Optimising mixed-grazing strategies for semi-arid Australian rangelands

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*Doctor of Philosophy*

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Abstract

Currently in semi-arid Australian rangelands properties produce mainly beef and wool on marginal lands. A major area of concern is grazing pressure. Kangaroos are considered to have a considerable impact on grazing pressure, and for that reason they are often considered pests by landholders. It has been thought that converting from farming European stock to native wildlife would have environmental benefits. The commercial benefits from the change are unclear. Through construction of a plant-herbivore model, the dynamics of cattle, sheep and kangaroo commodities are examined. Simulations were constructed so as to estimate the expected value for each and the correlation between the different commodities. Portfolio analysis using mean-variance, average value-at-risk, and multi-objective optimisation projects were used to analyse different allocations of forage to each herbivore. The effect of an enforced reduction in methane emissions is also explored. From the analysis it seems that diversification of herbivores (including kangaroos) is optimal on marginal lands, for the risk averse, and to reduce methane emissions.
Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, on whole or part, to qualify for another academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and ethics procedures and guidelines have been followed.

Paul D. Moloney
May 26, 2011
I would like to dedicate this work to my family and friends. Without their support this would never have happened.
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Chapter 1

Introduction and Literature Review

Historically pastoralists have seen native species as competition for the available biomass and therefore detrimental to their core business of maintaining a large, healthy herd of domesticated stock. In Australia the main native species that fall into this category are kangaroos and wallabies. Since the introduction of European settlement many species of macropod marsupials have declined in number, some to extinction. However, the larger macropods, Red Kangaroo *Macropus Rufus*, Eastern Grey Kangaroo *M. giganteus* and Western Grey Kangaroo *M. fuliginous*, have greatly increased in number (Calaby and Grigg, 1989; Dawson, 1995). In an attempt to control their numbers, culling and then harvesting for meat and skins has been allowed in most states. Since the 1980’s efforts have been made to increase the acceptance and scale of the kangaroo harvesting industry (Grigg, 2002), especially with regards to human consumption of their
meat. Their skins are highly sought-after due to their leather’s strong yet supple nature, while the meat is very lean and high in iron. Ecologically, the hoofed domestic species, introduced from Europe, break up the fragile rangeland soils much more than kangaroos and clearing of scrub and bushland has resulted in a loss of habitat for many other native species. Rather than seeing the possible ecological and economic benefits, landholder’s are reticent to diversify into kangaroos (Williams and Price, 2010). They generally see them as a pest that needs to be eradicated due to their impact on total grazing pressure (Pople and Grigg, 1999; Grigg, 2002).

1.1 Introduction

The Australian rangelands occupy nearly three quarters of the continent and are home to 2.3 million people. The rangelands are an economically important region to Australia, contributing more than 4 billion dollars of agricultural production as well as supporting substantial tourism and mining industries. They are also a major component of the natural resource base for Australia in terms of vegetation and biodiversity. Significant economic and social transformations are currently taking place in the rangelands and rangeland ecosystems are under pressure. Increasingly, there are constraints on and opportunities for development of the grazing and agricultural industries in rangelands. There are current and emerging tensions between grazing and the sustainability of natural resources, including biodiversity. Key questions that need to be addressed concern the nature of trade-offs, and their impacts concerning agricultural production and biodiversity.
Methods and strategies that jointly promote profitable grazing enterprises and sustainable use of the rangelands have therefore made fertile grounds for research.

Biodiversity monitoring and reporting is becoming an increasingly important component of policy development in Australia. Some enterprises hope to use biodiversity monitoring to showcase their improved environmental management of native plants and animals and the ecosystem services on which they depend (Smyth and James, 2004). However, there have been few incentives for rangeland graziers to implement management practices that would promote biodiversity without complementary increases in or maintenance of productivity or profitability (Anon., 2000). Most conservation objectives were seen as additional to sustainable production and pastoralists did not feel that they could deliver on these objectives.

The benefits of undertaking sustainable natural resource management (SNRM) activities are not always readily apparent for either production nor conservation purposes. This uncertainty in the outcome, as well as the long time lags involved for the activities to yield returns, reduces the perceived benefits of undertaking SNRM for all land managers. Overgrazing is one of the main causes of land degradation in the Australian rangelands (Anon., 2001). Solutions for addressing this problem have generally involved reducing stocking rates, but it is not clear how profitable grazing enterprises would remain under these reduced stocking conditions.

An alternative suggestion is that landholders could utilise commodities from both domestic livestock and wildlife (Grigg, 1987, 1989, 1995, 2002; Wilson and Edwards, 2008). Without bio-economic analysis of this alternative grazing sys-
tem it is unclear if grazing enterprises could remain profitable. By investigating the population levels and management strategies of both domestic livestock and wildlife, an optimal solution can be found in terms of the perspective of economic returns and risk (variability in returns). Because the optimal strategy can specifically include constraints for increasing biodiversity and conservation, adoption of the results of this work will not only give rise to increased economic benefits but also improvements in the sustainable use of the rangelands.

Attempts at modelling herbivore grazing in the Australian rangelands have usually considered either the ecology or the economics of the system (Tisdell, 1973; Collins and Menz, 1986; Caughley et al., 1987). However, management of grazing herbivores in the rangelands depends on considering both the ecology and economics of these systems. Informed decisions will most likely flow from studies that have explicitly integrated ecology and economics (Choquenot et al., 1998).

The thesis addresses the issue by examining what mix of grazing herbivores provide optimal trade-off between risk and return. In particular the focus is on the Maranoa region in southern Queensland (see Figure 1.1). The results of the models developed can inform graziers as to possible impacts of changing traditional grazing practices in the rangelands to include native species. This extends recent research into alternative harvesting strategies for kangaroos in the Australian rangelands since it integrates wildlife harvesting with domestic stock (cattle and sheep) grazing in an economically optimal way.
1.2 Literature Review

1.2.1 Kangaroos: Ecology and Harvesting

Kangaroo biology and ecology provides some interesting variations on domestic stock. For instance, red, and sometimes eastern grey, kangaroos use embryonic diapause. This means that they can carry a viable embryo in their uterus for many months whilst carrying pouch young. After post-partum mating (only days after the last birth) the development of the new embryo is limited to the blastocyst (pre-embryonic) stage and remains at this stage until either the pouch young is lost or lactation is reduced towards the end of pouch life for the current joey. The mean gestation time for red kangaroos is 33.2 days (with a standard deviation of 0.2 days), spending a further 235 days (on average, with a standard deviation of 2 days) before exiting the pouch permanently. The result of which is that there is usually only between 1-3 days between the permanent exit of one pouch young and the birth of the next (Dawson, 1995). This means that during relatively good times, each mature fecund female can produce 1.5 kangaroos per year. Another difference to standard domestic stock is that kangaroos, being non-ruminant forestomach fermenters, meaning they produce negligible amounts of methane (0.003t head$^{-1}$year$^{-1}$), which is a greenhouse gas (Wilson and Edwards, 2008). Compare this to cattle and sheep, which use enteric fermentation, that produce large amounts of methane (1.67t head$^{-1}$year$^{-1}$ and 0.14t head$^{-1}$year$^{-1}$ respectively) which accounts for 11% of Australia’s total greenhouse gas emissions (Wilson and Edwards, 2008). Hence, switching at least some production from cattle and sheep to kangaroos could result in a decrease in total greenhouse
gas emissions in Australia (Garnaut, 2008). More generally the native kangaroos have less of a negative impact on biodiversity compared to the livestock introduced by Europeans (Williams and Price, 2010).

Red kangaroos reach sexual maturity between 15 and 20 months for females and 24 and 48 months for males. Eastern greys take an average of 18 and 48 months for females and males respectively to reach sexual maturity. While males can continue to make significant contributions to breeding once mature, females tend to have reduced fecundity after the age of 9 years until becoming infecund by age 12 to 15 years. Mortality rates are high in juvenile kangaroos, particularly in males, and the reasons why are not fully understood (Dawson, 1995). In some areas 83\% of mature western grey kangaroo females may have pouch young, while only 27\% also have young at foot (YAF) (Arnold et al., 1991). The main factor for this high mortality is thought to be that as much of their nutrients go into formation of bone and muscle, little goes into fat, making them particularly susceptible to feed shortages. Another factor is predation by dingoes and foxes. More recent studies comparing densities either side of dingo fences suggest that predation by dingoes in semi-arid rangelands is more significant when an area is in drought when normally abundant prey, namely rabbits, are scarce (Newsome et al., 2001). Predation by foxes is found to be influential only in temperate areas (Banks et al., 2000).

Research has been conducted into the feed intake and preferences of kangaroos and domestic stock so as to enable them to determine the level of competition between species for the available food supplies. As a result functional responses of kangaroos, sheep and other animals in arid conditions have been estimated (Short,
The functional responses enables the estimation of feed intake given the biomass available, which is important in determining the effect of animals on the available plant life. It was found that kangaroos preferred young grass and green forbs due to the fact that they are easier to digest Caughley et al. (1987); Moss and Croft (1999); Davis et al. (2008). More recently Rafferty et al. (2010) studied the feed preferences of western grey kangaroos, comparing captive and wild populations. Also of importance is any competition for resources, where one species has a deleterious effect on another. While competition is possible between the red and grey kangaroos, the level of competition is unclear as they have different feed and microhabitat preferences (Dawson, 1995). Dudzinski et al. (1982) considered interaction between cattle and red kangaroos, and found that while both consume grass as the mainstay of their diet, the parts that they grazed differed, except in cases of extreme drought, and hence no competition occurred when cattle numbers are controlled. They also found a lack of facilitation, in that the kangaroos were not attracted to areas recently grazed by cattle. Dawson and Ellis (1994) looked at competition between kangaroos and sheep and found it only occurred during very dry winters. During these times sheep that grazed with kangaroos lost more weight and grew slightly less wool than sheep kept separate from kangaroos. Kangaroos in sheep-free paddocks had higher body weights, although their diets remained the same. A study of population dynamics in the semi-arid pastoral zones of South Australia by Jonzen et al. (2005) found, counter intuitively perhaps, that in their best models, sheep and cattle densities had a positive effect on the population growth rate of red kangaroos, even more than rainfall. They postulated this was due to the sheep and cattle acting as a surrogate for the availability of forage.
The functional response of kangaroos has been modelled using several methods. The functional response is the change in the population size as the density of its food changes. Bayliss (1985) calculated numerical response functions for both red and western grey kangaroos using both Michaelis-Menton and Ramp functions. Caughley et al. (1987) developed numerical response for red kangaroos using an Ivlev function to determine growth rate in response to available forage. Caughley’s numerical response has been used subsequently to explore population dynamics of kangaroos and possible effects of harvesting at different rates (Caughley et al., 1987; Bayliss and Choquenot, 2002). Alternative models have used rainfall (as a proxy to biomass) to predict population growth. These include stochastic Ricker models (Cairns and Grigg, 1993; McCarthy, 1996; Jonzen et al., 2005), and spatial kriging models (Pople et al., 2007). Rainfall was found to be insufficient to produce reasonable estimates for western grey kangaroos (Cairns et al., 2000). Bayliss (1985) noted that the rates of increase seemed to be dependent on the current age and gender structure of the population. Hacker et al. (2003) used a physiological structured population model to account for the influence on age and gender demographics in their model. A more detailed description of the mathematical models used occurs in Section 1.2.4.

Body condition reflects an animal’s nutritional state. It combines current and recent differences between required and available food. The body condition of kangaroos has been monitored and recorded most effectively using a kidney fat index (Caughley et al., 1987; Moss and Croft, 1999). This procedure requires the kidney to be assessed for the percentage of fat attached to the kidney. Moss and Croft (1999) determined that the amount of green grass biomass was the best
predictor for the body condition of red kangaroos. It was also noted that there was a lag between body condition and pasture biomass of approximately three months.

Kangaroos, unlike domestic stock and many other wild animals move freely between farms, national parks and other areas, due to their ability to jump fences. This leads to the situation of having free-roaming stock within demarcated ownership boundaries (Pople and Grigg, 1999). They will move between locations depending on the availability of food and water. A landholder who reduces their stocking rate of sheep, and hence increases the availability of food, is likely to receive an increase in their kangaroo numbers whether that was their desired outcome or not. Therefore, if a landholder wanted to increase their average kangaroo stocking rate, reducing its sheep stocking rate could increase the net kangaroo immigration onto the property Moloney and Hearne (2009). This may make the landholder in question unpopular with other landholders in the area, as they may believe that some of the enticed kangaroos may venture onto their property Pople et al. (2007). Conversely, a landholder may attempt to increase their stocking rate of sheep to create a net emigration of kangaroos, however, this could increase the risk of overstocking.

Habitat influences density and social groupings of western grey kangaroos (Coulson, 1993). McAlpine et al. (1999) investigated the effect of landscape structure on the density of red and eastern grey kangaroos, and common wallaroos in partially cleared semi-arid bushland in Queensland. They found linkages with the abundance of large kangaroo species and tree clearing practices, making it an important factor in conjunction with pasture productivity. Viggers and Hearne
(2005) monitored eastern grey kangaroos in south eastern Australia, particularly incursions from reserves onto farmland. They concluded that the kangaroos only dispersed where cover was available. Martin et al. (2007) argued that methodology used by Viggers and Hearn (2005) was flawed and insufficient data was obtained to draw their conclusions. These claims were rebutted in Viggers and Lindenmayer (2007), saying that the key claim, that landholders are at a disincentive to conserve remnant native vegetation, still held. Fukuda et al. (2009) found that fencing watering holes during a draught had little if any effect on the density of red kangaroos within 4km of the watering hole. Instead food availability was the main determining factor, as there is usually water within convenient reach of the kangaroos. Hence, fencing off watering holes during drought is not likely to have the desired effect of reducing kangaroo densities and allowing vegetation regeneration. The ideal free distribution \textit{IFD} is an ecological concept implying that animals will move between areas so that the ratio of animals to carrying capacity in each area will be equal (Fretwell and Lucas Jr., 1969). Coulson (2009) concluded that it is likely that the ideal free distribution holds for kangaroos in a review of the literature, but did note that further research through different management practices is required. Wiggins et al. (2010) investigated shifts in home range after (lethal and exclusion) interventions on two common macropod species in Tasmania, pademelons (thylagale billardierii) and red-necked wallabies (macropus rufogriseus rufogriseus). Their results conformed to predictions based on the ideal free distribution.

Plans are being investigated into how kangaroo harvesting can return some money to the landholders (Pople and Grigg, 1999; Baumber et al., 2009). Recent
research regarding sustainable harvesting and alternative management strategies for kangaroos has indicated that the integration of wildlife harvesting and traditional rangeland enterprises may not be straightforward (Hacker et al., 2003; Hacker and McLeod, 2003; McLeod et al., 2004; Baumber et al., 2009). For example, harvest strategies by individual kangaroo shooters may change the structure and dynamics of kangaroo populations to such an extent that they compromise other management goals, such as controlling total grazing pressure. Grazing pressure is the stress on vegetation, and therefore the ecosystem, from animal grazing.

Kangaroo (and wallaby) harvesting is controlled by state and federal governments. State governments set quotas and regulations that must be signed off by the Federal Government. This is due in part to the fact that as a native species, kangaroos (and wallabies) are under the protection of the crown. Each state has different protocols with regards to harvesting kangaroos and wallabies, what quotas are set and how the quotas are managed (Pople and Grigg, 1999). For instance, in New South Wales each region is given a quota of tags to be placed on each harvested kangaroo. These tags are then distributed to property owners, who can harvest (either themselves or engage external harvesters) until their tags are exhausted (Hacker and McLeod, 2003). In Queensland, each region is given a quota but it is the harvesters themselves who can purchase the tags, which can then be used to harvest kangaroos on private property (Office of the Queensland Parliamentary Counsel, 2010; Moloney et al., 2011). In addition to the quotas to control the off-take, there are also conditions that are meant to ensure a stable, genetically diverse kangaroo population. In Queensland, with similar conditions
elsewhere, these conditions include; minimum kangaroo densities; male off-take bias; and minimum weight limit for harvested kangaroos. Typically these are set to: a minimum kangaroo density of 2 kangaroos per km$^2$; a 70% male off-take bias; and a minimum live weight of 20kg or fully dressed weight of 13kg (Hacker et al., 2003; Office of the Queensland Parliamentary Counsel, 2010). Fully dressed refers to a carcass of a harvested macropod with the following parts removed: head; viscera; each forelimb from the elbow joint; foot of 1 hind limb, from a point below the tarsal joint; other hind limb from a point midway between the knee and ankle joints; tail (Office of the Queensland Parliamentary Counsel, 2010).

1.2.2 Portfolio Analysis, Multiple Objective Programming and Bioeconomics

Often the driving force behind change in agribusinesses derives from the perceived benefit of that change. To create that leverage there is a need to examine whether the inclusion of kangaroo harvesting within a mixed grazing strategy for their enterprise can be financially beneficial to pastoralists. The decision of which animal to stock and at what levels is analogous to the question of which shares should be invested in and to what degree. This problem of portfolio optimisation has had different techniques developed over time to analyse the best strategy for optimising the return on investment while accounting for the risk involved in the investment. One of the first ways used to analyse risk and return is classical mean-variance portfolio selection (Markowitz, 1952, 1991). The scenario is a limited amount of funds are to be invested in a variety of assets (Steinbach, 2001). The goal is that each asset is allocated funds, $y$, in such as way as to
Figure 1.1: The map of the macropod harvest zones in Queensland. The Maranoa region is between Charleville and Roma. This map is from the Queensland Government Department of Environment and Resource Management.
trade-off maximising performance, $\rho(y)$, and minimising risk, $R(y)$,

$$\max_y \pi \rho(y) - \frac{1}{2} R(y)$$

$$s.t.$$

$$e^T y = 1,$$

$$y \geq 0$$

where $\pi$ is the trade-off parameter, $e \in \mathbb{R}^n$ denotes the vector of all 1s and the budget equation $e^T y = 1$ specifies the initial wealth (without loss of generality set equal to one). This enables the conflicting objectives of maximising returns and minimising risk to be addressed and the set of pareto-optimal portfolios to be calculated.

But is risk defined? Markowitz (1952, 1991) suggested that variance as a proxy for risk in mean-variance analysis. However, it has been noted that variance is not an actual measure of risk, but a measure of uncertainty (Rachev et al., 2008). An alternative formulation more generally known as mean-risk analysis focuses on two main principles. Selecting the portfolio(s) with the minimum risk, given they meet a lower bound on expected performance. Selecting the portfolio(s) with the maximum performance, given they meet an upper bound on risk. Mathematically
these can be written respectively as

\[
\begin{align*}
\min_y & \quad R(y) \\
\text{s.t.} & \quad e^T y = 1 \\
& \quad \mu^T y \geq \mu_* \\
& \quad y \geq 0
\end{align*}
\]

(1.1)

and

\[
\begin{align*}
\max_y & \quad \mu^T y \\
\text{s.t.} & \quad e^T y = 1 \\
& \quad R(y) \leq R_* \\
& \quad y \geq 0
\end{align*}
\]

(1.2)

where \( R(y) \) is risk associated to that selection of portfolios, \( \mu_* \) is the lower bound on expected performance, \( \mu \) is the vector of expected performance for each portfolio, and \( R_* \) is the upper bound on risk. A number of different measures for risk aversion related to risk premiums are discussed by Pratt (1960) and Rubenstein (1973) amongst others. How risk should be measured is still debated with each method having its own strengths and weaknesses: asymmetric risk measures including expectation of loss and semi-variance (Harlow and Rao, 1989); risk models with higher moments (Kraus and Litzenberger, 1976); and, coherent risk measures (Artzner et al., 1997) have all been developed. Value at risk (VaR) is one of the most commonly used risk measures used in finance (Simons,
1.2 Literature Review

1996). Average value at risk (AVaR), also known as conditional value at risk and expected shortfall, is superior to value at risk as a measure of risk as shown by Palmquist et al. (2002). AVaR calculates the expected value of return given the return is in the lowest $\epsilon$ of the distribution, and can be calculated using:

$$\text{AVaR}_\epsilon(X) = \frac{1}{\epsilon} \int_0^{\epsilon} \text{VaR}_p(X) dp.$$  \hfill (1.3)

Multiple time period models in both discrete (Markowitz, 1991; Phelps, 1962) and continuous (Merton, 1971) time have been researched.

Using a mean-variance approach to analyse agricultural development was investigated by Freund (1956), Turvey et al. (1988), and more recently Theron and van der Honert (2003), where the emphasis was on gross margins and long-term wealth as well as Hearne et al. (2008) as it related to stocking rates in game ranches. In the present analysis, risk is defined as the variance in returns and risk aversion is the degree to which the landholder desires to minimise risk compared to maximising returns.

Clark (1990) introduces the idea of economically optimal, yet sustainable harvesting of populations, in effect maximising growth rates and then harvesting at a similar rate, often refered to as maximal sustainable yield (MSY). These models have included common populations such as fish (Pikitch et al., 2004). This idea was extended to finding the optimal two-species harvesting policies, on a Lotka-Volterra competitive model, by Mesterton-Gibbons (1996). He found that an optimal harvesting policy may drive one species to extinction given it is sufficiently easier to catch, even if the system would coexist in the absence of
1.2 Literature Review

harvesting. Conrad (1999) explored the idea of a marine sanctuary on neighbouring fishing grounds using diffusion of biomass finding variation in biomass was reduced. Skonhoft (2005); Skonhoft and Olaussen (2005) investigated the economic effect of moose migration, where migration is driven by seasonal factors. The analysis showed that neglecting migration can cause sub-optimal population sizes and substantial profit transfer among landholders. Skonhoft (2007) used biomass to look at the bioeconomics for a park agency and locals for land animals on conservation reserves and farmers in sub-Saharan Africa.

Kangaroos are like fish and moose in some aspects. They have a common population available to be harvested by many. They have areas of sanctuary where harvesting is not allowed. There are even boundaries of where certain groups can harvest and others can’t with international boundaries being akin to property boundaries. There are even harvest limits set. However, unlike fisheries, there is domestic stock to be considered as well, that we have a much greater control over. Unlike moose, there is competition for resources rather than predation on saplings for future logging. Skonhoft (2007) used a general model to look at mobile biomass, or a single species, without any captive stock. In the problem on interest there are both stock that is free-roaming across boundaries (that is publicly owned) as well as sedentary (privately owned) within the property competing for common forage.

1.2.3 Game Theory

The present situation is one where most of the power to influence commodity prices does not reside with the landholder. Game theory has investigated how
power influences operational decisions. In game theory it is assumed that each player acts rationally and therefore makes decisions about which strategy is optimal given the information they know. It has been used to explore decision making across many fields including: economics; computer systems; politics; and, genetics (Choi, 1991; Sumaila and Apaloo, 2002; Aliprantis and Chakrabarti, 2011).

A strategic game is one in which \( n \) players (labelled 1, 2,..., \( n \)) each have a strategy set (\( S_i, i \in \{1,2,...,n\} \)) and a payoff function (\( u_i, i \in \{1,2,...,n\} \)). All players choose simultaneously and independently their strategy (\( s_i \in S_i \)) and receives payoff \( u_i(s_1, s_2, ..., s_n) \) for each \( i \in \{1,2,...,n\} \) (Aliprantis and Chakrabarti, 2011). The concept of Nash equilibrium points (NEPs) Nash (1951) revolutionised strategic game theory. An NEP is where no single player can do better by changing their strategy while all other players play the same NEP. More formally this can be written as \((s_1^*, s_2^*, ..., s_n^*)\) is an NEP iff

\[
u_i(s_1^*, ..., s_{i-1}^*, s_i^*, s_{i+1}^*, ..., s_n^*) \geq u_i(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_n^*)
\]

\(\forall s_i \in S_i \) and \( i \in \{1,2,...,n\} \). Therefore it is possible to have multiple NEPs in a strategic game. If the strategy set \( S_i \) is an interval, the payoff functions are continuous and have second-order partial derivatives in the interior of \( S_i \) then \((s_1^*, s_2^*, ..., s_n^*)\) is the only interior NEP of the game iff,

1. Each \( s_i^* \) is in the interior of the interval \( S_i \).
2. \( \frac{\partial u_i}{\partial s_i}(s_1^*, s_2^*, ..., s_n^*) = 0 \) for each player \( i \).
3. Each \( s_i^* \) is the only stationary point of the function \( u_i(s_1^*, s_2^*, ..., s_n^*) \), \( s_i \) is in the interior of \( S_i \).
4. \( \frac{\partial^2 u_i}{\partial s_i^2}(s_1^*, s_2^*, ..., s_n^*) < 0 \) for each player \( i \).

Both cooperative and competitive (strategic) game theory has been widely used in economics and finance (Rosenthal, 1981; Sumaila and Apaloo, 2002). Game theory as an effective method has been used to describe and solve interaction mechanisms of the seller (landholder) and the buyer (processor) in a supply chain. For example, Yang and Zhou (2006) consider a two-echelon system with a seller and two competitive buyers where the seller has more power. They assume the product of one buyer is a substitute for the product of the other, therefore, their demand function follows the Bertrand model. The optimal wholesale price and quantity ordered are obtained under different scenarios. A similar model is presented in Chen et al. (2006) where they also consider the impact of transaction costs, while Yao et al. (2005) consider the impact of value adding in the demand function. Xiao and Qi (2008) and Yang and Zhou (2006) consider similar demand functions with the former offering two different quantity discounts, an all-unit quantity and an incremental quantity discount to the buyer. In addition, there are several which propose a supply chain context which incorporate elements of competition and cooperation between a seller and a buyer under non-cooperative and cooperative games (Esmaeili et al., 2009b,a). A significant shortcoming of all these models is that they only regard seller or buyer’s profits without considering any constraints. In other words, to avoid the confounding effect of constraints, they consider only a theoretical model.
1.2.4 Population Models

Population models in ecology can fall into several categories. There are many questions about the population to be answered to find the required category; is it measured in discrete or continuous time; is it age or size dependent; are stages discrete or continuous; are births a flow or a pulse; is there intraspecific competition; is there predation or interspecific competition; do the fertility and mortality rates change over time, stage or density; is the system deterministic or stochastic; are the genders significantly different; is spatial location important? Once these questions have been answered then an appropriate mathematical model can be selected. Whether it be a discrete system with a series of difference equations, a continuous process with ordinary differential equations, partial differential equations, or spatially distributed, there are various appropriate models to approximate the evolutionary behaviour of the population.

