school, a small primary school two houses from home. I was 4 and a half years old. A harrowing experience, having not attended kindergarten, I was somewhat ill-prepared. Excited, I assumed that school would be wonderful. So dressed in new clothes, shoes and carrying a red plastic lunch case with white handle, I happily set off on my first day. The following morning however, I wasn’t too pleased at the thought of having to go back, thinking that having been once, that would be enough. Wrong. The beginning of my journey towards a meaningful education had begun, albeit with some trepidation.

1.1 Setting the scene
My experience of school and learning mathematics was not particularly positive, and this has had great bearing on my work as a teacher. My philosophy about education and learning involves the desire to continually seek ‘better’ educational practices so that the schooling process is purposeful, meaningful, developmental and life-long. The fact that I am still learning and ‘going to school’ 42 years after that ‘first’ day is evidence of this.

I am not sure when I wanted to be, or even when I became a teacher. I do not recall ever not wanting to be one, and yet I don’t think I suddenly became a teacher upon graduation in 1984. And why, as a generalist primary school teacher, did I become a lover of mathematics learning? I struggled to understand mathematics throughout secondary school, and I had failed VCE (then the Higher School certificate) General Mathematics in 1981. Perhaps I needed to ‘fail’ to allow me to succeed. I am certain however, that very early on in my teaching career, I was ‘moved’ by the mathematics experiences of the students I taught, and I did not want to be intentionally guilty of ineffective practices.

The principal of the school where I taught upon completing my initial three year teaching diploma had a particular interest in mathematics education, and actively supported me to engage in ongoing teacher professional development. I participated in the Reality in Mathematics Education [RIME] (Lowe & Lovitt, 1984) initiative, and was invited by my principal to attend sessions where teachers consulted on the implementation of approaches.
to the teaching and learning of school mathematics. As I started to consider the completion of my fourth year of teacher training, it seemed logical to specialise in mathematics education. I completed a Graduate Diploma in Mathematics Education and then about a year later, commenced a Masters Degree by research. It was at this time that I also started a family and had the opportunity to work as a research assistant part-time at the Mathematics Teaching and Learning Centre, then at the Institute of Catholic Education (Christ Campus), now the Australian Catholic University (St Patrick’s Campus). I worked closely with many well-known mathematics teacher educators on, for example, research into multi-age teaching practices, and the use of open-ended questions (Sullivan & Lilburn, 1997) and problem solving approaches (Lovitt & Clarke, 1988a, 1988b). This research experience was deeply connected to my classroom teaching experience, and visa versa. The strong bond between research and practice was forged and was carried back into the classroom when I returned to teaching in early 2000.

I wanted the students with whom I worked to have positive and meaningful learning experiences that were contrary to my own. Upon reflection, I was an at-risk student. In my most recent teaching position as an upper primary teacher, I decided to completely cover the blackboard with wall carpet. There was little display space in my classroom and covering the blackboard this way meant I had more area on which to display student work. I used adhesive ‘velcro’ dots on the back of student work which could then be placed securely against the wall carpet. The principal of my school was bemused as to how I could “possibly teach mathematics without the use of the blackboard”. I am not sure why the study of mathematics was singled out specifically. It seems the assumption was made that the blackboard is the tool which assists the teacher to communicate mathematics to students. My preference was to work on the floor or seated at tables with the students, talking and discussing with the students and listening to their ideas and strategies. However, late in 2003 there was a flyer in my staff mailbox, about a full-time PhD opportunity at RMIT University. I was interested, and resolved to find out more. Suffice to say, I was fortunate to be able to take this next step in my own learning.

This research, *Constructing paths to multiplicative thinking: breaking down the barriers*, was conducted as part of the *Scaffolding Numeracy in the Middle Years 2003-2006* Project (SNMY). The SNMY Project was funded by the Australian Research Council Linkage scheme and directed by Professor Dianne Siemon at RMIT University. It was focussed on the efficacy of a new assessment guided approach to improving student outcomes in numeracy in Years 4 to 8 and included an APAI (Australian Post-Graduate Award Industry) scholarship. My
role within the SNMY Project was that of the APAI and I was specifically responsible for contributing to the design of suitable learning and assessment tasks, co-managing one of the three clusters or groups of primary and secondary schools, and exploring the possibility of working with at-risk students in Years 5 and 6.

The SNMY Project was prompted by previous research (e.g., Siemon, Virgona, & Corneille, 2001), which identified an 'eight-year' range in student mathematical achievement in any one year level in the middle years of schooling and highlighted the critical importance of finding ways to address the learning needs of at-risk students. This research found that student difficulty in numeracy was primarily characterised by an inability to engage in multiplicative thinking (Siemon & Virgona, 2001). As a consequence, the SNMY Project was aimed at developing and validating an evidence-based Learning and Assessment Framework for Multiplicative Thinking (LAF) and examining the extent to which student learning could be enhanced by more targeted teaching approaches.

The APAI study within the SNMY Project was designed to explore what could be achieved by a targeted intervention program involving a small number of students identified to be at-risk in terms of the LAF. An important component of this study was the identification of students’ strategies and beliefs about mathematics teaching and learning. The strategies were important to identify shifts in understanding. The beliefs were important to help explain any observed changes, given the recognised role of beliefs in students’ mathematical behaviour (e.g., Ernest, 1989; Spangler, 1992).

The SNMY Project and this study were designed to “contribute to a growing body of research” (e.g., Jacob & Willis, 2001; Siemon, 2003a) in relation to the development of multiplicative thinking in middle years students. As indicated above, these projects were prompted by previous research, in particular the findings of the Middle Years Numeracy Research Project [MYNRP] (2001). The MYNRP (2001), was “commissioned to identify and document in broad terms what works and does not work in numeracy teaching in Years 5-9” (Siemon, 2003a, p. 2). The MYNRP found that:

- there was a much difference in students’ performance within schools as between schools, that is, teachers make a difference;
- there was as much difference in student performance within year levels as across year levels 5-9; and that
- the areas most responsible for these significant differences in student performance were place-value, fractions and decimals, and proportional reasoning (Siemon et al., 2001, pp. 3-4).
In addition, the MYNRP reported that a further source of difficulty was their ability to justify their thinking and use mathematical forms of recording. Early diagnosis and intervention were recommended (Siemon, 2003a).

Central to the SNMY Project and the research reported here, is the importance of the development of multiplicative thinking and its significance in student numeracy performance. Multiplicative thinking has been defined as the capacity to work flexibly with concepts, strategies and representations of multiplication and division as they occur in a wide range of contexts (Siemon, 2004b). For example, a child may be given the task of determining the number of tiles needed to cover a larger area (Figure 1.1):

![Figure 1.1](image)

**Figure 1.1** How many grey tiles would be needed to cover the larger rectangle? (Adapted from Simon, 1995)

There are potentially three broad response types:

- ‘make all, count all’, for example, repeatedly tracing around the grey tile to reflect covering the larger rectangle and counting the ‘tiles’ individually by ones;

- some form of doubling (e.g., counting six tiles across, and thinking ‘double six, 12’ (for two rows) and ‘double 12, 24’ for four rows) or skip counting (e.g., 6, 12, 18, 24); and

- visualising the region in terms of columns and rows, for example 4 rows by six columns, thinking ‘4 sixes is 24, so 24 tiles in total’.

These are noted in increasing order of sophistication from additive to multiplicative. Multiplicative thinking would be indicated by the ability to recognise the application of multiplication, knowledge of multiplication facts and the ability to use this relevant to the situation in an efficient way. While additive strategies can lead to solutions, they become inefficient when working with larger whole numbers, rational number (fractions and decimals), algebra and problems involving proportion and ratio. These aspects of mathematics specifically appear with increasing complexity in the middle years mathematics curriculum (Year 5 to 9) and lay the foundation for further study in mathematics. However, these areas have been identified as being difficult for many students at this level (e.g., Clarke, Roche, & Michelle, 2007; Siemon et al., 2001). If reliance
on additive strategies is maintained, then working mathematically in these domains becomes tedious and ineffective.

Thinking multiplicatively is the key to working successfully in the areas outlined above. The diversity of student performance and extent of student difficulty with these areas, highlights the importance of addressing the learning needs of `at-risk’ students in the area of multiplicative thinking, as well as providing support for teachers in understanding multiplicative thinking and dealing with this level of difference in student performance.

The research reported here, *Constructing paths to multiplicative thinking: Breaking down the barriers*, is designed to explore what it would take to address the learning needs of students identified to be at-risk in a timely and effective manner. At-risk in this context, refers to students performing well below the level expected in relation to multiplicative thinking who relied on modelling and count-all strategies to solve relatively simple problems involving small whole numbers. As a consequence, an intervention study working with at-risk Year 6 students was planned and implemented within the overall SNMY Project. The following section outlines the structure of the thesis.

### 1.2 Organisation of thesis

The chapter to follow will explore the literature in relation to student difficulty with mathematics in the middle years, particularly in relation to multiplicative thinking. The first section will explore the background to and significance of this issue in relation to school mathematics. The remaining sections of Chapter 2 will examine the relevant literature in more detail. In particular, research related to the nature of multiplicative thinking and the specific difficulties experienced by students at this level. It will also examine what is known about effective teaching and learning practices in middle years mathematics, the theoretical orientations underpinning current practice, and the nature of teacher knowledge needed to teach effectively. Given the unique nature of students at this level of schooling, the literature on middle years students and the nature of at-riskness will be considered, together with an analysis of reported responses to at-riskness.

Chapter 3 will frame the theoretical rationale for the methodological approach taken by this intervention study. This will be followed by a description of the SNMY Project of which this research is a part. Then the research design will be described and justified and the three phases of the study, that is the Pre-intervention, Intervention and Post-intervention phases, will be detailed. The reader will be introduced to the students involved in the study and the sources of data used to understand their experience of
mathematics learning. Finally, the chapter will document the analyses used to answer the
questions posed by the research.

Chapter 4 will present the results derived from the analysis of the three interviews
administered to all students during the Pre-intervention (Pre-I) and Post-intervention
(Post-I) Phases. This chapter will also report the beginning and end SNMY Project
assessment results for the students involved in this study. The results of the Pre-I and Post-
I phases will be presented in Section 4.1 and 4.2 respectively. Within each, the results
derived from the different data sources will be discussed in turn. That is, the Drawing Task,
the Card Sort, and Multiplicative Task interviews and the SNMY achievement data
respectively.

Chapter 5 will describe and illustrate why the intervention program was successful.
The initial discussion will be framed by the importance of teacher knowledge, but will
acknowledge an under-represented area of the research literature, namely the role of
teacher identity in the teaching and learning of mathematics at this level. Attention will
then turn to the act of teaching, in particular, reinforce that reform oriented approaches
work for students at-risk, but also, to argue that something more is needed in responding
to the specific needs of at-risk middle years students. The critical need for building
effective relationships will be discussed, as without this, the benefits afforded by teacher
knowledge and progressive practice, are less effective. The chapter will conclude with a
metaphor for the dynamic of teaching that acknowledges the inter-relatedness of teacher
knowledge, teaching in action, and relationships.

Chapter 6 will bring the experience of this work to a close by making final
comments in relation to the research questions that framed this study. The chapter will also
acknowledge the limitations of the study with respect to generalisability, and discuss the
recommendations and implications of this research for future research and practice.
CHAPTER 2
LITERATURE REVIEW

In Chapter 1, the need to address the issue of significant under-achievement in mathematics in the middle years was introduced. My own background and experience, and how I came to be interested in the opportunity to work with the SNMY Project was also described. The review of the literature to follow will discuss the issues that surround student difficulty with mathematics in the middle years, particularly in relation to multiplicative thinking. In Victoria, it is generally accepted that the middle years include students from Year 5 through to Year 9, that is, students aged 10 to 15 years (Pendergast & Bahr, 2005). The purpose of this study is to investigate the efficacy of an intervention program designed to meet the needs of at-risk students in the middle years in relation to multiplicative thinking.

The first section will explore the importance and significance of school mathematics. The remaining sections of this chapter will elaborate on the sources of this problem:

- the demanding mathematics content expected at this level evidenced by student’s poor performance associated with multiplicative thinking (2.2)
- pedagogy, given that demanding mathematics demands quality teaching and teacher knowledge, yet procedural learning dominates classroom practice (2.3)
- students in the middle years themselves who, as they undertake transformation into young adulthood, have particular unique needs (2.4), and
- the nature and rationale of responses to at-riskness (2.5).

The chapter will conclude with a summary of the key issues and introduce the research questions (2.6).

2.1 Background

Competence in literacy and numeracy is essential for the pursuit of life-long learning and career opportunities. Studies have shown that individuals, without adequate skills in these areas are at a significant disadvantage in education, employment opportunities, earnings potential, social status and self-esteem. (Victorian Auditor-General’s Office (VAGO), 2009, p. 1)
2.1.1 The numeracy imperative

Major Australian and overseas school policy documents over the last twenty years or so have recognised the need for schooling to prepare individuals for meaningful personal, social, and work lives (e.g., Australian Association of Mathematics Teachers [AAMT], 1997; Australian Education Council, 1990; Department of Education, Training & Youth Affairs [DEETYA], 2000; Ministerial Council of Education, Employment, Training & Youth Affairs [MCEETYA], 2008a, National Research Council, 2002).

The National Statement on Mathematics for Australian Schools (Australian Education Council, 1990) provided schools with more than ‘what’ mathematics to teach. The Statement also recognised principles underpinning school mathematics that took account of attitudinal considerations, the nature of mathematical enquiry, and why mathematics is important for everyone. The study of mathematics held a place beyond school, in recognition of its potential “to enhance our understanding of the world and the quality of our participation in society” (Australian Education Council, 1990, p. 5).

By 1997, numeracy became a national priority, with improved numeracy achievement for all students a major goal. To this end, the Department of Employment, Education, Training and Youth Affairs [DEETYA], with the AAMT, and the Education Department of Western Australia, organised a national conference to focus on making this goal a reality. The resultant report, Numeracy = everyone’s business (AAMT, 1997) made a significant contribution in specifying current and future directions for numeracy education.

A common definition of ‘numeracy’ drawn from AAMT (1997) states that numeracy involves the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic); mathematical thinking and strategies; general thinking skills; and grounded appreciation of context. (p. 15)

This is consistent with a more recent definition (National Curriculum Board, 2009):

Numeracy is the capacity, confidence and disposition to use mathematics to meet the demands of learning, school, home, work, community and civic life. This perspective on numeracy emphasises the key role of applications and utility in learning the discipline of mathematics, and illustrates the way that mathematics contributes to the study of other disciplines. (p. 5)

The focus on numeracy is important as this has meant a shift away from a technique oriented curriculum to one that is ‘personal’ in fostering development of positive attitudes to learning and mathematics, the development of skills and concepts, and the “confidence and competence to make sense of mathematical and scientific arguments in decision-making situations” (Willis, 1990, p. 22).
2.1.2 Mathematics and numeracy

While the terms ‘mathematics’ and ‘numeracy’ have at times been considered synonyms, current policy and research documents acknowledge the important distinctions between the two (Commonwealth of Australia, 2008). For the purpose of this thesis, I am going to use the term mathematics to refer to the school mathematics curriculum, through which numeracy is largely achieved. The purpose of school mathematics, “...is to educate students to be active, thinking citizens, interpreting the world mathematically, and using mathematics to help form their predictions and decisions about personal and financial priorities. Mathematics also enables and enriches study and practice in many other disciplines” (National Curriculum Board, 2009, p. 5).

Over the 10 years spanning 1997 through to 2007, the Victorian government invested $1.2 billion in initiatives designed to improve literacy and numeracy outcomes (VAGO, 2009). During this time, Government and education systems had been committed to improving the educational achievement of all children. For example, in the report titled Numeracy: A priority for all (DETYA, 2000) frequent reference is made to the belief that all students can and should be able to have access to learning in literacy and numeracy, and therefore attain skills to prepare them for present and future life. Mighton’s (2003) book The myth of ability is representative of the belief that all students are capable and “can be led to think mathematically” (p. 5). At that time, Numeracy: A priority for all presented a National Literacy and Numeracy Plan. Among the various elements of the Plan (DETYA, 2000, p. 18), struggling students were recognised and early intervention advocated. This particular issue was targeted by the Commonwealth Government in the Mapping the Territory project (Louden, Chan, Elkins, Greaves, House, et al., 2000) that was commissioned to provide a national picture of how struggling students were supported in their learning in literacy and numeracy in normal school settings, and to identify the successful strategies.

More recently, The Melbourne Declaration on Educational Goals for Young Australians (MCEETYA, 2008a), reinforced the message about ‘numeracy for all’. Two major goals are identified, that “Australian schooling promotes equity and excellence” (p. 7), and that “…all young Australians become successful learners, confident and creative individuals, and active and informed citizens” (p. 7).

International efforts are complementary to our local endeavours. A vision for mathematics education was evident in the National Council of Teachers of Mathematics (NCTM) Principles and standards for school mathematics (2000). These Standards are framed against a perception of an exemplar classroom, where all students have access to quality
teaching and learning experiences, knowledgeable teachers, and a rich curriculum. Currently in the UK, the *Every Child Matters* initiative (Department for Education, 2010) provides a framework for improving outcomes for all students, in particular “to narrow the gap between those who do well and those who do not” (para. 4).

At the same time as this systemic focus on numeracy for all, evidence from research indicated that many students were not learning mathematics effectively. This prompted reflection upon, and reform of the teaching of mathematics, locally and globally (e.g., AAMT, 1997, National Board of Employment, Education & Training [NBEET], 1993; NCTM, 2000). The features of reform efforts will be described in 2.3.3. However, there remains a problem. Efforts to improve numeracy achievement have not resulted in marked improvement in literacy and numeracy across age-groups (VAGO, 2009). Where improvement was evident, this was for students in Prep to Year 2, however, “these were not sustained as they progressed through schooling” (VAGO, 2009, p. 2). This points to a significant problem with numeracy achievement in the middle years. This is critical for the following reason:

> Literacy and numeracy achievement are the strongest predictors of Year 12 completion and, the best transitions (from school) are achieved by those with high levels of numeracy (around 95% have good transitions), ahead of those with high levels of literacy (92%). The higher the level of literacy and numeracy the higher the probability of labour force participation and the lower the probability of unemployment. (Commonwealth of Australia, 2008, p. 1)

Specific research to inform teaching and learning in the middle years has emerged in the last twenty years (e.g., Willis, 1990; Slavin, 1993; Otero, 1999; Hill, Mackay, Russell, & Zbar, 2001). The following section will highlight the particular issues surrounding under-achievement in numeracy for students in the middle years.

### 2.1.3 Numeracy achievement of students in the middle years

Prior to 1994, there had been a dearth of research into middle years numeracy. The 1990’s defined ‘numeracy’, and what this meant in terms of school mathematics, in particular, specifying its role in preparing people for daily life whether in the home, at work or society itself (e.g., Australian Education Council, 1990; AAMT, 1997). The focus on numeracy has allowed research to focus on specific areas of need, resulting in attention being given to numeracy learning in both the early and middle years of schooling.

The Victorian *Early Numeracy Research Project* [ENRP] (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, et al., 2002) addressed the learning needs of children in the early years of schooling and identified a developmental framework in the
form of ‘growth points’ for understanding, student thinking and development, a structured
interview to provide a tool for assessment, and a teacher professional development
program aimed at addressing teacher confidence with and understanding of mathematics
(Clark, 2001). The framework of growth points enabled teachers to better understand the
stages that children undertake, and the interview identified what students knew and were
able to do. The outcome was that teachers were able to provide learning experiences for
their students more closely aligned to identified learning needs.

At this point, there was a lack of specific focus on the middle years, until the
Middle Years Numeracy Research Project [MYNRP] from 1999-2000. This research was
designed to improve the teaching and learning of numeracy in Years 5 to 9. Rich
assessment tasks informed by numeracy benchmarks at Years 5 and 7 in terms of number
sense, measurement sense, and chance and data, were used to assess close to 7000 students
to provide base-line achievement data, then a sample of 20 trial schools were selected to
explore what it takes to improve student numeracy outcomes (Siemon, Virgona, &
Corneille, 2001). The results of this project supported the findings of the ENRP, in that
student learning is improved when information from assessment is used to inform teaching.
However, unique to the MYNRP was the indentification of ‘hot spots’, that is, particular
areas of need. Among those, there were three critical findings relevant to the context of
this thesis.

Dealing with diversity: Results indicated that there is as much difference within years levels as
there is between Years 5 to 9 generally, that is, “in most Year 5 to 9 classes teachers can
and should expect a range of up to 7 school years” (Siemon et al., 2001, p. 3).
The primary reason for this diversity: The enormous range in student achievement largely due to
difficulty with multiplicative thinking.
Early diagnosis and intervention: Early diagnosis and intervention is needed to address student
diversity through the identification of key growth points in numeracy that extend those
identified by the ENRP, and determination of how best to support movement “from one
growth point to the next” (p. 4).

The Scaffolding Numeracy in the Middle Years Project [SMNY] prompted by the
findings of the MYNRP, also reported significant under-achievement of students in Years
4 to 8 (Siemon, Breed, Dole, Izard, & Virgona, 2006). Rich assessment tasks designed to
investigate multiplicative thinking were administered in May, 2004 to just over 1500
students in research schools. Results from this assessment indicated that, for each year level,
each zone of the *Learning and Assessment Framework for Multiplicative Thinking* (LAF) was represented by a proportion of students. This is shown in Table 2.1 below.

Table 2.1

<table>
<thead>
<tr>
<th>LAF Zone</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
<th>Year 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>25.1</td>
<td>11.8</td>
<td>3.4</td>
<td>5.3</td>
<td>3.6</td>
</tr>
<tr>
<td>Zone 2</td>
<td>25.1</td>
<td>18.8</td>
<td>8.3</td>
<td>9.0</td>
<td>4.1</td>
</tr>
<tr>
<td>Zone 3</td>
<td>18.3</td>
<td>19.8</td>
<td>11.1</td>
<td>9.0</td>
<td>8.2</td>
</tr>
<tr>
<td>Zone 4</td>
<td>17.8</td>
<td>20.4</td>
<td>19.3</td>
<td>24.0</td>
<td>22.1</td>
</tr>
<tr>
<td>Zone 5</td>
<td>6.8</td>
<td>9.4</td>
<td>15.5</td>
<td>10.1</td>
<td>15.9</td>
</tr>
<tr>
<td>Zone 6</td>
<td>4.4</td>
<td>10.5</td>
<td>19.9</td>
<td>13.3</td>
<td>19.5</td>
</tr>
<tr>
<td>Zone 7</td>
<td>2.5</td>
<td>8.0</td>
<td>19.6</td>
<td>24.5</td>
<td>19.0</td>
</tr>
<tr>
<td>Zone 8</td>
<td>0.3</td>
<td>1.3</td>
<td>3.1</td>
<td>4.8</td>
<td>7.7</td>
</tr>
</tbody>
</table>

Particularly relevant to this invention study, were the students in Year 5, as it would be from this group that the students would be selected to participate in the intervention prior to the transition to secondary school. Results indicate that 50% of students in Year 5 were located at Zones 1 to 3 of the LAF, 30% of students were located at Zones 4 and 5, and 20% of students were located at Zones 6 to 8. Given the Victorian curriculum (VELS, Victorian Curriculum Assessment Authority, 2007), it would be expected that students in Year 5 would be working at Zone 5, or above. This suggests that a large proportion of students are working well below expected, and therefore could be thought of as at-risk.

The magnitude of the problem is reflected in Australian performance in large scale international tests, such as *Third International Mathematics and Science Study* [TIMSS] (see [http://timss.bc.edu/](http://timss.bc.edu/)) and the *Program for International Student Assessment* [PISA] (see [http://www.pisa.oecd.org](http://www.pisa.oecd.org)). For instance, in the 2002 TIMSS study, 30% of Year 4 students were at or below the Low benchmark in mathematics (Australian Council for Educational Research [ACER], 2003). In terms of mathematical literacy, PISA results in 2006 indicate that 13% of Australian 15 year old students were below the minimum expectation and therefore judged to be at-risk (Masters, 2006). These results serve to reinforce that for a large proportion of students middle schooling is failing, and that this needs to be addressed as a matter of urgency.

So far, this review has highlighted that schools are faced with the task of meeting diverse learning needs of students in the middle years, particularly students who appear to experience significant difficulty. The following section will elaborate on one of the sources of this problem, that is, the demanding content associated with the mathematics expected at this level, in particular, multiplicative thinking.
2.2 Multiplicative Thinking

Mathematics is difficult. It is certainly among the most complex of human endeavours. As a school subject, it is often unmanageable. Much thought has been given over the years to the question of how it can be successfully taught in spite of the difficulty. (Sfard, 2003, p. 353)

There are two critical aspects that arise from Sfard’s statement, one relates to the difficulty of the subject and the other relates to the challenge of teaching it effectively. The latter will be address in section 2.3, however first it is important to explore what students in the middle years have difficulty with in relation to learning mathematics, that is, multiplicative thinking.

In terms of numeracy learning in the middle years, there has been a shift away from an emphasis on a curriculum that is content driven (Clarke, 2003) to a focus on the type of thinking necessary to engage meaningfully and successfully in middle year mathematics (Callingham, 2003; Siemon et al., 2001). The type of thinking referred to above is becoming commonly understood as multiplicative thinking. A definition of multiplicative thinking follows:

That is, thinking that is characterised by (i) a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole numbers, decimals, common fractions, and/or per cent), (ii) an ability to recognise and solve a range of problems involving multiplication and division including direct and indirect proportion, and (iii) the means to communicate this effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms). (Siemon et al., 2006, p. 113)

This section will explore what is meant by multiplicative thinking, then provide an overview of its development, from concepts in isolation, e.g., multiplication and division, and rational number, to more recent explorations of the transition from additive to multiplicative thinking.

2.2.1 What is multiplicative thinking?

Research into multiplicative thinking has emerged specifically over the last decade or so (e.g., Clark & Kamii, 1996; Jacob & Willis, 2001, 2003; Sullivan, Clarke, Cheeseman, & Mulligan, 2001; Siemon et al., 2006) building on earlier work in the ‘80s and ‘90s in relation to the concept of multiplication and its structures, together with children’s understandings (e.g., Vergnaud, 1988, 1994; Steffe, 1994). This earlier work is important as it has helped to establish agreement that multiplication is comprised of a range of interrelated concepts, rather than a single idea, as well as the developmental stages that children come to
understand over time. Vergnaud (1994) refers to this ‘range’ as the multiplicative conceptual field (MCF), that is:

... simultaneously a bulk of situations and a bulk of concepts. A concept is made meaningful through a variety of situations, and different aspects of the same concepts and operations are involved in different situations. At the same time, a situation cannot be analysed with the help of just one concept; at least several concepts are necessary. This is the main reason that researchers should study conceptual fields and not isolated situations or isolated concepts. (p. 46)

The ‘ingredients’ (Vergnaud, 1994) of the MCF include, but are not restricted to, multiplication and division, ratio, proportion, rate and rational numbers.

2.2.2 Understanding multiplication and division

Multiplication and division are complex operations that are a part of, but not restricted to, whole number concepts and operations that dominate the mathematics curriculum (Verschaffel, Greer, & De Corte, 2007). Anghileri and Johnson (1988) summarise the research focus in relation to multiplication and division across four main categories: ways of teaching multiplication and division, their structure and properties, children’s understanding of, and children’s use of algorithms.

Meanings for multiplication and division

Multiplication and division are not single concepts: multiplication has many different meanings, and there are two commonly recognised meanings for division. Because of this diversity, researchers use some form of grouping to frame their discussion (e.g., Anghileri, 1989; Greer, 1992, Nunes & Bryant, 1992, Quintero, 1986). There is no agreed way to classify the meanings for multiplication and division, for instance Nunes and Bryant (1992) acknowledge “a certain amount of controversy here. There is certainly much less agreement about how to classify types of multiplication and division problems than there is about addition and subtraction, and our own classification might not be universally accepted” (p. 142). They organise multiplicative situations under three broad headings, in contrast to Greer (1992), who offers a comprehensive matrix of ten different meanings, and indicates how each of these can be modelled by, multiplication, partition division, and quotition division. What is common across the literature, are meanings that involve:

- equal groups, for instance, “3 children each have 4 oranges. How many oranges do they have altogether” (Greer, 1992, p. 280)
- rate, for instance, “A boat moves 13.9 meters in 3.3 seconds. What is the average speed in meters per second” (Greer, 1992, p. 280)
• scale factor, for instance, “A piece of elastic can be stretched to 3.3 times its original length. What is the length of a piece 4.2 meters long when fully stretched?” (Greer, 1992, p. 280), and

• Cartesian product, for instance, “sets of shorts in 3 different colours and sets of shirts in 4 different colours...how many outfits” (Anghileri, 1989, p. 371).

Arrays are sometimes included as an aspect or meaning of multiplication (e.g., Anghileri, 1989), but there is also increasing evidence to support the importance of the array representation to support students’ understanding of multiplication and division, fractions and area (Barmby, Harries, Higgins, & Suggate, 2009, Outhred & Mitchelmore, 2004).

Arrays allow students to develop a deeper and more flexible understanding of multiplication/division, and to fully appreciate the two-dimensionality of the multiplicative process. … Arrays have enormous potential at the senior primary and secondary school levels to help strengthen students’ multiplicative thinking. (Young-Loveridge, 2005, p. 39)

The array is useful because it offers opportunity to: think about: the binary nature of multiplication, support the commutative and distributive properties, link to the multiplication of fractions (Barmby et al., 2009) and area (Outhred & Mitchelmore, 2004).

In relation to this, Battista (1999) and Battista, Clements, Arnott, Battista, and Barrow (1998) introduce the idea of spatial structuring of rectangular arrays:

Spatial structuring is the mental operation of constructing an organisation or form for an object or set of objects. It determines the object’s nature, shape, or composition by identifying its spatial components, relating and combining these components, and establishing interrelationships between components and the object. (Battista, 1999, p. 171)

The strategic use of arrays and regions will be described in the Chapter 5 in relation to the impact the intervention program had on student learning.

Sources of student difficulty

Students find multiplication and division difficult because of the complexity of this aspect of mathematics. Nunes and Bryant (1996) state that “multiplicative reasoning is a complicated topic because it takes different forms and deals with many different situations, and that means that the empirical research on this topic is complicated too” (p. 143).

Greer (1992) illustrates clearly why multiplication and division are difficult concepts by showing how 3 x 4 can be derived from a range of different multiplicative situations. For instance, problems involving equal groups, rate, multiplicative comparison, and Cartesian product. Anghileri (1989) makes the point that “children must not only
identify a correct procedure relating to the operation but must also decide the roles of the two numbers involved” (p. 372).

The mastery of the MCF (Vergnaud, 1994) requires an understanding of, and capacity to recognise, visualise, model or represent each multiplicative situation. Analysing student responses identifies *theorems-in-action* (Vergnaud, 1988) defined as “mathematical relationships that are taken into account by students when they choose an operation or sequence of operations to solve a problem” (p. 144). Quintero (1986) for instance, asked students (year or age of students not mentioned) to illustrate their understanding of “[a] store sells boxes of candy containing twelve pieces each. Mary bought four boxes of candy. How many pieces of candy did she get?” (p. 35). The students who understood the situation were able to show the relationship between the number of groups (multiplier) and the number in each group (multiplicand) by drawing four equal groups of 12. Students who did not understand the situation either represented the problem context, that is, Mary in the shop, or indicated “one or both of the elements ... but not the relationship between them” (p. 35).

The nature of students’ responses should be a starting point for instruction. Steffe (1994) who states that “[m]athematics learning consists of the accommodations children make in their functioning schemes as a result of their experiences” (p. 11). Quite pointedly, Steffe asserts that “[i]t is a drastic mistake to ignore the child-generated algorithm in favor of the ‘standard’ paper and pencil algorithms being taught in elementary schools” (p. 8).

Therefore, “the essential task at the beginning is not to provide ‘correct ways of doing’ but, rather, to guide children to find ways of operating to reach their goals” (Steffe, 1994, p. 5). What is suggested here is a tension between current school mathematics that focuses on teaching rules and procedures, and meaning making (e.g., Harel & Confrey, 1994; Sfard, 2003; Steffe, 1994). This raises questions in relation to teacher knowledge and pedagogical approaches as a source of difficulty that will be explored later in 2.3. Suffice to say at this point, that current approaches to teaching and learning mathematics involve the development of mathematical understanding (AAMT, 1997, 2006; NCTM, 2000). It is this belief that underpins much of the research to be explored in this and the following sections of this chapter.

### 2.2.3 Rational number

According to Behr, Lesh, Post and Silver (1983) there are at least 6 different meanings for rational number: “part-to-whole comparison, a decimal, a ratio, an indicated division
(quotient), an operator, and a measure of continuous or discrete quantities” (para. 6). Like multiplication and division, rational numbers involve numerous complex ideas.

Rational number concepts are a key aspect of multiplicative thinking (e.g., Behr, Wachsmuth, Post, & Lesh, 1984; Litwiller & Bright, 2002), and it is widely recognised that this is an area of considerable difficulty for students in the middle years (e.g., Behr et al. 1984; Clarke, Roche, & Mitchell, 2007; Siemon et al., 2001). Considerable attention has been given to the development of students’ fraction understanding (Keijzer & Terwell, 2001; Lesh, Behr, & Post, 1987; Smith III, 2002; Streefland, 1991; Taber, 2002).

Smith III (2002) offers a comprehensive description and illustration of how students develop fraction ideas, and summarises this growth broadly across two phases: “(1) Making meaning for fractions by linking quotients to divided quantities and (2) exploring the mathematical properties of fractions as numbers” (p. 7). He identifies what to look out for when working with students, and strongly advocates listening to what children say. Working closely and listening to students is evident in the research in this field.

Taber (2002) investigated students’ understanding of multiplication of fractions, in particular the misconception that multiplication makes bigger. Lesh et al., (1987) present results from a written test and interviews administered to Year 4 through to Year 8 students over a 20 week period. They found conceptual instability was an issue, with students attending to perceptual distracters, such as colour or shading and the way in which fractions were modelled. Keijzer & Terwell, (2001) describe a newly developed program of 30 lessons over the course of a school year for teaching fractions and detail one student’s growth in reasoning ability. The aim of the program was to negotiate and construct meaning, and develop fraction sense, including the language of fractions, the number line, comparing fractions, learning formal fractions. Underpinning the program was a belief in the rejection of instruction based on “meaningless following of rules of calculation” (p. 55).

In particular, teaching experiments and interviews with young students to Year 4 (Behr, Wachsmuth, Post, & Lesh, 1984; Empson, Junk, Dominguez, & Turner, 2005) and students in the middle years (Clarke et al., 2007; Pearn & Stephens, 2004) have identified understandings and misconceptions (Streefland, 1991).

The difficulties that students have with fractions is not surprising given the link to multiplication, as the expression of a coordinated relationship “between two discrete or continuous quantities” (Smith III, 2002, p. 3), and also division of wholes, continuous and discrete, that can be divided or partitioned into parts. Fractions are different from, and arguably more complex (Clarke, 2006) than natural numbers, where one number expressed
as a fraction, for example $\frac{3}{4}$, requires the co-ordinated understanding “that the meaning of the two numerical components is given by their position” (Smith III, 2002, p. 7). Like multiplication, “[m]uch of the confusion in teaching and learning fractions appears to arise from the many different interpretations (constructs), and representations (models)” (Clarke et al., 2007, p. 208). It is of concern that the learning of procedures sans understanding dominates classroom activity (Evans, 2005) and that “[t]his is especially so for the learning of fractions” (Keijzer & Terwell, 2001). The considerable research in this field “has had little impact on state and national curriculum documents and even less impact on classroom practice” (Clarke, 2006). This too, therefore is a reason that students in the middle years find this area so difficult.

**Intuitive models and representations**

The point was made earlier in this section about the importance of children’s informal understandings when building concepts for multiplication and division. This key message is equally relevant when considering children’s knowledge and use of fractions. The research in this field suggests that pedagogy should reflect: value in students’ intuitive models, the construction of meaning; development of number sense, exposure to a variety of contexts and models, the opportunity to talk and show understanding and approaches (Keijzer & Terwell, 2001), as well as recognise that children’s cognitive models grow from their intuitive models (Lamon, 1996). In particular, Mack (2001), who reported on her work with six students in Year 5, encouraged students to solve problems “… in whatever ways were meaningful to them … to communicate verbally their thought processes related to solution strategies…and to ask questions” (p. 273). In doing so, Mack reported that students continually returned to their initial understandings and reinforced that this “leads them to reconsider these understandings at opportune times for growth to occur” (p. 269).

Partitioning and unitising make explicit, and build upon, “the natural inclination to chunk quantities” (Lamon, 2002, p. 80), and has stemmed from the early work of Jere Confrey in relation to splitting. Confrey (1994) describes splitting as “as an action of creating simultaneously multiple versions of an original” (p. 292) and adequately represents a model of multiplication and division that connects directly with contexts students are likely to encounter.

Lamon (1994, 1996) reported on her work with students in Years 4 to 8 and the importance of using of partitioning using physical models and other representations. Partitioning, or creating equal parts, involves sharing to generate quantity, for example, the
sharing of four like pizzas among three people. Unitising involves the conceptualising of the amounts, before, during and after sharing, for example, using the previous example, 12 pieces are generated, giving 1 each plus a third (Lamon, 1996).

Partitioning offers “an operation that plays a role in generating each of those subconstructs” (Lamon, 1996, p. 192), those being, the number of shares (numerator) and the size of the shares (denominator). She concluded that, “[s]tudents need extensive presymbolic experiences involving these conceptual and graphical mechanisms in order to develop a flexible concept of unit and a firm foundation for quantification, to develop the language and imagery needed for multiplicative reasoning …” (p. 192). One such physical model is generated through paper folding.

Empson and Turner (2006) and Turner, Junk, and Empson (2007) have established that paper folding provides opportunity for students to recognise and reason about multiplicative relationships, and share their thinking and consider the ideas of others. They worked with students in Years 1, 3, and 5. An example of one such task is the determination of the sequence of folds required to generate a given number of equal parts. The authors reinforce in particular, that paper folding is not a trivial task (Turner et al., 2007) given that students were themselves able to make “powerful mathematical connections that aid multiplicative thinking” (p. 325). They reported that paper folding tasks presented multiple opportunities to test and revise ideas, that in turn, facilitated a shift away from additive thinking. For instance, thinking that each fold adds a fixed number of parts, to a shift away from additive thinking, evidenced by the following. The mathematical connections that were reported included: the link to multiplication, for instance, knowing 3 times 3 is 9 can be used to create thirds by thirds to give nine parts; the link to arrays, for instance to make 12 parts, based on seeing an image of 3 parts by 4 parts; the relationship between factors and products; and the link to fractions and notion of equivalence. The authors see “tremendous possibilities” (Turner at al., 2007, p. 326) and given diverse learning needs of students in the middle years, they argue that paper folding “may be an especially good task for supporting all students’ participation in communicating, justifying, and debating mathematical ideas” (p. 326). Partitioning strategies facilitated through paper folding experience will feature in Chapter 5 when the intervention program is discussed in detail.
2.2.4 The transition from additive to multiplicative thinking

Interest in the transition from additive to multiplicative thinking has grown. Identifying and understanding the developmental stages has come from syntheses of research (e.g., Jacob & Willis, 2003), individual research (e.g., Clark & Kamii, 1996), or a combination of the two (e.g., Siemon et al., 2006). In some instances, the developmental stages have been presented as hypothetical learning trajectories [HLT] (Simon, 1995) but these are also referred to as learning and assessment frameworks, that offer “mini theories of student learning in particular domains” (Siemon & Breed, 2006, p. 2). For instance, the Early Numeracy Research Project [ENRP] (Clarke, Sullivan, Cheeseman, & Clarke, 2000) identified a learning and assessment framework for particular mathematical areas, such as, counting and place-value, that included the identification of growth points for multiplication and division. In relation to students in the middle years, the SNMY Project (2006) identified a Learning and Assessment Framework for Multiplicative Thinking (LAF). The Project and the LAF will be described in detail in 3.2.1.

Although multiplicative thinking begins in the early years of primary school, the development of this thinking is slow (Clark & Kamii, 1996) and complex. Clark and Kamii (1996) report that 49 percent of fifth grade children demonstrated multiplicative thinking despite curriculum expectations at this level. It is important to note at this point, that Clark and Kamii’s definition of multiplicative thinking is not as expansive as the definition provided at the beginning of 2.2. They defined multiplicative thinking in terms of understanding multiplication as “the making of two kinds of relations” (p. 43) simultaneously. The fact that only 49% of students achieved this by Year 5 is therefore even more alarming.

Anghileri (1989) states that, “Understanding multiplication comes from the ‘unification’ of many schemas so that the child may recognise a mathematical (binary) operation whose application is appropriate for solving and representing a diverse range of tasks” (p. 384). The initial or early stages of the transition from additive thinking to multiplicative thinking is characterised by developmental phases that involve counting and addition (Anghileri, 1989; Clark & Kamii, 1996; Sullivan et al., 2001; Jacob & Willis 2001, 2003). In particular Jacob and Willis (2001) identified five stages, “one-to-one counting, additive composition, many-to-one counting, multiplicative relations and operating on the operator” (p. 306). The growth points for multiplication and division identified by the ENRP (Sullivan et al., 2001) also feature counting:
**Growth Point 0 - Not apparent**
Not yet able to create and count the total of several small groups

**Growth Point 1 – Counting group items as ones**
To find the total in a multiple groups situation, refers to individual items only.

**Growth Point 2 – Modelling multiplication and division (all objects perceived)**
Can successfully determine totals and shares in multiplicative situations by modelling.

**Growth Point 3 – Abstracting multiplication and division**
Can solve multiplicative problems, where objects are not all modelled or perceived. (p. 3)

Key to making the shift away from additive thinking is an abstracting stage where students are capable of solving problems without physical models. Sullivan et al., (2001) suggest that children need to develop appropriate mental models for various multiplicative contexts if they are to make this shift. This can be achieved by “specific activities prompting visualisation of multiplicative situations, broadly defined in groups, and arrays, multiplicative comparisons” (Sullivan et al., 2001, p. 8).

The literature reviewed so far is underpinned by a view of mathematics learning that involves more than doing mathematics, but rather about understanding. Ball (1990) states that “[h]elping students develop this kind of mathematical power depends on the insightful consideration of both content and learners … Figuring out how to help students develop this kind of mathematical knowledge depends on careful analysis on the specific content to be learned: the ideas, procedures, and ways of reasoning” (p. 2). The capacity to do this is reliant on the teacher, which brings into the discussion, teacher knowledge. Knowledge of content is not enough, what is also critical is “understandings of students themselves and how they learn” (Ball, 1990, p. 2). The next section will elaborate on an expanded view of teacher knowledge and current pedagogical practices.

### 2.3 Teaching Mathematics in the Middle Years

Mathematics is also not recognised as an easy subject to learn or to teach. (Commonwealth of Australia, 2008, p. 1)

Current views of teaching mathematics are underpinned by the importance of teaching and learning mathematics with understanding (Fennema & Romberg, 1999). Wood (1995) states that teaching “involves attempting to understand and negotiate meanings through communication” (p. 205). This section seeks to deal what it means to teach mathematics in the middle years. There are five parts to this section, first, perspective on teaching and learning mathematics will be addressed. This will be followed by the critical importance of teacher knowledge, the shift away from traditional teaching practices, and the role of assessment. The final part, will discuss issues peculiar to teaching middle years students.
2.3.1 Perspectives on teaching and learning mathematics

... it is our job to organise didactic situations and to experiment with them, both with the short-term objective of enabling students to develop new competencies and conceptions for immediate use and with the long-term perspective of offering a basis for concepts that will be essential a few years later. (Vergnaud, 1988, p. 143)

The research reported here is situated within a sociocultural view of learning. For example, Palincsar (1998) suggests that from a sociocultural perspective, “learning and development take(s) place in socially and culturally shaped contexts, which are themselves constantly changing” (p. 354). For the purposes of this research, the cultural context is the mathematics classroom and by sociocultural I refer to the interplay of culture and language on learning. This relationship implies an interactionist perspective which views cultural and social dimensions as “intrinsic to the learning of mathematics” (Voigt, 1995, p. 164), or as described by Ernest (1996), “persons in conversation” (p. 344).

The sociocultural view of learning has its roots in the theories of Vygotsky who determined that consciousness was the end product of socialisation and that interaction plays a crucial role in learning (Vygotsky, 1978). This supports Lerman’s view that social factors constitute learning and that learning is about ‘becoming’ (Lerman, 2001). The idea of ‘becoming’ has its basis in Vygotsky’s zone of proximal development (ZPD), where a space is nominally created as the distance between the individual’s ability without social support, and the potential that can be reached with support from the more learned others. For purposes of this research, social support is provided by the teacher and the social milieu is the learning environment. Lerman (2001) contends that interactions in the classroom setting should not be seen as “windows on the mind” (p. 89) but as evidence that discourse acts as a force to impel learners “forward into their increasing participation in mathematical thinking” (p.89). With this social view of learning in mind, it is natural then to locate this research within an ethnography methodology (Romberg, 1992) since the questions posed by the research required detailed documentation of the experience of students and teachers within the setting of the classroom.

Another way of thinking about teaching and learning is from a constructivist perspective (von Glasersfeld, 1987). Simon (1995) states that constructivism is underpinned by the view that, “we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge” (p. 115). In reference to students, Steffe (1994) explains, that:

They can, indeed be told to do something, but they cannot be told to understand. ... It can be summarised simply by saying that any “knowledge” that involves carrying out
actions and operations cannot be instilled ready-made into students or children but must, quite literally be actively built up by them. (Steffe, 1994, p. 4)

What is important to note, is that understanding is not, “built up from received pieces of knowledge” (Ernest, 1996, p. 336). The constructivist view signified a shift away from behaviourist views of learning evident in traditional teaching approaches concerned with transferring knowledge into the minds of students, where instruction included a reliance on texts, completion of pen and paper worksheets, and routine exercises where rules and procedures are followed that focus on the answer (Romberg & Kaput, 1999; von Glasersfeld, 1987).

The constructivist and socio-cultural views of teaching and learning are not in conflict with one another, but rather complementary views (e.g., Clarke, 2005; Cobb, 1994). Clarke (2005) explains that each view “provides coherent accounts and explanations for particular forms of learning in particular settings” (p. 73). These lenses have contributed to the significant body of work that have shaped how we understand growth in mathematical understanding. For instance, from the perspective of the individual, the learning process as described by Pirie and Kieren (1994) that features a lack of boundaries and the importance of ‘folding back’, that is “when a person makes use of current outer layer knowing to inform inner understanding acts, which in turn enable further outer later understanding” (Pirie & Martin, 2000, p. 131). From the perspective of ‘persons in conversation’ to borrow Ernest’s (1996) phrase, theories have emerged that preface the social. For instance, situated learning (Lave & Wenger, 1991) and communities of practice (Wenger, 1999), that positions “learning as social participation...being active participants in the practices of social communities and constructing identities in relation to these communities” (p. 4). To help understand the nature of this sociocultural activity more deeply, Rogoff (1995) offers three planes of analysis: apprenticeship, guided participation, and participatory appropriation. With apprenticeship, the focus is on the group oriented to a particular goal, and not necessarily framed by an “expert-novice dyad” (p. 143). Guided participation is the interpersonal and “stresses the mutual involvement of individuals and their social partners, communicating” (p. 146). Participatory appropriation is about the transformation of understanding.

When learning is viewed from a sociocultural position, it is possible to de-dichotomise teaching and learning. This is evident in Clarke (2005) who points to language as having an influence on how we view teaching and learning. To illustrate, he explains that the Dutch have one word that means both learning and teaching, this being *leren*. He suggests we might reframe the process as “‘teaching/learning’” (p. 74) as a way to
characterise classroom practice as shaped by “complementary actions of teacher and students” (p. 76).

Stigler and Hiebert (1998) state that, “[t]eaching is a complex system created by the interactions of the teacher, the students, the curriculum, the local setting, and other factors that influence what happens in the classroom” (p. 6). The behaviour of the teacher is influenced by their beliefs, knowledge and attitudes (Ernest, 1989). Pajares (1992) states that “(f)ew would argue that the beliefs teacher hold influence their perceptions and judgements, which in turn, affect their behaviour in the classroom” (p. 307). Researchers who have explored this area, have grappled with the scope and meaning of the term beliefs (Barkatsas & Malone, 2005; Leder, Pehkonen, & Torner, 2002b; Pajares, 1992; Wilson & Cooney, 2002), acknowledge tensions between espoused and enacted beliefs (Beswick, 2003; White, 2002), and describe typologies (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997; Barkatsas & Malone, 2005). There is no simple, consistent definition, however, I draw from Ernest’s model of knowledge, beliefs, and attitudes (1989) to describe beliefs as consisting of:

- Conception of the nature of mathematics
- Models of teaching and learning mathematics
- Model of teaching mathematics
- Model of learning mathematics
- Principles of education [sic] (p. 15)

What is relevant for this study is the link between teacher beliefs and teacher effectiveness. Askew et al. (1997) explored teacher beliefs and practices in relation to student attainment. They identified three belief ‘orientations’: connectionist, transmission and discovery and acknowledged that the teacher would not necessarily fit exclusively into one category only. However, they found that the students of connectionist teachers made the greatest gains. These teachers valued student reasoning and emphasised connections within mathematics.

2.3.2 Teacher knowledge

Teacher knowledge in relation to teaching mathematics is vital to student development and learning. Fennema and Franke (1992) remind us that “no one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms and ultimately on what students learn” (p. 147). Pateman (1989) adopted the stance that “it is not possible to engage in teaching mathematics effectively without some clear notion of the nature of mathematics and how that nature inevitably influences the act of teaching” (p. 12). Ma’s (2010) substantive investigation acknowledged the importance of profound teacher
understanding and student learning. To illustrate how teacher knowledge has been conceived, I will draw on two significant contributions in relation to teacher knowledge, that of Shulman (1886, 1987) and more recently, Ball, Thames, and Phelps (2008).

Since the mid-1980s, the place of teacher knowledge in understanding and professionalising the craft of teaching has grown, deepened and strengthened. Prior to this time, there was a strong emphasis on what teachers needed to know, predominantly in relation to subject matter. Shulman (1986) is credited for introducing the concept Pedagogical Content Knowledge (PCK) as a way focussing attention away from only considering the content or subject matter knowledge needed in order to teach. In particular, Shulman specified PCK as a particular form of content knowledge “which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching...that embodies aspects of content most germane to its teachability” (Shulman, 1986, p. 9). The notion of teachability is suggestive of some action or process. Shulman (1987) defined PCK as:

the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. (p. 8)

Shulman (1987) provided greater detail as to where PCK sits in relation to other categories of knowledge, including but not constrained to: content knowledge, pedagogical knowledge, and curriculum knowledge. What is of particular interest at this point, is Shulman’s Model of Pedagogical Reasoning and Action (1987) presented from the perspective of the teacher. Although Shulman does not link this model explicitly to PCK, there is an assumed relationship between the two (Wilkes, 1994). It appears that this was a first attempt to bring diverse aspects of teacher knowledge together with action, in recognition that teaching involves the exchange of ideas.

As we have come to view teaching, it begins with an act of reason, continues with a process of reasoning, culminates in the importance of imparting, eliciting, involving or enticing, and then it is thought about some more until the process can begin again. (Shulman, 1987, p. 13)

Shulman’s model (1987) identified six linked processes, the first two involve the teacher in reasoning about teaching that is yet to occur, the remaining four involve the act of teaching. The first begins with understanding defined in the model as Comprehension. This is followed by Transformation (that includes preparation, representation, selection, and adaptation), Instruction, Evaluation, Reflection, and New Comprehensions. The Instruction process indicates the commencement of the act of teaching. This is not elaborated upon in detail. As the title of this model suggests, these processes are both thought about and enacted.
through teaching. Shulman indicates that the processes are presented in sequence, however “they are not meant to represent a set of fixed stages, phases, or steps. Many of the processes can occur in different order.” (Shulman, 1987, p. 19). Essentially, Shulman started the conversation about teacher knowledge beyond just understanding the content that has continued in education and research circles to the present time.

Ball et al. (2008) acknowledge the influence of Lee Shulman and his colleagues by identifying their work has “been cited in more than 1200 refereed journal articles...the reach of this work, with citations appearing in 125 different journals, in professions ranging from law to nursing to business, and regarding knowledge for teaching students pre-school through to doctoral studies” (p. 392).

Since the later 1980s, considerable attention has been given to the role of teacher knowledge in practice (e.g., Fennema & Franke, 1992). Arising from this continued focus came the phrase mathematics knowledge for teaching (MKfT) as a way to link content, students and pedagogy (Moehr, 2006).

Despite the pervasiveness of the term PCK, Ball et al. (2008) state that PCK remains unspecified, hence their aim to address this over time (e.g., Ball & Bass, 2000; Hill, Rowan, & Ball, 2005, Ball et al., 2008). In contrast to the knowledge base of Shulman (1986, 1987) sourced in scholarship, materials and settings, research, and the wisdom of practice, Ball et al. (2008) focussed on the work of teaching “instead of starting with the curriculum, or with standards for students learning, we study the work that teaching entails” (p. 395). They define teaching as all that a teacher does to support student learning. The result is their model of the domains of mathematical knowledge for teaching. The diagram they present brings some of Shulman’s (1987) categories together with their own. This is shown below in Figure 2.1.
In the context of this study, teachers who understand multiplicative thinking and can distinguish between additive and multiplicative reasoning, are well equipped to help students (Clark & Kamii, 1996; Jacob & Willis, 2001). The ability to effectively communicate underpins these ideas. The Researching Numeracy Teaching Approaches in Primary Schools Project (RMIT University & DEET, 2003) identified twelve teacher scaffolding practices that “appear to be effective in improving student learning outcomes” and that these “essentially describe a range of communicative practices that teachers use to support students’ mathematics learning” (p. 1).

The implication here is that teacher knowledge about mathematics, and the ability to communicate effectively with students, is vital to improving student understanding and performance.

2.3.3 Traditional versus reform approaches
As mentioned briefly in 2.1.2, the shift away from traditional approaches to learning has emerged as the “movement to reform mathematics education that began in the mid-1980’s in response to the documented failure of traditional methods of teaching mathematics” (Battista, 1999, p. 3). The spear-head to this reform is embodied by AAMT (1997) and the NCTM (1995, 2000) which has “called for instructional approaches that move away from
learning by imitation and toward student mathematical understanding that is conceptually based” (Cooney, Sanchez, & Ice, 2001, p. 10).

In Australia, national policy stated that the improvement of teaching and learning of mathematics for all students would be achieved in part, through effective classroom practice (DETYA, 2000) that would feature “work in different contexts, as well as maximising opportunity for pupils to talk and be listened too, to receive feedback, to explain their knowledge, thinking and methods and to suggest alternate ways of tackling problems” (p. 37). This is consistent with the vision for school mathematics in the US (NCTM, 2000) that identifies six guiding principles, three of which relevant to this discussion are specified below:

- **Teaching**: …requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- **Learning**: Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- **Assessment**: … should support the learning of important mathematics and furnish useful information to both teachers and students.

It is no longer acceptable for teachers to spend the “bulk of their class time demonstrating procedures and supervising students while they practice those procedures” (National Research Council, 2002). Learning mathematics involves the integration of understanding, computing, applying, reasoning, and engaging (National Research Council, 2002). To illustrate the difference between traditional approaches and reform approaches, Lampert (2001) and Boaler’s (1997, 1999, 2000, 2002, & 2003) considerable contribution to this area will be highlighted.

I teach by engaging my students with the big ideas of the discipline as they work on problems and discuss the reasonableness of their strategies and solutions. Using problems, I teach them that they all can learn and that they can do it in school. (Lampert, 2001, p. 1)

Take the case of adding two related fractions, that is, $\frac{1}{2} + \frac{1}{6}$. A traditional approach might involve a teacher demonstrating to students how to complete this calculation, then giving them opportunity to practice the procedure with a series of similar questions. This is in contrast to the problem posed by Lampert (2001) that asked, “Some people conjectured on the quiz that $\frac{1}{2} + \frac{1}{6} = \frac{2}{8}$. Do you agree or disagree? Write your reasoning. Try to convince your group of what you think” (p. 341).

Over a three year period, Boaler (2002) examined the experience of teachers and students (aged 13 years at the commencement of the study) at two demographically similar
secondary or high schools in the UK, yet the approach taken by these schools differed greatly.

Amber Hill undertook a traditional approach to teaching via transmissive text-based instruction featuring short, procedural, closed questions. In contrast, Phoenix Park used reform approaches where the curriculum was devised by the teachers over time, and was project-based with an emphasis on open-endedness, problem solving, independence, and motivation. These projects could last 2-3 weeks.

Instruction at Amber Hill was broken up into manageable chunks and was extensively teacher directed. When students encountered difficulty, the students were provided with more structured guidance by the teacher. Discussion about strategies and why these would be implemented was not encouraged and lessons tended to commence with teacher presentations from the board followed by students being set to work. The students relied on following procedures and remembering what to do. Interviews with students revealed negative thought about mathematics, and observations of classroom activity indicated a lack of engagement.

At Phoenix Park, the openness of the projects offered opportunity to meet the individual needs of students. The teachers attended carefully and attentively to the ways in which the students learn and how they communicated. The students reported a positive view of mathematics. The teachers at the project-oriented school believed that all students at all levels could learn and “develop conceptual understanding of the mathematics with which they were engaged” (Boaler, 2002, p. 181). Boaler (2002) concluded that the students learned more effectively at Phoenix Park where the problem solving approach to learning took place:

The students at the two schools had developed a different kind of mathematics knowledge. The Phoenix Park students did not have a greater knowledge of mathematical facts, rules, and procedures, but were more able to use the knowledge they did have in different situations. … The Amber Hill students developed a broad knowledge of mathematical facts, rules, and procedures that they demonstrated in their text-book questions, but they found it difficult to remember these methods over any length of time, and they did no know enough about the different methods to base decisions on when or how to use or adapt them. (p. 104)

Despite call for reform (e.g., AAMT 2006, 2008; NCTM, 2000), middle years student performance in mathematics continues to be of concern. The levelling out of achievement of students in the middle years is well documented (e.g., Siemon et al., 2001; ACER, 2003, Hill & Russell, 1999). Earlier reference was made to student difficulty and poor performance, highlighting the need to assist students ‘at risk’. Research into literacy
and numeracy performance of middle years students, report consistent findings. Siemon et al. (2001) states:

There is a significant ‘dip’ in students numeracy performance from Year 6 to Year 7 which students do not appear to recover from until they reach Year 9. This supports the work of the MYRAD project [literacy] and confirms the need for a quite radical reappraisal of how the transition from primary to secondary school is managed and how learning is organised in the middle years of schooling. (p. 3)

The affective domain, including for example, motivation, engagement, confidence and attitude, are factors when considering student performance in the middle years (Hart & Walker, 1993; Hill et al., 2001; Sagor & Cox, 2004; Siemon et al., 2001). The latter research (Siemon et al., 2001) interviewed students (n=42) in Years 5 to 9 to determine views about their numeracy learning experiences, motivations, and reasons for disengagement. For instance, just over 80% of students indicated that mathematics was not their best subject, and just under 70% of students agreed that mathematics was difficult. However, enjoyment of mathematics was associated with active involvement, experience of success, problem solving, and a supportive relationship with the teacher. The experience of students who had fallen behind was also teased out. In particular, the study reported that these students have the desire to learn and believe that mathematics is important. However, from the students’ point of view, text-based approaches were not viewed as useful, success was closely aligned to engagement, and quality teacher explanations and relationships were “the most critical element in their learning” (Siemon et al., 2001, p. 56).

In terms of teaching multiplication and division, and fractions, the following three issues are raised by researchers: the teaching of algorithms, the important of meaning making, and difficulties particular to the learning of fractions. Each will be discussed in turn.

The teaching of algorithms

The teaching of algorithms is over-emphasised (Anghileri, 2001; Anghileri & Johnson, 1988; Clarke, 2005; Narode, 1993). Anghileri and Johnson (1988) support the recommendation that far less emphasis should be placed on computational algorithms than has traditionally been the case. Narode (1993) suggests this with greater strength and urgency, that algorithms supplant understanding, to the detriment of “flexible and creative thought”. To illustrate, Anghileri (2001) reported that “where the increases in the use of the algorithm were largest ... there were decreases in the number of correct answers” (p. 101). Similarly, Narode, Board and Davenport (1993) tell the story of ‘Jaimie’, a Year 2 student who after a
period of teaching was no longer inclined to offer a range of flexible mental strategies, but followed a single procedure that had been taught instead.

Rather than focus on teaching algorithms, the following advice is gleaned from the literature presented so far, that is, to: build on children’s intuitive approaches (Anghileri 2001; Mulligan & Mitchelmore, 1997), work with a range of situations or contexts together with materials so that the “language for discussion is acquired and an understanding of the concept will grow” (Anghileri & Johnson, 1988, p. 158), and “[i]f children are to retain their confidence in their invented strategies and see mathematical problem-solving as a progression in procedures, it is necessary that any more procedural algorithm is at least deferred” (p. 102).

**Meaning making**

The importance of student meaning making and understanding (e.g., Anghileri & Johnson, 1988; Quintero, 1986) and the importance of building on student’s intuitive models (Mulligan & Mitchelmore, 1996 & 1997) or schemas (Anghileri, 1989) as starting points for teaching is well recognised. An intuitive model is defined as “an internal mental structure corresponding to a class of calculation strategies” (Mulligan & Mitchelmore, 1997, p. 309). Mulligan and Mitchelmore (1997) reported the results of interviews with a sample of 60 students in Years 2 and 3. Correct responses identified three intuitive models for multiplication, that is, direct counting, repeated addition, and the multiplication operation, and one intuitive model for division, this being repeated subtraction.

Similarly, Anghileri (1989) uses the term *schema* to describe the strategies students use to solve a range of multiplication tasks. Students between the ages of 4 and 12 were interviewed. Anghileri reported that counting in some form dominated the students’ solution strategies, namely counting by ones, rhythmic counting, and skip counting. She noted that multiplication facts were used infrequently “even among older children who continued to use number patterns when the application of a single known fact would present a more economical solution” (p. 380). Children’s intuitive models are understood as having an influence on understanding complex multiplicative structures, and therefore, children’s informal understanding of multiplication and division need to be accounted for during learning opportunities (Mulligan & Mitchelmore, 1996 & 1997), however this is under-represented in the curriculum (Mulligan & Mitchelmore, 1997). Anghileri (2001) states that “[t]eachers need to help pupils to structure written recording of intuitive approaches that children understand, rather than teaching the algorithm” (p. 101).
Battista (1999) and Battista et al. (1998) interviewed students in Years 2 to 5 (aged 7 to 10 years) about their predictions as to the number of small tiles required to cover a certain larger area. These authors illustrate three broad levels of sophistication, and describe how to deal with issues students may have in relation to this. The strategies that students used included: counting imaginary or actual squares by ones, in terms of the number of columns and rows (not necessarily systematically), skip counting, repeated addition, or use of known facts. They assert that, in the past, it was assumed that students would naturally understand the row by column structure. It was found however, that the structure of the array needed to be conceived in the mind of the student, and were surprised by the number of students in Year 2 to 5 who had difficulty doing so (Battista, 1999). Therefore,

We cannot assume that students appreciate the row by column structure of arrays. …such structuring is not ‘in’ the arrays—it must be personally constructed by each individual. Consequently, traditional instructional treatments of multiplication and area need to be rethought. If students do not see a row-by-column structure in these arrays, how can using multiplication to enumerate the objects in the arrays, much less using area formula, make any sense to them? (Battista et al., 1998, p. 351)

Rather than assume that student will appreciate the structure of arrays, strategic use will provide the scaffold needed to support imaging (Young-Loveridge, 2005) that in turn will enable students to cross over the ‘abstracting’ barrier, that is, to solve problems without physical models (e.g., Sullivan et al., 2001).

**The difficulties with fractions**

As for the concepts of multiplication and division, it is important to build upon students’ informal knowledge (Mack, 2001) of fractions, use scaffolding practices and appropriate representations, such as paper folding (Empson & Turner, 2006; Turner et al., 2007) and partitioning (Lamon, 1996; 2002; Siemon, 2003b) so that students develop real conceptual understanding.

Despite what is known how students develop fraction ideas and implications for instruction, it is well known that students experience difficulty with fractions, especially those students in the middle years (e.g., Siemon et al., 2001). Pearn and Stephens (2004) state the importance of “appropriate images, actions and language to precede the formal work of fractions” (p. 430). What appears to be an issue that is consistent across research studies over a considerable period of time, is the dominance of whole number thinking (e.g., Clarke et al., 2007; Pearn & Stephens, 2004; Streefland, 1991). For instance,
Streefland’s (1991) long-term teaching experiment with the same group of primary children (Year 4 at the commencement of the study) over two years revealed students:

Making mistakes in honouring the concepts of fractions and ratio and proportion, for instance: \( \frac{1}{2} + \frac{1}{2} = \frac{2}{4} \) or, in other words, finding that \((2,4)\) is ‘equal’ to \((4,6)\) because of the same difference between the numbers. (Such errors are an indication that concrete sources for insight are absolutely indispensable). (p. 3)

Also, interviews with students in Year 4 conducted on 11 occasions during an 18 week teaching experiment (Behr et al., 1984) revealed that “many children gave evidence that they were still struggling to separate their thinking about fractions from their schemas for whole numbers” (p. 334). Whole number thinking also dominated Year 8 student thinking when asked to compare the size of two fractions (Pearn & Stephens, 2004).

Clarke et al. (2007) concluded that “a large, representative group of Victoria Grade 6 students do not generally have a confident understanding” (p. 214) of the range of fraction sub-constructs, such as part-whole, measure, division, and operator, nor the models or representations to illustrate or model these. The authors found that students had difficulty: naming fractions, forming mental images to support calculations, locating fractions on a number line, and recognising the equivalence of, for example, that \( \frac{6}{7} \) is the same as \( 6 ÷ 7 \).

**Summary of approaches to teaching**

So far this review has acknowledged that multiplication, division and fractions are a complex and diverse area of mathematics, and the considerable body of research over the last 30 years has provided insight into what is needed to enable students to learn. Clearly, given far ranging student difficulty, there is an apparent void between this knowledge and what happens in practice. For instance, the difficulties that students experience in relation to learning fractions can be minimised if learning experiences involve “providing students with opportunity to build concepts as they are engaged in mathematical activities that promote understanding” (Clarke et al., 2007, p. 207). Textbooks are also blamed for student misunderstanding, since the focus is often on procedure rather than conceptual understanding (Langrall & Swafford, 2000; Slovin, 2000). There is also the suggestion that the uptake of progressive reform orientation approaches is slow to reach classrooms. For instance, Anderson and Bobis (2005) have begun to explore primary teachers practices in relation to implementation of reform oriented approaches. They concluded that, “the majority of these teachers support reform-oriented teaching approaches that promote...
working mathematically in primary classrooms … careful reading of the open-ended responses suggests that this may not be what is implemented in practice” (Anderson & Bobis, 2005, p. 71).

The complex issues raised also have implications for teacher knowledge and practice. Siemon (2003c) indicates that where teachers have a clear understanding of the development of learning and locating where students 'are at', then “they are better able to make informed decisions about what targets to set and how to achieve them” (p. 5).

Students’ ideas and competencies develop over a long period of time. Teaching students at a particular grade requires that one have a fair idea of the steps that they may or may not have gone through and of the next and ultimate steps one would like them to reach. (Vergnaud, 1988, p. 142)

This is why it is so important to understand the developmental stages children undertake when learning mathematics. The following section will examine what is known about assessment, and the critical importance of accurately identifying student learning needs.

2.3.4 Assessment

In broad terms, assessment is “the process of evaluation through the use of a range of recording instruments” (Griffith & Kowalski, 2010, p. 10). Assessment “should model the mathematical activity that we value” (Clarke, 1995, p. 74). Historically, assessment has been generally conceived and popularly recognised as a way of measuring achievement, frequently in the form of tests or examinations (Clarke, 1996). This is known as assessment of learning. Where assessment is used to identify specific learning needs and target teaching, there is evidence of improved student learning outcomes. This is referred to as assessment for learning. Clarke (1996) captures the overall purposes of assessment to model, monitor and inform. This is consistent with assessment of learning and assessment for learning and the table below brings these ideas together.

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<tr>
<th>Table 2.2</th>
<th>Multiple purposes of assessment</th>
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<td>ASSESSMENT</td>
<td>Model</td>
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<td>Assessment of learning</td>
<td>• Curriculum</td>
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<td>Assessment for learning</td>
<td>• Exemplary practice</td>
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34
Stiggins (2002) argues for balance between assessment of and assessment for learning. There has been a general tension between quality control (assessment of learning) and quality assurance (assessment for learning) with a shift in focus from teaching to learning (Leahy et al., 2005). Each aspect of assessment will be discussed in turn.

**Assessment OF learning**

There has been and continues to be political and social interest in measuring the standard of student achievement. In Australia, National tests such as the National Assessment Program – Literacy and Numeracy [NAPLAN] (MCEETYA, 2008b) provide information about the standard of literacy and numeracy achievement in Years, 3, 5, 7, and 9. Results influence policy as well as assist schools and teachers to plan for both school and student improvement. It is recognised that how well students participate in school mathematics is a strong determinant of entry into further education and employment. Clarke (1996) recognises that

> It is through assessment that society provides or withholds its rewards…assessment, which has been used as a competency measure by many professions and education systems. Where statewide competency testing has been employed, a mathematics component is standard. (p. 329)

To determine global trends, there are international tests such as Third International Mathematics and Science Study [TIMSS] (see [http://timss.bc.edu/](http://timss.bc.edu/)) and the Program for International Student Assessment [PISA] (see [http://www.pisa.oecd.org](http://www.pisa.oecd.org)) that assess knowledge and application extensively across countries.

Test results can indicate levels of student achievement overall, and are intended to assist teachers evaluate the effectiveness of their teaching and systems review their curriculum expectations (Leahy, Lyon, Thompson, & Wiliam, 2005). In any given community, whether local or international, where school mathematics is valued, assessment of learning serves as a mechanism for accountability, that is, teachers are accountable for their teaching, schools are accountable to government, and government is accountable to the community (Clarke, 1996). However, this is only one aspect of why we assess.

**Assessment FOR learning**

The other much more common but less public aspect of assessment, are the practices teachers use to identify student learning needs, evaluate their own teaching in relation to student learning, and provide constructive feedback to students. Known as assessment for
learning, it refers to teachers and students who gather evidence about student understanding to inform future learning. Black and Wiliam (1999) summarise key research in this area, and identify 5 key factors for improved student learning:

- Provision of effective feedback to students
- Active involvement of students in their own learning and assessment
- Adjustment of teaching based on what assessments reveal
- Recognition that assessment influences student motivation and self-esteem
- The need for students to understand for themselves how to improve.

Reporting test scores to students does not lead to student learning (Wiliam, 2005). In particular, there is significant negative impact on low achieving students who interpret this to mean that they lack ability and therefore are not able to learn (Black & Wiliam, 1998). Teacher comments are critical to student learning when framed in such a way that they enable students to move forward. However, when scores and comments are communicated to students this negates the benefit afforded by the comments alone (Wiliam, 2005). Of course use of assessment for learning strategies will look different in different classrooms. However what is most likely to be consistent is that:

Students will be thinking more than they will be trying to remember something, they will believe that by working hard, they get cleverer, they will understand what they are working towards and will know how they are progressing. (Wiliam, 2005, p. 34)

More recently, with significant shifts in how learning is viewed, assessment is defined more broadly. Current views of assessment reflect that learning is both personally constructed and shaped by social practices, for example, students are more inclined to construct a response and work with others, as opposed to only relying on recall or remembering some fact or procedure (Clarke, 1996). This view is reflected in the *Standards for Excellence in Teaching Mathematics in Australian Schools* (AAMT, 2006). In relation to assessment, the *Standards* (AAMT, 2006) state that:

Excellent teachers of mathematics regularly assess and report student learning outcomes, both cognitive and affective, with respect to skills, content, processes and attitudes. They use a range of assessment strategies that are fair, inclusive and appropriate to both the students and the learning context. They maintain on-going, informative records of student learning outcomes that are used to map student progress and to plan appropriate future learning experiences. The excellent teacher of mathematics provides constructive, purposeful and timely feedback to students and their parents, and the school authorities, as required. (p. 4)

The extract above foregrounds student learning. Assessment for learning starts at the micro-level, that of the student and/or the school, and impacts significantly on teaching.
2.3.5 Teaching students in the middle years

Interest in middle years issues peaked around the late 1990s and into the 21st century (e.g., Barber, 1999; Earl, 2000) in response to research that highlighted student underachievement with literacy and numeracy. At the time, Barber (1999) stated that the middle years “is the key emerging question in the education revolution” (p. 1). He highlighted key deficits, in particular, that:

- the “foundations of learning are not provided properly” (p. 2)
- the transition from primary to secondary schooling is “little short of disastrous” (p. 2)
- curriculum, teaching and school organisation are inadequate, and
- students are “disaffected” and “bored” (p. 3).

In particular, the literature covers the nature of middle schooling (Luke, Elkins, Weir, Land, Carrington et al., 2003; Cuttance & Stokes, 2001; National Board of Education, Employment and Training [NBEET], 1992), transformation and change (Barber, 1999; Earl, 2000), ways to challenge and engage students (Callingham, 2003; Clarke, 2003; Otero, 1999; Vale, 1999), and exploratory programs (Anfara & Brown, 2000).

Chadbourne and Pendergast’s (2005) philosophy of middle school is “characterised as predominantly progressive, constructivist, outcomes-based, community oriented, developmentally responsive, student-centred, liberal reformist and contextually mediated – but neither completely nor exclusively so” (p. 22).

Jackson and Davis (2000) suggest that the heart of middle years education is about excellence, where all students learn to use their minds well, they learn through communication and collaboration, undertake meaningful problem solving, and are part of instructional programs where quality relationships between students and teachers are established.

Let us be clear. The main purpose of middle grades education is to promote young adolescents’ intellectual development. It is to enable every student to think creatively, to identify and solve meaningful problems, to communicate and work well with others, and to develop the base of factual knowledge and skills that is the essential foundation of these ‘higher order’ capacities. As they develop these capacities, every young adolescent should be able to meet or exceed high academic standards. Closely related goals are to help all students develop the capacity to lead healthy lives, physically and mentally; to become caring, compassionate, and tolerant individuals; and to become active, contributing citizens of the United States and the world. (Jackson & Davis, 2000, pp. 10-11)
Opportunity for learning needs to harness motivation and ‘connectedness’. The Organisation for Economic Co-operation and Development’s [OECD] International Program for International Student Assessment (PISA) found that student achievement is linked to the student taking a proactive role in the learning process, and in particular, looked at the role of motivation and strategies that students employ (Artlet, Baumert, Julius-McElvany, & Peschar, 2003).

Otero (1999) believes we need to ‘get the attention’ of our students and then usefully ‘focus’ this attention. This entails making links between effective teaching and learning and connecting learning with the social context in which we live (NBEET, 1993; Otero, 1999; Cuttance & Stokes, 2001). Learning and teaching are social activities (Stigler & Hiebert, 1998; Otero, 1999; Earl, 2000) and middle years students, as social beings, necessitate the connection of learning with social practices.

In terms of the teaching and learning of middle years numeracy, Siemon and Stephens (2001) identified a range of issues that included dealing with the degree of difference in student ability and connectedness, demanding content in relation to mathematics, use of “procedural, ‘surface’ based approaches” (p. 190) with the aim of getting the answer, and the lack of classroom dialogue that focussed on student thinking and reasoning. Therefore, the teaching and learning of numeracy in the middle years must address these. By way of example, Clarke (2003) shared some personal strategies under the following headings:

- Valuing and building upon students’ methods of solving problems (p. 100)
- Asking higher order questions (p. 101)
- Offering students choice and openness in mathematics activities and assessment (p. 102)
- Making hard decisions about less important content (p. 104)
- Building an element of challenge and excitement to enrich traditional content (p. 104)
- Recognising the effects of ability grouping mathematics (p. 105), and
- Developing and using rich assessment tasks (p. 106).

Pendergast (2005) suggests that the middle years have “come of age” (p. 10) given university commitment to the provision of specific middle years courses as part of their teacher education programs, as well as teacher textbooks on the subject (e.g., Bishop & Pflaum, 2005; Pendergast & Bahr, 2005). Yet, Jackson and Davis (2000) suggest while school organisational change is evident, “relatively little has changed at the core of most students’ school experience: curriculum, assessment and instruction” (p. 5). Australia participated in the TIMSS 1999 video study, a cross country investigation into Year 8
mathematics teaching practices (Hollingsworth, Lokan, & McCrae, 2003). The countries involved were Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland, and the United States. While there were consistencies across the countries who participated, 91% of Australian classroom lessons used a textbook or worksheet.

This advice about teaching middle years students is consistent with reform approaches to teaching and learning described in the previous section. The capacity for teachers to implement effective programs correlates positively with teacher knowledge (Ma, 2010). The following section will discuss the literature in relation to students in the middle years, particularly those at-risk.

2.4 Middle Years Students

Early adolescence has been described as “a fascinating period of rapid physical, intellectual, and social change” (Jackson & Davis, 2000, p. 6). Documents in relation to the middle years make frequent reference to adolescence as: (i) a time of physical and emotional change, (ii) growth towards greater independence and sense of identity, (iii) acceptance into the peer group, with specific shift from a home and parent focus, to that of peers and the community at large, and (iv) a development in thinking which becomes more reflective and abstract (Earl, 2000; NBEET, 1993; Pendergast & Bahr, 2005).

A specific youth culture has been identified indicative of the middle years (NBEET, 1992). Adolescents are ‘into’ building strong relationships with each other, music, television, mobile phones and the internet; fashion and money and ‘fitting in’ with the peer group. On a physical level, they are changing and developing into young adults. I only need to engage with my own teenage children to observe these issues in action. These characteristics impact on schooling and students’ capacity to learn, and raise issues of self-esteem, engagement, truancy, effective learning behaviours, diversity in student achievement and school attendance (NBEET, 1992). For instance, Earl (2000) describes the disparity in adolescent attention span: a short attention span for “many activities but (adolescents) are capable of concentrating on topics that interest them for long periods of time” (p. 6). Earlier in 2.1.3 the literature in relation to middle years student identified significant under-achievement in mathematics. Taken together, at-riskness and early adolescence, raise issues in relation to access and equity (Bishop & Forgasz, 2007) for all students. Local and international policy documents reinforce the message of ‘numeracy for all’ (see earlier 2.1.1), ultimately for the benefit of students’ participation in society.

Therefore, achievement is not the only purpose of schooling. Noddings (1988) states that
“moral education has long been and should continue to be a primary concern of educational institutions” (p. 228). She suggests that this is achieved through an orientation of caring in teaching. This is echoed by van Manen and Li (2002):

The pedagogical task of teaching seems to require qualities such as being able to really listen to children, having patience, being receptive, being in touch with students, understand how they experience things, caring for the young people with whom teachers share so much in their classroom life. (p. 224)

In addition to the academic needs of students, to be able to respond in this way, the psychological needs of students must also be met (e.g., Sagor & Cox, 2004). Sagor and Cox (2004) identify the basic psychological needs of at-risk students in terms of five “essential feelings” referred to as CBUPO (p. 5):

- Competence, through role of student as learner
- Belonging, determined by acceptance into the group
- Usefulness, through meaningful service
- Potency, through self-belief, and
- Optimism, through a positive life view based on previous experience.

The point is made also, that these are the needs of everyone, “that all humans want and need [is] to feel CBUPOs and feel it each and every day” (Sagor & Cox, 2004, p. 298). If we consider the at-risk student in their role of learner, they would be seen to be not very ‘competent’, and they know and feel it too. Immediately, the sense of CBUPO is compromised.

Research interest into the affective domain and student learning in mathematics “have always been central to the goals of mathematics education” (McLeod, 1994, p. 637). Mandler (1989) uses the terms affect and emotion interchangeably, and use is also made of the terms beliefs (e.g., Leder, Pehkonen, & Torner, 2002a; Shoenfeld, 1989), conceptions (Clarke, 1985), and perceptions (e.g., McDonough, 2002). Research in this field is important because of the recognition “that both the cognitive and affective aspects of learning are present when students construct mathematical understandings, that focussing on one without the other limits what teachers and researchers know about the teaching and learning of mathematics” (Hart & Walker, 1993, p. 23). The importance of effect is not only recognised in research, but the standards of professional organisations, such as NCTM (2000) and AAMT (2006). For instance, in the domain of professional knowledge AAMT (2006) confirms that excellent teachers of mathematics know “how confident [students] feel about learning mathematics’ (p. 2) and assessment involves “both cognitive and affective, with respect to skills, content, processes, and attitudes” (p. 3).
Studies have been conducted to find out what students think about learning mathematics, either as part of large scale studies (e.g., Siemon et al., 2001), systemic assessment (e.g., Hart & Walker, 1993), or research specifically designed to do so (e.g., McDonough, 2002; Stodolsky, Salk, & Glaessner, 1991). Stodolsky et al. (1991) interviewed 60 students in Year 5 over a two year period about their general views about mathematics and social studies. The interviews included students’ definition of the subject, their attitude to it, how they thought they learned the subject, and its accessibility. Given the focus of this intervention study, I will constrain the discussion to findings in mathematics. Findings suggest that students generally viewed mathematics as involving arithmetic operations with number. In terms of learning, students reported doing problems from the blackboard, worksheet or textbook. While mathematics was viewed as most important when ranked against other school subjects, it was not as well liked. Instances of dislike in mathematics were associated with “feelings of difficulty, failure, frustration and anxiety” (Stodolsky et al., 1991, p. 103). The importance of mathematics is consistent with the findings of the Middle Years Numeracy Research Project (Siemon et al., 2001) ten years later. In this study, middle years students, particularly those at-risk, “expect school mathematics to equip them for the future” (Siemon et al., 2001, p. 55), however the quality of teacher explanations is viewed as one of the most important factors in their learning. Teacher explanations will be addressed later in 5.2.1. The next section will define at-riskness, then consider the underlying theories of at-riskness.

2.4.1 What is at-riskness?
Sagor and Cox (2004) draw upon the work of Arthur Pearl (1972), to define ‘at-risk’ as follows:

Any child who is unlikely to graduate, on schedule, with both the skills and self-esteem necessary to exercise meaningful options in the areas of work, leisure, culture, civic affairs, and inter/intra personal relationships. (p. 1)

This definition is broad in that it is designed to apply across a range of contexts. Other terms or phrases associated with at-risk students are *underperforming students* (e.g., Commonwealth of Australia, 2008), *struggling students* (e.g., Adams, Girando, Gough, Palmer, & Sutton, 2000) and *vulnerable students* (e.g., Gervasoni, 2005). In terms of learning school mathematics, Groves, Mousley and Forgasz (2006) define students at-risk as “those identified as being in need of remediation in their numeracy learning, that is, their skills and knowledge are below expected of children of their age or at their year level” (p. 81). At this point, having mentioned the term remediation, it is sensible to highlight the difference
between this and intervention. Historically, remediation has been concerned with remedying faults, whereas intervention requires a cycle of activities from observations which lead to insights about the status of an individuals' knowledge that in turn need to be probed so as to be verified or amended before being built on to provide a means to new and deeper understanding. (Booker, 1999, p. 19)

In other words, remedying faults or deficits, that is, what is not known or is missing, versus building on from what is known.

2.4.2 Ways of thinking about at-riskness

The manner in which at-riskness is dealt with, is dependent on the perspective used to understand the cause or reason. For clarity and consistency I draw from the work of Sagor and Cox (2004) to illustrate at-riskness in physical, cultural, psychological, and cognitive terms.

There are two dominant theories of at-riskness summarised by Sagor & Cox (2004). The first is that the cause of at-riskness resides with the student and/or their environment. This is referred to as a clinical pathology. Within this perspective there are two models that connect with remediation as defined above:

- The medical model, where at-riskness exists because something is wrong with the student (which may or may not have been inherited), and a
- Cultural/environmental deficit, where at-riskness exists because the student has “caught a condition from their family or through interaction with their immediate environment” (Sagor & Cox, 2004, p. 18).

The second theory is that the cause of at-riskness is the result of an alterable institutional deficiency known as an institutional pathology. The two main institutions to which students belong, are family and school. The contention here is that a student is at-risk because the institution has provided improper or inappropriate treatment. These perspectives are not mutually exclusive, as environmental factors have an impact in both the clinical and institutional conditions.

For the purposes of the research reported here that focuses on at-risk students and their mathematics learning, it is now sensible to identify the theory to which it most closely relates. Westwood (1999) offers that:

In general it is not very useful to focus on so-called deficits with the learner when seeking to improve learning. It is of more value to improve the quality of instruction. (p. 216)
Therefore this study attends to the intervention offered by institutional pathology, in this case, schools. The following section describes some of the significant intervention strategies that have been implemented in schools to address the learning needs of at-risk students in mathematics.

### 2.5 Responses to At-riskness

Meeting the needs of at-risk students is recognised at the global and local levels as a priority for schooling (e.g., Louden et al., 2000; National Research Council, 2002). In meeting the needs of struggling students, Adams et al. (2000) suggest that schools need to be comfortable places for mixing with others, and engaging with worthwhile ideas, where students have access to frequent feedback. Schools should know their students as people, have classes that are safe, and celebrate errors that turn into learning.

In simple terms, when at-riskness is addressed in some way this is referred to as an ‘intervention’ or ‘remediation’. The table below (2.3) summarises the general processes for each of the three models.

<table>
<thead>
<tr>
<th>Medical model</th>
<th>Environmental /cultural deficit</th>
<th>Institutional pathology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the defect</td>
<td>Identify missing experience and or skills</td>
<td>Organisational practices produce differential treatment</td>
</tr>
<tr>
<td>Treat/cure the defect</td>
<td>Provide experiences</td>
<td>Differential treatment places students at risk</td>
</tr>
<tr>
<td>Student no longer at risk</td>
<td>Support the student during transition</td>
<td>Change organisational practices</td>
</tr>
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</table>

There are broadly speaking two ways in which at-riskness is addressed: through organisational means, that is, grouping students into classes according to a given criteria, and through the design and implementation of particular programs. Each response to at-riskness will be discussed in turn.
2.5.1 Ability-based organisational responses

Where specific programs are not implemented in schools (primary or secondary), to manage the range of ability in any one year level, including those at-risk, students are grouped or classed according to ability. The terms associated with this approach are ‘streaming’, ‘tracking’, and ‘setting’. This practice has been associated particularly with the study of mathematics and the subject of considerable research and debate (e.g., Bishop & Forgasz, 2007; Hallam, 2002; Wiliam & Bartholomew, 2004). Boaler (1997) offers historical developments of this approach dating back to the 1950s.

Linchevski and Kutscher (1998) suggest there are in essence two reasons schools adopt this approach. First, ability grouping is justified on the basis of the need to adapt “class content, pace, and teaching methods to students functioning on different levels” (p. 533). Second, in relation to the teaching of mathematics, this subject in particular is perceived to be more linear, and therefore difficult to teach to students of different abilities. Either way, it appears that this approach is selected for organisational and managerial reasons, as opposed to being grounded in effective practice.

Clarke & Clarke (2008) draw upon their lifetime of work as teachers, teacher educators and providers of teacher professional development, and conclude,

 Our observations, conversations with teachers and students, and our reading of the research literature have convinced us that a major impediment to the mathematical learning of students and their beliefs about themselves as mathematical thinkers is the widespread practice of ability grouping in mathematics. (para. 1)

This is consistent with earlier work of Wiliam and Bartholomew (2004). These authors state that in relation to achievement in mathematics it is not the school that makes the difference but the “set (group) you get put into” (Wiliam & Bartholomew, 2004, p. 10). Educators who have researched and/or synthesised the research in this area conclude that ability grouping or streaming does not impact positively on achievement for all students. In fact, it is agreed that the result is an increase in the gap between the high and low achieving students, and that there are gains for some students in the high or top groups (Boaler, 1997; Linchevski & Kutscher, 1998; Wiliam & Bartholomew, 2004). The Commonwealth of Australia (2008) make the following recommendation”

Recommendation 9:
That the use of ability grouping across classes in primary and junior secondary schooling be discouraged given the evidence that it contributes to negative learning and attitudinal outcomes for less well achieving students and yield little positive benefit for others, thus risking our human capital goals. [sic] (p. 49)
2.5.2 Intervention programs

Doig (2001) in his summary of Australian performance, practices, and programs in relation to mathematics learning in schools, generalises that intervention programs on the whole, have two components. They evolve on the basis of an identification phase, followed by a teaching phase. The nature of this teaching is derived from a research (literature and/or evidence) base designed to meet the individual needs of students.

In recent times, Australian intervention programs have focused attention on at-risk students in the early years of schooling. The basis for this is belief that the earlier the intervention, the more effective it will be. Such programs or initiatives include:

- **Year 2 Diagnostic Net** (Queensland) (QLD Department of Education and Training, 2010)
- **Early Years Numeracy** (Victoria) (DEECD, Clarke et al., 2000)
- **Mathematics Intervention** (Victoria) (Pearn, 1998), and
- **Maths Recovery Program** (New South Wales) (Wright, 1994).

Programs for students in the middle years of schooling exist but to a lesser extent. The **Building Accuracy and Speed in Core Mathematics program** [BASICS] (Byers, 2009) is one example. This program has been significantly influenced by the **QuickSmart program** (Graham, Pegg, Bellert, & Thomas, 2004).

Generally, there are two ways the learning needs of those students identified as at-risk have been addressed: withdrawal programs and within class programs. Many of the programs focus on students in the early years of school (Prep to Year 2) (e.g., Gervasoni, 2005; Pearn, 1998) and the later early years (Years 3 and 4) (e.g., Wright, Ellemor-Collins & Lewis, 2007). To a lesser extent there are some programs that work with students in the middle years (Year 5 to Year 9) (e.g., Byers, 2009; Graham et al., 2004).

Arising from the ENRP, came the **Extending Mathematical Understanding** (EMU) intervention program (Gervasoni, 2005). The ENRP framework of growth points was used to identify students “at risk of poor learning outcomes in mathematics” (Gervasoni, 2005, p. 34). The use of developmental frameworks is a common way for identifying students at-risk (Groves et al., 2006). The EMU intervention program focuses on students in the early years (Year 1 and 2) on the basis that the sooner the intervention, the greater the opportunity for the students to improve.

This age group and justification for early intervention is consistent with the earlier work of Pearn (1998) who developed the **Mathematics Intervention** program. Mathematics Intervention “features elements of both Reading Recovery (Clay, 1987) and Mathematics
Recovery (Wright, 1996) and offers students the chance to experience success” (para. 2). EMU involves students in Year 1 and 2, and specialist teachers who have participated in the associated teacher professional development. Underpinning the program is a constructivist perspective on learning, and accordingly problem solving and reflection was an important and regular part of the EMU sessions. In terms of mathematics content, number concepts that include counting, place value and the four processes are the focus. The program itself does not prescribe what to do, when, but is designed to be flexible so as to respond to individual student learning needs.

The Numeracy Intervention Research Project (NIRP) focusing on students in Year 3 and 4 is designed around the provision of pedagogical tools for intervention and the development of an instructional framework (Wright et al., 2007). The QuickSmart Program (Graham et al., 2004) follows a process of assessment, diagnosis, and remediation. Intervention programs have been influenced by and build upon:

- existing research, for example, the EMU program arose out of Gervasoni’s doctoral work as part of the ENRP (Clarke, 2001; Clarke et al., 2000);
- other programs, for example, Pearn’s (1998) Mathematics Intervention was influenced by “elements of both Reading Recovery (Clay, 1987) and Mathematics Recovery (Wright, 1996)” (para. 2);
- theories of learning, such as a constructivist perspective (Gervasoni, 2002); and
- research methodology, for example, the NIRP (Wright et al., 2007) which “adopted a methodology based on design research (Cobb, 2003; Gravemeijer, 1994)” (Wright et al., 2007).

The intervention programs largely withdraw students from the usual classroom settings, either in pairs (QuickSmart, Graham et al., 2004), or in small groups of no more than three (Gervasoni, 2005; Pearn, 1998). At this stage the documentation about the Targeted Early Numeracy Intervention Program [TEN] (NSW Department of Education, 2009) is a little unclear about the intent to withdraw students from their classrooms. In addition to working closely with at-risk students, the programs also:

- Have a teacher professional development and support component, where specially trained facilitators are in place to work with regular classroom teachers (Gervasoni, 2005; Pearn, 1998; NSW Department of Education, 2009), and
- Focus on aspects of Number, for example, addition and subtraction (TEN), automaticity and recall of basic facts (BASICS, QuickSmart), counting
(Mathematics Intervention, EMU), place value (NIRP, EMU), the four process (EMU, NIRP).

Program sessions tend to operate for no longer than half an hour (Pearn 1998, Gervasoni, 2005). However, TEN at this stage is looking at intense 10 minute blocks.

In summary, most of the early years intervention programs look to solid research about effective learning of numeracy, value frequent, meaningful interaction and dialogue between students and teachers, encouragement of diverse solutions, use of games, materials and models, and reflection. Students are often withdrawn from their classroom to work in pairs or in small groups. The tendency for intervention in the middle years is a linear sequential model for instruction. For instance, there are 3 levels of instruction for BASICS (Byers, 2009). At level 1, instruction focuses on basic rules, skills and concepts designed to enhance retention, recall and automaticity; at level 2, problem solving skills and strategies are developed, then at level 3, a hands-on enquiry-based approach with students working in small groups.

### 2.6 Summary

Recognition of the need and commitment to doing something meaningful for middle years students falling behind in mathematics is strong. The enormous range in student achievement in the middle years is well documented and largely due to difficult content in relation to multiplicative thinking. The capacity to address the needs of at-risk students in the middle years involves quality teacher knowledge and effective, progressive pedagogical practice. In addition to meeting the learning needs of students in relation to mathematics, the psychological needs are also critical. What middle years students think and feel is an important consideration in determining the best way forward. In 2006, ‘students at-risk’ was identified as a gap in Australian primary numeracy research (Groves et al., 2006). However, despite the various programs and initiatives in place to date, “there are no intervention programs to sustain literacy or numeracy support through the middle years of schooling for individual students in need” (VAGO, 2010, p. 45). The research reported here is one such study designed to address this gap.

This intervention study was guided by the following research questions:

- To what extent does an intervention program closely aligned to learning needs impact student outcomes in relation to multiplicative thinking?
- What strategies are used by at-risk students to solve tasks involving multiplicative thinking and to what extent are these impacted by the intervention program?
• What do at-risk students believe about learning mathematics and how are these beliefs impacted by their participation in the intervention program?
CHAPTER 3
METHODOLOGY

The research reported here was concerned with addressing the mathematics learning needs of at-risk Year 5 and 6 students. As previously stated, study was supported by an Australian Post-graduate Award Industry (APAI) which was included in the funding for the Scaffolding Numeracy in the Middle Years (SNMY) Research Project 2003-2006, an ARC Linkage study on the efficacy of a new assessment-guided approach to improving student numeracy outcomes in Years 4 to 8. This intervention study was designed to investigate the extent to which teaching, closely aligned to the learning needs of at-risk students in the middle years, would impact their achievement in relation to multiplicative thinking. In addition to this, the study was designed to identify the strategies that at-risk students use to solve a range of tasks involving multiplicative thinking, their beliefs about learning mathematics, and the extent to which these beliefs and strategies change as a result of their participation in the intervention.

This chapter will begin with the theoretical rationale for the methodological approach taken by this intervention study. This will be followed by a description of the SNMY Project of which this research is a part. Then the research design will be described and justified and the three phases of the study will be detailed. The reader will be introduced to the students involved in the study and the sources of data used to understand their experience of mathematics learning. Finally, the chapter will document the analyses used to answer the questions posed by the research.

3.1 Theoretical Framework

The intent of this section is to substantiate the methodological approach undertaken in this intervention study. The following premise underpins the research design: that learning is “socially and culturally situated, [and] the design of research studies needs to encompass participation in authentic and purposeful activities” (Pressick-Kilborn, Sainsbury, & Walker, 2005, p. 25).

As indicated in Section 2.3.1, the research reported here assumes a sociocultural view of learning (Lerman, 2001; Pressick-Kilborn et al., 2005; Wertsch, 1995). In the previous chapter, I explored the roots of this view of learning, the relationship between learning and culture, and the significance of language and communication in this
relationship. This is evidenced in this study by working directly with students in their own schools, where students and researcher are active participants in the enterprise. The notion of participation implies an interactionist perspective that asserts cultural and social dimensions as “being intrinsic to the learning of mathematics” (Voigt, 1995, p. 164). This idea of participation is illustrated for me by Stigler and Hiebert (1998), that “teaching in our view, is a cultural activity. It is more like eating family dinners than using the computer” (pp. 4-5). To take this idea further, it is hard to imagine a family dinner happening in silence. This analogy is illustrative of the idea that social factors constitute learning and that learning is about “becoming” (Lerman, 2001).

