Residual strength of composite laminates containing scarfed and straight-sided holes

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Abstract

This paper presents an investigation of the damage progression behaviour of composite laminates containing either straight-edged or scarfed holes. Test coupons, made of IM7/5250-4 carbon/BMI prepreg material, were tested under tension until failure. Hoop strains recorded by strain gauges located along the tapered face for laminates with scarfed holes indicate a much greater extent of damage progression than specimens containing straight-edged holes. The experimental results are utilised to assess the predictive capability of three residual-strength models, including an analytical cohesive zone model developed in this work, an analytical inherent-flaw fracture mechanics method and a finite element-based continuum damage model. Comparison of the experimental results with the model predictions reveals that the continuum damage model, calibrated using data from coupons with straight-sided holes, provides promising correlation with experimental results. The cohesive zone model is found to be capable of capturing the hole-size effect for straight-edged holes, and gives results of comparable accuracy to the continuum damage model, without requiring any finite element analysis.
1. Introduction

The size effect on the strength of composite structures is an important consideration in the design and analysis of composite structures containing stress concentrators (e.g., cut-outs, bolt holes, and in-service damage). In the case of open holes, this size effect, usually known as “hole size effect”, is normally accounted for using semi-empirical methods that are calibrated against strength pertinent to one particular hole size. The early work by Whitney and Nuismer [1] and Pipes et al. [2] yielded empirical models whose fitting parameters must be obtained by experiments for given stacking sequences and notch geometries. These types of semi-empirical methods require separate calibrations for specific configurations, such as the ratio of fastener load to bypass load, biaxial loading, stacking sequence of composite laminates, tapering geometry of countersunk fasteners, and out-of-plane bending [3].

The application of progressive damage modelling for in-plane ply damage has long been considered a prerequisite for laminate strength analysis, given the considerable underestimation of strength by the maximum stress or strain criteria. An approach applied by numerous authors involves a simple property knockdown strategy, where one or more composite failure criteria are monitored and linked to reductions in selected stiffness properties [4, 5]. However, it has been noted that model predictions depended strongly on mesh refinement, similar to that caused by damage localisation within a crack band defined by the mesh refinement [6, 7]. Recently, an approach implemented into the commercial finite element (FE) code Abaqus [8, 9] for in-plane loading has incorporated a crack-band model and continuum damage models for each failure process to simulate the stress-softening behaviour. This approach has been reported to yield significant improvements in removing the mesh
dependence [9]. A similar approach based on the LaRC04 failure criteria was applied to the analysis of open hole tension specimens, and was reported to be capable of capturing the size effect, particularly for small holes [10].

The cohesive zone model, originally developed to analyse elastic-plastic fracture of metallic structures, has been adapted to model compressive failure of notched composite laminates, because the micro-buckling band closely resembles a crack [11]. Recently the cohesive zone method has received renewed interest in simulating tensile fracture of the composite laminates [12-14]. Most studies so far have employed a linear softening model [13-15], as the shape of the cohesive law is known to have little effect on the residual strength of structures containing large crack-like damages or slits [16]. It is not clear, however, to what extent the shape of the cohesive law influences the predicted strength of laminates without pre-existing flaws.

Based on the concept of inherent flaws, Eisenmann and Rousseau [3] proposed an analytical method to incorporate the influence of laminate lay-up on the notch strength of composites. After accounting for other factors such as biaxial loading, fastener and hole configurations, this approach, named IBOLT, serves as the basis for designing bolted joints at Lockheed Martin Aeronautics (LM Aero) [3]. Recently Camanho et al. [10] reported that to accurately predict the notched strength of laminates the inherent-flaw based fracture mechanics approach would require the calculation of the length of inherent flaws for different notch geometries.

Scarfed and irregular holes are an important class of holes for aircraft structures. Here the term “scarfed holes” refers to cut-outs that have been bevelled at a shallow taper angle to a sharp tip, such as those formed for making scarf repairs [17, 18]. Due to the lack of a robust non-destructive technique to detect weak bonds, certification of adhesively bonded repairs to safety-critical structures often assumes that the adhesive bond is completely ineffective. To demonstrate compliance with this airworthiness
requirement, it is essential to determine the residual strength of a structure containing a scarfed hole only to ensure that it exceeds the design limit load. Other potential applications include designing irregular shaped slots or joggle features to accommodate conformal or flush mounted antenna, which are suited for applications where aerodynamic drag characteristics of standard blade antennas are not acceptable for operation or safety reasons. In this context, the scarfed-hole configuration represents a significant challenge for both FE analysis and analytical approaches, as the scarf region features not only a changing thickness, but also vastly different laminate properties.

