A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

Jane Burry BA Dip Arch AERB
School of Architecture and Design, Design and Social Context College
RMIT University
March 2011

LOGIC AND INTUITION IN ARCHITECTURAL MODELLING:
PHILOSOPHY OF MATHEMATICS FOR COMPUTATIONAL DESIGN
DECLARATION

Except where due acknowledgement has been made, the work is mine alone (see Section 16.0);

a. The work has not been submitted previously, in whole or in part, to qualify for any other academic award;

b. The content of the thesis is the result of work, which has been carried out since the official commencement date of the approved research program;

c. Any editorial work, paid or unpaid, carried out by a third party is acknowledged;

d. Ethics procedures and guidelines have been followed.

Signed:

Name: Jane Burry

Date:
{ACKNOWLEDGEMENTS}
I should like to acknowledge the input of my first and second supervisor, Prof Peter Downton, RMIT and Prof Michael Ostwald, University of Newcastle, both responsive and encouraging. I am particularly grateful for the speed and insight with which they have returned anything I have sent to them to read as well as their suggestions for new avenues. I have also had valuable meetings with Andrew Burrow, for some intellectual clarity on the nature of computing, Prof Leon van Schaik, RMIT, whose diagrams were an excellent guide to exiting the PhD maze, Hélène Frichot, RMIT, for reading suggestions and help with where not to go. Thanks to Dr Juliet Peers for her rigorous editing and discipline in bringing the dissertation to completion. Thank you to all the panel members who have provided feedback at Graduate Research Conferences, especially: Prof Ranulf Glanville, Prof Jeff Malpas, Prof Andrew Brennan, Mark Goulthorpe, Prof John Frazer, Paul Coates, A/Prof Greg Missingham, Zeynep Mennan, Dr Cameron Tonkenwise, Andrew Burrow, Dr Johann Verbeke, Prof Suzette Worden.

For opportunities for thoughtful and stimulating conversation, I thank all my colleagues, past colleagues and post graduates at SIAL, RMIT, especially Andrew Burrow, Yamin Tengono, Andrew Maher, Dr Dominik Holzer, Dr Paul Nicolas, Dr Marcus White, Dr Flora Salim, Alex Pena de Leon, Daniel Davis, Dr Lawrence Harvey, Dr Pia Ednie-Brown, colleagues at QUT University Prof John Frazer and Prof Robin Drogemuller, A/Prof Bharat Dave and Prof Tom Kvan at University of Melbourne, all of whom have presented new and interesting angles on Space, and Susu Nousala for patient and almost endless book loan.

I would like to thank the numerous wise and thoughtful people internationally that it has been a privilege to hear present, meet informally or interview during the PhD period with special acknowledgement for the influence of the thoughts of Chris Williams, University of Bath, Hugh Whitehead, Foster + Partners, Brady Peters, CITA, Copenhagen, Francis Archer, Arup, London.

I acknowledge Adam Corcoran’s expert graphical assistance to lay out the thesis.

Thanks, finally, are due to Mark Burry and our four supportive offspring for their tolerant attitude to cohabiting with this PhD over recent years amongst many other preoccupations.
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SUMMARY OF THE RESEARCH
This dissertation investigates the relationship between the shift in the focus of architectural modelling from object to system and philosophical shifts in the history of mathematics that are relevant to that change. Particularly in the wake of the adoption of digital computation, design model spaces are more complex, multidimensional, arguably more logical, less intuitive spaces to navigate, less accessible to perception and visual comprehension. Such spatial issues were encountered much earlier in mathematics than in architectural modelling, with the growth of analytical geometry, a transition from Classical axiomatic proofs in geometry as the basis of mathematics, to analysis as the underpinning of geometry. Can the computational design modeller learn from the changing modern history, philosophy and psychology of mathematics about the construction and navigation of computational geometrical architectural system model space?

The research is conducted through a review of recent architectural project examples and reference to three more detailed architectural modelling case studies. The spatial questions these examples and case studies raise are examined in the context of selected historical writing in the history, philosophy and psychology of mathematics and space. This leads to conclusions about changes in the relationship of architecture and mathematics, and reflections on the opportunities and limitations for architectural system models using computation geometry in the light of this historical survey.

This line of questioning was motivated as a response to the experience of constructing digital associative geometry models and encountering the apparent limits of their flexibility as the graph of dependencies grew and the messiness of the digital modelling space increased. The questions were inspired particularly by working on the Narthex model for the Sagrada Família church, which extends to many tens of thousands of relationships and constraints, and which was modelled and repeatedly partially remodelled over a very long period. This experience led to the realisation that the limitations of the model were not necessarily the consequence of poor logical schema definition, but could be inevitable limitations of the geometry as defined, regardless of the means of defining it, the ‘shape’ of the multidimensional space being created. This led to more fundamental questions about the nature of Space, its relationship to geometry and the extent to which the latter can be considered simply as an operational and notational system.

This dissertation offers a purely inductive journey, offering evidence through very selective examples in architecture, architectural modelling and in the philosophy of mathematics. The journey starts with some questions about the tendency of the model space to break out and exhibit unpredictable and not always desirable behaviour and the opportunities for geometrical construction to solve these questions is not conclusively answered. Many very productive questions about computational architectural modelling are raised in the process of looking for answers.
CHAPTER 1

INTRODUCTION
Abstract

Architectural modelling has moved from focussing on objects to focussing on systems. This results in more complex multidimensional model spaces that are arguably more logical, less intuitive spaces to navigate, less accessible to perception in the traditionally highly visual domain of architectural design. Such spatial issues were encountered much earlier in mathematics than in architectural modelling, with the growth of analytical geometry, a transition from Classical axiomatic proofs in geometry as the basis of mathematics, to analysis as the underpinning of geometry. Later a belief in the logical foundations of mathematics led to the intuition–logic debate of the late nineteenth, early twentieth century. What can be learnt from the changing modern history, philosophy and psychology of mathematics about the construction and navigation of computational geometrical architectural system model space? This question is amplified by reference to recent architectural project examples and to three more detailed modelling case studies. The spatial questions these examples and case studies raise are examined in the context of selected historical writing in the history, philosophy and psychology of mathematics and space. This leads to conclusions about changes in the relationship of architecture and mathematics, and reflections on the opportunities and limitations for architectural system models using computation geometry in the light of this historical survey. These are not so much detailed practical applications for modelling as useful framing of expectations of the place of the modeller and the role of action within the model. The judgement about the applicability of the philosophical mathematical thought to architectural representation is made through traditional scholarly research supported by reference to the selected architectural modelling examples.

Introduction

Robin Evans acknowledges, “Geometry has an ambiguous reputation, associated as much with idiocy as with cleverness”.1 He contrasts geometry that is largely stolid and dormant (the geometry of the shape of buildings and the shapes of their drawings on the page) with areas where geometry is active in what he calls the space between and the space at either end.2 “What connects thinking to imagination, imagination to drawing, drawing to building, and buildings to our eyes is projection in one guise or another, or processes that we have chosen to model on projection”.3 With the rise of the application of the logical relational model in architecture, with its potentially unlimited dimensionality, constructed with the support of digital computation, it might now be argued that projection may be on its way to join the reliable ranks of dead and dormant geometries that Evans identifies within the foundations of architecture. The machine is subsuming the conscious constructive engagement with projection in architectural representation. Physiological and psychological understandings of the basis of sight and visual perception have also moved beyond simple projection, with new topological interpretations

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2 Ibid.
3 Ibid., xxxi
of the organisation of the visual cortex and the physiological basis of direction finding, motion sensing and visual spatial perception.\(^4\) The sites of geometrical life and activity in contemporary architectural practice are moving into other, rather different, and more recently defined interstitial spaces.

There are both active and tacit changes to the potential geometrical complexity of architectural model space. Contemporary changes in architectural modelling techniques and the ways in which geometry is employed in models are bringing about a transition in spatial thought within design that broaches issues encountered much earlier in mathematics itself and in the philosophy of mathematics. The key question in this dissertation and its reason for the citation of some quite old references in the philosophy and psychology of mathematics is how useful these writings are to developing a better “designerly” understanding of the multidimensional model spaces constructed or encountered in contemporary architectural computational modelling.

Architectural propositions are no longer necessarily expressed in the first instance as two–dimensional inscriptions of projected three–dimensional geometrical objects. The traditional dress maker’s pattern translates the three dimensional intentions onto a two dimensional page of tissue thin paper but includes several variations through alternative traces for different sized or detailed garments. In a similar way, geometrical architectural models constructed using logical relations can imply an infinite field of possible three–dimensional configurations from a simple graph of relations. Usually we first see these possible configurations, or a few of them at least, translated for us to virtual manipulable three–dimensional images via our computer monitor. Thus we see geometrical instances derived from the model. But to “see” the model itself, we must resort to much more abstract representations: computer programming code, scripting language, diagrammatic graphs of nodes and edges, for instance.

Nigel Cross has written that the central concern of design is the conception and realization of new things and at its core is the language of modelling;\(^5\) this language is now changing. This change is orchestrated through tentative appropriation from mathematics and the prior experiences of computer science. This research tests the value of exposing certain historical philosophical aspects of this act of adoption, or tacit assimilation through the medium of digital technology.

Any geometrical relationships, formally expressed, may be used to link building function (“performance”) and building context to shape. Husserl defines geometry as “…all disciplines that deal with shapes existing mathematically in pure space–time.”\(^6\) Shape is never absent from architecture and thus Evans can write “geometry is in architecture.”\(^7\) Shape is far from all that we seek from architectural models but without shape, it is not architecture and can never be built.

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\(^6\) ‘Geometry (under which title … we include all disciplines that deal with shapes existing mathematically in pure space–time)’ Husserl, *The Crisis of the European Sciences and Transcendental Phenomenology*, Appendix VI *The Origins of Geometry*, 353.

The architectural model constructed as logical and geometrical relations over geometrical objects is invisible. It is in itself an extensive and, in general, geometrical space but it defies holistic representation as Euclidean means. For designers who rely heavily on their own powers of visualisation and intuitive qualitative spatial engagement it is challenging to know, let alone share, the space of the model. This cannot be mapped in any sensorially accessible fashion except through the sampling of individual instances of the geometry. It cannot be visualised meaningfully in three or four dimensions except through animating or imagining transformations along particular, selective motion and/or morphing pathways in the space.

**Ideation, problem solving, and mapping the conversation**

The shift in the active role of geometry is found both in the appropriation of geometry as idea and its engagement as problem solver for design resolution. In both respects, the system has moved to prominence over the object. Consider some of the audacious geometrical propositions of recent years in which geometry supplies the design idea: the proposals for the use of aperiodic, fractal tiling developed by Arup AGU for Liebeskind’s Victoria and Albert extension design, the fractal patterning at LAB/Bate Smart’s Federation Square, Foreign Office’s Yokohama Port building. In each there is the implication of continuous variation. The whole cannot be known all at once in its entirety. There is a bottom–up rather than top–down heuristic within the designed space. Consider some of the audacious geometrical propositions of recent years in which geometry solves the problem of design resolution: the idea of making use of the Pheaire Whelan foam model to realise the bubbles of the PTW/Arup Beijing Water Cube; the function–defined surface and dynamic relaxation used to find the shape and faceting of the glazed roof over the British Museum Great Court; and Foster + Partner’s use of the Torus patch to create buildable, apparently freeform curved surfaces that can nevertheless be resolved into flat quadrilateral panels. The intent is to generate form with apparently infinite variety, that cannot have its geometrical archetype read at a glance.

Thus, there is an active drive to greater complexity, for its interest, for its elegance and in response to fascination with the formal manifestations of the structures of natural systems. But there is also a more passive phenomenon; the opportunity to model variable relations and create dependencies between model components, rather than give the geometry a fixed, explicit identity and metrics. This results in model spaces that potentially run to thousands of dimensions if we accept Penrose’s definition that a model with five variable parameters is a five dimensional space.

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and the potential extrapolation of space into dimensions beyond the third.  

The multidimensional geometrical system model is already fulfilling its promise in architecture as an exploratory design tool. It supports modelling of what is known about the design, a map of the design conversation and to defer final decisions about shape and form, allowing these to be iterated, edited and replaced throughout the design process without always remodelling, while representing the known constraints and given relations. It is a useful way to explore options, limits, find better, more refined solutions. But it is also geometrically complex and potentially unpredictable in its variability and manifestations.

**The case for understanding the model space**

Interacting with such explicitly constrained and yet dimensionally unconstrained space is in itself a useful conceptual and pedagogical Odyssey for a discipline, which claims space and spatial organisation as its territory. By moving outside the former conventions of architectural practice, (which, it may be argued, have been highly mathematically reactionary since the discipline’s historical contribution to the science of projective and subsequently descriptive geometry), new territory is to be found. New territory here is new ways to understand and configure space, resulting in new forms of architecture. (Whilst this is still an area of transition in mainstream practice, there is now sufficient history of digital computing in architecture to look already for evidence of such a fundamental change.) Getting to know these model spaces, constructed from logical relations over geometrical objects may usefully release us from the ‘kenon’ or void, the ennui of continuous homogenous empty Cartesian space that Malpas identifies as our modern condition. It may provide useful insights into our own spatial and temporal experience beyond the most reductivist abstractions of this spacetime – our mysterious context. The space of the model that represents design deliberations, contingent and dependent choices and gradations is inevitably a complex one.

Design problems are wicked; they are not problems for which all the necessary information can ever be available to the designer. A solution comes early (one which ‘satisfices’); analysis follows. Beyond solution proposition, designers have always been explorers, generally searching within the bounds of explicit and tacit constraints. Even leaving aside

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the project-specific constraints of performance and context; aesthetic rule systems for order and patterning, styles and heuristics have governed this discipline of space making throughout its history. We may identify well-known examples such as Palladian villas or the designs of Durand’s École de Beaux Arts as searches within well-defined constraint spaces and many have done so, some have explored their translation to a more automated computational design setting.\(^\text{16}\)

In this sense, geometry has presented the components in the formal systems of architecture. But geometry is neither a formal nor a natural language. We need to question at what level it is a language at all.\(^\text{17}\) On the one hand, it undergoes multiple translations to access machine logic. On the other, as we explore multidimensional model space we must put in place ready translations back into the world of static Cartesian three-dimensional space that is so deeply learned and overlaid on our perception of ‘objects’ in the world that it is barely acknowledged as a representational convention at all.

Tacitly, in moving to a new and scientifically defined mode of practice, we have drawn into our discipline, through subterranean channels, language and concepts from mathematics and logic that we may not necessarily hear, see, or acknowledge explicitly in our discourse, that we are not called upon to account for. Infinity, for example, has been, in architecture, the boundless space extending out along the orthogonal implied axes of Mies van der Rohe's brick villa (1923), the flow of space along orthogonal planes – crystallised built abstract geometry. But now we are drawn into questions of countable and uncountable infinities. This seems to be outside the immediate scope of, or familiar territory of Euclidean geometry with which architectural propositions are habitually constructed. But it is implicated in our appetite for using computational systems to store and interrogate more complex architectural models.

While Richard Coyne writes that “Computers are merely a recent manifestation of a human will to dominate and to see everything technologically, in terms of causes, control, and domination”\(^\text{18}\), in other words, part of the mainstream Modernist program, they have also brought us intriguingly, as architects, into more imminent spaces. In such spaces, emergent and chaotic

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\(^{16}\) Hersey G. and Freedman, R. *Possible Palladian Villas*. Cambridge, Mass: The MIT Press, 1992. In this volume, for idea that the “notion of a recipe or algorithm that can generate plans, facades and designs for entire buildings, [evident in Vitruvius], was taken up in the Renaissance by Leone Battista Alberti and Leonardo and developed later by such diverse figures as Goethe, Monge, Froebel, Frege and Wittgenstein see: March, L. and Stiny G. *Spatial Systems in Architecture and Design: Some History and Logic in environment and Planning* B12, 1985, 31ff.


phenomena unfold within recursive systems in ways that were difficult to model earlier, except very obliquely and referentially. They are still difficult to translate into physical architecture without the use of digitally generated imagery and movement; or, perhaps, water as in the case of the carloratti associati and MIT’s Digital Water pavilion in Zaragoza. The system has stolen centre stage from the object.

**Motivation**

This research was motivated as a response to the experience of constructing digital associative geometry models and encountering the apparent limits of their flexibility as the graph of dependencies grew and the messiness of the digital modelling space increased. The chain of questions was inspired particularly by working on the Narthex model for the Sagrada Familia church, which extends to many tens of thousands of relationships and constraints, and which was modelled and repeatedly partially remodelled over a very long period. It achieved limited flexibility within important ranges but was challenged by the intent of a constrained but freely morphable world within which to hold design team conversations. This experience highlighted the difference between the fluidity of comparatively geometrically simple or small demonstrator projects, often encountered in research and pedagogy and the difficulty of maintaining the same fluency and meeting all the contingencies of a real world architectural project within a single large associative model. This led to the speculation that the geometrical or schematic approach was too adhoc, bottom-up and perhaps too inelegant to function well. Working from a more analytical geometrical approach, using mathematical functions to define form might lead to ‘cleaner’, more smoothly variable space, analogous to continuously differentiable functions.

The late realisation that the bifurcations, holes and limits of the model space were not necessarily the consequence of poor logical schema definition, but could be inevitable limitations of the geometry as defined, regardless of the means of defining it, lent mystery to the ‘shape’ of the multidimensional space being created. This led to more fundamental questions about the nature of Space, its relationship to geometry and the extent to which the latter can be considered simply as an operational and notational system. A particularly intriguing aspect was the apparent geometrical complexity of a system, built up from a series of very small, simple, ‘intuitive’ synthetic geometrical moves. Challenging the Inflexibility of Flexible Model has developed into a separate joint research project with other academics taking a more hands-on approach to different computational strategies for maintaining flexibility and interaction in complex system models, outside the scope of this doctoral research.

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Research Process

1. I investigated the literature of the interstitial space between architecture and mathematics. I conceived at first of the idea of an enzyme that facilitated their chemistry.

2. I investigated the opportunities of greater mathematical knowledge and content in architectural pedagogy by a) reviewing the mathematical content of projects in the Flexible 3D modelling course, (a generic elective to introduce digital parametric modelling for design to undergraduates and postgraduates) b) presenting a paper at a Mathematics in Education conference. I have also distributed questionnaires to 150 former students to gather their experiences of being introduced to computational geometrical system modelling for design.

3. I reviewed published architectural projects with a link to recent mathematical (or geometrical) discoveries or developments. This developed into the taxonomical study in which the projects were grouped together within six loosely mathematical themes and a visual glossary of mathematical terms and ideas. This was published in a co-authored book and is sampled in Chapter three. In the dissertation I have collapsed this into 5 themes and made a small selection of the projects for expansion as examples in Chapter 3.

4. I continued to experiment with modelling with the same basic brief, using different software and schematic approaches to explore the differences that different software ontologies and structures made and whether the comparative compact economy, tidiness and relative control of scripting impacted on the model’s geometrical flexibility.

5. I undertook continuous scholarly research in the following broad areas: architecture and geometry, architecture history and theory, architecture/science, geometry, mathematics and space including history of mathematics, philosophy, in particular philosophy of science and mathematics from the eighteenth to twentieth centuries, cognition, especially psychology of mathematics, history of computing and early system modelling in architecture, design and computation, design theory and education.

21 A software ontology is defined as a formal representation of knowledge as a set of concepts within a domain and the relationship between those concepts. It is used to reason over the entities in the domain and to describe the domain itself. "For AI systems, what “exists” is that which can be represented. When the knowledge about a domain is represented in a declarative language, the set of objects that can be represented is called the universe of discourse. We can describe the ontology of a program by defining a set of representational terms. Definitions associate the names of entities in the universe of discourse (e.g. classes, relations, functions or other objects) with human-readable text describing what the names mean, and formal axioms that constrain the interpretation and well-formed use of these terms. Formally, an ontology is the statement of a logical theory." "ontology." The Free Online Dictionary of Computing. Denis Howe. 12 Nov. 2010. <Dictionary.com http://dictionary.reference.com/browse/ontology>.

Figure 1: Starting point:
Literature review ‘Enzyme diagram’ of the reaction between architecture and mathematics.
These five research activities come together to, first, define the problem domain, that is identify certain issues of spatiality that are raised in design through the adoption of virtual computational geometrical modelling in architecture; second, provide selected historical mathematical and philosophical background to this freedom of geometrical construction of spatial design systems. Finally, the historical material is tested in the light of the modelling studies and experiences to reach some conclusions about the relevance of the historical thought in philosophy of mathematics to contemporary spatial understanding in design modelling process.

Not all of the material, exercises or directions represented in the research activities are directly represented or accounted for in the dissertation. In particular the research into the opportunities of greater mathematical content and knowledge in architectural pedagogy has informed the framing of the problem domain but was not pursued to conclusion in the final dissertation. Conversely, material in the literature review that was initially considered background has ascended to a central position in the argument.

The chapters that follow

Chapter 2 reviews the relationship between architectural modelling, geometry and mathematics. The geometria situ section considers the fundamental change that topology has wrought on representation of space since the seventeenth century. This includes its contemporary impact on architectural modelling, and, reciprocally, in this process, on architectural thought. It progresses to a review of analogue and early computational system modelling in architecture: the shift from object to system.

Chapter 3 takes examples of recent architectural exploration of geometrical spatial systems to inform both the designed space (aims and outcome) and the design space (means and process). It considers examples of architectural projects that appropriate from geometrical or mathematical discovery either for aesthetic design ideas, or for models for architectural problem solving, or a synergy of these two. It proposes a loose taxonomy of five mathematically inspired themes or clusters within which these projects can be grouped.

Chapter 4 lays out in greater detail three modelling examples that expose certain spatial conundrums encountered in computational geometrical system modelling. Two are personal modelling experiences, sub models from the Sagrada Família Narthex model; the third is an example from pedagogy applying mathematical functions. To expand and generalise these case studies I reflect on the design objectives for which students in a series of classes I have taught have chosen to apply computational geometrical system modelling, in what for many was their first encounter with computational system modelling. Using my observations and some of their responses to a qualitative questionnaire, I have reflected on the impact of their experience of computational system modelling on their design thinking.

Chapter 5 reconstructs a historical understanding of the philosophical relationship between representational and perceptual space, with particular reference to the place of geometry. It does this in order to investigate the how this
philosophy and psychology might elucidate the space of the architectural geometrical system model, with reference to the examples in the previous chapters. It starts from consideration of the schism identified between geometrical construction in the ancient or Classical idiom and the modernist approach led by Descartes, and how this definition of modernism is both taken up and countered within architectural design. This provides the background to Kant’s concept of ‘anschauung’, space and geometry as *a priori synthetic intuition*, sensibilities rather than deductions from experience or logic, and Poincaré’s subsequent geometries as conventions. This leads into the conflict between intuition and logic in late nineteenth century mathematics.

Chapter 6 examines the theme of intuition and logic within the history of mathematics in more detail and the implications for architectural design space. It explores the apparent cognitive parallels between the processes of design and mathematical discovery and the various interpretations of aesthetics in the two fields.

This leads to conclusions about the extent and the limits to which the history, philosophy and psychology of mathematics surrounding logic and intuition provide useful insights into the place of logic and of intuition in computational geometrical system modelling space in architectural design.
CHAPTER 2

ARCHITECTURE, MATHEMATICS AND SYSTEMS
**Introduction**

This thesis investigates what architects who are constructing virtual system models to represent the design can learn from earlier philosophy of mathematics regarding the place of logic and intuition in multidimensional model space. This is qualified by what they can learn that is relevant to ‘designerly’ knowledge of space. I will start by re-examining the relationship between architecture and mathematics as spatial pursuits. Starting from geometry I move to consider the more spatially abstract aspects of mathematics. The aim of this chapter is to re-examine the relationship between architectural modelling and geometry in order to start to identify the change in the ‘geometrical vocabulary’ of architectural modelling as it has made the transition from object to system modelling. It starts with a historical review of the designer’s and the mathematician’s approaches to spatial definition and differentiation and moves into the topological representations of space. Next, I review dynamical modelling approaches, starting with physical analogue, moving to early computational system modelling. This lays the historical groundwork for exposing, in the subsequent chapters, how modelling systems rather than objects leads to less intuitively accessible geometrical model spaces, which map less readily to perceptual space.

### 2.1 Architectural modelling and the virtual space of Geometry

The relationship between architecture and geometry is ancient and reciprocal. Blackwell wrote that geometry is the study of space and architecture is the creation of space. In this passage, Blackwell uses both the words geometry and architecture as disciplines, active pursuits, or lines of enquiry and not in the senses of treatises set down or buildings built. Here, just as in Blackwell, we will be more concerned with the term ‘architecture’ used to denote the act of designing, or in other words conceiving, developing and refining intention for organising space by building. But geometry in addition to meaning 1) its formal study, is also used in the sense of: 2) the basis of our conception and perception of spatial relations between things; 3) the several ‘systems that operate in accordance with a specific set of assumptions’ within which this is represented, and, finally, 4) as the static output information from dynamic system

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22 The geometrical vocabulary of architectural modelling is used here to signify geometry in the broadest sense. Languages, (symbolic systems for communication), have been developed for geometry but it is not this vocabulary but ‘vocabulary’ used as a convenient metaphor to encompass intuition, concepts, ideas, protocols, conventions, and applications of geometry, and not merely the groups of symbols used to communicate these diverse phenomena, that is investigated here. I do not share the view expressed by Greg Lynn and others that geometry itself is or has the characteristics of a language. The relationship of architecture and its modelling to geometry is not simply that of ‘user’. There is a history of interaction that is relevant to the ‘geometrical vocabulary’ of architectural modelling and to its extension through computation.


design models. In these several senses, geometry itself is the medium of representation of architecture.

Evans has written:

'Geometry is understood to be a constituent part of architecture, indispensable to it but not dependent on it in any way. The elements of geometry are thus conceived as comparable to the bricks that make up a house, which are reliably manufactured elsewhere and delivered to site ready for use. Architects do not produce geometry, they consume it.'

This statement is clear where it relates to the everyday practice of architecture, but in relation to the etymological and concrete origin of geometry, it appears an oversimplification. An alternative and compatible viewpoint is to adopt the metaphor of architecture and formal geometry sharing common ancestry.

Geometry itself is a word of Greek origin meaning ‘land–measurement’, ‘land–measuring’ or ‘earth measure’. Greek historian Herodotus (485–425 BC) is quoted as attributing the origin of geometry to the royal apportionment of land by equal measure to Egyptians and, in particular, the relationship of the revenue claimed by King Sesostris to the land area and any reduction in that area, duly measured by the King’s overseers, in the annual flooding of the Nile. This version of history links the origin of geometry to the concrete subdivision and organisation of space. In this sense, architecture and geometry are mutually implicated in their conception and development.

Both geometry and architecture as activities have the power to express and organize space by using representations outside the constraints of a direct mapping to the physical. The principle distinction between geometry and architecture lies in their levels of abstraction and generality. Geometry, as a pursuit, looks for generalities and, once established, (demonstrated or proved) offers them up as truths, (and for use). Architecture very selectively employs these general relationships constructively to underpin and create specific

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25 Refer to Chapter 3.
26 Evans, The projective cast: architecture and its three geometries, xxvi.
spatial relationships. But, I will show in chapter six, if we take several steps back from the proven truths that geometry proffers as the ultimate output of its pursuit, truths analogous to built buildings in architecture, the multi-stage process of moving from a loose connection of ideas to arriving at a proven generality, fully represented and communicated in mathematics may not be completely distinct from the multi-stage process of moving from a loose connection of ideas to the proven realisation of a unique building in architecture. It is possible that herein, at least in part, lies their unity in the matter of aesthetics. This is a different relationship from that portrayed by Evans’ geometrical ‘bricks’.

If surveying for spatial subdivision, valuation, and construction in the ancient world underpins geometry and architecture, projective geometry, born in the representational problems of architecture and pictorial space making becomes a fundamental gift to geometry itself. A system for representing objects in art and design spawns a more far-reaching system for the geometrical description of space.

‘While Fermat and Descartes were founding analytic geometry in the first half of the 1600s, Girard Desargues was developing a new branch of synthetic geometry called projective geometry. Renaissance artists and mathematicians had raised questions about drawing in perspective. These questions led Desargues to consider points at infinity and projections between planes.’\(^{32}\)

It is interesting that the focus on projection on the plane in art is mentioned here without referring to Desargues’ own professional interests. The significance of Desargues’ work in our context is not only his seminal contribution to mathematics: the pamphlet with which the modern study of projective geometry began, but his focus on practical applications as one who earned his living as a military engineer and architect. Written in French rather than Latin, full of metaphorical neologisms for the layperson that make it less translatable for mathematicians, the draft pamphlet is based on an in depth knowledge of, and original development of, ‘high’ Hellenic mathematics but grounded in the tradition of the ‘low mathematics’ of facilitating practical design and construction.\(^{33}\) However, Robin Evans\(^{34}\) writes that Giorgio Cucci has shown that the division between those for and against Desargues’ method for stonecutting, surprisingly, followed the lines of a class division between the few academic theorists who approved of it and a large body of builder–architects, associated with the Masonic tradition, who reacted with hostility to Desargues’ denigrating their established techniques and their native abilities. So, it appears that Desargues, despite his efforts in this respect, was not able to bridge the three epistemological worlds of architecture, mathematics and construction within his own time.


\(^{34}\) Evans, *The projective cast: architecture and its three geometries*, 203. (references: Giorgio Cucci, Philipe Delorme d’Architecture, Rykwert, J. On the oral transmission of Architectural Theory, AA files no. 6 (Spring 1084) 15–27)
Figure 2 Diagram of Desargue’s two triangle theorem and projective law of similar triangles.

For any two triangles for which each vertex and its mapping both lie on a line emanating from one projection point (blue dashed):

the three points of intersection of the equivalent edges of the triangles (red and yellow) always lie on a straight line (black).
Regardless of its early adoption or rejection as a method in architectural practice, it is one of the leading examples of architecture contributing to what Husserl refers to as the supertemporal nature of geometry,\(^{35}\) in this case, a question of concrete visual representation leading the development of the ideal (geometric) object, an acquisition that maintains its validity (in the light of later acquisitions in geometry), accessible to all.

Desargues’s development of projective geometry was the basis of further, paradigm shifting discovery in geometry following the work of Poncelet, another practical engineer, and others in the nineteenth century.\(^{36}\) It is closely related to Riemannian geometry\(^ {37}\) through the medium of the real projective plane.


Ibid., 356.

Georg Friedrich Bernhard Riemann (1826-66) German mathematician developed Riemannian geometry, also known as elliptical or spherical geometry. It is a non-Euclidean geometry. It contradicts Euclid’s parallel postulate, in which given a line \(l\) and a point \(p\) outside that line, there is exactly one line that passes through \(p\) that is parallel to \(l\). In elliptical geometry, there are exactly zero lines that pass through \(p\) parallel to \(l\). Imagine the lines of elliptical geometry as the great arcs or lines of longitude on the near spherical Earth. They all intersect at the poles. Imagine a triangle inscribed on the same globe between the cities of London, Berlin and Madrid. In contrast to a triangle created in the Euclidean plane, the sum of the angles subtended by the lines of the triangle between the three cities would be greater than 180°. The importance of non-Euclidean geometries in architectural representation has increased with the increasing facility that digital computation offers for modelling the non-planar.

The second clear example of an architecture or construction application–led development in geometry is the further development of projective geometry, that of descriptive geometry. Evans calls this ‘a mathematician’s generalisation of architectural drawing.’\(^ {38}\) Once more it is attributable to a French military engineer: Gaspard Monge, co–founder of the École Polytechnique who paved the way for more fundamental contributions by his pupils Charles–Julien Brianchon and Jean–Victor Poncelet. In this brief review of the historic relationship of architecture to geometry, and particularly in considering analytical geometry, it might be asked: why not write of architecture and the more general conception, mathematics?\(^ {39}\)

‘Geometry’ and ‘mathematics’ cannot of course be used synonymously. Of Mathematics, Auguste Comte wrote in 1851 that ‘The plural form of the name (grammatically used as singular) indicates the want of unity in its philosophical character, as commonly conceived.’\(^ {40}\)

\(^{35}\) Evans, The projective cast: architecture and its three geometries, 324.

\(^{36}\) Comte, A. The Positive Philosophy of Auguste Comte. New York:
plural\textsuperscript{41} and gathered from different activities. Geometry and arithmetic were two of the seven liberal arts. They belonged to the Quadrivium along with astronomy and music (the additional Trivium included grammar, rhetoric, and dialectic).\textsuperscript{42} The liberal in liberal arts implied the study of subjects, which, unlike architecture, are not necessarily directed to a profession. But mathematical studies were central also to the applied arts during the renaissance.

‘The sixteenth century Academy of Arts in Florence, for instance, was a kind of polytechnic college, where the teaching of mathematics was obligatory. Here mathematics was taught not in its abstract and pure form, but in its purposeful application as the leading science of the art of design (arti del disegno), which embraced all branches of the technique of arts and engineering.’\textsuperscript{43}

To geometry and arithmetic add algebra, and you have the basis of mathematics as it has developed since the Renaissance.

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Calvin Blanchard, 1855, 51.

\textsuperscript{41} The entry in the Complete Oxford English Dictionary is too long to quote here but from this source, the word has the combined Greek derivation from neuter plural τὰ μαθηµατικά, in the sense of ‘mathematical objects’ and feminine singular ἡ μαθηµατική, meaning ‘mathematical science’. The first recorded use in English is in the 1580s. While consistently plural in English it is used in both the plural and singular in French, so it is interesting that Comte, as a French philosopher should make this observation.

\textsuperscript{42} Pedoe, Geometry and the Liberal Arts, 11.


It is likely that geometry, arithmetic and algebra, while closely related, and progressively more interrelated in formal mathematics, have distinct origins, cognitively,\textsuperscript{44} philosophically and in the concrete world of descriptions and transactions.

Geometry and algebra were combined into analytic geometry in the first half of the seventeenth century by Pierre de Fermat and Rene Descartes. By asserting that any equation in two variables could be used to define a curve, they expanded the field of curves beyond those that could be constructed geometrically or mechanically. Fermat found tangents and extreme points of curves using what was essentially latter day calculus. Calculus developed rapidly in the second half of the 1600s. Isaac Newton and Gottfried Liebniz demonstrated its great power from their contrasting points of departure.\textsuperscript{45} For Comte this was ‘beyond all question, the loftiest idea ever yet attained by the human mind’\textsuperscript{46} as he carefully expounded the methods of calculus put forward by Leibniz, Newton and Lagrange.

‘Apart from its role in calculus, analytic geometry developed gradually. Analytic geometers concentrated first on giving analytic proofs for known results about lines and conics. Newton established the subject of analytic geometry in its own right when he classified cubics, a task beyond the power of synthetic– that is non–analytic – geometry’.\textsuperscript{47}


\textsuperscript{45} Bix, Conics and Cubics: a concrete introduction to algebraic curves, 3.

\textsuperscript{46} Comte, The Positive Philosophy, 72.

\textsuperscript{47} Bix, Conics and Cubics: a concrete introduction to algebraic curves, 3.
This point marks a bifurcation between synthetic and analytic representations of geometry and between artistic and architectural geometrical understandings or representations of shape and space and the further development of those of mathematical understandings. Already questions of intuition and perception are challenged by this development. Later statements such as ‘mathematicians can construct a continuous curve that has no tangent at any point’\textsuperscript{48} will leave many architects in no doubt of the reason for such a parting of ways.

There is space for an alternative view of the relationship of geometry and architecture beyond that of pure use or appropriation of one by the other. They are pursuits that may respond to some common historical and cognitive impulses and there has been symbiosis in certain limited areas of the complex boundary between the two. Architectural conception, representation and production are inconceivable in the absence of geometry in its broadest sense. Demonstrating this breadth of meaning, in some philosophical contexts geometry and space itself are treated apparently synonymously.\textsuperscript{49}

\subsection*{2.2 Architectural modelling and the virtual space of mathematics}

So what of the relationship between architecture and mathematics' other ‘components’? Plural and wanting in unity in its philosophical character mathematics may be, but it is not strictly meaningful to see the triumvirate of geometry, arithmetic and algebra as three stand alone contributors.

‘Arithmetic’ is also from the Greek, derived from the word αριθμός meaning the science of number, the art of computation by figures.\textsuperscript{50} Geometry may be scaleless, relationships defined by ratio – but metrics in architecture and the means to its realisation as building require number – quantified collections of discrete objects as well as dimension. Number, too, is a source of ideas. Its symbolic significance is without question in sacred architecture. Number, as well as relying on geometrical metaphor for its structure, can be the basis of space filling pattern and shape.\textsuperscript{51} Ratio and proportion are fundamental in the history of formal architectural space making and in vernacular building. It is hard to identify architecture’s unique reciprocal contributions to the development of arithmetic and number theory so perhaps the relationship between architecture and arithmetic can more easily be reduced to one of appropriation but for idea as often as for problem solving.

Finally, consider algebra: notation has also been a key innovation in the development of modern algorithms,

\begin{flushleft}
\textsuperscript{48} Hadamard, \textit{The Mathematician’s Mind}, 102.


\end{flushleft}
the adoption of a system of symbols that stand for both operators and for one or a set of values (variables) has reduced the weighty tomes needed to describe quite rudimentary mathematical relations expressed in language in the manner of the Greeks. Algebra is the primary meta–language of mathematics in which both geometrical objects and numbers are further abstracted and generalised. The formal (logical languages) of computer code are not algebra. They are in general procedural rather than the statements of relations that algebra provides. But algebra has provided the language in which to couch all the spatial, proximal and numerical relationships, which the algorithm–writing architect has in play. Its development marked the beginning of a progression to ever–higher levels of abstraction and generality in mathematics. Rocker writes:

"Today, when architects calculate and exercise their thoughts, everything turns into algorithms! Computation, the writing and rewriting of code through simple rules, plays an ever–increasing role in architecture."53

Architecture as an activity was ever a game of procedure rules, defined and redefined but expressing these rules in code rather than graphically makes space for much wider scope for algebraic relations. The reciprocity of algebra with synthetic geometry has been noted above. They are often two means of description of the same phenomenon, two paths to

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mathematical proof along which mathematicians have been known to divide on nationalistic lines.\textsuperscript{34} The topic of the synthetic versus the analytic and the history of formalism in opposition to intuitionism are large topics that I shall return to in chapters 5 and 6.

The relationship between architecture and mathematics, geometry in particular, is fundamental, and ancient. It is not possible to remove geometry from architecture and leave it standing but geometry does not exhibit the same dependence on architecture, except insofar as the intuitive understanding of Cartesian space and Phileban solids is supported by the empirical experience of the built environment. In this sense, perceived architectural space and representational mathematical space are mutually implicated in our experience and knowledge of the world. Similarly, Piaget has given us the reciprocal observation, that perceptual space is never free of mathematical representational space once its concepts are planted there.\textsuperscript{35} So far I have made reference to the connection of architectural modelling to geometrical and mathematical ideas that are metrical or connected to shape. In exploring the value of philosophy of mathematics to designerly understanding of virtual system model space, there is another development of modern mathematics that is key, which is topology.

2.3 Analysis situ: another way of knowing space

In a letter to Huygens in 1679, Leibnitz expressed the difficulties of being confined to quantitative metrical mathematical language when describing form.\textsuperscript{56} He specified the need for a different but geometrical analysis to convey the idea of situm (translated as situation or qualitative place relationship). This new geometrical language would be a shorthand, equivalent to algebra in its power to encapsulate but without its reference to metrics and measurement. This brought the term “analysis situ” into the lexicon and the need for topology into the world of ideas. However, the metaphor or instance that really brought the concept to life is an example from the built environment – Euler’s\textsuperscript{57} solution to the problem of the seven bridges of Königsberg (1736) – the question of whether it is possible to walk around the town, crossing each of the seven bridges once and once only. His network diagram\textsuperscript{58} that abstracts each of the crossings

\textsuperscript{34} The idea that the English have shallow, broad minds and prefer to construct their mathematical proofs while the French have deep narrow minds and prefer symbolic algebra – a tradition raised in conversation with Prof. Andrew Brennan Saturday 7th June 2008.


\textsuperscript{57} Leonard Euler (1707-1783).

\textsuperscript{58} The network diagram is a now familiar tool of graph theoreticians. Graph theory in mathematics is the study of graphs, structures to represent relations between certain objects. As a sub discipline it was born when Euler published his paper in 1736. Graphs are represented as a collection of vertices or nodes and a collection of edges that connect pairs of nodes. Graphs may be undirected, in which case there is no distinction made between the two nodes connected by an edge, or the edges many be directed from one node to another. The essential quality being represented is connectedness – they
to a curve of indeterminate length and the landings between crossings to vertices, or nodes, renders both the problem and the answer at once intuitive. It is clear, at little more than a glance that it is impossible to traverse all seven links between the four nodes in a single path without travelling along at least one link twice.

The matter of intuition in relation to visual readings and visualisation is intriguing. Here is an example where the more abstract topological representation is easier to read than either the three dimensional model of the city in Cartesian space or the sensorial, existential, event–based city of experience in which it might be necessary to wander for days trying the crossings in different sequences to satisfy the same conclusion empirically. Topological representations have been important in conceiving of the architectural disposition of adjacent spaces. They were adopted in a systematic way by the researchers applying computation to mathematical problems in architecture at the Cambridge Centre for Land Use and Built Form Studies (LUBFS) in the 1960s.

The arrangement are scaleless and non metrical. Apart from the direct mapping to spatial organisation, in architecture, graphs are a useful way of relating building services and systems, understanding complex construction processes, for viewing dependencies and/or constraints in digital geometrical models and for understanding the flow of data and generation of information in systems.

March, L. and P. Steadman. The Geometry of Environment An

problems that they took on, such as arrangement of rooms within a given perimeter, of spaces according to a given architectural program, or of activities within a given plan were not computer models of architectural geometry.

The focus was on the relationships between spaces, not on the forms of the spaces themselves. This focus had the practical advantage, at the time when graphical displays for computers were rare and expensive, that the output could be represented by tables or other simple alphanumeric outputs. Fitting into the category of enumeration of architectural possibilities, Philip Steadman’s 1973 “Graph Theoretic Representation of Architectural Arrangement” makes the case for the use of a graph of the adjacency of rooms to enumerate the possibilities and in some cases identify topological impossibilities. In practice it was only possible to compute exhaustive results for five or fewer rooms. Enumeration is only applicable to highly constrained problems, which is a serious limitation in the context of generally open–ended design questions. Or as Lionel March has written: ‘Digitization has its terrible limits, and it exacts a frightening intellectual price’.


Contemporary architectural relational geometric modelling uses graphs over geometrical relationships to construct a hybridised geometrical /topological spaces that can be thought of geometrically as many dimensional. These are spaces with shape at a conceptual level that map more directly to the form of built space, rather than purely topological spaces. The nature of this “shape”, however, may defy visualization. While the graphs in models will conform to certain theoretical archetypes in terms of their structure, whether directed or cyclical, for example, for representational purposes, the links in the graph operate at a higher level of abstraction than the architectural or geometrical meaning ascribed to them. In other words a link in the graph may represent a dependency between two geometrical objects but the specific objects and the nature of the dependency are not explicit in the existence of the link.