This implies the need for this research to involve participation in authentic activities in schools with students. Learning environments, as suggested by this participation, are unpredictable. Accordingly the approach needs to be flexible and adaptive to circumstances.

My view of learning resonates with the approach taken by Boaler (2002) described in the previous chapter (2.3.3). This approach will be discussed further in Section 3.4.2 when the Intervention Phase is described in greater detail. With this view of learning in mind, together with the desire to document the experience of at-risk learners in mathematics, the case for an educational ethnography or naturalist inquiry approach is now presented.

3.1.1 Ethnography

Ethnographic research in education has been used to “describe educational settings and contexts, to generate theory, and to evaluate educational programs” (LeCompte & Preissel, 1993, p. 8). This approach toward a better understanding of a social context, complements the view that learning is the result of enculturation into social practices (Pressick-Kilborn et al., 2005). An educational ethnographic method is reflected in my intention to deepen the understanding of multiplicative thinking and identify barriers in student progress towards it. This was undertaken by my regular school visits and the time I spent working with students. The ongoing monitoring of an intervention program, as well as working closely with the students in the context of the classroom, involved communication, interaction and listening carefully to what they had to say.
3.1.2 Design experiments

The research reported here, was initially conceived as a teaching experiment on the grounds that I wanted to experience “first-hand students’ mathematical learning and reasoning” (Steffe & Thompson, 2000, p. 267) as they made the transition from additive to multiplicative thinking. Romberg (1992) provides a useful summary of the approach:

hypotheses are first formed concerning the learning process, a teaching strategy is developed that involves systematic intervention and stimulation of the student’s learning, and both the effectiveness of the teaching strategy and the reasons for its effectiveness are determined. (p. 57)

In recognition of the strengths that the teachers, students and researcher would bring to the research, thought was given to using a multi-tiered teaching experiment (Lesh & Kelly, 2000). As suggested by the term ‘multi-tiered’, the researcher, teacher(s), and student(s) would be conceived as the tiers or layers committed to improving student learning. However, teaching experiment methodology involves significant collaboration between participants, in this instance, researcher, teachers, and students. As will be explained later (3.4.2), working closely with teachers was not possible in this instance, therefore other options were explored.

Case study (Stake, 1995; Merriam, 1998) and action research (Kemmis & McTaggart, 1988) were also considered, however I wanted to be directly involved with students. Clarke, (2005) inspired me to explore the meaning of the words teaching and learning. Clarke (2005) argued against the traditionally held view of teaching and learning as being dichotomous with “discrete activities sharing a common context” (p. 4). This brought to my mind a teacher ‘teaching’ and a student ‘learning’ in a place known as a classroom. However, if an integrated view of teaching and learning is posited, where one notion means the other, then learning is co-constructed by all participants, teacher and students, by virtue of their participation (Clarke, 2005). I wanted the opportunity to work closely with the students in manner similar to my approach to student learning when I was as a classroom teacher. As a result, the design for this research is more closely aligned with design-based research (Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) also referred to as design experiments.

The primary purpose of design experiments is to “study learning in context through the systematic design and study of instructional strategies and tools” (The Design-Based Research Collective, 2003, p. 5). The context in this case was the mathematics classroom in two of the SNMY Research schools, where I worked closely with a selected group of students to improve their capacity to think multiplicatively. This met an important criteria
of design experiment methodology, but it also needed to satisfy criteria of generating
theory targeting “domain specific learning processes” (Cobb et al., 2003, p. 9). In this case,
the learning process needed to shift students from additive to multiplicative thinking.

The research took the form of a design experiment through the creation of an
innovative educational environment, and simultaneous evaluation of the innovations
introduced in that environment (Brown, 1992). Whilst it is unwise to over-simplify features
of a design experiment methodology, key features are critical: periods of learning occur
over an extended period of time, the approach to learning is interventionist and innovative,
and the opportunity for retrospective analysis is facilitated by the maintenance of ongoing
records of the design process, all for the explicit purpose to better understand how to
enable at-risk students make the shift from additive to multiplicative thinking.

With respect to the design experiment specifically, I conducted a sequence of
teaching sessions with a small group of students, over an extended period of time (Cobb et
al., 2003). The teaching sessions were iterative requiring “systematic attention to evidence
about learning” (Cobb et al., 2003, p. 10). The program was interventionist and innovative
as the needs of at-risk students were addressed using a non-traditional pedagogical
approach (described in 3.4.2). The retrospective analysis occurred in the form of the daily,
post-session reflections and documentation of the intervention program provided the
ongoing record that was taken over the period of time.

For the purpose of this study, the iterative teaching sequences involved a sample
of at-risk students identified by the SNMY Project initial assessment data (April/May,
2004). Detail regarding the sample will be provided in section 3.3. In addition to working
closely with these students, data was also collected from a sample of successful students
working particularly well in the area of multiplicative thinking as I wanted to better
understand how at-risk students differed from those students who were successful in this
area of mathematics learning. These students became known as the ‘successful’ group.
Interviews with these students were included to identify their beliefs about learning,
teaching, and mathematics, but also the strategies they used to solve multiplicative tasks.
The involvement of the successful students also provided a bench mark to judge the extent
to which the at-risk students with whom I worked became more characteristic of the
successful group.
3.2 Research Design

This study is ethnographic in nature in its determination to describe, interpret and explain the experience of at-risk learners’ knowledge and thinking through the various student ‘voices’ that tell stories about mathematics and mathematics learning (LeCompte & Preissle, 1993). The student voice is reflected in the research questions and the manner in which the data was collected, analysed, and reported, namely through the administration of interviews with students and the implementation of the intervention program. In recognition of the relationship between learning and beliefs (Spangler, 1992), the interviews explored students’ beliefs about mathematics learning. The context of the research reported here is presented in the following section. The SNMY Project will be described in detail below.

3.2.1 Scaffolding Numeracy in the Middle Years Project 2003-2006

As mentioned previously, this intervention study was conducted as part of the SNMY Project (Siemon, Breed, Dole, Izard & Virgona, 2006), which collected data from 40 schools, 19 Research Schools, and a matched group of 21 Reference Schools. The Research Schools came from three groups of schools referred to as ‘clusters’, two clusters in Victoria and one cluster in Tasmania, where a cluster generally consisted of one secondary school and at least three primary schools.

The SNMY Project was guided by the following research questions:

- To what extent can we accurately identify key points in the development of multiplicative thinking and rational number beyond the early years?
- To what extent can we gather evidence about student achievement with respect to these key points to inform the development of a coherent learning and assessment framework?
- To what extent can authentic assessment tasks be developed and used to assess student performance against the framework?
- To what extent does working with the tasks and the knowledge they provide about student understanding assist teachers to improve numeracy performance at this level?
- What strategies and/or teaching approaches are effective in scaffolding multiplicative thinking and rational number understanding in the middle years?
- What are the key features of classroom culture and discourse needed to support/scaffold student’s numeracy-related learning at this level? (Siemon, 2003a, p. 5)

To address these questions, the SNMY Project was designed in terms of three overlapping phases. Phase 1 involved a synthesis of the research literature on multiplicative thinking to identify a broad hypothetical learning trajectory (Simon, 1995) which informed the development of a draft LAF, which was used to guide the design, trial, and refinement of multiplicative thinking assessment tasks and related scoring rubrics. In Phase 2, the assessment tasks were administered to Year 4 – 8 students in both Research and Reference schools in April/May 2004 and October/November 2005. The results of these assessments
were analysed using Rasch (1980) modelling to test and refine the LAF. Phase 3 involved Research School teachers and members of the research team working in study groups to design and trial a range of learning activities, referred to as ‘Learning Plans’, aimed at a specific zones of the LAF. Since Phase 2 was particularly relevant to my research, aspects of this phase are now explored in more detail.

The assessment task booklets consisted of one extended task and five supplementary tasks. Parallel versions of these were used in both 2004 and 2005 to control for practice effects and maximise the number of items that could be used to inform the development of the LAF. The extended tasks were *Tables and Chairs* (adapted from Callingham & Griffin, 2000) and *The Butterfly House* (adapted from Kenny, Lindquist, & Heffernan, 2002). The supplementary tasks from which five were chosen, were *Packing Pots, Adventure Camp, Tiles Tiles Tiles, Pizza Party, Speedy Snail, Canteen Capers, Swimming Sports and Stained Glass Windows* (see Appendix A, pp.2 -18). Research School teachers used scoring rubrics to assess student level of achievement. Reference School data was assessed by the research team. All tasks and associated scoring rubrics can be found in Appendix B (pp. 20 – 27).

Rasch (1980) analysis placed student performance and item difficulty on an interval scale. Content analysis of items largely confirmed the original hypothetical learning trajectory (see Siemon et al, 2006). The revised LAF comprised eight ‘zones’, or developmental levels from little/no understanding of multiplicative thinking to a well developed understanding of multiplicative thinking. Table 3.1 summarises the final LAF informed by the initial and final testing. The full version which includes specific advice for each zone about what content and strategies to consolidate and develop and what to introduce and establish is contained in Appendix C (pp. 29-36).
<table>
<thead>
<tr>
<th>LAF Zone</th>
<th>Description</th>
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<tr>
<td>1</td>
<td>Solves simple multiplication and division problems involving relatively small whole numbers but tends to rely on drawing, models and count-all strategies. May use skip counting for groups less than 5. Makes simple observations from data and extends simple number patterns. Multiplicative thinking (MT) not yet apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support more efficient calculation.</td>
</tr>
<tr>
<td>2</td>
<td>Counts large collections efficiently, keeps track of count but needs to see all groups. Shares collections equally. Recognises small numbers as composite units (e.g., can count equal groups, skip count by twos, threes and fives. Recognises multiplication needed but tends not to be able to follow this through to solution. Lists some of the options in simple Cartesian product situations. Some evidence of MT as equal group/shares seen as entities that can be counted.</td>
</tr>
<tr>
<td>3</td>
<td>Demonstrates intuitive sense of proportion. Works with useful numbers such as 2 and 5 and intuitive strategies to count/compare groups (e.g., doubling, or repeated halving to compare simple fractions). May list all options in a simple Cartesian production, but cannot explain or justify solutions. Beginning to work with larger whole numbers and patterns buts tends to rely on count all methods of additive thinking (AT).</td>
</tr>
<tr>
<td>4</td>
<td>Solves simple multiplication and division problems involving two-digit numbers. Tends to rely on AT, drawings and/or informal strategies to tackle problems involving larger numbers, decimals and/or less familiar situations. Tends not to explain thinking or indicate working. Partitions given number or quantity into equal parts and describes part formally. Beginning to work with simple proportion.</td>
</tr>
<tr>
<td>5</td>
<td>Solves whole number proportion and array problems systematically. Solves simple, 2-step problems using recognised rule/relationship but finds this difficult for larger numbers. Determines all options in Cartesian product situations involving relatively small numbers, but tends to do this additively. Beginning to work with decimal numbers and percent. Some evidence MT being used to support partitioning. Beginning to approach a broader range of multiplicative situations systematically.</td>
</tr>
<tr>
<td>6</td>
<td>Systematically lists/determines the number of options in Cartesian product situation. Solves a broader range of multiplication and division problems involving 2-digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Renames and compares fractions in the halving family, uses partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing capacity to work mentally with multiplication and division facts.</td>
</tr>
</tbody>
</table>
Solves and explains one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Solves and explains solutions to problems involving simple patterns, percent and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Constructs/locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically.

Uses appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations, fractions and decimals. Can justify partitioning, and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems.

The quantitative data generated by the student responses to the 2004 SNMY assessment round were available for the purpose of selecting students to participate in this intervention study. The 2005 data was used to evaluate any shifts in their thinking over time. Specific detail as to when this data was collected and how it was used is documented in section 3.5.3.

### 3.3 Sample

A criterion-based approach to selecting the sample for this study was taken (LeCompte & Preissle, 1993) because the attributes of the students needed for this study were determined in advance. Students were identified by the results from the SNMY initial assessment data, which indicated the LAF zone at which students were placed in relation to multiplicative thinking. The following section will illustrate the nature of the sample, then the procedure for selecting the students will be explained.

#### 3.3.1 Nature of sample

My research involved a sample of ‘performance outliers’, that is, students performing well outside what is generally expected for their year level in school mathematics. The groups fell into two different categories that are described as ‘at-risk’ (AR) or ‘successful’ (SC), that is, students performing well below expected curriculum levels and students performing well above expected curriculum levels respectively. For the purpose of this research, at-risk students were deemed to be those students in Years 5 to 8 whose performance on the initial SNMY assessment was at the lowest level of the LAF, that is, Zone 1. Students operating at Zone 1 of the LAF are able to solve simple multiplication and division problems involving relatively small whole numbers but rely on modelling, count-all strategies, and/or skip counting for groups less than 5 (see Table. 3.1). What is described
for this level is typically expected of students in Year 1 of primary school. The successful students were deemed to be those students in Years 5 to 8 whose performance on the initial SNMY assessment was at the highest levels of the LAF, that is, Zones 7 or 8. Students operating at these zones are able to solve a range of complex multiplication and division problems involving large whole numbers and fractions across a variety of contexts (see Table. 3.1).

Given that the intervention program meant working with students in an intense but regular fashion over an extended period of time, it was necessary to choose easily accessible sites that were in relative close proximity to the university. As a result, the at-risk students were identified first. The sample was drawn from the SNMY metropolitan Research School Cluster in Victoria. This cluster, comprised two secondary schools and six primary schools, located in Melbourne’s inner north, serving culturally diverse communities. At the beginning of the SNMY Project, all students in Project schools were assigned unique codes in accordance with RMIT University and the respective Department of Education (Victoria and Tasmania) ethics procedures. The codes were used to anonymously identify potential students for this intervention study. This process is documented in greater detail in 3.3.2. The sample was constructed on the basis of three groups: at-risk intervention students (ARI), at-risk non-intervention students (ARN), and successful (SC) students. It was expected that comparison across and between the two at-risk groups (intervention and non-intervention) and the successful group would identify the degree of similarity and difference between the students, as well as the impact of the intervention program on student learning outcomes. Where there is not need to distinguish between the ARI and ARN students, the students will be referred to as the AR (at-risk) students. The process of identifying and naming the three student groups is explained below. All Year 5 to 8 students whose performance on the initial assessment was judged to be at Zone 1 or 2 of the LAF were identified as being at-risk for the purposes of this study.

The at-risk non-intervention group (ARN)

Students in Years 5 in 2004 at Zone 1 of the LAF, who could potentially be tracked into Year 6 in 2005, were identified \((n=20)\) first. This was important as data collection was to span 2004 and 2005 and the design experiment was planned to occur in the 2005 school year. To expand the at-risk group further, students in Year 6 at Zone 1 \((n=8)\) and in Year 7 at Zones 1 and 2 \((n=5)\) were selected. Students in Year 8 in 2004 would leave the Project in 2005, therefore they were not included. In all, a total of 33 student codes were identified.
The at-risk intervention group (ARI)
This group comprised a subset of the AR group described above, that is, students in Year 5 in 2004 at Zone 1 of the LAF for the reasons described earlier. These students were involved in the intervention program that took place in 2005 and were selected from the ARN group on the basis of the greatest number of at-risk students attending the least number of schools. This would maximise the number of students involved in the intervention program and minimise the travel time. This group comprised five Year 6 boys and four Year 6 girls: Yousif (male), Hadi (female) and Douha (female) who attended school A, and Adir (male), Dean (male), Cansu (female), James (male), Ahmed (male) and Berrin (female) who attended school B. Pseudonyms have been used where permission was not given to identify a student by their actual name.

The successful group (SC)
The group of SC students were drawn from the same schools. The SC students were defined as students in Year 4 to 6, assessed at Zone 7 or 8 of the LAF for the same reasons described above. Initially, all students in Year 6 at Zone 8 of the LAF were identified ($n=5$). These five students came from three of the five sites, already supporting the at-risk (non-intervention) student group. To extend the size of this group further, students in Years 4 to 6, attending these sites or within the same classroom, and assessed at Zones 7 or 8 of the LAF were also identified ($n=14$). A total of 19 SC student codes were identified as a result of this process.

3.3.2 Sampling procedure
In accordance with RMIT University and the respective Department of Education (Victoria and Tasmania) ethics procedures, the 52 students identified above were known in terms of individual codes assigned for SNMY Project data identification purposes. Class lists matching research codes to student names were held by individual schools. The principals of the schools were approached to assist in communicating with the identified students and their parents regarding participation in this study. On this basis, the identified students were approached by school staff, with copies of the Plain Language Statement and the Consent Form in fulfillment of RMIT University ethics procedures.

Of the 33 at-risk students identified, 13 parents returned signed consent forms allowing their children to participate in the study. An additional student who had been
absent for the SNMY May 2004 Assessment, was included in this cohort on the
recommendation of the teacher who felt the student’s level of achievement was consistent
with the seven other at-risk students at the same school. Of this group of 14 students, nine
students in Year 5 attended two primary schools, a few kilometres apart, thus the at-risk
intervention group was formed. Of the 19 SC students identified, 14 parents returned
signed consent forms allowing their children to participate.

The ARI, ARN, and SC groups were formed toward the end of the 2004 and
beginning of the 2005 school years. Given this transition from one school year to the next,
seven students (1 at-risk student & 6 successful students) left the study in early 2005 as they
no longer attended SNMY Project schools due to relocation or transition from a primary to
a non-Project secondary school. Throughout 2005 however, the sample remained
consistent, with 14 at-risk students (9 ARI and 5 ARN) and 8 SC students. The ARI
students attended school A and school B. Table 3.2 below details the 2005 composition of
the at-risk groups and the successful group in terms of gender, year level and initial
assessment against the LAF.

<table>
<thead>
<tr>
<th>At-risk non-intervention</th>
<th>At-risk intervention</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Year Level (2005)</td>
<td>Initial LAF Zone</td>
</tr>
<tr>
<td>Female</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Male</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Female</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Male</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

### 3.4 Phases of the Intervention Study

In taking an ethnographic approach, the portfolio of evidence was collected in the field
over three distinct periods: a Pre-intervention Phase (December 2004 – May 2005), an
Intervention Phase (May 2005 – October 2005) and a Post-intervention Phase (November
2005 – December 2005). Data assembled during these phases enabled the participants and
researcher to mutually come to know the setting and the people who interacted within it. A
detailed account of each phase is given in sections 3.4.1 to 3.4.3 below.

#### 3.4.1 Pre-intervention Phase

When people meet for the first time, introductions are made. The Pre-intervention (Pre-I)
Phase reflected this, and its purpose was to get to know the students and the culture at
each of the school sites. Whilst I had access to the SNMY Project initial assessment task booklet for each of the students in my sample, I wanted to know more about the strategies they used in mathematics more generally, as well as their beliefs about mathematics learning. Research questions addressed during this phase were:

- What strategies are used by at-risk students to solve tasks involving multiplicative thinking and to what extent are these impacted by the intervention program?
- What do at-risk students believe about learning mathematics and how are these beliefs impacted by their participation in the intervention program?

A series of interviews were conducted during this phase with all students in the ARN, ARI, and SC groups. Each of these data sources, their purpose, and how and when they were administered, will be described in detail in 3.5.1. As in-depth information about student beliefs and understandings about mathematics, learning, and multiplicative thinking was required by this research (see 3.2), it was necessary to delve into both the affective and cognitive domains. The research literature has documented the nature of student beliefs in relation to mathematics (see 2.4). This intervention study is aligned with the view asserted by Spangler (1992):

> There appears to be a cyclic relationship between beliefs and learning. Students' learning experiences are likely to contribute to their beliefs about what it means to learn mathematics. In turn, students' beliefs about mathematics are likely to influence how they approach new mathematical experiences. (Spangler, 1992, p. 19)

Student beliefs about learning mathematics were elicited through student drawings (McDonough, 2002) and responses to card sorting activities (Pike & Selby, 1998). A semi-structured interview, the Multiplicative Task interview, was used to elicit the nature of strategies used by students to respond to mathematical tasks requiring multiplicative thinking. These interviews were underscored by the premise that learners are “active constructors of their own knowledge, motivations and self-perceptions” (Pressick-Kilborn et al., 2005, p. 27). These instruments are outlined in greater detail in section 3.5.

### 3.4.2 Intervention Phase

Acceptance of the social nature of learning necessitated that learning be researched in context (Lincoln & Guba, 1985). In this instance, the context was the primary school classroom, in recognition of its role in the enculturation of students into the practices of learning mathematics (Pressick-Kilborn et al., 2005). Hence, the purpose of this phase was to work closely with the nine ARI students to support the shift from relying on inefficient additive strategies to an increased capacity to think multiplicatively. The other main
intentions (as per Boaler, 2002) were to build student confidence, work mathematically, develop conceptual understanding, engage with problem solving, and improve communication skills (see 2.3). The Pre-I Phase interviews had confirmed that these students were unable to do this effectively. The research question addressed during this phase was:

- To what extent does an intervention program closely aligned to learning needs impact student outcomes in relation to multiplicative thinking?

The intervention program took place from May 2005 to October 2005, over an 18 week period. I visited each of the two schools (A and B), three times per week and worked with the nine ARI students in a group, three students at one primary school and six students at the other. Early in May, the three teachers concerned (MD, TP, and DC) and I met for an initial planning meeting. The agenda included planning a timetable for my visits, how we would involve each other in the teaching and learning process, and the identification of starting points for teaching based on information gained from the Multiplicative Task interviews and the initial SNMY assessment data. It was agreed that the intervention program would commence three weeks later, with visits scheduled three times per week, per school. The three teachers were happy to be involved in the intervention program. A flexible approach to planning and implementation was agreed upon so as to maximise their involvement, as well as maintain their other teaching commitments.

As indicated earlier, my original intention was to employ a teaching experiment approach (Lesh & Kelly, 2000). The three teachers and I had anticipated that we would meet regularly and that a collaborative approach to planning and teaching would be taken. Despite our best intentions, this did not happen as anticipated: MD was on Long Service Leave from Week 10 to 15 of the program, and as the Assistant Principal at her school, TP was released from her class frequently during my school visits. All three teachers were heavily committed to school-based and cluster-wide meetings and professional development. This made a collaborative approach between researcher and teacher very difficult. Justification of the design experiment methodology was given earlier in 3.2.3.

The teachers and I did however manage to organise approximately four to five occasions when I worked with each of the three teachers’ whole class, for an introductory game or activity. For the teacher, these sessions became an opportunity to engage in impromptu professional development as well as for me to become ‘part of the furniture’. When this occurred, the teachers would either work with me in leading the students through the activity or game or observe me taking the class. For example, a 2-digit number
line sequencing activity was conducted by pinning selected cards (numbered between zero and one hundred) to a rope (see Intervention Program, Post-session reflection for school B, Appendix D, p. 48).

The next section will describe the intervention program, however aims of the program will be provided first.

**The intervention program**

The intervention program was designed to build on from where the SNMY initial assessment and *Multiplicative Task* interviews had identified students were at in relation to the *Learning and Assessment Framework for Multiplicative Thinking* (LAF) (Siemon et al., 2006). That is, Zone 1 of the LAF, which is detailed below (see Appendix C, p. 29 for full version):

**Zone 1- Primitive Modelling**

Can solve simple multiplication and division problems involving relatively small whole numbers, but tends to rely on drawing, models and count-all strategies. May use skip counting (repeated addition) for groups less than 5. Can make simple observations from data given in a task and reproduce a simple pattern. Multiplicative thinking not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation.

The teaching advice identifies what needs to be consolidated/established:

- **Trusting the count** for numbers to 10 (eg, for 6 this involves working with mental objects for 6 without having to model and/or count-all). Use flash cards to develop **subitising** (ie, ability to say how many without counting) for numbers to 5 initially and then to 10 and beyond using **part-part-whole knowledge** (eg, 8 is 4 and 4, or 5 and 3 more, or 2 less than 10)
- **Simple skip counting** to determine how many in a collection and to establish numbers up to 5 as countable objects, eg, count by twos, fives and tens, using concrete materials and a 0-99 Number Chart
- **Mental strategies for addition and subtraction facts to 20** eg, *Count on from larger* (eg, for 2 and 7, think, 7, 8, 9), *Double and near doubles* (eg, use ten-frames and a 2-row bead-frame to show that 7 and 7 is 10 and 4 more, 14), and *Make-to-ten* (eg, for 6 and 8, think, 8, 10, 14, scaffold using open number lines). Explore and name mental strategies to solve subtraction problems such as 7 take 2, 12 take 5, and 16 take 9
- **2 digit place-value** – working flexibly with ones and tens, (by making, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming

The teaching advice also identifies what needs to be introduced/developed:

- **Doubling (and halving) strategies** for 2-digit numbers that do not require renaming (eg, 34 and 34, half of 46), build to numbers that require some additional thinking (eg, to double 36, double 3 tens, double 6 ones, 60 and 12 ones, 72)
- **Extended mental strategies for addition and subtraction**, use efficient, place-value based strategies (eg, 37 and 24, think: 37, 47, 57, 60, 61). Use open number lines to scaffold thinking
- **Efficient and reliable strategies for counting large collections** (eg, count a collection of 50 or more by 2s, 5s or 10s) with a focus on how to organise the number of groups to
facilitate the count (eg, by arranging the groups systematically in lines or arrays and then skip counting)

**How to make, name and use arrays/regions** to solve simple multiplication or sharing problems using concrete materials, and skip counting (eg, 1 four, 2 fours, 3 fours …), leading to more efficient counting strategies based on reading arrays in terms of a consistent number of rows (eg, 4 rows of anything, that is, 4 ones, 4 twos, 4 threes, 4 fours, …)

**3 digit place-value** – working flexibly with tens and hundreds (by making with MAB, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming …)

**Strategies for unpacking and comprehending problem situations** (eg, read and re-tell, ask questions such as, What is the question asking? What do we need to do? …). Use realistic word problems to explore different ideas for multiplication and division, eg, 3 rows, 7 chairs in each row, how many chairs (array)? Mandy has three times as many…as Tom…, how many … does she have (scalar idea)? 24 cards shared among 6 students, how many each (partition)? Lollipops cost 5c each, how much for 4 (‘for each’ idea)?

**How to explain and justify** solution strategies orally and in writing through words and pictures (important for mathematical literacy)

The 18 week intervention program with the ARI student group aimed to improve:

- student conceptual understanding of key ideas in relation to multiplicative thinking, to include development of efficient mental strategies, place-value understanding especially with respect to working with larger numbers, key fraction ideas, and contexts for multiplication and division
- student self-confidence with respect to learning mathematics and learning in general
- student capacity to work mathematically through problem solving in a variety of contexts using a variety of strategies, and
- communication skills, both oral and written, through frequent group discussion and sharing sessions and reflective writing.

These aspects were chosen on the basis of the literature reviewed earlier (2.3) in relation to the benefits of working this way (AAMT, 1997; Boaler, 2002, NCTM, 2000). The group of students at each site, was treated as a small teaching group. This was designed to reflect the practice of forming small groups of students with like-needs and engaging the students in purposeful, meaningful learning activities (Noddings, 1989). The teaching sessions were conducted either within the classroom during maths time or in a withdrawal area near the classroom. Each teaching session was informed by the previous session, and lasted approximately one hour. They were structured in the following way:

Introduction: A focus on reviewing one or more than one of the following: subitising and part-part-whole understanding, development of efficient mental strategies, consolidation of place value understanding often through the use of manipulatives
such as flash cards, playing cards, dice, MAB, games, number charts and puzzles. Generally this component of the session lasted 10 to 15 minutes.

*Learning Focus:* Further development of a core idea, for example, one or more of the following:

- mental strategies (shifting away from seeing numbers as a collection of ‘ones’, part-part-whole understanding, make to 10, working with place value parts, doubles and near doubles, extending these strategies to working with larger numbers)
- place value (beginning with two digit numbers, ‘make, name, record’ then consolidate through compare, order, count, rename),
- partitioning (whole number and fractions through practical sharing activities using discrete and continuous models), and
- key concepts of multiplication and division (exploring how these differ from concepts for addition and subtraction, with a shift away from the ‘groups of’ idea to exploring arrays and regions).

This part of the session tended to last 30 to 40 minutes.

*Conclusion:* A focus on sharing ideas and strategies through reflective journal writing and discussion. This component generally occupied the last 10 to 15 minutes of the lesson.

Table 3.3 below provides a summary of the program over the period of 18 weeks.
<table>
<thead>
<tr>
<th>Week</th>
<th>Learning focus</th>
</tr>
</thead>
</table>
| 1    | Mental images of number (subitising and part-part-whole)  
|      | Place value (2 & 3 digit numbers)  
|      | Efficient counting strategies (managing large collections) |
| 2    | Introduce the open-number line (addition & subtraction) and working with place value parts  
|      | Problem solving/number puzzles  
|      | Partitioning to sequence 2 digit numbers (rope and peg model) |
| 3    | Efficient mental strategy: ‘make to ten’  
|      | Number patterns on a 0-99 number chart  
|      | Problem solving/number puzzles |
| 4    | Efficient mental strategy: review  
|      | Place value (make, name, record) up to 4 digit numbers  
|      | Problem solving task boxes |
| 5    | Efficient mental strategy: ‘Doubles’  
|      | Formal recording for addition and subtraction (assessment)  
|      | Problem solving/number puzzles |
| 6    | Arrays in real world context  
|      | Efficient mental strategies and computation: review |
| 7    | Place value based strategies for mental computation  
|      | Explore array arrangements for small collections |
| 8    | ‘Working with arrays’ powerpoint: visualizing (construct, hide, reconstruct, name)  
|      | Commutativity |
| 9    | Problem solving in a measurement context: create a full-sized maths character  
|      | Estimation of and measuring height of character |
| 10   | Explore difference idea through height comparison  
|      | Rename height in diverse ways |
| 11   | Doubling strategy for 2’s facts  
|      | Introduce fractions through paper folding (halving, thirding) using continuous and discrete fraction models |
| 12   | Fraction as operator idea  
|      | Part to whole and whole to part fraction understanding |
| 13   | Place value review  
|      | ‘Ask, think, do’ problem solving strategy (fraction context) |
| 14   | Apply partitioning strategies to locate fractions on a number line  
|      | Fractions through paper folding (introduce fifthing strategy) |
| 15   | Doubling for 2’s, 3’s, 4’s and 8’s facts  
|      | Recognize and model multiplicative word problems |
| 16   | Construct region models for multiplicative situations  
|      | Explore different multiplicative problem types and strategies to solve them  
|      | Problem solving/number puzzles |
| 17   | Place value to thousands and beyond  
|      | Distinguishing between additive and multiplicative situations  
|      | Problem solving/number puzzles |
| 18   | Calculating efficiently, area of irregular arrays and ‘paint spills’. |
A detailed lesson plan was written just prior to each daily teaching session. The intervention program is elaborated in section 3.5.2 and the full program is included in Appendix D.

3.4.3 Post-intervention Phase
The purpose of the Post-intervention (Post-I) Phase was to document any change in student beliefs, understandings and knowledge in relation to mathematics and multiplicative thinking over a period of approximately 12 months. Therefore the interviews that were conducted during the Pre-I Phase were readministered in the Post-I Phase. I was particularly interested in the impact of the intervention program on the at-risk students’ performance in relation to the LAF, as well as the impact on their beliefs about learning mathematics. Research questions addressed during this phase were:

- To what extent does an intervention program closely aligned to learning needs impact student outcomes in relation to multiplicative thinking?
- What strategies are used by at-risk students to solve tasks involving multiplicative thinking and to what extent are these impacted by the intervention program?
- What do at-risk students believe about learning mathematics and how are these beliefs impacted by their participation in the intervention program?

3.5 Data Collection
Overall, three sources of data were available for analyses:

- Interviews: Two interviews were conducted with each identified student that explored their beliefs about the nature of learning mathematics well, and a third interview that explored the strategies these students used to solve multiplicative tasks
- Records: the intervention program, student learning journals kept by the nine at-risk-intervention students and descriptive documentation of my firsthand observations (Patton, 2002; Merriam, 1998) resulting from my work with these students in the form of post-teaching session reflections, and
- Quantitative data: SNMY Project initial (May 2004) and final (November 2005) assessment data locating participant student’s progress against the LAF.

Table 3.4 provides an overview of the data sources and when they were administered with the three student cohorts.
Table 3.4  
*Sources of data by Study Phase*

<table>
<thead>
<tr>
<th>Phase</th>
<th>Interviews (n=22)</th>
<th>Records</th>
<th>Quantitative</th>
</tr>
</thead>
</table>
| Pre-intervention (December 2004 – May 2005) | • Drawing Task  
  • Multiplicative Task  
  • Card Sort | • Field notes  
  • Student work  
  • Photographic record of drawing | SNMY initial assessment (May 2004) for ARI, ARN and SC students |
| Intervention (May 2005 – October 2005) | • Intervention program  
  • Student journals  
  • Post-teaching session reflections | | |
| Post-intervention (November, 2005 – December, 2005) | • Drawing Task  
  • Multiplicative Task  
  • Card Sort | • Field notes  
  • Student work  
  • Photographic record of drawing | SNMY final assessment (November, 2005) for ARI, ARN and SC students |

Each of these data sources, their purpose, and how and when they were administered, is described in detail below.

### 3.5.1 Interviews

Three separate interviews were used to collect information that was not necessarily discernable through observation or testing alone. Interviews, that is, conversations with a purpose, are an important aspect of qualitative research (Merriam, 1998). The purpose of the interviews was to reveal student beliefs about learning mathematics, and the strategies they selected when solving multiplicative tasks. It is noted at this point that a decision was made not to audiotape the interviews as I did not want to jeopardise the freedom in which the responses would be provided. Patton (2002) stresses that recording of interviews in this way

> must be used judiciously so as not to become obtrusive and inhibit…participant responses. A tape recorder is much more useful for recording field notes in private than it is as an instrument to be carried about at all times, available to put a quick end to any conversation. (Patton, 2002, p. 308)

There were three interviews, two interviews targeted the first purpose (beliefs) and one interview targeted the second purpose (strategies). Each interview was conducted with all 22 students on an individual basis. These are described below.
**Drawing Task Interview**

The purpose of this instrument was to determine individual student’s beliefs about learning, their beliefs about mathematics and what constitutes learning maths well for the reasons outlined at the beginning of this chapter. The *Drawing Task*, developed by McDonough (2002) was used to elicit student beliefs about effective learning environments in mathematics. This interview was conducted at the beginning and end of data collection so that student beliefs about what makes for effective learning could be inferred and any change in these beliefs documented. The first administration in the Pre-I Phase was a non-threatening way for me to engage with each student in a dialogue about mathematics. The Pre-I Phase *Drawing Task* interviews also contributed to the development of the *Card Sort* interview (to be described later in this section) and helped identify factors students believed support their learning of mathematics. The Pre-I Phase interviews were conducted over a two week period in mid-December, 2004, and a week in late March, 2005. The three month time span for these interviews was due to the sample being identified towards the end of the 2004 and beginning of the 2005 Victoria school years. This period included the six week Summer school holidays from late December through to early February. The students and their teachers were given time in February to settle into school and class routines. Two students who were either absent or not yet identified for the December 2004 and March 2005 interviews, were interviewed in late April and early May respectively. The Post-I Phase interviews occurred over a three week period in late November and early December, 2005.

The *Drawing Task* interview took approximately 30 to 35 minutes to complete and was conducted with students individually. A copy of the *Drawing Task* instrument is provided in Appendix E. Once the student was settled, the sentence, *think of a situation in which you are learning maths well, draw it* was presented verbally and in writing on a single sheet of A4 white paper. Time was given for the student to think about the ‘situation’ and address any questions or concerns about the task. The student was provided with a choice of drawing materials: black fine-line markers, coloured markers, pencils, and oil pastels with which to complete their picture. Once the student was satisfied with their picture, a discussion about their drawing took place. I took notes throughout our discussion about what the picture meant to the student. After the interview, each student drawing was photographed digitally. Field notes were then transformed into electronic annotations and were added to the digital photographs of student drawings. A sample from Ahmed (ARI,
Year 6) is included below. The complete interview record for each student is contained in Appendix F.

![Ahmed's drawing](image.png)

**Figure 3.1** Ahmed’s drawing and field note summary

**Extract from field notes (March 30, 2005):**

<table>
<thead>
<tr>
<th>8</th>
<th>I: Tell be about your drawing while you are colouring.</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>A: Sitting at my table by myself doing a maths challenge.</td>
</tr>
<tr>
<td>10</td>
<td>I: What's a maths challenge?</td>
</tr>
<tr>
<td>11</td>
<td>A: Like a test.</td>
</tr>
<tr>
<td>12</td>
<td>I: This helps you to learn maths well?</td>
</tr>
<tr>
<td>13</td>
<td>A: Yes.</td>
</tr>
<tr>
<td>14</td>
<td>I: Why does this help you?</td>
</tr>
<tr>
<td>15</td>
<td>A: I don't know...because I can learn.</td>
</tr>
</tbody>
</table>

**Drawing Task considerations**

Given that this interview was used to elicit student beliefs about learning mathematics well through drawing pictures, it was recognised that the *situation* could potentially be restricted to what students felt they were capable of drawing. It was expected that student drawings could include representation of:

- **objects**, such as tables, chairs, the whiteboard
- **people**, such as themselves and other students,
- **communication**, through speech bubbles, as well as
- **mathematics content**, for example, fractions and processes such as addition and subtraction.

My goal was to ascertain in depth views about learning mathematics, therefore the post-drawing discussion was important as it provided opportunity for me and the student to clarify and describe the ‘learning maths well’ *situation* in detail, rather than rely on the drawings in isolation. The student drawings and post-drawing discussion field notes, taken together constituted a way in which it was possible for me to identify factors associated with learning mathematics well.
The *Drawing Task* interview contributed to the design of the *Card Sort* interview. Both interviews elicited student beliefs about learning maths well, but did so in different ways. The *Card Sort* interview is described next.

**Card Sort Interview**

This interview was based on a card sort activity referred to as diamond ranking by Pike and Selby (1988). The purpose of this instrument was to elicit student beliefs about learning, their beliefs about mathematics and what constitutes learning maths well in a different yet related way to the *Drawing Task* interview. Card sorting tasks for research purposes are an effective elicitation and clarification tool (Friedrichsen & Dana, 2003). A semi-structured interview using a projection technique (Patton, 2002) where interviewees respond to a stimulus other than a question, was the basis on which this instrument was developed. This approach is particularly useful when interviewing children (Patton, 2002). Participatory methods such as this are employed to engage students’ views about issues, in this case, about learning mathematics well (Kellett & Ding, 2004). Pike and Selby (1988) note the potential of this type of card sorting activity for use with students:

> This activity helps students in an unthreatening way to clarify what their thoughts and feelings about a particular subject…underpinning the activity is the unspoken assumption that everybody has something relevant and valuable to bring to the discussion. (Pike & Selby, 1988, p. 134)

The card sorting activity also prompts consideration of aspects that may be difficult or cannot be easily represented in a drawing.

During this interview, stimulus words or descriptors were placed in a diamond shaped pattern on the basis of their perceived importance (see Figure 3.2). The two scheduled administrations of this instrument enabled comparison of beliefs over time. I conducted this interview with all students (ARN, ARI and SC, $n=22$) during the Pre-I Phase, over a two week period in mid to late May, 2005 and the Post-I Phase, over a three week period in late November and early December, 2005.

The descriptors for the *Card Sort* interview were created out of the initial analysis of student drawings and interview notes from the Pre-I Phase *Drawing Task* interviews. Students, having elaborated on their picture depicting a situation in which they were learning maths well, had identified key aspects they felt enhanced their learning of mathematics. The analysis involved reading and re-reading interview notes, and noting themes as they arose out of the data. Inductive analysis (Patton, 2002; Lincoln & Guba, 1985) revealed common messages or themes arising out of interview notes detailing student
elaborations of their pictures of learning maths well: the role of dialogue, the process and product of solving mathematical questions, the teacher and other people as a presence in the classroom, as well as affective factors, such as the importance of achievement, how they felt about themselves as learners and what they could do to improve. The Drawing Task themes were summarised in the form of 12 focus words or descriptors, to be introduced as the stimulus for the Card Sort: talking and discussing, problem solving, getting the answer, explanations, worksheets, the teacher, thinking, maths equipment, getting help, doing well, feeling good and group work. These descriptors were predominantly about ‘behaviours’ and are not considered to be mutually exclusive. There was a deliberate attempt to present the words in the Card Sort in language familiar to students. For example, the theme of ‘dialogue’ was represented in two cards, talking and discussing and explanations.

The interview was administered on an individual basis and each student interview took approximately 40-45 minutes. The 12 descriptors listed above were written on cards and presented to the student as being ‘about learning maths well’. An opportunity was provided for the student to read through the 12 cards and clarify what the words might mean. At this point, the student was asked if there were any words missing from the collection, using the question: Are there words or phrases about learning maths well that you think should be included? Blank cards were available so that any student nominated words, for example, ‘remembering’, could be recorded and included in the collection. The student was then prompted to select nine cards from the 12 card collection (or potentially more) on the basis of their relative importance to learning maths well. This was termed the initial sort.

After the nine cards were selected, the student was directed to rank them according to degree of importance. A diamond-shaped map was available on which to place cards. This sorting process was termed the diamond rank. A single card was placed at the first layer to indicate the most important, two cards were placed at the second layer, three cards form the third layer, two cards at the fourth layer and one card is placed at the fifth layer. Figure 3.2 below provides an illustration of this.

![Figure 3.2 The Card Sort map](image)

The interview was administered on an individual basis and each student interview took approximately 40-45 minutes. The 12 descriptors listed above were written on cards and presented to the student as being ‘about learning maths well’. An opportunity was provided for the student to read through the 12 cards and clarify what the words might mean. At this point, the student was asked if there were any words missing from the collection, using the question: Are there words or phrases about learning maths well that you think should be included? Blank cards were available so that any student nominated words, for example, ‘remembering’, could be recorded and included in the collection. The student was then prompted to select nine cards from the 12 card collection (or potentially more) on the basis of their relative importance to learning maths well. This was termed the initial sort.

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![Figure 3.2 The Card Sort map](image)
I made detailed notes throughout the interview, recording student responses on a record sheet designed for this purpose. The copy of the Card Sort proforma and recording sheet can be found in Appendix G. Where necessary, I clarified key points directly with the student at the time of the interview if I felt my notes were inadequate, for example, reading aloud the relevant section of my notes and asking the student if this was an accurate record of what they had said. This ensured that my notes were as accurate as possible. A 15-minute post-interview reflection allowed me to review and elaborate my notes further as necessary. These handwritten annotations were then transcribed into an electronic record for later analysis. An excerpt of Hadi’s interview record (ARI, Year 6) is included below in Figure 3.3. Complete interview records for each student interview are contained in Appendix H.

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

![Diamond ranking diagram]

3. Discussion
<table>
<thead>
<tr>
<th>Thinking</th>
<th>If you don’t think you don’t understand stuff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning</td>
<td>If you don’t learn you can’t do maths and learn things.</td>
</tr>
<tr>
<td>Explanations</td>
<td>If you don’t know something someone can tell you how to work it out because if you don’t know and they help you.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you don’t know ask someone and they help.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>You can talk about what you don’t know and what you do and this helps.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Helps in maths because you know how to work out problems…if you can’t solve problems, e.g., divided bys and times tables…need them to go to the bank and shops.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>If you don’t know it doesn’t matter because you can learn it, you feel good if you know something in maths.</td>
</tr>
<tr>
<td>Working out</td>
<td>If doing a problem you can try and work it out in different ways, like vertical (meaning the setting out).</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Asked H for examples: MAB, calculator, bead frame. If you don’t know how to do add, the equipment helps you ‘cos in a bead frame you put one down each time for as many as you need.</td>
</tr>
</tbody>
</table>

*Figure 3.3* Excerpt of Hadi’s Card Sort interview record
Card Sort considerations

As stated earlier, the Card Sort interview was developed to elicit student views about learning mathematics in a different manner to the Drawing Task interview. These two interviews complement each other for a number of reasons. The Drawing task posed an open-ended question about an ‘situation’ when learning maths well occurred. Initial analysis of the Drawing Task data led to the stimulus words or descriptors to be used in the Card Sort. This time however, the focus was no longer on the physical ‘situation’ as a whole, but predominantly on particular aspects considered in isolation, that students were required to sort in order of importance for learning mathematics well. In implementing the Card Sort interview, it was equally important to consider the reasons students gave for justifying the inclusion and ranking of certain words, as it was for the way in which the words were sorted and ranked (Friedrichsen & Dana, 2003).

Multiplicative Task interview

The purpose of this interview was to identify and track student thinking in relation to the strategies he/she employed across a range of multiplicative tasks, both familiar and unfamiliar. The administrations at the beginning and end of the study, provided a reference point at which the path to multiplicative thinking was to ‘begin’ and to track change and development over time. This interview was administered to all students (9 ARI, 5 ARN, & 8 SC students) on an individual basis during the Pre-I Phase, over a two week period in late April - early May, 2005 and the Post-I Phase, over a three week period in late November and early December, 2005.

This interview consisted of two parts: a reflective discussion of the student’s responses to familiar multiplicative tasks (Part A) completed in class time one week prior to the scheduled interview. This was followed by students working on a similar, yet unseen task in a ‘think aloud’ manner (Part B). The reflective aspect of the interview provided an opportunity to probe deeply into the strategies students used in the course of completing maths tasks on their own. The ‘think aloud’ aspect of the interview, provided opportunity to witness first-hand the strategies and approaches students used to solve tasks that were previously unseen, though not foreign to the students. The ‘think aloud’ approach (Patton, 2002) was used to “elicit the inner thoughts or cognitive processes” (p. 385) of the students, whilst completing tasks requiring multiplicative thinking. As the multiplicative tasks used at the Pre-I and Post-I Phases were different, versions of this interview differed therefore each phase will be treated separately.
Pre-intervention Phase Multiplicative Task interviews

The tasks designed for this interview were informed by the analysis of the identified at-risk student (intervention and non-intervention) responses to the SNMY initial assessment tasks (Appendix A). I reviewed the students’ written responses to the task items paying particular attention to items that indicated: an additive or non-multiplicative response, an incomplete or incorrect response, and an unclear or ambiguous response that might require further investigation. The SNMY task items fulfilling these response characteristics were *Butterfly House d* and *b, Packing Pots a, b and c*, and *Pizza Party a, b and c*. Parallel versions of these task items were created for teachers to administer prior to the *Multiplicative Task* interview, reflecting the same key multiplicative ideas: working with patterns, the ‘groups of’ idea for multiplication and division, simple proportion, Cartesian Product, and partitioning to describe and compare fractions. The two tasks used for this purpose during Part A of the Pre-I Phase interview were: *Patterns with tiles* (4 items) and *Footy lunch day* (3 items) (see Appendix I, pp. 229-233). These tasks were completed by all 22 students. The tasks *Patterns with tiles* and *Footy lunch day* are presented in full below.

<table>
<thead>
<tr>
<th>Multiplicative Task interview Part A task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Patterns with tiles</strong></td>
</tr>
<tr>
<td>Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.</td>
</tr>
<tr>
<td><img src="image" alt="Pattern with tiles diagram" /></td>
</tr>
<tr>
<td>a. How many times can this pattern be made with 28 black tiles, 21 grey tiles and 6 white tiles? <strong>Show all your working and explain your answer in as much detail as possible.</strong></td>
</tr>
<tr>
<td>b. This pattern uses 3 grey tiles for every 5 black tiles. How many black tiles would you need if you had 12 grey tiles? <strong>Show all your working and explain your answer in as much detail as possible.</strong></td>
</tr>
<tr>
<td>c. The art teacher orders 6 boxes of red tiles. Each box has 36 tiles. How many red tiles are there altogether? <strong>Show all your working and explain your answer in as much detail as possible.</strong></td>
</tr>
<tr>
<td>d. The art teacher needs 330 red tiles. How many boxes of red tiles does she need to order? <strong>Show all your working and explain your answer in as much detail as possible.</strong></td>
</tr>
</tbody>
</table>
Multiplicative Task interview Part A task

Footy lunch day
a. Show how you would share 2 large meat pies equally among 3 people?

Each person gets ..........................................................................

b. Jo ate $\frac{4}{9}$ of a large meat pie and Maggie ate $\frac{2}{3}$ of a large meat pie. Who ate the most pie? **Explain your reasoning using as much mathematics as you can.**

Jo ordered a sausage roll, a drink of cola and an icy-pole. What else might she have ordered? List all possibilities. **Show your working and explain your answer in as much detail as possible.**

<table>
<thead>
<tr>
<th>Food</th>
<th>Drink</th>
<th>Dessert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat pie</td>
<td>Cola</td>
<td>Ice-cream</td>
</tr>
<tr>
<td>Sausage roll</td>
<td>Lemonade</td>
<td>Icy-pole</td>
</tr>
<tr>
<td>Pastie</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To minimize interruption to the classroom program, these tasks were administered by classroom teachers at times that suited them in mid to late April, 2004. There were two versions of these tasks, one version for the ARI and ARN students and another for the SC students. There was no difference in the task items themselves, however the instruction to the students differed. I was particularly interested in identifying the range of efficient strategies that might be used by the SC group, so these students were directed specifically to use the most efficient method for solving each item that they could. Prior to each individual interview, I reviewed each student’s written responses and decided which items would be discussed during the interview. A minimum of two items for interview were chosen on the basis of at least one of a number of the following factors: evidence of inability to make a reasonable start in solving the problem, and/or the evident strategy was of interest, ambiguous, or needed further clarification.

Parallel versions of *Patterns with tiles* and *Footy lunch day*, were designed for Part B of the interview (Appendix I, pp. 229-233). A total of seven possible items were constructed: *Block Pattern a to d* aligning with the *Patterns with tiles* task items, *Sharing a & b*, and *Possible possibilities c* aligning with *Footy lunch day*. Support materials were prepared to assist students in the modelling of the problem. These included laminated squares to model the quilting squares for the *Block* problems (see Appendix I, p.231) and playdough (modeling clay) and icy pole sticks for the *Partitioning/sharing* problems. These tasks are presented in full below.
Block pattern
Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

```
  |   |   |
  |   |   |
  |   |   |
  |   |   |
```

a. How many blocks like this can be made with 32 black squares, 17 grey squares and 7 white squares? **Show all your working and explain your answer in as much detail as possible.**

b. This block uses 2 grey squares for every 6 black squares. How many black squares would you need if you had 6 grey squares? **Show all your working and explain your answer in as much detail as possible.**

c. The quilt will be made by sewing 25 of these blocks together. How many small squares will the quilt have all together? **Show all your working and explain your answer in as much detail as possible.**

d. If a quilt has 324 small squares, how many blocks of this pattern will be used? **Show all your working and explain your answer in as much detail as possible.**

---

Sharing part a…
a. How would you share the 2 sausages of playdough among 3 children?

```
  |   |   |
  |   |   |
  |   |   |
```

**How much does each person get?** …………………………………

Sharing part b…
b. If I have $\frac{4}{6}$ of the playdough sausage and you have $\frac{3}{4}$ of the playdough sausage, who has the most playdough? **Explain your reasoning using as much mathematics as you can.**

Possible possibilities part c…

<table>
<thead>
<tr>
<th>Bottoms</th>
<th>Tops</th>
<th>Hats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeans</td>
<td>Hoodie</td>
<td>Beanie</td>
</tr>
<tr>
<td>Shorts</td>
<td>Windcheater</td>
<td>Cap</td>
</tr>
<tr>
<td>Bordies</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. How many different outfits can I wear? List all possibilities. **Show your working and explain your answer in as much detail as possible.**

All students were interviewed approximately one week after the Part A tasks were administered. I had requested that students work on their own and that teachers should
avoid providing so much support that students who might not otherwise complete the task, were able to do so. Having done this, teachers contacted me via phone or email and mutually convenient times were arranged for the interviews. The staff at all school sites facilitated access to a withdrawal area in which to conduct the interviews.

Each student was interviewed individually in a quiet place within the school and each interview took approximately 40 minutes. The student was introduced to the interview’s purpose (for Part A) as “having a chat about their thinking behind their responses to the task they completed a week ago”. The student was first asked to read the question aloud, then retell the problem in their own words. This highlighted any issues relating to the reading and understanding of certain words as well as comprehension of the problem in general. Next, the student was asked to elaborate on what they had written in response to the task item and justify their thinking in relation to this response.

For Part B of the interview, each student was asked to complete at least one of the parallel tasks above in a think aloud manner. The student was asked to read aloud the question or stimulus and then retell the problem in their own words. If necessary, further discussion took place about the student’s understanding of the task. The student was subsequently invited to solve the problem and articulate their thinking out loud, that is, tell me what you are thinking and doing, as you do it. The support materials were available if necessary.

Throughout the interview I made notes. As indicated earlier, after each interview I spent 15-minutes reflecting on these, and elaborated on these while they were still fresh in my mind. Records took the form of the written student responses to the tasks and notes taken by me at the time. A sample is presented below. These notes, though not full transcripts, documented what I recorded at the time.

Example of field notes for Multiplicative Task interview

Hajar (ARN, Year 6)

Patterns with tiles
a. How many times can this pattern be made with 28 black tiles, 21 grey tiles and 6 white tiles?
Show all your working and explain your answer in as much detail as possible.
I: Can you retell the problem in your own words.
H: It is asking if the pattern can be made with 28 black tiles, 21 grey tiles and 6 white tiles.
I: Can you think of anything you could do now to answer the question
H: Add up all the tiles probably?
I: Would this be helpful…
I: What do we know?
H: How many black tiles, how many grey and how many white.
Sharing part a

a. How would you share the 2 sausages of playdough among 3 children?

Hajar partitioned two playdough sausages in response to the question.

H: I was going to cut into smaller pieces but I decided to make it easier
Hajar first cuts one sausage into 3 equal pieces and gave one piece to each, then repeated for the second sausage.

I: How much does each person get?

H: 2 pieces...probably a half, if I put them together.

I: Can you check?

Post-intervention Phase Multiplicative Task interviews

Student responses to selected SNMY final assessment task items were the focus for Part A, the reflective aspect of the interview. The class teacher administered SNMY final assessment tasks in late October 2005. Student responses were made available to me directly after administration, so that I could select items for discussion during Part A of the interview. Consistent with procedures for the Pre-I Phase interviews, specific items were selected on the basis of at least one of a number of the following factors: evidence of inability to make a reasonable start in solving the problem, and/or the evident strategy was of interest, ambiguous or needed further clarification. The SNMY final assessment tasks that prompted these response characteristics were 
Tiles, tiles, tiles, Stained glass windows, Tables and chairs and Butterfly house (see Appendix A, p. 2-18). To illustrate the types of items used in Part A of the interview, one of the tasks, Tiles, tiles, tiles is given below.

<table>
<thead>
<tr>
<th>Multiplicative Task interview Part A task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tiles, tiles</strong></td>
</tr>
<tr>
<td>Floor and wall tiles come in difference sizes. The basic tile is shown below.</td>
</tr>
<tr>
<td>![Basic tile image]</td>
</tr>
<tr>
<td>2 cm</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

a. How many basic tiles would be needed for an area of 6 cm by 4 cm?

b. How many basic tiles would be needed for an area of 27 cm by 18 cm?

c. If the length and width of the basic tile were increased by 2 cm, how many of the larger tiles would be needed to cover 1 square metre (100 cm by 100 cm)?

Show all your working so we can understand your thinking.

For Part B, the think aloud component of the interview, parallel versions of the extended task Tables and Chairs and the shorter task Tiles, tiles, tiles were designed: People sitting a – d and Tiles for the house a – c. These tasks are presented below and are also located in Appendix J (pp. 235-236).
### Multiplicative Task interview Part B task

**People sitting**

**a.** A rectangular table can seat 8 people. Draw one of these tables with the people sitting around it.

**b.** Draw a line of 4 of these rectangular tables placed end-to-end. How many people are able to sit at it?

**c.** How many people would be able to sit at 9 of these rectangular tables placed end-to-end?

**d.** How many of these rectangular tables would you need to seat 86 people?

---

### Multiplicative Task interview Part B task

**Tiles for the house**

Floor and wall tiles come in different sizes. The basic tile is shown below.

![Basic Tile](image)

**2 cm**

**3 cm**

**a.** Show or describe the area that 4 of these tiles would cover?

**b.** How many basic tiles would be needed for an area of 12 cm by 6 cm?

**c.** If the length and width of the basic tile were increased by 2 cm, would 120 of the larger tiles be enough to cover 50 cm by 50 cm? **Show all your working so we can understand your thinking.**

---

Consistent with the Pre-I Phase interviews, student responses to SNMY final assessment tasks for use during Part A of the interview, were analysed in order to prepare appropriate tasks for use during Part B of the interview. What was apparent in the at-risk student (intervention and non-intervention) responses to *Tables and chairs* and *Tiles, tiles, tiles*, were the diverse ways in which students responded to these task items: some responses indicated significant progress towards solving the task items efficiently, other responses indicated an inability to make a reasonable start to solve the problem, while other responses indicated use of a strategy that was vague or unclear. On this basis, the tasks *People sitting* and *Tiles for the house* were developed.

The final interviews were conducted in the same fashion as the initial interviews. The students’ classroom teachers administered the SNMY final assessment tasks. The teachers then contacted me via phone or email and mutually convenient times were arranged for the interviews. The staff at all school sites facilitated access to a withdrawal area in which to conduct the interviews.

Each student was interviewed individually in a quiet place within the school and each interview took approximately 40 minutes. The student was reacquainted with the interview’s purpose as “having a chat about their thinking behind their responses to the
task they completed a week ago”. For Part A of the interview, they were asked to read the question aloud, and retell the problem in their own words. This highlighted any issues relating to comprehension of the problem generally or particular words more specifically. I then asked the student to elaborate on what they had written in response to the task item and justify their thinking in relation to this response.

Each student was then asked to complete at least one of the unseen Part B tasks in a think aloud manner, again reading aloud the question or stimulus and then retelling the problem in their own words. If necessary, further discussion took place about the student’s level of understanding of individual words or the question itself. I then invited the student to solve the problem and articulate their thinking out loud, that is, *tell me what you are thinking and doing, as you do it.*

Consistent with the Pre-I phase interviews, throughout the interview I made notes. After each interview I spent 15-minutes reflecting on these, and elaborated on these while they were still fresh in my mind. Records took the form of the written student responses to the tasks and notes taken by me at the time.

### 3.5.2 Records associated with the Intervention Program

In ethnographic research, documents are traditionally viewed as part of the corpus of evidence in field research and constitute the “material culture”, for example, newsletters, curriculum material, and policy documents (Patton, 2002, p. 293). To understand the learning that took place during the Intervention Phase, it was necessary for me to maintain a record of the communication between myself and the students (Merriam, 1998). This manifested itself in three ways: documentation of planning, my reflections on students’ responses to the program, as well as the students’ responses to the program. As a result, there were three distinct records:

- detailed notes of the intervention program that were generated progressively via documentation of the teaching and learning sequences,
- my written post-session reflections, and
- workbooks maintained by students as they responded to the intervention program, that included work samples and written reflections on learning.

Each of these will be described in turn.
Notes from the Intervention Program

As indicated previously, the intervention program ran over the course of an 18 week period from May to October, 2005. During this time I worked three times per week with the nine ARI students. A detailed record of exactly what we did was maintained daily. The full intervention program is contained in Appendix D. An excerpt of the program is given below.

Intervention Program excerpt

SESSION 5: Thursday 2nd June, 2005

Focus
Efficient mental strategies for addition and subtraction
Towards a deeper understanding the place value system, ‘10 of these is 1 of these…’

INTRODUCTION ~ Mental strategies for addition, subtraction. Numbers in sequence

Resources
Jumbo playing cards, ace to 10
0-100 number cards

Purpose
Build on mental addition and subtraction strategies for numbers 1-10
Order numbers largest to smallest, smallest to largest for numbers under 100

Teaching and Learning Notes
Teacher and students sit in a circle. Teacher explains, “We are going to add 2 to the number shown on the playing card.” Teacher turns over card and models, “Eight and two more, ten.” Do orally as a group, repeat individually, moving round the circle of students. Repeat for ‘3 more’, then ‘1 less’, ‘2 less’. Hand out 6 to 8 number cards (showing a range of numbers under 100). Have students work together to order these from smallest to biggest, scramble and order from biggest to smallest. Encourage students to talk about their thinking as they complete the activity.

LEARNING FOCUS ~ Place value trading activity

Resources
Laminated place value chart showing ‘Thousands, Hundreds, Tens, Ones’, - one chart for each student

<table>
<thead>
<tr>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lots of MAB material
1 six sided dice* per pair of students
*can later be extended to dice with more than 6 sides

Purpose
To directly experience the structure of our base 10 number system
Understand the idea that ‘10 of these, is one of those’, e.g., 10 ones, is 1 ten

Teaching and Learning Notes
Students engage in trading activity with a partner, and take turns to throw the dice. If for example a 7 is the first throw, 7 ones are taken from the collection of MAB and placed in the ‘ones’ column. Repeat this process. As more than 9 ones is reached, students trade for a ‘ten’; e.g., 7 ones on the game board, a 5 is thrown, making 12 ones, trade 10 ones, so that 1 ten and 2 ones are now visible on the board. Trade for tens, hundreds as the collection grows. Teacher focuses the activity with key questions, “How many do you have now?”, “What number have you created?”, “Tell me about your number?”, “Can you say your number in another way?”

CONCLUSION ~ Reflective journal writing

Resources
Student work books

Purpose
End of week communication between teacher and students, to find level of student enjoyment and their perceptions of their learning.

Teaching and Learning Notes
Pose the following sentence starters:
The best thing about maths this week was… I learnt…
**Post-session reflections**

Observation, a necessary aspect of qualitative research (Patton, 2002), provided an opportunity for me as participant-observer to “represent a first-hand encounter with the phenomenon of interest” (Merriam, 1998). In line with the design experiment approach, the written observations assisted with the retrospective analysis. For this research, the phenomenon was the shift from additive to multiplicative thinking.

I observed the nine ARI students as we participated in the intervention program. I documented my thoughts daily, at the end of each learning session per school. I described what I observed and included what people said, student reactions to the session as well as my reactions. Below is an example of the post-teaching session reflection I wrote after session 9 on Tuesday 14th June in week 4 of the program for one site. The teaching focus had been a review of mental strategies, place-value understanding of four digit numbers, using ‘thinking strings’ and problem solving. An example of a ‘thinking string’ for 46 and 23 more might be “46, 56, 66, 69”. The ‘thinking string’ indicates the mental addition of 2 tens then 3 more.

**Example of a post-session reflection**

School B: Student absences meant I was able to spend time with Berrin and Ahmed on ‘make me’ cards, both a little unsure of make me 1 ten 9 ones. Berrin initially says 91, Ahmed then said 19. Spent time with other teen no’s. Berrin during cookie count unsure of 1 after 29. Used 0-99 chart as reference. Berrin and Ahmed decided that picture had 42 cookies and when it came to sharing Ahmed mentally calculated 22 each (close but couldn’t explain how). I: Ok let’s see if Ahmed is right. How will we check?
B: Share them out.
A and B do this by ones and kept loosing track of shares.
I: Can you think of a more efficient way?
B: By twos.
Which they did. They then counted their own share. Berrin with 20 and Ahmed with 22.
I: What’s the problem.
A: They’re not the same.
B: (asking Ahmed) Give me one.
Yes. Berrin beginning to keep track of numbers mentally, not necessarily reliant on recounting.

The post-session reflections written at the conclusion of all daily teaching lessons are contained in Appendix D.

**Student journals**

Prior to the intervention program, I purchased nine 96-page exercise books for students to record their work in. The provision of these workbooks served three purposes: to chronicle student responses to the intervention program to help track their progress over a given period of time, to keep our work together in the one place in a form that was easily
portable, and as a medium for communication between the student and myself. The journals were used for instances throughout the program that required the students to write or draw in some way and collected after each session to enable me to provide feedback as necessary. If a worksheet was used, these were pasted into the journal, though often the students maintained handwritten records. The students’ workbooks were retained as evidence for later analysis. Below is an excerpt of two consecutive pages from Cansu’s journal (Figure 3.4).

![Image of a journal page](image)

**Figure 3.4** Excerpts from Cansu’s learning journal (ARI, Year 6)

### 3.5.3 SNMY assessment tasks

Student responses to the SNMY Project initial and final assessments were accessed by this intervention study. As detailed earlier in section 3.3 of this chapter, the initial results identified both the at-risk students and SC students as being at the lower and upper ends of the LAF respectively. In addition to this, detailed scrutiny of the nature of student responses to task items informed the design of problems for administration during the Pre-I *Multiplicative Task* interview. The final SNMY Project assessment data, in addition to providing stimulus for the Post-I Phase *Multiplicative Task* interviews, provided a means of comparison against initial results to determine the extent of individual and cohort change in
relation to multiplicative thinking. All SNMY assessment tasks and scoring rubrics are located in Appendices A and B.

3.6 Analysis

The phenomenon addressed by this study is the transition from additive to multiplicative thinking in at-risk students in years 5 and 6. The study’s ultimate goal was to design, implement, and evaluate an intervention program in relation to multiplicative thinking, that was specifically designed to meet the needs of identified at-risk Year 5 and 6 students. In addition to data about the strategies students used to solve a range of tasks requiring multiplicative thinking, data relating to beliefs about teaching, learning and mathematics, came from a variety of sources and in a variety of ways (see 3.5.1). The following sections will explain the analysis that was used to interpret the interviews, the records associated with the intervention program, and the SNMY Project assessment data.

3.6.1 Individual Interviews

As described in 3.5.1 three individual interviews were administered in the Pre-I Phase and again in the Post-I Phase. Two of these interviews, the Drawing Task and the Card Sort were designed to elicit student beliefs about learning mathematics well. The Multiplicative Task interview identified the strategies students used to solve tasks requiring multiplicative thinking. How each of these interviews were analysed is described below.

Drawing Task interview

This data source required students to reflect on a situation in which they were learning maths well, then draw it. Following this, a discussion between myself and the student took place, where the student had opportunity to tell me about their picture. First layer analysis of drawings and interview notes collected during the Pre-I Phase employed an inductive approach to identify common elements (Lincoln & Guba, 1985, Merriam, 1998; Patton, 2002). There were three levels of analysis of the Drawing Task interviews: first, to inform the derivation of the Card Sort interview, second to identify features that students could actually draw, and third, to identify core ideas and relationships.

As indicated earlier, the initial analysis identified 12 focus words or descriptors: talking and discussing, problem solving, getting the answer, explanations, worksheets, the teacher, thinking, maths equipment, getting help, doing well, feeling good and group work. These words became the focus for the Card Sort interview. The process for determining these themes for the Card Sort was discussed in section 3.5.1.
To identify the range of features visible in the 22 student drawings, the features depicted in each diagram were described then grouped into four categories after extensive comparative analysis: objects, people, communication and mathematics content. For example, students drew typical classroom objects such as tables, chairs, worksheets, and the blackboard or white board. They also tended to include people in their drawings, often themselves, their friends or class mates and the teacher. In doing so, some students used speech bubbles with text to indicate classroom talk. They also indicated the nature of the mathematics content that they linked with learning maths well which generally included the multiplication tables, the four operations and fractions. A ‘student’ by ‘features’ matrix was constructed on the basis of scrutinising student drawings. An excerpt of the matrix generated from looking closely at the Pre-I Phase Drawing Task interview pictures is presented in Table 3.5 below. In the interest of clear and concise communication, this matrix will be presented as frequencies for each of the three student groups in Chapter 4.

### Table 3.5
*Pre-intervention Phase Drawing Task features matrix*

<table>
<thead>
<tr>
<th>Label</th>
<th>Student S1</th>
<th>Student S2</th>
<th>Student S3</th>
<th>Student S4</th>
<th>Student S5</th>
<th>Student S6</th>
<th>Student S7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OBJECTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tables</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>chairs</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White/blackboard</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>PEOPLE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>teacher</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>themselves</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>COMMUNICATION</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with others</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with self</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Subsequent analysis involved two distinct steps, one with the pictures and the other with the field notes, using a form of constant comparison (Patton, 2002). The photos of the student drawings were placed together in isolation from the related interview field notes. The photos of student drawings were then grouped according to common elements. The reasons for placing certain drawings together were noted. For example, speech bubbles were evident in the drawings of five students at the Pre-I Phase. These pictures were grouped together and were noted as forming the ‘dialogue’ element. Students working from a blackboard was evident in the drawings of six students at the Pre-I Phase. These pictures were grouped together and were noted as forming the ‘working from a blackboard’ element.
This grouping process continued until all student pictures had been ‘read’ and sorted. This sorting process of all student drawings was repeated a number of times until no new elements were detected.

The elements identified by the repeated sorting process were then grouped and organised in the form of a concept map. For example, the physical features of the classroom, could be grouped according to furniture, material, and people. Figure 3.5 below, an excerpt of the Pre-I Phase Drawing Task concept map, shows these elements.

![Concept map example](image)

**Figure 3.5** Concept map example

A similar process was undertaken with the interview field notes. The field notes were again carefully read and key elements identified. Those with similar key messages were grouped together. For example, two at-risk students noted the importance of finding out how they were doing. This element was labelled as ‘feedback’. This grouping process continued until all student field notes had been read and sorted. Again, this sorting process was repeated a number of times until no new elements were detected. The elements identified by the repeated sorting process were then grouped and organised in the form of a concept map.

The two concept maps generated separately via analysis of the pictures and interview field notes were then consolidated into one concept map. The purpose of this analysis was to illustrate the elements that emerged from the whole sample in response to the Drawing Task interviews (Jackson & Trochim, 2002). What resulted was a detailed cohesive picture of my interpretation of key messages arising out of the Drawing Task interviews (see 4.1.1 & 4.2.1).

**Card Sort interview**

The Card Sort interview required students to select nine words that they felt were important to learning maths well, and then to sort these according to their degree of importance for learning maths well against a diamond shaped pattern. There were five layers to the
‘diamond’ (see 3.5.1). A discussion then took place where students were asked to justify why they placed the words where they did. This data source was used twice, in the Pre-I Phase and again in the Post-I Phase. Two approaches to analysis were used: quantitative and qualitative. Each method will be described, beginning with the quantitative approach.

An Excel spreadsheet was used to enter the frequency each word was used at each layer of the card sort (Pre-I Phase, \( n=22 \); Post-I Phase, \( n=21 \)). This gave the total usage for each of the 12 words across the five possible layers. On this basis it was possible to calculate the percentage of the sample who accepted the word in their card sort and the percentage of students who did not. To ascertain the degree of importance for the words and phrases used by the student, the likelihood of each being placed at the first and second layers was calculated. This was calculated by summing the frequency for the first two layers and then dividing it by the frequency total for that word (that is, the sample space). For example, thinking was used by 21 of the 22 students in the Pre-I Phase Card Sort interviews. A total of 14 students had placed thinking at either the first or second layer of the sort. Hence 14 divided by 21 gave a likelihood of 67% for thinking being at the top two layers of the card sort for those students who had selected and used this word. This calculation was repeated for each word or phrase being placed at the bottom two layers (4th and 5th layers) of the card sort.

On the basis of these calculations, a ranked list of words from most important (with the greatest percentage of being placed at the top two layers) to least important (with the lowest percentage of being placed at the top two layers) for learning maths well was created. This was done for both the Pre-I and Post-I Phase administrations. Results of these analyses will be presented in the next chapter (4.1 & 4.2 respectively) in table form as well as descriptively in the form of a completed card sort.

Subsequent analysis of card sort field note summaries was conducted in the same way as described for the Drawing Task interview field notes (3.6.1). The Card Sort field notes were again carefully read and key ideas identified. Those with similar key ideas were grouped together. This grouping process continued until all student field notes had been read and sorted. Again, this sorting process was repeated a number of times until no new elements were detected. The ideas identified by the repeated sorting process were then grouped and organised in the form of a concept map.
**Multiplicative Task interview**

This interview involved students in a reflective discussion about their solution strategies to a range of already completed multiplicative tasks, as well the opportunity to complete an unseen though familiar task in a ‘think aloud’ manner. The field notes taken at the time of the Multiplicative Task interview were carefully read. Initially, in the case of the ARI students, analysis of the Pre-I Phase interviews identified what the students could and could not do. This resulted in the identification of starting points for teaching.

Subsequent analysis involved summarising the specific strategies used by each of the at-risk students (intervention and non-intervention) in response to each interview item. For comparison purposes, these summaries were recorded along side the specific strategies used by the SC students. For example, below is a summary of the specific strategies used by two ARI students during the Pre-I Phase interview for Patterns with Tiles a (section 3.5.1):

- Count by 3, seven times to get 21 tiles, count by 7 to 28 but unable to balance information and progress strategies to solution (Yousif p. 47 notes)
- Attempt to draw all, count all, (Dean p. 45 notes modelled pattern with components to represent tiles. Some students extend or build on the pattern to enlarge it, rather than repeat the pattern

The specific strategies for each task item were sorted and grouped according to common methods. For the example provided above, two different strategies were identified: skip counting and ‘make all, count all’. This comparative analysis was completed for all the Pre-I interviews and Post-I Phase interviews thereby making it possible to compare results of both administrations to identify student development over time in terms of multiplicative thinking.

### 3.6.2 Records associated with the Intervention Program

As indicated in 3.5.2, three records were generated throughout the Intervention Phase:

- the intervention program,
- post-session reflections, and
- workbooks maintained by students.

These records serve to cross reference the other sources of data collected and analysed throughout this intervention study, and will strengthen claims made throughout Chapters 4 and 5. For example, to illustrate the regular opportunity ARI students were given to document their thinking on given tasks, excerpts of their reflective writing pieces over time will be presented, together with my post-session reflections where I make specific comments about the students’ writing.
3.6.3 SNMY Project assessment data
As indicated earlier, the ARI, ARN and SC students completed SNMY Project assessment tasks in May 2004 and again in November 2005 (Appendix A). Scoring rubrics were used to assess their responses and locate their performance against the Learning Assessment Framework (LAF) (Appendix B). Comparison of performance against the LAF for each individual student for both administrations of the SNMY assessment tasks, enabled me to determine the degree of improvement made by the students.

3.7 Summary
This chapter has justified the theoretical rationale for the methodological approach taken by this intervention study. The research design was described and the phases of the study, the involvement of the participants, the data that was collected and how these were analysed, were detailed. The next chapter will report the results of the interviews administered in the Pre- and Post-I Phases, as well as the initial and final SNMY Project achievement data for the students involved in this research.
CHAPTER 4
RESULTS – BELIEFS & STRATEGIES

The previous chapter described the three phases of this intervention study within the context of design experiment methodology. The multiple sources of data, the purpose of each, how the interviews were administered and the data analysed, were described and justified in detail. This chapter will present the results derived from the analysis of the three interviews administered to all students during the Pre-intervention and Post-intervention Phases. This chapter will also report the beginning and end SNMY Project assessment results for the students involved in this study.

The literature review discussed the significance of the relationship between perceptions and behaviour in relation to learning mathematics (2.2.2.3). For the purposes of this research the terms beliefs and perceptions will be used interchangeably. In acknowledgement of this relationship, interviews were conducted to delve into student perceptions about the nature of mathematics and what it means to learn mathematics well as well as the strategies students use to solve multiplicative tasks. There were three interviews: the Drawing Task and Card Sort interviews that were designed to explore students’ views of learning mathematics well, and the Multiplicative Task interview that targeted the strategies students used to solve multiplicative problems. The interviews were administered to all students for whom Ethics forms had been completed (N=22). The sample comprised of three groups of students: the at-risk intervention students (ARI=9), the at-risk non-intervention students (ARN= 5) and the successful student cohort (SC=8). The interviews were documented and analysed as described in section 3.6.1.

The results of the Pre-intervention (Pre-I) and Post-intervention (Post-I) phases will be presented in Section 4.1 and 4.2 respectively. Within each, the results derived from the different data sources will be discussed in turn. That is, the Drawing Task, the Card Sort, and Multiplicative Task interviews and the SNMY achievement data respectively.

4.1 Pre-intervention Phase

The students involved in this study had been identified through the SNMY Project assessments as described in 3.2.1 and the three interviews were conducted between December 2004 and May 2005.
Although the results of the Drawing Task and Card Sort interviews were designed to explore students’ beliefs about learning mathematics well, they evoked qualitatively difference responses from the students. The descriptors used in the Card Sort were derived from the analysis of the Drawing Task interviews. The Drawing Task tended to reflect more public images of learning maths well as it located students in the social and physical space of the classroom. Whereas the Card Sort, which required the students to rank descriptors, encouraged students to reflect on what matters most in their mathematics learning. That is, the Card Sort interview involved more private views about learning maths well. This interview invited personal reflection independent of a physical image of themselves and/or others within a particular setting. The students were able to identify what matters to them and make judgements about what was least important and most important for learning maths well. This tended to generate a more private view of learning school mathematics although it did not necessarily exclude elements of the social and physical environment.

Accordingly, the results from the Pre-I Phase have been organised into four sections: perceptions about learning maths well in class, perceptions about what matters in learning maths well, the strategies students use to solve multiplicative tasks, and their achievement in relation to multiplicative thinking.

4.1.1 Perceptions about learning maths well in class
The purpose of the Drawing Task interview was to determine individual student beliefs about learning maths well. Students drew a picture based on the written stimulus, please think of a situation in which you are learning maths well and draw it. As indicated above, this invariably led students to locate themselves in the public space of the classroom.

The Drawing Task produced images of the students’ understanding of learning school mathematics which included a variety of elements such as furniture, the teacher, students, and teaching materials. The aspect of learning maths well called into mind images of how successful students perform in this environment.

What emerged from the student interviews regardless of cohort were three dominant themes about learning school mathematics: I learn maths well when I have the right answer, I learn maths well when I do it the right way, and I learn maths well when I talk about it with others. A distinguishing feature was the SC students’ capacity to articulate their thoughts in considerable more detail than the ARI and ARN students. Commonalities between groups will be described first. This will be followed by an elaboration on the manner in which students were able to communicate these ideas.
Commonalities in the Pre-intervention Phase

Student drawings about learning maths well and the related discussion elicited a range of beliefs about what was seen to be coincident with learning mathematics well. These were documented in both the students’ drawing and the field notes associated with the discussion. In addition to informing the development of the Card Sort interview, the analysis of the Drawing Task interviews (see 3.6.1) also identified both the specific elements that students chose to represent in their drawings, as well as the elements common to both the drawings and the field notes. The observable features that emerged from the student drawings will be presented first. This will be followed by the elements that emerged from the analysis of both the drawings and the associated field notes.

As Table 4.1 indicates, students tended to draw or identify a variety of classroom features across four main categories:

- **objects**, for example, items of furniture,
- **people**, for example, the teacher, themselves and/or other students,
- **communication**, for example, using speech bubbles to indicate talking with others, and
- **maths content**, for example, the multiplication tables.

Most of the student drawings ($n=19$) were dominated by images of the mathematics classroom. These included physical features such as typical school furniture and the curriculum content of mathematics classes. The student drawings generally depicted mathematics learning in relation to the four processes (addition, subtraction, multiplication and division). The students also showed people in their drawings. The teacher was frequently shown to be presenting work on the blackboard and/or communicating what to do and how to do it. It appeared that the students were all doing the same work. Table 4.1 below indicates the frequencies of these observable images by sample group.
Table 4.1

Pre-intervention Phase Drawing Task frequencies of observable features of student drawings

<table>
<thead>
<tr>
<th>Elements</th>
<th>ARI (n=9)</th>
<th>ARN (n=5)</th>
<th>SC (n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tables</td>
<td>6</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Chairs</td>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>White/blackboard</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Worksheets/books</td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Stationery</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Bed</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Computer at home</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>People</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher/adult</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Themselves (may incl. teacher)</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Student(s) (may incl. themselves)</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td><strong>Communication</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dialogue with others</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Dialogue with self</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Smiling</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Pointing/showing</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Hands up</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Maths Content</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number/4 processes</td>
<td>8</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Fractions</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Algebra</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The concept map presented in Figure 4.1, represents my interpretation of the elements students associated with learning maths well and how these elements relate to each other (see 3.6.1). The elements are indicated within the illustrative circles, and the arrows, show either how the element comprises other elements, or how this element might relate to other elements within the map. For example, the two main areas that were associated with learning maths well fell into two main groups: classroom and non-classroom situations. Within classroom situations, three sub-elements emerged: physical features, curriculum content related to mathematics, and ways of working. The themes do not appear directly in the concept map as they are over-arching notions derived from the drawings more generally. However, they are variously represented in the map as starred items. For example, in Figure 4.1, getting it right can be seen under the ‘feedback’ element where reference is made to correctness and success. Similarly, doing it the right way can be seen under the ‘teacher explanations’ element on the map where reference is made to step by step instructions.
Figure 4.1 Pre-intervention Phase Drawing Task concept map – elements associated with learning maths well
Student views in relation to the physical features of the school mathematics classroom and mathematics content generally represent what might be expected. However, the concept map also illustrates the complex nature of student beliefs in relation to the ways in which they engage with learning mathematics. Four ways of working were identified: the teacher and the whole class, the teacher with the individual, the student with other students and students working alone.

The key themes that emerged from the analysis of student drawings and associated field notes from the Drawing Task interviews are summarised by the following three statements: I learn maths well when I have the right answer, I learn maths well when I do it the right way, and I learn maths well when I talk about it with others. These statements were not uncommon nor unexpected, however the significance of these findings will be discussed in section 5.2.2. The word “it” in these statements is defined to indicate school mathematics. These themes relating to learning maths well are now discussed in detail. As mentioned earlier, pseudonyms have been used where appropriate.

Theme 1: I learn maths well when I have the right answer

Evidence about having the right answer emerged from the written and oral examples provided by the students. A consistent view held by the students interviewed was the belief that learning mathematics well was associated with correctly solving a maths question or by knowing the right answer. The maths question depicted was often in the form of a simple written algorithm. This theme was evident both visually, appearing in student drawings; and orally during the discussion that took place after the drawings were completed.

Having the right answer was related to students’ ability to solve and the act of solving computation questions by knowing the basic facts and providing correct answers. The students provided examples of closed questions involving addition, subtraction, multiplication and division. These were often presented on a blackboard or appeared as an algorithm on a worksheet. The following three responses from Adir (Figure 4.2, ARI, Year 6), Ibrahim (Figure 4.3, ARN, Year 6) and Chris (Figure 4.4, SC, Year 5) illustrate this. Each example provides an image of the student drawing and a sample of the field notes associated with the subsequent discussion. As described in 3.5.1, the field notes are not verbatim transcripts but records key comments made by the student at the time of the interview. In the extract, ‘I’ refers to the interviewer, ‘S’ refers to the student. In the following discussion, the numbers in the first column refer to the field note line number which may or may not be included in the extract.
First Adir draws a black/white board and his teacher writing on it. He says she is teaching us times. Adir then admits to not knowing ‘divided bys’ but knows times, plus and minus…

I: These three boxes, what are they?
S: Plus, times, minus.
I: Where are they?
S: In my head.
I: Where are you?
S: In three green figures and says they are ‘me learning times, me learning minus and me learning plus’.

In Adir’s drawing, written algorithms with correct answers appeared on the blackboard and on the student worksheets. These were identified by Adir as “times…divided bys…plus and minus”. His response suggests that learning maths well is associated with knowledge of these facts (lines 5-6). Dean (ARI, Year 6) also mentioned the need to study the times tables “until you know them” (see Appendix F, p. 109).
Ibrahim (ARN, Year 6)

Extract from field notes (10.12.2004)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>S: Times tables, first 8, 16 then I close book and I practice and I got better.</td>
</tr>
<tr>
<td>9</td>
<td>I: Where is this happening?</td>
</tr>
<tr>
<td>10</td>
<td>S: Our class…I showed the teacher my paper and she said ‘good’.</td>
</tr>
</tbody>
</table>

Figure 4.3 Ibrahim’s drawing and field note summary

Ibrahim also implied the need to know basic facts (lines 8-9) when learning maths well. He suggested that to achieve this, practice was necessary. The role of practice will be discussed in the next section in relation to the theme, ‘doing it right’.

Chris (SC, Year 5)

Extract from field notes (10.12.2004)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>I: Tell me about your drawing.</td>
</tr>
<tr>
<td>3</td>
<td>S: It’s when no one’s speaking, you can speak…but whisper…and when no one’s being silly…you can concentrate.</td>
</tr>
<tr>
<td>5</td>
<td>I: Who else needs to be around? What else do you need?</td>
</tr>
<tr>
<td>6</td>
<td>S: Someone next to you – to ask, or ask a teacher if you don’t understand.</td>
</tr>
<tr>
<td>7</td>
<td>I: If teacher/friend can’t help you what then?</td>
</tr>
<tr>
<td>8</td>
<td>S: If there’s a lot of questions you could skip it.</td>
</tr>
<tr>
<td>9</td>
<td>I: Where’s the maths in you drawing?</td>
</tr>
<tr>
<td>10</td>
<td>Chris explains that there are questions on paper with an example to help e.g., ‘213 divided by 3’.</td>
</tr>
</tbody>
</table>

Figure 4.4 Chris’ drawing and field note summary
Chris explained that the mathematics work represented in his drawing was him completing a “division sum” (lines 10-11). Like Ibrahim, Chris also suggested that practice, by doing more than one question (line 8), was associated with learning maths well.

Implicit in the theme about having the answer right, is the idea of success. For instance, Candi (ARN, Year 7) mentioned how happy he felt when he had improved his score from 52 to 79 correct when completing a time-restricted multiplication tables grid. Also, James (ARI, Year 6) expressed that he was “not good at maths” and Douha (ARI, Year 6) reported that she was not “good at (her) times tables” (Appendix F, p. 111-112). This suggests that learning maths well is indicated by being good at it. In addition to solving written algorithms, Chris’s response also included comments about his preference for quiet conditions to maximise concentration (lines 3-4). This will be discussed later in relation to the theme talking about it with others.

Theme 2: I learn maths well when I do it the right way
Evidence about doing it the right way also emerged from the written and oral examples provided by the students. In addition to the focus on having the right answer, many of the students consistently associated learning maths well with doing it a certain way by performing procedures as explained by the teacher. In the student drawings the teacher was frequently depicted and/or described as being positioned at the blackboard telling the students ‘what to do’ and ‘how to do it’, while the students were seated at tables completing the set work. In addition to Adir’s response shown earlier (Figure. 4.2), the following responses from Cansu (Figure 4.5, ARI, Year 6), Andrew (Figure 4.6, ARN, Year 7), Kalil (Figure 4.7, SC, Year 6) and Sandy (Figure 4.8, SC, Year 7) provide examples of this.

Cansu (ARI, Year 6)
Figure 4.5 Cansu’s drawing and field note summary

As suggested by the Pre-I Phase concept map presented earlier (Figure 4.1), learning maths well was associated with the need for students to both practice the work and ask questions. Cansu associates learning maths well with asking questions so she “can learn more” (line 4-5).

Andrew (ARN, Year 7)

Figure 4.6 Andrew’s drawing and field note summary

The students’ responses suggest that the ‘teacher’ is central to this theme about doing it right. Cansu and Andrew (Figures 4.5 & 4.6 above) reported that the teacher was
communicating to the whole class while standing near the blackboard. In the case of Cansu, even though other students are absent from her drawing, she mentioned them during the subsequent discussion and suggested that they would all be doing the same thing (Figure 4.5, lines 17-19). In both these students’ drawings, the teacher is located at the blackboard, and the interview discussion referred to all the other students completing the maths work in their workbooks.

Kalil (SC, Year 6)

Although the teacher is absent from Kalil’s picture, the teacher is directly implicated in his scenario throughout the discussion. In Kalil’s case, the teacher is “telling” and according to Kalil, when the teacher tells, “you learn” (line 9).
**Sandy (SC, Year 7)**

![Illustration of Sandy](image)

**Extract from field notes (10.12.2004)**

| 3 | S: This picture is what I see...she's (the teacher) teaching the basics first. Later going on to harder ones. Showing us...how to do fractions. |
| 4 | I: What is it about this situation that is helping you learn maths well? |
| 5 | S: Writing up the question and she goes through it...shows us step by step a few times so we remember it... |

**Figure 4.8 Sandy’s drawing and field note summary**

Sandy (and Andrew, see earlier Figure 4.6) explicitly detailed the teacher’s “how to” approach, the blackboard being the tool for communicating this to all students in the classroom. Learning maths well was associated with procedures, which could include the teacher's explanations of what to do and how to do it. Associated with this were student behaviours such as practising and asking questions. The students recognised that learning maths well could be hindered by not understanding the work. To address this, students recognised that interactions with others supported learning maths well.

**Theme 3: I learn maths well when I talk about it with others**

Although some students indicated that working alone was associated with learning maths well (for example, Chris, Figure 4.4, lines 3-4), and that classroom talk could be a source of distraction (for example, Ahmed, Figure 4.9, line 41), students commonly acknowledged that learning maths well was associated with talking about it with others. This dialogue occurred between the student and teacher (or other adults), as well as between the individual student and other students. The purpose of these interactions seemed to be linked to improving student understanding of what to do and how to do it.

Andrew’s drawing illustrated the teacher giving explanations to the whole class (see earlier Figure 4.6, lines 7-9, 21). He suggested that teacher explanations were provided as a way to address student difficulty. The following response from Ahmed (Figure 4.9, ARI,
Year 6) and Chris (SC, Year 5) (see Figure 4.4), suggest that learning maths well was also associated with the teacher being available to ask for and provide assistance.

*Ahmed (ARI, Year 6)*

![](image)

<table>
<thead>
<tr>
<th>Extract from field notes (30.03.2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 I: What do you like about maths?</td>
</tr>
<tr>
<td>31 S: Times, and plusses…and take away.</td>
</tr>
<tr>
<td>32 I: What do you use?</td>
</tr>
<tr>
<td>33 S: Hands, paper, that’s all.</td>
</tr>
<tr>
<td>34 I: Where does this happen?</td>
</tr>
<tr>
<td>35 S: In class.</td>
</tr>
<tr>
<td>36 I: Who else is there?</td>
</tr>
<tr>
<td>37 S: The whole grade does it.</td>
</tr>
<tr>
<td>38 I: What are they doing?</td>
</tr>
<tr>
<td>39 S: The same thing as me.</td>
</tr>
<tr>
<td>40 I: When you all do the same thing, this helps you to learn maths well?</td>
</tr>
<tr>
<td>41 S: If they’re doing something else and they talk, I get mixed up.</td>
</tr>
<tr>
<td>42 I: What else helps?</td>
</tr>
<tr>
<td>43 S: If you don’t know what to do go tell teacher.</td>
</tr>
</tbody>
</table>

*Figure 4.9 Ahmed’s drawing and field note summary*

Most students indicated during the interview discussion the importance of seeking assistance from the teacher when “you don’t understand” (e.g., Chris, Figure 4.4, line 6) but they also suggested that the teacher was not the only source of support.

When discussing learning maths well, students frequently commented that their friends and their parents were available to support their learning. This suggests that they recognise the need to interact with or have access to more knowledgeable others. The next two responses from Douha (Figure 4.10, ARI,
Year 6, and Hajar (Figure 4.11, ARN, Year 6) illustrate instances of students supporting each other.

**Douha (ARI, Year 6)**

![Douha's drawing and field note summary](image)

<table>
<thead>
<tr>
<th>Extract from field notes (30.03.2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 S: I forgot to do (draw) the teacher…she’s got a worksheet.</td>
</tr>
<tr>
<td>13 I: What helps you to do these worksheets well?</td>
</tr>
<tr>
<td>14 S: If 10 plus 9, I sometimes use my fingers or a scrap piece of paper and make 10 lines and 9 lines and count them up…</td>
</tr>
<tr>
<td>16 I: Are you working on your own?</td>
</tr>
<tr>
<td>17 S: When I do this?</td>
</tr>
<tr>
<td>18 I: Yes.</td>
</tr>
<tr>
<td>19 S: I sometimes have a friend to help and I help a friend.</td>
</tr>
<tr>
<td>20 I: Does that help you to learn maths well?</td>
</tr>
<tr>
<td>21 S: Sometimes, depends, if ‘clues’ then yes, but if she gives me the answer then no.</td>
</tr>
</tbody>
</table>

**Figure 4.10** Douha’s drawing and field note summary

**Hajar (ARN, Year 6)**

![Hajar's drawing](image)
Hajar draws a girl (herself) on the left of the page smiling and adds another girl ‘my friend’ playing dominos (fraction matching) then adds another friend to the picture. H: Sometimes the teacher puts us in mixed groups. I: What does this mean? H: Boys and girls. Hajar adds windows, doors and tells me ‘we’re at the front of the room near the blackboard’. Hajar tells me that it is better on the floor because the chairs aren’t always comfortable and that there is more space out the front. She adds desks and on the desks, draws pencil case, drink bottle and ruler/pen etc. H: When I work by myself I get mixed up…just a little mistake with the numbers.

**Figure 4.11** Hajar’s drawing and field note summary

Hajar’s (ARN, Year 6) response provides detail about the classroom environment in both her drawing and the discussion about her picture. She reports working in groups and the support that this gives her (line 15).

The following two responses from Hadi (Figure 4.12, ARI, Year 6) and Lynda (Figure 4.13, SC, Year 6) illustrate the role of parental assistance. While the mention of parental support in the context of the family home was not a common feature of the Drawing Task interviews, both these students described situations where mathematics work was provided by a parent for the student to complete at home.

**Hadi (ARI, Year 6)**

S: The ones I don’t know, I learn at home. I probe further about home and S tells me that her mum has a board and provides sums for her to do…

**Figure 4.12** Hadi’s drawing and field note summary
Then Lynda tells me about her dad - retired teacher - bringing home worksheets. …
S: My dad brings home worksheets from school, they’re helping me to practice.
I ask about the worksheets further, Lynda recalls that her dad teaches prep? 1? 2? And
Lynda is about 5/6 yr old. She thinks the sheet has 1 times and a ‘little bit of the 2 times
table’.
I: Why are you learning well in this situation?
S: Dad is helping me…
I: How?
S: Brings home, not just maths, but books, to help me read. When I started school – I’d
chat to him – about maths at school.

*Figure 4.13* Lynda’s drawing and field note summary

Students from all groups spoke of the importance of discussion that occurred while
seeking assistance during mathematics lessons. The interactions between students appeared
to be stimulated by the need for the answer. However, the students seemed to recognise
that being guided towards the answer was preferable to being told the answer. Responses
from Douha (see earlier Figure 4.10, ARI, Year 6), Kai (Figure 4.14, SC, Year 6), Adam
(Figure 4.15, SC, Year 6) and Alison (Figure 4.16, SC, Year 6) are examples of this. In
particular, Adam and Alison’s responses place emphasis on communication by sharing
ideas with other students (see Adam, lines 6-15) and debate (see Alison, lines 14-15). The
social nature of learning and its impact on student improvement will be addressed in
Chapter 5.
### Extract from field notes (10.12.2004)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>I: Why did this situation work for you?</td>
</tr>
<tr>
<td>7</td>
<td>S: If a plus b equals whatever, if a plus c equals whatever then this equals that. Because it also helps you…when you do stuff like that at secondary school so then Dad showed me a heap of problems.</td>
</tr>
<tr>
<td>8</td>
<td>Kai gave example on drawing x plus y plus z equals 10 etc.</td>
</tr>
<tr>
<td>9</td>
<td>I: But why?</td>
</tr>
<tr>
<td>10</td>
<td>S: Better with maths problems in equations that helps. Dad helped, showed me division with fractions.</td>
</tr>
<tr>
<td>11</td>
<td>I: Dad important, why?</td>
</tr>
<tr>
<td>12</td>
<td>S: He doesn’t tell me that answer, encourages me to find the answer.</td>
</tr>
</tbody>
</table>

*Figure 4.14* Kai’s drawing and field note summary

---

### Adam (SC, Year 6)
Without prompting, Adam, having read the direction at the top of the page, states ‘I think it should be fun so you can absorb it…’

S:…if they’re (students) happy and it sinks in…

S:…it’s better to be social about it so you can share what you’ve learnt and talked about and give it to someone else…

S:…it’s better to talk about it instead of (a teacher saying) shut up and be quiet…

He’s talking while drawing and I then ask him, where is this happening?

S: In a very happy classroom…hard to find…because people are either high on the social chain or low on the social chain.

I questioned Adam further about what he meant, he felt that ‘they either think that they’re too good’ (referring to those high on the social chain). He perceived himself low on the social chain and that people see him more as ‘a tool than a person’. People come to Adam for answers not help, but Adam would rather help. He expressed that it is rather ‘sad when students can’t do the work.’

**Figure 4.15** Adam’s drawing and field note summary

*Alison (SC, Year 6)*

The marks on the blackboard represent the teacher’s writing and a ‘diagram to help explain’. The teacher is shown pointing to the writing. Another student is added and a non-descript speech bubble is placed near the teacher’s mouth. Alison tells me the teacher is answering a question. Exercise books and pencils etc are added to the students.