The objective of the present work is to investigate the damage progression behaviour and predictive methodologies for composite laminates containing scarfed holes as well as straight-edged holes for comparison. Experiments were conducted of small coupons and large panels containing circular holes, straight or scarfed, of varying sizes. The results are compared to predictions of the ultimate strengths using a cohesive zone model, which is developed in this work for scarfed-hole laminates. The analysis is compared to the IBOLT inherent-flaw fracture mechanics approach developed at LM Aero [3], as an example of industry practice, and the continuum damage mechanics model implemented in Abaqus 6.7-1 [8, 9], as an example of commercial FE codes. A focus of the analysis is to assess these approaches from the point of view of calibration of material properties. In particular, the present investigation considers whether the approaches could use basic material properties and calibration with small coupon test data, and be applied for the analysis of large panels with both straight and scarfed holes.

2. Experiments and Results

Experimental specimens were tested in three configurations: open-hole tension (OHT) coupons, straight-hole panels and scarfed-hole panels. These configurations are summarised in Figure 1 and Table 1. The width of the large panels has been selected to provide adequate edge distance for the scarfed hole without exceeding the width limit of the testing machine. The specimens were all made of
IM7/5250-4 carbon/BMI unidirectional prepreg material, with basic material properties given in Table 2 [19]. The large panels were made of a “stiff” laminate \([45/0_2/-45/90]_3S\), whilst the OHT coupons were made of the same stiff laminate as well as a “soft” laminate \([45/90_2/-45/0]_3S\). Strain gauges were attached on the laminate surface as well as inside the hole edge for the straight-hole panel. The panels were loaded in tension to failure with a loading rate of 1 mm/min.

Table 1: Specimen details

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Hole diameter (D) (mm)</th>
<th>Width (W) (mm)</th>
<th>Length (L) (mm)</th>
<th>Stacking sequence</th>
<th>Number of specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>OHT coupon</td>
<td>6.35</td>
<td>31.75</td>
<td>200</td>
<td>([45/0_2/-45/90]_3S) ([-45/90_2/45/0]_3S)</td>
<td>5 (\text{OHT coupon}) 3</td>
</tr>
<tr>
<td>Straight-hole panel</td>
<td>50.8</td>
<td>500</td>
<td>515</td>
<td>([45/90/-45/0_2]_3S)</td>
<td>1</td>
</tr>
<tr>
<td>(\text{Inner diameter}=25)</td>
<td>25.0</td>
<td>250</td>
<td>515</td>
<td>([45/90/-45/0_2]_3S)</td>
<td>1</td>
</tr>
<tr>
<td>(\text{Inner diameter}=50)</td>
<td>500</td>
<td>515</td>
<td>([45/90/-45/0_2]_3S)</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Material properties for IM7/5250-4 carbon/BMI unidirectional tape [19, 20].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value (\text{GPa})</th>
<th>Property</th>
<th>Value (\text{mm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{11})</td>
<td>162</td>
<td>Ply thickness</td>
<td>0.13</td>
</tr>
<tr>
<td>(E_{22})</td>
<td>9.51</td>
<td>(\nu_{12})</td>
<td>0.32</td>
</tr>
<tr>
<td>(G_{12})</td>
<td>5.9</td>
<td>(X_T) [MPa]</td>
<td>2618</td>
</tr>
<tr>
<td>(G_{23})</td>
<td>3.3</td>
<td>(Y_T) [MPa]</td>
<td>66</td>
</tr>
<tr>
<td>(G_{31})</td>
<td>5.9</td>
<td>(S_{12}) [MPa]</td>
<td>103</td>
</tr>
</tbody>
</table>
The OHT and straight-hole panels exhibited a catastrophic fracture process, with failure occurring in net tension with some delamination around the hole edge. This behaviour is also seen in the strain gauge results of Figure 2(a), where the gauges on the panel surface show mostly linear deformation up to the ultimate load. Both the strain gauges (gauge length = 3.18 mm and oriented in the hoop direction) located inside the hole, however, indicate strong non-linearity at a strain of around 12,000 µε, followed by rapid increase at around 20,000 µε to finally exceed the gauge limit of 25,000 µε.

