Dynamically Reconfigurable Metamaterials using Pneumatics, Flexibility and Structural Nonlinearity

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Iryna Khodasevych.
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Abstract

Metamaterials are composites consisting of sub-wavelength resonant elements aiming to manipulate the material's electromagnetic properties. One of the advantages of artificially created materials over natural materials is the possibility to custom design and tune their properties as one desires. Metamaterials continue to draw the attention of the research community as new and significantly enhanced phenomena associated with them are discovered. Significant effort has also been devoted to integrating them with existing structures for potential applications in sensing, defence and next generation devices. Due to the resonant nature of the metamaterial elements, the desired properties are achieved only within a narrow frequency band. For various applications, it is desirable to be able to tune their frequency response. Although the connection between the modification of geometry of the resonators and resultant variations in their response individually and as an effective media has been extensively studied, the area of dynamic tuning could benefit from further investigation.

The major contribution made by this work includes investigation of real time tuning possibilities and developing new approaches for altering the shapes and orientations of metamaterial resonators, post fabrication, as means of widening flexibility in the design and improving variety of responses. A novel pneumatic switching approach is demonstrated for alteration of the shape of the resonators via addition or retraction of pneumatic elements, as well as application of this method to the realisation of a switchable graded index lens. Further, suspended resonators with mechanical degrees of freedom have been realised which allow shifts in their position and orientation leading to nonlinear effects. A new microfabricated mesh substrate with significantly reduced mass was developed. Embedding resonators into elastic substrates has also been explored for stretching and conformal adhesion purposes. Most of the work is for metamaterials operating in the microwave frequency range (GHz), except elastic metamaterial intended for far infrared (THz) frequencies.

In summary, metamaterial tuning approaches have been extended to dynamic manipulation of both shape and orientation of resonators providing greater flexibility and control over effective material parameters.
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List of Abbreviations

RF - radio frequency

GHz - gigahertz

THz, T-ray - terahertz

MEMS - microelectromechanical switch

SRR - split ring resonator

GRIN - graded refractive index

PDMS - polydimethylsiloxane

SU-8 - epoxy-based photoresist, www.microchem.com

AZ - photoresist trademark, www.azem.com

PMMA - polymethylmethacrylate

HDMS - hexadimethylsilazane

PET - polyethylene terephthalate

PEN - polyethylene naphthalate

VO₂ - vanadium dioxide

InSb - indium antimonide

GaAs - gallium arsenide

BST - bariumstrontium-titanate

UV - ultra violet

FCT - flip chip transfer

THz-TDS - terahertz time-domain spectroscopy

ANSYS HFSS - commercial software trademark
Chapter 1

Introduction

The materials we use have evolved from natural materials to artificially created composites. Each material is characterized by a substantial set of parameters, describing its chemical, mechanical and electromagnetic properties. Various chemical and mechanical processes have been developed to modify and improve on the properties of natural materials in order to better suit the requirements of various applications. Application of external factors results in variation of material parameters. This can be a desirable effect, enabling tuning of a particular parameter, but it may also be undesirable. In natural materials these effects are defined, as well as limited, by the chemical composition and atomic structure of the material.

In a new class of composite materials, called metamaterials, resonant inclusions are placed into a substrate material. The sizes of these inclusions are small enough such that the resulting media can be considered homogeneous, but large enough to manipulate their geometrical shape. They can be placed in a periodic arrangement mimicking a crystal lattice, and hence are often referred to as meta atoms. The resultant material parameters are defined to a lesser extent by the chemical composition and to a larger extent by the geometry and positions of the inclusions. This allows greater control over the material parameters via the design of these inclusions. Further, exotic properties that do not exist in natural materials can be achieved using metamaterials. The unconventional behavior of metamaterials allows creation of conceptually new devices like perfect lenses or even invisibility cloaks, as well as facilitates the discovery of novel physical phenomena. Scaling of these metamaterial inclusion designs allows development of devices operating in the less explored THz frequency range, where natural materials can be nonresponsive.

This thesis focuses on dynamic manipulation of the electromagnetic properties of metamaterials such as effective permittivity, permeability, refractive index and nonlinearity in order to improve the tunability of these parameters with respect to the frequency of the
incident radiation. Novel tuning approaches drawing from multidisciplinary knowledge are pursued to unlock the true potential of these new metamaterials.

Chapter two summarizes various approaches to tuning metamaterials ranging from electronic control, via addition of external devices, to manipulation of substrate properties, via heating or exploiting reconfigurability of liquid crystals, as well as structural reorientation of the resonators.

The third chapter is devoted to demonstrating the possibility of dynamic pneumatic tuning of fishnet metamaterials by changing the resonator geometry post fabrication. Manipulation of the resonators themselves is estimated to be one of the most effective approaches to tunability resulting in the broader operation frequency range. The study demonstrates a method to overcome the challenge of fixed metallic patterns within each unit cell by forming a united conducting pattern out of disconnected complex metallic shapes by using pressure.

In the fourth chapter, a pneumatic switching approach is applied to the realisation of a practical device: a graded index lens. Metamaterials can be made with significant variations of effective refractive index across the sample without adding complexity to their fabrication. The single layer planar design allows ultrathin lenses to be made. The ability to switch off the lens’s focusing action by turning the graded index metamaterial into an ordinary dielectric is demonstrated.

The fifth chapter explores metamaterial resonators suspended with mechanical degrees of freedom. Self action of the currents induced on the resonators lead to their reorientation. In doing so, nonlinear behavior was achieved without introduction of nonlinear media or nonlinear devices. In order to facilitate easier movement as a result of reduced mass, a new fabrication method for realising resonators on novel mesh substrate has been developed.

The sixth chapter explores metamaterials on elastic substrates. Elastic membranes capable of supporting metallic elements are required for further development of the pneumatic tuning concept. Elastic metamaterials were also suggested for strain sensing applications.

The final chapter briefly presents a summary of the findings of this thesis as well as suggestions for future work. Appendix A outlines the procedure for effective electromagnetic parameter extraction from simulated transmission and reflection through the structure. Appendix B describes parabolic reflector design for conversion of point source radiation into plane wave within tight space of parallel plate waveguide setup.
Chapter 2

Review of metamaterials

2.1 Introduction

In this chapter some of the theoretical metamaterial concepts are presented. The origins of various terms used to describe metamaterials are discussed. The mechanisms for achieving negative values of electromagnetic parameters in dispersive media are presented. A number of unusual effects taking place in metamaterial media and some applications taking advantage of these effects are overviewed. Some examples of practically realized metamaterials exhibiting negative values of electromagnetic parameters and using artificially engineered resonators are discussed.

Various approaches to metamaterial resonant frequency tuning are presented. These include incorporation of electronic devices into the metamaterials, manipulating dielectric properties of the underlying substrate via heating or infiltration, as well as mechanical reorientation of the resonators.

2.2 Metamaterial concept

The theoretical foundations for metamaterials were investigated in 1968, when Russian physicists Veselago [1] explored the dispersion equation:

\[ k = \frac{\omega}{c} \sqrt{\varepsilon_0 \mu_0} \]  

(2.1)

where \( k \) is the wave number, \( \omega \) is the frequency of the monochromatic wave, \( c \) is the speed of light and

\[ n = \sqrt{\varepsilon_0 \mu_0} \]  

(2.2)
is the refractive index. Veselago noted that the laws of physics do not forbid negative values of material parameters such as permittivity $\varepsilon_r < 0$ and permeability $\mu_r < 0$. As these parameters are in Eqs. (2.1) and (2.2) as a product, multiplication of two negative values would result in a positive product under the square root and would lead to a real propagation constant. So the name double negative media originated. If only one material parameter has a negative value, the resultant propagation constant is imaginary, which means the wave is evanescent in the media and will not propagate through to the other side. The implication of having negative values of $\varepsilon$ and $\mu$ in Maxwell’s equations and constitutive relations means that vectors of electric ($E$), magnetic ($H$) fields and propagation vector ($k$) form a left-handed coordinate system as shown in Fig. 2.1. Hence, such materials are also referred to as left-handed media. Such orientation of coordinate system leads to negative phase and group velocity as well as energy flow ($S$) in the direction opposite to the direction of wave propagation. This is referred to as backward wave propagation. However, due to energy considerations, simultaneously negative values of $\varepsilon$ and $\mu$ are only allowed in dispersive media where they are functions of the frequency:

$$W = \frac{\partial(\varepsilon\omega)}{\partial\omega} E^2 + \frac{\partial(\mu\omega)}{\partial\omega} H^2$$

(2.3)

where $W$ is the full energy of the electromagnetic field which is required to be positive.

Most natural materials have positive electromagnetic parameters $\varepsilon_r > 0$ and $\mu_r > 0$. However, negative permittivities $\varepsilon_r < 0$ are possible in plasmas. Negative permeability $\mu_r < 0$ is also possible in more complex anisotropic and gyrotropic materials such as plasmas in a magnetic field. However, no isotropic materials with simultaneously negative values of permittivity and permeability are known. These need to be created artificially.

![Fig. 2.1 Orientation of E, H, k, S vectors in (a) normal media (b) left-handed media.](image-url)
The effective permittivity can be engineered by manipulating the material response to the
electric component of the field. Metals can be considered plasmas of electric charges. The
response of metals to an electromagnetic field can be described by the Drude model [2-4]:

\[ \varepsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 - j\gamma\omega} \]  

(2.4)

where \( \gamma \) represents losses. Metals have negative dielectric permittivity for frequencies below
their plasma frequency \( \omega_{pe} \), which is usually very high (in the ultraviolet range). At
microwave frequencies the permittivity of metal can have very large negative values,
resulting in rapid attenuation of the incident wave. For metamaterial operation these values
should be reduced. Since the plasma frequency depends on the concentration of charges, it
can be reduced to the microwave range by 'diluting' the metal into an array of thin wires.
This modified plasma frequency can be altered by modifying the distance between the wires
and their cross section. As electrons are confined to the wires, the increase in self
inductance of the wires allows many orders of magnitude reduction in plasma frequency to
be achieved. However, this response is observed only when the electric field is aligned with
the wires, introducing anisotropy.

Engineering the effective permeability requires a more creative approach since very few
materials respond to the magnetic component of the field. Negative permeability can be
achieved by exciting resonant circular currents in subwavelength arrays of metallic loops,
with an incident electromagnetic wave with the magnetic field passing through the loop. The
effective permeability is described by the Lorentz formula [2, 4, 5]:

\[ \mu(\omega) = 1 - \frac{\omega_{pm}^2}{\omega^2 - \omega_0^2 - j\Gamma\omega} \]  

(2.5)

Here \( \omega_{pm} \) is the magnetic plasma frequency and \( \omega_0 \) is the resonant frequency of the LC circuit
where the current is induced. Both depend on the geometrical parameters and the distance
between the resonant elements. The magnetic plasma frequency is also affected by the
above mentioned electric response of the loops to the aligning electric field. The dissipation
parameter \( \Gamma \) is governed by the resistivity of the metal and should be small to allow the
permeability to reach negative values in a narrow region around the magnetic resonance.
For optimum performance of a metamaterial, the magnetic resonance frequency should be
slightly below the modified plasma frequency of the structure [2] in order to achieve similar magnitudes for the effective permittivity and permeability for better impedance matching.

So, metamaterials are artificially created composite structures, consisting of a set of closely spaced subwavelength resonators. They are often arranged in the form of a periodic lattice, with the repeated element referred to as the unit cell. However, it is a perceived homogeneity that is important rather than periodicity and periodicity is not strictly compulsory. Metamaterials are usually described in terms of effective parameters that represent them as equivalent homogeneous media. For the concept of effective parameters to be valid, the wavelength of the incident radiation should be much larger than the unit cell size, such that the field does not change significantly across each unit cell. The size of the resonator usually should not be larger than λ/10, however up to λ/3 is acceptable in some cases. The periodicity of much less than λ/2, which is usually used in photonic crystals and frequency selective surfaces exploiting Bragg diffraction, makes metamaterials distinctly different structures.

There is some disagreement between the scientists as to what structures to classify as metamaterials. The more strict classification is that the structure should be subwavelength in order to be described in the effective material parameters approach, and should have either ε < 0 or μ < 0 or both in the considered frequency range. A broader classification, however, includes also the structures with the effective parameters not found in nature, such as ε near zero, ε < 1, μ < 1 and abnormally high refractive index composite materials.

A number of unusual effects, not possible in natural materials, can occur in double negative metamaterials. One of them is negative refraction that could allow imaging resolution below diffraction limit under certain combination of metamaterial parameters, referred to as a ‘perfect lens’ [6, 7]. Negative refraction means that when a wave is incident on the interface between double negative and normal media, it would refract in the direction of the mirror image to the one predicted by the Snell’s law (Fig. 2.2). It should be noted that the phenomenon of negative refraction is not limited to metamaterials and occurs under specific conditions in structures with periodicity of λ/2 called photonic crystals. Another application made possible with metamaterials is an invisibility cloak [8]. Among more practical applications are metamaterial sensors, filters and absorbers.
2.3 Examples of metamaterials

The interesting derivations of Veselago [1] were long forgotten as no known natural material possessed simultaneously negative values of permittivity and permeability. Decades later in 2000, Sir John Pendry studied wire media behaving as a dispersive plasma and displaying negative permittivity [3, 9] and came up with the idea to artificially create media with negative permeability using split ring resonators [10] (Fig. 2.3 (a)). Double negative metamaterials consisting of an array of sub wavelength metallic split ring resonators and wires were experimentally demonstrated shortly thereafter [11, 12]. This is when the term "metamaterial" originated. It should be noted that similar structures, like omega particles and metallic helices, were investigated earlier using effective media approach and were referred to as composite media [13, 14]. However, while they possesed distinctive dispersive effective parameters, they lacked the regions of negative parameter values characteristic of metamaterials in the strict definition.

Fig. 2.2 Diagram of wave propagation showing negative refraction of the orange ray.
A split ring resonator acts as a resonant circuit when an electromagnetic wave falls on it with the gap acting as a capacitor and metalized ring acting as inductor. Since the first metamaterial was demonstrated for microwave frequencies, a number of different designs have been proposed to improve performance and enable the metamaterials to operate at higher frequencies. One of the first metamaterials resonating at terahertz frequencies was demonstrated by Yen et al. [15]. A number of various modified split ring resonator shapes have been investigated since [16], including simplified U-shape resonators [17-19] (Fig. 2.3 (b)). All of the THz structures were planar due to fabrication requirements. Generally these responded only to the electric component of the incident wave due to the illumination being normal to the substrate and hence the geometry of the resonators. To further advance metamaterials to near infrared optical frequency range, cut-wire pairs [20-22] (Fig. 2.3 (c)) and fishnet structures [23-34] (Fig. 2.3 (d)) were designed. The fishnet metamaterial consists of patterned metal layers, consisting of slab pairs and long wires, separated by a dielectric spacer. The fishnet is a planar structure, and can exhibit negative refractive index under normal illumination by an incident wave. The corresponding shapes with the underlying substrate represent the unit cell which is periodically repeated in two or three dimensions.

### 2.4 Tunable metamaterials

As the principle of metamaterial operation is based on the resonant response of the constituent elements, it exhibits unusual properties in a very narrow frequency band. Thus, for many of the proposed applications, it will be desirable to reconfigure the properties of the metamaterial to shift the frequency at which this resonance occurs.

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Fig. 2.3 Examples of metamaterial resonators: (a) split ring resonator, (b) U-shape resonator (c) cut-wire pair (d) fishnet structure.
2.4.1 Tuning by external devices

One of the approaches to tunability is to introduce external devices into the resonator. The simplest way is to introduce additional capacitors of different capacitance values into the resonator loop [35]. Placing a varactor diode across the gap of the resonator allows dynamic control over the resonant frequency via adjustment of the biasing current [36-39]. This method has also enabled nonlinearity in metamaterials [40-45]. The introduction of photodiodes into the resonator has enabled light controllable metamaterials [46-49].

Introduction of discrete electronic devices into the resonators limits the scalability of the material to higher operating frequencies, impacts manufacturability and introduces a lot of biasing wires into the structure. Also, there is a variation of parameters across different devices, which can cause unwanted variation in resonant frequency across the metamaterial elements. However, this method usually achieves a very broad tunability range.

2.4.2 Tuning by variation of substrate permittivity

Another approach to achieve tunability is to adjust the resonance of the split ring by changing the substrate permittivity since the ring is sensitive to the material in the gap. Tuning of the substrate permittivity can be achieved by heating the material [50, 51]. A vanadium dioxide (VO₂) substrate was used in Refs. [52-54] to trigger an insulator to metal transition and alter the conductivity of this semiconducting substrate, which can be achieved by heating. Usually heating to significant temperatures is required. Optical control of photoexcitation in silicon via laser irradiation is another option [55]. The change of substrate state in this case is temporary and once the illumination is removed, the structure reverts to its original condition. Electrically tunable substrates can also be used. Some examples include ferroelectric bariumstrontium-titanate (BST) [56], semiconducting gallium arsenide (GaAs) [57, 58] and indium antimonide (InSb) [59, 60].

Each of the above techniques requires substantial modification of the material surroundings or the use of additional complex equipment. The effect of changing the substrate permittivity in a very wide range from 2 to 14 has been numerically studied in Ref. [61]. However, the extent to which the dielectric properties of one type of substrate can be varied is just a fraction of this range and this limits the range in which practical structures can be tuned.
Tuning by addition of a thin film [62] or fluid [63] on top of the resonator layers has been explored for sensing applications. A number of metamaterials that can be tuned by infiltration with liquid crystals have been demonstrated. Liquid crystals can be reoriented by electrical or optical control, which causes a change of their refractive index [64]. Liquid crystals are usually used in fishnet metamaterials intended for optical frequency ranges. Due to fabrication and size limitations in optical fishnets, liquid crystals are usually used either as a superstrate layer, over the top of the structure [65], or the fluid is only infiltrated within the hole areas between the unit cells [66]. However, infiltration of whole area between fishnet layers has been theoretically investigated for optical frequencies in Ref. [67], but has only been practically demonstrated at lower operating frequencies in Ref. [68]. The possibility of ultrafast optical modulation of metamaterials with the help of liquid crystals has been proposed in Ref. [69]. Also, liquid crystals can be used to induce nonlinear responses in metamaterials [70, 71]. The application of liquid crystals to tuning other metamaterials shapes like split ring resonators [72, 73], omega-type inclusions [74, 75] and s-type inclusions [76] for GHz frequencies and nano strips [77] and nano split ring resonators [78] for infrared frequencies has also been investigated. The tunability range achieved by changing the substrate permittivity usually does not exceed 20%.