I asked Alison to tell me about her picture.

S: In a classroom and the teacher is explaining and some students are working, some are listening and some are listening while working. Some students are commenting (note student with hands up in the drawing) on what the teacher and other students have said…if they agree or not…by putting up their hand…

**Figure 4.16** Alison’s drawing and field note summary
Summary

In summary, the following themes resulted from analysis of the Pre-I Phase Drawing Task interviews from the AR (intervention and non-intervention) students and SC students. Students believed that they learnt maths well when they:

- have the right answer,
- do ‘it’ the right way, and
- talk about it with others.

What appeared to discriminate among the students, was the capacity of the SC students to articulate their thoughts in greater detail in comparison to the AR (intervention and non-intervention) students. The SC students also tended to represent a higher level of mathematics content. These differences are explored in the next section.

**Differences in student responses in the Pre-intervention Phase**

The SC students tended to articulate their thoughts and justify their opinions in greater detail than the AR students. Probing questions such as “How…?”, “Why…?” and “Tell me more about…”, were featured throughout the interviews with all students. The SC students demonstrated a greater capacity to elaborate on their thoughts and opinions as a result of my probing. The responses from Adam (see earlier Figure 4.15, lines 7-15) and Kai (see earlier Figure 4.14, lines 4-9) and Jessica (Figure 4.17 below), (SC students) are good examples of this.

*Jessica (SC, Year 6)*
Extract from field notes (10.12.2004)

1. S: What type of situation?
2. I: Depends on you.
3. S: Do you mean addition, subtraction etc?
4. I: Can be but doesn’t have to be.
   …
6. Think time by Jessica.
7. Then Joanne starts drawing.
8. I: Tell me about your picture.
10. I: Why is this situation helping you learn maths well?
11. S: I like working from whiteboard or blackboard in comparison to worksheet…more comfortable…I think the teacher knows what they’re doing where as on a worksheet it is hard to tell.
12. I: The teacher is important?
13. S: Yep because in order for me to learn I need someone who knows what they’re doing.

Figure 4.17 Jessica’s drawing and field note summary

In contrast, it was not unusual for responses from the AR students to include single word answers, very short sentences and/or an unwillingness to pursue a certain idea. Previously included responses from Adir (Figure 4.2), and Ahmed (Figure 4.9), and the responses from Berrin (Figure 4.18) and James (Figure 4.19) below, all ARI students in Year 6, exemplify this inability or reluctance to articulate reasons. For example, Adir and Ahmed (ARI, Year 6) could identify an activity, such as, the teacher is teaching, but when probed for more detail, they were unable to justify or clarify what this meant in relation to learning maths well. Typically, they would respond with “I don’t know”.

Berrin (ARI, Year 6)

Extract from field notes (8.12.2004)

5. She drew herself centred with a big smile on her face and draw plus, minus and times symbols around herself. She told me these were ‘plus, take away and time table’. Berrin then added ‘divided by’.
6. I: What are you doing with the plus, times, take away…
7. S: Learning them.
8. I: How?
9. S: In my head (mentions brain)… on a piece of paper…maths…
10. I: So you like working with maths on paper?
11. S: Yes.
12. I: Why does working this way work for you?
13. S: Because you learn more.
14. I: Why?
15. S: You can use your hands to count…use counters…blocks…little toys like farm toys…

Figure 4.18 Berrin’s field note summary
As might be expected, the other difference between the AR students and the SC students was the complexity of mathematics content they referred to. Some of the SC students referred to more complex ideas such as prime numbers and algebra (see Kalil, Figure 4.7 & Kai, Figure 4.14). The AR students typically referred to low level operations with number using single digits. For example, Douha’s response (see Figure 4.10, lines14-15) described counting on her fingers to calculate ‘10 plus 9’. Berrin’s response (Figure 4.18) suggests that learning maths well was viewed in terms of successfully solving simple addition and subtraction problems. She, like Douha, explained that “You can use your hands to count, use counters, blocks, little toys like farm toys” (line 17-18). This focus on addition and subtraction and associated strategies will be explored further in 4.1.3 and Chapter 5.

These differences almost certainly reflects the degree of students’ knowledge of mathematics and confidence when learning. The SC students are students who have achieved well in relation to the school mathematics expected for this level. In doing so, the students have access to, understand, and use language appropriate to this level of attainment suggesting that another explanation may be that the AR students simply do not have access to the language skills necessary to communicate their thoughts about learning maths well.

The next section will continue to explore the students’ perceptions of learning maths well by reporting the results of the Card Sort interviews.
4.1.2 Perceptions about what matters in learning maths well

The Card Sort interviews were designed to elicit student beliefs about learning mathematics well in a different but related way to the Drawing Task. The Card Sort interview required students to select nine descriptors they believed important for learning maths well and then rank these in order of importance in a five-layered diamond shaped pattern (see 3.5.1). A discussion then took place where students were asked to justify why they placed the words where they did. As mentioned earlier, this led students to focus on more personal or private views about learning maths well. Students were provided with 12 focus descriptors (talking and discussing, problem solving, getting the answer, explanations, worksheets, the teacher, thinking, maths equipment, getting help, doing well, feeling good and group work) but were also given opportunity to add words to this collection prior to sorting if they felt key ideas were missing. The interview was conducted with all 22 students.

The analysis of the Card Sort interviews (see 3.6.1) identified the descriptors most commonly ranked from most important to least important as well as the key reasons for what made these words important. Results indicate that the Card Sort interviews confirmed and extended the results of the Drawing Task interviews. Results in the Pre-I Phase suggest that all students interviewed recognise personal processes and actions lead to an outcome. The recognition of ‘process and actions’ reflect the Drawing Task theme I learn maths well when I do it the right way. The ‘outcome’ related to the Drawing Task theme I learn maths well when I have the right answer. There were also some interesting variations relating to the relative importance of having the right answer as well some differences between the AR students and the SC students.

The following sections will identify and illustrate similarities and variations (respectively) in the views of the ARI, ARN and SC students.

Common themes in the Pre-intervention Phase

The analysis of the Card Sort interviews (see 3.6.1) identified the descriptors most commonly ranked from most important to least important. In addition to this, a constant comparison and sorting process of the field notes identified key reasons for what made these words important. The ranked list of descriptors will be presented first, ‘1’ being most important through to ‘12’ being least important. This will be followed by the reasons that emerged from the analysis of the field notes.

While all 12 descriptors were selected at some point during the Card Sort interviews, thinking, talking and discussing and problem solving were selected by at least 19 of the 22 students
interviewed and feeling good, worksheets and getting the answer were discarded from the card sort by about 50% the sample. In terms of importance, the descriptors that were valued highly were thinking, talking and discussing and problem solving. The descriptors with low importance were getting the answer, doing well, worksheets and maths equipment.

Table 4.2 shows the frequency of descriptor use by card sort layer and the proportions of 1st/2nd layer selection (most important) in relation to the 4th/5th layer selections (least important) that were used to determine the overall ranking of each word in terms of importance.

**Table 4.2**
Pre-intervention Phase Card Sort frequency of focus word by layer & overall rank order for all students (n=22)

<table>
<thead>
<tr>
<th>Card Sort Layer</th>
<th>Total</th>
<th>Proportion</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>7</td>
<td>3</td>
<td>66.7</td>
</tr>
<tr>
<td>Explanations</td>
<td>1</td>
<td>2</td>
<td>53.3</td>
</tr>
<tr>
<td>The teacher</td>
<td>2</td>
<td>4</td>
<td>40.0</td>
</tr>
<tr>
<td>Problem solving</td>
<td>2</td>
<td>6</td>
<td>36.8</td>
</tr>
<tr>
<td>Talking &amp; discussing</td>
<td>1</td>
<td>6</td>
<td>35.0</td>
</tr>
<tr>
<td>Feeling good</td>
<td>1</td>
<td>2</td>
<td>30.0</td>
</tr>
<tr>
<td>Getting help</td>
<td>1</td>
<td>3</td>
<td>28.6</td>
</tr>
<tr>
<td>Group work</td>
<td>1</td>
<td>2</td>
<td>18.8</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>1</td>
<td>1</td>
<td>18.2</td>
</tr>
<tr>
<td>Doing well</td>
<td>1</td>
<td>1</td>
<td>15.4</td>
</tr>
<tr>
<td>Worksheets</td>
<td>0</td>
<td>1</td>
<td>10.0</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>0</td>
<td>1</td>
<td>6.7</td>
</tr>
</tbody>
</table>

Figure 4.20 below presents an alternate way of illustrating this ranked list in the form of a card sort arrangement. Those descriptors ranked beyond the fifth layer (rank 10, 11, 12) appear outside and below the card sort diamond.

**Figure 4.20** Pre-intervention Phase Card Sort: whole sample ranking (n=22)
At the time of the Card Sort interview administration, each student had the opportunity to indicate words relating to learning maths well that they believed were missing from the 12 descriptors provided. Not all students took this opportunity, however, 11 of the 13 students included their own descriptors in their card sort arrangement. Table 4.3 below, lists these by category, the students who used them, and the card sort layer where they were placed (indicated by the numeral in parenthesis after each student’s name).

### Table 4.3
**Pre-intervention Phase Card Sort: additional words**

<table>
<thead>
<tr>
<th>Student identified descriptor</th>
<th>ARI (n=9)</th>
<th>ARN (n=5)</th>
<th>SC (n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maths Content</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>The times tables</em></td>
<td>Berrin (1)</td>
<td></td>
<td>Kalil (3)</td>
</tr>
<tr>
<td><em>The four processes</em></td>
<td>Berrin (2), James (3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Behaviours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Show working out</em></td>
<td>Hadi (4)</td>
<td></td>
<td>Kai (1)</td>
</tr>
<tr>
<td><em>Learning</em></td>
<td>Hadi (2)</td>
<td></td>
<td>Kai (3)</td>
</tr>
<tr>
<td><em>Listening</em></td>
<td>Dean (1)</td>
<td></td>
<td>Kai (3)</td>
</tr>
<tr>
<td><em>Asking questions</em></td>
<td></td>
<td>Sandy (3)</td>
<td></td>
</tr>
<tr>
<td><em>Remembering</em></td>
<td></td>
<td>Kai (3)</td>
<td></td>
</tr>
<tr>
<td><em>Concentrating</em></td>
<td></td>
<td>Hajar (4)</td>
<td></td>
</tr>
<tr>
<td>Qualities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Being smart</em></td>
<td>Adir (4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Confidence</em></td>
<td>Adir (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Willingness</em></td>
<td></td>
<td>Adam (1)</td>
<td></td>
</tr>
</tbody>
</table>

Some students (3 ARI & 1 SC) felt that content in relation to school mathematics was missing from the Card Sort and as a result identified the ‘multiplication tables’ and one or more of the ‘four processes’ (addition, subtraction, division and multiplication). Personal qualities were identified by three students (2 ARI & 1 SC). In particular, Adir (ARI, Year 6) linked ‘confidence’ with persistence and ‘being smart’ directly with success or failure, and being able to answer a question quickly, for example,

> If you don’t be smart you fail, if you are smart you get to a higher level, and you know the answer straight away, quick (Adir, Appendix H, p. 179)

Adam (SC, Year 6) justified his selection of ‘willingness’ as most important in the following way:

> If you are not willing, it will take longer to learn, you will block out, tune out and talk to people (and become distracted). If this, then you are not learning because you are not listening. (Adam, Appendix H, p. 171)
An additional five students (3 ARI, 1 ARN, & 2 SC) identified behaviours associated with learning that in their view were missing from the given set: to show your working out, listening, asking questions, remembering, concentrating, and learning in general.

Student responses to the Card Sort interview will be presented in the next section to elaborate on the key message that emerged from the Card Sort interviews in the Pre-I Phase: for learning maths well, personal processes and actions are important. The student responses will also provide further evidence of the themes that emerged out of the Drawing Task interviews in the Pre-I Phase: I learn maths well when I have the right answer, do it right and when I talk about it with others.

Theme: When learning maths well, personal processes and actions are important

The Card Sort interviews provided students with the opportunity to identify what they believed to be associated with learning maths well and to rank these in order of importance. While similar themes emerged from both the Drawing Task and Card Sort interviews, the results of the Card Sort reveal that students recognise that personal processes and actions, such as thinking and explanations, are important for learning maths well. These are ‘personal’ in that they require personal effort to undertake, as opposed to more external actions, such as completing a worksheets or using maths equipment. To illustrate this point more clearly the ranked list of 12 descriptors are repeated below from most important to least important. The descriptors that signify various personal processes appear in italics and feature in the first half of the list.

- Thinking
- Explanations
- The teacher
- Problem solving
- Talking and discussing
- Feeling good
- Getting help
- Group work
- Getting the answer
- Doing well
- Worksheets
- Maths equipment

I had anticipated that getting the answer would be considered highly important for learning maths well. What I had not expected was the relative low importance of getting the answer in the Card Sort rank. A possible reason that getting the answer was ranked nine in the Pre-I
Phase Card Sort interviews is that the students interviewed tended to recognise the role of personal processes over actions in learning maths well. For example, behaviours such as thinking and talking and discussing and problem solving were viewed as important means by which to ‘get the answer’.

The result that thinking was at the top of the ranked list is interesting. When justifying inclusion of thinking in the card sort, students stated that thinking enabled “getting the answer” and helped to “work out the answer”. The purpose of explanations was similar to the purpose identified for thinking. The role of talking and discussing was to “get the answer”, “share the answer”, and “show other ways to get the answer”.

Students from all three cohorts spoke of ‘getting the answer’ when elaborating their reasons for their choice and placement of 10 out of the 12 descriptors, these being thinking, explanations, the teacher, problem solving, talking and discussing, feeling good, getting help, group work, worksheets and maths equipment. For instance, Yousif (see earlier Figure 4.21) referred to ‘getting the answer’ when justifying the placement of four other descriptors: problem solving, explanations, maths equipment and talking and discussing. Andrew (see earlier Figure 4.22) also made reference to ‘getting the answer’ when justifying the inclusion of thinking, getting help, explanations and talking and discussing. The descriptor getting the answer was included in the third layer of Alison’s card sort (see earlier Figure 4.23), and she also mentioned the answer when discussing the inclusion of thinking, group work, and worksheets.

This continues to support the notion that in terms of school mathematics, learning maths well is about getting it right, and that success is dependent on getting the answer or an answer. In addition to this, the students appreciate that personal effort is associated with learning maths well.

The following examples from Yousif (ARI, Year 6), Andrew (ARN, Year 7), and Alison (SC, Year 7), Figures 4.21-23 respectively, are presented to illustrate instances where at least two of the top four ranked descriptors overall (thinking, explanations, the teacher; and problem solving) appear in the first or second layers of their own Card Sort, and the four lowest ranked descriptors (getting the answer, doing well, worksheets and maths equipment) were placed on the bottom of the Card Sort or not used at all.

Each example provides an image of the student card sort arrangement and the field notes associated with the discussion. As described in 3.5.1, the field notes are not verbatim transcripts but records of key features and comments made by the students at the time of the interview. The comments made by students are italicised and the key features appear as
non-italicised text. In the second column, ‘I’ refers to myself as interviewer and ‘S’ refers to the student.

Figure 4.21 below illustrates Yousif’s ranking of the nine descriptors he selected.

**Yousif (ARI, Year 6)**

<table>
<thead>
<tr>
<th>First Layer</th>
<th>Second Layer</th>
<th>Third Layer</th>
<th>Fourth Layer</th>
<th>Fifth Layer</th>
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</thead>
<tbody>
<tr>
<td>Prob. solving</td>
<td></td>
<td></td>
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<tr>
<td>Thinking</td>
<td>Explanations</td>
<td></td>
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<tr>
<td>Maths equip.</td>
<td>Talk &amp; discuss</td>
<td>Group work</td>
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<tr>
<td>Doing well</td>
<td>Getting the answer</td>
<td></td>
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<tr>
<td>Worksheets</td>
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</tbody>
</table>

Extract from field notes (12.05.2005): Reason(s) for selecting

| Problem solving | *Cos helps you get the answer. I asked Yousif to elaborate about how this helps, but he had trouble, although mentioned work it out.* |
| Thinking | *So you get smarter and learn.* |
| Explanations | *If someone explains to you they explain the answer then you get the other answers.* |
| Maths equipment | *If M:AB, helps you get the answer.* |
| Talking and discussing | *About how you get the answer.* |
| Group work | *Help other people on your table or sitting next to, cos…* |
| Doing well | *So you learn more.* |
| Getting the answer | *To think how you are going to get the answer.* |
| Worksheets | *To understand, if you have worksheets, they help you to understand other things as well.* |

**Figure 4.21** Yousif’s card sort and summary discussion record

The first three layers of Yousif’s card sort included descriptors that indicate personal processes associated with learning maths well, such as thinking and problem solving. These are placed ahead of such descriptors that indicate the outcome or result of learning maths well, this being getting the answer. He also suggested that learning maths well involved ongoing improvement, that is, to “get smarter” and “learn more”. Yousif’s card sort record is consistent with the major themes of the Drawing Task that learning maths well is associated with getting it right, doing it right and talking about it with others. He also refers to other students and the role of interaction to help explain, and the assistance that is afforded by engaging in group work.

Andrew’s card sort is similar (Figure 4.22 below).
Andrew (ARN, Year 7)

- **Thinking**: Think of ways to get the answer or solve something.
- **Teacher**: You need a teacher there to be able to tell you what to do.
- **Problem solving**: Solving the problem helps you when you’re older or something like that.
- **Getting help**: Getting help from the teacher helps you get the answer.
- **Explanations**: You need to tell you the answer of whatever you’re going.
- **Maths equipment**: Actually using things to measure and do things instead of just worksheets.
- **Worksheets**: Help you with your thinking and solving things.
- **Talk & discuss**: If you discuss with people you can think of ways to get the answer so you need talking and discussing.
- **Getting the answer**: I like it when I get the answer to something hard.

**Figure 4.22** Andrew’s card sort and summary discussion record

Like Yousif, Andrew recognised the personal processes associated with learning maths well. Andrew’s card sort is also consistent with the three themes that emerged from the *Drawing Task* interviews: I learn maths well when I have the right answer, do it right and talk about it with others. He acknowledged the role of interaction with the teacher and other people in relation to learning maths well. Andrew explained that the teacher “tell(s) you what to do” and “helps you get the answer”.

Alison gives further evidence that personal processes such as *talking and discussing* and *thinking* are important for learning maths well.
Alison (SC, Year 7)

Extract from field notes (11.05.2005): Reason(s) for selecting

<table>
<thead>
<tr>
<th>First Layer</th>
<th>Second Layer</th>
<th>Third Layer</th>
<th>Fourth Layer</th>
<th>Fifth Layer</th>
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</thead>
<tbody>
<tr>
<td>Talk and discuss</td>
<td>Thinking</td>
<td>Explanations</td>
<td>Getting answer</td>
<td>Group work</td>
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<td>Prob. Solving</td>
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<td>Feeling good</td>
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<td>Teacher</td>
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<table>
<thead>
<tr>
<th>Reason(s) for selecting</th>
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<tbody>
<tr>
<td>Talking and discussing</td>
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<tr>
<td>Thinking</td>
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<td>Explanations</td>
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<td>Getting the answer</td>
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<td>Group work</td>
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<td>Problem solving</td>
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<tr>
<td>Feeling good</td>
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<tr>
<td>Worksheets</td>
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<tr>
<td>Teacher</td>
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</table>

Figure 4.23 Alison’s card sort and summary discussion record

Alison, in addition to supporting the themes that arose from the Drawing Task interviews: getting it right and doing it right, ascribed importance to interaction with others. The talking and discussing descriptor heads her card sort. She spoke of the importance of meaningful explanations so “you know what to do” and, asking questions when “you get too stuck with something”. Her descriptor, ‘working in groups’ was associated with talking and discussing in order to “get the answer”.

The next section will present the major differences found when comparing the responses from the AR (intervention and non-intervention) and SC students. These differences related to whether or not the descriptors maths equipment, worksheets and feeling good were included in the card sort.
Interesting variations in the Pre-intervention Phase

Despite the low rank of the getting the answer descriptor (ninth), all students mentioned ‘getting the answer’ in some way when justifying the selection and placement of other descriptors. This suggests that students appreciate that while getting the answer is a significant aim of school mathematics, they recognise that this requires personal processes. Also, differences arising out of the Card Sort interview analysis were the relative number of students in each group (ARI, ARN, & SC) who selected the descriptors, maths equipment, worksheets, and feeling good. Looking closely at where the differences occurred, they are not surprising. I expected the SC students, by virtue of their ability in mathematics to be more inclined to associate feeling good when learning mathematics, than those students who had difficulty. I expected that the AR students would associate the use of maths equipment as an aide to their work in mathematics more so than the SC students. However, given my observations of teaching practice over a considerable period of time indicating the prevalence of using worksheets as a part of instruction, I would have expected little if any differences among the student groups. The observations in relation to maths equipment, worksheets and feeling good will be discussed in turn.

Maths equipment

The inclusion of maths equipment appeared to be a difference between able and less able students. All 14 AR students included maths equipment as part of their card sort relating to learning maths well. In contrast, only one of the SC students did so. This is worthy of note. The Drawing Task interviews seemed to identify maths equipment most often in terms of items of stationery. Yet, the Card Sort interview stimulated discussion about maths equipment as manipulatives and/or mathematical models, such as counters and multi-base arithmetic blocks (MAB). All students who selected maths equipment tended to place this descriptor in the middle band of the card sort. The following indicative extracts of field notes illustrate this:

- If you use MAB, helps you get the answer (Yousif, ARI, Appendix H, p. 165).
- Doing decimals you have to make a model, the equipment is there to help you, comes in handy to do maths with (Douha, ARI, Appendix H, p. 168).
- Need it 'cos if you have to count something and you don’t have enough fingers or pencils, the equipment helps you. (Cansu, ARI, Appendix H, p. 174).
- Have to learn what it looks like, the equipment’s what to use, like abacus (Candi, ARN, Appendix H, p. 160).
- It helps you think more, if the question, e.g., 92 cubes and you took away 30 you can use maths equipment to help figure it out (Hajar, ARN, Appendix H, p. 169).
It is interesting to note that Cansu (ARI, Year 6), as indicated above, hinted at the use of a count-all strategy when using maths equipment. Earlier, when reporting the results for the Drawing Task, (see 4.1.2), strategies such as ‘modeling all’ and ‘counting all’ were identified. The nature of strategies will form an important part of the results for the Multiplicative Task interviews, as well as their significance as a barrier to the development of multiplicative thinking in Chapter 5.

Hadi and Ahmed (ARI, Year 6) also provided similar detail about the use of maths equipment, evident in the following extracts:

- If you don’t know how to do it, the equipment helps you ‘cos in a bead frame you put one down for as many. (Hadi, ARI, Appendix H, p. 166)
- If you have 10 plus 2 you put 10 of the equipment and put 2 more and it equals 12. Gives you help to do the plusses. (Ahmed, ARI, Appendix H, p. 176)

This ‘count-all’ approach will be discussed in depth in 4.3.1 when the results of the Multiplicative Task interviews are presented.

Lynda (SC, Year 6) interpreted maths equipment as a model of a mathematical situation presented in a problem or question. She stated that, “If you still don’t get the problem, (it) might be better if you saw the problem” (Appendix H, p. 163). She then stated that in order to get the answer, it was easier to ‘see’ addition, subtraction and division with counters than it was to see multiplication. Lynda gave the example of being able to put counters in groups to model division situations. This suggests that she does understand sharing, but does not have a mental or physical image of multiplication, such as arrays or regions and perhaps her understanding and use of multiplication is procedural.

As mentioned previously, in addition to modeling mathematical situations or problems, maths equipment was also interpreted as items of stationery by some of the AR students. These stationery items included pencils, pencil cases, erasers, paper, tins, chalk, glue sticks, and books. According to James, you use “Pencils to write out the answers, the rubber to rub out wrong answers and the calculator helps with the answer” (ARI, Year 6, Appendix H, p. 173). This suggests that the use of maths equipment is not necessarily seen as a strategy to assist students to understand concepts or develop efficient strategies.

Worksheets

Of the ten students who included worksheets in their card sort only two of these were SC students. On the whole, the eight AR students reported thoughts similar to the two SC students in relation to the importance of worksheets for learning maths well. In particular, worksheets were described as being repetitive, therefore assisting the students to
remember, and text based or written, indicating what to do and where to record the answer. For example:

Worksheets can help you work…times table and it’s on your sheet, you work it out and it helps you. It is good to get hard worksheets, if you haven’t done something for a while and you don’t remember. (Douha, Appendix H, pp. 167-168)

Cansu (ARI, Year 6) also commented on how the use of worksheets can provide opportunity to attempt a strategy before committing to writing the answer straight away:

…it if you get the worksheet and try yourself first before going to the teacher because it writes the question and you get scrap paper and write the finished answer on the sheet. (Cansu, Appendix H, p. 174)

This particular view is in contrast to the perceived importance of showing ‘the working out’. For example, Hadi (ARI, Year 6) identified ‘the working out’ as a descriptor missing from the initial 12 descriptors provided for the Card Sort Interview (see table 4.3). Dean (ARI, Year 6) suggested that without the work on the blackboard or on a worksheet, there would be no work to do (Appendix H, p. 180).

This suggests that most, if not all maths learning for Hadi occurred in part through the use of written scripts, for example, worksheets, that require interpretation, implementation of some process and completion by recording the result or answer. This is not to say that interactions with others did not occur in the course of introducing and working through the worksheet. However, there appeared to be few reports of situations where discussion and interaction with others were inspired by a non-written stimulus, for example an activity aimed at developing efficient mental strategies through using playing cards or dice. The importance of the latter way of working will be discussed in detail in Chapter 5.

Two SC students, Adam (Year 6) and Alison (Year 7), included worksheets in their card sort at the fourth layer. Adam suggested that worksheets have numerous similar questions and that this was important “because of the repetition so that it becomes second nature” (Appendix H, p.171). Alison built on this idea further, by suggesting that worksheets provide the basis for the work in school mathematics and the recording of the answer. She favoured the use of worksheets in preference to copying from the blackboard:

I prefer worksheets than copying things. You can put the answer next to it. (Alison, Appendix H, p. 157)
Feeling good

The SC students were more inclined to include feeling good in their card sort, in fact seven out of the eight SC students did so. By contrast, only two out of the nine ARI students and two of the four ARN students included this descriptor.

Alison’s card sort presented earlier (SC, Year 6, Figure 4.23) included feeling good at the 4th layer. She justified its placement in her card sort by indicating a relationship between feeling good and working well, “If you’re feeling bad you probably won’t work as good”.

Most of the SC students mentioned feeling good when you “know what to do” and “you get the answer” (for example, Sandy, Year 7, Appendix H, p. 162). It was also reported that you feel good when you “achieve more, (and) if you’re not right, at least you tried” (Lynda, Year 6, Appendix H, p. 163). Adam (SC, Year 6) referred to the impact on self-esteem in relation to feeling good in the following extract of field notes:

If you feel good then you want to do it more…self admiration…like a reward….work harder on things…do harder things…feel good about yourself…then you can say ‘I can do this’. (Appendix H, p. 171)

This extract is a powerful example of the contingent relationship between beliefs and action.

The four AR students (2 ARI and 2 ARN) who included feeling good in their card sort, shared ideas similar to those of the SC students. Sidona (ARN, Year 7), Hadi and Douha (ARI, Year 6) related feeling good to mathematical knowledge as some sort of cause-and-effect relationship, that is, you feel good when you know or can do more. Sidona stated, “If you don’t feel good, it’s hard to get all the answers correct” (Appendix H, p. 156). Ibrahim (ARN, Year 6) related feeling good to physical health:

Have to feel good because if you are sick and you’re not feeling well, the work will not be good…because you won’t be able to think if you’re sick and tired. (Appendix H, p. 183)

This suggests the need to feel physically well whilst doing the work. Douha and Hadi both viewed feeling good as the result of doing well in maths. Hadi (ARI, Year 6) expressed that “you feel good when you know something in maths” (Appendix H, p. 166). Douha stated that, “If you do the worksheet and get through it without help you feel good” (Appendix H, p. 168). I sensed that ‘feeling good’ was a consequence of mathematical success, which is linked to being a ‘knower’ and ‘doer’ of school mathematics. The idea of ‘feeling good’ is acknowledged by the AR students as a consequence of learning maths well, despite this being beyond their direct experience.
Summary

In the Pre-I Phase the results of the Card Sort interviews indicate that learning maths well was associated with doing it right, getting it right, and talking about it with others. However, the students also recognised the personal processes involved in learning maths well. That is, with respect to learning maths well, thinking, talking and discussing, and problem solving were regarded as more important than getting the answer, doing well, worksheets and maths equipment. Conversations with the students about their card sort revealed blurred boundaries between the descriptors. That is, the descriptors were not exhaustive or mutually exclusive. For example, in addition to talking and discussing, when justifying the use of a range of other focus descriptors, students consistently spoke about the need to have conversations with others. The getting the answer card was not the only descriptor that stimulated discussion about ‘the’ answer or ‘an’ answer.

The next section will shift away from student beliefs about learning maths and focus on the strategies that the students used to solve multiplicative tasks.

4.1.3 Strategy use

The purpose of the Multiplicative Task interviews was to identify the strategies students used to respond to a range of multiplicative tasks at the beginning and end of the study. The interview featured two parts: a reflective discussion of student responses to multiplicative tasks completed prior to the interview (Part A), followed by students working on a similar yet unseen task in a ‘think aloud’ manner (Part B). The interview was administered individually to all 22 students throughout April/May 2005.

As might be expected, the analysis of the Multiplicative Task interviews revealed distinct differences between the AR (intervention and non-intervention) students and the SC students in the Pre-I Phase with respect to the strategies used to solve multiplicative tasks. Again, where there is no need to distinguish between the ARI and ARN students, the students will be referred to as the AR (at-risk) students. The 14 AR students employed additive strategies when solving multiplicative tasks. By contrast, the eight SC students employed a range of efficient strategies. The strategies of the AR students will be described and illustrated first. This will be followed by a description of the strategies used by the SC students.
Pre-intervention strategies used by the at-risk student

Analysis of interview notes and student work samples revealed seven strategies commonly used by both AR student groups. These strategies were:

- ‘Make all, count-all’
- ‘When in doubt, add’
- Repeated addition
- Skip counting
- Additive partitioning
- Whole number thinking
- ‘Have a go’ approach to determining all options

The seven strategies listed above will be described and illustrated in turn. It is important to note that the first five of these additive strategies might be used quite reasonably to obtain correct solutions to problems involving small whole numbers in the early years of schooling (Prep to Year 4). However, the expectation of students in upper primary (Years 5 and 6) is to be able to work efficiently with large whole numbers and the operations of multiplication and division. An over reliance on the use of these five strategies becomes inefficient and lead to errors at this level. This in turn impacts negatively on student development of multiplicative thinking and ultimately contributes to a lack of progress in relation to school mathematics more generally.

The first of these strategies, ‘make all, count all’ is described below.

Make all, count all

This strategy is evident when students rely on physical models or drawing of all aspects of the problem and counted by ones to determine the solution.

To illustrate the strategy ‘make all, count all’ the following problems will be presented: Patterns with tiles a, Patterns with tiles b (both used in Part A, the reflective discussion aspect of the interview) and Block pattern a (used in Part B, the think aloud aspect of the interview). Where appropriate, the problem will be presented, followed by an indicative summary of student responses.

Consider the following problems, Patterns with tiles a (Figure 4.24) and Patterns with tiles b (Figure 4.25) completed as part of the Multiplicative Task interview Part A:
Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.

![Pattern with tiles](image)

a. How many times can this pattern be made with 28 black tiles, 21 grey tiles and 6 white tiles? **Show all your working and explain your answer in as much detail as possible.**

**Figure 4.24 Patterns with tiles a**

Dean (ARI, Year 6) attempted to draw the above pattern repeatedly until the given number of components were exhausted, that is, when he had ‘used’ all 28 black tiles, 21 grey tiles and 6 white tiles. He told me he could not make them all “fit”. Similarly, though more successfully, Hadi (ARI, Year 6) used the same drawing strategy and was able to solve the problem correctly. Adir and Ahmed (ARI students, Year 6) did not respond to this question until the interview, where they were given the opportunity to re-visit the task. Both students made use of manipulatives, such as tiles or counters to ‘make’ the patterns with the given number of components. They made the pattern five times and counted each pattern individually to determine the solution.

![Pattern with tiles](image)

b. This pattern uses 3 grey tiles for every 5 black tiles. How many black tiles would you need if you had 12 grey tiles? **Show all your working and explain your answer in as much detail as possible.**

**Figure 4.25 Patterns with tiles b**

Sidona (ARN, Year 7) and Douha (ARI, Year 6) both responded to *Patterns with tiles b* in a similar manner by attempting to replicate the pattern by drawing it. Sidona’s solution (Figure 4.26) is incomplete and inaccurate, whereas Douha’s solution (Figure 4.27) is complete and accurate.
Looking closely at Sidona’s pattern on the left side of Figure 4.26, it appears that she made numerous attempts to copy the pattern accurately and more squares than needed are evident. The pattern on the right of this figure is more true to the pattern presented in the problem. Use of the ‘make all, count all’ strategy in this instance did not support Sidona in solving the problem. The opposite is the case for Douha.

The Block task presented in Figure 4.28 below is similar to Patterns with tiles a and was used during Part B of the Multiplicative Task interview. Laminated squares were available to help model the problem if necessary, though there were insufficient tiles to support the ‘make all, count all’ strategy. This was deliberate. I wanted to see what students could do when the ‘make all, count all’ option was not seen to be supported.

**Block pattern**

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

```
[Diagram of a block pattern with 6 black squares, 2 grey squares, and 1 white square]
```

- How many blocks like this can be made with 32 black squares, 17 grey squares and 7 white squares? **Show all your working and explain your answer in as much detail as possible.**

**Figure 4.28 Block pattern a**

Yousif (ARI, Year 6) elected to use the laminated squares and attempted to use a ‘make all, count all’ strategy. He was able to make the pattern twice, stating, “You can only
do two times”. He then doubled the six black squares, then doubled again to determine 6 fours, but then lost track of what he was doing. James (ARI, Year 6) attempted to draw the pattern, his response is presented in Figure 4.29 below.

![Figure 4.29 James’s response Block problem a](image)

While James seems to have used a ‘make all, count all’ strategy, his picture suggests he may not have understood the problem. Instead of thinking, for example, “How many times can I make the pattern...?” and drawing the pattern repeatedly, he appears to have tried to ‘use up’ the 32 black squares, 17 grey squares and 7 white squares in one large block. At the time of the interview, he said it was “hard, there are too many, and won’t fit on the paper”. As discussed in Chapter 2, understanding tasks of this type requires a level of spatial reasoning and this is a known difficulty for AR students (2.3.5). This will be discussed further in the Chapter 5. The next strategy to be presented, ‘when in doubt, add’, is further evidence of a reliance on addition and of what students do when they are unsure how to solve the problem.

*When in doubt, add*

This strategy is evident when students decide to add all the numbers presented in the problem, irrespective of whether it was appropriate to do so or not.

This strategy was evident when students were responding to *Patterns with tiles a and b* (see earlier Figures 4.24 & 4.25), and *Patterns with tiles c* (Figure 4.30 below). Some of the AR students were unsure how to go about solving the problem even if they were able to correctly retell the problem in their own words, indicating an understanding of what the problem was about. The strategy used in these instances was to add all the numbers used in the question.
c. The art teacher orders 6 boxes of red tiles. Each box has 36 tiles. How many red tiles are there altogether? Show all your working and explain your answer in as much detail as possible.

Figure 4.30 Patterns with tiles c

Both Yousif (ARI, Year 6) and Candi (ARN Year 7) used the ‘when in doubt, add’ strategy to solve Patterns with tiles a by thinking 28, and 21 and six more is 51 tiles. Ahmed (ARI, Year 6) used this approach for his response to Patterns with tiles b by thinking 3 (grey tiles) and 5 (black tiles) are 8, and 12 more (grey tiles), is 20. This is correct, but for the wrong reason. Ahmed also used this strategy for Patterns with tiles c as did Candi (ARN, Year 7). Both Ahmed and Candi determined that the answer to Patterns with tiles c was 42 based on the thinking 6 plus 36 is 42. This reliance on addition suggests that this concept is something they understand and the associated calculation is one that they can implement successfully. This suggests that when the AR students are faced with a problem they are unsure about, they use what they know and can do, which is addition. The next strategy to be illustrated, ‘repeated addition’ also relies on addition.

Repeated addition

This strategy is evident, for example, when calculating 3 multiplied by 8, the student either thinks or writes 8 + 8 + 8 is 24.

Use of the repeated addition strategy was evident in student responses to Patterns with tile c (Figure 4.30 above) and Patterns with tiles d (Figure 4.31 below). To solve Patterns with tiles d information from Patterns with tiles c was needed. Students were reminded of this at the time of the interview as necessary.

d. The art teacher needs 330 red tiles. How many boxes of red tiles does she need to order? Show all your working and explain your answer in as much detail as possible.

Figure 4.31 Patterns with tiles d

For Patterns with tiles c, Sidona (ARN, Year 7) understood that 36 multiplied by 6 would be an appropriate strategy, but she was not sure that this would give her the right answer. She stated during the interview that, “I was thinking of different ways…but I thought it might be the wrong answer, so I plussed them together”. She had written 36 + 36 + 36 + 36 +36 + 36 and added all the ‘sixes’, then added all the threes.
Douha’s (ARI, Year 6) response to Patterns with tiles e was to repeatedly add 36, 6 times. What was unique about her response was the individual tally marks representative of the 6 ones in the number 36 and individual tally marks representative of the 3 tens in the number 36. She attempted repeat addition of six, six times first, and made a series of tally marks to help her work this out. It looked like this:

\[
\begin{align*}
\mid \mid \mid \mid \mid \mid + \mid \mid \mid \mid \mid + \mid \mid \mid \mid \mid + \mid \mid \mid \mid \mid + \mid \mid \mid \mid \mid + \mid \mid \mid \mid \mid = \\
\mid \mid + \mid \mid + \mid \mid + \mid \mid + \mid \mid + \mid \mid = 
\end{align*}
\]

She then used the same approach for the 3 tens, six times:

\[
\begin{align*}
\mid \mid \mid + \mid \mid \mid + \mid \mid \mid + \mid \mid \mid + \mid \mid \mid + \mid \mid \mid = 
\end{align*}
\]

Douha made various attempts to solve Patterns with tiles d (Figure 4.x) using this ‘tallying’ strategy. This is shown below in Figure 4.32.

![Figure 4.32 Part of Douha’s response to Patterns with tiles d](image.png)

This response suggests that Douha conceives of number in terms of ‘ones’ and is unable to work efficiently with meaningful chunks or ‘composite units’. She recognises that
a written algorithm is useful. It appears that the tally marks help her first to manage the number of ones, and then the number of tens. The importance of removing this barrier will be explored further in Chapter 5. At the time of the interview, Douha told me that she used this approach often. The following strategy to be illustrated is similar to repeated addition, this being skip counting.

*Skip counting*

This strategy is evident, for example, when calculating 8 multiplied by 6, the student either thinks or writes “8, 16, 24, 32, 40, 48”.

Although similar to repeated addition, this strategy is distinguished from repeated addition on the grounds that it was exhibited in the form of a mental strategy that could be articulated orally, or as a written counting sequence. Repeated addition was evident in the form of a written algorithm where the addition sign (+) was used.

The ‘skip counting’ strategy was used by the AR students in response to a range of tasks with varying degrees of success. Yousif (ARI, Year 6), in his response to *Patterns with tiles a* (see earlier Figure 4.24), used skip counting to determine the number of times the pattern could be made with the given number of coloured tiles. He counted by threes, “3, 6, 9, 12, 15, 18, 21”, (seven times, tapping his fingers one by one to keep track of the count) to work out the number of times possible to repeat the pattern with the 21 grey tiles, and counted by fives, to work out the number of times possible with the 28 black tiles. He was unable to work with the problem any further and made no further progress.

Hajar (ARN, Year 6) used this strategy in her response to *Block pattern a* (see earlier Figure 4.28) during the think aloud part of the *Multiplicative Task* interview. *Block pattern a* is a parallel version of the *Patterns with tiles a* task. Her thinking commenced with “how many sixes in 32?” so aloud she counted by six, “6, 12, 18, 24, 30…so five times with two left”. She then moved onto the 17 grey squares, but became confused and was unable to continue. The inability to arrive at a solution in the case of Yousif and Hajar described above may be related to the number of steps involved, as much as the inefficiency of the strategy used.

Hadi (ARI, Year 6), Hajar and Candi (ARN, Year 6 and Year 7 respectively) used skip counting to solve *Patterns with tiles b* (see earlier Figure 4.25). Their responses were based on the thinking that:

I know that 3 grey tiles make one pattern, 6 grey makes two patterns…so 3, 6, 9, 12. So you could make the pattern 4 times with 12 grey tiles and you would need 4 times the number of black tiles. 5, 10, 15, 20. 20 black tiles needed.
Skip counting helps to establish the idea of numbers (particularly, two, five and ten) as countable units which is important for the development of efficient mental strategies and initial place-value ideas. It is also an appropriate strategy to determine the amount in a large collection when physically counting. However, over-use of skip counting suggests that these students have not progressed beyond this to more appropriate and efficient models and strategies for multiplication, for instance array-based strategies. Like all the other strategies used by the AR students described so far, skip counting is also additive. The next strategy to be illustrated involves dealing with fractional shares in an additive way.

Additive partitioning
This strategy was evident when students were unable to describe shares in terms of the number of parts (numerator idea) and the size of the parts (denominator idea) and or show equal shares, for example, share 3 pizzas among 4

Common to the AR students was the inability to appropriately partition or show equal shares in response to the Footy lunch day a problem below (Figure 4.33).

Footy lunch day
a. Show how you would share 2 large meat pies equally among 3 people?

![Diagram of two meat pies divided into three equal parts](image)

Each person gets .................................................................

**Figure 4.33** Footy lunch day a

This task required students to partition or share the ‘meat pies’ equally, for example, three parts per pie and to indicate a fair share equivalent to 2 thirds of one meat pie. One solution is to give half a pie to each person then partition the remaining half into three equal parts. Specific examples of additive partitioning will be described next.

Hajar (ARI, Year 6) partitioned the two circles using repeated halving and a protracted sharing strategy. First, she divided each pie into 8 pieces providing a total of 16 pieces. She then ‘gave’ two slices to each of the three people, which left 10 remaining
pieces. She then ‘gave out’ another two slices to each person, so that four pieces remained, then one slice to each person, leaving one piece to share. This last piece she ‘cut’ into three “so it’s fair”. She concluded that each person got “3 pieces and 1 quarter”. The numbers “1, 2 and 3” were used to indicate which parts were allotted to each person (see Figure 4.34 below).

![Figure 4.34](image)

Figure 4.34 Hajar’s partitioning strategy, ARN, Year 6

What is interesting about Hajar’s response is the logical and systematic manner in which she was able to partition the two pies despite not recognizing the need for equal parts. She was unable to appropriately describe the share either informally or formally in terms of the equivalent of 2 thirds of pie. She described the share for each person as a count of three pieces (though in reality five equal sized pieces) and the remaining slice which she ‘cut’ into three, as “1 quarter”. This suggests that although physical sharing may be a part of her lived experience, she is yet to conceive of the share in terms of the ‘denominator idea’. That is, the number of parts names the part, for example, 8 parts are eighths. She has some sense of fraction language, for instance uses the word ‘quarters’, but misapplies this to the remaining small pieces.

Both Douha and James (ARI, Year 6) knew that each person would get “2 pieces of pie each” based on the reasoning that if each pie were cut into three, this would create 6 possible shares and these shared among 3, would provide 2 pieces each (Figures 4.35 & 4.36 respectively). However, these students were unable to divide or partition the circles appropriately into equal sized shares. Douha partitioned the first circle into four equal sized pieces using a halving strategy. Then she partitioned the second circle to create the remaining two pieces needed. Her response looked like this:
James’s response looked like this:

![Figure 4.35 Douha’s response to Footy lunch day a](image)

This is interesting because it appears as though Douha and James drew upon real-world knowledge and were able to imagine fair shares, but the act of representing and naming the share as grounded in school mathematics, proved problematic.

The *Sharing a task* (Figure 4.37 below) was included in the ‘think aloud’ part of the *Multiplicative Task* interview. Playdough, also commonly known as modeling clay was available during the interview.
Sharing part a...

a. How would you share the 2 sausages of playdough among 3 children?

How much does each person get? .........................................................

Figure 4.37 Sharing a

The actual playdough model appeared to support partitioning to show equal shares in a way that the diagrammatic form of the *Footy lunch day* a task didn’t. The AR students (3 ARI and 2 ARN) could physically roll the play dough into two sausages to model the problem relative to context. For instance, Dean (ARI, Year 6), Hajar and Ibrahim (ARN, Year 6) were able to cut the play dough sausages into approximate thirds and allocate two pieces for each of the three children. The students were unable describe this share more formally, such as, “2 thirds of the sausage for each of the children”. Douha’s (ARI, Year 6) response was similar to that of Dean, Hajar, and Ibrahim. However when asked if she could describe the shares in another way, her response was “two out of six”. This suggests that she is relying on whole number perception of parts as wholes that are no longer in relationship with the original. This is consistent with the suggestion made above relating to real-world context versus the school mathematics context. Working with physical models and real-world examples where students are informally making, comparing and describing shares is critical to the development of initial fraction ideas. This is what is expected in the early primary school years. The AR students have not progressed beyond this.

Cansu’s (ARI, Year 6) response was the least efficient of the strategies used by the students interviewed on this task. When she had read through the task, she said “I’d cut into little pieces and share them with three people”. When she decided to form sausage models with the play dough she made one cut, then made all subsequent cuts similar in size. This resulted in one sausage cut into seven pieces and the other sausage into eight pieces, giving 15 pieces. It did not concern her that she generated an unequal amount of pieces per sausage. She shared these 15 relatively even sized pieces out one by one so that each person got five pieces. This response seems to indicate that Cansu had transformed the continuous fraction models into a discrete collection of wholes that she could physically share out on the basis of a count of fifteen shared among five.

When given the opportunity to physically work with or imagine a real-world context, the AR students were able to informally describe equal shares, for instance, “three
pieces each”. This may be because sharing in this way is part of their lived experience. Given the use of additive partitioning strategies and the inability to formally describe equal shares, it was understandable that when interpreting fraction symbols, the AR students relied on what they know and understand, this being, whole number. This thinking in relation to their interpretation of fraction symbols is described in detail below.

*Whole number thinking*

This strategy is evident, for example, when the student states that 1 fifth is larger than 1 quarter because 5 is bigger than 4.

Responses from the AR students tended to fall into two groups: those students who were able to read the fraction notation in the *Footy lunch day* b task (Figure 4.38 below), as 4 ninths and 2 thirds and those students who were unable to do so.

<table>
<thead>
<tr>
<th>Footy lunch day</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. Jo ate ( \frac{4}{9} ) of a large meat pie and Maggie ate ( \frac{2}{3} ) of a large meat pie. Who ate the most pie?</td>
</tr>
</tbody>
</table>

**Figure 4.38** Footy lunch day b

In either case, the AR students were unable to work with the fraction notation meaningfully. It appears that the students were reading the numerator and denominator as whole numbers. For example, Cansu (ARI, Year 6) read 4 ninths as “four and half nine” and 2 thirds as “three two”. Sidona (ARN, Year 7) based her thinking on the reasoning that “9 is larger than 3 and four is larger than 2, so Jo ate more pie.”

Ahmed, Adir and Berrin (ARI, Year 6) were unable to read the fraction symbols. I read the question aloud to each of them at the time of the interview. Adir responded, “4 ninths? Never heard of it”.

Berrin, when completing the task, focused on the denominators, not as ninths and thirds, but as a count of nine and a count of three. She drew two circles to represent the meat pies and partitioned one pie into nine unequal parts and the other pie into three unequal parts. Her response is shown in Figure 4.39.
The *Sharing b* task (Figure 4.40 below) was included in the ‘think aloud’ part of the *Multiplicative Task* interview.

**Sharing b...**

b. If I have \( \frac{4}{6} \) of the playdough sausage and you have \( \frac{3}{4} \) of the playdough sausage, who has the most playdough?

*Explain your reasoning using as much mathematics as you can.*

**Figure 4.40** Sharing b

Six of the AR students (4 intervention and 2 non-intervention) used the play dough to physically form into two sausages to model the problem. The ‘I’ in the task refers to the researcher and the ‘you’ refers to the students. In response to the *Sharing b* task, Adir (ARI, Year 6) stated that “Margarita has more than me because Margarita has 4 and I have 3”. Adir focused on the count of parts (4 and 3) ignoring the relative size of the parts (sixths and quarters respectively). Similarly Sidona (ARN, Year 7) reasoned on the basis of whole number understanding, that since six is larger than four, a “sixth is bigger than a fourth and a fourth is bigger than third” so 4 sixths is larger. Andrew (ARN, Year 7) selected 3 quarters as being larger because 4 sixths “are smaller pieces”.

The fact that the AR students were unable to interpret and work with fraction symbols indicates that key initial fraction ideas were not fully established. The AR students appreciated the need for equal shares, and could work reasonably well with a count of parts (numerator idea). However, except perhaps Andrew who appreciated that sixths were smaller pieces, the AR students were yet to understand that as the number of parts increase the size of the part decreases, for example, 2 parts (halves) are larger than 4 parts (quarters), and the number of parts names the part, for example, 3 parts are *thirds*, 4 parts are *quarters*, 5 parts are *fifths*, and so on (denominator idea). These ideas need to be fully understood before recording fractions more formally.
Common to the strategies described so far, is an application of what the AR students know and understand well when constructing a response to a range of problems. This being addition. This suggests that leaving a task unanswered or blank is undesirable and that the students want to show what they know and can do and they will apply this even when inappropriate to do so. The next strategy reinforces the students’ willingness to provide a response, albeit an incomplete one.

‘Have a go’ approach to determine all options

This strategy is evident when students identify a few options for Cartesian product situations based on personal choice or a random, incomplete pattern.

This ‘have a go’ strategy became evident in response to the Footy lunch day c task (Figure 4.41 below). This problem deals with the Cartesian product or the ‘for each’ idea for multiplication as it requires students to think that ‘for each’ food option (of which there are there are three) there are two drink options (now six lunch options), and for each of these options, there are two dessert options, so a total of 12 different lunch combinations.

<table>
<thead>
<tr>
<th>Food</th>
<th>Drink</th>
<th>Dessert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat pie</td>
<td>Cola</td>
<td>Ice-cream</td>
</tr>
<tr>
<td>Sausage roll</td>
<td>Lemonade</td>
<td>Icy-pole</td>
</tr>
<tr>
<td>Pastie</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Jo ordered a sausage roll, a drink of cola and an icy-pole. What might she have ordered? List all possibilities. Show your working and explain your answer in as much detail as possible.

Figure 4.41 Footy lunch day c

All of the 14 AR students used a ‘have a go’ approach in an attempt to list all possibilities. None of these students were able to identify all 12 different options and only two students, Andrew and Sidona (ARN students, Year 7), listed eight of the 12 possibilities. Most of the students were able to indicate a couple of possibilities only. Adir’s (ARI, Year 6) response is indicative of this. He provided one menu option for each of the food items: “meat pie, lemonade, ice-cream”, and “sausage roll, cola, icy-pole”, and “pastie, icy-pole, cola”. Ahmed (ARI, Year 6) listed 6 single items of food: “pie, cola, icypole, pastic, sausage roll, lemonade”. Hajar (ARN, Year 6) and Berrin (ARI, Year 6) only indicated what they themselves would select for lunch. Dean (ARI, Year 6) had interpreted the question as “make up you own” lunch order. When asked to give the question another go, Dean was
only able identify three more additional options. James (ARI, Year 6) hinted at recognizing
the numerous possibilities by stating, “Oh man – it’d take too long!”

Summary
The strategies used by the AR students are characterized by a focus on additive thinking.
This is an aspect of school mathematics that they are confident with and successful in using.
However, when applied in contexts outside which addition is suitable and/or efficient, the
strategies do not necessarily support appropriate reasoning nor lead to correct solutions.
Many of these strategies, while appropriate in the early primary school years, are not
conducive to advancement in the middle years of schooling. The SC students on the other
hand present quite a different picture. Their strategies are illustrated and described in the
following section.

Pre-intervention strategies used by the successful students
This section will elaborate on the strategies used by SC students throughout the Pre-I
Phase Multiplicative Task interviews. Analysis of interview notes and student work samples
revealed five strategies. These were:

- Use known facts or procedures
- Use multiplication (as a special case of the strategy ‘use of known facts or procedures’)
- Multiplicative partitioning
- Rename fraction
- Systematic approach to determining all options

The strategy of ‘skip counting’ is noted here briefly because it was evidenced on one
occasion during the Multiplicative Task interviews with the SC cohort. Earlier in 4.1.3, ‘skip
counting’ with small numbers was illustrated as this was a strategy common to the AR
students. Lynda (SC, Year 6) on the other hand, managed this strategy with a two digit
number in her response to Patterns with tiles d (see earlier Figure 4.31). This task required
students to work out, if boxes of tiles come in quantities of 36, how many boxes are
needed for 330 tiles. Lynda counted by 36 because she had “forgotten how to do long
division”. The following number sequence was listed down the page: 36, 72, 108, 144, 180,
216, 252, 288, 324. Lynda noted that the art teacher would need 9 full boxes and 6 more
tiles to get to 330.

This suggests that like the AR students, there are times when it is necessary to
revert to a known, albeit less efficient, strategy. The five strategies listed above will be
described and illustrated in turn. The SC students’ use of known facts or procedures is presented below.

*Use known facts or procedures*

This strategy is evident when, for example, the students divide the total by the required number, for example, “64 divided by 8 is 8”, mentally or using written algorithms.

The SC students used division to respond to the *Pattern with tiles a, b, and d* (see earlier Figures 4.24, 4.25 & 4.31 respectively). In response to the *Pattern with tiles a* task, all eight SC students (1 Year 5, 5 Year 6, & 2 Year 7) “divided 28 by 5, 5 and 3 left over...divided 21 by 3, 7...” to determine that the pattern could be made 5 times, even though there were sufficient grey tiles for 7 repetitions and sufficient white tiles for 6 repetitions.

Sandy (SC, Year 7) used division to work out her response to *Patterns with tiles d*, even though most of the other SC students used multiplication. For example, Sandy wrote 330 divided by 36 to document her thinking. Her response (Figure 4.42) looked like this:

![Figure 4.42 Sandy’s response to Patterns with tiles d](image)

Lynda and Jessica (SC, Year 6) used quotition division to solve *Block pattern d* (Figure 4.43) below.

![Block pattern](image)

d. If a quilt has 324 small squares, how many blocks of this pattern will be used? Show all your working and explain your answer in as much detail as possible.
Both these students recorded and described the thinking that went with their recording in the same way. Figure 4.44 below illustrates Jessica’s (SC, Year 6) recording.

![Figure 4.44 Jessica’s recording for Block pattern d](image)

At the time of the interview, Jessica explained, “…that 9 goes into 3, can’t do. 9 into 32, goes 3 times and 5 left over. 9 into 54 goes 6, so the answer is 36.” Lynda, whose mathematical recording was the same as Jessica’s shown above, added the following written explanation to her working out:

I think that you can make this pattern 36 times with 324 small squares because if you need 9 for each whole then you divide 324 by 9 and that makes 36 according to the strategy I used.

Use multiplication
This strategy is evident, for example, when students calculate a total, they multiply the number of groups either mentally or using a written method, by the number in each group, for example, “5 boxes of 12 tiles, 5 times 12 is 60”.

This strategy was evident in SC students’ responses across a range of problems used in the Multiplicative Task interviews. Multiplication was applied in response to Patterns with tiles b (Figure 4.25) and its parallel version Block pattern b (Figure 4.45 below), as well as Patterns with tiles c, d and Footy lunch day c (Figures 4.30, 4.31 & 4.41 respectively).

Block pattern
Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

![Block pattern](image)

b. This block uses 2 grey squares for every 6 black squares. How many black squares would you need if you had 6 grey squares? Show all your working and explain your answer in as much detail as possible.

![Figure 4.45 Block pattern b](image)
In response to the *Block pattern b* task, Chris (SC, Year 5) and Kalil (SC, Year 6) recognised a factor increase of 3 for the grey tiles and applied this increase to the black squares, therefore “6 black squares multiplied by 3 is 18 black squares”. Jessica and Kalil (SC, Year 6) applied this strategy in response to the *Patterns with tiles b* task. Jessica explained that, “12 grey tiles meant that the pattern could be made 4 times, and 4 times 5 is 20”. These students’ responses indicate a sophisticated understanding of multiplication as they were able to recognise, apply and describe the proportion situation given in these tasks (see Appendix C, LAF, Zone 7, p. 35).

All SC students used multiplication to solve *Patterns with tiles c*. Adam’s response illustrated below appears as a standard vertical algorithm and is indicative of the responses made by all the SC students:

```
  36
x 3
216
```

Most of the SC students used multiplication to respond to *Patterns with tiles d*. The SC students who recognised the relationship between multiplication and division, knew that 360 divided by 36 would lead to a solution, yet they used a mental or written multiplication strategy rather than use a written method for division. Lynda (SC, Year 6) explained that she had “forgotten how to do long division”, and Chris (SC, Year 5) acknowledged that he “was not very good at division”. These students are presumably referring to the standard school-taught algorithm. The mental reasoning used by the SC students was based on the strategy ‘Think multiplication’. For example, ‘what by 36 will give me 330?’ and then known facts were applied. For example, Adam (SC, Year 6) explained that he “knew 10 by 36 is 360 and take off another box to ensure that you’re not ordering too much which is a waste of money”.

Jessica (SC, Year 6) chose to multiply 36 by 8 (288) as she knew that multiplying by 4 would not be enough. That is, approximation based on a reasonable estimate derived from sound knowledge of basic facts and the partition concept of division. She then added 36 to the total (324) to signify another box of tiles, so “9 boxes gives you 324 but you need another 6 tiles to get to 330”.

Multiplication also featured in the SC students’ responses to the Cartesian product task, *Footy lunch day c*. For example, Jessica (SC, Year 6) determined that there were 12 different possibilities based on the thinking that 3 food items multiplied by the 2 drink items multiplied by the 2 desert items, $3 \times 2 \times 2$, is 12. Lynda (SC, Year 6) focussed on the 4
options for having a meat pie and “that would be the same for the sausage roll and pastie, so 4 times 3 is 12”.

**Multiplicative partitioning**

This strategy is evident when students use appropriate partitioning strategies to show equal shares, for example, sharing 2 pizzas among 3

![Partitions of pizzas](image)

The SC students considered the need for equal shares when partitioning the fraction models in *Footy lunch day* a and b (see earlier Figures 4.33 & 4.38). All SC students indicated the equal shares for *Footy Lunch day* a in the same way and Lynda’s response (Figure 4.46 below) is used to illustrate this.

![Lynda's response](image)

**Figure 4.46** Lynda’s response to Footy lunch day a

The students were able to describe the share for each of the three people as “2 thirds of one pie”.

Two of the SC students used illustrations of their partitioning strategies to support their solution to *Footy lunch day* b (Figure 4.38). For example, Chris (SC, Year 5) drew two pies the same size to show 4 ninths and 2 thirds. He partitioned the first pie into nine equal parts, first partitioning into thirds (as shown above) then he partitioned each of these thirds into three, giving ninths. Along side this he showed 2 thirds on the second diagram and then selected 2 thirds as being bigger because “it looks more”. His response looked like this (Figure 4.47):
This illustrates that the SC students have developed advanced partitioning strategies but may not be able to generalise this to support reasoning when comparing two related fractions, such as 4 ninths and 2 thirds in the *Footy lunch day a* task. Instead they rely on an observed demonstration or they follow a rule or procedure for making denominators the same. This strategy for renaming fractions is described next.

*Rename fractions*

This strategy is evident when students are able to compare related and unrelated fractions by renaming, for example, halves renamed as quarters (related fractions), and quarters and sixths as twelfths (unrelated fractions).

For the tasks that required the students to compare the size of related and unrelated fractions, for example, *Footy lunch day b* and *Sharing b*, (Figures 4.38 & 4.40 respectively), the SC students described the “need to find a common denominator”. Sandy’s response (SC, Year 7), is indicative of the reasoning used by the other SC students. She explained that “it is harder to find the answer with different denominators so I converted to the same one”. I asked her how she did this. She explained that “I converted 2 thirds into ninths, by thinking 3 into 9 goes three times, 3 three’s are 9 and if 3 times 3 is 9, you have to times the 2 (numerator for 2 thirds) by 3, which is 3 times 2 equals 6. So 2 thirds is the same as 6 ninths, and you see which numerator is larger. Maggie ate most pie”. In a similar way, Kalil (SC, Year 6) wrote the following (Figure 4.48 below).

*Figure 4.47* Chris’ response to *Footy lunch day b*
The SC students were able to apply and describe a procedure for renaming fractions. What is unclear however, is whether they understood why this particular procedure worked. In retrospect, it would have been wise to probe their thinking and reasoning more deeply. Responses of this type, suggest that the SC students are recalling and using rules and procedures provided by their teacher.

*Systematic approach to determining all options*

This strategy is evident when students identify all options for a Cartesian product situation based on a systematic list, table, or tree diagram.

The SC students indicated all options for the Cartesian product tasks, *Footy lunch day* and *Possible possibilities* (Figure 4.49 below) by creating a table or tree diagram.

How many different outfits can I wear? List all possibilities. **Show your working and explain your answer in as much detail as possible.**

*Figure 4.49 Possible possibilities*

The SC students had access to representation strategies to model the Cartesian product situation in some form. In response to *Footy lunch day*, Alison’s (SC, Year 6) table shown below (Figure 4.50) highlights the methodical attention given to each of the food items in relation to the drink and dessert items.
Figure 4.50 Alison’s table in response to Footy lunch day c

Tree diagrams were also used by Adam, Kalil and Kai (SC, Year 6) in response to Footy lunch day c. Adam’s tree diagram is shown below in Figure 4.51.

Adam explained his tree diagram by stating, “four options for a meal with a meat pie, then there’s two other food options. So if there are 4 for the meat pie, there’s 4 for the
others, so 4 times 3 is 12.” The language he uses suggests some experience with problems of this type.

Summary

The range of strategies used by the SC students reflects an understanding of multiplication as it occurs in a variety of contexts. They were able to use language appropriate to the situation and/or the procedure they elected to use. The SC students were all well equipped to use the strategies described in this section, given that they understood the mathematical content sufficiently well. This points to the need for AR students to develop strong understanding of concepts before being introduced to formal procedures.

4.1.4 Achievement: SNMY initial data collection

As explained in the previous chapter (3.5.3), student responses to the SNMY Project Initial and Final assessments (see Appendices A - B, pp. 2-27) were available to support this intervention study. The initial SNMY Project results were used to identify the AR and SC student cohorts from the lower and upper ends of the Learning Assessment Framework (LAF) (Appendix C, p. 29-36) respectively. Rubrics were used to support the scoring of students’ responses (see Appendix B, p. 19) In addition, detailed scrutiny of student responses to the SNMY assessment task items informed the design of the Multiplicative Task interviews (3.5.1).

To illustrate differences in student achievement, responses from the AR students will be presented first. This will be followed by responses from the SC students.

Achievement of the at-risk students in the Pre-intervention Phase

The AR students who participated in this study were assessed to be at Zone 1 of the LAF for multiplicative thinking. This is well below the level expected for students of this age. Particularly evident in the students’ responses to the Project assessment tasks was the ‘make all, count all’ strategy as described in 4.1.3.

Andrew (ARN, Year 7) and Hadi’s (ARI, Year 6) responses to Butterfly house b (Appendix A, p. 5) clearly illustrate a ‘make all, count all’ strategy (Figures 4.52 – 4.53 respectively below).
Andrew wrote about his ‘make all, count all’ strategy in his response above (Figure 4.52) and it is rewritten here for clarity and emphasis:

I worked it out by putting the sixteen wings in four groups and (a) body (for) each four wings, and two feelers for one butterfly (Andrew, ARN, Year 7).

Similarly, Hadi drew the wings, bodies and feelers individually and attempted to group together four wings, one body and two feelers to represent one butterfly. This is indicated by the partially grouped second butterfly and subsequent strikethrough in her written response. Despite the fact that this task required the students to work with numbers less than 20, they relied on a ‘make all, count all’ strategy, and in the case of Hadi, this did not lead to a complete correct solution.

When faced with a task that used larger numbers, on the whole the AR students maintained a tendency to use this ‘make all, count all’ strategy. Andrew (ARN) and Hadi’s (ARI) response to Butterfly House c (Appendix A, p. 6) illustrates this (Figure 4.54 & 4.55 below).
The students’ responses above indicate that Andrew and Hadi attempted to draw all the components for the 98 butterflies and group them accordingly. To actually draw or represent the wings, bodies and feelers for 98 butterflies is tedious. This would require the students to separately manage the count by ones for 392 wings, 98 bodies and 196 feelers.

Butterfly House d (Appendix A, p. 6) is not quite as straightforward as Butterfly House b and c by asking ‘How many complete model butterflies could you make with 29 wings, 8 bodies, and 13 feelers?’. This task required students to work with quotition division to determine how many complete model butterflies are possible and decide what to do with left-overs. Figure 4.56 illustrates Andrew’s responses (ARN, Year 7) and Figure 4.57 illustrates James’s responses (ARI, Year 6).
Andrew’s strategy for this task is unclear. It seems as though there was some attempt to halve the number of wings, bodies and feelers. This suggests that some sort of sharing may have been attempted. Whatever the case, Andrew was unable to successfully complete this task. In contrast, James, used a ‘make all, count all strategy’ to draw the butterflies and count the components as they were ‘used’.

The examples presented in this section to illustrate the achievement of the AR students, is consistent with the description of Zone 1 of the LAF, where students:

Can solve simple multiplication and division problems involving relatively small whole numbers, but tends to rely on drawing, models and count-all strategies... Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support more efficient calculation. (Appendix C, p. 29)

The achievement of the SC students will be illustrated in the section to follow.

**Achievement of the successful students in the Pre-intervention Phase**

The SC students by comparison were assessed to be at Zone 7 or 8 of the LAF for multiplicative thinking. This is beyond what might be expected for students at this level. To allow comparison of the AR students responses in the previous section, responses from the
SC students to the same task items (*Butterfly House b, c, & d*, Appendix A, p. 5-6) will be used.

As expected, the responses from the SC students differed from those of the AR students. The SC students were able to recognise, solve and explain their solutions to a range of multiplicative problems, in particular, use multiplication and division as appropriate. The responses below from Jessica (SC, Year 6) and Kalil (SC, Year 6) to *Butterfly house b, c and d* are indicative of students at this level (Figures 4.58 – 4.59 respectively).

*Figure 4.58* Jessica’s response to Butterfly house b, c, and d
Figure 4.59 Kalil’s response to Butterfly house b, c and d

The examples presented in this section to illustrate the achievement of the SC students, are consistent with the description of

- Zone 7 of the LAF, where students are:
  
  Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. (Appendix C, p. 35)

- Zone 8 of the LAF, where students:
  
  Can use appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. (Appendix C, p. 36)

Summary
So far, this chapter has reported the results of the Pre-I Phase interviews and achievement results for the 22 students involved in this study. The next section will report the results for the Post-I Phase, and in particular highlight the significant change in the beliefs and achievement for the ARI students.

4.2 Post-intervention Phase

The Post-intervention (Post-I) Phase occurred in November to December 2005. The purpose of this phase was to determine the extent to which student perceptions and achievement had changed over the course of this research. Therefore, in addition to the
SNMY Project final assessment (November, 2005), the Drawing Task, the Card Sort, and the Multiplicative Task interviews were readministered at this time.

Consistent with organisation of section 4.1 of this chapter, the results of the Post-I Phase will be organised into four sections: perceptions about learning maths well in class, perceptions about what matters in learning maths well, the strategies students use to solve multiplicative tasks, and student achievement in relation to multiplicative thinking.

4.2.1 Perceptions about learning maths well in class
The Drawing Task interviews confirmed that the themes identified in the Pre-I Phase (December 2004 - May, 2005) were also present in the Post-I Phase. These themes were:

- I learn maths well when I have the right answer
- I learn maths well when I do it the right way
- I learn maths well when I talk about it with others

However, there were some important changes in the beliefs of the ARI students. About learning maths well which pointed to a shift in the way they participated in learning mathematics, placing particular emphasis on working as a group and sharing ideas with others. This shift was not evident in the ARN or SC students. Results for the ARN and SC students, consistent with the Pre-I Phase will be presented first. This will be followed by the shift in the ARI students at the Post-I Phase.

Little change in the beliefs of the ARN and SC students
What the ARN and SC students chose to draw in the Post-I Phase did not vastly change from the Pre-I Phase as indicated by Table 4.4 below.
Table 4.4
Post-intervention Phase Drawing Task: observable features of student drawings (frequencies)

<table>
<thead>
<tr>
<th>Features observed</th>
<th>ARI (n=9)</th>
<th>ARN (n=5)</th>
<th>SC (n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tables</td>
<td>7</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Chairs</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Cupboards</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>White/blackboard</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Worksheets/books</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Stationery</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Charts</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>People</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Themselves (may incl. teacher)</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Student(s) (may incl. themselves)</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Other student(s)</td>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>Communication</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dialogue with others</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Dialogue with self</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Smiling</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Pointing/showing</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Content</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number/4 processes</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Algebra</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Problem solving</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Strategies/ideas</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Assessment</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Also consistent with the Pre-I Phase is the concept map generated by the analysis of the Post-I Phase interviews (see Figure 4.60 below). Student inclusion of physical features of the mathematics classroom and mathematics content remain relatively consistent with the Pre-I Phase. In addition, the concept map also illustrates student beliefs in relation to the diverse ways in which they engage with learning mathematics, that is, by working alone silently, as well as by communicating with others for the purpose of sharing knowledge and strategies. Three aspects to this communication with others were identified: the teacher with the whole group, the teacher with the individual, and the student with other students. The themes do not appear directly in the concept map as they are overarching notions derived from the drawings more generally. However, they are variously represented in the map as starred items. For example, in Figure 4.60 having the right answer can be seen under the ‘feedback’ element on the map where reference is made to progress and success and the ‘multiplication’ element where reference is made to facts and automatic recall. In a similar way, talking about it with others can be seen under ‘explanations’ and ‘students with students’.
Figure 4.60 Post-intervention Phase Drawing Task interview concept map – elements associated with learning maths well
To illustrate the key similarities with the Pre-I Phase, indicative responses from two ARN students and two responses from the SC students will be presented.

Andrew and Hajar (ARN, Year 6)

In the Post-I Phase, Andrew (Figure 4.61, Year 7) and Hajar (Figure 4.62, Year 6), both ARN students, described learning maths well when the teacher explained what to do and how to do it.

Andrew (ARN, Year 7)

Extract from field notes (7.12.2005)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>I: What’s happening in your picture?</td>
</tr>
<tr>
<td>8</td>
<td>S: Mr G is helping us by doing it on the whiteboard first, then we know how to do it.</td>
</tr>
<tr>
<td>9</td>
<td>I: Then what happens?</td>
</tr>
<tr>
<td>10</td>
<td>S: After he tells us what to do, we do it or try and do it.</td>
</tr>
<tr>
<td>11</td>
<td>I: Point to the most important part of your picture.</td>
</tr>
<tr>
<td>12</td>
<td>S: The teacher ‘cos you need the teacher to tell you how to do it.</td>
</tr>
<tr>
<td>13</td>
<td>I: What other situations help you to learn best?</td>
</tr>
<tr>
<td></td>
<td>...</td>
</tr>
<tr>
<td>16</td>
<td>S: Knowing that other kids have to do it as well?</td>
</tr>
<tr>
<td>17</td>
<td>I: Why is that important?</td>
</tr>
<tr>
<td>18</td>
<td>S: ‘Cos we all have to do the same thing?</td>
</tr>
</tbody>
</table>

Figure 4.61 Andrew’s drawing and field note summary
Hajar (ARN, Year 6)

Figure 4.62 Hajar’s drawing and field note summary

<table>
<thead>
<tr>
<th>Extract from field notes (30.11.2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I: Tell me about your picture.</td>
</tr>
<tr>
<td>2. S: When Mr X gives us a sheet and we don’t understand and he stops us and explains it on the whiteboard instead of telling only me. Others might be stuck so he explains it to all of us. When I’m stuck I get out maths equipment, shapes and cubes to help me. Sometimes when I’m really stuck I use a calculator. When I am not allowed to use a calculator I get a piece of scrap paper and work it out on there.</td>
</tr>
<tr>
<td>3. I: What is it about this situation that is helping you to learn maths well?</td>
</tr>
<tr>
<td>4. S: The equipment mainly and Mr X explaining it. No point using a calculator, it’s only going to tell you the answer.</td>
</tr>
<tr>
<td>5. I: Why? Isn’t it better to just get the answer?</td>
</tr>
<tr>
<td>6. S: You have to work it out yourself and if you didn’t have a calculator and you needed to work something, you’d be stuck, you have to work it out on scrap paper.</td>
</tr>
<tr>
<td>7. I: Point to the part of your picture that is most important.</td>
</tr>
<tr>
<td>8. S indicates Mr X explaining the maths stuff on the whiteboard.</td>
</tr>
<tr>
<td>9. I: Why is that most important?</td>
</tr>
<tr>
<td>10. S: He explains it better…the equipment can help you, the better one is Mr X explaining.</td>
</tr>
</tbody>
</table>

The teacher was located at the blackboard, addressing all the students in the classroom. In Andrew’s view, the teacher is critical, “(because) you need the teacher to tell you how to do it” (Figure, 4.61, line 12). Hajar’s response supports the importance of teacher explanations. She explained that when she does not understand, the teacher “stops us and explains it on the whiteboard instead of telling only me” (Figure. 4.62, line 2-3). These responses suggest that the students are learning maths well by doing the same thing in the same way as everyone else in the classroom and they do not feel alone.

In the Pre-I Phase, the AR students were less able or inclined to provided detailed responses. However, Hajar’s (ARN, Year 6) response to this task in the Post-I Phase indicates a change in this regard. She provided a detailed and insightful view of learning
maths well (see for example, lines 1-6). It would be naïve not to acknowledge that maturation will naturally contribute to the development and progress made by students. Sections 4.2.3 and 4.2.4 will show that maturation and current teaching approaches cannot be relied upon to significantly shift AR students’ understanding nor impact on achievement and participation in learning mathematics.

Chris (SC, Year 5) and Kalil (SC, Year 6)

<table>
<thead>
<tr>
<th>Extract from field notes (9.12.2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
</tbody>
</table>

Figure 4.63 Chris’ drawing and field note summary

Kalil (SC, Year 6)
Consistent with earlier evidence, the role of teacher explanations in relation to learning maths well was prevalent in responses from the SC students. In the following examples, Chris (Year 5, Figure 4.63) and Kalil (Year 6, Figure 4.64) both acknowledge that teacher explanations contribute to students knowing what to do. Chris’ drawing is in two parts. In the first part, he shows himself asking for help and in the second part, as a result of the teacher explaining the question, his speech bubble indicates “I understand now”. Chris appeared to recognise the importance of understanding and his role in initiating help to ensure he does understand.

Kalil, in addition to not being distracted when he is working (Figure 4.64, line 13), felt that it was important to know what to do (line 15). This know-how is provided by the teacher who “explains how to do it” (line 21) and like Chris, Kalil acknowledged personal responsibility for his own learning.

Summary
Consistent with the Pre-I Phase interviews, the ARN and SC students, described learning maths well when they do it right, get it right and talk about it with others. This involved situations where students were solving written algorithms based on the explanations and procedures provided by the teacher. The key differences that emerged from the analysis of the Drawing Task interviews in the Post-I Phase were exhibited by the ARI students. The nature of this change will be illustrated in the following section.
Change in the beliefs of the ARI students

The ARI student drawings illustrated students working in groups and their drawings were less focused on showing the teacher telling students what to do and how to do it, or indicate activity involving low-level operations with number. In reporting about learning maths well in the Post-I Phase, the ARI students appeared to be drawing on their recent experience in relation to learning mathematics and a new theme emerged: I learn maths well when I work in a group and share ideas with others. This was evident in the responses from the ARI students but was not evident in the responses from either the ARN students or the SC students. To illustrate this shift for the ARI students in the Post-I Phase, when evidence is presented, both drawings from both Pre- and Post-I Phases will be included.

Theme: I learn maths well when I work in a group and share ideas with others

The idea of working in a ‘group’ where students had opportunity to share their ideas with others emerged from the Post-I Phase Drawing Task interviews of the ARI students. ‘Getting it right’ and ‘doing it right’ remained important for learning maths well, however, the ARI students’ participation in the intervention program provided them with alternate ways of working. Aspects of the intervention program featured in the responses from these students. For example, ARI students, Adir (Year 6, Figure 4.65), Ahmed (Year 6, Figure 4.66) and Cansu (Year 6, Figure 4.67) all refer to the intervention program in some way. Their view of ‘doing it right’ was different from previous responses and ‘getting it right’ was achieved in different ways. The intervention program was linked to this recent experience.

Adir (ARI, Year 6)

Pre-intervention Phase drawing
Adir acknowledged his own improvement in mathematics by stating “I’m getting better at it” (line 1) and “before I wasn’t good at times, but now I’m better at it” (line 14). This is consistent with the belief evident in the Pre-I Phase that learning maths well was a consequence of being good at it. When I probed further, he mentioned that I had helped him (line 17). He reported that he was now able to help his friends if they needed assistance (lines 1-3). His Post-I Phase picture shows four students sitting around a table. The teacher is not present in the drawing. In comparison, Adir's Pre-I Phase picture shows students sitting alone at their desks with the teacher at the blackboard.
Ahmed (ARI, Year 6)

Pre-intervention Phase drawing

Post-intervention Phase drawing

Extract from field notes (29.11.2005)

1  I: Tell me about your picture.
2  S: Whole class in this class (that is our intervention program group). You giving the
3     worksheets out and we’re doing them.
4  I: What is it about this situation that is helping you to learn maths well?
5  S: Like the second day we came to you, you understand it good to me.
6  I: So this picture is about the second day we got together?
7  S: Yes.
8  I: Point to the most important part of your picture.
9  S: (Points to all of it: group work, conversation).
10 I: Tell me more about what is happening.
11 S: You’re at the board and we are doing our work.
12 I: How do you feel now about learning maths?
13 S: Good.
14 I: Tell me more about how good you feel.
15 S: In class now, we do tables grid, and now I have my certificate.
16 I: Fantastic
17 S: And my division.

Figure 4.66 Ahmed’s drawing and field note summary
Ahmed’s Post intervention Phase drawing depicted our intervention group working together. Ahmed stated that “you understand it good to me” (line 5). He was referring to me and my capacity to help him understand the work that we did together. He told me with pride that he was awarded a certificate because of his improvement with tables facts (lines 15-17). When I asked Ahmed to indicate the most important part of his picture, he pointed to all of it (line 9).

Both Adir and Ahmed’s Pre-I Phase drawings show students sitting at tables on their own and both Post-I Phase drawings show students sitting with a group.

*Cansu (ARI, Year 6)*

![Pre-intervention Phase drawing](image)

![Post-intervention Phase drawing](image)
Cansu’s response indicates a situation where her regular classroom teacher is providing positive feedback about her improvement in mathematics that year. Cansu explained that she had improved in her Progressive Achievement Test in Mathematics (PAT Mathematics) (ACER, 2005) in comparison to her result at the beginning of the year (lines 13-14). She indicated that now she was “smarter” and that this improvement was due to working at home with her older brother as well the work we did together (lines 18-21). Consistent with earlier beliefs, asking the teacher questions was important for understanding (line 5).
While Hadi’s Post-I Phase response did not directly mention the intervention program, she illustrated a feature of the program, that is, using arrays to model multiplicative situations. Hadi has drawn herself out the front of the group at the blackboard or whiteboard, and she...
is showing her solution to the rest of the group. Like Adir, Hadi has indicated a change in participation. She has placed herself in a position of support to other students. This suggests that learning maths well was coincident with being knowledgeable and confident.

These results provide evidence of improved results for these ARI students, that is, they are now ‘doing it better’. Chapter 5 will draw out how targeted support throughout the intervention program contributed to this.

Summary
The purpose of the Drawing Task interviews was to identify student beliefs about learning mathematics well in recognition of the two-way relationship that exists between beliefs and behaviour. Administration of this interview in the Pre-I Phase and again in the Post-I Phase allowed comparisons to be made within and between groups. The results from the Post-I Phase Drawing Task interviews indicate the ARN and SC students held views about the nature of learning mathematics well that were consistent with the Pre-I Phase. These beliefs were summarised by the following three themes:

- I learn maths well when I have the right answer
- I learn maths well when I do it the right way
- I learn maths well when I talk about it with others

However, differences between these groups evident in the Post-I Phase, indicated a shift in beliefs held by the ARI students. These students represented situations where learning maths well was associated with working in groups and sharing ideas with others. They acknowledged an improvement in their own ability and confidence in learning mathematics. The improvement in their own learning reported by the ARI students is supported by the results of the Multiplicative Task interview (4.2.3) and SNMY assessment results (4.2.4).

The results of the Card Sort interviews will be presented in a similar way to the Drawing Task interview as the two instruments targeted student beliefs about learning maths well, yet did so in a different way.

4.2.2 Perceptions about what matters in learning maths well
The Card Sort interview identified the descriptors about what matters in learning maths well most commonly ranked from most important to least important. Consistent with Pre-I Phase interviews, students gave high importance to thinking and talking and discussing, and low importance to worksheets and getting the answer. There was a change in the way the maths
equipment descriptor was conceived, despite being ranked consistently low at both the Pre- and Post-I Phases. Explanations was not so important at the Post-I Phase and getting help and doing well became more important. At the Pre-I Phase, the overall theme identified was when learning maths well, personal processes and actions are important. While the views of the students remained stable over time, differences at the Post-I Phase can be attributed to shifts in the views of the ARI students. Table 4.5 below presents the frequencies of descriptor use for each of the five layers of the card sort by order of importance.

### Table 4.5
Post-intervention Phase Card Sort frequency of focus word per layer – all students (n=21)

<table>
<thead>
<tr>
<th>Card Sort Layer</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>Total</th>
<th>Proportion</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>8</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>20</td>
<td>70.0</td>
<td>1</td>
</tr>
<tr>
<td>The teacher</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>14</td>
<td>50.0</td>
<td>2</td>
</tr>
<tr>
<td>Getting help</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>7</td>
<td>0</td>
<td>18</td>
<td>44.4</td>
<td>3</td>
</tr>
<tr>
<td>Talking &amp; discussing</td>
<td>1</td>
<td>6</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>19</td>
<td>36.8</td>
<td>4</td>
</tr>
<tr>
<td>Problem solving</td>
<td>0</td>
<td>5</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>16</td>
<td>31.3</td>
<td>5</td>
</tr>
<tr>
<td>Doing well</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>12</td>
<td>25.0</td>
<td>6</td>
</tr>
<tr>
<td>Group work</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>0</td>
<td>3</td>
<td>12</td>
<td>25.0</td>
<td>7</td>
</tr>
<tr>
<td>Feeling good</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>14</td>
<td>21.4</td>
<td>8</td>
</tr>
<tr>
<td>Explanations</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>15</td>
<td>13.3</td>
<td>9</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>12.5</td>
<td>10</td>
</tr>
<tr>
<td>Worksheets</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>9</td>
<td>11.1</td>
<td>11</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>12</td>
<td>8.3</td>
<td>12</td>
</tr>
</tbody>
</table>

A new theme emerged relating to the importance of sharing strategies with each other for learning maths well. The section to follow will report the Card Sort interview results that were consistent with the Pre-I Phase. This will be followed by what had changed in the Post-I Phase.

**Consistent with the Pre-intervention Phase**

The table above (Table 4.5) presents the frequencies of descriptors and the rankings for thinking, the teacher, talking and discussing, problem solving, group work, feeling good, getting the answer, worksheets, and maths equipment were the same or similar (a change of + or – 1 in rank) to the Pre-I Phase at the Post-I Phase.

Figure 4.69 below presents the Post-I Phase ranked list in the form of the card sort arrangement. Those descriptors that were generally rejected, those being ranked tenth to twelfth, appear outside and below the card sort diamond (indicated by *). The Pre-I Phase card sort (Figure 4.70) is also included to allow immediate comparison.
The theme that emerged from the Pre-I Phase Card Sort interviews was the belief that when learning maths well, personal processes and actions lead to an outcome. This view was evident in the Post-I Phase for the ARN and SC students. Once again, the comments made by students are italicised and the key features appear as non-italicised text. In the second column, ‘I’ refers to myself as interviewer and ‘S’ refers to the student.
Candi (ARN, Year 7) acknowledged the role of talking to the teacher and other students (see reasons for selecting the teacher, talking and discussing, group work, explanations and thinking in Figure 4.71) yet this was in reference to determining how to work it out and whether the result is correct. Candi mentioned the ‘answer’ when justifying use of problem solving and also referred to the process of getting the answer by knowing what to do and how to do it. He warned that “if you work quickly you might get it wrong”.

**Figure 4.71** Candi’s card sort and summary discussion record
Kalil (SC, Year 6)

Extract from field notes (19.05.2005): Reason(s) for selecting

<table>
<thead>
<tr>
<th>Thinking</th>
<th>First Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Second Layer</td>
</tr>
<tr>
<td>Getting help</td>
<td>Third Layer</td>
</tr>
<tr>
<td>The teacher</td>
<td></td>
</tr>
<tr>
<td>Show working out</td>
<td></td>
</tr>
<tr>
<td>Talk &amp; discuss</td>
<td></td>
</tr>
<tr>
<td>Check your work</td>
<td></td>
</tr>
<tr>
<td>A quiet place</td>
<td>Fourth Layer</td>
</tr>
<tr>
<td>Feeling good</td>
<td>Fifth Layer</td>
</tr>
</tbody>
</table>

| Thinking | I put it at the top ‘cos if you didn’t you would know what you’re doing, e.g., 8 times 4, think compared to just saying 6, the first number that comes to mind. |
| Explanations | I put it there ‘cos you need explanations to get a clear idea of what you’re meant to do. You need someone to explain it to you. |
| Getting help | If you’re stuck ask for help, they’ll probably be able to help you. If it’s really hard ask ‘what does this mean?’ e.g., 5 squared and you don’t know what squared means. |
| The teacher | Will probably be the best person to ask for help and they explain what to do, talk to you about it, if someone is smart, they can give their ideas to some who aren’t so smart. |
| Show your working out | Because they (the teacher) say, ‘show your working’ so they know how you worked it out…where you might have made a mistake and help fix it. If you show your working you might get one point if you still make a mistake or 2 points if you’re right with showing working out. |
| Talking and discussing | Share ideas so those who aren’t so smart, will help them learn. |
| Check your work | Because if you’re doing a test or it’s really easy as sometimes happens to me, like 10 times 10, I put 10 and forget the extra zero, if I’d checked it I would have got it right. |
| A quiet place | It’s really hard to think and learn if the place is really noisy, (admits that this is a bit of a problem for him). Kalil spoke about respecting other students’ needs if he finished early. |
| Feeling good | It would be hard to work if you’re feeling sick, you might start…but have to keep stopping…or if you’re not happy you might think ‘I don’t want to do this’. I: What about students who don’t feel good because of the maths? S: The teacher, talk to the teacher about it and the teacher might be able to think of a way where they might be able to do it and feel good about it…could happen…but I haven’t seen it happen. |

Figure 4.72 Kalil’s card sort and summary discussion record

Like the example from Candi (ARN, Year 7), Kalil (SC, Year 6) also provided some evidence regarding the role of interacting with others. This was dominated by reference to understanding information and procedures, that is, knowing what you are doing, and what you are meant to do (see reasons for selecting thinking, explanations, getting help and the teacher (Figure 4.72). Kalil did make reference to sharing ideas as being useful for assisting less able students (see reasons for selecting talking and discussing in Figure 4.72). In relation to
learning maths well, Kalil nominated four of his own descriptors to the collection of 12: these being, *a quiet place, don’t rush, check your work* and *show your working out*. It appears that the SNMY Project emphasis on reasoning might have impacted on Kalil’s approach to learning mathematics.

**The Card Sort additional words: Observations at the Post-Intervention Phase**

As for the Pre-I Phase *Card Sort* interview, each student in the Post-I Phase had the opportunity to indicate descriptors related to learning maths well that they believed were missing from the 12 descriptors provided. At this time, 11 of the 13 students who indicated other words or phrases, chose to include their own in their card sort arrangement. Table 4.6 below, lists the descriptors, the students who used them and the card sort layer (1<sup>st</sup> – 5<sup>th</sup>) at which they were placed. The table below also identifies:

- new descriptors indicated by an asterisk (*),
- descriptors used in both the Pre-I and in the Post-I Phases, indicated by an ‘=’.

The following descriptors were unique to the Pre-I Phase, *the times tables, listening, asking questions, concentrating, and being smart* were not present in the Post-I Phase. The student-nominated have been grouped into four categories. In addition to the three categories used in the Pre-I Phase, these being, maths content, behaviours, and personal qualities, the learning environment emerged as a fourth category.
Table 4.6
Post-intervention Phase Card Sort: additional words

<table>
<thead>
<tr>
<th>Student identified descriptor</th>
<th>ARI (n=9)</th>
<th>ARN (n=5)</th>
<th>SC (n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maths content</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The four processes</td>
<td>Berrin (2), Dean (3)</td>
<td></td>
<td>Kai (2)</td>
</tr>
<tr>
<td>The problem*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Behaviours</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check your work*</td>
<td></td>
<td></td>
<td>Kai (3), Kalil (4)</td>
</tr>
<tr>
<td>Doing the work*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Show your working out*</td>
<td></td>
<td>Yousif (2)</td>
<td>Kai (4), Kalil (3)</td>
</tr>
<tr>
<td>Learning*</td>
<td></td>
<td>Hadi (2)</td>
<td></td>
</tr>
<tr>
<td>Right levels*</td>
<td></td>
<td></td>
<td>Adam (2)</td>
</tr>
<tr>
<td>Work with a friend*</td>
<td>Cansu (4)</td>
<td>Ibrahim (5)</td>
<td></td>
</tr>
<tr>
<td>Understanding*</td>
<td>Yousif (3)</td>
<td></td>
<td>Adam (1)</td>
</tr>
<tr>
<td>Competition*</td>
<td></td>
<td></td>
<td>Adam (4)</td>
</tr>
<tr>
<td>Remembering*</td>
<td></td>
<td>Yousif (4)</td>
<td></td>
</tr>
<tr>
<td>Qualities</td>
<td></td>
<td></td>
<td>Sandy (1), Sandy (2)</td>
</tr>
<tr>
<td>Confidence=</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Willingness=</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Environment</td>
<td></td>
<td></td>
<td>Kalil (4)</td>
</tr>
<tr>
<td>A quiet place*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students who nominated additional words in the Pre-I Phase did not necessarily do so in the Post-I Phase and if they did, these descriptors were not necessarily the same. As with the Pre-I Phase Card Sort interviews, three students (2 ARI and 1 SC) identified aspects relating to the mathematics curriculum. The four processes was identified by two ARI students and Berrin (ARI, Year 6) had identified and used this descriptor in the Pre-I Phase also. Kai (SC, Year 6) added the problem to his card sort justifying this by stating “without a problem maths wouldn’t need to be used”.

The repertoire of behaviours seen to be necessary for learning maths well were extended to include: check your work, doing the work, provide students with work appropriate to their right levels, work with a friend, and understanding. Cansu and Yousif (ARI, Year 6) had not identified their own descriptors in the Pre-I Phase but did so in the Post-I Phase. In particular, Yousif identified and included three of his own descriptors: understanding, show your working out and remembering. These words seem to relate to the intervention program which emphasised importance on conceptual understanding and sharing of problem solving strategies.

**What had changed**

The rank assigned to explanations was reduced in the Post-I Phase in comparison to the Pre-I Phase, by a drop from second to ninth in the list of 12 (see Table 4.5). The descriptors
getting help and doing well were ranked more highly at the Post I phase. In the Pre-I Phase, explanations was generally understood to mean communication by the teacher in relation to what to do and how to do it. The reduced importance given to explanations in the Post-I Phase may be attributed to the nature of instruction during the Intervention Phase and achievement of the ARI students. The ARI students who participated in the intervention program, experienced a pedagogical approach characterised by a sustained period of learning targeted at meeting their needs. It is therefore not surprising that getting help and doing well were of greater importance.

There was change too in the way the students conceived the maths equipment descriptor. Looking closely at what the students said at the time of the Card Sort in relation to these changes, it appeared that the more noticeable change occurred in the ARI students who participated in the intervention program.

Getting help
There was an increase in the number of students from all three groups who selected the getting help card and its perceived importance. In the Post-I Phase, where 21 interviews were administered, nearly all students (n=18) included getting help in their card sort. This increase of five students comprised two from each of the SC and ARI student groups and one student from the ARN group.

The distinction of interest in the Post-I Phase resides in the difference between the ARI and ARN students. Views expressed by the ARN students were consistent for both the Pre-I and Post-I Phase administrations. This being that the teacher is the person who provides the ‘help’ when students don’t understand. The messages communicated by the ARI students, evidenced some change in view. The students talked about other options for seeking help prior to approaching the teacher. This suggests that the ARI students were now more inclined to take individual responsibility for their own learning For example, Cansu (ARI, Year 6) who placed getting help in the third layer of her card sort, stated:

(I put it) third, because first you have to discuss, then solve it, then think, if you can’t do it get help. (Appendix H, p. 206)

Yousif (ARI, Year 6), like Cansu, recognised that asking the teacher for help when he did not understand was no longer the only option. He indicated that a friend and/or the use of maths equipment could provide the necessary support:

(Discuss) with someone if you don’t know what the answer is…they can help a bit then you can figure it out. … Using something to get your answer … like MAB, unifix. (Appendix H, p. 196)
Doing well

This descriptor became more important for learning maths well in the Post-I Phase with most of the students tending to place the doing well card at higher layers of the card sort. In the Pre-I Phase, doing well and getting the answer were ranked close together (ninth and tenth respectively). In the Post-I Phase, these descriptors were further apart (sixth and tenth respectively).

Consistent with the Pre-I Phase, doing well was associated broadly with by understanding what to do, doing your best, and practice. However, the nature of doing maths well was different in the responses of ARI students in comparison to the ARN students. For the ARN students this was something they acknowledged was important, whereas for the ARI students, this was something that they now ‘owned’, that is, it was no longer something desirable in principle, but something they now had personal experience of. For example, Douha (ARI, Year 6) stated that doing well:

Is important to me, when I am doing well I am proud of myself. If you put it in your mind you’ll do well instead of putting it to your friends. (Appendix H, p. 192)

And Cansu said that:

You have to practice a lot and you have to concentrate, solve it, do it by yourself...because I did really well in my maths and Dart test and I’m proud. (Appendix H, p. 206)

Success was now something that the ARI students spoke about in terms of their own experience. This will be confirmed later in this chapter when the results for the Multiplicative Task interviews and achievement data is presented.

Maths equipment

It was evident in the Post-I Phase that the view of maths equipment had changed significantly for the ARI students. In the Pre-I Phase the SC students had seen the potential for modeling mathematical situations with maths equipment, whereas the ARI and ARN students tended to think about maths equipment in terms of items of stationery. In the Post-I Phase, this view changed for the ARI students. Their view was more like that of the SC students, recognizing the potential of maths equipment as mathematical models. This is evident in the following extract of field notes:

Because sometimes if you’re having trouble, the teacher suggests get some equipment...and I can draw what I’m thinking…they help a lot when I’m having trouble (Douha, ARI, Appendix H, p. 192)

…get MAB to help with thousands, hundreds, tens. (Hadi, ARI, Appendix H, p. 194)
As mentioned earlier, a new theme relating to sharing ideas and strategies emerged from the Post-I Phase Card Sort interviews for ARI students only.

**Theme: Sharing ideas and strategies with each other is important for learning maths well**

This theme emerged from the ARI students as a result of the analysis of the Post-I Phase Card Sort interviews. It is consistent with the theme that arose out of the Drawing Task interviews at the same time, that students learn maths well when they work in a group.