In contrast, the scarfed-hole panel exhibits a more progressive failure, as evidenced by the small load drops occurring continuously throughout the loading. From the strain gauge data, as shown in Figure 2(b), it is clear that the scarfed-hole panel experiences sequential failure moving outwards from the taper edge. The failure at the hole edges does not seem to affect the far-field strain (at SG 1), which appears to increase linearly until failure. Therefore there is no noticeable load shedding from the failure of the plies at the tapered edge.

3. Analysis Methodology and Calibration

3.1. Inherent-flaw fracture mechanics

The inherent-flaw fracture mechanics tool (IBOLT) [3] performs a fracture mechanics-based static strength prediction for a rectangular composite joint element subjected to any combination of biaxial membrane loads, shear loads, and an off-axis bolt load. This method assumes that failure is due to unstable propagation of a pre-existent flaw of length \( a_0 \). The notched strength is given in terms of the un-notched strength, \( \sigma_n = \sigma_0 / f(a_0/R) \), where \( f(a_0/R) \) is a function developed from linear elastic fracture mechanics (LEFM) by Bowie [21] for a tensile coupon with a central hole of radius \( R \) having symmetric edge cracks. The inherent flaw size \( a_0 \) is determined in accordance with linear-elastic fracture mechanics, assuming that the total fracture energy of the laminate is a linear summation of the
fracture energies of the plies in the stack. Since all plies are assumed to experience the same opening
displacement, the fracture energies for the 45-degree and 90-degree plies can be related to that of the 0-
degree ply via the ratio of stiffness, i.e., $G_{45} / G_0 = E_{45} / E_0$, $G_{90} / G_0 = E_{90} / E_0$. The static tensile strength
(un-notched strength) of composites of varying stacking sequences is calculated using laminate theory
with the following relationship,

$$\sigma_0 = E e_{i1,t} = E \frac{X_T}{E_{11}}$$

(1)

where $X_T$ and $E_{i1}$ are given in Table 2, and $E$ denotes the laminate stiffness.

In this work, the inherent flaw fracture mechanics approach is only applied for the analysis of
straight-hole laminates, as no extension of this approach for the analysis of scarfed holes can be located
in the literature.

3.2. Abaqus Continuum Damage Model

The Abaqus in-plane progressive failure model for fibre-reinforced composites [8] incorporates
the phenomenological failure criteria of Hashin [22] and a crack-band approach [6] to reduce mesh
sensitivity. The response of the material is described by a continuum damage mechanics approach in
which the stiffness matrix is reduced according to the extent of fibre damage and matrix damage. The
evolutions of the damage variables are calculated using an equivalent stress versus equivalent
displacement relationship. An important feature of the Abaqus model is the incorporation of an element
characteristic length into the stress softening relationship to alleviate mesh dependency during material
degradation. For shell elements this characteristic length is the square root of the element area. The
material response remains linearly elastic up to failure initiation, after which point there is a linear
degradation up to the maximum equivalent displacement. In the degraded section, the material’s elastic
stiffness depends linearly on displacement. The area under the load-displacement curve is equal to the
energy dissipated by the particular damage mode. Several authors have proposed experimental and
analytical approaches for characterising these fracture energies [23, 24], which are based on empirical validation using experimental results and the standardised mode I fracture toughness [25].

An explicit solver is selected in this work because it is conditionally stable, based on a maximum time step. To simulate the quasi-static loading, the appropriate loading rate needs to be sufficiently slow so that dynamic effects could be ignored, yet high enough so that a solution could be achieved in a reasonable time. Standard shell elements are employed in the present investigation, and run-time accuracy and mesh sensitivity studies have been conducted. Although not presented in this paper for brevity, the numerical investigations reveal that an analysis time around 30 times the fundamental vibration period could reduce the error from dynamic effects to less than 2%. The mesh study indicates that the predicted strength is largely mesh-independent for elements within the range of 0.1 mm and 0.6 mm. Consequently the element size of 0.4 mm is selected in the subsequent FE analysis.

To model the scarfed-hole laminates, the scarf region, as illustrated in Figure 3, is padded with zero-stiffness plies so that the overall thickness remains the same. The added zero-stiffness plies have negligible stiffness and fracture energy. For the scarfed-hole models, a sequence of laminate materials is created with plies being gradually removed by substituting with zero-stiffness plies. The mesh is constructed such that the element edges coincide with the step length of the scarfed plies within the 3° scarf region.