### 2.4.3 Structurally tunable metamaterials

Structural tuning of metamaterials involves changing the mutual position of metamaterial elements, resulting in a resonant frequency shift because of the changed coupling between the elements [79, 80]. The effect of the lattice order on the metamaterial parameters has been theoretically studied by [81]. Experimental demonstration of a shift in resonant frequency as a result of changing the spacing between the split ring resonator layers has been reported by Shadrivov et al. [82]. Tuning by lateral displacement of metamaterial layers containing split ring resonators with respect to each other was theoretically proposed by Lapine et al. [83] and further analysed by Powell et al. [84]. Another work by Wang et al. investigated shifts in both lateral directions [85], suggesting both coarse and fine tuning options. Effects of separation and relative orientation have been experimentally studied at THz frequencies by Feth et al. [86]. Heat induced bending of the alternate strips containing resonators was shown to affect transmission through the structure at infrared frequencies [87].
Another approach to structural tuning is relative rotation of the layers containing resonators. As a result of coupling, symmetric and anti-symmetric modes can be induced on the twisted resonator pairs leading to a splitting of the resonant frequency. The effect has been theoretically predicted by H. Liu et al. [88] and further theoretically and experimentally investigated by N. Liu et al. [89, 90]. The results are summarized in Ref. [91]. Crossing of the modes was observed and theoretically explained in Ref. [92]. The effect of rotation on nonlinear response of the resonators was studied in Ref. [93]. Twisted resonators were proposed as candidates for the realisation of a sub-wavelength motor due to capability to provide tunable torque when illuminated with electromagnetic radiation [94].

More complex ‘pop up’ structures, consisting of rotating individual electric split ring resonators, have shown previously absent magnetic type resonant responses [95-97]. Simultaneously negative electric and magnetic response has been achieved using only split ring resonator shapes by leaving half of them in the plane and rotating the other half out of the plane with the help of cantilever legs. The angle of rotation could be controlled through the temperature of the sample. However, once rotated, the structure could not be reversed to the unrotated state. In the work of Bouyge et al. [98] split ring resonators partially attached to substrate, and serving as cantilevers, were electrostatically bent in order to tune their resonance. The resonators were coupled to transmission lines in this case.

The practical realisation of tuning via shifting metamaterial layers was demonstrated and involved a mechanical translation stage [99]. Moving parts of the resonator, rather than whole resonator relative to each other, enabled manipulation of the unit cell element itself [100]. Each side of a double gap split ring resonator was patterned on a different substrate layer using micromachining techniques. When the layers were moved relative to each other, the gaps of the split ring resonator increased, leading to a shift of resonant frequency. Shortening the gap by moving the sides of the split ring towards each other until they came into contact was also demonstrated, resulting in elimination of the resonance. It should be noted that only few of the demonstrated structures were dynamically tunable.

The approach of structural tuning, and in particular manipulation of the shape of the resonator itself, appears to allow the most substantial change to the metamaterial structure. Being one of the most recent developments in the field of metamaterial tunability, this approach could benefit from further investigation. Drawing on the knowledge from other disciplines often results in creative and unconventional structures. In particular, applying
pneumatic approaches to metamaterials would enable realization of novel and dynamically tunable metamaterial structures.
Chapter 3

Tuning metamaterials by pneumatic switching at GHz frequencies

3.1 Introduction

This chapter broadens the concept of structural tuning of metamaterials from mechanical translation of the elements relative to one and other, to the mechanical transformation of the elements constituting unit cells themselves. The tuning will be demonstrated on a fishnet metamaterial structure. The work draws on the concepts of microelectromechanical switching and pneumatic microoptics.

Previous theoretical investigations have shown that the geometry of a resonant metamaterial unit cell has a substantial effect on the resonant frequency of the structure. Theoretical study of the so called fishnet structure by Kafesaki et al. [23] analytically explored the effect of various parameter modifications on its resonant frequency. Later Ding et al. [101] conducted a parametric study of a polarization independent fishnet with respect to various geometrical parameters, such as fishnet slab width, neck width and thickness of the dielectric spacer. The results suggest that changing fishnet slab width results in the most substantial shift of the resonant frequency.

The challenge of applying this approach to tunability is the permanent nature of metallic patterns on the substrate after the fabrication is completed. Thus, in order to change the frequency of operation, another pattern with different dimensions must be fabricated, unless another tuning method is applied. However, it is known that metallization patterns can be manipulated with the help of microelectromechanical (MEMS) switching [102]. For some of the first applications of MEMS to metamaterials, switches were placed over split ring resonator loaded transmission line [103, 104]. Employing MEMS switches as a part of
split ring itself by placing it across the additional gap was demonstrated by Hand et al. [105]. Shifting the resonant frequency of a split ring between two states, as well as shortening the gap to eliminate resonant response was achieved. The use of MEMS technology for more sophisticated tuning of metamaterials has been theoretically proposed by Ekmekci et al. [106] and practically realized and further analysed by He et al. [107, 108]. In these works, the gaps on a multigap split ring resonator were selectively closed using MEMS switches. Multiple cantilever beams across the same gap were the most advanced application of the MEMS switching to split ring resonators [109-112]. Frequency shifts in the range of 1-2 GHz were achieved by these methods. A magnetically actuated, flexible cantilever which was placed over the split ring resonator gap was proposed as a structure that would not require a biasing network and was theoretically investigated for THz frequency range [113]. Microelectromechanical switching was applied to the split ring metamaterial resonator shapes only, and a small number of resonators were involved. A larger area electrostatic actuation of multiple patches on a single membrane was demonstrated by Sterner et al. [114] for beam steering with a high impedance surface. Each membrane contained a small number of elements. In order to operate over a larger area multiple switches and membranes were used. At the time of writing, it would appear that no work has been reported on reconfigurable microwave fishnets. Although microelectromechanical switching is commonly used for frequency selective surfaces, their principle of operation and periodicity relative to wavelength are distinctly different, making them another class of structures.

It is possible that more substantial variations in geometry of a metamaterial unit cell can be achieved by adding extra metallic elements where desired to an already existing structure through pneumatic actuation. By ensuring good electric contact, initially separate elements can be united into a more complex conducting structure. The significant advantage of pneumatic switching as compared to MEMS switches is that metallic elements of arbitrary shapes and numbers can be combined using the same or different actuating membranes. This can result in a wider range of tuning effects on the structure. A pneumatic actuation mechanism can occupy less space than MEMS switches, since pneumatic chamber walls and membranes can be made only a few micrometers thick. Hence, pneumatics is more suitable for scaling to higher operating frequencies when the space within unit cell and between neighboring cells becomes limited. Pneumatic structures can also maintain their state for some time when disconnected from the vacuum source, without requiring constant power.
supply, if a leak free valve is added. However, the major advantage of pneumatic actuation is the elimination of any metallic biasing network in the structure, which can interfere with the metamaterial operation and may be prone to damage in a harsh environment.

Pneumatic operation is widely used for pumping fluids through channels in microfluidics [115-117], as well as for reorienting mirrors [118] and tuning lenses [119, 120] in microoptics. However, the application of pneumatic switching to microwave elements is novel. The question of what is the pressure required to ensure satisfactory electrical contact between metallic elements is yet to be investigated. This chapter presents the experimental demonstration of a pneumatic switching technique on a fishnet metamaterial for the first time.

In this case, pneumatic actuation is used to switch the resonant frequency of the fishnet metamaterial between two operating states by changing the metallization pattern within its unit cell with the addition of extra elements to an initial existing structure. The approach could potentially be extended to multifrequency switching, however, continuous tuning would be challenging.

3.2 Pneumatically reconfigurable fishnet metamaterial

3.2.1 Design and assembly of the layers

The proposed pneumatically reconfigurable fishnet structure consisted of three layers. The central layer was the well known fishnet structure [23, 101], and its unit cell with all the geometrical parameters is shown in Fig. 3.1 (a). The fishnet used in this study was polarization independent due to the symmetric square shape of its slab of size $p$. It was assumed to be periodic in the x-y plane. The structure was illuminated by an electromagnetic plane wave falling perpendicular to it. The fishnet was fabricated by a conventional printed circuit board processes on both sides of a FR4 substrate with $t = 0.4$ mm thickness, with permittivity $\varepsilon_r = 4$ and loss tangent $\delta = 0.013$. The metal traces were formed from copper of 17 $\mu$m thickness.
Fig. 3.1 (a) Unit cell of the middle fishnet layer with the following parameters: continuous wire width $w = 1$ mm, slab length and width $p = 11.5$ mm, thickness of a dielectric substrate $t = 0.4$ mm, cell dimensions $15 \times 15$ mm (b) Unit cell of the additional square ring elements with the following parameters: size of a square ring side $ps = 13.5$ mm, width of a square ring $ws = 1$ mm, thickness of a substrate $ts = 0.1$ mm. (c) Multilayered pneumatic unit cell showing layer alignment. Photographs of the fabricated layers: (d) middle fishnet layer (e) layer with the square rings (f) staggered arrangement of the layers (g) sealed pneumatic fishnet.

Fig. 3.1 (b) depicts the unit cell of the additional elements used to modify the unit cell geometry of the fishnet. These additional elements were square rings with a side length larger than the width $p$ of the central square of the fishnet element. Two identical layers
with square rings patterned on one side only were fabricated using a flexible substrate (Rogers Ultralam 3850) with a thickness of \( t_s = 0.1 \) mm, dielectric constant \( \varepsilon_r = 2.9 \), loss tangent \( \delta = 0.0025 \), and a copper thickness of 17 \( \mu \)m. Photographs of the fabricated periodic structures of Figs. 3.1 (a) and (b) are shown in Figs. 3.1 (d) and (e) respectively.

Figs. 3.1 (c, f) depict the alignment of the layers for assembly of the reconfigurable fishnet structure. The markers were added to the fabrication mask and all the layers were cut to the same size using these markers, which ensured layer alignment when the layers were placed together. The middle layer with the fishnet from Fig. 3.1 (a) was placed between two layers patterned with the square rings oriented such that metallic sides faced the fishnet and rings were positioned symmetrically around the fishnet slabs. The three layers were then sealed together around the edges of the sheets using adhesive tape as shown in Fig. 3.1 (g). No separators were used between the layers. The layers serving as membranes as well as the FR4 substrate were fairly thin and semi-transparent when viewed with light transmitting through them, so the alignment was further confirmed by viewing through the structure. An outlet tube was connected to one corner of the structure and then to a vacuum pump for pneumatic operation. Due to the flexibility of the top substrate, the inhomogeneity introduced by the tube subsided within 1.5 cm of the connection. The overall dimensions of each layer were close to the size of an A4 sheet, containing 14 unit cells in the \( x \) direction and 19 unit cells in the \( y \) direction. The dimensions of the structure's unit cell were 0.36 \( \lambda \) in transverse direction and 0.024 \( \lambda \) in the direction of propagation.

### 3.2.2 Principle of operation

The fishnet was reconfigured via pneumatic actuation of the layers with the square rings using a vacuum pump. In the open switch state when no vacuum was applied an air gap existed between the square rings and the fishnet layer, as the flexible substrates were not perfectly flat. This small gap was sufficient to break electrical contact between the metallic elements. The combination of the large area and the small thickness of the structure aided in achieving a reasonably homogeneous air gap. The square rings were assumed to be at an approximate distance \( g = 0.2 \) mm from the fishnet (Fig. 3.2 (a)). In this open state, the only structure that exhibited a simultaneous electric and magnetic resonant response in the frequency range of interest was the middle fishnet layer. The square rings in isolation resonated at a frequency which was well outside of the considered range.
Fig. 3.2 Reconfigurable pneumatic fishnet structure in (a) switch open ($g = 0.2$ mm) and (b) switch closed ($g = 0$ mm) states.

By applying vacuum, the layers were forced together to form the closed switch state (Fig. 3.2 (b)), creating metallic contact between the square rings and the fishnet ($g = 0$ mm) in the neck areas. The square rings became part of the fishnet, changing the geometry to be equivalent to a fishnet with a larger metal slab size, and hence a different resonant frequency. The square rings became involved in conducting the antiparallel currents induced in the fishnet slabs. This resulted in a shift of the resonant frequency of the closed switch structure. The proposed fishnet did not have air holes in the substrate between the neighbouring unit cells as in a traditional fishnet. The absence of the air holes allowed addition of metallic elements to the surface and involvement of a larger dielectric area in capacitive behavior.

The two actuation states correspond to different unit cell geometries and thus different resonant frequencies of the structure.

\subsubsection{Transmission simulations}

In order to understand the effect of additional elements on transmission and tunability of the fishnet the performance of the fishnet structure with and without pneumatic elements
as well as slotted pneumatic fishnet having gaps in its slab were numerically studied and compared. All simulations were carried out using ANSYS HFSS software based on the full-wave finite element method. All structures were simulated as periodic and infinite in the x and y directions. A lossy metal model of copper with a conductivity of $5.8 \times 10^7 \text{ S}\text{m}^{-1}$ was assumed.

**Transmission through the fishnet without pneumatic elements**

First, a parametric study of tunability with respect to the slab width of the fishnet structure without pneumatic elements was conducted. Simulation results for the transmission through the fishnet structure from Fig. 3.1 (a) when varying slab width are shown in Fig. 3.3. As the width of the fishnet slab was decreased from 14.5 mm to 3.5 mm, which are the extreme values allowed for the cell size, the resonant frequency increased from 6.17 GHz to 12 GHz. Although varying $p$ in the region of smaller values produced a larger frequency shift, the resonant response of the structure became weaker and eventually disappeared at $p = 3.5$ mm. It can be also noted that at larger values of $p$ there is higher conductor loss.

It can be seen that by varying fishnet slab widths, a theoretically predicted tunability range of almost 100% could potentially be achieved, which is about five times wider than can be achieved by changing the electromagnetic properties of the substrate. This means that structural tuning of the geometrical parameters of the fishnet could be one of the most effective ways to change the operating frequency once the method is practically implemented.

**Transmission through the pneumatic fishnet**

It was predicted in the previous section that the resonant behavior of the fishnet should be modified significantly when the size of its slab was changed. It was proposed to achieve the required modification of the slab size using additional pneumatically actuated square ring elements placed on both sides of the central fishnet layer. It was expected that in switch open state with a gap between the ring and the fishnet assumed to be $g = 0.2$ mm the resonant response of the pneumatic structure should be similar to the response of the fishnet of the corresponding size without the additional ring elements as shown in Fig. 3.3. The central fishnet layer slab size $p = 11.5$ mm was used in the simulations. In the closed state with $g = 0$ mm the square rings were expected to form a connected conducting
Fig. 3.3 Transmission characteristics of fishnet structures with different slab widths $p$ at fixed other parameters.

structure equivalent to the fishnet with a larger slab size of $p = 13.5$ mm, coinciding with the outer size of the square ring element $ps$. The transmission through the fishnet depicted in Fig. 3.2 was simulated using parameters described in Figs. 3.1 (a, b).

Fig. 3.4 shows the simulated transmission through the reconfigurable pneumatic fishnet structure in switch open and switch closed positions. Two different resonance frequencies can be seen. The 7 GHz frequency corresponds to the switch open position and the fishnet slab width $p = 11.5$ mm. The 6.25 GHz frequency corresponds to the switch closed position when metal squares touched the fishnet and effectively increased its slab width to $p = 13.5$ mm. The results confirmed that it is possible to switch the fishnet's resonant frequency with the help of addition and retraction of pneumatic elements to the central fishnet layer. Addition of the square rings did not interfere with the operation of the fishnet in the switch open state.
Fig. 3.4 Transmission through the reconfigurable pneumatic fishnet structure in switch open ($g = 0.2 \text{ mm}$) and switch closed ($g = 0 \text{ mm}$) states.

**Effect of the gap between the pneumatic elements and the resonator**

The resonant response of the structure in the open switch state should be essentially the same as the fishnet layer without the metal rings present. It is important that the response of the structure is not highly sensitive to the spacing between the layers as this may not be possible to control precisely. It is expected that, beyond a certain separation, the distance between the ring and the fishnet will have a minimal impact on the response. To test this, simulations were done with incremental separations between the fishnet and the metal rings as shown in Fig. 3.5. Fig. 3.5 demonstrates that the distance $g$ between the layers with square rings from the main fishnet had a relatively minor effect on the resonant frequency of the structure. A resonant dip at around 7 GHz is seen in all curves. For smaller gaps the resonant dip deviated up to 0.1 GHz from the 7 GHz value due to increased coupling between the elements, however, as gap increased the resonant frequency tended to remain the same. The position of the transmission peak at 7.2 GHz, where the structure is usually intended for operation, remained relatively unaffected. Low sensitivity of the structure to the layer spacing is an advantage for frequency tuning because it eliminates the need for precise control of the gap size.
In spite of the observation that gap had very minor effect on the resonant frequency of the structure, Fig. 3.5 shows that the gap had significant effect on the transmitted power before and at the resonance. Controlling value of \( g \) may be useful for tuning the transmitted power level.

**Transmission through the square ring elements**

It was assumed that the square rings themselves should not resonate within the same frequency range as the resonance of the fishnet. To test whether this is true, simulations have been done for square rings only with the central fishnet structure removed. The layers were spaced at 0.4 mm from each other which would correspond to the switch closed position. The results are presented in Fig. 3.6 and no resonance is evident. The downward slope of the transmission curve towards the lower frequencies is also followed by the pneumatic fishnet and could be attributed to the filtering action of the wire-like grid array. It also indicates that the rings approach resonance shortly below 5 GHz, which was confirmed by simulations not presented here. The absence of any resonance of the rings over the range of 5 to 7.5 GHz proves that the rings themselves did not resonate in this frequency range and hence should not interfere with the operation of the fishnet in its open state.
Fig. 3.6 Transmission through the square ring layers only spaced at 0.4 mm (corresponding to switch closed position \( g = 0 \) mm).

**Transmission through a slotted pneumatic fishnet**

It is possible that the layers of pneumatic fishnet could potentially be fabricated with errors in sizes. This could result in a small gap being left between the ring and the central fishnet slab in the lateral direction. Since the ring would be short circuited to the fishnet ‘neck’ region in any case, it is expected that such a gap would have minimal impact on the transmission response of the fishnet in its closed state. To test whether the presence of such a gap would impact the transmission response of the fishnet, a structure with a gap along the perimeter of the slab, shown in Fig. 3.7, was also studied. Fig. 3.8 shows the transmission through the slotted fishnet formed by the middle fishnet layer and the square rings with the inner side size larger than the fishnet slab size, forming a 0.1 mm gap along the perimeter in the switch closed state. In the switch open position the resonant response was at the same 7 GHz as for the previous case when square ring sizes matched the fishnet slab size and did not form a gap. In the switch closed position, however, resonance shifted from 6.25 GHz to lower 6 GHz. This could be explained by an extra capacitance introduced by the gap. This result indicates that it is important to design the ring such that fabrication errors would not result in a gap and thus a small overlap between the ring and fishnet slab should be assured.
Fig. 3.7 Slotted pneumatic fishnet unit cell in switch close state \((g = 0 \text{ mm})\) with gap between square ring and fishnet \(s = 0.1 \text{ mm}\).