*Cansu (ARI, Year 6)*

![Diagram](image)
Extract from field notes (8.12.2005): Reason(s) for selecting

<table>
<thead>
<tr>
<th>Reason(s) for selecting</th>
<th>Reason(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking &amp; discussing</td>
<td>When you do maths…you have to talk and discuss otherwise won’t learn that much, when you discuss you get more ideas, more ideas, how to do it.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>I put it second ‘cos when you get a problem that you can’t normally solve, get scrap paper to do it there more ‘proplier’ (more properly). I: Why use scrap paper. S: Miss P says use scrap paper…if not enough room.</td>
</tr>
<tr>
<td>Thinking</td>
<td>When you normally do maths you have to think if you don’t you can’t do it.</td>
</tr>
<tr>
<td>Getting help</td>
<td>Third, because first you have to discuss, then solve it, then think, if you can’t do it, get help.</td>
</tr>
<tr>
<td>The teacher</td>
<td>If you still don’t understand from friends, go to the teacher to help you solve it. Friends first then the teacher.</td>
</tr>
<tr>
<td>Doing well</td>
<td>Have to practice a lot and you have to concentrate, solve it, do it by yourself. I: How do you feel then? S: Happy. I: Why? S: Because I did really well in my maths and Dart test and I’m proud.</td>
</tr>
<tr>
<td>Work with partner</td>
<td>Good to solve together, discuss it, different ideas you and your friend, do it together because you’re able to work more better and you’ll be better at working.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>If you don’t have worksheets you don’t have to work. I: We didn’t have worksheets but did we work? S: Yes. I: So what’s the difference? S: Mental maths answer questions on a sheet or questions on a blackboard.</td>
</tr>
</tbody>
</table>
| Explanations            | You don’t normally need explanations for maths. I: Why? S: ‘Cos it helps you but not that much ‘cos…not sure. I: Can you give me an example? S: When you’re doing work, doing mental maths, 6 times 2 and you don’t understand the question, go to the teacher and can help explain it to you. I probe for Cansu’s advice. S: ‘count by two’s, 6 plus 6’ and ‘double 6’.

**Figure 4.73** Cansu’s card sort and summary discussion record

Cansu (Figure 4.73) added *work with a partner* to the collection of 12 descriptors. Her phrase is consistent with the *Drawing Task* theme identified in the Post-I Phase: *I learn maths well when I work in a group*. She included her own nominated phrase, *work with a partner* in the nine words for the card sort, together with *problem solving* and *getting help*. Whilst, Cansu did not use *feeling good* in her card sort, ‘feeling good’ is implied in her justification of *doing well*:

(You) have to practice a lot and you have to concentrate, solve it, do it by yourself. I: How do you feel then? S: Happy. I: Why? S: Because I did really well in my maths and Dart test and I’m proud.

She reported a sense of pride in her achievement on an end of year numeracy and literacy test. Her reference to “solving it” was also mentioned when justifying interaction with others, through discussion where different ideas were shared (see *talking and discussing, getting help, and work with a partner*). She specifically mentioned ‘the answer’ once (see reasons for selecting *worksheets* in Figure 4.71) and more often referred to solving maths. Notice that *talking and discussing* is placed at the top of her card sort and she suggested that without this “you wont learn that much”. Douha (*ARI, Year 6*) in relation to *talking and discussing* mentioned:

…because when you need, talking and discussing with your friends, or table mates and work together…discuss, not tell the answer, just as the teacher would do. (Appendix H, p. 192)
Adir supports this idea through his justification of the group work descriptor:

Get a group and work together, all of you work together…If you work together and need help, a group can help. (Appendix H, p. 209)

This may be explained by the intervention program where there was a strong emphasis on dialogue as learning. When justifying the inclusion of explanations, Cansu stated that these were not vital but did not offer reasons why this might be the case. I suspect that given prior emphasis on explanations as providing advice as to what to do and how to do it, the experience of the intervention program provided a different way of working: explanations in the form of ‘telling’ did not occur in this way. This also suggests that perhaps they were no longer reliant on the teacher’s explanations and could do more for themselves and each other. The impact of the intervention program will form the basis of the next chapter.

Summary
Greater important was given to getting help and doing well as what matters in learning maths well in Post-I Phase Card Sort interviews. This was noticeably evident in the responses of the ARI students who frequently referred to the nature of their interactions with others which centered on sharing ideas and strategies as well as commenting on their own improvement.

The purpose of the Card Sort interview was to determine the degree of importance students assigned to given descriptors associated with learning maths well. Results for the Card Sort interviews confirmed the themes that emerged from the analysis of the Drawing Task interviews for the ARI group, with respect to:

- The Pre-I Phase: I learn maths well when I have the right answer, I learn maths well when I do it the right way and I learn maths well when I talk about it with others
- The Post-I Phase: I learn maths well when I work in a group and share ideas with others

For all 22 students interviewed at the Pre-I Phase, learning maths well is a consequence of being successful by knowing what to do, how to do it and to arrive at the answer. This view remained stable over the duration of this study for the ARN and SC students. Results of the Card Sort interviews in the Pre-I Phase identified that students recognised personal processes, such as thinking, for learning maths well. However at the Post-I Phase, the ARI students not only recognised the importance of personal contributions, but now valued them as they had become part of their personal repertoire.
These students acknowledged that interactions with others were important with respect to knowing what to do and how to do it. Even though getting the answer was rarely selected, students made frequent reference to this when justifying their selection of other descriptors seeing it as the outcome of learning maths well.

Section 4.2.3 will present the results of the Multiplicative Task interviews in the Post-I Phase. In particular, evidence will be presented of the shift in strategy usage made by the ARI students and their increased self knowledge and confidence in relation to multiplicative thinking.

4.2.3 Strategy use
This section will provide the results from the Multiplicative Task interview in the Post-I Phase. Once again, the SNMY Project assessments were used as the stimulus for the reflective discussion (Part A of the interview). The focus here will be on the differences in the strategies used in the Post-I Phase in comparison to the Pre-I Phase. Section 4.1.3 detailed seven strategies used by the AR students in the Pre-I Phase, such as ‘make all, count all’, skip counting and repeated addition. This was followed by a description of the five strategies used by the SC students in the Pre-I Phase, such as ‘using known facts’ and ‘use of multiplication’ across a range of problem contexts. At that time, the strategies used by the AR students were characterised by additive thinking and were in stark contrast to the multiplicative strategies used by the SC students.

Results of the Multiplicative Task interviews in the Post-I Phase indicate a marked shift in the strategies used by the 9 ARI students compared to the 5 ARN students whose strategies remained much the same. In particular, it was apparent that the ARI students were beginning to use some of the efficient strategies used by the SC students. The 5 ARN students maintained their reliance on inefficient strategies based on concepts for addition. The 8 SC students continued to use efficient strategies based on their concepts for multiplication and division. To illustrate what was consistent with the Pre-I Phase, strategies used by the ARN and SC students will be presented first. This will be followed by a description of the marked shift in the strategies used by the ARI students.

Strategies consistent with the Pre-intervention Phase
At the Post-I Phase, the strategies of the ARN students were consistent with those used in the Pre-I Phase. These were, make all, count all, when in doubt, add, repeated addition, and skip counting. To illustrate, Candi (ARN, Year 7, Figure 4.74) and Hajar (ARN, Year 6, Figure
4.75) used a ‘make all, count all’ approach in response to *Butterfly House b* and *d* (Appendix A, pp. 5-6).

![Figure 4.74](image1)  
**Figure 4.74** Candi’s response to Butterfly house b

Candi, in response to the *Butterfly House b* task (Figure 4.74) drew four complete butterflies, each with four wings and two feelers. Looking closely it is possible to see some single pen marks in the centre of the butterfly wings, suggesting that perhaps he counted by ones when completing the task.

![Figure 4.75](image2)  
**Figure 4.75** Hajar’s response to Butterfly house d

Upon initial inspection, it appeared that Hajar could possibly have reasoned her response mentally. However, at the time of the interview, Hajar in explaining how she worked out her response to *Butterfly House d* described a ‘make all, count all’ approach. She related that, “First I drew eight bodies and added four wings per body and two feelers and I think there were left-overs and then I counted the complete butterflies”.

Consistent with the Pre-I Phase interviews, the strategy of ‘skip counting’ featured in the strategies used by the ARN students. Examples from Andrew (ARN, Year 7, Figure 4.76) and Hajar (ARN, Year 6, Figure 4.77) illustrate this. During the interview, when responding to *People sitting b* (Appendix J, p. 236), Andrew skip counted by threes, saying “3, 6, 9, 12, 15, 18, 21, 24, 25, 26”. He tapped his fingers to keep track of the number of table sides. His written response is provided in Figure 4.76 below.
Andrew’s response to People sitting b
Hajar (ARN, Year 6) counted by twos in her response to Tables and chairs b (Appendix A, p. 2).

Figure 4.76

Hajar’s response to Tables and chairs b

I interviewed her on this task item as her written response was interesting: she had drawn four tables end to end and written the answer as ‘20 people’ based on the placement of two people for each table side. I asked her why she had placed two people per side of each table. She explained that “So more people could sit, no people to miss out”.

I wanted to know more about her reasoning, so the People sitting task (Appendix J, p. 236) was used for the ‘think aloud’ part of the Multiplicative Task interview. Again, she used skip counting, this time by a count of threes (Figure 4.78 below) for People sitting b.

Figure 4.77

Hajar’s response to People sitting b

When thinking aloud for part c of this task, Hajar said, “You could do the same as for b and add 5 more tables onto here…” After a period of silence, she said, “There would be 57 people. I added five more tables, three people on each side and one on the end. 25 people (from People sitting b) ‘cos you take one off’. Then she started counting, this time by ones, “25 … 26, 27, 28 … 29, 30, 31… 32, 33, 34 …” and so on. I did not expect to witness this protracted approach to manage counting by threes from 25 given the 18 month period between Multiplicative Task interviews in the Pre-I and Post-I Phase. The
demands of this task were not particularly high and I was surprised to see a lack of development in the strategies Hajar used given the 18 month intervening period.

The strategies used by the SC students were also consistent with those evident at the Pre-I Phase. These were, use known facts or procedures, use multiplication, skip counting, multiplicative partitioning, rename fractions, and systematic approach to determining all options.

Lynda (SC, Year 6, Figure 4.79) used division to support her reasoning for Tiles, tiles, tiles b (Appendix A, p. 18). This task required students to work out the number of 2cm by 3cm tiles that would be needed to cover an area of 27cm by 18cm. Her solution that 18 tiles were needed was incorrect and I interviewed her to find out more.

![Figure 4.79](image)

Lynda’s response to Tiles, tiles, tiles b

When I asked her to explain her thinking, Lynda said, “It’s 18 tiles, because 27 divided by 3 is 9 tiles down the side and 18 divided by 2 is 9 tiles across. So I added them 18.” At this point she stopped and told me to “Hold on.” Then she said, “That’s wrong, up across but not filling in the middle, so 9 times 9 is 81. Whoops!” This points to the value in drawing students’ attention to what they’ve done.

Alison (SC, Year 7) used multiplication to support her solution to Tiles, tiles, tiles c (Appendix A, p. 18). This task required students to work with the basic rectangular tile (3 cm by 2 cm) but increase its dimensions by 2cm (now 5 cm by 4 cm), then calculate how many of these larger tiles would be need to cover an area of one square metre. Her written response appears in Figure 4.80 below.
She explained that she “…added 2 centimetres to the length and width” of the original tile. Then “work out how many it would take” down both sides, “so 100 divided by 4 (25), then 100 divided by 5 (20). Then multiply them (20 and 25) to get the answer”. I asked her, why do you multiply and she told me, “That’s what you do for area, you multiply length by width”.

Unlike the ARN students, the strategies used by the ARI students in the Post-I Phase were more akin to the strategies used by the SC cohort. This is illustrated in the section to follow.

**Significant change in the at-risk intervention students**

Results from the *Multiplicative Task* interviews in the Post-I Phase indicated a significant shift in the strategies used by the ARI students. These students were less reliant on the inefficient additive strategies that featured in their Pre-I Phase interviews. The ARI students indicated a stronger understanding of multiplication by using strategies akin to those used by the SC students. Also, the ARI students tended to write less when composing their responses in the Post-I Phase. For example, Hadi (ARI, Year 6) used an efficient approach to solving *Butterfly House b* (Appendix A, p. 5). She justified her response by indicating “think times, 4 fours, 16, and 4 twos, 8”. Hadi’s response to this task in the Pre-I Phase (see Figure 4.53 earlier) indicated an inefficient make all, count all approach that did not support a correct solution.

James’s (ARI, Year 6) strategies also became more efficient. In particular, reasoning based on place-value parts with an increased capacity to hold information in his head. He recorded his solution for *Butterfly house c* (Appendix A, p. 6) without further justification of his reasoning. His response is shown below in Figure 4.81.
When asked about his thinking, he explained “because if you have only one body then 98 times 1 is 98. (For) two feelers, 98 times 2, 90 plus 90 equals 180, 8 plus 8 equals 16, so 196. (For the wings) I doubled the feelers”. This reasoning indicated that he worked with the number of tens, then the hundreds, then the ones: “90 plus 90 is 180, 100 plus 100, is 200, 200 plus 180 is 380. 6 and 6 is 12. So 380 and 12 is 392”. His justified his response to Butterfly House d (Appendix A, p. 6), by saying “you can’t have 8 butterflies even though you have enough bodies. I took 1 off the feelers because you can’t use it, so half of 12 is 6. 6 butterflies with enough wings”.

Consistent with this, is James and Adir’s (ARI, Year 6) response to Tables and chairs h (Appendix A, p. 2). Both students simply wrote “200” for their answer. Adir’s response is given in Figure 4.82 below.

Figure 4.82 Adir’s response to Table and Chairs h

Adir explained his thinking as “99 plus 99 is 198 and 2 more chairs. 200”.

ARI and SC students used similar strategies across the range of tasks used for the Multiplicative task interview, particularly for Tiles, tiles, tiles and Stained glass windows (Appendix A, p. 18 & 16 respectively). For example, Hadi and Yousif (ARI, Year 6) and Chris (SC, Year 5) and Sandy (SC, Year 7) all indicated the same albeit incorrect reasoning, thinking that two tiles can fit the area based on ‘double two is four and double three is six’. Yousif’s response is shown in Figure 4.83 below.
I was pleased to note Ahmed’s (ARI, Year 6) response to this task, the nature of which I would expect from the SC students. He indicates the correct reasoning for this task as shown in Figure 4.84 below.

He revealed that he drew the larger area first (6cm by 4 cm) then drew in the basic tile and “I saw that I could fit 4 in”.

**Summary**

The results from the *Multiplicative Task* interviews have indicated that in the Post-I Phase, there was a significant change in the strategies used by the ARI students. On the other hand, the strategies used by the ARN and SC students were the same as those used in the Pre-I Phase.

The next section will present the achievement data arising from the SNMY Project final assessments. These results show that the shift in the ARI students described above is also reflected in their improvement in LAF level, and that there was little change in achievement of the ARN and SC students.
4.2.4 Achievement: SNMY final data collection

As explained in the previous chapter (3.5.3), student responses to the SNMY Project Initial and Final assessments (see Appendix A, pp. 2-18) were available to support the intervention study. As described earlier in Section 3.3, the initial SNMY Project results identified the AR and SC students as being at the lower and upper ends of the Learning Assessment Framework (LAF) respectively. The student responses to the SNMY assessment task items were also used to inform the design of tasks for use during the Multiplicative Task interviews (3.5.1).

In this case, the final SNMY Project assessment data (November, 2005) was used to inform the Multiplicative Task Post-I Phase interviews and compare outcomes against initial SNMY Project assessment results. This comparison enabled me to determine the extent of individual and cohort change in relation to multiplicative thinking. This is presented in Table 4.7 below with average shift calculated to the nearest tenth. Project-wide SNMY assessment data will be considered later in this section.

Table 4.7
Intervention study change in student achievement in relation to LAF zones (n=22)

<table>
<thead>
<tr>
<th>Name &amp; 2005 Year Level</th>
<th>LAF Zone (Pre-intervention Phase)</th>
<th>LAF Zone (Post-intervention Phase)</th>
<th>Degree of shift (no. of LAF zones)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yousif (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Hadi (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Douha (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Ahmed (Year 6)</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Berrin (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Adir (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Dean (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Cansu (Year 6)</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>James (Year 6)</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ibrahim (Year 6)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Hajar (Year 6)</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Sidona (Year 7)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Andrew (Year 7)</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Candi (Year 7)</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lynda (Year 6)</td>
<td>8</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>Jessica (Year 6)</td>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Kalil (Year 6)</td>
<td>8</td>
<td>7</td>
<td>-1</td>
</tr>
<tr>
<td>Kai (Year 6)</td>
<td>8</td>
<td>7</td>
<td>-1</td>
</tr>
<tr>
<td>Adam (Year 6)</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Chris (Year 5)</td>
<td>7</td>
<td>6</td>
<td>-1</td>
</tr>
<tr>
<td>Alison (Year 7)</td>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Sandy (Year 7)</td>
<td>7</td>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>
As Table 4.7 illustrates, the nine ARI students, assessed to be at Zone 1 of the LAF in the Pre-I Phase made significant improvement in the Post-I Phase. The SNMY Project final assessments, located these students to be at either Zone 4 or 5 of the LAF in the Post-I Phase. In terms of the LAF, this is a shift of three or four zones. In contrast, the 5 ARN students made little, if any shift. Only one student, Andrew (ARN, Year 7) progressed from Zone 1 to Zone 3 of the LAF. Of the 4 remaining students, two students remained at Zone 1 and two students progressed one level to Zone 2 of the LAF. This is consistent with the overall SNMY Project shift of students initially assessed to be at Zone 1 of the LAF (Siemon et al., 2006).

Achievement data for the SC students was not expected to change greatly given that in the Pre-I Phase, they were assessed to be at the upper end of the LAF. The LAF does not indicate performance beyond Zone 8 therefore it is not possible to show an increase in student achievement. It would be reasonable to expect that the 4 SC students at Zone 7 initially might shift to Zone 8 in the Post-I Phase. However, these students did not make the shift to Zone 8. Two of these students remained at Zone 7 and the other two students regressed to Zone 6 and Zone 5 respectively. The four other SC students initially assessed at Zone 8 of the LAF either remained at that level or regressed one or two levels. This ‘ceiling effect’ and decline in performance may be related to the transition from primary school to secondary school where this ‘dip’ in performance is characteristic of students in the Middle Years (2.2.3). It may also reflect an over-reliance on well-rehearsed procedural skills as opposed to deep conceptual understanding.

The following sections describe the achievement for the ARN and SC students where little improvement was evident in the Post-I Phase. This will be followed by a description of the significant improved achievement levels for the ARI students in the Post-I Phase. Pre- and Post-I responses will be presented together for the purpose of comparison.

Post-Intervention Phase achievement of ARN and SC students

The ARN students’ responses to the SNMY Project final assessments were consistent with Zone 1 and 2 of the LAF (see Appendix C, pp. 29-30). On the whole, the SC students’ responses continued to reflect the use of multiplicative strategies consistent with the upper ends of the LAF, although three of these students regressed to either Zone 5 or 6 of the LAF.
Students at Zone 1 of the LAF tend to use ‘make-all, count-all’ strategies. In response to Butterfly house b, Andrew (ARN) in the Pre-I Phase clearly used a ‘make all, count all’ strategy. Andrew wrote about this strategy in his response below (Figure 4.85) and it is rewritten here for clarity and emphasis:

I worked it out by putting the sixteen wings in four groups and (a) body (for) each four wings, and two feelers for one butterfly. (Andrew, ARN, Year 6).

In the Post-I Phase, Andrew’s drawing of the four butterflies (above) suggested that he was still reliant on this strategy. Ibrahim (ARN, Year 6) also drew four butterflies when he responded to this item (Figure 4.86 below) at the Post-I Phase.
The strategies of ‘repeat addition’ and ‘skip counting’ were also used by the ARN students in the Pre-I Phase. Hajar appears to have attempted to use ‘skip counting’ for her solution to Butterfly house b (Figure 4.87 below) in the Post-I Phase.

```
3 butterflies
412
422
432
442
```

**Figure 4.87** Hajar’s Post-intervention Phase response to Butterfly house b

Andrew’s Pre- and Post-I Phase responses to Butterfly house c were interesting (Figure 4.88 below). While Andrew’s response in the Post-I Phase does not show continued reliance on a ‘make all count all’ strategy, his solution to this relatively straightforward problem is incorrect. One can assume that he doubled the number of bodies (98) to work out the number of feelers (196) but was unable to progress any further. What is surprising is that the 18 month period of learning between his first and final response did not equip him to solve this task item successfully.

```
<table>
<thead>
<tr>
<th>wings</th>
<th>bodies</th>
<th>feelers</th>
</tr>
</thead>
<tbody>
<tr>
<td>98</td>
<td>98</td>
<td>96</td>
</tr>
</tbody>
</table>
```

**Figure 4.88** Andrew’s Pre- and Post-intervention Phase response to Butterfly house c
Butterfly House d is not quite as straightforward as Butterfly House b and c. This task requires students to work with division to determine how many complete model butterflies are possible and decide what to do with left-overs. Figure 4.89 illustrates Andrew’s (ARN, Year 6) Pre- and Post-I Phase responses.

![Pre-intervention Phase](image)

**Pre-intervention Phase**

![Post-intervention Phase](image)

**Post-intervention Phase**

**Figure 4.89** Andrew’s Pre- and Post-intervention Phase responses to Butterfly house b

Andrew’s strategy in the Pre-I Phase is unclear. It seems as though there was some attempt to halve the number of wings, bodies and feelers. This suggests that some sort of partitive thinking relating to division may have been attempted. In the Post-I Phase, Andrew used a ‘make all, count all’ strategy where eight bodies were drawn, with 32 wings and 12 feelers. The additional feeler-less butterflies were crossed out to determine that six complete butterflies could be made.

**SC Students**

Responses to Butterfly House, b, c and d by Alison (SC, Year 7), Kalil, and Jessica (SC, Year 6), are included to illustrate the consistence of strategy usage by the SC cohort over the 18 month period.
Figure 4.90 below provides the response from Alison (SC, Year 7) to *Butterfly House b*. Her response suggests use of known facts which she has chosen to record in terms of division.

![Image of Alison's response](image1)

**Post-intervention Phase**

*Figure 4.90* Alison’s Post-intervention Phase response to *Butterfly house b*

Kalil’s response (Figure 4.91 below) used multiplication to complete his solution to *Butterfly house c* in particular, he multiplied by two for the number of feelers, then doubled this result using addition for the number of wings.

![Image of Kalil's response](image2)

**Figure 4.91** Kalil’s Post-intervention Phase response to *Butterfly house c*

Jessica used her understanding of division to produce an efficient solution to *Butterfly house d* (Figure 4.92 below). Near her calculations she wrote the following:

First I divided 4 into 29 (groups of wings), 1 into 8 (number of bodies) and 3 into 13 (groups of feelers). But I got difference answers for all of them. For bodies – 8. Wings – 7 (1 left over). Feelers – 6 (1 left over). I had to make complete butterflies, so I used the smallest number (feelers – 6 remainder 1) of any one of my ‘sums’. So my answer is 6 full butterflies.

![Image of Jessica's response](image3)

**Figure 4.92** Jessica’s Post-intervention Phase response to *Butterfly house d*
For the ARN and SC students, there was little improvement or change in achievement in relation to multiplicative thinking at the Post-I Phase. This strongly supports the results of the Post-I Phase *Multiplicative Task* interviews discussed earlier (see 4.2.3).

**Post-Intervention Phase achievement of the ARI students**

Post-I Phase results from the SNMY Project final assessments indicate that the ARI students shifted from Zone 1 of the LAF to Zones 4 or 5 of the LAF. The following images from students’ assessment booklets illustrate the difference between Pre- and Post-I Phase results.

*Figure 4.92* Jessica’s Post-intervention Phase response to Butterfly house d

*Figure 4.93* Hadi’s Pre- and Post-intervention Phase responses to Butterfly house b
Hadi and Cansu (ARI, Year 6) are no longer reliant on a ‘make all, count all’ strategy to respond to Butterfly house b (Figures 4.93 & 4.94 above). Hadi and Yousif’s response to Butterfly House c (4.95 & 4.96 below), looks much the same as Kasieal’s (SC, Year 6) response shown earlier (Figure 4.91). This supports the finding that the ARI students used strategies used by SC students evident in the Post-I Multiplicative Task interviews.
**Figure 4.95** Hadi’s Pre- and Post-intervention Phase responses to Butterfly house c

**Figure 4.96** Yousif’s Pre- and Post-intervention Phase responses to Butterfly house c
James’s Pre- and Post-intervention Phase responses to Butterfly house d

James simply wrote “6” for his response to Butterfly house d (Figure 4.97). I interviewed James about his response to this task as part of the Multiplicative Task Post-I Phase interview. I asked him to tell me how he arrived at this solution. He explained:

You can’t have 8 butterflies even though you have enough bodies, there’s not enough feelers. I took one off the feelers because you can’t use it, so 12. Half 12 is 6. So 6 butterflies with enough wings.

He then went on to indicate that you could make seven butterflies if you had one more feeler.

It was possible to track the progress of 19 SNMY project-wide AR (SNMY-AR) students, who were in Year 5 in 2004 and assessed to be at Zone 1 of the LAF but external to this study. The achievement results for this group of students in presented in the next section.

**Results for at-risk project-wide students**

In addition to students whose results were presented previously in Table 4.7, the 19 SNMY-AR students were consistent with the sample criteria for the 14 AR students who participated in this intervention study. Since the main focus of this research was the identification of the path towards multiplicative thinking taken by AR students, comparison of students of the same initial level without intervention was possible.

As Table 4.8 shows, the 19 SNMY-AR students initially assessed to be at Zone 1 of the LAF did not make the same degree of improvement as the ARI students.
Approximately one third of the SNMY Project AR students remained at Zone 1 of the LAF in the Post-I Phase, and approximately one third of the SNMY Project AR students indicated a shift of one zone of the LAF. Only three Project AR students made the same degree of improvement in achievement against the LAF as the ARI students.

Table 4.8 below indicates the degree of shift in terms of LAF zone made by SNMY Project AR students in comparison to the ARI students who participated in this study.

<table>
<thead>
<tr>
<th>Table 4.8</th>
<th>Comparative assessment data for Intervention Study ARI students (n=9) &amp; SNMY Project AR students (n=19)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LAF Zone (Pre-intervention Phase)</td>
</tr>
<tr>
<td>Intervention Study: ARI students</td>
<td>7</td>
</tr>
<tr>
<td>SNMY Project: AR students</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Results indicate that all 9 ARI students involved in this intervention study, improved by 3 or 4 zones of the LAF in the Post-I Phase. The students who did not participate in the intervention program, that is the ARN students and the SNMY Project-wide AR students, did not improve to the same extent, if at all. This indicates that the period of teaching that took place during the Intervention Phase made a difference to the learning outcomes of the 9 ARI students.

4.3 Summary

This chapter identified a range of beliefs held by the students who participated in this intervention study in relation to learning school mathematics well. Results indicated that although these beliefs remained relatively stable over the period of the research for the ARN and SC students, the ARI students not only identified the importance of working in groups and sharing ideas, they also indicated a change in their perception of themselves as learners.

The chapter also explored the strategies students used to work with multiplicative problems. Initially, the results indicated a stark contrast between the strategies used by the
AR students and the SC students. However, after the intervention there was a marked shift in this situation for the ARI students. The nine ARI students commenced their work with me using ‘make all, count all’ strategies suitable only for working with small numbers and appropriate in the first year of schooling. After the period of intervention, the *Multiplicative Task* interviews, and the SNMY Project final assessments revealed that the ARI students were using more efficient strategies based on a deeper understanding of multiplication and division. This degree of change was not evident in the ARN students.

The next chapter will discuss what made a difference to the learning outcomes of the ARI students who participated in the intervention program. In particular, how the social nature of learning, access to the Learning and Assessment Framework (LAF) for multiplicative thinking, and teacher identity impacted on both student identity and achievement in relation to the development of multiplicative thinking.
CHAPTER 5
DISCUSSION – THE INTERVENTION PROGRAM:
WHAT MADE THE DIFFERENCE?

The results of the SNMY final assessments indicated that all 9 at-risk intervention students (ARI) shifted from Zone 1 of the Learning and Assessment Framework for Multiplicative Thinking (LAF) (April/May, 2004) to Zone 4 or 5 of the LAF by the end of the intervention study (November, 2005). The fact that the 5 at-risk non-intervention (ARN) students from the same school cluster either remained at Zone 1 or made a shift to Zone 2 of the LAF suggests that the shift in performance of the ARI students was due to their participation in the intervention program. This is supported by the results of the Post-intervention (Post-I) Multiplicative Task interviews that showed that the ARI students were far more likely to use efficient multiplicative strategies than the ARN students. In addition, while student beliefs in relation to learning maths well remained relatively consistent across the ARN and SC (successful) sample groups, there was evidence of a shift in ARI student views about learning maths well, specifically, that they now recognised the importance of working in a group and sharing ideas with others. This belief is noteworthy because it accurately reflects the way in which the intervention program was implemented and suggests that working in groups in classrooms is more effective than working with individuals in withdrawal situations at this level. This challenges accepted views of intervention and traditional, lock-step classroom teaching practices in the middle years.

The over-arching message arising from this intervention study, is that it is not ‘too late’ to intervene in the later years of primary schooling. However, the reasons for this success differ from the reasons that interventions are successful in the early years (see 2.5.2). This chapter will reflect on the nature and implementation of the intervention program in light of the literature with a view to identifying and explaining what made the difference. Three broad but inter-related headings, based on the critical importance of teacher knowledge underpinning instruction (e.g., Shulman, 1987) and the instructional dimensions identified by Carpenter and Lehrer (1999) will be used for this purpose. Namely, what teachers need to know to teach mathematics effectively, that is, knowledge for teaching mathematics (5.1), the teacher’s capacity to act-in-the moment, that is, the act or performance of teaching involving engagement with tasks and tools (5.2), and the social
norms and values established in the group, that is, the nature of the relationships between all those involved in the enterprise (5.3).

But filtering through this, consideration will also be given to what appears to be a little known but, in this context, a very important acknowledgement by Shulman (personal communication, Boaler, 2003) that there are some missing elements from his *Model of Pedagogical Reasoning and Action* (Shulman, 1987) which are reported by Boaler (2003) as follows:

What is missing in that model? How must the model adapt? First we need more emphasis on the level of action, and the ways in which the transformation processes are deployed during interactive teaching, and how these are related, for example, to the elements of surprise and chance. Second, we need to confront the inadequacy of the individual as the sole unit of analysis and the need to augment the individual model with the critical role of the community of teachers as learners. Third, the absence of any emphasis of any emphasis on *affect, motivation or passion*, and the critical role of effective scaffolding in the teachers’ learning must be repaired. Fourth, we need to invite the likelihood of beginning, not only with text (standing for the subject matter to be taught and learned), but with *students, community, general vision or goal*, etc. (p. 15, my emphasis)

The initial discussion (5.1) will be framed by the recent work of Ball and her colleagues on mathematics knowledge for teaching (e.g., Ball et al., 2008) and Shulman’s (1986, 1987, 2004) views on pedagogical content knowledge (see 2.3.2). In particular, it will address two key but inter-related aspects of teacher knowledge, knowledge of mathematics for teaching, and knowledge of students’ mathematics learning.

Section 5.2 will be framed in terms of what is generally referred to as reform-based pedagogies (e.g., Boaler, 2002; Lester, 2007) but it will also consider what is needed to ensure these practices are effective with at-risk middle years students, particularly in relation to explicit practices and responsiveness.

Section 5.3 will spotlight the critical need for building effective relationships without which, the benefits afforded by teacher knowledge and progressive practice, are less effective. This section will also address recent concerns about the absence of the affective domain from considerations of what is needed to teach mathematics effectively (e.g., Garritz, 2010).

For simplicity, from this point, unless otherwise stated, the ARI students with whom I worked during the Intervention Phase will be referred to as ‘students’ and where identified by name, pseudonyms will be used.
5.1 Knowledge for Teaching Mathematics

An important reason for the success of the intervention program was teacher knowledge, particularly knowledge required to teach mathematics effectively. In terms of content, my knowledge of multiplicative thinking was well supported by the Learning and Assessment Framework for multiplicative thinking (LAF) (see Appendix C, p. 28). In terms of pedagogy, my teaching experience and professional reading over time, contributed to my knowledge of effective teaching practices.

The link between teacher knowledge and teacher effectiveness was discussed in Section 2.3.2. While researcher such as Ma (2010) and Hill et al. (2005) point to the strong positive relationship between teacher knowledge and student learning, the complex dimensions of which are acknowledged in the models developed by Shulman (1987) and Ball et al. (2008), they are careful to qualify the nature of this knowledge as knowledge for teaching mathematics.

In what follows, knowledge for teaching mathematics will be considered in terms of knowledge of mathematics for teaching (5.1.1) and knowledge of students (5.1.2). The first of these will consider the role teacher knowledge of key learning trajectories and effective pedagogy played in improved student learning. The second will consider the critical role of what Shulman (cited in Boaler, 2003) subsequently recognised as a missing element in his model of teacher knowledge. That is, teacher knowledge of students as individuals, their preferred ways of learning and interacting and their current levels of understanding and confidence.

It is argued that it was from this informed stance, that is, a knowledge of mathematics for teaching and critically deep knowledge of students that the intervention program was developed and implemented. Evidence from the Intervention Phase will be used throughout to support the claim that these two aspects of teacher knowledge are vitally important factors in the improvement made by the students.

5.1.1 Knowledge of mathematics for teaching

Middle years students are known to experience difficulty with multiplicative thinking (see 2.1.3). Therefore, having access to the very specific advice provided by the LAF was very important for the following reasons. The LAF contributed to my own knowledge for teaching mathematics in relation to multiplicative thinking. In terms of this intervention study, the structure of the LAF (Siemon, 2004, Siemon et al., 2006) supported the development and implementation of strategic learning tasks and sequences that took place
during the Intervention Phase. For each of the eight zones, the LAF identifies the key ideas and strategies that students would typically be comfortable with 50% of the time. It also provides advice as to what to *consolidate*/*establish*, that is, the key ideas and strategies within the child’s zone of proximal development, and what to *introduce*/*develop* (see Appendix C). The LAF allowed me to understand the potential learning trajectory not only in developmental terms but in terms of the mathematics content. The importance of access to this type of framework was discussed earlier in relation to understanding the transition from additive to multiplicative thinking (see 2.2.4).

However, how I implemented this advice was also critical. The measure of this is the achievement results for the students who participated in this intervention compared to the students who did not. All teachers who participated in the SNMY project had access to the LAF, but not all at-risk students project-wide made the same improvement (see 4.2.4). What is distinctive about this intervention is the sense I made of the very detailed information about *what* to teach provided by the LAF together with *how* I put this advice into practice. This illustrates the importance of the mathematics knowledge needed for teaching (see 2.3.2) which is a dynamic mix of content and pedagogical knowledge. The following evidence from Session 26 of the intervention program exemplifies this claim. The teaching episode was supported by Zone 2 of the LAF, in particular, the exploration of initial partitioning strategies for developing fraction understanding. The excerpt shows my understanding of the Zone 2 advice and how it was transformed in practice.

For instance, it is widely recognised that fractions are an important part of multiplicative thinking, yet is a problematic area of the curriculum for both students and teachers at this level (see 2.2.2). In terms of my work with these students, I went beyond standard classroom practice by making explicit connections to multiplication that were supported in strategic ways through meaningful models and representations (e.g., discrete, region, and number line models). I was aware of the importance of building on students’ intuitive understandings to enable them to connect meaningfully with more formal fraction ideas, such as the meaning and implications of fraction recording. I was also aware of the value in making connections to more generalisable ideas for multiplication, namely region models and the ‘for each’ idea”. This supports the link to the *factor idea* of multiplication (Siemon, 2003b) and ultimately the thinking required to mentally rename any fraction. For instance, renaming 4 fifths as fifteenths involves recognising that as the number of parts has been increased by a factor of 3, the number of parts (4) must also be increased by a factor of 3 on the grounds that for each fifth there are now 3 smaller parts (fifteenths), and
4 by 5 is 20. This link to the *for each idea* or *Cartesian Product* and the recursive nature of these ideas relates directly to Confrey’s (1994) notion of *splitting* and partitioning referred to in 2.2.3.

Partitioning and paper folding strategies discussed earlier in 2.2.3 were a starting point for teaching. The halving partitioning strategy is located within Zone 2 of the LAF:

- … through paper folding…apply thinking involved to help children create their own fraction diagrams. Focus on making and naming parts in the halving family…including mixed fractions…and informal recording…no symbols. (Appendix C, p. 30)

These fraction explorations through paper folding, and subsequent scaffolding of students’ creation of their own fraction diagrams is a direct consequence of my knowledge of teaching fractions. While paper folding is common to most approaches to teaching fractions, what is different here is the link I made to multiplication and the emphasis I gave to students exploring and creating their own fraction models and representation. As shown in Figure 5.1 the second and third region models illustrate how 2 parts (halves) by 2 parts (halves) gives 4 parts (quarters) and 4 parts (quarters) by 2 parts (halves) gives 8 parts (eighths) respectively. This in turn illustrates the inverse relationship between the number of parts and the size of each part.

*Figure 5.1* Successive halving

Also, with each successive halving of the region, there is an increase by a factor of 2 to create the new part. That is, for each half there is an increase by a factor of 2 to make quarters and for each quarter, there is an increase by a factor of 2 to make eighths.

The representations and models that were used and the teaching strategies that served to scaffold and link key ideas such as discussion, sharing ideas, and prompts are evident in the excerpt below. Following the excerpt is a discussion about the students’ responses to this aspect of the program.
SESSION 26: Monday 15th August, 2005

LEARNING FOCUS ~ Fractions: Paper folding partitioning strategies

Resources
Kinder squares
Paper strips about 30 cm long and 3 cm wide or so
Whiteboard/blackboard
Whiteboard markers/chalk

Purpose
Partitioning for equal parts (continuous fraction model)
Begin to explore denominator idea, how many, how much
Fraction comparisons

Learning Notes

We begin exploring ‘halving’ using kinder squares and paper strips. I encourage the students to halve in a variety of ways. The students then choose indicative examples of paper folding to paste in workbooks and label parts accordingly:

We discuss the relationship of parts to whole, the need for equal size and other observations the students can make.
I introduce ‘thirding’, “We’ve shown halves with paper folding, now let’s see if you can show thirds, by paper folding … (Some students interpret this as “easy” and proceed to fold and create quarters!). Then I ask “How many parts have you shown? “four” What do we call these, if there are four of them? “quarters” So how many parts do you think we’ll need if we want to show thirds? “three”. Ok, try again and fold to show three parts. Students found thirding more of a challenge.

I encourage students to keep persisting with folding to show thirds. We share successful strategies as discovered by students.
Students can ‘third’ kinder square and paper strips. Record, label and paste in workbooks.
We discuss comparisons between half and third and I encourage students to use these ideas to help with accurate ‘thirding’, that is, “a third is smaller than a half, find half, fold/draw a little less, then halve the remaining part.”
We apply ‘halving’ and ‘thirding’ thinking to the creation of our own fraction diagrams on the whiteboard or blackboard. I model the drawing of various shapes on the whiteboard as well as the thinking required for halving and thirding. I ask students to come to the whiteboard to show halves and then thirds themselves. I encourage the students to talk aloud about their thinking as they do so:
E.g., Partition to show halves…(repeat for thirds)

Post-session reflection
SCHOOL B: Students found thirding a challenge! And seemed confused with the language. ‘Half’ ok, but comparison between half and third unclear, e.g., third is smaller than a half.
SCHOOL A: Students were able to have a go at partitioning circles into halves and thirds. Discussed when we share things this shape in real life, they came up with pizzas, pies, cake. A challenge for them but trial and error on whiteboard led to improvement in equality of size of parts. When transferred this idea to paper
circles, used the strategy of placing a dot in the centre of the circle, then made useful marks on the edge of
the circle before marking in lines. See below:

I wanted students to understand how fractions are made and named, and we commenced
making fractions in the halving family on the grounds that doing this would help students
to understand the following key ideas:

- the need for equal parts
- the number of parts names the part, for example, five parts are called fifths, and that
- as the number of parts increases, the size of the part decreases.

Results from the Pre-intervention (Pre-I) Phase Multiplicative Task interviews confirmed that
the students did not understand these key ideas. They had used a range of additive
partitioning strategies (see 4.1.3) and interpreted fraction notation on the basis of their
whole number knowledge (see 2.3.3. and 4.1.3).

The exploration of halving through paper folding assisted the students to make and
name their own fraction diagrams in the halving family (Siemon, 2003b). For example, in
Figure 5.2, Ahmed indicates that despite partitioning in slightly different ways to make
quarters, (i.e., halving and halving length and width-ways, and halving and halving again
length-ways only), the quarters are the same.

Figure 5.2 Ahmed’s observations about quarters, Thursday Sept 8, 2005
Successive *halving* to create quarters, eighths, sixteens and so on, supports the idea that as the number of parts increases the size of the part decreases. Hadi’s strategy of halving to make and name quarters is illustrated in Figure 5.3 below.

![Figure 5.3 Hadi’s writing about fractions, Thursday Sept 8, 2005](image)

The use of paper folding as a basis for generating the thinking required to *make* and *name* fractions allowed the students to make complex observations themselves. This is illustrated below in Figure 5.4. When Ahmed and I were talking about his understanding of thirds, I wrote down what he told me. At the time, he noticed and explained that “one and half thirds will fit a half” and this is indicated in my handwriting in the figure below. I cannot be sure how he came to notice this, however, I suspect that frequent opportunity to partition many different paper models, create his own fraction drawings, and talk about his thinking contributed substantially to this. In order to maximize opportunity for making new connections and observations, there were times when I removed the requirement for the students to document their thinking in writing during our discussions. I did not want to add to the ‘cognitive’ load by having the students, think, talk and write at the same time.
Summary
This section has demonstrated the importance of knowledge of the ‘big ideas’ in relation to multiplicative thinking. Access to the LAF provided the necessary support to guide the transition from additive approaches to multiplicative thinking. I was also aware of the difficulty that students have, particularly in relation to fractions. I was informed about their understandings as well as their misconceptions. To address these, I knew of the importance of building on intuitive models, and in the case of fractions, through paper folding and partitioning to construct diagrams and line models. My knowledge of mathematics for teaching and knowledge of students’ learning allowed me to transform the LAF ‘text’ in ways that made the design and implementation of the intervention program so effective. This illustrates the nature and depth of knowledge required for teaching mathematics and has important implications for teacher professional learning.

5.1.2 Importance of knowledge of students
At this point, I would like to discuss one of the four elements that Shulman in Boaler (2003) acknowledged as missing from his model, that is, the importance of “beginning, not only with text (standing for the subject matter to be taught and learned), but with students, community, general vision or goals” (p. 15). Shulman (1987) had originally touched on this idea in a footnote, but had not given it primacy. Instead, Shulman’s (1987) model begins “with the assumption that most teaching is initiated by some form of ‘text’: a textbook, a
syllabus, or an actual piece of material” (p. 14). However the footnote suggests that:

Under some conditions teaching may begin with a “given group of students”. It is likely that at the early elementary grades, or in special education classes or other settings where children have been brought together for particular reasons, the starting point for reasoning about instruction may well be the characteristics of the group itself. There are probably some days when a teacher necessarily uses the youngsters as a starting point. (Shulman, 1987, p. 14, my emphasis)

For the purposes of this discussion, there are two points of interest in this footnote. Firstly, the idea that ‘some’ teaching on ‘some’ days can begin this way. Secondly, the suggestion that this only applies or is limited to the early years of schooling or in special education settings. It appears that Shulman’s (Boaler, 2003) missing element, that is beginning with students, is an acknowledgement of its importance for all teaching, and for all students, not just ‘some’ students in certain situations. I strongly align myself with this point of view. Beginning with where the students were at in relation to multiplicative thinking and being prepared to start there, is one of the critical reasons that the intervention program was successful.

Earlier in 2.2.2 the point was made about the importance of considering both content and learners (Ball, 1990). For this intervention study, the content was multiplicative thinking, and the learners were a group of students deemed to be at-risk. It was important for me to know not only their mathematical learning needs, but their social and emotional needs as well.

Recent focus on assessment for learning (see 2.3.4) has reinforced the view that students improve when specific learning needs are identified and addressed through focussed teaching (e.g., Black & Wiliam, 1998; Clarke et al., 2002; Wiliam, 2005). Shulman (2004) reminds us that, “teaching involves connecting not with their ignorance, but with their prior knowledge. Understanding the impact of prior knowledge on subsequent learning has been the most significant area of progress” (p. 131). Shulman (2004) describes what he terms a pedagogy of substance, and in doing so identifies the characteristics of an effective teacher. These characteristics include:

- Qualities, such as persistence and tenacity
- Understanding the subject matter and why it is important, and
- Being selective

But what resonates strongly with me is his mention of the “human connection… the connection with the lives and culture of the kids…” (Shulman, 2004, pp. 132-133). I will return to the importance of connecting with students in this fuller sense later in 5.3. At this
point, I will remain focussed on the role of understanding the learning needs of the students in relation to multiplicative thinking.

Knowledge of students is strongly recognised by professional associations, such as *Australian Association of Mathematics Teachers* (AAMT, 2006, 2008) and *National Council of Teachers of Mathematics* (NCTM, 2000) but perhaps not recognised to the same extent by researchers in their models of teacher knowledge (e.g., Ball et al., 2008).

Informal discussions with the classroom teachers of the students revealed that the teachers were aware these students were behind and that they experienced difficulty with mathematics. However, the teachers did not appreciate the extent or specific nature of this until their involvement in the SNMY Project and the initial testing results provided by the *Multiplicative Thinking* assessment tasks (MTATs). This highlights the need to ensure that ‘actual’ starting points for teaching are linked directly to the students, as opposed to the assumed learning needs suggested by some form of text, such as curriculum or program.

In the case of this intervention study, the LAF was a valuable and reliable diagnostic and planning tool that provided a research-based framework for me to match teaching to student learning needs. The time the students and I spent together over the 18 week period during the Intervention Phase, actually facilitated a shift in learning that was much greater than that achieved by the SNMY project overall (see 4.2.5). The LAF provides a basis for thinking about Hypothetical Learning Trajectories (Simon, 1995) for individual student and/or groups of students by identifying more closely what students already know, their critical areas of learning need, what might be done to address these needs, and where to go next (see 3.4.2). This supports the case made in 5.1.1 for the critical role of teacher knowledge. However, while access to the LAF is necessary, it is not a sufficient explanation for the significant improvement in student learning. Something more is needed.

**Knowing what students need**

Prior to working directly with the students, I had in-depth knowledge about their performance in relation to multiplicative thinking, as well as knowledge about their views about learning mathematics. This knowledge was derived from and supported by:

- the student responses to the SNMY initial assessment tasks,
- the Pre-I Phase *Multiplicative Task, Drawing Task* and *Card Sort* interviews, and
- the LAF for multiplicative thinking.
The SNMY initial assessment (May, 2004) had identified the students to be at Zone 1 of the LAF. The Pre-I Phase Multiplicative Task interviews confirmed this finding. I had evidence that these students:

- used a range of inefficient additive strategies to solve multiplicative tasks, and as a result,
- were well below where they should be in relation to the development of multiplicative thinking.

I was able to interpret and analyse student responses and what they meant for teaching. To illustrate and emphasise the inefficient nature of students’ work with tasks of this type, Hadi’s initial response to Butterfly House (Figure 4.55.) is re-presented as Figure 5.5.

**Figure 5.5** Hadi’s Pre-intervention Phase Butterfly House response

The tally marks suggest that there was an attempt to represent and count the number of components individually. I knew that I needed to focus my attention of developing the students’ knowledge of composite numbers, in particular to build mental objects for numbers to ten (as indicated by Zone 1 of the LAF). For example, in the case of the problem above, having the capacity to double 98 for the number of feelers and double this, for the number of wings would indicate the use of efficient mental strategies based on place-value understanding.

In the first couple of sessions of the program, I decided to introduce students to:

- **subitising** which is the ability to visualise the number of items in a small collection without counting the items one by one, and
- **part-part-whole knowledge** which is knowing the numbers one to nine in terms of their parts, for example, knowing 7 is 5 and 2, 3 and 4, 1 and 6, 3 less than 10, and so on.
This was critical as these underpin the additive strategies needed to develop confidence with working with numbers as composite units. For example, an excerpt from the second session of the program is given below.

**SESSION 2: Thursday 26th May, 2005**  
**INTRODUCTION ~ Part-part-whole understanding**

**Resources**  
Subitising cards  
Part-part-whole cards (tens frame model)

**Purpose**  
To model efficient ways of thinking about numbers, e.g., knowing all there is to know, e.g., 7; 3 and 4, 6 and 1, 5 and 2, 3 less than 10…  
To move children from thinking about numbers as a collection of ‘ones’

**Learning Notes**  
I work with the students sitting in a circle on the floor, reviewing the subitising cards. I flash through the cards at an appropriate speed and ask students to say aloud how many stickers they see. When it is my turn I do the same thing.  
The part-part-whole cards are then introduced, turned over slowly one by one in front of the students. A whole group discussion takes place. As each card is turned over, I ask, “What do you see?” - “Five.” - “What do you notice?” - “I can see a four and one more.”… “Five less than ten.”  
Students then are asked to select an appropriate number under 10. Students record in their workbooks everything they know about their chosen number, e.g., 6:

- 6 six
- It’s even
- 3 plus 3
- 8 take away 2
- 2 plus 4
- 6 is double 3
- 10 take 4

This work was well received, despite my initial concern that taking the students back to ideas that are normally established in the first year of schooling, would be demoralizing. To off-set this potential, I sat with the students on the floor, and participated in the activity in the same way I expected of them. I explained that I had made the cards myself, wanting something light and easy to transport on the back of my motorcycle. I modelled my thinking and reasoning, not once by way of example, but each time it was my turn. I was like them, and they were like me, in that our act of participation was the same. This is in
stark contrast to the practice of the teacher telling the students what to do at the front of the room, then setting the students to practice on their own (see 2.3.1).

What became very clear to me during these early sessions was that the students seemed to grasp the ability to *subitise* and develop *part-part-whole* knowledge far more quickly than I expected them to. Also, during these initial sessions I did not explicitly connect these ideas to addition and subtraction, however the students made the connection themselves as is evident in the following extract from Douha’s workbook (Figure 5.6). Douha recorded her understanding of the number 7 in terms of part-part-whole and included reference to subtraction.

![Figure 5.6 Douha’s understanding of 7, Thursday May 26, 2005](image)

I wanted to provide the students with the opportunity to apply their knowledge of part-part-whole relations in an alternate context, in this case, a puzzle style problem. I knew however that this might be potentially threatening to the growing confidence in the students. Therefore, I needed to find a way of removing this threat. The task that I had in mind, would typically be presented as a pen and paper task, and appears in the relevant excerpt of the program below.
SESSION 3: Tuesday 31st May, 2005

... Learning Notes

The task is to place the numbers 1 to 6 in each circle so that the sum of each row is the same. Tile/counters available for students to manipulate, removing the threat of having to commit to writing possible solutions down. Tiles can be manipulated and moved around on the circle problem sheet.

... I was aware that these students might struggle with the ongoing recording that might be required in the course of solving this task. Douha for instance, throughout the Pre-Intervention Phase interviews, as well as in the sessions to date, was very concerned that her handwriting and recording be correct, neat, and ‘done in the right way’. To remove the threat of having to undertake repetitive iterations of this task, I provided the students with a blank circle template of the puzzle, and 6 tiles, each tile with the numbers 1 through to 6 written on it. The students were then able to manipulate the tiles over the puzzle template, in various configurations, until a solution was found. If I observed them struggling, I was able to provide a hint, for instance, telling them the correct location for 2 of the tiles. The students recorded the solution in their workbook, once they had solved the puzzle with the help of the tiles and/or the hint. As it happened, we didn’t solve this task during session 3.

We worked on it once more in session 4, as well as some other modified versions of the task. My post-session reflection below indicates that two of the students elected to continue the problem in their own time at home. The excerpt below also indicates that the students were well-prepared to undertake modified versions of the task.

Excerpt from Post-session reflection (Session 4, Wednesday June 1, 2005)

... Cansu and Adir reported doing circle prob. At home! All students successful after being given the clue, ‘all sides 12’. Cansu and Adir able to make sides equal, 10, 11 and 12. Berrin, 12, Ahmed and Dean , 11 and 12.

It was important to provide the students with alternate ways of working to maximise task engagement. In the case of the example above, providing more than one opportunity to solve puzzles of this type meant that the students could build on their previous experience at the same time as I withdrew the degree of support.
5.1.3 Summary

The focus of this intervention was the student and their specific learning needs. In order to do this, mathematics knowledge for teaching guided by the ‘big ideas’ (see 2.2) in relation to multiplicative thinking and rational number proved to be critical. For teachers, this support comes in the form of the LAF that provides the basis for identifying where students are at in relation to the development of multiplicative thinking, and appropriate starting points for teaching. However this advice is not in the form of a tightly constructed set of instructions to be followed in a strict manner. There, the LAF alone is insufficient. For this study, the initial assessment process had identified the students to be a Zone 1 of the LAF and this enabled me to make a start with my planning. This purpose of the LAF is consistent with other learning and assessment frameworks, however the Learning and Assessment Framework for Multiplicative Thinking is unique in that it deals with the big ideas associated with the transition from addition to multiplicative thinking. The advice provided by the LAF works by locating where an individual student is at and scaffolds the developmental progression towards the development of multiplicative thinking.

What is critical is teacher knowledge, and attention to individual student’s learning needs. It is also critical for teachers to have the capacity to interpret student responses to identify what needs to be done and how, while at the same time being alert to possible behaviours that might get in the way of learning. It is therefore important, how this advice and knowledge of both content and students is transformed in practice. The next section deals with the pedagogical approaches implemented in the course of the intervention, in particular, reform practices (see 2.3.3) as well as some qualifying advice.

5.2 Teaching in Action

Explanations. We teachers – perhaps all human beings – are in the grip of an astonishing delusion. We think that we can take a picture, a structure, a working model of something, constructed in our minds out of long experience and familiarity, and by turning that model into a string of words, transplant it whole into the mind of someone else. Perhaps once in a thousand times, when the explanation is extraordinarily good, and the listener is extraordinarily experience and skillful at turning word-strings into non-verbal reality, and when the explainer and listener share in common many of the experiences talked about, the process may work, and some real meaning may be communicated. Most of the times, explaining does not increase understanding, and may even lessen it. (Holt, 1973, p. 164)

In this section the complex act of teaching that occurred during the intervention program will be examined with a view of identifying what else made a difference to the significant improvement of the students. The reform principles and approaches associated with
effective practice (see 2.3.3) particularly discussion, problem solving, constructive feedback, and group work were featured throughout the intervention program. The quote from Holt (1973) at the beginning of this section sums up my experience as a school student. Often the teacher’s explanation did not make sense. Romberg and Kaput (1999) suggest that traditional practices associated with teaching and learning mathematics are more the norm:

Traditional school mathematics has failed to provide students with any sense of the importance of the discipline’s historical or cultural importance, nor any sense of its usefulness. Is it any wonder that students dislike mathematics and fail to learn it? The premise of this book is that traditional teaching and learning of mathematics has not enabled students to learn mathematics with understanding. (p. 5)

I wanted to teach with understanding, for understanding. To do this, children’s “mathematics learning must be observed in contexts where we as teachers can intervene with the intention of influencing it” (Steffe, 1994, p. 13). Reform-oriented approaches typically offer more opportunity to intervene in constructive ways. These practices have featured in my teaching, both in my past and current teaching positions, and the impact of these in this context for this intervention study will be considered in 5.2.1.

In Section 5.2.2, I will turn my attention to the responsive nature of my approach which I believe is another reason these middle years students made such a significant improvement. The sense in which ‘explanations’ are framed in Holt’s quote above, is the antithesis of the approach I took when working with my students. The previous section highlighted the importance of teacher knowledge of mathematics and students. The students were my constant starting, and re-starting point for teaching. By ‘re-starting’ I mean that they were not my starting point once at the beginning of the intervention only. They were my constant frame of reference. My ongoing planning was highly dependent of the students’ actions and reactions to what we did. Taking this approach enabled me to provide quite targeted and explicit advice and explorations that enabled me to be particularly sensitive to their needs.

5.2.1 Reform-oriented approaches work

The success of the intervention program demonstrates that reform principles and approaches can be usefully applied to targeted intervention with students in the middle years. Reform approaches featured in 2.3.3, advocate the use and implementation of group discussion, co-operative group work, open-ended tasks, problem solving and investigations, and the provision of constructive feedback. The broad purpose is to teach for understanding (e.g., Fennema & Romberg, 1999). These approaches were undertaken
throughout this intervention and served to contribute to making a difference to the students a long way behind curriculum expectations for their level. Shulman (1987) referred to the performance of teaching as instruction, but provided little detail about what this entailed. Understandably so, as teaching is a challenge to write about. Shulman returned to the nature of instruction and identified it as another ‘missing element’ in Boaler (2003), referring to the “the ways in which the transformation processes are deployed during interactive teaching.” (p. 15).

The approach taken in this intervention is atypical of what we currently know about intervention programs which tend to focus on basic skills and deficit models, may involve individualized instruction, perhaps some work in small groups, and where the program itself tends to be linked more closely to curriculum than to students (see 2.5). This study changes this image. Reform approaches were successful in the context of this intervention because we worked in groups, and this was one of the aspects that students noticed most evident in their Post-Intervention Phase Drawing task interviews (see 4.2.1).

What follows is evidence from the Session 31 and 32 of the intervention program together with student responses designed to highlight four practices associated with reform approaches, that is, discussion, problem solving, constructive feedback and group work. Following this, the four approaches will be discussed in more detail.

SESSION 31: Monday 29th August, 2005

LEARNING FOCUS ~ Fraction review

“How many are hidden?”
I used tiles/counters (discrete fraction model) to set up the following 4 situations progressively one after the other. For each situation:
1. I ask the students, “I have hidden half the collection. How many tiles are under the card?”
   ![Illustration of half the collection]
   We offer our ideas to each other, sharing our responses and why, for instance, “I think there must be eight counters under the card, because half means two groups that are the same…”
2. “I have hidden 2 quarters of the collection. How many tiles are under the card?”
   ![Illustration of 2 quarters of the collection]
3. “I have hidden a quarter of the collection. How many tiles are under the card?”
   ![Illustration of a quarter of the collection]
4. “I have hidden a third of the collection. How many tiles are under the card?”
   ![Illustration of a third of the collection]
I then posed a task using continuous fraction representations and handed out to each student a small square labeled ‘1 half’ and a small rectangle labeled ‘1 third’. Their task was to paste each in the workbook and indicate what the ‘whole’ would be. Students talked to one another about their ideas and responded individually in their books.

Session 32: Thursday 1st September, 2005

LEARNING FOCUS ~ Problem solving strategy: Ask, think, do!

Resources
Formal and informal fraction notation cards
Ask think do problem solving guidelines (Siemon, 1990) (laminated cards)

Questions we can ask…

Things we can think about…

Things we can do…

Purpose
Teach strategy for problem solving in context of fractions

Learning Notes
First we play ‘Concentration’ with the fraction cards. Then I introduce the following situation to the students: “Sometimes we have to think about fractions when we do everyday things…” We allow time for discussion. I then present the following problem: I have six cups of milk. A muffin recipe uses 3 quarters of a cup of milk. How many times could I make the recipe before I run out of milk? I model the use of the ‘ask, think, do’ cards to guide our thinking about the problem. We engage in discussion and allow student to trial their ideas and share with others. Where necessary I actively model cups and the amount of milk. Students document their thinking and solutions in workbooks using words and pictures to illustrate.

Post-session reflection
SCHOOL B: James, Dean, Ahmed absent. Only worked with Cansu and Adir. Worked well at reading and naming formal fraction notation. Both made sensible decisions about placement of 3 digit numbers in the place value path game. And could generalise “I need to throw a 1 or 2 at least, to make a number that I need.” Needed to actively model and think out the problem. Will create a similar problem with support materials, e.g., cups and amounts of milk to support conceptualisation of the problem.
SCHOOL A: Douha and Hadi made some inefficient decisions about placement of 3 digit numbers in the place value path game (repeat Friday). They realised themselves though. Hadi and Yousif able to observe, “J, you can’t do anything”. Students able to argue according to place value understanding. Recapped make, show, record ideas to help equip students what they can think about and do, paper folding and drawing. Hadi drew 6 cups, showed quarters and could see 6 times. When reminded of left over milk, said “Eight”. Yousif (followed strategies of those around him) and Douha worked similarly but left over milk not as immediately conceptualised.

The importance of discussion

An important aspect of the program was the provision of ample opportunities for the students and I to talk about our thinking and solution strategies wherever possible. I know anecdotally that the amount of conversation time far outweighed the time spent writing or recording that tend to be more of a focus in typical intervention programs. These discussions described by Bostic and Jacobbe (2010) as “content-relevant conversations” (p. 34) provided a forum for sharing ideas and strategies and opportunity to learn from one another. More often than not, these conversations occurred while writing too. The
evidence from the Session 31 of the intervention program given above illustrates the discussion that occurred when the students and I were building on our work with fractions, by having the students engage with the ‘make, show, record’ activity. Below in Figure 5.7 is Hadi’s ‘make, show, record’ response for ‘one and 1 third’.

**Figure 5.7** Hadi’s ‘make, show, record’ for 1 and 1 third, August 29, 2005

I noted in my post-session reflection that the discussion the students had with each other contributed to their success with this task.

Discussion was important when the ‘ask, think, do’ model of the problem solving process was introduced (Session 32) for the muffin recipe problem. As noted in my post-session reflection, the students and I engaged in a discussion about the problem and how this problem solving strategy might assist in guiding the solution process. As part of this discussion, I acted out the use of the cups and the milk to support the conceptualization of the problem. How this approach impacted on the students’ work with this problem will be addressed in the section to follow in relation to problem solving. At this time however, the students were comfortable with the ‘talk’ that took place. However, this had not always been the case. I noted after Session 9, when Berrin and Ahmed had worked together at solving a cookie sharing problem, that Ahmed had difficulty explaining his thinking.
After Session 9, I resolved to give particular attention to thinking and reasoning for the remaining 4 sessions of the school term. For example, in Session 10, the students were presented with my thinking for ‘33 and 12 more’ in the form of a thinking string, and were asked, “If this is my thinking, what might the question have been?” This is illustrated in the following excerpt from the intervention program.

Excerpt from Intervention program (Session 10, Wednesday June 15, 2005)
INTRODUCTION ~ Think on the thinking string
Present a different a thinking string from the one used in the previous session. Ask, “If this is my thinking, what might the question have been?”

Have students create their own thinking string for ‘57 add 13 more’. This will allow the student opportunity to justify their thinking and make their thinking visible to the teacher. (Appendix D, p. 55)

My thinking string and the associated discussion, provided the students with a model for documenting their own reasoning strategies. Figure 5.8 below shows Douha’s thinking strings for 41 and 15 more and 72 and 11 more.

Figure 5.8 Douha’s thinking strings, Thursday June 16, 2005
Engaging in problem solving

Engaging in problem solving enabled the students to acquire the skills and confidence to tackle problems and explore possible solution strategies. Over the course of the intervention program the students became more resourceful when confronted with something new or different, such as puzzles and problems. Their initial response, for example, to a circle puzzle first introduced in Session 3, was “I don’t get it”. Then, during Session 11, I introduced the following problem (adapted from AAMT, 2005):

- “I have some pets at my house. I see 16 legs go past me. How many pets might I have?”

I encouraged the students to think beyond one animal type, for example, ‘dogs, dogs have 4 legs, so 4 dogs’; or ‘birds, birds have 2 legs, so 8 birds’. Though, as indicated in the excerpt below, the students were generally unable to do so.

Excerpt from Post-session reflection (Session 11, Thursday June 16, 2005)

…Both sites: Introduction activity a little confusing to student initially, with students saying, “I don’t get it”. … This evident also in leggy problem. Students tend to rely on sticking with single animal types e.g., 4 dogs, and 8 birds, Hadi, Josh, Ahmed and Adir beginning to explore combinations…(Appendix D, p. 57)

Later in the program, during Session 32, when I presented the muffin recipe problem involving fractions, the students ‘not getting it’ was not evident at all. The students had access to each other to discuss the problem and very quickly started to represent the problem visually in their workbooks. They drew on the partitioning experiences though paper folding and creating their own fraction diagrams we had explored in previous sessions, to create their own images of the ‘cups of milk’. These experiences and opportunity for discussion contributed to their success with the problem solving task. For example, Hadi’s strategy was to draw the six cups, then partition each of the cups into quarters.

Excerpt from Post-session reflection (Session 31, Monday August 29, 2005)

...Hadi drew 6 cups, showed quarters and could see 6 times. When reminded of left over milk, said “Eight”. Yousif (followed strategies of those around him) and Douha worked similarly but left over milk not as immediately conceptualised… (Appendix D, p. 81)

She then coloured 3 quarters of each cup in red and the remaining 1 quarter of each cup in green. She grouped three of the green quarters, twice to represent the other two times the recipe could be made. Hadi’s workbook response to this problem is included in the Figure 5.9 below.
Provision of constructive feedback

The on-going discussions that occurred during the intervention program provided the vehicle for the provision of constructive feedback at the point of need. For instance, after introducing partitioning strategies, such as ‘halving’ and ‘thirding’ through paper folding, we applied the reasoning strategies derived from the paper folding activities to create students’ own fraction diagrams. It was possible for three or four students at a time, to work directly on the whiteboard and share their thinking in a think-aloud manner with the whole group (including me). This strategy is noted in Session 26 of the intervention program:

- Extend ‘halving’ and ‘thirding’ strategies to fraction diagrams on the whiteboard or blackboard. Draw shapes on the whiteboard and ask students to use the whiteboard marker to show halves and then thirds. Encourage students to talk aloud about their thinking as they do so. (Appendix D, p. 72)

By working this way at the blackboard as opposed to, for example, completing the task with pen and paper on a worksheet, the students benefited in a number of ways. They did not have to commit permanently to each partitioning stroke they made. If they were inaccurate at partitioning for example, a circle into thirds, they could erase an initial attempt and erased and improved upon it. This supported them to be successful in terms of completing the task appropriately which in turn contributed to a sense of accomplishment and willingness to ‘have a go’. I was also able to provide advice and/or prompts about their partitioning strategies, as these were created which gave them the confidence to take risks knowing that they would be well supported and guided:

- …but trial and error on whiteboard led to improvement in equality of size of parts… (Session 26, Excerpt from Post-session reflection, Appendix D, p. 74)

The students, giving voice to their own reasoning, and having the opportunity to observe and listen to the strategies of the other students around them, enabled them to learn from each other:
• ... (students are) beginning to think, ‘third smaller than a half...’ and use this thinking for a variety of models ... (Session 29, Excerpt from Post-session reflection, Appendix D, p. 78)

One of Yousif’s responses for ‘thirds’ is shown in Figure 5.10 below.

![Figure 5.10](image)

**Figure 5.10** Yousif’s understanding of ‘thirds’ September 8, 2005

In contrast, James had been experiencing some difficulty creating his own fraction diagrams for thirds. This is shown in Figure 5.11 below.

![Figure 5.11](image)

**Figure 5.11** James’s thirds, pre- and post-feedback problem solution August 25, 2005

The image on the left of Figure 5.11 shows James’s attempt to construct his own fraction diagrams for thirds. It is clear that he understands the need for three equal parts when partitioning, but has difficulty when partitioning the circle. This attempt has been crossed out. I provided the following feedback to the James: “Try thinking back to the strategies you used when folding the paper circles to show thirds”. We talked about how a third was a little less than a half, which enable the first partitioning stroke (identified by the arrow on the image far right). His attempt was successful after this prompt.
Group work

Reform-based approaches work in the context of intervention with at-risk students in the middle years. What made the difference in this case was working in groups. This is what the students noticed most about our work together and was the most distinctive shift in beliefs in learning maths well. In particular, group work provided the context for meaningful discussion, supported problem solving opportunity, and the provision of constructive feedback ‘in the moment’. This suggests that intervention programs with middle years students need to be characterised by working in groups rather than one-on-one withdrawal situations that focus on basic skill development and automaticity. Reform practices are most effective where the focus is on a) understanding, and b) opportunity to work collaboratively and interactively in a group.

5.2.2 Responsiveness

So far, I have shown how the broad-brushed reform approaches discussed in section 2.3.3 featured throughout the intervention program and contributed to the significant improvement in student learning. Reform approaches focus on the activity of teaching in general terms. This intervention program involved more than this. Teaching-in-action is also about carefully targeted responses to particular, unique needs. These additional considerations are relevant to the middle years context. At-risk middle years students have unique academic and psychological needs (see 2.1.1 & 2.4 respectively) and assessment for learning practices make a difference to student learning (see 2.3.4). This intervention approach used reform practices in an ‘up-close and personal’ way to meet the students’ unique needs. This was achieved through:

- Contingent planning
- In-depth explorations of key ideas, and
- Sensitivity

Each of these considerations is described and illustrated in turn.

Contingent planning

An important feature of the intervention was that I did not plan too far ahead as opposed to intervention programs that tend to be highly prescribed. My planning, though supported by the LAF, was at the same time dependent on the students’ actions and reactions to the activities and discussions we had. In fact, I generally planned one session at a time on the basis of reflection upon the session that can just occurred. For instance, during Session 2, I
tipped in excess of 1500 counters in front of the students to observe the strategies they would use to count large collections (see Zone 1 of the LAF 1, Appendix C). We had started the session reviewing the subitising cards and I reminded them that our aim “was to not count by ones, but to work with chunks”. First, I asked the students to make a guess as to the number of counters on the floor. I was a little bemused by their estimates. Douha had suggested that there might be “one hundred counters?” in the pile. The students’ inappropriate estimates revealed to me a lack of experience with, or sense of large numbers. This was further confirmed by the students’ attempts to count a portion of the collection, one counter at a time. None of the students demonstrated the capacity to keep track of the count by making piles of five or ten counters, and then counting the piles. Dean restarted counting his collection a number of times as he kept losing his place.

I observed their strategies for a little while, then asked the students if they could think of a way to count their collection more efficiently. Hadi suggested they could make smaller piles. I supported this suggestion by saying “okay, give that a try”. Some students tried piles of 5 counters or 10 counters. The then skip counted by 5s or 10s as appropriate. I shared my observation of this strategy with the students by saying (pointing to Hadi’s arrangement), “See how this strategy allows us to count in chunks more efficiently than counting each counter one at a time?”

On this basis, I continued to focus on recognizing numbers as composite units by establishing subitising and part-part-whole understanding. As the students’ competence with these ideas increased they indicated readiness to build efficient mental strategies. I knew that this would support future work with concepts and strategies in relation to place-value, multiplication, division and fractions. In particular, the following strategies were developed:

- **Count on/ back from larger** (Session 3, Tuesday May 31, 2005), for example, 6 and 2, think “6, 7, 8”
- **Make to ten** (Session 6, Tuesday June 7, 2005), for example, 7 and 5, think “7 and 3,10, 10 and 2 more, 12”

Hadi’s reflection on session 2 is included below in Figure 5.12. She too notes the shift towards more efficient counting strategies.
I noted in my post-session reflection for Session 7 that Yousif’s confidence had increased.

Contingent planning meant that the learning needs of the students and the learning experiences offered were well matched, therefore offering the opportunity for frequent experience of success. In the case of Hadi, this helped to prompt the realization that what we were doing in our intervention sessions were transferrable outside of our time together. In the case of Yousif, his increased confidence prompted him to seek further challenges.

**Explicit and deep explorations in diverse ways**

The intervention program was not drawn from advice about teaching the mathematics curriculum for that year level. As mentioned previously, my planning was supported by the LAF and it was contingent on the students’ actions and reactions to each session at the time of implementation. This enabled me to jointly focus on both the student and the necessary content and was a major feature in the success of the intervention. The learning experiences were not just experiences for experiences sake, but carefully chosen and implemented for a specific reason. As a result I was able to work with ‘actual’ learning needs, as opposed to ‘assumed’ learning needs as nominated by year level curriculum expectations. I was prepared to go ‘way back’ to where LAF pointed me to, for example, subitising, part-part-whole ideas, and 2-digit place value. This is not what is generally expected of students at this level.

To illustrate when I mean by explicit and deep explorations in diverse ways, I refer to the work we did with arrays. In order to ensure that the students made a shift in thinking

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**Figure 5.12** Hadi’s journal reflection, Thursday May 26, 2005

To day in Maths we did estimating and numbers counting. We tried counting without using our fingers.

I really like counting without using our fingers because when Miss ima asks me questions I can answer them really fast.
from a ‘groups of’ way of thinking about multiplication and division to an array-based representation, I planned an initial exploration for Session 15 (see Appendix D, p. 62). I knew that working with array representations would enable the students to simultaneously co-ordinate the number of groups, the number in each group and the total, and help them recognise commutativity, a key feature in the development of more efficient mental strategies for multiplication and division. This was supported by knowing that when looking at groups of representations, it is not uncommon for students to believe that:

- four groups of three is more than three groups of four, because they are comparing the number of groups only, or that
- three groups of four is more than four groups of three, because they are comparing the number in each group only.

I did not want to tell the students that this was the case for the reasons I will discuss later in this section. Rather, I wanted to provide opportunity for the students to notice these key features for themselves. Arrays also provide the basis for making the links between multiplication and division as well as supporting for the development of efficient strategies for multiplication and division. For example, a collection of 12 can be seen to be shared among three, with four in each, or shared among four, with three in each. It is now possible to describe 12 in various ways: that ‘3 fours are 12’, ‘4 threes are 12’, that ‘12 divided by 3 is 4’ and ‘12 divided by 4 is 3’. Section 2.3.3 reinforced that a teacher cannot assume that students will naturally attend to the row by column spatial structuring of the array. For this reason, it was important for students to engage with the representation in diverse ways to foster the development of new mathematical knowledge (Carpenter & Lehrer, 1999). The language I used to describe the structure changed from the language associated with ‘groups of’ (e.g., 3 lots of 4) to ‘3 fours’. The emphasis is on the idea that 3 tells how many, and 4 tells how much. Used in this way, ‘3 fours’ emphasises numbers as composite units.

The making and naming arrays is introduced at Zone 1 of the LAF:

- **How to make, name and use arrays/regions** to solve simple multiplication and sharing problems using concrete materials, and skip counting (e.g., 1 four, 2 fours, 3 fours …), leading to more efficient counting strategies based on reading arrays in terms of a consistent number of rows (e.g., 4 rows of anything, that is, 4 ones, 4 twos, 4 threes, 4 fours, …) (Appendix C, p. 29)

Arrays were first introduced to the students in Session 15 and an excerpt of the program is included below.
Excerpt from Intervention Program (Session 15, Friday July 15, 2005)

**Focus**
Looking at arrays for multiplicative situations

... 

**LEARNING FOCUS ~ Looking at arrays**

**Resources**
Muffin tin
Counters/tiles
‘Star’ stickers

**Purpose**
To move from groups of idea for multiplication, to arrays (organisation of collection in terms of columns, rows, and the total)

**Learning Notes**
The students and I have a discussion about the organisation of the muffin tin. “What do you notice?” We explore ideas of rows, columns and total. The muffin tin makes 12 muffins arranged as 3 fours. I pose the question: “How many different muffin tin designs can we create?” Students are given ‘star stickers’ to record their individual results in their workbook. I encouraged students to explore all possibilities. Some responses might be:

* * * * * *

* * * * *

* *  *

*  *

* * *

* * *

*  *

(Appendix D, p. 63)

A section from Cansu’s workbook is included below in Figure 5.13 to show how she recorded one of the muffin tin options.

*Figure 5.13* Cansu’s arrays, July 15, 2005
This was first session on arrays and what I did not expect so soon, was that the students themselves would notice the idea of commutativity and be able to rename the groups. We had been walking around the school looking for arrays in the environment (e.g., staff pigeon-holes, arrangement of windows and student art work on display, and drainage grates). We talked about and described what we saw, for example, a window arrangement was named as 2 fours. Depending on where the students were standing, the same example of an array could be described differently. Looking down on a rectangular arrangement of pavers, one student said, “If I stand here the array is different to the array when I stand here”. The commutativity of arrays is evident in the excerpt from Cansu’s workbook above. Cansu has identified that 4 threes is the same as 3 fours, and that this array can also be described as 2 sixes.

The questions I asked at this time in the program helped the students to look deeply at the arrays. I was interested in what they noticed, not what I could potentially tell them. I asked questions like, “What do you notice?” with emphasis on the you (the student), and “How many … can we create?” with emphasis on the we (the group). Throughout the intervention program questions like this occurred with the deliberate intent to value the students’ own knowledge, understandings and strategies. Indicative examples of the questions that I asked throughout the intervention program are listed below, for example:

- What do you see? What do you notice?
- How do you know?
- How many … do you think we might have there?
- Can you think of a better way?
- Ok, have we finished?
- Tell me about your number?
- What patterns can you see?
- What…have we made now?
- How will we check?
- Can you think of a more efficient way?
- What shall we do next?
- What is another way to describe this?

To appreciate the row by column structure of the array, the students had more than one opportunity to make, name, and record arrays. This is documented in the excerpts to follow. First they made as many different arrays as they could for collections with 1 through to 12 counters. For example, a collection of 8 counters could be organised in four different arrays, these being, 1 eight, 8 ones, 2 fours, and 4 twos.
Excerpt from Intervention program (Session 18, Friday July 22, 2005)

... 

LEARNING FOCUS ~ Arrays investigation

Resources
Collection of counters/tiles
Large sheets of butcher's paper
‘circle’/‘dot’ stickers

Purpose
Engage students in the systematic recording of small collections in the form of arrays
Opportunity to discover the difference in variety of responses possible for odd, prime and even numbers

Teaching and Learning Notes
Students work with collections of 1 through to 12 counters. For each focus number, the students first create all the arrays possible for that number with tiles/counters. Then they record these using the stickers on the butcher's paper. These create a number of pages to be compiled into an ‘array’ book. As students work, I discuss what they notice about the arrays in relation to the total collection, e.g., the arrays for a collection of 7 will differ from a collection of 9 or 12. We discuss why. …

(Appendix D, p. 63)

I then became interested in the extent to which the students could reconstruct and name arrays from a given stimulus. I used a PowerPoint presentation to show a series of arrays, then hide the array away. The students’ task was to reconstruct the array from their memory.

Excerpt from Intervention program (Session 20, Thursday July 28, 2005)

... 

LEARNING FOCUS ~ Working with arrays PowerPoint presentation

Resources
Computer/laptop
Counters

Working with arrays PowerPoint presentation*

*Stimulus for ideas came from Bobis et al. (2004), p. 158-159.

Purpose
Reconstruct arrays from visual stimuli

Learning notes
I open the Working with arrays PowerPoint presentation. When the first array is shown, I decided how long the children could view it. Initial discussion took place as to the array’s multiplicative nature, 3 fives, fifteen. The next slide asks ‘make it from memory’. I then gave the students ] to reconstruct the array, now no longer visible, using counters. The third slide reinforces the first slide visually but now has an appropriate description. This is repeated 7 times with different arrays. An example of a set of 3 slides is presented below.

slide a

Make it from memory…

slide b
Without exception, all students were able to re-create the arrays accurately for all arrays presented in the slideshow. This indicated to me that they had created a mental image of the array in their mind and could distinguish between 3 fives and 5 threes even though they knew the product to be the same.

Work with arrays (and later, regions) continued in various ways from this point throughout the intervention program during Sessions 16, 18, 20-21, 30, 35-43. During these sessions we:

- explored where arrays occurred in the school environment, such as staff pigeonholes and the manner in which art work was displayed in the corridors
- explored the systematic recording of small collections (numbers one to twelve) as an array, for example, a collection of nine could be constructed as 1 nine, 9 ones, and 3 threes
- visualized, constructed, and named arrays presented in a PowerPoint presentation (this is illustrated in the excerpt from the intervention program presented below)
- developed efficient strategies for knowing multiplication facts
- used concrete materials to model multiplication word problems, and
- used efficient strategies for solving problems where arrays and regions were clearly defined as well as when partially visible.

These quite specific, explicit-to-purpose, diverse approaches were used to complement the more open-ended, problem solving strategies that also featured in our work together. I refer back now to the quote by Holt (1973) about explanations at the beginning of this section. Explanations which are only offered in one way without recognising students’ level of understanding are highly likely to be ineffective. It is the quality of the explanations that count. Previous research (e.g., Siemon et al., 2001) indicates that at-risk students appreciate the importance of explanations, but not in the way that Holt describes.
I did not explain by telling the students what to do, when to do it, or how, based on curriculum expectations at this level. I did however connect with their level of understanding by making this my starting point for instruction frequently, and communicated for understanding in diverse ways using various models and representations, through my prompts and questions, provision of constructive feedback, and making explicit to the students, the purpose of particular activities, for instance to ‘work with chunks (composite units) rather than counting by ones’, or ‘this activity will give you the chance to use an image in your mind’s eye’. This aspect of my approach reminds me of Shulman’s *pedagogy of substance* (2004) where he refers to the work of Jaime Escalante and the film *Stand and deliver* (Musca & Menendez, 1988). This prompted me to watch the film having not seen it in a while. I was struck by a particular scene when Escalante has his students chanting over and over a rule involving positive and negative numbers. He then stops, waits, then asks “Why?” A teacher who asks his/her students ‘why?’, wants his/her students to understand. The ‘why’ was not a mystery to the students who participated in the intervention. As previously mentioned, I communicated the purpose of our activities to the students. Exploring and knowing why contributed to the students’ understanding. Ahmed summed this up during the Post-I *Drawing Task* interview when he gave me some positive feedback by saying, “you understand it good to me”.

**Sensitivity**

My approach was also finely-tuned, that is, taking account of the fine detail in relation to the student and his/her learning needs. This aspect contributed to the success of the intervention because the learning needs of the students were being addressed explicitly. This close attention helped to let the students know that I really cared for them. While this was prompted by my knowledge of the LAF and my understanding of the literature surrounding middle years and at-riskness, it was also deeply connected to my lived experience of adolescence. What springs to mind is the act of fine-tuning a TV to get the sharpest picture. In the context of my intervention this ‘fine-tuning’ is the ‘sensitivity’ – the act of being alert or sensitive to any subtle shifts in thinking, behaviour, attitude, that might warrant a certain response, change in direction, shift in of approach, or a different task.

I was able to be sensitive to the needs of the students by communicating with them in a way that was mutually respectful. The conversations we had, both written and oral, allowed a trusting communicative relationship to grow. This contributes to the feeling of optimism in CUBPO (Sagor & Cox, 2004) making it ‘safe’ to take risks. At the
commencement of the intervention program, I was aware of student beliefs about mathematics as well as their performance. The initial Drawing Task and Card Sort interviews revealed that their views about learning mathematics involved situations where the teacher was somewhat removed from the students. For instance, the teacher was depicted as standing at the blackboard, giving instructions about what to do and how to do it. The other students in the classroom were all working on the same activities. The students also reported that if they needed assistance, then they would seek out the teacher. “If you need help then go ask the teacher” was a common statement. The students did not indicate instances of the teacher actively pursuing the views or learning needs of the students. It appears that it was left for the students to initiate contact with the teacher if they had difficulty with the work. Most at-risk students would want to avoid identifying themselves as needing help, therefore it is highly unlikely that they would seek help in this context and risk exposure.

The following example illustrates an instance of a personal connection between Ahmed and myself. The strength of Ahmed’s feelings was made known to me early in the Intervention Phase at the conclusion of our eighth session together. On this occasion, I had asked all the students to draw a face to show how they felt about their maths learning and complete the sentence starter “I feel……because …” (Siemon, 1986; Smith, 2003). After the students had finished writing, I collected their books and the students went off to do something else. I read through their responses quietly by myself then wrote back to them individually. This was something I did each time the students wrote this way. I was startled however, when I opened Ahmed’s book to find two conflicting responses, illustrated below in Figure 5.14. The angry face was the first image I saw in his workbook, and the second image was over the page. I did not expect such a harsh reaction and was bewildered that Ahmed thought that what we had been doing together was “too hard”. I was seriously concerned and started to reflect on what we’d done so far that might have created this reaction.
My response at the time to Ahmed’s journal entry is included below in the relevant excerpt from the Post-session reflection.

**Excerpt from Post-session reflection (Session 8, Thursday June 9, 2005)**

…Evidence of developing positive attitude reflected in journal writing. Ahmed writes though, “angry cos its too hard” but then states that maths is his favourite subject. Have a chat to him about this!

*Inserted after I spoke to Ahmed:* At close of next session I asked Ahmed about his journal writing. I ask him why he felt angry? He tells me he is angry because he can’t do it then tells me he is referring to his work in the classroom, and gives the example of when doing a test, and you can’t ask for help. I ask if he is angry when we work together? He says no! Regarding maths as his favourite subject, he tells me of the importance of maths in daily life, e.g., shopping.

(Appendix D, p. 53)

After discussing the issues directly with Ahmed the following day, I was very relieved to know that our time together was not the stimulus for his anger. Although I was gratified that he trusted me well enough to respond in this way, what concerned me greatly was that he felt this way in the first place.

This was a significant experience for me and reinforced my conviction that students not only need to have the opportunity to communicate and reflect on their learning experiences, but someone to care enough to find out more and act positively on this basis.
I cared very much about the thoughts and feelings of these students. To this end, this form of two-way written communication featured regularly throughout the intervention program. Not only did I comment on or highlight their mathematical skills, but also on their own personal qualities in relation to their own observations. The provision of meaningful feedback (see 2.3.4) can make a difference to student learning (e.g., Black & Wiliam, 1999). The list below is an indication of the nature of my responses to their writing, for example:

- Well done Yousif!! You’ve created an interesting list here. I look forward to next week. (Yousif’s workbook, May 26, 2005)
- A great thing to notice…Yes Cansu, you are right! There are lots of different ways to solve maths problems. (Cansu’s workbook, June 9, 2005)
- I am glad you enjoyed the game. You and X worked well keeping track of who had what. I like the way you are always willing to have a go. (Yousif’s workbook, June 21, 2005)
- I am pleased that you were able to think of and use a strategy to help you (Yousif’s workbook, May 26, 2005)
- I’m please you remembered to mention working backwards!! (Yousif’s workbook, October 14, 2005)
- This is fantastic Hadi. I am proud that you understand more about solving problems! Well done! (Hadi’s workbook, October, 14, 2005)

One of the first things that the students would do upon being handed their own workbook, was to open it up and read what I had written to them. Being sensitive to the students allowed me to sustain a personal relationship with them all which encouraged them to take risks, ask questions, and persist long after they might otherwise have done.

5.2.3 Summary

The way in which this intervention was enacted was through the implementation of effective teaching approaches endorsed by reform (e.g., Lester, 2007), but also a sense of responsiveness that occurred ‘in the moment’. Discussion and problem solving should feature as a regular part of classroom activity as it enhances student engagement and “promotes conceptual understanding of mathematical ideas and focuses on the connections between mathematical concepts” (Bostic & Jacobbe, 2010, p. 37). Given this, it is imperative that group work be the modus operandi. It is also important that the feedback given to students be constructive, framed in such a way as to facilitate learning (see 2.3.4), rather than comment on it in a vacuous way, for instance, ‘good work’ or ‘well done’ (Black & Wiliam, 1999). Statements like these must be qualified. The results of this study clearly suggest that a flexible, student-centred approach to planning is needed. One that is based on meeting the students actual learning needs as opposed to learning needs determined by curriculum, and not planning too far ahead to allow student responses and reactions to
help drive ‘where to’ next in consultation with the LAF. This is not normally a feature of intervention programs. This might be considered a bold move for any teacher in any school system that values maintaining standards according to the prescribed curriculum. I will address this issue further in Chapter 6.

As mentioned previously, the quality of teacher explanations is an issue for middle years students. In my view, the communication of concepts and strategies facilitated through explicit and in-depth exploring of key ideas contributed to the success of the intervention program. The students were well informed about what we were doing and why. This was made possible through:

- explicit guiding questions and prompts rather than explaining in meaningless ways
- opportunity for exploration of multiplicative situations that stimulated the student’s own constructions and noticing, and
- prudent use of appropriate materials, models and representations helped to support the students in breaking the abstracting barrier (Sullivan et al., 2001).

Finally, moment by moment sensitivity to where the students are at builds the opportunity for personal relationships to be established contributing to identity and agency of the students.

5.3 Relationships

The work of teaching is at least twofold. Teachers teach subject matter. Teachers also deal with students as whole human beings and need to respond to them as emotional, moral, social, and cultural as well as cognitive beings. Two often, these two dual obligations are analyzed separately by educational theorists…Teachers, however live these commitments simultaneously. (Rosiek, 2003, p. 411)

If we consider the dualities that Rosiek speaks of above, the first two sections of this chapter, the role of teacher knowledge and how this is transformed in practice, address the first duality of the act of teaching. In this section on relationships, I will focus on the second duality which in my view holds the key to the success of the intervention program. I have already touched on some aspects of this in the discussion above. The building of relationships and community are critical, as without this, the benefit afforded by teacher knowledge and progressive, considered practice are less effective. At the core of this, is teacher agency. This will shape the initial discussion, in particular the importance of the teacher’s sense of agency and her relationship to mathematics and her students. These relationships enabled the teacher to be actively present in the act of teaching. Attention will
then shift to how the sense of belonging through participating together supported the ongoing success of the students, provided opportunity for their voices to be heard, and the cohesion of the group, the building of personal mathematics identities, and ultimately the success of the intervention program.

5.3.1 Teacher agency and identity

In the sections above I have addressed knowledge of content, knowledge of teaching and knowledge of students. However, there is a knowledge that is not necessarily written about in the same way, and this knowledge powerfully contributed to the success of the intervention program: the knowledge of self. Teacher knowledge of self has an importance place with respect to this intervention study. There needs to be a fine balance between all aspects. All aspects need to be kept in play and none should necessarily take precedence as is suggested by Mathematics Knowledge for Teaching e.g., Ball et al, 2008).

Shulman, in Boaler (2003) acknowledged “the absence of any emphasis on affect, motivation or passion”. This section aims to address this gap. This section will take two parts. Firstly by reflecting on how I see myself as a teacher and how this contributed to the success of the intervention program. Secondly, in acknowledgement of the increased professional interest in the emotional and spiritual aspect of teaching (e.g., Brown, 2009; Day, 2005; O’Reilley, 1998) I will discuss the importance of presence in the act of teaching which facilitated a strong sense of community, feeling of belonging, which impacted on student identity.

My sense of agency and identity

Shulman (2004) speaks of the need to “educate by teaching and not just by selection” (p. 129). In other words, educating all students, not only the students who are capable or who should be learning mathematics. This stance is coming from an ethical point of view as well as an educative one, “a sense of obligation, commitment, moral and ethical, as well as pedagogical responsibility, to teach all the youngsters who are entrusted to us” (Shulman, 2004, p. 129).

Teachers with a strong sense of personal agency have the potential to engender agency in others, conversely teachers “who have a low sense of instructional efficacy believe there is little they can do ” (Bandura, 1977, p. 240). I have worked with teachers who have expressed the belief that ‘there are simply some children who do not understand mathematics, and never will understand mathematics’. The likely consequence of this for at-risk students is disengagement with both the teacher and mathematics. I believe the
opposite is true. Underpinning this belief is how I see myself as a teacher, and how I position my capacity to make a difference. This is consistent with Stein, Remillard, and Smith (2007) who state that a “significant component of teachers’ professional identity is how they construct their roles in relation to students” (p. 354). This was key to the success of the intervention program as I believed in myself and in students.

What has contributed to how I see myself as a teacher is a genuine knowledge of the ‘big ideas’ and how they connect, a rich repertoire of teaching strategies, and care for individual students. All aspects of the student (whole person, cognition, emotional needs) are attended to. It is not sufficient to only deal with or attend to deficits in a student’s mathematical knowledge, but also to “the ways that students think about themselves in relation to mathematics” (Cobb, Gresalfi, & Hodge, 2009, p. 40). I have a strong sense of my capacity to change student thinking and improve learning outcomes. I believed in two things, first, in my ability and second, equally in the ability of the students. My approach to intervention is no different from my approach to teaching and it is intimately related to the ‘person-ness’ of myself as teacher and the ‘person-ness’ of the students. I shared aspects of my life telling the students about my five children and my husband, our pets including some pet pythons. I was interested in their lives outside school and we would commence each session with easy informal conversations about up-coming events, social and sport activities.

Underpinning my approach is a love of mathematics and love for students (Brown, 2009). In essence, this characterises the nature of my relationship with them both. My identity is about being a caring, passionate, concerned person who has struggled with mathematics in the past but has worked hard to come to understand and love mathematics. I really understood the experience of these children and was empathetic to their situation. I believed that I could and would make a difference. Stein et al. (2007) define identity “as a construct [that] integrates what one knows, feels and is inclined to do in a particular context into an inseparable whole, making activity, rather than knowledge, central. While knowledge is often viewed as static, identity is active” (Stein et al., 2007, p. 354). I have tried to create images of what this ‘looks’ like to me (see Figures 5.15 – 5.18). The image in Figure 5.15 below is my representation of identity.
The image is deliberately non-uniform to indicate that identity is not static and defined, but rather active as suggested above. I see it as fluid and malleable, shaped by many things. Each identity is unique. The next image in Figure 5.16 attempts to show how I hold mathematics (Barton, 2009) and this is indicated in red.

Mathematics as a field of endeavour exists outside of my identity in its own right. However, I am connected to this in that I ‘hold’ (Barton, 2009) mathematics in a certain way, as part of my identity. I was present at Bill Barton’s keynote (Barton, 2009) when the idea of holding mathematics was first illustrated to me. I hold mathematics in a loving way which I inevitably communicate in and through my teaching, the love goes with it. To help illustrate what I am trying to say, the image in Figure 5.17 is an attempt to illustrate what this might mean in relation to my perception of typical mathematics classrooms. I have in mind a classroom where teaching involves telling and showing the students what to do with little regard for their learning needs or deep understanding of mathematics (e.g., Bostic &
This traditional view has also come from my teaching and research experience in classrooms, my own learning experiences, as well as a parent listening to the stories my teenage children told. The ‘M’ is mathematics in red, the ‘I_s’ is the identity of the student, and ‘I_T’ is the identity of the teacher.

Figure 5.17 My representation of typical teacher and student identity in relation to mathematics

This image above acknowledges that perhaps mathematics and the way it is ‘held’ is not necessarily integrated or seamless. It is held apart as an objectified entity. The arrows suggest distance between the student, the teacher, and mathematics. This would present a contrasting view of connectionist teachers (Askew, 2008). How I imagine what I know, feel, and am inclined to do is presented in Figure 5.18 below. The dynamic ellipse (indicated by the arrows) is for me the dynamic of teaching, that integrates my identity and how I hold mathematics, and the impact on students through our relationship. It is important to notice the key connection, that is, the closeness of the teacher and student identities that is not present in the image above. This intersection shown in purple will be elaborated further in 5.4 when I bring the nature of this intervention together.
I have struggled with the difficulty of writing about love and passion in a thesis that aims to be as objective as possible about what made the difference. The word ‘passion’ or being ‘passionate’ was one of the words used most frequently in the AAMT & Monash University (2001) research study on the Professional Standards. My own 15 year old daughter was intrigued one day when I was sorting and classifying aspects of the Standards in an effort to make connections and see the links between them. She watched me for a while and spent some time reading them. I asked her out of interest, “What is missing?” She replied, “Love of students and love of maths”.

How I see myself as a teacher led me to do certain things. In particular:

- I shared stories of my life and was interested in theirs
- I participated in the learning activities in the same way as the students, and
- Planned for frequent, sustained experience of success.

These considerations contributed to the cohesion of our group. The intervention program placed emphasis on community, participatory approaches and contingent practices that nurtured and supported the students in diverse ways. The students felt safe to take risks knowing that their identities would not be threatened.
Presence in the act of teaching

…the pedagogical task of teaching involves knowing the importance of “being there” or “availability” for students when it matters (van Manen & Li, 2002, p. 222).

Another, culturally important aspect of relationships, presence in the act of teaching. Van Manen and Li (2002) suggest that teacher knowledge is pathic “to the extent that the act of teaching depends on the teacher’s personal presence, relational receptiveness, tact for knowing what to say and do in contingent situations…” (my emphasis, p. 217). This leads me to O’Reilley’s (1998) sense of mindfulness, “the Buddhist practice of simply being there, with a very precise and focused attention, listening, watching” (p. 3), and spirituality in Brown (2009) who believes, “[t]he grace of being in the moment, the energetic present, is spirituality to me” (p. 148).