Initially we shall discuss unstructured population models. These are models where the population(s) can be considered a homogeneous group without losing too much information. Even before Malthus (1798) the idea of exponential growth in populations being bounded by some external factor due to intraspecific competition had been discussed (Seidl and Tisdell, 1999). Verhulst (1838, 1845) was the first to give this idea an equation, that of logistic growth,

\[
\frac{dN}{dt} = rN \left( \frac{\kappa - N}{\kappa} \right), \tag{1.4}
\]

where \(N\) is the population, \(r\) is the relative growth rate (birth rate - death rate), and \(\kappa\) is the carrying capacity. Since then, the idea of carrying capacity has
### Notation Definition

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time since the model initiated.</td>
</tr>
<tr>
<td>$S$</td>
<td>The set of species, numbered $i = 1, 2, ...s$.</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of individuals in a species of animal.</td>
</tr>
<tr>
<td>$V$</td>
<td>Total available biomass.</td>
</tr>
<tr>
<td>$r$</td>
<td>Relative growth rate (births - deaths).</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Carrying capacity of the species.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The prey’s growth rate under no predation.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The predation rate.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The predator reproduction rate per prey eaten.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The predator mortality rate.</td>
</tr>
<tr>
<td>$\alpha_{ij}$</td>
<td>Affect the population of species $j$ has on the population of species $i$.</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>The saturation rate of grazing for herbivores.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The half-saturation constant (amount of available vegetation where herbivores intake is halved).</td>
</tr>
<tr>
<td>$\xi$</td>
<td>The vegetation to herbivore conversation rate.</td>
</tr>
<tr>
<td>$\chi$</td>
<td>The zero population growth herbivore consumption rate.</td>
</tr>
<tr>
<td>$a$</td>
<td>Age of the cohort.</td>
</tr>
<tr>
<td>$\omega$</td>
<td>The maximum age of survivorship.</td>
</tr>
<tr>
<td>$n_{x,t}$</td>
<td>Number of females of stage $x$ at time $t$.</td>
</tr>
<tr>
<td>$B_t$</td>
<td>Number of female births at time $t$.</td>
</tr>
<tr>
<td>$l_a$</td>
<td>Fraction of newborn females surviving to age $a$ (survivorship function).</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Number of females born to a female of age $a$.</td>
</tr>
<tr>
<td>$x$</td>
<td>Stage of the individual.</td>
</tr>
<tr>
<td>$\mu(x,t)$</td>
<td>Mortality rate for individuals of stage $x$ at time $t$.</td>
</tr>
<tr>
<td>$b(x,t)$</td>
<td>Birth rate for individuals of stage $x$ at time $t$.</td>
</tr>
<tr>
<td>$g(x,t)$</td>
<td>Growth rate function for individuals of stage $x$ at time $t$.</td>
</tr>
</tbody>
</table>

**Table 1.1:** Definitions of symbols used in the population models.
changed from an immutable constant upper limit to the population, possibly unknown, to a more abstract one of maximum density a range is capable of supporting (Dhondt, 1988), possible of changing with time and environment.

Simple logistic models for population growth have some intrinsic flaws. Exogenous environmental forces can alter the carrying capacity, $\kappa$, the relative growth rate, $r$, or a lag-factor in response time. The possible existence of tipping points, where $\kappa$, $r$ or a lag-factor are altered once a certain population has been reached (Seidl and Tisdell, 1999). Different models and methods for calculating carrying capacity were analysed by McLeod (1997), showing that complex characteristics, uncertainties and stochastic environments cannot be effectively modelled using this approach, unless it was used for determining short-term potential densities as a function of resource availability rather than long-term equilibriums. For a more complete look at logistic type models see Banks (1994).

Functional response can be considered the rate at which a species consumes resources, given the availability of those resources. Holling (1959, 1965) introduced three types of functional responses. Type I is a linear response, consumption is directly proportional to the availability of the resources (e.g. Lotka-Volterra model). This is not always realistic, so sometimes, consumption is capped when the species is satiated. A Type II functional response is hyperbolic, the speed at which the consumption rate increases decreases as it approaches the satiation asymptote (e.g. Rosenzweig-MacArthur model). A Type III functional response is sigmoidal, reflecting inefficient foraging at low resource densities. Crawley (1992) discusses a fourth functional response, where the rate of consumption decreases with higher resource densities due to prey interference or toxicity. Fig-
1.2 Literature Review

Figure 1.2 displays the behaviour of the four different types of function responses.

![Functional response diagram]

Figure 1.2: Plot comparing the four types of functional responses. Each has the same satiation level.

Interaction between competing species in a bounded system was first modelled by Lotka (1925) and Volterra (1926) (commonly now known as the Lotka-Volterra models) who studied a predator-prey scenario and follow the equations,

\[
\frac{dN_{\text{Prey}}}{dt} = N_{\text{Prey}} (\alpha - \beta N_{\text{Predator}}) \quad (1.5)
\]

\[
\frac{dN_{\text{Predator}}}{dt} = -N_{\text{Predator}} (\gamma - \delta N_{\text{Prey}}). \quad (1.6)
\]

Equations 1.5 and 1.6 represent the change in the number of prey and predators, respectively, where \( N_{\text{Prey}} \) and \( N_{\text{Predator}} \) are population of prey and predators with \( \alpha, \beta, \gamma \) and \( \delta \) being the parameters for the interaction between the two species. It can be easily shown that these populations reach equilibrium when either both species are extinct, or \( N_{\text{Prey}} = \frac{\gamma}{\delta} \) and \( N_{\text{Predator}} = \frac{\alpha}{\beta} \). Subsequent models included density dependence, alternative functional responses, intraspecific and interspecific competition and facilitation between multiple species. The equations for
intra- and interspecific competition are,

\[
\frac{dN_i}{dt} = r_i N_i \left( \frac{\kappa_i - \sum_{j=1}^{s} \alpha_{ij} N_j}{\kappa_i} \right),
\]

(1.7)

where the \( i \)th equation in the system of \( s \) equations for a competitive Lotka-Volterra model with \( s \) species. \( N_i, \kappa_i \) and \( r_i \) are the population, carrying capacity and relative growth rate for the \( i \)th species and \( \alpha_{ij} \) is the effect population of species \( j \) has on the population of species \( i \).

The Lotka-Volterra equations are known to be unrealistic in their oscillations, due to their sensitivity to perturbations (Brauer and Castillo-Chavez, 2001). The stability near the equilibrium points, when considering just two species, can be determined by the eigenvalues of the Jacobian matrix from the system of equations (linearisation). If the eigenvalues are: real and negative, the equilibrium is a stable node; real and positive, the equilibrium is unstable; real and of opposite sign, the equilibrium is a saddle point; complex with negative real part, the equilibrium is a stable focus (spiral in); complex with positive real part, the equilibrium is an unstable focus (spiral out); purely imaginary there is a centre, stable or unstable focus, when considering two species (Kot, 2001). However, Smale (1976) proved that with a large number of species (\( s \geq 5 \)) then the system could take on any dynamical behaviour.

Grazing (or plant-herbivore) systems have been characterised as a variation on predator prey interaction (Edelstein-Keshet, 1986). The Rosenzweig-MacArthur system (Equations 1.8 and 1.9) is one of the earlier and still dominant plant-herbivore models. It includes logistic density dependency within vegetation and
hyperbolic function response in the herbivore (Rosenzweig and MacArthur, 1963). In the system $\zeta$ can be interpreted as the saturation grazing rate per capita of herbivore, $\theta$ is the half-saturation point, $\xi$ is the vegetation-herbivore conversion rate and $\chi$ is the consumption rate required to maintain the current density. Turchin and Batzli (2001) argued that while the Rosenzweig-MacArthur system is appropriate for interaction where the plant’s biomass is accessible to the grazer, this is not the case for many perennial grasses and sedges, where at least 80% of their biomass is underground. When the latter case is true, initial recovery from grazing is much quicker than the logistic density dependence response. To counter this it is suggested (Turchin and Batzli, 2001; Turchin, 2003) that an initially linear regrowth model (Equation 1.10) is more appropriate and should replace Equation 1.8.

\[
\frac{dV}{dt} = r_v V \left( 1 - \frac{V}{\kappa_v} \right) - \frac{\zeta V N}{\theta + V} \tag{1.8}
\]
\[
\frac{dN}{dt} = \xi N \left( \frac{\zeta V}{\theta + V} - \chi \right) \tag{1.9}
\]
\[
\frac{dV}{dt} = u_v \left( 1 - \frac{V}{\kappa_v} \right) - \frac{\zeta V N}{\theta + V} \tag{1.10}
\]

Age or stage structured population models are appropriate when populations are heterogeneous. Difference equations can be used to model populations most effectively where the organism can be grouped into non-overlapping groups or generations measured over discrete time (Smith and Keyfitz, 1977; Kot, 2001; Tuljapurkar and Caswell, 1997). This could be due to the adults dying and are replaced totally by their progeny after some fixed interval, individuals undergo abrupt changes, or progress through series of discrete stages. The linear difference
equation takes the general form as shown in Equations 1.11 and 1.12 and include such famous progressions as the Fibonacci sequence (when \( l_a = 1, m_1 = m_2 = 1 \) and \( m_a = 0 \) otherwise).

\[
B_t = \sum_{a=1}^{t} B_{t-a}l_a m_a + G_t \tag{1.11}
\]

where

\[
G_t = \sum_{a=1}^{\omega} n_{a,0} \frac{l_{a+1}}{l_a} m_{a+1} \tag{1.12}
\]

While the section of the model from Equations 1.11 and 1.12 is concerned with only the next generation, the age distribution of the population can be modelled as an extension of this by retaining the information via matrices

\[
n_{t+1} = Ln_t \tag{1.13}
\]

where

\[
L = \begin{bmatrix}
F_0 & F_1 & F_2 & \cdots & F_{\omega-2} \\
0 & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & P_{\omega-2} & 0
\end{bmatrix} \tag{1.14}
\]

\[
P_a \equiv \frac{l_{a+1}}{l_a} \tag{1.15}
\]

\[
F_a \equiv P_a m_{a+1}. \tag{1.16}
\]
Matrices models that progress the population from one time step to the next are often referred to as Leslie matrix, named after Leslie (1945), who popularised their use. In practice these models often had constant state variables like birth and mortality rates to allow for easier computations. The advent of computing and the increase in its processing power has enabled extensions of these discrete models where these state variable can change with time (Caswell, 2001).

Alternatively, a structured population may have continuous time, and in this case either ordinary differential equations or partial differential equations, PDEs, are used. McKendrick (1926) originally used PDEs to model age-structured populations, but this approach was not popularised until the later work of von Foerster (1959). This work was later extended to include classification by size or physiological age by Sinko and Streifer (1967, 1969) amongst others. The general form of the equation is,

\[
\frac{\partial n(x,t)}{\partial t} = -\mu(x,t)n(x,t) - \frac{\partial g(x,t)n(x,t)}{\partial x} \quad (1.17)
\]

\[
n(0,t)g(0) = \int_{0}^{\infty} b(x,t)n(x,t)dx \quad (1.18)
\]

\[
n(x,0) = n_{x,0} \quad (1.19)
\]

where Equation 1.18 is the boundary condition relating to the rate of recruitment of individuals of stage-0 (new born) and Equation 1.19 is the initial condition at
time \( t = 0 \). This reduces to the McKendrick-von Foerster equations,

\[
\frac{\partial n(a,t)}{\partial t} + \frac{\partial n(a,t)}{\partial a} = -\mu(a,t)n(a,t) \tag{1.20}
\]

\[
n(0,t) = \int_{0}^{\infty} b(a,t)n(a,t)da \tag{1.21}
\]

\[
n(a,0) = n_{a,0} \tag{1.22}
\]

when \( x \) represents age, as \( a = x \) and \( g(x) = 1 \). Subsequently work has been carried out into the well-posedness and stability analysis and parameter estimation. More recently sensitivity equations for the initial conditions and various rates used in the equations have been studied by Banks et al. (2009), while Liu and He (2009) investigated stability in size-structured populations with resource dependencies and inflow of stage-0 individuals from external sources.

While the first-order partial differential equation from this form of structured population model may not be too difficult to solve in itself, the boundary condition complicates things quite a bit. Accurate numerical solution of the PDE models from Equation 1.17 and 1.20 can be difficult (Gurney and Nisbet, 1998). To counter this issue de Roos (1988); de Roos et al. (1992) formulated a method that not only provided a tool for numerical study of PDE models referred to as physiological structured population models, PSPMs, (alternatively referred to as Escalator Boxcar Train (Murray, 1993)) but could also be used as a population model in its own right. The idea is that, rather than simulating the dynamics of the density function \( n(x,t) \) the PSPMs follow the progress of cohorts, mutually exclusive and exhaustive groups of width \( \delta t \) over the interval \((0, t + \delta t]\), that make up the entire population. Births are continuous so members of a new cohort,
n_0(x_b, t) are accumulated over $\delta t$ from reproduction in the other cohorts. Note that births from the cohorts go into the "new born" cohort rather than their own cohort. Also, as cohort membership is decided by when the individuals are born, these cohorts are in effect isolated, cannot increase in population and decrease in population only through mortality. If interval $\delta t$ is small enough, then the individuals in the cohort can be characterised by their average. In effect it is like sending a person, every $\delta t$ to monitor a single cohort's development over time. They monitor age, size, mortality and births (that do not enter their cohort, but the $n_0$ cohort). The equations for this model are:

\[
\begin{align*}
\frac{dn_i(t)}{dt} &= -\mu(\sigma_i(t)) n_i(t); \quad (1.23) \\
\frac{d\sigma_i(t)}{dt} &= g(\sigma_i(t)); \quad (1.24) \\
\frac{dn_0(t)}{dt} &= -\mu(x_b)n_0(t) - \frac{\partial \mu(x_b)}{\partial x} + \sum_i b(\sigma_i(t), t) n_i(t); \quad (1.25) \\
\frac{d\pi_0(t)}{dt} &= g(x_b)n_0(t) + \frac{\partial \mu(x_b)}{\partial x} \pi_0 - \mu(x_b)\pi_0; \quad (1.26)
\end{align*}
\]

where $\sigma_i(t)$ is the $i^{th}$ cohort’s age, $x_b$ is the age of the youngest in the "new born" cohort, and $\pi_0$ is the average age of the "new born" cohort. For more details on the models discussed above see either Murray (1993), Tuljapurkar and Caswell (1997) or Kot (2001).

1.3 Thesis Format and Objectives

Currently in semi-arid Australian rangelands properties produce mainly beef and wool on marginal lands. A major area of concern is grazing pressure. Kangaroos
are considered to have a substantial impact on grazing pressure, and for that reason they are often considered pests by landholders. It has been thought that converting from farming European domesticated stock to native wildlife would have environmental benefits. The perceived benefits include: restoration of native ecosystems; decreasing greenhouse gas emissions; and public health (substitution of other meat products for low fat, high iron, kangaroo meats). The commercial benefits from the change are unclear. These thoughts lead the three key questions that inform this thesis. Can the inclusion of kangaroo commodities increase resilience to landholders in semi-arid regions of Australia? If kangaroos were to be encouraged on one property, would this have detrimental impact on neighbouring properties? What impact would a requirement to reduce greenhouse gas emissions have on the viability of including kangaroo commodities?

Through construction of plant-herbivore models, the dynamics of cattle, sheep and kangaroo commodities are examined. Simulations were constructed so as to estimate the expected value for each and the correlation between the different commodities. Portfolio analysis using mean-variance, average value-at-risk, and multi-objective optimisation projects were used to analyse different allocations of forage to each herbivore, with and without methane emission reduction requirements. From the analysis it seems that diversification of herbivores (including kangaroos) is optimal on more marginal lands, for the risk averse, and to reduce methane emissions.

If landholders can see a financial benefit from diversifying the commodities they produce then the environmental benefits could be a consequence. Chapter 2 uses a simplified scenario of price changes and fecundity to explore mixed-grazing
strategies. It investigates reducing the risks involved in farming in a semi-arid rangeland in Australia through the inclusion of kangaroos. If this is not the case in the simplified scenario, then it is unlikely that diversification will be useful in a more detailed model.

Currently the kangaroo meat processors are in a dominant position, able to determine prices and quantities. What would happen if that were to change? The supply chain between landholder and processor using a game theoretical approach is explored in Chapter 3. Does migration have an effect on the landholders’ willingness to supply beef and kangaroo meat? What difference does a power imbalance between landholder and processor make to the scenario?

Allocating forage to different species is only possible if the population size of each species can be controlled, otherwise competitive exclusion and migration could override the allocations. The effect of kangaroos, a mobile species that can cross boundaries and cannot always be explicitly controlled, needs to be explored. Chapter 4 investigates the dynamics between vegetation and herbivores. Differential equations are used to analyse: the effect of mobility on MSYs; and the impact from neighbours in the form of large national parks and similarly sized commercial properties. What are the best options when considering interacting properties? When does the mobile herbivore dominate the captive herbivore?

Mitigation through income from kangaroos could help alleviate financial downturn experienced during drought. Added to this kangaroo numbers rapidly increase after a drought has broken, and harvesting could financially counter re-stocking costs of domestic animals (Dawson, 1995). For this reason developing an understanding of the herbivores reactions to weather conditions via a plant-
herbivore model is required. Chapter 5 discusses the construction of a PSMS for kangaroo populations, underpinned by a pasture growth model calibrated to the area of interest. The model reacts to daily weather, predicting beef, wool, and kangaroo production. Issues related to its use and efficacy are noted.

Chapter 6 is the culmination of the research carried out in the previous chapters. Results from the simulated property and portfolio optimisation combine to examine the risk associated with different mixed-grazing options. Pareto-optimal efficient frontiers are constructed and conclusions are drawn about the extent of diversification. The effect of methane emissions reduction on the portfolio is also examined.

The conclusion discusses the finding of the research and possible consequence that follow. It also considers the effect of changes to the situation as well as further research that is required in the area to better inform some of the parameters and assumptions used in the models underlying this thesis.
Chapter 2

Initial Exploration of Viability of Mixed-Grazing

Currently landholders see kangaroos as a pest and of little to no value. Therefore the possible benefits replacing some cattle and sheep production with native herbivores is not realised. If landholders can see a financial benefit from diversifying the commodities they produce then the environmental benefits could be a consequence. As a first step it makes sense to explore whether it is possible that inclusion of kangaroos into the commodities that a property produces has some benefit to the landholder. Of particular interest is reducing the variability of relative returns. Do the relative returns of the property become more resilient to external forces when the commodities harvested are from both kangaroos and domestic stock? Also of interest is any possible difference in strategy, dependent on how marginal the land is. In particular, does the amount of money invested affect the optimal grazing strategies? The data used in the scenarios explored
in this chapter are limited to average fecundity for kangaroos, as actual rates for different years are not known. If it can be shown that mixed-grazing is economically beneficial to landholders in the simplified case, then further analysis is warranted.

2.1 Formulation and Assumptions

Classical mean-variance portfolio selection involves a scenario with a limited amount of funds to be invested in a variety of assets (Steinbach, 2001). The goal is that each asset is allocated funds, \( y \), in such a way as to trade-off maximizing returns, \( \rho(y) \), and minimizing risk, \( R(y) \):

\[
\max_y \quad \pi \rho(y) - \frac{1}{2} R(y) \quad \text{(2.1)}
\]

\[
s.t.
\]

\[
e^T y = 1 \quad y \geq 0,
\]

where \( \pi \) is the trade-off parameter, \( e \in \mathbb{R}^n \) denotes the vector of all 1s and the budget equation \( e^T y = 1 \) specifies the initial wealth (without loss of generality set equal to one).

Analogous to the problem stated in Equation 2.1 is that of allocating the overall stock level to different species of domestic and native herbivores. This is done so as to trade-off the competing objectives of maximizing the relative return on investment and minimizing the risk involved in the investment. In Australia stock levels can be compared via units of dry sheep equivalents, independent of
time. A dry sheep equivalent (dse) standardises the feed requirements of different animals across different species, using a 50kg, non-lactating Merino ewe as the standard for comparison. Hence, a dse of 3 equates to an animal that requires three times the feed of the aforementioned ewe (Millear et al., 2003). So, the overall stock level can be defined as the total amount of dse that an area supports. Therefore the budget equation is replaced by the idea of overall stocking rate, $K$, in dse. Portions of the overall stock level are allocated to each species of interest, $k_i$, $i \in S$ in dse, where $S$ is set of domestic and native herbivores. Obviously the proportion of the overall stock level allocated to each species will effect the populations, $x_i$, $i \in S$. While the expected return is dependent on the prices, $p_i$, and fecundity, $f_i$ of the species.

Consider a typical property in central Queensland of 200km$^2$ supporting 12,000 dse or 60dse/km$^2$. Of interest is determining the proportion of the overall stock level that each species will be allocated. Without loss of generality it can be argued that proportional stock allocation would allow for scaling to similar availability of forage or property size. This would lead to a formulation similar to Equation 2.1 to allocate the proportions $q_i = k_i/K$, $i \in S$ in the following problem:

$$\max_q \frac{(1 - \lambda)q^T \rho}{\rho^*} - \frac{\lambda q^T \Sigma \rho q}{\Sigma^* \rho^*} \quad (2.2)$$

s.t.

$$e^T q = 1$$

$$q_i \geq 0 \ \forall i \in S,$$
2.1 Formulation and Assumptions

where the $\rho$ terms are derived from the expected proportional increase in value from one year to the next of the species and $\rho^*$ is the maximum expected return possible and $\Sigma^*_\rho$ is the minimum variance for a given fixed cost. To balance the competing objectives of maximizing return and minimizing risk the $\lambda$ term has been included as a measure of the degree of the investors risk aversion, 0 for no risk aversion (only concerned with the expected return), to 1 being completely risk averse (only wanting to minimize the fluctuations in returns). The benefit of this definition of risk aversion is in its intuitive nature. Risk will be measured using the variance of returns. While different measures can be used, an area attracting much discussion (Steinbach, 2001), variance will produce a result with a broad basis (Markowitz, 1991) and is commonly understood, whilst retaining the quadratic nature of the objective function.

To calculate the returns it is clear that both the fecundity and increase in price need to be included. The value of a population of animals from a given species can be calculated by multiplying the price of each animal by the number of animals owned. After one year the change in the value of the population would be due to changes in both price and population growth. Hence it can be easily shown that the return on an investment after one year is given by,

$$\rho_i = \frac{p_i x_i (1 + \Delta p_i)(1 + f_i) - p_i x_i}{p_i x_i} = \Delta p_i + f_i + \Delta p_i f_i$$  \hspace{1cm} (2.3)

where $p_i$, $\Delta p_i$ and $f_i$ are the price, change in price and fecundity respectively of species $i \in S$.

This formulation only includes the amount invested in stock. The value of
2.1 Formulation and Assumptions

the property, associated with land value and facilities, should also be considered in the investment amount. The inclusion of a term to account for the non-stock investment, say $NSI$, into the amount invested gives a new measure of growth, $r_i$ (Equation 2.4, a variation on Equation 2.3),

$$r_i = \frac{p_i x_i (1 + \Delta p_i)(1 + f_i) + q_i NSI - (p_i x_i + q_i NSI)}{p_i x_i + q_i NSI} = \frac{\Delta p_i + f_i + \Delta p_i f_i}{1 + \frac{d_i NSI}{Kp_i}},$$  

(2.4)

where $d_i$ is the dse for species $i \in S$ as $x_i = \frac{a_i}{d_i}$. Note that Equation 2.3 is the special case of Equation 2.4 when $NSI=0$. The result of the inclusion of non-stock investment means that it is now a case of optimising

$$\max_q \frac{(1 - \lambda) q^T r}{r^*} - \frac{\lambda q^T \Sigma_r q}{\Sigma_r^*}$$

(2.5)

s.t.

$$e^T q = 1$$

$$q_i \geq 0 \ \forall i \in S.$$

It has been assumed that there is no substitutability of the commodities. Hence, the decision of the landholder will not affect the overall market and prices for each commodity. This simplification seems reasonable given the focus on a small region implementing a mixed-grazing strategy. It is also assumed that land holders have sufficient financial resources and can actively control the animal numbers on their properties. In reality this is straightforward for cattle and sheep assets, as they have the ability to buy or sell livestock, and fencing maintains an effective boundary for these species. However, kangaroos do not
belong to the land holder. They are under the care of the Federal Government, meaning direct purchase and sales of kangaroos is not an option. To further complicate controlling the kangaroo population, properties can be thought of as having porous borders, with kangaroos easily jumping standard fencing. This presents the possibility of enticing kangaroos to an area from neighboring land if their numbers need to be increased. Unfortunately this also means a chance of losing kangaroos to "greener pastures". To address the issue of migration the property is considered to be part of a cooperative with common stocking levels on neighboring properties. With common stocking levels, there should be no net migration as all pastures would be equally attractive. Currently kangaroo harvesting cooperatives are being considered in at least two regions, one of which is the Maranoa (Baumber et al., 2009). It is also assumed that the State’s quotas for the number of kangaroos permitted to be harvested would not limit the harvesting of kangaroos on the property. From 2001 to 2007, nationally an average of 69.5% of the quota was utilized (Anon., 2009).