2.4 Geometries, Groups and Sets

During the nineteenth century mathematicians arrived to a potentially infinite hierarchy of different geometries, spaces with different geometrical truths. Manuel Delanda describes in some detail the progression from Gauss’s application of differential geometry to consistent intrinsic description of surface to the extension of such intrinsic description from the two dimensional space of surface to n–dimensional space by Riemann. Gauss is also credited with developing, although not publishing, parts of the mathematics of group theory. In 1872, Felix Klein used group theory to tackle the problem of categorising and characterising the multiple different geometries in his influential manifesto known as the “Erlangen program”. A group is a set closed under a binary operation satisfying three particular axioms.

63 Non–Euclidean geometry is revisited in Chapter 5.
From the mid 1870s to the mid 1880s, Cantor was also developing the basis of Set theory. Up until this time the concept of ‘set’ had been considered a simple one pertaining to finite sets and it had been in implicit use dating back to the Greeks. Cantor moved infinity from its place as a philosophical concept into the sphere of concrete mathematical relations. In showing that some infinities are larger than others, some are bounded, the distinction between countable and uncountable sets, he established set theory as a foundational theory of modern mathematics. To appreciate the complexities of determining the criteria that define the members of a set, we have only to dwell for a few moments on Russell’s paradox, presented in challenge to the set theory of Richard Dedekind and Gottlob Frege in 1901: the set that contains exactly the sets that are not members of themselves.68 (Consider this paradox for a moment if you are not familiar with it. If this set is, by definition, a member of itself, it is excluded from membership of itself, yet, if by definition it is not a member of itself, it exactly fits the definition and is thereby a member of itself...) Most of the work of the architectural design system modeller involves explicitly defining and editing the definition of sets. The domain of each variable is a set (possibly an infinite set such as the set of all positive real numbers or a random real value between 0 and 1); a list or an array may hold the objects whose collection is also a set. Computational design system modellers define sets and subsets, intersections and unions of sets without necessarily being able to conceptualise the members or the full formal implications of their relationships. Rather, the set definitions are the translation of linguistically framed design intentions to formal statements in the computer program or architectural model.

2.5 Examples of analogue system modelling in architecture

System modelling in engineering has a long and venerable history. Physical analogue modelling has also made an important contribution in architecture. It is clear that modelling systems as opposed to objects in architecture is not exclusive to the recent period following the development of affordable personal computing and powerful graphical computing interfaces, nor in fact to the period of the development of electronic computation at all.

To Antoni Gaudí we can attribute an important innovation in architectural system modelling. Not the introduction of the hanging chain or catenary (formerly caternaria) for finding the form of masonry structures per se;69 but the systematic use of

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68 Also identified although not published by Ernst Zermelo in 1900.

69 In 1670 Robert Hooke had proposed the following problem to the then ten–year–old Royal Society: what is the ideal form of an arch and what force does it exert against its abutments. In 1671 he announced that he had discovered the answer but did not reveal it. In 1679, he inscribed the answer to this question in an appendix to his Description of Helioscopes as the anagram ‘abcccddeeeefggiiiiiiiiillmmmmnnnnnooop rrsstttttt—u u u u u’. His executor provided the solution only after his death in 1705 as “Ut pendet continuum flexile, sic stabit contiguum rigidum inversum – As hangs a flexible cable, so inverted, stand the touching pieces of an arch.” There
the principle to create a dynamic structurally equilibrated model for a whole building structure in three dimensions. This is an important progenitor of the use of digital computation to find complex, structurally responsive surface shape. The funicular or hanging model for the Colonia Güell church is an extraordinary piece of analogue computing in which lengths and weights of the small bags of sand representing the loads can be adjusted to allow the simultaneous design intervention to the shape of the church and immediate feedback in regaining structural equilibrium after change and in which the shape is responsive to changes in the loading. It represents what Axel Kilian has described as a bidirectional constraint system. The large physical hanging model as a scale representation of not only form but also forces are much more ancient structures that exhibit arches with a close approximation to the shape taken up by the uniform chain hanging suspended in a curve from its two ends. The hanging chain gave its Latin name to the mathematical curve the ‘catanaria’ or now, ‘catenary.’ This curve along the line of tensile force in the chain, when inverted represents the shape of the line of compressive force in a masonry arch of uniform thickness or loading. In 1691, Leibnitz, Christiaan Huygens and Johann BernoulI derived the equation for this shape in response to a challenge by Jakob BernoulI. It is given by \( y = acosh\left(\frac{x}{a}\right) \), the shape and curvature varying according to change to the parameter \( a \). Galileo had noted that the catenary shape was very close to a parabola, particularly for low curvature where the hanging angle is less than 45 degrees. (Galileo Galilei (1914). *Dialogues concerning two new sciences.* Trans. Henry Crew & Alfonso de Salvio. Macmillan. 149, 290. (Galileo Galilei, Discorsi e dimostrazioni matematiche, intorno à due nuove scienze, 1638.) Galápagos makes expressive and structurally ingenious use of both the catenary and parabolic forms in his architecture. I will show in the project work in the appendix that the quadratic, the form of the parabola equation, \( y = ax^2 + bx + c \) is also present in other relationships in the geometrical sequences in the Sagrada Familia church.

The Colonia Güell church designed by Antoni Gaudí for Count Eusebi de Güell in Santa Coloma de Cervelló, near Barcelona (1898, 1908-1915) of which only the crypt was built. The form of the church was determined through the use of the funicular model to find the directions of the lines of force and construct columns vaults and walls in response. The design was developed by photographing the hanging model, inverting the photograph, and drawing over it in charcoal and gouache. Website last accessed 22nd December 2010: http://www.greatbuildings.com/buildings/Colonia_Guell.html

in the structure is a challenging artefact to build and to use iteratively. Each change in the geometry or loading can propagate changes across many links of the structure and could require lengthy hand adjustments back and forth across the network to regain equilibrium – at which point the form may have deviated significantly from the intention. It is significant and very influential as a responsive system design model linked to physical, material parameters (density and mass) and gravity constructed and operated without automated computation.  

This use of funicular models for vaulted structures found further application for the design of thin concrete shells, notably Heinz Isler’s use of wet cloth hanging under its own weight and then frozen for inversion. Taking up the vaulting tradition, in the field of engineering, during the mid twentieth century, we see the work of Félix Candela, Heinz Isler, Frei Otto, Pier Luigi Nervi, Eduardo Torroja, exploit the form–finding potentiality of thin concrete shells and membranes – empirically discovered shapes and shapes following the rules of analytic geometrical structural forces. They are working in the empirical tradition of the inventive dynamic funicular form–finding system of Gaudí and, as engineers, combining empirical discovery or verification with analytical methods for description.

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73 Heinz Isler (1926-2009) structural engineer and artist, first presented his novel techniques for form-finding thin shell surface shapes in 1959 at the first congress of the International Association for Shell Structures (IASS), organised by Spanish structural engineer Eduardo Torroja. (Shell structures had up until that time mainly conformed to simply describable mathematical forms such as spheres, conoids and hyperbolic surfaces). Isler showed 39 forms found using earth mounds, inflated rubber membranes and hanging cloths.
The use of natural statics in physical modelling to simulate the performance of structure under loading is an example of a multi dimensional parametric model space, albeit analogue in representation and limited in parameters and constraints. The mathematical generalisation of the surfaces created in the physical model provides a parallel understanding of the space but has to be worked hard to include all the parameters and constraints to accurately simulate the material system.

2.6 Early computational system modelling in architecture

There are two aspects of early computational system modelling in architecture worth considering in this context. The first is the pioneering work in conceptualising and prototyping early Computer Aided Design (CAD) systems. The second is the scientific and mathematical framework of architectural “problem solving” within which architectural design research was first introduced to the electronic computer.

The first aspect, takes us to Cambridge Massachusetts in the early 1960s where Steve Coons, Professor in the Mechanical Engineering Department and the PhD student in electronics, Ivan Sutherland met and were in discussion at MIT. In 1963, Coons defined the objectives of a CAD system: to support the design process by assisting creative thinking, to increase productivity, and improve communication and collaboration within design teams through well designed human–computer interaction. This collaboration within the team and with the machine he conceived of and described as a “design conversation”. Visualization was considered key to this interaction. First, the system must offer visualization using graphical representations as well as structural abstractions using the symbolic languages that represent the ideas within the design. Second, it must have analysis tools able to perform all the computations required for the design process, including structural, mechanical services, electrical analyses and any other analytical processes needed to validate and optimize the design. Third it should be a platform that promotes ease of collaboration, enabling the same model to be accessed and modified by multiple designers in different locations simultaneously. Finally, a CAD system should be highly generic to accommodate creative activities across multiple discipline domains within the same system, and potentially within the same design model. In 1963, this was a highly forward–looking manifesto on just about every count. The internet itself, let alone synchronous online collaboration, was not even close to reality. ARPANET (Advanced research Projects Agency Network), the internet’s forerunner, was only introduced five years later in 1968. Coons’s manifesto offered an early glimpse of the potential to take the digital design model into a space that transcended existing protocols and went beyond anything that simply emulated or streamlined contemporaneous practice. It also made reference to parametric design descriptions, demonstrated in the work

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of Ivan Sutherland, through his ‘Sketchpad’ program built as part of his PhD program in the same year.

Sketchpad has been described as one of the most influential computer programs ever written by an individual. Although executable versions were limited to a customized TX–2 machine in the MIT Lincoln Laboratory, the influence from the ideas it prototyped was far reaching, showing a level of intuitive human computer interaction, unfamiliar in commercially supported computer use in architectural design for decades subsequently. A movie made at the same time, demonstrating its use, had a wider distribution and one might draw a parallel with the magnitude of the impact of the black and white images of Mies van der Rohe’s Barcelona Pavilion within architecture itself. It was one of the first graphical interfaces and used the light pen, a forerunner of the mouse, to allow users to point at and interact with objects directly on the screen in combination with programmed buttons. The author describes his system as using ‘drawing as a novel communication medium for a computer’ and says that: ‘The Sketchpad system makes it possible for a man and a computer to converse rapidly through the medium of line drawings’. Sketchpad was designed to be a completely generic design system and also as a program to bring computer programming within the reach of the designer or draftsperson through drawing as a programming language. Written language is used only in legends (keys). It introduced concepts of, ‘subpicture’ (which we might call replication or reuse of any object), ‘definition copying’ (copying attributes such as length of a line), ‘linking’ and topological consistency (geometrical associativity) and ‘constraints’. It could introduce precision to a rough sketch, allowed the user to build their own objects for parametric instantiation with variable geometry, when instantiated into different contexts, and update many instances of an object by editing the master object in the drawing. It also introduced structural analysis into the drawing environment by using Sketchpad as an input and output program for other computation programs.

It is clear that in both Coons’ manifesto for the characteristics of a CAD system, and Sutherland’s prototype graphical drawing program, Sketchpad, the underlying design concepts and generic geometrical and topological ideas were leading the thinking. However, Sutherland acknowledges that it was through the actual implementation that the fundamental differences between a conventional and a computer drawing system really became evident. In particular, that what he calls ‘the strong conditions notion’ which stimulated the

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75 Biographical note: Ivan Sutherland, BSc in Electrical Engineering from Carnegie Institute of Technology, 1959, MSc in Electrical Engineering from California Institute of Technology, 1960, PhD supervisors at MIT: Professors Claude E. Shannon and Marvin Minsky, PhD research carried out in the Lincoln Laboratory where Sutherland also worked as a staff member during the summers of 1960–1962.


77 Ibid., 9.

78 Ibid., 17.
conventional tools of drafting (T–squares and set–squares, for example) while included, was not adequate or sufficient for the recursively extensible world of computer drawing.  

At the same time that the ‘design conversation’ between human designer and computer was under such inventive scrutiny at MIT, there were different, but related paradigm shifts afoot within architecture and the design research culture both within the same town and across the Atlantic in the University of Cambridge. There, the Cambridge University’s first Professor of Architecture, Leslie Martin had been leading the School of Architecture, in newly academic and ‘scientific’ directions since 1956. The year after Coons published his specifications for a CAD system and Sutherland published on Sketchpad, in 1964, Christopher Alexander published ‘Notes on the Synthesis of Form’, the book from his PhD research at Harvard 1958–62.  

This followed his (Alexander’s) Cambridge mathematics degree and a hastily completed architecture degree (1956–58), concurrent with, fellow mathematically inclined student, Lionel March, who was also one of Martin’s first student cohort at Cambridge. Both Alexander and March proceeded from Cambridge to the ‘Joint Center for Urban Studies at Harvard and MIT’. March recalled that once Alexander subsequently moved to Berkeley, he (March) was left as the sole architect at the Joint Center amongst the (mainly) lawyers, economists and political scientist engaged in urban studies.  

March returned to Cambridge, reported to Leslie Martin on the work of the Joint Center. In 1966 they received substantial funding from the Centre for Environmental Studies (a combination of Ford Foundation and British Government money) and the centre for Land Use and Built Form Studies (LUBFS) was started at Cambridge (later the Martin Centre). Martin led the centre for the first two years from 1967 and March from 1969 to 1973. This was the post war world of late modernism in architecture. Sean Keller has written of “a positivist revival” in architecture (coming at least ten years after the death of positivism in philosophy and science). This was driven by the technological and social developments of the second world war and post war period: the introduction of computers, the cultural dominance of science, the spread of mathematical methods in the social

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82 Ibid., 144.

83 Karl Popper and William Van Orman Quine are credited with having killed logical positivism in philosophy in the 1950s (Keller, Systems Aesthetics, 5.) but the similar line of argument: that no amount of analysis will ever positively yield the geometry of a building – it will only rule it out, (Ibid., 75) was not adopted in the Cambridge architectural research of the 1960s.
Chapter 2 | Architecture, mathematics, systems and aesthetics

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sciences, the centralization for planning and architects' own professional anxieties. 84 This positivist revival led to 'quantitative approaches to architectural programming and rigorous formalized approaches to design methodology'. 85 Whereas in the pre war period, there had been an emphasis on buildings that 'looked' scientific, Le Corbusier's ocean liners and grain silos, and the German Neue Sachlichkeit, for instance, the quest was now on in the post war period for a deeper structural logic for functionalism, one that explicitly disregarded 'appearances'. 86 Bob Maxwell noted that a serious attempt to apply functionalism would require analytical tools more penetrating than an eye for a fine building. 87 The RIBA Oxford conference in 1955 had moved British architecture and its former trade skills into the universities in a committed way; Liverpool was the forerunner having had a dedicated School of Architecture since 1893 offering full time degrees since 1911. Architects had also moved wholesale into work in the public domain (45% were in public service in Britain in 1955). 88 In response to the wartime and immediate post war perception of the profession as superfluous aesthetes they had moved into areas of strategic and economic planning. 89

The new functionalist quest found a voice in Alexander's Notes on the Synthesis of Form. Despite its promise that

'in an ideal design solution, the organizational structure of the requirements and that of the resulting object would be identical', this thesis while it raised the value of systematic contextual analysis, failed ultimately, as acknowledged later by the author, 90 to proffer any real connection to formal solutions from the analysis advocated in his design method. The emphasis was on finding a 'formal picture' that retained only the 'abstract structural features' of the design problem, in this case using Boolean Set theory. The program was the basis of the functional decomposition of the problem. 91

While Alexander had left Cambridge in haste (compressing his second and third year studies into one) after confronting what he saw as the 'absurdity' of Martin's position in drawing on the formalism of constructivism, early twentieth century painters such as Mondrian and architects Theo van Doesburg, Ludwig van der Rohe, 92 it is interesting to see that his abstract (perhaps Kantian) notions of form and matter seem to epitomise the tendency that the Cambridge based research first took under Martin's stewardship to 'drift from buildings to the abstractions of geometry and mathematics'. 93

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84 Keller, Systems Aesthetics, 5.
85 Ibid., 3.
86 Ibid., 9.
87 Ibid., 17.
88 Ibid.
89 Ibid., 16.
90 In the Preface to the paperback edition of 'Notes' in 1971, Alexander already refutes his own mathematical methods, and writes in support of piecemeal development of diagrams in a natural way based on designer experience. Keller provides a very detailed critique of the inconsistencies in Notes on the Synthesis of Form with regard to the "troubled connection between the logical analysis and formal synthesis" (Keller, Systems Aesthetics, 68–78, 73.)
91 Alexander, Notes on the synthesis of Form, 129.
The work of the centre for Land Use and Built Form Studies (LUBFS)

In leading LUBFS, Lionel March’s mission was to produce an architectural science that would correct the profession’s obsession with appearances and enable design problems to be solved by mathematical methods. The expressive aspect of architectural form was completely and consciously disregarded. March and the group had access to electronic computers. This access was still a very privileged position outside military and major science organisations and it ‘made quantitative analysis of “all the possible variants” seem like a newly realizable goal’. Architectural research was expanding in scope within a functionalist paradigm, from purely technical issues such as lighting and heating to the relationship of the general organisation and planning of spaces to user needs. This expanded field became known as environmental design. Just as Alexander’s early work had been predicated on the new promise of the electronic computer to solve any problem that could be quantified, in his case, the rather arbitrary selection of 141 criteria for the design of an Indian village was a number too large to have its linkages sorted manually, yet small enough to be handled within a reasonable amount of computer time on by an IBM 7090. The early work of LUBFS, too, exhibited independent mathematical computational criteria underlying the design science rhetoric. The computer had replaced the machine as the modernist icon in architecture.

It is interesting, if not paradoxical, to contrast the approach of the electrical and mechanical engineers and AI pioneers at MIT (Coons, Sutherland et al) working, in their own words, to bring computation into conversation with the designer, with the Cambridge research architects who were working to reshape design to fit their understanding of the quantitative mathematical computational paradigm!

In 1971, the LUBFS manifesto became explicit in a special issue of Architectural Design edited by Lionel March, Marcial Echeñique and Peter Dickens and dedicated to the work of the Cambridge Laboratory. Architecture and planning needed new mathematical foundations as part of the wider “structural revolution” in social and behavioural sciences based on the new awareness of “systems and structures”. It rejected the contribution of intuition as first, unequal to the complexity of the post war

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94 Anecdotally, Australian architectural education closely followed these cultural shifts. Peter Downton: “In 1973, I started teaching in a 4th year design studio at University of Melbourne and we tutors were explicitly forbidden from passing comment on the aesthetics of anyone’s design. This was very different to my undergraduate experience at the same place from 64–8. Melbourne University had a computer. I learnt Fortran 4 in 1968 and did other programming classes as a research fellow in 73/4 on an ARGC grant.”


97 Keller, Fenland Tech, 50.

98 Keller, Systems Aesthetics, 29.


100 Keller, Fenland Tech, 51.
context and second, private, opaque and unaccountable. ‘Draughtsmanship is a drug’ expressed their revolutionary position: that the long history of drawing could be replaced by mathematics.  

Following in Alexander’s footsteps, this architectural science focused on problems of spatial arrangement: arrangement of rooms within a given perimeter, adjacencies of subdivisions of space, taking a purely topological rather than geometrical approach and drawing on ideas from graph theory, operations research, and electronics. Enumeration was a strategy that searched for a list of all possible configurations meeting given criteria. Philip Steadman’s 1973 ‘Graph Theoretic representation of Architectural Arrangement’ is a good example of this. It first represents rooms as nodes in all possible planar non-directed graphs of, for instance, a six–room configuration, and then computes which of these graphs can produce a planar (plan) arrangement of contiguous rectangular rooms within a rectangular boundary. For each room layout, there is a single topological graph, although its representation or shape on the page can vary. For each graph there may be multiple possible room layouts. In subsequent work it was found that with as few as eight rooms, this results, after thousands of instructions and significant computing time, in the enumeration of the 5,124 possible topologically distinct plans. The number of possible graphs rises steeply with the number of nodes, particularly past 6, and, at the time, these large numbers often represented impossible computer run times. Steadman acknowledges that the computer loses out to the heuristics of the human brain in respect of its ability to manipulate pattern and topology (but defends its speed and memory capacity for large volume activity such as near exhaustive enumeration). The computer’s strength in enumeration but weakness in short cuts to analysis through pattern manipulation was similarly demonstrated by the case of the computer pitted against the Grand Masters in chess. While the computer could compute many powers of ten more potential moves per second than the Grand Master, the Grand Masters had, at least until 1997, tended to win the game.

Optimisation was the second, higher goal of LUBFS: automating the selection of best–fit configurations to given criteria.

But in 1972, the detailed review of Philip Tabor and his student Tom Willoughby of previous work and their own in the ‘circulation’ or ‘activity–location’ problem (how to minimize the distances travelled between activities in a building or complex) led to a critique that concluded that quantitative architectural approaches were largely impossible. They identified many ultimately qualitative, subjective and cultural issues in the mix, such as changing patterns of use and difficulty applying meaningful weighting to different types of traffic within buildings. Tabor exercised Alexander’s clustering diagrams but identified their inevitable distortions and returned to exploring building types – slab, courtyard,

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101 Ibid., 275.
103 A planar graph is one which can be arranged such that none of the curves between the nodes cross – thus it can represent simple room or building adjacencies.

Figure 6 Philip Steadman: some possible dissections of the rectangle up to 6 (not exhaustive)
tower etc., which had earlier been regarded as a historicist tool for criticising quantitative methodology.

By the early 1970s March himself had to address the realisation that the mathematical method was not leading to architectural results and did so in the form of a new manifesto in the introduction to ‘The Architecture of Form’. He opens with a critique of Alexander as the naive scientific methodologist and constructs in place of this methodology a new one based on typology, evolution and value judgements, carefully distinguished from science. Many of the tenets from the earlier manifesto remained in place: explicit public process, rejection of artistic intuition, mathematical models to ensure an inclusive approach to multiple criteria, and the conception of architectural projects as too complex to be handled in traditional ways. Mathematics and the computer were to continue to keep architecture serious as a discipline in the post-war societal climate.

March made much of the idea that his own interests were aesthetic– excited by the beauty of the ordering in the system, rather than the object, and made reference to programmed composition in which the results cannot be foreseen and the idea of the same structure lying behind many different appearances. This aesthetic sensibility is perhaps illustrated in his representation of the Seagram Building in 1972 by the hexadecimal number $10283F00F2$ and Le Corbusier’s Maison Minimum as the less than minimal: $F280F71280F032F$. Only through decoding can we start to make an aesthetic evaluation of these number representations, perhaps adopting Hardy’s evaluative criteria. ‘the beauty of a mathematical theorem depends a great deal on its seriousness, as even in poetry the beauty of a line may depend to some extent on the significance of the ideas which it contains’. How much of the idea of each building and at what depth and degree of folding is it really captured in these numbers?

Some further elucidation of March’s particular sense of the aesthetic of mathematics within architecture is possible from reading his review of ‘Mathematics and Architecture since 1960’ written in 2002 in Nexus Network Journal. Early in the piece he gives the caveat that ‘of course every mathematical model abstracts from actuality and only deals with a limited number of factors and assumptions. In my view, such models are useful in questioning our prejudices and sharpening our understanding as long the limitations are taken fully into account.’

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107 Ibid., 172–6.
108 A hexadecimal number is a number in the base 16. In this case the numbers use the symbols 0–9 and letters A–F as the 16 number symbols from 0 to 16. As an example A5E23 is the number $(10 \times 16^4) + (5 \times 16^3) + (14 \times 16^2) + (2 \times 16) + 3 = 679459$
Many of his examples are the macro studies of land use – looking at different urban typologies, making reference to the Catalan civil engineer Ildefons Cerda, responsible for the ‘polycentric egalitarianism’ of the chamfered cornered gridded expansion of the city of Barcelona in the nineteenth century. Examples of March’s own projects are the abstract macro studies of different grid typologies for cities, learning from Singapore’s ‘grossly inefficient buildings’ and Manhattan’s ‘towers on tiny footprints’. Courtyard planning, based on the rapidly diminishing width of the bands in the Fresnel square with increasing square size, is found to be a very effective way to achieve efficient density while maintaining open space. The larger bocks reduce the density of roads…but this results in wider roads with more lanes to carry the same traffic. The LUBFs study for rebuilding Whitehall proposed large stepping building to maintain sun and sky angles to the interiors. Using George Polya’s Enumeration Theorem, the nine square grid leads to 126 distinct courtyard house configurations (choosing five spaces from nine: \(9!/5!4!\)), although many of the 126 are the same under symmetry – right and left handed or rotated. March moves on to the work of George Stiny (and Terry Knight) on shape grammars with Stiny’s words: ‘a design is an element in a \(n\)–ary relation among drawings, other kinds of descriptions, and correlative devices’ and ‘a relation containing designs is defined recursively in an algebra that is the Cartesian product of other algebras’. This choice of quotations seems to illustrate the reductionism of these exercises to highly abstract, logic–based geometrical combinatorial game play, a tidy problem–solution–founded aesthetic sensibility. Although with regard to computation satisfying the criteria for a Turing Machine using shapes in place of symbol strings or other primitives, March writes ‘that the repercussions of “seeing” shapes in a calculation liberates us from the norms of thinking with symbols, words and numbers.’

This liberation theme is also present when he writes of ‘Mathematical processes of thought used to provide counter–examples to the conventional wisdom’, a mode of thought that Alfred North Whitehead called “speculative”. However, the example used to illustrate this: perimeter development as opposed to centralized blocks, seems too classically logically exclusive to constitute what would generally be considered in contemporary design practice as speculation. This is tempered by his reference to Froebel’s sense of beauty as abstract designs that exhibited strong symmetries ‘yet he would break the symmetry of one design and, through a series of moves, transform the design into a novel one.’ March, however, counters this immediately by pointing out the inescapability of symmetry in contemporary mathematics in which symmetry is a relative concept and

112 George Pólya’s (1887-1985) Enumeration Theorem (1937), also known as the Redfield–Pólya Theorem is a theorem in combinatorics ‘generalizing the number of orbits of a group acting on a set’. An example is to consider permutations of a number of colours among a number of beads on a necklace. It is used in chemistry.

113 Ibid., 29.

114 Ibid.

115 Ibid., 19.
asymmetry, called the identity, acts like multiplying by one and counts as a unique form of symmetry.\footnote{Ibid., 20.}

This account of the early computational system modelling in architecture exposes two approaches. The first approach is that of the mathematically grounded, electrical engineer and mechanical engineer working to bring design conversations to the medium of computing and seeing the need for a graphical language of representation and programming to do this. In other words, these engineers sought to bring mathematics to a high level synthetic geometrical interface; closer to the visual world and traditional design interaction. The second approach is that of the architects with a special reverence for the depth of abstract mathematical relations and their power in relation to computing, but with little conception of how to apply that outside of analysis in a way to inform synthetic formal design propositions. In the first approach, the spatial intuition of the design modeller is privileged and exploited as the route to constructing logical relations in the model. In the second approach the logicist view of mathematics is privileged as a necessary framework into which architectural modelling must conform. This conformity is seen as necessary if design is to benefit and be informed or driven by the newly expanded possibilities for analysis of brief and context enabled by computation. The second approach is rapidly found to be unsustainable by its own proponents, explicitly undermined by the work of Tabor and Willoughby in 1972.

2.7 From giving form to the system to giving system to the form

There are other threads to follow in the history of system modelling in architecture. The formalist Peter Eisenman arrived in Cambridge shortly after Alexander and claims to have written his own PhD thesis, ‘the Formal Basis of Modern Architecture’, in furious reaction to being shown an early draft of Alexander’s by Colin St John Wilson.\footnote{Keller, S.B., ‘Systems Aesthetics: Architectural Theory at the University of Cambridge, 1960-75.’, in Architecture, Landscape Architecture, and Urban Planning (Cambridge: Harvard University, 2005), 59-60. (Retrieved April 28, 2010, from Dissertations & Theses: Full Text. (Publication No. AAT 3173946)).} Keller writes that Eisenman exemplified the attempt to discover laws of architecture within architecture, of which mathematics and systems theory provided the dominant models and metaphors. Certainly with the benefit of hindsight and subsequent generations of application of mathematical computational thinking in form generation, Eisenman’s work does appear highly metaphorical in its system theory references. It appears now as a vital formative or influential bridge between the systems methodologists and the applications of systems to expressive formal and figurative ends that were to follow. Some of the very early CAD systems operated within highly constrained geometrical systems in which predefined units could be placed within a predefined grid and subject to the basic Euclidean shape preserving transformations – translation,
rotation and mirroring.\textsuperscript{118} Eisenman’s explorations were closely in parallel with such developments.

In Alexander’s and the LUBFS centre’s early work, the architectural modelling vocabulary is extended into the space of pure topology and this extension presented, from the start, a difficulty in translation back into formal architectural proposals within figurative or physical space. But this difficulty was in large part attributable to the purely analytical problem framing as opposed to synthetic solution framing underlying the topological structures. The way that Sutherland’s Sketchpad still reaches out to designers cogently from the extracts of the movie of its use mounted on Youtube,\textsuperscript{119} emphasises, the critical role of the development of the graphical interface to a synthetic engagement with electronic computation that was neither abstractly analysis–led nor abstractly metaphorical.

Computer power’s adherence to Moore’s law,\textsuperscript{120} and the development of the graphical user interface has been paralleled by huge shifts in architectural culture and theory over the intervening decades since the late 1960s, early 1970s. The committed investigation of the computer as a potential tool for the description or representation of formal and geometrical novelty or complexity in architecture occurred during the 1990s at a time when spatial adventurism was already epitomised by projects with no particular attribution to computation for their systems of formal generation. Two completed buildings at Weil am Rhein underline this point well: Frank Gehry’s Vitra Design Museum, 1989 and Zaha Hadid’s first built work, Vitra Fire Station, 1994.

In the nineties, whether or not the computer offered new levels of inventiveness (creative visionaries certainly aspired to this, and to some degree found it), in the right hands, it offered better and much quicker ways to find and represent more possible solutions, a more advanced iterative conversation, offering the potential of greater levels of formal sophistication and refinement. This is not the ‘enumeration’ of the early era, simply moved to an expanded geometrical field, but exploration: a more productive conversation between human and computer in which the designer’s meta description could automate the visual/figurative/graphical descriptions.

Pioneering work in appropriating software from engineering for manufacturing\textsuperscript{121} \textsuperscript{122} and from the film industry\textsuperscript{123} started

\begin{itemize}
  \item\textsuperscript{119} http://www.youtube.com/watch?v=USyoT_Ha_bA http://www.youtube.com/watch?v=mOZqRJzE8xg Alan Kay’s demonstration: Last accessed 6 May 2010.
  \item\textsuperscript{120} Moore’s law, named after Gordon E. Moore, cofounder of Intel, and coined, around 1970 by the Caltech professor and entrepreneur Carver Mead, describes the long-term trend in computer hardware. The number of transistors that can be placed inexpensively on an integrated circuit has doubled approximately every two years for more than half a century and it is not anticipated to stop until 2015 or later. Computer processing speed, memory capacity, number and size of pixels in a display at a given time and for a given cost are all closely related to Moore’s law.
  \item\textsuperscript{121} Burry, M.C. The Expiatory Church of the Sagrada Familia, London: Phaidon Press Ltd, 1993.
  \item\textsuperscript{122} Burry, M. ‘Parametric design and the Sagrada Familia’. Architectural Research Quarterly, 1, (1996), 70–81.
  \item\textsuperscript{123} Lynn, G. Animate Form. New York: Princeton Architectural Press, 1998.
\end{itemize}
to demonstrate the potential to represent continuously variable design proposals as opposed to discrete solutions. Now, rather than the topology of the organisational diagram, it was the topology of associations between geometrical objects and relations that was being mapped, both to time and to other external variables.

The interest in computing as a formal design tool with which to manipulate geometry (rather than merely computers to emulate the drawing board with some minor efficiency gains)\(^{124}\) gained ground through the nineteen nineties. With the interest in the creative geometrical use of computing, there was also a growing interest in geometry and shape in architectural form making per se.

The freeform “vermiform” blobs born of Non–Uniform Rational Basis splines (NURBS)\(^{125}\) in solid modelling programs were symptomatic of the growing interest in surface shape, though not necessarily of a deep or sophisticated interest in surface description in all cases. (I will illustrate this distinction through the examples in the next chapter.)

Recapitulation: Chapter 2

In this chapter I have traced examples of the history of the transition from object to system modelling in architecture. In starting to address the question of whether and how the philosophy of mathematics can inform a designerly understanding of this transition and the nature of the modelling spaces into which it moves design modellers, I have revisited the relationship of architecture to geometry, geometry to mathematics and architecture to mathematics.

I have supported the argument that the relationship between architectural modelling and geometry cannot always be reduced to one of use of geometry as an ordering device and spatial notational system by architecture. While architecture cannot be conceived in the absence of geometry in its broadest and most intrinsic sense (a sense that will be explored further in Chapter 5), the historical development of formal geometry has drawn on practical design representation for the early development of projective and descriptive geometries, for instance. In this sense, geometry and design have shared common figurative modes of representation of possible relationships in the physical world.

\(^{124}\) During the first half of the nineties, at least, even the most ‘advanced’ offices still had competing teams of computer and manual drafters documenting the same project in parallel as a belt and braces approach to risk mitigation.

\(^{125}\) Non Uniform Rational Basis Splines are curves and surfaces that began development in the 1950s to introduce precision to the description of freeform surfaces such as ship’s hulls and car bodies that were otherwise referenced from one off models. Pierre Bezier an engineer at Renault and Paul De Casteljau at Citroën worked simultaneously on this problem. In computer graphics splines with control points off the curve are today recognised as Bezier splines. They moved from the CAD packages of carmakers to become ubiquitous in computer–aided design, manufacturing and engineering. A NURBS curve is defined by its order, a set of weighted control points and a knot vector. They usefully provide a single mathematical form that can be used for both mathematically–defined and free–form shapes. They are also economically stored in numerous industry standard formats. They have arguably had a significant influence on architectural aesthetics particularly during the period of steep uptake of digital CAD in the discipline and practice during the late 1990s and early 2000s.
The transition from object to system modelling in architecture starts with physical analogue models. But with their limited variables and particular material characteristics, not easily reducible to pure geometrical representation, these are not direct forebears in the development of computational modelling paradigms. Interesting contemporary digital modelling is influenced by the will to introduce more physical world constraints. The value of having gravity within the digital model when designing for a world with gravity, for instance, should not be underestimated.\(^{126}\) The different streams of thought in early computational design modelling range from the introduction of drawing as a computer programming language to support design conversations, to a call for an anti-figurative, anti-formal revolution in architectural conception, to fit what was perceived as the quantitative numerical demands of the process of computation and the analytical demands of a logical scientific approach. We have seen that when this last was overturned from within its own ranks, aesthetics emerged as the central issue while mathematical aesthetics remained confounded with architectural aesthetics. I will return to the relationship between mathematical and architectural aesthetics in Chapter 6.

This chapter has reviewed the relationship of mathematics to both object modelling and early system modelling in architecture. In the latter, mathematics is seen on the one hand as a way to facilitate high level intuitive design interaction with electronic computation, and on the other, as an empiricist way to bring analytical information to architectural design decision making. The problems with this latter approach nurtured a return to aesthetics as the central concern of architecture, including architectural system modelling. This brings the system modellers more into line with the architectural Zeitgeist of the late 1960s and early 1970s: the first wave of postmodern reaction to the aesthetic reductivism of mainstream modernism and its failure to account for either the complexity of cultural history or urbanism.\(^{127}\) But I have noted that the aesthetics heralded in by Lionel March in the case of the LUBFs system modelling stream was in itself a reductivist interpretation of mathematical aesthetics, with the concept of symmetry at its heart. In chapter six I will explore the aesthetics of mathematics from a very different standpoint, from the point of view of its process affinity with design.

In the next chapter I will take the opposite point of departure in reviewing examples of architectural projects since the late 1990s that have developed an architectural aesthetic based on a mathematical idea, in general a system-modelling paradigm. In this case the architecture becomes a conduit for the expression of recent mathematical discovery, or invention, through its appropriation as pure idea or for model problem solving or a synthesis of these two.


SYSTEM MODEL EXAMPLES IN RECENT ARCHITECTURE


Introduction

The transition from modelling objects to modelling systems and the role that computation has played in making system modelling accessible and extensible in design has allowed architects to engage a renewed interest in generic mathematical ideas and systems. The idea of modelling using explicitly stated variable parameters, constraints and geometrically expressed relations is inherently an appropriation from mathematics. (There were always variable geometrical parameters, constraints and relations in the process of architectural design implicit in sketches and usually progressively understated in the architectural drawing.)

Computational system modelling has started to open up the exploration of other recent (post seventeenth century and even late twentieth century) mathematical ideas to architectural design. These ideas are accessible in a more operational, less metaphorical way than in the past. This chapter does not yet directly address the question of the value of philosophy of mathematics to the architectural modeller but lays the groundwork for new ways of considering geometrical space in architecture through engaging systems.

The chapter is principally a review of recent architectural exploration of geometrical space to inform both the designed space (aims and outcome) and the design space (means and process). It considers very select recent examples of architectural projects that appropriate from geometrical or mathematical discovery either for design ideas, or for models for architectural problem solving, or a synergy of these two and that rely on system modelling to explore these relationships. It proposes a loose taxonomy of five mathematically inspired themes or clusters within which these projects seem naturally to group.

3.1 ‘Mathematical’ Themes

The use of computation as a creative tool for expressive geometrical form making or form–finding from the 1990s onwards led to a renewed interest in the appropriation of mathematical ideas for their expressive power as well as their problem solving potential. A number of dominant themes have emerged within this trend.

Five broad ‘geometrical and mathematical’ themes I have identified that have occupied architects exploring the potential of mechanised computation to extend the creative formal output of system modelling are: 1) Surface; 2) Chaos, Complexity and Emergence; 3) Packing and Tiling; 4) Optimisation; 5) Topology. The interest in topology in architecture, I have divided into two subsets: static iconic representation of ideas from topology, such as knots and non-orientability, and kinetic architecture that responds to information from data streams. These are broad categorisations identified among recent architectural projects that either: overtly appropriate ideas developed in the

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129 This can be witnessed in the work of such architects and academics as Asymptote, Mark Burry, Bernard Cache, dECOi architectes, Foreign Office Architects, Gehry and Partners, Greg Lynn, Marcus Novak, NOX, to name a few of the early innovators.
discipline of geometry, or: make novel use of geometry for architectural problem solving, using computational modelling for representation. To be very clear on this point, these are not topics taken from geometry and investigated within architectural modelling, but rather topics that emerged from my search among architectural projects.

Surface description is an area in which computation has invited major shifts in architectural modelling paradigms. These divide into definition of surface shape, structural optimization of surface shape and subdivision of surface for description and construction.

The three related themes of Chaos, Complexity and Emergence are another – although there are fewer examples of the translation of the virtual opportunities for systems that exhibit these characteristics into built architecture. The interest in complexity science and fractal geometry in architecture dates back to the 1980s soon after the translation of Benoit Mandelbrot’s essay Les Objets fractals: Forme, Hasard et dimension (1975) to the English Fractals: form, chance and dimension, and its subsequent replacement by the book The Fractal Geometry of Nature (1977). Peter Eisenman adopted the idea of fractal geometry in metaphorical and iconic ways in his 1980s work: the Biocenter for the University of Frankfurt, for example, that while it has no obviously fractal aesthetic or extrinsic reading, makes reference to an abstract idea of fractal ordering in its design process as described by the architect (Figure 29). The use of computation has taken this into much more literal models of fractal and other recursive systems.

The mathematics of three-dimensional Packing and Aperiodic Tiling has made an appearance in architectural design thinking and representation. The Graph theoretical work of Philip Steadman also solves certain tiling problems in converting topological networks into credible plans of rectangular rooms within a rectangular boundary (Chapter 2: Figure 6). But the recent interest in tiling feeds on the architectural expression of certain discoveries within mathematics itself, in particular the property of aperiodicity. Optimization remains an obvious way to exploit the capacity of the computer to test and compare a large number of alternatives rapidly and its use has spread from structural analysis to other performative applications as well as the purely geometrical.

Topology underlies all relational, computationally supported, geometrical modelling, but topology has also been a dominant expressive idea in the architecture of this period and iconic representation of such concepts as knots and non-orientable surface are to be found among recent projects. ‘Kinetic informationscapes’ is not a term appropriated from

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131 Mandelbrot himself combines intuitive metaphorical description with highly mathematical text and makes clear in his introduction which sections of the book are targeted at which of a number of potential readerships, gathering in this way a broadly catholic following.


133 Ibid.,192.
mathematics or the biological sciences but a further subset of architectural projects concerned with topology in the sense of graph theory, the linking of the information interpreted from large data bases or data streams to an architectural response. It draws heavily on mathematics in linking physical inputs to virtual process and physical outputs. It concerns projects in which the dynamic or kinetic system model linking the context and built work becomes the architecture itself, maintaining its dynamic interactive character when realised. This contrasts with the dynamic system model used to explore and seek out the static architectural solution.

A search for projects since the nineteen nineties that apply recent, or at least post seventeenth century, mathematics uncovered a very limited number of architectural projects that consciously appropriate ideas from mathematics. The work of Melbourne-based practices Minifie Nixon\textsuperscript{134} and Ashton Raggatt MacDougall, during the period that Paul Minifie worked for that practice, exemplifies this approach. Minifie Nixon’s professed focus on technique in architectural production per se is underpinned by a consistent appropriation of well–defined figurative ideas from twentieth century mathematics to explore their application in architectural space making. The Advanced Geometry Unit in Arup have also been involved in a series of projects that adopt this approach. Within the group the work of Francis Archer and Daniel Bosia are notable in this respect. Cecil Balmond’s book Informal\textsuperscript{135} eloquently depicts the high level architectural and structural spatial narrative within which such investigations unfold.

It is easier to find architectural projects that make novel use of geometry for architectural problem solving, using computational system modelling for representation. This is a more expedient recourse to mathematics to support architectural system modelling. This expediency can, nevertheless, be a matter of ‘fit’ between the architectural objectives and an appropriate analogous mathematical context for architectural problem solving. The Beijing Water cube is a good example. The Weaire–Phelan packing model, is a computationally derived mathematical model related to empirical research in crystallography and a significant landmark in the history of difficult proofs in polygonal packing problems.\textsuperscript{136} It is closely emulated in order to achieve

\textsuperscript{134} This practice is now Minifie van Schaik.


\textsuperscript{136} Sir William Thomson (later Lord Kelvin) (1824–1907) tackled the problem “What partitioning of space into equal volumes minimizes their surface area?” in 1887. His answer was accepted for over a century. The unit cell described by Kelvin for this foam of uniform bubbles was a form of truncated octahedron to which Kelvin gave the longer name of tetrakaidecahedron. It was one of the 13 Archimedean solids. This solution, informed by his knowledge of crystallography, survived as the packing model that gives the best solution to this question until 1993 when Robert Phelan and Denis Weaire reopened the search. Robert Phelan began his research at Trinity College Dublin in 1993. His work was to explore the Kelvin problem and variations on the theme using Brakke’s Surface Evolver computer program. He joined a group with a background in solid state and materials science who already had some hunches about what types of structures might compete with Kelvin’s that already manifest in nature. Phelan started with the covalent bonding structure of clathrates compounds in
Figure 7 Kelvin and Weaire Phelan packings and the double façade of the Beijing Olympic pool by PTW Architects and Arup.

Kelvin's packing model using regular truncated octahedra

The Weaire Phelan packing combines two irregular dodecahedra and six Triacontahedra with very particular matching rules.
an irregular ‘bubble’ aesthetic and a structural rationale for the double skinned walls of the Olympic swimming pool building. A more expansive survey of recent architectural projects that fall within the six broad themes and their significance to the question of working in ‘difficult’ system model spaces in architectural production follows.