Throughout the first two sections of this chapter, I have illustrated the nature of the intervention program and my responsive participation. This has been evidenced in the frequent mathematical discussions we had, the games we played, the problems we worked on together and the provision of constructive feedback ‘in the moment’. I was active and attentive to the students’ needs at all times. This notion of presence comes in more than one form: I cared for the students and wanted the students to do well, but I was also able to communicate at their level in meaningful ways and scaffold important mathematical ideas. However ‘being there’, in the same room as the students does not necessarily constitute teacher presence. To illustrate, some years back I had the opportunity to visit some schools and observe mathematics teaching in middle years classes. On this particular occasion, I sat up the back of the classroom and chatted to some of the students intermittently throughout their lesson. I happened to be positioned in a seat near two boys. They were quietly working on the exercises from their textbook and one student who indicated that he was not having any trouble with the work, was assisting the student sitting next to him who was. The teacher stayed at the front of the room, near the blackboard for the duration of the class. At the conclusion of the lesson, the teacher and I spoke. She said to me, ‘I notice you were chatting to ________ and ________. Yes, he has a lot of difficulty with the work’. What struck me, was that the teacher had identified that this student struggled, and even though the teacher was physically present in the room, there was no attempt made to approach this boy in any way at all. This is an example of being there, being physically present but without presence.

There were many times too when there was a sense of presence in between the teaching sessions when I was not actually working with the students at all. I may have still
been in the school and writing up my post-session reflections, or off-site at home or at work, undertaking preparations and planning for what we were going to do next based on responding to their learning needs. So, while I was not physically there with the students, there was still a sense of presence in play. An example of this is when I responded to their writing in their workbooks after our sessions had concluded. For example, James had missed a couple of our sessions. When he returned on the 21st of June, about a third of the way through the Intervention Phase, James wrote the following in his journal (Figure 5.19). As was always the case, I collected their books and wrote back to students individually. My response to James is also evident below in the figure.

![Figure 5.19 A personal response to James](image)

His regular teacher had made me aware of some significant difficulties he was having at home, and I wanted him to know that he was a valued member of our group and that he was ‘missed’ when he was not at school.

Around the same time, I was informed that Douha was going to have to start wearing glasses and was she quite nervous about it. I chatted to her about this, and told her that I wore glasses too, but wore my contact lenses when I rode my motorcycle. I asked her about the frames and what they were like, and told her about mine. I resolved to drive the car to our next session and wear my glasses too. I wanted her to feel as positive as possible about her new eyewear. She was delighted. Students want to be seen, students want to be noticed: ‘A common complaint of students is that teachers do not care about them’ (van
Manen & Li, 2002, p. 223). Students also want to be treated respectfully. Siemon et al. (2001) suggested that the disengagement that at-risk students feel “may have as much to do with their perceptions of how they are treated by their teachers as the particular nature of the teaching practices used” (p. 107).

This idea of presence helped to establish a sense of belonging. The next section will show how this personal approach enabled the students to build their own sense of mathematical identity and agency, where their voices were heard, they were supported to ‘have a go’ and experience success. This ultimately contributed to their confidence and competence as learners of mathematics.

5.3.2 Establishing a sense of belonging

This intervention established the students and I within a community of practice (Wenger, 1999). This is not new in the literature per se, however this approach is not necessarily advocated in intervention programs. Lave and Wenger (1991) state that as “an aspect of social practice, learning involves the whole person; it implies not only a relation to specific activities, but a relation to social communities – it implies becoming a full participant, a member, a kind of person” (p.53).

I was very aware of my leadership role as ‘more learned other’ (Vygotsky, 1978) and that I implemented both reform-based and quite explicit, direct scaffolding practices for the benefit of the students. However, from the outset, the creation of a community of learners was important and this influenced the way in which the students and I engaged with one another. Previously presented excerpts from the intervention program have indicated instances of the students and I working, talking, playing games as a group, often sitting in a circle on the floor. While I led or directed the activities, for instance the use of the subitising and part-part-whole cards, I also participated as if I were a student in the group. This empowered the students to actively contribute to the group and to the activity. This links directly to the importance for middle years students to feel a sense of belonging (Sagor & Cox, 2004), and this is an important reason for the success of the intervention given the students willingness to engage with all aspects of the program and me. Working this way respected the identities of the students. The sense of belonging was created though: our participation together, the opportunity for success to breed success, and the space for voice. Each of these aspects will be described and illustrated. This section will conclude with evidence of our connection as a group which enabled students’ personal mathematical identities to grow.
Collaborative participation

Initially, my participation provided a model for the type of response that would be appropriate, but also my continued participation in this way helped to give students a sense that we could, and would, operate together. I was well aware that the students held the belief that learning maths well was associated with explanations often provided by the teacher who was positioned at the blackboard telling the students what to do and how to do it (see 4.1.1). This view appears to be consistent with the acquisition metaphor of learning (Sfard, 2003) “where a person who learned something new has been said to acquire a new concept or procedure” (p. 355). The student drawings that resulted from the Pre-I Phase Drawing Task interviews frequently depicted the teacher standing at the blackboard (see for example Figure 4.2 & 4.5). My view of learning is dominated by the participation metaphor of learning (Sfard, 2003) where the learner is a part of “mathematizing community” (p. 355).

Throughout the intervention, I wanted to subtly challenge the notion that the blackboard was a teaching tool for the purpose of teacher to student communication only, the place where the teacher explains what to do and how to do it. Over the years in my work as a teacher, I have become increasingly less reliant on the use of the blackboard as a teaching tool (see Chapter 1). I provided the students themselves with the opportunity to be positioned at the blackboard so that they would consider it beyond a tool for the teacher, and articulate their thinking and reasoning strategies to themselves, the other students and myself. In our intervention, the blackboard (or whiteboard for that matter) was a tool for student to teacher communication, student to student communication, and student to group communication. Earlier in 5.2.1 when discussing constructive feedback, I gave the example of the students practicing their partitioning strategies at the blackboard or whiteboard. They were encouraged to explain out loud the thinking about their partitioning strategies at the same time.

On another occasion, I modelled a particular place-value activity for the first time in Session 38 (Appendix D, p. 91-92), a game that was conducted by the ‘game leader’ (myself initially) while standing with various clues that were documented on the blackboard. However, from Session 39 onward, the students took turns at being the game leader. This positioned the student leader at the blackboard. During these times, I participated in the activity in the same way as the rest of the students, relinquishing the teacher role to the individual student, and placed myself in the position of student. The students were enthusiastic in both roles as ‘participant and ‘game leader’ and made frequent requests to
play the game. This supports Sagor and Cox’s (2004) view that ‘role’ is intimately connected to belonging, and making contributions to the community establishes feelings of usefulness, and ultimately creates a positive sense of identity and agency.

**Success breeds success**

One of the barriers preventing at-risk student success with mathematics tasks at this level was their reliance on low-level inefficient additive strategies. Prior to the commencement of the intervention program, the students used strategies such as ‘make all, count all’ when responding to a range of problems. The point was made earlier (4.1.3) that an over reliance on this strategy in response to problems typically presented to students in upper primary (Years 5 and 6) often results in errors. In terms of failure, Gilpin (2010) makes the point that “[s]tudents and adults tend to shy away from situations in which failure is probable. Failure can often deal a devastating blow to the fragile psyche of an adolescent” (p. 21).

The primitive additive strategies described in detail in 4.1.3 were not used by the SC (successful) students. The SC students used a range of efficient mental and written strategies based on their understanding of multiplication and division. As a result, I determined that the students needed to experience success and that this could be achieved by developing the use of efficient mental computation strategies. Sagor & Cox (2004) state that, the “bricks that make up the foundation upon which self-esteem is built are feelings of competence” (p. 39). The importance of success connects directly with the feeling of competence and usefulness, two of the essential feelings of CBUPO: competence, belonging, usefulness, potency and optimism (see 2.4).

The following two excerpts of my reflections are presented to illustrate the growing experience of success for Douha, and how this impacted on her emotionally. She, like the students commenced the program using primitive ‘make all, count all’ strategies. I noticed during our fourth session, that Douha counting by ones to ‘add 2 to the numbers shown on the playing card’ (Appendix D, p. 46). By Session 14, I noted a difference in Douha’s approach.

**Excerpt from Post-session reflection (Session 14, Thursday June 23, 2005)**

…Spent one on one time with Douha with 2, 4, 6 game. This allowed opportunity for utilising known facts for 10, e.g., 6 and 4, 10, 10-4, 6 and apply these to 20, 20-6, 14 etc. Noticed Douha was less reliant on using her fingers and that her responses were becoming more automatic. She appears genuinely surprised when she is quick and right. Douha quickly solves the number puzzle. … (Appendix D, p. 62)

**Excerpt from Post-session reflection (Session 27, Thursday August 18, 2005)**

… Developing efficient doubling strategies for larger numbers. Douha a little slower that the others, but developing efficiency! Haven't noticed her using her fingers for weeks. SHE WAS HEAVILY RELIANT ON THIS! … (Appendix D, p. 76)
Upon writing the post-session reflection in August, I wanted to share what I noticed with Douha. I took her aside after our session the following day, and asked her if she had noticed that she was no longer counting by ones using her fingers? She was obviously delighted with this feedback as her whole face broke into a smile, with the hint of tears. I told her I was really proud of her. Then later, after Session 40 (Appendix D, p. 95), Douha wrote:

- …they were so much fun I didn't have trouble doing them…I loved the weird shape a because it was so much fun and because I finished first.

Prior to the intervention program, Douha was rarely the first to finish activities and tasks in maths classes. The success she achieved over the duration of the program allowed this situation to change. She became more confident to tackle new things and was less inclined to be concerned about her work or her recording being neat, or correct straight away.

**Space for voice**

The opportunity for the students to undertake written reflections in their workbook opened up spaces for their voice to be heard. This contributed to the success of the intervention program because I was well informed about their current thoughts, feeling and strategies. This idea is supported by O’Reilley (1998) who states that writing “can create a spacious moment” (p. 6). The students had the confidence to tell me what they were having difficulty with as well as when they were doing well. I was well aware of the coping strategies that at-risk students use to avoid confrontation (e.g., Sagor & Cox, 2004). Typically, at-risk learners are avoiders, they are distrustful, and low in self-confidence. The excerpt from James’ journal below in Figure 5.20 shows that he is no longer feeling like this. He states confidently that he is finding mathematics easier and he is becoming more efficient. Yet, he trusts me enough to divulge what he is still struggling with.
Earlier in Figure 5.14, Ahmed indicated that he felt safe enough very early on in the intervention to share his anger with me (see 5.2.2).

**Connection as a group**

The trust the students had in me, my attention to their care, and our participation as a group established a sense of belonging. This was important because we all wanted to ‘be there’. I spoke earlier about teacher presence, however, the students’ sense of *presence* also contributed to the program’s success in the same way. When I arrived each time, the students greeted me warmly and enthusiastically. The students wanted to attend these sessions. Berrin wrote the following in her journal about one month after the program had started: “The best thing was the Number Chart. I was go good at that. I like coming with you. Thank you for learning stuff to us. I learned lots of stuff with you”. On another occasion, having arrived for Session 35, the school was in the midst of a whole-school outdoor Football Day activity. It had been re-scheduled for earlier than planned, as the weather was going to become inclement. This impacted on the running of my session. All the students and staff in the school were outside participating in various ball games. Adir saw me arrive and when I went outside to join them and say hello, he wanted to leave the games and come to work with me. I was delighted that working together took preference over sports!

Our connection as a group was evident in the drawings they produced in the Post-I Phase *Drawing Task* interview. Before the intervention had taken place, for example, Adir and Cansu had drawn a teacher at the blackboard, Dean and Yousif had drawn themselves
and a blackboard, and Ahmed had drawn himself doing some work at his seat. However, students in group formation featured in the Post-I Phase drawings of Adir, Cansu, Ahmed and Douha (refer back to 4.2.1). The change in these drawings is consistent with the illustrations produced by Shelley, a Year 8 student in the study reported by Bishop and Pflaum (2005). These authors speak to the notion of community and belonging, and illustrate what this means for Shelley who had drawn two pictures, one to show a time of engagement, the other to show a time of disengagement. Engagement was indicated by a group of students sitting in a circle where they are “safe to take risks” (Bishop & Pflaum, 2005, p. 10). Disengagement was indicated by the teacher at the board and the students are separate from one another.

Not only did these students improve mathematically, there was a change in their view of themselves. They indicated feelings of potency, one of the essential feelings of CBUPO (Sagor & Cox, 2004). For instance, Ahmed who felt angry at the beginning of the intervention when work in mathematics classes was too hard. At the conclusion of the intervention program, Ahmed wrote in his workbook a simple though powerful statement about himself: “I’m born to do math”. It seems that one for the reasons for this shift in his view is evident in the positive feedback he gave me during his Post-I Drawing Task interview. His words were, “you understand it good to me”.

Hadi’s participation in the intervention contributed to her being empowered in her regular classroom situation. The excerpt from her workbook shown in Figure 5.21 shows enjoyment, but also a greater self-awareness of the efficient strategies she is now using, and improved understanding of problem solving outside our sessions together.

Figure 5.21 Hadi’s positive view of herself, October 14, 2005
Her view of being empowered is also present in her Post-I Phase *Drawing Task* image shown in Figure 5.22. Upon initial inspection, it looks as though she has drawn the teacher at the blackboard teaching the students who are sitting on the floor. However, when I asked her about her drawing, she indicated that the person at the board was her because “I had the answer and I’m giving it out in front of the class”. This is a powerful illustration of her growing sense of identity and agency in relation to school mathematics.

*Figure 5.22* Hadi: “I had the answer and I’m giving it out in front of the class”

The students evolved over the course of the intervention from little or no sense of agency to much higher levels which enabled them to feel confident and competent in their mathematical endeavours.

### 5.3.3 Summary

Despite the focus on timely intervention in the early years (see 2.5.2), intervention with students in the final year of primary school is not too late. This is indicated strongly by the measurable improvement that the students in Year 6 made in relation to their assessment against the LAF. Traditionally, intervention programs have targeted students in the Early Years, for example, Reading Recovery (Clay, 1987) and Maths Recovery (Wright, 1994), the underlying premise being that the earlier the intervention the better. While I do not dispute the appropriateness of early intervention, I fully endorse that intervention in the middle years of schooling is both necessary and valid. Other programs have advocated working with students one-on-one, through a course of intensive basic skills training or delivery of heavily prescribed content, and/or the use of systematic streaming. The approaches and programs associated with these ideas were discussed at length earlier in 2.4.2. The findings of this study suggest that intervention with students in the middle years requires...
knowledgeable teachers in terms of content and pedagogy. However, this alone is not sufficient to making a difference in student learning. The teacher must have a strong sense that they can work together with students, to make a difference. Hattie (2003) states that “schools barely make a difference to achievement … it is what teachers know, do and care about which is very powerful in this learning equation” (p. 2). The act of teaching is about enacting passion, and “promoting responsiveness to students' emotional experience of learning” (Rosiek, 2003, p. 400). This was a critical feature in the success of this program.

5.4 Conclusions

To bring all aspects of the intervention together, this section will introduce a new way of describing the dynamic that brings the importance of relationships together with knowledge for teaching mathematics and teaching in action.

In order to do this, I would like to describe the art of French polishing as I have experienced it. Not as a practitioner, but as an observer, sitting quietly in our workshop at home a number of years ago, mesmerised by my husband Marty's prowess with this dying craft. He learned the art of French polishing from his father here in Australia, and his father Aubrey learned it from Marty's grandfather in England. Shellac imbued on a hand-held rubber skates across the surface of the timber to be polished. This process occurs a number of times. The aim is to leave no discernible marks. It is a must that the end result is for the beauty of the timber to shine through. The layers of polish should be invisible. The phrase that is used to describe what the polisher is aiming for is to "keep the wet edge alive". I have tried to locate this phrase somewhere in a book, however, it seems to me that it has evolved through an oral tradition. But keeping the wet edge alive is what creates the seamless body of polish over the expanse of the piece of furniture being restored. A complex series of circles and loops, working progressively over the surface, maintains this 'wet edge'. A stroke of virgin polish must connect directly with the wet edge of an already applied stroke, and this must be maintained through the application of that particular coat. Failure to do so, results in marks, gaps, lines, and ultimately cracks.

This intervention study is about keeping the wet edge alive, however, in this case it is in reference to the dynamic of teaching. Earlier in Figure 5.18 I indicated an initial attempt at explaining and visualizing this dynamic. The model in Figure 5.23 is a more developed preliminary image to help frame the aspects of this dynamic in relation to my approach.
The success of the intervention program was shaped by my knowledge for teaching mathematics (indicated above as area 1), teaching in action (indicated as area 2), and the relationships that were established (indicated as area 3). The overlay of the circles for each of these aspects is designed to show their inter-connectedness of the three domains. It is possible to foreground one and background the other two as the organisation of this chapter reflects. However, teaching is dynamic. This most critical action is added to the complete model in Figure 5.24 below. The arrows represent ‘keeping the wet edge alive’ by the circular movement of the edges or intersection of each domain as they interact seamlessly with each other. Much in the same way as the French polisher does with the shellac. This creates the new space (indicated by the number 4), the space in which learning and growth of the whole person occurs. The learning and growth is not the exclusive domain of the student. This space represents the learning and growth of the teacher. In this case, me. This space is ‘mutual’ and lies at the heart of teaching in this way. *Keeping the wet edge alive* is about keeping the opportunities for learning open and accessible at all times.
This interpretation challenges the component view of the knowledge needed to teach mathematics effectively (e.g., Ball et al., 2008) and suggests that we need to work in new ways to explore and make sense of exactly what it is that informs and motivates effective teachers of mathematics in-the-moment.

I have learned a great deal in the pursuit of seeing this research come to a close. The words of Shulman (2004) are in mind, and I can honestly say that I am a “teacher who is in love with the knowledge that [I have] acquired” (p. 137). The next chapter will bring the experience of this work to a close by making final comments, acknowledging the limitations of the study, and outlining some recommendations and implications for teacher professional learning and further research.
CHAPTER 6

CONCLUSIONS & RECOMMENDATIONS

This intervention study was designed to investigate the extent to which teaching, closely aligned to the learning needs of at-risk students in the middle years, would impact their achievement in relation to multiplicative thinking. In addition to this, the study was designed to identify the strategies that at-risk students use to solve a range of tasks involving multiplicative thinking, their beliefs about learning mathematics, and the extent to which these beliefs and strategies change as a result of their participation in the intervention.

This study was supported by an Australian Post-graduate Award Industry (APAI) scholarship as part of the *Scaffolding Numeracy in the Middle Years* (SNMY) Research Project 2003-2006, an ARC Linkage study on the efficacy of using an assessment-guided approach to improving student numeracy outcomes in Years 4 to 8. The SNMY Project was prompted by previous research (e.g., Siemon et al., 2001) which identified that the major reason for the very significant difference in mathematics achievement in the middle years was almost entirely due to students’ capacity for multiplicative thinking and highlighted the critical importance of finding ways to address the learning needs of at-risk students. The SNMY Project was aimed at validating an evidence-based *Learning and Assessment Framework for Multiplicative Thinking* (LAF) and examining the extent to which student learning could be enhanced by more targeted teaching approaches informed by the advice provided by the LAF. This intervention explored what could be achieved by a targeted intervention program involving a small number of students identified to be at-risk in terms of the LAF.

The research design was underpinned by the view that learning is “socially and culturally situated, [and] the design of research studies needs to encompass participation in authentic and purposeful activities” (Pressick-Kilborn et al., 2005, p. 25). Design experiment methodology was appropriate given the desire to create an innovative educational environment, and simultaneously evaluate those innovations (Brown, 1992).

The three research questions posed by this study were addressed over three phases, a Pre-intervention Phase (December 2004 – May 2005), an Intervention Phase

- To what extent does an intervention program closely aligned to learning needs impact student outcomes in relation to multiplicative thinking?
- What strategies are used by at-risk students to solve tasks involving multiplicative thinking and to what extent are these impacted by the intervention program?
- What do at-risk students believe about learning mathematics and how are these beliefs impacted by their participation in the intervention program?

Interviews were conducted during both the Pre-I and Post-I Phases: these were the Drawing Task, Card Sort and Multiplicative Task interviews. The sample comprised ‘performance outliers’ at Year 6, that is, students performing well outside what is generally expected for their year level in school mathematics. The groups fell into two broad categories: ‘at-risk’ (intervention [ARI] or non-intervention [ARN]) and ‘successful’ [SC]). That is, students performing well below expected levels and students performing well above expected levels respectively. The at-risk students were selected from those students whose performance on the initial SNMY assessment was at Zone 1. Students operating at Zone 1 of the LAF are generally able to solve simple multiplication and division problems involving relatively small whole numbers but rely on modelling, count-all strategies, and/or skip counting. The successful students were those students whose performance on the initial SNMY assessment was at the highest levels of the LAF, that is, Zones 7 or 8. Students operating at these zones are more likely to be able to solve a range of complex multiplication and division problems involving large whole numbers and fractions across a variety of contexts (see Table. 3.1). Comparison across and between the two at-risk groups, ARI and ARN, and the SC group determined the degree of similarity and difference between the students, as well as the impact of the intervention program on student learning outcomes.

The results associated with the research questions defined for this study were presented in Chapter 4 where it was reported that all 9 at-risk intervention students (ARI) shifted from Zone 1 of the LAF (April/May, 2004) to Zone 4 or 5 of the LAF by the end of the intervention study (November, 2005). The 5 ARN students either remained at Zone 1 or made a shift to Zone 2 of the LAF despite the fact that their teachers had access to the LAF and some access to a visiting SNMY research team member. This suggested that the shift in performance of the ARI students was due to their participation in the intervention program. This is supported by the results of the Post-I Multiplicative Task interviews that showed that the ARI students were far more likely to use efficient multiplicative strategies
than the ARN students. In addition, while student beliefs in relation to learning maths well remained relatively consistent across the ARN and SC groups, there was evidence of a shift in ARI student views about learning maths well, specifically, that they now recognised the importance of working in a group and sharing ideas with others.

Chapter 5 reflected on the nature and implementation of the intervention program in light of the literature to identify and explain what made the difference. This was considered in terms of three inter-related aspects, teacher knowledge, teaching in action, and the critical importance of relationships. A metaphor, keeping the wet edge alive, was used to help explain the inter-relatedness of these three aspects and the dynamic of teaching.

This chapter will bring the study to a close by addressing the research questions framed by this research and acknowledging the limitations of the study (6.1), and will conclude with recommendations for future research, professional learning, and practice (6.2).

### 6.1 Conclusions

It is not too late to intervene in the middle years, however successful at this level requires more than the in-depth teacher knowledge described by Shulman (1987). Teacher knowledge must also encompass knowledge of self that embraces a love of mathematics and the joy of learning, and a knowledge of students that goes beyond the cognitive domain. Successful intervention requires the ongoing support of a community of learners, responding mindfully in the moment, through genuine engagement with student identities with passion and love. Reform approaches work in the context of intervention with at-risk middle years students where there is a focus on working in groups. However, there is an important place for responsiveness that involves explicit explorations of key ideas in diverse ways. This section will address the main research question and two sub-questions framed by this intervention study.

#### 6.1.1 Research question 1

*To what extent does an intervention program closely aligned to learning needs impact student outcomes in relation to multiplicative thinking?*

This research has shown that teaching closely aligned to the learning needs of students in the middle years makes a significant difference to outcomes in relation to multiplicative thinking with all the at-risk intervention (ARI) students achieving at expected levels by the
end of the intervention program. As indicated above, by the end of the intervention the students were at or working towards Zone 5 of the LAF. A summary of Zone 5 is included below:

Solves whole number proportion and array problems systematically. Solves simple, 2-step problems using recognised rule/relationship but finds this difficult for larger numbers. Determines all options in Cartesian product situations involving relatively small numbers, but tends to do this additively. Beginning to work with decimal numbers and percent. Some evidence MT being used to support partitioning. Beginning to approach a broader range of multiplicative situations systematically. (Siemon et al., 2006)

However, three aspects in relation to student learning are critical:

- In-depth teacher knowledge of content and students
- Flexible and effective pedagogy, and
- Mutually endorsing and respectful relationships.

Mathematics knowledge for teaching must be guided by the ‘big ideas’ in relation to multiplicative thinking and rational number (see 2.2) as this supports making links within and across mathematical ideas. The LAF for multiplicative thinking is invaluable for assessment and planning purposes. It provides the basis for identifying where students are at in relation to the development of multiplicative thinking and appropriate starting points for teaching. The advice provided by the LAF scaffolded the development of multiplicative thinking for the students involved in this study. Rather than planning for instruction, based on curriculum expectations, planning should be based on meeting the actual learning needs students.

This study supports Hattie’s (2003) claim that the teacher remains the most important factor in student learning (e.g., Hattie, 2003). It is the teacher who needs to know and deeply understand the content required for students to make progress. It is the teacher who decides how best to transform students’ experiences of that content to maximise their opportunities for learning. But above all it is the teacher’s capacity to engage with learners as valued individuals in a community of enquiry that is needed to make a difference at this level of schooling. Teaching is about enacting passion, and being responsive to the students’ emotional and cognitive experience of learning (Rosiek, 2003). This is increasingly being recognised as the spiritual dimension of teaching practice (e.g., O’Reilley, 1998).

The significant improvement in ARI student outcomes as a result of the intervention program supports the efficacy of reform-based teaching approaches (e.g.,
Boaler, 2002; Lester, 2007). In this case, the improvement in student learning outcomes can be attributed to the following reform-based practices:

- group work: this established and maintained a sense of community, which encouraged risk taking, built students’ self confidence and increased their preparedness to ‘have a go’,
- discussion and problem solving: afforded student engagement which promoted learning mathematics with understanding (e.g., Fennema & Romberg, 1999), and
- the provision of timely constructive feedback to students: lead to an increased self-awareness of what was needed to maximise learning by building on what was known (e.g., Black & William, 1998).

In addition to the general strategies and approaches endorsed by reform, the results of this study have demonstrated that at-risk middle years students benefit from:

- an unrelenting focus on important mathematics, through
  - explicit, guiding questions and prompts rather than explanations that do not account for students’ prior knowledge and experience, nor passive participation,
  - opportunity to explore multiplicative situations that stimulate student constructions and noticing, and
  - prudent use of appropriate materials, models and representations in diverse ways; and
- attention to moment-by-moment sensitivity to where students are at, emotionally and cognitively to enable personal relationships to be established which build trust and a willingness in the students to participate fully.

Current models of teacher knowledge do not necessarily account for knowledge of the students in this way. This is an important area that must be considered when planning interventions at this level.

6.1.2 Research question 2

*What strategies are used by at-risk students to solve tasks involving multiplicative thinking and to what extent are these impacted by the intervention program?*

Prior to the intervention, the at-risk students used a range of inefficient additive strategies. These were:

- ‘Make all, count-all’
- ‘When in doubt, add’
• Repeated addition
• Skip counting
• Additive partitioning
• Whole number thinking
• ‘Have a go’ approach to determining all options

An over reliance on the use of these strategies is inefficient and leads to frequent errors at this level. This in turn impacts negatively on student feelings of potency (Sagor & Cox, 2004) as well as on the development of multiplicative thinking and ultimately contributes to a lack of progress in relation to school mathematics more generally. For instance one of the tasks required the students to work out how many blocks for a quilt could be made with a given number of squares (see Figure 4.28 earlier). To illustrate, James’ response to this Block problem task is presented below in Figure 6.1

![Figure 6.1 James’s response Block problem a](image)

However, after the intervention program, results from the Multiplicative Task interviews and the SNMY final assessment indicated a significant shift in the strategies used by the students with whom I worked closely. They were less reliant on the inefficient additive strategies that featured in their Pre-I Phase interviews and initial SNMY assessments. The students also indicated a stronger understanding of multiplication by using strategies akin to those used by the SC students (see 4.2.3). Prior to the intervention, when responding to Butterfly house e (Appendix A, p. 6) it was not uncommon for the students to attempt to draw and count all the components for 98 butterflies. To illustrate the shift, James’ efficient response to this task after our time together is shown below in Figure 6.2.
James’s response presented above suggests that he used an efficient mental strategy. This example is in stark contrast with earlier attempts where he may have needed to model all and count all. The more sophisticated strategies associated with an increased capacity for multiplicative thinking cannot be taught in isolation. They emerge as a result of an increased understanding of concepts and strategies that are carefully scaffolded over time. It is the teacher’s attentiveness to, and ability to interpret what is needed that makes the difference.

6.1.3 Research question 3

What do at-risk students believe about learning mathematics and how are these beliefs impacted by their participation in the intervention program?

Prior to the Intervention Phase, the themes that emerged from analysis of the Drawing Task interviews for the ARI students, ARN students, and SC students indicated a belief that learning maths well involved:

- having the right answer,
- doing ‘it’ the right way, and
- talking about it with others (that is, asking the teacher or a neighbor to help when required).

These beliefs were enlarged upon by the results of the Card Sort interview that indicated that students value the personal contribution they make to learning maths well. That is, with respect to learning maths well, thinking, talking and discussing, and problem solving were regarded as more important than getting the answer, doing well, worksheets and maths equipment.

Consistent with the Pre-I Phase interviews, the beliefs of the ARN and SC students remained stable across both Drawing Task and Card Sort interviews. These students for both Pre-I and Post-I Phase interviews tended to draw on experience of learning maths by solving written algorithms based on the explanations and procedures provided by the
teacher. The images and examples from these students suggest they are drawing from their experience of learning mathematics using traditional approaches. To illustrate, Hajar’s (ARN, Year 6) Post-I Phase Drawing Task image is presented in Figure 6.3 below.

Figure 6.3 Hajar’s Post-I Phase Drawing Task interview image (ARN student, Year 6)

However, for Post-I Phase interviews, there was a shift in the beliefs held by the students who participated in the intervention. These students represented situations where learning maths well was associated with working in groups and sharing ideas and strategies with others. This contrasts to the theme that emerged prior to the intervention in relation to talking about it with others that involved asking the teacher or a neighbour for assistance when required. The students acknowledged an improvement in their own ability and confidence in learning mathematics. This was reflected in the results of the Card Sort interviews where greater importance was given to getting help and doing well. The ARI students frequently referred to the nature of their interactions with others which centered on sharing ideas and strategies, as well as commenting on their own improvement. This suggests that they recognised and valued their role and the role of others. I have included Adir’s Post-I Phase Drawing Task interview image in Figure 6.4 below.
Asking students what they think is an important aspect of intervention at this level, and provides insight into students’ attitudes which may in turn help to explain certain behaviour and orientations to learning. This helps the teacher to be well-informed about the whole person who is also a learner of mathematics. The students who participated in this intervention indicated increased feeling of CBUPO (Sagor & Cox, 2004). Asking students what they think, invites students to join the conversation about their own learning. Middle years students have been described as “the missing voice of school reform” (Bishop & Pflaum, 2005, p. 2).

6.1.4 Limitations
Given the desire to undertake in-depth explorations of student learning, the study was necessarily limited in its scope and design to a relatively small number of students. I came to this intervention with my personal, teaching, research, and teacher education experience (mentioned in Chapter 1). I was able to transfer research and reflection into practice. I believed that effective teaching practice endorsed by reform would be effective in the intervention context and that the establishment of community would be critical. I brought my own experience of failure of school mathematics with me. I did not have a pre-conceived notion as to what the intervention program would ‘look like’. The program grew over time, based on the particular needs of the students that I was working with. Given such a personal approach it is hard to comment on the generalisability of the outcomes this study has achieved. Also, the program itself was highly dependent on the particular students involved, so the program itself has limited direct transferability. Further research is needed to determine what would be needed to support classroom teachers to achieve the same result. However, having said this, there are some important characteristics that are
suggested by this study that have important implications for classroom practice, professional learning, and further research. These will be discussed in the next section.

6.2 Recommendations & Implications

The results of this intervention study strongly suggest that interventions in students’ learning at this level of schooling need to go beyond prescribed programs to approaches that identify particular learning needs, and respond to these by taking account of the whole person. Intervention is teaching, and teaching is deeply personal. The following section will summarise the key dimensions for intervention in the middle years with at-risk students and will make recommendations for classroom practice, professional learning and policy, and further research.

6.2.1 Classroom practice

This intervention study has described the dynamic of teaching through the metaphor of *keeping the wet edge alive* of which there are three critical aspects: in-depth teacher knowledge of content and students, flexible and effective pedagogical practices, and mutually endorsing and respectful relationships. These three aspects have strong implications for classroom practice.

Teacher knowledge

- *Knowledge of content:* Teachers should be guided by the ‘big ideas’ in relation to multiplicative thinking. For instance, the type of knowledge supported by the *Learning and Assessment Framework for Multiplicative Thinking* (LAF). The structure of the LAF (Siemon, 2004; Siemon et al., 2006) identifies the development and implementation of strategic learning tasks and sequences to support teacher understanding of the potential learning trajectory not only in developmental terms but in terms of the mathematics content.

- *Knowledge of students:* The results of this study suggest that it would be worthwhile using the SNMY Project assessment materials with all students in the middle years to locate where students are at in relation to the development of multiplicative thinking. Use of the tasks, scoring students’ responses, and planning for teaching based on the LAF will support teachers’ capacity to interpret students’ responses to relevant tasks and inform their mathematics knowledge for teaching. In addition,
strategies for determining students’ beliefs and values should be implemented so that teachers are aware of students’ emotional needs.

**Teaching in action**

- **Reform-based approaches**: This intervention study found that reform-based approaches were successful in meeting the needs of at-risk middle years students. For instance, engaging in meaningful dialogue, problem solving, the provision of constructive feedback to students, and group work. Working this way helps to establish a sense of community and feelings of CBUPO, that is, competence, belonging, usefulness, potency, and optimism (Sagor & Cox, 2004) which is critical for student engagement and a deep understanding of mathematics.

- **Responsiveness**: Contingent planning that is highly responsive to individual learning needs was a key factor in the success of the intervention program. This suggests that teachers should resist the pressure to rely on curriculum expectations at this level as the sole determinant of what they will teach, and when. An unerring focus on the student and planning informed by diagnostic assessment, and a knowledge of the ‘big ideas’ and developmental frameworks, such as the LAF, is needed to address the learning needs of all students more appropriately. This approach ensures that students are engaged in meaningful relevant mathematical activity as opposed to simply ‘doing’ tasks.

**Relationships**

- **Sense of belonging and community**: The shift in beliefs and achievement of the ARI students attest to the importance of creating a sense of belonging by building positive relationships with students, working in groups, and implementing learning activities closely matched to student learning needs. All students, especially those at-risk, must regularly experience success which results in an increased sense of identity and agency.

6.2.2 Professional learning and policy

The recommendations that have been presented so far are not highly visible in practice generally. Hattie (2003) suggests that the answer lies in the person who gently closes the classroom door and performs the teaching act—the person who puts into place the end effects of so many policies, who interprets these policies, and who is alone with students during their 15,000 hours of schooling. I
therefore suggest that we should focus on the greatest source of variance that can make the difference – the teacher. (p. 2-3)

This study points to the critical importance of in-depth teacher knowledge of students, effective pedagogy, and the creation of respectful relationships. This has clear implications for professional learning and recommendations for policy development.

**Professional learning**

Meeting the learning needs of at-risk middle years students in relation to the development of multiplicative thinking requires a knowledge and understanding of the ‘big ideas’, and the capacity to transform the advice provided by the LAF through teaching. This can only be achieved with extensive support for targeted, ongoing, and effective professional development for teachers, particularly in relation to scaffolding the shift from additive to multiplicative thinking.

Attention has been given to the importance of teacher knowledge as defined by Shulman (1986, 1987) and Ball et al. (2008). However, equal standing must be given to teacher knowledge of *self* and teacher knowledge of *students* in ways that go beyond the cognitive. This requires an expanded view of teacher knowledge that has been endorsed by others in more recent times (e.g., Garritz, 2010; Rosiek, 2003; Zembylas, 2007). Zembylas (2007) argues from the perspective that:

> teacher knowledge is a form of knowledge ecology – a system consisting of many sources and forms of knowledge in a symbiotic relationship: content knowledge, pedagogical knowledge, curriculum knowledge, knowledge of learners, emotional knowledge, knowledge of educational values and goals and so on. (p. 356)

Garritz (2010) suggests that the affective domain has been “forgotten and diminished” (para. 4). Underpinning this way of thinking is a genuine sense of efficacy (Bandura, 1977). Teachers must believe in their own capacity to make a difference to the learning experiences of all students and believe in the capacity of all students to learn. It would be useful for teachers to be supported to explore their own values, beliefs, and orientations toward teaching mathematics and their role as teachers through professional reading and activity designed to stimulate reflection and make links to practice. For instance, professional learning in the field of identity, knowledge of self, and emotional intelligence. Further, appreciating the value and role of deep knowledge of students as ‘people’ will encourage teachers to explore a range of strategies that give students a voice in the conversation of learning.
The experience of this study strongly suggests that intervention in the middle years cannot be packaged as a pre-conceived set of instructions independent of the students. Teachers at this level need to make considered responses to the large range of students’ learning needs through contingent approaches to planning. This requires professional learning in the field of differentiated instruction (e.g., Boaler, 2002; Tomlinson, 2001), that is, effective ways to meet the range of diverse learning needs of students in the middle years, through the use of rich tasks, group work, questioning, grouping strategies, and classroom management and organization.

It has been suggested that ‘traditional’ approaches to teaching mathematics dominates the school landscape (e.g., Anderson & Bobis, 2005). This study has shown the effectiveness of reform-based approaches in the context of intervention with students in the middle years. Teachers need to be fully supported to adopt progressive, effective teaching approaches endorsed by reform (e.g., Fennema & Romberg, 1999; Lester, 2007), and subject associations (e.g., AAMT, 2006; 2008; NCTM, 2000).

**Policy**

The experience and findings of this study demonstrate the critical importance of identifying the ‘actual’ learning needs of students and using these as starting points for teaching. Assessment *for* learning is widely recognised (e.g., Black & Wiliam, 1998, 1999), and in Victoria it is an expectation of all schools (e.g., Department of Education and Early Childhood Department, 2010). However, assessment *for* learning is not well supported because of competing pressures to teach and report student achievement in terms of curriculum expectations. If, as a matter of policy, the school system seriously valued meeting students’ actual learning needs as opposed to maintaining standards according to the prescribed curriculum, this would reduce the reliance on assessment *of* learning and help legitimise differentiated teaching.

This study also suggests that adoption of a stronger system-wide stance on teacher efficacy in recognition that teachers with a deep sense of agency and identity, who ‘hold’ mathematics and learning in a certain way (Barton, 2009), make a significant contribution to student learning. This might mean, for example, that systems give consideration to including personal attributes in their statements of professional standards.

Another important consideration is support for the work that teachers do in classrooms. Initially, this might be achieved through mentoring but ultimately it may require the provision of specialist mathematics teachers. The results of this study indicate
what is possible with an ‘extra pair of hands, a heart, and mind’ in the context of busy classroom practice. Although many schools provide specialist teachers in the area of the performing and visual arts, and languages other than English, this is not the usual practice in mathematics or literacy. Instead of working with all children in Prep to Year 6 (which is the norm for the more customary specialist) a mathematics specialist could work only with those students most in need of support. I am thinking of support for at-risk students in a similar way to that reported here, but it could also apply to students working well above their expected level of schooling. This would necessitate a significant financial commitment by education authorities and would need to be supported through effective professional development. For instance, I worked with 9 students three times a week in 2 schools over an 18 week period, approximately equivalent to two Victorian school terms. It might be possible for one full-time specialist teacher to work with more at-risk students than I did. I would welcome the opportunity to mentor a small group of teachers to explore and expand their teacher knowledge, undertake mindful teaching practice, and build meaningful caring relationships with students as they undertake intervention themselves with their own students.

6.2.3 Further research
To support teacher knowledge of content in relation to the development of multiplicative thinking further research is needed to extend the LAF beyond Zone 8. Given that there are students in Years 5 and 6 at the upper zones of the LAF, a deeper understanding as to what is required beyond these zones is required to meet the needs of our successful students. It would also be useful to trial additional items at all zones of the LAF to provide multiple assessment forms in addition to the two options that are currently available. This would add to what we already know about the development of multiplicative thinking and contribute to the current work on mapping the development of rational number (e.g., Confrey, Maloney, Nguyen, Mojica, & Myers, 2009).

Further research is also needed to explore whether the approach to intervention reported in this study can be usefully applied with students in Years 7-9.

There has been an increased focus on testing teacher knowledge, particularly in relation to mathematics knowledge for teaching (e.g., Ball et al., 2008). However, further research is warranted to explore new ways of conceptualising the work that teachers do that goes beyond knowledge of content and students in relation to that content. An
important outcome of this research would be the development of new tools to evaluate teacher self-efficacy, agency, and quality relationships.

The metaphor of *keeping the wet edge alive* has been used to describe the dynamic of teaching by bringing teacher knowledge, teaching in action, and relationships together. Further research is needed to determine whether this approach to intervention can support the work of teachers in making a difference to the learning needs of their students and how this might contribute to the development of richer theories about the important but highly complex work that teachers do.
REFERENCES


annual conference of the Mathematics Education research Group of Australasia Vol. 2 (pp. 843-852). Adelaide: MERGA.

Australian Mathematics Teacher, 61(3), 34-40.

CONSTRUCTING PATHS TO MULTIPLICATIVE THINKING: BREAKING DOWN THE BARRIERS

APPENDICES

Margarita H. Breed

SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE SCHOOL OF EDUCATION COLLEGE OF DESIGN & SOCIAL CONTEXT RMIT UNIVERSITY

June, 2011
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APPENDIX A

SNMY PROJECT ASSESSMENT TASKS
TABLES AND CHAIRS …

Dean's community is planning a street party. They have lots of small square tables. Each table seats 4 people like this:

The community decides to put the tables in an end-to-end line along the street to make one big table.

a. Make or draw a line with 2 tables. How many people will be able to sit at it?

b. Make or draw a line of 4 tables. How many people will be able to sit at it?

c. Make or draw a line of tables that would seat 8 people. How many tables are needed?

d. Make or draw a line of tables that would seat 12 people. How many tables are needed?

e. Make or draw a line of tables that would seat 20 people. How many tables are needed?
f. Fill in the shaded boxes to show your results so far.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

g. Can you find another way to describe your results so far? Show this in the space below.

h. The community can borrow 99 tables. How many people could they seat using 99 tables placed end-to-end? **Show your working and explain your answer in as much detail as possible.**

i. The community can borrow rectangular tables that seat 6 people. Draw one of these tables showing the people sitting around it.

j. Draw a line of 5 of these rectangular tables placed end-to-end. How many people will be able to sit at it?
k. Explain what happens to the number of people as more rectangular tables are placed end-to-end. Describe or show your findings in at least two ways.

l. How many people could be seated if 46 of these rectangular tables were placed end-to-end? Show your working and explain your answer in as much detail as possible.

m. How many of these rectangular tables would you need to place end-to-end to seat 340 people? Show your working and explain your answer in as much detail as possible.
BUTTERFLY HOUSE...

Some children visited the Butterfly House at the Zoo. They learnt that a butterfly is made up of 4 wings, one body and two feelers.

While they were there, they made models and answered some questions.

For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

________________ wings
________________ bodies
________________ feelers

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?
c. How many wings, bodies and feelers will be needed to make 98 model butterflies. Show all your working and explain your answer in as much detail as possible.

_______________ wings
_______________ bodies
_______________ feelers

d. How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers? Show all your working and explain your answer in as much detail as possible.
e. To feed 2 butterflies the zoo needs 5 drops of nectar per day. How many drops would they need each day for 12 butterflies? **Show all your working and explain your answer in as much detail as possible.**

f. How many butterflies could you feed with 55 drops of nectar per day? **Show all your working and explain your answer in as much detail as possible.**

g. How many butterflies could you feed with 135 drops of nectar per day? **Show all your working and explain your answer in as much detail as possible.**
h. Model butterflies can be made with wings, grey, brown or black bodies and either long or short feelers. How many different model butterflies are possible? **Show all your working and explain your answer in as much detail as possible.**

i. In addition to either grey, brown or black bodies and either long or short feelers, model butterflies can also be made with either all yellow, all blue or all red wings. How many different model butterflies can be made now? **Show all your working and explain your answer in as much detail as possible.**
ADVENTURE CAMP ...

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again.

The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

<table>
<thead>
<tr>
<th></th>
<th>Rock Wall</th>
<th>Canoeing</th>
<th>Archery</th>
<th>Ropes Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 5</td>
<td>15</td>
<td>18</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>Year 7</td>
<td>19</td>
<td>21</td>
<td>38</td>
<td>22</td>
</tr>
</tbody>
</table>

Camp Reefton Thursday Activities

a. What can you say about the choices of Year 5 and Year 7 students?

b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director’s statement? **Use as much mathematics as you can to support your answer.**
CANTEEN CAPERS…

The school canteen makes great lunches. They offer 2 choices of rolls, 4 choices of filling and 3 choices of drink.

<table>
<thead>
<tr>
<th>Rolls</th>
<th>Fillings</th>
<th>Drinks</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>Peanut butter</td>
<td>Orange Juice</td>
</tr>
<tr>
<td>Multi-grain</td>
<td>Salad</td>
<td>Apple Juice</td>
</tr>
<tr>
<td></td>
<td>Cheese and vegemite</td>
<td>Cola</td>
</tr>
<tr>
<td></td>
<td>Tuna and lettuce</td>
<td></td>
</tr>
</tbody>
</table>

For each question, show your working and explain your answer in as much detail as possible.

a. Claire ordered a peanut-butter roll and a drink. What might she have ordered? List all possibilities.

b. There are 23 students in Claire’s class. If everyone ordered a roll with a filling and a drink, could they all have a different lunch order?
FENCING THE FREEWAY…

To reduce the number of animals killed on the freeway, a fencing company has been asked to fence some sections of the freeway.

The company knows that they need 1 post for every 2 metres of fencing with an additional post at the end. So a fence 4 metres long will require 3 posts.

For each question show all your working and explain your answer in as much detail as possible.

a. How many posts will be needed for a 12 metre fence?

b. How many posts will be needed for a 116 metre fence?

c. A fence uses 85 posts. How long is the fence?

d. The free-way builders asked the fencing company to build a rectangular yard for their vehicles. They wanted the area of the yard to be 48 square metres. What options might the fencing company consider? Which one do you think they would choose and why?
FILLING THE BUSES …

A school is planning a bus trip to the swimming pool for the school sports.

There are 489 students and 24 teachers at the school and each bus can hold 45 passengers. Everyone must wear a seat-belt.

For each question, show all your working and explain how you got your answer in as much detail as you can.

a. How many buses will be needed to carry all the students and teachers to the pool?

b. Make a plan for the most likely way for the students and teachers to travel to the pool by bus. Show how many students and teachers will be on each bus.
MISSING NUMBERS …

a. These numbers have been left off the number line. Without using a ruler, draw lines from each fraction to the number line below to show where it belongs. Try to be as accurate as you can.

\[
\begin{array}{cccc}
1.5 & \frac{3}{4} & 0.2 & \frac{5}{3}
\end{array}
\]

\[
\begin{array}{cccc}
0 & & & 2
\end{array}
\]

b. For each fraction explain why you located it where you did.

\[
\begin{array}{c}
1.5
\end{array}
\]

\[
\begin{array}{c}
\frac{3}{4}
\end{array}
\]

\[
\begin{array}{c}
0.2
\end{array}
\]

\[
\begin{array}{c}
\frac{5}{3}
\end{array}
\]
PACKING POTS

Jim works in a plant nursery. He has been asked to put all the small pot plants into trays to take to the market.

For each question, show all your working and explain your answer in as much detail as you can.

a. To stop the pots from moving around Jim packs them into the tray as tightly as possible. How many pots will he be able to fit into this tray?

b. Jim was given some black trays that each contained 35 pot plants. How many plants would be contained in 3 of these trays?

c. How many plants would be contained in 14 of the 35-pot trays?

d. If Jim had 560 plants, how many of the 35-pot trays would he need?
PIZZA PARTY …

a. Show how you would share four pizzas equally among three people.

Each person gets ..........................................................

b. Show how you would share 3 pizzas equally among 4 people.

Each person gets ..........................................................

c. Mathew ate \( \frac{3}{8} \) of a large pizza. Glen ate \( \frac{4}{16} \) of a large pizza. Who ate the most pizza? Explain your reasoning using as much mathematics as you can.
STAINED GLASS WINDOWS...

Stained glass windows can be made using small triangles.

This stained glass window is made from four small triangles joined together. It is 2 triangles wide at the base and 2 triangles high.

a. How many small triangles will you need if your window is to be 4 triangles wide and 4 triangles high?

b. Part of the stained glass window shown below, is hidden by a sign. How many small triangles were needed to make this window?

Community Centre

c. How would you advise a friend on how to work out the number of small triangles that would be needed for a window 26 triangles wide?
SWIMMING SPORTS

The following times were recorded for the Under 14 Freestyle. The times are shown in seconds.

<table>
<thead>
<tr>
<th>Lane 1</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
<th>Lane 6</th>
<th>Lane 7</th>
<th>Lane 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>39.26</td>
<td>45.7</td>
<td>44.3</td>
<td>38</td>
<td>37.24</td>
<td>41.08</td>
<td>39.4</td>
<td>40.84</td>
</tr>
</tbody>
</table>

a. Complete the following table to show who came first second, third etc

<table>
<thead>
<tr>
<th>Place</th>
<th>Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td></td>
</tr>
<tr>
<td>Third</td>
<td></td>
</tr>
<tr>
<td>Fourth</td>
<td></td>
</tr>
<tr>
<td>Fifth</td>
<td></td>
</tr>
<tr>
<td>Sixth</td>
<td></td>
</tr>
<tr>
<td>Seventh</td>
<td></td>
</tr>
<tr>
<td>Eighth</td>
<td></td>
</tr>
</tbody>
</table>

b. In the State Final of the Under 14 Breaststroke, Sally improved her personal best time by 2%. What was her time in the State Final if her previous personal best was 52 seconds exactly? **Explain your reasoning using as much mathematics as you can.**
TILES, TILES, TILES...

Floor and wall tiles come in difference sizes. The basic tile is shown below.

2 cm

3 cm

a. How many basic tiles would be needed for an area of 6 cm by 4 cm?

b. How many basic tiles would be needed for an area of 27 cm by 18 cm?

c. If the length and width of the basic tile were increased by 2 cm, how many of the larger tiles would be needed to cover 1 square metre (100 cm by 100 cm)?

Show all your working so we can understand your thinking.
APPENDIX B

SNMY PROJECT ASSESSMENT TASK
SCORING RUBRICS
### TABLES AND CHAIRS...

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE:</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct (6)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b. Incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct (10)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>c. Incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct (3)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>d. Incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct (5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>e. Incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct (9)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>f. No response or incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct (all gaps filled correctly)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>g. No response or incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Results shown as a list or set of ordered pairs, or described additively, eg, “It goes up by 2 each time”</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Results shown as a graph or expressed as a rule in words and/or symbols, eg, “The number of people is double the number of tables plus 2 more.”</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>h. No response or incorrect</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Incorrect but working and/or explanation indicates attempted use of general rule, or correct (200 people) with no working and/or explanation or evidence of an additive approach, eg, all tables drawn or all ordered pairs listed</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Correct (200 people), working and/or explanation indicates general rule recognized in some way, eg, “I doubled it and added 2”</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>i. No response or incorrect (eg, does not show 6 people and/or table not rectangular)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Correct, ie, rectangular table and either 2 people on longer sides and 1 on each end (arrangement A), or 3 on each of the longer sides (arrangement B)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>j. No response or incorrect for the arrangement indicated in the previous question</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Diagram correct and number of people appropriate for the arrangement shown (ie, 22 for arrangement A or 30 for arrangement B)</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
### Constructing paths to multiplicative thinking: Appendices

| k. | No response or incorrect, eg, explanation irrelevant and/or findings not shown in any meaningful way | 0 |
|    | Recording (ie, explanation and/or mode(s) of representing findings) indicative of a systematic but inefficient approach consistent with arrangement shown earlier, eg, list or diagram, or “It goes up by 4s” (or “6s” if arrangement B) | 1 |
|    | Recording describes relationship between people and tables in words, eg, “You times the number of tables by 4 and add 2” (arrangement A), or findings shown in a way that indicates an understanding of the general rule, eg, the inclusion of a larger number of rectangular tables and the corresponding number of people in a table or list | 2 |
|    | Recording describes relationship between people and tables expressed in symbols or equations consistent with arrangement shown earlier, eg, p=6n for arrangement B | 3 |

| l. | No response or incorrect | 0 |
|    | Incorrect but working and/or explanation indicates use of general rule consistent with table arrangement, or correct (186 for arrangement A or 276 for arrangement B) with no working and/or explanation or evidence of an additive approach (eg, makes all, counts all or an inefficient repeated addition strategy) | 1 |
|    | Correct (186 for arrangement A or 276 for arrangement B), working and/or explanation indicates efficient strategy based on general rule | 2 |

| m. | No response or incorrect | 0 |
|    | Incorrect but working and/or explanation indicates use of general rule, eg, divide by 6 for arrangement B, but no appreciation that an extra table is needed for remaining number of people | 1 |
|    | Correct (85 tables for arrangement A or 57 tables for arrangement B) with no working and/or explanation or evidence of an additive approach, eg, all tables drawn or all ordered pairs listed | 2 |
|    | Correct (85 tables for arrangement A or 57 tables for arrangement B), working and/or explanation indicates rule recognized in some way, eg, “I took of 2 and divided it by 4” for arrangement A | 3 |

### BUTTERFLY HOUSE …

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE:</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (28 wings, 7 bodies, 14 feelers)</td>
<td>1</td>
</tr>
<tr>
<td>b.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (4 butterflies)</td>
<td>1</td>
</tr>
<tr>
<td>c.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Partially correct with some indication of multiplicative thinking (eg, multiplication algorithm attempted), or correct but evidence of additive thinking, eg, 98+98+98+98</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>All correct (392 wings, 98 bodies, 196 feelers) with evidence of multiplicative thinking, eg, algorithm applied correctly or efficient computation strategies such as doubling or renaming (eg, 400-8 for 4x98)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><strong>d.</strong></td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (6 butterflies) but working and/or explanation indicative of additive thinking (e.g., make-all, count all strategy), or incorrect with some indication that the task has been understood in terms of multiplication or division</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (6 butterflies) with clear explanation in terms of other body parts, eg, “Can’t be 7 because not enough feelers.”</td>
<td>2</td>
</tr>
<tr>
<td><strong>e.</strong></td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (30 drops) but working and/or explanation indicates an additive approach (e.g., counts all, 5+5+5+5+5+5 or uses successive doubling strategy), or incorrect with some indication that the task has been understood in terms of multiplication or division</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (30 drops) with clear explanation and/or working which indicates an appreciation of proportional relationships (e.g., “for each group of 2, zoo needs 5 drops, 6 groups of 2, so 30 drops needed”)</td>
<td>2</td>
</tr>
<tr>
<td><strong>f.</strong></td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (22 butterflies) but working and/or explanation indicates an additive approach (e.g., counts all, 5+5+5+...), or incorrect with some indication that the task has been understood in terms of multiplication or division</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (22 butterflies) with clear explanation and/or working which indicates an appreciation of proportional relationships (e.g., “5 drops feed 2 butterflies, 55 is 11 times 5, so there must be 2x11 butterflies”)</td>
<td>2</td>
</tr>
<tr>
<td><strong>g.</strong></td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (54 butterflies) but working and/or explanation indicates an additive approach (e.g., counts all, 5+5+5+...), or incorrect with some indication that the task has been understood in terms of multiplication or division</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (54 butterflies) with clear explanation and/or working which indicates an appreciation of proportional relationships (e.g., see above)</td>
<td>2</td>
</tr>
<tr>
<td><strong>h.</strong></td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (6 butterflies) but no working and/or explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (6 butterflies), working and/or explanation indicates an additive approach (e.g., draws all, counts all, not particularly systematic)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (6 butterflies) with clear explanation and/or working which indicates an appreciation of Cartesian product or “for each” idea (e.g., tree diagram, systematic list)</td>
<td>3</td>
</tr>
<tr>
<td><strong>i.</strong></td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (18 butterflies) but no working and/or explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (18 butterflies) but working/explanation indicates an additive approach (e.g., draws all, counts all, not particularly systematic)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (18 butterflies) with clear explanation and/or working which indicates an appreciation of Cartesian product or “for each” idea (e.g., tree diagram, systematic list)</td>
<td>3</td>
</tr>
</tbody>
</table>
## ADVENTURE CAMP …

<table>
<thead>
<tr>
<th>TASK</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect statement</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>One or two relatively simple observations based on numbers alone, eg, “Archery was the most popular activity for both Year 5 and Year 7 students”, “More Year 7 students liked the rock wall than Year 5 students”</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>At least one observation which recognises the difference in total numbers, eg, “Although more Year 7s actually chose the ropes course than Year 5, there were less Year 5 students, so it is hard to say”.</td>
<td>2</td>
</tr>
<tr>
<td>b.</td>
<td>No response</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect (No), argument based on numbers alone, eg, “There were 21 Year 7s and only 18 Year 5s”.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (Yes), but little/no working or explanation to support conclusion</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (Yes), working and/or explanation indicates that numbers need to be considered in relation to respective totals, eg, “18 out of 75 is more than 21 out of 100”, but no formal use of fractions or percent or further argument to justify conclusion</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Correct (Yes), working and/or explanation uses comparable fractions or percents to justify conclusion, eg, “For Year 7 it is 21%. For Year 5, it is 24% because 18/75 = 6/25 = 24/100 = 24%”</td>
<td>4</td>
</tr>
</tbody>
</table>

## CANTEEN CAPERS …

<table>
<thead>
<tr>
<th>TASK</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect with no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect but recognises that there is more than one option, or correct (6 options) with little/no working and/or explanation to support conclusion</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (6 different options), working and/or explanation indicates answer arrived at additively, eg, 6 options listed or drawn fairly randomly</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (6 options), working and/or explanation clearly indicates systematic approach and/or recognition of Cartesian Product idea, eg, “It’s 2 x 3 because she has 2 choices of roll and for each one she has 3 choices of drink”</td>
<td>3</td>
</tr>
<tr>
<td>b.</td>
<td>No response or incorrect with little/no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (Yes), but little/no working or explanation to support conclusion, eg, drawing or list not systematic, not clear that Cartesian Product idea seen as relevant</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (Yes), working and/or explanation clearly supports conclusion, eg, systematic drawing or list, and/or recognition of 24 options in terms of 2x4x3</td>
<td>2</td>
</tr>
</tbody>
</table>

## FENCING THE FREEWAY …

<table>
<thead>
<tr>
<th>TASK</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (7 posts), but little/no working or explanation to support conclusion</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (7 posts), working and/or explanation clearly supports conclusion, eg, drawing and/or explanation given in terms of pattern, divide by 2 and add 1</td>
<td>2</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

<table>
<thead>
<tr>
<th></th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (59 posts), but little/no working or explanation to support conclusion</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (59 posts), working and/or explanation clearly supports conclusion, eg, response indicates use of pattern, divide by 2 and add 1</td>
<td>2</td>
</tr>
<tr>
<td>c.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (168 metres), but little/no working or explanation to support conclusion</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (168 metres), working and/or explanation clearly supports conclusion, eg, response indicates use of pattern, subtract 1 multiply by 2</td>
<td>2</td>
</tr>
<tr>
<td>d.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>At least one correct option, but little/no working or explanation to support conclusion, eg, “8 metres long and 6 metres wide because it’s the best”,</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>One or more correct options with working and/or explanation to support conclusion, eg, “4 metres long by 12 metres long, because it would fit beside the freeway and the trucks could park one behind the other”</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Two or more correct options (eg, 24m x 2 m, 12m x 4m, 8m x 6m, …), working and/or explanation clearly supports conclusion, eg, “8 metres by 6 metres as this would give them more room to move the vehicles”</td>
<td>3</td>
</tr>
</tbody>
</table>

**FILLING THE BUSES …**

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect with no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect (eg, 11 or 11.4 buses), working and/or explanation indicates that multiplication/division is needed but not all information taken into account, eg, need to include teachers and students in total, or remainder is not interpreted relevant to context</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (12 buses), little/no working and/or explanation, or evidence of an inefficient additive strategy, eg, 45+45+45+ …</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (12 buses), working and/or explanation indicates efficient multiplicative strategy, eg, 10x45=450, 1 more bus for 495, so 12 needed</td>
<td>3</td>
</tr>
<tr>
<td>b.</td>
<td>No response or irrelevant, eg, drawing of bus with passengers, no indication of how students and teachers might be allocated.</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Plan shows correct number of buses but little/no consideration given to the likely distribution of students and teachers</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Plan is more systematically, thoughtfully presented in relation to context, eg, recognises need for some teachers to be on each bus</td>
<td>2</td>
</tr>
</tbody>
</table>

**MISSING NUMBERS …**

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect ( use overlay provided, most outside ± 4 mm)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>At least 2 correctly located (within ± 4 mm, where 1.5 = 11.7 cm, 3/4 = 5.85 cm, 0.2 = 1.56 cm and 5/3 = 13.0 cm)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>At least 2 correctly located (within ± 3 mm)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Three or more correctly located (within ± 3 mm)</td>
<td>3</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

<table>
<thead>
<tr>
<th>b.</th>
<th>No response or inadequate, eg, “I just guessed”</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>At least two responses provided. Explanations refer to estimating, eg, I estimated a half and it said it was a bit less”. Little/no evidence that a systematic partitioning strategy was used.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>At least 2 responses provided. Explanations indicate a partitioning strategy of some sort, eg, “I halved it to get 1, then I halved it again to get ½ then I halved that to find ¼”</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Two or more responses provided. Explanations indicate the systematic use of partitioning strategies and/or thinking derived from known relationships, eg, ¼ is 0.75, 5/3 is 1.66… etc</td>
<td>3</td>
</tr>
</tbody>
</table>

**PACKING POTS …**

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE:</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect (eg, pots drawn fairly randomly, “I just guessed”)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect, but a evidence of a systematic attempt to draw the pots as an array, or correct (24 pots), but explanation refers to counting all pots</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (24 pots), drawing and/or explanation indicates that array notion has been employed and/or multiplication used to determine total, eg, 4x6=24</td>
<td>2</td>
</tr>
<tr>
<td>b.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect but working and/or explanation indicates that multiplication is needed, or correct (105 pots) with little/no working and/or explanation or response indicates repeat addition or count all, eg, “I just added”, 35+35+35=105</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (105 pots) working and/or explanation indicates that an efficient calculation has been used, eg, the double and 1 more group mental strategy or 35x5=105</td>
<td>2</td>
</tr>
<tr>
<td>c.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect but working and/or explanation indicates that multiplication is needed, or correct (490 pots) but working and/or explanation is unclear or indicates repeat addition or the inefficient use of doubling/other strategy</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (490 pots) working and/or explanation indicates that an efficient calculation has been used, eg, a mental strategy such as “10 trays is 350, another 4 trays is 140, so 490 altogether”, or 35x14=490</td>
<td>2</td>
</tr>
<tr>
<td>d.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect but working and/or explanation indicates that division attempted, or correct (16 trays) but working and/or explanation is unclear or indicates inefficient strategy</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (16 trays) working and/or explanation indicates that an efficient calculation has been used, eg, a mental strategy such as “35 by 10 is 350, 5 more is 175, so 15 trays will take 525, enough for 1 more tray, so 16 trays “, or 560 ÷ 35 = 16</td>
<td>2</td>
</tr>
</tbody>
</table>
## PIZZA PARTY …

<table>
<thead>
<tr>
<th>TASK</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect partitioning</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Appropriate partitioning but each person’s share either not described or described informally, “A bit more than a whole pizza” or “a whole pizza and 1 piece more”</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Appropriate partitioning with clear indication of each share expressed as a fraction in words or symbols eg, 1 and a third or 1(\frac{1}{3})</td>
<td>2</td>
</tr>
<tr>
<td>b.</td>
<td>No response or incorrect partitioning</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Appropriate partitioning but each person’s share either not described or described informally, “They each get a bit less than a whole pizza” or “3 pieces”</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Appropriate partitioning with clear indication of each share expressed as a fraction in words or symbols eg, 3 quarters or (\frac{3}{4})</td>
<td>2</td>
</tr>
<tr>
<td>c.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Correct (Mathew) with little or no working or explanation to justify conclusion</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (Mathew) with clear explanation and/or working based on a comparison of parts, eg, recognition that (\frac{4}{16}) is equivalent to (\frac{2}{8}) or (\frac{3}{8} = \frac{6}{16})</td>
<td>2</td>
</tr>
</tbody>
</table>

## STAINED GLASS WINDOWS …

<table>
<thead>
<tr>
<th>TASK</th>
<th>RESPONSE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect with no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (16 triangles), with evidence of additive reasoning based on drawing and counting</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (16 triangles), with evidence of multiplicative reasoning based on (4 \times 4)</td>
<td>3</td>
</tr>
<tr>
<td>b.</td>
<td>No response or incorrect with little/no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (81 triangles), with evidence of additive reasoning based on drawing and counting, or inappropriate use of area formula, eg, (L \times W)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (81 triangles), with evidence of multiplicative reasoning based on pattern, eg, (9 \times 9)</td>
<td>3</td>
</tr>
<tr>
<td>c.</td>
<td>No response or incorrect</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Advice based on additive thinking, eg, “1 less each time you go up”</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (advice based on rule, eg, (26 \times 26))</td>
<td>2</td>
</tr>
</tbody>
</table>
### SWIMMING SPORTS …

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE:</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or largely incorrect (4 or more misplaced)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Partially correct (only 1 to 3 misplaced)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (1st to 8th identified correctly as Lane 5, Lane 4, Lane 1, Lane 7, Lane 8, Lane 6, Lane 3, and Lane 2 respectively)</td>
<td>2</td>
</tr>
<tr>
<td>b.</td>
<td>No response</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect but working and/or explanation indicates some attempt to find 2% of 52 seconds using place-value or an appropriate calculation, eg, 1% identified but not doubled or 2% found but added to 52 seconds</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (50.96 seconds) but little or no working or explanation to justify conclusion</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (50.96 seconds), reasoning based on appropriate calculation (eg, 52 x 0.98) or place-value (eg, 1 hundredth of 52 is 0.52 so 2% is 1.04, take 1.04 from 52)</td>
<td>3</td>
</tr>
</tbody>
</table>

### TILES, TILES, TILES …

<table>
<thead>
<tr>
<th>TASK:</th>
<th>RESPONSE:</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>No response or incorrect with no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect (2 tiles), reasoning based on perceived relationship between dimensions eg, “2 goes into 4, 2 times and 3 goes into 6, 2 times” or incorrect drawing, or correct but little/no working or reasoning</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (4 tiles), with appropriate diagram and/or explanation</td>
<td>2</td>
</tr>
<tr>
<td>b.</td>
<td>No response or incorrect with little/no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Incorrect (9 or 18 tiles), reasoning based on factors as above, or correct (81 tiles) but little/no working/explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correct (81 tiles), with appropriate diagram and/or evidence of additive strategy, eg, count all or skip count</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (81 tiles), with appropriate diagram and/or explanation indicating multiplicative reasoning, eg, factors used appropriately</td>
<td>3</td>
</tr>
<tr>
<td>c.</td>
<td>No response or incorrect with little/no working and/or explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Some attempt, eg, dimension of larger tile (4cm by 5cm) indicated and/or incomplete solution attempt, eg, attempt to draw all</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Incorrect, calculation based on incorrect dimension of larger tile, eg, 4cm by 6cm, but supported by correct reasoning of the area required; or correct (500 tiles), with little/no explanation</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Correct (500 tiles), supported by appropriate diagram and/or explanation based on appropriate diagram or computation strategies</td>
<td>3</td>
</tr>
</tbody>
</table>
APPENDIX C

LEARNING ASSESSMENT FRAMEWORK FOR MULTIPLICATIVE THINKING (LAF)
### Learning & Assessment Framework for Multiplicative Thinking

<table>
<thead>
<tr>
<th>Zone 1 – Primitive Modelling</th>
<th>Teaching Implications</th>
</tr>
</thead>
</table>
| Can solve simple multiplication and division problems involving relatively small whole numbers (e.g., Butterfly House parts a and b)*, but tends to rely on drawing, models and count-all strategies (e.g., draws and counts all pots for part a of Packing Pot). May use skip counting (repeated addition) for groups less than 5 (e.g., to find number of tables needed to seat up to 20 people in Tables and Chairs). | **Consolidate/establish:**
| Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation. | **Trust the count** for numbers to 10 (e.g., for 6 this involves working with mental objects for 6 without having to model and/or count-all). Use flash cards to develop subitising (i.e., ability to say how many without counting) for numbers to 5 initially and then to 10 and beyond using part-part-whole knowledge (e.g., 8 is 4 and 4, or 5 and 3 more, or 2 less than 10). Practice regularly.
| Can make simple observations from data given in a task (e.g., Adventure Camp set) and reproduce a simple pattern (e.g., Tables and Chairs a to e). | **Simple skip counting** to determine how many in a collection and to establish numbers up to 5 as countable objects, e.g., count by twos, fives and tens, using concrete materials and a 0-99 Number Chart.
| Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation. | **Mental strategies for addition and subtraction facts to 20** (e.g., Count on from larger (e.g., for 2 and 7, think, 7, 8, 9), Double and near doubles (e.g., use ten-frames and a 2-row bead-frame to show that 7 and 7 is 10 and 4 more, 14), and Make-to-ten (e.g., for 6 and 8, think, 8, 10, 14, scaffold using open number lines). Explore and name mental strategies to solve subtraction problems such as 7 take 2, 12 take 5, and 16 take 9. Practice (e.g., by using Number Charts from Math300).
| **2 digit place-value** – working flexibly with ones and tens, (by making, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming, see Booker et al, 2004). Play the ‘Place-Value Game’ (see Support Materials on the CD-ROM). | **Introduce/develop:**
| Doubling (and halving) strategies for 2-digit numbers that do not require renaming (e.g., 34 and 34, half of 46), build to numbers that require some additional thinking (e.g., to double 36, double 3 tens, double 6 ones, 60 and 12 ones, 72). | **Doubling (and halving) strategies** for 2-digit numbers that do not require renaming (e.g., by doubling 24, double 3 tens, double 6 ones, 60 and 12 ones, 72).
| Extended mental strategies for addition and subtraction, use efficient, place-value based strategies (e.g., 37 and 24, think: 37, 47, 57, 60, 61). Use open number lines to scaffold thinking. | **Extended mental strategies for addition and subtraction**, use efficient, place-value based strategies (e.g., 37 and 24, think: 37, 47, 57, 60, 61). Use open number lines to scaffold thinking.
| Efficient and reliable strategies for counting large collections (e.g., count a collection of 50 or more by 2s, 5s or 10s) with a focus on how to organise the number of groups to facilitate the count (e.g., by arranging the groups systematically in lines or arrays and then skip counting). | **Efficient and reliable strategies for counting large collections** (e.g., count a collection of 50 or more by 2s, 5s or 10s) with a focus on how to organise the number of groups to facilitate the count (e.g., by arranging the groups systematically in lines or arrays and then skip counting).
| How to make, name and use arrays/regions to solve simple multiplication or sharing problems using concrete materials, and skip counting (e.g., 1 four, 2 fours, 3 fours …), leading to more efficient counting strategies based on reading arrays in terms of a consistent number of rows (e.g., 4 rows of anything, that is, 4 ones, 4 twos, 4 threes, 4 fours, …). | **How to make, name and use arrays/regions** to solve simple multiplication or sharing problems using concrete materials, and skip counting (e.g., 1 four, 2 fours, 3 fours …), leading to more efficient counting strategies based on reading arrays in terms of a consistent number of rows (e.g., 4 rows of anything, that is, 4 ones, 4 twos, 4 threes, 4 fours, …).
| 3 digit place-value – working flexibly with tens and hundreds (by making with MAB, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming – see Booker et al, 2004). | **3 digit place-value** – working flexibly with tens and hundreds (by making with MAB, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming – see Booker et al, 2004).
| Strategies for unpacking and comprehending problem situations (e.g., read and retell, ask questions such as, What is the question asking? What do we need to do? …). Use realistic word problems to explore different ideas for multiplication and division, e.g., 3 rows, 7 chairs in each row, how many chairs (array)? Mandy has three times as many…as Tom…, how many … does she have (scalar idea)? 24 cards shared among 6 students, how many each (partition)? Lollipops cost 5c each, how much for 4 (‘for each’ idea)? | **Strategies for unpacking and comprehending problem situations** (e.g., read and retell, ask questions such as, What is the question asking? What do we need to do? …). Use realistic word problems to explore different ideas for multiplication and division, e.g., 3 rows, 7 chairs in each row, how many chairs (array)? Mandy has three times as many…as Tom…, how many … does she have (scalar idea)? 24 cards shared among 6 students, how many each (partition)? Lollipops cost 5c each, how much for 4 (‘for each’ idea)?

* Please note, the problems referred to in italics and their associated scoring rubrics can be found in either the Support Materials or the Assessment Materials for Multiplicative Thinking sections of the CD-ROM.
### Zone 2: Intuitive Modelling

Trusts the count for groups of 2 and 5, that is, can use these numbers as units for counting (eg, Tables & Chairs, Butterfly House), counts large collections efficiently, systematically keeps track of count (for instance may order groups in arrays or as a list) but needs to ‘see’ all groups (eg, Tiki, Tiki, Tiki a, or for Butterfly House, may use list and/or doubling as follows:

- 2 butterflies 5 drops
- 4 butterflies 10 drops
- 6 butterflies 15 drops
- 12 butterflies 30 drops

Can share collections into equal groups/parts (eg, Pizza Party a and b).

Recognises small numbers as composite units (eg, can count equal groups, skip count by twos, threes and fives)

Recognises multiplication is relevant (eg, Packing Pots c, Speedy Snail) but tends not to be able to follow this through to solution

Can list some of the options in simple Cartesian Product situations (eg, Canton Capers a)

Orders 2 digit numbers (eg, partially correct ordering of times in Swimming Sports a)

Some evidence of multiplicative thinking as equal groups/shares seen as entities that can be counted systematically

---

### Consolidate/establish:

**Ideas and strategies** introduced/developed in the previous level (see above)

### Introduce/develop:

**More efficient strategies for counting groups** based on a change in focus from a count of equal groups (eg, (1 three, 2 threes, 3 threes, 4 threes, …) to a consistent number of groups (eg, 3 ones, 3 twos, 3 threes, 3 fours, …) which underpin the more efficient mental strategies listed below and ultimately lead to the factor-factor-product idea

**Array/region-based mental strategies for multiplication facts to 100** eg, doubling (for 2s facts), doubling and 1 more group (for 3s facts), double doubles (for 4s facts), relate to tens (for 5s and 9s facts) and so on (see Thue’s More to counting Than Meets the Eye (see Papers and Presentations on the CD-ROM); Booker et al, 2004)

Efficient strategies for solving problems where arrays and regions only partially observed

**Commutativity**, by exploring the relationship between arrays and regions such as 3 fours and 4 threes. Play ‘Multiplication Toss’ (see Support Materials on the CD-ROM)

**Informal division strategies** such as think of multiplication and halving (eg, 16 divided by 4, think: 4 ‘whats’ are 16? 4; or half of 16 is 8, half of 8 is 4)

**Extended mental strategies for multiplication** (eg, for 3 twenty fives, Think: double 25, 50, and twenty five more; 75) and use place-value based strategies such as 10 groups and 4 more groups for 14 groups

**Simple proportion problems** involving non-numerical comparisons (eg, If Nick mixed less cordial with more water than he did yesterday, his drink would taste (a) stronger, (b) weaker (c) exactly the same, or (d) not enough information to tell)

**How to recognise and describe simple relationships** and patterns (eg, ‘double and add 2’ from models, diagrams and tables; or notice that a diagonal pattern on a 0-99 chart is a count of 11, 1 ten and 1 ones)

**Language of fractions** through practical experience with both continuous and discrete, ‘real-world’ fraction models (eg, 3 quarters of the pizza, half the class), distinguishing between how many and how much (eg, in 2 thirds the numeral indicates how many, the name indicates how much)

**Halving partitioning strategy**, through paper folding (kinder squares and streamers), cutting plasticine ‘cakes’ and ‘pizzas’, sharing collections equally (counters, cards etc), apply thinking involved to help children create their own fraction diagrams. Focus on making and naming parts in the halving family (eg, 8 parts, eighths) including mixed fractions (eg, “2 and 3 quarters”) and informal recording (eg, 3 eighths), no symbols

**Key fraction generalisations** – that is, that equal parts are necessary and that the number of parts names the part
### Zone 3: Sensing

Demonstrates intuitive sense of proportion (eg, partial solution to *Butterfly House*) and partitioning (eg, *Missing Numbers*)

Works with ‘useful’ numbers such as 2 and 5, and strategies such as doubling and halving (eg, *Packing Pots* and *Pizza Party*)

May list all options in a simple Cartesian product situation (eg, *Canteen Capers*), but cannot explain or justify solutions

Uses abbreviated methods for counting groups, eg, doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions (eg, *Pizza Party*)

Beginning to work with larger whole numbers and patterns but tends to rely on count all methods or additive thinking to solve problems (eg, *Stained Glass Windows* and *Tiles, Tiles, Tiles*)

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### Consolidate/establish:

#### Ideas and strategies introduced/developed in the previous level (see above)

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### Introduce/develop:

**Place-value based strategies** for informally solving problems involving single-digit by two-digit multiplication (eg, for 3 twenty-eights, THINK, 3 by 2 tens, 60 and 24 more, 84) mentally or in writing

Initial recording to support place-value for multiplication facts (see Booker et al, 2004 and *Then’s More to Counting Than Meets the Eye*)

More efficient strategies for solving number problems involving simple proportion (eg, recognise as two-step problems, What do I do first? Find value for common amount. What do I do next? Determine multiplier/factor and apply. Why?*)

How to rename number of groups (eg, think of 6 fours as 5 fours and 1 more four), Practice (eg, by using ‘Multiplication Toss’ (see Support Materials on the CD-ROM)). Rename composite numbers in terms of equal groups (eg, 18 is 2 nines, 9 twos, 3 sixes, 6 threes)

**Cartesian product** or for each idea using concrete materials and relatively simple problems such as 3 tops and 2 bottoms, how many outfits, or how many different types of pizzas given choice of small, large, medium and 4 varieties? Discuss how to recognise problems of this type and how to keep track of the count such as draw all options, make a list or a table (tree diagrams appear to be too difficult at this level, these are included in Zone 5)

How to interpret problem situations and solutions relevant to context (eg, Ask, What operation is needed? Why? What does it mean in terms of original question?)

Simple, practical division problems that require the interpretation of remainders relevant to context

**Practical sharing situations** that introduce names for simple fractional parts beyond the halving family (eg, thirds for 3 equal parts/shares, sixths for 6 equal parts etc) and help build a sense of fractional parts, eg 3 sixths is the same as a half or 50%, 7 eighths is nearly 1, “2 and 1 tenth” is close to 2. Use a range of continuous and discrete fraction models including mixed fraction models

**Thirding and fifthing partitioning strategies** through paper folding (kinder squares and streamers), cutting plasticine ‘cakes’ and ‘pizzas’, sharing collections equally (counters, cards etc), *apply thinking involved to help children create their own fraction diagrams (regions) and number line representations* (see Siemon (2004) *Partitioning – The Missing Link in building Fraction Knowledge and Confidence* (see Papers and Presentations on the CD-ROM)). Focus on making and naming parts in the thirding and fifthing families (eg, 5 parts, fifths) including mixed fractions (eg, “2 and 5 ninths”) and informal recording (eg, 4 fifths), no symbols. Revise key fraction generalisations (see Zone 2), include whole to part models (eg, partition to show 3 quarters) and part to whole (eg, if this is 1 third, show me the whole) and use diagrams and representations to rename related fractions

**Extend partitioning strategies** to construct number line representations. Use multiple fraction representations

**Key fraction generalisations** – the greater the number of parts, the smaller they are, and conversely, the fewer the parts the larger they are.
### Zone 4: Strategy Exploring

Solves more familiar multiplication and division problems involving two-digit numbers (eg, Butterfly House c and d, Packing Pots c, Speedy Snail a)

Tend to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations (eg, Packing Pots d, Filling the Bases a and b, Tables & Chairs g and h, Butterfly House b and g, Speedy Snail c, Computer Game a, Stained Glass Windows a and b). Tend not to explain their thinking or indicate working.

Able to partition given number or quantity into equal parts and describe part formally (eg, Pizza Party a and b), and locate familiar fractions (eg, Missing Numbers a)

Beginning to work with simple proportion, eg, can make a start, represent problem, but unable to complete successfully or justify their thinking (eg, How Far a, School Fair a and b)

### Consolidate/establish:

**Ideas and strategies** introduced/developed in the previous level (see above)

### Introduce/develop:

**More efficient strategies for multiplying and dividing larger whole numbers** independently of models (eg, strategies based on: doubling, renaming the number of groups, factors, place-value, and known addition facts,

- eg, for dividing 564 by 8, THINK, 8 what’s are 560? 8 by 7 tens or 70, so 70 and 4 remainder.
- eg, for 3908 divided by 10, RENAME as, 390 tens and 8 ones, so 390.8)

**Tenths as a new place-value part**, by making/representing, naming and recording ones and tenths (see Booker et al, 2004), consolidate by comparing, ordering, sequencing counting forwards and backwards in ones and/or tenths, and renaming

How to partition continuous quantities more generally using the **halving, thirding, fifthing strategies** (see Siemon (2004) Partitioning – The Missing Link in building Fraction Knowledge and Confidence (see Papers and Presentations on the CD-ROM)), eg, recognise that sixths can be made by halving and thirding (or vice versa), tenths can be made by fifthing and halving etc, use this knowledge to construct fraction diagrams (eg, region models) and representations (eg, number line) for common fractions and decimals including mixed numbers

**Informal, partition-based strategies for renaming simple unlike fractions**, eg, recognise that thirds and fifths can be renamed by thirding and then fifthing (or vice versa) on a common diagram, eg.

<table>
<thead>
<tr>
<th>thirds (3 parts)</th>
<th>fifths (5 parts)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Link to region model of multiplication (in this case 3 fives, or 3 parts by 5 parts) to recognise that thirds by fifths are fifteenths, so 2 thirds can be renamed as 10 fifths and 4 fifths can be renamed as 12 fifteenths. Use strategies to informally add and subtract like and related fractions

**Key fraction generalisations** - that is, recognise that equal parts are necessary, the total number of parts names the part, and as the total number of parts increases they get smaller (this idea is crucial for the later development of more formal strategies for renaming fractions (see Zone 5) which relate the number of parts initially (3, thirds) to the final number of parts (15, fifteenths) in terms of factors, that is, the number of parts has been increased by a factor of 5)

**Metacognitive strategies** to support problem comprehension, problem representation, strategy monitoring/checking, and interpretation of outcomes relevant to context (see Siemon and Booker (1990) paper on Teaching and Learning For, About and Through Problem Solving (see Papers and Presentations on the CD-ROM))

**Simple proportion problems that introduce techniques for dealing with these situations** (eg, find for 1 then multiply or divide as appropriate, using scale diagrams and interpreting distances from maps)
Zone 5: Strategy Refining

Systematically solves simple proportion and array problems (eg, Butterfly House a, Packing Pots a, How Far a) suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions (eg, School Fair a, Spicy Snail b)

Able to solve simple, 2-step problems using a recognised rule/relationship (eg, Fencing the Freeway a) but finds this difficult for larger numbers (eg, Tables & Chairs k and l, Tiles, Tiles, Tiles c, Stained Glass Windows c)

Able to order numbers involving tens, ones, tenths and hundredths in supportive context (Swimming Sports a)

Able to determine all options in Cartesian product situations involving relatively small numbers, but tends to do this additively (eg, Canteen Capers a, Butterfly House l and i)

Beginning to work with decimal numbers and percent (eg, Swimming Sports a and b, Computer Game b) but unable to apply efficiently to solve problems

Some evidence that multiplicative thinking being used to support partitioning (eg, Missing Numbers b)

Beginning to approach a broader range of multiplicative situations more systematically

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<table>
<thead>
<tr>
<th>Consolidate/establish:</th>
<th>Introduce/develop:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideas and strategies introduced/developed in the previous level (see above)</td>
<td>Place-value ideas and strategies for 5 digits and beyond if not already developed and decimal fractions to hundredths (see partitioning below) including renaming</td>
</tr>
</tbody>
</table>

**Flexible, meaningful and efficient strategies** for multiplying and dividing by multiples of ten (eg, 2.13 by 10, THINK, 21 ones and 3 tenths, 21.3)

The **area idea** to support multi-digit multiplication and formal recording (see Booker et al, 2004) and more efficient strategies for representing and solving an **expanded range of Cartesian product problems** involving three or more variables and **tree diagram representations**

Formal terminology associated with multiplication and division such as factor, product, divisor, multiplier and raised to the power of …. Play ‘Factor Cross’ game (see Support Materials on the CD-ROM). Use calculators to explore what happens with repeated factors eg, 4 x 4 x 4 x 4 ..., factors less than 1, and negative factors.

Informal, partition-based strategies for renaming an expanded range of unrelated fractions as a precursor to developing an efficient, more formal strategy for generating equivalent fractions (see below), eg, explore using paper folding, diagrams and line models how sixths and eighths could be renamed as forty-eighths but they can also be renamed as twenty-fourths because both are factors of 24

The **generalisation for renaming fractions**, that is, if the number of equal parts (represented by the denominator) increases/decreases by a certain factor then the number of parts required (indicated by the numerator) increases/decreases by the same factor

**Written solution strategies for the addition and subtraction of unlike fractions**, eg, think of a diagram showing sixths by eighths … forty-eighths … Is this the simplest? No, twenty-fourths will do, rename fractions by inspection

\[
\begin{align*}
\text{Total number of parts increased by a factor of 3, so parts required} \\
\text{increased by a factor of 3} \\
7 & \rightarrow 9 \\
\frac{3}{8} & \rightarrow \frac{9}{24}
\end{align*}
\]

\[
\begin{align*}
\text{Total number of parts increased by a factor of 4, so parts required} \\
\text{increased by a factor of 4} \\
-3 & \rightarrow 20 \\
\frac{5}{6} & \rightarrow \frac{20}{24}
\end{align*}
\]

9 twenty-fourths can’t take 20 twenty-fourths, trade 1 one for 24 twenty-fourths to get 6 and 33 twenty-fourths, subtraction is then relatively straightforward

Explore **link between multiplication and division and fractions** including decimals (eg, 3 pizzas shared among 4, 3 divided by 4 is 0.75 etc) to understand **fraction as operator idea** (eg, % of 120, 75% of $48, 250% of 458,239). Use ‘Multiple Patterns’ (see Support Materials on the CD-ROM). Establish benchmark equivalences (eg, 1 third =33⅓ %)

**Metacognitive strategies** to support problem comprehension, strategy monitoring/checking, and interpretation of outcomes relevant to context (see Siemon and Booker (1990) paper on Teaching and Learning For, About and Through Problem Solving (see Papers and Presentations on the CD-ROM))

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Zone 6: Strategy Extending

Can work with Cartesian Product idea to systematically list or determine the number of options (e.g., Canteen Capers b, Butterfly House i and b)

Can solve a broader range of multiplication and division problems involving two digit numbers, patterns and/or proportion (e.g., Tables & Chairs b, Butterfly House f, Stained Glass Windows b and c, Computer Game a and b) but may not be able to explain or justify solution strategy (e.g., Fencing the Freeway b, Fencing the Freeway d, and Swimming Sports b, How Far b, Speedy Snail b)

Able to rename and compare fractions in the halving family (e.g., Pizza Party c) and use partitioning strategies to locate simple fractions (e.g., Missing Numbers a)

Developing sense of proportion (e.g., see relevance of proportion in Adventure Camp b, Tiles, Tiles, Tiles b), but unable to explain or justify thinking

Developing a degree of comfort with working mentally with multiplication and division facts

Consolidate/establish:

Ideas and strategies introduced/developed in the previous level (see above)

Introduce/develop:

Hundredsths as a new place-value part, by making/representing, naming and recording ones, tenths, and hundredths (see Booker et al, 2004), consolidate by comparing, ordering, sequencing counting forwards and backwards in place-value parts, and renaming. Link to %

How to explain and justify solution strategies for problems involving multiplication and division (see Multiplication Workshop (see Papers and Presentations on the CD-ROM)), particularly in relation to interpreting decimal remainders appropriate to context, e.g.,

- How many buses will be needed to take 594 students and teachers to the school Speech night, assuming each bus hold 45 passengers and everyone must wear a seatbelt?

More efficient, systematic, and/or generalizable processes for dealing with proportion problems (e.g., use of the ‘for each’ idea, formal recording, and the use of fractions, percent to justify claims), e.g.,

- Jane scored 14 goals from 20 attempts. Emma scored 18 goals from 25 attempts. Which girl should be selected for the school basketball team and why?

- 6 girls share 4 pizzas equally. 8 boys share 6 pizzas equally. Who had more pizza, the girls or the boys?

- 35 feral cats were found in a 146 hectare nature reserve. 27 feral cats were found in a 103 hectare reserve. Which reserve had the biggest feral cat problem?

- Orange juice is sold in different sized containers: 5L for $14, 2L for $5, and 500mL for $1.35. Which represents the best value for money?

More efficient strategies and formal processes for working with multiplication and division involving larger numbers based on sound place-value ideas, e.g., 3486 x 21 can be estimated by thinking about 35 hundreds by 2 tens, 70 thousands, and 1 more group of 35 hundred, ie, 73,500, or it can be calculated by using factors of 21, ie, 3486 x 3 x 7. Two digit multiplication can be used to support the multiplication of ones and tenths by ones and tenths, eg, for 2.3 by 5.7, rename as tenths and compute as 23 tenths by 57 tenths, ie, 1311 hundredths hence 13.11. Consider a broader range of problems and applications, e.g.,

- Average gate takings per day over the World Cricket cup Series

- Matt rode around the park 8 times. The odometer on his bike indicated that he ridden a total of 15 km. How far was it around the park?

- After 11 training sessions, Kate’s average time for 100 metres butterfly was 61.5 seconds. In her next 2 mails, Kate clocked 61.21 and 60.87 seconds. What was her new average time?

Integers using real-world examples such as heights above and below sea-level, temperatures above and below zero, simple addition and difference calculations

The notion of variable and how to recognise and formally describe patterns involving all four operations. Use ‘Max’s Matchsticks’ (see Support Materials on the CD-ROM) to explore how patterns may be viewed differently leading to different ways of counting and forms of representation.
Zone 7: Connecting

Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording (e.g., Filling the Buses a, Fencing the Freeway d, Packing Pots d).

Can solve and explain solutions to problems involving simple patterns, percent and proportion (e.g., Fencing the Freeway c, Swimming Sports b, Butterfly House g, Tables & Chairs g and l, Speedy Snail c, Tiles, Tiles, Tiles b and c, School Fair a, Stained Glass Windows a, Computer Game b, How Far b). May not be able to show working and/or explain strategies for situations involving larger numbers (e.g., Tables & Chairs m and k, Tiles, Tiles, Tiles c) or less familiar problems (e.g., Adventure Camp b, School Fair b, How Far c).

Locates fractions using efficient partitioning strategies (e.g., Missing Numbers a).

Beginning to make connections between problems and solution strategies and how to communicate this mathematically.

<table>
<thead>
<tr>
<th>Consolidate/establish:</th>
<th>Introduce/develop:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideas and strategies introduced/developed in the previous level (see above)</td>
<td>Strategies for comparing, ordering, sequencing, counting forwards and backwards in place-value parts, and renaming large whole numbers, common fractions, decimals, and integers (e.g., a 3 to 4 metre length of rope, appropriately labelled number cards and pegs could be used to sequence numbers from 100 to 1,000,000, from -3 to +3, from 2 to 5 and so on). The metaphor of a magnifying glass can be used to locate numbers involving hundredths or thousandths on a number line as a result of successive tonpling (see Siemon (2004) Partitioning – The Missing Link in building Fraction Knowledge and Confidence (see Papers and Presentations on the CD-ROM)). An appreciation of inverse and identity relations, eg, recognise which number when added leaves the original number unchanged (zero) and how inverses are determined in relation to this, eg, the inverse of 8 is -8 as -8 + 8 = 0 and 8 + -8 = 0. In a similar fashion, recognise that 1 is the corresponding number for multiplication, where the inverse of a number is defined as its reciprocal, eg, the inverse of 8 is (\frac{1}{8}).</td>
</tr>
<tr>
<td>Index notation for representing multiplication of repeated factors, eg, (5 \times 5 \times 5 \times 5 \times 5 = 5^6)</td>
<td>A more generalised understanding of place-value and the structure of the number system in terms of exponentiation, eg, (10^3, 10^4, 10^5, 10^6, 10^7, 10^8)…</td>
</tr>
<tr>
<td>Strategies to recognise and apply multiplication and division in a broader range of situations including ratio, proportion, and unfamiliar, multiple-step problems, eg, Orange Juice task (see Support Materials on the CD-ROM)</td>
<td>How to recognise and describe number patterns more formally, eg, triangular numbers, square numbers, growth patterns (eg, 'Garden Beds' from Maths 300 and 'Super Market Packer' from Support Materials on the CD-ROM)</td>
</tr>
<tr>
<td>Notation to support general arithmetic (simple algebra), eg, recognise and understand the meaning of expressions such as (x+4, 3x, 5x^2), or (\frac{x-1}{3})</td>
<td>Ratio as the comparison of any two quantities, eg, the comparison of the number of feral cats to the size of the national park. Recognise that ratios can be used to compare measures of the same type (eg, the number of feral cats compared to the number of feral dogs) and that within this, two types of comparison are possible, for instance, one can compare the parts to the parts (eg, cats to dogs) or the parts to the whole (eg, cats to the total number of cats and dogs). Ratios can be also used to compare measures of different types, ie, generally described as a rate (eg, the number of feral cats per square kilometre). Ratios are not always rational numbers (eg, the ratio of the circumference of a circle to its diameter).</td>
</tr>
<tr>
<td>Strategies for recognising and representing proportion problems involving larger numbers and/or fractions (eg, problems involving scale such as map calculations, increasing/reducing ingredients in a recipe, and simple problems involving derived measures such as volume, density, speed, and chance)</td>
<td></td>
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### Zone 8: Reflective Knowing

Can use appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals (eg, *Adventure Camp b, Speedy Snail b*)

Can justify partitioning (eg, *Missing Numbers b*)

Can use and formally describe patterns in terms of general rules (eg, *Tables and Chairs, m and k*)

Beginning to work more systematically with complex, open-ended problems (eg, *School Fair b, Computer Game i*)

### Consolidate/establish:

Ideas and strategies introduced/developed in the previous level (see above)

### Introduce/develop:

#### A broader range of multiplicative situations

eg, problems involving the calculation of area or volume, derived measures and rates, variation, complex proportion, and multiple step problems involving large whole numbers, decimals and fractions, eg,

- Find the volume of a cylinder 4 cm in diameter and 9 cm long.
- Find the surface area of a compound shape
- Foreign currency calculations
- Determine the amount of water lost to evaporation from the Hume Weir during the summer.

#### Strategies for simplifying expressions

eg, adding and subtracting like terms, and justifying and explaining the use of cancellation techniques for division through the use of common factors, eg

\[
\frac{42a}{7} = 6a \quad \text{because} \quad \frac{42a}{7} = \frac{7 \times 6a}{7} \quad \text{and} \quad 7 = 1
\]

Algebraic reasoning and representing multiplicative relationships, eg,

- If 2 T-shirts and 2 drinks cost $44 and 1 T-shirt and 3 drinks cost $30, what is the price of each?
- 5 locker keys are returned at random to the students who own them. What is the probability that each student will receive the key that opens their locker?
- A mad scientist has a collection of beetles and spiders. The sensor in the floor of the enclosure indicated that there were 174 legs and the infra-red image indicated that there were 26 bodies altogether. How many were beetles and how many were spiders?
- 365 is an extraordinary number. It is the sum of 3 consecutive square numbers and also the sum of the next 2 consecutive square numbers. Find the numbers referred to.

#### Strategies for working with numbers and operations expressed in exponent form, eg, why \(2^1 \times 2^2 = 2^3\)

More abstract problem solving situations requiring an appreciation of problem solving as a process, the value of recognising problem type, and the development of a greater range of strategies and representations (eg, tables, symbolic expressions, rule generation and testing) including the manipulation of symbols
APPENDIX D

INTERVENTION PROGRAM & POST-SESSION REFLECTIONS
WEEK ONE

SESSION 1: Wednesday 25th May, 2005

Focus

Supporting efficient mental images of number (beyond a collection of ‘ones’)
Efficient computation and counting strategies
Evaluation of place-value understanding
Building personal confidence and self-esteem as a mathematical thinker
Developing mathematical language through opportunity for oral discussion and reflective writing

INTRODUCTION ~ Images of number

Resources

Subitising cards (envelopes with round stickers placed in various configurations, from simple to more complex)

Purpose

To explore the ways in which students view number in small collections
Challenge the current strategy of thinking of numbers in terms of ‘ones’

Learning Notes

Teacher works with the students sitting in a circle on the floor. The subitising cards are turned over relatively slowly one by one in front of the students. A whole group discussion takes place. As each card is turned over, the teacher asks, “What do you see?” - “Five.” - “How do you know?” - “I can see a group of three and two.”
LEARNING FOCUS ~ Place value understanding

Resources
MAB materials

Purpose
To check students’ place value understanding, in particular, irregular numbers such as 17 (the 7 is heard first) and numbers with internal zeros such as 107

Learning Notes
‘Make, Name, Record’ modeled by teacher using the MAB materials and the whiteboard to model recording.