2.2 Illustration

Data were collected from The Australian Bureau of Agricultural and Resource Economics 2010, which is available to the public, and data from kangaroo harvesters, information not available in the public domain (T. Garrett, pers. comm., 2008). Data relating to the prices of beef, kangaroo meat, lamb and wool as well as fecundity rates for cattle and sheep was collated from 1988 to 2005 for the Charleville-Longreach region of Queensland. The price data are expressed in
Illustration

2008-2009 Australian dollar values in terms of the revenue produced per animal (See Table B.1). These data were then used to produce matrices for the estimated mean, $E[r]$, and covariance, $\Sigma_r$, for the percentage return per year. It should be noted that in this region sheep are stocked for their wool, with only small numbers of lambs being sold for meat, while kangaroo harvesting, with no return to the landholders, has been established for some time. The $dse$ values were calculated using this information and a Queensland Government conversion chart (Millear et al., 2003) for the cattle (9 $dse$) and sheep (1 $dse$) with the rate often used by landholders for kangaroos (0.7 $dse$). This is an upper bound to the estimate for the kangaroos (Grigg, 2002; Munn et al., 2009). These parameters give a scenario more likely to favor the status quo of domesticated livestock, due to the low $dse$ estimates for domestic stocks and a high $dse$ estimate for kangaroos.

The pareto-optimal solutions for the special case with non-stock investments not included ($NSI = 0$), are shown in the form of the efficient frontier (Figure 2.1). The pareto-optimal solutions where calculated in Mathematica 8.0 (Wolfram Research, Inc., 2010) using the function ”Solve” for a sequence of risk aversion values. For cases with low returns, it can be seen that changing the stance to increase the expected relative return incurs a relatively small increase in the standard deviation (Figure 2.2). However, as the return becomes greater the increase in the risk grows at a faster rate. The solutions in the efficient frontier correspond to the allocation of forage (Figure 2.3), clearly favoring kangaroos when risk aversion is low ($\lambda < 0.24$), with sheep only being considered once $\lambda > 0.24$ and cattle only for moderate to high risk aversion, $\lambda > 0.6$.

If non-stock investment is now considered ($NSI > 0$), not only is there a
2.2 Illustration

Figure 2.1: The efficient frontier for the optimal solutions to the forage allocations as shown by the expected percentage return on investments versus its risk, here measured as the standard deviation so that the units are equivalent.

Figure 2.2: Plot of expected relative return and the standard deviation of the relative return as risk aversion, $\lambda$, varies, excluding non-stock investments.
reduction in the expected proportional returns as one would anticipate, but also the variance. This seems to be due to the fact that the larger and more expensive species have smaller variances and are hence less risky. In terms of worth per $dse$ cattle are the most expensive whilst kangaroos are the least. As the property values increase, the slopes of the efficient frontiers are also decreasing, meaning that reducing the risk has less of an impact on the expected return. In terms of financial resilience, let us say a landholder is trying to minimize the frequency of poor returns. In one scenario a loss, on average, approximately once every 5 years became once every 250 years when a more risk averse position was taken. That was on a property with moderate value ($NSI = $2,500,00$) and the risk aversion measure went from $\lambda = 0$ to $\lambda = 0.5$. The return was reduced by 45.1%, but the stocking investment was also significantly reduced by some 66.9%. The scenario assumed that returns were normally distributed and only optimal solutions were considered.
One obvious reason non-stock investments will differ between properties is quality of the pastures and availability of water. The greater the non-stock investments the more expensive species begin to be favored because their return on total costs increases relative to the less expensive species (Figures 2.4 and 2.5). From a non-stock investments of $\text{NSI} = 0$ (refer to Figure 2.3) to $\text{NSI} = 2, 168, 350$, when considering purely the returns ($\lambda = 0$), a strategy of solely stocking kangaroos is favored, due to their higher fecundity rate. At higher non-stock investments though, as seen in Figures 2.4a and 2.4b, where $\text{NSI}$ is $2,500,000$ and $5,000,000$ respectively, cattle are favored when maximizing returns, due to their higher price per $dse$.

No matter the non-stock investments, once the landholders become even moderately risk averse ($\lambda > 0.25$) a mixed strategy is preferred. This point is illustrated in Figure 2.4, where it is clear to see that when greater non-stock investment is required the forage allocated to cattle increases, reducing the kangaroo allocation as the sheep allocation plateau. Comparing the low (2.5a) and higher (2.5b) risk aversion strategies for different $\text{NSI}$ reiterates the preference for mixed-grazing at both higher levels of risk aversion and $\text{NSI}$.

It is also noted that there is some disagreement in the literature as to the true value of the kangaroo’s $dse$, with values from 0.15 to 0.7 quoted by different sources, the most recent of which suggests $\sim 0.4$ (Munn et al., 2009). If this value is changed from the 0.7 used above, then the resulting optimal scenarios change significantly (Figure 2.6). A smaller value of kangaroo $dse$ reduces the resources allocated to cattle or even eliminates cattle from the optimal solution altogether, reducing the diversification down to just sheep and kangaroos.
Figure 2.4: Plot of proportion of resources allocated to each species as risk aversion, $\lambda$, varies given a non-stock investment of $2,500,000$ and $5,000,000$. 

(a) $NSI=$2,500,000

(b) $NSI=$5,000,000

(c) Legend

- Kangaroo Allocation
- Sheep Allocation
- Cattle Allocation
Figure 2.5: Plot of proportion of resources allocated to each species as non-stock investment, NSI in thousands of dollars, varies given a risk aversion of $\lambda = 0.2$ and $0.8$. 
2.3 Discussion

Biodiversity benefits are not always easy to quantify or justify on the bottom line, however, the possible financial pay-offs of diversifying the commodities produced

Figure 2.6: Plot of proportion of resources allocated to each species as the value of kangaroo dse varies given a non-stock investment of $5,000,000 and a risk aversion measure of $\lambda = 0.2$ and 0.8
by the enterprise can be quantified. From this analysis it can be seen that, given the correct circumstances, active management of kangaroos could be a viable way of increasing the resilience of the land financially as well as ecologically. Diversification in semi-arid Australian rangelands seems to result in less volatility in returns on average. This is the case even where there is some uncertainty with regards to the true nature of the comparative impact of kangaroos. Different combinations of these parameter values, which seem to cover the plausible spectrum, all resulted in some form of diversification along the efficient frontier. Furthermore, as the land becomes more marginal, and decreases in value, there seems to be benefits with respect to returns from the inclusion of kangaroo harvesting.

Additionally it is noted that several practical difficulties need to be overcome before the systems considered in our model can be implemented. The relationship and distribution of returns between landholders and harvesters needs to be reviewed. Will harvesters continue to operate as they are currently but with some return to the landholders or will they work for the cooperative? Compared to beef sales, kangaroo meat has a very low market share and therefore elasticities with beef are not clear. Greater acceptance for kangaroo meat within Australia and internationally could affect its demand. If diversification into kangaroos was to increase, there may be a point were some of the assumptions may need to be altered. Further discussion on some of the issues can be found in Cooney et al. (2009).

Anecdotal evidence suggests that during times of hardship caused by drought, kangaroo harvesting becomes easier as they are forced to leave their sheltered bushland for clearings in search of any available forage. It is exactly at this time
that landholders future incomes are decreased due to a reduction in their stocking rates of domestic species and the need to buy extra fodder. The affect on kangaroos during drought is an increase in mortality, after a lag of several months, which disproportionately affects juveniles and the elderly kangaroos (Caughley \textit{et al.}, 1987). Mitigation through income from kangaroos could help alleviate some of this financial down turn. Added to this is the rapid increase in kangaroo numbers after a drought has broken, where harvesting could financially counter restocking costs of domestic animals (Dawson, 1995). For this reason developing this problem to include results from a better understanding of the herbivores reactions to weather conditions via a plant-herbivore model would be of some use. An extended \textit{GRASP} model will be used later to capture the transient affects of drought and post-drought recovery. Allocating forage to different species is only possible if population size of each species can be controlled. Otherwise competitive exclusion and migration could override the allocations. The effect of kangaroos (a mobile species that can cross boundaries and cannot always be explicitly controlled) is analysed later. Other areas in need of investigation include: differentiating the food types and food preferences of the species; considering the variance in kangaroo fecundity; and supply and demand in a meat supply chain. These areas may show further benefits. Although there are practical difficulties still to be overcome, the analysis strongly suggests that mixed-grazing involving kangaroos possibly offers benefits and is worth further investigation.
Chapter 3

Meat Supply Chain with Migration

Currently the kangaroo meat processors are in a dominant position, able to determine prices and quantities. What would happen if that were to change? This chapter focuses on the landholder and processing aspects in a supply chain to determine optimal stocking and harvesting rates. The landholder (seller) wishes to maximise their return whilst maintaining a sustainable property. They harvest kangaroos, the free-roaming stock, as well as the domestic stock. Whereas the meat processor (buyer) is trying to meet demand while minimising their costs. Several meat supply chain models are proposed and solved. The seller’s model determines the optimal quantities of sheep and kangaroo to offer so as to maximise their income based on buying price. The property’s carrying capacity, animal fecundity and mortality and (in the case of kangaroos) migration are taken into account. The optimal buying price and order quantity are determined for these
substituted goods to minimise the processor’s purchase cost where the order quantity is a function of buying price. The interactions between the processor and the landholder are modelled by both non-cooperative and cooperative games. The non-cooperative aspect is a seller-Stackelberg scenario, where the seller has more power than the buyer. In addition Pareto efficient solutions to the cooperative game model are provided.

### 3.1 Notation and Problem Formulation

This section introduces the notation and formulation used in the supply chain problem. Specifically, all decision variables, input parameters and assumptions underlying our models will be stated.

#### 3.1.1 Decision Variables

The decision variable are the parameters that the landholder and processor have control over. Table 3.1 lists these variables and states which player has direct control of the variable.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_s$</td>
<td>The buying price of sheep (processor’s decision variable)($/kg).</td>
</tr>
<tr>
<td>$p_k$</td>
<td>The buying price of kangaroo (processor’s decision variable)($/kg).</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>The fraction of sheep to sell (landholder’s decision variable)(dimensionless).</td>
</tr>
<tr>
<td>$\gamma_k$</td>
<td>The fraction of kangaroo to sell (landholder’s decision variable)(dimensionless).</td>
</tr>
</tbody>
</table>

Table 3.1: Decision variables used in the supply chain problem.
3.1.2 Input Parameters

The number of sheep ($N_s$) and kangaroos ($N_k$) per square kilometre is affected by many considerations. These include maximal growth rates of sheep ($\nu_s$) and kangaroos ($\nu_k$) comprising births and deaths per capita per year. The cost of maintaining and harvesting a sheep or a kangaroo is $c_s$ and $c_k$ respectively. The property can only hold a limited number of sheep and kangaroo ($\kappa_s$ and $\kappa_k$ respectively) known as the carrying capacity. Competition for the forage between these species also needs to be considered. The effect of a single kangaroo on the sheep population is given by $\alpha_{sk}$, while the reverse relationship is $\alpha_{ks}$. The rate at which the kangaroos migrate per year ($\tau^\dagger$) needs to be considered due to the free-roaming nature of kangaroos as well as the kangaroo density of the world outside the property relative to the property ($\phi$). All of these affect the rate of change in the sheep ($\dot{N}_s$) and kangaroo ($\dot{N}_k$) populations.

Note, as the prices are based on $$/kg the average weight of harvested sheep ($w_s$) and kangaroos ($w_k$) is needed as part of the objective function.

For the processor, the ordering quantity of sheep and kangaroo are based on Yang and Zhou (2006) as follows:

$$quant_i = D - a_ip_i + bp_j; \quad i, j \in \{s, k\}, i \neq j. \quad \text{(3.1)}$$

$D$ represents the demand for sheep and kangaroo if their prices are zero ($D > 0$), $b$ is the degree of substitutability between the two goods (the substitutability coefficient of the two products ($b > 0$)), $a_i$ represents the measure of sensitivity between the amount ordered and price for good $i$ ($a_i > b$), $quant_s$ and $quant_k$ are
### 3.1 Notation and Problem Formulation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_s$</td>
<td>The stocking rate of sheep ($\text{sheep/km}^2$).</td>
</tr>
<tr>
<td>$N_k$</td>
<td>The stocking rate of kangaroo ($\text{kangaroo/km}^2$).</td>
</tr>
<tr>
<td>$\dot{N}_s$, $\dot{N}_k$</td>
<td>The rate of change for sheep and kangaroo respectively ($\text{animal/km}^2/\text{year}$).</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>The maximal growth rate of sheep ($\text{sheep}^{-1}\text{years}^{-1}$).</td>
</tr>
<tr>
<td>$\nu_k$</td>
<td>The maximal growth rate of kangaroo ($\text{kangaroo}^{-1}\text{years}^{-1}$).</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>The maximum stocking rate of sheep the property can maintain ($\text{sheep/km}^2$).</td>
</tr>
<tr>
<td>$\kappa_k$</td>
<td>The maximum stocking rate of kangaroo the property can maintain ($\text{kangaroo/km}^2$).</td>
</tr>
<tr>
<td>$\alpha_{sk}$</td>
<td>The impact on sheep of each extra kangaroo ($\text{sheep/kangaroo}$).</td>
</tr>
<tr>
<td>$\alpha_{ks}$</td>
<td>The impact on kangaroos of each extra sheep ($\text{kangaroo/sheep}$).</td>
</tr>
<tr>
<td>$\tau^\dagger$</td>
<td>The rate at which kangaroos can migrate to and from the property ($\text{years}^{-1}$).</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The comparative kangaroo density of the national park when compared to the property (dimensionless).</td>
</tr>
<tr>
<td>$c_s$, $c_k$</td>
<td>The cost of maintaining and harvesting sheep and kangaroo respectively ($$/\text{kg}$).</td>
</tr>
<tr>
<td>$w_s$</td>
<td>The average weight of the harvested sheep ($\text{kg/sheep}$).</td>
</tr>
<tr>
<td>$w_k$</td>
<td>The average weight of the harvested kangaroo ($\text{kg/kangaroo}$).</td>
</tr>
<tr>
<td>$\text{quant}_s$, $\text{quant}_k$</td>
<td>The quantity of sheep and kangaroo ordered (kg).</td>
</tr>
<tr>
<td>$D$</td>
<td>The total demand for sheep and kangaroo if their price were zero (kg).</td>
</tr>
<tr>
<td>$a_s$, $a_k$</td>
<td>The measure of sensitivity between the amount ordered and its price for sheep and kangaroo respectively ($\text{kg}/($/$\text{kg})$).</td>
</tr>
<tr>
<td>$b$</td>
<td>The degree of substitutability between the commodities ($\text{kg}/($/$\text{kg})$).</td>
</tr>
</tbody>
</table>
| $Z_1$, $Z_2$ | The cost and profit equations for the processor and landholder respectively ($\$)$.

**Table 3.2:** Input variables used in the supply chain problem.
the ordering quantity of sheep and kangaroo respectively.

The rate of change in the sheep population, \( \dot{N}_s \), is based on the Lotka-Volterra competition model (Kot, 2001) with includes harvesting and takes the form,

\[
\dot{N}_s = \nu_s N_s \left( 1 - \frac{N_c + \alpha_{sk} N_k}{\kappa_s} \right) - \gamma_s N_s
\]

(3.2)

where the first term is for the growth of the sheep population and the second term is for the harvest rate of the sheep. The growth term allows for intra- and inter-specific competition between sheep and kangaroo. The harvest term just specifies what fraction of the current sheep population being harvested at any given time.

Analogous to Equation 3.2 is the equation for the rate of change for the kangaroo population, with the addition of a term for the possible migration of kangaroos to and from the property and national park,

\[
\dot{N}_k = \nu_k N_k \left( 1 - \frac{N_k + \alpha_{ks} N_s}{\kappa_k} \right) - \gamma_k N_k.
\]

(3.3)

The migration term depends on the numbers of sheep and kangaroos in the property as well as the kangaroo density in the national park and needs to calculate the number of kangaroos wishing to enter (or leave) the property from the national park. To this end, \( \text{Migration}(N_s, N_k, \phi) = 0 \) when the relative densities of the two regions are equal \( \left( \frac{N_k + \alpha_{ks} N_s}{\kappa_k} = \phi \right) \) and the number migrating must be relative to the size of the properties carrying capacity for kangaroos \( (\kappa_k) \) as well as the fraction of those whom wish to migrate who actually migrate \( (\tau^\dagger) \). The diffusion equation for the migration rate can then be given as,
3.1 Notation and Problem Formulation

\[
\text{Migration}(N_s, N_k, \phi) = \tau^\dagger \kappa_k \left( \phi - \frac{N_k + \alpha_{ks}N_s}{\kappa_k} \right) = \tau^\dagger (\phi \kappa_k - (N_k + \alpha_{ks}N_s)).
\]  

(3.4)

So combining Equations 3.3 and 3.4 will give the equation for the rate of change for the kangaroo population on the property,

\[
\dot{N}_k = \nu_k N_k \left( 1 - \frac{N_k + \alpha_{ks}N_s}{\kappa_k} \right) - \gamma_k N_k + \tau^\dagger (\phi \kappa_k - (N_k + \alpha_{ks}N_s)).
\]  

(3.5)

3.1.3 Assumptions

The proposed models are based on the following assumptions:

**Assumption 1.** The target market of the processor includes customers who consume both sheep and kangaroo meat.

**Assumption 2.** The ordering quantities of sheep or kangaroo, is dependent on the buying price (pricing) of both goods and the property of the substituted products.

**Assumption 3.** The kangaroo population will endeavour to spread itself across the region so as to even out the fraction of the carrying capacity used.

**Assumption 4.** The rate of change for sheep and kangaroo numbers is a competitive model as defined by Equation 3.2 and 3.5.
3.1.4 The Buyer’s Model Formulation

The processor’s objective is to determine the buying price and the ordering quantity of sheep and kangaroo such that the purchasing cost is minimized. The ordering quantity of sheep and kangaroo are influenced by the buying price of sheep and kangaroo according to our assumption. The processor’s purchase cost is:

\[
Z_1(p_s, p_k) = w_sp_s quant_s + w_k p_k quant_k = w_sp_s(D - a_sp_s + b_p_k) + w_k p_k(D - a_k p_k + b_p_s).
\]

(3.6)

It can be shown that \( Z_1(p_s, p_k) \) is a convex function under the condition that \( 4a_s a_k w_s w_k - ((w_s + w_k)b)^2 > 0 \) with respect to \( p_s, p_k \). Hence, the first order condition of \( Z_1(p_s, p_k) \) with respect to \( p_s, p_k \) determines that \( p^*_s, p^*_k \) minimize \( Z_1(p_s, p_k) \) where:

\[
p^*_s = \frac{2Da_k w_s w_k + Db(w_s + w_k)w_k}{4w_s w_k a_s a_k - (w_s + w_k)^2 b^2}, \quad \quad (3.7)
\]

\[
p^*_k = \frac{2Da_s w_s w_k + Db(w_s + w_k)w_s}{4w_s w_k a_s a_k - (w_s + w_k)^2 b^2}. \quad \quad (3.8)
\]

3.1.5 The Seller’s Model Formulation

The landholder’s objective is to determine the optimal offering of sheep and kangaroo such that the profit is maximized. Thus, the landholder’s profit is:
3.1 Notation and Problem Formulation

\[
Z_2(\gamma_s, \gamma_k) = w_s(p_s - c_s)\gamma_s N_s + w_k(p_k - c_k)\gamma_k N_k
\]

subject to \[
\dot{N}_s = 0 \quad (3.9)
\]
\[
\dot{N}_k = 0
\]

By considering constraints, \(\gamma_s, \gamma_k\) yields:

\[
\gamma_s = \nu_s \left( 1 - \frac{N_s + \alpha_{sk}N_k}{\kappa_s} \right) \quad (3.10)
\]
\[
\gamma_k = \nu_k \left( 1 - \frac{N_k + \alpha_{ks}N_s}{\kappa_k} \right) + \frac{\tau^\dagger(\phi\kappa_k - (N_k + \alpha_{ks}N_s))}{N_k} \quad (3.11)
\]

Since \(\gamma_s, \gamma_k\) are functions of \(N_s\) and \(N_k\) for the sake of integrity, we will use \(Z_2(N_s, N_k)\) instead of \(Z_2(\gamma_s, \gamma_k)\). Substituting Equations 3.10 and 3.11 into Equation 3.9, the problem transforms into an unconstrained model of two variables \(N_s\) and \(N_k\). It can be shown that \(Z_2(x, y)\) is a concave function with respect to \(N_s\) and \(N_k\), when \(\alpha_{sk}\alpha_{ks} > 2\), which may not be the case, or more generally when

\[
(\alpha_{sk}\varpi_s)^2 + (\alpha_{ks}\varpi_k)^2 > 2\varpi_s\varpi_k\kappa_s\kappa_k \quad (3.12)
\]

where \(\varpi_s = (p_s - c_s)\nu_s w_s\) and \(\varpi_k = (p_k - c_k)\nu_k w_k\). Therefore, the optimal solution, \(N_s^*\) and \(N_k^*\) can be found by taking the derivative with respect to \(N_s\) and \(N_k\) such that:
3.2 The Seller-Stackelberg Model

This section considers the interaction between landholder and processor as a seller-Stackelberg game, where the landholder as a leader has the initiative and can enforce the strategy on the processor as a follower. The leader makes the first move and the follower then reacts by playing the best move consistent with available information. The objective of the leader is to design their move in such a way as to maximize their revenue after considering all rational moves the follower can devise (Basar and Olsder, 1999). Therefore, in our model, the processor, obtains the optimal buying price \( p^*_s, p^*_k \) according to the processor’s model, which is given by (3.7) and (3.8) respectively. The seller then maximize their profit \( Z_2(N_s, N_k) \), based on the pair \( p^*_s, p^*_k \). Thus, the problem reduces to

\[
Z_2(N_s, N_k) = w_s(p_s - c_s)\gamma_s N_s + w_k(p_k - c_k)\gamma_k N_k
\]

subject to

\[
\begin{align*}
p^*_s &= \frac{2Da_kw_s w_k + Db(w_s + w_k)w_k}{4w_s w_k a_s a_k - (w_s + w_k)^2 b^2} \\
p^*_k &= \frac{2Da_sw_s w_k + Db(w_s + w_k)w_s}{4w_s w_k a_s a_k - (w_s + w_k)^2 b^2}
\end{align*}
\]
Substituting all constraints into the objective function, the above seller-Stackelberg problem reduces to optimizing an unconstrained nonlinear objective function.

### 3.3 The Cooperative Game

The landholder and the processor may be able to increase their profits by choosing their policies in a cooperative way. For example, a manufacturer may build an exclusive product for a reseller using the profit sharing approach. This way, the partnership can be a win-win situation for both parties. In a cooperative game, the landholder and the processor work together to determine pricing and equal values for ordered and offered quantities in a Pareto efficient way. A solution is Pareto efficient when there is no other solution where one party can maintain its current profit while the other party attains a higher profit; i.e., when the gain by one party can be made only at the expense of the other party. Such cooperation is carried out through the joint optimisation of the weighted sum of the landholder’s and processor’s objective functions, i.e., the set of Pareto efficient solutions can be characterised by maximising (Esmaeili et al., 2009b):

\[
Z = \lambda Z_2 - (1 - \lambda)Z_1, \quad 0 < \lambda < 1,
\]

that is,

\[
Z = \lambda (w_s m_s \gamma_s N_s + w_k m_k \gamma_k N_k) + (\lambda - 1) (w_s p_s (D-a_s p + b_p) + w_k p_k (D-a_k p + b_p))
\]

(3.16)
The first order condition for maximizing $Z$ with respect to $p_s$ yields:

$$\frac{\partial Z}{\partial p_s} = 0 \Rightarrow \lambda = \frac{w_s(D - 2a_sp_s) + (w_s + w_k)bp_k}{w_s(\gamma_sN_s + D - 2a_sp_s) + (w_s + w_k)bp_k},$$

which shows that $\lambda \in (0, 1)$. First order conditions with respect to $p_k$, $N_s$ and $N_k$ further yield:

$$p_k = \frac{w_k \lambda \gamma_k N_k + (\lambda - 1)(w_k D + (w_s + w_k)bp_s)}{2w_k a_k (\lambda - 1)},$$

$$N_s = \frac{\kappa_s \left( w_s \nu_s \mu_s - w_k \alpha_k \tau^\dagger m_k \right) - N_k \left( \frac{w_s \alpha_{sk} \nu_s \mu_s}{\kappa_s} + \frac{w_k \alpha_k \nu_k m_k}{\kappa_k} \right)}{2m_s w_k \nu_s},$$

$$N_k = \frac{\kappa_k \left( w_k (\nu_k - \tau^\dagger) m_k - N_s \left( \frac{w_s \alpha_{sk} \nu_s \mu_s}{\kappa_s} + \frac{w_k \alpha_k \nu_k m_k}{\kappa_k} \right) \right)}{2m_k w_k \nu_k}.$$

Pareto efficient solutions can be obtained through a negotiation between the landholder and the processor over a fixed $p_s$, i.e. Equations 3.17 to 3.20 are solved simultaneously to obtain $\lambda^*$, $p_k^*$, $N_s^*$, and $N_k^*$ for a fixed $p_s$. The other approach is to assume $\lambda$ and solve Equations 3.18 to 3.20 for $p_s$ and other variables.

The results show that the buying price in a cooperative game is less than that in the non-cooperative game. Let $p_{2C}^*$ and $p_{2N}^*$ be the optimal buying price in a cooperative and non-cooperative game respectively, i.e. $p_{2C}^*$ is given by Equation 3.18 and $p_{2N}^*$ by Equation 3.8. We obtain

$$p_{2C}^* = \frac{\lambda \gamma_k N_k}{2(\lambda - 1) a_k} + p_{2N}^*$$

where the first term of the equation is negative $\lambda \in (0, 1)$ and therefore $p_{2N}^* > p_{2C}^*$. 
3.4 Computational Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_s$</td>
<td>0.45 sheep$^{-1}$ years$^{-1}$</td>
<td>$c_s$</td>
<td>$0.3/kg$</td>
</tr>
<tr>
<td>$\nu_k$</td>
<td>0.58 kangaroo$^{-1}$ years$^{-1}$</td>
<td>$c_k$</td>
<td>$0.1/kg$</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>30 sheep/km$^2$</td>
<td>$w_s$</td>
<td>45 kg/sheep</td>
</tr>
<tr>
<td>$\kappa_k$</td>
<td>27 kangaroo/km$^2$</td>
<td>$w_k$</td>
<td>25 kg/kangaroo</td>
</tr>
<tr>
<td>$\alpha_{sk}$</td>
<td>0.4 sheep/kangaroo</td>
<td>$D$</td>
<td>50 kg</td>
</tr>
<tr>
<td>$\alpha_{ks}$</td>
<td>3 kangaroo/sheep</td>
<td>$a_s$</td>
<td>22.5 kg/($/kg$)</td>
</tr>
<tr>
<td>$\tau\dagger$</td>
<td>3.6%/year</td>
<td>$a_k$</td>
<td>25 kg/($/kg$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1</td>
<td>$b$</td>
<td>0.5 kg/($/kg$)</td>
</tr>
</tbody>
</table>

Table 3.3: Parameter values used in numerical analysis.