A total of fifty two projects from practices around the world were studied, a selection of built, as yet unbuilt and a very small number of more speculative academic examples where these last represented appropriations not matched among real world projects. The research included detailed interviews with the architects and other design specialists. A visual glossary was developed, expanding the 6 topics into a total of 58 mathematical subtopics that were addressed in the projects. For this thesis it is not appropriate to revisit the entire taxonomy of projects or themes. A slightly reduced version has been published in the course of the research. I will draw very selectively on

which the bonds can be envisaged as foam cells. Most of the rings of bonds on the sides of the cages are fivefold, creating pentagonal faces. It is a regular assembly of two types of irregular polyhedral cell with respectively 12, and 14 faces, combined in the ratio or 2:6 in a repeating unit of eight polyhedra. It turned out to have a cell surface area for volume that was 0.3% less than the venerable conjecture of Kelvin.

The breakdown of countries of the projects studied: Abu Dhabi: 3; Australia: 11; Austria: 1; China: 2; Egypt: 1; Finland: 1; France: 1; Germany: 1; Japan: 2; Netherlands: 5; Qatar :1; Taiwan: 1; United States: 5; United Kingdom: 10.

A co–authored book has been published in the course of the research. It contains brief descriptions of the majority of these projects, grouped after six introductory thematic essays and this resource for a small number of projects that illustrate the transition to system modelling and its significance for the adoption of spatial paradigms from mathematics. The selected projects are not intended to be exhaustive or representative, merely exemplary. The subjective selection is made on the lines of projects in which the modelling technique is deeply implicated in the architecture and makes unconventional demands on the translation between the logically structured representational space and the perceptual space of the designers.

3.2 Surface

“Consider surfaces not as boundaries of bodies, but as bodies of which one dimension vanishes.” … Carl Friedrich Gauss

Surface is a primary concept and space–making medium for architecture. It is the means of enclosure – architecturally, space is wrapped in surface, whether closed or open. Extrinsic surface shape, granularity, materiality, translucency, colour, scale defines and differentiates the space.

In the natural sciences, surfaces are the boundaries of matter, the interface between solid or liquid matter and the gaseous elements or space. These surfaces are complex and dynamic


at molecular scale. In some models we may find that surface disappears along with any clear boundary condition at this scale leaving only variable gradients in a unified world of matter\textsuperscript{140} while others represent these variations through complex continuous surface descriptions.\textsuperscript{141} Designers tend to engage with more abstract and idealised surface descriptions. In this sense they are working conceptually closer to the impossibly, less–than–gossamer–thin surfaces of Gauss and the mathematicians. Surface in architecture is predominantly a geometrical idea.

It is appropriate that this section should open with Gauss, who brought to surfaces consistent intrinsic description, independent of the surface embedding in three–dimensional


Riemann followed his professor with the extension of such intrinsic description from the two dimensional space of surface to n–dimensional space.

The intensive investigation of surfaces followed hard on the heels of the mastery of curvature, its quantification and the means to measure changing curvature, which are associated in modern times with the introduction of the calculus in various guises in the seventeenth century.

Curvature is a slightly fast and loose term in mathematics. It divides into extrinsic curvature and intrinsic curvature. A straight line has zero curvature, which fits with our intuition. A circle has constant curvature equal to the inverse of the radius. As the radius increases the curvature of a larger circle as opposed to a smaller circle reduces. In order to measure the curvature of a one–dimensional curve, it must be embedded in two or three dimensions. For this reason its curvature is termed extrinsic (it is only known from outside the space of the object). Intrinsically every one–dimensional curve is intrinsically equivalent to a straight line. Surfaces, on the other hand, have extrinsic curvature that is known when they are embedded in a three dimensional space but their curvature is also intrinsic. Convex surfaces like spheres have positive curvature, saddle surfaces like hyperboloids have negative curvature and planes have zero local and global curvature. Surfaces can also be characterised as having single curvature (cylindrical and conical surfaces) or double curvature (spherical, toroid, hyperbolic and parabolic surfaces, for instance, that cannot be unrolled flat without “stretching” and shrinking the surface.) The members of a small subset of the set of doubly curved surfaces are ruled surfaces, for which a swept straight line generates the surface.

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142 Gaussian curvature is an intrinsic measure of surface curvature. It is the product of the two principal curvatures of the surface at a point.
Non–Euclidean geometry

A second important idea that impacts on the study of surfaces is non–Euclidean geometry. This refers to the break with Euclid’s planar geometry in the nineteenth century and is also an area strongly associated with the work of Gauss. One of Euclid’s postulates had always caused headaches for mathematicians. The fifth or parallel postulate implies that for a given infinite line and a point off the line there is one and only one line through the point that is parallel. In 1830 and 1832 Lobachevsky and Bolyai respectively independently published their discovery of hyperbolic geometry in which there are many parallel (non intersecting) lines through a given point P to a line L that does not pass through P (Figure 10 ii. Top surface). The hyperbolic plane can be imagined as a saddle shaped surface in three dimensions. Another form of non–Euclidean geometry is Elliptic geometry, also known as Riemannian geometry after Bernhard Riemann, which gives rise to the Real Projective Plane. In this geometry there are no lines through a point P.
The Real Projective Plane is also described as the set of points on the unit sphere in which every point and its antipodal point are indistinguishable, or as Jeffry Weeks has put it, glued together. The third definition is more technical, \( P(x, y, z) \) is equivalent to \( P'(x', y', z') \) if and only if there is a non-zero real number \( a \) such that \( P = a \cdot P' \). Finally, we can visualise the real projective plane by progressively considering non-orientable surfaces.

The relatively familiar Möbius strip can be thought of as a flat sheet in which the relative direction of two opposite edges has been reversed before being glued together. In the Klein surface, the other two edges of the rectangular sheet have also been glued together but without introducing a second twist. The Real Projective Plane has each pair of opposite edges of the rectangular sheet reversed or twisted before gluing. Thus it cannot be embedded in three dimensional space or represented through immersion without self intersection. (Real 3–space is the three dimensional Euclidean space in which every point is described by a triple of real numbers describing three dimensions each in the direction of one of three mutually perpendicular real lines intersecting at an origin.)

**Figure 12**
Diagrams of embedding and immersion
that are parallel to a line \( L \) that does not pass through \( P \). In the simplest case, lines in this case are the great arcs of the sphere (Figure 10 ii lowest surface). Non–Euclidean geometries have some very intriguing space–making potential in architecture. The best–known application of hyperbolic geometry in art is in the work of M. C. Escher where it is used to represent spaces that confront our intuition.

**Embedding and immersion**

Embedding is taking an object and placing it in a space so that its topological qualities are preserved. In the case of graphs this is maintaining connectivity. A simple graph can be embedded onto a sphere without any crossings. A more complex graph will produce crossings on a sphere so needs a hole to maintain connectivity. The number of holes or handles the graph needs to be embedded into a space relates to its genus. The three dimensional representation of the Klein surface commonly known as the Klein bottle is the best known illustration of the immersion of a higher dimensional surface in three dimensions. It cannot be represented without self–intersection, that is, it cannot be embedded in three dimensions. There are many other examples of surfaces of this kind such as the Boys surface and other representations of the real projective plane. The diagrams below illustrate the concepts of embedding and immersion by showing network diagrams that can and cannot be embedded in surfaces of different genus.

**Minimal Surfaces**

This chapter also includes the application of so–called minimal surfaces. These are the surfaces, which are defined as having a mean curvature of zero. This does not mean they are necessarily planar but that the positive and negative curvature might be thought of as cancelling out. They are stable surfaces of low energy and include but are not limited to surfaces of minimal surface area between given boundaries. Their most familiar manifestation is in the soap film surfaces that form when dipping a wire into soap solution. The classic examples of minimal surfaces are catenoids, helicoids and the Enneper surface (which self–intersects in three dimensions). Then there is the more recently discovered Costa minimal surface, described topologically as a thrice–punctured torus.

**Functional surfaces**

The common thread of the ‘mathematical’ or functional surfaces in this chapter is that they can be described in analytical geometry. While they belong to generic families that vary parametrically, the characteristics of their shape matters. This is different from a topological engagement with surface in which topologically homologous (equivalent) surfaces may be very different in shape. In architecture, functions are a useful and potentially very precise and economical ways of designing, modifying, communicating the shape of a profile.

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144 Unless otherwise stated, the term ‘function’ in this thesis takes its general meaning from mathematics:

a.) Also called correspondence, map, mapping, transformation. A relation between two sets in which one element of the second set is assigned to each element of the first set, as the expression \( y = x^2 \); operator.

b.) Also called multiple–value function. A relation between two sets in which two or more elements of the second set are assigned to each element of the first set, as \( y^2 = x^2 \), which assigns to every \( x \) the two values \( y = +x \) and \( y = -x \).

c.) A set of ordered pairs in which none of the first elements of the pairs appears twice.” (Dictionary.com Unabridged (v 1.1). Random House, Inc. 05 Nov. 2008.)
or surface. For mathematicians visualising functions may be a useful aid to understanding the characteristics, singularities, or periodicity of a function. I include here projects in which the height of each point on the surface, its z coordinate, is a function of its position in the x and y directions. Architects and mathematicians have worked from well-known functions and iteratively edited the algebra to refine and sculpt quite specific complex shapes.145 As a function expresses dependence between two quantities, functions can also be used to create quantitative dependencies between shape and spatial organisational characteristics of the architecture on the one hand, and external inputs such as incidence of sunlight or imposed gravity and wind loads. Functions are also used to try and digitally emulate the shape and behaviour of physical materials.

In 1986, a very elegant two volume compendium recording surviving physical plaster surface mathematical models in university and museum collections was published, edited and part authored by Gerd Fischer.146 The first volume contains 132 full page photographs of solid plaster surface models as well as a few examples made from wood, wire mesh, and string surface models of ruled surfaces. They are divided into the chapters: Analytical Geometry; Algebraic Surfaces; Differential Geometry; Convex Bodies of Constant Width; Regular Star Polyhedra, Models of the Real Projective Plane and Functions. The second has commentaries on the models by multiple authors, all mathematics professors from German universities. It includes in

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the chapter, Models of the Real Projective Plane, for instance, a model of the Boy surface from 1903. This is a surface obtained by the immersion of the real projective plane in 3–dimensional space.\textsuperscript{147} At the time it was known that the projective plane cannot be embedded as a smooth surface in 3–space. This is because any smooth closed surface in 3–space divides the space into an interior part and an exterior part. This implies that it is orientable but the real projective plane is not orientable. The Real Projective Plane IP2 cannot be embedded in the 3–dimensional space IP3.\textsuperscript{148} To visualize it in three–dimensional space, we have to content ourselves with a less than smooth surface, for instance, one in which parts of the surface penetrate one another. Another parameterization of this surface\textsuperscript{149} discovered by German

\textsuperscript{147} ‘Just as the adjunction of imaginary numbers simplified the study of algebraic equations, so the creation of the projective geometry by Gérard Desargues, by the adjunction of elements at infinity to an affine plane, simplified certain problems of intersection by cancelling the concept of parallelism. While the use of projective geometry dates back to the early seventeenth century, the first appearance of the real projective plane as a surface is attributed to August Ferdinand Möbius.’ (Apery, R. Models of the Real Projective Plane: Vieweg, 1987, 13)


\textsuperscript{149} Wolfram Mathworld, Boys Surface last accessed on 7th May 2010 at: http://mathworld.wolfram.com/BoySurface.html

\textsuperscript{150} The eversion of the sphere is the process of turning it inside out in three–dimensional space, with self intersections but avoiding any creasing. An explanatory movie produced at the Geometry Center at University of Minnesota by Bill Thurston, Silvio Levi, Della Maxwell and team was accessed at http://video.google.com/videoplay?docid=--
describe with such clarity that he could instruct models to be made. The eversion of the sphere is a good example of a process that is well represented visually through digital animation (for the less clear inner sighted than Bernard Morin). But the plaster, wood, wire, and string models are evocative frozen moments in a continuum, with potential to give by inference, or sequence, a more general topological or parametric understanding of surface ‘types’. It is a moot point whether this understanding is equally accessible to both a mathematically literate audience and a spatial design audience.\footnote{\textsuperscript{151}}

Moving to the first surface example, or case, I return to the exploration of minimal surfaces, that is, surfaces that when they occur in nature, do so by minimising the surface energy, mathematically, surfaces of zero mean curvature. In this case it is the recent discovery of the Costa Hoffmann Meeks surface through highly theoretical computational exploration in 1984 that provides the raw material for novel architectural expression.

**Australian Wildlife Health Centre – Minifie Nixon: Costa minimal surface\textsuperscript{152}**

Apery’s book was a source of inspiration to Paul Minifie of Minifie Nixon for the practice’s architectural exploration of ‘mathematical’ surface. Surfaces built for architectural enclosure in three dimensions present even greater challenges than mathematical models created in the homogeneous plastic medium of gypsum plaster, or using strings that can pass in space to represent intersecting surface. At the Australian Health Wildlife Centre, Minifie Nixon gave architectural expression to the mathematics of surface by using the relatively recently discovered and highly sculptural Costa minimal surface. This

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\textsuperscript{152} A minimal surface is defined as a surface of zero mean curvature. A plane fits this description. Other well known examples that have equal opposite overall curvature are the catenoid (formed by sweeping a catenary curve around an axis); the helicoid (swept out by a simultaneously rotating and translating line); the Enneper surface, and the more recently discovered Costa–Hoffman–Meeks surface, but there are many more. The investigation of minimal surfaces started with the work of Lagrange (1736–1813) when he posed the question of whether there existed for every arbitrarily complicated boundary curve, one surface of least area. Soap films on a deformable wireframe boundary proved a wonderful medium in which to investigate this question. Minimal surfaces include but are not limited to surfaces of minimal area. The sphere although it represents the minimum surface area for a given volume is not a minimal surface according the mathematical definition as it has everywhere equal positive curvature.
was a piece of chronological symbolism akin the use of the Iron atom for the form of the Atomium at Expo in Brussels in 1958 where it represented an iron crystal in the age of the new science of strong materials. Minifie Nixon adopted the cutting edge in surface geometry description. Celso J Costa in his PhD thesis in 1982\textsuperscript{153} and subsequent publication in 1984\textsuperscript{154} described a genus one minimal surface with two ends asymptotic to the two ends of a catenoid and a middle end asymptotic to a plane. David Hoffman and William Meeks proved the global embeddedness for the Costa surface, and generalized it for higher genus.\textsuperscript{155} 156 It is a surface with three punctures, no boundary and it does not intersect itself. Until Costa’s discovery, the only other known complete minimal embeddable surfaces in R3 with no self–intersections were the plane (genus 0), catenoid (genus 0 with two punctures), and helicoid (genus 0 with two punctures), and it was conjectured that these were the only such surfaces.\textsuperscript{157} An extensive computational ‘search and test’ process was carried out ahead of formal mathematical proofs that this is indeed a member of the family of minimal surfaces. In the Minifie Nixon architectural model a computational search was carried out within a constraint model that also parameterised the architectural context – constraining the overall size of the surface boundary and the location of the triple punctures in the surface. The openings on one side of the surface are distributed as roof lights around an internal ambulatory and the openings seen from the other, within a central courtyard that they open to the external air, providing a solar chimney to ventilate the building. This is an intriguing example of a model that even when its final form has been found, resolved for structural support and fabrication and built,


156 Genus is the maximum number of simple closed curve cuts through a figure without leaving the resulting manifold disconnected. (This also often equates to the number of holes, for instance a sphere is genus 0 and a donut and a coffee cup are genus 1).

Figure 17 The Australian Wildlife Health Centre, entrance, cupola viewed from the courtyard and from the ambulatory, cross section, plan and site plan
persists in challenging perceptual understanding in the space, so unfamiliar is the geometry of the surface.

While the convoluted shape of the Costa surface could be fabricated using tensile fabric construction with a three dimensionally curved steel support structure also of very unfamiliar form, the use and description of curved surfaces in architecture has more often been constrained by constructional considerations – the need to use sheet cladding material or resolve the surface into planar quadrilateral panels for glazing for example.

The Disney Concert Hall – Gehry Partners LLP: single curvature/developable surfaces

Dennis Shelden says: ‘developable surfaces sneak up on you.’\(^{158}\) He is speaking of the paper modelling design process initiated by Frank Gehry in Gehry Partners design studio and the tacit constraint system of the medium in which you work, in this case: paper and its bending behaviour. A developable surface has, by definition, zero Gaussian curvature at every point on its surface. Gaussian curvature of a point is the product of the two principal curvatures on the surface. If one of these is a straight line, with zero curvature, the product is too. Gehry Partners, developed a particular approach to laying sheet materials, that, like paper, more or less admit curvature in one direction only over complex curving surfaces. Or, to put it more carefully, of working through the translation from physical paper and sheet material form finding models into digital space for developed design and rationalisation and description for construction and back to the physical space of sheet materials for construction.

Shelden’s 2002 PhD thesis is an in depth snapshot of particular aspects of this process.\(^{159}\) It moves between space, which is palpably physical and builderly in its description, and space, which is categorically mathematical in its use of language, concepts and notation for its description and definition. From the space of project testing of novel CAD strategies for digital design development, documentation and fabrication including the constructional and organisational constraints being met, it changes gear at page 119 to ‘lift the hood’ on the mathematical space. The description of the nature of a parametric mathematical space begins, ‘A space in mathematical terms is just an ordered set of variables e.g. \((x_1, x_2, x_3, x_4, … x_n)\)^{160} and moves smoothly through Vectors, Vector fields, Coordinate fields and Frame fields before moving to the question of Mappings and manifolds.

‘The concept of a manifold allows a rigorous definition of geometric objects and their occupancy in space through mappings between a local coordinate system, intrinsically defined on the spatial object and Euclidean in nature, and some extrinsic, containing space.’


\(^{160}\) Ibid., 119–129.
This is accessible to intuition and imagination – taking an object from one space and deforming it to an equivalent existence in another. But ‘Typically we will be interested in mappings that present derivatives at least up to some finite order’, is already more difficult; that is, trying to visualise a mapping itself as a differentiable geometric entity. Thus the cognitive or intuitive shift between physical and mathematical visualisation is already challenging. This is because ‘the nature of alpha as a function and as a curve in space is indistinguishable from a mathematical perspective.’ In his introduction Shelden highlights the dilemma around the role of computer–based methodologies in a fundamentally tactile, evocative process. ‘CAD strips away ambiguity, producing definitive geometric forms that “leave little to the imagination.”’

After revealing in some depth alternative approaches to generating surfaces assured geometrically of being developable, he writes, ‘The common characteristics of both approaches – reliance on differential geometry heuristics and constructs – have limited capabilities for representing the behaviour of physical materials and operations.’ This is a prelude to presenting a number of material simulation approaches. The reality is that real modelling paper in the world is rarely found to have perfect single curvature. We come full circle to Husserl’s ideality of geometry. But on the way, Shelden has taken us step by step into n–dimensional modelling space.

Shelden considers that surfaces aren’t just surfaces, they are fields embedded in R3. What their interpreters care about is the Gaussian curvature: the derivatives in two dimensions mapped to their equivalents in three space (a manifold, or Cartesian product space). They must consider the reason that two ants drift apart when walking in parallel on a curved surface. The three dimensional parametric surface can be seen as a subset of the product of a 2 space and a 3 space. These surfaces use implicit rather than explicit functions. Each point on the surface has an x, y and z parameter that is a function of the u and v parameters of the surface. It is potentially useful in terms of mathematical manipulation to see this space as a point in 5 space.

161 Ibid., 23.

162 Ibid., 203.

While broad ranging discoveries in the context of making the round trip between physical and digital representations occur in application to a large number of Gehry Partner projects over time, the Disney concert Hall is an interesting case because it spans two distinct periods in the practice history. The practice won the competition in 1988 and the first design development phase was in 1992 when numerically controlled milling technologies were available but computing performance and information distribution technologies were still limited. The building was at that time to be clad in CNC milled stone panels and economic constraints were to be met through modelling in which a variable distribution of planar, conic and free form panels was possible. In fact conical panels proved to have no economic advantage in milling time, wastage etc. over free form.
When the project was revisited and subsequently built from 1998 onwards, many lessons had already been learnt on the execution of the Guggenheim at Bilbao. The cladding material was changed from stone panels to stainless steel sheet and the surface description from conical to developable ruled surface, without altering the shape but allowing a different geometrical resolution corresponding to a different strategy for the sub-framing.

Figure 19 shows two surfaces – one a collection of planar conical and cylindrical patches with tangent continuity at their straight line boundaries. The second is a surface defined by a swept straight line between two curves. The trick to understand is that a ruled developable surface cannot be found between just any two curves. In this case I have cheated by generating the surface from one curve and deriving the second curve from the developable surface swept out by the straight line travelling along the first.

The sinuous metallic wrapping surfaces leading people into curving Baroque interiors were becoming the Gehry signature of the second period. To realise these surfaces with the scale, structure and constructability necessary for a building of this stature was a journey into the geometry of surface itself. The surfaces have their own inherent characteristics and onto this a different constructional geometry must be mapped that makes sense of the unitary building elements that have to come together according to different ordering rules to support and shelter the building and conform to the conventions of gravity and directionality in the physical world.
The physical modelling must adhere to a language of sheet materials but computational surfaces are nothing like the physical. You can work with paper unaware of the constraints that are being imposed on you. It is in this way that developable surfaces (or surfaces with another mathematical overlay) can sneak up on you, as quoted above. Fortuitously, the characteristics of the developable surfaces (single curvature and able to be unrolled onto the plane) can be exploited directly for finding supporting structures (using the straight lines and opposing single curves) and for constructability in the building (sheet materials that only need to be subjected to single curvature, maintaining their flat dimensions without plastic deformation, trimming or pleating). A developable surface is one in which the Gaussian curvature is everywhere zero. As Shelden noted, it is impossible to construct a developable surface from [the rulings between] two arbitrary curves in space [in the general case.] Although it may be possible to describe a combination of ruled surfaces with shared adjacent edge ruling lines. In this case it is likely that there will be discontinuities at the joints analogous to the folds, wrinkles or buckles that would occur for a physical sheet material forced over two arbitrary curves in space. Moreover, he points out that the ruled surface constructed by ruling lines between two curves inevitably assumes that the surface also has two straight edges. Physical sheet materials like paper can be seen curved along all four edges and this points to their shapes being describable by a collection of joined developable regions rather than a single developable surface.

The Sagrada Família Church – Antoni Gaudí – recent and ongoing work

For one such geometrical protocol in the history of architecture, we can turn once more to Antoni Gaudí. He is seen to be a master of freeform ‘plastic’ sculptural architecture, as in the undulating stone facades of the Casa Mila 1906–1912. Perhaps partially because this way of working was so challenging for communication and unacceptably costly, in his magnum opus, the Sagrada Família he developed a highly codified geometrical system for the complex surfaces of the building. This system is based on combining a palette of ruled hyperbolic surfaces, the hyperboloid of revolution of one sheet, the hyperbolic paraboloid and the helicoid.

The principle currency of design was scaled plaster models made by exploiting the use of a swept straight line and precise curved templates to produce the doubly curved surfaces in wet plaster. For construction, the geometry can similarly be exploited in the stone cutting – arriving at the surface through cutting a series of straight lines between marked templates. The complexity lies in intersecting adjacent surfaces in space. Whether by intention or otherwise the use of the hyperboloid and paraboloid, surfaces of negative curvature, make reference to the ‘discovery’ of non-Euclidean geometry by Lobachevski, Bolyai and Gauss respectively in the early nineteenth century. These are surfaces for which the angles of a triangle inscribed on the surface, will have a sum of less than 180 degrees and for which Euclid’s 5th ‘parallel’ postulate does not hold. They are also surfaces with a closer visual resemblance to organic surface geometry: the hyperboloid to the shaft of a bone, the paraboloid to the web between the fingers or tree.

roots, for instance, than more familiar traditional architectural
surfaces, vaults of consistent positive curvature, based on arcs,
spheres or cones. These three surface types represent a vastly
increased variation in possible surface shape over, for instance,
segments of the sphere which has only one shape parameter,
the radius, which controls its (constant) curvature and three
parameters controlling its relative location in space. The
hyperboloid has three parameters controlling its shape as well
as a further 6 controlling its relative position in space and its
relative orientation or rotation about three axes.

Gaudí combined these surfaces through iterative plaster
design modelling at 1:25 and 1:10. Only a small
proportion of the church was built during his lifetime
and none of that built work yet employed the system
of hyperbolic surfaces that he developed during the last
12 years of working on the design for the church. Any
graphical process using drawings or traits, if it existed, was
lost when the church was sacked and on site drawings and
photographs burnt during the Spanish Civil War. Plaster
modelling continued after his death and resumed with the
laborious restoration of the smashed models after the war.
The plaster modelling tradition, handed down between
generations of modellers working on the site included the
synthetic geometrical methods for creating the curved
shape of the zinc templates for the plaster hyperboloids.
In 1979, preparations for detailing the upper nave for
the continuing construction included laborious graphical
 technique, plotting adjacent hyperboloid surfaces using
their rulings to identify the precise surface parameters
and locate surface intersections to create the information
for the plaster surface assemblies. An academic research
partnership started in 1990 to investigate the translation
of the process to a digital computational environment.\textsuperscript{165}
Once the means were uncovered, all the parameters
throughout a whole window assembly could then be in
play in the digital model simultaneously, putting heavy
cognitive demands on the modeller.\textsuperscript{166} The task was to

\textsuperscript{165} Mark Burry, who had worked on a scholarship from the
Temple Sagrada Família Junta in 1979 to unravel the upper
nave design intentions, returned as an academic at Victoria
University, Wellington, initially to take up a two- year
research contract between the church and the university
that was subsequently renewed and extended to include the
involvement of UPC University in Barcelona. Refer to: Burry,
M., ‘Mathematics and Architecture: Gaudí Innovator’, in
Mathematics and Culture I (v.1), ed. Emmer, M. (Springer,
2004).

\textsuperscript{166} This is significant topic in its own right. The modelling could
not be undertaken at the time using personal computers and
architectural software. It required a Sun workstation running
find a geometrical model that at once resolved all the measured surfaces from a restored historic plaster model into precise ruled hyperbolic surfaces, circular and elliptical hyperboloids of revolution of one sheet, hyperbolic paraboloids, and planar facets as inscribed decoration. But they must at the same time produce curves and points of intersection between adjacent surfaces that conformed to Gaudí’s overall composition of the facades, viewed in two dimensions. The triple points between three adjacent doubly curved surfaces are particularly critical in this respect. For this work a hill–climbing algorithm was first used to progressively test and find solutions that more closely approached the ideal, successively altering the geometric surface parameters. The digital model assemblies were solid modelled using one of the earliest applications of parametric solid modelling in architecture, borrowed from shipbuilding and aeronautical engineering, by sculpting the surfaces from conceptual solids. The technique borrowed from the process of sculpting the stone itself, using, in Unix–based aeronautical parametric software, at high cost. The geometrical technique developed by Mark Burry was based on Boolean subtraction of multiple parametric hyperbolic solids from a virtual wall. It was scripted within the software allowing maximum flexibility and interactivity. “Matching” of geometrically constructed surfaces to reconstructed measured historical plaster surfaces was partially achieved using hill–climbing optimization algorithms developed by Peter Wood at Victoria University Wellington to minimize the distance between measured and geometrically constructed surfaces through iterative adjustment of the parameters. Refer to: Burry, M., ‘Mathematics and Architecture: Gaudí Innovator’, in Mathematics and Culture I (v.1), ed. Emmer, M. (Springer, 2004), 155–164.
In this case, algorithmic Boolean subtraction to progressively remove solid hyperboloids, revealing the curves and points of intersection between overlapping subtractions. In this project, it was Gaudí who, from 1914, appropriated well-defined surface geometry, the specific geometries for their shape versatility and the qualities of strength and light reflection that they introduce. But continuing into the twenty first century, it is the geometry itself, which has allowed its systematic and precise ongoing resolution for construction using first graphical and, much more productively, digital computational technique.

**Surface examples discussion**

Surface description and resolution for construction in architecture has become a large topic. The pursuit of architecture is primarily one of defining and communicating intention for built space. Architects are constrained to employ the surfaces that they can define and describe. This has not been a particularly onerous constraint for the architect working sculpturally. Just as the sculptor is able to find complex curved surface shape as the boundary to a mass of wet clay, or through highly controlled subtraction from the surface of a block of stone, architects have been able to sculpt physical models representing complex, curved or organic intentions.

But when whole buildings take on sculptural surfaces, a constraint system, or constructional heuristic predicated on structural or cladding economy or even just the demands of communicability of the surface description demand a geometrical protocol. We are not yet at the point of whole CNC production of whole ‘freeform’ building envelopes although there is experimental and prototypical work in this

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167 Gaudi almost certainly began to work with the suite of hyperbolic surfaces before he dedicated himself full time to the Sagrada Família church design from 1914. (He had been the architect of the Temple Sagrada Família since 1883). In the porch to the crypt of the Colonia Güell (built between 1908 and 1914) there are already interlocking hyperbolic paraboloids used in the same sinuous way as in the porch to the western transept of the Sagrada Família church.
domain, for instance the ground breaking work of Enrico Dini’s D Form towards 3D printing whole buildings.\textsuperscript{168}

The examples I have selected illustrate a range of surface issues and interests. In Minifie Nixon’s AWHC, the surface is defined by the class of surfaces of zero mean curvature, it is selected as an icon of contemporary mathematical discovery and for the challenging visual and spatial gymnastics of its extrema. The scientific description provides the idealized architectural model description, which must then re-enter the world of physics for cutting and fabrication as a taut tensile structure. Dennis Shelden’s consideration of the translation of curved paper surfaces to abstract geometrical description and back into the material space of moderately deformable materials uses the constraint of single curvature or developability but also uncovers the limitations of applicability of such idealized geometrical constraint models to real material behaviour. The surface description of the upper nave windows for the Sagrada Família church, conforms to a geometrical constraint system – the use of ruled surfaces – a family of surfaces that provides, in profuse combination, a broad spectrum of spatial and shape opportunities, and wonderful diffusing reflective properties for light and sound. Their ruled or straight line in the surface property provides opportunities for combining them with other surfaces on common straight lines and for sculpting stone by cutting straight lines from point to point. This last advantage is undermined by the developments in file to factory CAD–CAM techniques of stone cutting that have progressed in parallel to other computing innovations on the project. Each hyperboloid surface has 9 parameters, the space of possible intersections of two neighbouring hyperbolic surfaces is thus a space of $9 \times 9 = 81$ dimensions, or $9^3 (729)$ dimensions for the space of the possible intersections between three, although only a few solutions result in the triple point. This is by way of making the distinction between the space of the surface – which can be defined mathematically in terms of its $u$ and $v$ coordinates and the model space of possible surfaces with its degrees of freedom.

Because architecture is never without surface, the projects within the other ‘geometrical’ themes will also touch on specific aspects of surface description.

3.3 Chaos, Complexity and Emergence

A key idea in complexity theory is that of small, simple parts, which are replicated, combined or changed, following simple rules. After a number of iterations, the result is a diverse system whose future state is not easily predictable. The system itself gives back new information from simple inputs. In architecture, this idea provides opportunities for analogical inference directly from other processes, including self–organizing systems in nature that result in spatial form and materiality.

Although recent interest in complexity science and fractal geometry in architecture was awakened by Mandelbrot’s writing and its subsequent translation into English in the late 1970s, the underlying ideas in mathematics and physical and biological science go back much further. Mandelbrot himself, in late editions of The Fractals Form Chance and Dimension added a postscript on forgotten early applications of Cantor Sets. The year after Georg Cantor published the Cantor triadic Set in 1883, Poincaré found an application for it in his theory of automorphic functions. The Cantor set is a set of points lying on a line segment. One example is the Cantor ternary set created by removing the middle third of the real numbers between 0 and 1, then the middle third of each of the resulting line segments and the middle thirds of the four resultant lines … recursively ad infinitum. Within mathematics such sets have very significant deep properties that are beyond the scope of this brief introduction.

Fractal dimension is the name of a concept also known as the Hausdorff–Besicovitch dimension. The word ‘fractal’, from the Latin adjective fractus meaning ‘irregular or fragmented’ and related to frangere meaning ‘to break’ is Mandelbrot’s own neologism. In his own words, his essay “proposes new solutions to very old problems with the help of mathematics that is very old too, but that had not been used in this fashion – with the exception of Brownian motion.” The problem as he presents it is the geometrical description of spatial patterns in nature that are either too irregular or fragmented for Euclidean description: coastlines, clouds, trees … He points out the short comings of topology for the study of such aspects of form – each coastline being topologically equivalent to a circle by its topological description. The Hausdorff dimension increases as the shape becomes more fragmented at smaller scale. Self–similarity is also a characteristic of fractal geometry,

the same shape manifesting at many different scales. Mandelbrot’s work is highly significant for bringing hitherto difficult mathematical concepts to a broad readership including the creative and social arts. In the 1980s, architect Peter Eisenman adopted the idea of fractal geometry in metaphorical and iconic ways in his Biocenter for the J. W. Goethe University in Frankfurt. While the building has no obviously fractal aesthetic or extrinsic reading, it makes reference to an abstract idea of fractal ordering in its design process. Michael Ostwald made a detailed study of

the ‘Appropriations of Theory between Architecture and the Sciences of Complexity’ during the period of 1980s and 90s with a significant focus on the work of Coop Himmelblau, Kazuo Shinohara, Ushida Findlay and Peter Eisenman.173 While the work of these architectural practitioners and theoretician teachers clearly exhibits an interest in self–similarity, recursion and scaling applied in architectural composition and design process, the literal creation of virtual complex systems was to be the work of a later generation of designers. In the 1980s buildings, the references to complexity are concrete and transparent: the pavilion shapes, relative scale and delicate spiral arrangement in the plan of Ushida Findlay’s Kaizankyo House, for example.

Architecture has also taken inspiration from chaos theory.174


174 Chaotic systems have a number of general characteristics. They are nonlinear; they are deterministic (rather than probabilistic – there are underlying rules that every future state of the system must follow); they are sensitive to initial conditions; they exhibit sustained irregularity, order in disorder. The
Among the burgeoning restlessly emergent algorithmically generated systems of form in architecture of two decades later, of which Biothing’s The Invisibles Interactive installation (2003) is still arguably one of the most powerfully troubling encounters for being just outside real experience of the biological world, it is not yet easy to identify the successful physical built manifestations. They either remain virtual and cinematic or suggest screens or material schemas for architectural enclosures of delicately variable ‘porosity’. The closest is the challenging construction of façade systems with morphological variance of cells across the field of their instantiation, Asymptote’s Yas hotel as a recent example and we will consider some immediate precursors in the section on optimization. In exhibitions of experimental construction, for instance the Morpho ecology studio in the DRL at the Architectural Association, this line of experimentation has been in manifest evidence. The Kinetic informationscapes section will include architecture where system dynamics translate to kinetics in the physical domain.

The modern understanding of chaos is largely attributable to the work of Edward Norton Lorenz. In the 1960s he created a simplified computer model representing the air flows causing weather. It was a recursive system with a number of variables and could be left to run overnight. While attempting to repeat a particular cycle he discovered the significance of small changes to starting values when an issue with his system caused a small change to the number of decimal places given to a particular starting value. Over many iterations this resulted in a dramatically different state of the system. Chaotic systems are characterised by patterns that may appear similar but never precisely repeat. Deterministic means that each event is determined precisely by what went before – although they may appear random.

Federation Square 2002: bottom–up fractal facades

The principal example here sits right between the two phenomena: the clear allusion to complexity within highly controlled authorial compositional technique and the algorithmic system that generates either recursive self–similarity or chaotic emergence.

This is LAB architects’ Federation Square. The façade system for Federation Square is highly restrained, constrained primarily by constructability ahead of describe–ability. In this way it starts from the architecture, the designed space ahead of the design or model space. For this reason, I consider the building as the representation of the system. It is redolent with possible solutions within the system description but it was the building that was modelled, not the system in this case. The possible detailed configurations were represented and tested in a three dimensional explicit digital model as part of the design process. There was no sophisticated computational approach to the generation of alternative configurations so the number of iterations that could be explored in the design process was necessarily limited by time to represent them. But it is a story that reflects at every level the shift in geometry from top–down to bottom–up, encapsulated by complexity theory and fractal geometry. The research for the design of the building led to the investigation of geometrical patterns that allowed for repetition (in terms of constructional elements), but differentiation of the composed surfaces of the building. For the architects, the fractal self–similarity of the panels became a vital quality in achieving coherence and difference to the facades.\footnote{Donald Bates, ‘Surface Strategies’ in \textit{the Architectural Review, Australia}, issue 090 October 2004, 106–110}
facades of the buildings that define the public spaces there is an almost iconic representation of self-similarity: the $1:2:√5$ triangles that combine in $5$s to create larger triangles and hence, five of these into the next scale of the same proportion, are an easily intellectually graspable and simply construct-able motif that is nevertheless combined in ways that generate relentless difference, and absence of repetition across the whole site.

Geometry, 'the measure and image of a sensate world' and geometry, 'the conceptual ordering which affirms its relevance in spite of the sensory world' are almost tangibly present and experienced through the senses as much as through the intellect.

Deleuze distinguished between diversity and difference: ‘Difference is not diversity. Diversity is given, but difference is that by which the given is given.’ Gregory Bateson also made the well-known statement that ‘information is the difference that makes a difference’. It is the subtleties of

177 Ibid.
179 Bateson, G. Steps to an Ecology of Mind, Chicago, University of Chicago Press, 2000, 459. Consider also contemporaneous concepts (Bateson originally published this work in 1972) such as Jacques Derrida’s ‘différance’ (first used in 1963, (Derrida, J. Cogito and the History of Madness. From Writing and Difference. Trans. A. Bass. London & New York: Routledge, 1978, 75)) which conflates the meanings of difference and deferral. It suggests that the meaning of terms in writing or language is found less in the direct relationship of ‘signifier to signified’ than in their difference from other terms; for instance the meaning of ‘house’ is given more
difference that make the experience of the built Federation Square project an enriching one. There is a strong sense of simple underlying order that cannot be taken in and rationalised in a top–down hierarchical manner. The game is clear, but the permutational opportunities are beyond immediate conception, in this case using colour, materiality, building shape and the distribution of openings in the facades. Everywhere there is difference, everywhere there is conformity to the geometrical schema, if there is diversity it is not the diversity of randomness and maximum entropy – precisely through it’s difference from the terms ‘shed’, ‘mansion’, ‘cottage’ than through its association with the image of some archetypal house. Moreover, absolute meaning is always out of reach or deferred, for instance when we look up a term in the dictionary, for precision, this then requires reference to the dictionary definitions of all the other terms given in the definition and, so on ad infinitum in an endless regression. Similarly, the meaning of any term in a text may be substantially modified by other terms yet to follow it. This deferral component in Derrida is not alien to Zeno’s paradoxes: *Achilles and the Tortoise* or the *Dichotomy paradox*. In a race against the much slower tortoise, Achilles gives the tortoise a head start of 100 metres. While Achilles runs this 100 metres, the tortoise has covered another, shorter distance, say 10 metres. While Achilles runs this 10 metres, the tortoise has once more covered a shorter distance, say 1 meter, while Achilles runs this 1 metre….etc. In other words the information from difference may bring things closer (the aim of Bateson’s hunter’s gun to the centre of its target, the meaning of Derrida’s word to exactly what it stands for) but never to arrive at a coincident point. These philosophical difficulties in reconciling the continuous and the discrete in mathematics and the contest of plurality and change against holism and stasis in philosophy will be revisited in chapters 5 and 6.

Chaos Complexity and Emergence examples discussion

Chaos, Complexity and emergence as one particular broad manifestation of the shift in attention from modelling objects to modelling systems presents particular perceptual challenges in the representation of architecture. In this case the model may adopt the embryological paradigm in which information is couched in the starting point and algorithm but the actual future manifest states of the organism are tacit or virtual and subject to environmental influence as they unfold. In fractal geometry, we cannot simultaneously perceive self-similarity at all the scales implicit in the model. In Delanda’s reconstruction of Deleuze’s world, the mechanism–independent commonality of singularities in relation to soap bubbles and sugar crystals, flood the mind with morphing images. Yet something is hidden in the specificity of the image– there is no visual generalisability to all the systems given as examples to illustrate the idea of Deleuze’s multiplicities. Deleuze treads in Descartes’ steps in seeking to replace the ancient philosophical concept of essence to create definitions built on the (in Deleuze’s case, morphogenic) process that gives rise to the thing ahead of the static essentialist account of the thing itself. But there is a trap: to revert inadvertently to essentialism to describe the essence of the morphogenic...
process and “multiplicities are introduced to break this cycle”. The topic of the modern transition away from essentialism is considered in more detail in Chapter 5 of the dissertation.

### 3.3 Packing and Tiling

The choice of Federation Square as the example project for the previous section on Chaos complexity and emergence, provides a natural segue into the topic of tiling. Tiling has had a fundamental place in architectural space making and surface treatment for thousands of years. It is in most manifestations entirely to do with system and very little to do with object other than the object or “cell” of the tile itself as a base unit. Tiling is the arrangement of tiles in a field.

From the ninth century, Islamic surface treatment started to exhibit the use of multi–level geometric design and at the height of the development in the fourteenth and fifteenth centuries; self–similarity is exhibited in both eastern (Persia and neighbours) and western (Morocco and Andalusian) traditions. In the Alhambra we can find up to seven fold symmetry. Tiling spaces are potentially very complex, Bonner extends the principles identified in fourteenth and fifteenth century Islamic self similar tiling to both aperiodic tiling and to spherical and hyperbolic surface tiling.

Tiling in a purely mathematical sense is to do with arranging regular figures in the plane in ways that leave the least or most regular gaps. Packing is the three dimensional equivalent: for instance, the study of packing is born of the investigation of the best way to organise spheres in space to achieve the closest packing, that is, the least left over space between. Aperiodicity is a particular property of a set of tiles. It is the property of only permitting a complete tiling of the plane that cannot map to itself through translation.

There are many sets of tiles that permit an aperiodic tiling but many fewer that only permit aperiodic tiling. The most familiar tiling pattern – the grid of square tiles is a simple example of periodic tiling; you can map the tiling onto itself through any number of different translations. Just as Mandelbrot brought wide attention to the significance of the Cantor Set and Hausdorff dimension, through his ‘invention’ of fractal geometry, so Roger Penrose, brought the nature of aperiodicity to light through his discovery of very small sets of tiles with this property and their publication in his 1974 paper: ‘The role of Aesthetics in Pure and Applied Mathematical Research’. They were known earlier as non–Wang tiles in response to a conjecture by Hao Wang in 1961 that the tiling of the plane by a set of tiles is decidable only if at least one periodic solution exists. Other names associated with the discovery of aperiodic tilings are Robert Berger, Donald Knuth, Raphael Robinson and Robert Ammann.

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182 Ibid.