On the other hand, this outcome could be intentionally utilised by leaving large enough gap to avoid overlap.

The introduction of this small gap appears to have had a significant effect on the closed state resonance frequency. It is expected that the resonant frequency would depend on the width of this gap. In order to estimate the effect of the gap on the resonant frequency, simulations were conducted with incrementally increasing gaps. In each simulation the outer square ring size was held constant at \(ps = 13.5 \text{ mm}\) and hence increasing the gap resulted in narrowing of the ring. The results of these simulations are presented in Fig. 3.9, which shows even further shift of the resonant frequency towards the lower values for larger gaps, however, the resonant response disappeared for very large gaps.

The results suggest that the fishnet slab does not need to be continuous in order to operate as a resonant element. Having a small gap along the perimeter of the central slab even improves the tunability range. So, potentially, multiple distinct rings might be used in the structure for multi frequency tuning via switching of individual elements.
Fig. 3.8 Transmission through the slotted pneumatic fishnet structure with gap between square ring and fishnet $s = 0.1$ mm in switch open ($g = 0.2$ mm) and switch closed ($g = 0$ mm) states.

Fig. 3.9 Transmission through the slotted pneumatic fishnet for various gaps $s$ between square ring and fishnet (switch closed position ($g = 0$ mm)).
**Comparison of tunability ranges for various fishnets**

Fig. 3.10 summarizes the results from Figs. 3.3, 3.4 and 3.8 representing transmission through the fishnet without pneumatic elements (Fig. 3.3), the reconfigurable pneumatic fishnet with addition of pneumatic elements not resulting in the gap along the perimeter of the central fishnet \( (s = 0) \) (Fig. 3.4) and the slotted version of pneumatic fishnet with the gap \( (s = 0.1 \text{ mm}) \) (Fig. 3.8) along the perimeter correspondingly. Comparison between these three structures can be drawn. Theoretically varying the slab width of the fishnet without the pneumatic elements predicted the lowest amount of the frequency shift.

The resonant frequencies for continuous fishnets were 6.95 GHz for \( p = 11.5 \text{ mm} \), corresponding to switch open state of pneumatic version and 6.3 GHz for \( p = 13.5 \text{ mm} \), corresponding to switch closed state. Addition of pneumatic elements increased the frequency shift from 6.25 GHz to 7 GHz for the corresponding switching states. The lower

![Graph showing transmission through fishnets](image-url)

**Fig. 3.10** Comparison of transmission through the fishnet without pneumatic elements, the pneumatic and the slotted fishnet structures. Solid lines are for switch open states and dashed lines are for switch closed states.
transmission level through pneumatic fishnet in switch open state can be associated with the gaps between the layers possibly causing multiple reflections rather than with the loss in the substrates serving as membrane, as no reduction in transmission level is seen in the case of switch closed state. Leaving gaps along the perimeter of the fishnet slab increased the frequency shift as a result of switching even further from 6 GHz to 7 GHz.

It can be concluded from the comparison that the pneumatic fishnet had a wider spread between the resonant frequencies in the switching states representing different slab sizes than the fishnets of the corresponding sizes without pneumatic elements. Using a slotted pneumatic fishnet with the gaps along the perimeter further improved the predicted tunability range.

Although in this study only two particular states were presented as an example, the trends can be applied to other combinations of slab and square sizes utilizing full potential of the possible tunability range. In all further studies the original pneumatic design with $s = 0$ will be used.

### 3.2.4 Field and current distributions

It is expected that investigation of the field and current distributions on metallic surfaces may provide further insight into the operation of the structure. To visualise the fields and currents, the geometry of Fig. 3.2 was simulated in both open and closed configurations. With the fishnet configured in the open position, the fields were analysed at a frequency of 7.2 GHz, the frequency just above the resonance, corresponding to maximum transmission. Figs. 3.11 (a, b) show the distribution of the magnitude of the total electric field and the vector current density on the metal surface of the middle fishnet structure at zero phase in the switch open position at 7.2 GHz. The maxima of the induced electric field were located at the corners of the structure. The currents flowed through the middle of the fishnet slab and through the necks in the opposite direction. The points where the currents met served as an origin of the displacement current through the capacitive dielectric spacer to the second fishnet layer where similar currents flowed in direction antiparallel to the first layer thus closing the inductive-capacitive loop around the magnetic field as shown in Figs. 3.11 (i, j).
Fig. 3.11 Electric field and current distributions for middle fishnet (a, b, e, f) and square rings (c, d, g, h) in switch open (a, b, c, d) and switch closed (e, f, g, h) positions at frequencies just above the corresponding resonances. Magnetic field (i) and current (j) distributions in y-z plane (side view) for the structure in switch open position.

Figs. 3.11 (c, d) show the response of the square rings, suspended above the fishnet, also at 7.2 GHz. Some charges were induced along the inner contour of the ring because of the proximity to the middle metal layer and were not caused by the resonance of the ring. Weak non-resonant current flowed on both layers of rings in the same direction.

To examine the fields and currents on the structure in the closed position, the shift in resonant frequency was taken into account and the analysis was performed at 6.4 GHz, the frequency just above the new resonance. Once the contact between metallic elements was established in the switch closed position, the maximums of the electric field shifted from the corners of middle fishnet layer towards the corners of the square rings as shown in Figs. 3.11
(e, g). Such a field distribution and resonant frequency corresponds to a fishnet structure with larger slab size, confirming the hypothesis that the square rings behave as part of the fishnet. The current on the central fishnet (Fig. 3.11 (f)) was similar to the open switch case, however, the current on the square ring (Fig. 3.11 (h)) was significantly different. Similarly to the electric field, the current on the ring appeared to be a continuation of the current distribution on the middle fishnet, having large values on the side and neck areas. The current on the second layer of the square rings in this case flowed in the direction antiparallel to the current of the first layer of the rings in agreement with the currents on the corresponding side of the middle fishnet layer.

Field and current distributions on the surfaces of middle fishnet and square rings demonstrated no involvement of the rings in the fishnet operation in switch open position and rings becoming part of fishnet in switch closed position.

3.2.5 Effective parameter extraction

The effective electromagnetic parameters of the reconfigurable pneumatic fishnet structure were extracted from the simulation data using the procedure explained in Appendix A. Effective electromagnetic parameters imply that the structure is three dimensional and can be considered as a bulk material. As fishnet is an anisotropic structure, the presented effective parameters are valid strictly along the direction of normal incidence of radiation. For the purpose of parameter extraction the structure was assumed to be periodic in the direction of propagation with period $z = 3$ mm. This period defined the distance to which transmission and reflection coefficients were de-embedded in simulations. Choosing another periodicity would alter the values for effective electromagnetic parameters.

The real part of the effective permittivity is plotted in Fig. 3.12 and shows the Drude type antiresonant response of thin wires of the fishnet, and is negative in the frequency range up to the plasma frequency of the wire. At this plasma frequency, the effective permittivity crosses zero and becomes positive at higher frequencies. The real part of effective permeability response of the fishnet slabs, shown in Fig. 3.13, is of Lorentz type and is negative in the narrow frequency range around the resonant frequency of the structure. The refractive index shown in Fig. 3.14 has negative values for both switch positions.
Fig. 3.12 Effective permittivity extracted from simulations in switch open ($g = 0.2$ mm) and switch closed ($g = 0$ mm) positions.

Fig. 3.13 Effective permeability extracted from simulations in switch open ($g = 0.2$ mm) and switch closed ($g = 0$ mm) positions.
Fig. 3.14 Effective refractive index extracted from simulations in switch open \((g = 0.2 \text{ mm})\) and switch closed \((g = 0 \text{ mm})\) positions.

The effective parameters of the reconfigurable fishnet structure show resonances and negative values at two distinct frequencies corresponding to switch open and switch closed positions. This shows that an apparent negative refractive index should be achievable and that this should be tuned in frequency when using pneumatic actuation.

3.2.6 **Measurements of resonant frequency shift**

Transmission through the pneumatically reconfigurable fishnet structure was measured in a free space setup using two microwave horn antennas connected to the input and output of a Wiltron 37269A vector network analyzer (Fig. 3.15). The bottom antenna, hidden under the foam cube supporting the fishnet, was the transmitting antenna and the top horn was the receiving antenna. The area around the periphery of the fishnet was masked using a metallic aperture to prevent diffraction around the outside of the sample. One corner of the fishnet was connected to a vacuum pump via flexible tube for pneumatic operation.
Fig. 3.15 Reconfigurable pneumatic fishnet in free space measurement setup.

**Switch open position**

Fig. 3.16 (a) shows the simulated and measured transmission through the reconfigurable fishnet structure in switch open position. Good agreement between the predicted and measured positions of resonance is observed. The measured structure resonated at a frequency close to 7 GHz. The slight deviation of the experimentally determined resonant frequency from the theoretically predicted value can be explained by a possible misalignment between the layers of the structure and variations in size due to fabrication errors. The higher transmission values below resonance for all experimental results can be explained by finite size of the fabricated sample as compared to infinite simulated structure. The additional resonant dip seen at 5.7 GHz originates from the resonance of the rings only. As discussed in section 3.2.3, the rings were expected to resonate shortly below 5 GHz and out of the considered range, however, placing the central fishnet layer between them has somewhat altered their resonant frequency. Although the structure is traditionally operated in transmission mode, the simulated reflection is also shown in Fig. 3.16. The result confirms the hypothesis that when the vacuum is not applied, there is not sufficient electrical contact.
Fig. 3.16 (a) Simulated and measured transmission through the reconfigurable three layered fishnet structure in switch open position \((g = 0.2 \text{ mm})\) and (b) in switch closed position \((g = 0 \text{ mm})\). The dotted line represents the simulated reflection.
between the metallic elements and the resonant frequency corresponds to the fishnet slab size \( p = 11.5 \text{ mm} \).

To ensure a total absence of contact between the layers the structure can be inflated by applying positive pressure. Even though the desired operation was already achieved without inflation, further tests with a slightly inflated structure were conducted. The distance between the layers with square rings and the main fishnet was assumed to be \( g = 0.5 \) mm in this case. The transmission results are shown in Fig. 3.17. The structure still resonated at around 7 GHz confirming that in the switch open state the square rings have a very minor effect on the resonant frequency and affect only losses in the structure. An increased gap did not significantly shift the resonance frequency.

**Switch closed position**

Fig. 3.16 (b) presents the simulated and measured transmission through the fishnet structure in the switch closed position \( (g = 0 \text{ mm}) \). Again, good agreement is observed between prediction and measurement. The resonance has shifted to around 6.25 GHz from the 7 GHz obtained in the open state. The frequency response of the switchable fishnet in
the switch closed position corresponded to the resonant behavior of the fishnet with a slab of the larger size $p = 13.5$ mm. The results support the hypothesis that when the metal square rings touch the fishnet, they form a connected conducting structure effectively changing the fishnet geometry and resulting in an increase of the slab width, and hence a lower operating frequency. Higher loss seen in the measurement results compared to the simulations can be due to the quality of the electrical contact. Using metals which are less susceptible to oxidization and employing hermetic packing to remove impurities could improve the performance. Upon contact with the fishnet, the square rings impacted on the circular currents, induced by the incident magnetic field, which in this case flowed along the square rings as well as the central part of the fishnet slab. Also, due to the fact that there was continuous dielectric substrate between the fishnet cells, adding the metal ring increased the effective area of the capacitive slab and hence the overall resonant response of the structure moved to a lower frequency.

When disconnecting the fishnet structure from the pneumatic pump and allowing air to fill the space between the layers of the structure, the metallic contacts between the square rings and fishnet were disconnected and the structure reverted to the original resonant frequency of 7 GHz. Thus, negative pressure forcing metal elements against each other is essential for good electric contact. A pressure of 0.15 bar in switch closed position was used to achieve sufficient electric contact. The switching was performed a number of times and no noticeable shifts in the resonant frequencies of either state were observed.

### 3.3 Discussion

Pneumatic technology offers great flexibility and efficiency in a variety of switching configurations. On each side of the presented fishnet structure, 266 elements were switched simultaneously using a single membrane with 1.5 mm spacing between square ring elements and a 0 mm gap between square ring and middle fishnet.

The square ring shapes of the additional elements were more complex than the rectangular shorting patches traditionally used to close gaps in microelectromechanical configurations. Pneumatic technology can be used as a tool for uniting a number of metallic elements into a more complex conducting structure.
The pneumatic method is scalable to higher frequencies, where micromechanical switches with biasing and actuation network are too large to fit into the structure. On the other hand, pneumatic membranes separating neighboring chambers can be made fairly thin.

This approach can be extended to other microwave applications where changing the metallization pattern without the addition of biasing networks which could interfere with the electromagnetic response of the structure can be beneficial.

Among the drawbacks of pneumatic operation is slow switching speed. Also, this method does not allow continuous tuning; however, many microwave applications require operation just at a fixed number of discrete frequencies.

It was proven that using the pneumatic approach, sufficient electric contact could be achieved between the metallic elements so that they could be considered to be a connected metallic structure of differing geometry thus manipulating the metallization patterns post fabrication.

### 3.4 Suggestions for future work

One of the advantages of the demonstrated pneumatic actuation is greater flexibility in the shape and positions of the reconfigurable elements. Switching of connected or disconnected elements of even more complex shapes, like meander or dendritic structures, might be performed using a single membrane. Pneumatic switching could be used to add elements into tightly confined spaces, as well as in a densely packed arrays. Also, configurations when one element connects to two or more initially disconnected elements placed around it could be explored. Non uniform placement of switchable elements could be used for creation of graded profile structures.

In this work, simultaneous actuation of all pneumatic elements was used. Placing a pneumatic chamber around each unit cell would enable independent actuation of each resonator. Theoretical investigations suggest that structure with gaps along the perimeter of the fishnet slab behaved as if it was continuous. These gaps would allow space for pneumatic chamber walls between a number of square ring elements of different sizes within a single unit cell that could be employed separately or in combination, creating a multitude of tuning options (Figs. 3.18 (a, b)). In this case the membrane should be made out
of flexible or stretchable substrate, such as PDMS, for example, capable of bending into the pneumatic chambers. Realization of this more complicated multi-resonant structure will require a more sophisticated multichamber integrated actuation and control mechanism.

### 3.5 Conclusions

In this chapter a new method of pneumatic switching was used to reconfigure a fishnet metamaterial structure between two resonant frequencies. Pneumatic operation was applied to bring two outer layers patterned with metal square rings in contact with a middle layer, which had the form of a traditional fishnet. The potential of using pneumatic switching for uniting a number of metallic elements into a connected conducting structure has been experimentally demonstrated. Pneumatic techniques were used to modify the geometry of the resonant elements in each unit cell by changing the metallization pattern post fabrication. The proposed pneumatically reconfigurable fishnet metamaterial structure has been realized, and operation at two different frequencies in switch open and switch closed states corresponding to different fishnet geometry configurations was confirmed. A large number of complex elements were placed on a single membrane in close proximity to each other and switched simultaneously without introduction of interfering metallic biasing lines into the structure.
Pneumatic actuation might be explored as a switching tool for nontrivial element shapes and multiconnected structures. Individual control of each switchable element, as well as non uniform element placement could be used for creation and reconfiguring of graded profile structures.
Chapter 4

Application of pneumatic switching to metamaterial lenses at GHz frequencies

4.1 Introduction

Lenses have been invented and used since ancient times and have existed in biology for far longer. They are indispensable part of various imaging systems. The materials used for lenses have made a long journey from traditional glass to less intuitive aperture arrays and metamaterial structures. As humanity explored new frequency ranges, like radiofrequencies, the demands on lens design and materials has constantly changed. Usually, the approaches are not directly transferable to new operating conditions and require innovative thinking. Remarkably, there are still areas, like the terahertz frequency range, which are in the early stages of research and open vast possibilities for exploration of new devices and materials.

Tunability of the lenses has always been one of the most sought after features. Variable focal lengths are required in microscopes and other imaging equipment. For radio frequency (RF) imaging, beam deflecting phase arrays are commonly used in radar scanning systems. These systems also require tuning of the operation frequency. It may be beneficial to switch the focusing action off for stealth purposes. Adaptive refocusing becomes necessary for tracking of moving targets. Compactness and weight reduction are also important factors, especially at low frequency ranges.

This chapter applies the pneumatic tuning approach introduced in Chapter 3 to demonstrate a practical device, specifically a switchable pneumatic metamaterial graded index lens for the microwave frequency range. This chapter is organized as follows. First an overview of graded index lenses made of natural dielectric materials and metamaterials and their design guidelines are given. Then, a range of reported graded index metamaterial
lenses and tunable graded index metamaterial lens options are discussed. The novel pneumatically tunable graded index lens is then presented including design, realization and characterization. The chapter then concludes with suggestions for future work and a summary of the findings of this investigation.

4.2 Introduction to graded index lenses

4.2.1 Graded index concept in dielectric lenses

Focusing action can be described as transforming plane wave into a converging cylindrical wave. Consider propagation of a plane wave through the lens spanning BC in Fig. 4.1. The path lengths of the rays AO and CO travelling towards the focal point O through the center and the edge of the lens correspondingly differ by the amount [121]

\[ CD = l - f = \sqrt{x_{\text{max}}^2 + f^2} - f \]  

(4.1)

where \( x_{\text{max}} \) equals to the radius of the lens, \( f \) is the focal length of the lens, which also equals to the radius of the curvature of the converging cylindrical wave. It is assumed that there is no variation of field along the \( y \) direction. The corresponding phase difference, as a function of position \( x \) along the lens, that the rays would have acquired while travelling through the air after exiting the lens is:

\[ \varphi_a(x) = \frac{2\pi}{\lambda} \left( \sqrt{x^2 + f^2} - f \right) + 2\pi m \]  

(4.2)

where \( \lambda \) is the incident wavelength, \( m \) is an arbitrary integer.