3.4 Computational Results

In this section, we present numerical examples which are aimed at illustrating some significant features of the models established in previous sections. We will also perform sensitivity analysis of two main parameters of these models. We note that Examples 1 and 2 below illustrate the seller-Stackelberg and cooperative models respectively. In the examples, we set the biological and economic parameters as shown in Table 3.3. These figures are based on research presented by Caughley et al. (1987) and Pople and Grigg (1999) and have some degree of variability associated.

3.4.1 Numerical Examples

Example 1 The seller-Stackelberg model produces the following optimal values for our decision variables: $p_s^* = $1.129/kg; $p_k^* = $1.143/kg; $\gamma_s^* = 0.3740; $\gamma_k^* = 0.3488; $quant_s^* = 25.17kg; $quant_k^* = 24.85kg; $N_s^* = 0 sheep/km$^2; $N_k^* = 12.66$ kangaroo/km$^2; $\gamma_s^* N_s^* = 0$ sheep/year/km$^2$; and $\gamma_k^* k^* = 4.416$ kangaroos/year/km$^2$.

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The corresponding landholder’s profits and processor’s purchasing costs are $Z_2^* = 115.125/km^2$ and $Z_1^* = 1907.09/km^2$ respectively. This gives us a baseline to compare the cooperative solution against in the next example.

**Example 2** We obtained Pareto efficient solutions by assuming that the landholder and processor has negotiated an agreement on the pricing of sheep $p_s = 0.7/kg$. Equations (3.18) - (3.20) are then used to obtain $p_k^*, N_s^*$, and $N_k^*$. The final solutions are as follows: $p_k^* = 0.5199/kg; N_s^* = 3.599$ sheep/km$^2$; $N_k^* = 6.402$ kangaroo/km$^2$; $Z_2^* = 62.65/km^2$; and $Z_1^* = 1572.6/km^2$. Since, in the seller-Stackelberg model, the landholder has more power; we would expect that the profit of the landholder in the seller-Stackelberg be greater than in the cooperative model. However, the processor’s purchasing costs in the seller-Stackelberg are greater than in the cooperative model. Therefore, the processor prefers the cooperative model as it reduces the purchasing costs compared to the cooperative model. More generally an increase in the price of sheep, $p_s$, results in an increase in the price of kangaroo $p_k$ until $p_s > 0.9988/kg$ when the value of $p_k$ starts to decrease, as seen in Figure 3.1. It can also be seen that the landholder’s profit is maximised ($Z_2 = 86.0.2/km^2$) at a similar value of $p_s = 1.001/kg$ (see Figure 3.2). These both occur after the number of sheep stocked drops (Figure 3.3) and kangaroo immigration starts to increase (Figure 3.4) due to the reduced density on the property. This reduced density on the property allows the landholder to harvest the “free” kangaroos that migrate.
3.4 Computational Results

Figure 3.1: The optimal price for kangaroo meat, $p_k$, dependent on the price of sheep meat, $p_s$, using the cooperative model.

Figure 3.2: The optimal profit for the landholder, $Z_2$, dependent on the price of sheep meat, $p_s$, using the cooperative model.
3.4 Computational Results

Number of sheep stocked

![Graph showing the relationship between the price of lamb and the number of sheep stocked.](image)

**Figure 3.3:** The optimal sheep stocking rate, $N_s^*$, dependent on the price of sheep meat, $p_s$, using the cooperative model.

Immigrating kangaroos/year/sq.km

![Graph showing the relationship between the price of lamb and the number of kangaroos immigrating.](image)

**Figure 3.4:** The number of kangaroos that immigrate dependent on the price of sheep meat, $p_s$, when the cooperative model is optimised.
3.4.2 Sensitivity Analysis

We investigate the effects of parameters $\phi$ and $\tau^\dagger$, two parameters related to the landholder’s model. The reason for choosing these parameters in particular is due to the uncertain nature of the migration rate, $\tau^\dagger$ and the impact of changing the stocking density on the property compared to the outside environment could have important consequences for landholders. All parameters are fixed as in the previous examples ($p_s = 0.8$) but we allow $\phi$ and $\tau^\dagger$ to vary. Results of these sensitivity analyses are summarised in Figures 3.5 to 3.8. It can be seen that, as the comparative density, $\phi$, has a positive relationship with the optimal price of kangaroo, $p^*_k$ in Figure 3.5, landholder’s profit, $Z^*_2$ in Figure 3.6, the sheep stocking rate, $N^*_s$ in Figure 3.7, and not surprisingly the migration numbers, Figure 3.8. Therefore, by controlling their own total grazing pressure with the domesticated animals and harvesting regime of the free-roaming stock, and making their land more desirable for kangaroos to immigrate it could actually improve their financial position.
3.4 Computational Results

Figure 3.5: The optimal price for kangaroo meat, $p_k$, dependent on the comparative density of kangaroos, $\phi$, using the cooperative model.

Figure 3.6: The optimal profit for the landholder, $Z_2$, dependent on the comparative density of kangaroos, $\phi$, using the cooperative model.
3.4 Computational Results

Number of sheep stocked

![Graph showing the optimal sheep stocking rate, \( N^*_s \), dependent on the comparative density of kangaroos, \( \phi \), using the cooperative model.](image1)

**Figure 3.7:** The optimal sheep stocking rate, \( N^*_s \), dependent on the comparative density of kangaroos, \( \phi \), using the cooperative model.

Immigrating kangaroos/year/sq.km

![Graph showing the number of kangaroos that immigrate dependent on the comparative density of kangaroos, \( \phi \), when the cooperative model is optimised.](image2)

**Figure 3.8:** The number of kangaroos that immigrate dependent on the comparative density of kangaroos, \( \phi \), when the cooperative model is optimised.
Finally, the effect of the migration parameter $\beta$ on results for the cooperative model is investigated. Varying this parameter from $\tau^\dagger = 0$ to $\tau^\dagger = 0.125$ changes the kangaroo immigration significantly, from 0 to 1.184, compared to the standing stocking rate of 7.690 and 7.297 respectively. Consequently the landholder’s profit, $Z_2^*$, increased by 2.4%, due to a 10.8% increased harvesting rate for kangaroos, $\gamma_k$, paired with a 5.1% decrease in the kangaroo stocking rate $N_k$. However, the effect on the other variables is negligible, generally changing the no migration result by less than 1.8%. Hence, while this term may not be known, the accuracy of this term will have a minimal impact on the optimal solution.
Chapter 4

Plant-Herbivore Modelling

Allocating forage to different species is only possible if population size for each species can be controlled. Otherwise competitive exclusion and migration could override the allocations. The effect of kangaroos (a mobile species that can cross boundaries and cannot always be explicitly controlled) needs to be explored. Landholders have concerns that kangaroos negatively impact their operation through foraging on their properties (Pople and Grigg, 1999; Grigg, 2002). However, unique opportunities and challenges present themselves if the landholder is willing to diversify their commodities. Issues related to population dynamics on and between properties and regions is the focus of this chapter. What is the effect of a species that can circumvent property boundaries and do not belong to the landholder? Can decisions taken by the property managers (at least theoretically) control both native and domestic stock? How will neighbour’s management goals affect each other? In this section theoretical models are used to explore these questions.
The initial models deal only with herbivores. It is used to investigate different harvest regimes, the effect of mobile herbivores, national parks and neighbouring commercial properties. These models can help explain some of the dynamics in the system, laying the foundation for the plant-herbivore system. The models are then expanded to include vegetation and its effect on the system. When modelling a plant-herbivore system the type of model used will affect the results and therefore conclusions. For ease of comparison between species dry sheep equivalents will be used to measure the population of each herbivore. The models are for a square kilometre of semi-arid grazing land. The models use differential models with both continuous time and population, on an average $km^2$ representative of the system.

The notation and definition to be used in the plant-herbivore models is collated in Table 4.1.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>The set of animal species use in the models.</td>
</tr>
<tr>
<td>$c, k, s$</td>
<td>The notation for cattle, kangaroo, and sheep respectively.</td>
</tr>
<tr>
<td>$N_i$</td>
<td>The population of species $i \in S$ in $dse/km^2$.</td>
</tr>
<tr>
<td>$V$</td>
<td>The amount of vegetation, the total standing dry matter in $kg/km^2$.</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>The maximal growth rate of species $i \in S$.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>The initial regrowth rate of the total standing dry matter.</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>The harvest rate for this species.</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>The minimum population level before harvesting occurs for the species.</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>The constant that converts harvest rate from the no minimum population limit case to the limit case.</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>The carrying capacity of herbivores or total standing dry matter per $km^2$.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The proportion of the carrying capacity used in the region surrounding the property.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>The parameter controlling the speed at which kangaroos transfer between regions.</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>The saturation rate of grazing for herbivores in $kg/dse/year$.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The half-saturation constant (amount of available vegetation where herbivores intake is halved) $kg/km^2$.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>The vegetation to herbivore conversation rate in $dse/kg$.</td>
</tr>
<tr>
<td>$\chi$</td>
<td>The zero population growth herbivore consumption rate in $kg/dse/year$.</td>
</tr>
</tbody>
</table>

*Table 4.1:* Symbols used in the herbivore model.
4.1 Modelling a Herbivore Population

Consider a herbivore population whose growth rate is affected by the carrying capacity of the region which they inhabit. An initial logistic model is constructed to model the population of a herbivore. Obviously the population is limited by factors such as area and availability of forage and water. Movement of herbivores to and from the region will be considered. As well as this, several harvesting regimes will be analysed.

4.1.1 A Herbivore in Isolation

It is widely known (Clark, 1990) that given the formulation for the change in population as

\[
\frac{dN}{dt} = \nu N \left(1 - \frac{N}{\kappa}\right) - H(N)
\]  

(4.1)

where, \(H\) is the amount harvested, then the maximum sustainable yield (MSY) will be achieved when

\[
N = \frac{\kappa}{2}
\]  

(4.2)

Typically the harvest is either a constant \((H_1(N) = h)\) or a proportion of current population \((H_2(N) = \gamma N)\). In this section these two options as well as a third option are explored. The third option \((H_3)\) reflects that under certain conditions it is not viable for harvesters to seek kangaroos, one of the herbivores of interest (see Equation 4.3). Benefits of the \(H_3\) formulation for harvesting are: that it reverts to equilibrium quicker after perturbation (i.e. it is more stable) than \(H_1\) or \(H_2\) as shown later; and that the likelihood of local extinction is reduced,
as below the threshold \( (\eta) \), no harvesting takes place (Maynard Smith, 1974; McNair, 1986), effectively providing a refuge for the herbivores. Given that the government control harvesting it is an important consideration, as it is within the governments power to halt macropod harvesting (Office of the Queensland Parliamentary Counsel, 2010).

\[
H_3(N) = \gamma^\dagger \max\{N - \eta, 0\}
\] (4.3)

The harvest formulation from Equation 4.3 retains the property that the MSY occurs at half the carrying capacity when \( \gamma \) is chosen as to maximise the sustainable yield and \( 0 < \eta < \frac{\kappa}{2} \) (see Appendix A.1). At this point it is noted that the optimal harvests (assuming \( N > \eta \)) are equal for all three regimes and an arbitrary choice for \( \gamma^\dagger \). Let \( \gamma^{\dagger*} = \varsigma \gamma^* \), where \( \varsigma \geq 1 \) is an arbitrary scaling parameter, then with the previous sentence,

\[
H_2^*(N^*) = H_3^*(N^*)
\]

\[
\gamma^* N^* = \gamma^{\dagger*} (N^* - \eta^*)
\]

\[
\gamma^* N^* = \varsigma \gamma^* (N^* - \eta^*)
\]

\[
\eta^* = \frac{N^* \varsigma - 1}{\varsigma}
\] (4.4)

inferring that \( \gamma^{\dagger*} \) and \( \eta^* \) would be an optimal harvesting regime under \( H_3 \), given \( \gamma^* \) is the optimal value under harvesting regime \( H_2 \). When \( \varsigma = 1 \) then the variable harvesting regimes are equivalent \( (H_2 = H_3) \). In general, the effect of \( \varsigma \) is to control the speed at which the system is returned to equilibrium. The larger \( \varsigma \) the steeper the return gradient. However, in practice, if \( \varsigma \) is too large it
4.1 Modelling a Herbivore Population

Harvest equation \( N^* \) \( H^* \) \( \gamma^* \) \( \eta^* \) Effect of perturbation of \( \varepsilon \) from \( N^* \)

\[ h \times \text{(constant)} \]
\[ \frac{\kappa}{2} \quad \frac{\kappa \nu}{4} \quad \frac{\nu}{2} \quad \frac{\nu \varepsilon}{2} \quad \frac{-\nu \varepsilon}{2} \]

\[ \gamma N \]
\[ \frac{\kappa}{2} \quad \frac{\kappa \nu}{4} \quad \frac{\nu}{2} \quad \frac{\nu (\kappa + 2 \varepsilon)}{2\kappa} \]

\[ \gamma \max\{N - \eta, 0\} \]
\[ \frac{\kappa}{2} \quad \frac{\kappa \nu}{4} \quad \frac{\nu \varsigma}{2} \quad \frac{\nu (\varsigma - 1)}{2\varsigma} \quad \frac{\nu (\kappa^2 - 4 \varepsilon^2 - 2\kappa \varsigma \max\{0, \varepsilon + \frac{\kappa}{2\varsigma}\})}{4\kappa} \]

Table 4.2: Maximum sustainable yield properties for the three different harvesting regimes.

can cause instability as it can be prone to "overshooting" the equilibrium. The values \( H^* \), \( \gamma^* \) and \( \eta^* \) that maximise the different harvest equations are given in Table 4.2.

Looking at the linearisation of each differential equation it is clear that the variable harvesting regimes are stable to perturbation \( (\varepsilon) \) from the equilibrium, as long as \( \varepsilon > -\frac{\kappa}{2} \). The constant harvesting regime is only stable if \( \varepsilon > 0 \). Furthermore, it can be shown that harvesting of the form in Equation 4.3 reverts back to the equilibrium quicker than the proportional harvest regime, which is in turn quicker than the constant harvest regime (when it is stable). Figure 4.1 illustrates this point clearly.

The value of \( \varsigma \) will directly affect the speed at which the population returns to equilibrium. The larger \( \varsigma \), the quicker it takes to regain equilibrium (as shown in Figure 4.1). However, if \( \varsigma \) is too large, then there is a possibility that the system will fluctuate if the carrying capacity was to vary. From this point forward the constant harvesting regime will be ignored, due to its instability to perturbations.
4.1 Modelling a Herbivore Population

It is also clear that \( H_2(N) \equiv H_3(N) \) when \( \varsigma = 1 \), hence, effectively \( H(N) = H_3(N) \) hereafter.

4.1.2 Defining Herbivore Migration

As mentioned previously (Section 1.2.1) kangaroos can move freely between private property, national parks and other areas. For the purposes of the thesis, the movement from one region to another will be referred to as migration. The ability to migrate between property boundaries requires the addition of an extra term in the population rate of change of kangaroos. It has been noted that the primary influence on kangaroo density in a spatial context is food availability (McAlpine et al., 1999; Fukuda et al., 2009). Assume that the theory of ideal free distribution (IFD) (Fretwell and Lucas Jr., 1969) holds in this scenario. The IFD implies that animals will move between areas so that the ratio of animals to carrying
capacity in each area will be equal. Works by Coulson (2009); Wiggins et al. (2010) concluded that the macropods in their study tended to follow the IFD. For the purposes of the thesis the net movement will be referred to as migration, even though it may not be strictly migratory behaviour. The equation for the migration of kangaroos to the property and external areas would take the form,

\[
\text{Migration} = \frac{1}{\tau} \frac{\kappa N_{\text{ext}} - \kappa_{\text{ext}} N}{\kappa + \kappa_{\text{ext}}}
\]

(4.5)

where \(\tau\) controls the speed at which the population moves between regions, \(N_{\text{ext}}\) and \(\kappa_{\text{ext}}\) are the population and carrying capacity for external areas respectively (see Appendix A.2 for a proof of this equation).

4.1.3 A Herbivore on a Property Bounded by a National Park

For the sake of simplicity the first scenario considered is where the kangaroo population external to the property is very large (say a large national park or other unharvested kangaroo population) and therefore the actions of an individual property owner does not affect the external population. In effect it can be considered that the external carrying capacity is infinite and that the ratio of
kangaroos to carrying capacity is $\phi \left( = \frac{N_{\text{ext}}}{k_{\text{ext}}} \right)$. Hence, Equation 4.5 becomes,

$$\lim_{k_{\text{ext}} \to \infty} \text{Migration} = \lim_{k_{\text{ext}} \to \infty} \frac{1}{\tau} \frac{k_{\text{ext}} N_{\text{ext}} - k_{\text{ext}} N}{k + k_{\text{ext}}}$$

$$= \lim_{k_{\text{ext}} \to \infty} \frac{1}{\tau} \frac{k_{\text{ext}} N_{\text{ext}} - k_{\text{ext}} N}{k_{\text{ext}} + k_{\text{ext}}}$$

$$= \lim_{k_{\text{ext}} \to \infty} \frac{1}{\tau} \frac{\phi k - N}{k_{\text{ext}} + 1}$$

$$= \frac{\phi \kappa - N}{\tau}$$ \hspace{1cm} (4.6)

Therefore combining Equations 4.1, and 4.6 results in,

$$\frac{dN}{dt} = \nu N \left( 1 - \frac{N}{\kappa} \right) - H(N) + \frac{\phi \kappa - N}{\tau}$$ \hspace{1cm} (4.7)

The solution that maximises harvest in the scenario described by Equation 4.7 result in the following MSY,

$$H^* = \begin{cases} \frac{\kappa \phi}{\tau}, & \nu \tau \leq 1 \\ \frac{\kappa (1 + \nu^2 \tau^2 + 2 \nu \tau (2 \phi - 1))}{4 \nu \tau^2}, & \nu \tau > 1 \end{cases}$$ \hspace{1cm} (4.8)

The condition $\nu \tau \leq 1$ can be interpreted as when the intrinsic growth is less than or equal to the fraction of herbivores that migrate. That implies that local population is dominated by the external population. Hence, the harvest rate is the quantity of herbivores that would migrate given no herbivores on the property. Therefore, the landholder’s best option is to de-stock completely and concentrate purely on harvesting, allowing as many herbivores to migrate as possible. This
4.1 Modelling a Herbivore Population

scenario is highly unlikely to be the case in reality. The other part of the equation is when the intrinsic growth rate is greater than the fraction of herbivore migration.

Focussing on \( \nu \tau > 1 \) it can again be shown that the harvest regime is stable when \( \varepsilon > -\frac{\kappa}{2} \). The equilibrium point for the harvesting regime is,

\[
N^* = \begin{cases} 
\frac{\kappa(\nu \tau - 1)}{2 \nu \tau}, & \nu \tau > 1 \\
0, & \nu \tau \leq 1 
\end{cases}
\] (4.9)

The optimal harvest rate and minimum harvest level for the variable harvest regimes are,

\[
\gamma^* = \frac{1 + \nu^2 \tau^2 + 2 \nu \tau (2 \phi - 1)}{2 \tau(\nu \tau - 1)} \quad (4.10)
\]

\[
\eta^* = \frac{\kappa(\nu \tau - 1)(\varsigma - 1)}{2 \nu \tau \varsigma} \quad (4.11)
\]

It should be noted that when infinite migration is included, the optimal harvesting solution (when \( \nu \tau > 1 \)) results in the stocking level \( N^* \) to be greater than when the property is considered to be isolated. This fact is easily verified given,

\[
\frac{\kappa(\nu \tau - 1)}{2 \nu \tau} = \frac{\kappa}{2} - \frac{\kappa}{2 \nu \tau} < \frac{\kappa}{2}
\]

which shows that the difference in stocking rate under infinite migration is \( \frac{\kappa}{2 \nu \tau} \) lower than under isolation. As \( \tau \) (and hence \( \nu \tau \)) increases, the optimal harvest and optimal herbivore population under infinite migration tend towards their respective solutions under isolation (Equations 4.12 and 4.13). These ideas are
4.1 Modelling a Herbivore Population

represented visually in Figure 4.2.

\[
\lim_{\tau \to \infty} \frac{\kappa (1 + \nu^2 \tau^2 + 2 \nu \tau (2 \phi - 1))}{4 \nu \tau^2} = \frac{\kappa \nu^2 \tau^2}{4 \nu \tau^2} = \frac{\kappa \nu}{4} \tag{4.12}
\]

\[
\lim_{\tau \to \infty} \frac{\kappa (\nu \tau - 1)}{2 \nu \tau} = \frac{\kappa \nu \tau}{2 \nu \tau} = \frac{\kappa}{2} \tag{4.13}
\]

Figure 4.2: The optimal herbivore population and harvest rate under the migration from an infinite external population is compared to the optimal herbivore population and harvest rate under isolation (no migration).

Furthermore, it can be shown that, whilst

\[
\phi > \frac{2 \nu \tau - 1}{4 \nu \tau}
\]

the optimal harvest is larger during infinite migration compared to no migration. This is an extreme case, where the external world has effectively infinite population willing to migrate to the property. This is realistic for a property that is relatively small compared to a large national park on its boundary.
4.1.4 A Herbivore on Two Properties with Common Boundary

The next scenario involves two properties with porous borders. Of interest is the impact that the properties can have on each other. Assume that Property 1 uses an optimal harvest regime at all times. Can the management regime of Property 2 have a detrimental effect, intentional or not, on the other property? If it does, then resistance to change may increase. The first Property 2 regime analysed is a constant stocking rate. That will give the equation of interest as follows,

$$\frac{dN_1}{dt} = \nu N_1 \left(1 - \frac{N_1}{\kappa}\right) - H(N_1) + \frac{N_2 - N_1}{2\tau} \quad (4.14)$$

The solution to the optimisation of $H(N_1)$ and $N_1^*$ are given in Equations 4.15 and 4.16, while the harvest parameters under the variable harvest regimes are in given in Equations 4.17 and 4.18. Similar to Section 4.1.3, when the herbivores that migrate dominate the internal growth on Property 1 ($2\nu\tau \leq 1$) then the individuals that migrate are harvested $\left(\frac{N_2}{2\tau}\right)$. If that were the case then Property 1’s harvest would rely purely on Property 2’s stocking level. For the rest of this
4.1 Modelling a Herbivore Population

section it is assumed that \(2\nu\tau > 1\).

\[
H^* = \begin{cases} 
\frac{\kappa(2\nu\tau - 1)^2 + 8\nu\tau N_2}{16\nu\tau^2}, & 2\nu\tau > 1 \\
\frac{N_2}{2\tau}, & 2\nu\tau \leq 1
\end{cases}
\] (4.15)

\[
N_1^* = \frac{\kappa(2\nu\tau - 1)}{4\nu\tau}
\] (4.16)

\[
\gamma_1^* = \frac{\kappa(2\nu\tau - 1)^2 + 8\nu\tau N_2}{4\kappa\tau(2\nu\tau - 1)}
\] (4.17)

\[
\eta^* = \frac{\kappa(2\nu\tau - 1)(\varsigma_1 - 1)}{4\nu\tau\varsigma}
\] (4.18)

The equilibrium points are stable for variable harvest regimes. The partial derivative of the growth rate is negative, when the equilibrium solutions are substituted in, as seen in Equation 4.19. Equation 4.19 clearly shows that as \(\varsigma\) increases the system returns to equilibrium faster.

\[
\frac{\partial}{\partial N_1} \left( \frac{dN_1}{dt} \right) \bigg|_{N_1 = N_1^*} = -\frac{\varsigma}{4} \left( \nu \left( \frac{8N_2}{\kappa(2\nu\tau - 1)} + 2 \right) - \frac{1}{\tau} \right)
\] (4.19)

From Equation 4.16 it is clear that the optimal stocking level for Property 1 does not depend on Property 2’s stocking level. Therefore Property 1 should maintain the same population of herbivores, only altering their harvesting parameters as Property 2 changes their herbivore population. It can also be seen that as in Section 4.1.3 the optimal stocking level under migration is smaller than if the property was isolated. While the \(N_1^*\) may be less than \(N^*\), the harvest is greater, given that Property 2 does not reduce their stocking level below \(\frac{\kappa}{2} - \frac{\kappa}{8\nu\tau}\) (see Appendix A.3 for proof). This can be interpreted as, whilst Property 2 has a population of herbivores close to or above half the carrying capacity
(\(N_2 \geq \frac{\kappa}{2} - \frac{\kappa}{8\nu\tau}\)) then Property 1 can attract herbivores over the porous borders. Even though the Property 1 herbivore level is constant (\(N_1^* = \frac{\kappa}{2} - \frac{\kappa}{4\nu\tau}\)) their harvest will increase as Property 2 increases its herbivore population.