184 Ibid., 12.

Roger Penrose took inspiration from Johannes Kepler, the 17th century astronomer and astrologer, who had explored tilings built around pentagons in his book Harmonices Mundi. Tiling the plane with pentagons leaves gaps and Penrose proposed to fill these with three other shapes: a star, a boat and a diamond. By publishing his tile set with an accompanying rule set about their matching and adjacency, he ensured that their tiling was aperiodic. To understand the novelty of this proposal for an aperiodic set of 4 tiles, it is important to understand that this was a descent from previous much larger sets. In 1964, Robert Berger produced an aperiodic set of 104 different tiles (the smallest to date). In 1971 Raphael Robinson simplified his proof and found a set of just six tiles. Penrose went on to find two other sets of aperiodic tiles that used just two tiles, one consisting of a kite and a dart and the second of just two rhombuses with similar length sides but different angles. Although there is no translational symmetry – the meaning of aperiodic is that the pattern cannot be shifted and mapped onto itself in its entirety – any bounded region of the pattern, no matter how large, will be repeated an infinite number of times within the tiling.

This last condition calls upon set theory to elucidate the various natures of infinities and their comparative sizes and characteristics, as indeed, does much of the work of the computational design modeller.

**Storey Hall Annex and Refurbishment 1994**

Ashton Raggatt MacDougall (A–R–M), at the time that Paul Minifie was working for the Melbourne–based architectural practice built a mildly irreverent tribute to this area of geometrical discovery that was nevertheless said to be well–received by Penrose on his subsequent visit to Melbourne. Howard Raggatt reports that Penrose politely pointed out the architect’s “mistakes” in the tiling system but seemed interested, rather than inflamed by the architectural interpretation that introduced and embraced “holes” created by mis–orienting some of the Rhomboid tiles. The principle tiling used both on the façade to the 1994 extension to the nineteenth century Storey Hall and in decorative applied profusion on the ceiling and walls inside the hall itself is the combination of two Rhomboids, Penrose’s simplest and most elemental tiling. This is elaborated in the decorative architectural schema by introducing layered relief, organising the pentagonal tiling onto larger scale triangular facets or tiles that contribute to a (flawed) pentagonal tiling at much larger scale in the ceiling tiling and light panels. The use of a palette
of greens and yellow oxides produces the impression of foliage, turning the hall itself into a forest bower and, in the use of green verdigris bronze tiling on the façade, of a creeper that Raggatt says ‘is set to take over our consciousness’ in the city. This is an overt appropriation and celebration of the culture of mathematical discovery in a centre for learning, a hall that is now a prominent RMIT university lecture hall.\textsuperscript{187} There were many symbolic and process–driven agendas behind the architectural design of the extension and refurbishment by A–R–M (amongst which the expression of the mathematical ideas may be counted by the architects the least significant). But, it is the reference to the tiling system, which is in many ways the abiding and iconic connector of the space to the bottom–up organic and incomplete systemic nature of the city and ultimately to contemplation of the infinite. In this sense the use of the mathematical system in this project is much more significant for its expressive application than for any problem–solving agenda.

\textbf{Daniel Libeskind’s V & A proposal 2002: Ammann and fractals}

Daniel Libeskind’s unbuilt competition winning proposal for the extension of the Victoria and Albert museum in London is redolent with mathematical references and manifestations. The walls of the building itself are generated by a chaotic spiral: the spiral of history; a spiral in which not only the radius but also the centre shifts as the building rises out of the ground plane. The fractile is a term coined by the designers for the scaleless, self similar patterning of the building surface.

\footnote{The hall itself behind its classical façade was originally opened by the Hibernian Australasian Catholic Benefit society in 1887.}
deploying Ammann\(^\text{188}\) aperiodic tiling that would tile the planar surfaces of the building infinitely, unceasingly, evenly, although without overall pattern repetition, were it not disrupted by a fractal tile subdivision.

The Arup AGU group proposed a number of possible tilings including the use of

\(^{188}\) Robert Ammann (1946–1994) was a retiring amateur mathematician who independently discovered five sets of aperiodic tiles which were later published in Grünbaum, B. and Shephard G.C., Tilings and Patterns, Freeman, NY 1986 and for four of which he later published proofs in collaboration with the same authors. In an earlier letter written in 1975 he revealed his discovery of aperiodic set of two tiles and a foursome of ‘golden rhombohedra’ that formed aperiodic tilings in three dimensions. The best known Ammann tile set combines a slim rhombus with a square tile. One of the interesting phenomena exhibited in this tiling in common with the Penrose Rhombus tiling is called Ammann bars. Certain patterns of line segments on the tiles result in straight lines of infinite length that run through the tiling. Intriguingly, these lines are parallel and the separation of neighbouring lines consists of one of two dimensions. If the unit dimension is assigned to the smaller of these, the second is the golden section number Phi. If the numbers ‘1’ and ‘0’ are assigned to these two dimensions, their sequence, the consecutive separations of the parallel Ammann bars, is found to correspond to the Golden String.
Robert Ammann’s aperiodic tiling. Daniel Libeskind liked the rectilinear nature of the Ammann tiles. The tiling was enriched tectonically by the designers through creating a fractal from it by doing what is called a selective subdivision. In the Ammann set used there are three differently shaped tiles. Each one of those 3 tiles can subdivide perfectly into copies of the same 3 tile shapes, scaled down exactly by the ratio of the Golden Ratio. The ‘selective’ subdivision means subdividing the tiles in this way but choosing to subdivide some and not others as the subdivision proceeds. Each time a particular one of the set of three is created, the ‘R’ tile, the subdivision is stopped for that tile. This creates a fractal which is particularly rich because it, too, is aperiodic and does not repeat in a way that would allow it to be mapped to itself through translation. This is the geometrical underpinning of the ‘fractile’.

Just as there are infinite straight, parallel diagonal lines through the vertices in a simple repetitive square grid, Robert Ammann discovered that there are infinite, straight parallel lines through the vertices of his aperiodic tiling, now known as Ammann bars. These lines are spaced at a distance of either 1 unit apart or Phi, the Golden section ratio apart in the same units. The sequence of the spacings of adjacent lines is aperiodic, a never repeating string known as the Golden String. If you represent Phi by aught this gives a binary sequence, such as 0110011 etcetera to represent the sequence of line spacing running in two different directions through the 2D tiling. This Golden String is an irrational number like Pi that when expressed as a number is an infinite sequence of digits and will never allow the prediction of the full sequence of digits through repetition in that sequence.

The fractal subdivision results in variable density of tiles and lines at different locations across the tiling pattern. When the pattern is wound onto the spiral walls, you get different densities at different locations in the building. By having the unfolded building slide over the fractal tiling pattern, Daniel Libeskind was able to choose where these areas of greater density should occur. Finally, how should the graphical representation of the fractal be translated to the fabrication of the physical tiling itself? It was not practical to use different tiles at all the different sizes. The answer
was to raise the tiles in a relief in which the depth of relief represents the different scales in the fractal.

**Battersea Power station crystal theatre proposal: Danzer packing and Ammann lines**

This proposal for the foyer of a 4000 – 5000 seat hexagonal auditorium sunk in the south park area in the grounds of the long obsolete Battersea power station gave full rein to the Arup AGU group to extend ideas that had already been introduced in earlier projects. This was a master plan relatively unconstrained by site and contextual exigencies or budget (which has not been built – the site was subsequently sold on). Set against the monumental language of the Gilbert Scott power station building, the proposed theatre and surrounding car parking is buried beneath a shifting sloping landscape with the translucent crystalline eruption of the theatre roof and foyers from the landscape, a mysterious and energy–focussing counterpoint.

The form and setting out of the theatre and the car park combine a number of ideas appropriated from science with respect to the mathematical packing structures found in crystals and quasicrystals. Whereas in the Victoria and Albert museum project Ammann’s aperiodic tiling was deployed to tile the plane, in this project the Ammann planes were combined with Danzer aperiodic packing, moving the ideas of non–repeating pattern fully into three dimensions. Quasicrystals exhibit long–range orientational order but no translational symmetry. Taking the traditional practice of designing a building on a grid generated by three sets of intersecting rectilinear planes, in this design, a new type of grid is generated with many more planes and unpredictable intervals between them.

The Golden String is used to set up the set out grids for the building. This is non–periodic. Although recognizable patterns re–occur the sequence is non–repetitive and when viewed locally, appears unpredictable. There are two modules in the pattern. A ‘short’ module will always be flanked on either side by a ‘long’ but it is unpredictable whether ‘long’ modules will appear in pairs or individually. The short and long modules in the Golden String occur in the ratio of 1 to Phi, or the Golden Ratio. The resulting grids share the characteristics of musical patterns, being clear and understandable while remaining unpredictable and engaging.

challenging mathematical problems on aperiodicity in response to the 1984 discovery of quasicrystals. Quasicrystals are the name given to alloys discovered by Shechtman with a novel kind of structure, intermediate between crystalline and amorphous. They exhibit long–range orientational order but no translational symmetry. Fivefold and even icosahedral (20–faced regular polyhedron) symmetry is observed leading to the conjecture that the so–called “golden rhombohedra” might provide a geometric explanation, analogous to the Penrose tiling in the plane. Working from the idea that the long–range order in the quasicrystals must stem from local conditions, Danzer found families of tetrahedral prototiles, which become aperiodic when subject to appropriate matching rules. In analogy to the Ammann bars found in two-dimensional aperiodic tiling, there are continuous infinite planes in the space of the three dimensional Danzer tiling.

189 Ludwig Danzer, mathematician, convex and discrete geometrician (1928–) has had a broad spread of mathematical interests but a particular passion for tiling theory. He has worked extensively on aperiodic tilings including collaborations with Brank Grunbaum and Geoffrey Shephard and was one of the first mathematicians to seriously study
The facetted crystalline patterns of the roof and foyer to the auditorium can be imagined as being set out from a dodecahedron. The dodecahedron has twelve faces, six parallel pairs of faces. There are fifteen ways to select a combination of two out of these six. You have a choice of six planes for the first choice and the five remaining for the second choice. Half the possible combinations are equivalent, just chosen in the opposite order so the number is \((6 \times 5) / 2 = 15\). Fifteen new planes are set up, each bisecting one pair of dodecahedron planes. These fifteen planes are intersected with Ammann planes, which occur at aperiodic intervals in space and the result is used as the structural setting out grid for the building including the proposed glass space frame of the theatre foyer roof.

The car parking, landscaping and rest of the site is set up on an 18m equilateral triangular grid with one axis aligning with the power station and with the idea of rotational opportunities. The 120° angles are very close to those between the dodecahedron faces. The regular periodic crystalline nature of these areas is set against the quasi crystalline structure of the building. The building and car parking and hence the crystalline and quasi crystalline grids are separated by deep vertical chasms, bridged at entry points.

**Packing and tiling examples summary**

While the geometrical and mathematical implications inherent in tiling and packing systems are deep and troublingly extensible, we imagine into, as yet, unexplored spatial regions, their application in architectural projects seems comparatively concrete. The abstract system is not the tiling system itself but the model of possible tiling systems, which according to their rules and the degree of stochastic selection or determinism, may be as emergent and unpredictable as any other complex system. Just as the physical act of tiling is itself largely recursive – after initial setting out, each tile or row of tiles must be laid in relation to those that have gone before. It is a highly constrained area of architectural modelling, particularly where confined to the plane. The extension to space division in three dimensions in the Battersea Crystal theatre proposal, while fascinating as an experiment takes the formal imposition of the geometrical system on the architecture to an extreme degree, although arguably no more deterministic that the traditional tartan grid on space planning. What can be said of the models in this section is that they are reasonably homogeneous as descriptions of prescribed subsets of the overall architectural organisation and have been able to be refined to quite well defined problem solving domains.

### 3.5 Optimisation

In Chapter 2, there is a note that while enumeration of possible states of a model was an early focus for architectural computation, particularly in relation to the work the LUBFs group in Cambridge, the higher goal was optimization.

The word optimal, an adjective from the noun optimus has its origins in nineteenth century biology. Its meaning is most favourable or desirable and it speaks loudly of best, of a single goal within the world of the ordered and rational Cartesian scientific search for truth. It referred to the best conditions of, let us say, light, temperature, altitude for an organism to prosper.
The process of optimization describes the synthetic search for this best state within a model, (or the better states, where the process includes trade–off) whether of a biological system, or of an architectural or structural system, usually under a restriction or a set of restrictions, implied or expressed. These restrictions or conditions, the way that the optimal goal has been defined, and the nature of the model will all determine the outcome. In other words, no matter how deterministic the procedure or algorithm of optimization may be, the optimal state is always relative to the details of the system in which it is sought. For instance the most materially economical structure may be nudged towards an architectural form that is favoured for other reasons through judicious changes to the overall structural context of loads and constraints within which is the model is optimised.

In architecture, optimization has been used as a ‘form–finding’ tool. Starting with the model in a state that is an approximate fit for purpose, adjustments are made to the geometry, to the form, and assessed for whether or not they have moved the overall performance closer to the goal. The changes are made iteratively and may or may not be recursive, looking for incremental improvement on the new temporary state of the model. Electronic computation offers high speed, automated ways of deploying different families of algorithms to undertake this reversible sculpting or form finding exercise.

The art of architecture always engages, at some level, the search for an optimal formal, spatial, constructional answer to diverse aesthetic and performance measures, or a knowing compromise amongst these. Trying to formalise this play–off between very different performative impulses in design, to search for several types of best where each factor affects the others, is complex. It is known as multi–criteria optimization. It may be defined by a Pareto optimization, a state in which one thing can only improve at the expense of another. However, this rate of inverse change may not be the same for all states of the model.

Gaudí’s hanging models and some of the other physical models to which brief reference is made in the previous chapter are analogue optimisers. Gaudí was a virtuoso in minimising the use of material to carry structural load. This is epitomised by the resonant single brick columns carrying the floors above in pure vertical compression in the Santa Teresa convent and the inclined columns of the Colonia Güell chapel, hewn back to the line of force to express their lack of redundancy.

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Chapter 3 | System model examples in recent architecture

Structural economy is one of the more familiar applications for optimization processes in architecture. However, it is far from the only application. It is applied to other performative goals: building program and space planning in the historical Cambridge examples, potentially, purely geometrical goals and environmental performance goals such as minimising building energy use or maximising the use of natural daylight in contemporary design research models.\(^\text{191}\)

Mathematically, optimization methods can be crudely divided into stochastic and deterministic approaches.

Stochastic is derived from the Greek term stocha\'zesqai, to aim or to shoot with a bow at a target, in such an activity there is a spread of arrows, some of which hit the bulls eye or come close. A sequence that combines a random or probabilistic component with a selective process so that only certain outcomes of the random can prevail is said to be stochastic. In stochastic methods of optimization the current state does not completely determine the next. The same process run repeatedly under the same conditions will not necessarily arrive at exactly the same outcome.

In deterministic methods of optimization there is no randomness. Values are assumed to be precise. Each state and operation determines exactly the next state. So the same optimization routine repeated with exactly the same starting conditions for the same number of iterations will result in exactly the same outcome each time.

British Museum and the Smithsonian courtyard roofs: one architect, one problem, two models for optimisation

The glazed roof over the Great Court at the British museum designed by Foster + Partners posed a complex but well defined set of geometrical and structural challenges. The courtyard is a 73m by 97m rectangular with the circular reading room placed 3m off centre within the space. Fosters initially proposed a continuous curved structure supported off the existing walls springing from low arches along each boundary and arching up over the space between the boundaries and the reading room. While the arch along each edge had some structural advantages, the decision was taken to target a continuous horizontal boundary around both the rectangular courtyard and the circular building. Chris Williams from University of Bath worked to develop the detailed design of the roof shape and panellisation. The shape was first defined mathematically as a surface on which the nodes of the steel grid would lie in order to start exploring specific solutions. The height of this surface above its boundary corner height, \(z\) was a function of \(x\) in the easterly direction and \(y\) in the northerly direction. The origin of this coordinate system lay on a vertical line through the centre of the Reading Room.

This function was \(z = z_1 + z_2 + z_3\); where \(z_1, z_2,\) and \(z_3\) are each built up from their own fundamental function. The first of these, \(z_1\), provides the correct change in level between the


Figure 36 British Museum Great Hall roof: plan geometry, function without corner singularity, function with singularity, level change function, construction detail and views of the built roof
rectangular boundary around the Great Court and the Reading Room. \( z_2 \) and \( z_3 \) make \( z \), the surface height, equal to zero around the rectangular and circular boundaries respectively.\(^{192}\)

A surface singularity was needed in each corner to accommodate the condition that the rectangular boundary should be on sliding supports to avoid transferring horizontal thrust to the existing historic walls. A function without the curvature singularity at the corners would inflect to be horizontal at the corners, like the end of a ski jump and create forces that could not be resisted by the tension in the edge beam. A formula was needed to create a cone on its side with its apex in the corner as part of the surface.\(^{193}\)

Determining the form of the structural steel grid on this surface went through many stages before arriving at the final solution. The starting point was a simple diagram in which equally spaced points along the rectangular boundary of the Great Court are joined to equally spaced points on the circular Reading Room boundary by radial lines. These lines were divided by a variable number of points at equal spacing and these “dots” joined to form the structural grid. This produced a grid with discontinuities especially in the diagonal direction. These were gradually removed by applying a process known as dynamic relaxation, a process invented by Alister Day.\(^{192}\)

This process involves solving non-linear equations through repeated application of an algorithm until the component of friction tangential to the surface applied to each of the structural nodes by imaginary strings to its four nearest neighbours converges at zero. The whole mathematical grid was run through 5000 cycles before the process was judged to have converged. Constants in the algorithm could be varied to accelerate the speed of convergence but the slower the process and the larger the number of cycles to convergence, the more numerically stable it was.

Apart from some mirror symmetry in the overall composition, all the panels are unique in shape and size, while constrained to conform to a visually similar range of dimension and angle. The weighting functions were chosen to control the maximum size of glass panel. The relaxation was carried out on a finer grid than the actual steel members. The principal optimization routine in this process is dynamic relaxation – iteratively finding what might be called the most even distribution of nodes across the surface.

The Great Court roof is a functional surface controlled by the expert function writer, programmer and engineer applying the application of dynamic relaxation in the person of Chris Williams, to resolve well-defined problems in consultation with the design team.\(^{194}\) A different process was developed for Foster and Partner’s design of the glazed roof of the equally grand space of the courtyard of the Smithsonian Institution building that houses the National Portrait Gallery and the American Art Museum. This subsequent design was evolved from a design...
sketch of a sinuous, undulating surface, which formed three continuous domed areas with curved valleys in between. Both the London and the Washington structure are free-form in the sense of having been arrived at through design processes other than geometrical pre-rationalization even though they are described algebraically.195 With the later Smithsonian design, the idea of creating a tool that permitted live ‘haptic’ redefinition of the surface shape was included as goal for the in house Specialist Modelling Group in Foster + Partners. The preliminary design won an invited competition in 2004.

Whereas the British Museum canopy was supported on existing walls, the Smithsonian roof had to be completely free of the one sandstone and three granite walls of the historic former Patents Office, and absent from the protected views of the building from the street. The competition design supported the roof on eight new independent supporting columns, which also became the valleys and drainage points in the undulating roof form. The columns collect the rainwater. The design was driven by acoustic and solar considerations. In the competition proposal, this translated to a diagonal grid of deep steel fins that twisted differentially in space to occlude light over the domed areas, while becoming normal to the glazed surface close to the columns where the structural forces were greatest. The acoustic material was inside these deep perforated triangular sections.

In the post-competition design, the beams retained their subtle, changing field-of-wheat twist across the domes and valleys, but became universally normal to the undulating surface. The solar occlusion was simplified using glass coatings, and the design emphasis shifted to giving visual access to broad tracts of sky. This reinforced the proposition of large, quadrilateral glazing panels on a diagonal grid, projected onto the flowing design surface.

195 Whitehead, H. & Peters, B. Geometry, Form and Complexity in Littlefield, D., ed., Space Craft: Developments in Architectural Computing (London: RIBA Publishing, 2008): 22–25. This method contrasts with a number of other projects by the same architects, in which complex curved surface shapes were predefined as a composition of arcs, torus patches or sheared cones to simplify the shape definition, setting-out and process of constructing the surface from planar quadrilateral panels for instance: Sage, Gateshead, GLC.
Unlike the British Museum roof, where the shape and its panellization were refined principally in response to shape and structural optimization, the design process here allowed for the ‘manual’ alteration of the roof shape through simple controls to explore hundreds of different versions of the form for a range of different performance issues. All of the constraints and decisions were encoded into control geometries in a computer program by Brady Peters of Foster + Partners’ Specialist Modelling Group, a programme that was changed and added to throughout the design process.
As built, the Smithsonian canopy is complex: every node, twisted beam and glazing–panel shape is unique, yet all were generated from the same simple rules. Using quadrilateral panels on a free from rather than, for instance, toroid roof means the four corner vertices of the panels cannot all lie on the surface, as in theoretically possible for the triangular panels of the Great Court roof.

While the overall shape criteria for the roof and its components were encapsulated in a model of thousands of lines of code, significantly this was dealt with in modules. Thus shape and detail changes to each of the structural mullions between the glazed panels and the panels themselves could be dealt with independently of the overall roof shape model and the roof repopulated with the edited components to examine overall assembly issues. Similarly the roof shape could be altered. The model attempted to minimise what have been called “long chain dependencies”.

Models in which the graph of relations is very extensive and every geometrical variable depends on every other are notoriously brittle. A modular approach is one way to limit the impact of decisions in one domain on the viability of the geometry in all the others. From the model it was also possible output different types of model at different levels of detail, for instance the three dimensional geometry model for use in acoustic analysis software must be a very simple geometrical abstraction compared to that including the description of the gutter details between panels.

**Pinnacle tower: constructional heuristics**

Another project, which accepts a ‘snakeskin’ solution outcome in order to be able to make use of quadrilateral panels on a curved surface is KPF’s Pinnacle tower façade. The overall building shape in this case was not free form but conclusively pre–rationalised: a triangular plan to fit the site, tapered vertically by up to 2.5°.
Interference Diagram
Solution Schemes

**Mathematical Scheme**
Tangent between circle and ellipse plus stationary node defines the solution plane. Algebraic form of a quartic equation.

**Dynamic Relaxation Scheme**
Particle spring simulation approximates solution obtained by springs’ equilibrium nodes.

**Numerical Computation Scheme**
Abstract numerical solver based on a divide and conquer scheme. Solution found within tolerance 1.0e-7.
The acute angles of the triangular plan became rounded to maintain usable interior space, and shear cones were used to unite the differentially sloping faces (which were circular arcs in plan rather than elliptical, as the plan of an inclined cone would be). The building can be resolved in two dimensions into simple arcs and tangential lines. A large number of parametric variants of this form were explored for their sculptural qualities in the site context.

The aspect of the design relevant to the reflection on models for optimization is the resolution of the flume-shaped outer wrapping façade into regular rectilinear panels, with the mullions offset in each storey to accommodate the reducing width. The building has a double façade. The façade is composed of a single, flexible module type. There is an upright (internal) frame on the slab edge; the external frame is registered in space off the internal frame. Both panels are rectangular and of regular size all along the façade. Internal panels form the building enclosure and include opening windows, while external panels, which lap in a ‘snakeskin’ pattern, provide weatherproofing and allow for natural ventilation, even at high level.

An optimization programme targeted the tightest packing of the external panels with the smallest opening between the lapped panels, and the best visual continuity in their orientation.

The first experiment forced a constant distance between the internal and external panels, and between the external panel and maximal volume envelope. This procedure resulted in a high level of collisions between panels in the external skin, especially in areas of high curvature.

The second system replaced the constant distance by a constant angle constraint between the internal and external panels, which produced some solutions in which the panels did not collide. It also resulted in a 1.23 per cent increase in gross floor area as a result of the 19.05 per cent reduction in the overall cavity volume between the two skins.

The third and final version employed a heuristic, which used the gap between adjacent external panels, rather than any constant relationship between internal and external panels. It was based on imagining a person positioning each panel in relation to the preceding one, measuring a particular set of distances. Interestingly, the results from this much less constrained model were not only better in terms of clash avoidance, gross internal floor area, and cavity reduction, but also had the advantage of being non-panel specific, and allowing easy substitution of a new module with detailed variations. Significantly, it was also much more transparent to the collaborating designers and open to design intervention.

Sidra Trees: growing architecture

This example of optimization is based on the principle of form-finding by a computational sculptural process using the subtraction of the least stressed material in a materially homogeneous block in order to find the most efficient resultant form to carry loads. The most general name for this process is ‘evolutionary structural optimisation’ (ESO) but it has been refined to calculate and respond to both compressive and tensile forces and to both subtract material in low strain regions and add material in high strain regions (multidirectional evolutionary structural optimisation).
In this way it is closely analogous to natural processes such as bone growth.

The Sidra tree, a plant native to Qatar that flourishes in the country’s unforgiving desert climate, inspires Arata Isozaki’s design for the 250m–long entrance to a new convention centre in Doha, home to the Qatar Foundation, which includes a 2,500–seat theatre, an exhibition hall and banqueting facilities. A traditional source of nourishment and medicine, the Sidra tree also symbolizes knowledge of the divine. The exact nature of the voluptuous, tree–like shape of the building is far from arbitrary, at least structurally. Design engineer Professor Mutsuro Sasaki evolved the form using an optimization method known as extended evolutionary structural optimization (eeso). For a particular set of site, material and structural loading criteria, this method achieves the best, most efficient mechanically performing shape, using the least material possible.

The top of the ‘tree’ was constrained to remain perfectly flat. Two support points on the ground, 100m apart, were also architectural givens. The trees were sculpted by a computational cycle of calculation and excision, with the tree forms evolving from a hypothetical block of virtual material, using structural finite element analysis to identify and subtract the most structurally redundant areas, and then repeating the process on the residue many times over. While the pure form adopts the structural strategy of real trees – shedding redundant material and gathering substance where it is needed to resist force – its materialization and realization outside an evolutionary biological setting becomes an intriguing challenge.

In moving from the pure form to the built artefact, Büro Happold’s Software Modelling Analysis Research Technologies group (SMART) carried out further integrated optimization to the free–form tree structure, incorporating new constraints related to the geometry, structural stiffness and fabrication. For the construction, the SMART team resolved the design of the complex surface into 6mm–thick superficial steel panels, almost all of single curvature, while maintaining the organic profile of the trees. Concealed within the trunk and branches, which are up to 7m in diameter, is a simplified core structure with an octagonal cross section that follows the centre lines. This profile is composed of flat steel sheets.

Further rigorous geometrical optimization was needed to maintain the sheets of the octagonal core as close as
possible to the finished panelled design surface, while maintaining their perfectly flat (rather than warped) profile over long lengths. This kept their transportation and fabrication costs within budget. Plate thickness was also optimized to minimize the weight of the structure. Each component is unique and needed to be oriented correctly; the modelling included individual identification, tagging for fabrication, and checking for connectivity between adjacent panels.

Strategically placed movement joints with gasket seals and insulation of the trees enabled the building to avoid stress caused by very high temperature gradients. The structure had to be precambered by as much as half a metre, the effect of which had to be accounted for in the geometry of the model. The modelling was integrated through a central 3D modelling hub, to integrate geometry development with analysis, optimization and machined output.
Finally, here is one other example that illustrates the concept of global and local optima.

**I Project: Island City Central Park Green house roofs and sensitivity analysis**

The thin “free-form” concrete shell roofs for Toyo Ito’s design for the green houses were conceived as part of a continuous wave formation through an undulating landscape of the park as part of the development on reclaimed land in Hakata Bay. They were to be lightly and amorphously wrapped by their concrete shells. Professor Mutsuro Sasaki, author of the form of the Sidra Trees for Isosaki’s Qatar project, introduced a different optimization process in this case, for finding the greenhouse roof shape, which he calls evolutionary shape design by means of sensitivity analysis.197 This is based on the acknowledgement that mechanical performance and geometrical shape are intimately interrelated in a way that means that even for a loose free form shape there will be variants that represent local minima. These are minima in terms of structural strain and hence the amount of material needed to resist the forces in the structure in an the overall landscape of possible shape variants. Thus the green house shapes are set up through topological description that can vary and morph in geometrically constrained ways in a search for good structural ‘sweet points’ or singularities. In general, a structure, which is shaped to transmit loads axially rather than through bending moments, has the most efficient load transmission and lowest strain energy in the overall structure. Sasaki portrays this shape design technique as a two way system in which the computer calculates structurally optimal or low strain energy points in the model that may deliver hitherto unforeseen shapes within the design constraints and the designer can edit the design shape inputs affecting the structural performance and nearby structurally optimal shapes in the model.198

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197 Sensitivity analysis is the study of how variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources in the input of a model. The aim is to identify the relative weighting of sources of uncertainty. This understanding of the response to changes in its inputs is often obscured in mathematical models, but it is important in making correct and meaningful use of the model. In this architectural/structural example, a global minimum may represent the most structurally efficient shape, but other local minima may represent very good solutions that are much closer to the design shape and to meeting all the other design criteria. It is also important to understand the shape of a minimum—how flat or well-defined relative to the surrounding landscape.

Optimisation examples discussion

In this section I have reviewed a very small selection of projects employing a variety of optimization techniques in architectural models for form finding. The first two: the Roofs of the Great Hall in the British museum and the Smithsonian courtyard used, in the first case, an algebraic surface description to optimize the surface shape to meet all the geometrical design criteria (in particular to create an upward curving domed form that met all the existing straight and cylindrical walls in a horizontal boundary condition that would not transfer lateral loads). In the second case of the Smithsonian roof a manually tweak–able B–spline surface meeting certain constraints, in particular with respect to its support points and valley positions was the model. Computational optimization is used in both models to derive the surface subdivisions, locating structural steel members and glazed panels. The first aimed to keep all the nodes in the surface but distributed them using a relaxation algorithm to provide the best visual and structural triangulation of the surface. The second aimed to minimise the out–of–surface deviation of diagonally opposite corners of the quadrilateral panels. In both cases while computational models were used to achieve optimal outcomes a further level of highly specific local detailing was required to accommodate the high level of variation manifest in the panels, their junctions and their fit across the surfaces. Neither of the resulting panellization patterns could have been worked out manually without the use of computation using the same criteria within a reasonable period of time. This is due to the sheer number of iterations and the high level of interdependence of all the nodes and panel positions across the whole surface.

In the second example, the façade cladding of the Pinnacle tower, the best outcome is ultimately achieved by adopting a procedural heuristic akin to the sequential placement of the façade panels during construction. In this way each panel position is optimised individually and sequentially relative to its immediate neighbour in a way that not only results in an outcome that is viable and buildable but is found to achieve the best overall conditions for reducing and regularising the gap between the internal and external façade and maximising the usable floor area. This last is intriguing because it counters the ‘black–box’ model of optimization. This is the model in which there are clear inputs, goals and variables, and outputs that may be universal or local best results, but between inputs and outputs a slightly mysterious intermediate procedural stage that is relatively obscure to the designer. Through a series of iterations, alternatives are sampled until the sequential outcomes no longer show any numerical improvement against previous iterations or until a previously set number of iterations has been completed.

The third example, the Sidra Tree entrance to the Qatar Education city Convention Centre illustrates the use of a very different optimization approach. It starts with the concept of a structure as a continuous material whole. Through its division into finite elements, analysis of the structural strain in each part of it and removal of the least strained material in a recursively iterative way, it produces a structurally optimal 3D solid sculpture. This computational approach to structural optimization, in this case subtractive, (although, as noted, evolutionary structural optimization has also been developed to be additive) is like the hand of the sculptor working with stone, or exploiting the plasticity of clay. The example of the Sidra
trees illustrates how the realisation at architectural scale cannot necessarily at this time pursue the sculptural 3D homogeneous paradigm into detail design. In this case, it is resolved into framing and skin in another process that completely undermines its original structural rationale.

Each of these examples deals with the form resolution of a part of a building, albeit the most architecturally significant or novel part of each building. None of the models links the geometry of the whole building; each operates within and in relation only to its immediate context. Intriguingly, although the procedure of optimization is, in each case, highly logical and numerical, the apprehension of its meaning in terms of architectural form making is highly visual and intuitive. The geometry is understood in terms that are largely subjective and qualitative. The British Museum Roof poses an exception in which the geometry, the shape, is defined algebraically, albeit variably in relation to its constraints. In the others, the shape is understood synthetically as a combination of certain constraints, ‘meta shape’ conditions and either plastic deformation or component ‘fit’ within the given boundary conditions.

It is interesting that in this most computationally intensive approach to system modelling the visualization of the system, of the model itself is comparatively unproblematic. The number of criteria against which the form is optimized is generally limited, the number of variables is limited, the goals are well defined and well understood in natural language. These are highly constrained models for refining an organizational system that is already envisaged and well defined. They fit well to abstract mathematical models and processes by which natural phenomena have been described and from which the optimal concept is etymologically and conceptually derived. They are also prototypical of simple abstractions of computation itself. A procedure that accepts inputs in the form of arguments and delivers outputs. In most optimization routines there are repeated interim outputs that are used recursively as inputs in the following step of the loop. Visualization of the model, the understanding of its function is dynamic and morphogenetic. In all the examples I have given, the variation in form, corresponding to the steps in the optimization process is continuous or sequential rather than discrete.

3.6 Topology

Topology is a way of considering the Greek concept of Topos: the place, the space and everything that is in it. It belongs to architecture and dwelling in very many ways. The freedoms it affords as a more generalised framework than geometry have received much greater attention and appreciation in post digital design times in architecture. It has also been known as rubber sheet geometry. I have already touched on topology in Chapter 2, first of all in reference to its origins as an idea from Leibniz and the illustration of its value by Euler through his bridges of Konigsberg. The birth of topology has also been attributed to Henri Poincaré upon publication of his book, adopting Leibniz’s name for this study for the title: Analysis Situs. In the surfaces section in this chapter, topological qualities of surfaces: orientability and non–orientability, for example, are discussed. Topology covers multiple different spatial and mathematical definitions but the essential qualities that distinguish it from space according to other mathematical conventions, are that it is non–metric,
independent of measured dimension, angle and even ratio and proportion. It is concerned with proximity and connectivity. It encompasses the abstract graph representations of relationships between entities, maps of relations, and also the mappings of points from one space to another. Topological transformations that leave a form invariant are those that maintain connectivity and the proximity of points. They need not preserve shape in any metrical sense. A sphere is equivalent to a cube or any other simple closed surface but it is not equivalent to a torus or its well known homological equivalent, the coffee cup, as it is necessary to create a hole involving cutting the surface and re-gluing to move from the sphere to the torus. There are other essential topological qualitites such as compactness199 and convexity.200

What is it about topology that captured the architectural and architectural criticism imagination in more recent decades? Arguably, it is the ability to model processes of continuous transformation, outside metrical constraints. In her introduction to Architecture and Science in 2001, titled The Topological Tendency in Architecture, Guiseppe Di Cristina wrote:

"The action of mixing, in a continuous and cohesive way, different forces internal and external to the architectural object in accordance with a logic of ‘gratification’ rather than conflict [as in deconstruction] has resulted in the adoption of pliant systems – that is, flexible and changing systems – in response to the various contextual, programmatic, structural and other requirements of the project."201

So the topological nature of the system facilitates both its heterogeneity (Di Cristina’s ‘forces internal and external to the architectural object’) and its continuity. There is an implication of plastic or elastic flexibility and, in contrast to Alexander’s promise of the mysterious logical deduction of form from rigorous analysis of program implicit in his Notes on the Synthesis of Form, Di Cristina’s description of the ‘Topological Tendency’ keeps form to the fore in the idea of the ‘architectural object’ as the central entity operated upon by the system.

“What most interests architects who theorize about the logic of curvilinearity and pliancy is the meaning of the ‘event’, ‘evolution’ and ‘process’, that is, of the dynamism that is innate in the fluid and flexible configurations of what is now called topological architecture.”202 In this statement Di Cristina daintily steps around the expression of curvilinearity and pliancy in architectural form, leaving only architectural thought and theory in this realm somewhere between fluid mechanics, and Morse’s mathematical description of singularities in mathematical functions.

199 Compactness in topology means intuitively that taking an infinite number of steps within a space will bring you close to some other point in the space, thus closed and bounded spaces like a rectangle or disc in the plane or sphere are compact while an infinite line or plane are not, nor is a disk with missing points.

200 Convexity – a space is convex in topology if for every straight line segment joining two points within the space, every point on the line segment lies within the space. For instance, a disc is a convex set of points but a torus or any topologically homologous space is not.


202 Ibid., 8.
“The very architecture of topological deformations, over and above any attempts at formal and spatial dynamism, goes beyond the defined form and reveals the qualitative space of spatial relationships. And the topological space, that of spatial relations, is directly connected to man’s existential dimension. In fact, according to studies in psychology, and above all those of the celebrated Swiss psychologist Jean Piaget, the topological properties of space are connected with man’s sensible experience; our actions and our experiences of the physical environment comprise a spatial dimension according to the properties of vicinity, opening, interior, etc, so topological concepts are also existential concepts. That is to say topological space corresponds to the space of existence.”

This is a huge claim in a paragraph of Di Cristina’s that runs too far ahead of the argument in this dissertation, but I will return to the place of topology in human development and spatial thought. We can draw from this quotation from Cristina the suggestion that the dynamic (topological) model of spatial relations has the potential to create a more immanent and intuitive working space for the designer than the static object model in which these variable relations are also present, but only through extension in imagination. Whether this state of immanence, of tapping into the existential, is really possible within the constraints of logically organized, geometrically programmed modelling systems is a question that will be explored further. To take Piaget’s observations about the relationship of the human mind to its external reality, however constructed in the mind, and invert it to encompass the generation of possible external realities from the existential mind through the medium of topology is a large step involving many assumptions. As we shall see, although in the nineteenth century topology becomes a contextual set for many of the other conventions of geometry, more constrained in their symmetries, it is nevertheless in and of itself a mathematical convention and we will never absolutely escape from the implication of Gödel’s second incompleteness theorem that a system of proof will not be able to prove its own internal consistency. Immanence and the existentialism of the human mind are not subject to such tests of consistency in the mathematical sense, so it is unlikely that any mathematical spatial convention can indeed correspond to the space of existence as Di Cristina claims. Nevertheless, the power of computation to extend connectivity within a topologically organised model space undoubtedly affords certain freedom of transformation that may be seen as less of an abstraction of lived space than the static geometrical conventions of object modelling.

1. Static iconic architectural representations of ideas from topology

The work of some of the contributors Di Cristina’s Architecture and Science, have been included in other sections of this dissertation than “Topology”; for instance, Gehry and Partner’s work in the section of this chapter on Surface description, Daniel Liebskind’s Extension to the V&A in Packing and Tiling, the work of Lars Spuyboek and NOX in Kinetic informationscapes. This serves to illustrate how pervasive and ubiquitous the “topological tendency” is in the foundations of recent methods of design modelling and representation. In this sense it a difficult concept to

203 Ibid., 12.
pin, in any figurative sense, to particular project exemplars. Nevertheless, in this the first of two project sections for the topic of topology, some buildings for which the principle organising ideas come from topology, and have been applied in an iconic or emblematic way, follow.

**The National Museum of Australia**

“We wanted to deeply problematize the idea of a single singularity, of an intersection.” 204

Tangled Destinies is the title given to the monograph on the design of the National Museum of Australia. 205 It is this tangle, the inseparability of histories from different cultural perspectives and the impossibility of simplifying the relationship between them that dominates the conceptual design of the museum. The site is the prominent Acton Peninsular on Lake Burley Griffin in Canberra.

The geometry of threads that entwine and tangle in a series of knots is one among a range of devices in the highly wrought assembly of cultural allusions, architectural motifs and visual puns 206 that tell Australian history not a consensual national story but as many stories tangled together. The tangle of threads becomes the central organizing device for the distribution of the buildings, internal and external spaces and the journey through the exhibitions.

204 Howard Raggatt at the offices of ARM, 26th February 2009.

There is a second twist in the mathematical/geometrical telling of the Australian story in this building and that is the significance given to what is not there. As a pathway taken by visitors through the museum, the thread becomes a huge virtual twisted extrusion of a regular pentagon. This five-sided figure is generally only seen where the thread cuts through the building envelope as it enters the interior. The thread is experienced as space but not seen except in the residual built fabric after its knotted tangle has been subtracted from the building mass.

Howard Raggatt mischievously summarizes the design of the Main Hall in the museum in terms reminiscent of Dr Seuss as “knot in a box”. “This is a knot and no one can read it as a
Where the knotted thread crosses the bounding box and its virtual presence materializes. Roof lights take on the actual rather than subtracted form of the thread as it passes through the top of the box. During design their number was reduced from eight to six and the knotted thread had to be re-tied to reduce the number of intersections. Similarly the warped surface of the twisted pentagonal section thread could not be reproduced in glass and the knot must be untwisted to achieve flat glazed surfaces.

Neither mathematics nor digital process were starting points for the conceptual design of this building but it is only through the use of computation that these convoluted representations of meaning could adopt their knotted topology and assert their negative presence in the spatial sequence of the Museum.

Knots have been part of life and culture since prehistoric times but their theoretical description only entered mathematics through Gauss’ work in the nineteenth century. In a mathematician’s knot, the two ends are joined together. A knot is a topological embedding of a circle in 3-dimensional Euclidean space $\mathbb{R}^3$ but they are generally represented by two dimensional diagrams in which the crossings are represented. One knot may have several 2-D diagrams. The adoption of the knot in the National Museum of Australia design is another example of the appropriation of a relatively young mathematical idea, translating it to a formal design technique to give it architectural spatial expression. It is a double reference. It is used both to signify the knotted nature of the history represented in the building and, arguably, also as an icon of the mathematical idea itself, expressed through experimental formal design technique. This is seen elsewhere in the work of ARM of this period and in the subsequent work of Minifie Nixon.

**Klein Bottle House**

McBride Charles Ryan’s Klein Bottle House started out its design life as a spiral of spaces gathered around a central area in a family home. But a spiral has a distinct start and end point. Once these two were collapsed together into a continuous eternal journey, the Klein surface emerged from the process and once recognised as such, could be exploited for its conceptual magic. The Klein surface (or bottle – perhaps an etymological confusion of the German flache (surface) and flasche (bottle)) is one continuous, two-sided, non-orientable surface with no distinction between inside and outside. The spaces around the central area in the house and the proximity between occupants remained but the fit to both the site and circulation settled comfortably into the eternal form. This form was found infinitely distortable to fit the cusp of the steep change in slope on the site.

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207 Howard Raggatt at the offices of ARM, 26th February 2009.

208 Knot equivalence: one knot is topologically equivalent to another if you can make the knot without cutting the other knot. Knot addition: knots can be added together by cutting the two knots and splicing the ends together without introducing any new crossings. Prime knots: A prime knot is one, which can not be created through addition of two other knots.
A folded origami model provided the clues to making the house buildable from planar surfaces. While the Klein bottle itself is defined by the continuity of the single sided surface, the contrasting colours of the two sides of the origami paper provided the first clues to the use of colour to differentiate the parts of the building at a local level. The architects see the red platform as the metaphorical ship in the (Klein) bottle. The house emerges from the model – through a cycle of digital and physical modelling process.

This is a much more concise and spatial use of the concept of the eternal loop than UN Studio’s Möbius house in which the Möbius
band is a literal pathway through the space. In the Klein house, the sequence of spaces themselves become the Klein bottle and the proximity of the occupants and their heart(h)–hugging activities becomes a matter of topological proximity as much as metrics.

**Taichung Opera House**

For the Taichung opera house Toyo Ito adopts the theme of continuous infinite and free flowing spaces interrupted by a bounding box. In early design, the extended internal surface differentiating the spaces within the opera house starts life as a series of intersecting catenoids. The surface form is powerfully visually suggestive of families of mathematical surfaces seen in other contexts: minimal triply periodic surfaces create infinite continuous gridded spaces that interlock on either side of a continuous convoluted surface. (Such hyperbolic surfaces intriguingly offer useful representations of curvature in condensed matter at atomic and molecular scale.)