In order to focus, all the rays have to arrive at point O in phase, which leads to requirement for an inverted phase profile distribution within the lens material, also referred to as phase retardation, in order to compensate for different path lengths in the air [122-124]:

\[ \varphi_a(x) = -\varphi_a(x) = \frac{2\pi}{\lambda} (f - \sqrt{x^2 + f^2}) + 2\pi m \]  

(4.3)

The larger wave front retardation through the middle of the lens leads to cylindrical phase profile at the exit of the lens.
Another way to describe phase advance in the material is \[125\]

\[ \varphi = knd \]  \hspace{1cm} (4.4)

where \( \varphi \) is the phase accumulated during propagation, \( k = 2\pi / \lambda \) is the wave number, \( n \) is the refractive index of the medium and \( d \) is the thickness the wave has travelled through. In a traditional curved lens, the required variation in phase shift at different locations is acquired due to change in the thickness \( d \) across the lens (Fig. 4.2).

Graded refractive index lenses (GRINs) have uniform thickness \( d \) and the phase shift is acquired due to a change in the refractive index \( n \) along the span of the lens (Fig. 4.3). These flat lenses are free from spherical aberrations present in curved lenses. In graded index lenses, the refractive index value changes parabolically from a maximum along the axis of the lens to a minimum at the sides. The refractive index profile is described by the following equation \[121\]:

\[ n = n_0 \left( 1 - \frac{A}{2} x^2 \right) \]  \hspace{1cm} (4.5)
Fig. 4.2 Traditional convex lens.

Fig. 4.3 Graded refractive index lens.
where $n$ is refractive index at a distance $x$ from the optical axis, $n_0$ is the refractive index on the optical axis, $\sqrt{A}$ is called the gradient index constant. A graded index material can be thought of as consisting of thin layers, each with constant refractive index (Fig. 4.4). The ray refracts at the interface between each layer until it reaches the angle of total internal reflection of one of the layers at which point it is reflected back. If the layers are imagined to be infinitesimally thin, than the ray will appear to continuously bend as it propagates. Although rays falling on the lens at different angles travel different path length, they arrive at the focal point at the same time because they have travelled with different velocities.

GRIN lenses are usually cylindrically shaped and are used for coupling light into optical fibers or transmitting images through endoscopes. A ray incident on the lens follows a sinusoidal path (Fig. 4.5 (a)), which is characterized by the pitch parameter $P$, corresponding to one period variation in the ray trajectory along the axis:

$$P = \frac{2\pi}{\sqrt{A}}$$

(4.6)

A quarter-pitch length could be interpreted as analog of the focal distance (Fig. 4.5 (b)).

Traditionally graded index material, for example in the case of an optical fiber, is fabricated using a vapor deposition method [125]. The process relies on chemical reactions of specific gases (like SiCl$_4$ and GeCl$_4$) in a furnace to form various glasses. It is a tedious and expensive process of slowly building up the material layer by layer. Another process used to manufacture graded index lenses is high-temperature ion exchange within the glass, where ions in the glass are partially exchanged with silver or lithium ions via diffusion.

With a typical gradient index constant of $A = 0.33-0.7$, variation of refractive index across the lens is just few percent with the variation of $\Delta n = 0.02$ in optical fibers and up to $\Delta n = 0.145$ in cylindrical lenses. For applications where compactness is necessary, as well as at lower microwave frequencies, it is not possible to have long cylindrical lenses, so the thickness needs to be reduced. However, in order to maintain the phase difference required for focusing, significantly greater variation of refractive index is required, as follows from Eq. (4.4). This poses a challenge because of the limited variation possible with natural materials, which also need to be sufficiently transparent in the intended frequency range.
Fig. 4.4 Bending of the ray as it passes through graded index multilayer structure.

Fig. 4.5 (a) Path of rays through the graded index lens: focusing by (b) 0.25P and (c) 0.23P lenses.
4.2.2 **Graded index metamaterial and plasmonic lenses**

Metamaterials, being engineered artificially, offer great flexibility and allow larger variation of their properties than possible in natural materials. The refractive indices of metamaterials can be varied to a much greater extent than natural materials, making it possible to vary the phase shifts, experienced by the wave by hundreds of percent by adjusting the geometry of constituting unit cells. Also, exotic properties such as refractive indexes below one, near zero and negative, can be achieved. This could allow switching the lens from focusing (convex) to defocusing (concave). Metamaterial refraction is based on the resonant response of its unit cells. At frequencies close to the resonant frequency of the cell, the transmitted wave will experience a significant phase shift. By using unit cells of different geometry, and hence resonant frequency, the amount of phase shift through each element can be controlled, making it possible to create a parabolic phase profile required for lens operation. Also, graded index metamaterials can be fabricated through the same process as homogeneous metamaterials, adding no complexity in production.

The first metamaterial lenses reported consisted of the same elements, were planoconcave and achieved focusing by recreating traditional curved lenses of variable thickness [126-128]. The first graded index microwave metamaterial was demonstrated in 2005 by Smith et al. [129]. Variation in refractive index was achieved via removing different depths of the substrate in the vicinity of the split ring resonators using micromilling, thus shifting their resonant response. Multiple layers of such resonators were used to demonstrate deflection of the incident beam.

Lenses utilizing negative refractive index metamaterials can be considered due to the possibility to control permeability as well as permittivity independently and achieve good matching to free space across the lens. Both split ring resonators and continuous wires were required for the designs of Greegor et al. [130] and Driscoll et al. [131]. Impedances of $Z=1$ across the lens and large refractive index variations from -1 to -2.6 were achieved. The latter group used advanced fabrication techniques that allowed abandoning of time consuming 'wine-crate' assembly of the resonator layers. These designs were also multilayered. Broadband metamaterial lenses based on square shaped elements [132] and dielectric rod arrays [133] have also been proposed. An interesting version of the lens consisting of an array of split ring resonators patterned on the inner surface of the parallel plate waveguide
has been demonstrated [134]. Graded index metamaterials can also be applied to beam-scanning antennas [135]. A metamaterial lens based on l-beam elements and operating at infrared frequencies has been demonstrated [136].

Plasmonic lenses based on aperture arrays were more compact as they were single layered. They used metallic rods [137], slots [138, 139], square [122], circular [140-142] and cross shaped [124, 143] apertures of varying sizes cut in a metal plate. The principle of operation of these plasmonic structures is different from metamaterials. Phase builds up as the wave propagates through the slit, which behaves like a waveguide, so thicknesses comparable to wavelength in the direction of propagation are usually used to achieve large phase shifts. However, once phase dependence is determined, there are many similarities in lens design.

A number of beam bending techniques involving unit cell geometry modifications have been suggested for plasmonic [123] and metamaterial lenses [135], however, these were not dynamically tunable.

4.2.3 Tunable metamaterial and plasmonic lenses

In addition to possibility of significantly larger parameter variation, metamaterials offer greater flexibility in the ways they can be tuned. Along with temperature variation, mechanical transformation and incorporation of miniature electronic components can be used.

In conventional optics changing of the focal length is achieved by moving a combination of multiple lenses along the optical axis [144]. The focal length of the combined system depends on the focal lengths of each lens $f_1, f_2$ and the distance between them $d$:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad (4.7)$$

Traditional glass lenses are quite thick with fixed focal lengths. Space is required between them to allow for their movement. This contributes to the bulkiness of the system.

Numerous approaches have been developed to tune the focal length of the lens itself. As follows from the formula for biconvex lens:
\[ f = \frac{R}{2(n-1)} \]  

(4.8)

the focal length of the lens can be varied by changing either the radius of curvature \( R \) or the refractive index \( n \) of the material that the lens is made from. Varying the focal length by changing the curvature of the lens has been demonstrated for THz [145] and optical frequency ranges [146, 147]. The reported THz lens consisted of a polyethylene polymer shell filled with optically transparent fluid, in this case medical white oil. Other liquids like water, glycerin or ethanol can also be used. The optical lenses used PDMS infiltrated with calibrated refractive index optical fluids. The volume of the fluid pumped into the elastic shell defined the radius of curvature of the lens and could be easily modified. However, the approach is less suitable for larger sized lenses, required for lower operating frequencies, since gravitational pull on the fluid would result in less spherical and more pear shaped lens, leading to distortions.

Another approach to change the focal length would be to modify the refractive index of the material that the lens is made from. This method relies on the possibility to change the refractive index of natural materials under certain conditions, for example, by applying voltage or heating. Tunable liquid crystal microlenses have been demonstrated [148-152] and these have relied on a capability of liquid crystals to change their refractive index as a result of molecule reorientation when voltage was applied. By adjusting the polarity of the voltage applied to the electrodes, a graded index profile could be created. However, the resulting change in refractive index was rather small, with [149] reporting \( \Delta n \sim 0.285 \), limiting the extent of focal length tuning. That is why liquid crystal microlenses usually include a layer with traditional curved lens that serves to provide initial focusing.

There are very few examples of real time tuning of the focal length of metamaterial or plasmonic lenses. The simplest reconfigurable metamaterial lens that has been reported consisted of metal rods inserted different distance into the substrate with high permittivity [153]. The refractive index was controlled by readjusting the height of the rod in the dielectric within each unit cell. Both deflecting and focusing structures were demonstrated with just 4 to 7 unit cells across the lens aperture.

One reported tunable plasmonic lens used high temperature heating [154]. The lens consisted of an array of subwavelength air microslits, of various widths, machined into a slab
of indium antimonide (InSb) semiconductor. The permittivity of InSb changes with temperature, which leads to a change in the phase profile and this shifted the focal point closer to the lens.

Reconfiguring of a graded index of metamaterial via voltage induced heating of vanadium dioxide (VO₂) substrate, supporting split ring resonators, was recently proposed [155]. The effect of temperature change on the properties of the metamaterial as a whole was shown to be substantially greater than the effect on the natural substrate material. However, no temperature tunable metamaterial lens has been demonstrated yet.

One of the advantages of metamaterials, especially at microwave frequencies where large unit cell sizes are possible, is that other devices like diodes can be incorporated into the structure. This allows expanding the possible tuning options. For example, recent reports show that introducing photodiodes into the metamaterial allowed control of the phase profile along the structure with the help of an array of light emitting diodes [156]. Tuning the deflection angle of the metamaterial reflector using the above method has been demonstrated, as well as operation in focusing and defocusing states. The method was dependent on the tuning capabilities of the photodiodes, which were quite modest and required fairly bulky and individually realized interfaces to the discrete diode elements.

To sum up, although substantial research has been done in the area of tunable uniform metamaterials, the area of graded index metamaterials, and tunable lenses in particular, is largely unexplored.

4.3 Pneumatically switchable metamaterial lens

This section proposes a new structure which is a pneumatically switchable graded index metamaterial lens consisting of a one dimensional array of split ring resonators. A parabolic phase profile and corresponding effective graded refractive index should be possible using resonators of differing geometry. This investigation attempts to realize a reconfigurable graded index lens, which is very thin, consisting of one layer of resonators. The intended lens operating frequency is 9 GHz.

A distinctly different feature of this lens, compared to previously demonstrated metamaterial lenses, is the possibility to switch off the focusing action by eliminating resonant response of the split rings. This effectively turns the graded index metamaterial
into uniform dielectric with properties close to those of the substrate itself. Switching between focusing and non-focusing states should be enabled by building on the work described in the previous chapter using pneumatically actuated metal patches that are pressed against the gaps of the resonators as the pressure in the pneumatic chamber is reduced. The patches could short circuit the gaps of the split rings and eliminate the resonant response, thus evening out the phase difference between the elements across the lens. The entire array could be actuated simultaneously by placing shorting patches on the same membrane. Pneumatic switching would not require metallic biasing wires, thus the pneumatic chamber would be electromagnetically transparent and would not interfere with the operation of the lens.

4.3.1 Ultrathin design: electric vs. magnetic resonators

Metamaterial graded index lenses that have been reported in the literature have exploited magnetic resonators and have often had a multilayered structure. In magnetic resonators the resonance has been induced primarily by the H component of the incident electromagnetic field. This requires the resonators to be oriented along the direction of wave propagation, which is referred to as side irradiation, such that the H field can pass through the loop formed by the resonator and induce circular current (as illustrated in Fig. 4.6 (a)). As a result, the minimum achievable thickness of the layer equals to the unit cell size in the direction of propagation, which is slightly larger than the width of the resonator. An electric field directed across the gap also contributes to the current flow, making the structure generally anisotropic, however, the resonators are referred to as magnetic in order to distinguish from the structures not driven by the magnetic field at all. A medium made of such resonators is known to exhibit effective permeability $\mu_{\text{eff}} < 0$ in the frequency range near the resonance.

Electric resonators are excited only by the E component of the incident field. Resonators should be oriented normal to the direction of wave propagation and such, that the E field is directed across the gap in the ring inducing similar circular current as in the case of magnetic resonator but driven by the induced change variation across the gap (as illustrated in Fig. 4.6 (b)). Such positioning of the resonators allows the realization of planar, ultrathin lenses, which are easy to fabricate by standard photolithography techniques. The thickness of the lens in this case is limited by the thickness of the substrate used, which can be an order of
Fig. 4.6 (a) Magnetic and (b) electric metamaterials orientation diagram showing current induced at the resonance and minimum layer thickness.

magnitude less than the unit cell size in the lateral direction defined by the periodicity of the structure. A medium made of such resonators is known to exhibit effective permittivity $\varepsilon_{\text{eff}}<0$ in the frequency range near the resonance. This is the orientation that was selected for the proposed lens.

Natural dielectric lenses, as well as plasmonic lenses, require the wave to travel a certain distance in order to accumulate the required phase shift. These distances can be on the order of a wavelength or more. For the microwave frequency range, this can be many centimeters. Unlike plasmonic or natural media, for resonant elements such as metamaterial split rings, the phase shift occurs across the thickness of the resonator itself, which is on the order of micrometers for the same microwave range, enabling ultrathin lens design.

4.3.2 **Switchable split ring resonator design and effect of pneumatic elements position on the resonators**

For the proposed lens design, one of the classic metamaterial resonator shapes that is symmetrical and has a gap located in the center, was chosen [16, 157]. Fig. 4.7 illustrates one of the resonators of the array, consisting of resonators of various widths. Copper split ring resonators were located on one side only of the commercial microwave Rogers duroid substrate. The polarization direction of the incident fields are shown with arrows. To enable
switching of the resonators, additional copper patches on a flexible substrate serving as a membrane were suspended above each split ring with metal facing the resonator. Fig. 4.7 (a) illustrates this switchable structure configured in the open state. Patches can be pneumatically actuated to short-circuit the gaps in split ring resonators. Fig. 4.7 (b) illustrates the same switchable structure as Fig. 4.7 (a), but pneumatically actuated to be in the closed state.

The effect of introducing a metal patch in close proximity to the split ring resonator was investigated by simulating the structure. The layer with the patch was placed at different distances $gl$ from the layer with the resonator. A resonator with $wx = 6$ mm, $wy = 8$ mm was chosen. Resonators of the other sizes showed a similar trend. Fig. 4.8 shows the simulated transmission results for the spacings from $gl = 0.1$ to 3 mm.

![Fig. 4.7 Unit cell of the metamaterial lens array with the following geometrical parameters:](image)

- split ring resonator height $wy = 8$ mm, width $wx = 8$ mm, trace width $wt = 1$ mm, gap $g = 0.4$ mm, gap length $lg = 2.4$ mm, duroid $\varepsilon_r = 2.2$, thickness $t = 0.5$ mm, unit cell dimensions 12 x 12 mm.
- Membrane parameters: patches size $1 \times 2.4$ mm, membrane $\varepsilon_r = 2.9$, thickness $tl = 0.1$ mm, at distance $gl = 1$ mm from the split ring, total thickness $d = 1.6$ mm (a) open state ($gl = 1$ mm) (b) closed state ($gl = 0$ mm).
It can be noted that starting from the separation of \( gl = 2 \) mm and further, the patch had no effect on the resonant behavior of the split ring. Conversely, when the spacing was small, the shift of the resonant frequency was very significant for small variation in spacing. The closer the patch was to the resonator, the larger was the shift of the resonant frequency. This is as expected, as capacitive coupling between the patch and the split ring become stronger at these small spacings. The consequence of this effect for the lens is that if the gap is less than \( 2 \) mm when open, small errors in positioning of the actuation layer could lead to significant deviations of the designed phase shift through the structure. Thus, this coupling effect defines the limit to which the thickness of the pneumatic lens can be reduced.

There is a significant difference in the effect of the patch on split ring resonator in switch the open position as compared to fishnet from Chapter 3. In Chapter 3, the field was concentrated between the layers of the fishnet, and thus the patch had only a minor effect of the structure's resonant frequency, despite of the fact that it was of a larger size. In the case of the split ring resonator of the current investigation, at small separations the patch would be within the resonant field extending from both sides of the gap (Fig. 4.9 (a)). However, the strong dependence of resonant frequency on the layer spacing potentially offers an additional parameter for frequency tuning applications. If it were possible to achieve high precision control of the membrane position via variation of the pressure, this parameter could be used for resonator tuning and phase profile creation as well.

The separation of \( 1 \) mm was selected as a tradeoff between the desire to minimize the thickness of the lens and keeping errors in the phase shift to the acceptable level.

### 4.3.3 Effect of resonator geometry variation on phase shift

In order to focus, a plane wave should experience a larger phase shift when travelling through the middle of the lens than at the edges. The amount of phase shift through different parts of the lens can be engineered by adjusting the geometry of the resonators. The strength of the focusing power of the lens depends on the maximum achievable phase difference between the elements across the lens. The phase shift at a particular frequency depends on the resonant frequency and the resonant frequency can be adjusted by adjusting the length of the metal ring making up the split ring. Since the split rings considered thus far have been square, there are two axes on which length can be varied. It was decided to vary only the width (x axis) and leave the height (y-axis) constant. This width
Fig. 4.8 Dependence of split ring resonant behavior on layer spacing $g_l = 0.1$-3 mm for the resonator with $w_x = 6$ mm, $w_y = 8$ mm.

Fig. 4.9 (a) Side view of the electric field distribution between the resonator and the patch (b) top view showing enhanced filed in the gap of split ring resonator for $w_x = 6$ mm, $w_y = 8$ mm at $f = 8.85$ GHz.
can be no larger than the width of the cell size of the array and can be no smaller than the gap feature in the centre of the resonator. Resonators were simulated with ANSYS HFSS software, using a standard approach by assuming the structure is infinite and periodic in the x and y directions. Perfect electric conductor (PEC) and perfect magnetic conductor (PMC) boundary conditions were used. The incident electromagnetic plane waves were represented by a wave port.