So far in this section only a constant herbivore population on Property 2 has been investigated, implicitly assuming their management regime could maintain that population and they had no interest in maximising their own harvest. The next scenario is where both properties are dynamically linked. Hence, the equations governing the system are Equation 4.14 and 4.20,

\[
\frac{dN_2}{dt} = \nu N_2 \left(1 - \frac{N_2}{\kappa}\right) - H(N_2) - \frac{N_2 - N_1}{2\tau}
\] (4.20)

Assume that the land holder of Property 1 knows what is happening on Property 2 and vice-versa (perfect information in Game Theory parlance). The harvest on Property 1 while at equilibrium results in the following harvest,

\[
H(N_1) = \frac{\kappa N_2 - 2\nu\tau N_1^2 + \kappa N_1 (2\nu\tau - 1)}{2\kappa\tau}
\] (4.21)

given one of the following conditions hold,

\[
N_1 = N_2 \text{ or } \left(0 < N_j < N_i < \kappa \text{ and } \nu > \frac{\kappa(N_i - N_j)}{2\tau N_i (\kappa - N_i)}\right), i, j = 1, 2, i \neq j
\] (4.22)

This leaves us with two scenarios, either the properties regimes are equivalent (cooperation) or one property has a smaller stocking level (competition). If the
4.1 Modelling a Herbivore Population

regimes are equal, then it is clear that,

\[ H(N_1) = \frac{\nu N_1(\kappa - N_1)}{\kappa}, N_1 = N_2 \]  \hspace{1cm} (4.23)

which, when optimised \(w.r.t.\) \(N_1\), gives,

\[ H(N_1^*) = \frac{\nu \kappa}{4} \]  \hspace{1cm} (4.24)

\[ N_1^* = \frac{\kappa}{2} \]  \hspace{1cm} (4.25)

\[ \gamma_1^* = \frac{\nu}{2} \]  \hspace{1cm} (4.26)

\[ \eta_1^* = \frac{\kappa(\varsigma - 1)}{2\varsigma} \]  \hspace{1cm} (4.27)

Obviously an analogous result holds for Property 2. It should be noted that this \(MSY\), population and regime parameters are the same as in the isolation case from Section 4.1.1. This is to be expected, as when both properties are using identical regimes, there will be no net migration, so in effect it would be as if they were in isolation. If the properties do not set their populations to be equal then Property 1 would maintain their herbivore population at the level suggested in Equation 4.16. Substituting this into Equation 4.20 then Property 2’s harvest rate is given by

\[ H(N_2) = \frac{\kappa(2\nu \tau - 1)}{8\nu \tau^2} + N_2 \left( \nu - \frac{1}{2\tau} \right) - \frac{\nu N_2^2}{\kappa} \]  \hspace{1cm} (4.28)

It can be shown that Equation 4.28 is non-negative whilst

\[ N_2 \leq \frac{\kappa}{2} + \frac{\kappa(\sqrt{4\nu^2 \tau^2 - 1} - 1)}{4\nu \tau} \]  \hspace{1cm} (4.29)
4.1 Modelling a Herbivore Population

and therefore this condition limits the stable population on Property 2 whilst Property 1 is using their competitive strategy. If $N_2$ is greater than the upper limit of Equation 4.29 the herbivores are migrating quicker than they are being born. If Property 1 is using their optimal competitive strategy, maximising Equation 4.28 w.r.t. $N_2$ gives Property 2’s optimal competition strategy,

\[ H(N_2^*) = \frac{\nu \kappa}{4} - \frac{\kappa}{16 \nu \tau^2} \tag{4.30} \]

\[ N_2^* = \frac{\kappa (2 \nu \tau - 1)}{4 \nu \tau} \tag{4.31} \]

Under competition it is clear from Equation 4.30 that Property 2 has a reduced harvest when compared to the cooperative case. Furthermore, as shown in Appendix A.3 if Property 2 uses $N_2 = N_2^*$ then Property 1’s harvest will also be inferior to the equal density case. Moreover, it can be shown that $N_1 = N_2 = N_2^*$ is a Nash Equilibrium Point (NEP) for the competitive game (Appendix A.4). Hence, the properties co-operating and maintaining equivalent herbivore populations produces the optimal strategy. The effect of cooperative and competitive strategies (where Property 2 acts first) are illustrated in Figure 4.3. Their individual harvests will be greater than if they try and compete with each other. In effect, with competition they are both trying to entice herbivore immigration whilst deterring emigration resulting in lower herbivore populations and harvests. Ironically, the populations and harvests are equal to each other, but lower than under cooperation. As would be expected, as the amount of migration decreases to approach no migration ($\tau \to \infty$) the competitive solutions approach the cooperative (and therefore isolation) solution.

Alternatively, it could be thought of as one property and the decision is
Figure 4.3: The harvest rates for Property 1 and 2 under competition as well as cooperative as Property 2 changes their herbivore population as a fraction of $\kappa$. The optimal harvest rate under isolation is included for comparison.

whether to have one large paddock (cooperative case) or two smaller paddocks with different herbivore populations (competitive case). The combined harvest rate for the separate paddocks, each with carrying capacity $\kappa$, is the sum of Equations 4.15 and 4.30, and is equal to:

$$\frac{\nu\kappa}{4} - \frac{\kappa}{16\nu\tau^2} + \nu N - \frac{\nu N^2}{\kappa}. \quad (4.32)$$

Equation 4.32. The single large paddock harvest rate will just be double that of Equation 4.24. Here $N$ represents the herbivore population with regards to a second separate paddock and half the single large paddock, akin to Property 2. The harvest rate for the single paddock is larger when

$$\frac{\kappa(2\nu\tau - 1)}{4\nu\tau} < N < \frac{\kappa(2\nu\tau + 1)}{4\nu\tau}.$$  

Since both arrangements maximise when $N = \frac{\kappa}{2}$, which is inside the interval, the single large paddock is superior (as illustrated in Figure 4.4).
4.1 Modelling a Herbivore Population

Figure 4.4: The harvest rates for combined separate (competitive) paddocks and the single large paddock (cooperative) as Property 2 changes their herbivore population as a fraction of $\kappa$. 

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**Figure 4.4:** The harvest rates for combined separate (competitive) paddocks and the single large paddock (cooperative) as Property 2 changes their herbivore population as a fraction of $\kappa$. 

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4.2 Modelling Two or More Herbivore Populations

Consider two or more herbivore populations whose growth rates are affected by the carrying capacity of the region in which they inhabit. The resources that limit the carrying capacity are now utilised by each herbivore in competition or facilitation. As particular interest is in the interaction between cattle, sheep and kangaroos on a commercial property, the herbivores are measured in the standardised units $dse$ as typically used in Australia. The growth components of each differential equation take the form of the well known competitive Lotka-Volterra equations. The harvesting regimes are the same as those used in Section 4.1.2, with harvesting occurring only when the given population is above a predetermined mark. These considerations result in the following differential equation to be used for each herbivore’s population growth;

$$\frac{dN_i}{dt} = \nu_i N_i \left(1 - \sum_{j \in S} \frac{N_j}{\kappa}\right) - \gamma \max\{N_i - \eta_i, 0\} + \text{Migration Rate}_i \quad (4.33)$$

where the migration rate is set to zero when migration is not possible. That formulation (Equation 4.33) implies that we are considering a scenario where there are no niches. Gause’s exclusion principle or the principle of competitive exclusion states that species occupying the same niche cannot coexist (Gause, 1934; Hardin, 1960). However, more recent ideas have called this ”Law of Ecology” into doubt, allowing for species co-existence without niche separation (Rastetter and Ågren, 2002).
4.2 Modelling Two or More Herbivore Populations

With different species in the model, the task of maximising becomes more difficult. In particular, what are we going to maximise, the total number of dse, the weight harvested, or the revenue created? These questions are better answered in Chapters 2 and 6. Suffice to say, that without a migratory herbivore, analytically the two herbivores interact similarly, with analogous solutions. In turn, which ever metric is optimised, you could think of it as allocating a proportion of the available total standing dry matter to each herbivore.

If the idea of allocating a proportion of the available forage to each given species is possible, then the species need to be able to co-exist. Hence, it needs to be shown that the species can co-exist in predetermined proportions, with and without the influence of migration.

4.2.1 Two or More Herbivores in Isolation

This scenario examines a property, with two or more herbivore species grazing on the same plants. The herbivores are restricted to the property, as there is no migration. If each species had the same maximal growth rate (\(\nu_i\)) then they could be considered as one species for population density purposes. It would then follow that the overall harvest would be optimised in the same way it was optimised in Section 4.1.1. This is due to the fact that under the optimal harvest regime no one species could outgrow another species. The herbivore population size of each herbivore (\(N_i^*\)) would be the optimal overall population size (\(N^*\)) multiplied by the proportion of carrying capacity allocated to species \(i\), \(\omega_i\), where \(\sum_{i \in S} \omega_i = 1\). The optimal minimum harvest levels (\(\eta_i^*\)) would also be in the same proportions of \(\eta^*\). The proportional harvest rates for each species would be identical, \(\gamma^*\).
4.2 Modelling Two or More Herbivore Populations

Hence, the property managers desired allocation could be achieved by setting the minimal harvest levels for each herbivore as such,

$$\eta_i = \omega_i \eta^*$$  \hspace{1cm} (4.34)

In reality, it is unlikely that the population dynamics of each species was identical.

In the more realistic case where the maximal growth rates of each herbivore are not equal, similar allocations may be possible for either the overall population or the overall harvest. If the desire is to have the overall population proportions allocated via $\omega_i$ then Equation 4.34 should still be followed. Due to the different maximal growth rates, the amounts harvested will not be in the same ratio. It is most probably the amount of harvest that is of importance for meat production. To make sure the harvest amounts are in the correct proportions, the fraction of the optimal density and the minimal level for harvesting would be determined by,

$$\omega_i = \frac{\omega^\dagger_i \prod_{j \in S, j \neq i} \nu_j}{\sum_{k \in S} \left( \omega^\dagger_k \prod_{j \in S, j \neq k} \nu_j \right)}$$  \hspace{1cm} (4.35)

where $\omega^\dagger_i$ is the desired proportion of the overall harvest to come from herbivore group $i$. This allocation structure maintains the optimal utilisation of the carrying capacity as determined in Section 4.1. Making the following substitutions (Equations 4.36 to 4.38) into Equation 4.33 shows that it is an equilibrium
4.2 Modelling Two or More Herbivore Populations

solution and stable.

\[ N_i = \frac{\kappa \omega_i}{2} \]  \hspace{1cm} (4.36)
\[ \gamma_i = \frac{\nu_i \zeta}{2} \]  \hspace{1cm} (4.37)
\[ \eta_i = \frac{\kappa (\zeta - 1) \omega_i}{2\zeta} \]  \hspace{1cm} (4.38)

Therefore, in a closed system, if the desired proportion (either of overall population or of amount harvested) are known, then the solution given above will utilise the carrying capacity optimally. To illustrate this Figure 4.5 shows the effect (over time) on three theoretical herbivore populations. The herbivores represent a collection of species with high, moderate and low maximal growth rates. The equilibrium solutions under the two methods of proportioning give very different results. These differences are most notable between the high and low maximal growth rate herbivores. Note for instance that when the target is the harvest proportions the population of the "low" group is greater than the others (Figure 4.5b), even though it has the lowest harvest proportion (Figure 4.5d) and vice-versa for the "high" group. Please note this is not the same as optimising the revenue, profit or some other objective. It is optimising the amount the system can produce sustainably.
Figure 4.5: An illustration of the effect of different proportions for the population and harvest over time. The proportions are 0.3, 0.5 and 0.2 for the high, moderate and low maximal growth rate herbivores respectively.
4.2 Modelling Two or More Herbivore Populations

4.2.2 Two or More Herbivores on a Property Bounded by a National Park

Now consider a system where one herbivore can migrate to and from the property and the external world, but others are bounded by the borders of the property. By looking at the scenario of a small property bounded by a large national park the extremes can be analysed. Of particular interest is when the captive herbivores are dominated by the region external to the property. Initially consider the case where there are only two herbivores, the first captive, and the second mobile. The equations governing this system will then be of the form of Equation 4.33 where the migration rates for the two herbivores are,

\[
\text{Migration Rate}_1 = 0 \quad (4.39)
\]

\[
\text{Migration Rate}_2 = \begin{cases} 
\frac{\phi \kappa - (N_1 + N_2)}{\tau} & N_2 \geq 0 \text{ OR } N_1 \leq \phi \kappa \\
0 & \text{otherwise}
\end{cases} \quad (4.40)
\]

respectively. The alteration to the second migration rate from that used in Section 4.1.3 is due to the fact that you cannot have the mobile herbivore leave the property when there are none left on the property. It can then be shown that maximising the sustainable harvest rate of the captive herbivore can be achieved under the following conditions:

1. The mobile herbivore’s population on the property is zero and the density of the captive herbivore is equal to the external density.

OR
2. The population of the mobile herbivore is less than the carrying capacity and the harvest rate for the mobile species must be less than the rate at which the mobile species would grow in the absence of the captive species.

AND EITHER

- The external density is greater than the internal density of mobile herbivores AND EITHER the external density is less than one OR the harvest rate of the mobile herbivore is greater than the migration rate would be if the property was at the carrying capacity.

OR

- The external density is one or more AND the harvest rate of the mobile herbivore is greater than the migration rate would be if the property was at the carrying capacity.

OR

- The density of the mobile herbivore on the property is greater than the external density AND the mobile herbivore is harvested.

If the first of these conditions are met then the system acts as if there is no migration as internal and external densities are equal. In reality this is unlikely to happen, as the carrying capacity would not be constant, and the herbivores are likely to have different growth rates. Of the second set of conditions the most likely conditions to exist are when the external density is greater than the internal density of mobile herbivores. An external density less than the carrying capacity is plausible, if not expected under normal circumstances. Also a harvest rate for
the mobile herbivore being greater than the migration rate when the property is at carrying capacity means that the mobile harvest is high enough to stop the property being dominated by the mobile herbivore. The following is the harvest regime for the optimal captive herbivore harvest.

\[
H(N_1^*) = \begin{cases} 
\frac{\nu_1 \phi \kappa (1 - \phi)}{\nu_1 (\gamma_2 N_2 \tau + \kappa (1 - \phi)) (\nu N_2^2 \tau + N_2 \kappa (1 + \gamma_2 \tau - \nu_2 \tau) - \kappa^2 \phi)} \\
\frac{\nu_1 \phi \kappa (1 - \phi)}{(\kappa + \nu_2 N_2 \tau)^2} 
\end{cases}, \quad \text{Condition 1}
\]

\[
N_1^* = \begin{cases} 
\frac{\kappa \phi}{\kappa^2 \phi - N_2 (\nu_2 N_2 \tau + (1 + \gamma_2 \tau - \nu_2))} \\
\frac{\kappa \phi}{\kappa + \nu_2 N_2 \tau} 
\end{cases}, \quad \text{Condition 1}
\]

\[
\gamma_1^* = \begin{cases} 
\frac{\nu_1 (1 - \phi)}{\nu_1 (\gamma_2 N_2 \tau + \kappa (1 - \phi))} \\
\frac{\nu_1 (1 - \phi)}{\kappa + \nu_2 N_2 \tau} 
\end{cases}, \quad \text{Condition 2}
\]

The effect of the external density \((\phi)\) and the speed of migration \((\tau)\) are illustrated in Figures 4.6 and 4.7 respectively. It is noted that: when the external density is much lower than the internal density there is no internal mobile population as shown in Figure 4.6; after the external density exceeds the "no mobile species internally" threshold, the captive population decreases, while the mobile population increases as shown in Figure 4.6 and; when the external density exceeds the "mobile population dominates the internal dynamics" threshold, migration outpaces harvesting as shown in Figure 4.7. The "no mobile species internally" threshold is likely to be exceeded unless the landholder is severely
over-stocking the captive species, while the "mobile population dominates the internal dynamics" threshold is not likely to be exceeded unless the property is either severely under-stocked.

![Figure 4.6: The stable populations of two herbivores (the first captive the second mobile) as the density of the external mobile herbivore changes.](image)

### 4.2.3 Two or More Herbivores on Properties with Common Boundaries

Attention is now turned to the other migration scenario. Consider two properties with common boundaries and three herbivores, only the third of which is mobile. The migration rate of the mobile herbivore onto Property 1 is given by,

\[
\text{Migration Rate}_{1,3} = \begin{cases} 
\frac{N_{2,1} + N_{2,2} + N_{2,3} - (N_{1,1} + N_{1,2} + N_{1,3})}{2\tau} & \text{Condition 3} \\
0 & \text{otherwise}
\end{cases}
\]

\[(4.45)\]
4.2 Modelling Two or More Herbivore Populations

![Graph](image)

**Figure 4.7:** The harvest of two herbivores (the first captive the second mobile) and the migration rate, as the density of the external mobile herbivore changes.

where

\[ \text{Condition 3} \equiv N_{j,3} \geq 0 \text{ AND } N_{3-j,1} + N_{3-j,2} + N_{3-j,3} < N_{j,1} + N_{j,2} + N_{j,3}, j \in \{1, 2\} \]

Essentially this only allows emigration from Property 1 when mobile herbivores are present and immigration to Property 1 if they are present on Property 2. An analogous equation to Equation 4.45 is used for Property 2.

Through similar arguments to Section 4.1.4 the cooperative game arrives at solutions for optimal use of the carrying capacity as the isolation case. Hence, as long as the optimal strategies given in Section 4.2.1 are used, then the property owners can set their own internal proportions of each herbivore species. Therefore, the decisions of which species to stock at which levels will not affect the other property under cooperation. As an illustration, Figure 4.8 shows a scenario where the properties work in cooperation. Using the Equations 4.35, 4.37 and 4.38 the ratio of each herbivore are 5:3:2 and 3:6:1 for Property 1 and 2 respectively.
4.2 Modelling Two or More Herbivore Populations

The initial boundary conditions are non-optimal, however, the system is stable, converging to the desired solutions.
Figure 4.8: An illustration of two properties in cooperation stocking three herbivore species. Two are captive and one mobile herbivore are used. Initially the grazing is not optimal, but optimal harvest regimes are used.
4.3 Modelling Vegetation and a Herbivore Population

So far the models have used a constant carrying capacity. However, the number of animals an area can sustain depends (to some degree) on the availability of vegetation. Does having the herbivore population dependent on the available vegetation impact on the previous conclusions? Therefore the next models of interest involve herbivores and the available forage or $tsdm$. As discussed in Turchin (2003) and Rockwood (2009), herbivore grazing can follow two standard forms; grazing the whole plant, including roots; or grazing on a plant where part of the plant has refuge, invulnerable to grazing. It can be argued that the grazing on grasses by cattle, sheep and kangaroos falls into the latter category. Therefore, a standard set of equations for modelling the dynamics of a grazing system is an initially linear growth model for the forage (Equation 4.46), with herbivore growth dependent on the availability of forage with a Type II functional response (Equation 4.47). Together these equations are known as the herbivory-regrowth model (Turchin, 2003).

\[
\frac{dV}{dt} = \nu \left(1 - \frac{V}{\kappa V}\right) - \frac{\zeta V N}{\theta + V} \quad (4.46)
\]

\[
\frac{dN}{dt} = \xi N \left(\frac{\zeta V}{\theta + V} - \chi\right) \quad (4.47)
\]
4.3 Modelling Vegetation and a Herbivore Population

4.3.1 Vegetation and a Herbivore in Isolation

As in Section 4.1.1 the first scenario considered is where the herbivore is bounded by impermeable fences. Harvesting is included in the variable form \( H_3(N) \) where there is a minimum population level required prior to harvesting commencing. This results in the modification of Equation 4.47 so that it becomes,

\[
\frac{dN}{dt} = \xi N \left( \frac{\zeta V}{\theta + V} - \chi \right) - \gamma \varsigma \max\{0, N - \eta\} \tag{4.48}
\]

Maximising the harvest component whilst Equations 4.46 and 4.48 are set to zero results in an optimal solution to the isolated vegetation-herbivore system. This solution is

\[
H(N^*) = \frac{\xi v(\kappa \zeta + \theta \chi - \kappa \chi - 2\kappa \varphi_1)}{\kappa \zeta} \tag{4.49}
\]

\[
V^* = \frac{\theta \chi}{\varphi_1} \tag{4.50}
\]

\[
N^* = \frac{v(\theta \zeta + \kappa \varphi_1 - \theta(2\chi + \varphi_1))}{\kappa \zeta \varphi_1} \tag{4.51}
\]

\[
\gamma^* = \frac{\xi \varphi_1(\kappa \zeta + \theta \chi - \kappa \chi - 2\kappa \varphi_1)}{\theta \zeta + \kappa \varphi_1 - \theta(2\chi + \varphi_1)} \tag{4.52}
\]

\[
\eta^* = \frac{v(\varsigma - 1)(\theta \zeta + \kappa \varphi_1 - \theta(2\chi + \varphi_1))}{\kappa \zeta \varphi_1 \varsigma} \tag{4.53}
\]

\[
\varphi_1 = \sqrt{\frac{\theta \chi(\zeta - \chi)}{\kappa}} \tag{4.54}
\]

when \( \chi < \frac{\zeta \kappa}{\theta + \kappa} \). The condition can be interpreted as the amount of forage per herbivore to maintain the status quo must be less than the amount consumed per herbivore when the vegetation is at its carrying capacity. A perfectly logical condition. The equilibrium solution is asymptotically stable (see Appendix A.5.
for proof), as illustrated in Figure 4.9. It should also be noted from Figure 4.9 that using a minimum population level before harvesting ($\varsigma > 1$) allows the system to return to equilibrium quicker and without the large oscillations when compared to purely proportional harvesting ($\varsigma = 1$).

Figure 4.9: The population dynamics of a grazing system under two different harvesting regimes; no minimum herbivore population ($\varsigma = 1$); and a minimum herbivore population ($\varsigma = 2$). The forage, herbivore population and harvest rate under isolation (no migration) are shown for each harvesting regime.

4.3.2 Vegetation and a Herbivore with National Park Boundary

Using the $IFD$ as stated in Section 4.1.2, it is the density of the animals that is of interest. In the scenario where only carrying capacity is considered, this density was with respect to the carrying capacity. Hence, in this scenario where forage is considered, it makes sense to use the density of animals with respect to the $tsdm$. Therefore, the general migration rate (Equation 4.5) can be modified by replacing carrying capacity $\kappa$’s with $V$’s for $tsdm$. The resulting migration rate
4.3 Modelling Vegetation and a Herbivore Population

under infinite external population is,

\[
\text{Migration rate} = \frac{\phi V - N}{\tau} \quad (4.55)
\]

The inclusion of the (infinite) migration rate into Equation 4.48 gives the system of differential equations using Equations 4.46 and 4.56.

\[
\frac{dN}{dt} = \xi N \left( \frac{\zeta V}{\theta + V - \chi} \right) - \gamma \varsigma \max\{0, N - \eta\} + \frac{\phi V - N}{\tau} \quad (4.56)
\]

The overall harvest rate of the system is maximised with respect to \( N \) and \( \gamma \) when,

\[
H^*(N) = \frac{V^2(\zeta \kappa \phi + \nu \varphi_2) + \nu V(\theta + \theta \xi \tau \chi - \kappa \varphi_2) - \theta \kappa \nu(1 + \xi \tau \chi)}{\zeta \kappa \tau V} \quad (4.57)
\]

\[
N^* = \frac{V(\kappa - V)(V + \theta)}{\zeta \kappa V} \quad (4.58)
\]

\[
\gamma^* = \frac{\theta \kappa \nu(1 + \xi \tau \chi) - \nu V(\theta + \theta \xi \tau \chi - \kappa \varphi_2) - V^2(\zeta \kappa \phi - \nu \varphi_2)}{\nu \tau(\kappa - V)(V + \theta)} \quad (4.59)
\]

\[
\eta^* = \frac{V(\kappa - V)(V + \theta)(\varsigma - 1)}{\zeta \kappa \varsigma V} \quad (4.60)
\]

\[
\varphi_2 = 1 - \xi \tau(\varsigma - \chi) \quad (4.61)
\]

given combinations of conditions like: the amount of vegetation present is greater than that required for zero population growth; the rate of vegetation growth, without loss to herbivore, must be smaller than the amount of vegetation required to maintain the external population; the current population is less than or equal to the external population density. The equations that maximise the harvest under equilibrium can be obtained, but they are too large to be included here.
4.3 Modelling Vegetation and a Herbivore Population

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu )</td>
<td>0.65</td>
<td>The value used in Hacker et al. (2003) and within the range used in Caughley et al. (1987).</td>
</tr>
<tr>
<td>( \nu )</td>
<td>80</td>
<td>Calculated using formulation from Turchin (2003).</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>2000</td>
<td>Upper limit based on tsdm estimates from Caughley et al. (1987).</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.15, 0.3, 0.6</td>
<td>An average and high density using data from Dawson (1995) (kangaroos/tonne).</td>
</tr>
<tr>
<td>( \tau )</td>
<td>10</td>
<td>Estimates of roughly 10% annual migration in Viggers and Hearn (2005).</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>290</td>
<td>Ad liberum grass consumption for a 30kg kangaroo using Equation 5.14 converted to kg/year.</td>
</tr>
<tr>
<td>( \theta )</td>
<td>58.2</td>
<td>The tsdm where consumption is halved using Equation 5.14.</td>
</tr>
<tr>
<td>( \chi )</td>
<td>145</td>
<td>Calculated as half ( \zeta ) as done in Turchin (2003).</td>
</tr>
</tbody>
</table>

Table 4.3: Parameter values used in the herbivore model.

They also include similar conditions and different solutions depending on growth rate of the herbivore compared to the migration rate, similar to those found in Section 4.1.4.

To illustrate the types of solutions that occur the parameters have been estimated and used in the calculations (see Table 4.1). Please note that these values are correct in order of magnitude, and are used to enable the illustration rather than as a definitive source. The high external density was selected so that immigration did not quite dominate the property.

From Figure 4.10 it can be seen that the equilibrium solutions are affected by the harvest proportion. It is noted that as the harvest proportion increases the different variables plateau. This is due to the fact that as the proportion
harvested increases, the amount harvested converges to the migration rate, as all 
the herbivores on the property are harvested \((N \to 0)\). Similar to the results from 
Section 4.1.3, when \(\gamma\) is relatively small the herbivore population on the property 
is larger than the external density and emigration occurs (Figure 4.10d). While 
losing herbivores through emigration is not generally desirable, it is possible that 
the optimal harvesting regime may have this occur. The benefit of harvesting from 
a higher population on the property \((N)\) outweighed the leaking of the herbivores 
(Figure 4.10c). At both the low and average external densities \((\phi=0.15 \text{ and } 0.3)\) 
the optimal equilibrium harvest proportion occurred when the migration rate was 
negative (migration rate of -0.018 and -0.008 respectively).