3.3. Cohesive Zone Model

Considering a composite laminate with a circular hole, as illustrated in Figure 4, the length of the cohesive zone at a given applied stress can be determined by the condition of zero-singularity at the end of the cohesive zone. The stress $\sigma_{ho}(t)$ at location $t$ from hole edge is given in Ref. [26]. The weight function $G(t,a)$ for a pair of point loads acting on symmetric cracks of length $a$ emanating from
A circular hole has been provided by Newman [27]. For a given applied stress, the cohesive zone length can be obtained by iteratively solving the cohesive equation. As the applied stress increases, the cohesive zone, starting from zero length, extends until it reaches a critical length. At this length, the applied stress attains its maximum value [13], which is the ultimate strength of the laminate. The self-consistency condition for the cohesive zone model implies that the cohesive strength $\sigma_o$ must be equal to the un-notched strength of the laminate, otherwise the model would not be able to simulate the strength at very small hole sizes.

To investigate the influence of the shape of the cohesive law, the strain-softening law is represented as a simple power relationship. The cohesive stress $\sigma_{coh}$ at some opening displacement $\delta$ is $\sigma_{coh} / \sigma_o = (1 - \delta / \delta_c)^n$. Here $\sigma_o$ is the critical (maximum) laminate strength and $\delta_c$ is the critical opening displacement. The parameter $n$ defines the shape of the cohesive law. The limiting case of $n = 0$ corresponds to the elastic-perfectly plastic behaviour assumed in the original Dugdale model [28]. For a given total fracture energy $G_{total}$ and the un-notched laminate strength $\sigma_o$, the critical opening displacement is $\delta_c = (1 + n)G_{total} / \sigma_o$.

To illustrate the effect of the cohesive law shape, represented by the parameter $n$, on the notch strength, Figure 5 displays the predicted strengths, normalised by the un-notched strength, of the stiff laminate for various values of exponent $n$. Here the 0-degree ply fracture energy $G_0$ was kept constant at 100 kJ/m$^2$. The results in Figure 5 indicate that the shape of the cohesive law significantly affects the predicted strength by up to 40% when the exponent $n$ varies from 0 to 2. Therefore it is generally necessary to completely characterise the power law cohesive relationship to enable robust application of the cohesive zone model. To avoid the need for complete characterisation of the cohesive law experimentally, the present work adopts a simplified approach by focusing on only two cohesive relationships: non-hardening ($n=0$) and linear degradation ($n=1$).
Elastic analysis of a panel with a scarfed hole reveals a significant difference to the case of the straight hole in terms of stress or strain concentration. As shown in Figure 6, while the strain concentration at the vicinity of a straight hole agrees well with the analytical solution for a circular hole, the scarfed hole produces a much higher strain concentration. It is interesting to observe that the distribution of the hoop strain can be described by the same shape as for a circular straight hole; the influence of tapering in the scarf basically causes a proportional increase in the hoop strain, which can be described by the following expression, denoting the distance away from the hole edge as \( t = x - R \),

\[
e_{yy}(t) = \frac{K_e}{3} \left[ \frac{1}{2} \left( \frac{1}{1 + t/R} \right)^2 + \frac{3}{2} \left( \frac{1}{1 + t/R} \right)^4 \right] \frac{\sigma_y}{E_y},
\]

where \( E \) signifies the stiffness of the composite laminate in the load-carrying direction. The results presented in Figure 6 show that the strain concentration factor reaches approximately 7.0 for a scarfed hole with a 3° scarf angle. Furthermore, the results in Figure 6 reveal that the strain distribution in a composite laminate is approximately the same as in an isotropic panel with the same tapering angle.

Having established that the strain distribution close to the edge of a scarfed hole can be well approximated by Equation (2), the cohesive zone model is now extended to the scarfed-hole configuration. To achieve this extension requires the results of elastic strain distribution from the FE models. Similar to the FE analysis, the scarfed region is treated as a composite section as illustrated in Figure 3. The effective stiffness within the scarfed region becomes

\[
E_t(t) = \frac{t \tan \alpha}{h} E_{yy}(t).
\]