Examples given to students orally, to ‘Make, Name, Record’ using the MAB materials. Teacher asks for example, “Make me, 1 ten and 7 ones.”

Discussion initiated by teacher regarding “10 of these is 1 of these”, that is, 10 ones is 1 ten.

CONCLUSION ~ ‘Place value path’ game

Resources
Place value path (laminated playing board) for each student
2 x ten-sided dice per pair of students

Purpose
To check students’ place-value understanding, for numbers 1 to 100 in relation to order and sequence

Learning Notes
Students work with a partner to play the game. Teacher evaluates student thinking in relation to placement of numbers within the path.

Each player has a game sheet and takes it in turn to throw 2 ten-sided dice. The numbers are used to create 2-digit numbers, eg, a 5 and 2 could be recorded as 52 or 25.

Players record their numbers in the most appropriate position between 0 and 100.

If numbers cannot be placed, the player misses a turn.

The winner is the first to fill all places.
SCHOOL A: Lina tells me what other students in the grade are working on, addition and subtraction of decimals using money. I shared place value path game with MD and suggested that the game would be suitable if path is from 0 to 1, using decimals (hundredths and tenths).

Thought subitising cards would appear too basic for upper primary children. Students responded well to discussion and worked enthusiastically.

Open Morning at SCHOOL A so half the grade out of the grade supporting school tours. I worked with the remaining 11 students. This meant less active modelling of 2, 3, and 4 digit numbers by AR students. This was still a good introduction for these students as they commenced working with me, and the other students in the grade can view me as part of their classroom.

SCHOOL B: Ahmed absent – cut on his eye at recess. Intervention program likely to coincide when TP is out of the room as AP (5 of the 6 AR students in TP’s grade). Therefore, likely to impact on degree of teacher/researcher collaboration with teaching and resultant professional growth.

James perhaps better equipped re numeracy in cp to other 5 students. Demonstrates some efficient mental strategies and part-part-whole understanding. Not as reliant on counting ones when ppw models are more complex.

All students enthusiastic. Capable of make, name, record using MAB to thousands therefore consolidate then move onto larger numbers. Think a generic checklist with student names/columns/comments will be a useful record keeping tool

Will be appropriate to follow language and modelling of today with more structured opportunity for recording tomorrow.

SESSION 2: Thursday 26th May, 2005

INTRODUCTION ~ Part-part-whole understanding

Resources

Subitising cards

Part-part-whole cards (tens frame model)
**Purpose**

To model efficient ways of thinking about numbers, eg, knowing all there is to know, eg, 7: 3 and 4, 6 and 1, 5 and 2, 3 less than 10...

To move children from thinking about numbers as a collection of ‘ones’

**Learning Notes**

Teacher works with the students sitting in a circle on the floor, reviewing the subitising cards. Flash through the cards at an appropriate speed and ask students to say aloud how many stickers they see.

The part-part-whole cards are introduced, turned over slowly one by one in front of the students. A whole group discussion takes place. As each card is turned over, the teacher asks, “What do you see?” - “Five.” - “What do you notice?” - “I can see a four and one more.”… “Five less than ten.”

Students select an appropriate number under 10. Students record in their workbooks everything they know about their chosen number, eg, 6:

- 6 six
- It's even
- 3 plus 3
- 8 take away 2
- 2 plus 4
- 6 is double 3
- 10 take 4

**LEARNING FOCUS ~ Counting large collections**

**Resources**

Many counters or tiles

**Purpose**

To observe and explore the strategies students use to count items in large collections

**Learning Notes**

Students sit in a circle and teacher tips the counters/tiles out in front of them. Teacher asks, “How many counters do you think we have here?” Discuss and record estimates (black/whiteboard or chart). Students record their estimates in their workbooks.

Focus is now on, how can the amount of tiles be determined: “How can we work out how many counters we have here?” Discuss with students possible strategies and select a strategy where each student gets a pile of counters to count.

Teacher observes what the students do…count by ones? lose track of the count? keep track by recording progressive totals? place counters in piles? If the students are using inefficient strategies, ask, “Can you think of a better way to count these?”

When count is complete, share totals and record on a chart, near the initial estimates already recorded. Students estimate again, as now they have a sense of the numbers they are dealing with.

Teacher asks, “Ok, have we finished? Do we know how many we have?” Discuss appropriate strategies for finding the total and implement as appropriate.
Discuss observations of their first and second estimates in light of the grand total.

**CONCLUSION ~ Journal reflection**

Students reflect on their learning by writing about ‘today’s’ experience. Sentence starter provided if necessary: “In maths today we…”, eg,

> *Today in maths we did estimating and numbers counting. We tried counting without using our fingers…*

Teacher responds to communication by writing back to students for them to review during the next session.

SCHOOL B: Chicken scramble became more of a mini investigation. Estimate. Group solution to finding total. Writing up results. Noticed Dean asking “What’s after 190? 199? Berrin unable to count by 10’s therefore reinforce this with her individually. Dean of own accord, kept track of progressive totals, by writing these down. I modelled piles of ten for Berrin.

Students’ estimates generally way out! Adir closest with 1000, other estimations ranged between 100 – 700. There were in fact approx 1500 counters. I modelled addition to find total on the whiteboard.

We looked at part-part-whole (ppw) representation on the tens frames and I modelled all we could say about 7. Each student was given an envelope to prepare their own ppw list. Noticed Berrin didn’t have an intuitive sense of doubles and seemed not to sense the relationship to subtraction, eg, Berrin says “4 take away four, four.”

SCHOOL A: Hadi creates piles of 5 (to keep track of count) cf Yousif and Douha, who both count by ones.

Douha becomes quite anxious when she realises her estimate is way out. I give reassurance. Students given 2nd opportunity to second guess their estimates after list of individual piles are given. Hadi and Yousif able to suggest that if you add subtotals this will give you the total.

I noticed that MD had reworked place value path game to support students in her class. Some students had 0 to 1, some had 0 to 2. Evidence of meeting individual needs and willingness to try something new.

Note: students at both schools even after two sessions appear to be quicker in their immediate recognition of numbers to 6 without counting.

**WEEK TWO**

**SESSION 3: Tuesday 31st May, 2005**

**Focus:**

Move from analogous models of number (●●●)to working with numeral symbols (３)

Build confidence in working with numbers mentally

Introduce the open-number line as a visible thinking tool

**INTRODUCTION ~ Mental strategies for addition**

**Resources**

Jumbo playing cards (picture and joker cards removed, ace represents one, 1)

**Purpose**

To build confidence in students’ ability to work with numbers mentally (in relation to addition and subtraction)

Apply part-part-whole understanding


**Learning Notes**

Teacher and students sit in a circle. Teacher explains, “We are going to add 2 to the number shown on the playing card.” Teacher turns over card and models, “Eight and 2 more, ten.” Do orally as a group, repeat individually, moving round the circle of students, teacher included. Teacher observes what students do, do they take their time? Do they appear to say the first thing that comes to mind? Do they use their fingers? Reinforce that accuracy and confidence is the key, not speed, “Say your answer when you are sure.” As student confidence increases, ask students to add 3, take 2, take 3 to the shown card. Although speed is not the focus, the rate at which cards are shown can increase covertly as necessary.

**LEARNING FOCUS ~ Making thinking visible**

**Resources**

Rope
Clothes pegs
Number cards (0-100)

*This week’s football scores* teacher designed worksheet

**Purpose**

Engage in problem solving
Scaffold the open number line through the rope/peg activity
Provide students with a visible thinking tool (open number line) to support mental calculations

**Learning Notes**

Teacher selects five or so number cards, eg, 50, 77, 13, 95, 31. Students peg these appropriately on the rope (0 and 100 placed at either end to the rope) modelling a number line. Discuss and elicit student sense of sequence, eg, 50, “Why did you place the 50 card where you did?” and “What were you thinking about when deciding where the 95 was going to go?”

Teacher models use of open-number line to solve, if Hawthorn scored 70 and Fremantle scored 124, “How much did Freo win by?”

Students refer to *This week’s football scores*… sheet. Discuss highest and lowest scores and solve, Who had the best win? using the open number line as a visible thinking tool.
**CONCLUSION ~ Number Puzzle: circle problem 1**

**Resources**

*Circle problem* 1 sheet for each student  
6 tiles/counters each numbered 1 to 6

**Purpose**

Problem solving and applying mental strategies focused on so far

**Learning Notes**

The task is to place the numbers 1 to 6 in each circle so that the sum of each row is the same. Tile/counters available for students to manipulate, removing the threat of having to commit to writing possible solutions down. Tiles can be manipulated and moved around on the circle problem sheet.

SCHOOL B: Ahmed able to jump ten on number line, didn't think to work 1 back. James ab. This will be an issue throughout. Dean takes time to understand mathematical situations, game with ‘2 more’, ‘1 more’, totals of sides the same for the circle problem. Dean could make to nearest ten, 95 to 100 and 10 and 8 more on the no. line. Berrin provide feedback to TP re Berrin for non intervention days. Difficulty with add 2 to playing cards, count by tens and for 15, add 8, counted by ones. Cansu, on no. line could make one jump of ten only. Adir emerging efficient strategy. Can articulate “win by…”, can make to nearest 10 from 76 to 80, 4, 80 to 89, 9, so 13 point win.

SCHOOL A: Yousif appeared to work independently to complete 4 no. lines. Count of tens and larger count, 4 tens evident, eg, 59 to 79, and back 2. Hadi can bridge 100 from 95 by 10 to 105 and 10 more, 115. Douha can bridge 100 from 95 by 10 to 105, found 10 more, 115, a little less intuitive.
In general, these three students caught onto open number lines more quickly than those at SCHOOL B. Will endeavour to implement ‘car prices’ activities while SCHOOL B student finish no. line work tomorrow.

Conducted 1-99 rope sequencing activity with the whole grade. Most students attended to the 0 or 100 end, and were able to approximate 48 as being close to 50 so half way. Generally unable to partition a third of the way for 33, or 2 thirds of the way for 67 etc. tended to want to estimate jumps of one!

SESSION 4: Wednesday, 1st June, 2005

INTRODUCTION ~ Mental strategies for addition, subtraction. Numbers in sequence

Resources
Jumbo playing cards (picture and joker cards removed, ace represents one, 1)
Rope and pegs
Number cards 0-100

Purpose
Working mentally with small numbers
Model the number line with the rope and pegs and build partitioning skills in relation to sequence of number 0-100.

Explain thinking for an open-ended number line question

Learning Notes
Teacher and students sit in a circle. Teacher explains, “We are going to add 2 to the number shown on the playing card.” Teacher turns over card and models, “Eight and two more, ten.” Do orally with the whole group, repeat individually, moving round the circle of students. Repeat for ‘3 more’, then ‘1 less’.

Using 0-100 number cards, select 5 or so cards, eg, 48, 67, 12, 74, 89. Use the rope to model a number line, either tied to cupboard doors or have two students hold the rope taut at each end. Peg the 0 card and the 100 card to each end of the rope. Give each student a card and ask them to peg to the rope, where they think the ‘number would live on this number line’. Ask each student to explain why they placed the number where they did.

Draw up an open number line on the whiteboard, eg:

```
  4  3 tens  6
  56  60  90  96
```

Have the students consider the open number line suggested above. Ask, “If this is my thinking, what might the question have been?” Discuss possible ways of interpreting the number line.

LEARNING FOCUS ~ Footy scores (Moreland PS) Car prices (Brunswick N PS)

Resources
This week’s football scores worksheet (SCHOOL B) see previous session
Car prices teacher designed worksheet (SCHOOL A)
Purpose
(SCHOOL B) consolidate understanding of place value and part-part-whole through the open number line
(SCHOOL A) evaluate extent of understanding of 5 digit place value.

Learning Notes
At SCHOOL B allow students to use the football scores data to work out for each game, who won and by how much. Encourage use of open number line to show efficient thinking.
At SCHOOL A students work in pairs or small groups to discuss which are the cheapest and most expensive vehicles. Look at the 8 cars advertised on the left hand side of the page, order from cheapest to most expensive.

CONCLUSION ~ Number Puzzle: circle problem 1 continued
Time is given at this point, for students to solve the puzzle. To assist students with this, provide some hints as to the placement of two of the six numbers. Record in their workbooks when correct solution determined.
SCHOOL A: bring Tallangata L5 LP for MD.
All students quicker with mental strategies of add 1, 2, and 3 and take 1. Douha tendency to count by ones, evident when explaining her solution strategies. Whole class rope sequencing activity with no.s 0-50. Car prices activity, students able to determine the cheapest/most expensive and order. Students able to solve circle problem after clue of total of sides. Very pleased with themselves.
SCHOOL B: Rope tied to cupboard doors and cards 0 to 50 spread on the floor for students to select from and sequence. Worked with Berrin to match names and symbols for multiples of ten, sequence and count orally, asked what no. comes after, played concentration.
Cansu and Adir reported doing circle prob. At home! All students successful after being given the clue, ‘all sides 12’. Cansu and Adir able to make sides equal, 10, 11 and 12. Berrin, 12, Ahmed and Dean , 11 and 12. During intro game, students supportive of Berrin taking her time. Initially guessed, 2 and 3 more, 8! Modelled counting on strategy and as a group we did this orally with her. After a couple of goes she had a turn on her own. When successful a quiet “yes!” from Adir who was pleased with Berrin’s success. Adir has been vocal and enthusiastic from the start. Had a tendency to ‘take over’. Today was more supportive by not butting in and giving others time to think for themselves.

SESSION 5: Thursday 2nd June, 2005
Focus
Efficient mental strategies for addition and subtraction
Towards a deeper understanding the place value system, ‘10 of these is 1 of these…’

INTRODUCTION ~ Mental strategies for addition, subtraction. Numbers in sequence

Resources
Jumbo playing cards, ace to 10
0-100 number cards

Purpose
Build on mental addition and subtraction strategies for numbers 1-10
Order numbers largest to smallest, smallest to largest for numbers under 100
Learning Notes
Teacher and students sit in a circle. Teacher explains, “We are going to add 2 to the number shown on the playing card.” Teacher turns over card and models, “Eight and two more, ten.” Do orally as a group, repeat individually, moving round the circle of students. Repeat for ‘3 more’, then ‘1 less’, ‘2 less’.

Hand out 6 to 8 number cards (showing a range of numbers under 100). Have students work together to order these from smallest to biggest, scramble and order from biggest to smallest. Encourage students to talk about their thinking as they complete the activity.

LEARNING FOCUS ~ Place value trading activity

Resources
Laminated place value chart showing ‘Thousands, Hundreds, Tens, Ones’, - one chart for each student

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Lots of MAB material
1 six sided dice* per pair of students
*can later be extended to dice with more than 6 sides

Purpose
To directly experience the structure of our base 10 number system
Understand the idea that ‘10 of these, is one of those’, eg, 10 ones, is 1 ten

Learning Notes
Students engage in trading activity with a partner, and take turns to throw the dice. If for example a 7 is the first throw, 7 ones are taken from the collection of MAB and placed in the ‘ones’ column. Repeat this process. As more than 9 ones is reached, students trade for a ‘ten’; eg, 7 ones on the game board, a 5 is thrown, making 12 ones, trade 10 ones, so that 1 ten and 2 ones are now visible on the board. Trade for tens, hundreds as the collection grows. Teacher focuses the activity with key questions, “How many do you have now?”, “What number have you created?”, “Tell me about your number?”, “Can you say your number in another way?”

CONCLUSION ~ Reflective journal writing

Resources
Student work books

Purpose
End of week communication between teacher and students, to find level of student enjoyment and their perceptions of their learning.
Learning Notes

Pose the following sentence starters:

*The best thing about maths this week was… I learnt…*

SCHOOL B: When I arrived at school today, TP tells me that my work with Berrin is featuring in her class reflective learning journal. She writes about learning to add 1, 2 to numbers and her ppw understanding of the number 4. She also writes, ‘we love Margarita’.

When Playing add 2, 3 take 2, Adir is enthusiastic to try adding 5! Car prices extended activity too difficult, eg, 48 900 add 410 to price. Moved onto place value trading game straight away. Revisit concept later. Trading game allowed students to see 10 of this is 1 of these.

SCHOOL A: Modified session at SCHOOL A more successful, left out ‘car prices’ entirely to explore place value trading game fully. All students able to order 8 random 2 digit no.s from smallest to largest. Students able to name numbers modelled throughout the trading game. Changed 6 sided dice to 10 sided dice to allow student to build collection a little quicker.

Yousif able to add potential ones, eg, 8 to 6 ones, left 4 and picked up a ten. Encouraged Hadi and Douha to not count their ones by ones. Hadi evidence of seeing 5 without counting 5 and Douha counting by 2’s.

WEEK THREE

SESSION 6 & 7: Tuesday 7th June & Wednesday 8th June, 2005

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<th>Focus</th>
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<tbody>
<tr>
<td>‘Make to 10’ mental strategies for addition</td>
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<tr>
<td>Consolidate part-part-whole understanding</td>
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<tr>
<td>Number patterns on a 0-99 number chart</td>
</tr>
<tr>
<td>Strategies for problem solving</td>
</tr>
</tbody>
</table>

INTRODUCTION ~ ‘Make to ten’ strategy for addition

Resources

Tens frames (blank)
Counters
Jumbo playing cards, ace to 10
Part-part-whole envelopes

Purpose

Consolidation of efficient mental strategies, extending to ‘make to ten’

Explore all the different ways of thinking and knowing numbers

Learning Notes

Teacher shows a number under 10 on the tens frame, eg, 7, using counters. Ask students, “How many counters do we need to make 10?” “How do you know?” Discuss.

Move onto the same idea using the playing cards. Teacher with playing cards, and students sit in a circle. Teacher turns over a card, and models the make to ten strategy, eg, 8 card is shown, “8 and 2 needed to make to 10”. Work through the deck, with students responding as a group and then individually as confidence increases.

*All I know about… Students select a number under 10 and document in their workbooks, all they know about that number. Teacher can attend to the ways in which students write this, formally, informally, in words or symbol and the diversity of their response.*
LEARNING FOCUS ~ Number chart activities

Resources

Overhead Projector

Overhead transparency of 0-99 Number chart

Portable 0-99 Number chart (Wed)

Laminated 0-99 number charts, one per student

Supply of blutak

Regular and irregular sized overlays for both individual and portable charts. The overlays hide numbers from view.

Place value charts and MAB (Wed.)

Purpose

To explore the number patterns, the count of ten, from zero and other starting points

Learning Notes

(Tuesday) The 0-99 Number chart is placed on the OHP. Discuss, “What patterns can you see?” Work with students orally with the count of 10, starting at 10. Move to other starting points, eg, 5, 15, 25…so on. Anticipate possible misconception, that this is a count of 5, not a count of 10. Some students may have difficulty with this idea. It is not unusual that students at this level, interpret this sort of count as a count of 5 (determined by the repeated 5 at the end of each number, vertically down from 5).

(Tuesday) Allow students to explore these ideas through individual Number chart activities. Each student has a laminated chart and some small regular and irregular overlays. Blutak overlay to the chart. Ask “What numbers are hidden? How do you know?” record in workbooks. Swap overlays among the students as time permits. Note: Regular overlays are easier than irregular overlays.

(Wednesday) Small group oral discussion using the portable 0-99 Number chart and appropriate overlays. Follow up count of ten misconception from Tuesday. Model count of ten using place value charts and MAB: show 2, with 2 ones in the ones column. Add 1 ten to the tens, ask “What number have we made now?” (12), add another ten to the tens, “What number have we made now?” (22), add another ten to the tens…and so on. Repeat with different starting numbers.

CONCLUSION ~ Number Puzzle: Circle problem 2

Resources

Circle problem 2 sheet for each student

6 tiles/counters numbered 0, 2, 4, 6, 8, and 10
Purpose
Problem solving and using mental strategies
Build on the successes students felt for solving circle problem 1

Learning Notes
The task is to place the 6 numbers, one number/tile in each circle so that the sum of each row is the same (Solutions possible: 12, 14, 16, 18). Tile/counters available for students to manipulate, removing the threat of having to commit to writing down possible solutions. Tiles can be manipulated and moved around on the circle problem sheet. Hints can be given to scaffold as necessary.

7th June, SCHOOL B: Adir absent. All students except James needed visual support of tens frame to make turned over card to 10, eg 8, see tens frame, see 2 missing, make to ten, 2. Dean and Ahmed less reliant on this, however the time they took and nods of the head consistent with a count by ones. Berrin despite model had no idea. Used tens frame model and counters with her throughout.

All students unfamiliar with 0-99 chart and the OHP (Under-utilised resource in the school?) Could count by tens starting at 0, more difficult at various starting points.

Individual work with green shapes over parts of charts worked well. I chose shapes (degree of difficulty) in line with student ability. They then selected other places on the chart to place the shapes and recorded these in their workbooks. They swapped shapes around. With James I extended his shape off the chart (beyond 100) and he was successful.

Students having seen circle problem before were well equipped to make a start despite the change in numbers. Students have requested that we play place value path again soon!

7th June, SCHOOL A: Students caught on quickly to make to 10. When looking at 0-99 chart students were able to observe count by ones -> but not convinced of count by ten down the chart, they tell me, 34, 44, 54 count by 4. I ask are you sure? Convince me! Thought it necessary to model count by 10 using MAB and p value chart, 7, show 7 ones, place a ten in tens place, 17 and so on.

8th June, SCHOOL A: Played place value game with whole grade. Noticed Hadi inaccurate when adding to find total, esp. at hundreds to thousands. Make to ten activity with playing cards, alerted me to trying 0-99 cards and make to next ten, 25, 5, 30, also 0-20 cards make back to ten.

Yousif queried me, “Will we be getting harder stuff?” growing confidence and comfort despite some issues re understanding count of tens down the 0-99 chart. Douha needed clues to support total of 12 for circle prob. Groups discovered that totals, 12, 14, 16, 18 possible. Move group onto 1, 3, 5, 7, 9, and 11 for circle prob.

8th June, SCHOOL B: Make to ten too difficult for Berrin, but accessible to the others (esp. James then Adir, Dean, Ahmed) Cansu tends to rush. I supported Berrin with tens frames. Have these ready for her. Students enthusiastic to work on circle prob. But found this more difficult. I’ve prepared hints for SCHOOL B students and left hints off for SCHOOL A students.

SESSION 8: Thursday 9th June, 2005

INTRODUCTION ~ Circle problem discussion

Resources
Student workbooks
Tiles and problem sheet as appropriate
Our solutions sheet showing previous solutions

Purpose
Sharing through discussion of solutions strategies
Provides a springboard for another variation of the same problem

Learning Notes
Small group sharing of previous problem solutions strategies. Handout previous solutions sheet. This is to support students’ solution to the use of the numbers, 1, 3, 5, 7, 9 and 11 using the same circle problem arrangement. Record in student workbooks as appropriate.

LEARNING FOCUS ~ Place value – make the largest number

Resources
10 sided dice

Make the largest number chart (on blackboard or whiteboard), see example below:
Place value path game board, one per student

**Purpose**

Use knowledge of place value to create the largest/smallest number in the context of a game

Problem solving strategies based on place value understanding

**Learning Notes**

(SCHOOL A) Played *Make the largest number* with all students in MD’s grade. Children draw up the chart in their maths book. Teacher models play on the black/whiteboard. The aim is to arrive at the greatest total and if, for example, a 9 is thrown, discuss “Where is the most sensible place to put it?” and “Why?” Teacher throws a 10 sided dice 10 times, and students place these numbers in the top 10 places of the chart. Therefore, a range of numbers are created. These numbers are added to find grand total. When ready, teacher asks “Who thinks they have created the greatest number?” Follow up with, “Who has a number that is bigger/greater than that?” Continue as necessary until the largest number is found. Repeat game on a new chart where the aim is for the smallest number to be created.

Play Place value path game. Each pair of students has a game board and 2 ten sided dice. Take turns to throw the two dice. If a 4 and 6 is thrown, the number 46 or 64 can be placed in the path. Continue until all places in the path are exhausted. Note: the same number can not be placed, twice. If a space is not available, eg, 46 to be placed, and 42 and 48 are next to one another, then 46 can not be written, therefore miss a turn.

CLOSE ~ Reflective journal writing

**Resources**

Student workbooks

**Purpose**

End of week communication between teacher and students

Communication of student enjoyment and perceptions of their learning

**Learning Notes**

Draw a face to show how you feel:

I feel……………..because…..
Ask students to write about something new they have learnt.

SCHOOL B: Calmer more successful. All students and Berrin with a little more support able to solve new circle prob. Students feeling good about their achievements. They enjoyed the p value path game. Berrin needs work on consolidating place value and sense of order and sequence. Other students making sensible decisions and using appropriate strategies. Dean and Berrin unable to add to find total for the other place value game. Totally engaged and willing to solve prob. Task boxes. Evidence of developing positive attitude reflected in journal writing. Ahmed writes though, “angry cos it's too hard” but then states that maths is his favourite subject. Have a chat to him about this! Inserted after I spoke to Ahmed: At close of next session I asked Ahmed about his journal writing. I ask why? He tells me he is angry because he can't do it then tells me he is referring to his work in the classroom, and gives the example of when doing a test, and you can't ask for help. I ask if he is angry when we work together? He says no! Re maths as favourite subject, he tells me of the importance of maths in daily life, eg, shopping]

SCHOOL A: All students choose to try the circle prob. Without the hint. All students successful in the given time, so quicker than the first time they were given a problem of this type. Encouraged student to find other solutions, and they were happy to explore this idea. Yousif teaches place value path to another student. Douha making more place value appropriate choices for her placement of numbers, talking her ideas through, I can place x here or if I get x. re truth tiles, at least four solutions requested, Yousif and his partner want to find at least 8!

WEEK FOUR
SESSION 9: Tuesday 14th June, 2005

Focus
Review of mental strategies to date, with 0-20 cards
Place value: “Make, name record” four digit numbers
Introduce a ‘thinking string’
Problem solving strategies (thinking and recording)

INTRODUCTION ~ Review of efficient mental strategies

Resources
0-20 Number cards
Whiteboard and whiteboard markers

Purpose
To review mental strategies focused on so far (ie, part-part-whole, make to 10)
Apply strategies to numbers greater than 10 (to include make to 10)

Learning Notes
Use the number cards to work with the students in a small group. Turn the cards over and have students respond according to: 2 more, 3 more, 2 less, 3 less, make to ten, or make to 20 as appropriate.
Present a thinking string (on whiteboard/blackboard) similar to the one below:
Discuss with the students, “If this is my thinking, what might the question have been?” “What does my thinking show?”

**LEARNING FOCUS ~ Problem solving task boxes**

* *Mathematics Task Centre Project*

**Resources**

‘Truth Tiles’

‘Farmyard Friends’

‘Nim’

‘Cookie Count’

**Purpose**

To engage in non-pen/pencil problem solving tasks

**Learning Notes**

Students work with a partner to solve the tasks. Teacher has opportunity to observe and scaffold problem solving strategies attempted and used by the students, as well as the ways in which students work collaboratively together.

**CONCLUSION ~ Make, name, record**

**Resources**

‘Make me’ cards 2, 3 and 4 digit numbers, eg,

<table>
<thead>
<tr>
<th>Make me…</th>
<th>Make me…</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 hundreds</td>
<td>1 ten</td>
</tr>
<tr>
<td>9 tens</td>
<td>9 ones</td>
</tr>
</tbody>
</table>

MAB material

**Purpose**

To check students’ understanding of *teen* numbers, and numbers with internal zeros (to thousands)

**Learning Notes**

Students work with a partner to *make* the numbers with the MAB according to the ‘Make me’ cards. Encourage students to *name* by saying aloud, these numbers and *record* them in the workbooks. Watch for students who write 1 ten and 9 ones as 91; and numbers with internal zeros, eg, 3085, as 385.

* Students at SCHOOL B worked with 2 digit numbers and students at SCHOOL A were able to extend this work to 4 digit numbers

Student absences meant I was able to spend time with Berrin and Ahmed on ‘make me’ cards, both a little unsure of make me 1 ten 9 ones. Berrin initially says 91, Ahmed then said 19. Spent time with other teen no’s. Berrin during cookie count unsure of 1 after 29. Used 0-99 chart as reference. Berrin and Ahmed decided that picture had 42 cookies and when it came to sharing Ahmed mentally calculated 22 each (close but couldn’t explain how). I: Ok let’s see if Ahmed is right. How will we check?

B: Share them out.

A and B do this by ones and kept losing track of shares.
I: Can you think of a more efficient way?
B: By twos.

Which they did. They then counted their own share. Berrin with 20 and Ahmed with 22.
I: What’s the problem.
A: They’re not the same.
B: (asking Ahmed) Give me one.

Yes. Berrin beginning to keep track of numbers mentally, not necessarily reliant on recounting.

SCHOOL A: All students worked well on problem solving tasks. Hadi and Douha took a while to sort out their 48 cookies to share. Hadi mentally halved 40 and suggested 20 each then halve the 8 left. Next question requires sharing among 4. Douha and Hadi recognise the need to halve their share of 24. Hadi able to mentally calculate half of 20, 10, half of 4, 2, so 12 each. They’ve come a long way!

Yousif aware that once farm yard problem solved, not necessarily all constraints fulfilled. When playing Nim, Yousif quickly switched onto strategy that would enable him to win.

All students able to make, name, record 4 digit no’s. Need to check Douha and Hadi re 3085, internal zeros. Able to rename 2 and 3 digit no’s with MAB orally and then renaming on whiteboard. Yousif able to add 10, 100 and 1000 to regular examples with trading/renaming place value part, eg, 3462, 4372, 3562, 4462.

SESSION 10: Wednesday 15th June, 2005

INTRODUCTION ~ Think on the thinking string

Present a different a thinking string from the one used in the previous session. Ask, “If this is my thinking, what might the question have been?”

Think…33, 43, 45

Have students create their own thinking string for ‘57 add 13 more’. This will allow the student opportunity to justify their thinking and make their thinking visible to the teacher.

LEARNING FOCUS ~ Problem solving task boxes continued

Additional resource

Number chart activity 1
Learning Notes

Continue as stated for session 9, with solving the four task problems. When finished, students complete *Number chart activity 1*. To complete this number chart the students need to fill the open cells with numbers in sequence, by ones. This chart begins with 1 and ends with 100. For the first line, students will need to fill in “2, 4, 5, 6, 7, 8, 9, 10”. The number 3 does not need to be written as this cell is black. Initially, students will complete the grid horizontally counting by ones. Teacher encourages students to work the chart vertically, so that students use the more efficient count of ‘ten’, from various starting points. As confidence increases, students begin to self-correct when irregular patterns emerge from their responses.

**CONCLUSION ~ What might the headings be?**

**Resources**

Subitising envelopes  
Part-part-whole cards  
Number cards

**Purpose**

Discussion and justification of reasoning, through “Convince me…” of what might the headings for the table be?

**Learning Notes**

Set up the following situation on the whiteboard (The idea is for students to distinguish between ‘odd’ and ‘even’):

| ? | ? |

One by one add some data to the columns in the form of the resources listed above. (Place even number representation on one side and odd representations of number on the other). Ask students if they can work out what might be appropriate headings for the two columns. When they offer a suggestion, ask, “I’m not sure, convince me!…”

SCHOOL A: thinking strings still a bit of a challenge, give a few examples for student to do, continue this tomorrow. No. chart 1 may scaffold add 10. Checked again 4 digit place value make, name, record. Students able to compare and make clear observations about which 4 digit no. larger and why, > introduced.

SCHOOL B: Make to 10 greater ease and becoming semi-automatic. Checked (with success) make, name, record 3 digit nos with internal zeros.

**SESSION 11: Thursday 16th June, 2005**

**INTRODUCTION ~ An open ended question…**

**Resources**


Laminated 0-99 number chart  
Whiteboard markers
Purpose
Give students opportunity to give more than one answer when problem solving

Learning Notes
Present the question, “If this shape covers the number 43, what might the other numbers be?”

Provide the student with the laminated number charts and whiteboard markers to assist in the solutions. Record their thinking and responses in their workbooks.

LEARNING FOCUS ~ Problem solving - A ‘leggy’ investigation

Resources

Purpose
Opportunity for systematic thinking and recording when solving an investigative problem

Learning Notes
Pose the problem:

“I have some pets at my house. I see 16 legs go past me. How many pets might I have?”

Encourage students to think beyond one animal type, eg, dogs, dogs have 4 legs, so 4 dogs or birds, birds have 2 legs, so 8 birds. Look for students who will think of more than one type of animal at home and record accordingly.

Both sites: Introduction activity a little confusing to student initially, with students saying, “I don’t get it”. Once they had one solution the other solutions followed. Students enthused to find as many as possible, Noticed students saying numbers aloud, 33, 43, 53, 52 etc. Not yet evidence of systematic thinking and/or recording. This evident also in leggy problem. Students tend to rely on sticking with single animal types eg, 4 dogs, and 8 birds, Hadi, James, Ahmed and Adir beginning to explore combinations.

WEEK FIVE
SESSION 12 & 13: Tuesday 21st June and Wednesday 22nd June, 2005
INTRODUCTION ~ Doubles strategy

Resources
Jumbo playing cards (ace to 10)
‘2, 4, 6’ game board
counters
3, 6 sided dice per pair of students

Purpose
Introduce ‘doubles’ strategy
Practice efficient mental computation strategies in a game situation
Learning Notes

Work with students in a small group and introduce the idea of doubling. Turn over a playing card, e.g., 4, say “4, double 4, 8”. Have students respond as a group first as cards are displayed, then move around the circle with students responding individually.

Introduce and play 2, 4, 6 game.

You need:
- 3 dice
- Playing board
- Set of tiles/counters in two colours
- Partner

Throw 3 regular 6-sided dice (or ten-sided when more confident)
Use the three numbers to create a sum mentally using +, -, X, ÷
E.g., 3, 5, 6 could be 3 + 5 is 8, 8 – 6 is 2… or 5 x 3 is 15, 15 + 6 is 21
Place a tile on the “2” on the playing board.

The aim is to have three of your tiles in a row

LEARNING FOCUS ~ Review of numeration and strategies

Resources
3 share’n’fit puzzles (add, take 2, 3; doubles, near doubles) see Learning Notes for example
Number patterns activity card
0-99 chart available
Formal recording (addition and subtraction) activity card
MAB and place value charts available
Open number line activity card

Purpose
Review key ideas developed to date
Check understanding of formal recording, trading/regrouping

Learning Notes

Rotate the activities around the group of students. Keep anecdotal notes as to their responses, triumphs, difficulties.

The Number patterns activity card (see below), asks students to explain the rule for a variety of pattern situations involving 2 and 3 digit numbers.

* 50, 60, 70, __, __,…
* 6, 16, 26, __, __ 56, __,…
* 11, 21, __, 41, __, __, 71…
* 143, 153, 163, __, __, __,…
<table>
<thead>
<tr>
<th>95, 85, __, __, 45, __...</th>
</tr>
</thead>
<tbody>
<tr>
<td>98, 87, 76, __, __, ...</td>
</tr>
</tbody>
</table>

The *Formal recording* activity card (see below) deals with addition and subtraction involving 2 digit numbers and includes an open question.

<table>
<thead>
<tr>
<th>* 5 6 * 7 3 * 1 3 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 2 3 8 4 4 7 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>* 9 7 * 8 3 * 7 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 2 5 - 4 1 - 5 9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>* 3 ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 1 ?</td>
</tr>
<tr>
<td>? 2</td>
</tr>
</tbody>
</table>

What might the missing numbers be?

The *Open number line* activity card (see below) asks students to ‘show’ their ‘thinking’ for addition and subtraction situations, eg, 47 and 24; 56 take 37.

Use open number lines to show your thinking...

*33, add 14
* 47, add 24
* 61, add 38
* 27, add 71
* 79, take 23
* 56, take 37

The ‘share’n’fit’ puzzle can be reconstructed either in pairs or individually. The nine cards are cut, for students to reassemble, see example below:
CONCLUSION ~ Number puzzle 1

Resources

Number puzzle 1 sheet
Counters (labelled 1-6)

Purpose

Problem solving
Application of computation strategies and logical thinking

Learning Notes

Students have access to 6 counters, labelled 1 to 6. These are placed in the circles or squares so that the numbers placed in the circles add up to the number in the square:

SCHOOL B: Dean and James absent. Ahmed away for part of session. Not convinced 6, 16, 26 is a count by 10. Berrin had some difficulty with open number lines, unable to relate to place value parts. Cansu a little confused that 27 and 13 was equivalent to 27 + 13. Knows 1 and 8, 9, but didn’t use this to solve 91 and 8, counted by ones. Adir able to rename for subtraction formal recording and very willing to do lots more. Confident in knowing the count of 10 for different starting points.
SCHOOL A: Yousif successful with number lines for addition, used moving forward notation for subtraction situations. Addition formal recording ok. taught him re grouping for subtraction with the MAB. Hadi successful with open number lines and addition and subtraction formal recording. Douha, similar to Yousif re open number lines, mixes strategies for + and -. Notices diagonal pattern of 11 on number chart.

SESSION 14: Thursday 23rd June, 2005
Last session for Term Two.
INTRODUCTION ~ share’n’fit and ‘2, 4, 6’
Play these with the students (See previous session for details).
LEARNING FOCUS ~ Create! Using digits 0 to 9!

Resources
Counters labelled with the digits 0-9

Purpose
Work with place value ideas

Learning Notes
Work with students in a small group with access to the labelled counters. Work through the following questions (on chart, whiteboard, blackboard) as a group and discuss notions and ideas as appropriate:

Use the digits 0 – 9 to make…
- Biggest 2 digit number
- Smallest 2 digit number
- Smallest odd number
- Biggest 3 digit number with 9 in the tens place
- Biggest even 4 digit number with 4 in the thousands place
- Biggest odd number with 7 in the thousands place
- Smallest even number, bigger than 30 000

CONCLUSION ~ Number chart activity 2 and journal writing

Resources
Number chart activity 2 one per students
10 sided dice

Purpose
To encourage children to work with the chart vertically, to reinforce the count of ten and potentially go beyond 100

Learning Notes
Students complete the Number chart, however the starting number is determined by a roll of the 10 sided dice. The starting number must not be 1. This will enable the chart to go beyond 99. Observe how students handle this.
Number Chart Activity 2

Open opportunity to compete journal writing content driven by student choice.

SCHOOL A: Spent one on one time with Douha with 2, 4, 6 game. Allowed opportunity for utilising known facts for 10, eg, 6 and 4, 10, 10-4, 6 and apply these to 20, 20-6, 14 etc. Noticed Douha was less reliant on using her fingers and her responses were becoming more automatic. She appears genuinely surprised when she is quick and right. Douha quickly solved the number puzzle. Reviewed subtraction with MAB model and language one on one with Douha while other students solving prob.

SCHOOL B: Will allow student to continue working with activity cards and allow absent student yesterday to catch up. Set Berrin up with make me cards, name, name, record 2 digit no’s. Used open number lines for difference. Dean absent again (missed Tuesday and Wednesday). Adir’s journal writing communicates enjoyment of all tasks. Berrin communicates challenge involved with share n fit, being good at no. chart 2 and ‘learned lots of stuff with you’. Cansu reiterates what we did during the session. Dean talks of fun and enjoyment.

Term Three 2005 commences

WEEK SIX

SESSION 15: Friday 15th July, 2005

Focus

Looking at arrays for multiplicative situations

INTRODUCTION ~ Sum and difference

Resources

1-10 number cards

Share’n’fit puzzle

Purpose

Revise addition and subtraction strategies from last term

Understand the language of sum and difference

Learning Notes

Using the 1-10 cards, teacher works with the students in a small group. Two cards are present at a time. Teacher models first the idea of sum then difference idea, eg, if the 7 and 3 card is presented, the sum is 10 and difference is 4.

Students work in pairs to reconstruct the ‘share’n’fit’ puzzles.
LEARNING FOCUS ~ Looking at arrays

Resources
Muffin tin
Counters/tiles
‘Star’ stickers

Purpose
To move from groups of idea for multiplication, to arrays (organisation of collection in terms of columns, rows, and the total)

Learning Notes
Have a discussion about the organisation of the muffin tin. “What do you notice?” Explore ideas of rows, columns and total. The muffin tin makes 12 muffins arranged as 3 fours. Pose the question: “How many different muffin tin designs can we create?” Students are given ‘star stickers’ to record their individual results in their workbook. Encourage students to explore all possibilities. Some responses might be:

* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *

CONCLUSION ~ Number chart activity 3

SCHOOL B: Students beginning to quickly work with ppw relationships when mentally calculating + and -, eg, 8 and 5, 8 make to 10 and 3 more, 13. Re arrays: student beginning to see and describe different arrangements. Today I allowed student to model ‘groups’ of as well as array idea. Students able to informally orally describe. Ahmed tried some uneven arrangements. No. chart 3 presented Dean with a challenge as he had trouble with one after 109, 110. Worked orally with him. Cansu confused, jumped to 200 and 300. Check this. Orally ok when I questioned her.

SCHOOL A: MD’s birthday surprise cut into time today. Played Greedy Pig with the whole class. Card activities, students building speed and ability to articulate ppw relationships. Arrays well understood.
WEEK SEVEN
SESSION 16: Monday 18th July, 2005
INTRODUCTION ~ Sum and difference

Resources
1-10 number cards
Cards labelled ‘tens’
‘Share’n’fit’ activity

Purpose
Work with addition and subtraction strategies for numbers under 100
Understand the language of sum and difference

Learning Notes
Using the 1-10 cards and ‘tens’ cards, teacher work with the students in a small group. Two number cards are present at a time and ‘tens’ cards placed next to the number cards, eg,

Teacher models first the idea of sum then difference idea, eg, if the 9 and 2 card are presented, the sum is 11 tens, one hundred and ten and difference is 7 tens, seventy.

Students work in pairs to reconstruct the ‘share’n’fit’ activities.

LEARNING FOCUS ~ Array trek!

Resources
n/a

Purpose
To explore the existence of arrays in the real world

Learning Notes
Walk around the school with the students and look for evidence of arrays in their school environment. Look out for the way in which art work is displayed, aspects of construction, eg, drainage grills, windows, paving and the like. Discuss. Children may naturally notice that the arrays change according to where they stand, that is a 3 fours arrangement can appear as 4 threes, depending on their perspective (where they stand). When back in the classroom, have students record 3 different examples from their walk. Encourage them to define ‘array’ in their own words and label columns, rows and total.

Adir absent. Talk to TP re Berrin going to Turkey for a lengthy time! Dean has requested what’s my number activity. Tomorrow’s session check methods, thinking and understanding of formal recording of addition and subtraction. Work individually with students.

SESSION 17: Thursday 21st July, 2005

Work with students individually to check their understanding of formal recording and thinking when providing written solution to addition and subtraction algorithms.

The students complete the sheet, as per below:
The teacher, completes the same sheet, noting the thinking, responses and strategies used by the students as they complete the tasks.

See notes from each individual interview.

SESSION 18: Friday 22nd July, 2005

INTRODUCTION ~ Blind noughts and crosses

Resources
Whiteboard/blackboard
Markers/chalk
2 players
2 recorders

Purpose
Variation of the traditional noughts and crosses game adding aspect of visual imagery

Learning Notes
A noughts and crosses game is presented with grid references:

```
3   
2   
1   
A   B   C
```

Select 4 students, 2 to record and 2 to play. The recorders stand on either side of the chart. The players stand in front of the chart, with their back to it so they cannot see the progress of play. Before play begins, ask students to study the chart momentarily to orient.
themselves to it. Discuss grid references if necessary. The players are allocated either noughts or crosses and the recorders, write these according to the players’ direction, eg place a cross in B3. As in the traditional game, the winner is the player who has 3 tokens in a row.

**LEARNING FOCUS ~ Arrays investigation**

**Resources**
- Collection of counters/tiles
- Large sheets of butcher’s paper
- ‘circle’/’dot’ stickers

**Purpose**
Engage students in the systematic recording of small collections in the form of arrays

Opportunity to discover the difference in variety of responses possible for odd, prime and even numbers

**Learning Notes**
Students to work with collections of 1 to 12, taken one at a time. For each focus number, first create all the arrays possible for that number with tiles/counters. Then record these using the stickers on the butcher’s paper. These will create a number of pages to be compiled into an ‘array’ book. As students work, discuss what they notice about the arrays in relation to the total collection, eg, the arrays for a collection of 7 will differ from a collection of 9 or 12. Discuss why.

**CONCLUSION ~ What’s my number?**

**Resources**
- 0-99 number chart laminated, one per student
- whiteboard markers

**Purpose**
Explore properties or number

Problem solving

**Learning Notes**
Each student has a chart and whiteboard marker. Sitting back to back, so that the partner’s chart is hidden, each student selects a number and circles it with the marker. Students take turns to ask questions about each other’s number. The answers to the questions should be answerable with a ‘yes’ or a ‘no’. Encourage useful questions such as, “is your number even?” “is your number larger than 50?” In comparison to questions like ‘is your number 47?’ have students keep track of the clues gained by marking their chart appropriately or making notes on the space available of the chart.

**WEEK EIGHT**

**SESSION 19: Monday 25th July, 2005**

**INTRODUCTION**
Repeat introduction from Session 16.
LEARNING FOCUS

Students are given time to finish array investigation from previous session.

SCHOOL B: Students still struggling with 10 tens, 100 and 2 more tens, 12 tens, 120. Cansu able to predict array will be an odd shape.

SCHOOL A: cancelled Gracie with me, but too sick to continue.

SESSION 20 & 21: Thursday 28th July and Friday 29th July, 2005

INTRODUCTION

Repeat Introduction from Session 16.

LEARNING FOCUS ~ Working with arrays powerpoint presentation

Resources

Computer/laptop

Counters

Working with arrays powerpoint presentation*

*Stimulus for ideas came from Bobis et al. (2004), p. 158-159.

Purpose

Reconstruct arrays from visual stimuli

Learning Notes

Open the Working with arrays powerpoint presentation. When the first array is shown, the teacher decides how long the children can view it. Initial discussion takes place as to the array’s multiplicative nature, 3 fives, fifteen. The next slide asks ‘make it from memory’. Time is then given to the students to reconstruct the array, now not visible, using counters. The third slide reinforces the first slide visually but now has an appropriate description. This is repeated 7 times with different arrays. An example of a set of 3 slides is presented below.

slide a

Make it from memory…

slide b
CONCLUSION ~ Reflective journal writing

Students complete a front cover of their ‘arrays’ book and include some information in their own words about arrays.

SCHOOL B: Police in schools 10-11 every second Thursday. Took Ahmed first and worked through introduction to the arrays powerpoint presentation. Then worked with Adir, Cansu, Dean and Berrin (it’s her last day before going to Turkey – due to return October 10). James absent. Arranged to play class game in TP’s room – Greedy pig before Intervention session.

Played two games of greedy pig with TP’s class. Students very enthusiastic, hungry for more wanted to keep playing. Will play with DC’s class next week.

Students finished array’s book. As yet to label, describe arrays. James absent a significant amount of time but caught up quickly. Other students presented ‘all they know’ on their front covers. Noticed Cansu counting by ones when calculating progressive totals during greedy pig game.

Friday’s session at SCHOOL B only (SCHOOL A Imax excursion). Gives SCHOOL B students chance to catch up.

WEEK NINE

Focus

Number: Making links to measurement
Creativity
Estimation skills

SESSION 22 & 23: Monday 1st August and Thursday 4th August (SCHOOL A)/Friday 5th August (SCHOOL B), 2005*

* Curriculum days/incursion interrupts program, therefore 2 days at each site this week.

INTRODUCTION ~ Relationship between addition and subtraction

Resources

0-100 number cards
unifix, tiles, counters as necessary

Purpose

Consolidate mental computation strategies for subtraction

Learning Notes

Session 17 interviews identified students unable to connect relationships 10 and 5, 15, so 15 take 5, 10. Working with students in a small teaching group. Students have their workbooks with them on the floor. Start with the number card ‘10’ in the middle of the circle, ask students to mentally add 1, “eleven”; add 2, “twelve”; add 3, “thirteen” and so on to add 9, “nineteen”. Then record in their workbooks. Repeat orally for the number 20
and ask students to mentally take 1, “nineteen”, take 2, “eighteen” and so on to take 9, “eleven’. Students then choose a tens number, eg, 40 and record subtraction pattern in their workbooks.

**LEARNING FOCUS ~ Introducing ‘our maths dudes’ project**

**Resources**
- Metre rulers
- Chalk
- Large sheets of butcher’s paper
- Masking tape

**Purpose**
- Link number ideas through the area of measurement
- To facilitate the *difference* idea (for subtraction)
- Problem solving and creativity
- Estimation skills

**Learning Notes**
- Tuning in, how long is 1 metre?
- Draw a chalk line on the carpet. Ask students to estimate where a meter away would be. Students mark their estimates with chalk. Then measure to check. Discuss over/underestimates as appropriate. Some students unfamiliar with the concept of a metre. Explain that the meter will be important when completing the activities over the next few sessions.
- Set the task of creating a large piece of paper that will accommodate both their own height and width.

**CONCLUSION ~ Discussion**

Engage in an open discussion about the day’s session, share problems, strategies and results.

**SCHOOL B:** Arrived to see opera incursion “Hansel and Gretel” about to start at 9.30 – 10.30. Cut into time, however sat with teachers and students (as part of the furniture) and watched presentation with them. I think it is important that Intervention students see me as very much part of their school culture. Sat with DC.

Students at **SCHOOL B** appeared very unfamiliar with metre in terms of a unit of measurement generally and relationship to cm’s as well as using the ruler itself.

**SCHOOL A:** Students more in tune with metre and estimates relatively accurate. Could cope with measuring and deal with the concept of 1m and a bit, in cms.

**SCHOOL B:** Curriculum day at **SCHOOL A** so only **SCHOOL B**. Played greedy pig with DC’s class. Noticed many students in DC’s grade, counting by ones, so I talked about doubles, near doubles, ppw relationships. DC tuned into this and began advising individual students as she roamed around.

Worked with Intervention students (James and Dean absent) on their maths character. Problem posed of putting/tapping enough paper together that would sufficiently accommodate their height and width. Adir, Ahmed and Cansu a little unsure. Adir then led the idea of putting three sheets together, which Ahmed found to be too much for him (as he is significantly shorter). They were unsure of how to go about checking that the length of paper would accommodate their height. Discussion followed. They decided lying on their paper
was a good strategy. I asked them to give their character and personality and a mathematical name. Cansu knew immediately that ‘arrays’ would be part of her name and became Angela Arrays. She decorated the arms of character accordingly. Ahmed called his ‘Mathematics’. I suggested using this to be a first and second name, his therefore became Mathe Matics. He placed a region model on his character’s shorts and told me “I’ve shown 3 sixes” and he was right. Asked also if he could give his character a world champion in maths gold medal! Of course he could. All students cut these out. Used the computer to create name labels for their character. Bring digital camera for photos!

WEEK TEN
SESSION 24: Monday 8th August, 2005
INTRODUCTION ~ Pairs with a difference

Resources
Number cards to 20
Counters/tiles, unifix as necessary

Purpose
Investigate pairs of number under 20, with a difference of 7, then 8 then 9

Learning Notes
Pose the question for numbers 20 or less, “what are the pairs of numbers that have a difference of 7?” Students have access to manipulatives as appropriate. Discuss what they notice and the patterns that this generates. For example, 1,8; 2,9; 3,10…each number increases by 1. Students record in their workbooks (for pairs of numbers with a difference of 7, then 8 and 9).

LEARNING FOCUS ~ Creation of our maths dudes

Resources
Large paper from previous session
Coloured markers, pencils etc
Scissors

Purpose
To design a mathematical ‘dude’ for height comparison

Learning Notes
The students are to design a ‘maths dude’ character based on their own body. Once the large paper is ready to accommodate their own height and width, students work with a teacher/partner to outline their body shape on the paper. They then design their ‘dude’ clothing and give their ‘dude’ a personality and name. Then cut their ‘dude’ out.
SCHOOL B: Adir absent and Cansu involved in Music lesson. Took this as opportunity for Dean and James to catch up and create their characters. James said as he is cutting his out, “He’s still being born”. Both Dean and James worked efficiently to create paper (and check size) to accommodate as necessary.

James names his Arnold Maths Dude and displays a maths necklace around his neck, Dean names his Mr Numbers and displays a maths is fun logo on his t shirt, Adir names his Crazy Maths, Cansu names hers Angela Arrays with arrays for decoration. Ahmed names his Mathe Matics and includes shapes and region model (6 sixes) as decoration.

SCHOOL A: Yousif names his 50plus50, Hadi names her bodmas Brandy and Douha Rita Mitre.

SESSION 25: Thursday 11th August, 2005

~ Maths Dude Height Investigation ~

Resources
Cut out *maths dudes*
Rulers, 1m and/or 30 cm

Purpose
Organise mathematical information
Practice skill of estimation
Using appropriate tools to gather information
Rename, order and sequence measurement in metres and centimeters, eg, 1 m and 43 cm

Learning Notes
Teacher explains, “Today we are going to investigate the height of our maths dudes, but first we’re going to estimate how tall we think they are”. Discuss with students the best way to organise the information (eg, table format) and what the estimates will be based on (eg, their own height).

“What should we do next?” Measure them. Each student is responsible for measuring their own maths dude. Allow students time to experiment with the best way for doing this, then come together as a group to share their results.
The table below can be completed:

<table>
<thead>
<tr>
<th>name</th>
<th>height estimates</th>
<th>actual height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bodmaths Brandy</td>
<td>1 metre and a half</td>
<td>158 cm</td>
</tr>
<tr>
<td>50plus50</td>
<td>1 metre and 45 cm</td>
<td>149 cm</td>
</tr>
<tr>
<td>Rita Metre</td>
<td>1 metre and 36 cm</td>
<td>148 cm</td>
</tr>
<tr>
<td>Div 2</td>
<td>1 metre and 40 cm</td>
<td>148 cm</td>
</tr>
<tr>
<td>Value</td>
<td>1 metre and 28 cm</td>
<td>145 cm</td>
</tr>
</tbody>
</table>

Opportunity for discussion/working with:

- the different ways that these measurements can be recorded, that is, renaming. Eg, 158 cm, 1 m 58 cm, 1.58m etc
- height differences, eg, 3 cm difference between ‘Bodmas Brandy’ and ‘Value’
- placing measurements on a number line (rope and peg activity)

SCHOOL A: Students responded in much the same way as at SCHOOL B. Douha showed the height of Rita Metre as 1.48 and when asked, she said “cms”. I am alerted to moving to fractions and decimals with these students. Noticed also that Yousif measure his dude with 1 one metre ruler and 2 30cm rulers. NEED efficient strategies. Print out photos for students to record and write reflections in their workbooks. Number line activity not done at SCHOOL A as measurements too close. SCHOOL A looked at height difference instead. Students had little trouble with this.

SCHOOL B: Students knew that their dudes were based on their own bodies. Used this to help judge estimates. All students could connect 1m 69cm and 169 cm but not necessarily 1.69m. Students used 1m rulers to measure their dudes. Some students when measuring their dude’s height, tried to hold the cut out up against the wall. Then decided lying down was better. Once each was measures, student recorded their results on the whiteboard in their own way. Students could order these heights in order from shortest to tallest. Measurements placed on an open number line. Orienting discussion about half way to assist their placements.

Cansu placed 156 and 140 cm reasonably well. 139 placed some distance from 140. And 163 and 169 further apart. Dean, no real sense, placed 156 less than half way along. Adir 150 and 140 placed appropriately (other measurements not so). James recommended that 150 would be useful reference point. Attended all measurements appropriately.

**WEEK ELEVEN**

**SESSION 26: Monday 15th August, 2005**

**Focus**
Doubling
Doubling strategies for multiplication facts
Partitioning for fraction understanding, halving and thirding

**INTRODUCTION ~ Doubles**

**Resources**
Number cards to 50
Cuisenaire material
Purpose
Efficient strategies for doubling larger numbers using place value understanding
Utilise doubling strategies for 3s facts

Learning Notes
Teacher and students in small circle. Warm-up activity with number cards to ten, doubling number shown. Extend number cards to 50. Teacher can model thinking using place value understanding, for example, 22 is turned over, “double 2 tens, forty, double 2 ones, 4, so forty four.”

Introduce ‘double and one more group’ for 3s facts. The cuisenaire material helps students to see this relationship. Have material on the floor in the middle of the circle of students. Ask them to show and describe “2 of anything” using the material. This can be 2 ones, 2 twos, 2 threes or any 2 up to 10. Extend this idea and ask students to show and describe “3 of anything”. Discuss, What do they notice about ‘three somethings?’ 3 somethings is 2 somethings and 1 more something.

LEARNING FOCUS ~ Fractions: Paper folding partitioning strategies

Resources
Kinder squares
Paper strips about 30 cm long and 3 cm wide or so
Whiteboard/blackboard
Whiteboard markers/chalk

Purpose
Partitioning for equal parts (continuous fraction model)
Begin to explore denominator idea, how many, how much
Fraction comparisons

Learning Notes
Explore ‘halving’ using kinder squares and paper strips. Encourage students to halve in a variety of ways. Choose indicative examples of paper folding to paste in workbooks and label parts accordingly:

1 half

1 half

1 half, 1 tells me how many, half tells me how much

Discuss relationship of parts to whole, equal size and other observations the students can make.

Introduce ‘thirding’. We’ve shown halves with paper folding, now I’d like you to show thirds, by paper folding … (Some students may interpret this as “easy” and proceed to fold and create quarters!). If this is the case, ask students how many parts have you shown? “four” What do we call these, if there are four of them? “quarters” So how many parts do you think we’ll need if we want to show thirds? “three”. Ok, try again and fold to show three parts. Students will find thirding more of a challenge.
Encourage student to keep persisting with folding to show thirds. Share successful strategies as discovered by students.

Students can ‘third’ kinder square and paper strips. Record, label and paste in workbooks.

Discuss comparisons between half and third. Encourage using these ideas to help with accurate ‘thirding’, that is, a third is smaller than a half, find half, fold/draw a little less, then halve the remaining part.

Extend ‘halving’ and ‘thirding’ strategies to faction diagrams on the whiteboard or blackboard. Draw shapes on the whiteboard and ask students to use the whiteboard marker to show halves and then thirds. Encourage students to talk aloud about their thinking as they do so:

Partition to show halves…(repeat for thirds)

SCHOOL B: Students found thirding a challenge! And seemed confused with the language. ‘Half’ ok, but comparison between half and third unclear, eg third is smaller than a half.

SCHOOL A: Students were able to have a go at partitioning circles into halves and thirds. Discussed when we share things this shape in real life, they came up with pizzas, pies, cake. A challenge for them but trial and error on whiteboard led to improvement in equality of size of parts. When transferred this idea to paper circles, used the strategy of placing a dot in the centre of the circle, then made useful marks on the edge of the circle before marking in lines. See below:

SESSION 27: Thursday 18th August, 2005

INTRODUCTION
Repeat Introduction for Session 26

LEARNING FOCUS ~ Measurements on a number line

Resources
Rope
Pegs
Cards labelled with height (based on worksheet)
Maths Dude sequencing worksheet

**Purpose**
Apply partitioning strategies on a number line in a measurement context

**Learning Notes**
Tuning in, students and teacher work with the same measurements described on the worksheet and peg these to the rope, one end labelled 1 m, 100 cm and the other end labelled 2 m, 200 cm. Encourage students to share their thinking verbally and discuss with the group.

Students complete the worksheet and paste into their workbooks.

**Maths Dudes**
A group of children created their own 'Maths Dude' and measured their character's height. The results are presented below:

<table>
<thead>
<tr>
<th>Name</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rudolf Ruler</td>
<td>153 cm</td>
</tr>
<tr>
<td>Casey Calculator</td>
<td>189 cm</td>
</tr>
<tr>
<td>Peter Problem</td>
<td>125 cm</td>
</tr>
<tr>
<td>Sam Shape</td>
<td>133 cm</td>
</tr>
<tr>
<td>Kim Think</td>
<td>170 cm</td>
</tr>
</tbody>
</table>

They placed their results on an open number-line. Complete the number-line below by writing the names in the right boxes

**CONCLUSION ~ Photo opportunity**

**Resources**
Digital camera

**Purpose**
Provide stimulus for reflective journal writing

**Learning Notes**
Use digital camera to take photos of students sitting with their 'maths dude' (as permission allows) or a photo of their created 'dude'. Print these pictures for students to use as stimulus for reflective journal writing the following day.
SCHOOL B: Students worked well, efficiently and steadily solving doubles for no’s under 50. James and Adir confident to try no’s >50. Cansu unable to see and confidently work out double 39, think double 40, 80 take 2, 78.

Maths dude worksheet encouraged student to read carefully. Initially most thought to write measurements, not write the names in. Rope modelling prior to written sheet, successful scaffold.

SCHOOL A: Developing efficient doubling strategies for larger numbers. Douha a little slower that the other, but developing efficiency! Haven’t noticed her using her fingers for weeks. SHE WAS HEAVILY RELIANT ON THIS! Could conceptualise and verbalise the cuisenaire model for 2 and 3’s facts.

No, line scaffold. As I start setting up the rope, Yousif comments, “a number line”. Students suggest marking half way point to help them orient placement of the cards. They also suggest half again for quarter of the way and 3 quarters of the way. I asked students to explain their thinking for the placement of 2 names on the number line. All students able to explain well and clearly in terms of order, though not necessarily in relation to partitioning.

SESSION 28: Friday 19th August, 2005

INTRODUCTION ~ Doubling to 100

Resources
Number cards to 100

Purpose
Practice using place value understanding to double larger numbers

Learning Notes
(See Session 26)

LEARNING FOCUS ~ Reflective journal writing

Resources
Printed photos taken previous session

Purpose
To describe their thinking, strategies throughout the maths dude work

Learning Notes
Students can paste photos in their workbooks then write about the procedure for their character’s creation, the strategies they used for measuring them and what they used to do so.

CONCLUSION ~ Number chart 4

* Roll 2 ten sided dice to create a starting number greater than 50
SCHOOL B: Students working well at mental addition of no’s to 50, with James and Adir trying no’s to 100. Students very quiet and patient as each student worked out their answer. Students enjoyed writing, seeing and using their photos of dudes in their workbooks. Re No. chart 4, all students now able to work down the chart in tens, work back and pick up their mistakes. Dean a little unsure of 10 more than 103.

Checked understanding of half using partitioning strategies with Adir and Cansu on whiteboard using circle, square, rectangle, no. line and discrete models. Half ok. However Adir couldn’t see

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 ½ ??</td>
<td>2</td>
</tr>
</tbody>
</table>

When moving on, they anticipated “quarters”, but unable to halve and halve for 4 even parts. Ahmed who joined us at this point, said “I know” and partitioned correctly.

Tested student understanding of sharing collections with counters on the floor. Quite shaky, although Cansu could see that a collection of 12, shared between 4 people would be 3 because 6 each only enough for 2 people, so 3 for 4 people.

WEAK TWELVE

**Focus**

Fraction as operator idea
Whole to part fraction understanding
Continuous and discrete models

SESSION 29: Thursday 25th August, 2005

**INTRODUCTION ~ Halving**

**Resources**
Number cards to 100

**Purpose**
Efficient strategies for halving larger numbers using place value understanding

**Learning Notes**
Teacher and students in small circle. Warm-up activity with number cards to 100, doubling number shown. Extend activity to halving numbers shown on cards. Teacher can model thinking using place value understanding, for example, 84 is turned over, “halve 8 tens, forty, halve 4 ones, 2, so forty two.” Regular examples with even tens and ones before irregular examples with odd numbers in place value parts.

**LEARNING FOCUS ~ Exploring ideas with discrete and continuous models**

**Resources**
Counters/tiles
Kinder square and paper circles

**Purpose**
Explore ‘half of’, ‘third of’ a given collection
Consolidate halving, thirding and halving again to make quarters
To make, name, record fractions to help generate students’ own fraction diagrams
Learning Notes
Use tiles/counters to work with students in efficient strategies for showing ‘half of 12…18…24…48’ then ‘third of 27…12…18…21’. Discuss with students the contexts where this sort of thinking might be useful.

Make, name, record: halves, thirds and quarters using paper folding techniques from earlier session (make out of kinder square or paper circles), discuss description of the parts (name) and label these (record). See if students discover half and half again for quarters. Paste examples in their workbooks. Then these students can create their own fraction diagrams based on their paper folding work.

CONCLUSION ~ Fractions matching activity

Resources
Fraction cards (halves, quarters and thirds) in two forms, eg, ¼ and 1 quarter

Purpose
Match formal and informal ways of recording fractions

Learning Notes
With students and teacher working in a small group on the floor, place the 28 different cards, face up on the floor. Ask students if they can find two cards that match in some way, ie, different but mean the same thing? Continue until all cards are paired up and discuss observations, conceptions and misconceptions as they arise.

SCHOOL B: Students succeed at halving even tens and ones, eg, 44, 22. They were certainly less confused when halving and thirding small collections. Were able to articulate necessity for equal shares and halving produces 2 shares and thirding produces 3 shares. Made connections between paper folding and fraction diagrams. They are beginning to think, ‘a third is smaller than a half…’ and use this thinking for a variety of models.

SCHOOL A: Students succeeded at mentally halving as above but also no’s in 30’s, 50’s. Strategy used, eg, 54, half 50, 25 (hold) half 4, 2, 25 (retrieve) and 2, 27! Students made comparisons of half and third with continuous and discrete models. Students able to observe, halving: 2 parts, equal, 1 cut/fold; thirds: 3 parts, equal, 2 cuts/fold. Aware of continuous and discrete situations.

SESSION 30: Friday 26th August, 2005
INTRODUCTION ~ Multiplication Toss

Resources
Cuisenaire materials
1 cm grid paper
2 ten-sided dice per pair of students
pen or marker

Purpose
Work with arrays to support efficient understanding of multiplication facts
Cohesion between groups, number of groups and the total, eg, 3 fives, 15
Commutativity, eg, 3 fives, 15 and 5 threes, 15
Learning Notes

Tuning in: Throw 2 dice, for example 4 and 3, and ask students to show 3 fours or 4 threes with the cuisenaire. Discuss total for both (12). Explain that this will help them to record when we play the ‘multiplication toss’ game to follow.

Introduce ‘multiplication toss’ activity. ‘Multiplication toss’ is played with 2 dice and 1cm grid paper for each student. Students work in pairs or in threes. Take turns to throw the two dice. If for example a 6 and 5 is thrown, the student can choose to record 6 fives or 5 sixes on the grid paper in the form of a region:

Students continue to take turns, filling in the grid paper as efficiently as possible. When there is less room or it is strategically better to fill a certain space, the region can be split. For the example here, 6 fives, could be split.

The teacher observes the strategies students use. Some students, when they play for the first time, will not take too much notice of how they complete the grid, resulting in a number of odd shaped spaces to fill. As students’ experience with the game increases, a more strategic approach is taken. When ready, students can ‘record’ their array within each array they create, in the following ways: 6 fives, 6 by 5, 6 x 5, or 6 x 5 = 30.

LEARNING FOCUS ~ If ? then what’s the whole?

Explore the following part-whole fraction situations as appropriate. Use paper for continuous situations and tiles/counter for discrete situations. Take each situation one at a time and have children discuss thoughts and ideas with each other and the teacher.

If ✖️ is 1 half, show the whole…

If ✖️ is 1 third, show the whole…

If ❌ is half the collection, what would the whole collection look like?
If \( \bullet \) is a third of the collection, what would the whole collection look like?

**CONCLUSION ~ Review**

Review partitioning strategies, naming and recording by ‘make, show, record’

- 3 quarters,
- 2 thirds, and
- 1 and a half

Students make using paper squares, circles and/or paper strips (paste in their books), show by creating their own fraction diagrams, and record in more than one way, eg, 1 and a half, 1 ½, 3/2…

SCHOOL B: Make, show, record: Cansu unsure of quarters, Adir a few attempts at thirds, Ahmed, too many parts for quarters at first, then ok. James recorded informally. Dean all ok.

SCHOOL A: all students ok.

**WEEK THIRTEEN**

**SESSION 31: Monday 29th August, 2005**

**INTRODUCTION ~ Games**

Play ‘Multiplication Toss’ (see previous session)

Play ‘concentration’ using formal/informal fraction number cards (see session 29).

However, this time the cards are placed face down. The students take turns to turn over 2 cards, hoping to find a match. If a match is not found, the cards are turned back over and hidden. If a match is found, the pair is kept and the student has another turn. Continue until all pairs are found.

**LEARNING FOCUS ~ Fraction review**

‘Make, show, record’

- 1 third
- 2 quarters
- 1 and 2 thirds
- 4 thirds

‘How many are hidden?’

Use tiles/counters to set up the following 4 situations.

1. I have hidden half the collection. How many tiles are under the card?

2. I have hidden 2 quarters of the collection. How many tiles are under the card?

3. I have hidden a quarter of the collection. How many tiles are under the card?
4. I have hidden a third of the collection. How many tiles are under the card?

The pose a task using continuous fraction representations and hand out to each student a small square labeled ‘1 half’ and a small rectangle labeled ‘1 third’. Students’ task is to paste each in the workbook and indicate what the ‘whole’ would be. Students discuss their ideas and respond individually in their books.

SCHOOL B: Cansu and James absent. All students ok with make, show, record, 1/3 and 2/4 (Ahmed tells me halve it twice.) 1 2/3 and 4/3 presented more of a challenge. Deans recognises that 2 pieces of paper is needed for 1 2/3. I needed to model thinking for 4/3.

Dean becoming more confident with bridging 100 (really weak initially). Held off problem solving task until all students present. Students requested No chart 4. Adir decided to start at a number greater than 100. Ahmed and Dean happy to keep number greater than 50 less than 100.

SCHOOL A: Incidental teaching of 8's (double, double, double) and 9's relate to tens. Students working flexibly with table facts in multiplication toss. Re make, show, record 1 2/3 and 4/3; students coped via discussion with each other. I was able to guide their ideas compared to modelling of thinking for students at SCHOOL B.

Session 32: Thursday 1st September, 2005

INTRODUCTION ~ Extended place value path

Resources

3 ten sided dice per pair of students

Place value path game board but with the 0 and 100 changed to 100 and 1000 respectively

Purpose

To check students’ place-value understanding, for numbers 100 to 1000 in relation to order and sequence

Learning Notes

Students work with a partner to play the game. Teacher looks for student thinking in relation to placement of numbers within the path.
LEARNING FOCUS ~ Problem solving strategy: Ask, think, do!

Resources
Formal and informal fraction notation cards
Ask think do problem solving guidelines (laminated cards)

<table>
<thead>
<tr>
<th>Questions we can ask…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Things we can think about…</td>
</tr>
<tr>
<td>Things we can do…</td>
</tr>
</tbody>
</table>

Purpose
Teach strategy for problem solving in context of fractions

Learning Notes
Play ‘Concentration’ with the fraction cards.

Then introduce the problem to the students: “Sometimes we have to think about fractions when we do everyday things…” Allow time for discussion. Present the following cooking problem:

Problem…
1 have six cups of milk. A muffin recipe uses 3 quarters of a cup of milk. How many times could I make the recipe before I run out of milk?

Teacher uses the ‘ask, think, do’ cards to guide student thinking about the problem. Engage in discussion and allow student to trial their ideas and share with others. Where necessary actively model cups and the amount of milk. Students document their thinking and solutions in workbooks using words and pictures to illustrate.

James, Dean, Ahmed absent. Only worked with Cansu and Adir. Worked well at reading and naming formal fraction notation. Both made sensible decisions about placement of 3 digit numbers in the place value path game. And could generalise “I need to throw a 1 or 2 at least, to make a number that I need.” Needed to actively model and think out the problem. Will create a similar problem with support materials, eg, cups and amounts of milk to support conceptualisation of the problem.
SCHOOL A: Douha and Hadi made some inefficient decisions about placement of 3 digit numbers in the place value path game (repeat Friday). They realised themselves though. Hadi and Yousif able to observe, “J, you can’t do anything”. Students able to argue according to place value understanding. Recapped make, show, record ideas to help equip students what they can think about and do, paper folding and drawing…Hadi drew 6 cups, showed quarters and could see 6 times. When reminded of left over milk, said “Eight”. Yousif (followed strategies of those around him) and Douha worked similarly but left over milk not as immediately conceptualised.

WEEK FOURTEEN
Session 33 & 34: Monday 5th September, 2005 & Thursday 8th September, 2005

INTRODUCTION ~ Fractions rope & peg sequencing activity

Resources
Kinder squares
Rope
Pegs
Fraction notation cards (halves, thirds, quarters)

Purpose
Practice partitioning fractions (halves, thirds and quarters from 0 to 2) using the physical number line model (rope)
Scaffold student thinking to abstract notion of an open number line in the written form

Learning Notes
Tuning in: discuss with students an efficient strategy for creating quarters. Encourage students to have a go, via paper folding and have them explain their thinking, eg, “half and half again will create quarters”. Explain that this thinking might be useful when doing the next activity…

Select appropriate fractions for students to place on the rope with pegs, eg, $1 \frac{1}{4}$, $2/3$, $1 \frac{1}{2}$. Set up the rope and conduct the activity with the small group. Ask students to explain their thinking as they think about where to ‘place’ their fraction card, eg, I know that $2$ thirds is less than one…

LEARNING FOCUS part a ~ Another recipe problem

Introduce the following problem to the students. Can be written up on white/blackboard:

I have 4 cups of milk. The muffin recipe needs half a cup of milk. How many times can I make the recipe before I run out of milk?

Teacher uses the ‘ask, think, do’ cards to reinforce student thinking about the problem. Engage in discussion and allow student to trial their ideas and share with others. Students can make use of that fact that this is a similar problem from a few sessions prior, so what did I do/think then…? Where necessary actively model cups and the amount of milk.

Extension idea: have students write their own problem, modelled on this one, for the teacher to solve.

LEARNING FOCUS part b ~ Partitioning review

Resources
Coloured paper (kinder squares, circles, streamers)
**Purpose**

Review children’s understanding of partitioning and reasoning about fractions in relation to ‘number of parts, name of part’, eg, five parts, makes fifths

**Learning Notes**

Work with students individually. Ask them to partition through paper folding, halves, thirds, quarters. Paste into workbooks and label appropriately. Then, use these strategies to guide their thinking when partitioning on the open number line.

**LEARNING FOCUS part c ~ Paper folding for fifthing**

**Resources**

Kinder squares
Paper strips about 30 cm long and 3 cm wide or so
Whiteboard/blackboard
Whiteboard markers/chalk

**Purpose**

Introduce fifths

Partitioning for equal parts (continuous fraction model)
Begin to explore denominator idea, how many, how much
Fractions comparisons

**Learning Notes**

Revise ‘halving’ and ‘thirding’ using kinder squares and paper strips.

Introduce ‘fifthing’. We’ve shown halves, quarters and thirds with paper folding, now I’d like you to show fifths, by paper folding …Have students experiment with this. Ask while they do so, “how many parts are you trying to create?” – “Five”. Students will find this a considerable challenge. Teacher can model and guide where necessary.

Students can ‘fifth’ kinder square and paper strips. Discuss what they notice about the size of the parts, that is, “a fifth is smaller than a quarter”. (Opportunity to compare previous partitioning explorations so pasting in their workbooks is useful here.) Teacher can reinforce that this observation is very useful when it comes to ‘fifthing’…

Think: “A fifth is a little bit smaller than a quarter. So find a quarter (half and half again) and a little less, then halve and halve what remains.”

![Diagram of fifthing process](image-url)
Record, label and paste in workbooks.

Extend ‘fifthing’ to faction diagrams on the whiteboard or blackboard. Draw shapes on the whiteboard and use marker to show fifths. Encourage students to talk aloud about their thinking as they do so:

Partition to show fifths:

CONCLUSION ~ Partitioning an open number line

Ask students to prepare an open number line in their workbooks, labelling one end zero and the other end 2.

List the fractions on the white/blackboard.

\[
\frac{1}{2} \quad \frac{2}{3} \quad 1 \frac{1}{4} \quad 1 \frac{1}{2}
\]

Ask students to ‘place’ these on their number line and explain in the written form, why they placed them where they did.

SCHOOL A: No Intervention Friday SCHOOL A (Athletics carnival). So 2 sessions per site this week.

Check student ability to read formal fraction notation – all ok. Then partitioning fraction cards on a rope with pegs. Match strategies and thinking for half (and half by half gives quarters) and thirds, to help partition no. line. Links with thinking still tenuous therefore continue to provide student with experiences. Hadi attending to denominator only, eg, 2/3, thirds and placed 2/3 at 1/3. Hadi placed 1 ¼ half and half again after 1 (correct). Yousif re 2/3 compared it to 1, 1 ¼ and 11/2 knew it was smaller, 1 ½ was half way between 1 and 2. Douha still struggling with 2/3 as “is half of 0 and 1, but 1 ½ and 1 ¼ placed appropriately.

SCHOOL B: Only completed number line activities. Dean absent. Students more successful with rope and pegs placing halves and quarters, not necessarily able to apply to abstract notion of no. line. Thirds still presenting conceptual difficulty in both forms, pegs and rope and open number line. Therefore more partitioning with paper folding. No difficulty with the notion of 1, half way between 0 and 2. Cansu needed significant support with number line (proportion/ratio in terms of sequence not yet consolidated) but knows half and half again gives quarters. Adir showing development of mathematical thinking in ability to explain partitioning strategies, developing a sense of order and sequence.
SCHOOL B (8.9.2005) worked with students individually. Partitioning to show halves, quarters and thirds. Paper folding and discussion of what they know, tell me and I record for them.

Cansu paper folds ok, beginning to apply paper fold strategy to partitioning number lines. We marked them labelled no. lines. All students responded orally while I recorded using fraction notation. (See workbook).

Adir as above but the notion that fractions live in between whole numbers a bit of a challenge. We tried recording no's on no. line and he tells me it’s “a bit hard” (see workbook).

Ahmed initially thought that these quarters:

![Quarters](image1.png)

Were larger than these quarters:

![Quarters](image2.png)

Even though same paper used. We tested it. Ahmed was also able to observe that 1 ½ thirds is the same as a half.

James inefficient folding strategies for quarters, then did a roll fold. Thirding still a bit of a challenge although whole numbers with fractions to the no. line was fine, I suspect he plays “dumb” at times?

**WEEK FIFTEEN***

*one session at each site this week. End of term three. Whole school and class activities impact on scheduled time. Last session for Term Three.

**SESSION 35: Thursday 15th September, 2005**

**Focus**

Establishing understanding of multiplication through various contexts: equal groups, rate, comparison, array and Cartesian product

**INTRODUCTION ~ Doubling for 2s, 3s, 4s and 8s facts**

**Resources**

Number fact flash cards, eg, 2 x 4, 3 x 4, 4 x 5, 8 x 7

Cuisenaire material

**Purpose**

Efficient strategies for number facts related to doubling

**Learning Notes**

Teacher asks students to show with the cuisenaire, 2 fives. Ask students, “What is another way of describing this?” – “double five, so 10”. Repeat for 3 sixes. Ask, “What is another way of describing this” – “double 3, 6 and 1 more three, 9”. Repeat this scenario with 4 sevens. Ask, “What is another way of describing this?” - “Double 7, 14 and double again, 28”. Repeat for 8 fours. Ask, “What is another way of describing this?” – “Double 4, 8, double again, 16, double again, 32”. Show using the cuisenaire for each example.
Teacher then introduces the flash card to respond to as a group. Flash cards have examples of 2s, 3s, 4s, 8s facts. Encourage students to use doubling strategies in response to the questions on the cards.

**LEARNING FOCUS ~ Multiplication word problems**

**Resources**

*Multiplication word problems* displayed on A4 sized cover paper

Cuisenaire material

Counters /tiles

**Purpose**

To check students comprehension of various multiplicative situations

Identify students’ chosen method of solving the problems

**Learning Notes**

There are 10 *Multiplication word problem* cards, 2 problems (generally one simpler [S], one a little more difficult[D]) for each of the following concepts for multiplication: equal groups, rate, comparison, array and Cartesian product. These are presented below:

(Equal groups)

*There are 6 tables in the classroom with 5 children at each table. How many children are there altogether? [S]*

*There are 8 tables in the restaurant with 14 people seated at each table. How many people are there altogether? [D]*

(Rate)

*If you need 5c to buy chocolate frogs at the milkbar, how much money would you need if you wanted 8 frogs? [S]*

*It's your birthday and you want to buy lollies to share with your classmates. If you need 9c to buy a lollipop, how much money would you need if you wanted 26 lollipops? [D]*

(Comparison)

*Mike has 7 marbles, Jo has 5 times as many. How many marbles does Jo have? Jo has 5 gel pens, but Mike has 4 times as many as Jo. How many gel pens does Mike have?*

(Array)

*There are 4 rows of chocolates in the chocolate box with 9 chocolates in each row. How many chocolates are there altogether? [S]*

*There are 6 rows of roses at the garden centre with 14 roses in each row. How many roses altogether? [D]*

(Cartesian product)

*The shop sells ice-cream. You can choose between a single or double cone, and have strawberry, vanilla or caramel ice-cream. How many different choices do you have? [S]*

*You've bought some new clothes, a pair of jeans, a pair of shorts and 3 t-shirts. How many different outfits can you wear? [D]*

Present the problem cards to the students (select an appropriate set of five). Students share the cards around until they have solved each of the five problems. The students can work on their own or in pairs, each recording their thinking and solutions in their workbooks.
Teacher takes the opportunity to discuss with students their interpretation of the problems and their method of choice with respect to solution. Materials are available to assist with modelling where necessary.

SCHOOL A: Students found multiplication problems initially hard to conceptualise, eg Yousif ‘table’ problem, needed clarification of 8 tables but thought 2 people at each. This may be because of the picture. Helped to re-read and chat about the scenario. Douha tended to want to add 26 and 9 for ‘birthday’ problem. Took her back to 9c for 1, how much for 2? Took coins from my purse to help. Hadi set up (vertically) 14 x 8 for ‘table’ problem.

Since level of difficulty a challenge for SCHOOL A students decided to provide simpler situation for SCHOOL B tomorrow (hence the 10 problems).

General comment: This week lacked fluidity. My health Monday! (I was unwell) Footy day at SCHOOL B. Whole school activity changed to 9.45 am to suit the weather. Better to strike while the iron is hot. Adir still wanted to come with me though, when he saw that I had arrived! Despite the sporting activity. SCHOOL A students were planning a surprise party for ‘Rachel X’ MD’s replacement whilst on LSL. This meant that I worked with SCHOOL A students on Thursday and SCHOOL B on Friday.

SCHOOL B: Choc frog problem: Ahmed and Cansu counted by 5, eight times. Adir 4 x 8, 40, think he meant to write 5 x 8. Box Choc problem: Ahmed, 4 x 9, double 9, 18, double 18, 36. Adir and James , 4 x 9. Cansu like Ahmed, and drew an array. Marbles problem: Ahmed, 5 sevens are 35, Adir, 5 x 7, James, by counting. Cansu worked with me, drew an array and had to chat about “ 5 times as many”. Classroom problem, Ahmed and Adir, 5 x 6, James, by counting, and Cansu counting by 5, six times. Icecream problem, Ahmed, double and single cone, three choices each so 6. Adir started systematic list – helped by Cansu (unfinished) recorded 6 (I suspect from hearing other students’ answers. James initially reticent to record thinking to keep track, mentally knew 6 but felt 12 could be possible with flavour combinations on the cones.

Term Four 2005 commences
WEEK SIXTEEN
SESSION 36: Thursday 6th October, 2006
INTRODUCTION ~ Working mentally
Resources
0-10 number cards
Purpose
Revisit efficient strategies for add 3, take 3 and doubles
Learning Notes
Work with small group of students with teacher flashing through the number cards, asking students to answer orally ‘add 3’, ‘take 3’, ‘double’ the numbers shown. Teacher presents cards at an appropriate speed.
LEARNING FOCUS ~ Constructing models of multiplicative problems
Resources
Cuisenaire material
Purpose
To construct region models for multiplicative situations
Learning Notes

Teacher presents the alternate set of 5 problem cards from the previous session. Students are asked to construct models of each problem situation using the cuisenaire material. Discuss the models that the students create, eg,

*There are 6 rows of roses at the garden centre with 14 roses in each row. How many roses altogether?*

![Diagram of roses]

Ask students what they see – “6 tens, so 60 here and 6 fours, 24, so 84 altogether.”

**CONCLUSION ~ Number chart activity funny shape a**

**Resources**

*Number chart activity funny shape a* for each student

**Purpose**

Reinforce count of ten from different starting points

Students to write about how they completed the task, what was easy, what was difficult.

**Learning Notes**

Present the irregular shaped number chart activity. Ask students to complete by placing the appropriate numbers in the white squares. Students may want to ‘write in’ the missing spaces and/or work the chart horizontally. Watch what students do and discuss as necessary. Ask, “Is there a more efficient way to fill in the white spaces?”

When students have completed the task, ask them to reflect on the activity by writing down how they did it, what was easy, and what was hard.

*Number Chart Activity 5 ‘Funny Shape a’*

![Diagram of number chart]

SCHOOL B: Adir returned to PIS became silly/disruptive. Berrin returns from Turkey. Appeared relatively confident with doubles yet add 3 less secure, eg, 7 and 3 “7, 8, 9”. Berrin found modelling of multiplication problems hard to model (we used Cuisenaire), eg, 5 Cuisenaire rod for 5 cent frog but then pulled out 8 eights to model the frogs! Ahmed used Cuisenaire to model groups of idea for table problem cp using
Constructing paths to multiplicative thinking: Appendices

Cuisenaire to model region idea. For marbles, Ahmed showed 7 ones. When I asked if he was finished, he took a seven rod, I reinforced, Jo has five times as many" and he was ok from there. Cansu modelled appropriately, but counted by ones to determine total. I asked if this was the most efficient way? So when working with 5 nines, we looked at double, double and 1 more. When modelling 14 flowers in 6 rows:

SCHOOL A: Modelling of problems, with an emphasis on justifying their models orally.

SESSION 37: Friday 7\textsuperscript{th} October, 2005
INTRODUCTION ~ Circle problem 3
Resources
\textit{Circle problem 3} worksheet
Tiles and counters numbered 1 to 7 if necessary
Purpose
Mental strategies
Problem solving
Observe reliance on needing to manipulate tiles
Learning Notes
Present the problem to students: Place the numbers 1 to 7 (one of these numbers will not be used) so that each row adds to the same number (totals 7, 8 and 9 are possible). Keep totals secret from the students initially and when they come up with a solution, ask, “Is that the only one? See if you can come up with a different one.”

[Diagram of circle problem 3]

LEARNING FOCUS ~ How would you solve…?
Resources
\textit{Multiplication word problem} cards
Cuisenaire
Purpose
Documentation of student thinking and interpretation of multiplicative contexts
Learning Notes

Using the *multiplication word problem* cards from the previous 2 sessions, ask students to write what they would do if they had a pen and paper to help them with the solutions. Ask, “What would you write down to help you solve the problems?” Discuss with individual students, pairs or as a group as issues arise. Look for incorrect interpretations of problem (eg, subtraction), or use of repeated addition or evidence of multiplicative thinking.

Then, have students create their own region using the Cuisenaire material and think about “What might the situation be about?” This will tune students into the think board activity to come in session 39.

**CONCLUSION ~ Number chart activity 5 funny shape b**

Number chart activity funny shape b

* throw a 10 sided dice to create your starting number

SCHOOL B: Circle problem, students able to recognise that 6 counters needed, and solved successfully. Berrin used pairing strategy from Dean eg, 6 and 2 and confidently placed them on the circles. In describing arrays, students unable to confidently describe their situation. I modelled first. Seemed to attend to total or no. of rods only, eg, 3 tens (so 3 orange Cuisenaire), 30 “I bought 30 bugglegums” or I bought 3 icecreams. To address this, have to model situations with tiles/counters.

SCHOOL A: circle problem solved easily by all.

Worked with the group using the whiteboard, on create own array what might situation be about. Students offered suggestions, eg, 26 by 9. I noted absence of repeated addition!

Saturday 8th October: Note: Potential impact of Ramadan which commenced for Muslim students on October 5, 2005. Students are fasting, nil orally from sun up to sun down which means no sustenance at school. TP and DC tell me student absences may be an issue during this time. Absences an issue with some students at SCHOOL B anyway. (See field notes greater detail).

**WEEK SEVENTEEN**

**SESSION 38: Monday 10th October, 2005**

**INTRODUCTION ~ Place value hangman**

**Resources**

White/blackboard

Markers/chalk

Multiplication facts handout
Purpose
Problem solve in a place value context (to thousands)

Learning Notes
Set up the following scenario on the board:

_________   _________   _________   _________

To represent the places of a four digit number.

Have in mind a particular number, eg, 4572 (unknown to students at this stage). Students need to ask questions about the placement of numbers from 0 to 9 and the value that they might hold. Students may ask questions such as, “Is there a 5 in the tens place?” Teacher responds with “Yes there is a five, but not in the tens place”. This can be noted on the board as 5, with a tick next to it. If a student correctly indicates the right number in the right place, this is written on the appropriate line:

_________   _________   _________   _________

5

If a student asks a question about a number that is not used at all, eg, “Is there an 8 in the ones?” this is noted with a cross next to it, eg, 8x. In this instance, the hangman drawing is begun. Students need to solve the number, before a complete hangman drawing is presented. Each line of the hangman drawing represents each ‘no’ response to a student’s question:

Hand out the following sheet for students to have as reference. Students decorate chart with their name, so that these can be laminated and returned to them at the following session.
Constructing paths to multiplicative thinking: Appendices

Multiplication Facts

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LEARNING FOCUS ~ Mathematical models

Resources

Tiles/counters

*Multiplication Scenario* cards

Purpose

Students interpret *Scenario* cards and present these visually in the form of arrays

Learning Notes

Students share a *scenario*, one between two, and work individually to create models in the form of arrays, using tiles/counters the following situations:

(SCHOOL B)

*The room has some chairs set up for a meeting. There are 6 rows with 8 chairs in each row. How many chairs are there?*

*A packet of tim tams has 11 biscuits. How many biscuits in 5 packets?*

*I have 5 neopets. My friend has 3 times as many. How many does my friend have?*

*An octopus has 8 legs. How many legs for 7 octopuses?*

Students work their way through each scenario and discuss with each other and the teacher the most efficient way for showing visually.

(SCHOOL A) Students are presented with 4 problems and their task is to identify the multiplication problem.
A packet of stickers has 5 sheets. Each sheet has 14 stickers. How many stickers are there in total?

A furniture maker needs to make 25 stools. A stool has 3 legs. How many legs will need to be made?

Jane brings 32 freddo frogs to school. She gives out 27. How many does she have left?

I have 24 textas. My friend has 14. How many do we have altogether?

Students share their thoughts and the teacher asks students to be convincing in justifying their thinking in relation to which problems are multiplicative and which aren’t, why?

CONCLUSION ~ Circle problem 3, a variation

Place the numbers 7 to 13 (one number will not be used) so that each row adds to the same number.

SCHOOL B: Students generally well equipped to solve circle problem without support tiles. Cansu counted by ones with fingers. Ahmed said “I don’t get it”. Needed some support, eg, listing numbers and crossed them off as they were used. Review of multiplication problems using laminates grids successful. Students able to draw in area model on grid. Some used multiplication chart to check totals. No one here appeared to count by ones. Students able to construct own arrays and create thinkboard graphic organiser to show their thinking. Dean, 4 arms each on 2 people but drew 8 arms on 2 people. I asked him to check his picture and he was able to self correct. Ahmed, Cansu and Adir all fasting consistently throughout Ramadan and definite drop in focus and concentration noticed. Ahmed and Adir rolling on floor and tossing equipment. Cansu listless. Berrin not fasting all the time.

SESSION 39: Thursday 13th October, 2005

INTRODUCTION ~ Place value hangman

See the previous session for detail. This time the focus is on individual students leading the game in the role of the teacher.

LEARNING FOCUS ~ Array think board

Review problems from previous session. Students are then asked to create their own array and complete the following thinkboard:
A student’s response might look like this:

There were 5 people they all had five textas each.

SCHOOL A: Yousif leads place value game showed excellent understanding for 5 digit numbers and presented clues etc clearly. Student playing showed understanding by selecting and using appropriate clues.

Students worked at finding centre to set up their thinkboard. Douha very concerned about getting it in the middle. I checked their drafts from Monday and will keep them especially with regard to Yousif who still had trouble conceptualising his array situation. The thinkboard harmonises context, symbols, language and models.

SESSION 40: Friday 14th October, 2005

INTRODUCTION ~ Place value hangman

This game is played at the beginning of the next four sessions at the request of students. They enjoy leading the game and each have a turn as ‘teacher’. The game is gradually extended to millions as appropriate.

LEARNING FOCUS ~ Identify the type of problem

(SCHOOL B) Identify the multiplication problem

A packet of stickers has 5 sheets. Each sheet has 14 stickers. How many stickers are there in total?

A furniture maker needs to make 25 stools. A stool has 3 legs. How many legs will need to be made?

Jane brings 32 freddo frogs to school. She gives out 27. How many does she have left?

I have 24 textas. My friend has 14. How many do we have altogether?

Students are presented with these problems, two are multiplicative. Have a variety of materials at hand, for students to access if necessary. Their task is to determine which of the four problems are multiplicative. Students articulate their thinking and the teacher pushes the students to “be convincing!”
Concluding paths to multiplicative thinking: Appendices

(SCHOOL A)

Present the following Good Questions (Sullivan & Lilburn, (2004)).

Make up a sentence with each word having the same number of letters. How many letters are there altogether?

What might the missing numbers be?

\[ \square \times \square = 36 \]

At a party the lollies were shared. Each person got 3. How many people at the party and how many lollies?

What are some answers you can make to multiplication questions using the numbers 3, 4 and 5 once each?

Students are presented with these problems. Have a variety of materials at hand, for students to access if necessary. Their task is to provide more than one solution. Students articulate their thinking and the teacher pushes the students to “be convincing!”

CONCLUSION ~ Reflective journal writing

Teacher asks student to “Look back over the last week…” and reflect on their experiences and to write what comes to mind.

SCHOOL B: re problem differentiation, read them as a group. Ahmed when faced with the subtraction problem says “there’s five left”. I ask, “What did you do to solve it?” Ahmed says take away. This helped to focus the others as to the type of problem. When the two multiplication problems were identified, I asked, “how can we solve them?” eg, stool problem Cansu suggested drawing the stool 25 times. The efficiency of this strategy was discussed. “What else can we do?” 25 times 3 was suggested. “OK! Do it”. Sticker problem appeared more difficult to understand. Students tried repeated addition. “How many 14’s here?” Students tell me 5. I say, “So 5 fourteen…solve more efficiently?” 15 times 5 suggested. I then worked with them orally, me recording, other repeated addition situations. What is a more efficient way to solve…and so on.

SCHOOL A: Worked through the open questions and students responded in their workbooks. Re journal writing, student encouraged to look back on the last week and reflect on their learning. Douha, Hadi and Yousif wrote insightful comments. Yousif able to compare his work in relation to others. Hadi now understands word problems. Douha felt good because she was able to be the first finished!

WEEK EIGHTEEN

SESSION 41: Monday 17th October, 2005

INTRODUCTION ~ Place value hangman to millions

Students enjoy playing and leading this game. At their request, this game is played frequently from now on.

LEARNING FOCUS ~ How many tiles?

Resources

1 cm grid paper (laminated)

various shaped paper cut-outs to be the mats

Purpose

Use efficient strategies for multiplication

Learning Notes

Teacher introduces the problem, “How many tiles are covered by the mat?”. The 1 cm grid paper represents a tiled floor and the paper cut-outs represent floor mats. Teacher blutaks a regular sized mat to the floor, and poses the question, “How can we find out how many tiles are covered by the mat?” – “Count them?” – “How?” – students are then
encouraged to talk about their strategies. Lead (tacitly) them to the most efficient way, that is, the number of rows, columns, then multiply. Students are then given their own floor and a selection of mats to work with. Provide blutak so that they can place their mats securely to the floor. Students record their methods and solutions in their workbooks.

*How many tiles are covered by the mats?*

---

**CONCLUSION ~ What's my number?**

**Resources**

- 0-99 number chart laminated, one per student
- Whiteboard markers

**Purpose**

- Explore properties of number
- Problem solving

**Learning Notes**

Each student has a chart and whiteboard marker. Sitting back to back, so that the partner’s chart is hidden, each student selects a number and circles it with the marker. Students take turns to ask questions about each other’s number. The answers to the questions should be answerable with a ‘yes’ or a ‘no’. Encourage useful questions such as, “is your number even?” “is your number larger than 50? In comparison to questions like ‘is you number 47?’ have students keep track of the clues gained by marking their chart appropriately or making notes on the space available of the chart.