But the stretching and morphing of the surface to house the variety of scales and types of space in the building means that the resulting surface is not everywhere funicular or catenary in shape and the shell will have out-of-surface forces under gravity load. This is accommodated by having a double surface. Each cement shell is everywhere equal in thickness but the spacing between the two surface layers varies to accommodate the varying deviation of out-of-plane forces. This is an example of ambitious free form surface definition and topological rubber sheet thinking.

To develop a digital model of the surface as one continuous entity, a smoothing subdivision algorithm, programmed by the Arup’s Advanced Geometry Group (AGU) using Rhino 3D™ as the visualisation interface is used. This was first developed by the AGU for the continuous roof and walls.

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of the Arnhem Central Station, designed by UN Studio. It interpolates between neighbouring vertices in the surface mesh to even out curvature in the surface. Those vertices coinciding with the exterior box remain on the box and are only allowed to perform a 2D smoothing of the curve of the opening on the facade. The wireframe of the edge curves of the doubly curved surface and the corner points is then sufficient for the structural software to generate a so called Coons patch which is translated into information that can be executed instantly in a Finite element package for structural analysis.\footnote{210 Meredith, M. (2008). ‘Taichung Metropolitan Opera House’. \textit{From Control to Design: Parametric/Algorithmic Architecture}. A. L. Michael Meredith, Mutsuro Sasaki, Actar: 54–59.}

\textbf{Arnhem Central}

Arnhem Central is a transport interchange planned for a total capacity of 108,800 journeys per day, journeys by bus, by train, by taxi, bicycle, by car and on foot. It is, by definition, and by diagram, a network of nodes and edges, in the tradition of Leonhard Euler‘ Seven Bridges of Königsberg – a problem of routing and connections. This idea of a continuous and coherent journey with any of many starting and end points is fundamental to moving people through such interchange.

In this project it is translated into another topological manifestation: the idea of a continuous non–orientable surface. Cecil Balmond wrote: ‘we drew a line that moved up from the foundations to loop and coil over space.’ To keep the curvature as a natural consequence of the concept, the roof walls and floors were merged into one network.\footnote{211 Balmond, C., 1943– , C. Brensing, and J. Smith. \textit{Informal}. Munich; New York: Prestel, c2002, 349.}

Thus the building is a knot and a surface that is governed not as a smooth differentiable manifold with a shape governed by measurable curvature, but a topological diagram made matter. There is a process by which this transformation is affected. However, the idea continues to inhabit the domain of connections and proximities rather than shape and measurement. The process acquires for the building a fixed shape, measurable curvature and surface thickness – in this case governed by repeated structural analysis and adjustment of an initial free form surface in–filled between the “strings” of the knot. Once the surface is ‘an optimum engineered surface, intact with the geometric definitions of its free edges,’ the strings can disappear. \footnote{212 Ibid., 363.}
The surface shapes clearly remain frozen flows, not surfaces derived from geometrical orderings or from material response to structural forces. These are contrived flows representing the design problem in a pure, idealised, geometrical space, free from gravity and divergent influences of the physical world. They are topological in their conception.

**Mobius bridge**

One of the design criteria for Hakes Associates design of their competition–winning footbridge across the Avon between the Norman ruins in Castle Park and a historic brewery catacomb on the other side was that the bridge be structurally independent of the riverbanks. It should be a freestanding figure, a structurally hermetic form, at least with regard to lateral forces, a bridge that would appear to float on the river between the banks. This is achieved with the ultimate hermetic geometrical figure, the simplest and best–known non-orientable surface: the Möebius band. This is the surface created in topology by gluing two opposite ends of a rectangular surface, switching their orientation to create a twist. In this way a continuous one–sided surface is generated.

This leads to the form of the bridge, which is a very elegant continuous gestural curve, a twisted asymmetrical figure of eight. The deck is braced by a minimal crossing point in the figure of eight loop where the curve that sweeps up from the pier to become the compression arch that supports the slim cables to the deck, meets the deck itself.
The bridge is passively pre-stressed by the arch on the Park side of the crossing being set higher than intended and allowed to drop under gravity. This increases tension, torsion and strength in the bridge structure.

This project is included for its pure representation of the foundational idea of the non-orientable surface rather than for its design modelling system. It was developed in early design through a series of paper models with small variations and studied largely as an object, albeit for its structural behavior and stiffness with a variety of proportions and crossing points.
2. Kinetic informationscapes (the topological tendency embodied)

The project examples in the topology section so far exemplify static architectural interpretation of important topological ideas, expressed largely through the organisation of surface. Kinetic informationscapes, by contrast, are not confined to the static but include kinetic transforming architecture that responds to input information streams. Kinetic informationscapes are not really different from topology in architecture but an extension of its expression. They are founded on the topology of their graphs or networks of relations. These are the relations between input data streams, or databases, interpreted for difference or variation between the data and translated to output characteristics or behaviour of the architecture. In their interactivity, they dare to investigate more closely the dynamism of what Di Cristina referred to as the space of existence.

Data (or datum, singular) has its etymological roots in things, which are given. The etymological root of information is knowledge communicated. While ‘scape is an abbreviation of landscape, a painterly word that came into English from Dutch at the end of the sixteenth century along with developing Dutch mastery in painting their own rapidly changing landscapes. Just as in the sixteenth and seventeenth centuries, techniques were being developed for rendering the visual, and with it other resonances of place, in luminous landscape paintings, designers of Kinetic informationscapes now investigate diverse means of rendering the given and the knowledge communicated through the variations within the given, within sensually enlivening and cogitatively accessible

scapes of their own. This can include representing what cannot be seen such as the visual representation of changing ambient conditions, atmospheric changes. What distinguishes these (media)scapes from the possible virtual worlds of digital gaming, for instance, is their immanent relationship to a referenced world outside. Generally there is a near real-time mapping between the represented/interpreted and the representation. Not all Kinetic informationscapes are virtual digital environments; many re-represent information from the physical world in the physical or sensory domain as the three following examples illustrate. Kinetic informationscapes are dynamic places in which spatial representation extends to more than three static Cartesian dimensions; where change and, at the very least, the fourth temporal dimension is engaged in a time-based spatial representation.

Digital computation has greatly extended the scope for the programmed linking of events to spatial configuration. It offers representation of an event-based understanding of time and space free from the constraints of the pre-choreographed linear sequence familiar from film. As Di Cristina inferred, movement and change in virtually represented and physical space can now be linked to unlimited other changing forces in the world. To represent a system using a database and programmed links between objects and phenomena within it as a possibility for designing multi-dimensional spaces is clear. Digital networking invites us to live in a space that is more closely aligned with the mathematical idea of multidimensional

space. Of multidimensional space, I have noted that Roger Penrose writes that a space that has five variable parameters is a five dimensional space and this is extensible to any number of dimensions.²¹⁴ But Lefebvre notes the rift that has developed between the precisely defined mathematical spaces including the possibility of spaces with an infinity of dimensions and space as social or physical reality.²¹⁵ This question of the segregation of the notion of space into mathematical space and space itself (or not) is one that I will return to in chapter 5.

I have written at the beginning of this chapter that I am treating architectural projects since the 1990s as the exemplars for the recent extension of the geometrical vocabulary of architectural modelling. This is a time period that corresponds to what in Chapter 2 I referred to as the shift ‘from giving form to the system to giving system to the form’. In other words this is a period in which there has been a marked interest in computation as a means for form finding and novel formal expression in architecture. Ironically the use of the terms form, formal and formalism in architecture are almost antonymic to their use in mathematics and its philosophy. Here I am using form in the architectural sense, concerned with shape and outward appearance. There is an important progenitor of the datascape from an earlier period. It is a pre choreographed performance and thus misses the essential nature of datascape that link incoming data and dynamic response in real time but nevertheless engages the senses and emotions in architecture brought to life through other informational channels.

It is the Philips Pavilion by Le Corbusier, Iannis Xenakis and Philips and the Poème électronique by Le Corbusier, Louis Varèse and Xenakis.

**Philips pavilion**

The pavilion was conceived as a 480 second performance to 500 standing audience immersed and effectively bombarded by imagery, light and sound reflected and distorted by the hyperbolic geometry of the interior in which they find themselves. It is remarkable for the coherence of the conception of an experience in which the architecture, film and sound are co–curated as elements of the composition rather than receptacle and performance. The architect’s commission spanned both architecture and the performance, including the design and content of the 8 minute film and sound track. It is surprising to discover, therefore that the film in its seven sections: Genesis, Matter and Spirit, From darkness to dawn, Manmade gods, How time moulds civilization, Harmony and To all mankind, was developed in relative isolation from the composition of the sound, coordinated solely through their precise 480 second total track. It corresponds to the surreal–influenced approach of the frames and their transitions as a series of “shocks”, syntax carrying much of the meaning, no avoidance of awkward juxtapositions. The film was composed from black and white stills, orchestrated by dramatic rhythm changes between cadences, down to one second sequences in the most rapid staccato sections such as Matter to Spirit. The final section becomes a promotional sequence for the architect (rather

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Figure 47 Images of the Philips Pavilion: a. The completed Pavilion, b. Early wire and gauze model, c. plywood model at 1:10, Delft 1957, d. plaster model 1:10 linked to strain gauges for structural measurement, e. levels for simulating lateral loads, f. pre stressing almost complete, g. Xenakis’s sound routes, h. Poème électronique: skeletal hand in part 3 From Darkness to Dawn. (all images from: Treib, M. Space Calculated in Seconds: the Philips Pavilion, Le Corbusier, Edgard Varese Princeton: Princeton University Press, 1996.)
than Philips) and his architectural and ideological proposals for a brighter future. While the film and imagery was black and white and Marc Treib speculates on the likely technical as well as consistency of content reasons for this in 1958, it was superposed with simultaneous projections called “Trictrous”, effectively blocks of colour and colour profiles through stencils. Treib, M. *Space Calculated in Seconds: the Philips Pavilion*, Le Corbusier, Edgard Varese Princeton: Princeton University Press, 1996, 139.

Philips prowess was displayed in the automation of the whole and coordination of different projection sources with the sound. The colour stencils must coincide with dark areas and moments in the image projection.

Philips were also intimately involved in the realisation of the innovative form and structure of the pavilion, as well as its acoustic performance. Le Corbusier was effectively the design architect while Philips, through the coordination of Louis Kalff, appointed the construction team and their consulting engineers. Moving from the French company Eiffel, who had proposed a steel framed building to the Belgian company Strabed, who believed that the pavilion could be realised as pre cast concrete shell panels reinforced by concrete ribs at much lower cost. The structural design, with a heavy impost of double wind loading imposed, could not be determined mathematically and proceeded through the time–honoured process of measurement of physical models and prototypes. The earliest were wire and gauze, then plaster, then plywood. Systems of strain gauges, horizontal level systems for simulating wind load, large numbers of bags of sand to simulated gravity loads (reminiscent of Gaudi’s hanging model

Finding the geometrical form of the pavilion envelope is also interesting, grounded no doubt in Le Corbusier’s long standing interest in geometry. Soon after gaining control of the commission in 1956, he wrote to the rectors of the university and technical institute in Zurich to request books illustrating three dimensional representations of mathematical functions. The young Greek architect, mathematician and musician Iannis Xenakis was appointed project architect and is attributed with the decision to employ ruled surfaces, specifically the hyperbolic paraboloid for the form of the pavilion. We can note that the design team were comparatively unconstrained geometrically or technically by their pre–digital design context. Physical analogue modelling provided the analysis tools and Philips, while intimidated by the prospect that the American pavilion would display the earliest colour television, an area in which they lagged, were forerunners in sound and acoustic correction in difficult performance spaces.

What the electronic prowess of 1956–58 did not yet entertain was direct interaction between audience or ambient environment and the performative environment. Yet, as Treib has pointed out, even sophisticated film audiences were yet to experience the jump cuts of Jean–Luc Godard, for example in *Breathless* (1959), so the experience of the filmic collage of the Poème électronique would likely have left its audiences feeling far from purely passive receivers.

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217 Ibid.
Marc Treib’s account of the design of the pavilion, retold largely from the letters of the protagonists, portrays a process at times as frenetic, rhythmic and arrhythmic as the Poème électronique itself, and it is an engaging read.

**Aegis Hyposurface**

The Aegis Hyposurface (dECOi 1998–) was conceived as a wall that would respond to physical presence of people and their activities in its vicinity in dramatic and ultimately unnervingly interactive ways. Commissioned as an installation for the long wall at the entrance to the Birmingham Hippodrome, the challenge was taken up to animate the wall itself. Drawing on ideas for reconfigurable moulds, it developed as a facetted aluminium skin articulated by rubber squids between its reflective facets but the original vision which won the competition was an animated video in which a smooth almost liquid skin was sent rippling by diverse stimuli such as the footsteps of a passerby registered as plop, plop effects on the vertical surface above their head. A stream of cyclists engendered a more complicated effect, and applause from an audience registered as a series of mad fibrillations surging and ebbing exactly in sync with the sound volume from within the auditorium.

Mathematicians, consulted by dECOi architects, proffered algorithms that would emulate the mouse running under the rug, or the circular ripples spreading out from a stone dropped into the water. Sprites might dart across leaving a wake behind them, or a point disturbance might spiral away from the centre. The design task became one of choreography combining and animating the effects offered by the mathematicians and programmer within a narrative overlay likened by Mark Goulthorpe to contemporary ballet.

By the time the wall reached the 2001 Cebit show at Hannover the 10 metre by 3 metre unit had absorbed the attention of a team dispersed around the world including ballistics experts,
hydraulics experts a material and structural team at Arups and mechatronics experts who designed and built the control unit at Deakin University in Australia.218 In addition to its pre-choreographed high art responses it could enact video, moving text and shudder to the beat of those dancing around it.

The animation continued as a tool during design development – attempting a simulation of the real time behaviour of the wall. The spatial challenges included understanding the potential clashes of the corners of the triangular facets as they moved in and out on pistons whose neighbouring actions might combine in any of a spectrum of combined actions. The signal time for the activation of each piston in the array was crucial as was the actual piston action in time and space. The tuning continued through a series of increasingly ‘lifelike’ prototypes in trying to return the mechanical realisation to the ephemeral fluidity of the original conception.

The journey from the animation of rubber sheet geometrical surface to the discrete actions of the parts of the mechanical assembly, massaged back to fluidity in the ultimate realisation was achieved through tuning the actuation. It was also achieved through the theatre of the effects themselves – the complexity of the possible effects whose momentum and meaning transcended the medium. To animate, from animus, is to bring to life. The test of success is the goose bumps that the live facsimile engenders when it is the child of an array of 1000 hydraulic actuators or pistons under the command of a large digital control unit capable of interpreting diverse real time inputs – movement, sound, video, as well as pre-choreographed sequences. It is genuinely interactive and animate in a less than predictable way.

**Digital Water pavilion**

The Digital Water pavilion (Carlorattiaassociati and MIT Medialab, opened 2008) has an even more impressive array of mechanical and in this case hydraulic components in order to produce an equally surprising effect by taking the medium of water – familiar in sheets and sprays and chaotic flows in waterfalls and their pools and etching and sculpting it into geometric and ordered patterns as the walls of the pavilion. Once again the temporal dimension is harnessed in the performance, in this case by subverting the process of printing. While the printing press automated the process of recording and making knowledge static and permanent on linen or paper as the receiving space and the ink carrying the information to its surface, in the pavilion walls the water fulfils the role of paper, ink and information. The inscription can be changing and transient or apparently static like a still presented on an endlessly scanning cathode ray monitor. Setting aside the technical achievement of successfully bringing together the informatics and mechatronics at this exquisite level of resolution, the conceptual combination of water droplet pixels using gravity as their change agent to run an endless picture show through a high precision fountain is a remarkable

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218 The Aegis Hyposurface team is listed here: [http://www.sial.rmit.edu.au/Projects/Aegis_Hyposurface.php](http://www.sial.rmit.edu.au/Projects/Aegis_Hyposurface.php)

innovation in combining technological models. Like Aegis, the choreography becomes the architecture and it displays virtuosity in a range of effects both pre-choreographed and interactively responsive to people in it.

**Spoorg**
Servo’s Spoorg (or Semi–Porous Operable ORGanism), is a system of decorative plastic cells with photo sensors, speakers, wireless radio microcontrollers all embedded in hollow regions of the cells. The green cells respond to interaction with both their neighbouring cells and changes in ambient conditions to alter their transparency, contribute to a changing ambient soundscape. They are static but subtly changing in a manner appropriate to the plant world on which they draw for inspiration. NOX architect’s Freshwater Pavilion also uses changing sound and images to
Figure 50 Servo’s Spoorg installation.
animate the watery interior while the static architecture uses devices like changing sloping floors and never horizontal or vertical reference planes to further destabilize the interactive environment through the movement of the human actors. Thus not all these Kinetic informationscapes involve kinetic architectural structure, however all engage some form of dynamic response and feedback loop to link the architecture to its changing environment, the behaviour of its users, or other external drivers. All represent a graph of relations between inputs (with ever-changing values) and outputs also represented as change whether as form-change, colour, sound, patterns mapped in water, or surface depth.

**Topology examples discussion**

Topology is the foundation of all computational system modelling in architecture. The projects in the first part of this section are selected as expressions of a topological ideal or icon – the knot, the compact, non-orientable surface that continues forever. They express the primacy of the idea of connectedness over composition. In practice, they are constrained to metrical space, like all built architectural systems. The implied variability and independence of metrics inherent in the mathematical idea must be eschewed early in the execution of the precise building description for construction. Nevertheless, the underlying ideas, which underpin their design process, emphasize the mutability of the model, not conceptually predicated on or constrained by precise shape or measurement. Examples of this are the blurring of vertical and horizontal structure in Arnhem, the endless circulation in the Klein house, which nevertheless achieves the closeness and intimacy of a holiday house within a geometrical paradigm that has no scale or metrics. The more fundamental impact of topology and topological thinking in architectural modelling is that described by Di Cristina as the topological tendency. This is the capacity within the model system for linking diverse parametric drivers to diverse parameters and relations in the graph of dependencies to create a dynamic world, which is not apprehended as form or object but rather as behaviour and cause and effect. This aspect is more manifest in the realisation of the second set of examples of design projects, which I have called kinetic informationscapes.

Mark Goulthorpe et al have written of the transition from autoplastic (determinate) space to alloplastic (interactive, indeterminate) space, a new species of reciprocal architecture. There is a long history of animating architectural space, principally through the creative use of lighting and sound, in some cases through mechanical interaction. Here, I have focussed on the overall system of drivers and responses rather than the animate nature of the result per se. Replicating the animate characteristics of the organic in physical constructed artefact is still a significant material and mechanical challenge. There are many systems that could have been taken to exemplify the phenomenon of kinetic informationscape, not all kinetic in the sense of changing architectural form. The adoption of an intrinsically fluid medium of interaction (water) for its principal kinetic component by the carlo ratti associati / MIT Digital Water Pavilion represents considerable inspiration in circumnavigating the ongoing aesthetic shortcomings of mechanical systems for embodying animate responses in responsive architecture.
Recapitulation: Chapter 3

This chapter has reviewed, through example, the contemporary architectural interest in mathematical ideas and systems, engendered in part through computation and, in particular, through the graphic representational capacity of computers in recent decades. In particular it illustrates the way in which the interest in systems manifests in many cases in a highly operational way in the models. This provides architectural context for considering the changes in geometrical space in architectural modelling brought about through engaging systems.

The singular selection of examples has been chosen to develop a loose taxonomy of five mathematically inspired themes or clusters within which these projects seem naturally to group.

The chapter is a survey of projects within these five thematic groups using dynamic system modelling approaches for formal and affective architectural ends. Many of the examples are informed or inspired specifically by ideas appropriated from geometrical discovery and invention, many use it for problem solving, and in the best tradition of design synthesis it serves both to inspire and to solve in numerous cases. The dynamic space of the model is a model of the design parameters and constraints within which the design is refined and its nature iteratively better understood. Its dynamism accommodates the push and pull of the search within a design space for different alternatives and for better fit.

What all the thematic subsets of projects have in common, from a modelling point of view, is the strong underlying ideas about a dynamic system driving the design, whether of surface, chaos, tiling, optimization or topological description. This distinguishes them from the general use of geometry in models simply to solve local problems or define objects through static shape characteristics.

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219 In the limited and prototypical case of responsive kinetic informationscapes this extends in a literal way to the architecture itself.
CHAPTER 4

INSIDE THE MODEL SPACE: BIFURCATIONS AND HOLES
Introduction

This chapter examines some examples from architectural system models that I have constructed and the geometrical spatial questions that these presented. I will also draw on examples from classes I have taught. In this way I aim to expose my motivation for seeking to contextualise the complex spaces created through computational geometrical modelling processes. These spaces challenge the architect’s control and mastery of their geometrical construction of the design proposition in ways that the drawing or object model do not. They expose philosophical questions about the relationship between geometry and space and place of the subject in relation to them that lead to the philosophy of mathematics as a domain in which to look for answers.

Many contemporary architectural models replace explicit single instance geometry by geometry defined by explicit variables. They move architectural conception from Platonic idealism to morphogeneticism and, in doing so, promise smooth transformative spatial qualities. But they are frequently insufficiently homogeneous in their geometrical representation to fulfil this apparent smoothness of space. The singularities and discontinuities that have enlivened the spatial writing of Bernard Cache\textsuperscript{220} and Gilles Deleuze\textsuperscript{221} are commonplace in the complex geometrical spaces created through mapping design intentions as a graph of geometrical relations. These spaces are understood in different ways – through the medium of logic expressed in natural language, pseudocode, scripting or code, through diagrammatic topological mappings of relationships and dependencies but they also exist as synthetic, geometrical constructs and impinge on the most intuitive understanding of space and matter. They are always phenomenal, as in cognizable by the senses, rather than archetypal, as in conceptual, ideal or the form, the original model, from which a particular type of thing or shape of thing is derived. They do not refer to idealised form because of their immanent propensity for change. This formal change is encoded logically with the possibility of both continuous and discrete variation.

In each of the examples chosen, a system is modelled rather than an object or a single, possible design solution. While this represents expanded opportunities for the use of models or custom design tools for ‘economical’ design exploration within a domain, it also leads to working in geometrically complex and potentially unpredictable space. Significantly it involves working with models that are not visible or necessarily, readily visualisable as models in their entirety.

4.1 ‘Models for...’ and ‘models of...’

In architecture models are simultaneously a mode of engaged thought and a means to communicate intent. This has elsewhere been expressed as ‘models for’ and ‘models of’


respectively. However there is a distinction between a static geometric model that represents a single iteration, a frozen or even ‘final’ moment in the design process and a responsive model that can be changed formally or qualitatively in answer to new input information, changed or refined intent, streaming data, or simply the adjustment of the relative influence of each of many design drivers.

**Digital–analogue**

Computers are based on digital processes. Digital means literally ‘of or pertaining to a digit or finger.’ In computers, data takes the form of a finite sequence of bits that can be coded as a natural number. Digital systems are systems of representation that use discrete (discontinuous) values that are usually (but not always) expressed numerically. Based on the elemental logic of true or false, open or closed, this allows a highly generic description of anything as a simple series of binary values for numbers and basic operations.

Thanks firstly to Leibniz’s early observation and use of the idea that mathematical problems could be broken down to require only the operation of addition to automate computation; to Alan Turing, Kurt Gödel and Alonzo Church who worked in the 1930s on the question posed by David Hilbert: what does it mean to be computable; this digital system of representation has led to the ultimate generic machine that can be turned to any use; locating stars, and facilitating design.

The tools and tool design that designers engage in using ‘digital tools’ are at a level that is distinctly analogical rather than discrete as ‘digital’ implies. Toolmakers designing tools to design with are generally operating at a specialised level compared to Turing’s original conception of a generic coding and decoding machine. A number represented in the

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223 The term ‘bit’ refers to the smallest basic unit of information storage and communication, a binary digit, taking the value of one or zero, in digital information theory. It is first used by Claude E. Shannon in his paper in 1948 *A Mathematical theory of Communication* but Vannevar Bush also referred to the “bits of information” that could be stored on the punch cards for his mechanical computation devices in 1936.


225 Leibniz’s mechanical calculating machine, introduced in 1673 demonstrated the principle that multiplication and division can be achieved through addition and subtraction respectively – represented through parallel rotating rollers that added the partial products – ultimately addition alone is needed if the rollers always revolve in one direction only.

226 Alan Turing (1912-1954), English mathematician, logician, cryptanalyst and computer scientist; Kurt Gödel (1906-1978) Austrian logician, mathematician and philosopher; and Alonzo Church (1903-1995), American mathematician and logician; David Hilbert (1862-1943) German mathematician.

computer is a discrete entity, no matter how long the binary sequence that defines it. Similarly, as Benoit Mandelbrot has pointed out, air is a discrete system at molecular scale.\textsuperscript{228} However, at the level at which we interact with the computer, our perception of the continuity of the systems we build can be indistinguishable from our analogue perception of the air we breathe as a continuous medium. The representation of our intentions as a few simple operations on numbers is as well concealed as the billions of molecules that dance in intermittent contact with us in the air of our immediate surroundings.

So, in consideration of terms such as ‘digital architecture’ or ‘digital design’, the conceptual relevance to architectural design of the ‘digital’ or discrete nature of the operation of the computer as a machine (processor/hardware)\textsuperscript{229} that manipulates data according to set of instructions (algorithms/software) appears questionable except insofar as the computation of unambiguous geometrical boundaries is critical to functionality. At the same time it is difficult to overstare the influence of computation as a process on both design process and architecture as cultural expression.

**Model making tools**

Models when they are constructed using geometrical and logical relations as their medium, that is without the scalar properties of physical materials, or constrained to represent a single instance of an idea, can be nested structures. The principle model making tool is another model. The static geometrical instance of shape and form is one output of a model of geometrical relations between objects. The relations represent design intent and constraints defined in the design context. I choose to call this a relational model.\textsuperscript{230} These relations themselves may be built up out of geometrical objects and methods defined and given symbolic and iconic representation through programming in software. I call this the software model. Each proprietary and open source software has a specific ontology, the result of a series of decisions and negotiations by software engineers. There is nothing intrinsic about geometrical operations in particular software. The manipulation of these objects and methods may be governed by parameters and relations defined by functions constructed by the user and drawing on analytical geometrical history and convention. I call these the mathematical models. These may represent many small sub–models within the relational model, each potentially

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\textsuperscript{229} The Turing machine was not hardware as we understand it but a theoretical idea about a processing schema.

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\textsuperscript{230} The term “relational model” commonly refers specifically to Codd’s model for database management (Codd, E.F., ‘A relational model of Data for Large Shared Data Banks’, in *Communications of the ACM* (1970).) This is a logical model in the terms of mathematical model theory. While the general relevance of model theory to consideration of architectural modelling is not outside the scope of this thesis and while Codd’s model and its variants may not be completely irrelevant to the specific design of software which supports architectural modelling, the term relational model within this thesis, unless otherwise specified, refers to high level geometrical and attribute relations between objects representing physical or virtual architecture within a spatial model, and not to Codd’s model.
several layers deep. Each of these models that I have typed is a tool for constructing one or more of the others. The model making tools in computationally–supported geometric modelling for design are the meta–models and sub–models (or systems). Geometry is the medium and digital computation provides a system within which to deploy these tools. Picking up Seymour Papert’s cry, that digital computation is “to enhance thinking”; I adopt the position that geometry is a form of thought.

4.2 First example: Simple Algebraic Engagement

Simple algebra is a useful adjunct in design modelling. In this example it is used to define a sequence of relationships concisely. The example is the growth algorithm for the steps in the model of the crowning element of the portal to the west transept of the Sagrada Família church in Barcelona. By comparison with most of the surface geometry in the church, this element appears geometrically simple; it is a rising multi–tier staircase of rectilinear blocks sitting like a pediment above a colonnade of much more organic appearance and a frieze of cupped hexagonal prisms.


Figure 52a Plaster model of the colonnade built and revised many times during the 1980s by a team led by Cardonner. Examples of Cardonner’s sketches. An overlay of a digital version of Cardonner’s model on the original photograph illustrating the changes (continued over page).
Figure 52b
Chapter 4 | Inside the model space: bifurcations and holes

Figure 53 Diagram overlay on the photograph indicating the relationships between the parts of the Colonnade, frieze and cornice.

Frieze of hexagonal prisms above the columns, inclined to the front (picture) plane and vertically. In elevation arrayed between straight pitch lines, hexagons ‘kissing’ and each second hexagonal lower apex apparently coinciding with the axis of a column.

Nine columns on each side of the colonnade, highly sinuous and bone-like, variable inclination in the front plane, shadows and intercolumnation.

Basalt cornice finished at the edge with ruddle surface as also seen in the porch of the Colonia Guell crypt.

‘Crestaria’: stepping pediment with hyperbolic finials on relatively rectilinear steps. Each layer of steps fits between straight pitch lines in the front plane but the step length changes, diminishing from the centre to the edge. The step intervals appear to have no relationship to the hexagonal frieze or the columns below them.

Pre-existing columns construction completed up to basalt cornice in the 1970s.
after Gaudí’s death. Here the prisms of the frieze appear independent in their spacing from the column spacing below and the stepping pediment has become much more elaborate and layered, steeper and higher. The organic columns of the Gaudi drawing have also become highly articulated and baroque. The contemporary design process and the digital system modelling to support it must mediate these sources aiming to return the design to a closer facsimile or interpretation of the 1917 photograph of the drawing as the primary evidence. This information is given to set the scene for the type of flexibility, variable iterations and support for deliberations that might be expected of a contemporary digital design model for this proposal.

The stepping cornice can be seen to be composed of repeating units of three to five steps laterally across the composition from front to back, or roughly normal to the front plane. Each of these successive units is deeper and higher in roughly equal proportion to the one that preceded it as you move up the sloping cornice from the outside to the centre and within the unit the steps ascend from the front to the centre and descend again towards the back. The exact alignment of the steps from one unit to the next varies as you ascend.

By making reference to the only solid piece of historical evidence for the geometry, the surviving photograph of the drawing of the elevation of the façade made at the time Gaudí completed the last proposal for this elevation, it appeared that the position and distribution of the steps in this ascending stepping giant’s causeway was not closely related to the changing intercolumniation below nor to the distribution of the hexagonal figures in the frieze. This greatly simplified the interface between this element and the rest of the assembly in the relational model. A simple constraint system could be set up to fit the element to the lower and upper limits of its ‘site’ in the model, maintain the linear pitch lines through the staircase, ensure the vertical coincidence in the height of certain repeating patterns of steps through the assembly and marry this with the curved profiles of the steps in plan.

Geometric Schema
There were many ways to structure this but the decision was taken early to provide a framework of three dimensional curves based initially on parabolas (quadratics) in plan and straight lines in elevation / vertical section. This is consistent with and based on an analysis of the plan of the lower part of the façade, already built and with the use of second order surfaces and conic sections throughout the building.

Onto this framework a system of variable (parametric) growth would be introduced (points of intersection on the curves) that could vary in type and rate of change, in other words, it could conform to various mathematical growth functions as well as being parametrically variable for each. This is a simplified diagram showing one of the 3D curves and its intersection with parallel planes at variable intervals.

233 While it was my responsibility to construct the parametric model of the assembly, the architectural schema (as opposed to its detailed geometrical interpretation within the model) was the product of the input the design team, notably from the Director of the technical office at the Sagrada Familia church. I was initially responsible for detailed geometrical analysis of the photograph of Gaudi’s drawing to inform the relationships between the component elements of the assembly.
In this process a parameter would also be introduced for varying the number of instances, or number of stepping units, within a given overall length (this length also parametrically variable) for the whole assembly.

This would maintain the maximum flexibility for fine-tuning the best fit for the stepping assembly to the graphical primary evidence in a number of different ways once the model was completed.

Here, geometrical construction is employed to set up the primary relationships, or what has been named in the chapter introduction, ‘the relational model’. This particular model is a relational sub–model of a larger relational assembly. In other words, every part of the model is linked to every other through a hierarchical graph of relations but the stepping pediment is linked to the rest of the model through a relatively simple relational interface.
The position of the front of each step was found to be statistically a quadratic function of the step number. Form:

\[ a + b \cdot N + c \cdot N^2 \]

- \( a \) is the start point relative to other geometry, turned out to be 0.1,
- \( b \) and \( N \) are variables that can have their values changed in a spreadsheet,
- \( c \) is calculated automatically: \( c = ((L-a)/N-b)N \)

This seemed in accord with the parabolic plan.

The Parameters of the instance illustrated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>Angle relation to hexagons (deg)</td>
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</tr>
<tr>
<td>Lowest plane offset (mm)</td>
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<td>Highest plane offset (mm)</td>
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</tr>
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<td>( N )</td>
<td>20</td>
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<tr>
<td>( b ) (mm)</td>
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<td>Height of top points (mm)</td>
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<tr>
<td>Height third points (mm)</td>
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<tr>
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<tr>
<td>Inclination 4th points (deg)</td>
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<tr>
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<td>Finpype2_03 upstand ratio</td>
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<tr>
<td>Inclination of 2nd points (deg)</td>
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</tr>
</tbody>
</table>
Software model - collection of objects, methods etc.

Figure 56: Small sample part of the graph to illustrate the dependencies described.
It is built onto a plane, whose parameters are controlled in common with the hexagonal prism frieze below and a start point and end point similarly linked to the geometry of the frieze. Its plan shape is also controlled by a series of parabolas whose parameters are linked to the parameters of the equivalent plan geometry for the frieze.

The curves that provide the geometrical foundation for the assembly can move anywhere in space, change their overall length through a single linked parameter. This accommodates possible changes to the site survey information or decisions about the relationship to the whole façade including the built part below. There are angle parameters for each of the lines governing the pitch of any particular ascent, and the plan shape of the curves can change through the tangent angles and shape parameter of the parabolas. The stepping units are understood to be parallel to one another but the angle of the plane governing their collective interfaces to the ordinal planes can vary parametrically. The base of the whole ‘crestaria’ assembly conforms to a plane parallel to the top of the tilted frieze of hexagonal prisms.

This is the basic schema of the relational model. There is another level of modelling which will now be described in more detail that defines the variable growth of the stepping units. This is what is referred to in the chapter introduction as a ‘mathematical model’.

**Plug in growth algorithm**

In analysing the intended relationships represented in the photograph of the Gaudí drawing, two big questions remained open. From the photograph it was difficult to count definitively how many of the repeating stepping units occurred from base to top, and, closely related to this question, the dimensions of the steps in width, depth and height increased from the base to the top in a way that was clearly not linear. Measurements from a high resolution scan of the photograph of the step heights and depths were each plotted against a step number \( n \) from 1 to \( N \). (The \( x \) and \( y \) coordinate values for the edges of each step in the photograph, by comparison, would give the linear result of the pitch line.) Curve fitting software gave a good fit for a quadratic equation. The principle aim in the context of the project was to arrive at a three dimensional form that fitted well the form shown in the photograph of the Gaudí drawing. The short term tactic in the design process was to slot a working growth algorithm into the digital model that satisfied the overall constraint criteria, could be tweaked to adjust the distribution of the steps for a better fit and importantly could be replaced, like a modular component, if new evidence pointed to a different function type or a less geometrically regular pattern. The result was a very flexible configuration that met all the external constraints but could change its rate of growth or the overall number of steps within viable geometric limits when combined with the values of the other parameters defined in the basic geometric schema.
cardoner plaster model measurements

lengths of the crestaria steps plotted against the number of the step from the centre

1. the step lengths in the plaster model built by cardoner's team during the 1980s were measured from its digital facsimile and plotted against the number of the step as counted from the centre. (upper blue curve).

2. the difference between the length of the adjacent steps was also plotted and the difference between the differences also against the step number. (pink and yellow curves). (the first point can be disregarded in every case - it is merely the reference.)

3. the lower graph shows the figures for the step lengths taken from the notes for the same model - similar to the blue curve above but smoother than that for the empirical measurements from the plaster model as executed.

Figure 57a: Graphs of the growth in the step sizes in the 'crestaria' (continued following page).

<table>
<thead>
<tr>
<th>Lengths (plaster-digital)</th>
<th>Differences</th>
<th>Difference change</th>
<th>Cardoner notes</th>
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<tr>
<td>2.938</td>
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<td>3</td>
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</table>

Figure 57a: Graphs of the growth in the step sizes in the 'crestaria' (continued following page).
Chapter 4 | Inside the model space: bifurcations and holes

**Figure 57b**

1917 PHOTO OF GAUDÍ DRAWING MEASUREMENTS

Lengths of the Crestaria steps plotted against the number of the step from the centre:

1. The step lengths in the 1917 photograph were measured and plotted against the number of the step as counted from the centre. (upper blue curve).

2. The difference between the length of the adjacent steps was also plotted and the difference between the differences also against the step number. (pink and yellow curves).

(The first point can be disregarded in every case - it is merely the reference.)

This reinforced the idea that the growth of the steps in the Crestaria as drawn conformed loosely to a second order parabolic function.

<table>
<thead>
<tr>
<th>Lengths of steps(y)</th>
<th>Differences</th>
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<td>0.57</td>
<td>0.065</td>
</tr>
<tr>
<td>0.505</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4 | Inside the model space: bifurcations and holes

Figure 58 diagram illustrating step numbering and the meaning of the quadratic relationship.
The algebra

A simple quadratic equation is used.

\[ \ln = a + bn + cn^2 \]

where:

- \( \ln \) is the distance of the front of each step in the direction of their parallel risers from a starting point,
- \( n \) is an integer from 1 to \( N \) assigned to each consecutive step,
- \( LN = a + bN + cN^2 = \) the total length of all the steps combined in the same direction = external parameter
- \( a = 0.1 \) (through empirical trial – it controls the starting point),
- \( c = ((LN-a)/N-b)/N \)
- \( N \) is the total number of steps and
- \( b \) is the “growth hormone”

both the last two variables were controlled in a spreadsheet.

This is a mathematical series in which the position of each successive riser is \( \sum_n \ln \)

How the variation manifest – what you see

Other functions were also trialled by way of comparison with the quadratic, for instance sine and cosine functions to see the effects. In the case of a sine function, rather than growing progressively from the base towards the centre of the colonnade, the steps could increase in height and width and then reduce and then increase again, depending on the wave length compared to the overall “crestaria” length. Despite these changes the edges of the steps still conform to the pitch line in elevation and parabola in plan, rather like the effect of pulling curtains along a rail, whereby areas may become more or less densely pleated without leaving the rail.

The purpose of this little mathematical sub-model

What was the purpose of this particular variability? In this case it was to be able to “fit” the steps of the “crestaria” as well as possible to the 1917 photograph of the Gaudí model. While it was simple to measure pitch lines in elevation and possible to infer the plan shape from the built porch and towers above which and in front of which the colonnade is to be built along with the shadows cast in the drawing, it was very difficult to intuit the pattern of growth in the steps. Through measurement, one function type was proposed, but this seemed sufficiently questionable, given the quality of the photographic evidence to warrant a very open approach to both the “form” and parameters for this part of the model.

Viability – why it only varied smoothly locally

The form of the steps could vary smoothly within a fairly limited range of the variable ‘\( b \)’ (the variable I have called the growth hormone) for any given value of \( N \) to yield geometries that were viable within all the other constraints of the system.

---

234 Crestaria is the Catalan name given to the stepping pediment.

235 In this case the term ‘form’ is used according to a second definition, that is, in opposition to content as in ‘form and content’ or to matter in ‘form and matter’. It refers to the structure or framework of the model within which specific geometries, relations or algorithms are enacted.
Theoretical stepping profiles for the Crestaria from a 2D parametric profile model.

The three-dimensional model cannot accommodate this range of values or functions and maintain its integrity.

Figure 59: Two dimensional diagrams of different functions controlling the growth in the steps.

As the differential of the sine function, unlike the quadratic, is not linear, nor is the pitch line in this version of the steps profile.
(There are also "loops" in the example illustrated, where the progressive values actually diminish before growing again - these can be eliminating by selecting the right constant at\(a\sin(n)\).)
The arrays of steps in the 'Crestaria' follow a series of curves which are parabolic in plan.

The curve at the back has the least curvature and results in small slippages between adjacent step edges. As this curve is varied parametrically the point at which these slippages alter from stepping out (top ellipse) to stepping in (lower ellipse) also alters.

This can cause the geometrical shape description to fail.

Figure 60: Image showing the stepping back of the model changing with the parameter controlling the curvature in plan. Perspective view (above) and parabolas overlaid in plan (below).
The algebraic expression indicates only the distribution of the parallel planes between two externally defined limits. It gives no indication of the wider implications of altering this distribution for other aspects of the schema and the resultant three-dimensional geometry. Redistributing the planes, redistributes the front edges of stepping units along a curve. On the slow concave curve of the back of the assembly, this may change whether each unit steps in or out from the one below in plan. This has the potential to cause the surfaces of the steps to self intersect — a non-viable result. This is what is meant by “viable within all the other constraints of the system.” This is one particular type of “model failure” or discontinuity; perhaps the most basic that is discussed in the conclusions to this chapter.

Summary — first example — simple algebra
This sub model is very simple mathematically and very flexible internally in two dimensions, that is with respect to the variable depth of the steps and the variable growth pattern in the depth of the steps. However, when combined with all the other constraints in the 3-Dimensional model it becomes very brittle. What do the viable solutions look like as a space of possibilities? This is very difficult to know because it is so sensitive to all possible changes all over the parent model of the Crestaria and even changes in the values of parameters external to the Crestaria, for instance changing the inclination of the plane interface with the hexagonal prisms or the start and end points of the Crestaria. A space has been created — the space of concretely possible (viable) solutions for the crestaria shape. This space itself has a shape but it is not 3-dimensional and it is very difficult, if not impossible to know what it is. We can find even more constrained solutions within this space, for instance, using optimisation to find the solution with the smallest difference between the depth of the lowest step and the depth of the highest step. But the computer does this not by surveying the whole space of viable solutions but by repeatedly sampling it and varying the inputs offered to it, then re-sampling and following pathways that show an improvement in output values according the optimisation criteria.

4.3 Second example: Curious Bifurcation
This second example is also taken from my experience of constructing the same extensive relational model for use to reverse engineer Gaudi’s as yet unbuilt design proposal for the upper part of the of the Passion façade to the western transept of the Sagrada Familia church.

A major element of this assembly is a colonnade of slightly gaunt bone–like columns. The geometric schema for these intertwined columns was a combination of an elliptic hyperboloid of one sheet for the central trunk of the column combined with eight paraboloids — four paraboloid branches at the top and four interleafed paraboloid ‘roots’ at the base — married to it along the straight lines common to these two different types of ruled surface. Similarly each column bonds with its immediate neighbouring columns through shared ruling lines on the overlapping hyperbolic paraboloid branches. There are precedents for this lapping branch

236 Column geometrical schema designed and defined by Mark Burry.
relationship at the Sagrada Familia and in the porch of the crypt of the Colonia Güell chapel.

This schema was developed in response to a detailed understanding of the geometric codex that Gaudi developed for the design for the sculptural surfaces of the church as well as direct reference to the photograph. In particular, the drawn columns in the photograph, which while portrayed with a powerfully emotive quality of line and shadow, appear much simpler, in their form and geometry, than those later developed in the plaster model by Cardonner and collaborators in the 1980s. Returning to this simplicity motivates the contemporary schema.