As previously noted (Section 4.1.3), when the external density increases the 
proportion of the population harvested \((\gamma^*)\) increases (see Figure 4.11d). In par-
ticular, when the external density increases, immigration begins to dominate the 
properties’ optimal solution. The harvest increases (Figure 4.11c) by relying on 
encouraging immigration (Figure 4.11e). This is managed through lowering the 
properties herbivore density and increasing the amount of vegetation (as seen in 
Figures 4.11b and 4.11a respectively) and then effectively harvesting all immi-
grants as they enter the property. When the external density is approximately 
0.4 herbivores per tonne of forage the optimal harvesting regime results in neutral 
migration (see Figure 4.11e). When comparing the internal and external densities 
of herbivores per tonnes forage it can be seen (Figure 4.11f) that the relationship 
is not linear, but a negative convex curve, until there are no herbivores on the 
property \((N^* = 0)\). This is to be expected given the external density remains 
constant and therefore larger in relative terms when the external density is larger.
4.3 Modelling Vegetation and a Herbivore Population

Figure 4.10: An illustration of the effect of different harvesting proportions ($\gamma$) for low, average and high infinite external densities ($\phi$). Assumed parameters are given in Table 4.1.
4.3 Modelling Vegetation and a Herbivore Population

Figure 4.11: An illustration of the effect of infinite external densities ($\phi$) on the optimal harvesting regimes. Assumed parameters are given in Table 4.1
4.3 Modelling Vegetation and a Herbivore Population

So far the optimal dynamics when \( tsdm \) is considered is analogous to the straight carrying capacity case. The infinite migration scenario is useful to see some of the dynamics when a small property is bounded by a large national park.

### 4.3.3 Vegetation and a Herbivore on Two Properties with Common Boundary

The finite migration cases importance is in analysing the effect one property can have on its neighbour. Similar to Section 4.1.4 scenarios will look at optimal strategies on Property 1 given Property 2 maintains a given regime, a cooperative regime and a competitive regime. The differential equations governing the system are

\[
\frac{dV_i}{dt} = \nu \left( 1 - \frac{V_i}{\kappa V} \right) - \frac{\zeta V_i N_i}{\theta + V_i} \\
\frac{dN_i}{dt} = \xi N_i \left( \frac{\zeta V_i}{\theta + V_i} - \chi \right) - \gamma_i s_i \max\{0, N - \eta_i\} + \frac{V_i N_{3-i} - V_{3-i} N_i}{\tau (V_i + V_{3-i})}
\]

(4.62) \hfill (4.63)
Solving Equation 4.62 set to zero with respect to $N_i$ gives the equilibrium population size dependent on the vegetation available on the property (Equation 4.64).

\[
N_i = \frac{v(\kappa V - V_i)(\theta + V_i)}{\zeta \kappa V V_i} \quad (4.64)
\]

\[
\gamma_i = \frac{\varphi_{3,i} + \xi(\kappa - V_i)(\zeta V_i - \chi(\theta + V_i))}{(\kappa - V_i)(\theta + V_i)} \quad (4.65)
\]

\[
\eta_i = \frac{v(\kappa V - V_i)(\theta + V_i)(\varsigma - 1)}{\zeta \kappa V_i V_i} \quad (4.66)
\]

\[
H(N_i) = \frac{v(\varphi_{3,i} + \xi(\kappa - V_i)(\zeta V_i - \chi(\theta + V_i)))}{\zeta \kappa V_i} \quad (4.67)
\]

\[
\varphi_{3,i} = \frac{(V_{3-i} - V_i)(\theta V_i V_{3-i} - \kappa(\theta V_{3-i} + V_i(\theta + V_{3-i})))}{\tau V_{3-i}(V_i + V_{3-i})} \quad (4.68)
\]

Therefore the equilibrium herbivore population on either property can be written in terms of just their respective vegetation levels. If Property 2 maintains a given vegetation level, that implies a certain level of herbivores. Using this premise, the harvest rate for Property 1 and 2 can be found and equations for the values of $\gamma_1$, $\gamma_2$, $\eta_1$, and $\eta_2$ found in terms of $V_1$ and $V_2$ (Equations 4.65 and 4.66). Equations 4.64 to 4.66 can be used to calculate the harvest rate (Equation 4.67).

The harvest rate for Property 1 can then be maximised given the vegetation level on Property 2. Due to their length they have not been included. An illustration of the system designed to maximise Property 1’s harvest rate based on the vegetation level is captured in Figure 4.12. Note that the herbivore population on Property 1 is fairly consistent despite the vegetation level on Property 2. Also note that migration rate is generally small compared to the harvest rate. Figure 4.12 highlights the fact that under the initially linear regrowth model, when vegetation density (compared to the carrying capacity of the vegetation) is low, then herbivore population is large. Theoretically as $V_i \to 0$ then the equilibrium
solution infers $N_i \to \infty$.

\[
K_4 \frac{K_2^3}{\theta + \kappa} V^2
\]

\[
\text{Figure 4.12: An illustration of Property 1 maximising their harvest rate dependent on the vegetation level of Property 2. Assumed parameters are given in Table 4.1}
\]

Consider the scenario where both properties co-operate, in effect joining their two properties. The optimal harvesting regime for the combined property is the same as a single property in isolation for the reasons discussed in Section 4.3.1. Therefore, the optimal harvest regime for each property (under cooperation) is given in Equations 4.49 to 4.54. If the properties are in competition then the harvest rates (given perfect knowledge) for each property is given by Equation 4.67. Figure 4.13 illustrates the case where the two properties are in competition, cooperation and isolation. Using the parameterisation from Table 4.3, it can be seen that one property always has a lower harvest rate compared to the optimal cooperative strategy (Figure 4.13a). Even when the harvests of both properties are combined, cooperation provides the greatest harvest rate (Figure 4.13b). It can be shown that this holds more generally when $\chi < \frac{\zeta \kappa}{\theta + \kappa}$.
4.3 Modelling Vegetation and a Herbivore Population

![Graph of harvest rates for Property 1 and 2 under competition as well as cooperative as Property 2 changes their herbivore population. This is then compared to the optimal harvest rate under isolation.](image)

**Figure 4.13:** The harvest rates for Property 1 and 2 under competition as well as cooperative as Property 2 changes their herbivore population. This is then compared to the optimal harvest rate under isolation.
4.4 Discussion

The models used in this chapter give insight into some of the dynamics of a deterministic plant-herbivore system. It has been shown theoretically that stable solutions for plant-herbivore models, excluding and including migration, can be found. Moreover, these can be optimised in terms of their harvest, including herbivore preferences for the landholder.

When the property is isolated (herbivores are kept within its boundaries) harvests are optimised by leaving half the herbivore population untouched. The minimum level before harvesting in this scenario can be based purely on the carrying capacity and optimal growth rate of the forage. This fact causes competing needs when optimising revenue based on herbivore harvest and wool production. However, it is possible to control the herbivore populations so that a given proportion of the forage allocation is utilised by each species. Suppose a landholder wishes to allocate half their forage to wool production, two-fifths to beef production, and the remainder to kangaroo meat production. Then Equations 4.34 suggest harvest rates equal to the maximal growth rates for each species and setting refuge levels at an eighth, a tenth, and a fortieth of the carrying capacity respectively for sheep, cattle and kangaroos.

When the property is not isolated the outcomes are affected by what is happening in the external environment as well as internally. The relationship between the total $dse/km^2$ and that external environment can have a large effect on optimal strategies. Migration rates also have an impact on decision making. When neighbouring properties use the optimal harvesting strategy, their herbivore pref-
erences should not affect the neighbours. Also, under the optimal harvesting strategies it is possible to have different combinations of herbivores as desired by the landholder. These two points infer that landholders’ decisions will have limited bearing on other properties, whether they utilise kangaroos or not.

The population models used in this chapter have been useful in exploring the effect of mobile species on internal populations and stability. However, the simplifications implicit in these models used do not account for external forces on the carrying capacity or available forage. A model that does account for the effects of weather and environment is contained in Chapter 5.
Chapter 5

Population Modelling with

\textit{GRASP}

Mitigation through income from kangaroos could help alleviate some of the financial down turn during drought. With rapid increase in kangaroo numbers after a drought has broken, harvesting could financially counter restocking costs of domestic animals. Therefore the kangaroos’ ability to react to changes in weather could be used to mitigate the effect of droughts and post-drought recovery in marginal areas. For this possibility to be explored fully a model that responds to environmental factors such as weather and soil condition needs to be used. The dynamics are important. When forage is scarce, and fodder has to be bought to maintain stock, the pest value of kangaroos is at its highest. When forage is plentiful landholders are not concerned by kangaroo numbers. There is so much forage it cannot be efficiently utilised. In particular the model needs to capture some of the nuances of kangaroo ecology conveyed in Section 1.2.1.
A model to predict biomass changes over time for Australian conditions was initially developed by Neil Flood and John Carter as the AussieGRASS model in 1995. Since then it has developed into what is known as GRASP today, which is what is used to estimate available biomass. GRASP can simulate the effect of weather, soil condition, stocking rates for either cattle or sheep and the associated production of beef or wool (Littleboy and McKeon, 2005). Part of the specified conditions of use of the GRASP code was that the main program itself was not to be altered, but changes were allowed to be made via adding sub-programs that can be included or excluded as required by the users. It is via these sub-programs that we constructed our kangaroo model. Unfortunately, this limited the ability of the kangaroo model. As a result, the entire GRASP program was re-coded for use in Mathematica.

The notation used in the kangaroo population model is collated in Table 5.1.

5.1 The Kangaroo Population Model

The kangaroo population model is based on a previous physiological structured population model used in Hacker et al. (2003). It also includes the effect of the environmental conditions on the development and mortality of the kangaroo population. This enables dynamic feedback into the system so as to better simulate the effect of nutritional intake on the mammals. This approach has been used as we believe that the resilience of the plant-herbivore system is an important factor in the decision making process. If the non-traditional livestock can increase in numbers quickly after drought, their presence may help the pastoralist recover
5.1 The Kangaroo Population Model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{f,i}, N_{m,i}$</td>
<td>The population of cohort $i$ of females and males respectively.</td>
</tr>
<tr>
<td>$V$</td>
<td>The total standing dry matter (vegetation) available.</td>
</tr>
<tr>
<td>$age_i$</td>
<td>The mean age of the members of cohort $i$.</td>
</tr>
<tr>
<td>$\mu(age_i, V)$</td>
<td>The mortality rate based on the age and forage available.</td>
</tr>
<tr>
<td>$\text{Harv}(gender, age_i)$</td>
<td>The harvest rate based on the age and gender of the cohort.</td>
</tr>
<tr>
<td>$\text{Migration}(gender, age_i)$</td>
<td>The migration rate for that age and gender of the cohort.</td>
</tr>
<tr>
<td>$Wt(gender, i)$</td>
<td>Average weight of the animals in that cohort, by gender.</td>
</tr>
<tr>
<td>$g(gender, age_i, V)$</td>
<td>The function of weight gain given the forage available and the gender and age of the animal.</td>
</tr>
<tr>
<td>$b(age_i, V)$</td>
<td>The birth rate for that group, given their ages, and available forage.</td>
</tr>
<tr>
<td>$T_n$</td>
<td>The point at which the $n^{th}$ new cohort is established.</td>
</tr>
<tr>
<td>$s$</td>
<td>The primary sex ratio at birth.</td>
</tr>
<tr>
<td>$\text{Intake}(V, Wt)$</td>
<td>The function for the daily intake of forage for a member of the group, given their weight.</td>
</tr>
<tr>
<td>$\text{cond}(gender, i)$</td>
<td>The condition of the group, has a delayed effect included.</td>
</tr>
<tr>
<td>$\text{delay}$</td>
<td>The time delay for the groups condition.</td>
</tr>
<tr>
<td>$\text{satiation}$</td>
<td>The amount of available forage required for the animal to be satiated.</td>
</tr>
<tr>
<td>$\text{peaten}(t)$</td>
<td>The proportion of the total desired forage actually eaten.</td>
</tr>
<tr>
<td>$\text{desire}(t)$</td>
<td>The total amount desired to be eaten based on the available forage.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The overall harvest rate for the species.</td>
</tr>
<tr>
<td>$\text{refuge}$</td>
<td>The minimum kangaroo density.</td>
</tr>
<tr>
<td>$\text{pref}_{gender}$</td>
<td>The harvest bias for the given gender.</td>
</tr>
<tr>
<td>$H(gender, i)$</td>
<td>An indicator function for if the animal is harvestable.</td>
</tr>
</tbody>
</table>

Table 5.1: Symbols used in the kangaroo population model.
5.1 The Kangaroo Population Model

quicker financially.

The \textit{PSPMs} can be derived heuristically (de Roos et al., 1992). Consider the change in population of animals. Suppose that the population is age-structured, age is continuous and that there is competition for a dynamically varying food supply. Then, by forming equal length cohorts, based on when the animal was born, and generalising the age-structured Leslie model, the \textit{PSPMs} follows each age cohort as time passes. The population of each cohort increases via births and immigration and decreases with deaths and emigration. As each cohort is based on the age of the animal, births only affect the most recently formed cohort. New cohorts are formed after a set amount of time, usually based on the animals’ reproduction cycle. At this point the cohort numbering is changed, increased by one. This makes their cohort number represent their age in terms of the time between birth measurements. The model tracks not only the population, but also the size of each cohort. The current population is described via Equations 5.1 to 5.4. Please note that having different groups by gender allows for use of the fact that females and males are harvested at different rates and ages.

\[
\frac{dN_{f,i}}{dt} = -\mu(\text{age}_i, \text{cond}(f, i)) N_{f,i} - \text{Harv}(f, i) + \text{Migration}(f, i) \quad (5.1)
\]

\[
\frac{dN_{m,i}}{dt} = -\mu(\text{age}_i, \text{cond}(m, i)) N_{m,i} - \text{Harv}(m, i) + \text{Migration}(m, i) \quad (5.2)
\]

\[
\frac{dW_{t,f,i}}{dt} = g(f, \text{age}_i, \text{cond}(f, i)) \quad (5.3)
\]

\[
\frac{dW_{t,m,i}}{dt} = g(m, \text{age}_i, \text{cond}(m, i)) \quad (5.4)
\]

In addition to the equations above, Equations 5.5 to 5.8 are boundary conditions.
5.1 The Kangaroo Population Model

required for each new cohort. These equations relate to the establishment of the new cohorts (births) in the next time period. At the beginning of each time period the existing cohorts must also be updated as given by Equations 5.9 to 5.12. In the following equations $T_{n+1}^-$ and $T_{n+1}^+$ represent the time just before and just after $T_{n+1}$.

\[
N_{f,0}(T_{n+1}^+) = \sum_{i=0}^{Q-1} b(\text{age}_i, \text{cond})N_{f,i}(T_{n+1}^-) \quad (5.5)
\]

\[
N_{m,0}(T_{n+1}^+) = sN_{f,0}(T_{n+1}^+) \quad (5.6)
\]

\[
W_{t,f,0}(T_{n+1}^+) = W_{t0} \quad (5.7)
\]

\[
W_{t,m,0}(T_{n+1}^+) = W_{t0} \quad (5.8)
\]

\[
N_{f,i+1}(T_{n+1}^+) = N_{f,i}(T_{n+1}^-) \quad (5.9)
\]

\[
N_{m,i+1}(T_{n+1}^+) = N_{m,i}(T_{n+1}^-) \quad (5.10)
\]

\[
W_{t,f,i+1}(T_{n+1}^+) = W_{t,f,i}(T_{n+1}^-) \quad (5.11)
\]

\[
W_{t,m,i+1}(T_{n+1}^+) = W_{t,m,i}(T_{n+1}^-) \quad (5.12)
\]

The age related mortality of has been previously modelled (Hacker et al., 2003) using a Weibull survival function (Equation 5.13), where \(AGE\) is the random variable for the age at which a kangaroo dies. It has been assumed that female and male kangaroos have the same age related mortality. However, it is noted that male kangaroo mortality is thought to be higher during the ages of 3 to 5 year old (Dawson, 1995). The functional response (the level of grazing dependent on the forage available) has been estimated (Caughley et al., 1987) by Equation 5.14. The functional response is used not only to determine the
amount of forage consumed by a kangaroo at the current level of total standing dry matter, but also its condition. The condition of the cohort of kangaroos is estimated via a goal gap formulation (Equation 5.15), with the instantaneous condition (Equation 5.16). There is a delay term in the differential equation as it has been noted that the change in condition of kangaroos has an approximately 3 month delay related to a change in forage (Caughley et al., 1987; Dawson, 1995; Moss and Croft, 1999; Bayliss and Choquenot, 2002).

\[ S(age) = P(AGE > age) = e^{-(0.614age)^{0.428}}, \]  
(5.13)

\[ \text{Intake}(V, Wt) = 0.0623(1 - e^{-\frac{V}{30}})Wt^{3/4} \]  
(5.14)

\[ \frac{d\text{cond}(gender, i)}{dt} = \frac{\text{condT}(t) - \text{cond}(gender, i)}{\text{delay}} \]  
(5.15)

\[ \text{condT} = \frac{\text{peaten} \times \text{greeneaten} \times \text{Intake}(V, Wt)}{0.88 \text{eaten} \times \text{Intake}(\text{satiation}, Wt)} \]  
(5.16)

\[ \text{peaten}(t) = \frac{\min\{V, \text{desire}(t)\}}{\text{desire}(t)} \]  
(5.17)

\[ \text{desire}(t) = \sum_{gender} \sum_{i=0}^{Q} \text{Intake}(V, Wt_{gender,i})N_{gender,i} \]  
(5.18)

As can be seen in Figure 5.1, once the available forage is greater than 300kg/ha the amount eaten plateaus. This can be thought of as the satiation level, the point at which the kangaroos appetite is satisfied. The satiation level was then compared to the actual amount eaten, both in terms of total and green forage, to determine the instantaneous condition. It is noted that the proportion of their diet that is green (alive) is important in determining kangaroo condition (Moss and Croft, 1999).

The condition is used as a proxy in the mortality and fecundity functions.
5.1 The Kangaroo Population Model

Figure 5.1: The function response for a 20\(\text{kg}\), 30\(\text{kg}\) and 60\(\text{kg}\) kangaroo. That is the amount eaten (kg) dependent on the available forage (kg/ha).

The better the condition of the kangaroos the lower the mortality and higher the fecundity (Caughley et al., 1987; Dawson, 1995; Moss and Croft, 1999; Pople and Grigg, 1999). For this reason, both the fecundity and mortality functions have the average fecundity and mortality multiplied by different functions of the current condition.

The modelled mortality (Equation 5.19) of the kangaroos is the product of the mortality due to age and the effect of condition on mortality. Mortality due to age is derived as the hazard rate related to the survivorship equation (Equation 5.13). The mortality related to condition is derived through the following arguments. When the condition is 1 (the kangaroo’s hunger is sated) then the mortality should be average and therefore the multiplier should be 1. When the condition is close to 1, it can be assumed that the mortality is still approximately average and hence the multiplier should be relatively flat, with a slight negative slope.
5.1 The Kangaroo Population Model

However, when the condition is above 1 (eating more than satiation levels) then the mortality should be smaller than average. According to Moss and Croft (1999) when vegetation is plentiful (and therefore intake is very close to the horizontal asymptote) then the mortality from pouch young to weaned is 85%. Alternatively, it is argued that as condition decreases, mortality increases. Furthermore, the rate at which mortality increases also increases as the condition of the kangaroo gets further from the satiation level. Over 4 months when food intake was at 25 – 50% of ad libitum levels, the mortality rate was such that 40% of kangaroos died (Caughley et al., 1987). Given those conditions a piece-wise function, based on two cubic functions both having a point of inflection at (1,1), was constructed. This function is shown inside the brackets in Equation 5.19 and Figure 5.2.

\[
\mu(\text{age, cond}) = \frac{0.34736}{\text{age}^{0.572}} \left( 1 - (\text{cond} - 1)^3 \right) \times \begin{cases} 
232 & , \text{cond} \leq 1 \\
33033.5 & , \text{cond} > 1 
\end{cases} \tag{5.19}
\]

The fecundity of kangaroos is modelled (Equation 5.20) as the product of the average fecundity given the female’s age and multiplier related to their condition. When considering the reproduction cycle of the kangaroo it has been noted that they can reproduce once every 8 months (Caughley et al., 1987; Hacker and McLeod, 2003). This equates to a possible average of 1.5 young at foot per year per fertile female. Female kangaroos start reproducing from around 2 years of age and continue until 12. They are most productive from 4 to 10, with roughly 80% having pouch young (Arnold et al., 1991). When their condition is too poor, kangaroos will either continually replace dead pouch young or have
5.1 The Kangaroo Population Model

Mortality multiplier

![Graph showing mortality multiplier vs condition.](image)

**Figure 5.2:** The mortality multiplier due to condition. When appetite is sated (condition is 1) the mortality is average. As condition decreases or increases, the mortality rate is increased and decreased respectively.

Extended embryonic diapause ([Caughley et al., 1987; Dawson, 1995](#)). In effect no successful births happen when condition is poor ([Moss and Croft, 1999](#)) or forage is scarce (less than 95kg/ha).

\[
b(age_i, \text{cond}) = \begin{cases} 
0.4(age_i - 2) & 2 \leq age_i < 4 \\
0.8 & 4 \leq age_i < 10 \\
0.4(12 - age_i) & 10 \leq age_i < 12 \\
0 & \text{otherwise}
\end{cases} 
\]  

(Kangaroo (and wallaby) harvesting is controlled by state and federal governments. State governments set quotas and regulations that must be signed off
by the Federal Government. This is due in part to the fact that as a native species, kangaroos (and wallabies) are under the protection of the crown. Each state has different protocols with regards to harvesting kangaroos and wallabies, what quotas are set and how the quotas are managed (Pople and Grigg, 1999). In Queensland regions are allocated quotas that the harvesters purchase, which can then be used to harvest kangaroos on private property (Office of the Queensland Parliamentary Counsel, 2010; Moloney et al., 2011). In addition to the quotas to control the off-take, there are also conditions that are meant to ensure a stable, genetically diverse kangaroo population. These include: minimum kangaroo densities; male off-take bias; and minimum weight limit on harvested kangaroos. Typically these are set to; a minimum kangaroo density of 2 kangaroos per km$^2$; a 70% male off-take bias; and a minimum live weight of 20kg or fully dressed weight of 13kg respectively (Hacker et al., 2003; Office of the Queensland Parliamentary Counsel, 2010). The kangaroo processors for human consumption have been known to set a higher minimum fully dressed weight (T. Garrett, pers. comm., 2008).

The harvest model for kangaroos therefore needs to include these conditions. There is a minimum density of kangaroos which must be met otherwise harvesting cannot commence. The harvest rate is a proportion of what can be harvested. To maintain the male harvest bias a stoichiometric formulation is used. Harvesting only occurs periodically, in this case on a monthly basis.
5.2 Issues with GRASP

Several issues were discovered with the operation of GRASP.

- The grass basal area can be set to a constant or as a function of evapotranspiration. However, changes made to grass basal area, seemed to have
5.2 Issues with *GRASP*

no effect on *tsdm*. This component was included in the re-coded model.

- Similarly tree basal area is constant and changes seemed to have little if any bearing on the *tsdm*. For that reason, it was left out of the re-coded version. It could be argued that much of the land is pasture with trees only at the edges of paddocks.

- Frost was capable of causing the *tsdm* to reach zero after only one or two events. Using the original weather data there was such an event, one cold day and the entire *tsdm* was set to zero. When cattle was included, regrowth was eaten and reasonable levels of *tsdm* were achieved only after complete de-stocking. In the re-coded model the temperature limit at which *tsdm* loss was total was lowered sufficiently so as to exclude this event.

- Rain water that cannot be taken into the top layer of soil (runoff) is just lost to the system.

- The default setting for livestock is a constant stocking rate. This was not appropriate as the kangaroo population was dynamic. Therefore the stocking regime option which sets the next years (domesticated) stocking rate based on previous pasture growth and herbivore intake was selected.

- *GRASP* converting between between *beasts/ha* and weaner equivalents when calculating the amounts of each forage type eaten. This may be acceptable when the conversion rates between sheep and cattle are known. However, as stated earlier, this figure is in dispute Grigg (2002); Munn et al. (2009). Therefore it is more appropriate to work entirely in kangaroos for the kangaroo simulations.
After much effort it was decided that to enable GRASP to be extended to include kangaroo grazing, re-coding would be required if it were to be used. AP-SIM, a program based on GRASP was considered. However, within the program it is not possible to construct a dynamic herbivore and the base code is not available. For these reasons the decision to re-code GRASP was made. The extended GRASP model was coded in Mathematica (Wolfram Research, Inc., 2010) and took several months (base GRASP was over a hundred pages of code). This meant that a thorough understanding of the processes used in GRASP has been gained.

5.3 GRASP Simulations

GRASP uses diurnal weather data to predict total standing dry matter, animal weight gain, wool production and abundance. It is therefore important to be able to generate weather data to populate the simulations. Weather data from 1970 to 2008 were obtained from the Queensland Department of Environment and Resource Management (DERM) for Mitchell, Queensland. The data are from the DataDrill interpolations based on the SILO weather data sets from the Bureau of Meteorology. The data from DataDrill were designed for use with GRASP (Jeffrey et al., 2001).