Fig. 4.10 shows the simulated transmission and phase shift through the equal height split ring resonators of the largest and the smallest widths, plotted with solid lines, as well as the transmission and phase shift for elements with intermediate width sizes used in construction of the lens array, plotted with dashed lines. The dotted lines in Fig. 4.10 show the simulated transmission and phase shift through the same resonators when the gaps are short circuited by metal patches (as shown in Fig. 4.7 (b)). The characteristic resonances in both phase and amplitude have been eliminated in the 6-10 GHz range. A small amount of phase difference was still present due to different sizes of the shorted elements. Transmission through the short-circuited resonators was 2-3 dB lower than in the open state. In the closed state, the structure was rather a patterned composite dielectric in the considered frequency range than a metamaterial because of the absence of the resonant response. The extracted effective refractive indexes for the shorted resonators, shown in Fig. 4.11, are almost constant in the considered frequency range, confirming this suggestion.

The vertical line at 9 GHz in Fig. 4.10 depicts the chosen lens operating frequency. This frequency was selected taking into account the following considerations. The largest phase shift occurs at the resonant frequency; however, to ensure high transmission through the lens, each resonator must be detuned from its resonance. On the other hand, moving too far from the resonance eliminates the required phase differences between the resonators. At 9 GHz the transmission through the resonators was very high, with the difference in transmission through the elements of the largest and smallest sizes not exceeding 3 dB. This corresponds to 50% difference in field amplitude level at the exit of the lens and was deemed acceptable for the metamaterial lens design. The maximum phase difference between the resonators at the selected operation frequency was 1 radian.

Other geometrical parameters of the split ring, such as gap features lg and g and trace width wt could also be used for implementation of a parabolic phase profile, achieving a similar maximum phase difference. However, simulations indicated that the variation of
these parameters resulted in less consistent strength of the resonance. Changing the resonator height simultaneously with the resonator width was also considered as illustrated in Fig. 4.12. According to simulations, this would result in a larger phase difference of 1.5 radians between the largest and the smallest elements. However, there were significant differences in transmission levels at the corresponding operation frequency (shifted to 11 GHz as a result of different resonant frequencies of the split rings), making this approach unsuitable.

### 4.3.4 Phase profile design

The required phase shift as a function of the lens width (Fig. 4.13) can be calculated from equation similar to Eq. (4.3) introduced at the beginning of the chapter:

\[
\Delta \phi = 2\pi \left( f - \sqrt{x^2 + f^2} \right) / \lambda + \phi_0 \tag{4.9}
\]

where \( f \) is the focal length of the lens, \( \lambda \) is the incident wavelength, \( \phi_0 \) is the phase shift through the centre of the lens and \( \Delta \phi \) is the required relative phase shift at the distance \( x \) from the center of the lens. Since the maximum achievable phase difference \( \phi_0 \) between the resonators was determined to be 1 radian in previous simulations (Fig. 4.10 (b)), the focal length and the width of the lens were selected to fit within the measurement setup dimensions of 35 x 35 cm. This resulted in a 13 unit cell design which was 15.6 cm wide. Fig. 4.13 shows the required parabolic phase profile along the length of the lens calculated using Eq. (4.9). The parabolic phase profile was approximated with the discrete staircase profile, where each step corresponds to a resonator cell.

The relative phase shift of each element of the lens operating frequency of 9 GHz as a function of resonator width was simulated and is presented in Fig. 4.14. The phase shift was acquired from simulations as in Fig. 4.10 (b) by referencing the phase shift of each element to that of the element with least phase shift, which was the largest of the resonators. Using Figs. 4.13 and 4.14 the required width for each resonator in the array could be determined.
Fig. 4.10 (a) Transmission (b) phase shift through the split ring resonators with widths $wx = 6$ mm and $wx = 10.5$ mm with height $wy = 8$ mm. Dotted lines are for same elements shorted with the metal patch. Dashed lines are for resonators with intermediate $wx$ values. The blue vertical line indicates the intended lens operation frequency.
Fig. 4.11 Effective refractive indexes for shorted resonator structures.

Fig. 4.12 Transmission (solid line) phase shift (dashed line) through the split ring resonators of variable width and height. The blue vertical line indicates corresponding lens operation frequency.
Fig. 4.13 Parabolic phase profile along the 15.6 cm width of the lens with staircase approximation.

Fig. 4.14 Dependence of the phase shift on the split ring resonator width at the lens operation frequency of 9 GHz.
Some researchers argue that effective parameter extraction should not be used for single layered structures since the concept of effective media assumes that the structure can be considered as a bulk and calculation of the effective media parameters introduces ambiguity as it depends on the unit cell periodicity in the direction of propagation as the effective thickness. For this reason the lens was designed as a phase shifting array based on the data directly available from the simulations, rather than through extraction of effective refractive index parameters. Also, the structure was inhomogeneous with respect to the direction of propagation, which would further complicate an attempt to extract the effective parameters.

For the purpose of comparing the metamaterial lens to a dielectric lens made of natural materials, but with the same phase profile and hence focusing power, the required variation of refractive index was estimated based on well known relationship between the refractive index and the phase acquired during propagation through the corresponding material as mentioned in Eq. (4.4). The equivalent refractive index variation across of the lens as a function of position could be approximated from the phase shift using [125]:

$$\Delta n = (\Delta \phi / d) (\lambda / 2\pi)$$  \hspace{1cm} (4.10)

and assuming the material thickness $d$ is the thickness of the physical structure. It follows that $\Delta n = 3.3$ would be required to achieve the same phase difference with natural dielectric material (Fig. 4.15). Note that the initial value of $n$ can be selected arbitrarily, since only the difference of the refractive indexes features in focal length calculations. Lenses with various initial values of $n$ are expected to achieve similar focusing action.

The thickness of the proposed lens was only 0.015 $\lambda$. Addition of the pneumatic switching membrane would increase the thickness to 0.048 $\lambda$ in open state (Fig. 4.16), which is considerably less than previous metamaterial and plasmonic lenses.
Fig. 4.15 Parabolic refractive index profile along the 15.6 cm length of the equivalent dielectric lens.

Fig. 4.16 Lens thickness vs. wavelength schematic.
4.3.5 Fabrication and layer assembly options

The lens was fabricated using standard printed circuit board photolithographic techniques. The resonator array was realized on Rogers RT/duroid 5880, while the shorting patches were realized on much thinner Rogers Ultralam 3850. Fig. 4.17 shows the fabricated layers of the lens. In order to assemble the lens the layers were aligned so that metal patches were located over the gaps of split ring resonators and sealed around the edges of the lens using adhesive tape (Fig. 4.18). An outlet tube of 3 mm diameter was connected to one corner of the lens and led to a vacuum pump for pneumatic switching.

To ensure uniform distance between the layers, a 1 mm thin acrylic frame was used as a spacer around the perimeter of the lens as shown in Figs. 4.19 and 4.20 (a). A prototype was also realized with the frame removed as illustrated in Fig. 4.20 (c). This should result in a different shorting membrane behavior in the closed state. Figs. 4.20 (b, d) show the side view of the lens with the vacuum applied to framed and unframed versions. Although the frame would ensure better control over the layer spacing, in this case the membrane with patches had to be fairly flexible in order to bend into the structure. Also, the shorting elements had to be located along the middle of the membrane, since this was the only part of the membrane that came into contact with the bottom layer. With the frame removed, the top layer was not required to be flexible any more, since the operation relied on the flexibility of the bonding tape acting as the flexible element. The entire surface area of the top layer came into contact with the bottom layer allowing more versatile choice of resonator gap and patch positions.

4.3.6 Whole lens numerical simulations

In order to accurately predict the behavior of the lens, the whole structure was simulated in ANSYS HFSS software based on the full-wave finite element method. Only the metamaterial layers were simulated with frame and seal being neglected. The lens was simulated in a parallel plate waveguide replicating measurement setup with perfect electric conductor boundaries located above and below the lens with a separation of 12 mm. At a width of 4.68 \( \lambda \) and numerical aperture \( NA = 0.32 \) this lens is a microlens and diffraction effects are expected to be significant. As suggested in the literature [158-160], for lenses of such small numerical aperture, the focal point is expected to be up to 75% closer than predicted by geometrical optics as described by Eq. (4.9). Figs. 4.21 (a) and (b) shows simulated field
Fig. 4.17 Photograph of the layers before assembly.

Fig. 4.18 Lens assembly schematics.

Fig. 4.19 Frame placed on the resonator array.

Fig. 4.20 Side view of framed (a, b) and unframed (c, d) assembly options in open (a, c) and closed (b, d) states.
intensity distributions for the lens in open and shorted positions respectively in response to a normally incident plane wave at 9 GHz. Focusing is seen at 10 cm from the lens in the open state. No focusing is seen when the lens is shorted.

Figs. 4.22 and 4.23 show 3D plots of intensity distributions with respect to frequency in cross sections along the direction of propagation 'AO' and through the focal plane 'BC' correspondingly as shown in Fig. 4.24. The plots show that the focusing effect is also present in a narrow range of frequencies $\Delta f \approx 0.1$ GHz around the designed frequency. The intensity distribution in the focal plane cross section at the operating frequency of 9 GHz is plotted separately in Fig. 4.25. As expected focusing occurs 75 % closer to the lens than estimated with geometrical optics. An elongated focal spot which is typical for the lenses of small width can be seen. The focal spot is around 6 cm wide, so this lens does not allow subdiffraction limit resolution. Generally, the theoretical subdiffraction resolution limit due to the discrete structure of metamaterial lenses, including perfect lens possessing negative refractive index $n=-1$, is around $5a$, where $a$ is the period of the structure [161].

Some hot spots due to diffraction can be seen in the space between the lens and the focal spot. These arise because of fairly coarse stepwise approximation of the parabolic phase profile introducing phase errors. As expected of resonant structures, focusing is observed only in a narrow frequency range.

![Simulation](image.png)

*Fig. 4.21 Simulated field intensity distributions at 9 GHz for the lens in (a) open ($g_l = 1$ mm) and (b) shorted ($g_l = 0$ mm) states.*
Fig. 4.22 Intensity distributions vs. frequency for cross section passing through the middle of the lens along the direction of propagation AO as shown in Fig. 4.24.

Fig. 4.23 Intensity distributions vs. frequency in the focal plane BC as shown in Fig. 4.24.
Fig. 4.24 Schematic of the cross sections.

Fig. 4.25 Intensity distribution in the focal plane at 9 GHz.
4.3.7 Measurements of field intensity distributions

As an initial test, the response of a single cell of the lens was measured with a free-space transmission system by masking the other cells with a metal plate. Then the air was pumped out of pneumatic cavity and the response was measured again. It was found that an internal pressure of 0.2 bar was sufficient to eliminate the resonant features indicating effective shorting of the rings in the lens array. The time required for switching was on the order of a few seconds, which is quite slow and could be considered a shortcoming of the pneumatic approach. The suggestions for reducing the response time to less than a second would be to improve on air tightness of the lens and connecting tube and using positive pressure to revert the lens back to the focusing state.

To experimentally measure the behavior of the lens, a parallel plate waveguide was used with spacing between the plates 12 mm and field mapping area of 35 x 35 cm (Fig. 4.26) [41]. The bottom plate of the waveguide was located on a computer-controlled translation stage and held a source monopole antenna, producing a cylindrical wave. A specially designed parabolic reflector was designed and implemented to convert this cylindrical wave into the plane wave. The metamaterial lens was placed in front of this parabolic reflector as shown in Fig. 4.26. Due to the limited space in the setup, the reflector was designed to produce a plane wave within as close 5 cm distance from the source. The details of the design are described in Appendix B. The receiving probe was fixed to the stationary top plate of the waveguide. A microwave absorber was placed along the sides of the parallel plate waveguide in order to minimize reflections. The transmitting and receiving probes were connected to the output and input ports of the Rohde and Schwartz vector network analyser. The lens was connected to a vacuum pump (not shown on the photo), placed near the moving stage for real time switching.

The electric field distributions for both open and shorted states were measured with 2 mm resolution. Both framed and unframed versions of the lens were tested. Fig. 4.27 (c, d) presents the measured field intensity distributions at 9 GHz in open and shorted positions for the framed version and Fig. 4.27 (e, f) is for the unframed version. Simulations were also replotted for comparison purpose (Fig. 4.27 (a, b)). Focusing is seen at around 10 cm from the lens for open state and no focusing is seen for shorted state. Some inhomogeneity could be attributed to the deviation of the incident angle from the normal incidence, inhomogeneity of the layer spacing as well as layer displacement. When the pneumatic...
pump was engaged and disengaged, the lens switched reversibly between shorted and open states without any noticeable change in performance. Measured results for both versions of the lens are in good agreement with simulated results, demonstrating real time switching of the focusing behavior of the lens.

### 4.3.8 Other operation regimes

The behavior of the lens was measured at frequencies far from the designed operation frequency and a number of other interesting operation regimes were observed. A selection of these are presented in Fig. 4.28 for the lens in open state. Fig. 4.28 (a) shows maxima at 9.62 GHz due to diffraction effects (this is also predicted for a shorted lens at this frequency). Fig. 4.28 (b) shows the lens operating at 9.18 GHz in a highly transparent mode with a narrow beam of radiation due to good impedance matching to air across all elements. Fig. 4.28 (c) shows an inverted focusing pattern at 8.7 GHz with a dark spot due to high resonant absorption of the central elements where previously focusing has been seen. Finally, Fig. 4.28 (d) shows the beam divided in three at 8.5 GHz due to the resonant absorption of the intermediate width elements between the centre and the edge of the lens.
Fig. 4.27 Simulated (a, b) and measured (c, d, e, f) field intensity distributions at 9 GHz for framed (c, d) and unframed (e, f) versions of the lens open ($g/l = 1$ mm, focusing) (a, c, e) and shorted ($g/l = 0$ mm, not focusing) (b, d, f) states.
Fig. 4.28 Alternate operation regimes in open state: (a) focusing caused by diffraction (b) narrow beam with high transmission (c) minima at 10 cm (d) beam split in three parts.

4.4 Suggestions for future work

As future work, it may be possible to achieve a full π phase shift by stacking three resonator layers. This could enable the construction of a Fresnel zone plate configuration as well as help reducing the focal spot size. The focal spot size $d$ can be found using the formula [121]

$$d = 1.22\lambda f / D$$

(4.11),

where $D = 2x_{\text{max}}$ is the aperture (width) of the lens. Traditional approach of reducing focal spot size by increasing the aperture of a dielectric lens also results in a larger required phase
difference between the center and the side of the lens due to increased thickness at the center. However, this approach is not suitable for the one layer metamaterial lens demonstrated in this chapter, since the maximum achievable phase shift is fixed by resonator geometry. The alternative route to increasing the phase difference for metamaterial lenses would be to stack the resonator layers. It should be noted that it would not be possible to simply stack these resonators directly on top of each other as it is expected that the resonators would interact. Hence a gap of around 1-2mm, similar to the separation required between the resonator and shorting patch in the open state would be required between each layer. Further, employing electrically smaller multiloop resonators could help reducing discrete step size and improve the accuracy in parabolic profile approximation leading to a better focusing behavior.

The graded index concept could be extended to two dimensional structures by placing resonators in a circular arrangement. Also, polarization independent lenses could be designed by using polarization independent variations of split ring resonators with symmetrically located gaps (Fig. 4.29).

By employing split ring resonators with multiple gaps, switching between various focal lengths and operation frequencies could be achieved. A multiple gap modification of the designed lens (Fig. 4.30) for switching between operation frequencies was also investigated. Each resonator had an additional two gaps on the sides and the membrane was prepared with a corresponding set of patches to short these gaps. When the side gaps were shorted the lens was similar to previous design. When the side gaps were open the triple gap resonators would have higher operating frequencies, hence the focusing action of the lens was expected to shift to higher frequency as well. However, this prototype could not be tested since the operating frequency of triple gap lens was outside the measurement range of the equipment used. More sophisticated design with independent actuation for each resonator could result in even more variety of responses (Fig. 4.31).

With more precise control of air pressure in the chamber continuous tuning of the focal length from convex to concave, including no focusing state, could be possible by controlling the separation between the patch and the split ring resonator gap and thus affecting the coupling between them (Fig. 4.32). Further, if a leak free valve were introduced at the pneumatic port, the lens could be set to maintain either open or shorted state without requiring external power associated with the pump.
Fig. 4.29 Examples of symmetric polarization independent multigap resonators [16].

Fig. 4.30 Photo of a multigap lens sample.

Fig. 4.31 Schematic of actuation with independent control of each unit cell.

convex

no focusing

concave

Fig. 4.32 Schematic of pressure controlled continuous tuning.
4.5 Conclusions

In this chapter a pneumatically switchable graded index metamaterial lens operating at GHz frequencies has been designed, implemented and characterized. A graded parabolic phase profile has been designed using split ring resonators of different widths. By exploiting the electric resonance of split rings an ultrathin lens of only 0.015 \( \lambda \) was achieved which can focus a normally incident plane wave.

An experimental demonstration of switching between focused and unfocussed states of the lens using pneumatically actuated metal patches was presented. On the application of negative pressure, the metal patches were pressed against and short-circuited the gaps in the split ring resonators to eliminate the resonant response and corresponding phase difference between the elements at the edges and in the center of the lens. Two versions of the actuation structures were tested, both showing good agreement with the predicted behavior for each. A number of further tuning possibilities including stacked, multigap, polarization independent resonators which could be controlled independently and with variable pressure were discussed. Overall, this chapter has shown that it is indeed possible to realize a metamaterial graded index lens and that pneumatic tuning is a promising technique for the reconfiguration of this structure.
Chapter 5

Tuning nonlinearity in metamaterials

5.1 Introduction

Nonlinearity of materials is an important phenomenon that has to be taken into account for applications in optics. Natural materials exhibit nonlinear properties only at high intensities of incident radiation. Since metamaterials allow control and tuning of various material parameters, research has been done to investigate their nonlinear properties. The range of frequencies where a nonlinear effect is measurable has been extended to microwaves. As was described in Chapter 2, a nonlinear response in metamaterials is usually achieved via introduction of nonlinear devices, such as varactor diodes [40-45], or using nonlinear media as a substrate [70, 71]. Recently, a new concept of achieving nonlinearity in metamaterials without introduction of nonlinear elements has been introduced [162]. This magnetoelastic metamaterial relies on resonator movement to achieve nonlinear frequency tuning in response to changes in the incident power level.