**Finding the lines of surface intersection – (a mathematical model)**

The difficult part of assembling the column form: locating the straight lines on these surfaces and their key intersections had already been solved and coded in a function through Mark Burry’s collaboration with Peter Wood, Wellington based engineer and programmer. This function could be called within the parametric (relational) model that I was engaged in constructing of the whole colonnade assembly. The function, developed by Peter Wood, defined algebraically an area of an infinite hyperbolic surface (the high curvature area near the plane of the extrema of the revolved hyperbola). The parametric inputs were the major and minor axis lengths of the throat ellipse and of an ellipse on a plane parallel to the throat plane and offset by a given length. This elliptical hyperboloid of revolution of one sheet was cut by any plane (not necessarily parallel to the throat plane) and from two points on the intersection curve of the surface and plane, a third point on the surface could be found, such that the line from each of the two given points to this third, discovered, point lay in the surface. The function identified the third point on the surface such that the lines constructed to this from the two given points lie in the surface. It was originally called from the Rhino 3D modelling software and constructed the lines there. Within the larger parametric model in which
the individual columns would have to conform parametrically to their individual geometric contexts, it could be called with in CATIA software and construct the line there.

**Interfacing with the relational model**

Within the larger relational model of the whole assembly, I constructed another schema (the next Russian doll outside the given internal column schema) to determine the specific inputs from the context model that determined the unique position, orientation and inclinations of each individual column in context. Each column when located in the colonnade in the larger relational model must relate to the hexagonal prism frieze above and the sloping basalt cornice below.

The centre point “P” of the “throat ellipse” of the column’s hyperboloid was a point in space in the larger model. This
point was defined as lying on a three dimensional curve, “C”, similar to those described in the crestraria schema, a straight line of variable pitch in elevation combined with a parametrically variable shaped quadratic curve in plan. These column centre points could slide up and down this three dimensional curve C in relation to individual neutral reference points found by intersecting the vertical central planes of the hexagonal prism above each column in the frieze with the curve C (Figure 65).

From this, the central axis of each column was defined as the line through the column centre point P, already described, and through a second “sliding” point H located on the lower edge of the corresponding hexagonal prism in the frieze (Figure 65).

Finally, the direction of the semi major axis of the ellipse at the centre of the hyperboloid was defined as normal to the column axis itself and normal to the tangent to the curve for the centre points projected on the plane through the centre point and normal to the column axis (Figure 64). Thus each column in the array varied in ‘plan’ orientation in response to the tangent to the curve. The inclination of the column axis in the (global) front plane was controlled by both the centre point position (point P Figure 66), in turn controlled by the pitch of the curve, and according to the horizontal angle of rotation of the hexagonal prisms in plan view.

237 “plan” in relation to the individual columns internal coordinate system, not the global coordinate system of the model.
Figure 64 Integrating the column hyperboloids in the larger model: orientation and aspect ratio

1. Inclined planar parabolic curve, along which columns are arrayed (red).
2. Curve normal to inclined planar parabolic curve in its plane (red).

Semi-major axis of the elliptical hyperboloid
Definition:
1. Normal to the inclined parabola on which all the column centres points lie (field geometry)
2. Normal to the central axis of the column hyperboloid (cell geometry)

3. Normal curve projected onto the plane normal to the column axis (depicted by grey circle).

4. Aspect ratio and radii of elliptical hyperboloid controlled by parametric affine transform along the direction of the projected line and its normal in the central plane (semi minor axis).
1. The width of the nth hexagonal prisms ($W_n$) instantiated onto its top front point (cyan) and a direction of extrusion is determined as follows:

$$W_1 = \frac{LN}{2N^+ (N \times \text{growth})}$$

$$W_n = W_1 \times \frac{(N + (n \times \text{growth})/N)}{N}$$

2. The position and direction of the column central axis is determined as a line between the column centre point (a sliding point on a 3D projected conic (magenta) and the axis top point (a sliding point on the lowest edge of the hexagonal prism (brown)).

Figure 65: diagram of locating a second point on the inclined column axis (axis of revolution for the parent circular hyperboloid)
Setting Up the 3D curve and variable intervals for the hexagonal prisms and columns

The variable intervals between the (green) planes are set up according to the following formula in which the distance of each normal plane through a point along a line from a startpoint P1 is defined by ‘ln’.

\[ ln = \frac{LN}{(2^N + N^{growth})} \times (2n + (n^2 \times growth/N)) \]

- \( N \) = total number of intervals between planes
- \( n \) = consecutive integers assigned to each plane from 0 to 16
- \( LN \) = total length of the line L
- \( ln \) = the distance of the point + plane from P1 on line L
- ‘growth’ = factor by which interval increases from lowest to highest

The green planes are intersected with a curve (magenta) that is parabolic in the XY planar view and a straight line in the YZ planar view. The resulting points of intersection define the top front points of the hexagonal prisms in the frieze.

The actual rate of growth of the hexagons for a fixed number of hexagons within a fixed interval is governed by the variable ‘growth’.

The same set of planes is intersected with another inclined conic curve to define reference points for the ‘centre points’ on the column axes. The actual column centre points can slide relative to these reference points along the conic to vary the inclination of the columns as viewed in the YZ front and XZ side planes.

Figure 66: diagram to show the definition of the column centre points.
Figure 67 The colonnade in plan and elevation showing how the semi major axis of the elliptical section of each column is oriented normal to the parabola

Plan and front elevation showing:

1. The column ellipses arrayed on the conic (crimson curve)
2. The parallel hexagonal prisms rotated 3 degrees (1 shaded grey)
3. the varying inclination of the column axes linked to both (short red lines)
4. top and base hyperbolic paraboloids in plan (blue)
Figure 68: Images of the model in the front plane and side plane with the column axes dotted in.
Figure 69: The sequence of column dependencies:

1. The central axes between points on the central curve and the under edge of the hexagonal prisms,
2. Elliptical hyperboloids of revolution of one sheet instantiated on the axes and with their major axes normal to the tangent point on the curve,
3. Hyperboloids trimmed by the hexagonal prisms and the irregular basalt surface at their base,
4. Trimmed by opposing generatrix lines in the surface generated from points on the trimmed edge curves,
5. With some of the hyperbolic paraboloid ‘branches’ and ‘roots’ instantiated on the shared lines of intersection.

The inclination of the column axis in the (global) side plane, or relative to the depth of the colonnade is also determined by both the position of the centre point on the curve, dependent on the plan shape of the curve, and the vertical angle of rotation of the hexagonal prism. Since these prisms were not arrayed normal to the cardinal planes, the combined inclination results were quite unpredictable, controlled by distance parameters along the curves rather than angles relative to the cardinal axes. The net result was a column array with the columns splayed progressively outwards from centre to extremity and leaning progressively into the elevational picture plane, the effect haptic, and somewhat irregular in the manner portrayed in the 1917 photograph. The parameter values were ultimately adjusted manually and individually until consensus was reached regarding the correspondence of the model to the understanding of the intention of the photographic evidence.
Contextual trimming of the elliptical hyperboloid

1. irregular basalt surface at the base
2. extended lower planes of the inclined hexagonal prisms of the frieze at the top.
3. the resulting shape

Edge points for ruling line intersections with paraboloid 'branches'.

4. four points created on the trimmed top edge of the hyperboloid - they are located in each of four quadrants of the curve but are defined parametrically to slide on the edge curve.
5. these points are projected onto the central normal plane in the direction of the column axis.
6. the same procedure is followed for the base curve (division into four quadrants and the creation of four sliding points on the edge curve, which are projected onto the central plane.
7. from each of these projected points two tangents are drawn to the ellipse of the intersection of the central plane with the elliptical hyperboloid.
8. the points of intersection between the tangents are identified (four new points from the top edge and four from the base curve - only one is shown.)
Through each point of intersection between tangents (pt 3) a line is drawn parallel to the column axis that intersects the hyperboloid surface in two points (pts 4 + 5) (black). Through one of these two points lines can be drawn on the surface in the original generatrix points on the top edge curve (red).

This procedure is repeated four times to find all the generatrices (ruling lines) passing through the four top points. An equivalent procedure is followed to find opposing ruling lines through each of four points on the base edge curve.

Onto this framework, I instantiated the elliptical hyperboloid model for the column ‘trunks’. It had its own internal parameters as described in 2.3.1 that controlled its aspect ratio, size and curvature.

I intersected these hyperboloids with and trimmed them with the lower planes of the corresponding hexagonal prism at the top and the irregular sloping basalt cornice at their bases. On these curves of intersection of the trimmed top and base edges of the hyperboloids, points that could slide on the curves were established using a system of ratios from reference points that varied for the upper and lower curve. These were the points from which the paraboloid ‘branches’ and ‘roots’ would be constructed, from which ruling lines in the hyperboloid surface were found.

**Paraboloid branches**

Having brought the model to this state: an array of trimmed hyperboloids of variable girth, curvature, aspect ratio and central axis inclination, controllable along the curve C, or the underside of the hexagonal prism, I could call Peter Woods COM function through a VBA interface in the relational model to find lines in the surface from the chosen points on the curves of intersection with the cornice at the base and the prisms at the head.

The COM function had until this point been tested by Mark Burry and Peter Woods on a simple explicit prototypical hyperboloid model in Rhino 3D, varying angle and position of the planes trimming the hyperboloid and the positions of the given points on the trimmed ‘edge’ of the hyperboloid surface. The script would be run, the function called, the third point calculated for each two given points, and lines would
appear in the hyperboloid surfaces: a series of long ‘V’s, four from the top edge interlaced with four from the base edge of the hyperboloid. Each ‘V’ linked two given edge points to the calculated third point on the surface. (Figure 71)

These lines on the hyperboloid surface would be used to construct the hyperbolic paraboloid branches. An area of a hyperbolic paraboloid can be defined as the ruled surface between two non–coplanar lines. Each of the ‘V’bs on the hyperboloid provided one of these two lines for a hyperbolic paraboloid branch. The second non–coplanar line was defined according to the pre–established schema in the following way. Taking the case of the top branches, there were four points around the trimmed top edge curve of the hyperboloid trunk, two of these points more to the front, two more to the back of the column as viewed from the front plane. A line was constructed from a front point on one column to the nearest rear point on its higher neighbour and this was repeated between each pair of neighbouring columns up the colonnade. (At the base, lines were constructed with the opposite relationship).

On this line, two points were created controlled by variable ratios along the line, the first for example ¼ of the way along the line, the second ¾ of the way along the line from the lower column end. Now a second line was constructed from a front point on the top edge of the lower column to the point ¾ of the way up the first new line and a third line from a front point on the higher column to the point ¼ of the way up the line. Each of these latter two lines provided the second non–coplanar line defining a hyperbolic paraboloid branch on the column from which they originated. The two paraboloids intersected, sliding past one another along the first of these three lines: the line linking the two neighbouring columns. (Figure 72)

**Model failure**

As the main relational model that I was assembling grew and developed, it was central to the collaborative process of moving towards consensus and design resolution within the small design team.

The form of the assembly continually morphed through repeated variation of many of the parameter values. For instance, the inclinations of the individual columns was varied many times, the girth and aspect ratios were varied collectively and individually,

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238 The relationship of interlocking ‘branches’ interfacing on shared straight lines was given as part of Mark Burry’s geometric schema for the columns and their relationship one to another.
1. Elliptical hyperboloids, with four sliding points on the trimmed top edge (red) and four on the base curve. The intersection point of each pair of surface ruling lines from the top red points is shown in green.

2. The generatrices or ruling lines on the surface (two opposing lines from each edge point) are identified and used to further trim the elliptical hyperbolic surface. The red lines are indicative of lines drawn between edge points of adjacent columns which will form the shared interface of two neighbouring hyperbolic paraboloid ‘branches’.

3. The paraboloid side ‘branches’ are defined by four straight edge curves: two hyperboloid generatrices or ruling lines forming a ‘v’ on that surface, a segment of the line linking points on adjacent columns and a fourth line closing the figure.

4. The front and back paraboloid ‘branches’ at the top of the columns also use two (red) points on the trimmed hyperboloid edge, the intersection point of the generatrices. Their fourth outlying point slides along the lowest edge of the hexagonal prism. There is a similar but opposing organisational strategy for the ‘roots’ at the base of the columns.
the angle of inclination of the prisms in the frieze were varied in plan and side elevation. The overall length from the top to base of the colonnade was reduced to improve the alignment with the existing inclined columns below the cornice. The pitch angles of the hexagonal frieze and the crestaria were adjusted. All these variations were carried out to try and achieve a closer alignment with the original Gaudí photograph and consensus among the design team about contentious details of the design.

Almost inevitably, in this process, some of the column geometry would fail. Closer inspection revealed that some of the junction lines (calculated by the COM function) between the hyperboloids and hyperbolic paraboloids no longer lay on the hyperboloid surface.
Figure 74 Diagrammatic representation of the hierarchy of the schema for the columns (continued opposite)

Model circular hyperboloid of revolution of one sheet. Parameters: central radius top + base radius height

Scale hyperboloid in one dimension normal to the central axis to create an elliptical hyperboloid

Column axes in master model: column centre points line/vector from centre point, normal to both axes and centre points curve

Instantiate elliptical hyperboloids aligning semimajor axis (scaling direction) with normal line/vector

Trim in situ elliptical hyperboloids top and base

Hexagonal prisms: lower planes irregular sloping basalt surface

Hyperbolic paraboloids each from two skew lines, perfectly intersecting both hyperboloid and hyperbolic paraboloid of adjacent column in straight lines.
Model circular hyperboloid of revolution of one sheet. Parameters:
- Central radius
- Top + base radius
- Height

Scale hyperboloid in one dimension normal to the central axis to create an elliptical hyperboloid column axes in master model column centre points.

Line/vector from centre point, normal to both axis and centre points curve.

Instantiate elliptical hyperboloids aligning semimajor axis (scaling direction) with normal line/vector.

Trim in situ elliptical hyperboloids top and base hexagonal prisms: lower planes irregular sloping basalt surface.

Create points on top and base rims of trimmed hyperboloids.

Project points onto the plane through centre point and normal to axis.

Tangents from projected points to ellipse (intersection between plane and hyperboloid).

Intersection points between adjacent tangents.

Lines through intersection points parallel to axes.

Lines intersected with hyperboloid surface (two resulting points.)

2 points on each line nominally 25% and 75%.

Line joining each point to unselected front or rear point on adjacent hyperboloid rims.

Tangents from projected points to ellipse (intersection between plane and hyperboloid).

Intersection points between adjacent tangents.

Lines through intersection points parallel to axes.

Lines intersected with hyperboloid surface (two resulting points.)

Ruling line in hyperboloid surface between rim point and one of intersection points on surface.

Hyperbolic paraboloids each from two skew lines, perfectly intersecting both hyperboloid and hyperbolic paraboloid of adjacent column in straight lines.
Forensics
The script to locate the point and lines was running on cue, the reaction in the main program was working. The reaction, in this context, was a feature of the parametric software that allowed the script to be rerun to call the COM function without destroying all the ‘downstream’ parametric geometry dependent on the lines produced in this process: such as the hyperbolic paraboloids and the trims and joins linking this part of the model into one continuous stone surface for the whole assembly. The script ran, the reaction worked, yet the problem persisted.

Finding the lines of surface intersection – (another mathematical model)
Over this period, I now redeveloped and refined parts of the model structure, including rebuilding the columns from first principles using synthetic geometrical tools in the main parametric program. This meant that there was no longer any need to call the COM function to construct the lines on the surface. They were now found through synthetically constructed associative geometry. The geometric sequence of this process produced two points on the hyperboloid surface, one of which lay on two straight lines on the surface passing through two chosen points on the surface boundary. The second point could, it was assumed, be disregarded. “Never assume”, a familiar aphorism from architectural professional practice lectures.

Interpretation
This was the slow beginning of a realization about the wider geometrical context. To try and define the conditions when each one of the two points should be selected, I first listed all the parameters, a change to the value of which would change the column shape. Without trying too hard or moving too far from the immediate vicinity of the column, I listed forty-six. Comprehension started to dawn. This was essentially a forty-six dimensional space. It was also not a smooth continuous space but a space with discontinuities and singularities and, in this case, a bifurcation in the geometry. As parameters were changed to subtly alter the shape and inclination of the leaning columns, the points that created the intersection of yore now came to mind. By carefully observing what was happening as other parameters in the model were changed, through their impact, changing the shape and proportions of the columns, I observed that it was not always the same one of the two points on the particular column that generated lines lying on the doubly curved surface. However, only one of these two points ever satisfied this criterion for a given instance. Investigation revealed that the original COM function also generated both of these two points but Peter Wood had also assumed that the point furthest from the originating points on the edge of the surface could be disregarded in the algorithm as this was never required in the initial geometrical examples tested. Only through extensive design variation of the column shapes and situations in the larger model were instances produced that required the second further point to generate the generatrices.
creases between trunk and branch would slide smoothly across the hyperboloid surface until a critical point was reached, at which time, this intersection point would abruptly jump to a completely different point on the surface.

This simple geometrical model constructed through the synthesis of Euclidean and conic elements had become complex and bifurcated, through the dependencies created between the geometrical objects in the space. This is a very different type of “model failure” or discontinuity from that described in the first example, more difficult to conceptualise but potentially more fundamental. It is a product of neither the functioning of the software, nor its customisation but a geometrical result

of the geometrical relationships being constructed. It occurs in the multidimensional geometry of the design space rather than in the three dimensional geometry of the aspects of the model instance that can be visualized.

### 4.4 Third example: Smooth periodic space

This third example looks to a more holistic approach to representing space algebraically. It is taken from a senior undergraduate student project. The context was an experimental research class bringing together architecture and civil engineering students to find ways to collaborate on design. The aim was to avoid the dual traps of structural pre–rationalisation (here’s a fine structural system – can you design using it?) and structural post rationalization (please make this design stand up now). Could they negotiate a third way, ‘co–rationalization’ where the architectural design and structural consideration were concurrent in the design process?

The semester long course was divided into three parts. In the first part, architect–engineer student partnerships explored technique, in the second, application of the techniques developed, in the third part they were to start over but this time site their design in a specific and challenging context.

**Phase II Mathematical derived surfaces**

I describe the work of one particular partnership, who came together in the second (application) phase of the program. An architecture student who had explored shell structures, continuous surfaces, structural optimisation and empirical structural testing in Phase I was now teamed with an engineering student who saw his own strength in mathematical understanding. The ‘architect’ immediately responded to his partner’s interest by adopting some challenging, mathematically derived surfaces. A number of equations were selected including a combined Jacobi elliptic function and hyperbolic cosine function. They were chosen on criteria of aesthetics and spatial potential from a library of surfaces they had assembled: surface models of regions of various mathematical surfaces and the functions of which each represented an instance. Through very simple manipulation of these ‘found’ surfaces – Booleans to create edge boundaries and openings and differential

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241 In the early stages of considering the Philips Pavilion design, Corbusier wrote to the rectors at the university and technical institute in Zurich to request a similar catalogue of geometrical surfaces and their appearances for architectural use. (Treib, M. *Space Calculated in Seconds: the Philips Pavilion, Le Corbusier, Edgard Varese* Princeton: Princeton University Press, 1996) I don’t believe that the students were aware of the parallel.
\[ x(u,v) = cn(v) + u \]
\[ y(u,v) = sn(u) - v \]
\[ z(u,v) = cn(u)/\cos(v) \]

**Station Equation**

**Resultant Surface**

**Booleaned and scaled Surface**

**Figure 77**: Railway station and high rise design proposals using surface defined by function. (image by Steven Swain)

**Interpreted Train Station - roof with skylights**

This surface was then scaled \( x \times 5, y \times 5, z \times 3 \) to achieve the desired profile for a long span structure. The surface was then sliced at \( z = 13 \) removing the top portion to create skylights for the interior volume.

\[ X(u,v) = cn(v) + u \]
\[ Y(u,v) = sn(u) - v \]
\[ Z(u,v) = cn(u)/\cos(v) \]

**Interior**
scaling – a series of form articulations of program were suggested, an interpreted railway station roof, a sinuous high rise building development.

The most compelling was the use of a surface in its most raw state as the shell structure of the Hybrid Cathedral. In this proposal the surface mediated between a soaring sacred space of monumental proportions at the heart, and multilevel apartments nestled in the sinuous peripheral undulations of a gamma function mediating between the two programs. At this stage, the partnership modified the surfaces from their surface library through small changes to the coefficients in the function and through cutting the surface in different ways, resulting largely in differential scaling and making openings.
Phase III Surface responding to site –
Geelong Bypass Inhabitable Bridge

The third phase introduced site and location, subtly inverting
the program–seeking form experimentation earlier. It led
this same partnership to the development of an inhabitable
bridge that could be mathematically defined. Architectural
and structural parameters were identified as they embarked
on writing an equation that would satisfy both parties and the
program that they had jointly defined. At this stage, a much
more intense interaction with the mathematics unfolded. The
detailed development of the bridge geometry was much more
sophisticated than the surface manipulations of the earlier
projects. Whereas the student designers had initially varied the
surface shape through simply editing the variables within the
original function, an effect equivalent to differential scaling and
trimming using external algorithms in modelling software, in the
bridge project they engaged in detail with tailoring the algebraic
surface description to the parametric performance drivers for the
bridge shape. Once again they started from a given function:

\[ X(u,v)=\operatorname{cn}(ff(v))+(cc(u)) \]
\[ Y(u,v)=\operatorname{sn}(ee(u))–(dd(v)) \]
\[ Z(u,v)=\operatorname{cn}(aa(v))x \cosh(bb(u)) \]

This was close to the function developed for the hybrid
cathedral with the u and v coordinates reversed in the Z
access. It was originally derived from a given Jacobi Elliptic
function from 3d–Xplormath library of functions and
selected without any deep mathematical understanding
of the properties of the function on the basis of the visual
understanding of the graphical representation of the surface.

A publicised but short–lived proposal to divert the Geelong
freeway across the entrance to Corio Bay was reawakened
to advance the concept of a single mathematically
controlled surface as structure, rich space–defining
boundary and interface between monumental scale and
domestic infill. 242 The real world requirements were to
maintain and span the dredged shipping channel, also
allowing small craft to pass between Corio Bay and
its parent Port Philip Bay, maintaining the tidal flow.
The design must also observe the spatial, gradient and
curvature constraints of the freeway and separate the
habitation with its services and access roads from the
freeway. These requirements intensified the quest to
develop the relationship with the surface equation that
would allow detailed manipulation of the parameters
without relinquishing the emergent qualities and
aesthetic coherence of the surface itself. It introduced all
the architectural dialectics around the intensity of the
experience of crossing the bay at the historic fording point
and the iconic and environmental impact of the bridge as it
reshaped the view and context for Geelong. It also engaged
with the specific engineering challenges of exceptionally
long spans, building in water, and site conditions at the
springing points.

Corio Bay is an offshoot of Port Philip Bay in Victoria
Australia. It is a natural a sheltered harbour for the city of
Geelong but its narrowed mouth is closed from Port Philip
by shallow water over a naturally tidally exposed sand bank.
A deep shipping channel has been dredged through the centre
of the sand bank, which was otherwise traversable at low tide
from Point Henry at the Geelong end to Avalon in the north.
This ancient crossing point was the proposed site.
Figure 79: Bridge based on a Jacobi Elliptic function (image by Steven Swain)

**Top Chord Equation**
\[
x(u,v) = 20(\sin(v 0.25)) + 0.5(u) \\
y(u,v) = 20(v) \\
z(u,v) = cn(5(v)) \cosh(0.45(u)) 0.1 \left( \sin \left( \frac{v}{3} \right) \right) + 20 e^{-\left( \frac{0.5 vv}{2} \right)}
\]

**Bottom Chord Equation**
\[
x(u,v) = 20(\sin(v 0.25)) + 0.5(u) \\
y(u,v) = 20(v) \\
z(u,v) = cn(5(v)) \cosh(0.45(u)) 0.1 \left( \sin \left( \frac{v}{3} \right) \right) + 15 e^{-\left( \frac{0.5 vv}{2} \right)}
\]
The brief
The student team speculated that the private–public partnership would mean that the small–scale private housing on the bridge would contribute to funding the huge causeway, bridging over a shipping canal at its midpoint.

Their own brief for this project provided a short list of very clear critical parameters: constraints on the gradient and curvature of the freeway over the top of the bridge, structural bays between piers supporting the bridge, the great height and span of the bridge over the shipping canal. There were also more qualitative drivers: achieving height, shape and curvature in the undulations of the surface between structural piers suitable to accommodate the waterside housing on the bridge. It was quickly clear that their cursory engagement with the mathematical functions generating the surfaces in phase II would not be sufficient to create a surface that would meet all the criteria for the bridge.

Experimentation
This excited a period of experimentation in which they started to understand the function better through more direct engagement, superposing new functions that provided detailed surface articulation, allowed control of differential spacing between the structural piers, appropriate curvature of the bridge in plan to meet the springing points set up by the approach roads on either side of the bay.

Simply editing the variables within the original function had a similar impact to scaling the surface using external software algorithms; for instance, it altered the distribution of bridge piers but continued to create repetitive, regularly–spaced piers.

To be able to create the large opening for the shipping canal but find more optimised structural intervals for the other parts of the bridge it would be necessary to add a second function to disrupt the rhythm. Various functions were overlaid, some causing too much disruption and surface distortion. Finding a satisfactory addition through empirical experimentation imbued a situated awareness of the power of superposition of different functions, and it was possible in the same way to overlay a fine grain to the surface, a detailed level of surface undulation or corrugation for combined aesthetic and structural opportunity.

Refinement
The formula was then simplified in experimentation to find out how to control the level of detail and hierarchy of peaks, calibrating it to control the height of the peaks in the undulating surface (varying this in relation to the width of the bridge and spans) and a further function superposed to vary the height of these peaks. By this stage, the designers had entered or immersed themselves in equation or function building as their design environment. At each iterative step, the form elegance and subtlety of the model increased with the increasing control and mastery over its potential to vary. In order to curve the bridge in plan into the sinuous ‘S’ needed to meet the freeway routing at each abutment, and make the crossing at the old fording route, some of the existing components of the function could be used but had first to be rearranged and separated or their impact altered through denomination. The peaks then had to be controlled in a way that specifically reduced their height at the springing, where short piers were required, and at the main shipping canal, where the vast span would require stiffness but all
Figure 80: Some examples of surface experiments: variants and their algebraic descriptions. (image by Steven Swain)
possible reduction in the weight. This variation could be periodic but the period relative to the pier intervals needed to be controllable in a specific way. This required the further superposition of a specific function for \( u \) and \( v \) in \( Z \). Although the form mastery now extended to understanding how to vary not simply the parameters but the function itself, the means to arch the bridge deck following a specific curve from springing to springing was not yet clear.

**Solving the arch**
The source of the original kernel of the function and surface led to Paul Bourke at that time in the Astro Physics department of Swinburne University (now at University of Western Australia). With Paul’s help the function was rewritten in a way that clearly parametrised it for the variables already identified and an additional Gaussian function now gave the arch to the road to allow it also to rise up 70m over the shipping channel from its low lying springing points.

The designers could now rewrite the equations satisfied by the \( x, y \) and \( z \) values of each \( u,v \) point on the surface with the list of variables shown graphically in Figure 82.

**Inhabitation**
A lower deck was needed below the freeway to provide access to the inhabited pier shells. For this the same functions could be used with a small change to the value of the last variable: altering one variable in the short Gaussian expression.

**Outcome**
In summary, every aspect of the bridge is periodic, determined by its tidy three line function but the superposition of these periodic ‘behaviours’ is spatially subtle and variably aligned with programmatic constraints. Its description is simple and simply conveyed or transmissible; its spatial manifestation rich and animalistic.

Now they had an undulating shell structure, highly organic and variable in its form, several kilometres long. It could morph in response to a specific set of drivers and be transmitted between design participants in three short lines of function. It was this notational economy that ultimately delighted the protagonists and made them feel that they were somewhere on the track to revealing the secrets of a shared or co–rational design process lying between architectural and structural engineering design.
In the context of architectural borrowing, inheritance, deep inspiration from science and mathematics, what is the significance of the experimental application of the discoveries of Carl Gustav Jacob Jacobi around 1830 and Gaussian number theory developed in the closing years of the eighteenth century in a joint architecture engineering studio in the early twenty first century?

Perhaps it is Antoine Picon’s hypothesis that it is the similarity of operation between science and architecture that at certain points makes the relationship most productive. Picon and Ponte also write of ‘a new type of connection between architecture and science’ for which ‘the computer, of course, is central’.243

Chapter 4 | Inside the model space: bifurcations and holes

Freeway Surface Equation

\[
\begin{align*}
x(u,v) &= 20(\sin(v \cdot 0.25)) + 0.8(u) \\
y(u,v) &= 20(v) \\
z(u,v) &= cm(10(v)) \cdot \cosh(0.55(u)) \cdot 0.1 \left( \left( \frac{3}{2} \right) \cdot \sin \left( \frac{v}{1.5} \right) \right) + 20 \cdot e^{\left( \frac{0.85 \cdot x}{4.4} \right)}
\end{align*}
\]

Lower Level Equation

\[
\begin{align*}
x(u,v) &= 20(\sin(v \cdot 0.25)) + 0.8(u) \\
y(u,v) &= 20(v) \\
z(u,v) &= cm(10(v)) \cdot \cosh(0.55(u)) \cdot 0.1 \left( \left( \frac{u}{2} \right) \cdot \sin \left( \frac{v}{1.5} \right) \right) - 15 \cdot e^{\left( \frac{0.85 \cdot x}{13.13} \right)}
\end{align*}
\]

Figure 83 Adding the other decks (image by Steven Swain)
*Figure 84*: Image of the inhabited bridge from the water (image by Steven Swain)
What is compelling about mathematical surface definition or generative processes that bear a metaphorical resemblance to the ‘laws of nature’? Clearly, there is a rationalist drive to define design objectives as a rule set controlling the configuration of space and form. This is a way to gain greater efficacy from the technology – using computation to achieve a set of complex spatial or geometrical objectives simultaneously through the definition of their relations. Then there is the matter of beauty. There is the rational scientific idea that underlying natural beauty is a profound system of law–abiding relationships. By reconstructing a closely analogous system, not only the source but the resulting sensory delight will be rediscovered. There is also the distinct question of mathematical beauty: the authors’ delight in a bridge of great spatial and programmatic complexity from a three–line function. In this project there is a distinct sense of seeking to address design, sculptural and mathematical beauty simultaneously. I will return to the topic of design and mathematical aesthetics in chapter 6. For the protagonists the beauty of the notational economy of a complicated and articulated structural shape defined by three lines of algebra was what Marcus Novak has earlier referred to as ‘transmissibility’, its ability to be passed between disciplines and design activities without any loss or degradation of information.

**Limits and schisms**

The sections describing the first two modelling examples each conclude with the conditions for a model limit or schism.

This third example belongs in a particular tradition of concrete shell design. In this case the shape has not been optimised for structural performance within the scope of this one semester research studio in the manner of precedents from masters such as Candela, Nervi, Isler. It conforms to rule of thumb calculations for spanning, cantilevering, curvature. One might speculate that its next design iteration might be to mediate between finite element analysis and the surface mathematics. The shape is also unconstrained with respect to construction, components and fabrication and assumes advanced computer numeric control fabrication techniques and technologies. In this sense it is not so different from some of the preceding real world examples – consider the Sidra Trees at the Doha Convention Centre and their level of post rationalisation once designed in detail for economic and technically realisable construction. But it is a conceptual design model.

In this way it is not directly comparable to the highly constrained real world examples in either Chapter 3 or the first two sections of this chapter. More importantly its economy and mathematical beauty relies on a certain level of homogeneity. This is regardless of how the function might be constructed to combat the visual perception of spatial consistency or repetition and to introduce the appearance of a higher fractal dimension.

To what extent could the first two examples in the chapter have been modelled using this approach? It would be dangerous to assert that those hybrid geometrical relationships could not be recreated algebraically in this way. There are notable precedents of quite precise shape reproduction from physical models in architecture described this way.\(^{244}\)

\(^{244}\) Frank O Gehry ‘horse head’ sculpture as the DG Bank
This is rather a point of reference. A design space generated from an analytical geometrical description in contrast to the earlier examples of tacit design spaces resulting from hybrid synthetic geometrical and contextual relationships.

4.5 “The Problem”

This chapter describes three different responsive architectural models that can be changed spatially or qualitatively in answer to new input information from changed or refined intent or the adjustment of the relative influence of each of many design drivers. They have been constructed using digital computation as the system for deploying geometry, the modelling medium. The models have sub models, each a tool for the construction of the greater model.

Two of these models are also sub models within the same larger model. They have been chosen to illustrate not only different approaches to working with geometry but also two very different types of discontinuity with the space, variously described as “the model space”, the “search space” or the “design space”. The third model is more speculative. It illustrates a third position in regard to using geometry in architectural modelling where the formal geometrical description leads the design in a generative way. In this third case the “design space” is concisely notated mathematically.

Each of these models is a system. They are dynamic in three-dimensional space. They have many variable parameters. The model space is thus mathematically many dimensional.

This means that, even though the model may have been constructed through a series of simple synthetic geometrical moves, the space of possible variations articulated through the geometric dependences is difficult to conceptualise and impossible to visualize. This is equally true of the space of the functions in the example from pedagogy but it is possible that the homogeneity of conforming to a relation defined by a simple function allows imagination a better foothold in grasping the shape morphing pathways through the various parametric manipulations in this case.

In order to model effectively, it appears that this space, the space of possible variations, the model space, the search space, the design space is the space into which we must effectively project ourselves.

Can we do this? And if so, how do we do it?

In order to use these rich and dynamic types of model as effective tools of design, do designers need enhanced formal mathematical knowledge? Or is the knowledge of this type of space gained through direct empirical engagement as in the case described of the student process in this chapter – neither student had a high level of specialist mathematical knowledge? Is it even reasonable to consider this as space at all? Is it the space that Blackwell?
says architects “create” or is it a mathematical conceit, such as those that Le Febvre claims were invented by mathematicians: “an ‘indefinity’… of spaces”.248

In the first two model examples the models have limited ranges of viability they break and require remedial remodelling at certain points in the cycle of variation. They break not through poor use of the technology but through geometrical paradox in the first case and bifurcation in the second. Could the same dependent objects have been represented differently mathematically and avoided these “holes” in the design space. Or must we regard them as the inevitable consequence of a highly constrained geometrical design brief?

The third model is constructed directly from an algebraic description of surface – is this an innately more homogenous and predictable construct or only in so far as the algebra itself is kept within well understood territory? Could any design brief be approached in this way? For instance, the particular hybrid geometries of the Sagrada Família Narthex model are considered to respond to Gaudí’s intense observation of natural form and growth, to the technologies and techniques he employed for design modelling, and to support the stereotomy-based description and communication of doubly curved surfaces for stone cutting. Can these hybrid geometries be reproduced in the same economical function notation? Would this make the space of the dependencies any more homogeneous and predictable? Presumably not if exactly the same combination of conditions were to be met.

Hugh Whitehead249 considers that “long chain dependencies” are to be avoided. Thus in order to maintain the fluency and agility of a design model, it is necessary to be selective in the relationships that are established between its components, avoiding over constraint and maintaining schisms and spaces for change in the overall ordering.

The problem of designing the model is like the problem of design itself. A balance must be struck between using the algorithms to maintain the integrity of the network of relationships defined through design decision making and maintaining the agility and usefulness of the model which may, for this reason, represent only a small component or component behaviour or trade off of the whole design system.

4.6 Why are these model examples generalizable?

This question arises in relation to all three examples. The first two examples come from a very particular context in which the emphasis is to reverse engineer, or discover, a geometrical schema from Gaudí’s pre–extant, though, like all designs, incomplete design, understood to have adopted a particular geometrical codex.250 The system modelling of the perceived

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250 Burry, M., Coll, J., Goméz, J. *La Sagrada Família: de Gaudi al CAD*. Barcelona, Spain: Ediciones Universitat Politècnica de
geometrical criteria for this design reveals certain conundrums of geometrical space and I am claiming that these are generalisable to geometrical system modelling in general.

In the academic case study, geometry is taken up in quite a counter-spirit. Rather than geometry in the service of a particular spatial and form goal, well-defined geometry itself is appropriated as form and subverted algebraically to meet the performance criteria expressed as parameters.

Playing the devil’s advocate, the particular emphasis on geometry in all three case studies could be questioned; the first two because they are located in the context of a building which is known to have been a laboratory for bringing particular geometries to the service of a unique nature-inspired form and structural essay, by a singular architect of genius, Gaudí. The third might be dismissed also as singular because it is a piece of research into the application of geometry in architecture and engineering not only for the generation of form novelty but as a shared currency of exceptional notational economy.

So, do the model spaces generated in these processes have relevance to wider modelling application in architecture? How is this modelling process extensible to a more general design context? I argue that these examples exemplify more general design modelling challenges and provide an ideal laboratory in which to expose and investigate them for three principal reasons.

The first reason is that design is characterised by contextual constraints. This is one its chief distinguishing characteristics from fine art. Design is the ‘useful’, ‘social’ art. Many design processes start in a very open framework and become progressively more constrained. Constraints are uncovered as performance criteria for different aspects of the design become progressively more explicit, particularly in design for construction, which involves diverse disciplines and criteria. Spatial and form goals and constraints will be known from the outset in terms of conforming to a legislated planning envelope or deemed to satisfy configurations for fire escape distances. The art of architecture may however manifest as divergent, or at least very rapidly changing spatial proposals in the earliest modelling exercises.

The modelling examples from the design for construction of the Narthex of the Sagrada Família church are exceptionally constrained from an early stage in the sense that there is already a spatial, formal and aesthetic outcome targeted, in so far as Gaudí’s design for the church is defined and communicated through the surviving evidence. There is just a shortfall in information about the process and procedures to realise that goal.

Research case studies in general are chosen to minimise the number of unknowns or variables in order to focus clearly on the central problem. In scientific research this is done by holding those variables not in question constant and, through particular experimental conditions, accessing particular behaviour of the object of study. In this light, this is a strong case study as the emphasis is on finding a

model schema to refine a given design towards a relatively well–defined goal. The emphasis is on modelling effectively, using geometry not generatively nor experimentally but responsively and artisanally.

The second reason is that the emphasis in recent practice, on realising the construction of “difficult” geometries in culturally significant and iconic buildings that were rarely considered previously suggests that examples such as these do now exemplify more general design modelling.

The third reason is that the structural issues about space raised in these three examples are not confined to the production of “difficult” or unfamiliar geometries in architecture. They are deeper problems that relate to the nature of geometry and definitions of space, in particular modelling dynamic systems rather than static three-dimensional objects or instances. This is an occupation confronted more recently in architectural design modelling than in simulative and predictive modelling of observed natural systems or mechanical and chemical engineering modelling of kinetic machinery and synthetic systems.

The first example above describes the use of algebra to make a very specific component of a larger model. This model was built to reverse engineer some specific static geometry by trying to discern the underlying pattern, ‘the pattern that connects’. We aimed to achieve authenticity while interpreting the intentions of Gaudí, probably using approaches that were different to his. There was no requirement for the stepping assembly to be infinitely morph–able, simply to be able to concertina gently within a given but still variable space to achieve a better correspondence with a historical photograph. Moving outside the viable range for the key variables would soon ‘break’ the geometry. This is because the other components of the geometry were created synthetically by intersecting planes and curves and applying surfaces to the results. It takes little to create non–viable intersections or steps following curves in plan that step out from a neighbour where previously they stepped in. So while the form varies smoothly within this small region of the design space, it, nevertheless, has complex boundaries and discontinuities through its heterogeneous nature.

The second example was perhaps the most surprising.

While building up a variable design model from Euclidean

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Hensel et al have adopted the term in a somewhat different sense. In this context, I am specifically considering the heterogeneity of the clashing surface and assembly geometries in the composition of the Narthex colonnade which is much more geometrically diverse than some other parts of the church where the geometries are married more seamlessly – the branching columns in the nave to the vault geometry, for example, or the array hyperboloids and interstitial planes in the upper nave window assembly. However, I consider that many of the more general challenges encountered in dynamic virtual modelling originate in the difficulty of translation between system description and instance representation and between geometrical instance and subdivision for construction in physical space. In this sense the intentions of the model are often very heterogeneous in their spatial nature.
elements and conics, their cumulative effect when related within a single system is to exhibit behaviour attributable to “bad smooth functions”, mathematical behaviour defined only in the twentieth century.

The difference between the third example and the previous two is clear. The principle spatial device in this proposal was a continuous periodic surface (albeit based on a Jacobi Elliptic function which is doubly periodic and meromorphic). The space is thus defined by a mathematical function, rather than resulting in one by hybrid means. Mathematically at least, it is a more homogeneous space. Nevertheless, the form is not only driven by parameters given by the design of the bridge but highly varied and surprisingly animalistic in its local manifestation. This is a space that can be controlled top–down through editing a function to meet the local formal and performance criteria. In this case even when needing to add the internal service road and the lanes on the freeway in order to be able to create renders of the bridge with indications of scale and use, it was found more successful to add these using the function than to try and add them externally in a modelling program. In this sense the space maintains its underlying homogeneity.

4.7 Pedagogical examples and feedback

In 2007, as part of the PhD work, I wrote a paper for the International Journal of Architectural Computing titled ‘Mindful Spaces: Computational Geometry and the Conceptual Spaces in which Designers Operate’. This was an opportunity to reflect on six semesters of teaching the Flexible 3D Modelling for Design and Prototyping course at RMIT. The students in this course are drawn from a wide selection of core disciplines across the university, including architecture and design students from the host School, but also automotive, aerospace and civil engineering students, business students, urban design students, media and communication, and fine arts students. The paper argued that while extending the opportunities for design iterations in a variety of types of design project that the students undertook, the main contribution of the course had been the extension of the students’ own concept of the space in which they design through the hands on experience of constructing flexible computational system models of the designs. The paper makes reference to Lawson’s ‘primary generators’ and his quotation of Darke who writes that designers often tend to latch on a simple idea very early in the design process to narrow down the range of possible solutions. The difference in the sketch stage for

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258 Darke, J., “The Primary Generator and the Design Process”, in
computationally supported associative geometry model compared to a more conventionally represented design idea is that this early simple idea should be not be represented as a shape or a map but an action – what should the design model be able to do? How should it be able to change?

The paper reported on a taxonomy of types of projects for which the students had used parametric (computational system) design modelling in the course. The list was as follows: (1) What could my design become (the development or elaboration of an earlier proposal using geometrical associativity); (2) Responding to Environmental Drivers (linking the parameters of the design model to the incidence and direction of sunlight, to temperature, to wind speed, for example) (3) Emulating the Behaviour of Physical Materials (one of the harder challenges – CAD software is not predicated on even the perceptually simple dimensional constraints of a shape-changing handkerchief); (4) Mass Customisation; (5) Dynamic Systems (kinetic or changing responsive architecture and artefacts similar to those designated “Kinetic Informationscapes” in Chapter 3). Most of the projects that the students had proposed in order to develop and test their Flexible modelling understanding and skills had conformed to one of these five categories.

I considered that the students were engaged in three distinct computational geometrical design spaces: (1) the visualization, (2) the database, and (3) the spreadsheet. The defining distinctions between these three were as follows.