School B: Irregular mats more difficult for all students, regular examples, students generally used L by W or 10, 8 times, 10, 20, 30…80. Cansu used the strategy of making the mat regular and taking off the excess. This strategy became a model for other students.
SESSION 42: Tuesday 18th October, 2005
INTRODUCTION ~ Place value hangman to millions
Student requested activity

LEARNING FOCUS ~ How many tiles?

Resources
Teacher constructed ‘tiled floor and mats’ worksheet

Purpose
Revise and document strategies for the previous session

Learning Notes
Teacher prepares a worksheet based on the work from the previous session. Each student is presented with four coloured mats pre-pasted on 1 cm grid paper. The task is to determine how many tiles are covered, and for students to explain their thinking for each solution. Worksheet is pasted in their workbooks.

How many tiles are covered by the mats?
CONCLUSION ~ Cross puzzle

Use the numbers 1 to 9 so that each row has the same total

Present this puzzle to the students with a little less scaffolding than previous sessions. Observe what students say, think, do. Guide as appropriate.

SCHOOL B: James evidence of multiplicative thinking and could explain. Dean also but got a bit lost when reviewing and writing about his strategies. Adir also found irregular ones harder to explain. Ahmed said loudly “I got it” when he solved the puzzle and was obviously delighted with himself. Students who didn’t have time to start puzzle elected to take it away with them.

SCHOOL A: Hadi wanted to work “longer” today. We finished 10 min early yesterday and wanted more. Students able to clearly explain and when checking no. in columns/rows, counted by 2’s.

SESSION 43: Thursday 20th October, 2005

Last intervention program session.

INTRODUCTION ~ Number chart activity funny shape c

Number Chart Activity 5 ‘Funny Shape c’

* throw a 10 sided dice to create your starting number

LEARNING FOCUS ~ Paint spill

Resources

1 cm grid paper (laminated)
various shaped laminated paint spill cut-outs
sheets of a4 1cm grid paper
Purpose

Use efficient strategies for multiplication

Learning Notes

Implement this activity in much the same way as for the tiles and mats activities (sessions 41 and 42). This is similar, but requires a greater degree of strategic problem solving. Observe what students do: are they put off by the irregularity of the paint spill, does this inadvertently mean they will go back to count all strategies?… The students may wish to use the a4 1 cm grid paper and trace around each paint spill onto it. Encourage the students to think about their strategies and implement as appropriate.

How many tiles will need cleaning?

CONCLUSION ~ reflective journal writing

Stimulus for writing today is the following sentence starter:

To work out how many tiles would need cleaning I….

SCHOOL B: Students re funny shape, Adir needed to fill in cells not there to keep track. Berrin who has missed many sessions, could keep track of no’s in non existent boxes. More comfortable by ones but beginning to use 10’s. Ahmed and Cansu both worked the chart in both ways. Re paint spill, Dean marked off 5 sections in a paint spill but then counted by ones, cp previous day. James marked out one 3x3 array and counted the rest. Combined written and mental strategies for total. Cansu marked off 2 section approx. half way through spill. No evidence of dots to mark the count by ones, written strategy 24 plus 30 to find total. Ahmed marked off spill into 10+ sections, arrays, but dots in the cells to keep track of the count. Adir marked off sections and those appearing the same size, approximated. Berrin despite missing sessions coped quite well. Made sensible arrays familiar to her.

SCHOOL A: Douha and Hadi efficiently worked the chart in both directions, keeping track of missing cells mentally. Re paint spill, Yousif, marked off arrays, totals in centre. Douha similarly but with some evidence of dots to keep track of the count. Mental strategies for 80 and 28, 108. Hadi as above, but could attend to half
tiles knowing 2 halves was a whole. First spill marked off in rows. I encouraged her to use a more efficient strategy like we did the other day? We had a discussion re the difference in student answers and what might account for that??? They noticed that spills smaller in size had consistently similar answers compared to the larger paint spills, there was a greater degree of difference.
APPENDIX E

THE DRAWING TASK INTERVIEW
Think of a situation in which you are learning maths well. Draw it.
APPENDIX F

DRAWING TASK INTERVIEW RECORDS
PRE-INTERVENTION PHASE DRAWING TASK INTERVIEW RECORDS
Drawing Task Interview notes – Pre-intervention Phase

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<th>Moreland PS</th>
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<tr>
<td>Adir Year 5</td>
<td>At-risk intervention (funded DNI?)</td>
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Field notes (8.12.2004)

1. After reading the nature of the drawing task, Adir mentions ‘plus, minus’ and perceives himself as ‘not a very good drawer’. He also thought that a grey lead pencil would be handy.
2. First Adir draws a black/white board and his teacher writing on it. He says she is teaching us times. Adir then admits to not knowing ‘divided bys’ but knows times, plus and minus.
3. I: How is she teaching you?
4. A: I dunno
5. I: These three boxes, what are they?
7. I: Where are they?
8. A: In my head.
9. I: Where are you?
10. Adir then draws in three green figures and says they are ‘me learning times, me learning minus and me learning plus’.
11. Adir in the course of the interview, states that maths is fun, easy. He perceives himself as not being good at English, but good at maths. Adir likes school and learning, and that learning is important so you ‘can be smart and get a good job’.

Adir admitted to finding English more difficult (17). He has depicted a situation where the teacher is standing at the blackboard, teaching the students (4-5) who are seated at tables. There is a focus on the four processes (1, 5-6, 10, 14-15) and they are written as closed questions on the blackboard. According to Adir, these algorithms also live in the mind (12) of the student. This situation could be interpreted as a traditional transmissive view of learning. Adir communicated a positive view of learning and told me that ‘maths is easy and fun’ (16) and important for the future (18). It was interesting to note that he reported finding maths easy despite being assessed well below expected.
Ahmed did not participate in the SNMY May assessment. He was overseas at the time. DC his teacher last year and in 05 selected him and his parents gave permission. DC feels he would benefit from the work we'll do together.

We engaged in an orienting chat about my work and the task for today. Had trouble reading the word ‘situation’. After clarification seemed comfortable with the task and commenced drawing.

Draw table, chair, worksheet/book with a person sitting at a table.

I: Tell be about your drawing while you are colouring.
A: Sitting at my table by myself doing a maths challenge.
I: What’s a maths challenge?
A: Like a test.
I: This helps you to learn maths well?
A: Yes.
I: Why does this help you?
A: I don’t know…because I can learn.
I: What are you learning about?
At this point I tuned him in to the task and reminded him about what he’d told me so far.
I: What’s on the maths challenge?
A: Like 2 times 2 is 4.
I: Is that the only thing on the sheet?
A: Like 20 questions.
I: How do you feel when you are doing it?
A: Good.
I: Is there anything missing from your picture?
A: The hands (missing in his picture).
I: Anything else?
A: No.
I: What do you like about maths?
A: Times, and plusses…and take away.
I: What do you use?
A: Hands, paper, that’s all.
I: Where does this happen?
A: In class.
I: Who else is there?
A: The whole grade does it.
Ahmed had chosen to draw himself working on his own doing a ‘maths challenge’ (9) as he called it. All the students are doing the same work (39) individually answering twenty multiplication algorithms (21) on a worksheet (23). Students are given feedback about their performance on these questions approached through whole class correction, where students correct each other’s work (46-47). The successes and failures of students would therefore be transparent to each other. Ahmed’s view of mathematics work focussed on written algorithms and the four processes (21, 31). It is implied that working in ways other than described, is a source of distraction (41). This student however, acknowledged positive feelings associated with the work (24-25) and that the teacher is there as support (43) when something is not understood. When prompted to think of anything that might be missing from picture, Ahmed could think no further than to suggest that his hands were missing (26-27).

| 38 | I: What are they doing? |
| 39 | A: The same thing. |
| 40 | I: When you do all the same thing, this helps you to learn maths well? |
| 41 | A: If they’re doing something else and they talk, I get mixed up. |
| 42 | I: What else helps? |
| 43 | A: If you don’t know go tell teacher. |
| 44 | I: Then what happens? |
| 45 | A: (tells that students are given more time…work might be swapped for correction. Teacher calls on students for the answers. If someone is incorrect, then the teacher corrects the situation). |

Field notes (8.12.2004)

1. Berrin had trouble understanding the phrase learning maths well. We chatted about learning other things well and compared it to not learning those things well, e.g., swimming. She seemed satisfied with that and continued with the task.
2. Berrin asked about ‘plus signs’.
3. She drew herself centred with a big smile on her face and draw plus, minus and times symbols around herself. She told me these were ‘plus, take away and time table’. Berrin then added ‘divided by’.
4. I: What are you doing with the plus, times, take away…
5. B: Learning them.
6. I: How?
B: In my head (mentions brain)… on a piece of paper… maths…
I: So you like working with maths on paper?
B: Yes.
I: Why does working this way work for you?
B: Because you learn more.
I: Why?
B: You can use your hands to count… use counters… blocks… little toys like farm toys…
I found Berrin needed to be probed further as she had a tendency to give closed, non-elaborative answers.

Initially, Berrin had trouble comprehending the concept of *learning maths well* so we discussed this idea further (1-3). She pictured herself with a smile on her face (5). She displayed a strong focus on learning the four processes (4-7) evident both in her picture and verbally. She had difficulty elaborating further as to how learning took place. She suggested that learning involved thinking (11) written responses (11) with the possibility of using maths equipment (17) suggesting the need to work with models to represent mathematical situations. According to Berrin, she felt working this way meant gaining knowledge – ‘because you learn more’ (15).

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<th>Moreland PS</th>
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<tr>
<td>Cansu Year 5</td>
<td>At-risk intervention</td>
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Field notes (8.12.2004)

1. Cansu was communicative, willing and pleasant. She had recently returned from a five month stay in Turkey.
2. I: Tell me about your drawing.
3. C: Our teacher is teaching us fractions. I put my hand up to ask a question so I can learn more. I’ve shown my table with maths… pencil case, pencils, maths book.
4. I asked about the maths books as she said it was a text but didn’t know the title, ‘Not sure of the name’.
5. I probed further about the importance of asking questions.
6. C: When you ask a question you get more information about maths…
7. Cansu said she drew the teacher and showed herself wanting to ask about different ‘maths styles’
8. I: What do you mean about different maths styles?
9. C: Like hippopotamus or something like that, my brother says it is too hard.
10. I: Maybe you mean hypotenuse?
Cansu acknowledged that she has some difficulty learning mathematics (20-21). The situation that she drew involved the teacher positioned near the blackboard teaching fractions (4). Cansu had access to maths books possibly a text book (6-7) and other writing material (5). This seems to suggest a traditional transmissive view of learning. Though she hadn’t drawn them, other students, doing the same thing as her, were considered to be present in her situation. Cansu indicated the importance of asking questions (4, 9) as the goal is to gain more information (9). She demonstrated an awareness of other aspects of mathematics, such as geometry (13, 14) through discussion with her brother in secondary school, who finds geometry difficult (13).

Field notes (8.12.2004)

1 I observed Dean to be pensive and thoughtful, carefully considering his answers to my questions without necessarily answering quickly.
2 D: Can it be up in the classroom?
3 I: Can be but doesn’t have to be.
4 Dean commenced by drawing a blackboard with times algorithms displayed. He drew a small table with a worksheet reflecting what was written on the blackboard.
5 He then drew a thinking bubble from a smiling figure (representing himself). The bubble says ‘time to learn’.
6 After Dean had finished his picture I asked ‘Is there anything missing?’
7 D: No.
8 I: What is it that makes you learn maths well in this picture?
9 D: If you want to learn you have to study.
10 I: What’s study?
11 D: Like you work on something.
12 I: Do you need to use anything?
13 D: You need paper for times tables until you know them…pencil to write…that’s it.
Dean had drawn a blackboard with multiplication algorithms on it (5) and this is repeated in the worksheet (6) placed on the desk. It appeared that Dean believed in the value of writing answers to these questions until they are known (16). The teacher is not present in his picture and wasn’t mentioned in the dialogue. Dean’s view of learning seemed to reside more with himself, through thinking (7) and having to work at maths through studying (12-14). This appears to be in contrast to a view where the teacher is primarily responsible for student learning.

SCHOOL B PS
28.4.05
James Year 6 At-risk intervention

Field notes (28.04.2005)
1 I met James for the first time this year on this day. He appeared reluctant and nonchalant with an apparent closed body language (slumped in his chair, arms crossed, lack of eye contact).
2 James tells me that he’s not good at maths though. I tell him that that does not matter.
3 J: Can’t draw it.
4 I: Tell me about it then? Where is it happening?
5 J: Classroom.
6 I: Classroom with you in it, what else?
7 J: Chair….desk.
8 I: Perhaps start drawing that. (James does and he then draws items on the desk)
9 I: Tell me about what’s on the desk.
10 J: Papers, pencils.
11 I: Tell me more about the papers and pencils.
12 J: You use them.
13 I: What for?
14 J: Work.
15 I: Do they help?
16 J: Yes.
17 I: How?
18 J: To finish it.
19 I: What?
20 J: The work.
21 I: What might be on the paper?
22 J: Maths, English or something.
23 I: Does English help with maths?
J: No.
I orientate James to the original purpose of the task, ie, situation in which you are learning maths well.
I: Is there anything missing from the situation?
J: The class
I tell James that he can write this down if he likes and ask him to think of three or four more things to add.
He indicates the teacher and other kids. It is then that he starts to offer more detail. He tells me that teachers teach you what to do, the classroom is where you work and the other kids do the same thing ‘as me’.

James appeared resistant and reluctant to communicate. He appeared to hold a negative attitude toward his ability in mathematics (1-3, 4). Before he commenced drawing, I probed further about what the situation might be about. His response focussed predominantly on the environment, that is, where the work is done (35), the furniture, writing equipment and the ‘work’ itself (8, 10, 17-23). According to him, the aim is ‘to finish it’ (21). He acknowledged that the class, who all do the same work (34), and the teacher who teaches what to do (35), were missing from his drawing.

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<th>North Brunswick PS</th>
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<td>Douha Year 6</td>
<td>At-risk intervention</td>
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Field notes (30.03.2005)

1 Really enthusiastic and eager to begin working with Miss MD and myself. Very chatty. During our chat she tells me she’s not good at her tables and is sometimes too embarrassed to ask for help. She seems eager to start learning.
2 Demonstrated an understanding of the task. First she drew addition algorithms ‘good at these’. I encouraged D to draw more of the ‘situation’. Like H before her, asked if it was ok…Can I … a number of times. This seems commensurate with lacking in confidence? I tell her it is all ok with me.
3 (Then she draws 4 tables, children in groups). D…this is my table here and I’m going to draw a piece of paper with the addition sums on it.
4 D: Is that how you want it?
5 I: I want it how you want it.
6 D: I forgot to do the teacher…she’s got a worksheet.
7 I: What helps you to do these worksheets well?
8 D: If 10 + 9 I sometimes use my fingers or a scrap piece of paper and make 10 lines and 9 lines and count them up.
Douha expressed eagerness to begin learning with me the following term (1, 3). She chatted easily and admitted to not being good at her times tables (2) and is often too embarrassed to ask for help (3). She appeared to lack confidence (6, 7, 10) given that she asked a lot of questions about whether her ideas were acceptable for the task. Her situation showed a classroom with students sitting in table groups (8). They were working on a worksheet of addition algorithms (4, 9) which she reported being ‘good at’ (4-5). The teacher was mentioned as a presence in the situation(12). When I probed further as to what helped her learn, Douha explained her ‘make all, count all’ approach (14-15) and also acknowledged that friends can help if they provide guidance, not simply tell the answer (19-21).

North Brunswick PS
30.03.05
Hadi Year 6

At-risk intervention

An orienting discussion about the drawing task, with H reading aloud the instructions. She tells me she ‘doesn’t get it’. I ask her what specifically? ‘Situation’. We talk about a time she can recall when she was learning maths well. As she begins, she asks lots of questions as to whether it is ok for her to:
Can I...?
Can I...?
Can I...?
Shall I write the answers?
I reinforce that she’s in charge and she can show her response in any way she likes.
I encourage H to draw what the learning maths well situation looks like. She had only listed some aspects. H adds a figure at a desk (later determined as the teacher) and another desk with two students. She asks ‘Shall I tell you?’ H elaborates on the ‘Around the world’ game.
The teacher is asking questions and the students have to answer, first person to answer gets to move on. If you get all the way round you get 50 points. H tells me she has nearly made it all the way twice and enjoys the game. I probe as to how the game helps her to learn maths well.
Hadi initially asked a number of questions about whether her ideas were appropriate for the task. At first, she simply listed some ideas, such as ‘BODMAS’, an addition sum and a multiplication sum. I encouraged her to draw more about what the situation ‘might look like’ (9-11). She added students sitting at desks (11) playing ‘around the world’ (13). This game involved gaining points if individual students responded with the correct answer. Hadi explained that what she doesn’t know, she learns at home (18) with assistance from her mother who provides more algorithms for her to do (19). It is interesting to note that the teacher or school-based assistance or support was not mentioned, but home was. In general, much of what Hadi spoke and drew about, was in relation to rules, procedures and closed questions (21, 22).

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**Brunswick South West PS**  
**Yousif Year 6**  
**30.03.2005**  
**At-risk intervention**

**Field notes (30.03.2005)**

| 1 | MD tells me he is a little reticent about the whole experience. I was mindful of his sensitivities and provided lengthy think time if necessary. We had a chat prior to task to break the ice a little. Y not especially talkative. Read the DRAWING task and seemed to get the idea. Y proceeds to draw himself near/at a table then adds a blackboard with division and addition algorithms/questions on it. He tells me he’s doing sums, at his table, from the blackboard. |
| 2 | I: Is there anything missing? |
| 3 | Y: (adds a book and pencil) |
| 4 | I: What else is around? |
| 5 | Y: Other kids learning maths in the grade. |
| 6 | I probe further about what else? |
| 7 | Y tells me about books and charts after further probing. I ask him to show me examples and he points to the tables chart above the blackboard. |
| 8 | I: Do these help you to learn maths well? |
| 9 | Significant wait time provided but Y could not elaborate further. I didn’t push. |
MD tells me Yousif is a little reticent about being involved in my visits. I was mindful of his sensitivities and provided lengthy think-time if necessary. We had a chat prior to task to break the ice a little (1-4). The situation that Yousif represented is one where the students, including himself (4, 10) are seated at tables (4) apparently working on some addition and divisions algorithms (5-6) displayed on a blackboard. He mentioned other aspects of the environment (12) but is unable to elaborate further as to how these help him to learn maths well (15).

Field notes (11.05.2005)

1 Andrew and I engage in an orienting discussion about the task and Andrew questions me about what I expect. He seems happy to proceed when I say ‘you can’t make a mistake with this task’. He draws the picture shown above.
2 I: Tell me about your picture.
3 A: I am sitting with students at tables and Mr G is teaching us maths.
4 I: Tell me more about ‘Mr G teaching us maths’.
5 A: He shows us how to do the maths question.
6 I: Tell me more about him showing you…
7 A: If I don’t get it he shows me other ways to do it.
8 I: What are you doing in your picture?
9 A: Listening to what he is saying.
10 I: In this situation, how are you feeling?
11 A: dunno…focussed.
12 I: Tell me more about the work you are learning…what’re you doing?
13 A: Can’t remember.
14 I: What is it about this situation that is helping you to learn maths well?
15 A: Mr G.
16 I: What’s the best thing about Mr G’s class?
17 A: He shows us ways to do it on the board.
18 I: So he supports you.
19 A: Yes.
20 I: Tell me more about the other kids in your picture.
21 A: They’re focussed also…listening…not chatting.
22 I: Is that important?
23 A: That they’re listening? Yes.
Andrew has drawn himself in a secondary mathematics classroom. The students are sitting at desks and the teacher standing near the blackboard is teaching (5). Andrew described a procedural approach to teaching with the teacher showing and talking about how to do the question (7, 15) and when difficulty arose, showed different ways on the board (21). Andrew and the other students pay attention (11, 25) are focussed (11-15, 25) and work without chatting (25). Learning maths is viewed as being useful in the future (29).

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<td>Candi Year 6</td>
<td>At-risk non-intervention (BSC in '05)</td>
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Field notes (8.12.2004)

1. Candi is from Indonesia and came here in 03 and spent time at the Language Centre where he tells me he made ‘new friends’. He tells me about Arabic class and his teacher ‘Miss G’. He is a little vague but tells me that the class is “like normal” and spoke English.
2. C: We were practising ‘time table’ with Miss G.
3. I: Where is Miss G?
4. C: Not sure…three weeks ago…
5. His drawing is about a tables grid and the grade five students have 6 mins to complete it and grade sixers have 5 minutes
6. I: Tell me about your drawing.
7. C: I was doing time table grid…practice a lot…at the start I feel nervous then I do it, it was correct..79 I was happy. I wrote in my Learning Journal ‘I achieve my goals’…and the teacher was happy because I achieve my goals….my lowest score…mmm…52…Miss G tell me easy way…like
8. But unfortunately was unable to express further. Might be a good idea to follow this up.

Candi drew a picture of himself practising the times tables (5, 11) and described feeling happy when success was achieved (12-13). Candi’s situation involved a teacher, who described ways to makes things easier (14) and is pleased with his progress (13). The use of a learning journal is mentioned as evidence of encouraging students to reflect on their learning (12-13). Nervousness is mentioned in relation to completing the times tables grid (11), I suspect due to poor performance and low scores (12-14), but happiness, when he had done well (12).
Field notes (8.12.2004)

1. General discussion about the task and I used the analogy of looking through the window, what would I see…
2. H: Not a very good drawer
3. (But I think she is!)
4. H: That’s a blackboard. (Hajar is writing English, Maths, House points).
5. She draws a girl (Hajar) on the left of the page smiling and adds another girl ‘my friend’ playing dominos (fraction matching) then adds another friend to the picture.
6. H: Sometimes the teacher puts us in mixed groups.
7. I: What does this mean?
9. Hajar adds windows, doors and tells me ‘we’re at the front of the room near the blackboard’. Hajar tells me that it is better on the floor because the chairs aren’t always comfortable and that there is more space out the front.
10. She adds desks and on the desks, draws pencil case, drink bottle and ruler/pen etc.
11. H: When I work by myself I get mixed up…just a little mistake with the numbers.
12. Hajar tells me about the resources in the room and that her teacher (Maureen) ‘likes maths, I don’t’.

The physical appearance of a classroom is well represented in the drawing by Hajar: the blackboard (5), the teacher (8) who ‘likes maths’ (16), and other students (6-7). In particular, Hajar, who doesn’t like maths, and some friends are playing a fraction matching game with dominos (6-7). They are seated on the floor, which is more spacious and comfortable than sitting in chairs (12-13). The way in which the teacher grouped the students is apparent, in particular mixed gender groups (8-10). I was familiar with the grouping strategies used by Hajar’s teacher and believe Hajar would have been used to working in various group arrangements. Hajar admitted to doing better, when she doesn’t work on her own (15).
Brunswick South West PS  10.12.04
Ibrahim Year 5  At-risk non-intervention

Field notes (10.12.2004)

1  Orienting chat about the task.
2  Ibrahim is drawing himself with teacher and speech bubble saying ‘excellent’.
3  Ibrahim is saying ‘Yes’.
4  I: Why?
5  Ib: Because I get helped.
6  I: Help?
7  Ib: Times tables, first 8, 16 then I close book and I practice and I got better.
8  I: Where is this happening?
9  Ib: Our class…I showed the teacher my paper and she said ‘good’.
10  I: Good, how did you fell?
11  Ib: Good…I feel proud of myself.
12  I: What else helps you to learn maths well.
13  Ib: Not sure.

Ibrahim’s picture looks as if it was drawn by a much younger student. He had drawn himself and the teacher, who is providing Ibrahim positive feedback (2-3, 9). He mentioned in particular, the teacher assisting him with his knowledge of table facts (5), through what appeared to be a skip counting approach for the 8s facts (7). The strategy for learning the multiplication facts, of closing the book and practising (7) led me to think that Ibrahim may have been skip-counting or reciting the facts in his head. He identified using this approach as a source of improvement (7) and with improvement, a sense of pride (11).
Constructing paths to multiplicative thinking: Appendices

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<td>Sidona Year 6</td>
<td>At-risk non-intervention (BSC in '05)</td>
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Field notes (8.12.2004)

1. Discussion about the terms situation, learn, maths well. Sidona tells me that ‘I learnt maths on the computer…playing games…
2. I: Where?
3. S: At home.
4. I: You can draw that.
5. She draws a computer screen with ‘7 x 3’ a keyboard, mouse and Sidona smiling.
6. All of this is placed in an imagining bubble
7. I: Tell me about your picture.
8. S: Imagining playing game on the computer. Might teach me maths if I get it wrong or right.
9. I find out that she is talking about games on CD.
10. I: How does the computer help you?
11. S: Doesn’t tell you the answer…try again…teaches you times tables.
12. I: How?
13. S: Puts up questions and tells you if you are wrong and until you get them right.

Sidona has located her learning maths well situation at home (4), where she is playing a maths game on the computer (2, 6) designed to teach the times tables (18). The program gives immediate feedback, if her answer is wrong, the program gives the instruction to try again. This aspect is identified as helping her to learn maths well (14-15, 20).
Field notes (8.12.2004)

1. Without prompting, Adamander, having read the direction at the top of the page, states ‘I think it should be fun so you can absorb it…’
2. ‘…they’re happy and it sinks in…’
3. ‘…it’s better to be social about it so you can share what you’ve learnt and talked about and give it to someone else…’
4. ‘…it’s better to talk about it instead of (a teacher saying) shut up and be quiet…’ He’s talking while drawing and I then ask him, where is this happening. His response: ‘In a very happy classroom…hard to find…because people are either high on the social chain or low on the social chain. When I questioned Adamander further about what he meant, he felt that ‘they either think that they’re too good’ (referring to those high on the social chain). He perceived himself low on the social chain and that people see him more as ‘a tool than a person’. People come to Adam for answers not help, but Adam would rather help. He expressed that it is rather ‘sad when students can do the work…it is almost as if you can’t be socially healthy and smart at the same time…” (admitting that this may be a ‘bit of an exaggeration’). He then went on to tell me the value of competitions in maths because ‘hormones kick in and it is a natural reaction to be competitive’. Adam tells me about table squares grid game where he competes with himself and others of similar ability, being both helpful and encouraging…” I didn’t win”…and that this doesn’t matter.

The culture of the classroom and social status (11, 14-15) was the focus of our discussion. Predominantly, Adam spoke of the value of communication for learning mathematics well (4-6, 12-13), in particular, to share what has been learnt (4-5) and discussed (4). As a capable student, I sensed a certain level of frustration with regard to other students who seek Adam’s assistance. Adam would like to help, not just give the correct answer. (12-13). Additionally, Adam believed that learning maths for him, should be enjoyable, as this would enable students to absorb it (2-3). The value of personal and group competitiveness was also mentioned as being helpful and encouraging (16-19).
Initially Alison was concerned as to the standard of her picture and rubbed about and started afresh. As her second attempt progressed, I told her it was ‘looking good’ so as to encourage her to continue.

Alison first drew a blackboard, then table, two students seated facing the blackboard with their hands up. The teacher (smiling) and writing was drawn on the blackboard. The marks on the blackboard represent the teacher’s writing and a ‘diagram to help explain’. The teacher is shown pointing to the writing. Another student is added and a non-descript speech bubble is placed near the teacher’s mouth. Alison tells me the teacher is answering a question. Exercise books and pencils etc are added to the students.

I asked Alison to tell me about her picture.

A: (sic) In a classroom and the teacher is explaining and some students are working, some are listening and some are listening while working. Some students are commenting (see hands up) on what the teacher and other students have said…if they agree or not…by putting up their hand…On the table are pens, pencil cases, rulers ‘they all need rulers’.

I: Why?

A: Rule up margins, measure things and draw lines…and a maths book.

I: What type of book?


Dialogue between teacher and students is represented in this picture. The happy teacher (5), is explaining (12) the work and answering students’ questions (9), and is also allowing students to comment on (14) and judge (15) what the teacher or other students have said. In addition to explaining the work orally, the diagram on the blackboard is there to support student learning (7). The students were equipped with books and writing materials (9-10, 15-20) in this situation.
Field notes (10.12.2004)

1. No trouble with nature of task, Chris thought then drew.
2. I: Tell me about your drawing.
3. C: It’s when no one’s speaking, can speak…whisper…when no one’s being silly…you can concentrate.
4. I: Who else needs to be around? What else do you need?
5. C: Someone next to you – ask, or ask a teacher if you don’t understand.
6. I: If teacher/friend can’t help you what then?
7. C: If there’s a lot of questions you could skip it.
8. I: Where’s the maths in you drawing?
9. C: If there’s a lot of questions you could skip it.
10. Chris explains that there are questions on paper with an example to help eg ‘213 divided by 3’.

This student believed in the benefit from a quiet classroom (3) as this facilitated greater ability to concentrate (4). Learning maths well according to Chris meant doing the work alone and asking for assistance from the teacher or friend when ‘you don’t understand’ (6). If help is not available, Chris suggested that an alternative might be to skip some of the questions (8) if there were many of them. The maths work appeared to be in the form of numerous closed questions, eg, a division question, written down on paper as a worksheet (10).
Field notes (10.12.2004)

| 1 | Orienting chat ‘can it be at my old school?’ ‘Homework?’ ‘Can I label it?’ |
| 2 | ‘Happened, might happen?’ |
| 3 | I: Tell me about your drawing. |
| 4 | K: I used algebra…no one knew what I was writing. |
| 5 | The chicken problem (weight of three chickens) was set for homework. |
| 6 | I: Why did this situation work for you? |
| 7 | K: If a plus b equals whatever, if a plus c equals whatever then this equals that. |
| 8 | Because it also helps you…when you do stuff like that at secondary school so then |
| 9 | Dad showed me a heap of problems. |
| 10 | Kai gave example on drawing x plus y plus z equals10 etc. |
| 11 | I: But why? |
| 12 | K: Better with maths problems in equations that helps. Dad helped, showed me |
| 13 | division with fractions. |
| 14 | I: Dad important, why? |
| 15 | K: He doesn’t tell me that answer, encourages me to find the answer. |
| 16 | I: At school? |
| 17 | K: Problems aren’t as hard, equations I can do…just learnt to do division really |
| 18 | quickly…just knowing if I don’t understand then ask the teacher who won’t tell, but |
| 19 | guide you. |

Kai’s situation depicted a time when the maths he had done, was inaccessible to the other students (4). He had used algebra to solve a maths problem that was set for homework (5). He presented algebraic thinking (7) as being useful when solving various problems (10, 12) as well as for further schooling (8). Both his father and the teacher, featured as figures of support in his learning (8, 12-15) as they don’t provide the answers (15) but guidance to find the answer (18-19).

Brunswick South West PS 10.12.04

Kalil Year 5 Successful

Field notes (10.12.2004)

| 1 | Orienting chat using window, what would I see analogy. Kalil is meticulous and |
| 2 | gives attention to detail (and takes his time with drawing). |
| 3 | KS: Blackboard, we’re learning about prime numbers and I’d be here. |
| 4 | I: Tell me about your drawing. |
| 5 | KS: Me sitting meant to be more tables in the background…our teacher is telling us |
| 6 | what prime numbers are. |
The blackboard is a dominant feature of this student’s picture. Kalil is learning about prime numbers in this situation (3), based on what the teacher is telling the students (5-6, 9). According to Kalil, learning is happening because the subject matter is new to the students (8) and because the teaching is imparting this information (9). Kalil mentioned that it was unusual for him to have difficulty when learning maths(10) but when he did, he seeks assistance from people around him (11).

Field notes (10.12.2004)

1 I: Discussed nature of task. Lynda asked numerous clarifying questions. ‘Do I draw the problem’, ‘In school?’ ‘Or somewhere else?’ In particular, explained the term ‘situation’. I: Wherever you like.
2 L: Then Lynda tells me about her dad - retired teacher - bringing home worksheets.
3 I: Do I have to draw the surroundings?
4 L: Give me a sense of them.
5 I: Tell me about your drawing.
6 L: My dad (worksheet from school – times tables) helping me to practice.
7 I: I ask about the worksheet further, she thinks dad teaches prep? 1? 2? And she is about 5/6 yr old? Not sure. She thinks the sheet has 1 times and a ‘little bit of the 2 times table’.
8 I: Why are you learning well in this situation?
9 L: Dad is helping me…
10 I: How?
11 L: Brings home, not just maths, but books, help me read. When I started school – you’d chat to him – about maths at school.
12 I: Talking important?
13 L: Explaining helps me.
14 I: What if explaining doesn’t work?
15 L: I try to figure it out on my own.
Lynda gives the example of not understanding division straight away nor ‘carrying’ in grade 1. She was confused.

This situation illustrated a parent’s role in student learning (4, 7, 9, 10, 14). The student’s father, a retired teacher has provided worksheets for Lynda to do at home (4, 9, 16). Lynda explained that her father provided assistance (14) through giving explanations (19). If for some reason, his explanations were not useful, Lynda would persevere on her own (21). She illustrated this by saying that she did not understand division or the procedure of ‘carrying’ straight away (22-23).

Brunswick East PS  
Jessica Year 5  
10.12.04  
Successful

Field notes (10.12.2004)

1 J: What type of situation?
2 I: Depends on you.
3 J: Do you mean addition, subtraction etc?
4 I: Can be but doesn’t have to be.
5 Orienting discussion takes place.
6 Think time.
7 Then Joanne starts drawing.
8 I: Tell me about your picture.
9 J: Whiteboard with addition and subtraction sums using fractions.
10 I: Why is this situation helping you learn maths well?
11 J: I like working from whiteboard or blackboard in comparison to worksheet…more comfortable…I think the teacher knows what they’re doing where as on a worksheet it is hard to tell.
12 I: The teacher is important?
13 J: Yep because in order for me to learn I need someone who knows what they’re doing.
14 I: Where are you in this picture?
15 J: (adds herself)
16 I: Are you the only one in the picture?
17 J: Probably not…mostly kids in my class, friends and the teacher.
18 I: Joanne recognises that we all think different things.

The maths work shown in this situation is presented on a whiteboard (9) as addition and subtraction of fractions algorithms(9). Jessica explained that she prefers to work off the
whiteboard because to her, the teacher’s knowledge is more credible in this form (11-12), in comparison to work on a worksheet where ‘it is hard to tell’ (13). What was also important to learning maths well, was working with a more competent other, this could be the teacher (14), though not necessarily (15).

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<thead>
<tr>
<th>Brunswick East PS</th>
<th>10.12.04</th>
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<tbody>
<tr>
<td>Sandy Year 6</td>
<td>Successful (BSC in ’05)</td>
</tr>
</tbody>
</table>

Field notes (10.12.2004)

1. Orienting discussion takes place.
2. I: Tell me about your drawing.
3. S: This picture is what I see…she’s teaching the basics first. Later going on to harder ones. Showing…how to do fractions.
4. I: What is it about this situation that is helping you learn maths well?
5. S: Writing up the question and goes through it…shows us step by step a few times so we remember it.
6. I probe about what is maths. Her response encompasses a lot of subheadings, fractions 4 symbol things, numbers usually, a subject like English, useful for learning – shopping. I probe further. Building houses perimeter, area amount of timber, we use it all our lives.

Sandy’s view of learning is through the provision of developmental activities. She gave a fractions example, where the teacher is teaching the ‘basics first’ (3) with a view to giving harder ones later (4). The maths work, which featured numbers and the four processes (9-10), was in the form of questions that are written (6) then reviewed (6), with the teacher showing the solution process, step by step (6). According to Sandy, this process means that the students will recall what to do (7). She considered maths a subject to be studied at school but recognised that maths was also useful for daily life (11).
### POST-INTERVENTION PHASE DRAWING TASK RECORDS

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<th>SCHOOL B</th>
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<tbody>
<tr>
<td>Adir grade 6</td>
<td>At-risk intervention</td>
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</tbody>
</table>

#### Field notes (29.11.2005)

1. A: I’m doing times and plusses like 5 times 5 is 25…work…I’m getting better at it.
2. If my friends need help, I help, if I need help, they help. If don’t know answer, tell the teacher.
3. I: What else is shown in your picture?
4. A: I’m working at maths.
5. I: What are you doing?
6. A: Working…out the answer.
7. I: What’s at your table?
8. A: Work, friends, working out, tables and chairs. (Adir’s picture shows times tables).
9. I: Are these charts?
10. A: Yes.
11. I: What is it about this situation that is helping you to learn maths well?
12. A: Because before I wasn’t good at times but now I’m better at it, at maths, before not good, now better.
13. I: What has made you better?
14. A: Before not good, I need help but now better…you help me.
15. I: So how do you feel about maths now?
16. A: Good and smart and…yes.
17. I: When did you notice this?
18. A: Before…
19. I: About how long ago?
20. A:…3 months.
21. I: So this year?
22. A: Yes…last year not good, but now my mum and you, Miss P, my friends help me.
23. I: That’s great!!! Thanks!

The focus of our post-drawing discussion was Adir’s perception of his own improvement (1, 14-15, 17, 19). He had drawn himself and his friends working at calculating the answer (7) to the maths (5) work. The worksheet in the picture showed multiplication algorithms with ‘ticks’ to indicate that the answers were correct. There were tables facts charts on display in the classroom. Adir identified that the work we did together during the intervention program, his class teacher TP, his mother and his friends were sources of his
improvement (17, 25-26). He appeared to feel empowered to help other students as well as being helped by other students (2).

Ahmed Year 6

At-risk intervention

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>I: Tell me about your picture.</td>
</tr>
<tr>
<td>2</td>
<td>A: Whole class in this class (that is our Intervention group). You giving the</td>
</tr>
<tr>
<td>3</td>
<td>worksheets out and we’re doing them.</td>
</tr>
<tr>
<td>4</td>
<td>I: What is it about this situation that is helping you to learn maths well?</td>
</tr>
<tr>
<td>5</td>
<td>A: Like the second day we came to you, you understand it good to me.</td>
</tr>
<tr>
<td>6</td>
<td>I: So this picture is about the second day we got together?</td>
</tr>
<tr>
<td>7</td>
<td>A: Yes.</td>
</tr>
<tr>
<td>8</td>
<td>I: Point to the most important part of your picture.</td>
</tr>
<tr>
<td>9</td>
<td>A: (Points to all of it).</td>
</tr>
<tr>
<td>10</td>
<td>I: Tell me more about what is happening</td>
</tr>
<tr>
<td>11</td>
<td>A: You’re at the board and we’re doing our work.</td>
</tr>
<tr>
<td>12</td>
<td>I: How do you feel now about learning maths?</td>
</tr>
<tr>
<td>13</td>
<td>A: Good.</td>
</tr>
<tr>
<td>14</td>
<td>I: Tell me more about how good you feel.</td>
</tr>
<tr>
<td>15</td>
<td>A: In class now, we do tables grid, and now I have my certificate.</td>
</tr>
<tr>
<td>16</td>
<td>I: Fantastic</td>
</tr>
<tr>
<td>17</td>
<td>A: And my division.</td>
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</tbody>
</table>

Ahmed has created a picture that represented the work that we did together during the intervention program. He identified that I assisted him very early in the program and helped him to understand (5). He had drawn our small group of at-risk students and mentioned (though not drawn) that I am at the board and the students are working (3, 11). He reported feeling good about learning maths (13). Ahmed illustrated that this had impacted on his work in the classroom with DC, with respect to gaining a multiplication facts certificate (15). When asked to indicate the most important part of his drawing, he indicated ‘all of it’ (9). It is interesting that no one feature stood out, but that the situation may have been valued as a cohesive whole.
Field notes (29.11.2005)

1. B: I draw Mrs P, she learning us times table, plus and these, (Indicates drawing of MAB) thousands, hundreds and tens.
2. I: What is it about this situation that is helping you to learn maths well?
3. B: To get better at times tables…
4. I: Why is that?
5. B: Because when you go to high school you have to be quicker and answer quicker, and quicker and quicker.
6. I: Point to the most important part of your picture.
7. B: (points to times tables).
8. I: Why did you choose times tables?
9. B: Because times tables, more important to learn quicker.
10. I: What are you doing in your picture?
11. B: I am doing my work.
12. I: Where is this happening?
13. B: At table.
14. I: Tell me more about you at your table.
15. B: Got my maths book, times table chart and pencil case…
16. I: Tell me more about the times table chart?
17. B: Have to learn up to 100.
18. I: What does the chart look like, where is it from?

Knowing the multiplication facts was a key concern for Berrin (1, 4, 9, 11, 17). In particular, she emphasised that speedy recall of table facts was important, especially when going to high school (6-7, 11). Given that secondary school orientation day was scheduled for approximately one week after this interview, this may be the reason that this was in the forefront of her mind. She had drawn her classroom teacher, TP teaching addition, multiplication and place value (1-2). Berrin who is smiling, was shown to be doing her work at her table (13-15). She had access to her maths book, tables chart and writing equipment (17).
I: Tell me about your picture?
C: First I was thinking...of when I was doing my PAT maths test and Mrs P says to me 'you're doing really well'.
I: So what is about this situation that is helping you to learn maths well?
C: (at this time) I could ask Mrs P if I didn’t understand, and I had the times table chart that you gave me.
I: How did the chart help?
C: For the times table, the PAT test had lots that I didn’t understand and I ask Mrs P if it’s ok to use my chart and she said yes.
I: Ok tell me more about what you’ve drawn.
C: Chairs, tables, my Pat test paper, pens...and my maths chart.
I: So how do you feel about maths now?
C: That I achieve a lot, Mrs P tells me I got 31 (on PAT test) and at the beginning of the year I got 7.
I: Why do you think you improved so much?
C: I was practicing with my brother at home...
I: Tell me more...
C: My brother asks me questions, times and divided by’s and when we have homework and it has problem solving, my brother used to help me.
I: Why do you think you changed this year?
C: I’ve gone smarter...because I’ve been practicing with you.

A key theme for this interview with Cansu, was the degree of improvement she had made that year. The picture depicted Cansu doing her PAT maths test recently administered by TP as part of her assessment and reporting procedures (2). Cansu reported that she scored poorly at the beginning of the year but that her end of year result was a significant achievement (13-14). She believed that assistance from her brother (16, 18-19) and the work that we did together during the intervention program (21) were the reasons for this improvement.
Field notes (29.11.2005)

1. I: Very detailed Dean, tell me about your picture.
2. D: When it was with you…we were working together learning our times tables and you got us to write some things down in our books…Some one is asking you for help. You say ‘ok’ and someone is saying ‘cool’.
3. I: How were we learning our tables?
4. D: You wrote them up on the board and then you asked us to write them in our books and work out the answers.
5. I: How did you work out the answers?
6. D: We work out the answer by seeing how much 2’s in 5, 4’s in 5 (see picture for algorithms on blackboard).
7. I: What did you use to work them out?
8. D: Mmmm…
9. I: Any equipment?
10. D: No.
11. I: So what is it about this situation that is helping you to learn maths well?
12. D: Because you were…a maths teacher and because of the sums on the board, and we just worked it out with your help.
13. I: I can’t remember ever putting sums on the board like that…did we do that often?
15. I: Can you point to the most important part of your picture, what did you point to?
16. D: I pointed to the books…the work books you gave us to write in, because they’ve got the information and the answers in it.

Dean chose to depict a situation from our intervention program (2). He mentioned that ‘we were working together learning our times tables’ (2) and this involved writing in ‘our books’ (3). A student is apparently asking for my help (3, 17) and this help is appreciated (4). I was considered helpful because of my ‘expert status’ as a maths teacher(16) from a university. What is interesting though, is that what is described as happening, did not happen in this way. Dean reported that I wrote maths questions as algorithms as illustrated, and that I had asked the students to write these in their books and work out the answers (6-7). We did not work this way at all. The focus for learning the multiplication facts was on efficient strategies, for example, double and one more group for the 3’s facts. These strategies were modelled and done mentally, and on no occasion did students copy algorithms from the board to be answered in the written form. I was a little confused by this perception at the time of the interview and decided to explore whether Dean
acknowledged use of maths equipment (11-13). Dean said ‘no’ (14) even though cuisenaire rods were used to model various strategies. I suspect that what has prevailed in his mind, is what he has been used to, most of his maths learning life, and not the experience of something quite different over the course of the 18 week intervention program.

<table>
<thead>
<tr>
<th>SCHOOL B</th>
<th>8.12.2005</th>
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<tbody>
<tr>
<td>James Year 6</td>
<td>At-risk intervention</td>
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</tbody>
</table>

Field notes (8.12.2005)

1 Sat down with James at 10am. When initiating this instrument, James says, ‘Can’t think of anything’. We decide that we’ll try and pick up on it after recess.
2 Implement card sort instead. James in the last week, under pressure of a confidential nature, so I am not surprised at his ‘closed’ approach. He hasn’t appeared himself today. Significant issues at home. James comes in after recess happy to do Drawing Task.
3 I: Tell me about your picture.
4 J: Person with a pencil on the workbench and he’s doing his work.
5 I: And he’s learning maths well?
6 J: Yes.
7 I: How do you know?
8 J: Dunno, make it look like it.
9 I: What is it about this situation that is helping you to learn maths well?
10 J: It’s like he’s never done it, the teacher tells him what to do and he’s learning.
11 I: So the teacher is important?
12 J: Sort of…shall I draw the teacher?
13 I: Yes you can if you like.
14 J: What’s the teacher done to help?
15 J: Tells you what to do and how to do it.
16 I: What’s the guy doing?
17 J: His work.
18 I: What type of work?
19 J: Maths.
20 I: What type of maths?
21 J: Any maths…dunno.
22 I: Point to the most important part of your picture.
23 J: (indicates the maths, that is, the work sitting on the desk).
24 I: Is this the only way to see maths work, on paper?
25 J: No, other ways, draw it on a board, don’t know…in your head.
26 I: Tell me more about maths in your head?
27 J: Picture it in your head and try and work it out…it’s like thinking.
As I got to know James, I understood that it was not unusual for James’s mood to dictate how he interacted with other people. His attendance at school or lack thereof, was of concern to his teacher and the school principal. This interview, is illustrative of the degree of reserve apparent in the manner in which James responded to certain situations (1-5). He had depicted a student that he referred to in the third person (7, 13) and made it ‘look like’ he was learning maths well (11). His picture showed a student at a desk with a pen or pencil in his hand, writing on what appears to be a worksheet (7). These aspects must to some extent, be significant in James’s mind, since it was his intention to make it appear as though the student was learning maths well. James suggested that learning something new was important (13) and that the teacher’s role was to tell the student what to do (13) and how to do it (18). When prompted about the significance of the teacher and the maths work itself, his views are somewhat ambivalent (15, 20-24), though James indicated that the work was the most important aspect of his drawing (26). I wanted to know if this was the only way in which to view mathematics work, and was a little heartened when James acknowledged maths activity as a presence in his mind (28), that could be visualised and involve thinking (30).

<table>
<thead>
<tr>
<th>SCHOOL A</th>
<th>28.11.2005</th>
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<tbody>
<tr>
<td>Douha Year 6</td>
<td>At-risk intervention</td>
</tr>
</tbody>
</table>

Field notes (28.11.2005)

1. Douha begins to tell me about a dice game she played with another student and ‘I did well with that’.
2. I: Tell me about your picture.
3. D: Partner work…I asked if we could work in the back room. I was working with XX and we had to do 100 throws and see what number was least likely to come up, with two dice and…
4. I: That’s what you’ve drawn?
5. D: Yes.
6. I: Tell me what you’ve drawn in your picture.
7. D: Me and XX there, Hadi and XX and XX at the table…with worksheets and dice.
8. I: Is there anything missing from your picture?
9. D: Oh, then we had to use a calculator to do our score.
10. I: So what is it about this situation that’s showing you learning maths well?
11. D: ‘Cos I like graphs, working on them and I’m probably the best in these…times and division I have trouble with sometimes.
12. I: Can you point to the most important part of your picture?
13. D: The work.
Douha had selected to draw a situation in which she had performed well (1-2). This involved working with a partner on a Chance activity set by MD (5-6). Students in this situation could access appropriate tools (10-11, 13) such as a worksheet to record their findings, and calculator to work out the score. When asked to select what was significant in this situation, Douha focuses on what she liked to do, in this case, graphs (15-16). A cause and effect relationship between liking the activity and doing well was described. Multiplication and division are mentioned as being problematic at times (15-16). According to Douha, what is most important in the picture is the ‘work’ (18) and that working with a partner as well as working alone has its benefits (23-24). Conversely, working independently can mean greater concentration and less distraction (26).

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**SCHOOL A**

<table>
<thead>
<tr>
<th>Date</th>
<th>Grade</th>
<th>Intervention</th>
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<tbody>
<tr>
<td>28.11.2005</td>
<td>6</td>
<td>At-risk intervention</td>
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</tbody>
</table>

**Field notes (28.11.2005)**

1. H: I did arrays…teacher asks me to do the sum…I had the answer and I’m giving it out in front of the class.
2. I: What are you adding now? (Hadi draws more detail in).
3. H: Chairs for the people.
4. I: What people?
5. H: Classmates.
6. I: What is it about this situation that is helping you to learn maths well?
7. H: Because when you do, like dots for times, I get them easier.
8. I: Do you mean arrays?
9. H: Yes, two times 6…when I draw it I find it easier than just with numbers.
10. I: What’s the most important part of your picture?
12. I: Why is this the most important?
What is interesting about the situation that was drawn by Hadi was a model of 2 sixes in array form (1). Additionally, Hadi depicted herself sharing her answer with the rest of the group out the front (1-2) of a group of students. Other student drawings had typically placed the teacher in this position. Hadi explained that visualisation helps her to understand concepts more easily (8, 10) in comparison to abstract representation with symbols (10). According to Hadi the drawing on the board and the answers are what is important in this situation (12) as these represent what is needed in everyday life (14).

Field notes (28.11.2005)
1 Y: I’ve drawn me sitting down…have a book in front of me and the teacher is writing sums on the board.
2 I: Tell me about the book that’s in front of you.
3 Y: A maths book.
4 I: What sort of maths book?
5 Y:…to write on…
6 I: Tell me about the sums on the board.
7 Y: She’s, the teacher’s, writing sums on the board and I’m answering them in my book.
8 I: Where is this happening?
9 Y: In the class.
10 I: Who else is there?
11 Y: Classmates (Yousif starts to add pupils to his picture).
12 I: What are they doing?
13 Y: They’re writing sums up as well.
14 I: What is it about this situation that is helping you to learn maths well?
15 Y: ‘Cos that’s how we learn with the class and sometime we do graphs.
A traditional classroom is presented in this picture and throughout the interview discussion. Yousif had shown himself (1-2) and other students (11, 13) answering algorithms (2, 8, 15) in their maths books (1, 4, 6, 9). When asked to elaborate further about what about this situation was helping him to learn maths well, Yousif stated that this is the way we learn (17). The mention of graphs is almost an afterthought (17). The most important part of his picture was the notion of ‘learning’ (19) communicated though the blackboard (21) and Yousif’s position in the room (21). I sensed that Yousif’s view of himself as a learner was something that he could manage (21) rather than something outside his control.
Andrew had selected to draw himself learning best, when learning algebra (6). His teacher is teaching the students at the board while the other students are seated in a horseshoe table arrangement. A procedural view of learning is described: the teacher does the work in front of the students first, to enable the students to understand how to do the question (8). The students then attempt the work themselves (10). According to Andrew, two factors are important: the teacher, because the teacher tells the students how to do the work (12) and the fact that Andrew is not alone, that all the students have to do the same thing (18).

Field notes (7.12.2005)

1 I: Tell me about your picture.
2 C: My maths test…doing it really well and my teacher came and collected my test.
3 I: When was this?
4 C: Few weeks ago here at school with Miss S.
5 I: What is it about this situation that is helping you to learn maths well?
6 C: Because I try really best at this test.
7 I: How did that make you feel?
8 C: Great.
9 I: Why?
10 C: Because I feel happy.
11 I: What made you happy?
12 C: I think I will make a good report, good maths.
13 I: So you did well on the test?
14 C: Yes, ‘cos at home I practice it.
15 I: What types of things did you practice?
16 C: Like had to do algebra and fractions.
17 I: Can you give me an example?
18 C: Not sure…
19 I: Point to the most important part of your picture.
20 C: (indicates the test).
21 I: What else helps you to learn maths well that is not in your picture?
22 C: The teacher, help me doing this, she tells what is algebra and what is fractions.
23 I: What else helps?
24 C: My parents…they do example…to make me understand how to do it.
Candi has drawn a situation in which he had done well when completing a maths test. According to Candi, this test is the most important part of his drawing. He reported that this situation represented himself really trying his best through doing extra work at home with assistance from his parents and he felt 'good' because of it. He believed that doing well on the test would enable him to produce a good end of year maths report. He mentioned practising fractions and algebra but couldn’t elaborate further with specific examples. Interestingly, Candi acknowledged the support of the teacher, who explained what algebra and fractions are however, reported that his parents are the ones who help him to understand.

### CWPS

<table>
<thead>
<tr>
<th>CWPS</th>
<th>30.11.2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hajar grade 6</td>
<td>At-risk non-intervention</td>
</tr>
</tbody>
</table>

### Field notes (30.11.2005)

1. I: Tell me about your picture.
2. H: When Mr O gives us a sheet and we don’t understand and he stops us and explains it on the whiteboard instead of telling only me. Others might be stuck so he explains it to all of us. When I’m stuck I get out maths equipment, shapes and cubes to help me. Sometimes when I’m really stuck I use a calculator. When I am not allowed to use a calculator I get a piece of scrap paper and work it out on there.
3. I: What is it about this situation that is helping you to learn maths well?
4. H: The equipment mainly and Mr O explaining it. No point using a calculator, it’s only going to tell you the answer.
5. I: Why? Isn’t it better to just get the answer?
6. H: You have to work it out yourself and if you didn’t have a calculator and you needed to work out something, you’d be stuck, you have to work it out on scrap paper.
7. I: Point to the part of your picture that is most important.
8. H indicates Mr O explaining stuff on the whiteboard.
9. I: Why is that most important?
10. H: He explains it better…the equipment can help you, the better one is Mr O explaining.
11. I: When is this happening?
12. H: (He) Always does it when we have maths but if it’s a maths test, you have to do it yourselves. But nearly always like this.
13. I: What else can you tell me about your picture?
14. H: Probably maths books, maths dictionaries. If you don’t know what a cube is, it’ll tell you…and doing work by myself, I learn maths better.
It appears that teacher explanations were a key aspect to Hajar’s situation about learning maths well (3-4, 8, 15, 17). When an area of difficulty arose for one or more students, the teacher is reported to stop the entire group and explain the difficulty to the whole class (2-4). In addition to teacher explanations, Hajar had other strategies for understanding the work: using maths equipment (4-5, 8, 17) including the use of a calculator (5) and working out on a scrap piece of paper (6). She stressed the importance of working things out for yourself, not simply relying on being told an answer by the calculator or another student (8-9, 26-27). Working independently was viewed as necessary in test situations (20-21) but also to encourage understanding (26).

Ibrahim’s responses were shallow and reflected disinterest in the activity. His classroom teacher reported that this was indicative of his response to school in general at this particular time of the school year. Ibrahim drew a naïve picture of himself doing maths (2)
at home (6), completing multiplicative questions (12) as set out in a maths book (10).

Ibrahim was unable or unwilling to elaborate why this situation was helping him to learn maths well. Suffice to say that the situation he had elected to represent was not at school.

<table>
<thead>
<tr>
<th>BSC.</th>
<th>7.12.2005</th>
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</thead>
<tbody>
<tr>
<td>Sidona Year 7</td>
<td>At-risk non-intervention</td>
</tr>
</tbody>
</table>

Field notes (7.12.2005)

1. I: Tell me about your picture.
2. S: Me doing maths on paper.
3. I: What sort of maths?
5. I: Can you give me an example?
6. S: 5 times 5, 6 plus 1, 9 divided by 18 or something.
7. I: What is it about this situation that is helping you to learn maths well?
9. I: How does the sheet help you?
10. S: Has problem solving...gets your mind working...if you're in the holidays you're doing nothing in your head, will feel like, empty.
11. I: Is it better when your head is full?
12. S: Yes.
13. I: Why?
14. S: Because if it's not full you don't think very much.
15. I: Why is thinking important?
16. S: If you don't think you don't know nothing.
17. I: Point to the most important part of your picture.
18. S: (indicates the worksheet).
19. I: Is there anything else that helps you to learn maths well that is not in your picture?
21. I: Where is this situation happening?
22. S: In school in my classroom.
23. I: What type of situation helps you to learn maths well?
24. S: When I'm alone in the classroom with a teacher to help so I don't get distracted.
25. I: Does that happen often?
26. S: Yes, used to, I used to go to special English classes (Saturday morning) don't have it anymore.
27. I: Would this have worked for maths classes do you think?
28. S: Yes.
Sidona had drawn herself doing a mixed set of maths questions (4-6) presented on a worksheet (2, 8). According to Sidona, the worksheet (19) is the most important part of her picture. She described this as ‘problem solving’ (10) even though maths questions of this type are contrary to my view of problem solving. I suspect the questions are closed, for example, 5 times 5 (6) and make little cognitive demand. Sidona believed that this situation depicted thinking (10-11) described as a ‘head’ that is ‘full’ (11, 15), that lead to factual and procedural (6) knowledge (17). As with her first interview, Sidona also suggested the value of computer maths games (22) and working with a teacher in isolation from other students and distractions (26).

Field notes (7.12.2005)

1. I: Tell me about your picture.
2. A: Last time I drew two people, this time more people, a story is being told, subtle about maths. I remember in prep, Mr T was a great teacher, everyone enjoyed his stories. This situation you’ve got to do while students are young and influential.
3. I: Are you represented in this picture?
4. A: Not really but I could be, I remember that’s how…the next year we had sheets, easy, I went and did them easily…this same method (story telling) could be used for operations (four operations).
5. I: But only with young children?
6. A: Yes.
7. I: Why?
8. A: They believe everything you say, if they believe it, they apply it/adopt the method in everyday life.
9. I: Tell me about the label ‘filling up’.
10. A: Can’t get over-full, when children are young they enjoy learning, after about grade 2 kids don’t enjoy it that much, will get out of it if they can…stories give ‘down time’ cos you can’t work all the time.
11. I: Why have you chosen to draw a situation from when you were very young?
12. A: Because it’s my best memory of maths…other memories boring or unpleasant.
13. I: Can you give me an example of boring?
A: Revision, I know it’s meant to hammer it in, but some children don’t need it. I know that they should try and meet individual needs. If you had a group of very smart boys, they’d grow with their competitiveness.

I: An example of unpleasant?

A: A few years ago, all had to do Maths Challenge. I didn’t want to do it ‘cos I wasn’t good at times tables, I hated it…I was hoping that there wouldn’t be time for me…I’d get embarrassed.

I: Tell me about the label ‘sharing’.

A: Important to share what you know, sharing is… like, variety is the spice of life, this is an important part of it. You need it, if you don’t share, you’re greedy, but it can help people work things out.

I: Is there anything else about learning maths well that you haven’t yet mentioned?

A: Table square worked well last year, this year not so much, perhaps we started doing them too late. Depends on the grade, feeds on competitive nature.

I: Table square is?

A: 12 by 12 grid, randomly calling out numbers between 1 and 12, placed down the side of the grid, and the numbers 1 to 12 placed across the top. Complete it for your times tables. If you don’t complete it in 10 mins you say 10 plus. ‘It really works!’ like weightwatchers. (Adam tells me he improved 5 mins over the course of the year). Some people didn’t improve much, because they didn’t have much to improve on.

I: What about students who did have to improve a lot?

A: Everyone improved, the harder you try…don’t even have to practise at home, I didn’t, I improved 5 minutes.

Adam recalled the first drawing task interview (2) where communication between two students was illustrated, but on this occasion a larger group was represented (2). The focus was on the teacher’s ability to tell stories (2, 4, 7, 16) recalled from a time when Adam was very young (4, 15). Adam believed that telling stories served two purposes: for enjoyment (3-4) and to teach through sharing (14, 29-31). He considered it greedy not to share (30).

Enjoyment of learning apparently runs out after about Year 2, according to Adam, when students are no longer easily influenced (4, 12-13). The use of worksheets, correlated in Adam’s mind with the provision of little challenge (7). He described a positive memory (19) through the story telling skills of the teacher, then elaborated on what he considered boring or unpleasant (19): revision (21) and embarrassment (27). Adam reported that revision for some students is unnecessary (21) with the implication that teachers should try to meet each student’s individual needs (22). Adam recalled feeling embarrassed at a time when knowledge of multiplication facts was not a strength (26), and participation in a tables challenge activity (25) would make this weakness known to everyone in the group (25-26).
<table>
<thead>
<tr>
<th>Field notes (7.12.2005)</th>
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<td>20</td>
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</tbody>
</table>

Alison had depicted a student, possibly herself, working or copying from a mathematics textbook (2). According to Alison, the textbook is the most important part of the drawing (6) because you don’t have to rely on the work being presented on the blackboard (9) and you can work with friends (8). Working with friends encouraged communication and this helps the students to understand (15-16). Generally, Alison reported finding the maths work easy (9, 11) in particular, fractions and the using the four processes (13). Whilst the teacher isn’t in the drawing, Alison acknowledged that teachers are critical (18) even if this isn’t obvious (18-19). Alison had recognised the relationship between the teacher’s explanations and the student doing the work (21-22).
I: Tell me about your picture.
C: Well me, and I don’t understand what they mean by the question. I ask the teacher and then in the second picture, I’m thinking ‘I understand now’ because the teacher has explained the question to me.
I: What is it about the situation that’s helping you to learn maths well?
C: Well if you need help you need to ask someone otherwise you wont understand, you wont know what to do and probably get it wrong. If you understand you’re learning maths well and you’ll probably get the question right.
I: Point to the most important part of your picture.
C: “Excuse me I need help” and the teacher says, “I’ll help you”.
I: How would you name, describe this part?
C: ‘Speaking out to understand’.
I: Where is this happening?
C: In a normal classroom.
I: Is this happening to you?
C: Yes.
I: Lots?
C: A fair bit.
I: So what else helps to learn maths well that isn’t in this picture?
C: ‘Not too much noise…a pencil that’s sharpened…most maths you have to write out the answer. If you don’t have a pencil that’s sharp or a pen with ink, the teacher might not be able to understand what you’ve done. Some…a friend who you can ask, and to learn maths well the question has to have information in it, not just how many 2’s in 2, 470.”

A simple comic strip approach has been taken to depict this situation across two time frames. Chris had shown himself initially not understanding the question, and therefore asked the teacher for assistance (2-3). Seeking assistance was seen as most important (10) to learning maths well. The second picture showed Chris understanding the question as a result of the teacher’s explanation (3-4). Chris advised that if you need help, ask, because when you understand, you are learning maths well (6-8) and more likely to get the question right. Failure to do so, would lead to lack of understanding, therefore resulting in getting the question wrong (6-8). He described this as ‘speaking out to understand’ (12). It appeared that in this situation the onus for learning was on the student. The mathematics itself was hinted at being more like a problem than closed questions (6-8) to be answered.
procedurally or via recall. Peace and quiet and having writing materials in good working order are other aspects to learning maths well (920-21).

<table>
<thead>
<tr>
<th>BSWPS</th>
<th>9.12.2005</th>
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<tbody>
<tr>
<td>Kai Year 6</td>
<td>Successful</td>
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<table>
<thead>
<tr>
<th>Field notes (9.12.2005)</th>
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</thead>
<tbody>
<tr>
<td>1 K: Can’t think…</td>
</tr>
<tr>
<td>2 I: Why?</td>
</tr>
<tr>
<td>3 K: Can’t think of a situation where I’m learning maths, something new…does it</td>
</tr>
<tr>
<td>4 have to be new?</td>
</tr>
<tr>
<td>5 [Kai explains that learning to him means new knowledge, he seems to think that the</td>
</tr>
<tr>
<td>maths he does now, he has already learnt.]</td>
</tr>
<tr>
<td>7 I: Not necessarily.</td>
</tr>
<tr>
<td>8 K: Do you mind if I draw something, a front cover of something I’ve done?</td>
</tr>
<tr>
<td>9 I: No, not at all. (Kai draws).</td>
</tr>
<tr>
<td>10 I: Tell me about your picture.</td>
</tr>
<tr>
<td>11 K: So basically Muddy Maths, Paul and I did for a maths talent quest, we won a</td>
</tr>
<tr>
<td>12 credit and I got to go to an award ceremony at Latrobe Uni. Based on building a</td>
</tr>
<tr>
<td>13 mud brick seat which (Kai tells me was vandalised).</td>
</tr>
<tr>
<td>14 I: Tell me more about the maths involved.</td>
</tr>
<tr>
<td>15 K: We had to do three sections. Introduction, Section 1, which was sizes of mud</td>
</tr>
<tr>
<td>16 bricks of the seat by me. (We discussed the height, width, length, multiplied all</td>
</tr>
<tr>
<td>17 together then we got area or volume. Then we did it for the mud brick seat which</td>
</tr>
<tr>
<td>18 was trickier). Then had section 2 by Paul, the layers of the seat. We both did the 3rd</td>
</tr>
<tr>
<td>19 one which was making a lego model of the seat 2 times bigger. In the 3rd section if</td>
</tr>
<tr>
<td>20 you times height, width, length, thickness, two for each of them...</td>
</tr>
<tr>
<td>21 I: What is it about this situation that is helping you to learn maths well?</td>
</tr>
<tr>
<td>22 K: Wasn’t really helping, but showing how we did learn it, algebra, volume, area, lots</td>
</tr>
<tr>
<td>23 of numbers being multiplied by lots of other numbers, multiplication and division.</td>
</tr>
</tbody>
</table>

Learning maths means learning something new (3-6). Kai implied that recently the maths he has done was no longer new to him, therefore his wasn’t learning maths well (5-6, 22). He decided to draw the front cover of the work he and a partner submitted to a maths talent quest (11) based on the building of a mud-brick seat (13). I asked him to elaborate on the mathematics involved in this investigation: working out the volume of the seat (15-18), the construction of the seat in terms of layers of bricks (18), and the making of a lego model based on an enlargement of the seat (19-20). Kai reported using algebra as well as other aspects of multiplication and division (22-23) as part of this project.
Kalil has elected to draw about a situation in which he was learning something new: algebra (1-2). He identified that a desire to know something made it easier for him learn (3, 8). Other students and the teacher, though not present in the drawing, are considered part of the situation (5-6). If students are working without distracting each other (9, 13, 25-26).
Kalil reported doing more work, more easily. Also, teacher explanations (11), knowing what to do and how to do it (16-21) were important to learning maths well. The whiteboard was identified as being the most important part of his picture as that is where the work is located (15).

### Field notes (30.11.2005)

1. I: tell me about your picture.
2. L: Learning maths well...someone will explain, they may have a different process or a different answer and they explain how they've done it...and it would be another way to do it. And this gives you more chances of getting in right if you work it out in two different ways.
3. I: Is that what your picture is showing?
4. L: Someone explaining something, and I'm writing it down to see if I got the same answer using a different process.
5. I: What is it about this situation that's helping you to learn maths well?
6. L: I think that...I'm learning that there's more than one process and there's different ways of doing things, and your way may not always work, if you know different ways, you can check.
7. I: Point to the most important part of your picture.
8. L: (Indicates that) all pretty important. If there was no-one next to me, I'd be just me writing and not necessarily listening to anything. And if I wasn't there, then there'd be no reason for the talking.
9. I: Where is this happening?
10. L: Probably at school 'cos that's where I do maths, where there's a range of people.
11. I: Is this pretend or real?
12. L: Real, people at school have explained stuff, your group, mates, friends...work like this not all the time but a bit.
13. I: What advice would you give to help other students to learn maths well?
14. L: Read questions really well, listen really carefully and try different ways if you know them.

Participation in discussion with other people (18, 20) enabled Lynda to hear about alternate ways of working (2-5). Her picture was about someone explaining and Lynda is comparing this with her own work (7-8). Lynda explained further that learning involved more than one definitive process and method (10-12). Working independently meant that there was reduced opportunity for listening and talking to others (14-16). I asked Lynda about what
advice she would give to other students if they wanted to learn maths well (22). She responded that reading carefully, listening and trying more than one way (23-24) was important.

In this situation the student and the teacher are both at the blackboard (2) with other students watching (2-3). This situation according to Jessica enables student to have access to feedback (3-4, 16) as well as to review procedures before they make mistakes (4-5, 17, 24). This is an apparent hypothetical situation (7-11), one in which Jessica thinks should occur to enable students to learn something new (13). Jessica reported that this approach...
would improve student confidence (15), and listening skills(15). The most important aspect of her picture is herself actually engaging with the work at the whiteboard (21, 25).

<table>
<thead>
<tr>
<th>BSC</th>
<th>13.12.2005</th>
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<tbody>
<tr>
<td>Sandy Year 7</td>
<td>Successful</td>
</tr>
</tbody>
</table>

Field notes (13.12.2005)

1. I: Tell me about your picture.
2. S: I like doing algebra so I think I’m doing well when I’m learning it.
3. I: Where is this happening?
4. S: At school or with the tutor I go to.
5. I: When do you work with your tutor?
6. S: Saturday’s.
7. I: Why are you working with a tutor?
8. S: Mum wanted me to.
9. I: Well, I think you’ve done well in maths…
10. S: I think…Mum wants me to train for McRob’s.
11. I: So what is it about this situation that is helping you to learn maths well?
12. S: I listen a lot so I know what to do.
13. I: What else, you mentioned liking algebra?
14. S: Yeah I find it kind of simple.
15. I: If this situation is at school, what are you doing?
16. S: A worksheet I was given.
17. I: By whom?
19. I: What else is happening to help you learn maths well?
20. S: She’s a good explainer.
21. I: What other qualities are needed to learn maths well?
22. S: I don’t know…
23. I: You mentioned liking it, being a good listener and the teacher being a good explainer…
24. S: mmm
25. I: Point to the most important part of your picture.
27. I: Why is that?
28. S: Because I’m working and I seem willing to do it.
29. I: What else is important to you learning maths well that is not in your picture?
30. S: I don’t know.
Sandy believed that if she likes the work, for example, algebra, then she’s learning maths well (2). The situation she had depicted included working on a worksheet provided by a teacher (16-18) at school or her Saturday morning tutor. Sandy works with a tutor on Saturdays in preparation for acceptance at a private girls’ school. Listening to a teacher’s explanation (20) is identified as an important strategy for knowing what to do (12). The most important part of her picture is Sandy, writing (27) because this means she is willing and working (29).
APPENDIX G

THE CARD SORT INTERVIEW
The cards

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>(free response)</td>
<td>(free response)</td>
<td></td>
</tr>
</tbody>
</table>

Word/concept orientation
Ask student to read through 12 cards. Check whether there is any word that the child does not understand. Discuss if necessary.
Ask, *Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*
Record if appropriate.

1. Initial sort:
Ask, *When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?*
Record on Card sort record sheet.

2. Diamond ranking:
Ask, *Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.*

Have map of Diamond ranking pattern available.

Record ranking on Card sort record sheet.

3. Discussion
Ask, *Tell me about your ranking…*
Record discussion on Card sort record sheet.
Card Sort record sheet

Name: | Year:  
--- | ---  
School: | Cohort:  

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
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<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
3. Discussion

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
APPENDIX H

CARD SORT INTERVIEW RECORDS
PRE-INTERVENTION PHASE CARD SORT INTERVIEW RECORDS

Name: Sidona (JV)  Year: 7  
School: BSC 11.05.05  Cohort: At-risk non-intervention

Word/concept orientation

Comments: encouraged her not to discard her words

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Workhseets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td><strong>concentrating</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important? Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Workhseets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td><strong>concentrating</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

Group work

Getting help  thinking

Feeling good  concentrating  Maths equipment

teacher  Prob. solving

Doing well
3. Discussion
Sidona…(JV)

<table>
<thead>
<tr>
<th>Group work</th>
<th>‘Cos sometimes when you work in a group it could help you</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>If you need help you can always ask someone about it.</td>
</tr>
<tr>
<td>Thinking</td>
<td>‘Cos if you don’t think you don’t get the answer.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>If you don’t feel good it’s hard to get all the answers correct</td>
</tr>
<tr>
<td>Concentrating</td>
<td>If you don’t concentrate it’s hard to do your work.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Help you to do your work, help you measure and stuff like that.</td>
</tr>
<tr>
<td>Teacher</td>
<td>If you need help it’s best to ask a teacher. (“Is that easy?” – sort of depends what it is.)</td>
</tr>
<tr>
<td>Problem solving</td>
<td>It’s like doing your work, if you’re problem solving you can always like add in your head.</td>
</tr>
<tr>
<td>Doing well</td>
<td>The only reason I put that last is ‘cos it doesn’t matter if you’re doing well or not, as long as you tried.</td>
</tr>
</tbody>
</table>

**Name:** Alison (JV)  
**Year:** 7  
**School:** BSC 11.05.05  
**Cohort:** SC

**Word/concept orientation**

*Comments:*  
Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td><strong>Working together</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**1. Initial sort:**  
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?  
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Working together</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Talk and disc</th>
<th>thinking</th>
<th>explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting answer</td>
<td>Group work</td>
<td>Prob. Solv’g</td>
</tr>
<tr>
<td>Feeling good</td>
<td>worksheets</td>
<td></td>
</tr>
</tbody>
</table>

Alison…(JV)

3. Discussion

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Need to talk and discuss, so you know what you’re doing. If you just put stuff on the board it is difficult ‘cos you need to discuss stuff. It’s a bit like ‘explanations’.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>You need to think, ‘cos if you don’t you can’t do anything. ‘Cos when you do maths like 2 plus 2 you need to think 4.</td>
</tr>
<tr>
<td>Explanations</td>
<td>If things aren’t explained properly you won’t be able to do it. You need explanations so you know what to do.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>Well that’s the main thing about what you’re doing. You can think and stuff but you need to get the answer.</td>
</tr>
<tr>
<td>Group work</td>
<td>Working in groups you can talk and discuss. You can teach people stuff and work together to get the answer.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Problem solving is part of maths. You need problems solving in everyday life.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>You work better if you’re feeling good, if you’re feeling bad you probably won’t work as good.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>I put down worksheets because I prefer worksheets than copying things. You can put the answer next to it.</td>
</tr>
<tr>
<td>Teacher</td>
<td>If you get too stuck with something you can get help from the teacher, like with all that stuff (indicates cards). They are there to ask.</td>
</tr>
</tbody>
</table>
Name: Andrew (JV)  
Year: 7  
School: BSC 11.05.05  
Cohort: (At-risk non-intervention)

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

```
thinking

Prob solving  Teacher
Getting help  explanations  Maths equip
worksheets  Talk & discuss
Getting the answer
```
Andrew… (JV)

3. Discussion

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Teacher</th>
<th>Problem solving</th>
<th>Getting help</th>
<th>Explanations</th>
<th>Maths equipment</th>
<th>Worksheets</th>
<th>Talking and discussing</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of ways to get the answer or solve something.</td>
<td>You need a teacher there to be able to tell you what to do.</td>
<td>Solving the problem helps you when you’re older or something like that.</td>
<td>Getting help from the teacher helps you get the answer.</td>
<td>You need, to tell you the answer of whatever you’re going.</td>
<td>Actually using things to measure and do things instead of just worksheets.</td>
<td>Help you with your thinking and solving things.</td>
<td>If you discuss with people you can think of ways to get the answer so you need talking and discussing.</td>
<td>I like it when I get the answer to something hard.</td>
</tr>
</tbody>
</table>

**Name:** Candi  
**Year:** 7  
**School:** BSC 11.05.05  
**Cohort:** (At-risk non-intervention)

Word/concept orientation

Comments:

_Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?_

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:

_When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?_

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

teacher

Getting help explanations

Talk & discuss Maths equip Prob. solving

Group work thinking

Doing well

Candi… (JV)

3. Discussion

<table>
<thead>
<tr>
<th>Teacher</th>
<th>It’s important ‘cos the teacher will tell you what to do, if the teacher is not there you can’t do it.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>If you are stuck on your maths you ask the teacher for help, if the teacher is busy you can ask your friend.</td>
</tr>
<tr>
<td>Explanations</td>
<td>The teacher has to explain the maths, what to do.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>The students talk, last time the students and the teacher had to discuss what we had to do like in ESL.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Have to learn what it looks like the equipment’s what to use, like abacus.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>You have to learn to solve the mystery, like you have to crack the password.</td>
</tr>
<tr>
<td>Group work</td>
<td>You in group, tell them what to do and what you’re doing, what you’re studying.</td>
</tr>
<tr>
<td>Thinking</td>
<td>You have to think what you do next, if you don’t, like, if you get something wrong.</td>
</tr>
<tr>
<td>Doing well</td>
<td>Sometimes you do well, if it’s easy you do well, you’re good at it.</td>
</tr>
</tbody>
</table>

Name: Sandy (JV) Year: 7
School: BSC 11.05.05 Cohort: (At-risk non-intervention)

Word/concept orientation

Comments:
Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Feeling good</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Group work</td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Feeling good</td>
<td>Getting help</td>
</tr>
<tr>
<td>questions</td>
<td>Group work</td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.

3. Discussion

<table>
<thead>
<tr>
<th>Thinking</th>
<th>You need to think a lot to get the answer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking and discussing</td>
<td>Important to talk and discuss because it will help you to get the answer.</td>
</tr>
<tr>
<td>Teacher</td>
<td>You should ask a teacher if you don’t understand that’s important and they will help as well.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Are important to help you understand.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you don’t understand it you need help, if you don’t get help you wont get the answer so it’s important to get help.</td>
</tr>
<tr>
<td>Questions</td>
<td>You should ask questions 'cos it’s really important 'cos they will show you a way to do the problem easier.</td>
</tr>
</tbody>
</table>
Doing well | I put it there ‘cos it’s good to do well it shows you understand and everything. (You feel confident, so you wouldn’t mind doing more).
---|---
Getting the answer | Not near the top, ‘cos when my teacher marks my work, she marks the working, so that’s important that you understand.
Feeling good | Not really important, you just feel good ‘cos you’ve got the answer. It is important but not compared to these (indicates words above).

**Student card sort record sheet**

**Name:** Lynda  
**Year:** 6  
**School:** BEPS 13.05.05  
**Cohort:** (SC)

**Word/concept orientation**

Comments: read/understand cards ok.

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well*</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. **Initial sort:**

*When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?*

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

![Diamond ranking diagram]

1: All pretty important, will be a little bit difficult…(and therefore Lynda appeared to make careful considered choices)

Lynda… (MB)

3. Discussion

<table>
<thead>
<tr>
<th>Thinking</th>
<th>You should think first about the problem, put into other words to make it easier and if you already know what to do, you can think about the answer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>If you can’t put it into your own words, you may need someone to explain it to you.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>If you’ve thought and had an explanation and you’re still unsure, you can talk about it to help with what to do.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>When you’re feeling good about yourself you can achieve more, think bad and then you won’t. Feel better you feel good about what you do even if you don’t get it right, at least you tried.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>If you still don’t get the problem, might be better if you saw the problem eg, addition or subtraction with counters (easier to see with + and – and division in comparison to x to help get the answer. Lynda gives the example of being about to put counters in groups to model division situations.</td>
</tr>
<tr>
<td>Teacher</td>
<td>You could ask the teacher, they might give the answer, but put you on the right path to get close or even get it right. Kind of like getting help which is the next one. If someone just gives you answers you’ll never learn.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you ask someone else, if they don’t give you the answer, they explain the problem. When you read it in your head you may not see the problem but when someone explains you may see if it.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>If you’re working on one special problem and others, then you can compare your work, you might understand one type of problem and can use the process to help you with the one you are stuck on.</td>
</tr>
<tr>
<td>Group work</td>
<td>Hearing different opinions and how they got the answer, and different techniques, so you can use whatever works for you.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Yousif</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL A 12.05.05</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: read/understand ok

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

“12 words ok”

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

Prob. solving

Thinking

Explanations

Maths equip.

Talk & discuss

Group work

Doing well

Get the answer

worksheets
Yousif… (MB)

3. Discussion

Yousif found it hard to give detail to his answers. Was like getting blood from a stone.

<table>
<thead>
<tr>
<th>Problem solving</th>
<th>‘cos helps you get the answer. Asked Y to elaborate about how helps, had trouble, although mentioned “work it out”.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>So you get smarter and learn.</td>
</tr>
<tr>
<td>Explanations</td>
<td>If someone explains to you, they explain the answer then you get the other answers.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>If MAB, helps you get the answer.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>About how you get the answer.</td>
</tr>
<tr>
<td>Group work</td>
<td>Help other people on your table or sitting next to, cos…</td>
</tr>
<tr>
<td>Doing well</td>
<td>So you learn more.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>To think how you are going to get the answer.</td>
</tr>
<tr>
<td>worksheets</td>
<td>To understand, if you have worksheets, they help you to understand other things as well.</td>
</tr>
</tbody>
</table>

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Hadi</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL A</td>
<td>12.05.05</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: read/understand ok except for ‘explanations’ brief chat about that

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Learning *</td>
<td>Working out*</td>
<td>Teaching*</td>
</tr>
</tbody>
</table>

1. Initial sort:

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Learning</th>
<th>explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>Talk &amp; discuss</td>
<td>Prob. solving</td>
</tr>
<tr>
<td>Feeling good</td>
<td>Working out</td>
<td>Maths equip.</td>
</tr>
</tbody>
</table>

Appears to make careful choices

Hadi… (MB)

3. Discussion

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Learning</th>
<th>Explanations</th>
<th>Getting help</th>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Feeling good</th>
<th>Working out</th>
<th>Maths equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you don’t think you don’t understand stuff.</td>
<td>If you don’t learn, you can’t do maths and learn things.</td>
<td>If you don’t know something, someone can tell you how to work it out because if you don’t know and they help you.</td>
<td>If you don’t know, ask someone and they help.</td>
<td>You can talk about what you don’t know and what you do and this helps.</td>
<td>Helps in maths because you know how to work out problems…if you can’t solve problems, eg, divided bys and times tables…need them to go to the bank and shops.</td>
<td>If you don’t know it doesn’t matter because you can learn it, you feel good if you know something in maths.</td>
<td>If doing a problem you can try and work it out in different ways, like vertical (meaning the setting out).</td>
<td>Asked H for examples: MAB, calculator, bead frame. If you don’t know how to do it the equipment helps you ‘cos in a bead frame you put one down for as many.</td>
</tr>
</tbody>
</table>

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Douha</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL A 12.05.05</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: Read/understand ok except for explanations, maths equipment. Had a brief chat about these. Douha suggested that MAB was an example of maths equipment.
Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

Douha tells me that there’re pretty much all there…

**1. Initial sort:**
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

**2. Diamond ranking:**
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

Douha… (MB)

**3. Discussion**

<table>
<thead>
<tr>
<th>Doing well</th>
<th>It's better if you do well, to get a good report, good education, for high school.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>If you’re doing worksheets and they are hard and you think about it, you get it done.</td>
</tr>
<tr>
<td>Teacher</td>
<td>Because if you’re in need of help the teacher is always there, need someone to talk to, the teacher.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>Worksheets can help you work, eg, times tables and it’s on your sheet, you work it out and it helps you. It is good to get hard</td>
</tr>
</tbody>
</table>
worksheets, if you haven’t done something for a while and you don’t remember.

Talking and discussing
If you talk and discuss with someone at your table you can probably get help, not the answer, but talking and discussing it with you.

Feeling good
If you do the worksheet and get through it without help you feel good. I: What if you ask for helps can you still feel good? D: Yes you can still feel good about yourself.

Explanations
‘cos if the paper if it doesn’t have what you have to do, the explanations at the top of it are better. I: These are written explanations, are there other sorts? D: If you’re working on the floor with a teacher, they explain what you have to do.

Maths equipment
Doing decimals you have to make a model, the equipment is there to help you, comes in handy to do maths with.

Group work
I put this least ‘cos you don’t always need a group to help you get the work done, sometimes, not all the time. Eg, table groups with large pieces of paper, you can share ideas for maths and other things.

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Hajar</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: CWPS 13.05.05</td>
<td>Cohort: (At-risk non-intervention)</td>
</tr>
</tbody>
</table>

**Word/concept orientation**

Comments: Read/understand ok

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

1.Initial sort:

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

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<tr>
<th>Talking and discussing</th>
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<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>concentrate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

- Getting answer
- Maths equip.
- Talk & discuss
- Prob. solving
- Getting help
- Group work
- Thinking
- Concentrate
- Doing well

Hajar… (MB)

3. Discussion

| Getting the answer | If you use equipment and talk and discuss…if you don’t get the answer…what’s the point of doing it. It helps you learn for when you are older to answer maths questions and stuff. |
| Maths equipment | It helps you think more if the questions, eg, 92 cubes and you took away 30 you can use maths equipment to help figure it out. I: Can you give me some examples of types of maths equipment? Pencils, paper, tins, chalk, glue sticks, books. At the end of the discussion Hajar tells me that computers and calculators also help to learn maths well. |
| Talking and discussing | Helps you to understand the question ‘cos if you don’t understand, and you talk about it with your group you can understand it better. |
| Problem solving | You can write the question on paper, you can use divided bys and times and stuff…helps you getting the answer. |
| Getting help | If you don’t know what the question means you can ask someone (student teacher, teacher, children in the grade, friends). I: What if you don’t get help? You try and solve it on paper or a chalk board. |
| Group work | Discussion in a group is better than by yourself. Sometimes you are more comfortable talking with friends than the teacher for “some reason”…more comfortable with friends. Helps you understand more of the question, problem solve together and answer together. |
| Thinking | When you think of the question you do it in your head. Adam is very smart. |
| Concentrate | If you concentrate you probably get more of the answer than if you’re talking (ie not paying attention). When the teacher asks, you have to say you were talking and you wont get as much done. |
| Doing well | You have to do well, like the answer, you have to think while you do it, to do well, without rushing. Have to do well if the teacher is going to mark your work. |
Student card sort record sheet

| Name: Adam | Year: 6 |
| School: CWPS 20.05.05 | Cohort: (SC) |

Word/concept orientation

Comments: read and understand ok.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Being willing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adam tells me that some of these aren’t important, eg, doing well, has not much to do with ‘actual’ learning; and getting the answer, that understanding is more important.

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Worksheets</td>
<td>Teacher</td>
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<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
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<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Being willing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
### 3. Discussion

<table>
<thead>
<tr>
<th>Constructing paths to multiplicative thinking</th>
<th>Appendixes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adam …</strong></td>
<td></td>
</tr>
<tr>
<td><strong>3. Discussion</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Being willing:</strong></td>
<td>If you are not willing it will take longer to learn, you will block out, tune out and talk to people (and become distracted). If this then you are not learning because you are not listening.</td>
</tr>
<tr>
<td><strong>Talking and discussing</strong></td>
<td>Sharing ideas and strategies, other ways to do things eg, rounding and different strategies. You can work on one strategy and get faster (and add it to your skills).</td>
</tr>
<tr>
<td><strong>Group work</strong></td>
<td>Mainly this is about sharing which is similar and equally important to talking and discussing, to give you ideas, help and strategies.</td>
</tr>
<tr>
<td><strong>Explanations</strong></td>
<td>Really important, if you don’t know how to do something, there’s no point doing it. Explanations, you have to know how to start, important to start easy and do things of (increasing complexity). Explanations are essential at the start, till you don’t need much, if at all. I: Can you see a time when you wont need explanations? A: Yes…in about year 11, 12…</td>
</tr>
<tr>
<td><strong>Getting help</strong> (made the point that this was strongly linked to explanations)</td>
<td>Essential if you need help, similar to explanations. Everyone needs to get help occasionally…not everyone understand straight away.</td>
</tr>
<tr>
<td><strong>Thinking (allows “second nature” to occur)</strong></td>
<td>Nature of thinking is situation and person dependent. Thinking involves looking out for the clues. Because if you’re not thinking…unless you’re…eg, adding 3 digit numbers can become second nature as a result of repetition…but then repetition can be come unnecessary, there are some things that you don’t need to repeat (hard to give eg)…depends on the person.</td>
</tr>
<tr>
<td><strong>Feeling good</strong></td>
<td>If you feel good then you want to do it more…self admiration…like a reward…work harder on things…do harder things…feel good about yourself…then you can say “I can do this”. I: Some children don’t feel good about their maths learning, what can be done to help? A: Find out what they know and build up on it</td>
</tr>
<tr>
<td><strong>Worksheets</strong></td>
<td>These are important because of the repetition so that it becomes second nature which refers to thinking</td>
</tr>
<tr>
<td><strong>Teacher</strong></td>
<td>Not very important but you have to have it somewhere to get…(points to getting help). It is half as important as worksheets. If a teacher takes too big a role (and Adam moves teacher card further up the diamond)...it needs to be pretty much ‘you’, if not it's like it becomes his achievement, not mine to claim.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: James 12.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL B</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: read/understand ok.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>+ x - ÷</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

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<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>+ x - ÷</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
James… (MB)

3. Discussion

<table>
<thead>
<tr>
<th>Getting help</th>
<th>Just help. I don’t know how to do some that are hard.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems solving</td>
<td>So you can learn…so you’re not dumb when an adult.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>Has the writing and you have to do it.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>About finishing, finishing your work. I: Important to finish your work? J: Yes so you don’t get into trouble and stuff.</td>
</tr>
<tr>
<td>+ - x ÷</td>
<td>J gave examples of each, I: doing that helps? How? J: So you’re not dumb when you’re big, an adult.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Eg? Pencils, rubber, calculator (couldn’t give other). Pencils to write out the answers, the rubber to rub out wrong answers and the calculator helps with the answer.</td>
</tr>
<tr>
<td>Group work</td>
<td>Friends…get help…</td>
</tr>
<tr>
<td>Thinking</td>
<td>Makes you smart…think hard about the answers…</td>
</tr>
<tr>
<td>Doing well</td>
<td>Makes you good at it.</td>
</tr>
</tbody>
</table>

Student card sort record sheet

| Name: | Cansu 12.05.05 | Year: | 6 |
| School: | SCHOOL B | Cohort: | (At-risk intervention) |

Word/concept orientation

Comments: read/understand ok

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<tr>
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</thead>
<tbody>
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<td>Teacher</td>
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<tr>
<td>Thinking</td>
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<td>Getting help</td>
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<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
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</table>

1. Initial sort:

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

### Explanations

Talk & discuss

Prob. solving

thinking

Maths equip.

Group work

Teacher

Getting help

Worksheets

Cansu… (MB)

3. Discussion

<table>
<thead>
<tr>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>You need explanations to do the work in maths…if you don’t you can’t do it good.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>If you don’t solve it yourself you won’t learn anything, try to solve it first then ask the teacher.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>With friends in groups to get their opinion you have to try and think if you’re right or wrong, you have to talk and discuss, be more better if 4 or 5 in a group and one says it’s right and the other don’t, this helps arrive at an answer.</td>
</tr>
<tr>
<td>Thinking</td>
<td>Third because if you don’t you will get nothing right…if you think better…you can find a different way to solve it.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Need it ‘cos if you have to count something and you don’t have enough fingers or pencils, the equipment helps you.</td>
</tr>
<tr>
<td>Group work</td>
<td>Because to get a different person’s opinion, different answers from everyone, get to work with friends…develop more…get answers and non-answers.</td>
</tr>
<tr>
<td>Teacher</td>
<td>If you can’t get help/understand it/ solve it the teacher tells you what it means and how to do it better to solve it quickly and easily.</td>
</tr>
<tr>
<td>Getting help</td>
<td>This is good ‘cos if you tried solving or you didn’t understand, get help from a friend, teacher or someone smart in the grade.</td>
</tr>
<tr>
<td>worksheets</td>
<td>I put this least ‘cos if you get the worksheet and try yourself first before going to the teacher because it writes the question and you get scrap paper and write the finished answer on the sheet.</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Ahmed 12.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL B</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: had trouble reading and understanding the words: equipment, group, and explanations. Discussion took place. *Used ‘explanations’ in sort, when it came to telling me about his ranking, he forgot what it meant and therefore, changed it for doing well.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<td>Feeling good</td>
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<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>#Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>#Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>*Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
**Ahmed… (MB)**

### 3. Discussion

<table>
<thead>
<tr>
<th>Problem solving</th>
<th>Tells you about the work, before they start, tells what you're doing.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group work</td>
<td>To help each other. Why? To finish quick, they think and get the answers.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>Talk about the work...because if you can’t read and write the teacher can explain it.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>That’s where you’ve got to write it…</td>
</tr>
<tr>
<td>Getting help</td>
<td>Put your hand up and teacher comes and tells you where you need help…’cos if you don’t you wont know what to do…have to get help.</td>
</tr>
<tr>
<td>Thinking</td>
<td>After getting help, when the teacher goes you think about it and get the answer.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Eg, if you have 10 + 2 you put 10 of the equipment and put two more and it equals 12. Gives you help to do the plusses.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>When you finish, correct the work, you find out whether you get it right or wrong.</td>
</tr>
<tr>
<td>Explanations (forgot what it meant and changed it for…) doing well</td>
<td>If you're doing well, you learn everything, you have to learn everything. I: Tell me more about ‘everything’, is it important to learn everything? A: On that day for that day.</td>
</tr>
</tbody>
</table>

---

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Berrin 12.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL B</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

**Word/concept orientation**

Comments: read/understand ok, however reviewed ‘explanations’ and ‘discussing’

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>+ x - ÷</td>
<td>Times tables</td>
<td></td>
</tr>
</tbody>
</table>

**1. Initial sort:**

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Times tables</th>
<th>+ x - ÷</th>
<th>Prob. solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>Worksheets</td>
<td>Doing well</td>
</tr>
<tr>
<td>Group work</td>
<td>Maths equip.</td>
<td></td>
</tr>
<tr>
<td>Talk &amp; discuss</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Berrin… (MB)

3. Discussion

<table>
<thead>
<tr>
<th>Times tables</th>
<th>Some people don’t know their times tables, you have to learn them, when in high school the teachers says quickly and you don’t have time to count your fingers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ x ÷</td>
<td>You have to learn all of them. When you have a test all those stuff is on the test.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Sometimes teacher gives problem solving to learn more about problem solving…so you don’t have to count…</td>
</tr>
<tr>
<td>Thinking</td>
<td>If you don’t know the answer you have to think.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>To get better at maths…learn…read it, and read the information on the worksheet.</td>
</tr>
<tr>
<td>Doing well</td>
<td>You have to try your best to all the maths and try to do it yourself and this means doing well.</td>
</tr>
<tr>
<td>Group work</td>
<td>Getting along with the group…talk together…so we don’t have fights…sometimes people “butt in” (co-operation)</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Involves all maths stuff, books, ruler, pencil case, counters maths cards, blocks, dice…to improve yourself…get better…to learn.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>When the teacher and others talk…gets loud…if you want help go to the teacher (communication)</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Adir 12.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL B</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: orienting chat about ‘discussing’, ‘explanations’ and ‘equipment’.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Thinking</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Doing well</td>
<td>Being smart</td>
<td>Confidence</td>
</tr>
<tr>
<td>The working out</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Helping others</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Thinking</td>
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<td>Group work</td>
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<tr>
<td>Doing well</td>
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<td>Confidence</td>
</tr>
<tr>
<td>The working out</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Helping others</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
3. Discussion

<table>
<thead>
<tr>
<th>Teacher</th>
<th>‘cos if you don’t have a teacher you can’t learn maths…no-one to teach you to learn…help you.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>You persist in your work…in working. I: How does confidence help? (unable to articulate)</td>
</tr>
<tr>
<td>Problem solving</td>
<td>If you don’t know 8 x 8 ask the teacher and friends or if friends don’t know…get paper…work it out.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>Whether answer is right or wrong (feedback)</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Play with equipment you learn to play games with friends, mum and dad.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>Paper to figure it out…like a test…no-one should copy (therefore own work important).</td>
</tr>
<tr>
<td>Being smart</td>
<td>If you don’t be smart you fail, if you are smart you get to a higher level, you know the answer straight away, quick…10 sec or one minute.</td>
</tr>
<tr>
<td>Helping others</td>
<td>If a friend doesn’t know, you help, you don’t give the answer you help (and tell if necessary).</td>
</tr>
<tr>
<td>Group work</td>
<td>“group 3” talk together, get the answer…</td>
</tr>
</tbody>
</table>

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name:</th>
<th>Dean 12.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School:</td>
<td>SCHOOL B</td>
<td>Cohort: (At-risk intervention)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: discussion about ‘equipment’. I asked Dean to give me some examples of maths equipment “pencil, pen, calculator, rubber, liquid paper”. No other equipment mentioned.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>listening</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.