The aim of this chapter is to produce an optimized and microfabricated version of the recently demonstrated magnetoelastic metamaterial that would allow making a large number of resonators with high precision and low parameter variation across multiple elements, as well as using a wider range of more complex split ring resonator shapes. While in previous chapters the resonator shape was modified by moving additional elements and joining them with the resonator, in this chapter, position and orientation of the resonators was changed. Suspended split ring resonators with mechanical degrees of freedom were explored to allow shifts in the resonator position and orientation to be induced by incident radiation leading to nonlinear effects. Optimization of design for microfabrication led to a modified concept using gravity instead of elasticity as a restoring force. Requirement of mass reduction resulted in development of novel light weight mesh substrate.
A nonlinear system is the system that does not satisfy the superposition principle, where the output is not directly proportional to its input. In a nonlinear media the electric polarization $P$ depends on the electric field $E$ in following way [125, 163]:

$$P = \varepsilon_0 \chi^{(1)} \cdot E + \varepsilon_0 \chi^{(2)} : EE + \varepsilon_0 \chi^{(3)} : EEE + \cdots$$  \hspace{1cm} (5.1)

where $\varepsilon_0$ is the vacuum permittivity and $\chi^{(n)}$ is the $n$-th order of the electric susceptibility tensor of the medium. However, very high intensities of the incident field are necessary to observe this nonlinearity. Such intensities on the order of 1 GW can be achieved with pulsed lasers. Hence, the phenomenon is usually only considered in nonlinear optics, where it is known as Kerr effect, which is a change in the refractive index of a material in response to an applied electric field. In terms of refractive index

$$n = \sqrt{1 + \chi} \cong n_0 + n_2 I$$  \hspace{1cm} (5.2)

where $n_0 = \sqrt{1 + \chi^{(1)}}$ is the linear refractive index and $n_2 = 3 \chi^{(3)}/8n_0$ is the second-order nonlinear refractive index, and $I = |E|^2$ is the intensity of the wave. The values of $n_2$ are relatively small for most materials, on the order of $10^{-20}$ m$^2$ W$^{-1}$ for typical glasses.

Some of the applications of nonlinearity include second and multiple harmonics generation [164], self-phase modulation, self-focusing, mode-locking and soliton propagation. For example, an intense Gaussian beam of light will produce a change in the medium’s refractive index similar to the intensity distribution in the cross section of the beam, resulting in graded index lens profile. This would cause the beam to focus itself as it propagates through the medium.

Significantly stronger nonlinearities can be designed in metamaterials allowing nonlinear effects to be induced by modest powers and microwave frequencies. Such a capability opens possibilities for various nonlinear experiments and applications.
5.3 Nonlinear magnetoelastic metamaterial concept

Recently the novel concept of a magnetoelastic nonlinear metamaterial was introduced [162]. The work took further the approach of structural tuning via changing mutual orientation of metamaterial elements. The principal difference from the previous work was that the resonators were not restricted by a rigid substrate, but were allowed to move in response to the incident electromagnetic radiation. For the purpose of theoretical investigation, it was imagined that the resonators were embedded in some elastic medium as shown in Fig. 5.1.

The principle of operation is as follows. Magnetic resonators were employed, which means that they were excited by the magnetic component of the incident field and oriented such that magnetic field passed through the rings which would excite circular currents flowing in the same direction on both rings. As is known from Ampere’s law, current carrying wires attract each other. This attraction between the rings would compress the elastic material which would resist this compression providing the restoring force as per Hooks law

$$F = kx$$ (5.3)

where \( k \) is the elastic constant of the media and \( x \) is the displacement. When the forces balance each other, a stable position is reached where spacing between the resonators is changed. Any displacement of the resonators results in change of the resonant frequency of the system due to altered mutual coupling.

Fig. 5.1 (a) Magnetoelastic metamaterial operation schematic (b) Experimental testing in the waveguide schematic.
The resonance frequency of the structure could be tuned by adjusting the power level of incident field itself. Altering the incident power resulted in different current values, and consequently different resonator spacing in the equilibrium position. This resonant frequency shift depended on power level in a nonlinear way. So, the system was capable of complex nonlinear behaviors without the introduction of traditional nonlinear materials or devices. The structure was also capable of bistable behavior, exhibiting hysteresis loops in certain operation regimes.

However, the required elastic constant of the substrate was very low with stiffness coefficient \( k = 0.44 \text{ mN/m} \). Although there are some elastic materials that are compatible with photolithography, they are too stiff and usually have high loss. For the experimental demonstration of [162] the elastic material was substituted with a spring. The resonators were made of copper wires bent to form a shape of split rings and attached to a thin plastic transparency sheet for stability. They were connected by a spring (keratin filament) and suspended in the waveguide as shown in Fig. 5.1 (b) at 1 mm separation from each other free to move in response to the incident radiation. The induced currents resulted in attraction which displaced the resonators. An elastic spring provided a restoring force, preventing resonators from collapsing onto each other. The system displayed a non-linear frequency shift of 13 MHz with respect to changes in incident power in the range of 1 W.

5.4 Nonlinear gravitational metamaterial

Following the initial demonstration of [162] this section aims to design this type of metamaterial suitable for fabrication by standard photolithography process and incorporating an integrated suspension mechanism. The high precision of microfabrication allows repeatability of resonator geometrical parameters and hence the resonant frequency.

Since elastic materials with suitable elastic constants are not available and springs significantly complicate the design as they are not suitable for production of large areas of material and limit scalability of the structure, it is proposed to remove the elastic material or spring and use only the force of gravity as a restoring force. This led to modified gravitational metamaterial concept employing rotational movement of the resonators, however, the capability of nonlinear frequency tuning was preserved.
5.4.1 **Analysis of forces acting on the resonators**

First the directions of force involved in the movement of arbitrary shaped split ring resonators in the nonlinear operation regime were theoretically investigated in order to develop guidelines for the resonator and suspension system design. Fig. 5.2 shows the diagram of forces acting on the resonators. The side view is shown, where rings on a substrate are suspended in the waveguide in close proximity to each other so that the metal coated sides face each other and the gaps are located at the top of the rings. A suspension rod passes through the middle of the resonators.

Initially the rings hang parallel to each other. The force of gravity balances the rings by keeping them in a stable equilibrium corresponding to the lowest position of the center of gravity in order to minimize the potential energy. At the resonant frequency, the incident magnetic field passing through the rings induces strong circular currents that attract each other with the Ampere force. In a general form, this force is a function of the current at each point along the conducting loop and the mutual loop orientation [165, 166]:

![Diagram of forces acting on the resonators]

Fig. 5.2 Schematics of the forces acting on the resonators.
\[ \vec{F}_i = \frac{\mu_0 I_1 I_2}{4\pi} \int \int d\vec{r}_2 \times \frac{d\vec{l}_i \times \vec{r}}{r^2} \]  

(5.4)

where each line integral is carried out over the corresponding current circuit, \( I_1 \) and \( I_2 \) are RMS values of the currents induced on the rings, \( \vec{r} \) is a unit radius vector. The calculation can be broken down into two steps: first calculating the magnetic field created by the loop 1

\[ \vec{B}_1 = \frac{\mu_0 I_1}{4\pi} \int \frac{d\vec{l}_1 \times \vec{r}}{r^2} \]  

(5.5)

and then integrating the field along the loop 2

\[ \vec{F}_i = I_2 \int d\vec{r}_2 \times \vec{B}_1 \]  

(5.6)

By approximating the loops as circular and parallel to each other simplified formula can be derived [165]:

\[ B = \frac{\mu_0 I a^2}{2(d^2 + a^2)^{3/2}} \]  

(5.7)

d is the distance between the loops, \( a \) is the radius of the loop. However, the force dynamically changes when resonators move and mutual orientation of the loops has to be taken into account. The calculations are further complicated by changed mutual inductance of the loops affecting the magnitudes of the currents.

The maximum of the current distribution is along the bottom side of the rings. The translational movement of the rings along the suspension rod needs to be restricted to prevent them from slipping towards each other and sticking together, which can be done by cutting grooves in the support rod. The current induced magnetic force applied to the bottom of the rings rotates each ring towards the other around the corresponding suspension axis \( O \), created by the groove, with moment

\[ M_i = F_i l_i \]  

(5.8)

where \( l_i = OB \) is the moment arm. The response time of this mechanical movement is much larger than the period of incident wave oscillations, hence can be considered quasistatic. Also, electrical charges of equal signs are induced on each ring in the gap area on the top of the resonators. The charges repel each other and further assist the movement. This rotation
forces the centre of gravity $C$ to rise and to move sideways. Force of gravity serves as the main restoring force and is given by

$$F_g = mg$$ (5.9)

where $m$ is the mass of the suspended structure including the split ring and the substrate, $g$ is the gravitational constant. This force acts on the moment arm $l_g = OA$ and rotates the ring in the opposite direction with moment

$$M_g = F_g l_g$$ (5.10)

in order to return it to initial state with the lowest position of the centre of gravity $C$. As the ring rotates, the moment arm of the Ampere force $l_I$ reduces and the moment arm of the gravitational force $l_g$ increases. The equilibrium is achieved when

$$M_I = M_g$$ (5.11)

The friction and air resistance forces also oppose the movement of the resonators and thus present a minimum power threshold for nonlinear action. In this study, it is assumed that their magnitudes were sufficiently small to be neglected. The magnitude of the Ampere force and correspondingly the angle of rotation is controlled by the power of the incident radiation that influences the magnitude of the induced currents. The amount of incident power needed to induce sufficiently strong currents for the rings to move, can be reduced by minimizing the effect of gravitational force via reducing mass of the structure.

### 5.4.2 Optimized resonator with integrated suspension design

Fig. 5.3 (a) shows the proposed split ring resonator design with an optimized and integrated suspension mechanism. Since the force of gravity depends on mass, one of the thinnest and lightest microwave substrates was used. The substrate chosen for the design was copper clad Rogers Ultralam 3850 material with dielectric constant $\varepsilon_r = 2.9$ and loss tangent $\delta = 0.0025$ available in thicknesses of 0.1 mm and less. Its density of 1.4 g/cm$^3$ is lower than other available circuit materials and allows reducing the mass of the structure. Also areas of substrate were removed leaving just enough to provide structural support. This can be done using either laser or mechanical milling.
Fig. 5.3 (a) Split ring resonator design: ring side size $w = 9$ mm, trace width $wt = 0.2$ mm, substrate size $a = 10$ mm, ring width $wr = 1$ mm, suspension height $hs = 1$ mm, counterbalance height $hc = 1.1$ mm, thickness $t = 0.1$ mm (b) Schematic of the resonators in the waveguide, two resonators suspended at $d = 1$ mm apart from each other.

By reducing the moment arm of the gravitational force, the lever effect is magnified. To achieve this, the centre of gravity needs to be close to the axis of rotation. For this reason, it is beneficial to have the axis of rotation in the middle rather than at the top of the structure. Thus, a horizontal strip of substrate serving as a suspension mechanism was placed in the middle of the structure.

However, one must ensure that the centre of gravity of the resonators is below the axis of rotation, otherwise stable suspension can not be achieved. So, the suspension strip was counterbalanced by an extra section of substrate at the bottom of the resonator. By varying the width of this strip the amount of restoring force can be controlled.

In order to reduce the friction due to surface roughness, the area of contact with the support was minimized by using tooth shaped pillars to contact the Teflon support rod. Grooves were defined in teflon support along the plane of the resonators to fix the separation between them and prevent them from sliding towards each other.

Resonators were fabricated using standard printed circuit board photolithography (Fig. 5.4). The unwanted areas were cut out.
5.4.3 Angle dependence simulation

The response of the structure at different angles of rotation was simulated in order to estimate the potential frequency shift that could be achieved if the rings would rotate completely and make contact. For the separation of 1 mm between the pair of resonators maximum angle each ring could rotate before the bottom sides touch is around 5 degrees. The simulation shows that the corresponding frequency shift was up to 400 MHz (Fig. 5.5). Increasing the separations between the resonators would leave more room for rotation. However, the larger the separation, the weaker is the interaction between the currents.

Fig. 5.4 Photo of the resonator fabricated on commercial microwave substrate.

![Photo of the resonator](image)

Fig. 5.5 Dependence of transmission through the pair of the resonators on the angle of rotation.

![Graph showing transmission vs frequency](image)
5.4.4 Measurements of nonlinear response of the resonator pair

A pair of the resonators was tested in WR187 rectangular waveguide at low (-5 dBm, 0.316 mW) and high (29.8 dBm, 950 mW) incident power levels (Fig. 5.6) using Wiltron 306B vector network analyzer. A resonant frequency shift of 20 MHz was observed. At low power levels, the induced currents were too weak to cause any noticeable movement of the resonators. A power of 1 W was sufficient to rotate the resonators towards each other and a frequency shift could be registered. A change in resonant frequency of the metamaterial resulted in different effective parameters, and thus different refractive index.

To ensure that the shift is indeed caused by the change in mutual position of the rings and not by change in size due to heating, the same test was performed on single ring and no frequency shift was observed when switching to higher power level.

The amount of registered frequency shift suggests that resonators rotated less than a degree. To further improve performance of the structure it was decided to create new microfabricated mesh substrate that would allow to dramatically reduce the mass of the resonator.

![Graph showing transmission characteristics of nonlinear metamaterial measured at low and high input power levels.](image)
5.5 Gravitational metamaterial on mesh substrate

5.5.1 Resonator and mesh design

It is proposed to use microfabrication techniques and traditional photoresist called SU-8. It can be patterned with complex shapes such as mesh with high resolution. The SU-8 3050 photoresist with dielectric constant $\varepsilon_r = 3.2$, loss tangent $\delta = 0.033$ and thickness of 100 $\mu$m was patterned with equilateral triangles of 0.5 mm height and 40 $\mu$m thick beams (Fig. 5.7 (a)). The unexposed photoresist was removed during fabrication.

Fig. 5.7 (b) shows the proposed resonator on the mesh substrate. The shape of the structure has been changed to be hexagonal to better overlap the resonator with the mesh. The material of the resonator was changed to gold due to ease of fabrication. The circumference of the hexagonal split ring resonator was of similar length to the previous square shape, so that the resonant frequency would be in the same range. The hexagonal structure uses the least material to create a lattice of cells within a given volume.

![Diagram](image)

Fig. 5.7 (a) Schematic of the mesh design (b) split ring resonator with the following parameters: hexagon side $wh = 6.46$ mm, gap $g = 1$ mm, trace width $wt = 0.2$ mm, counterbalance height $hc = 0.5$ mm.
This honeycomb structure is the strongest geometry as it provides high compression and shear resistance to the material overall. It is commonly used in packaging. However, a triangle is the most rigid shape and is best for minimizing bending and local distortions. It is widely used in civic structures. Combining two structures would improve the strength of the substrate. Based on the volume of the removed material mesh substrate was estimated to be 75% lighter than the solid version.

5.5.2 Fabrication

Since the trace width of the resonator was larger than the mesh beam width, the gold was patterned first and then SU-8 mesh was created on top. This approach is different from the methods used for perforated SU-8 membrane for gold mirror in [167] and silver hole arrays on SU-8 substrate in [168, 169], where the mesh and overlaying metal sizes coincided. Fig. 5.8 shows the fabrication steps. In order to fabricate the resonator first the silicon substrate was spincoated with a 950 PMMA-A4 sacrificial layer 300 nm thick and softbaked at 180°C for 90 seconds. The seed layer of Ti/Ni/Au (50/25/30 nm) was deposited. Then the layer was coated with HMDS and AZ 4652 (8 μm), softbaked at 100°C, UV light exposed and developed. 8 μm of gold was electroplated into the AZ mould. The AZ was then stripped with acetone.

![Fabrication steps diagram](image)

Fig. 5.8 Fabrication steps diagram.
To make the mesh on top of resonator SU-8 3050 photoresist was spincoated to 100 μm thick and softbaked at 65°C for 5 minutes, 95°C for 10 minutes, 65°C for 5 minutes. To define the mesh pattern, the SU8 was exposed to UV light through printed transparency mask aligned with the resonator, then postbaked at 65°C for 5 minutes, 95°C for 10 minutes and, 65°C for 5 minutes to minimize stress. Then SU-8 was developed to reveal the mesh pattern. Finally the sacrificial PMMA layer was dissolved with acetone to release the SU-8 mesh with resonator on it. The seed layer was etched away as well. These fabrication procedures were carried out by RMIT University staff member.

Figs. 5.9-5.12 show the fabricated structures. First, only the mesh without the resonator was made (Fig. 5.9 (a)). Samples with beam widths of 40-140 μms in steps of 20 μm were made in order to estimate the strength and flatness of the structure. The advantage of the microfabrication is that all of the mechanical features could be formed in a single fabrication step during exposure through the mask. The unexposed areas were removed during the developing stage, so no milling was required afterwards. All the samples were successfully fabricated demonstrating free standing hexagonal shapes with removed inner parts, defined suspension beam and pillars. It was decided to proceed with the mesh beam width of 40 μm as this should be the lightest.

Fig. 5.9 (b) shows a gold split ring resonator on a mesh of SU-8. Fig. 5.10 (a, b, c) shows the enlarged corner, resonator gap and pillar areas. The 200 μm trace width of the resonator was wider than 40 μm mesh beam width, so the metal layer was partially in the air, however, it was still mostly flat throughout the resonator, with occasional minor buckling evident. Samples with other mesh sizes were also made. Fig. 5.11 shows images of the structures with mesh beam sizes of 100 μm and 140 μm.

The mesh was sufficiently robust to provide structural support for the resonators and withstand handling. Fig. 5.12 shows the resonator bent into semicircular shape without breaking, which is remarkable since the SU-8 material is not elastic. After release, the structure assumed initial flat shape, no cracks on either substrate or metal were seen.
Fig. 5.9 Photos of the fabricated mesh substrate with 40 μm beam width: (a) without the resonator (b) with the resonator.

Fig. 5.10 Enlarged photos of various resonator areas: (a) corner (b) gap (c) suspension pillar.

Fig. 5.11 Photos of the fabricated resonators on substrates with mesh beam width of (a) 100 μm and (b) 140 μm.
5.5.3 *Simulation of electromagnetic properties*

Removing areas of the substrate also had a positive effect on the electromagnetic properties of the structure, since the effective permittivity of the mesh is expected to approach free space values. In order to explore the electromagnetic properties of the mesh substrate, resonators of the same size, but on different substrates, were simulated with the ANSYS HFSS software. Fig. 5.13 shows comparison of transmission through the pair of the resonators of the same size but on different substrates: in free space; on solid SU-8; and on mesh SU-8 with 40 μm beam width, all suspended in WR187 waveguide. It can be seen that in the case of the mesh SU-8 substrate, as the mesh became thinner, the performance of the structure approached the case when the resonators were in free space. A higher resonant frequency and reduced losses were predicted compared to the case with a solid SU-8 substrate. It was calculated that effective permittivity of the 40 μm beam mesh was $\varepsilon_{\text{eff}} = 1.39$ compared to $\varepsilon_r = 3.2$ for solid SU-8.

This substrate can be thought of as analogue to foam used in microwave measurements to support the structure without affecting its electromagnetic performance. This SU-8 mesh substrate concept could be utilized for other RF applications, or could be reduced in dimensions to operate at THz frequencies and may find applications in other fields as well.