“The visualization is an immediate space that engages at least one of the five senses in ways closely analogous to movement in physical space – it is more literally three dimensional than natural space and framed, pictorial, isometric rather than perspectival but importantly kinetic – the viewpoint can move in real time. This is the space that links to the innately topological space of our visual cortex but possibly the one that reveals least about the true topology of the model. This space is the most intuitive to read but also for this reason the most potentially deceptive – we may read patterns that are counter to underlying relations and miss those that are there.

The database is the space with the most significant content and the hardest to read or conceptualise. Relations are invisible, like parameters, they are represented symbolically, and syntactically. They become apparent only through change (changing parameter values, changing the relations themselves.) We cannot read them through the visualization when it is in one particular state. They are the formalization of design intentions. Their hierarchy can be graphically represented. It is, arguably, in this space that the design can most meaningfully be described as topological.

The spreadsheet is a natural- and formal-language space were we see values in a table, a collection, we can sort it and order it various ways, and we can map its symbols to the visual geometrical objects (sometimes they are helpfully represented as annotations within the visualization). This can be regarded as the combinatorial space – a space for interrogation and for creating algorithms relating geometrical objects.”

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259 Burry, J.R., ‘Mindful Spaces, Computational Geometry

New Directions in Environmental design Research EDRA 9, ed. Rogers and Ittleson (1978).
My argument is that this tripartite computational geometric space is, in a sense, expansive of the novice designer’s conceptual space. The interrelationship of the three is a generative, exploratory space in which to work. This is illustrated by the delight and surprise expressed by the student when their first prototypical system model is in place and, in response to varying a parameter value, resolves itself into a geometrical composition that they do not feel that they themselves have literally built.

In this paper, I conclude that the construction of computational system models at the very earliest sketch design stage does appear to be inhibitory to rapid investigation of ideas, simply through the time taken and the complexity of committing to parametric definitions and relations while modelling. For this to be valuable, the interface to the tool would have to be very immediate and make few demands on the designer for the reflective construction of a graph of relations. On the other hand, simple associative geometrical models are quickly useful for testing early ideas, as soon as it is possible to posit a few definitive characteristics of the domain to test. They can provide a relatively intuitive interface for empirical exploration once constructed. But the greatest value of this for pedagogy lies in its capacity to expand the cognitive spaces in which students design.

In 2007 I also presented a conference paper titled ‘Mathematical Relations in Architecture and Spatial Design’ at a conference on Maths Education. The paper also used the examples of the Flexible 3D Modelling course and the outputs of the joint architecture-engineering studio described in Chapter 4, as vehicles to reflect on the changing status of geometry and mathematics as both entry requirements and subjects in architectural education.

Attending this conference was also an opportunity to contextualise mathematical approaches in design education within the general field of mathematics education. Within formal mathematics education in most of the developed world, debates rage, the so-called “Maths Wars”, while post and late high school interest in mathematics and levels of proficiency decline. There is a powerful aspiration towards “a fundamental change from the passive and de-contextualised absorption of mathematical knowledge and skills acquired and institutionalized by past generations toward the active construction in a community of learners of meaning and understanding based on the modelling of reality.” At the same time evidence is widely presented that it is inefficient and ineffective to give learners their head in finding their own problem solutions at the expense of securely imparting standard proven algorithms in the time-honoured way.

Design education sits firmly in the aspirational camp with project-based learning a dominant paradigm, in which learner


designers define their own problems and search for their own solutions. Their use of formal mathematics is incidental and of the ‘just-in-time’, response-to-need type. This is largely ad hoc and relies on individual interest in seeking levels of sophistication in modelling and formal generative tasks. However, it is also potentially exemplary in deploying graphical and time-based means of representation, engaging with available digital tools to link mathematical modelling to formal and spatial simulation and representation. In educational terms, this relies on a high level of general ability amongst students and their previous “acquisition of the mathematical disposition” (to quote the mathematics education reformists) during primary and secondary education.

In 2010, I followed up these reflections with a qualitative questionnaire seeking the students’ own appraisal of their experience of ‘flexible modelling’ in the ‘Flexible 3D Modelling’ course. It was sent to students of the course from 2004 – 2010. Many of the responses focussed closely on the evaluation of the specific software for the student’s needs. Many were not sufficiently expansive to provide much obviously valuable feedback. This may indicate that the research could have benefitted from a more expertly constructed series of questionnaire questions, following a pilot. However, as the response rate was low – largely a factor of the difficulty of successfully contacting former students at some distance in time after their participation in the course – it was only practical to make one approach to the group. There were 20 respondents. The tone of the responses divided very noticeably on disciplinary lines among the students. While the aim was to elicit their reflections on their generic experience of system modelling as opposed to modelling static geometry, their responses were generally software-specific. The software that they were using was originally developed in an engineering and manufacturing context, before being customized for construction design, so, unsurprisingly, the level of enthusiasm was least measured among those from engineering disciplines. Not only are engineers more at home with spreadsheets, graphs, text and diagram-based media as both inputs and outputs, but the base ontology of the software is likely to conform more closely to traditional bottom-up component-led processes. The basis of assessment was also clearly different between disciplines with a marked emphasis on what they perceived as utility among the engineers and creative freedom among architecture and related disciplinary students (landscape, industrial design, etc.)

One engineering student wrote: the “software was far more complex than I had imagined CAD software could possibly be…I believe that the modelling experience helped me to develop a number of skills that were not specifically related to the class. Modelling in 3D certainly helped to improve my spatial thinking in a design sense…I liked the concept of the tree [representation of the design history choreographed by the designer in their model], although at times I found that some of the rules that govern it’s hierarchy and how it worked to be...

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263 The questionnaire, as approved by the RMIT ethics committee and distributed, is attached as an appendix to the dissertation.
impractical and frustrating. As my skills progressed, I found that it was important to minimise the complexity of the tree, and to plan ahead...I believe that the process of designing and modelling systems instead of objects in architecture requires a hybrid approach that fundamentally changes the very essence of the social class based profession that architecture is. Unfortunately many recent architectural projects and proposals that utilise technical ideas like associative geometry seem to be designed off a metaphor of the concept and in reality are just cultural objects where the designer has failed to embrace the poetics of the technical object.” In this instance, interestingly the questionnaire has become a vehicle for open interdisciplinary critique. With a similarly positivist outlook, an electronic engineering student wrote, “I have nothing to compare it to [no previous digital geometrical modelling experience], however if I were to undertake a modelling process again it would certainly not be using static models… Scripting helped me out of one particular bind. Design reuse is always handy.” The response of the engineering students also tended to emphasize procedure over more existential spatial understanding of the model system. A student of Advanced manufacturing and Mechatronics wrote, “…overall this was a very powerful tool for engineering, being able to shift an angle or length of a beam, without having to change other aspects of the model also, and just having the model work itself out.” But in a more philosophical tone, a civil engineering student contributed, “the program made perfect sense to me once I was used to it. The geometry and parameters were probably the most … direct representations of what I had worked out on paper… I really liked the idea of designing the underlying form and then building geometry rather than the form itself. Being able to manipulate them a thousand ways made everything so much more interesting. Being able to print off a specific iteration is extremely useful as making comparisons becomes a lot easier.”

There were more reserved responses from some of the architecture students. One confessed “I found it incredibly difficult to break down the hierarchy of the problem in a way that it could be modelled in the software. With explicit modelling it is easy to be quite experimental, without even understanding or articulating the problem.” Another noted, “I thought that the Excel© relationship was extremely divorced from the actual modelling experience. It was good to have a play with [the software] but I found it to be quite constraining compared with my experience of [another parametric software]. Working with integrated analysis might be where it really shines.” A third wrote that working in a flexible (system) modelling environment as opposed to creating digital solid models or surface models using explicit geometry was “not too different, although the interface and workflow was slightly inefficient and difficult to work with when translating the design intent from sketch space to geometry space” but “the most positive aspect was the flexibility in the sketch space of [the program]”. The architectural education of these students places heavy emphasis on early conceptual design and there is a high expectation of loose-fit development of ideas at sketch stage. Some of the responses from the architecture students were more akin to those of their engineering colleagues. “It’s a different way of thinking. You focus more on parameters and the rules, setting up all the background work. Then your design develops from that. In the end, it allows you to see how
changing certain variables has certain effects on the model as a whole. I found the studio both interesting and useful. Even though I might not be using Parametric Design now, I can still think of my design as a breakdown of data and requirements, which build up to the final model – each having a cause and effect on each other.”

Two landscape architecture students wrote about associativity in the model but with opposite points of view on the value of having everything linked within the model. The response: “The flexibility offered was a pleasant surprise, in comparison to traditional modelling. The ability to generate various forms with a strict logic behind it [sic] relatively easily without having to recreate the model over and over was a definite plus…Being able to plug in and change inputs via spreadsheets was a huge help…” By contrast, a fellow landscape architecture student wrote: “It was a bit difficult to get my head around the way the software works. I found that I had to keep going back and changing things, which then affected how the rest of the design turned out, as it was all connected. Using equations was also quite a difficult approach for me”.

Finally at the opposite end of the spectrum of responses from those of the engineers, an industrial design student noted frankly, “It seemed quite clinical and mathematical. Aesthetics were considered at the very end of the project. Emphasis should have been placed on experimentation and exploration and building a system rather than anything to do with modelling. A 3D model may just have been the vehicle.”

Clearly these responses were very individual, depending on the modelling tasks the respondents had set themselves, as well as their own background experience, expectations and the skills and aptitudes they brought to the class. There is a pattern across the responses: acknowledgement of a paradigm shift. For most of the students, the change from the modes of (static) representation with which they were familiar to constructing a design system model meant more than simply familiarising themselves with approaching their design representation using a new software. Some acknowledge that the principal activity becomes developing the right heuristic. For the engineering students who have been educated in a problem-solving framework, the broad possibilities of the software in this respect can be very liberating and liberalising. In some ways the parametric freedom allows them to work less precisely and open up the problem for exploration in a more conceptual way. For the architecture and design students, accustomed to working in a proposition or solution-led way where it is important to be able to represent the first idea very immediately and suggestively but not necessarily with any precision, the task of thinking analytically to construct the heuristic rules and process, could be a chore that they saw as a barrier or impediment to their design workflow. All the observations must be tempered by the condition that all the respondents were approaching this part of their learning as novices. But it does suggest that an introduction to some programming or precise but flexible mode of representation, harnessing computation, preferably at the level of programming at an early stage of design education could have a significant impact on its assimilation and on the possibilities for design outcomes from the recipients.

Building parametric models using proprietary software must be seen as a particular activity. Many of the students extended
their scope through scripting which gave them variously access to more emergent processes, greatly increased productivity, economy, and better, more direct ways to express intentions. The holy grail of the spatial designer who can combine a strong sense of phenomenal and projected spatiality with a high level of programming skill has had currency at least since the 1960s. This person can still shine as an anomaly in the design community but perhaps the numbers are rising.

Megan Yakeley wrote in her PhD dissertation in 2000 about using computer programming to develop a personal design process. Within a programming course (outside the design studio) that she ran for several years at MIT School of Architecture and Planning she identified five stages of development in understanding the design process. These were: (1) designing for an imagined future user of the students’ computer code, (2) seeing themselves as the future user of the final code product, (3) seeing the process as design, not code, emergent rather than imposed rules, (4) recognition of the parallels between coding and designing processes. In her conclusion, Yakeley writes:

“The course was a transformative educational experience for the students. They began with an assumption about computer programming as a tool, and ended with the realisation that through learning techniques in the course they had transformed their own understanding of their personal design processes.”

The particular workflow challenges (schematising the performance of the model in relation to the design, constructing histories, stating parameters and geometrical relationships, interacting with an excel spreadsheet through a slow connection to the modelling program) can and are being tempered in the development of different software. Interface design can and is ameliorating some of the overtly procedural nature of the workflow and what the design students, as novice flexible computation modellers, experienced as an awkward relationship to aesthetics (their direct phenomenal engagement with what they are making). Clearly the stage of fluency that they have attained in their own learning and adaption impacts the level at which the procedural aspect of the modelling is assimilated rather than, firstly supporting their design as a tool and secondly parallel to or mirroring their design process. This assimilation is in a sense a measure of their ability to conceptualise and operate in multidimensional model space.

Recapitulation: Chapter 4

Relational models are systems that may result in geometry and geometrical behaviour beyond, or meta- to the geometry used to construct them. There are many ways to approach this meta design of the system. This chapter illustrates and contrasts three, giving reasons behind the different approaches and highlighting the constraint and behavioural differences between them. Finally, it makes reference to the introduction of relational geometrical model through projects and formal procedure in design pedagogy and records some of the reactions of students to this experience.

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265 Ibid., 237.
In order to build models, the geometry or the geometrical construction used is atomised into a series of procedures that can be enacted or encoded. These may be individually well understood and result in a shape outcome that can be viewed and understood in two, three, or four dimensions. However, in this procedure, a system- or meta-geometry is constructed that is not equally perceptually accessible. The modeller becomes subjected to, rather than fully determining, all the resultant behaviour of the system when using and manipulating the model for design purposes. The model is used to explore the design space but also sometimes woefully constraining of that space.

What is the cognitive, philosophical or perceptual framework for understanding the unseen geometrical and behavioural characteristics of the system model space, once created?

I know that the surprising nature of mathematical or geometrical deterministic space is not unique to working in design. Lorenz was surprised by the impact of a changed decimal place in a starting point in his system model of weather patterns. His surprise led to the development of Chaos theory. This is another example of computation opening up the operational modelling sphere across many fields, in empirical, analytical science no less, and probably arguably earlier, than in design and also of the generic nature of the spatial questions raised by the system model.

Where should I, as a designer, go to try and equip myself with an improved contextual understanding? This leads in the next chapter to those who have made an in depth philosophical study of space and geometry.
Introduction

This Chapter starts to explore the question of what contemporary architectural modellers can learn about virtual space from earlier philosophy and discussion in mathematics. In the previous chapter I have exposed, through example, some problems that computational geometrical system modelling presents for a simple Newtonian understanding of space. In the earlier chapters I have also come up against the question of a possible dichotomy of mathematical space and the space of existence. Design modellers to some extent inhabit both simultaneously.

In the following pages I will highly selectively review the relationship between representational, sensory, and perceptual space in the philosophy of mathematics. I seek, in order to test, definitions for the place of geometry in spatial representation and spatial perception that provide the ground for the nineteenth and early twentieth century debate about the place of intuition and logic in mathematics. I explore the divide between, on the one hand, the idea of design as a holistic phenomenological engagement with space and, on the other, the place of pure geometry as a privileged but incomplete description of spatial phenomena. In particular this is related to the digital modelling environment, in which pure geometry and logical relations over geometry are the principal spatial modelling media. The relationship of geometry and space is viewed through a historical, philosophical lens. The term intuition is in its turn examined in the context of the philosophical history of geometry and space. Similarly the historical portrayal of ‘Representative (meaning ‘sensory’) Space’, as oppositional or complementary to Geometrical Space is considered critically. I aim to clarify the distinction between a concept of sensory space and the more central idea of perceptual space that is profoundly significant for the design process.

Finally, I will evaluate the usefulness of the earlier writing in philosophy and psychology for developing a ‘designerly’ understanding and engagement with the geometrically challenging model spaces in contemporary architectural computational modelling. My aim is to test the value of these modes of thought about space for understanding challenges of contemporary system modelling approaches in design.

5.1 Design Space

Previous chapters explored some architectural models that use computation to construct a dynamic system. The model does not represent a particular architectural proposal but a defined set of proposals. It is a proposition and the proposition is a set of relationships that might be fulfilled in the design outcome. The variation in the forms that the design instances of this model can take depends very much on the detail of the model construction – the form of the system. This model with its field of possible outcomes has been described as a space: the design space, the search space, the model space.266

Is this ‘space’, which evades conceptualization and defies visualisation truly Space? The medium of space–making here

is geometrical and logical relations.\textsuperscript{267} While other object qualities that are not considered immediately geometrical may be central to the subject of the model: colour, material texture, light, weight… these must be represented as attributes of geometrically defined objects and relations. Particularly in modelling relations between objects there is scope for an understanding of space outside a geometrical framework. Are the non–geometrical qualitative relations truly spatial? How general or particular in relation to space is the geometric description or understanding?

Here I address the topic of the relationship between geometry and space. I do this with a view to establishing firstly, whether design space in digital dynamical system architectural models fulfils the defining characteristics of space itself, outside a narrow formal definition belonging to the discipline of mathematics,\textsuperscript{268} and secondly, the nature of human engagement in such a space, if such it is. In the introduction I have written that the complex multi–dimensional spaces of computational geometrical system modelling are arguably more logical, less intuitive spaces to navigate, less visible, less accessible to perception than object modelling spaces. Here I will test these assertions in relation to human and cognitive positions on space and geometry in the history of philosophy of mathematics. This tests how the philosophy may inform an understanding of the designers’ potential to project themselves into the virtual space of the system model. It contributes to the idea that there are innate capacities not always tapped as a result of the structures of practice and that it is possible to learn to exercise an innate sense as these structures change.

5.2 Space

Architectural modelling is about translating ideas, intentions and constraints into spatial propositions; making space. The model itself when it is a relational or computational geometrical system model created to represent the design process, or aspects of it, is also a space. Geometry plays a part in both constructing and understanding these spaces. This space is not neutral but charged by particular understandings of the nature of space itself. For this reason, I will include some philosophical background to the question of the nature of space and more particularly the philosophical understanding of the relationship of geometry and mathematics to \textit{spatial knowledge} about the true nature of external reality that will inform the understanding of the relationship of geometry and mathematics as it has developed to \textit{spatial construction}.

\textbf{Space for Newton, space for Leibniz}

Newton’s conception of space as absolute is useful in distinguishing the movement of bodies or matter in space relative to one another from their motion relative to space itself. It is a view that gives to space existence independent of whether there is any matter in it or not and a powerfully objective viewpoint to the observer. Leibniz has a relativistic

\begin{itemize}
\item \textsuperscript{267} Logical relations: Boolean operators, conditionals if…then, for loops, or the simple geometrical associations: all this geometry moves when this point moves, this subunit belongs in this way to this surface.
\item \textsuperscript{268} LeFebvre, H. \textit{The Production of Space}. Translated by Nicholson Smith, D., 1991, 2.
\end{itemize}
view that space is defined by the relationships between objects, their distance and direction from one another. These two views can be characterized as firstly: space as a fundamental structure of the universe that has dimension, in which objects are located and separated, have size and shape and through which they can move and secondly: as part of an abstract mathematical conceptual framework together with number and time, within which we can compare and quantify the distance between objects, their shapes, and relative motion. The first is a container through which things can move; the second is not. Both are abstract in the sense that they are not accessible to the senses or perception. Yet both pertain to the existence of an independent external world outside immediate human thought.

External Reality

Within the history of philosophy the independent external world or reality outside human thought is neither a given, nor its nature uncontested. Morris Kline in his book, Mathematics and the Search for Knowledge, opens with the question, “Is there an External World?” This echoes Bernard Russell’s title ‘Our Knowledge of the External World’ (1914) from which he quotes, “Philosophy, from the earliest times, has made greater claims and achieved fewer results than any other branch of learning.”

The Ancients

From this unpromising starting point, we proceed to discover from Kline that Heraclitus (circa 500BC) does not deny the existence of an external world but claims that everything is constantly changing – you cannot step twice into the same river – so our knowledge of the external world no longer pertains in the next instant. This dynamical foment view of reality gives dynamic system modelling more currency as representations of external reality and projected external realities than idealised static geometrical representations. This is particularly true of emergent model structures. However the rates and scale of change are very significant in finding the appropriate models and appropriate changes on which to focus. Spontaneous change in architecture may be slow in relation to human lifetimes as, the Lamp of Memory of which Ruskin writes or more rapid, as in the contemporary turnover of office fit outs, or real time when we conceptualise kinetic architecture or concern ourselves with material events at the interface with atmosphere at the scale of the molecular. It is intriguing to try and marry up Heraclitus’s external reality with the human, and evidently animal, capacity for perception of stable objects among all the non-repeating sensory inputs to our perceptions as Rene Thom and others have done. The world of Epicurus (371–270 BC) is, by contrast, unchanging. An early

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271 Kline, M. Mathematics and the Search for Knowledge, 4.


advocate of conservation of matter/energy, Epicurus entreats us to trust the senses which tell us that matter exists, motion occurs and the true realities are bodies of atoms in the void. For Plato (427–347 BC) the senses are unreliable and deliver a motley changeable experience; for him the true world is the world of ideas. This is a world readily taken up in architecture, particularly in twentieth century mainstream modernism. Architecture itself belongs variously to the imagination, the world of ideas and of partial representation ahead of translation to physical artefact, accessible to the senses. Plato is an advocate of pure reason in accessing the truth and, for him, the truth, as expounded in *The Republic*, is reliant on the highest order of knowledge that is sensible to humanity; that is mathematics. His dismissal of information attained through the senses is absolute: “we must use the blazonry of the heavens [merely] as patterns to aid in the study of those realities, if we are to have a true part in the science of astronomy.”

Aristotle (384 -322 BC) refutes Plato’s idealism by accepting the existence of a real world external to humanity and maintains that our ideas about the world are obtained by abstracting from it ideas that are common to various classes of material objects, such as triangles, spheres, foliage and mountains.

Kline points out that we might be inclined to dismiss the Greek philosophers, living as they did in a pre–scientific world that greatly emphasized the value of mathematics. However, it is important to keep in sight the foundation of all Greek mathematics, which lies in Geometry – their mathematical world was a constructed one, largely figurative. Greek algebra was limited by ‘unsuitable’ notation but the relationships in factorizing \((a+b)^2\) to \((a^2 + b^2 +2ab)\) represented as areas for example, were well known and understood through geometrical representation (Fig. 85 shows a coin with this representation). Why is this significant for design? David Lachterman has identified a shift in the meaning of construction between ancient and modern mathematical thinking that can be loosely summarised as a change from the reconstruction of given objects to the novel and progressive construction of new problems. This question of construction is significant because design takes place through constructing solutions, it is found to be solution–focused rather than problem–focused, but constructive rather than rigorously
This highlights both the roots that are shared by the activities of spatially subdividing and building in the world, on one hand, and arrival at formalised mathematical understandings on the other (as discussed in Chapter Two).

The Moderns

There is a renewal of interest in the physical world with the birth of modern science in the Renaissance. René Descartes (1596–1650), French philosopher and geometer, is at heart an Aristotelian and Scholastic thinker, although he sees his role as subverting Scholasticism. Seeing the logical possibility that all his beliefs are false he seeks a bedrock for truth in the one fact he can be sure of: Cogito, ergo sum (I think therefore I am). As Descartes understands God to be perfect, He would not be a deceiver, thus the material universe cannot be a deceit. I will argue that his construction of problems in mathematics is much closer to the preoccupations of architectural system modelling – the power of the model to produce – than the Classical construction of geometry as proof of the truth of relations.

Descartes’ contemporary, Thomas Hobbes (1588–1679), English philosopher, responds in his Leviathan (1651) to contemporaneous knowledge acquisition in science and mathematics with a purely physical/mechanical explanation of the external world: external bodies press against our sense organs and produce sensations in our brain. All substance that gives rise to ideas is material. Mathematical activity of the brain detects regularities and produces genuine knowledge of the physical world, thus mathematical knowledge is truth. Contemporaries, even mathematicians, interpret this as a dangerous knowledge hierarchy in which knowledge of mathematics is necessary for a knowledge of philosophy and knowledge of philosophy is, in turn, necessary for knowledge of religion. Moreover the idea of the mind as a mass of matter acting mechanically is offensive to those with a more metaphysical conception of mind; as it has remained until much more recently. Mathematics as an innate activity of the physical brain, as the first means of ordering and creating knowledge as well as the filter for genuine knowledge (as opposed to the direct input from the senses) appears close to the Platonic lineage – yet the primary material for interpretation is not from an ideal but directly from physical sensation.

John Locke (1632–1704), English philosopher and physician, in his Essay ‘Concerning Human Understanding’ (1690) starts from a similar position to Hobbes before him but in contrast to Descartes portrays the mind as starting as a blank slate. There are no innate ideas in humans; all ideas come from experience. He is an empiricist. However, although the mind cannot invent or frame a simple idea, it does have an innate capacity for constructing complex ideas by reflecting on, comparing and uniting simple ones. For Locke, mathematical knowledge is real

277 Kline, Mathematics and the Search for knowledge, 6.
even though it consists of ideas. Physical sensory knowledge is unreliable but a physical world possessing the properties described by mathematics does exist, as do we, and God. Locke, like Descartes, discards all secondary properties. The world is colourless, soundless, senseless, restricted to the motion of meaningless bodies. This primacy of mathematics and abstract mathematical extension in considering questions of space is startlingly prevalent in classical and modernist philosophy, within or without a materialist framework. As viewed from a postmodernist standpoint, this might be understood for its pragmatism and usefulness rather than its truth, veracity or reliability. But the materialist and mathematical arguments of both Hobbes and Locke, are attacked by Bishop George Berkeley (1685–1783), Anglo-Irish philosopher, who placed existence firmly back in the Mind of God. He attacked mathematics, a powerful hegemony in the eighteenth century, through questioning the credibility of instantaneous rates of change, importantly infinitesimals, a young concept, introduced by Leibniz as the underlying concept for his calculus. Infinitesimals were not like real or Archimedean numbers. They were effectively without dimension in the numbering system – no sum of infinitesimals ever reached a real number. In Chapter 6, I shall touch on how they reappear in twentieth century mathematics. Thus Berkeley will accept mathematics as handed down in truth about the world from philosophy and ultimately from the Mind of God but treats novel mathematical invention, conception or representation as subversive or heretical.

David Hume (1711–1776), Scottish philosopher, historian and economist, goes even further than the earlier British empiricists Hobbes and Locke. In Treatise of Human Nature (1739–1740) he consigns mind as well as matter to fiction. We perceive only impressions, and ideas are just faint effects of impressions, he applies scepticism to both mind and matter. He also demolishes the security of mathematical truth – representing mathematical statements as tautology, ‘3 x 3’ being simply another name for ‘9’, rather than a meaningful relationship. His work has been profoundly influential on diverse other areas through, for instance, its reading and influence on his friend, moral philosopher and economist Adam Smith (1723-1790), British naturalist, Charles Darwin (1809-1882) and British biologist Thomas Henry Huxley (1825-1895) and Immanuel Kant (1724-1804) German Philosopher, geographer, anthropologist.

Immanuel Kant, in the Aesthetic in his Critique of Pure Reason, 1781, gave us the proposition that our minds possess, independently of experience, the forms of space and time. He called these forms “Anschauung” translated to English as intuition, suggesting “Our intellect does not draw its laws from nature but imposes its laws on nature.” As expanded below, Kant was an empirical realist and transcendental idealist. While he was not closely aligned with empiricists such as Hobbes, Locke and Hume, he accepts what we see and experience as real, not a representation – and does not concern himself with reality at this level.

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Diversity

The principle purpose of the brief selective synopses above of the contributions of classical and modernist philosophers to the question of external reality is to illustrate the diversity and development of positions on questions of space. *Models* are measured attempts to represent external reality or, more accurately, selective aspects of it and, in the case of architectural design, as in other disciplines, aspects of possible projected realities. It is useful to understand the philosophical and historical context within which we construct these models. The profound influence of these thinkers and their understandings of reality are delivered not only directly through their writing, but also indirectly via their influence in the natural sciences, pure and applied mathematics, political science, and design. In modernist philosophy, mathematics remains central firstly as the true source of knowledge but also via the Empiricist and Sceptical tradition as the human sensibility through which truth is apprehended as well as, in the Cartesian tradition a constructive arena within to construct new knowledge. System modelling has grown out of this history. In order to be able to examine the relationship between geometrical and perceptual space and its place in architectural computational geometrical system modelling, this context needs to be considered.

Philosophy

“Is not the whole of philosophy like writing in honey? It looks wonderful at first sight. But when you look again it is all gone. Only the smear is left.” Einstein


We can read in the abbreviated history of modern Western philosophy above its propensity to take on firstly, the conflict between church and science and secondly the dialectic of rationalism versus empiricism. I have quoted Russell’s dismissal of the impact of philosophy relative to the magnitude of its claims. Leibniz considered that philosophy could teach us the *necessary truths* as opposed to the mere contingent truths of fact. The truths of philosophy hold independent of all the facts we may know. It is only by thought that philosophic truth can be known.

Leonard Nelson also writes: “To put it negatively, philosophy owes nothing to the senses, it is free of all sensible knowledge. Factual knowledge all rests, in fact, on sensible intuition, and can never be obtained by pure thought. Philosophical knowledge, by contrast, is derived from thought alone. That is what makes it philosophical.”

285 and further: “Philosophic skill is really skill in abstraction”.

This makes the eschewal of secondary properties related to the senses (colour, for example) by modern philosophers such as Locke, less puzzling. But Nelson’s separation of philosophy from the perception and sensory knowledge makes the division of the first section of Kant’s Critique of Pure Reason, Transcendental Doctrine of the Elements into the Aesthetic and the Logic curious to the non–philosopher. “For Kant,


285 Ibid., 5.

286 Ibid., 8.
aesthetic considerations are ones pertaining to our senses – to what we see, hear, feel, taste and smell – and have nothing to do with the artistic questions which would now be called aesthetic”. 287 But the Aesthetic centres not on problems about the senses but about space and time. 288

5.3 The spatial claims of modernism (how it differs from classical knowledge)

I will take a step back before moving into any detailed consideration of the possible implications of the content of Kant’s Aesthetic for the directions in which computationally-supported architectural modelling has been leading contemporary design thought in architecture. It is important to try to understand the modern context for Kant’s revolution and its influence on mathematical and philosophical spatial thinking since the end of the eighteenth century. David Lachterman constructs with some care the argument that what lies at the heart of the distinction between ancient and modern (post Cartesian) thought are questions of ‘construction’ itself. 289 At the core of modernist thought is the act or process of making and the power invested in the individual through this creativity. According to Lachterman:

“A fairly direct line runs from the “construction of a problem” (Descartes), through the “construction of an equation” (Leibniz) to the “construction of a concept” (Kant)”. 290

Modernist construction emphasizes the activity of creation itself; the central ontological question is the activity of construction as the proof of the existence of mind. Constructability gives geometry its value but as a system for further making, for further feats of the mind. This is in marked contrast to the constructability in Euclid’s Elements that gives the proof of existence of the mathematical entities themselves. 291

Thus, the transition from the emphasis on the given object in geometry to the actively engendered generative system in mathematics is seen to occur in Descartes’ work in the early seventeenth century.

The common use of the term ‘construct’ for almost any abstract structure, such as, concepts, architectural theories, systems, worlds and “the world”, is, according to Lachterman, not only an index of Kant’s philosophical triumph (which I will return to in the next section) but also, and principally, “the outcome of a signal alteration in the way mathematics itself is practiced and understood in the early modern, pre-Kantian period”. 292

Lachterman also quotes Schelling’s lectures in the philosophy of art where he writes that in the ancient world, everything is eternal, lasting, imperishable and the universal concept of the genera and the individual coincide, whereas

288 Ibid., 4.
290 Ibid., vii.
291 Ibid., xii.
here – in the modern [post Cartesian] world –, the ruling order is *variation and change*, and the individual gains new significance. In this sense, evolutionary thinking about existence (continual progression and responsive change) predates its detailed biological formulation in the nineteenth century in both art and science.

The spectral understanding of ‘construction’ (even with regard to just problems, equations and concepts) is highlighted both by Vico’s issue with Cartesian method, the perception of a conflict between synthetic (Euclidean) methods and analytical geometry; and by Kant’s observation that the *philosopher’s* transcendental concepts, unlike those of the *geometer*, can never thoroughly determine a singular object, independently of some a posteriori perception. Design, as an activity, sits on the other side of mathematics from philosophy in this spectrum, straddling the construction of (1) objects in thought, also potentially inaccessible to perceptual knowledge (except via metaphorical or linguistic representation in early design), (2) perceptually accessible objects, and (3) the eventual reapprehension of the representations of these objects, a posteriori. It has been said that in architectural design, first the solution is built, and then the problem is defined. Nevertheless, the construction of a problem in a sense very close to that of Descartes, (though perhaps less ‘pure’ in the search for one ubiquitous extensible method) is frequently at the heart of the iterative design process, especially for new or novel geometrical situations. The structure of the Beijing Watercube, is, once more, a good example of this. The geometrical problem was reconstructed several times, modifying the goals and constraints before the right packing problem for the particular fabrication, cost and structural context was constructed. I am using the term ‘object’ above in the loose sense of *the object of the act of thinking and making* – not to the exclusion of spatial arrangements that might not immediately be described as ‘objects’ in the everyday lexicon of design usage. Architecture is linked to geometry in the ancient world through reverence for the *pre-given object* and through *beauty*. Architecture is linked to geometry in the modern world, through the partially analytical *process of construction* and through *interest*.

One of the central arguments here is that the correspondence between modern geometry and modern architecture is substantially anachronistic or at least asynchronous. The modernist break from the past occurs much earlier in mathematics and philosophy than in architecture. The modernist shift may not be evident in the architecture of the early seventeenth century. It does begin to appear the writing of Viollet le Duc, in the nineteenth century, is strongly evident in the architecture of Antoni Gaudí by the early twentieth century but it is consummated most explicitly in the world where the virtual spatial *domain* of the expert mathematical mind is partially represented in the spatial representational possibilities of extensible digital computation by the late twentieth century. Paradoxically, much of the architecture that is considered to be the product of high

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293 Lachterman attributes to Friedrich Schlegel the naming of the shift from “the Beautiful” as the principle of antiquity to the “interesting” as the principle of modernity. Lachterman, *The Ethics of Geometry*, 3.
[architectural] modernism reverts to the ancient geometrical context and values of the given object and beauty.

Frédéric Migayrou in 'non-standard orders: nsa codes' aims to expose the imposition of standardisation and the industrial norm, or type, on Modernisme in architecture, particularly following the formation of the Congrès International d’Architecture Moderne (CIAM) at Chateau de la Sarraz, Switzerland 1928. Migayrou’s overall objective is to locate ‘non-standard architecture’ as the resumption of earlier (pre-CIAM) modernist ideas and ideals that were partially subverted by the idea of standardisation for industrial production. The essay is written originally in introduction to the non-standard architecture exhibition, which Migayrou curated in the Centre Pompidou, Paris in 2003. Migayrou acknowledges the mathematical connotation of the term non-standard with reference to the work of Abraham Robinson’s non standard analysis in mathematics. (I will explore the implications of Robinson’s work further in the following chapter in the section 6.2 Intuitionism, formal and informal.)

Of the spatial and philosophical nature of Robinson’s proposition, Migayrou writes:

"Beyond a mere debate between mathematical formalism and intuitionism, non-standard analysis posits a dynamic structuralism, an abstract semantics that underpins the interrelation between phenomena and meaning."

In other words, Migayrou takes the mathematician’s space as the point of departure for exploring the possibility of the definition or specification of a non standard architecture. There is another ingredient: resistance to the normative or standardising forces of production. He quotes Bernard Cache:

“Although the rise of the standardised object did lead to the idea of variability, this notion remained limited to a repetition of type, a mismatch between the real aesthetic determination of variation circumscribed by the avant–gardes and industrial production limited by the Taylorism of the series...There are no longer pre-established functions requiring a form; we have only the occasional functions of fluctuating forms.”

This reference could be to the Taylorism of Brook Taylor (1685-1731), English mathematician best known for Taylor’s theorem and Taylor series (the representation of a mathematical

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296 Congrès International d’Architecture Moderne (CIAM) 1928-1959 a series of events and congresses around the world organised by the leading architects of the time with the aim of proselytizing the principles of the Modern Movement in all the main domains of architecture, including landscape, urbanism and industrial design. The first congress was organized by Le Corbusier and Sigfried Gideon

297 Abraham Robinson (1918 – 1974), Jewish mathematician, born in Germany, immigrated to British Mandate of Palestine, 1933, subsequently in London, Toronto, Jerusalem and finally, University of California, Los Angeles. His Non-Standard Analysis is an approach to Calculus that reintroduces the idea of infinitesimals, originally proposed by Leibniz.

298 Migayrou, non-standard orders: ‘nsa codes’, 17.

function as an infinite sum of terms calculated from its derivatives at a single point). The function referred to in the Cache quote however refers to use of the designed object, not to the mathematical functions that the Taylor series is used to approximate. This charged ambiguity (between references to mathematical/geometrical ideas in the ‘generation of infinite forms’ and ideas about rationalism in ‘standard’ modern architecture and industrial production) continues on the next page where Taylorism appears again in Migayrou’s essay:

“the Taylorism of the Gilbreths does not assert a simple mechanisation of the body; it is, to use the words of Siegfried Giedion, “the intervention of the machine in the very substance of both the organic and inorganic”.

One must assume that the Taylor in this quotation is Frederick Winslow Taylor (1856-1915), the American mechanical engineer who sought to improve industrial efficiency, (coined the father of scientific management), contemporary of fellow advocate of scientific management, and motion study pioneer, Frank Bunker Gilbreth (1868-1924) and his wife Lillian Moller Gilbreth (1878-1972) engineer, PhD, industrial psychologist. Taylor and the Gilbreths diverged in their foci – the Gilbreths focused on minimizing motion (and, allegedly, on worker welfare), Taylor, on the stopwatch, speed of production (and, allegedly, on profit.)

In this way Migayrou’s essay is assembled playfully, collage-wise; fertile in ideas, highly populated rather than authoritative. Nevertheless, the Schelling’s art-philosophical definition of the motifs of modern as variation and change are consistently evident in Migayrou’s exploration of the non-standard in architecture. Moreover, geometry is the main link between (1) the rejection of norm in early architectural modernism and (2) the rejection of norm in the non standard manifestations of the post-digital era.

Migayrou writes that the “mathematization that allows one to keep Gestaltung within a rational and geometric framework flows directly from J.L.M. Lauweriks… searching for a geometric transcription of the world inspired by theosophist doctrine.”

300 Migayrou, non-standard order: ‘nsa codes’, 19.

301 Siegfried Gideon (1888 – 1968), Bohemian born Swiss architectural historian and critic, author of ‘Space Time and Architecture’ (1941) and ‘Mechanization Takes Command’ (1948). Migayrou refers to the “still neo-Kantian aesthetic positioning of Gideon…” (Migayrou, non-standard orders: ‘nsa codes’, 23.)

302 Migayrou, non-standard orders: ‘nsa codes’, 22.

303 Gestaltung. The first English translation of Gestaltung in the Beolingus online German-English dictionary is fortuitously ‘construction’ which brings us tidily back to Lachterman’s thesis on the distinction between ancient and modern geometry. Gestaltung is also understood as ‘design’, ‘arrangement’, ‘formation’, ‘configuration’ so we should perhaps take it here to mean the vagaries and specificities of the design process.

304 Johannes Ludovicus Mathieu (Mathieu) Lauweriks (1864 - 1932) was a Dutch architect and architecture professor. Park and monument to the fallen design during 1st World War: Weltkriegsdenkmal 1915. The geometrical inventiveness of the Weltkriegsdenkmal is constrained to the plan – a mirror symmetrical pattern of curving quasi-organic forms.

305 Migayrou, non-standard order: ‘nsa codes’, 25.
Of the work of Raoul Hausmann, Migayrou writes: “each form was a frozen moment-image participating in the creative aura of the atmosphere (fluidum), a component idea for Hugo Haring, Mies van der Rohe and Le Corbusier that had to contain the entirety of the normative immobility of the type and an opening to vitalism.”

Migayrou also writes of the order of movement and inflection, the early expressionism origins of the Werkbund, played down in later histories of modernism. He links the influence of Henri Poincaré’s *Analysis Situ* to the comprehension of a mathematical continuum and new cognitive domain bringing together Duchamp, Van Deosburg, Mondrian, Malevich, El Lissitzky and Buckminster Fuller such that “one has trouble understanding how the questioning of standardization of normalization of production never reappeared…” But he finds that “Normativity remained riddled with the relativism of the Gestaltung, which perhaps allows us to restore the post-war work of Le Corbusier to this continuity, i.e., of Ronchamp or the *Poème électronique* and the Philips Pavilion.”

This lateral exploration into the possible relationship of modern architecture to modern and ancient geometry (according to the Lachterman’s proposed division) has arrived at a staging point of continuing ambivalence. The ambivalence lies in Migayrou’s architecture and architect examples, which (and who) broadly appear to retain their allegiance to both ancient and modern epistemologies. There is also ambivalence in the interpretation of the intent of the earlier examples of pre-CIAM, pre-automated-computationally-supported architectural work and the extent to which their authors have the means to embrace the variation and change, the pathway to the new, inherent in the modernist and post-modernist geometrical oeuvre. Catalan architect, Antoni Gaudí remains an exception, working as he did primarily in three dimensions and able to fully embrace morphogenetic ideas using second order curved surfaces as an apparent development of freeform representations relating geometry closely to observation of natural form.

**Ratio in geometry and arithmetic: the issue of homogeneity**

I will continue to explore the ancient – post-Cartesian divide more closely. Lachterman devotes a long chapter to the questions related to what construction involved for the Greek geometers, how its contribution to the articulation of geometry as a science is evaluated by the geometers themselves and by philosophers. This includes questions about analysis in Greek geometry and the locale of Greek geometry, “something that may be foreign to the modern conceptions of extension and space”. The theory of logos, or ratio, which Euclid defines as a sort of relation in respect of size between two magnitudes of the same kind, is central, although Euclid’s definition is widely criticised subsequently as metaphysical, rather than mathematical,
given that nothing depends on it. Euclid is quoted from Book V of the Elements as saying that it is necessary to speak of *homogeneous magnitudes* because magnitudes, which are not homogeneous, have no ratio to one another. “For a line no more stands in a ratio to a plane-surface that a plane-surface does to a body; however a line is comparable to a line” etc. In other words, a ratio is never reduced to a [rational or irrational, real or imaginary] number”. Mediaeval commentators take from this that all straight lines in respect of one another are of one genus, curves of another, etc. But the Archimedean Axiom discriminates between two subclasses of a single genus on this basis, for instance a finite and an infinite line cannot stand in a ratio to one another even though they are of the same genus. Euclid himself ‘opens the door a crack’ to transgression of homogeneity when he introduces the technical operation alternando (a:b::c:d, then a:c::b:d) without calling attention to the homogeneity requirement for all four terms. This also raises the question of the naturalness of the three dimensionality of classical geometry: if I multiply a line segment by itself three times, I can compare the volume of cube $a^3$ to the volume of cube $b^3$ but the same technique or operation produces $a^6$ just as easily. Compounding of ratios is fundamental to post-Euclidean mathematics, but little commented on, Lachterman notes, despite forming the basis of an intriguing cross over between arithmetic (number) and geometry.\textsuperscript{312}

Descartes advises Desargues that he could make the demonstrations of his new projective geometry "more trivial …by using the terms and calculus of arithmetic…for there are many more people who know what multiplication is than know the compounding of ratios …”.\textsuperscript{313} It seems that there is a shared conviction that we draw our understanding of ratio and “being in the same ratio” from the case of numbers (even though, in the context of geometrical homogeneity, number is just one genus) so any extension of the understanding of ratio has to be made rigorous on the basis of arithmetic.\textsuperscript{314} But another set of issues arises in relation to Euclidean principles (which do not contain the principle of continuity), and Zeno’s paradox (consider Achilles and the tortoise).\textsuperscript{315} These resurface in the work of Weierstasse\textsuperscript{316} and Dedekind\textsuperscript{317} in the attempt to “ground a complete theory of real numbers as ratios of magnitudes”.\textsuperscript{318} An infinitely divisible continuum cannot be reconstituted out of an (denumerably) infinite number of discrete and indivisible points. Dedekind’ solution, building on the work of others, is to assume that the geometrical line is essentially analogous

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313 Ibid., 38.
314 Ibid., 42.
315 Zeno’s paradoxes: Achilles and the tortoise is the first of these: In a race against the much slower tortoise, Achilles gives the tortoise a head start of 100 metres. While Achilles runs this 100 metres, the tortoise has covered another, shorter distance, say 10 metres. While Achilles runs this 10 metres, the tortoise has once more covered a shorter distance, say 1 meter, while Achilles runs this 1 metre….etc.
316 Karl Theodor Wilhelm Weierstrasse (1815 – 1897), German mathematician, sometimes called ‘father of modern analysis’.
317 Richard Dedekind (1831 – 1916), German mathematician, abstract algebraist and number theorist.
318 Lachterman, *The Ethics of Geometry*, 43
\end{flushright}
to an ordered arrangement of “numbers”, each of which is uniquely definable on the basis of other “numbers”. This is a very modern transition (the possibility of slippage between a line and an infinite sequence of numbers), expedient, and clearly incompatible with Euclid’s homogeneity criterion.