where \( E_{yy}(t) \) denotes the extensional stiffness of the laminate in the loading direction. Application of laminate theory to the scarfed region of the laminate, neglecting secondary bending associated with the local non-symmetric lay-up, yields the stiffness \( E_{yy}(t) \) for a given stacking sequence. Due to the inhomogeneity over the scarfed region, the weight function for the case of a circular hole in a
homogeneous material is no longer valid. In the present investigation, a new canonical expression of
the weight function is developed for a 3° scarfed hole with the aid of FE analysis. The same approach
described below can be employed to generate solutions for other scarf angles if needed. After
accounting for the changing sectional thickness shown in Figure 3, the hoop stress at a distance \( t \) ahead
of the hole edge is

\[
\sigma_{\theta\theta}(t) = \frac{t \tan \alpha}{h} E_{yy}(t) \varepsilon_{\theta\theta}(t),
\]

where \( \varepsilon_{\theta\theta}(t) \) is given by expression (2) in terms of the applied stress \( \sigma_s \). By recourse to Castigliano’s
theorem (e.g., Ref. [29]) the crack face displacement at \( t \) is given by

\[
\begin{align*}
    u_y(t) &= \int_0^{t} \frac{h}{E_{yy}(a')a'tan \alpha} K(a')G(t,a')da'.
\end{align*}
\]

Therefore the weight function for a scarfed hole can be expressed in terms of the opening displacement

\[
G_i(t,a) = \frac{E_{yy}(a')a'tan \alpha}{K(a)h} \frac{\partial u_y(t,a)}{\partial a}.
\]

To fully characterise the weight function, the FE method is used to obtain the crack opening
displacement for various crack sizes. The non-dimensional crack opening displacements are presented
in Figure 7. As expected, the maximum opening displacement increases with crack size. Furthermore, a
crack emanating from a scarfed hole opens more than its straight-hole counterpart by 25 to 85 per cent.
The maximum crack opening displacement, normalised by \( a \sigma_s / E \) as shown in Figure 7(a), has been
found to depend parametrically on the ratio \( a / R \). The shapes of crack opening displacement, as
displayed in Figure 7(b), however, are almost identical to that pertinent to a straight hole.

Consequently, the results presented in Figure 7 can be described by

\[
u_y(t,a) = \frac{u_{\text{max}}}{u_{\text{max}}} \sqrt{1 - t/a},
\]

where \( u_{\text{max}} \) is the maximum crack opening displacement at \( t=0 \). The following expression provides a
good fit for \( u_{\text{max}} \), as displayed in Figure 7(a),
\[ u_{\text{max}} = \left( A + B \frac{a}{R} + C \frac{a}{R} \right) \frac{\sigma_{\text{y}} a}{E}, \]  

(8)

with the non-dimensional coefficients \( A, B, \) and \( C \) being 21.4, -23.1, and 8.06. Substituting expressions (7) and (8) into (6) yields the weight function for a symmetric crack emanating from a scarfed hole with \( \alpha = 3^\circ \). The contribution of the cohesive stress to the stress-intensity factor is given by a similar expression as equation (8), after accounting for the reduced thickness in the scarfed region,

\[ K_{\text{coh}} = \int_0^a \sigma_{\text{coh}}(t) \frac{t \tan \alpha}{h} G_t(t, a) dt. \]  

(9)

Here the cohesive stress \( \sigma_{\text{coh}}(t) \) relates to the stiffness \( E_{\text{yy}}(t) \), and ultimate failure strain \( \varepsilon_{\text{ULT}} \), and the fracture energy via

\[ \sigma_{\text{coh}}(t) = E_{\text{yy}}(t) \varepsilon_{\text{ULT}} \left( 1 - \frac{\delta}{\delta_c} \right)^n, \]  

(10)

with

\[ \delta_c = (1 + n) \frac{G_{\text{total}}(t)}{E_{\text{yy}}(t) \varepsilon_{\text{ULT}}}. \]  

(11)

By applying the zero-stress-intensity condition the length of the cohesive zone can be determined iteratively so that the cohesive law is satisfied along the crack. The crack face opening displacement at distance \( t \) is given by a similar expression as (5), after combining the contributions of the prospective stress and the cohesive stress,

\[ \delta(t) = 2 \int_0^a \frac{G(t, a') h}{a' \tan \alpha} da' \int_0^t \frac{t' \tan \alpha}{h} \left[ \varepsilon_{\text{yy}}(t') - \varepsilon_{\text{ULT}} \right] G_t(t', a') dt'. \]  

(12)

Similar to the straight-hole case, iterative solution is necessary to determine the residual strength by gradually increasing the cohesive zone length: at a given cohesive zone length iterative calculations are carried out until the cohesive law (10) is satisfied.
3.4. Model Calibration

Calibration for all three analysis approaches is conducted using the experimental results for the OHT coupons (with 6.35 mm straight hole), where results for both the stiff laminate (mean strength = 688 MPa) and soft laminate (mean strength = 473 MPa) are considered. The results of this calibration are presented for all approaches in Table 3.