The goal was to be able to generate new weather data sets for use in the simulations. They would mimick the distributions of rainfall, temperature, evaporation, radiation and vapour pressure. Initial success was had modelling the rainfall. It used a combination of a three state Markov model, to determine
5.3 *GRASP* Simulations

rain state (no rain, start raining event, continue raining, or end rain event), and separate Weibull distributions for each month’s daily intensity dependent of the rain state. Unfortunately evaporation, radiation and vapour pressure where not readily modelled. It was decided that an alternative method for generating new weather periods should be used. New weather data were generated for each month by bootstrapping. The data for each month were randomly selected, with replacement, from all the corresponding months in the original data. For instance, the first three months of a new weather data set may be January from 1980, February from 2003, March from 1997. This technique was used to generate 1000 instances of new weather data, each lasting 20 years.

Simulations for cattle, sheep and kangaroo were then carried out separately, using *GRASP* and the same weather data. In effect it was assumed that the property was only stocking a single species in isolation. Each simulation used the same set of parameterisations for each species. Parameters related to pasture production were identical across all species. The initial total biomass was set to 1000kg/ha, divided into each group using the ratios found after running *GRASP* without grazing over the original data set (0.128:0.295:0.031:0.542 for green leaf: green stem: dead leaf: dead stem). The initialisation parameters for the re-coded *GRASP* that differ from the default settings are stated in Appendix B.2.

Once each fifteen year simulation was completed, results from the first five years were removed as an initialisation period. This approach falls in line with the suggested approach for *GRASP* (Littleboy and McKeon, 2005). The statistics of interest were then recorded for each simulation for each animal. These statistics (where applicable) were the mean yearly population, weights harvested per year,
wool clips per year, and mean and standard deviation \textit{tsdm}.

To illustrate what was generated by the extended \textit{GRASP} program the data produced for one weather simulation sequence for each herbivore in Figure 5.3. Note that as expected, the kangaroo population falls sharply when condition is too low for too long (Figure 5.3a). Also note that commodity production is not entirely in lock step. Finally, it is clear that by looking at the \textit{tsdm} under each herbivore, that different amounts of available forage are utilised, with kangaroos the most and cattle the least.

\section*{5.4 Discussion}

After some trials and tribulations \textit{GRASP} was extended to include kangaroo grazing and population dynamics. The results from the simulations highlight the fact that kangaroo population size can dramatically change depending on the environmental conditions, a feature which may be exploited to mitigate lost production during and post-drought. This seems to enable them to utilise the available forage to a greater degree when conditions change. Production levels for each animal have different lags to external events. This could be partially due to the fact that domestic stock was only bought and sold annually. To see if the differences in commodity production enhance resilience an agribusiness need to be explored. These herbivore simulations are integral to the portfolio analysis conducted in Chapter 6, which will demonstrate to what degree mixed-grazing can improve resilience.
Figure 5.3: The results from the same weather data of extended GRASP.
Chapter 6

Results and Analysis

This chapter is the culmination of the ideas explored in the previous chapters. Is mixed-grazing (including kangaroos) feasible from a financial point of view? Chapter 2 explored the idea of allocating the available forage in such a way as to optimise the trade off between possible revenue and consistency by using a multi-objective optimisation problem \( (MOP) \). This used historical data from a number of sources to estimate the covariance in revenue between cattle, wool and kangaroo production over time. The production rates of the different commodities were considered known and constant. Having a better understanding of the dynamics of the production of the commodities could influence the results of the portfolio optimisation. Chapter 4 developed the analysis of the dynamics between herbivores and the environment. It suggested that the actions of neighbours would not affect properties, whilst they all used an optimal harvesting regime. This is important as it can help allay landholder fears that an adjacent property ”encourage” kangaroos could negatively affect their property. Also it
addresses the concern mentioned in Section 2.1 that porous borders could result in a net loss or gain of herbivores. Hence a landholder can choose which ratio of herbivores to stock or harvest without impinging on neighbours. Admittedly nature reserves are not "optimally managed" for harvesting commodities, and therefore could possibly influence outcomes on any property. Chapter 5 introduced the plant-herbivore model GRASP and additional components designed to model the kangaroo population. This enables a better understanding of the dynamics of the different herbivores in the environment and the quantity of the commodities they produce.

Combining sections from the previous three chapters resulted in running simulations of different weather patterns. The data generated, can then be used in a MOP to construct an efficient frontier. Landholders can use the efficient frontier to help decide on the future of their enterprise. What combination of cattle, sheep and kangaroos will suit them given their level of risk aversion? Finally, what impact would the enforcement of methane emission reduction have on the optimal strategies?

6.1 Simulation Results

There are several components that require Monte Carlo simulations for use in the portfolio optimisation. The extended GRASP model is run with different simulated weather events to generate commodity production data. The data on the price (or change in price) of the commodities needs to be simulated. Using a combination of these results the relative returns can be simulated to inform the
Table 6.1: Symbols used in portfolio optimisation.
6.1 Simulation Results

Table 6.2: Overall mean yearly herbivore density, commodity production and $tsdm$ from the extended GRASP model for each herbivore.

<table>
<thead>
<tr>
<th>Mean Density (Hervibore/ha)</th>
<th>Commodity Production (kg/ha)</th>
<th>$tsdm$ (kg/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cattle</td>
<td>0.077</td>
<td>11.172</td>
</tr>
<tr>
<td>Sheep</td>
<td>0.410</td>
<td>0.9644</td>
</tr>
<tr>
<td>Kangaroo</td>
<td>1.694</td>
<td>0.4998</td>
</tr>
</tbody>
</table>

6.1.1 GRASP Simulation Results

A summary of the overall results of the simulations are given in Table 6.2. There are several notable points related to these results. Firstly, the production of beef is vastly greater in quantity than either of the other commodities produced, by a factor of over 10. This implies, that unless beef is much cheaper than either wool or kangaroo meat, it would seem that beef production would result in the greatest returns. Secondly, the relationship between the herbivore densities should relate to the $dse$’s mentioned earlier. The extended GRASP program keeps track of the quantity of each herbivore in terms of average cattle and sheep and monthly cohorts of kangaroos.

GRASP has been validated for biomass, beef and wool production in the Maranoa region (Littleboy and McKeon, 2005). Unfortunately kangaroo population numbers have not been monitored in any detail in the area, making validation of the kangaroo model difficult. By looking at the kangaroo density and comparing it to the sheep density the effective kangaroo $dse$ can be estimated. If the
6.1 Simulation Results

Simulated \textit{dse} is not similar to the expected kangaroo \textit{dse} it would be evidence that the model is invalid. Using the sheep as the standard, that gives a ratio of 5.32:1:0.24 for cattle:sheep:kangaroo quantities. However, the ratio to cattle (6.1 for a 200kg weaner as used in \textit{GRASP}) and kangaroo (\sim 0.35) seems low compared to its usual \textit{dse} (Millear \textit{et al.}, 2003; Munn \textit{et al.}, 2009). These compared the amount eaten ad libitum and do not include any spatial measurement (sheep/ha for instance). However, in the simulations the \textit{tsdm} is per hectare and the mean differs for each species. Converting the ratios to include the mean available forage as well results in a ratio of 4.74:1:0.32. The kangaroo conversion factor is in line with the most recent estimates and therefore support the model. Conversely the cattle conversion factor is low. The original \textit{GRASP} model has been validated for both cattle and sheep, and that part of the program was not changed.

6.1.2 Pricing Simulation

Historical data on the prices of cattle, sheep, wool and kangaroos was collated from \textit{ABARE} (2010) and kangaroo harvesters (T. Garrett, pers. comm., 2008). Using these data statistics on the measures of interest were estimated. The statistics included means, standard deviations and covariances for the relative change in the price of cattle, sheep and kangaroos as well as the ratio of wool price to sheep price (see Table 6.3). Multivariate regression, with time and annual rainfall as covariates, found no significant models. A Shapiro-Wilk multivariate normality test was conducted and the findings did not reject the assumption that the data were multivariate normal (\textit{p}-value=0.1575). Hence the Cholesky factor
6.1 Simulation Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Cattle</th>
<th>Kangaroo</th>
<th>Sheep</th>
<th>Wool Price</th>
<th>Sheep Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0110</td>
<td>0.0659</td>
<td>0.0894</td>
<td>0.1956</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.1149</td>
<td>0.1874</td>
<td>0.3736</td>
<td>0.1007</td>
<td></td>
</tr>
<tr>
<td>Covariance matrix</td>
<td>0.0132</td>
<td>0.0060</td>
<td>0.0074</td>
<td>-0.0029</td>
<td>-0.0029</td>
</tr>
</tbody>
</table>
<pre><code>                                                             | 0.0060 | 0.0351   | 0.0058 | -0.0043    | -0.0143     |
                                                             | 0.0074 | 0.0058   | 0.1396 | -0.0143    | 0.0101      |
                                                             | -0.0029|-0.0043   | -0.0143| 0.0101      |             |
</code></pre>

Table 6.3: Summary statistics estimated from data from ABARE and kangaroo harvesters.

of the covariance matrix was used to generate the multivariate random numbers as per Gentle (2006). That is, let $Y$ be a vector of independent identically distributed (i.i.d.) standard normal random variables, and let the matrix $A$ be the Cholesky factor of the covariance matrix. Then, $AY + \mu \sim N(\mu, \Sigma)$.

6.1.3 Relative Returns Simulation

The relative return on investment is calculated as the change in value from one year to the next of stock plus the value of production, relative to the original value (Equation 6.1).

$$R_i = \frac{\text{Value}(t+1, i) - \text{Value}(t, i) + \text{Production}}{\text{Value}(t, i) + q_i \text{NSI}}$$  \hspace{1cm} (6.1)
6.1 Simulation Results

When the commodity produced is meat (as in the case of cattle and kangaroo) then Equation 6.1 can be re-written to take the form of

\[
R_i = \frac{ValW_i(t+1)(Wt_i(t) + Harv_i(t)) - ValW_i(t)Wt_i(t)}{ValW_i(t)Wt_i(t) + q_iNSI}
\]

\[
= \frac{ValW_i(t)(1 + P_i)Wt_i(t)\left(1 + \frac{Harv_i(t)}{Wt_i(t)}\right) - ValW_i(t)Wt_i(t)}{ValW_i(t)Wt_i(t) + q_iNSI}
\]

\[
= \frac{(1 + P_i)\left(1 + \frac{Harv_i(t)}{Wt_i(t)}\right) - 1}{1 + \frac{q_iNSI}{ValW_i(t)Wt_i(t)}}
\]

\[
= \frac{P_i + H_i^\dagger + P_iH_i^\dagger}{1 + L_i}
\] (6.2)

where \(H_i^\dagger = \frac{Harv_i(t)}{Wt_i(t)}\), \(L_i = \frac{NSI}{ValA_i(t)k_i}\) and \(i \in \{c, k\}\). It is noted that this is similar to the arguments made in Chapter 2.1 for Equation 2.3 the calculation of relative return. When considering the case of the percentage return from a product not related to the weight of the animal (sheep) the equation becomes,

\[
R_s = \frac{ValA_s(t)A_s(t)(1 + P_s) - ValA_s(t)A_s(t) + WWoolPrice(t)}{ValA_s(t)A_s(t) + q_iNSI}
\]

\[
= \frac{P_s + WW^\dagger}{1 + L_s}
\] (6.3)

where \(W^\dagger = \frac{WoolPrice(t)}{ValA_s(t)A_s(t)}\).

\(Q\) is the set of simulated observations of \(R_c\), \(R_k\), and \(R_s\) generated through the combination of the extended \textit{GRASP} model and commodity prices. The extended \textit{GRASP} simulations to generate \(H_c^\dagger\), \(H_k^\dagger\) and \(W\). The price simulations gave \(P_c\), \(P_k\), \(P_s\) and \(W^\dagger\). The current values ((\textit{ABARE}, 2010) and T. Garrett, pers.
6.2 Portfolio Optimisation Using the Simulated Results

As mentioned previously, landholders are not purely interested in maximising their return. They also have to weigh up the risks involved in each grazing strategy. To this end three techniques for optimal portfolio allocation are investigated, an efficient frontier constructed and their results compared. The classical mean-variance (M-V) portfolio optimisation method uses variance as a proxy for risk. Given the minimum acceptable expected return, this selects the portfolio with the least variation. The average value-at-risk (AVaR) portfolio optimisation method defines risk as the average loss in the worst $\epsilon$% of cases. Given the minimum acceptable expected return, this method selects the portfolio that minimises the AVaR$_\epsilon$. The multi-objective optimisation problem (MOP) method used uses a risk aversion metric to trade-off minimising variance (a proxy for risk) and maximising return.

6.2.1 Mean-Variance Optimisation

The benefits to the M-V approach relate to ease of use and understanding. Computationally it is easy to solve the quadratic optimisation. The idea of allocating resources is readily accepted. Both measures (expectation and standard devia-
6.2 Portfolio Optimisation Using the Simulated Results

tion) are known to many people and can be explained with relative ease. The efficient frontier (the set of pareto-optimal solutions) gives a clear visual interpretation to the balance between return and a proxy for risk. Using the formulation given in Equation 1.1 and the notation in Table 6.1 the M-V requires the solution of the following problem,

$$\min_{q} \quad q^T \Sigma_R q$$

\[ s.t. \quad e^T q = 1 \]

$$\mu_R^T q \geq \mu^*$$

$$q \geq 0$$

where $\mu_R$ and $\Sigma_R$ are estimated from $Q$ and given below.

$$\mu_R = \begin{pmatrix} 0.08018 & 0.00578 & 0.03118 \\ 0.00142 & 0.00017 & 0.00048 \\ 0.00017 & 0.00030 & 0.00002 \\ 0.00048 & 0.00002 & 0.00091 \end{pmatrix}$$

$$\Sigma_R = \begin{pmatrix} 0.08018 & 0.00578 & 0.03118 \\ 0.00142 & 0.00017 & 0.00048 \\ 0.00017 & 0.00030 & 0.00002 \\ 0.00048 & 0.00002 & 0.00091 \end{pmatrix}$$

Solving the optimisation problem 6.4 for different values of $\mu^*$ (minimum acceptable relative return) gives the efficient frontier. Figure 6.1a shows the M-V efficient frontier, while Figure 6.1b shows the associated allocation. It is clear from Figure 6.1 that as the variance, and therefore relative return, increases the preference changes from kangaroos to cattle. Sheep allocation stays relatively stable until it is replaced by cattle at the higher variance levels. It is also noted that at either end of the variance scale not all herbivores are allocated to the
6.2 Portfolio Optimisation Using the Simulated Results

property.

![Efficient Frontier](image1)

![Allocation](image2)

![Legend](image3)

**Figure 6.1:** The efficient frontier of mean-variance plane. The shaded regions show how the allocation of forage is aligned to the variance.
To put these results in context, consider a landholder who has a $200\text{km}^2$ property and wishes to have an expected relative return of 7%. Their best option (in a $M-V$ sense) would be an allocation of $162.9\text{km}^2$, $28.4\text{km}^2$ and $8.7\text{km}^2$ to cattle, sheep and kangaroo respectively. The expected return for this allocation would be $630,332$ given an investment of $9,004,740$ with a standard deviation of $296,470$.

There is one possible problem with the $M-V$ approach in this case. To consistently give the best return for the same variance the underlying distribution of $Q$ should be multivariate normal. This property is known as second-order stochastic dominance ($SSD$) (Rachev et al., 2008). Using a Shapiro-Wilk multivariate normality test on $Q$ gave a $p$-value $= 0.038$, which means at the usual significance level ($0.05$) the assumption that $Q$ is multivariate normal is rejected. Therefore the $M-V$ portfolio optimisation is unlikely to be $SSD$. Hence, it is possible the best return for the same variance has not been selected.

6.2.2 Average Value at Risk Optimisation

An alternative to classifying risk as variance is using a measure like average value-at-risk ($AVaR$, also known as conditional value at risk). $AVaR$ is $SSD$ without requiring multivariate normal returns (De Giorgi, 2005). $AVaR$ is based on the expected return given that the return was in the lowest $\epsilon$ of the distribution. If the distribution is not known, the $AVaR$ of a single return can be estimated from
6.2 Portfolio Optimisation Using the Simulated Results

a sample via the Equation 6.5 (Rochafellar and Urasev, 2000).

\[
\hat{AVaR}_\epsilon(r) = \min_{\vartheta \in \mathbb{R}} \left( \vartheta + \frac{1}{n\epsilon} \sum_{j=1}^{n} \max\{-r_j - \vartheta, 0\} \right)
\]

(6.5)

where \( \vartheta \) is an auxiliary variable. Optimising the AVaR for a portfolio can be done via the following optimisation problem (Palmquist et al., 2002).

\[
\begin{align*}
\min_{q, \vartheta, d} & \quad \vartheta + \frac{q^T e}{n\epsilon} \\
\text{s.t.} & \quad -Qq - \vartheta e \leq d \\
& \quad e^T q = 1 \\
& \quad \mu^T_R q \geq \mu_* \\
& \quad q \geq 0, d \geq 0, \vartheta \in \mathbb{R}
\end{align*}
\]

(6.6)

where \( d \) is a vector of auxiliary variables and \( \vartheta \) is the additional parameter coming from the minimisation formula. Solving the optimisation problem 6.6 for different values of \( \mu_* \) gives the efficient frontier. Consider the AVaR at a tail probability of \( \epsilon = 0.2 \). That is the (relative) expected loss given that the return is in the bottom 20% of portfolio returns. Put another way, on average, every five years you would expect to have a loss this large. Figure 6.2a shows the AVaR\(_{0.2}\) efficient frontier, while Figure 6.2b shows the associated allocation. It is clear from Figure 6.2 that as the AVaR\(_{0.2}\), and therefore relative return, increases the preference changes from kangaroos to cattle. Sheep allocation stays relatively stable until it is replaced by cattle at the higher variance levels. It is also noted that at either end of the variance scale not all herbivores are allocated to the property.

To put these results in context consider a landholder who has a 200km\(^2\) property
6.2 Portfolio Optimisation Using the Simulated Results

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**Figure 6.2:** The efficient frontier of mean-risk plane. The shaded regions show how the allocation of forage is aligned to the risk.
and wishes to have an expected relative return of 7%. Their best option (in a AVaR sense) would be an allocation of 161.3km$^2$, 33.1km$^2$ and 5.6km$^2$ to cattle, sheep and kangaroo respectively. The expected return for this allocation would be $629,890 given an investment of $8,998,430 with an AVaR of $175,666.

### 6.2.3 Multi-objective Optimisation Problem

Alternatively, the problem can be formulated as a MOP. The MOP formulation allows for greater diversity in what can be considered in the objective function. Using a similar formulation to that in Section 2.1 an objective function that trades off risk and return is,

$$
\min_{q, \vartheta, d} \lambda \left( \vartheta + \frac{q^T e}{n \epsilon} \right) \frac{AVaR^*}{\mu^*} - (1 - \lambda)\mu^T q
$$

subject to

$$
-Q^T q - \vartheta e \leq d
$$

$$
e^T q = 1
$$

$$
q \geq 0, d \geq 0, \vartheta \in \mathbb{R}
$$

where AVaR$^*$ is the minimum AVaR and $\mu^*$ is the maximum relative return. The benefit of this formulation is twofold:

- It allows for the use of a single risk aversion parameter ($\lambda$) to determine the best grazing strategy.

- The risk and return components of the objective function are relative to their optimal values.
Comparison to the minimum risk and maximum return results in the strategy only changing once the relative reduction in risk outweighs the increase in return and vice versa. This is clearly illustrated in Figure 6.3. Think of the AVaR efficient frontier for the portfolio (Figure 6.2a). What the MOP then does is uses that as its feasible region for the optimisation of the linear objective function. Therefore the solutions must come from the vertices (and technically the edges) of the feasible region. As the risk aversion ($\lambda$) changes, it is only when the gradient of the objective function $\left(\frac{\lambda}{1-\lambda}\right)$ equals a gradient of the feasible region that the solution changes. When the gradients are equal, the allocation could take any point on that edge. The MOP effectively compresses the possible allocations. In this case that compression results in only three allocations, low, moderate and high risk aversion as shown in Figure 6.3 and Table 6.4.

**Figure 6.3:** The allocation of the property to each species as the landholder’s risk aversion changes. See Figure 6.2c for legend.
6.2 Portfolio Optimisation Using the Simulated Results

<table>
<thead>
<tr>
<th>Outcome (Units)</th>
<th>Risk Aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low ($0 \leq \lambda \leq 0.356$)</td>
</tr>
<tr>
<td>Investment ($)</td>
<td>9,090,560.</td>
</tr>
<tr>
<td>Return ($)</td>
<td>728,154.</td>
</tr>
<tr>
<td>AVaR 0.2 ($)</td>
<td>209,360.</td>
</tr>
<tr>
<td>Cattle (ha)</td>
<td>200.0</td>
</tr>
<tr>
<td>Sheep (ha)</td>
<td>0.0</td>
</tr>
<tr>
<td>Kangaroo (ha)</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 6.4: The investment, return, AVaR and herbivore land allocation under the MOP for a 200 km² property. Money in 2009/2010 dollars.

6.2.4 Portfolios Considering Methane Emissions

A consideration for landholders in the future may be carbon emissions. Cattle and sheep produce substantially more methane than kangaroos (Wilson and Edwards, 2008). Using average densities and emissions kangaroos emit less than a tenth and a twenty-fifth the methane of sheep and cattle respectively. Therefore, if emission from agriculture were to be considered, then kangaroo harvesting may become more appealing. Garnaut (2008) gave a target of 10% reduction carbon emission from 2000 levels by 2020.

Assuming the property was exclusively using cattle (the highest emitter of methane). Then using the 10% reduction target for an AVaR portfolio optimisation just requires the optimisation problem 6.6 to include another constraint,

\[ \text{Methane}^T q \leq 0.9 \text{Methane}_c \]

where \( \text{Methane} = \{ \text{Methane}_c, \text{Methane}_k, \text{Methane}_s \} \) is the average methane lev-
6.2 Portfolio Optimisation Using the Simulated Results

...emissions per hectare using each species exclusively. Reducing the methane emissions has also changed the optimal portfolios significantly. The efficient frontier for this new problem shows that the returns have been reduced compared to the solutions in Section 6.2.2 (see Figure 6.4a). The allocation of forage to each species has substantially changed (see Figure 6.4b). Note, at either end of risk aversion, that all three herbivores are now allocated forage and that there is a section where the pareto-optimal solution does not include any kangaroo allocation. This is due to the fact that while sheep methane emissions are much higher than kangaroo emissions, they are still under half the emissions of cattle per hectare per year. Sheep also achieve higher relative returns than kangaroo and so provide a better option for reducing emissions for the moderately risk averse. When relative returns are more important, kangaroos’ low emission offset the much higher returns available through cattle allocation.

To put these results in context consider a landholder who has a 200 km$^2$ property and wishes to have an expected relative return of 7%. Their best option (in a AVaR sense) with at least a 10% reduction in methane emissions would be an allocation of 158.8 km$^2$ and 41.2 km$^2$ to cattle and sheep respectively, with no kangaroo allocation. The expected return for this allocation would be $629,156 given an investment of $8,998,950 with an AVaR of $104,371 and methane emissions of 22.8 tonnes/year (an 11.4% reduction).
Figure 6.4: The efficient frontier of mean-risk plane including a 10% reduction in methane emissions from a cattle only base. The shaded regions show how the allocation of forage aligned to the risk.
Presently it is unclear if a 10\% reduction in carbon emissions will be enforced on agriculture in Australia. Therefore it may be more of a choice that a landholder may wish to consider their carbon emissions. The relative level of methane emissions can be dealt with via an \textit{MOP}. Including a term minimising the methane emissions (compared to an exclusively cattle emission) results in the following \textit{MOP},

\[
\min_{q, \vartheta, d} \lambda_A \left( \vartheta + \frac{q^T e}{n} \right) \frac{\lambda_R \mu^T_R q}{\mu^*} + \frac{\lambda_M \text{Methane}^T q}{\text{Methane}_c} \quad (6.8)
\]

\[
s.t. \quad -Qq - \vartheta e \leq d
\]

\[
e^T q = 1
\]

\[
\lambda_A + \lambda_R + \lambda_M = 1
\]

\[
q \geq 0, d \geq 0, \vartheta \in \mathbb{R}
\]

where \(AVaR^*\) is the minimum \(AVaR\), \(\mu^*\) is the maximum relative return and \(\lambda_A\), \(\lambda_R\) and \(\lambda_M\) are the preference weighting for minimising risk, maximising relative returns and maximising the relative decrease in methane emissions respectively. Two scenarios are used to illustrate the effect of including methane emission reduction on expected relative returns.

The first scenario involves an equal split of preferences between maximising returns and emission reduction and minimising risk. The resulting regime on a 200\(km^2\) cattle property has: an investment of $8,935,790; a reduction in methane emissions of 21.6\%; an expected return of $552,341 (6.2\%); an \(AVaR\) of $77,616 (0.1\%); and allocation of 133.4\(km^2\), 50.8\(km^2\) and 15.8\(km^2\) to cattle, sheep and kangaroo respectively. Note that the reduction in methane emissions quite large.
The second scenario involves exploring the effect of the preference for reducing methane emissions ($\lambda_M$), where the remaining preferences ($\lambda_R$ and $\lambda_R$) are split in a ratio of 3:1 respectively. It can be seen, when methane emission reduction is preferred, that expected relative revenue falls (see Figure 6.5a) and the kangaroo allocation increases (see Figure 6.5b). For most methane emission reduction preferences ($\lambda_M < 0.625$) the solution is actually the same result as in equal weighting scenario.
Figure 6.5: The expected relative returns against the preference for methane emission reduction. The shaded regions show how the allocation of forage aligned to the preference for methane emission reduction. The remainder of the preferences are shared between maximising returns and minimising risk in a ratio of 3:1.
6.3 Discussion

According to ABARE (2010) average rate of return since 1985 is 7.34% with a standard deviation of 7.65%. For the same expected relative return the efficient frontier using the $M-V$ has a standard deviation of (3.42%). The $AVaR$ for the average rate of return over the same period was estimated at 5.35%, compared to 1.44% with the portfolio optimisation simulation. Either portfolio method resulted in a reduction in the risk for the average property in the region.