![Bent resonator on SU-8 mesh substrate with 40 μm beam width.](image)

Fig. 5.12 Bent resonator on SU-8 mesh substrate with 40 μm beam width.
Fig. 5.13 Simulated transmission characteristics of the resonator pair on different substrates.

5.5.4 Measurements of the resonant frequency of individual resonators

The resonators were tested in WR187 rectangular waveguide. First the effect of mesh beam width on the transmission through the structure was studied on single resonators of the same size but on mesh substrates with beam widths 40-140 μm as shown in Fig. 5.14. As can be seen from the photo, stable suspension was achieved due to the action of the gravitational force acting to bring the resonator to the state with minimal potential energy, which is vertical. As mentioned earlier, adding a counterbalance to the bottom of the resonator is vital for stabilizing the structure and preventing it from falling over. The stability of the suspension was also reflected in the transmission measurements, showing no deviations from the initial result over time. The transmission measurement results presented in Fig. 5.15 support the hypothesis that as beam width of the mesh is reduced the properties of the substrate approach that of the free space. The resonators on the mesh with narrower beams showed higher resonant frequency and quality factor. The simulations of the same size resonators in free space and on solid SU-8 substrate are given for comparison (shown with dashed lines).
Fig. 5.14 Photos of single resonator suspended in the waveguide.

Fig. 5.15 Measured single resonator transmission characteristics for different mesh beam sizes. Simulations for free space and solid SU-8 cases shown in dashed lines for comparison.
5.5.5 Measurements of nonlinearity of the resonator pair

A pair of the resonators was suspended on a teflon support with grooves at a distance of 2 mm from each other in a rectangular WR187 waveguide and tested at low (10 dBm, 10 mW) and high (30 dBm, 1 W) incident power levels (Fig. 5.16) using Wiltron 37269A vector network analyzer. The resonators should be identical in order to resonate at the same frequency. However, as an outcome of the fabrication, no identical mesh size resonators were available. When placed in close proximity to each other resonators respond collectively to the incident radiation due to coupling effect. However, preliminary measurements of resonator pairs with mesh widths of 40, 100 and 140 μm revealed that their resonant frequencies are too far from each other resulting in split resonance response. During the individual resonator resonant frequency measurements it was found that one of the resonators with mesh width of 60 μm resonated at the same frequency as the resonator with the mesh width of 100 μm due to fabrication outcome that resulted in reduced metal trace width (Fig. 5.17). It was decided to proceed with the measurements using these two resonators of different geometry but similar resonant frequencies as they produced single resonant frequency response as a pair. For high incident power level measurements vector network analyser excitation signal was amplified by a signal amplifier with a maximum output power of approximately 32 dBm. The variation in the output power of the amplifier was 0.2 dB within resonators operating frequency range. A resonant frequency shift of 24 MHz was observed (Fig. 5.18), which is larger than 13 MHz in initial demonstration [162]. The frequency shift was repeatable when power was switched between high and low levels. Fig. 5.19 shows dependence of resonant frequency shift on incident power level. As the mechanical response of the resonators was much slower than the frequency of electromagnetic oscillations, some time was required for resonators to settle into their final configuration. When doing the measurements, a few minutes were allowed for the resonators to settle before proceeding to the next power level. Larger frequency shifts occurred at lower power levels compared to the initial demonstration in [162] in spite the resonators being further away from each other. So, reduction of power level required to tune the structure has been achieved. The measurements were repeated for a separation 1.5 mm between the resonators (Fig. 5.20). A significantly larger resonant frequency shift of 126 MHz in the opposite direction was registered. This implies that bottom of the resonators repelled each other, which would happen when the currents are antiparallel.
Fig. 5.16 Photos of two resonators suspended in the waveguide at 2 mm apart.

Fig. 5.17 Transmission characteristics for single resonators of different geometries used for nonlinear measurements.
Fig. 5.18 Transmission through the pair of resonators at low (10 dBm) and high (30 dBm) incident power levels.

Fig. 5.19 Dependence of resonant frequency shift on incident power level.
Fig. 5.20 Transmission through the pair of resonators at low (10 dBm) and high (30 dBm) incident power levels.

The lighter substrate minimized the force of gravity in resisting the electromagnetic movement of the structures. Improvement in the performance of the structure was seen, however it did not reach its full tunability potential. Increasing the Ampere force via more sophisticated resonator design capable of supporting larger currents may further improve the performance of the structure. This may be the topic of further investigations.

It should be noted that in this work, the vector network analyser excitation signal was used instead of signal generator in the initial demonstration. As the vector network analyser performs a frequency sweep, the power is delivered to the resonators at their resonant frequency only for a fraction of the time. Matching the rate at which frequency of the incident radiation is changed to the speed of mechanical movement of the resonators, as well as using identical resonators, might dramatically improve the results.

Among the shortcomings of this particular tuning method the following should be mentioned. The structure cannot be used on tilted surfaces and is sensitive to vibration. The substrates were not entirely flat. Although this did not have a noticeable effect on the single resonator, it could affect the resonant frequency of the pair. This could be improved by better control of film stresses during fabrication. The structure is not intended for fast switching, the frequency shift happens within few seconds, which is, however, suitable for many applications.
5.6 Conclusions

In this chapter, a nonlinear gravitational magnetoelastic metamaterial based on self-reorienting resonators has been designed, realized and characterized. The resonant frequency of these resonators was tuned by the power level of the incident wave without use of inherently nonlinear devices. Forces involved in the movement of the suspended resonators with mechanical degrees of freedom were theoretically investigated. Methods for optimising the design to minimise the gravitational restoring force were proposed.

The design was realized on commercially available as well as custom made substrates compatible with standard photolithographic fabrication techniques. An SU-8 photoresist mesh substrate with 40 μm features has been produced by microfabrication techniques. A reduction of 75% in mass of the substrate, as well as effective electromagnetic properties close to free space, have been achieved. As a result, measurements demonstrated improvement in the nonlinear performance of the structure by larger shift of the resonant frequency in response to reduced level of incident radiation power. Matching the rate of the incident radiation frequency change to the speed of mechanical movement of the resonators and using identical resonators might further reduce the amount of the required power.

Further reduction of operating power level could enable practical nonlinear experiments and applications at RF frequencies. Power levels of 30 dBm is a little extreme. A reduction to 20 dBm power would make a number of experiments more practical. It may be possible to further engineer these structures to achieve even lower power requirements. If this can be done it may even be conceivable to create nonlinear structures for THz frequencies, where powerful radiation sources are not available.

The high resolution patterning capabilities of microfabrication will enable manipulating the mechanical and electromagnetic features of the substrate as well as shape the metallic coating for the production of arrays of resonators in nonlinear volumes of a metamaterial. The ability to engineer the linear and nonlinear behavior of the metamaterials will enable exploration of materials with graded characteristics.
Chapter 6

Elastic fishnet metamaterial for THz frequencies

6.1 Introduction

In Chapter 3 and 4, a pneumatic switching approach for metamaterial tuning has been discussed. It was mentioned that greater flexibility in the design could be achieved by individual switching of each unit cell. This method requires not only flexible, but also an elastic membrane that could bend into the small space of the unit cell. Also, investigating the behavior of resonators embedded into elastic medium could further the research on magnetoelastic metamaterials discussed in chapter five.

Flexible metamaterials are mostly intended for terahertz (THz) operating frequency range. Terahertz radiation, also known as low infrared, can be used in biomedical imaging, since it is nonionizing and thus it is safer than conventional X-ray computed tomography [170, 171]. Other applications include security screening, package inspection and manufacturing quality control [172]. THz radiation was largely unexplored until recent years because it is difficult to find natural materials that respond to THz frequencies. Metamaterials, unlike natural materials, exhibit strong resonant behavior at THz frequencies [173, 174]. Another promising areas for flexible metamaterials are flexible electronics [175], sensors [176, 177], lab-on-a-foil systems [178].

Since the first demonstration of metamaterials for microwave frequencies, the majority of metamaterials have been fabricated on rigid substrates. However, with the growing demand for metamaterials that can be wrapped around objects or attached to curved surfaces, some flexible structures have been produced. Among them are split-ring structures on polyimide [179, 180] and parylene [181]. Also flexible devices, such as metamaterial
strain sensors [182, 183] on Kapton tape (polyimide based) and metamaterial absorber on polyimide [184] have been demonstrated. Further, a multilayered metamaterial with refractive index as high as 27 has been constructed based on polyimide substrate as well [185]. Another materials used as flexible substrate were polypropylene [186], silk [187], which is also biocompatible, and polyethylene naphthalate (PEN) [188], which was used to construct multi-layer metamaterial in order to achieve broadband frequency response. Polyethylene naphthalate was also used for optical frequency range metamaterial intended for chemical and biological sensing [189]. In these works traditional metamaterial structures, such as split-ring resonators, were used with substrate layer thicknesses around 5-16 μm for parylene and polyimide and 80-125 μm for silk and polyethylene. Ultrathin structures have been achieved on silicon nitride (Si₃N₄) membranes 0.4-1 μm thick [190, 191]. Another multilayer structure involved metallic crosses embedded in benzocyclobutene [192]. Unlike split ring resonators, this structure demonstrated negative refractive index behavior. An example of continuous flexible single layered metallic grid-like structure fabricated on thin SU-8 membranes and operating at optical frequencies was demonstrated in Ref. [193]. A flip chip transfer (FCT) technique was used in Ref. [194] to relocate a tri-layer metamaterial absorber from quarts to polyethylene terephthalate (PET) substrate with potential to extend its application to other flexible substrates. The latest work demonstrated fishnet metamaterial on 381 nm thick polyimide substrate, designed to have near zero refractive index and matching to free space at 1.55 μm wavelength [195]. A roll up InGaAs/GaAs/Ag multilayers for optical frequencies have been proposed in Ref. [196].

Most of the demonstrated flexible materials were, however, not elastic. This means they cannot be stretched. Elasticity is required for the operation of various metamaterial based strain sensors. These rely on modification of the unit cell via increasing the gap sizes in double gap split ring resonators as a result of stretching the substrate. One report has presented results using 1 mm thick polydimethylsiloxane (PDMS) as a substrate for single layer of split ring resonators which allowed to stretch the samples by a remarkable 50% without destruction of metallic elements, with reversible tuning obtained for up to 10% stretching [197]. Stretching can be also used for manipulation of metallization pattern, since gold, commonly used for THz metamaterials, retains its conductive properties when stretched up to 10% [98-200]. The effects of stretching of metallic parts of split ring resonator by 5% have been theoretically investigated in Ref. [201] and tuning effect of this approach was confirmed.
An elastic fishnet design has not been demonstrated yet and the effects of stretching on its tunability and structural integrity have not been investigated. In this chapter, the electromagnetic properties of polydimethylsiloxane (PDMS) are characterized and it is used as a free-standing substrate for the realization of elastic fishnet metamaterial at terahertz (THz) frequencies. PDMS adds a degree of freedom to terahertz metamaterials with the potential for tuning by elastic deformation. With suitably designed initial parameters, fishnet material offers a unique opportunity, as it could be turned from single negative to double negative just by stretching. PDMS is a low cost, highly elastic, biocompatible material that is already widely used in microfluidics and nanofabrication [115-117, 202]. Also, the low surface energy of PDMS allows conformal adhesion to curved surfaces. Suitability of PDMS as metal carrying membranes for pneumatic switching has been demonstrated in Ref. [203]. The thickness of the material for the structure was chosen such as to allow future incorporation of pneumatics and microfluidics.

In this chapter, an elastic THz fishnet metamaterial is presented including design, realization and characterization. The fishnet fabrication has been done by another student at RMIT University and the THz measurements were carried out by the collaborators from the University of Adelaide. Tuning possibilities via elastic deformation and fluid infiltration of the fishnet metamaterial are theoretically explored as suggestions for future investigation.

6.2 Elastic fishnet metamaterial

In this work, a fishnet metamaterial structure is used as it is generally comprised of two metal layers without vertical interconnects, which is suitable for planar processing. Moreover, in the terahertz frequency regime, the fishnet unit cell dimensions are in the micro scale and compatible with standard microfabrication techniques. Unlike planar arrays of split-ring resonators, fishnets are capable of exhibiting negative permittivity and negative permeability simultaneously, achieving the negative index of refraction. Fishnet structures also allow realization of polarization insensitive designs [101].
6.2.1 **PDMS characterization at THz frequencies**

As a silicone polymer, PDMS is known to have stable dielectric properties and low transmission loss in optical frequency range [204], however, it is not well studied at THz frequencies. In this work, the dielectric properties of PDMS were investigated using a free-space coupled terahertz time-domain spectroscopy (THz-TDS) system [171], equipped with a photoconductive antenna for T-ray generation and electro-optical detection. A mode-locked Ti:sapphire laser with a central wavelength of 800 nm, a pulse duration of <100 fs, and a repetition rate of 80 MHz was used. The system generated pulsed T-ray radiation in the frequency range of 0.05 to 2.8 THz, with a maximum dynamic range of 30 dB. The collimated beam diameter was approximately 10 mm.

The tested PDMS samples were two sheets of 375 μm and 960 μm thickness respectively. Reference measurements were conducted at room temperature. The index of refraction and the absorption coefficient for both thicknesses were determined, both having similar characteristics. The refractive index and absorption coefficient for the 960 μm thick PDMS sheet are shown in Fig. 6.1.

The absorption coefficient of 20 cm\(^{-1}\) at 1.0 THz compares well with 30 cm\(^{-1}\) for polyimide [180]. Using these results, the dielectric constant and loss tangent of PDMS were estimated as \(\varepsilon_r = 2.35\) and \(\delta = 0.02-0.04\), respectively, across the 0.2 to 2.5 THz band. This relatively low absorption of PDMS is significant to a reduction of loss in metamaterials.

6.2.2 **Fishnet design and simulations**

Using the determined dielectric properties, the fishnet structure was simulated in ANSYS HFSS software. The proposed fishnet metamaterial consisted of patterned metallic layers embedded in PDMS. Fig. 6.2 shows one unit cell of the structure, which was assumed to be periodic and infinite in the x-y plane. This fishnet was independent of polarization of the incident wave due to its symmetric square shape. The direction of propagation of the incident wave was normal to the plane of the structure. The thicker outer PDMS layers were required to prevent tearing during stretching and handling of the material.

The fishnet exhibited a magnetic resonance at 2.1 THz and an electric resonance at 2.9 THz as shown in Fig. 6.3. Although the behavior of the structure is determined by the collective response to both electric and magnetic components of the incident field, response
to one of the components is greatly enhanced when the current path associated with it results in resonant behavior, while the contribution of response to the other component is diminished. So, the resonances are referred to as magnetic or electric after the component of the incident field mostly contributing to the response, as explained in the reference [23].

Fig. 6.1 Dielectric properties of a 960 μm thick PDMS sample in the terahertz region: (a) index of refraction and (b) absorption coefficient.
Fig. 6.2 Fishnet unit cell with the following parameters: cell dimensions $70 \times 70 \times 510 \ \mu m$, $p = 50 \ \mu m$, $w = 5 \ \mu m$, $t = 10 \ \mu m$, $d = 250 \ \mu m$ for both outer PDMS layers.

Fig. 6.3 Simulation results comparing the terahertz response of a single and double metal layer structures.
The magnetic resonance originates from the antiparallel currents induced in the first and second layers of the structure by the magnetic field passing between the metal layers [23]. The surface currents on the top and the bottom metal layers of the fishnet at 2.1 THz are presented in Fig. 6.4. The currents form an inductive-capacitive LC loop within each unit cell, with the spacer between the layers acting as a capacitor and metal slabs acting as inductors (effective LC circuit model available in reference [23]). This response of the slab pair results in a negative permeability of the material. If the resonance were magnetic in nature, excitation of these surface currents in the fishnet is impossible if the loop were to be broken by removing the second metal layer. As a demonstration, the simulation was repeated for only a single layered structure, and the response is presented as a dashed line in Fig. 6.3. The disappearance of the 2.1 THz resonance for the single layer confirms its magnetic nature. So, it is essential to use double layered structure with closely spaced layers in order to achieve magnetic resonance leading to negative effective permeability of the material. This resonance is also accompanied by the Drude-like response of the thin wire grid to the electric component of the field resulting in the negative permittivity below the corresponding plasma frequency. If the frequency ranges for negative permeability and negative permittivity responses overlap than negative refractive index behavior is expected.

The electric resonance, occurring for a double layered fishnet at 2.9 THz, is largely unaffected by the removal of the second layer. In this case, the electric field induces currents oscillating along the metal slabs in the same direction for both layers, as shown in Fig. 6.5. After removing the second metal layer, the currents are still induced on the first layer resulting in the electric resonance that has shifted to 2.8 THz that can clearly be seen in Fig. 6.3. The current path length is shorter at electric resonance, hence the frequency is higher than at magnetic resonance. This resonance can be considered as a different mode and is not of interest for negative refractive index operation.

6.2.3 Effective electromagnetic parameters

The effective permittivity and permeability of the fishnet were extracted from the simulation results using the procedure described in Appendix A. The effects of the external PDMS layers were removed from the transmission and reflection coefficients via de-embedding to reveal only the response of the fishnet. The structure showed negative permittivity below 1.7 THz and negative permeability at 2.1 THz as shown in Fig. 6.6.
Fig. 6.4 Distribution of the surface currents of the (a) top and (b) bottom metal layer of the fishnet near the magnetic resonance at 2.1 THz.

Fig. 6.5 Distribution of the surface currents of the (a) top and (b) bottom metal layer of the fishnet near the electric resonance at 2.9 THz.

Due to anisotropy of the fishnet the extracted effective parameters should not be used for directions of wave propagation other than normal.

The fishnet metamaterial is sensitive to the dielectric properties of the surrounding media. Embedding a metal grid into the thick outer PDMS layers resulted in reduction of its electric plasma frequency, which coincides with the point where real part of the effective permittivity crosses zero. This put the negative permittivity region below the negative permeability region and prevented negative refraction.
However, if desired, the structure could be optimized to exhibit double negative behavior if the shift in plasma frequency were corrected by increasing the width of the wires. Elastic fishnets with different neck widths $w$ were simulated with all the other parameters the same as in original fishnet from Fig. 6.2. It was determined that the elastic fishnet with neck width $w = 35 \mu m$ would exhibit simultaneously negative values of permittivity and permeability in the frequency range near the magnetic resonance, leading to negative refractive index. The predicted transmission through this fishnet is presented in Fig. 6.7 and the effective permittivity, permeability and refractive index are presented in Figs. 6.8-6.10 correspondingly. Note that the resonant frequency has shifted from 2.1 THz for fishnet with $w = 5 \mu m$ to 2.55 THz as a result of increase of the neck width to $w = 35 \mu m$.