Table 3: Fracture parameters for IM7/5250-4 laminate (calibrated against OHT strength of the stiff laminate containing 40% 0° plies and 40% ±45° plies).

<table>
<thead>
<tr>
<th>Models</th>
<th>Fracture Parameters (kJ/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inherent flaw</td>
<td>$G_0 = 385$</td>
</tr>
<tr>
<td></td>
<td>$G_{45} = 44$</td>
</tr>
<tr>
<td></td>
<td>$G_{90} = 44$</td>
</tr>
<tr>
<td>Abaqus continuum damage</td>
<td>$G_{f\beta} = 110$</td>
</tr>
<tr>
<td></td>
<td>$G_{f\sigma} = 25$</td>
</tr>
<tr>
<td></td>
<td>$G_{m\beta} = 0.15$</td>
</tr>
<tr>
<td></td>
<td>$G_{m\sigma} = 0.45$</td>
</tr>
<tr>
<td>Cohesive zone (n=0)</td>
<td>$G_0 = 100$</td>
</tr>
<tr>
<td>Cohesive zone (n=1)</td>
<td>$G_0 = 270$</td>
</tr>
</tbody>
</table>

For the inherent-flaw method, the fracture energy $G_0$ is determined based on matching the stiff laminate experimental strength, and the parameters $G_{45}$ and $G_{90}$ are determined using the stiffness ratios as previously discussed. The inherent flaw size $a_0$ is estimated to be 2.2 mm.

For the Abaqus damage model, the parameters required include the in-plane fracture toughness of a ply (crack is perpendicular to fibre direction) under tension and compression, $G_{f\beta}$ and $G_{f\sigma}$, and the interlaminar (crack being parallel to fibres) fracture toughness, $G_{m\beta}$ and $G_{m\sigma}$, of a ply. Predictions are made for the tensile strength of the OHT coupons made of stiff and soft laminates, varying the fibre tension fracture toughness $G_{f\beta}$ until good agreement is reached with the experimental results. The results are displayed in Figure 8(a), indicating that a $G_{f\beta}$ value of 110 kJ/m², gives excellent agreement for the stiff and soft laminates. This value also compares well with values reported by other authors for
other carbon-fibre polymer composite material systems [23, 24]. The values of $G_{fc}$, $G_{mt}$ and $G_{mc}$ are taken from literature for similar material systems [23, 24].

For the cohesive zone models, the calculated OHT strength of the stiff laminate displays a strong dependence on the 0-degree ply fracture energy $G_0$. According to the results presented in Figure 8(b) and seen in Table 3, the non-hardening law ($n=0$) and linear degradation law ($n=1$) require fracture energies of 100 kJ/m$^2$ and 270 kJ/m$^2$, respectively, to correctly predict the mean experimental OHT strength.

4. Analysis Results and Comparison with Experiment

4.1. Straight-hole laminates

Analyses are carried out using all three approaches for the OHT coupons and panels containing straight holes. Additional results for quasi-isotropic OHT coupons with the same material system are taken from Iarve et al. [30], and compared to predictions with the three approaches. The results of all analyses of straight-hole laminates are summarised in Figure 9.

For the inherent flaw method, the results in Figure 9 show that the approach provides satisfactory accuracy with quasi-isotropic and stiff laminates for hole sizes not too different from the calibrating configuration ($D = 6.35$ mm). However, for laminates containing holes several times larger than the calibration geometry, the approach significantly under-predicts the strength as displayed in Figure 9(b). This prediction of a very strong size effect and significant underestimation of residual strength away from the calibration geometry is similar to the trend observed by Camanho et al. [10]. The experimental results for the quasi-isotropic laminates in Figure 9(a) show that the stacking sequence affected the laminate strength, though this effect is not able to be predicted by the inherent flaw method.
The Abaqus FE model slightly over-predicts the tensile strength for both quasi-isotropic and stiff laminates as shown in Figure 9, and is found unable to predict the effect of stacking sequence for the quasi-isotropic laminates. Despite this, the Abaqus model gives close predictions for both laminates at all hole sizes investigated. Figure 10 shows the fibre failure damage index in a 0° ply for the straight-hole panel. The damage index value, $d$, varying between 0 and 1, indicates that the predicted final fracture is inclined at an angle of about 11° to the horizontal direction, which is somewhat less than the experimental value of 22°. The calculated strains at the hole edge are also plotted together with the experimental results in Figure 2(a), where both the maximum strain and the linear/non-linear regions of the curves demonstrate that the model captured the experimental behaviour well. Compared with the inherent-flaw fracture mechanics approach, the Abaqus model provides an improvement in accuracy.