![Figure 6.6: The efficient frontiers (in terms of risk) of both the $M-V$ and $AVaR$ approaches.](image)

It is clear, when the comparing the $M-V$ and $AVaR$ efficient frontiers (Figure 6.6), that as expected the maximum relative returns are equal. However, it is noted that the minimum expected relative returns are quite different, with the $AVaR$ the larger of the two. Simplifying the choice for the landholder through the $MOP$ resulted in three scenarios, two of which had mixed-grazing profiles.

Consideration of methane emission reductions affected the allocation of forage. When a methane emission reduction target is set to 10% as cited by Garnaut
(2008) the result for a landholder only concerned with expected relative returns has: their expected relative return reduced from 8.0% to 7.2%; their AVaR reduced from 2.3% to 1.9%; and kangaroos are now included in the optimal portfolio. If the landholder is particularly concerned with reducing their methane emissions, then kangaroos are an important part of the solution. However, this comes at considerable expense in substantially reduced expected relative revenue.

The analysis in this chapter highlights several points. Cattle give the best return to the point of excluding sheep and kangaroo if maximising returns is the only issue. When minimising risk (whichever way it is measured) a combination of cattle, sheep and kangaroos is preferred. However, in the majority of cases the reduction in risk would be considered by most to be too small compared to the reduction in expected relative return. Hence, it is unlikely in the current circumstances that landholders would convert much if any of their land over to kangaroo production. If carbon reduction measures need to be taken by the landholders in the future two results are clear for those focussing on expected relative returns: the expected relative returns will be reduced; and, kangaroos harvesting would increase to counter-balance the high emission herbivores.
Chapter 7

Conclusion

The present view amongst the majority of landholders in Australia is that macropods (kangaroos and wallabies) are pests that increase grazing pressure without any financial return to them. Hence, landholders allow harvesting to occur on their properties in order to reduce their numbers. They receive no explicit financial compensation from the harvesters, seeing it as pest control. However, it has been argued that converting some production to kangaroo commodities would have beneficial ecological and environmental consequences. Native species, including macropods, cause significantly less damage to fragile soils and the creation of wooded refuges would enhance kangaroo numbers and increase biodiversity. Kangaroos emit significantly less methane (a greenhouse gas) than either cattle or sheep (0.18% and 2.14% per head of their emissions respectively). Logically then, if it can be shown that diversifying commodities through mixed-grazing (including kangaroo) can be beneficial financially, the ecological and environmental gains would follow consequentially.
In the semi-arid rangeland of the Maranoa region of Queensland issues of grazing pressure and land degradation are even more important. In this environment enhancing a property’s resilience is essential. Resilience can be improved through sustainable natural resource management and reducing financial risk. One way to analyse different stock options and their associated risks is through portfolio optimisation. Portfolio optimisation endeavours to select the strategy with the least risk for a given return. Each commodity is allocated a fraction of the available resources. In this case that resource can be considered the available forage, land or total grazing pressure. An exploratory portfolio analysis (see Chapter 2) showed that when the non-stock investment ($NSI$) was large the portfolio’s preference was for herbivores of greater value. When the $NSI$ for a property is low or the landholder was moderately risk averse mixed-grazing involving all species was optimal. Effectively, kangaroo allocations were higher on more marginal properties.

Allocating forage to different species is only possible if the population size of each species can be controlled, otherwise competitive exclusion, where one species dominates another, and kangaroo population movements could override the allocations. Also, concerns were raised during meetings with landholders in the Maranoa region about the effect of encouraging kangaroos on neighbouring properties. The effect from kangaroos (a mobile species that can cross boundaries and cannot always be explicitly controlled) was explored in Chapter 4. It concluded that, as long as steps are made to use the forage efficiently, then kangaroo movement between properties should not present a problem. Moreover, properties that border a national park or similar unharvested area could benefit from the kan-
garoo movement. Additionally, as long as forage is utilised efficiently and there
is refuge from harvesting, then competitive exclusion should not occur. It was
ultimately shown that a cooperative approach produced better results, an impor-
tant consideration given a kangaroo harvesting cooperative is being established
around Mitchell in the Maranoa region.

It is thought that many landholders over-estimate the impact kangaroos have
on grazing pressure. This is evidenced in the landholder’s belief that the impact
of a kangaroo is 70 – 80\% of that of a sheep. More recent studies (Munn et al.,
2009) as well as the kangaroo population model used in this thesis (see Chapter 5)
have the impact of at about half that rate. This point should be brought to the
attention of landholders. A better understanding of the impact of kangaroos may
lead to different management strategies by landholders.

The analysis of the ecological and economic model emphasised several features
outlined below. The amount of meat produced per hectare of kangaroo on average
was quite small when compared to beef or even wool production (Table 6.2).
Hence, if maximising returns is the only issue, cattle produce the best return
to the point of excluding sheep and kangaroo. The model reflects that in the
majority of cases the reduction in risk is too small to consider the inclusion of
kangaroos. That is, only for the risk averse was a combination of cattle, sheep
and kangaroo preferred. Hence, it is unlikely in the current circumstances that
landholders would convert much, if any, of their land over to kangaroo production.

For this conclusion to change substantially, one or more modifications to the
current state-of-play regarding kangaroos, agribusinesses and government policies
would be required. These modifications include:
• Reduce methane emission - A requirement to reduce methane emissions would increase the viability of inclusion of kangaroo because their methane emissions are very small relative to cattle and sheep (see Section 6.2.4).

• Increase the price paid for kangaroo meat - Kangaroo meat pricing is much less than that of other commodities produced. When NSI required is large, allocations of animals of low value (even if their reproduction rate is high) are reduced. If the price of kangaroo was to increase, its viability would improve.

• Place a value on kangaroo skins for the landholder or harvester - Currently the value of the kangaroo to a landholder or harvester does not seem to include a price for the skin (McLeod et al., 2004). The skin is quite valuable in itself. If some of this value was to be reflected in the price paid for kangaroo carcasses, the allocations for kangaroo would increase.

• Reduce the high rate of juvenile mortality in kangaroos - Kangaroo juvenile mortality is so high that even though they can breed quickly, the quantity harvestable is relatively small. It is possible that the kangaroo juvenile mortality rate could be decreased, however, as kangaroos “belong” to the government and not the landholder, it is unclear as to why landholders would incur the expense given they are not the legal owner of the animal. If more young-at-foot survived, the population demographics would change and increase the quantity of harvestable kangaroos.

• Increase the rate at which kangaroos gain weight - The rate at which kangaroos put on weight is much slower than cattle. This is to be expected, as
beef cattle are domesticated and have been selectively bred for weight gain. However, increased rate of weight gain should not arise through increased levels of fat as one of the key selling points of kangaroo meat is the fact that it is very lean. If kangaroos gained weight at a faster rate it would be more viable for landholders to diversify into kangaroo commodities.

- Amend policies concerning kangaroo population management - Live kangaroos cannot be legally bought and sold in Australia. Therefore, increasing population size on the landholder’s property must be via births, enticement to immigrate, or reduction in harvest. Any of these options take time, unlike domestic animals, which can be readily bought and sold. If policy changes, then kangaroos could be farmed in a more traditional sense, kept within boundaries and owned by the landholder. That may allay some concerns landholders’ have about diversifying into an animal that they cannot own or control.

The list above addressed changes that may increase the uptake of landholders diversifying into kangaroo commodities. If diversification becomes more viable then the following are practical difficulties needing to be addressed: the relationship between processors and harvesters; the relationship between harvesters and landholders; and, limitations of harvest quotas. Considering the relationship between processors and harvesters, all the power is with the processors. They set the prices and can enforce requirements above that required by the government. Working together to ensure supply and quality could be beneficial to both parties. The relationship between harvesters and landholders also needs consideration. Currently harvesters operate across many properties without explicit
financial return to the harvesters and landholders. Harvesters need permission to enter properties and landholders could harvest for themselves. Alternatively, cooperation between landholders and harvesters could guarantee access and quality of supply over a wider range of properties. Harvest quotas may also need to be redefined in the future. Greater acceptance for kangaroo meat within Australia and internationally could affect its demand, and this would impact its price and in turn, the incentive to diversify. If diversification into kangaroos was to increase, there may be an issue with harvest quotas being reached. If the quota is reached, there is no incentive for the landholders to diversify.

Further research needs to be carried out on the speed at which kangaroos resettle in different areas. Understanding when and how quickly kangaroo mobs relocate is key to including migration into the extended GRASP model. That in turn would enable the construction of a spatial model. The model requires validation, but it is impracticable at present due to the substantial time and money that would be required to gather the data. Other areas for continuing research into mixed-grazing include the effect on the environment under different strategies. The MOP methodology would be key in the inclusion of biodiversity and conservation objectives.

A model for the dynamics of the plant-herbivore system and the economics that underlie grazing in semi-arid Australia has been created. Analysis of this model shows that under the current circumstances, combining native species into a mixed-grazing regime is preferable for the risk averse, more marginal land, or if greenhouse gas emission reduction is required. If returns are considered more important, then domestic stock is dominant. When forage is used efficiently
or landholders co-operate in setting stocking rates, a mobile species (such as kangaroo) should not impact neighbouring properties.
Appendix A

Proofs and Lemmas
A.1 Proof of MSY for alternative harvest function

Theorem 1. When maximising the maximum sustainable yield (MSY) for a species its rate of change determined by

$$\frac{dN}{dt} = \nu N \left(1 - \frac{N}{\kappa}\right) - \gamma \max \{0, N - \eta\}$$

then the MSY is when the population is half the carrying capacity and the harvesting variables are;

$$\gamma = \frac{\nu \kappa}{2(\kappa - 2\eta)}$$

given that \(\nu > 0\) and \(0 < \eta < \frac{\kappa}{2}\).

Proof. For the harvest to be sustainable, implies the rate of change is zero,

$$\frac{dN}{dt} = \nu N \left(1 - \frac{N}{\kappa}\right) - \gamma \max \{0, N - \eta\} = 0.$$

Solving this equation for \(N\) gives the possible solutions,

$$N = 0, \kappa, \text{ or } \frac{\kappa \nu - \gamma \kappa \pm \sqrt{\kappa^2(\gamma - \nu)^2 + 4\nu \kappa \gamma \eta}}{2\nu}.$$

The solutions \(N = 0\) or \(\kappa\) relate to when no harvesting occurs \((N < \eta)\). Therefore, it is only the last solutions that are of interest. Looking at the derivative of the differential equation \((w.r.t. \ N)\) and substituting in the last two equilibriums
gives,

\[
\pm \sqrt{\gamma^2 + \nu \gamma \left(\frac{4\eta}{\kappa} - 2\right)} + \nu^2 + \begin{cases} 
0, & \nu \kappa \pm \sqrt{\kappa (\gamma^2 \kappa + 4\nu \gamma \eta - 2\nu \gamma \kappa + \nu^2 \kappa)} > \gamma \kappa + 2\nu \eta \\
\gamma, & \text{otherwise}
\end{cases}
\]

It is clear that when considering the parameters are only positive, the first of those expressions is negative (as long as \(0 < \eta < \kappa\)), while the second is positive. Hence, the third of the original equilibria is stationary. Substituting the stationary equilibrium solution into the harvesting component and then maximising \(w.r.t. \eta\) gives the following solution for the optimal \(\eta\),

\[
\eta^* = \begin{cases} 
\frac{2\gamma \kappa - \nu \kappa}{4\gamma}, & 0 < \eta < \frac{\kappa}{2} \\
0, & \text{otherwise}
\end{cases}
\]

Substituting this equation back into the harvesting component results in,

\[
H^* = \begin{cases} 
\frac{\nu \kappa}{4}, & 0 < \eta < \frac{\kappa}{2} \\
0, & \text{otherwise}
\end{cases}
\]

Note that the harvest equation is independent of \(\gamma\). Hence, the values of \(\gamma\) and \(\eta\) that maximise the harvest are determined by the other, conditional on \(0 < \eta < \frac{\kappa}{2}\). Therefore, the optimal harvest is when,

\[
\gamma^* = \begin{cases} 
\frac{\nu \kappa}{2(\kappa - 2\eta)}, & 0 < \eta < \frac{\kappa}{2} \\
0, & \text{otherwise}
\end{cases}
\]
A.2 Proposition of Migration equation

Substituting either $\gamma^*$ or $\eta^*$ into the stationary equilibrium solution gives the population for optimal harvest (MSY) as,

$$N^* = \frac{\kappa}{2}$$

A.2 Proposition of Migration equation

**Proposition 2.** Assuming that the ideal free distribution (IFD) holds, then the rate of migration to Region 1 from Region 2 follows the equation

$$\text{Migration} = \frac{1}{\tau} \frac{\omega_1 x_2 - \omega_2 x_1}{\omega_1 + \omega_2}$$

where $\tau$ controls the speed at which the population moves between regions, $x_1$ and $x_2$ and $\omega_1$ and $\omega_2$ are the populations and carrying capacities for Region 1 and Region 2 respectively.

**Proof.** The IFD infers that the ratio between population and carrying capacity in the regions and the overall ratio should be equal.

$$\frac{x_1 + x_2}{\omega_1 + \omega_2} = \frac{x_1}{\omega_1} = \frac{x_2}{\omega_2}$$

Concentrating on Region 1, the above implies

$$x_1 = \omega_1 \frac{x_1 + x_2}{\omega_1 + \omega_2}$$
Formulating the rate of change in migration using a goal-gap formulation gives the following migration equation

\[
\text{Migration} = \frac{1}{\tau} \left( \frac{\omega_1 x_1 + x_2}{\omega_1 + \omega_2} - x_1 \right) = \frac{1}{\tau} \frac{\omega_1 x_2 - \omega_2 x_1}{\omega_1 + \omega_2}
\]

where \( \tau \) controls the speed at which the population moves between regions. Repeating the process for Region 2 shows that there is a conservation of population (emigration equals immigration).

\[\Box\]

A.3 Proof of conditions for greater harvest with finite migration

**Theorem 3.** Let two properties have equal carrying capacity and logistic herbivore growth rates. If they have porous borders, migration of herbivores between the properties is governed by the IFD. When optimal harvesting regimes are used on Property 1 and if \( 2\nu\tau > 1 \) and \( N_2 > \frac{\kappa(4\nu\tau - 1)}{8\nu\tau} \), then

\[H^*_\text{finite migration} \geq H^*_\text{isolation}\]

**Proof.** From Table 4.2 the optimal harvest under isolation is,

\[H^*_\text{isolation} = \frac{\kappa\nu}{4}\]
whilst from Equation 4.15 the optimal harvest under finite migration is,

\[ H^*_\text{finite migration} = \frac{\kappa(2\nu\tau - 1)^2 + 8\nu\tau N_2}{16\nu\tau^2} \]

when \( 2\nu\tau > 1 \). Rearranging \( H^*_\text{finite migration} \) to get the \( H^*_\text{isolation} \) term isolated,

\[ \frac{\kappa(2\nu\tau - 1)^2 + 8\nu\tau N_2}{16\nu\tau^2} = \frac{\kappa(1 - 4\nu\tau) + 8\nu\tau N_2}{16\nu\tau^2} + \frac{\kappa\nu}{4} \]

Now

\[ \frac{\kappa(1 - 4\nu\tau) + 8\nu\tau N_2}{16\nu\tau^2} \geq 0 \]

if

\[ N_2 \geq \frac{\kappa(4\nu\tau - 1)}{8\nu\tau} = \frac{\kappa}{2} - \frac{\kappa}{8\nu\tau} \]

So if as \( N_2 \) no more than \( \frac{\kappa}{8\nu\tau} \) below \( H^*_\text{isolation} \) then,

\[ \frac{\kappa(1 - 4\nu\tau) + 8\nu\tau N_2}{16\nu\tau^2} + \frac{\kappa\nu}{4} \geq \frac{\kappa\nu}{4} = H^*_\text{isolation} \]

Hence,

\[ H^*_\text{finite migration} \geq H^*_\text{isolation} \]

under the given conditions.
A.4 Proof of the value of the Nash Equilibrium Point for a herbivore with finite migration

Theorem 4. Let two properties have logistic herbivore growth, harvest is the MSY rates and porous borders, where migration of herbivores between the properties is governed by by the IFD. Then the Nash equilibrium point for the competitive game is when

\[ N_1 = \frac{\kappa_1 (\nu \tau (\kappa_1 + \kappa_2) - \kappa_2)}{2 \nu \tau (\kappa_1 + \kappa_2)} \]

and

\[ N_2 = \frac{\kappa_2 (\nu \tau (\kappa_1 + \kappa_2) - \kappa_1)}{2 \nu \tau (\kappa_1 + \kappa_2)} \]

Proof. Given that the harvest is the only pay-off and it is MSY, then the pay-off for each property (player) is

\[ u_i(N_1, N_2) = \nu N_i \left( 1 - \frac{N_i}{\kappa_i} \right) + \frac{\kappa_i N_{3-i} - \kappa_{3-i} N_i}{\tau (\kappa_1 + \kappa_2)} \]

Then

\[ \frac{\partial u_i}{\partial N_i}(N_1^*, N_2^*) = \nu \left( 1 - \frac{2N_i^*}{\kappa_i} \right) - \frac{\kappa_{3-i}}{\tau (\kappa_1 + \kappa_2)} = 0 \]

when

\[ N_i^* = \frac{\kappa_i (\nu \tau (\kappa_1 + \kappa_2) - \kappa_{3-i})}{2 \nu \tau (\kappa_1 + \kappa_2)} \]

is the only stationary point of the function. Finally

\[ \frac{\partial^2 u_i}{\partial N_i^2} = -\frac{2 \nu}{\kappa_i} \]
Hence, the criteria are met to conclude that \((N_1^*, N_2^*)\) are the only NEP of the game. 

A.5 Proof of stability for single harvested grazer system without migration

**Theorem 5.** Given grazing system with a single harvested grazer and single vegetation that operate under a linear initial regrowth model,

\[
\frac{dV}{dT} = v \left(1 - \frac{V}{\kappa}\right) - \frac{\zeta VN}{\theta + V} \\
\frac{dN}{dT} = \xi N \left(\frac{\zeta V}{\theta + V} - \chi\right) - \gamma N
\]

, then the equilibrium solution, strictly in the first quadrant, is stable.

**Proof.** Introducing a change of variables to the grazing system such that;

\[V \equiv \theta x, \quad N \equiv \frac{\theta vy}{\zeta}, \quad T \equiv \frac{t}{v}\]

gives the following rescaled system,

\[
\frac{dx}{dt} = \left(1 - \frac{x}{\theta \kappa}\right) - \frac{xy}{1 + x} \\
\frac{dy}{dt} = \frac{\xi y}{v} \left(\frac{xy}{1 + x} - \left(\chi + \frac{\gamma}{\xi}\right)\right)
\]
Another substitution to simplify constants of:

\[ a \equiv \frac{\chi + \gamma}{\zeta}, \quad b \equiv \frac{\xi \zeta}{v}, \quad c \equiv \frac{1}{\kappa}, \quad d \equiv \frac{1}{\theta} \]

results in the rescaled system being rewritten as

\[
\begin{align*}
\frac{dx}{dt} &= \frac{(d - cx)(1 + x) - xy}{1 + x} \\
\frac{dy}{dt} &= by\left(\frac{xy}{1 + x} - a\right)
\end{align*}
\]

The vegetation isocline is when \( \frac{dx}{dt} = 0 \) and implies that is when

\[ y = \frac{(d - cx)(1 + x)}{x} \]

While the herbivore isocline is at \( \frac{dy}{dt} = 0 \) and implies that is when

\[ y = 0 \text{ or } x = \frac{a}{1 - a} \]

This gives us two equilibriums, but only one of interest (strictly in the first quadrant), as a solution without herbivores is not going to be optimal in terms of harvest in a closed system. Therefore the solution of interest is

\[ (x_1, y_1) = \left( \frac{a}{1 - a}, \frac{d(1 - a)}{a} + d - c - \frac{ac}{1 - a} \right) \]
A.5 Proof of stability for single harvested grazer system without migration

Rewriting the scaled system can ease the analysis, so,

\[
\frac{dx}{dt} = f(x)[g(x) - y], \\
\frac{dy}{dt} = by[f(x) - a]
\]

where,

\[
f(x) = \frac{x}{x + 1}, \quad g(x) = \frac{(d - cx)(1 + x)}{x} = \frac{d}{x} + d - c - cx
\]

Hence the Jacobian matrix for the rewritten system is,

\[
\begin{bmatrix}
    f'(x)[g(x) - y] + f(x)g'(x) & -f(x) \\
    byf'(x) & b[f(x) - a]
\end{bmatrix}
\]

Given that \( f(x_1) = a, \) \( g(x_1) = y_1 \) substituting the equilibrium solution \((x_1, y_1)\) into the Jacobian matrix, it simplifies to,

\[
\begin{bmatrix}
    ag'(x_1) & a \\
    by_1f'(x_1) & 0
\end{bmatrix}
\]

The resulting characteristic equation is therefore

\[
\lambda^2 - ag'(x_1)\lambda + abyf'(x_1)
\]

By the Routh-Hurwitz criterion, the coefficients of \( \lambda \) must be positive for the equilibrium to be stable. As \( a, \) \( b, \) \( f'(x_1), \) and \( y_1 \) are all strictly positive (while \(-1 < x_1 < \frac{d}{c}\)), stability is inferred when \( g'(x_1) < 0. \) Simplifying \( g'(x_1) \) and also
reverting back to the original coefficients gives,

\[ g'(x_1) = -c - \frac{d(1-a)^2}{a^2} = -\frac{\theta(\gamma + \xi\chi)^2 + \kappa(\gamma + \xi(\zeta - \chi)^2)}{\theta\kappa(\gamma + \xi\chi)^2} \]

which is strictly negative. Hence, the equilibrium solution strictly in the first quadrant, \((x_1, y_1)\), is stable. \(\square\)
Appendix B

Data Tables
<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
<th>Fecundity</th>
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<tbody>
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<td></td>
<td>Cattle</td>
<td>Kangaroo</td>
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<td>1988</td>
<td>641.67</td>
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</tr>
<tr>
<td>1989</td>
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</tr>
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</tr>
<tr>
<td>1993</td>
<td>580.69</td>
<td>9.24</td>
</tr>
<tr>
<td>1994</td>
<td>620.84</td>
<td>9.53</td>
</tr>
<tr>
<td>1995</td>
<td>510.06</td>
<td>9.47</td>
</tr>
<tr>
<td>1996</td>
<td>419.34</td>
<td>11.75</td>
</tr>
<tr>
<td>1997</td>
<td>481.27</td>
<td>16.87</td>
</tr>
<tr>
<td>1998</td>
<td>572.46</td>
<td>19.88</td>
</tr>
<tr>
<td>1999</td>
<td>641.61</td>
<td>14.82</td>
</tr>
<tr>
<td>2000</td>
<td>709.53</td>
<td>12.96</td>
</tr>
<tr>
<td>2001</td>
<td>794.98</td>
<td>15.76</td>
</tr>
<tr>
<td>2002</td>
<td>698.07</td>
<td>14.43</td>
</tr>
<tr>
<td>2003</td>
<td>605.67</td>
<td>13.02</td>
</tr>
<tr>
<td>2004</td>
<td>692.08</td>
<td>15.67</td>
</tr>
<tr>
<td>2005</td>
<td>721.70</td>
<td>21.16</td>
</tr>
</tbody>
</table>

**Table B.1:** The data relating to the value and fecundity of the different species considered in the model.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green leaf</td>
<td>128</td>
<td>Ratio after running GRASP over original weather data set.</td>
</tr>
<tr>
<td>Green stem</td>
<td>295</td>
<td>Ratio after running GRASP over original weather data set.</td>
</tr>
<tr>
<td>Dead leaf</td>
<td>31</td>
<td>Ratio after running GRASP over original weather data set.</td>
</tr>
<tr>
<td>Dead stem</td>
<td>542</td>
<td>Ratio after running GRASP over original weather data set.</td>
</tr>
<tr>
<td>Litter</td>
<td>400</td>
<td>Value after running GRASP over original weather data set.</td>
</tr>
<tr>
<td>$SW_1$</td>
<td>19.7</td>
<td>Value after GRASP over original weather data set.</td>
</tr>
<tr>
<td>$SW_2$</td>
<td>44.1</td>
<td>Value after GRASP over original weather data set.</td>
</tr>
<tr>
<td>$SW_3$</td>
<td>76.0</td>
<td>Value after GRASP over original weather data set.</td>
</tr>
<tr>
<td>$SW_4$</td>
<td>110</td>
<td>Value after GRASP over original weather data set.</td>
</tr>
<tr>
<td>frost_kill</td>
<td>-5</td>
<td>Lowered so that total tsdm event does not occur.</td>
</tr>
<tr>
<td>target_Util</td>
<td>0.3</td>
<td>Quoted as typical by the MLA (2010).</td>
</tr>
</tbody>
</table>

**Table B.2:** The parameterisations used to initialise the GRASP model that differ from the parameterisation.
Glossary

*ad liberum* - means "as desired".

*AVaR* - average value-at-risk is the expected loss, given the loss in the bottom $\epsilon$ of returns.

*dse* - dry sheep equivalent are the standard animal unit used in Australia.

*GRASP* - is a computer package used to calculate forage and stocking rates in semi-arid regions of Australia.

*grazing pressure* - is the stress on vegetation, and therefore the ecosystem, from animal grazing.

*IFD* - ideal free distribution is an ecological concept implying that animals will move between areas so that the ratio of animals to carrying capacity in each area will be equal.

*macropod* - are marsupials belonging to the macropodidae family, including kangaroos, wallabies, wallaroos and pademelons.

*maximal growth rate* - is instantaneous rate of growth rate under ideal conditions.

*migration* - for the purposes of this thesis it defines the process of mobile herbivores moving from one property or region to another.

*mobile herbivore* - is a herbivore that is not confined to a property, but can move freely between properties.

*MOP* - multi-objective optimisation problem.

*M-V* - mean-variance portfolio optimisation.

*NSI* - non-stock investment is the amount of money invested not including the value of the stock.

*off-take bias* - is where one section of the population is harvested at a higher rate.

*PSPM* - physiological structured population model.
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