![Graph showing effective permittivity and permeability](image)

Fig. 6.6 (a) Effective permittivity and (b) permeability of the fishnet in PDMS.
Fig. 6.7 Transmission through the double negative fishnet in PDMS with $w = 35 \, \mu m$.

Fig. 6.8 Effective permittivity of the fishnet in PDMS with $w = 35 \, \mu m$. 

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Fig. 6.9 Effective permeability of the fishnet in PDMS with $w = 35 \mu m$.

Fig. 6.10 Effective refractive index of the fishnet in PDMS with $w = 35 \mu m$. 
6.2.4 Fabrication

For fabrication of the fishnet, standard microfabrication techniques have been adapted to flexible device fabrication [175, 202, 203]. The fishnet has a total of 5 layers, with alternating layers of PDMS (3 layers) and patterned metal (2 layers).

Initially, a silicon wafer was cleaned with solvents and coated with 20 nm of aluminum deposited at room temperature by electron beam evaporation. This layer prevents permanent adhesion of the subsequent PDMS layer to the silicon substrate. A 250 µm thick layer of PDMS was then spin coated onto the wafer.

Metal layers forming the fishnet were deposited on the PDMS coated wafer using electron beam evaporation at room temperature. Alternating layers of chromium (20 nm) and gold (200 nm) were deposited with three layers of chromium encapsulating two layers of gold, resulting in a 460 nm thick metal layer. The alternating layers minimize thin film stress, and also the use of chromium as the outer layer improves adhesion to PDMS. The metal layers were patterned using standard photolithography techniques. Chromium was etched using a mixture of perchloric acid and ceric ammonium nitrate, with the gold etched in aqua regia (mixture of nitric and hydrochloric acids in water).

Then a 10 µm thick layer of PDMS was spin coated onto the first metal fishnet layer to serve as the dielectric separating the two fishnet layers. Following the curing of this layer, the multi-layer metallization and etching process was repeated to define the second fishnet layer, with an alignment process during photolithography. The fabrication process was completed by spin coating and curing a 250 µm thick PDMS layer. The resulting 5-layered symmetric structure was peeled off from the supporting silicon substrate to realize a free standing flexible fishnet metamaterial as shown in Fig. 6.11.

The elasticity of PDMS poses a challenge in the alignment of layers during the patterning of the second fishnet layer with respect to the first. One edge of the wafer comes into contact with the photolithography mask first, before the rest gradually comes in contact. This typically results in a 2-5 µm misalignment error in the multilayer elastomer samples. Fig. 6.11 (b) shows the fabricated fishnet structure with 3 µm misalignment in both x and y directions. The influence of this misalignment on the transmission was determined by HFSS modeling as shown in Fig. 6.12. The misalignment caused a slight splitting of both the stop
band (indicated by the arrows in the figure) and also the following pass band. Despite this slight splitting, the structure showed good tolerance to misalignment, since the magnitude and position of the transmission band remained almost the same. A slight shift occurred due to the change in capacitance of the spacer as a smaller overlap area was involved in the resonance.

The fabricated fishnet withstood handling and bending very well despite being a continuous metallic structure.

### 6.2.5 Measurements of transmission through the elastic fishnet

Measurements have been done using the same setup as for the characterization of the simple PDMS sheets described in Section 6.2.1. Fig. 6.13 compares the simulated and measured results for transmission through the fishnet structure. Good agreement between theoretical and experimental results is observed. A resonant dip is seen at 2.1 THz as predicted by simulations. Splitting due to the slight misalignment is present at 2.0 THz as also per theoretical predictions. The simulated results provide the best fit to the experimental results for a loss tangent of PDMS equal to 0.02, validating the low loss characteristics of the elastomeric PDMS substrates.
Fig. 6.12 Transmission through aligned and misaligned by 3 μm fishnet structures.

Fig. 6.13 Simulated and measured terahertz response of PDMS-based fishnet structure.
6.3 Exploration of methods for tunability

6.3.1 Effect of elastic deformation

As it was mentioned earlier, gold can be stretched by 10% without losing its conductive properties [198-200]. The effects of strain on metallic parts of split ring resonator have been theoretically investigated [201]. When stretching discontinuous structures like arrays of unconnected split ring resonators, stretching of the substrate leading to larger gaps is observed, while there is almost no stretching of the metal itself. In the case of continuous structure like a fishnet, the metallic wires themselves are stretched and the shape of metallic elements is altered. This is expected to lead to more significant effects on the tunability of the structure. The fishnet metamaterial offers a unique opportunity, as it can be turned from single negative to double negative just by stretching.

The opposite action of compression is also a possibility. This method may be less likely to compromise the integrity of the structure. To investigate the expected effect of compressing the fishnet, a compressed geometry was modeled in ANSYS HFSS. For the simulation a 5% reduction in both fishnet slab as well as neck length in the direction of the compression was assumed. The thickness of dielectric was not changed as it usually has very minor effect on tunability. The fishnet model was compressed along the direction of the E field as it should be more sensitive to modifications along the direction of electric field, than magnetic field. Fig. 6.14 presents the transmission response when the fishnet is compressed by 5%. It is clear that the magnetic resonance at 2.1 THz should shift to approximately 2.2 THz. The extracted effective parameters, shown in Figs. 6.15-6.17, also show a similar shift of the resonant features. Compression would remove polarization independent nature of original structure, so differently polarized components of the incident wave would propagate differently through the structure. This could allow dynamic modulation of incident radiation.

In order to test the effects of stretching and compression experimentally, specialized measurement apparatus would be required in order to induce precise stretching and compression. Unfortunately, such apparatus was not available at the time that these experiments were conducted, however, this would be interesting to pursue for future work.
Fig. 6.14 Transmission through elastic fishnet in relaxed and compressed by 5% states.

Fig. 6.15 Effect of compression on permittivity of elastic fishnet.
Fig. 6.16 Effect of compression on permeability of elastic fishnet.

Fig. 6.17 Effect of compression on effective refractive index of elastic fishnet.
6.3.2 **Effect of fluid infiltration**

There have been some investigations into infiltration of metamaterials and submerging them in fluids for biosensing applications [205-208]. Split ring resonators have been considered [209, 210]. It might be expected that larger frequency shifts, and thus higher sensitivity, should be possible with fishnet structures because the field is concentrated between the layers facilitating stronger interaction with infiltrating fluid than split ring resonators.

A brief theoretical investigation of the tunability of the elastic fishnet metamaterial embedded in PDMS of Section 6.2 was conducted. The investigation assumed that it would be possible to infiltrate all of the volume between the two metal layers with commercially available dielectric fluids [211]. These fluids were assumed to be electromagnetically transparent. Fig. 6.18 presents the predicted response when infiltrated with fluids with dielectric constants of \( \varepsilon_r = 1.69 \) and \( \varepsilon_r = 2.2 \). A substantial shift of the resonant frequency of the structure from 2.1 THz to 2.3 THz has been predicted.

![Fig. 6.18 Transmission through fishnet infiltrated with various fluids.](image-url)
In order to practically realize such a fishnet that could be fluid infiltrated, mechanical supports with fluid channels of various sizes should be defined within the structure using another material, such as SU-8. While not inherently elastic, these layers could be thin enough so that they do not significantly impact the flexibility of the top and bottom PDMS layers. The design and realization of such a structure is proposed for future work.

6.3.3 Suggestions for future work

Elastic metamaterials could serve as a platform for developing various tunable functionalities. PDMS as an elastic material allows fabrication of a network of membranes and pneumatic chambers supporting various switchable elements. The pneumatic switching capabilities described in Chapter 3 could be extended to individual unit cell switching as well as switching within unit cell itself. Also, compactness of pneumatic approach could allow scaling pneumatic metamaterials to THz frequency range.

Another area for exploration could be using elasticity for strain sensing and tuning purposes. Investigations are also being conducted into the use of flexible metamaterials as waveguide coatings that can be bent and wrapped around traditional materials to modify the guiding properties of the structure.

Another direction could be to build on the microfluidic capabilities of PDMS and manipulate the properties of the structure by infiltrating it with fluids with different permittivities via the network of channels. Sensitivity of the structure to the permittivity of the material between metallic layers makes it suitable for biosensing.

The magnetoelastic concept explored in Chapter 4 could be practically realized if the porosity of PDMS could be such that its elastic constant was reduced to the required value.

6.4 Conclusions

This chapter has demonstrated the fabrication and characterization of a fishnet metamaterial operating in the terahertz frequency range using the flexible elastomer PDMS for all dielectric layers. A strong magnetic resonance was observed. The fishnet, embedded in thick PDMS outer layers, showed high transmission values in the pass bands, highlighting the low propagation loss of the PDMS material. The structure exhibited high tolerance to layer misalignment, maintaining the position and strength of the resonances with only slight
splitting of the resonances observed. A theoretical investigation of utilization of microfluidic and elastic capabilities of PDMS to enable fluid infiltration and elastic deformation of fishnet metamaterials for tuning, wireless strain sensing and biomedical analysis applications was also reported. Possible avenues for future work include using PDMS for pneumatic tuning of metamaterials, further exploration of elastic metamaterial properties and reaction of fishnet metamaterials to infiltration with biological substances.
Chapter 7

Conclusions

The aim of this thesis was to explore dynamic frequency tuning possibilities for metamaterials by developing new approaches for altering the shapes and orientations of the metamaterial resonators, post fabrication. Manipulating the geometrical parameters of metamaterial inclusions leads to tunability of the electromagnetic properties of the media, such as effective permittivity, permeability, refractive index and nonlinearity, allowing custom design of material properties. In the first instance, a novel pneumatic approach was used to alter the conducting patterns and continued to reorientation and elastic deformation of the resonators.

The pneumatic switching approach combined concepts of microelectromechanical switching and pneumatic microoptics. Variation of the metallic patterns of the resonators themself was estimated to have one the most substantial effects on the tunability of the structure, resulting in a broader operating frequency range. It has been demonstrated that it is possible to overcome the challenge of fixed metallic patterns, post fabrication, by using pneumatic pressure. Initially disconnected complex metallic shapes were united into conducting patterns, demonstrating switching between two metamaterial operating frequencies. Pneumatic switching eliminated the interfering biasing network and offered more versatility in the shape and proximity of the actuated elements.

The pneumatic switching approach was then applied to a practical device - an ultrathin graded refractive index lens. Metamaterials can offer significant variations of effective refractive index across the sample without adding complexity to fabrication. The possibility of switching between a metamaterial lens focusing state and no focusing state, when graded index metamaterial was turned into ordinary dielectric by eliminating resonant response, has been successfully demonstrated.
Next, metamaterial resonators that can reorient in response to the very field with which they resonate were explored demonstrating nonlinear frequency shifts as a result of attractive forces between currents induced on resonators suspended with mechanical degrees of freedom. These structures exhibited nonlinear behavior at power levels significantly lower than required in optics. A new concept of gravitational metamaterials has been introduced and forces involved in the movement of the resonators have been analysed. The design has been optimized for microfabrication. As a result, a novel mesh substrate with significantly reduced mass and permittivity approaching that of air was developed.

Finally, a fishnet metamaterial embedded into an elastic substrate was explored for stretching, conformal adhesion and infiltration purposes. The elastic metamaterial has been fabricated and characterized showing strong resonance in the terahertz frequency range and low propagation loss. Theoretical investigations confirmed tunability of the structure as a result of elastic deformation and fluid infiltration.

In many of the demonstrations in this thesis, the resonator elements were tuned on mass with a single mechanism. In future, work individual actuation of each resonator within the material, as well as multiple elements within one resonator, could lead to multifrequency switching. With more sophisticated control of air pressure, continuous or at the very least multi-state tuning may be possible. This could be especially beneficial for graded index lens operation, allowing tuning of the focal length all the way from convex to concave via the no focusing state. Further reduction in power level, required for nonlinear operation, could extend the nonlinear applications to frequencies where powerful radiation sources are not available or are impractical. Elastic and microfluidic capabilities of metamaterials could be further explored for terahertz imaging, strain sensing and biosensing.

The implications of this work are widening flexibility in metamaterial design and improving a variety of metamaterial responses. The new concepts could potentially lead to novel devices capitalising on cross-disciplinary capabilities. The techniques are not limited to the field of metamaterials and could potentially be applied in other areas where changing of metallization patterns or lightweight substrates are required.
Appendix A

Electromagnetic parameters extraction

Metamaterials can be assigned certain frequency dependant effective electromagnetic parameters of a bulk material, like permittivity and permeability, which can be determined from plane wave reflection and transmission coefficients. Homogeneous materials are usually characterized by impedance $z$ and refractive index $n$. The electric permittivity and magnetic permeability have very simple relationship with these quantities:

\[
\varepsilon = \frac{n}{z} \quad (A.1)
\]

\[
\mu = nz \quad (A.2)
\]

A well defined refractive index can usually be determined even for non-continuous structures like photonic band gap materials. Impedance, on the other hand, requires more stringent conditions, since it depends on the overall size and surface termination of the structure. Impedance can be unambiguously defined when periodicity of a non-continuous material is much less than the wavelength of the incident radiation, and this defines the limit of homogenization. Metamaterial inclusions are usually required to be on the order of $\lambda/10$ in size. However, for fishnet structures, a larger size of $\lambda/3$ is still considered acceptable.

The following expressions relate transmission and reflection coefficients to the refractive index and impedance [5, 212]:

\[
T = \frac{1}{\cos(nkd) - \frac{j}{2}\left(\frac{z}{z} + \frac{1}{z}\right)\sin(nkd)} \quad (A.3)
\]

\[
\frac{R}{T} = \frac{j}{2}\left(\frac{z}{z} - \frac{1}{z}\right)\sin(knd) \quad (A.4)
\]

where $k=\omega/c$ is the wave number of the incident wave, $d$ is the thickness of the material.
Reverting the expressions leads to

$$z = \pm \sqrt{\frac{(1 + R)^2 - T^2}{(1 - R)^2 - T^2}}$$  \hspace{1cm} (A.5)

and

$$n = \frac{1}{kd} \cos^{-1} \left[ \frac{1}{2T} \left(1 - (R^2 - T^2)\right) \right] + \frac{2\pi m}{kd}$$  \hspace{1cm} (A.6)

where \( m \) is an integer. The expressions are functions with multiple branches. The selection of the correct branch is guided by the following considerations. Based on causality for passive materials \( \text{Re} (z) > 0 \) and \( \text{Im} (n) > 0 \) must be selected. For

$$\text{Im}(n) = \pm \text{Im} \left( \frac{\cos^{-1} \left[ \frac{1}{2T} \left(1 - (R^2 - T^2)\right) \right]}{kd} \right)$$  \hspace{1cm} (A.7)

the root that results is positive solution should be selected at each frequency. This also defines the sign of

$$\text{Re}(n) = \pm \text{Re} \left( \frac{\cos^{-1} \left[ \frac{1}{2T} \left(1 - (R^2 - T^2)\right) \right]}{kd} \right) + \frac{2\pi m}{kd}$$  \hspace{1cm} (A.8)

which may have negative regions. For large thicknesses \( d \), branches for different \( m \) can lie very close to each other. In order to identify the correct branch, a number of thicknesses need to be solved. The correct branch should result in the same value of \( n \) for different thicknesses. However, for very thin structures, with \( d \ll \lambda \), \( m = 0 \) is usually the correct branch.

For the purpose of effective parameter extraction, the phase of the transmission and reflection coefficients obtained from simulations has to be de-embedded from the ports of the simulation setup to the boundaries of the studied material. For magnetic split ring resonators, oriented parallel to the direction of propagation, the thickness of one layer is taken as equal to the unit cell size \([213, 214]\). The effective thickness selection is further discussed in relation to inhomogeneous materials in Ref. \([213]\). When working with single
layer structures, oriented perpendicular to the direction of wave propagation, like fishnet metamaterials, some ambiguity arises in defining the thickness of the material [215, 216]. As effective parameters represent the properties of bulk media, the thickness of the material in this case is taken as periodicity in the direction of propagation for multilayered media constructed from a number of single layers under consideration. Further analysis of multilayered structures can be found in Ref [217]. Selecting different periodicity results in different effective parameters. Some researchers disagree with this approach and argue that effective parameters extraction for a single layer structure is not meaningful [218, 219].

It should be noted that metamaterials are usually anisotropic and their electromagnetic parameters in general form would be described by tensors. The effective parameters discussed above would represent one of the components of the tensor corresponding to the selected direction of propagation. More general method based on Bloch mode analysis for effective material parameters determination has been proposed recently [220-222].
Appendix B

Parabolic reflector design

The reflector was designed using the equation for the parabola:

\[ z = \frac{x^2}{4f} \]  \hspace{1cm} (B.1)

where \( f \) is the focal distance of parabola. The reflector was flat along \( y \) axis (the height of the waveguide), so its focus was not a point, but a line parallel to \( y \) axis.

Because of the constrains of the measurement setup, the aperture of the designed reflector and source to reflector position needed to be much less than the recommended diameter to wavelength ratio of several wavelengths. Here \( h = 12 \) cm is the height, \( f = 3 \) cm is focal distance, \( d = 2x = 24 \) cm is aperture size of the main reflector. The source was shielded by a secondary parabolic reflector of smaller size to prevent interference of the circular wave propagating from back of the source with the plane wave leaving the main reflector. The parameters of a secondary reflector are \( f_2 = 5 \) mm, \( d_2 = 1 \) cm, \( h_2 = 1,25 \) mm. The source was placed into the focal spot of both reflectors. The shematics of the double reflector is shown in Fig. B.1.

Fig. B.1 Schematic diagram of the double parabolic reflector.
Fig. B.2 shows simulations of the electric field distributions for the setups with, and without the second reflector and measurements for the double reflector setup. Without the second reflector, a wave front curvature of around 10 degrees could be observed at the reflector aperture and a similar curvature was still present at 20 cm distance from the aperture (Fig. B.2 (a)). The second reflector significantly reduced the curvature of the wave front of the exiting wave, allowing the lens to be placed closer to the source. A flat wave front could be seen as close as 5 cm from the source (Figs. B.2 (b, c)). The field across the source was assumed to be 1V in the simulations. The reflector was cut out from foam to the required shape and lined with copper tape along the corresponding surfaces. Fig. B.2 (c) shows the measured electric field distribution of the double reflector setup, featuring flat wave front profile in agreement with the simulation.

![Fig. B.2 Electric field distributions in (a) simulations for single reflector (b) double reflector setup (c) measurements for double reflector setup.](image-url)
References


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