For the cohesive zone model, the predictions by both cohesive laws are in reasonable agreement with the experimental results for holes within the range of 2.5 mm and 12.7 mm. Nevertheless, both cohesive laws, calibrated using data for the stiff laminate only, are found to achieve good agreement with results from the quasi-isotropic laminate, confirming that the linear summation of the fracture energy could satisfactorily account for different laminate lay-ups or ply percentages. Applying the cohesive zone model to panels with large holes (25 mm and 50 mm in diameter), as shown in Figure 9(b), reveals that the non-hardening law ($n=0$) gives a better agreement with experimental results than the linear degradation cohesive law ($n=1$). This result is rather unexpected, considering that the vast majority of research on cohesive zone modelling of composites has employed the linear degradation cohesive law almost as default.

To elucidate the reason for the observed superior performance of the non-hardening cohesive law, the hoop stresses determined by the Abaqus model are plotted in Figure 11 near the vicinity of the hole edge. Although the Abaqus model assumes a linear degradation in the continuum damage model, the hoop stress is seen to remain almost constant over the region where damage occurs (i.e., where plies
are stressed beyond the failure strain), due to the relatively large value of fracture energy. It can be expected that a different damage model, closer to a non-hardening cohesive law, may produce an equal or improved correlation with experimental results. This will be the subject of future research.

4.2. Tapered-hole laminates

Analyses are carried out using the Abaqus and cohesive zone models for the panels containing tapered holes, and the results are summarised in Table 4. The results show that the Abaqus model under-predicts the strength for both hole sizes, though provides reasonable predictions, particularly for the larger hole size panel. The fibre fracture damage index is given in Figure 12, and shows that the Abaqus model predicts the crack to extend at an inclined angle of approximately 11°, compared with the experimentally observed value of 22°. This demonstrates that the Abaqus model is capable of capturing the asymmetric crack path through the tapered region, though the difference between the predicted and measured angle may be attributed to the coupling between different stress components in the Hashin damage model.

From Table 4, the cohesive model is seen to under-predict the strength for both panels, though matched the prediction of the Abaqus model for the smaller hole size panel. This is significant, as it demonstrates that the analytical model, which took minimal effort to calibrate and seconds to run, gives comparable accuracy to the complex and computationally expensive FE analysis. As in the case of straight-hole panels, the difference between the cohesive zone model and the FE model is likely due to the difference in the shape of cohesive law used.

Table 4: Tensile strength of panels with circular scarfed holes

<table>
<thead>
<tr>
<th>Inner diameter</th>
<th>[45/0₂/-45/90]₂S</th>
<th>[45/90/-45/0₂]₃S</th>
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<tbody>
<tr>
<td>Experiment</td>
<td>420</td>
<td>410</td>
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<tr>
<td>Inner diameter = 25 mm</td>
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<tr>
<td>Inner diameter = 50 mm</td>
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5. Discussion

Given that a main objective of the present work is to investigate analysis approaches with calibration for one coupon configuration only, the results shown in Figure 8 indicate that the OHT strength results have a relatively weak dependence on the fracture energy. This is largely due to the catastrophic nature of failure for these specimens, where the behaviour is dominated by the initiation of fibre fracture, and post-initiation damage properties have little effect on the total strength. This suggests that the OHT specimen may not be the most suitable configuration to calibrate the ply fracture energies. Furthermore, these OHT results are only shown to be suitable for calibrating the fibre tensile failure, and further assumptions are necessary to compute the other required fracture energies. As discussed previously, other researchers have proposed techniques for obtaining fracture energies, which include experimental characterisation, analytical relations or relation to the standardised mode I and II fracture toughness.