Dean… (MB)

3. Discussion
Had trouble articulating deeper elaboration of his responses at some times.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Listening</td>
<td>If you don’t listen you won’t know what to do*.</td>
</tr>
<tr>
<td>Teacher</td>
<td>‘Cos teacher tells you what to do*. Important because it affect understanding and ability to do the questions</td>
</tr>
<tr>
<td>Thinking</td>
<td>You’ve got to think about the work. I: Can you tell me more about that? D: if you don’t you won’t know what to do and understand it.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>You need worksheets ‘cos if you don’t have worksheets you can’t do it. I: Can you do maths without worksheets? D: Unless the teacher writes it up on the board.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Helps you when you are older to work things out. I: only when you are older? No when you are little as well. Why? …</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you need help ask someone or a teacher</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>Get other people’s ideas about the worksheet. How does this help? If you make a mistake their idea might be better for it.</td>
</tr>
<tr>
<td>Explanations</td>
<td>The teacher tells you how to do the worksheet. I: Is that the only time there are explanations? D: No with other subjects areas as well, eg, English and Integrated.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>If you don’t have the equipment you won’t be able to do the work.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Chris 19.05.05</th>
<th>Year: 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: BSWPS</td>
<td>Cohort: (SC)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: Read/understand ok.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
### 3. Discussion

Chris…

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Because you need to think about it too, like, if a question…sometimes quickly, but you need to think to get harder answers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>If you don’t have help at all, you will not know what to do and you’ll just sit there.</td>
</tr>
<tr>
<td>Explanations</td>
<td>(eg, written on the sheet, or oral from other students and the teacher) Sometimes if you need a bit more clarification, if you need more information or if you don’t understand, explanations would be good.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>That…you could…good to get the answer but if you try and the answer is wrong, then at least you tried. Sometimes the working out is more important.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>You need sometimes just to share answers…to show each other different ways you can answer this question.</td>
</tr>
<tr>
<td>Doing well</td>
<td>Pretty important because you need to do well to understand and know how to do this and other questions.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>(If there’s a problem and it needs solving and finding a solution) Pretty good to have problem solving and you need a solution and problem because if there’s an error in a question it’s good to know how to solve it ‘cos then you can work out the question the proper way.</td>
</tr>
<tr>
<td>Group work</td>
<td>Good to be in a group sometimes, then you can share answers and ways of solving and also if you need help, someone might be able to help, or if someone else is stuck, then I can help.</td>
</tr>
<tr>
<td>Teacher</td>
<td>Last because instead of a teacher to show you, proper way, might have other students or text book. If you need help, other students can help…but it’s good to have help.</td>
</tr>
</tbody>
</table>

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Ibrahim 19.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: BSWPS</td>
<td>Cohort: (At-risk non-intervention)</td>
</tr>
</tbody>
</table>

**Word/concept orientation**

Comments: read/understand ok.
Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Maths text books</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.

3. Discussion
Ibrahim…

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Feeling good</th>
<th>Doing well</th>
</tr>
</thead>
<tbody>
<tr>
<td>Because you have to think if you are going to do something.</td>
<td>Have to feel good because if you are sick and you’re not feeling well, the work will not be good…because you wont be able to think if you’re too tired and sick.</td>
<td>You'll do well if you’re doing the work…if you can’t do it – do bad…’cos you wont get the answer and you will fail.</td>
</tr>
</tbody>
</table>

| Problem solving | Every Monday we do problem solving because good for you because you think. I: What does this problem solving time look like, what would I see? Ibrahim: Sheet with maths stuff and it’s called problem solving. |
Teacher
You need the teacher so you can learn, schools, need a teacher to tell you what to do to help you. If no-one to help you then you wont know what to do.

Getting the answer
If you do good you get the answer, if you feel good you get the answer, if you think you get the answer. I: Is getting the answer the aim of maths? Ib: Yes. I: Can you do good, feel good and think and not get the answer? IB: Sometimes you can, sometimes you can’t.

Talking and discussing
Discussion and talking to friends, teacher…can’t sit and wait for the teacher to come to you. Can’t be slack or lazy*, you have to make the effort.

Maths equipment
You need, if you don’t have pens, pencils, books how can you do your maths? I: Can you give me some other examples of maths equipment? Ib: rubber, whiteout if make mistake, pen/texta

Group work
Sometimes good, sometimes bad. Good if you are in a group with people who work, bad if you with a slack* group.

*Ibrahim recognizes that being slack/lazy is not good. I: Do you try hard not to be slack? Ib: Not try hard, try not to be slack sometimes.

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Kai 19.05.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: BSWPS</td>
<td>Cohort: (SC)</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments: read/understand ok.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>If you don’t get answer, don’t feel bad</td>
<td>remembering</td>
<td>learning</td>
</tr>
<tr>
<td>listening</td>
<td>Showing working out</td>
<td>Not using maths equipment</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
### Talking and discussing
- Problem solving
- Getting the answer

### Explanations
- Worksheets
- Teacher

### Thinking
- Maths equipment
- Getting help

### Doing well
- Feeling good
- Group work

<table>
<thead>
<tr>
<th>If you don’t get answer, don’t feel bad</th>
<th>remembering</th>
<th>learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>listening</td>
<td>Showing working out</td>
<td>Not using maths equipment</td>
</tr>
</tbody>
</table>

Kai...

### 2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

```
+-----------+
| Learning  |
+-----------+
| Feeling good |
| Thinking |
| Remembering* |
| Showing working out |
| Listening |
| Problem solving |
| Talking & discussing |
| Explanations |
```

* If it were possible to put remembering on the same level as feeling good and thinking, Kai would.

Kai...

### 3. Discussion

<table>
<thead>
<tr>
<th>Learning</th>
<th>Is basically the main thing you need to get out of maths, a lot of things today are about maths, eg, money/change/shops/chess.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feeling good</td>
<td>If you don’t feel good in maths you won’t feel confident to use it in the real world. Basically this is just about confidence.</td>
</tr>
<tr>
<td>Thinking</td>
<td>You need to think to work out your maths. Had it as most important but then decided Learning was more important and swapped it.</td>
</tr>
<tr>
<td>Remembering</td>
<td>If there were three slots for the second level ‘remembering’ would go there, you have to remember stuff, remembering shows you paid attention and that you feel good about your maths.</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

<table>
<thead>
<tr>
<th>Showing working out</th>
<th>Important because if you don’t get a question right, if you show your working out you can see that you’ve worked hard and even if it is wrong you still get marks.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Listening</td>
<td>Also very important ‘cos if you can’t listen, can’t learn, if you can’t learn, you can’t remember, if you can’t remember, you can’t feel good about your maths.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>(can be equations, word problems, can be done in History and English). It’s one of the main things you learn in maths, it’s just like writing in English, be like, one of the main work you do. Problem solving is problem solving because you are solving problems.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>Use this if you are not sure if you are right, you can talk to person or teacher and discuss, see what you’ve done wrong. Bit like thinking almost, but you have 2, 3 lots of minds to think with.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Pretty important, bit like show your working out, but use your voice, not paper. If a problem is wrong you can explain and get marks and they can help you understand.</td>
</tr>
</tbody>
</table>

**Student card sort record sheet**

| Name: Kalil | Year: 6 |
| School: BSWPS | Cohort: (SC) |

**Word/concept orientation**

Comments: read/understand ok.

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Feeling good</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Times tables</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. **Initial sort:**
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
### Constructing paths to multiplicative thinking: Appendices

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher*</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

* only need if you don't know something

#### 2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.

![Diamond diagram]

#### 3. Discussion

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Getting the answer</th>
<th>Show working out</th>
<th>Times tables</th>
<th>Explanations</th>
<th>Doing well</th>
<th>Problem solving</th>
<th>Teacher</th>
<th>Talk &amp; discuss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>Because usually need to do a lot of thinking in maths, if you don’t think, you probably get it wrong.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Getting the answer</td>
<td>Equally important (to show working out) sometimes teacher will focus us on show working because they can see how you think. But, the answer is important because you know you’re thinking the right thing. Show your working helps to keep track of thinking and tell you where you have gone wrong.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Show working out</td>
<td>If you don’t know your tables a lot of things will be a lot harder for you eg, multiplication and division, a lot of problem solving is about multiplication.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Times tables</td>
<td>If you don’t know your tables a lot of things will be a lot harder for you eg, multiplication and division, a lot of problem solving is about multiplication.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>Sort of like ‘working out’ but more telling them what you did compared to showing your working.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing well</td>
<td>If you are not doing well then you probably wouldn’t do much of the words above.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Problem solving</td>
<td>If you don’t solve any problems when you older you will get stuck on...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
things and ask yourself “What do I do? What do I do?” You won’t know what to do. Kalil then gave an example of sharing two cakes equally among 4 people, problem solving is about real things that could happen.

Teacher
If there is no teacher and you had no knowledge then you wouldn’t get anywhere. Teacher can look at what you’re doing, mistake, wrong? (ie feedback).

Talking and discussing
After done all the work, sort of like correcting at the end. Talking and discussing to share difference so that one person might really think, use calculator, and if you talk and discuss, this can help understand how to answer.

---

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Jessica 26.5.05</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: BEPS</td>
<td>Cohort: (SC)</td>
</tr>
</tbody>
</table>

**Word/concept orientation**

Comments: Read and understand ok.

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*

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<th>Problem solving</th>
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</thead>
<tbody>
<tr>
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<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

Words “All there”.

**1. Initial sort:**

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
### 2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

#### Feeling good

#### Thinking

#### Explanations

#### Getting help

#### Talk & discuss

#### Prob. solving

#### Teacher

#### Doing well

#### Group work

---

### Jessica

#### 3. Discussion

<table>
<thead>
<tr>
<th>Feeling good</th>
<th>You have to feel good to know what to do otherwise you’ll not do well, not understand, not concentrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>Thinking about what you’ll be doing leads onto explaining. You have to think about how to solve and different ways so you know there’s different ways/options and find a way easy for you.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Explaining to people who are listening, know what you’re doing and you know what you’re doing, so if people listening notice problems they can help.</td>
</tr>
<tr>
<td>Getting help</td>
<td>Always good if you are stuck and you want to know what to do, get more information, what things are about...later on in life/school. You don’t have to ask for help again, unless you forget.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>It’s good to talk about the problems you are having with maths, where you went wrong...so, bit like getting help, you can, helps with vocabulary and you wont have to discuss it next time.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Jessica tells me that problem solving is like you write it in words eg word problems, but says it is more likely about solving problems eg, 53 x 2 ie sums, algebra and fractions. You need to know how to solve to actually solve problems. Susan, (last year’s teacher and a SNMY Project teacher) says that there isn’t a job that doesn’t involve maths...a lot of things Susan told us stayed with me.</td>
</tr>
<tr>
<td>Teacher</td>
<td>Always good for giving help, for talking with the teacher that teacher knows a lot about you and how you think. They can give work and that improves understanding.</td>
</tr>
<tr>
<td>Doing well</td>
<td>This is very important to me, I’m academic, because it gives you confidence, if you don’t...getting the answer isn’t very important, it’s how, not the answer, near least important, if you don’t...it’s not vital</td>
</tr>
<tr>
<td>Group work</td>
<td>It’s good to work in a group if you don’t like people in the group…maths using other kids/people’s ideas to solve the problem, so you can get easier strategies and get to know other people better.</td>
</tr>
</tbody>
</table>
**POST-INTERVENTION PHASE CARD SORT INTERVIEW RECORDS**

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Douha</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL A 28.11.2005</td>
<td>Cohort: At-risk intervention</td>
</tr>
</tbody>
</table>

**Word/concept orientation**

Comments:

_Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?_

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td></td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Working well</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Initial sort:**

_When you think of learning maths well, out of these 13 cards, select 9 to show what you think is most important?_

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td></td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Working well</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Diamond ranking:**

Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

```
<table>
<thead>
<tr>
<th>Feeling good</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher</td>
</tr>
<tr>
<td>talking &amp; discussing</td>
</tr>
<tr>
<td>Maths equipment</td>
</tr>
<tr>
<td>Prob. solving</td>
</tr>
</tbody>
</table>
```
### 3. Discussion (Douha)

<table>
<thead>
<tr>
<th>Feeling good</th>
<th>I put that there because you should feel good about yourself…starting to get through it and it’s not that hard you feel good about yourself</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher</td>
<td>He or she can help you…they’re there to help you…say if you’re in need of help they can explain, get it through to you, help you.</td>
</tr>
<tr>
<td>Doing well</td>
<td>Is important to me, when I’m doing well I am proud of myself. If you put it in your mind you’ll do well instead of putting it to your friends (ie being distracted).</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>I put it fourth because when you need, talking and discussing with your friends, or table mates and work together…discuss, not tell the answer just as the teacher would do.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>Can come in handy sometimes because I find them easier, help me more than working from the board. Worksheets are better. I: How are they different? D: Not really that different…not sure why…doesn’t make a big difference. I: What does make a big difference? D: I just think on a worksheet I do better than in my maths book. I: can you give me an example? D: The dice activity (as described in DRAWING TASK instrument) ‘cos I understood what’s going on</td>
</tr>
<tr>
<td>Thinking</td>
<td>Because if there was…(time passes). I: What are you thinking about when you included thinking in your sort? D: Me thinking. When I think really hard about my work I get through it quicker than I usually do.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Because sometimes if you’re having trouble, the teacher suggests get some equipment…and I can draw what I’m thinking…they help a lot when I’m having trouble.</td>
</tr>
<tr>
<td>Getting help</td>
<td>It’s not that important…but it’s good, if you’re in trouble, get help…if you’re not having trouble it’s best to do it yourself.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Not sure why…I thought of (problem solving) as getting a scrap piece of paper and do the problem on that, helps…I start to know what I’m doing, so I leave the scrap and work on the sheet. May have to use scrap…if not much space on the sheet. I: What if there is plenty of space on the sheet, would you still use scrap paper? D: Yes…because I’m used to it.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

**Name:** Hadi  
**Year:** 6  
**School:** SCHOOL A 28.11.2005  
**Cohort:** At-risk intervention

**Word/concept orientation**

Comments:  

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<thead>
<tr>
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<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Thinking</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Doing well</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Initial sort:**  
When you think of learning maths well, out of these 14 cards, select 9 to show what you think is most important? * Hadi organised these in an array of 3 threes

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
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<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Thinking</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Doing well</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Diamond ranking:**  
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

```
                    thinking
                   /       \      
Talking & discussing /           \     /  
Maths equipment     Problem solving    Doing well
                   |                   |
Explanations        Getting help       
                   |                   |
Feeling good
```
### 3. Discussion (Hadi)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>I put it first...if you don't think what to do, you won't do well, if you don't try, there's no point to learning.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking and discussing</td>
<td>I put it there because if you don't understand or it's too hard, go to someone else to help you.</td>
</tr>
<tr>
<td>Learning</td>
<td>I put that next because every time you learn new things, helps you in your maths, like arrays with times tables.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>If something's too hard, just get maths equipment to help you...if you're having trouble, say like counting, get MAB to help with thousands, hundreds, tens.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Important 'cos you usually do it everyday, it's important outside of school, like shopping...</td>
</tr>
<tr>
<td>Doing well</td>
<td>'cos in maths if...’cos it's important to do well in maths so you can learn stuff. I: Why? H: 'cos when you learn stuff you try hard.</td>
</tr>
<tr>
<td>Explanations</td>
<td>If you don't understand, someone can help you in an easier way, the teacher or someone else.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If something's too hard or if you don't understand I ask my friend or teacher to help me.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>The rest (of the words) are about learning maths. This one is about yourself...you have to feel good to learn...it helps you, if you're not well...you don't pay attention.</td>
</tr>
</tbody>
</table>

### Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Yousif</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: SCHOOL A 28.11.2005</td>
<td>Cohort: At-risk intervention</td>
</tr>
</tbody>
</table>

### Word/concept orientation

**Comments:**

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card? *carefully considered and made 4 additions to the words.*
1. Initial sort:
When you think of learning maths well, out of these 16 cards, select 9 to show what you think is most important? *again, careful culling from 16 to 9
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Working good</td>
<td>The working out</td>
<td>Remembering things</td>
</tr>
<tr>
<td>Understanding</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern. *quite a bit of swapping around

3. Discussion (Yousif)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Because when you think…get a sum…you have to think before you can do it.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The working out</td>
<td>You’ve go to do the working out before the answer…see if the answer’s right or not. Rough copy, scrap paper to show how you’re making the answer.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>If you don’t get the answer right you have to do it again until you get it.</td>
</tr>
<tr>
<td>Understanding</td>
<td>You have to understand…if it’s too hard you ask the teacher, a friend, classmate.</td>
</tr>
</tbody>
</table>
## Constructing paths to multiplicative thinking: Appendices

### Talking and discussing
With someone if you don’t know what the answer is…they can help a bit then you can figure it out.

### Explanations
If you don’t get it…get someone or the teacher to explain it to you…so you get the answer right.

### Maths equipment
Using something to get you’re answer…like MAB, unifix. I: Now these don’t talk to you and say: hey the answer is 43, what do they do to help? Y: They help by getting the answer, they help by putting them somewhere, in groups or something.

### Remembering things
If you have a sum from last time, you can remember it (the answer) then you can get the answer and you don’t have to work it out again. I: Can you give me an example? Y: Nine times nine.

### Getting the answer
Getting the answer right. I: Do you always have to be right? Y: Not all the time. I: Can you give me an example? Y: When the teacher says it doesn’t matter if you can’t do it.

### Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Jessica</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: BEPS 30.11.2005</td>
<td>Cohort: SC</td>
</tr>
</tbody>
</table>

### Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

### 1. Initial sort:

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

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<tr>
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<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>
2. **Diamond ranking:**
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Feeling good</th>
<th><strong>Feeling good</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking &amp; discussing</td>
<td><strong>Talking &amp; discussing</strong></td>
</tr>
<tr>
<td>thinking</td>
<td><strong>thinking</strong></td>
</tr>
<tr>
<td>Group work</td>
<td><strong>Group work</strong></td>
</tr>
<tr>
<td>Explanations</td>
<td><strong>Explanations</strong></td>
</tr>
<tr>
<td>Getting help</td>
<td><strong>Getting help</strong></td>
</tr>
<tr>
<td>Problem solving</td>
<td><strong>Problem solving</strong></td>
</tr>
<tr>
<td>The teacher</td>
<td><strong>The teacher</strong></td>
</tr>
<tr>
<td>Doing well</td>
<td><strong>Doing well</strong></td>
</tr>
</tbody>
</table>

3. **Discussion (Jessica)**

<table>
<thead>
<tr>
<th>Feeling good</th>
<th>You have to feel good about what you do otherwise you’ll have no interest and won’t concentrate. In maths you have to know what it’s all about ‘cos every job has maths.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking and discussing</td>
<td>This is really important, it’s good, like now, so you know why you did something and discuss what you did, ‘cos if you’re right or wrong you know what to do or not do next time, shortcuts to make it quicker, easier.</td>
</tr>
<tr>
<td>Thinking</td>
<td>You have to think before you do something. If you jump straight in you can get muddled up or not understand, you have to think it through in your head.</td>
</tr>
<tr>
<td>Group work</td>
<td>Because it’s good to work with a team and learn from and co-operate from and with people, ‘cos you’re not always going to get your own way, or for things to be up to you, working with people can help you.</td>
</tr>
<tr>
<td>Explanations</td>
<td>‘cos it’s good to explain what and why you did something, it leads to your own understanding. You can notice if something’s wrong.</td>
</tr>
<tr>
<td>Getting help</td>
<td>Because it’s good to, and not to so you figure it out for yourself, but help helps. If you don’t you could continue not to understand and not know what to do, it can help summarize, make it easier.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Instead of a set question, it’s good to understand what to do in a realistic point of view. I: Example? J: Lady went to the milk bar and bought 1lt bottle of milk for $2…</td>
</tr>
<tr>
<td>The teacher</td>
<td>The teacher is not up the top with getting help ‘cos you can get help from other people, friends. The teacher teaches stuff you need to know, to teach you,…to teach</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

Doing well | Just as long as you understand, it doesn’t matter if you get something wrong, understanding is important…If you don’t understand, you can’t do it the next time and remember it, like (the four process) dividing for percentages, if you have no understanding of division then you wont understand percentages, decimals, fractions.

Student card sort record sheet

| Name: | Lynda | Year: | 6 |
| School: | BEPS 30.11.2005 | Cohort: | Successful |

Word/concept orientation

Comments:

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:

*When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?*

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:

*Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.*
3. Discussion (Lynda)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>I put thinking up the top because it's important to think about what you're doing instead of doing the first thing that comes to mind, you might get it wrong.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking &amp; discussing</td>
<td>It’s good ‘cos you can compare with other people’s answers and how you got it, to identify what’s right and what’s wrong.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Hearing what others think is right or definitely right is important, you might not’ve understood the question. An explanation helps you know what to do.</td>
</tr>
<tr>
<td>Group work</td>
<td>When working in a group you can all compare answers and work together and find out about problems. ‘Cos it also improves relationships and know how to, you may not talk to someone often and you might learn something from them.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>Because you can, feel good about yourself and you concentrate more. If you’re not happy, not as much effort, if you’re feeling good you feel better about yourself</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>It's visual and you can see what you’re working with, counters, blocks, for example if you’re doing area, you can make shapes to see how much area it takes up, see what you’re doing.</td>
</tr>
<tr>
<td>The teacher</td>
<td>If you’ve thought, and had explanations, been talking and discussing, and you still don’t know, ask the teacher so you have more of an idea.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you still don’t understand after many processes, ask your friends, they can explain in words you understand and ways they know you will understand.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>‘These are good ‘cos you can put what you know into practice. On a worksheet you show your working out and see what you are doing. Reading the questions is part of the process.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Hajar</th>
<th>Year: 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>School: CWPS 30.11.2005</td>
<td>Cohort: At-risk non-intervention</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

The teacher

Talking & discussing Maths equipment

Thinking Worksheets Problem solving

Explanations Getting help

Doing well
### 3. Discussion (Hajar)

| The teacher | I put the teacher first because he explains the work, I understand the teacher. |
| Talking and discussing | If you are stuck, talking and discussing with friends |
| Maths equipment | I put it third because it helps me when I am stuck instead of asking the teacher…if the teacher’s busy and I don’t want to interrupt him I use maths equipment. |
| Thinking | You need to think about what you’re doing, focus on your work. (Thinking) will help you…you can get paper and write what you’re thinking so you don’t forget and it’ll help you. |
| Worksheets | I put it here because instead of Mr O using the whiteboard, uses worksheets, you can do (these) quietly, they can help you learn…like times tables…if you don’t know them the sheet may have some information to help you. |
| Problem solving | Normally I answer before I write on the sheet (get scrap paper to write on in case I forget) then Mr O tells us the answers and if I’m wrong I put a cross and if I’m right I tick it. |
| Explanations | I learn more by people explaining stuff to me, Mr O explains it clearly. If he uses words I don’t know, I get a dictionary and look them up. |
| Getting help | I get help from Mr O, my friends, maths dictionaries…’cos I understand the teacher and my friends. They (my friends) use words I understand and know what they mean. |
| Doing well | ‘Try to do your best when doing maths, you might get it right, if not you’ve done your best and tried hard. Trying your best at everything in maths. |

### Student card sort record sheet

| Name: Ahmed | Year: 6 |

### Word/concept orientation

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*
1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

3. Discussion (Ahmed)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Because you have to think about it, if you don’t, have to ask the teacher.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>If you think and you still don’t understand it you get help. I: Why do you bother to ask for help? A: ‘Cos if you don’t, you don’t know what you’re doing.</td>
</tr>
<tr>
<td>The teacher</td>
<td>Then tell the teacher if you don’t know…the teacher’s there to help you know.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>If teacher says, like it’s a test and can’t ask the teacher, you can use equipment. I: Is that the only time? A: Dunno…</td>
</tr>
<tr>
<td>Worksheets</td>
<td>‘Cos if you don’t know maths, they give you a sheet and you have to learn about it.</td>
</tr>
</tbody>
</table>
### Explanations
The teacher explains it because before you do maths the teacher, explains, because if you don’t how to do it.

### Talking and discussing
In groups talk to each other to get…you know how to do it.

### Problem solving
Like you don’t know how to do it and you get problems and you have to solve the question.

### Group work
If someone doesn’t know how to do it, in a group the people who know tell the people who don’t know.

---

#### Student card sort record sheet

<table>
<thead>
<tr>
<th>Name</th>
<th>Dean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>6</td>
</tr>
<tr>
<td>School</td>
<td>SCHOOL B 8.12.2005</td>
</tr>
<tr>
<td>Cohort</td>
<td>At-risk intervention</td>
</tr>
</tbody>
</table>

#### Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Times tables + - x ÷</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Initial sort:**

When you think of learning maths well, out of these 13 cards, select 9 to show what you think is most important?

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Times tables + - x ÷</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

```
The teacher
Thinking
Worksheets
Maths equipment
Times tables + - x ÷
Problem solving
Getting help
Getting the answer
Feeling good
```

3. Discussion (Dean)

<table>
<thead>
<tr>
<th>The teacher</th>
<th>Without the teacher, we won’t know what to do and children can ask questions about the worksheet.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking</td>
<td>Then we think, if you don’t think you won’t understand the worksheet.</td>
</tr>
<tr>
<td>Worksheet</td>
<td>Because they’ve got all the questions in them. I: How do they help? D: Bring it near you and read it then you think about the work and then you get maths equipment and work out what sort of question it is and try to solve it. If you don’t understand ask the teacher, write down your answer then you feel good afterwards.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>If you don’t have equipment, you won’t be able to do it. I: Can you give me an example? D: Calculator, pen, pencil, eraser, sharpener and glue.</td>
</tr>
<tr>
<td>Times tables + - x ÷</td>
<td>Without these there wouldn’t be much of maths. I: What about when we did our maths dudes? D: Yes but all I keep thinking about is paper to draw on.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Because you have to try and work out the answer. I: What is problem solving? D: Is something where you try to work something out.</td>
</tr>
<tr>
<td>Getting help</td>
<td>Don’t understand the question put your hand up, wait till the teacher comes and ask about the question. I: Is that the only way to get help? D: Calculator, ask a friend how to do it.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>When you get the answer, and it’s wrong, you still tried, that’s the main thing.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>After you’ve finished the work, you feel good ‘cos you’ve finished it. I: Because Oh thank goodness it’s over? D: Sometimes…but it’s fun and you enjoyed it and you feel good afterwards.</td>
</tr>
</tbody>
</table>
**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Cansu</th>
<th>Year: 6</th>
</tr>
</thead>
</table>

**Word/concept orientation**

Comments:

_Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?_

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td>Getting help</td>
<td></td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Work with a partner</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**1. Initial sort:**

When you think of learning maths well, out of these 13 cards, select 9 to show what you think is most important?

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td>Getting help</td>
<td></td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Work with a partner</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**2. Diamond ranking:**

Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

1. Talking & discussing
2. Problem solving
3. Thinking
4. Getting help
5. The teacher
6. Doing well
7. Work with a partner
8. Worksheets
9. Explanations
3. Discussion (Cansu)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking &amp; discussing</td>
<td>When you do maths...you have to talk and discuss otherwise won’t learn that much, when you discuss you get more ideas, more ideas, how to do it.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>I put it second 'cos when you get a problem that you can’t normally solve, get scrap paper to do it there more ‘propier’ (more properly). I: Why use scrap paper? C: Miss P says use scrap paper...if not enough room.</td>
</tr>
<tr>
<td>Thinking</td>
<td>When you normally do maths you have to think, if you don’t you can’t do it.</td>
</tr>
<tr>
<td>Getting help</td>
<td>Third, because first you have to discuss, then solve it, then think, if you can’t do it, get help.</td>
</tr>
<tr>
<td>The teacher</td>
<td>If you still don’t understand from friends, go to the teacher to help you solve it. Friends first then the teacher.</td>
</tr>
<tr>
<td>Doing well</td>
<td>Have to practise a lot and you have to concentrate, solve it, do it by yourself. I: How do you feel then? C: Happy. I: Why? C: Because I did really well in my maths and Dart test and I’m proud.</td>
</tr>
<tr>
<td>Work with partner</td>
<td>Good to solve together, discuss it, different ideas, you and your friend, do it together because you’re able to work more better and you’ll be better at working.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>If you don’t have worksheets you don’t have to work. I: We didn’t have worksheets but did we work? C: Yes. I: So what’s the difference? C: Mental maths answer questions on a sheet or questions on a blackboard.</td>
</tr>
<tr>
<td>Explanations</td>
<td>You don’t normally need explanations for maths. I: Why? ‘Cos it helps you but not that much ‘cos...not sure. I: Can you give me an example? C: When you’re doing work, doing mental maths, 6 time 2 and you don’t understand the question, go to the teacher and can help explain it to you. (I probe for Cansu’s advice: ‘count by tow’s’, 6 plus 6’ and ‘double 6’).</td>
</tr>
</tbody>
</table>

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Berrin</th>
<th>Year: 6</th>
</tr>
</thead>
</table>

Word/concept orientation

Comments:

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*
1. Initial sort:

When you think of learning maths well, out of these 14 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

2. Diamond ranking:

Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

3. Discussion (Berrin)

<table>
<thead>
<tr>
<th>Take away, plus, divided by</th>
<th>Have to learn them lots of times to get improved…because you have to get more quicker, not slow, put up hand quicker.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem solving</td>
<td>Have to add, times, plus and you have to put the one up the top, don’t know what it’s called, important ‘cos in a test and you have to try and do it. I: can you give me an example? B: 2 plus 6 is 8, 1 plus 8 is 9.</td>
</tr>
<tr>
<td>Times tables</td>
<td>So if the teacher asks a times table numbers and you have to be quicker, when you go to high school you learn them lots and lots of times and primary schools.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>Some teachers give worksheets to get better to learn better and get better at it. I: How do you know when you learn better? Teacher tells me. I: By talking to you? Yes. I: Is this the only way? B: Yes.</td>
</tr>
</tbody>
</table>
Maths equipment: Marbles, cars, pencils, tens thousands and helps me better at maths…to count and get divided by’s and other stuff.

Doing well: Get better at all the maths, divided by, times, times table.

Talking & discussing: If you don’t know anything about maths talk to the teacher, and tell you what to do. I: Is that all that talking and discussing does? B: Telling you what to do? Yes.

Explanations: Teacher helps a little bit and tells you what to do. I: Is the teacher the only one to give explanations? B: Yes only the teacher.

Group work: You work in a group and they help you sometimes to get better at maths.

---

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Adir</th>
<th>Year: 6</th>
</tr>
</thead>
</table>

**Word/concept orientation**

**Comments:**

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Solving out a problem</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**1. Initial sort:**

When you think of learning maths well, out of these 13 cards, select 9 to show what you think is most important?

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>The teacher</th>
<th>Getting help</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Explanations</th>
<th>Problem solving</th>
<th>Group work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Talking &amp; discussing</th>
<th>Maths equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Getting the answer  | |
|---------------------| |

3. Discussion (Adir)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Have to think first, if you don’t think you might get wrong answer, like 5 plus 5 is 2, it’s not, it’s 10, you’re not thinking properly. You put any answer on the sheet…you’re not smart and you don’t get smarter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher</td>
<td>If you don’t know the answer ask the teacher. I: Why else is teacher important to learning maths well? A: If you don’t have a teacher, who’ll help you learn at school? Maybe mum can’t talk English properly, so teacher at school helps.</td>
</tr>
<tr>
<td>Getting help</td>
<td>First ask a friend for help, if they don’t know the answer or can’t help, ask the teacher, if they don’t know, get blocks to help work out the answer.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Tell you what the work is all about …teacher explains, help you work out how to do it and what to do.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Is when something’s a bit hard, and you have to think, or tell the teacher (Adir gets a bit vague here, finding it hard to articulate significance).</td>
</tr>
<tr>
<td>Group work</td>
<td>Get a group and work together, all of you work together. I: Why is this important? A: If you work together and need help, a group can help.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>With friends about the work, because you get the answer quick. I: Is only the answer important? A: No, thinking, like 30 add 10, add 5, use blocks, think to work it out.</td>
</tr>
<tr>
<td>Maths equipment</td>
<td>Working with dice, blocks, boards (games) if you don’t know the answer you can count the blocks by 2’s, 4’s, helps you.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>You know the answer 'cos the blocks help you do it. Papers to work it out on, draw pictures or use numbers.</td>
</tr>
</tbody>
</table>
Constructing paths to multiplicative thinking: Appendices

Student card sort record sheet

<table>
<thead>
<tr>
<th>Name:</th>
<th>James</th>
<th>Year:</th>
<th>6</th>
</tr>
</thead>
</table>

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<thead>
<tr>
<th>Talking and discussing</th>
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<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Pencils</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 13 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

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<td>Group work</td>
</tr>
<tr>
<td>Pencils</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

J: Perhaps pencils is part of maths equipment

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

Getting the answer

Thinking

Group work

Talking & discussing

Worksheets

Problem solving

Getting help

Feeling good

Maths equipment
### 3. Discussion (James)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting the answer</td>
<td>And you finish when you get the answer. I: Why is that important? J: `cos you finish your work.</td>
</tr>
<tr>
<td>Thinking</td>
<td>You have to think to get the answers and then you learn. I: Tell me more about learning? J: So you get smart…important. I: Why? J: When older doesn’t take long time to work things out.</td>
</tr>
<tr>
<td>Group work</td>
<td>Work together to work out the answer. I: Why? J: So you can learn to work with others (James is not sure why). Like it sometimes. (James prefers group work). I: Why is better than say working by yourself? J: Dunno…just like it.</td>
</tr>
<tr>
<td>Talking &amp; discussing</td>
<td>Like in a group, talking about the answer and trying to work out the answer.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>You need the paper so you can do the work. I: Is this always the case? J: No, but you need it. Need if you don’t have it you wont know where to put the answer or what the question is.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>You work out the answer, like the same as thinking. I: Can you give me an example? J: One person has $5 and another has $25, try to plus is and see what the answer is, `cos there could be maths problem solving on the worksheet.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you’re having problems ask for help so you can work it out. I: Who do you get help from? J: The teacher. I: Is that all? Yes.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>If you’re feeling sad, you wont feel good about the work. If you feel good about the work, you’re happy doing it.</td>
</tr>
</tbody>
</table>

---

**Student card sort record sheet**

| Name: Adam | Year: 6 |
| School: CWPS 7.12.2005 | Cohort: SC |

**Word/concept orientation**

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?
1. Initial sort:
When you think of learning maths well, out of these 15 cards, select 9 to show what you think is most important?
Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

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</tr>
</thead>
<tbody>
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<td>Teacher</td>
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<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Competition</td>
<td>Right levels*</td>
<td>Understanding</td>
</tr>
</tbody>
</table>

* involves the right work provided for the right groups of people

2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.
3. Discussion (Adam)

<table>
<thead>
<tr>
<th>Understanding</th>
<th>If you understand the concept of whey then you can find any answer…do anything with it.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>If you don’t…stay behind…won’t work properly in maths, effect work in high school, this leads to understanding also.</td>
</tr>
<tr>
<td>Right levels</td>
<td>If you have the right groups with the right work, everyone gets ahead and move onto higher groups.</td>
</tr>
<tr>
<td>Explanations</td>
<td>If you have explanations, helps them understand, if they understand, it goes back up to here</td>
</tr>
<tr>
<td>Talking &amp; discussing</td>
<td>Can open up explanations that you learn from other people, to lead to understanding.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>Helps you express how you feel…feel good, you work better, they’ll be open to talking and discussing, to lead to explanations, to lead to understanding.</td>
</tr>
<tr>
<td>Thinking</td>
<td>To develop thoughts, and if you share it, explanations, talking and discussing, to lead to understanding.</td>
</tr>
<tr>
<td>Competition</td>
<td>With competition, it brings out the best in you, like a dangling carrot, incentive, talking and discussing, explanations, understanding.</td>
</tr>
<tr>
<td>Group work</td>
<td>Can lead to talking and discussing, explanations, understanding, a much more direct path.</td>
</tr>
</tbody>
</table>

Student card sort record sheet

| Name: Kalil | Year: 6 |

Word/concept orientation

Comments: notices that all words or most of them relate to learning anything, not just maths

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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</thead>
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<tr>
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<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>A quiet place</td>
<td>Don’t rush</td>
<td>Check your work</td>
</tr>
<tr>
<td>Show your working out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Initial sort:
When you think of learning maths well, out of these 16 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

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<thead>
<tr>
<th>Talking and discussing</th>
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<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>A quite place</td>
<td>Don't rush</td>
<td>Check your work</td>
</tr>
<tr>
<td>Show your working out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

```
Thinking
Explanations
The teacher
Getting help
Show your working out
Talking & discussing
Check your work
A quiet place
Feeling good
```

3. Discussion (Kalil)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>I put it at the top ‘cos if you didn’t you wouldn’t know what you’re doing, eg, 8 times 4, think compared to just saying 6, the first number that comes to mind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>I put it there ‘cos you need explanations to get a clear idea of what you’re meant to do. You need someone to explain it to you.</td>
</tr>
<tr>
<td>Getting help</td>
<td>If you’re stuck, ask for help, they’ll probably be able to help you. If it’s really hard ask ‘what does this mean?’ eg, 5 squares and you don’t know what squared means.</td>
</tr>
<tr>
<td>The teacher</td>
<td>Will probably be the best person to ask for help and they explain what to do, talk to you about it, if someone is smart, they can give their ideas to some who aren’t so smart.</td>
</tr>
<tr>
<td>Show your working out</td>
<td>Because they (the teacher) say, ‘show your working’ so they know how you worked it out…where you might have made a mistake and help fix it. If you show your working you might get one point if you still make a mistake or 2 points if you’re right with showing working out.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>Problem solving</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Check your work</td>
<td></td>
</tr>
<tr>
<td>A quiet place</td>
<td></td>
</tr>
<tr>
<td>Feeling good</td>
<td></td>
</tr>
</tbody>
</table>

Talking and discussing: Share ideas so those who aren’t so smart, will help them learn.

Check your work: Because if you’re doing a test or it’s really easy as sometimes happens to me, like 10 times 10, I put 10 and forget the extra zero, if I’d checked it I would have got it right.

A quiet place: It’s really hard to think and learn if the place is really noisy, (admits that this is a bit of a problem for him. Kalil spoke about respecting other students’ needs if he finished early).

Feeling good: It would be hard to work if you’re feeling sick, you might start…but have to keep stopping…or if you’re not happy you might think ‘I don’t want to do this’. I: What about students who don’t feel good because of the maths? K: The teacher, talk to the teacher about it and the teacher might be able to think of a way where they might be able to do it and feel good about it...could happen...but I haven’t seen it happen.

---

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Kai</th>
<th>Year: 6</th>
</tr>
</thead>
</table>

**Word/concept orientation**

**Comments:**

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<tbody>
<tr>
<td>Explanations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing well</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Showing working out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Initial sort:**

When you think of learning maths well, out of these 15 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

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<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td><strong>Worksheets</strong></td>
<td><strong>Teacher</strong></td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Showing working out</td>
<td>The problem</td>
<td>Double checking</td>
</tr>
</tbody>
</table>
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Group work</th>
<th>The problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Double checking</th>
<th>worksheets</th>
<th>Feeling good</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Doing well</th>
<th>Showing working out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

3. Discussion (Kai)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>The mind is the main tool for doing maths.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The problem</td>
<td>Because without a problem maths wouldn’t be need to be used.</td>
</tr>
<tr>
<td>Group work</td>
<td>‘Cos sometimes only your thoughts are needed but you can compare with others and realise ‘I’ve made a mistake’ also if you have no ideas, bit like thinking, but more people to think with.</td>
</tr>
<tr>
<td>Double checking</td>
<td>You might get the answer the first time, if you check and it’s wrong, you can change it, it's not too late and get it right. If it's right it’s still good to double check.</td>
</tr>
<tr>
<td>Worksheets</td>
<td>Worksheets and the problem should be linked, the main thing the worksheets is to give the explanation of the problem.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>‘Cos if you don’t feel good, if you’re feeling negative, you’re not going to do as well.</td>
</tr>
<tr>
<td>Doing well</td>
<td>Linked with showing working out, especially if you are asked to, if you don’t do well I can see a good understanding and still get marks for that answer</td>
</tr>
<tr>
<td>Showing working out</td>
<td></td>
</tr>
<tr>
<td>The teacher</td>
<td>Is important but not as important as other things, if you’ve got a good teacher it eases you into maths, a strict teacher will make you feel negative and that’s why it links with feeling good. The teacher is only good it you’re starting out in maths, ‘cos you can get maths from the worksheets.</td>
</tr>
</tbody>
</table>
**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Ibrahim</th>
<th>Year: 6</th>
</tr>
</thead>
</table>

**Word/concept orientation**

Comments:

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*

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<td>Maths equipment</td>
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</tr>
<tr>
<td>pen</td>
<td>Maths books</td>
<td>Friends</td>
</tr>
<tr>
<td>family</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**1. Initial sort:**

*When you think of learning maths well, out of these 16 cards, select 9 to show what you think is most important?*

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

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<td>family</td>
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</tr>
</tbody>
</table>

**2. Diamond ranking:**

*Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.*

* seemed to place these without much thought

```
Doing well

Problem solving
Thinking

Getting the answer
Talking & discussing
Group work

Feeling good
Getting help

Friend
```
3. Discussion (Ibrahim)

| Doing well | Top ‘cos you have to do good, if you don’t you get in trouble. |
| Problem solving | Next, in maths you solve some things…yes. |
| Thinking | Got to think to get the answer. I: Only to get the answer? Ib: Yes that’s all. |
| Getting the answer | Got to get the answer. I: Why? Ib: If don’t, bad, don’t do your work. |
| Group work | {(silence) I probed without success.} |
| Feeling good | If you don’t you can’t concentrate, then you can’t do it. |
| Getting help | From the teacher if you don’t understand what to do. |
| Friends | Don’t really need them…if you don’t have friends you don’t do it. |

Student card sort record sheet

| Name: Chris | Year: 5 |

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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</tr>
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<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

<table>
<thead>
<tr>
<th>Getting help</th>
<th>Thinking</th>
<th>Feeling good</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group work</td>
<td>explanations</td>
<td>Doing well</td>
</tr>
<tr>
<td>Talking &amp; discussing</td>
<td>Getting the answer</td>
<td></td>
</tr>
<tr>
<td>Maths equipment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Discussion (Chris) ~ explain/justified from bottom of diamond rank and worked upwards

| Getting help | If you’re stuck on something and you just sit there, you should if you can get help…from friends or the teacher…but if you don’t it’s a waste of time. I: Is time the only thing you waste? C: No you waste your learning. |
| Thinking | Well you need to think to answer the question…well with some you know them off by heart but some questions you need to think about them to answer them. |
| Feeling good | If you’ve tried and someone says it’s wrong, then you won’t feel good about being put down like that, but you’ve tried and maybe you just didn’t understand. |
| Doing well | It’s pretty important to do well, but alright if you don’t do well but good if you try your hardest, because you don’t want to go home after a test and say ‘I don’t know why I didn’t answer that question’ then you won’t regret not trying your hardest. |
| Explanations | Sometimes you need explanations to show, explain what’s going on…sometimes it’s easier to have explanations so it’s less confusing. |
| Group work | Some people are good by themselves, don’t need to work in a group. |
| Getting the answer | Don’t always have to get the answer, try to show the working out even if you’re not right. |
Talking & discussing | Sometimes you don’t have to talk and discuss, some people are good at doing it themselves.
---|---
Maths equipment | Don’t need maths equipment that much, just you’re head and maybe something to write answer with.

**Student card sort record sheet**

| Name: Andrew | Year: 7 |

**Word/concept orientation**

**Comments:**

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*

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<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>Doing work</td>
<td></td>
<td></td>
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</tbody>
</table>

1. **Initial sort:**

*When you think of learning maths well, out of these 13 cards, select 9 to show what you think is most important?*

Indicate by **highlighting** the 9 to be included in the next step (Diamond ranking).

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<tr>
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</tbody>
</table>

2. **Diamond ranking:**

*Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.*

Getting help

Doing well  The teacher

Getting the answer  Thinking  Problem solving

Doing work  Feeling good

Talking & discussing
### 3. Discussion (Andrew)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Getting help</strong></td>
<td>If you don’t get it (understand the work) you need to tell someone. I: Why? A: o you get it next time say if you have a test or something. I: Tell me more about ‘get it’. A: It’s right, correct. I: What is right, correct? A: What you’re doing.</td>
</tr>
<tr>
<td><strong>Doing well</strong></td>
<td>You need to do well for things, to succeed in life. I: How does this relate to learning maths? A: It helps when you are older. I: In what way? A: If you’re like a builder, measure stuff, for a job.</td>
</tr>
<tr>
<td><strong>The teacher</strong></td>
<td>They need to teach what to do. I: How do they help you to learn maths well? A: They teach you how to do maths. I: How? A: If there’s no teacher, you wouldn’t know how to do it.</td>
</tr>
<tr>
<td><strong>Getting the answer</strong></td>
<td>Get the answer so you know how to do it and it helps you. I: Helps with? A: By getting smarter if you know how to do it.</td>
</tr>
<tr>
<td><strong>Problem solving</strong></td>
<td>How to…I: What is problem solving? A: Working out ways how to solve problems. Make you think of ways, other ways to get the answer.</td>
</tr>
<tr>
<td><strong>Doing work</strong></td>
<td>Helps you, shows you ways, getting the answer, makes you smarter as well. Doing the maths. I: How do you feel about doing the maths? A: What you learnt you put down on the sheets, shows you what you know and don’t know.</td>
</tr>
<tr>
<td><strong>Feeling good</strong></td>
<td>Need to feel good about doing maths. It makes you feel better. I: Why is that important? A: To make you learn better so you know how…better to feel good than feel bad. Confident.</td>
</tr>
<tr>
<td><strong>Talking and discussing</strong></td>
<td>Different ways of getting the answer. I: How does talking and discussing work? A: Get other people’s opinions and yours and others…see if they’re similar…think of ways they’ve got, how they got it.</td>
</tr>
</tbody>
</table>

---

### Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Candi</th>
<th>Year: 7</th>
</tr>
</thead>
</table>

#### Word/concept orientation

Comments:
Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
<td>Worksheets</td>
<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
<th>Getting the answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations</td>
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</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

3. Discussion (Candi)
The teacher
Need the teacher to help us doing the work so we know what to do. I: Why is knowing what to do important? C: To help you learn. I: Learn what? Important things about maths work. I: Can you give me an example? C: Time, plus, division.

Talking & discussing
You need to talk to a person, like teacher, my friend, ‘is it right? Is it how you do it?’ If you know, you can work really well.

Getting help
Getting help to do algebra. I: Why? C: Important to learn so you work it, how you do it and explain how.

Problem solving
If you have a problem with maths you have to solve it…so you know the answer.
Maths equipment
When you get to class you need equipment to do the work. I: Can you give me an example? C: Protractor, calculator, ruler, pencil to write or shade it…think that’s it.

Group work
Important because you share the information. I: Why is that important? C: To hear what they say then you write it.

Explanations
Like you explain to people how they ride a bike, how they do this worksheet, so they know how you do it.

Thinking
Is important so you think about how you do it, explain it and really think about it to do the maths. I: Where does this thinking happen? C: In the class. I: Who controls it? C: Yourself.

Doing well
Important for me to do work. If not doing well, you rush and finish, if you work quickly you might get it wrong. I: Any other reason? C: To try something new…that’s it.

---

**Student card sort record sheet**

<table>
<thead>
<tr>
<th>Name: Sandy</th>
<th>Year: 7</th>
</tr>
</thead>
</table>

**Word/concept orientation**

Comments:

*Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?*

<table>
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<tr>
<th>Talking and discussing</th>
<th>Problem solving</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>Teacher</td>
</tr>
<tr>
<td>Thinking</td>
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<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
<tr>
<td>willingness</td>
<td>confidence</td>
<td></td>
</tr>
</tbody>
</table>

**1. Initial sort:**

When you think of learning maths well, out of these 14 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).
2. Diamond ranking:
Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.

- Confidence
- Getting help
- Willingness
- The teacher
- Talking & discussing
- Feeling good
- Thinking
- Doing well
- Explanations

3. Discussion (Sandy)

<table>
<thead>
<tr>
<th>Confidence</th>
<th>Most important because you won’t get anywhere if you don’t think you can do it.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Getting help</td>
<td>Because if you don’t understand, get help or else you’re most likely to get it wrong.</td>
</tr>
<tr>
<td>Willingness</td>
<td>You have to be willing, if you don’t, you won’t want to do it and you won’t get a good result.</td>
</tr>
<tr>
<td>The teacher</td>
<td>Is important because they’re the one explaining. Important that you listen so you know what to do, they’re there to help you.</td>
</tr>
<tr>
<td>Talking &amp; discussing</td>
<td>Also important if you’re talking and discussing, other people can help you with your work. I: Who? S: Friends, teachers. I: How do they help? S: If you don’t understand, talk about it.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>Like confidence, if you feel good you will get through it, compared to feeling bad, you won’t get anywhere. I: Is having goals important? S: Yes I: Do you have some? S: I don’t think so.</td>
</tr>
<tr>
<td>Thinking</td>
<td>Important ‘cos if you think about the question properly and read it over, most likely get it right. I: can you do this and still get it wrong? S: Yes. I: So why else is thinking important? S: Helps you understand the problem. I: Where does the thinking happen? S: In a problem solving question. I: Can you give me an example? S: If someone bought 5 apples, ate 3…</td>
</tr>
<tr>
<td>Doing well</td>
<td>If you’re working and you think you’re doing well, probably get it correct, ‘cos it’s good to have a positive mind</td>
</tr>
<tr>
<td>Explanations</td>
<td>In this, out of these 9 words, least important, they help you understand the question. I: Where do explanations come from? S: The strategy or technique for doing a problem. I: Who provides them? S: Teachers usually. I: Anyone else? S: Friends, people around.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

<table>
<thead>
<tr>
<th>Name: Alison</th>
<th>Year: 7</th>
</tr>
</thead>
</table>

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<tbody>
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<tr>
<td>Thinking</td>
<td>Maths equipment</td>
<td>Getting help</td>
</tr>
<tr>
<td>Doing well</td>
<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:
When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?
Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

<table>
<thead>
<tr>
<th>Talking and discussing</th>
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<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

2. Diamond ranking:
Now I'd like you to put them in order, from most important to least important, in this diamond shaped pattern.

Thinking

Group work

Talking & discussing

Getting help

The teacher

Explanations

Getting the answer

Problem solving

Feeling good
3. Discussion (Alison)

<table>
<thead>
<tr>
<th>Thinking</th>
<th>You need to think to learn, can’t learn without thinking. Why? A: Because it just works like that. Can’t get something into your head unless you think about it.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group work</td>
<td>Help each other and when working together, talk it out and think which one, like, mmm, the answer.</td>
</tr>
<tr>
<td>Talking and discussing</td>
<td>You have to, similar to group work, ‘cos talking about it, stuff, when you work with people, understand stuff and help them.</td>
</tr>
<tr>
<td>Getting help</td>
<td>Like if you don’t understand, get help, or else you’ll never understand something, explain so you know what it means.</td>
</tr>
<tr>
<td>The teacher</td>
<td>Because they teach you what you need to know. If they’re not there you wouldn’t understand it much or at all.</td>
</tr>
<tr>
<td>Explanations</td>
<td>Because that’s linked to the teacher and getting help, so explanations help make sense of it in your head. If you don’t understand, you need to have it explained to you.</td>
</tr>
<tr>
<td>Getting the answer</td>
<td>When you get the answer, that’s the main thing you set out to do. Unless algebra, or not so far, we’re simplifying the algebra not solving it, we’re getting the question.</td>
</tr>
<tr>
<td>Problem solving</td>
<td>‘That’s, everything in maths is problem solving. You need to solve the problem otherwise what would be the point in doing it. I: So what is the point? A: The point of learning maths is solving the problem, at school and outside of school.</td>
</tr>
<tr>
<td>Feeling good</td>
<td>You need a good attitude if you like it, have a better attitude, therefore you’re more likely to learn.</td>
</tr>
</tbody>
</table>
Student card sort record sheet

<table>
<thead>
<tr>
<th>Name:</th>
<th>Year:</th>
</tr>
</thead>
<tbody>
<tr>
<td>School:</td>
<td>Cohort:</td>
</tr>
</tbody>
</table>

Word/concept orientation

Comments:

Some people say that these words are about learning maths well, are there any words missing that you feel should be written on a card?

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<td>Feeling good</td>
<td>Group work</td>
</tr>
</tbody>
</table>

1. Initial sort:

When you think of learning maths well, out of these 12 cards, select 9 to show what you think is most important?

Indicate by highlighting the 9 to be included in the next step (Diamond ranking).

2. Diamond ranking:

Now I’d like you to put them in order, from most important to least important, in this diamond shaped pattern.
APPENDIX I

MULTIPLICATIVE TASK INTERVIEW
PRE-INTERVENTION PHASE TASKS
PATTERNS WITH TILES…

Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.

![Pattern with tiles]

a. How many times can this pattern be made with 28 black tiles, 21 grey tiles and 6 white tiles? Show all your working and explain your answer in as much detail as possible.

b. This pattern uses 3 grey tiles for every 5 black tiles. How many black tiles would you need if you had 12 grey tiles? Show all your working and explain your answer in as much detail as possible.

c. The art teacher orders 6 boxes of red tiles. Each box has 36 tiles. How many red tiles are there altogether? Show all your working and explain your answer in as much detail as possible.

d. The art teacher needs 330 red tiles. How many boxes of red tiles does she need to order? Show all your working and explain your answer in as much detail as possible.
FOOTY DAY LUNCH...

a. Show how you would share 2 large meat pies equally among 3 people?

Each person gets .................................................................

b. Jo ate 4/9 of a large meat pie and Maggie ate 2/3 of a large meat pie. Who ate the most pie? Explain your reasoning using as much mathematics as you can.

c. For Footy Day, the school organised a special lunch. They offered 3 choices of food, two choices of drink and two choices of desert.

<table>
<thead>
<tr>
<th>Food</th>
<th>Drink</th>
<th>Dessert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat pie</td>
<td>Cola</td>
<td>Ice-cream</td>
</tr>
<tr>
<td>Sausage roll</td>
<td>Lemonade</td>
<td>Icy-pole</td>
</tr>
<tr>
<td>Pastie</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Jo ordered a sausage roll, a drink of cola and an icy-pole. What else might she have ordered? List all possibilities. Show your working and explain your answer in as much detail as possible.
Block Pattern

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

a. How many blocks like this can be made with 32 black squares, 17 grey squares and 7 white square? **Show all your working and explain your answer in as much detail as possible.**

b. This block uses 2 grey squares for every 6 black squares. How many black squares would you need if you had 6 grey squares? **Show all your working and explain your answer in as much detail as possible.**

c. The quilt will be made by sewing 25 of these blocks together. How many small squares will the quilt have all together? **Show all your working and explain your answer in as much detail as possible.**

d. If a quilt has 324 small squares, how many blocks of this pattern will be used? **Show all your working and explain your answer in as much detail as possible.**
Sharing

a. How would you share the 2 sausages of playdough among 3 children?

How much does each person get? .................................................................

b. If I have $\frac{4}{6}$ of the playdough sausage and you have $\frac{3}{4}$ of the playdough sausage, who has the most playdough? **Explain your reasoning using as much mathematics as you can.**

Possible possibilities

<table>
<thead>
<tr>
<th>Bottoms</th>
<th>Tops</th>
<th>Hats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeans</td>
<td>Hoodie</td>
<td>Beanie</td>
</tr>
<tr>
<td>Shorts</td>
<td>Windcheater</td>
<td>Cap</td>
</tr>
<tr>
<td>Bordies</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. How many different outfits can I wear? List all possibilities. **Show your working and explain your answer in as much detail as possible.**
APPENDIX J

MULTIPLICATIVE TASK INTERVIEW

POST-INTERVENTION PHASE TASKS
TILES FOR THE HOUSE...

Floor and wall tiles come in difference sizes. The basic tile is shown below.

2 cm

3 cm

a. Show or describe the area that 4 of these tiles would cover?

b. How many basic tiles would be needed for an area of 12 cm by 6 cm?

c. If the length and width of the basic tile were increased by 2 cm, would 120 of the larger tiles be enough to cover 50 cm by 50 cm?
   
   Show all your working so we can understand your thinking.
PEOPLE SITTING …

a. A rectangular table can seat 8 people. Draw one of these tables with the people sitting around it.

b. Draw a line of 4 of these rectangular tables placed end-to-end. How many people are able to sit at it?

c. How many people would be able to sit at 9 of these rectangular tables placed end-to-end?

d. How many of these rectangular tables would you need to seat 86 people?
APPENDIX K

CARD SORT INTERVIEW CONCEPT MAPS
PRE-INTERVENTION PHASE CARD SORT CONCEPT MAP
Constructing paths to multiplicative thinking: Appendices

POST-INTERVENTION PHASE CARD SORT CONCEPT MAP
APPENDIX L

MULTIPLICATIVE TASK INTERVIEW

SUMMARY OF STRATEGIES
PRE-INTERVENTION PHASE MULTIPLICATIVE INTERVIEW STRATEGY SUMMARY

PATTERNS WITH TILES...

Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.

![Pattern with tiles]

a. How many times can this pattern be made with 28 black tiles, 21 grey tiles and 6 white tiles? Show all your working and explain your answer in as much detail as possible.

Task advice

This task requires students to think multiplication or division for this situation to determine the number of times the pattern can be constructed with the equipment ‘provided’, eg, divide 28 by 5, 21 by 3, 6 by 1 (although unnecessary in most cases) or use known multiplication facts. Students need to take into account the three pieces of information, to then provide a solution. Answer: 5 times.

<table>
<thead>
<tr>
<th>Strategies/Responses used by At-risk (intervention and non-intervention)</th>
<th>Strategies/Responses used by Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>▪ When in doubt add, so 28 and 21 and 6 is 51</td>
<td>▪ Divide all part by appropriate number and may show shortfall. (Jessica, Lynda)</td>
</tr>
<tr>
<td>▪ Knew 6 tiles for white tiles only</td>
<td>▪ Determine number of sets possible for each (Adam)</td>
</tr>
<tr>
<td>▪ Count by 3, seven times to get 21 tiles, count by 7 to 28 but unable to balance information and progress strategies to solution (Yousif p. 47 notes)</td>
<td>▪ Division and repeated subtraction.</td>
</tr>
<tr>
<td>▪ Attempt to draw all, count all, (Dean pg 45 notes at part a of interview, modeled with components and through researcher prompting, saw the pattern )some students extend the pattern rather than repeat the pattern</td>
<td></td>
</tr>
<tr>
<td>▪ Halve 28 is 14 but during interview decided that was fruitless, reliant on halving strategy despite being unnecessary in that instance (Cansu pg 46 notes)</td>
<td></td>
</tr>
<tr>
<td>▪ Sense of 5’s to 28 and 3’s to 21, referred to white tiles only.</td>
<td></td>
</tr>
<tr>
<td>▪ Drew all, count to solve successfully (Hadi)</td>
<td></td>
</tr>
<tr>
<td>▪ Source materials to make all count all</td>
<td></td>
</tr>
</tbody>
</table>
(Ahmed and Adir initially no response to task. Ibrahim see pg 54 notes re significant assistance from student teacher)

- Drew 2 diagrams (James pg. 60 notes).
- Attend to white tiles only, so 6 times.
- ‘I can’t work it out’ (Hajar)
- No initial response from Berrin so at part a of interview attempt to use equipment. Had trouble repeating pattern more than once)

**PATTERNS WITH TILES…**

Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.

![Pattern with tiles](image)

b. This pattern uses 3 grey tiles for every 5 black tiles. How many black tiles would you need if you had 12 grey tiles? **Show all your working and explain your answer in as much detail as possible.**

**Task advice**

This proportion task requires students to manipulate the ‘for each’ idea for multiplication in this case 3 grey tiles for every 5 black. Determine the factor increase and apply this to the black tile will determine problem solution. 3 grey tiles to 12 grey tiles is a factor increase of 4, therefore 5 black tiles increased by the same factor. Answer: 20 black tiles.

<table>
<thead>
<tr>
<th>Strategies/Responses used by At-risk (intervention and non-intervention)</th>
<th>Strategies/Responses used by Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer: ‘Seven times’ but forgot why, however during interview, with tile support material was guided to think: Knows 3 grey makes one pattern, 6 grey makes two patterns, count by 3 to 12, 4 times, so count by 5 4 times is 20 (Hajar)</td>
<td>12 grey tiles meant that the pattern could be made 4 times, and 4 by 5 is 20 black tiles (Jessica, Lynda)</td>
</tr>
<tr>
<td>When in doubt add, so 3 and 5 and 12 which is 20! Right answer for wrong reason (Ahmed)</td>
<td>Table form of relationship between 3 grey and 5 black each time the pattern is made: 3, 5; 6,10; 9,15; 12, 20 and stopped at 12 grey tiles.</td>
</tr>
<tr>
<td>Count by 3 to 12, so four times, then 5 by 4 is 20. (Hadi, Candi self corrected during interview part a, and then employed this strategy)</td>
<td>3 by 4 is 12 so 5 by 4 is 20 (Adam, Kalil)</td>
</tr>
<tr>
<td></td>
<td>2 extra black for every 3 grey, 4 lots of grey, so 4 by 3 grey is 12 and 4 by 2 (2 extra black) is 8, 12 + 8 is 20.</td>
</tr>
<tr>
<td></td>
<td>Balanced 3+3 is 6, 6+3 is 9, 9+3 is 12 with (directly under) 5+5 is 10, 10+ 5</td>
</tr>
</tbody>
</table>
Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.

![Pattern with tiles]

e. The art teacher orders 6 boxes of red tiles. Each box has 36 tiles. How many red tiles are there altogether? **Show all your working and explain your answer in as much detail as possible.**

**Task advice**

To calculate the total number of red tiles for 6 boxes of 36 tiles, multiply 36 by 6. Answer 216.

**Strategies/Responses used by At-risk (intervention and non-intervention)**

- When in doubt add, 36 and 6 is 42 (Ahmed, Candi pg 63 notes)
- Tally marks to represent each tile (Cansu no response other than tally marks at part a of interview, inefficient make all, count all strategy with intuitive sense of multiplication/division needed, pg 46 notes)
- Sense of multiplication or division involved but unable to conceptualise.
- Repeated addition of 36, 6 times. Had done 6 tally marks for the 6 in 36 and three tally marks for the 3 in 36 (Douha pg 49 notes)

**Strategies/Responses used by Successful**

- Multiply 36 by 6 using formal multiplication algorithm (Jessica, Lynda, Adam)
- Calculates 36 by 6 mentally (Kai, Sandy pg 59 notes)
- Mental 6 by 2, 12, 3 by 6, 18 and 1, 19 so 192 (self corrects at part a of interview: Alison)
Knew 36 by 6 would be appropriate but chose repeated addition instead (Sidona pg 57 notes)
Mental strategy: add 6 30’s, 180, then 6 sixes, 36, add 180 and 36, 216 (James pg 61 notes)
36 by 6 = 216 (Hajar)
Repeated addition 36, 6 times, 216 (Adir)
No response Berrin at part a of interview, too difficult (no need to proceed)

PATTERNS WITH TILES...

Some children are making patterns with square tiles in an art class. To make this pattern you need 5 black tiles, 3 grey tiles and 1 white tile. It looks like this.

![Tile Pattern](image)

d. The art teacher needs 330 red tiles. How many boxes of red tiles does she need to order? Show all your working and explain your answer in as much detail as possible.

Task advice

This partitive division task requires students to divide 330 by 36 mentally or algorithmically, think multiplication, 36 by what will give 330, or use known facts, that is, “I know 36 by 10 is 360”. Context of the situation is attended to. Nine boxes will leave the art teacher short and 10 boxes will give the art teacher more than necessary. Some students may think that nine boxes and ordering single tiles will suffice.

<table>
<thead>
<tr>
<th>Strategies/Responses used by At-risk (intervention and non-intervention)</th>
<th>Strategies/Responses used by Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer: ‘9 and 1 half’ for 330 red tiles: “I think I share between something”, however at interview was asked to have another go at solution, student asked if ‘you could times 330 by 36?’ (Hajar)</td>
<td>Knew 330 divided by 36 necessary, but choose multiplication, 36 by 8 as an estimate and add more boxes until sufficient number reached.</td>
</tr>
<tr>
<td>You have 36 and times it with 330.</td>
<td>‘Forgot’ how to do long division so count by 36 to desired number (Lynda)</td>
</tr>
<tr>
<td>Attempt to count by 36 to 330 (Ahmed, Hadi,</td>
<td>Knew 10 by 36 is 360 and take off another box to ensure not ordering too much which is a ‘waste of money’ (Adam)</td>
</tr>
<tr>
<td>Repeated addition trial and error until close (Douha pg 49 notes)</td>
<td>What by 36 is 330?, tried 9 and checks using multiplication, decides 10 boxes with 30 tiles left.</td>
</tr>
<tr>
<td>330 divided by 6 is 220 (three into 6 is two etc.)</td>
<td></td>
</tr>
<tr>
<td>Attempt to use strategy from part c,</td>
<td></td>
</tr>
</tbody>
</table>
but decides “too hard”.
- No response (Adir)
- No response Berrin at part a of interview, too difficult so didn’t proceed
- If 6 is 201 (previous question) then one more box 7 is probably enough (Sidona, pg 58 notes)
- If each box has 1 tile, then 330 boxes, if box has 330 tiles, 1 box…36 in each “too hard” James pg 61 notes.
- Part c, 6 boxes estimated 3 more, 9, 9 by 39…(Andrew pg. 64 notes.

- 36 by 8 and add one more box, 9 boxes and 6 extra tiles bought singly (Jessica)
- 36)330?? “not very good at division. At part a of interview, what? by 36 is 330, tried nine times 36 (pg 52 notes)

FOOTY DAY LUNCH…

a. Show how you would share 2 large meat pies equally among 3 people?

Each person gets ……………………………………………………………..

Task advice
Requires students to partition into meaningful shares, that is, thirds per pie and allocate a fair share to 3 people. Answer: 2/3 of one meat pie.

<table>
<thead>
<tr>
<th>Strategies/Responses used by At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Partitioned to have 16 pieces, gave 3 people 2 slices, 10 slices left, gave another 2 slices each, 4 left, then three people get one and the last piece cut into 3 so it is fair. (Hajar)</td>
<td>Partitions each pie into three and each person gets 2 thirds of one pie (Jessica, Lynda, Adam, Alison)</td>
</tr>
<tr>
<td>Inappropriate partitioning but able to describe share informally as 2 pieces each (Douha pg 49 notes)</td>
<td>…or 1 half and 1 third of 1 half.</td>
</tr>
<tr>
<td>Attempts to partition each pie into three for the three people Ibrahim pg 54 notes)</td>
<td></td>
</tr>
<tr>
<td>Partitions three (unequal) pieces per pie and 2 pieces each (Ahmed, Berrin)</td>
<td></td>
</tr>
<tr>
<td>Partitions as above, nos 1, 2, 3, 1, 2, 3 so 2 pieces (Adir)</td>
<td></td>
</tr>
</tbody>
</table>

245
FOOTY DAY LUNCH…

b. Jo ate \( \frac{4}{9} \) of a large meat pie and Maggie ate \( \frac{2}{3} \) of a large meat pie. Who ate the most pie? **Explain your reasoning using as much mathematics as you can.**

Task advice

This task deals with comparing related fractions. Think \( \frac{2}{3} \) is the same as \( \frac{6}{9} \), and \( \frac{6}{9} \) is more than \( \frac{4}{9} \), so Maggie ate most.

### Strategies/Responses used by At-risk (intervention and non-intervention)

- Unable to read \( \frac{4}{9} \) as 4 ninths and \( \frac{2}{3} \) as 2 thirds, draws diagram to show Jo eating 1 quarter and Maggie eating 1 half (Ahmed)
- 4 ninths unknown to student, “Never heard of it” (Adi)
- Jo ate 4 and Maggie ate half
- Able to read 4 ninths and 2 thirds, but unable to say how answer derived other than ‘from fractions’ (Dean)
- Reads 4 ninths and *four and a half nine* and 2 thirds as *three two*. (Cansu sense of equal shares, pg 46 notes)
- Responds a half is bigger than a half: Self corrects response on paper during interview, tried a number line to partition to help with solution but unable to determine line parameters (Hadi pg 49 notes)
- 9 is larger than 3 and 4 is larger than 3, so whole number understanding applied to fraction ideas (Sidona)
- Created a fraction model to show 4 ninths and another showing 2 quarters (nor 2 thirds)
- Partitions appropriately and informal description of shares (Candi)
- Reads \( \frac{4}{9} \) as 4 ninths appropriately but doesn’t understand “4 ninths and stuff”.
- Jo ate most cos 4 out of 9, Maggie only ate 2 out of 3 (Hajar)
- Berrin couldn’t interpret fraction notation, at part a of interview unable to partition

### Strategies/Responses used by Successful

- Convert to thirds, 2 thirds of 9 is 6 so Jo needs another 2 ninths to eat the same as Maggie's (2 thirds) so Maggie ate more (Jessica)
- Convert denominator to ninths and compare (Sandy, Lynda, Adam)
- Find a common denominator, so convert to ninths, nine is divisible by 3 and 4 ninths can’t be renamed as thirds evenly.
- Convert to thirds and compare, so 2 thirds Maggie's and \( \frac{11}{3} \) Jo (4 divided by three as \( 1 \frac{1}{3} \))
FOOTY DAY LUNCH...

c. For Footy Day, the school organised a special lunch. They offered 3 choices of food, two choices of drink and two choices of desert.

<table>
<thead>
<tr>
<th>Food</th>
<th>Drink</th>
<th>Dessert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat pie</td>
<td>Cola</td>
<td>Ice-cream</td>
</tr>
<tr>
<td>Sausage roll</td>
<td>Lemonade</td>
<td>Icy-pole</td>
</tr>
<tr>
<td>Pastie</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Jo ordered a sausage roll, a drink of cola and an icy-pole. What else might she have ordered? List all possibilities. **Show your working and explain your answer in as much detail as possible.**

**Task advice**

This problem deals with the Cartesian product or ‘for each’ idea for multiplication and can be solved in a number of ways: a systematically drawn exhaustive list, work out the number of possibilities for meat pie with a drink and desert, of which there are 4 different options, and multiply this by 3 (for the three food choices), or see 3 (food) by 2 (drink) by 2 (dessert) is equal to 12.

<table>
<thead>
<tr>
<th>Strategies/Responses used by At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>▪ Writes option that was personally chosen (Hajar, Berrin at part a of interview only able to identify 3 more possibilities)</td>
<td>▪ 3 by 2 is 6; 6 by 2 is 12 (Jessica)</td>
</tr>
<tr>
<td>▪ Initially interprets question as ‘make up your own’, then during interview was only able to determine 5 options (Dean)</td>
<td>▪ See 4 options for meat pies, which will be the same number of options for pastie and sausage roll so 4 by 3 is 12.</td>
</tr>
<tr>
<td>▪ Non-systematic, only able to see a minimal number of possibilities or options.</td>
<td>▪ A line with all the options, showing the different ones each time, then 4 by 3 is 12.</td>
</tr>
<tr>
<td>▪ Indicates one or two options only.</td>
<td>▪ Tree diagram (Adam, Kalil)</td>
</tr>
<tr>
<td>▪ Three options one for meat pie, sausage roll and pastie (Adir)</td>
<td>▪ 4 by 3 is 12 (Lynda)</td>
</tr>
<tr>
<td>▪ 6 single items listed (Ahmed)</td>
<td>▪ 4 options for each of the food items, 4+4+4 is 12.</td>
</tr>
<tr>
<td>▪ 8 options listed (semi-systematic).</td>
<td>▪ 12, kept tally marks progressively for each food picture (Chris).</td>
</tr>
<tr>
<td>▪ “oh man…it’d take too long” therefore appears to recognise a number of options possible.</td>
<td></td>
</tr>
</tbody>
</table>
Block Pattern

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

![Block Pattern Image]

Task Advice:
This task requires students to think multiplication for this division situation to determine the number of blocks that can be constructed with the given number of squares, eg, divide 32 by 6, divide 17 by 2 or use known facts. Students need to take into account all constraints to provide solution.

a. How many blocks like this can be made with 32 black squares, 17 grey squares and 7 white squares? **Show all your working and explain your answer in as much detail as possible.**

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• How many sixes in 32…6, 12, 18, 24, 30 5 times, 2 left, then moved onto 17 grey on her own (Hajar)</td>
<td></td>
</tr>
<tr>
<td>• Uses equipment, 2 times…Y. looks at 6 black, double, double to get 4 times, then got lost (Yousif pg 48 notes)</td>
<td></td>
</tr>
<tr>
<td>• This is hard, too many to fit on paper – see response in file and pg 62 notes (James)</td>
<td></td>
</tr>
</tbody>
</table>

**Summary: skip count, doubling, too hard**
Block Pattern

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

![Block Pattern Image]

b. This block uses 2 grey squares for every 6 black squares. How many black squares would you need if you had 6 grey squares? Show all your working and explain your answer in as much detail as possible.

Task Advice

This proportion task requires students to manipulate the ‘for each’ idea for multiplication, in this case, 2 grey squares for every 6 black squares. Determine the factor increase and apply this to the black squares will determine solution. Factor increase of 3 therefore 6 black squares increase by the same factor. Answer: 18 black squares

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 3 blocks needed because 6 grey and 2 for each pattern. I: How many black? Candi: 18...3 by 6 is 18</td>
<td>• Retells situation, draws 3 blocks for 6 grey, so 3 by 6, 18, applies factor increase Chris pg 52 notes</td>
</tr>
<tr>
<td><strong>Summary: for each, applying intuitive sense of factor increase</strong></td>
<td>• 2 grey all the time for each, for each grey 3 black, 6 by 3, 18 Kalil pg 53 notes.</td>
</tr>
<tr>
<td></td>
<td><strong>Summary: factor increase</strong></td>
</tr>
</tbody>
</table>
Block Pattern

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

![Block Pattern Diagram]

c. The quilt will be made by sewing 25 of these blocks together. How many small squares will the quilt have all together? Show all your working and explain your answer in as much detail as possible.

Task advice

Block Pattern

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.

![Block Pattern Diagram]

d. If a quilt has 324 small squares, how many blocks of this pattern will be used? Show all your working and explain your answer in as much detail as possible.
Task advice
This partitive division task requires student to divide 324 by 9 mental or algorithmically, think 9 whats are 324? Or use known facts. Answer: 36 blocks.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>•</td>
<td>• Retell ok, 324 divided by 9 used guzinta eg, 9 into 3 can’t do...(Lynda and Jessica)</td>
</tr>
<tr>
<td></td>
<td>• 9)324 algorithm with detailed explanation using region model (Kai pg 51 notes)</td>
</tr>
<tr>
<td></td>
<td><strong>Summary: guzinta division with some detail with region model</strong></td>
</tr>
</tbody>
</table>

Sharing
a. How would you share the 2 sausages of playdough among 3 children?

\[ \text{How much does each person get?} \]

Task advice
Requires student to partition into meaningful shares, that is thirds per sausage and allocate a fair share to 3 children. Answer 2 thirds of the sausage of play dough.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Partitions ok, but unable to describe shares (Hajar)</td>
<td>•</td>
</tr>
<tr>
<td>• 6…no 3…no 2 self corrects, initially unable to partition into thirds (dean pg 45 notes)</td>
<td></td>
</tr>
<tr>
<td>• I’d cut into little pieces then share with 3 people (Cansu pg 46 of notes)</td>
<td></td>
</tr>
<tr>
<td>• With play dough partitions into thirds ok, so 2 each. I: can you say another way? Douha: 2 out of 6 pg 50 notes.</td>
<td></td>
</tr>
<tr>
<td>• Uses playdough and partitions into 3 physically shares out to 3 and says 2 (Ibrahim)</td>
<td></td>
</tr>
</tbody>
</table>
Summary: partitioning ok, unable to describe shares other than “2”, need to model sharing

Sharing

b. If I have \(\frac{4}{6}\) of the playdough sausage and you have \(\frac{3}{4}\) of the playdough sausage, who has the most playdough? **Explain your reasoning using as much mathematics as you can.**

Task advice

This task deals with comparison of unlike fractions. Students need to rename and then compare. Answer: the student has the most sausage (9 twelfths larger than 8 twelfths)

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Playdough used, inappropriate. Partitioning, not linked to commonly understood pictures (Ahmed pg 41 notes)</td>
<td>• Find common denominator first…6 by 4, 24, what I do to the denominator I do to the numerator (Adam pg 36 notes)</td>
</tr>
<tr>
<td>• Margarita has more then me because M has 4 and Adir has 3 (whole no. understanding). Partitioning with playdough successful but did not help to support a change in initial thinking (Adir pg 43 notes).</td>
<td>• Convert to the same denominators…lowest common multiple…12 (Sandy pg 59 notes)</td>
</tr>
</tbody>
</table>
| • Attends to numerator only ? partitioning playdough, whole number description of shares. “2” (Berrin pg 44 notes) | **Summary: rename procedurally?**  
**Common denominator as twelfths of twenty-fourths.** |
| • 4/6 bigger 4 being number of people and 6 the number of pieces (Hadi pg 48 notes) | |
| • Sixth bigger than fourth, fourth bigger than third, so 4/6 (Sidona pg 58 notes) | |
| • \(\frac{3}{4}\) is more cos 4/6 are smaller pieces, ‘show me’…(Andrew pg 66 notes) | |

Summary: attention to numerator and/or denominator in relation to whole number understanding, creative ways of making sense of numerator and denominator!
Possible possibilities

<table>
<thead>
<tr>
<th>Bottoms</th>
<th>Tops</th>
<th>Hats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeans</td>
<td>Hoodie</td>
<td>Beanie</td>
</tr>
<tr>
<td>Shorts</td>
<td>Windcheater</td>
<td>Cap</td>
</tr>
<tr>
<td>Bordies</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. How many different outfits can I wear? List all possibilities. **Show your working and explain your answer in as much detail as possible.**

**Task advice**

This problem deals with the Cartesian product or for each idea for multiplication. Can be solved a number of ways, eg, all options listed systematically, tree diagram or multiplicative relationship: 3 by 2 by 2. Answer: 12.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>•</td>
<td>• Alison tells me all possibilities for jeans…4, then those for each of the bottoms, so 4 by 3, 12 (Alison pg 57 notes)</td>
</tr>
<tr>
<td></td>
<td><strong>Summary: 4 by 3, 12.</strong></td>
</tr>
</tbody>
</table>
POST-INTERVENTION PHASE MULTIPLICATIVE INTERVIEW

STRATEGY SUMMARY

TABLES AND CHAIRS …

Dean's community is planning a street party. They have lots of small square tables. Each table seats 4 people like this:

The community decides to put the tables in an end-to-end line along the street to make one big table.

a. Make or draw a line with 2 tables. How many people will be able to sit at it?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• 8 ‘people on the sides’ (but 8 tables drawn) Ibrahim</td>
<td>• 3 by 2, 6 Kai</td>
</tr>
</tbody>
</table>

b. Make or draw a line of 4 tables. How many people will be able to sit at it?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 12 tables and 12 people drawn (Ibrahim)</td>
<td>•</td>
</tr>
</tbody>
</table>

c. Make or draw a line of tables that would seat 8 people. How many tables are needed?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• Drew 2 tables apart to seat 8 people (Hajar)</td>
<td>•</td>
</tr>
</tbody>
</table>

g. Can you find another way to describe your results so far? Show this in the space below.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• For every table you add 2 chairs, tables times 2, + 2 equals chairs (Adam)</td>
<td>•</td>
</tr>
</tbody>
</table>
h. The community can borrow 99 tables. How many people could they seat using 99 tables placed end-to-end? **Show your working and explain your answer in as much detail as possible.**

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Double 99, 198 plus 2 on each end (James)</td>
<td>• Worked out people for 20 tables, expanded to 80 tables added 19 tables so 200 people (Kalil)</td>
</tr>
<tr>
<td>• 200 based on 99 + 99, 198 and 2 more (Adir)</td>
<td>• 248 based on think 100 tables, 100 div by 4, 25, 25 times 10, 250, take 2 (Jessica)</td>
</tr>
<tr>
<td>• 200 (no recall as to how, evidence of drawing (Berrin))</td>
<td>• 99 + 99 is 199 so 201 people (Andrew)</td>
</tr>
<tr>
<td>• 99 + 99 is 199 so 201 people (Andrew)</td>
<td>• 200 (no recall as to how, evidence of drawing (Berrin))</td>
</tr>
</tbody>
</table>

i. The community can borrow rectangular tables that seat 6 people. Draw one of these tables showing the people sitting around it.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Draws 6 tables (Hadi)</td>
<td>•</td>
</tr>
</tbody>
</table>

j. Draw a line of 5 of these rectangular tables placed end-to-end. How many people will be able to sit at it?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 5 tables, see how many I can fit (Hadi)</td>
<td>• 5 new tables is 10 old tables, old 10 + 1 is 11, 11 by 2, 22 (Kai)</td>
</tr>
<tr>
<td>• 6, 2 tables, 12 and 18 (6, 3 tables) 30 (Cansu)</td>
<td>•</td>
</tr>
</tbody>
</table>

k. Explain what happens to the number of people as more rectangular tables are placed end-to-end. Describe or show your findings in at least two ways.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• As table added, 4 more people (Ahmed)</td>
<td>• The people lose 2 seats every time you put tables end to end (Kai)</td>
</tr>
</tbody>
</table>
1. How many people could be seated if 46 of these rectangular tables were placed end-to-end? Show your working and explain your answer in as much detail as possible.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Based on 7 tables, 30 people, 14 tables, 60 people...42 tables, 180 people and 3 tables, 12 people so 192 (James)</td>
<td>• Based on thinking of square table with four people at extrapolated to this new table, 186: old table by 9, 20 people take 2 off ends, 18 people. 9 of these, 5 times, 45 tables and 1 more table, so 18 by (double 5), 180 plus extra table, 184 plus 2 on ends, 186 (Kalil)</td>
</tr>
<tr>
<td>• 2208 no recall how, maybe I 'timesed' it (Hajar)</td>
<td></td>
</tr>
<tr>
<td>• 23 at 46 tables (Dean no recall)</td>
<td></td>
</tr>
<tr>
<td>• Attempt to multiply 46 by 6/2436 scratched out, also a pick of 46 tables with statement 194 people can sit at it (Andrew)</td>
<td></td>
</tr>
</tbody>
</table>

m. How many of these rectangular tables would you need to place end-to-end to seat 342 people? Show your working and explain your answer in as much detail as possible.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• 342 take 2 (340) div by 2, 85 (Adam)</td>
<td>• 342 take 2 (340) div by 2, 85 (Adam)</td>
</tr>
<tr>
<td>• Answer taken from 46 tables -&gt; 186 people with a table showing table/people added singly so 86 with two extra seats (Jessica)</td>
<td>• Answer taken from 46 tables -&gt; 186 people with a table showing table/people added singly so 86 with two extra seats (Jessica)</td>
</tr>
<tr>
<td>• Progress towards solution based on take 2 from 342, 340. halve it, 170 so 170, 170 and 1 is 342 (Chris)</td>
<td>• Progress towards solution based on take 2 from 342, 340. halve it, 170 so 170, 170 and 1 is 342 (Chris)</td>
</tr>
</tbody>
</table>
BUTTERFLY HOUSE...

Some children visited the Butterfly House at the Zoo. They learnt that a butterfly is made up of 4 wings, one body and two feelers.

While they were there, they made models and answered some questions.

For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

   _____________ wings
   _____________ bodies
   _____________ feelers

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• 19 wings, 7 bodies, 14 feelers, based on 4 by 7 uses fingers (Hajar)</td>
<td>•</td>
</tr>
</tbody>
</table>

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
</table>
| • Think times, 4 by 4, 16; 4 by 2, 8 (Hadi)  
  • 3 b/flies based on incorrect skip count by 4s, 4, 8, 12, 15 so not enough (Hajar) | • |
c. How many wings, bodies and feelers will be needed to make 98 model butterflies. **Show all your working and explain your answer in as much detail as possible.**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>wings</strong></td>
<td></td>
</tr>
<tr>
<td><strong>bodies</strong></td>
<td></td>
</tr>
<tr>
<td><strong>feelers</strong></td>
<td></td>
</tr>
</tbody>
</table>

**At-risk (intervention and non-intervention)**

- 98 by 1, 98 by 2, 98 by 4 calculated mentally, 90 + 90, 180, 100 + 100, 200, +180, 380; 6+6, 12, 380 + 12, 392 (James)
- 98 by 4, 98 by 98 for bodies (9604), 98 by 2 (Hajar)

**Successful**

- 6

---

d. How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers? **Show all your working and explain your answer in as much detail as possible.**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At-risk (intervention and non-intervention)</strong></td>
<td><strong>Successful</strong></td>
</tr>
<tr>
<td>Times everything by amount you need to make b/fly (Hadi)</td>
<td>6 taking lowest of 6, 7, 8 possible sets of components (Adam)</td>
</tr>
<tr>
<td>29 by 4, 116 (Yousif)</td>
<td></td>
</tr>
<tr>
<td>Can’t have 8 even if enough bodies, took 1 off feelers because you can’t use it, so half 12, 6 b/flies with enough wings (James)</td>
<td></td>
</tr>
<tr>
<td>\ simply restate information with nothing else (Cansu)</td>
<td></td>
</tr>
<tr>
<td>Drew 14 butterflies (Adir)</td>
<td></td>
</tr>
<tr>
<td>Stated 6 (used scrap paper and drew all to count six complete) (Hajar)</td>
<td></td>
</tr>
<tr>
<td>6 b/flies, drew 8 bodies, 4 wings on each and 13 feelers (not all fit) so 6 complete (Ahmed)</td>
<td></td>
</tr>
<tr>
<td>Draws 8 b/flies based on 8 body info (Ibrahim)</td>
<td></td>
</tr>
</tbody>
</table>
TILES, TILES, TILES…

Floor and wall tiles come in different sizes. The basic tile is shown below.

![2 cm tile](image)

3 cm tile

a. How many basic tiles would be needed for an area of 6 cm by 4 cm?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Guessed 4 (Douha)</td>
<td>• 2 tiles based on double 2, 4 and double 3, 6 (Chris)</td>
</tr>
<tr>
<td>• Drew 4 by 6 to see how many tiles fit (Hadi)</td>
<td>• 2 tiles to fit 6 cm (Sandy)</td>
</tr>
<tr>
<td>• 2 + 2, 4, 3 + 3, 6, so 2 times. (Yousif)</td>
<td></td>
</tr>
<tr>
<td>• Drew 2 tiles as above for increase to 4 cm so 2 tiles (wrote 4 tiles so right for wrong reason) (James)</td>
<td></td>
</tr>
<tr>
<td>• Area for 6 by 4, 24 cm (Berrin)</td>
<td></td>
</tr>
<tr>
<td>• Calc perimeter (Candi)</td>
<td></td>
</tr>
<tr>
<td>• 2 + 2, 4, 3 + 3, 6, so 2 times, at part a of interview decided to calc perimeter better (Sidona)</td>
<td></td>
</tr>
</tbody>
</table>

b. How many basic tiles would be needed for an area of 27 cm by 18 cm?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• 81 because 3 by 9, 27, 2 by 9, 18, 9 by 9, 81 (Berrin)</td>
<td>• 2 by 9, 18, 3 by 9, 27, 9 by 9, 81 (Kalil)</td>
</tr>
<tr>
<td>• Drew 27 by 18 so 5 tiles (Ahmed)</td>
<td>• 29 div by 2, 13.5, 18 div by 3, 6, 13 by 6, 78; 6 by half, 3, 78 whole tiles and 3 divided into half (Kai)</td>
</tr>
<tr>
<td>• Calculated perimeter (Candi)</td>
<td>• 27 div by 3, 9 and 18 div 2, 9 so 9 and 9, 18 (Jessica and Lynda)</td>
</tr>
<tr>
<td>• 3 by 9, 27 and 2 by 9, 18 so 18 (Sidona)</td>
<td>• Evidence of thinking 3 by 9 is 27 and 2 by 9, 18 only (Sandy)</td>
</tr>
<tr>
<td></td>
<td>•</td>
</tr>
</tbody>
</table>
c. If the length and width of the basic tile were increased by 2 cm, how many of the larger tiles would be needed to cover 1 square metre (100 cm by 100 cm)?

Show all your working so we can understand your thinking.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Attempt to div 100 by 5 and 100 by 4 but no further (Dean)</td>
<td>• 20 tiles down 25 across, 20 by 25, 500 (Adam, Alison)</td>
</tr>
</tbody>
</table>

STAINED GLASS WINDOWS...

Stained glass windows can be made using small triangles.

This stained glass window is made from four small triangles joined together. It is 2 triangles wide at the base and 2 triangles high.

a. How many small triangles will you need if your window is to be 4 triangles wide and 4 triangles high?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Started to draw (Douha)</td>
<td>• Drew 4 across and 4 up noticed the same as squaring, 4 by 4, 16 (Kai)</td>
</tr>
<tr>
<td>• 8 triangles based on 4 + 4 (Dean)</td>
<td>• 4 triangles (at part a of interview notices more needed so needed to draw…”16” wouldn’t you do 4 by 4? (Lynda)</td>
</tr>
</tbody>
</table>
b. Part of the stained glass window shown below, is hidden by a sign. How many small triangles were needed to make this window?

![Stained Glass Window]

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Counted (Douha)</td>
<td>• Due to SRI (a) 9 up and 9 across, 81 (Lynda)</td>
</tr>
<tr>
<td>• 9 by 9 (Hadi)</td>
<td></td>
</tr>
<tr>
<td>• Worked out no. of tiles covered by sign “38” (Cansu)</td>
<td></td>
</tr>
</tbody>
</table>

C. How would you advise a friend on how to work out the number of small triangles that would be needed for a window 26 triangles wide?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• Times 26 by itself (Douha)</td>
<td>• Width by height = triangles (Adam)</td>
</tr>
<tr>
<td>• Part b) 9 by 9, 81 but c) blank because height not given and couldn’t apply thinking for b) (Ahmed)</td>
<td>• From a) and b) 26 by 26 (Lynda)</td>
</tr>
</tbody>
</table>

PEOPLE SITTING …

a. A rectangular table can seat 8 people. Draw one of these tables with the people sitting around it.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• As expected (Cansu, Hajar, Ahmed, Andrew, Ibrahim)</td>
<td>• As expected (Kalil, Jessica)</td>
</tr>
<tr>
<td></td>
<td>• Two options square table with 2 each side and rectangular table with 3 along long sides and 2 at the heads.</td>
</tr>
</tbody>
</table>
b. Draw a line of 4 of these rectangular tables placed end-to-end. How many people are able to sit at it?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
<th>Successful</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Draws, skip count by 3s to 24, 25, 25 (Cansu and Andrew, pg 51 notes)</td>
<td>• 4 tables by 6, 24 and 2 on end (Kalil)</td>
</tr>
<tr>
<td>• Initially draws table separately…count by 4 based on 2 people per side to 16 and 2 more, 18 (Adir)</td>
<td>• Pg 28 notes interest working out for options a) and b) (Kai)</td>
</tr>
<tr>
<td>• Draws, wants one person per side…2 people…3 too squashy (when pressed) skip count 3s, 24, 26. (Hajar)</td>
<td>• 3s to 12, 12 by 2, 24 and 2 more, 26 (Jessica)</td>
</tr>
<tr>
<td>• Draws tables one at a time, crossing out person at end as each table is added (Ibrahim, pg 64 notes)</td>
<td></td>
</tr>
<tr>
<td>• 14 based on rectangular table placed end to end longways, so three at each end and one per side, 3, 3, 4, 4 (Ahmed)</td>
<td></td>
</tr>
</tbody>
</table>

c. How many people would be able to sit at 9 of these rectangular tables placed end-to-end?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• Draws, 9; 1, 2, 3…count by 3…3 nines “You can by 6s too…6, 12…54, 55, 56 (Cansu)</td>
<td>• 9 by 6, 54 and 2 more, 56 (Kalil)</td>
</tr>
<tr>
<td>• Count by 2s to 36, 38 (Adir)</td>
<td>• a) 6 + (7 by 4) + 6….40 b) 7 + (6 by 7) + 7, 56 (Kai)</td>
</tr>
<tr>
<td>• Adds 5 more table, count by ones, 25, 26, 27, 28…(Hajar pg 24 notes)</td>
<td>• 9 by 3, 27, 27 by 2, 54, and 2 more, 56 (Jessica)</td>
</tr>
<tr>
<td>• Initially 9 by 8 then 9 by 5 cos 3 on one side as table is placed “we don’t need them” (Ahmed)</td>
<td></td>
</tr>
<tr>
<td>• Draws, counts as puts people in, 56 by ones (Andrew)</td>
<td></td>
</tr>
</tbody>
</table>

d. How many of these rectangular tables would you need to seat 86 people?
Constructing paths to multiplicative thinking: Appendices

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>• Knew division appropriate (Andrew)</td>
<td>• 86 take 2, 84, 84 div by 6, 16 (Kalil pg 25 notes)</td>
</tr>
<tr>
<td>• Applies option a) results for c) above, division for option b) (Kai pg 28 notes)</td>
<td>• Applies option a) results for c) above, division for option b) (Kai pg 28 notes)</td>
</tr>
<tr>
<td>• Applies option a) results for c) above, division for option b) (Kai pg 28 notes)</td>
<td>• Applies option a) results for c) above, division for option b) (Kai pg 28 notes)</td>
</tr>
<tr>
<td>• 86 take 2, 84, 84 div by 6, 14 (Jessica)</td>
<td>• 86 take 2, 84, 84 div by 6, 14 (Jessica)</td>
</tr>
</tbody>
</table>

TILES FOR THE HOUSE...

Floor and wall tiles come in different sizes. The basic tile is shown below.

2 cm

3 cm

a. Show or describe the area that 4 of these tiles would cover?

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</thead>
<tbody>
<tr>
<td>• Draws 4 of the tiles above and immediately describe 8 by 3 (Douha Hadi, Yousif, Dean)</td>
<td>• Draws 2 different ways to cover the same, 3 by 8 and 4 by 6, “24 cm sq” (Lynda, Sandy)</td>
</tr>
<tr>
<td>• Draws two tiles as above by 2 tiles, labels 2, 2 across, 3, 3 down the sides (James)</td>
<td></td>
</tr>
<tr>
<td>• Draws 4 tiles, with unusual labelling (Berrin, pg ? notes)</td>
<td></td>
</tr>
<tr>
<td>• Comprehension difficulty, I rephrase, draws each tile separately ‘area’ unfamiliar to him (Candi, pg 48 notes)</td>
<td></td>
</tr>
<tr>
<td>• 4 of these, 2 plus 3, 5, 5 by 4 is 20. I: Draw it for me? 3)8 (Sidona, pg ? notes)</td>
<td></td>
</tr>
</tbody>
</table>

b. How many basic tiles would be needed for an area of 12 cm by 6 cm?
Uses a) need another 2 across to make 12, then another set to make 9 down, count 6, 6, 12 (Douha, Yousif)

“Make above bit a bit longer, 3 cms” add 2 tiles across top (Dean)

Had not notion of what 3/8 in relation to a) (Sidona pg 48 notes)

12 div by 2, 6; 6 div by 3, 2…try another way 12 div by 3, 4; 6 div by 2, 3, 6 by 2, 12 so 3 by 4 (Lynda)

12 div by 3, 4; 6 div by 2, 3; 3 by 4 12 (Alison, Chris)

2 into 12, 6; 3 into 6, 2, 6 by 2, 18 (miscalculation) Sandy

c. If the length and width of the basic tile were increased by 2 cm, would 120 of the larger tiles be enough to cover 50 cm by 50 cm?

Show all your working so we can understand your thinking.

<table>
<thead>
<tr>
<th>At-risk (intervention and non-intervention)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Uses part a of interview recall of Tiles, Tiles, Tiles c), 500 tiles for 1 m sq is half 100 by 100 (tiles tiles tiles c) so 250 tiles needed therefore 120 is not enough (Adam pg 21 notes)</td>
<td>Uses part a of interview recall of Tiles, Tiles, Tiles c), 500 tiles for 1 m sq is half 100 by 100 (tiles tiles tiles c) so 250 tiles needed therefore 120 is not enough (Adam pg 21 notes)</td>
</tr>
<tr>
<td>Divide 120 by 4 and 5, work out area 50 by 50, 2500 “no don’t think enough” (Lynda)</td>
<td>Divide 120 by 4 and 5, work out area 50 by 50, 2500 “no don’t think enough” (Lynda)</td>
</tr>
<tr>
<td>Based on tile 5 by 4 need 10 tiles down, 4 cm doesn’t work equally…4 by 12, 48 one more tile needed, 13, 13 by 10, 130, 130 tiles, too many (Alison)</td>
<td>Based on tile 5 by 4 need 10 tiles down, 4 cm doesn’t work equally…4 by 12, 48 one more tile needed, 13, 13 by 10, 130, 130 tiles, too many (Alison)</td>
</tr>
</tbody>
</table>