The results of the previous section demonstrated that the Abaqus progressive damage model is able to qualitatively capture the phenomenon of an inclined crack path, which is observed in experiments. Although a direct comparison of the experimental and numerical crack path angles may be overly simplistic, particularly given the jagged and complex nature of the experimental crack path, the difference between the experimental and numerical angles suggests that some aspect of the continuum damage model fails to capture the complex coupling between various stress components during damage initiation and growth. Further work is required to ascertain whether this phenomenon is dominated by the initiation function or the stress-softening law. Separately, failure criteria that are based on stress or strain invariants have also been proposed [31, 32], which have demonstrated

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<tbody>
<tr>
<td>Abaqus</td>
<td>310</td>
<td>360</td>
</tr>
<tr>
<td>Cohesive zone</td>
<td>310</td>
<td>300</td>
</tr>
</tbody>
</table>
successes in predicting damage initiation. These recent improvements can be incorporated into a strength-based damage initiation model.

The stress-softening damage progression law may also require further study so that it will more accurately represent the experimental behaviour. As shown in Figure 2(a), the linear/non-linear regions of the experiment and numerical strain results are in good correlation, indicating that the initiation of damage in the model compared well with the experiment. However, the numerical predictions of strain at the hole edge show a higher maximum strain for a given far-field strain. This indicates that the numerical model may have had more rapid stiffness degradation than what occurred in the experiment, where a reduction in stiffness led to an increase in strain. This behaviour may be more accurately captured by a different stress-softening law, which remains an active area of research.

The analysis in this work has been focused on the in-plane failure mechanisms such as fibre fracture and matrix cracking. However, limited extent of delamination has been observed in the present investigation, consistent with observations on failure of notched composites [33, 34]. It may be necessary to incorporate delamination modelling into the analysis approach, for example with the implementation of cohesive elements. This would necessitate a three-dimensional approach to modelling, where the single layer of shell elements will need to be replaced by either continuum shell elements or solid elements. This is the subject of further work as it involves significant pre-processing and the extension of the progressive damage model from 2D shell elements to 3D elements. The delamination models themselves also require separate calibration.

6. Conclusion

In this work the progressive failure of composite laminates containing holes of various size and shape has been investigated by experimental testing, computational simulation and analytical modelling. The results showed that the inherent-flaw fracture mechanics approach, once calibrated at
one hole size, under-predicts the residual strength of panels containing larger holes. The continuum damage model in Abaqus and the cohesive zone model with non-hardening cohesive law are shown to give reasonable predictions for both quasi-isotropic and stiff laminates at various hole sizes, following calibration at one hole size.

Acknowledgements

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References


34. Lagace, P.A. *Notch sensitivity and stacking sequence of laminated composites*. in *Composite materials: testing and design (seventh conference)*, ASTM STP 893. 1986: ASTM.
(a) Notations for a straight hole in laminate; a cross-section is shown.

(b) Notations for a scarfed hole in a laminate

Figure 1: Geometry and notations for notched laminate: (a) straight hole, (b) schematic of a scarfed hole showing the cross-section.
Figure 2: Load and strain results for (a) straight-hole panel and (b) scarfed-hole panel. The composite panel is 500 mm wide and made of $[45/90/-45/0/0]_3S$. The inner hole diameter is 50 mm.
Figure 3: Modelling of scarfed section using laminate comprising of zero-stiffness elements.

Figure 4: Cohesive zone model for a finite-width composite laminate with a circular hole.
Figure 5: Influence of the cohesive law exponent on notch strength (Fracture energy of 0-degree ply is 100 kJ/m$^2$)

Figure 6: Elastic strain at the edge of straight and scarfed holes (panel height is equal to width).
Figure 7: Crack opening displacement of a crack from a scarfed hole: (a) crack-mouth opening displacement (b) normalised crack face opening displacement (inner hole radius 25 mm)
Figure 8: Calibration of fracture energy values with OHT experimental results.
Figure 9: Comparison between experimental results and predictions for straight-hole specimens; (a) quasi-isotropic laminate (experimental data taken from Ref. [30]) (b) stiff laminates.
Figure 10: The 50 mm straight-hole panel. Top: The experimental fracture path. Bottom: Abaqus analysis, showing the fibre failure damage index in a 0° ply.
Figure 11: Stress distribution from Abaqus solution near the vicinity of a straight-hole edge, showing a near-constant hoop stress over the damaged region.
Figure 12: The 50 mm inner diameter scarfed-hole panel. Top: The experimental fracture path. Bottom: Abaqus analysis, showing the fibre failure damage index in a $0^\circ$ ply.