The ancient theoretical and the modern problematical

A strong component of the ancient –modern divide within Lachterman’s argument is based on temporality. Speusippus tells us that knowledge is timeless and therefore the implication that a construction brings something into being that was not before should be discounted…it is better to say that all these things (such as isosceles triangles) are and we observe the coming-into-being in construction in the manner of recognizing what is. This is a theoretical rather than a problematical treatment. It is a position reinforced in Euclid’s Elements by the pervasive use of the perfect passive imperative tense: “let it have come about that”…”let there be a triangle with…” There is a prior need to be acquainted with the nature of a figure before commencing construction, in Euclid, and the use of language is sensitive to the nature of the particular figure. It is, by Lachterman’s account, well aligned with Plato’s doctrine of the Mathematicals, seeking acknowledgement that indefinitely many intelligible instances of each geometrical kind are sufficiently related that no accident of graphical representation will distort their shared nature.

By contrast, Descartes’ approach is entirely problematic. He is concerned with making visible the mechanics and operative arts, concealed in the received arts and sciences. There is a shift in attention from the natural to the artificial languages, to artfully devised images. From now on nature is to be measured by its accessibility to artifice “All things which are artificial are natural as well.” Paul Valery said of Descartes, “…in everything, he took his Self, of which he was so powerfully aware, as the point of origin of the axes of his thought.” Of all the possible definitions and understandings of “modern” (since Cassidorus in the 6th century, for whom it meant to imitate or emulate the ancients), Descartes took the most modern meaning: the novel, universal and irreversible change from the ancients and their traditions. Descartes generalized, by reducing all problems to a single type of problem, with its associated constructive solution: “It is only necessary to follow the same course in order to construct all problems, more and more complex, ad infinitum.” This course is to find roots, which are actually line segments that can be drawn, the lengths of which determine the distances of points from previously selected principal lines or axes and thereby the graph of the curve on which all the relevant points fall. To draw these lines or the curves they define is designated “constructing problems.” Descartes discovers a strict correlation between the number of lines involved in the problem, the degree of the equation of the curve on which the points lie (degree = highest exponent) and the degree of the simplest...
curve that can be used in actually constructing the locus defined by the corresponding equation.\textsuperscript{325}

“Descartes’ ‘Geometry’ is devoted exclusively to solving problems and not at all to proving theorems, and, underlying the procedures he follows, is the desire to exhibit \textit{virtuosity} – that is, by increasing the number of unknowns (thus rendering the problem and the equations seemingly more complex) and simultaneously decreasing the number of presupposed theorems, Descartes will succeed in finding a solution by the shortest and most artful or ingenious path”.\textsuperscript{326} As an echo of this Cartesian zeal, the pursuit of virtuosity per se is, as an idea, is very familiar in design from the recent years of computational architectural modelling fuelled by a strong collective belief in the potential power of computation as a tool or technique for generating the genuinely new.

\textbf{Modernism, the mechanical, and computational design modelling}

Descartes’ emphasis on problems (rather than theorems) is closely linked with the promotion of \textit{analysis} or the ‘art of invention’, an art celebrated at the expense of Aristotelian syllogistic logic. Syllogistic logic, in Descartes’ view, through the synthesis of two knowns for deductive purposes, produced nothing new, a mere rearrangement of knowledge, whereas the ‘art of invention’ produced genuinely novel knowledge and when re-enacted for the benefit of others, allowed them to participate in the discovery.\textsuperscript{327} These aims are echoed in the work of the programmer developing the CAD program or 3D modelling program that will bring them or other designers into participation in invention and the necessary steps to go from known geometrical territory to new, programmatically defined fields of possible design solutions. But the virtual workspace of the computer program exhibits issues familiar from this account of the divide between classical and modern mathematical thought. The issue of \textit{geometrical homogeneity} that roots the model components to clearly understood, stable geometrical primitives or objects, understood as instances of type, sails close to the Classical forms and essences that Descartes sought to banish – the logic of the program may enforce a level of geometric homogeneity through equilibrated units. Yet, the overall appetite of the computational designer is for the fluid interface between the geometrical and arithmetical, after such modernist thinking as Dedekind’s, that allows the anchor to the physical and perceptual to dissolve and reform according to virtuosity, economy and deftness of production. It is born of the eternal (modernism has no end point and new dawns) modernist impulse. The elevation of the role of the machine in the design process demonstrates the continuity of modern thinking since the seventeenth century. Isaac Newton in his preface to his ‘Principia’ states: “Geometry is nothing but that part of universal mechanics which exactly proposes and demonstrates the act of measuring.”\textsuperscript{328} Of the Classical position, by contrast, Lachterman writes: “I have said that mechanical constructions have been especially conspicuous in \textit{modern} accounts of the Greek tradition…” yet, “All the constructions in the Elements

\begin{itemize}
\item \textsuperscript{325} Ibid.
\item \textsuperscript{326} Ibid., 148-150.
\item \textsuperscript{327} Lachterman, \textit{The ethics of Geometry}, 152.
\item \textsuperscript{328} Newton, I. \textit{Philosophiæ Naturalis Principia Mathematica}, 1687
\end{itemize}
are in fact performable using only ruler and compass; … not only do Euclid and his commentator Proclus say nothing about this, but on the evidence of the text, further restrictions on the allowable use of these simplest instruments were part of Euclidean strategy.”

“Plutarch on two occasions has Plato reproaching Archytas, Eudoxus and Menaechmus for devising “instrumental and mechanical constructions” in response to the Delian Problem…” - the noetic and aesthetic are sharply divided in orthodox Platonism.

Descartes separates curves into geometrical and mechanical curves. For him, only the former are to be received into geometry. His criterion for geometrical curves is that they are created by only one continuous motion or if several motions succeed one another in a way that the later motions are “completely regulated by those that precede them”. This more or less corresponds to Leibniz’s distinction between algebraic curves, in the equations of which only rational numbers can appear as exponents, and transcendental curves for which the equation include irrational or indeterminate exponents, for instance: $y^{\sqrt{2}} + y = x$.

While Descartes was thoroughly familiar with some of these transcendental curves – he studied the logarithmic curve – he excluded them from the body of geometrical knowledge both on kinematic grounds and because the instrument used to generate the relevant curve is more complex, algebraically, than the curve it generates. Classical geometry uses the same terms (geometrical and mechanical) but quite different criteria. Lachterman links the interplay of continuity and punctual discreteness in Descartes’ geometrical deliberations to the “continuous and nowhere interrupted motion of thinking” in Descartes’ modernism – or, Cartesian minds always on the move.

329 Lachterman, The ethics of Geometry, 71.
330 Ibid., 73. Noesis means the exercise of reason, while Aesthetics refers broadly to the sensory, or concrete. The Delian problem is the problem of doubling the cube, or, of taking a cube of given side length L and volume V and finding the side length of the cube of volume 2V in terms of L. This was a very difficult pure and applied problem in Classical geometry insoluble through the use of compass and ruler. Plutarch (46-120 CE), Greek Historian from close to Delphi, Plato (427-347 BC), Classical philosopher and mathematician, Archytas (428-347 BC), Classical philosopher, mathematician, astronomer, statesman, Eudoxus of Cnidus, (408-347 BC) astronomer, mathematician and student of Plato and Menaechmus (380-320 BC), mathematician and geometer, friend of Plato and possible discoverer of conic sections.
331 Lachterman, The ethics of Geometry, 171. The exponent is the index in superscript to the right of the base number that defines how many times that base number is multiplied by itself (or the number of times that the ratio is compounded to use the Classical language) e.g. in $2^3$, 3 is the exponent, 2 is the base. (The ratio 2 is compounded 3 times, in this example.)
332 Lachterman, The ethics of Geometry, 170.
333 Ibid., 173.
Descartes, dimensionality, homogeneity and what is natural

Michael Stifel 1486 – 1567, Augustian monk and mathematics professor, is said to have introduced the practice of setting all the terms of an equation, known and unknown on one side and setting them equal to zero. This is a very novel practice in the context of the Greek conviction that ratios are always relations between instances of *manyness* (geometrical or numerical). Similarly, given the ultimate materiality of architectural design preoccupations, this is a potentially uncomfortable organisational or notational device for contemporary architectural modellers, were it not now so familiar through general mathematical education. It underscores the separation of the mathematical generation of the models from the substance of what they are ultimately intend to represent.

Descartes is responsible for divorcing dimensionality and homogeneity from the figural attachments they enjoy in the Euclidean tradition. For Lachterman this impact is incalculably more important than Descartes’ computational stratagem (to be able to find curves, or values for unknowns in an equation). Descartes subverts what is called here the “pre-theoretical” or “natural” understanding of dimensionality as intrinsically characterizing genera of magnitudes such as one-dimensional lines; two dimensional plane figures; three dimensional “solids”. All Cartesian magnitudes are homogeneous as dimension no longer belongs to these “shapes” but a numerical indicator of the sequential order and relative measurement of the terms in an algebraic equation. At the same time he both pays lip-service to homogeneity and retains the traditional nomenclature of “squares”, “cubes” and so on for these abstract relations. This is framed as a pedagogic strategy. His students are drawn into the new conception of dimensionality (a quantitative field closed under permissible operations) and simultaneously held comfortably by the old visual or spatial understanding for which the multiplication of a square by a line segment, for instance has no meaning. At the same time Descartes writes to Mersenne in 1638: “the whole of my physics is nothing other than Geometry.” He distinguishes between “abstract geometry” and a different sort of geometry to explicate the phenomena of “nature”. So his new geometrical homogeneity and dimensionality is not divorced from aesthetics or the description of the physical world. Yet not every solution to an algebraic equation is constructible to Descartes. “Only ‘true’ and ‘false’ roots are admissible as authentic solutions to geometrical problems since there is no ‘space’ in the local expanse determined by the principal lines [in Cartesian Space] in which ‘imaginary’ roots can be inscribed”.

335 Ibid., 164.

336 Ibid., 167.

337 Marin Mersenne (1588 – 1648), French theologian, philosopher, mathematician, and music theorist. Mersenne Primes (a prime number (positive integer) that is one less than a power of two (2\(^n\)-1). Mersenne was an important figure in the birth of the science of acoustics and maintained a key correspondence network between other scientists and mathematicians across Europe at the time.


339 Imaginary roots are complex numbers, not real numbers, which offer solutions to an equation. For instance, the equation \(x^2+1=0\) has the roots \(i\) and \(-i\) where \(i\) is \(\sqrt{-1}\). The square root of minus one is an imaginary number.

So it is into this post-Cartesian modern world, in which heterogeneous mathematical entities can be compared in algebra, and dimensionality already sits ambiguously between its “natural” attribution to figurative shapes or forms and a more abstract, extensible and exchangeable mathematical currency, that Immanuel Kant (1724 – 1804) is born. It is, moreover, a world in which what is and what is known (united in ancient and mediaeval thought) have already been torn asunder into an epistemological foundation located in the human as subject and an ontological foundation located in the world as object.

5.4 Immanuel Kant, Space and Time: the theory of Pure Intuition

In the Aesthetic in his Critique of Pure Reason, 1781, Kant gave us the proposition that our minds possess, independently of experience, the forms of space and time. He called these forms intuition, suggesting “Our intellect does not draw its laws from nature but imposes its laws on nature.”

Kant’s analysis of knowledge from sensible intuition led him to an understanding of a distinction between what he called the matter and the form of sense-intuition. Objects affecting our senses, give us only perceptual qualities:

As objects of pure intuition we have on the one hand space, and on the other, time. Space and time are not actual things, but are what make existence possible for spatio–temporal things. ‘Things’ pre–suppose Space and Time, nor are Space and Time mere relationships between things, for spatial and temporal relationship exist only where Space and Time are presupposed.

Kant overturns both Newton’s theory of absolute space (the space of the objective observer that exists independently of the things in it) and Leibniz’s theory of space and time based on the relationships between things. But while space and time are, in Kant’s construction, the foundations, which make it possible for a world of the senses to be, mathematics (based on ‘pure intuition’) remains the basis for its assessment. Moving smoothly from space to the propositions of geometry as though there were little distinction between the two, he wrote:

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343 Nelson Progress and Regress in Philosophy From Hume and Kant to Hegel and Fries, 118. Pure intuition is neither empirical (pure = not empirical) nor from thought (intuition = not from thought, or at least not derived through logical process). (Ibid.,123.)

344 Ibid., 119–123.
“Geometry is a science which determines the properties of space synthetically, yet a priori.”

For insight into Kant’s statement that geometry and space conform to synthetic a priori intuition, refer to Philip Kitcher’s paper *Kant and the Foundations of Mathematics.* Kant’s strict division of analytic and synthetic is important because whereas Leibniz viewed knowledge from intuition as being confused and needing deducing from conceptual knowledge to make it clear, Kant saw these two types of knowledge as so different in origin, that it is impossible to move from one to the other. Thus the division between analytic and synthetic is definitive. Analytic knowledge is derived from logic. Metaphysics, for Kant, has a set of rules quite distinct from those of logic. Objects and systems, from the Kantian viewpoint, are not so much constructed by use of geometry; geometry is the necessary intuitive context in which objects and systems can be conceived.

But Kant also wrote that:

“Geometrical principles are always apodictic, that is, united with the consciousness of their necessity, as, ‘Space has only three dimensions.’”

Were such truths empirically derived it would only be possibly to say, “so far as hitherto observed, no space has been found which has more than three dimensions.” This implies Kant’s adherence to the three dimensions of space as necessary truth, for, in order for other types of space to exist, geometrical knowledge would have to be a posteriori, or, in other words, empirical or from perception.

Whether Kant would have seen the truths of hyperbolic and Riemannian geometry as no less self-apparent and inherently

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345 Synthetic vs Analytic. Synthetic statements are those that introduce new information in the predicate. In their antonym, analytic statements, the predicate can be derived from the concept alone. For instance ‘the model is morphogenetic’ is synthetic as the idea of morphogenetic cannot be derived analytically from the idea model but ‘the shortest distance between two points is a straight line’ is only synthetic as long as a straight line is in fact distinct from the shortest distance between two points. If ‘is a straight line’ can be deduced from ‘the shortest distance’, it is analytic. That the geometrical understanding of space is synthetic, rather than analytic, is significant in separating the foundations of mathematics from the process of logical deduction. Analytic judgments can be formed from concepts alone; synthetic ones cannot.

346 A posteriori vs a priori. A posteriori knowledge comes from experience and is based on the perception of some object or phenomenon. A priori knowledge is independent of experience, and so is not based on perception at all. Thus synthetic a priori intuition, the nature, for Kant, of spatial knowledge and the foundations of geometry, is knowledge, which cannot be logically deduced, is not derived from empirical experience, and is not from thought.


348 Kitcher, P., “Kant on the Foundations of Mathematics”, in Philosophical Review (1975). Kitcher writes “The heart of Kant’s views on the nature of mathematics is his thesis that the judgments of pure mathematics are synthetic a priori.” Kitcher then proceeds to break down this single thesis of Kant’s into two sub-theses, one metaphysical, the other epistemological.

349 Ibid., 117.


351 Ibid., 69.
true than other geometrical understandings were they offered to him and whether higher dimensional spaces would belong, for him, to the analytic rather than synthetic are questions for speculation or philosophical interpretation.

By Kant’s proposition of pure intuition, synthetic a priori intuition we know certain spatial and geometrical ‘truths’ neither empirically nor analytically by derivation from others, but simply because we know them. Put crudely, because space and the fundamental axioms of geometry are an inherent facet of our way of knowing the world, we cannot think outside them. The example in the Aesthetic in The Critic of Pure Reason is the fact that three lines can enclose a planar space, while two cannot. The knowledge from Euler’s graph of the Königsberg bridges appears similarly intuitive. We know at once that we can cannot travel continuously along the edges of the network diagram and pass each once and once only.

Euler’s graph, credited with beginning Graph Theory may, however, deal with geometrical definitions outside the immediate scope of Kant’s definition of a priori intuition. The Space of which Kant is writing can be taken to be Euclidean geometrical space, homogeneous, three-dimensional, just as his Time is a continuous linear sequence. Did he consider a broader topological spatial framework? Although he wrote in 1781 and Euler of his seven bridges in 1736, it is possible that he did not.

While it is a potential paradox that a priori intuition might include ‘discoveries’ or ‘constructions’ yet to be, Kant conveniently gives us the line “Intuition takes place only in so far as the object is given to us” which leaves room to imagine that the geometries that burgeoned in the nineteenth century might also be the subject of pure intuition rather than logical inference. This offers the potential too for aspects of Hyperbolic and Riemannian geometries and the greater geometrical framework of Group Theory also to belong to the space of Kant’s pure intuition, once given.

Was the Space of which Kant wrote, the continuous, homogeneous, three dimensional space of object modelling (and figurative geometrical proofs) or can it be considered also to encompass the spaces in which systems are modelled with their potentially bifurcated, multi-dimensional geometry and propensity for both discrete and continuous behaviour?

Design, as an activity, whether of architecture, graphics or political systems, unlike pure mathematics, must remain concretely grounded and maintain a direct mapping between its models and the modelled: their imagined realization in the world. It is hard to conceive of the absence of intuition in the barely mediated leap back and forth between the imagined ‘real’ or signified world and the model or signifying context.

But, this comparison of design intuition with the translation of Kant’s use of ‘anschauung’ as intuition might need to be qualified following careful consideration of his choice of term. The computational design model may perform synthetically (provide design solutions) and analytically (performance of


353 Ibid., 5

354 The comparison of ‘design intuition’ with the translation of Kant’s use of ‘anschauung’ as ‘Intuition’ must be tempered by the more careful consideration of his choice of term in the paragraph that follows.
evaluations). The underlying logic of the model comes from the analysis of the context but its implementation is primarily synthetic and largely intuitive. The lengths to which the analogy between design modelling and mathematics can be extended must also be tempered by the knowledge that design is a search for form and fit and mathematics for truth or consistency.

**Intuition**

Norman Kemp Smith translated the German word *anschauung* by the term “intuition” in Kant’s work. In contemporary usage, the English translation is “idea”, “opinion”, “view” from the etymology looking at. Yet Nelson affirms that intuition in this context means ‘not from thought’. These are ideas that we believe, opinions that we hold, views that we have, that, according to Richard Robinson, we cannot help but hold. They come not from thought but from some more fundamental source.

Kemp Smith’s use of the term *intuition* in English to translate Kant’s use of “anshauung” is contentious. This is a type of knowing that neither requires deductive reasoning and analysis, nor comes from experience. We can argue long and inconclusively about how close that lies to the common contemporary meaning of the English word *intuition*. What *anschauung*, as used by Kant and *intuition* in contemporary English usage almost certainly have in common is reference to knowing in the absence of analytical process, but whether both exclude a role for empirical experience is more questionable.

Kant writes, “By means of sensibility, therefore, objects are given to us, and it alone furnishes us with intuitions; by the understanding they are thought, and from it arise conceptions. But all thought must directly, or indirectly, by means of certain signs, relate ultimately to intuitions; consequently, with us, to sensibility, because in no other way can an object be given to us.”

He also writes: “The undetermined object of an empirical intuition (intuition that relates to an object by means of sensation) is called phenomenon. That which in the phenomenon corresponds to the sensation, I term its matter; but that which effects that the content of the phenomenon can be arranged under certain relations, I call its form.” [my italics]

Thus, in Kantian terms, our spatial intuition divides into matter and form. Hence, perhaps in this philosophical framework there is not too much distinction between the space of the world that we perceive and the space that we create by design, or as Evans would have it, by projection. At some level both conform to the matter of phenomenon. On the other hand, this division into matter and form fits conveniently the underlying nature of the virtual system model. The structured relations of the model are its form, the instances or manifestations that can be seen, held in space or otherwise witnessed via sensation are, in deference to Kant, its matter. Each of these domains engages our intuition: presents aspects of the spatiality of the model that we cannot help but know.

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355 “Anschauung”. Beolingus online dictionary, ©TU Chemnitz 2006-2010

356 “Anschauung”. Collins Concise German dictionary.

357 Walsh, *Kant’s Criticism of Metaphysics*, 11.


359 Ibid., 21.
Empirical realism: transcendental idealism

Kant is an empirical realist and a transcendental idealist. He criticized Locke, Hume and Berkeley who separated what we can know of an object (via the senses) from its essential existence or nature. He considered that they were talking about a representation of the object. For Kant the human is at the centre, he starts with the notion that what you see is real because that is all you know. What the thing is really is a metaphysical question. He believes that it is an empirical mistake to see the world as ideal. This is very far from the Platonic position. This is also very distant from the traditional practices of classical and post renaissance architecture in which the space and form are idealized, and, through representation, the ideal is known empirically in parallel to empirical knowing of a building in all its sensory profusion and confusion. This is also relevant to architectural modelling and the problem of apprehension of a model space or system in which the geometrical complexity denies figurative knowledge of the system, or comprehensive predictive knowledge of the character of the space in which the designer is working. Effectively the totality of the system and the character of the overall space in which they are working may be, for the designer, in the sphere of that which cannot be known through the organisation of its relations (Kantian form–intuition) or empirically via sensation (Kantian matter–intuition), and thus in the realms of metaphysics. This overview knowledge is in the realms of the ideal. While, within this Kantian framework, the manifest instances that can be represented figuratively in images, video, three-dimensional prototypes and built systems, and the form, represented symbolically through graphs, code or other symbols, is the real model. This avoids risking the empirical mistake of idealising the model. We can also regard the model as a possible world, albeit in general, a highly constrained one. However this construction does not fully take account of the human-centred operational spatial domain offered by Descartes liberation of dimensionality from homogeneity constraints and the fluid interplay between geometry arithmetic and algebra (which he regarded as purely representational rather than a third topic in its own right).

Space and geometry

As the propositions of geometry are “recognized synthetically à priori, and with apodictic certainty”, where is it, Kant asks, that they, with this inherent certainty, come from? He looks for a possible analytical route in logic. How is it that we know that two lines cannot enclose a space or that three can? We cannot deduce such a fact from the conception simply of a straight line and the number 3 but are “forced to have recourse to intuition, as in fact geometry always does.” But, what kind of intuition? It cannot be by empirical, a posteriori intuition from experience because experience never can furnish a universally valid and unquestionably true proposition. Returning to the triangle, he argues that "if the….triangle….were something in itself, without relation to you the subject; how could you affirm that that which lies necessarily in your subjective conditions in order

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361 Ibid.
to construct a triangle, must also necessarily belong to the triangle itself? For to your conception of three lines you could not add anything new; which therefore must necessarily be found in the object ... the object is given before your cognition, not by means of it.”

What does the Kantian view of space mean for the creation of architectural space?

Firstly Kantian space and geometry are inseparable. They are both given by à priori synthetic intuition. Geometrical truth cannot be derived analytically, or through logical process. It is inseparable from Space, neither are things in themselves but rather the intuitive basis on which the existence of spatio–temporal phenomena are possible. It is on the basis of the received truth of geometrical propositions that we proceed to “construct” architectural space (and geometrical axioms). Objects and systems are not so much constructed by use of geometry; geometry is the necessary context in which objects and systems can be conceived. In this definition, architecture, even when reendowed with all its philosophically secondary properties: colour, movement of light, quality of sound, lightness or weightiness, is nevertheless inconceivable in the absence of geometry just as it is impossible to ‘create’ space in the absence of space. So does Kant’s Aesthetic provide a useful framework within which to elucidate the virtual space of architecture system modelling?

Firstly, the idea of space, and by implication, geometry, as the necessary context for phenomena to exist, rather than things in themselves, provides a partial explanation for the difficulty of the design modeller’s immanence in the space of the system model and the challenge of attaining an objective overview or reasoning over the model, within the model. The system model by contrast to the object or single instance model is a potentially extensible microcosm of space. It is a representation of space but simultaneously space, less easily constrained to include only predictable states than an object or ‘phenomenon’.

Secondly, Empirical Realism implies the unreachability of the ideal. Within this doctrine, the perfect model with absolute correspondence to the signified, and highly controlled behaviours developed through mathematical and logical means is an ideal, belonging to the realm of metaphysics.

Thirdly, it opens a quandary with respect to whether this is an edifice of thought that rests firmly and exclusively on the “apodictic certainty of three dimensions.” In this case it would clearly have limited application to considering systems and system models as defined in contemporary mathematical theory.

363 Second-order cybernetics “also known as the cybernetics of cybernetics, investigates the construction of models of cybernetic systems. It investigates cybernetics with awareness that the investigators are part of the system, and of the importance of self-referentiality, self-organizing, the subject-object problem, etc. Investigators of a system can never see how it works by standing outside it because the investigators are always engaged cybernetically with the system being observed; that is, when investigators observe a system, they affect and are affected by it.” http://en.wikipedia.org/wiki/Second-order_cybernetics last accessed on 13th February 2011.

362 Ibid.
5.5 Poincaré, Kant, and architectural system modelling

At the beginning of the twentieth century the renowned mathematician Henri Poincaré turned to the issues raised above with interpreting Kant’s synthetic a priori intuition in its application to geometry, to which geometry and how. In Science and Hypothesis he writes that were the geometrical axioms synthetic à priori intuitions, as Kant affirmed, there would be no non–Euclidean geometry. He wrote that the geometrical axioms were neither synthetic à priori intuitions nor experimental facts. They were conventions. One geometry could not be more true than another, it could only be more convenient. However, Poincaré’s position in the intuitionism–logicism debate is not consistent. In Science and Method, subsequently, he dismantles the logicists’ assertion that all mathematical truths can be demonstrated using the principles of logic without making a fresh appeal to intuition. The logicists were those who worked to reconcile the whole of mathematics with logic, notably Dedekind and Frege, but in the case of Poincaré’s argument specifically Courant and Peano. Logicists were concerned with the primacy of deductive reasoning.

As noted in Chapter 2, the primacy of deductive reasoning was also central to the logical positivists in architecture who followed much later in the early application of computational design to architecture in the 1960s. By his own subsequent analysis, Christopher Alexander’s Notes on the Synthesis of Form, 1964, while it raised the value of systematic contextual analysis, failed ultimately, to proffer any real connection to formal design solutions from the analysis advocated in his design method. Deductive reasoning is one part of the cyclic conversations that characterise design process. Critical analysis informs the iterative pathway of design propositions and refinement but it is not the foundation of synthetic proposition. Designers employ geometry that they know. Kant suggests that, at base, this knowledge is founded on axioms that are not derived through logic but known a priori and synthetically. Poincaré extends this to examine the need for “fresh recourse to intuition” in the synthetic and analytic work of the mathematician in their combinatorial work finding and proving new relations.

What part does intuition play in representing design intentions as geometrical constructions? Modelling designs as systems is akin to constructing problems in mathematics.

366 Ibid., 149.
367 Richard Courant (1888-1972) German mathematician, and Giuseppe Peano (1858 - 1932), Italian mathematician.
The model becomes a search for unknown values, which are like Descartes roots and an investigation of a design domain, which is similar to his curves. Design models represent both more general spatial design intentions – there may be tacit reference to sensorial space beyond the purely abstract rendition through geometrical objects and relations – and they represent, through example or instance, more general relationships and axioms from geometry itself. In this way there is reciprocity. When we make geometrical mappings, even models for the purpose of architectural, engineering or fashion design, these mappings represent certain pre–existing geometries and mathematical relations as well as representing buildings or apparel as geometrical maps.

Particular practices adopt geometrical representational conventions, consensual conventions which may become ossified and cease to explore in any way either the development or application of geometry to practice per se. The plan–section–elevation fragmentation of a building in the architectural technical drawing in order to better represent the whole is a good example of such a convention. Robin Evans highlights its fragmented nature and holistic intention through an interesting comparison with the intentions of Cubist painters. The various projections used to map the world are another example. Other innate capacities with regard to representing spatial phenomena to ourselves are not necessarily tapped as a result of the structures of practice. By implication, shifts such as the transition from object to system modelling may bring about the exercise of untapped innate spatial capacities.

The truth of geometrical axioms and the way of knowing such truths matters in so far as it affords predictability or reliability to the behaviour of models in which they are applied. It matters also in selecting the appropriate geometrical representation and hierarchy of relations. We must make a “fresh appeal to intuition”, to borrow Poincaré’s words, not only to demonstrate mathematical truths but in order to apply them selectively in the constructive arts. Kant’s synthetic a priori judgment applies to absolute truths, sure foundations, exemplified by his example of space enclosed by three lines but never by two. Empirical experience reinforces but never furnishes this same certainty for all cases. Design, as an activity,

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Christopher Tyler and Amy Ione identify many facets of cubism (their pun). Inaugurated by Pablo Picasso and Georges Braques between 1907 and 1914, Cubism emphasized the flat, two-dimensional surface of the picture plane. These painters rejected the traditions techniques of perspective, foreshortening etc. in favour of an approach of spatial deconstruction. The paintings of Analytical Cubism 1909-1912 show the breaking down of the surface into its constituent facets. Synthetic Cubism (after 1912) emphasized the combination, or synthesis, of forms in the picture. In a general sense, there is a Cubism of attempting to show all sides of the depicted objects at once.
is not concerned with this level of generality. It is much more involved in finding the appropriate propagation of geometrical ideas as a good fit to a particular context or a particular abstraction or mapping of a particular proposition, scenario or phenomenon. This fit can be tested empirically without any reference to the generality of the relations invoked or the nature of their ‘truth’ or conditional consistency.\textsuperscript{372}

However in practice, in modelling complex systems – computational geometrical system models – we are restricted to \textit{locally–derived} empirical knowledge of the geometry of the model. We are unable to predict the truth or consistency of the geometry as defined in the logic of the model for every particular hypothetical state of the system or instance of the geometry. We are working in this context with the idea of infinite possibility. David Hilbert suggests that the infinite itself may be seen as a pure idea.

“The role that remains for the infinite to play is solely that of an idea – if one means by an idea, in Kant’s terminology, a concept of reason which transcends all experience and which completes the concrete as a totality – that of an idea which we may unhesitatingly trust within the framework erected by our theory.” \textsuperscript{373}

For Kant, ideas are concepts of reason that transcend experience, rational rather than empirical. The twentieth century mathematician, Hardy, endows mathematical ideas with a transcendent quality, of divine or worldly provenance, yet allowing them to be empirically discovered.\textsuperscript{374} Design, by contrast, is unlimited and hybrid in sourcing ideas – it is as legitimate to move from the concrete and empirical sensory understanding of existing objects and processes in the world as to deploy pure geometrical or abstract symbolic strategies in establishing ideas. In representing them through the construction of models, and particularly with regard to system models there are choices to be made about whether to follow the pathway of rationalism and imposition of form on the system or to adopt primarily the investigative demeanour of the empirical experimenter and discoverer.

\section*{5.6 The New Geometries}

\textbf{Poincaré explanation and relationship to Euclidean}

In Science and Hypothesis, in his chapter on non–Euclidean geometries Poincaré\textsuperscript{375} reminds us that all treatises of geometry begin with the enunciation of indemonstrable axioms upon which every other theorem ultimately rests. He says that some of these are really propositions in analysis rather than propositions in geometry. For instance, ‘things which are equal to the same thing are equal to one another.’ These he views as \textit{analytical à priori} intuitions. Then there are three fundamental geometric propositions common to most treatises. (1) Only one line can pass through two points; (2) a straight line is the shortest distance between two points and (3) through a point only one parallel can pass through.
be drawn to a given straight line. As noted in Chapter 3, proof for the third of these, Euclid’s fifth postulate, was long sought in vain. Poincaré writes, “Finally at the beginning of the nineteenth century, and almost simultaneously, two scientists, a Russian and a Bulgarian, Lobachevsky and Bolyai, showed irrefutably that this proof is impossible.”\(^{376}\) It was only by negating the parallel postulate that Carl Friedrich Gauss, Nikolai Lobachevsky and Janos Bolyai discovered their alternative geometries, opening the field for new definition of geometry. Poincaré goes on to explain Lobachevsky’s assumption that several parallels can be drawn through a point to a given straight line while retaining all other axioms of Euclid. (Gauss’s assumption that the sum of the angles of a triangle is less than two right angles was equivalent.) Poincaré again, “From these hypotheses he deduces a series of theorems between which it is impossible to find any contradiction, and he constructs a geometry as impeccable in its logic as Euclidean geometry.”\(^{377}\)

Following in these footsteps, “…it was not long before a great step was taken by the celebrated memoir of Riemann, entitled: *Über die Hypothesen welche der Geometrie zum Grunde liegen.*\(^{378}\) This little work has inspired most of the recent treatises … among which I may mention those of Beltrami and Helmholz.”\(^{379}\)

Poincaré describes briefly the unbounded space of Riemann’s spherical geometry, which has no boundaries but is nevertheless finite: the tour of the sphere can be made. Saccheri had dismissed the possibility that no two lines are parallel, as contradictory to the second postulate based on the assumption that straight lines may be extended to an *infinite* extent. Riemann, while in agreement with regard to Euclid’s intent in this respect, argues that this does not follow from the postulate. A piece of straight line may be extended *indeinitely*:

“… we must distinguish between unboundedness and infinite extent ... The unboundedness of space possesses ... a greater empirical certainty than any external experience. But its infinite extent by no means follows from this.”\(^{380}\)

Circles (or any closed curve or surface) are of finite extent but continue indefinitely. This is the basis of Riemannian geometry.

So are there now an infinite number of possible geometries? Riemann’s work, of which what is commonly known as *Riemannian geometry* is only one example, constructs an infinite number. Lie’s theorem limits the number of geometries compatible with his premises, which are:

1. space has \(n\) dimensions
2. The movement of an invariable figure is possible
3. \(p\) conditions are necessary to determine the position of this figure in space.

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\(^{376}\) Ibid., 36.

\(^{377}\) Ibid., 37.

\(^{378}\) Translates as: ‘On the hypotheses that underlie geometry’.

\(^{379}\) Poincaré, *Science and Hypothesis*, 36.

By limiting \( n \), \( p \) is further limited.

Where does Kant’s pure intuition sit in relation to these geometries, which have been discovered or uncovered as the result of a change to a basic Euclidean axiom? While it seems the predicate is implicated here in a way that would bring their discovery at least into the realm of analytic propositions, in Kant’s terminology, nevertheless they seem to have, once known, a fully embodied spatial manifestation in which to understand the transformations under which figures remain invariant, rather as one understands that three lines enclose space and two do not.

If metrical [Euclidean] geometry is the “study of solids” and projective geometry is the “study of light”\(^{381}\), the non–Euclidean geometries, are the study of that equally ephemeral phenomenon: surfaces. Spatial design is most concerned neither with solid objects, nor with the passage of light per se, but with the boundaries and containment and opening of space and the sensory impact of their interaction with the play of light, sound and touch.\(^{382}\) Surface is the liminal condition and the space–defining idea, the representation of which has most preoccupied architects and spatial designers.

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5.7 Geometrical and Sensory Space

Geometrical Space and Representative Space: a physical theory

Returning to the question of an implied dichotomy between mathematical space and the space of external reality, this is a philosophical discussion that is relevant to architectural modelling approaches. Design is inherently bound to the concrete, the definition and description of artefacts and systems for the external world. This is a constraint that is absent from pure mathematics. Architectural system modelling, however abstract, nevertheless maintains its close mapping between the virtual formal world of the system model and the phenomena and relations or systems represented.

In his chapter on Space and Geometry Poincaré draws a distinction between Geometrical Space and Representative Space, the latter being the “framework of our representations and sensations”.\(^{383}\) At the same time he acknowledges that some hold the view that they are the same space. His definitions are close to Kant’s distinction between the form and matter of Pure Intuition. On the topic of “the convergence of space in narrowing Western thought”, Jeff Malpas writes much more recently of the way in which space is increasingly tied to physical extension.

“This can be seen…in the way in which the Greek notions of *topos* [space] and *chora* [place] have gradually been eclipsed in the history of philosophy, …by the concept of *kenon* or *void*. Nothing but an empty but open space – and it is

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\(^{381}\) Poincaré, *Science and Hypothesis*, 59.

\(^{382}\) “All material in nature, the mountains and the streams and the air and we, are made of Light which has been spent, and this crumpled mass called material casts a shadow, and the shadow belongs to Light.” - Louis Kahn. Kahn, Louis I. *Louis I. Kahn: Complete Work 1935–1974* ed. Heinz Ronner, Sharad Jhaveri, Princeton Architectural Press, 1987.

\(^{383}\) Ibid., 51.


**Figure 86**: Diagram of Geometrical and Representative Space after Poincaré

- **Intuition**
  - Geometrical Space
    - space of the geometers object of geometry
    - continuous
    - infinite
    - three dimensions
    - homogeneous (all pts the same)
    - isotropic
  - Representative space
    - framework for sensations and representations
  - Visual Space
    - non-homogeneous
    - physiological reasons
  - Tactile
    - aggregate of motor sensations, association of ideas - sense of direction without which as many dimensions as muscles
  - Motor space
    - not homogeneous (all pts not same)
    - not isotropic
    - not three dimensions

- **Design**
  - Projection
    - gives first two dimensions
    - Projective geometry = study of light
  - binocular muscular accommodation
    - gives third dimension
precisely this idea that lies at the heart of thinking about space in the work of Descartes and Newton. Thus, with Newton we arrive at an understanding of space as a single, homogeneous and isotropic ‘container’ in which all things are located, and even though modern cosmological physics no longer understands space in the terms developed in Cartesian and Newtonian thinking, still the idea of spatiality as primarily a matter of physical extendedness remains”.

This physical extension is understood through metrical geometry. Malpas continues to develop the distinction between space and place: “physical theory alone seems to have no need for a concept of place beyond the notion of simple location…The crucial point about the connection between place and experience is not, however that place is properly something only encountered ‘in’ experience, but rather that place is integral to the very structure and possibility of experience” (with reference to Martin Heidegger). Malpas’s philosophical understanding of space and place is “not a world of empty space or inhuman objects in a realm of purely subjective sensation or ‘sense–data’”.

Conversely, Poincaré is concerned with the relationship between physical experience on one hand and space as an abstract mathematical concept over which we reason on the other. But his treatment is expressly physical in both cases and in both cases turns to the descriptive language of space for science: that is, geometry.

Place is an important consideration in relation to the situatedness of the architectural system modeller within their model, and, by implication, what it represents in the imagined ‘real–world’. The model place also differs from both empty space (Poincaré’s Geometrical Space) and the space of physical sensation (Poincaré’s Representative Space). The physical dimensions of the design space may be at times closer to Malpas’s observations of the necessity of place for experience than Poincaré’s dissection of either geometrical Newtonian space, or the immediate space of sensory interpretation. The example of the successive heuristics for the automation of the façade cladding design of Pinnacle Tower in which the most successful iteration was also the closest to the act or sequence of construction illustrates this point well. In the exploratory virtual modelling work for the detailed resolution of the Sagrada Família church there is a stage of moving from the shape and surface definitions to variable computational geometrical system modelling of subdivision for stone cutting and assembly that similarly engages a shift from one type of spatial engagement to another more embodied one, defined by a new set of constructional constraints and the projected sequence of fabrication and construction.

**Poincaré’s Geometrical Space**

Of space that is the object of geometry, Geometrical Space, Poincaré states that it is: continuous; infinite; of three dimensions;
homogeneous, (that is to say that all its points are identical to one another); and isotropic. Space of four or more dimensions does not go unacknowledged, but for this argument he is concerned with the space that he believes we can represent to ourselves.\footnote{Ibid., 51.} This is instantly recognisable as the endless void that Malpas attributes to Descartes and Newton. The significance of this uniform abstract clinical space for being able to dissect the three static dimensions of architectural object propositions to two and three-dimensional descriptive geometrical representations is clear and persists in the basic framework of the CAD modelling systems. But I have shown through example that the architects working through computational geometrical design system modelling are already looking beyond and outside the confines of this space both for ideation and for spatial problem solving. In reality this rigid concept of a geometrical space constrained to three homogeneous dimensions is potentially problematic for conceiving of the heterogeneous dynamic system. Logical relations are not constrained in the same way. Conception within a 3D geometrical framework potentially limits opportunity. The system almost inevitably has claim to more dimensions and is difficult to represent adequately exclusively in this construct.

**Poincaré’s Representative Space: visual**

Visual space is central to architecture and visualisation is arguably the primary mode by which architecture comes into being. I will examine this argument more closely in relation to Geir Kaufmann’s detailed comparison of the place of the image compared to language in thought. Visualisation is also the name we have given to representation of the imagined and modelled through manipulable images in the Graphic User Interface of the computer.

By comparison with his Geometrical Space, Poincaré’s Visual Space is not homogeneous. Perception of the third dimension he reduces to a neurological sensation associated with the muscular effort of accommodation of distance in the lens of the eye and to parallax: the convergence of the visual impression from two eyes. These, he says are muscular sensations quite different from the visual sensations which have given us the concept of the first two dimensions. One assumes that he means by the visual sensations giving the first two dimensions: projection, the image overlaid on the receptors on the retina, in accord with the classical physics of optics. Poincaré writes:

“Nothing prevents us from assuming that a being with a mind like ours, with the same sense organs as ourselves, may be placed in a world in which light would only reach him after being passed through a refracting media of complicated form. The two indications which enable us to appreciate distances would cease to be connected by a constant relation. A being educating his senses in such a world would no doubt attribute four dimensions to complete visual space.”\footnote{Ibid., 54.}

This adaptive nature of visual perception is subsequently well demonstrated through psychological experiments combining novel combinations of visual and tactile sensation. Gregory Bateson notes that “there is no free will against the immediate commands of the images that perception presents to the “mind’s eye”. But through arduous practice and self
correction, it is possible to alter those images”. Bateson uses an example of this elsewhere in Mind and Nature to illustrate the concept of “orders of recursiveness”. His example is the difference between improving aim with a rifle and with a shot gun. The rifle marksperson uses the sights to repeatedly correct the current error before firing, the shotgun marksperson has to compare the difference between outcomes after firing in the light of the information about the visual and tactile experience of aiming and firing and use all this to recursively self correct and learn how to aim to hit the target – the whole operation is in question every time he shoots. The Marksman shoots at a new target (third round) and then should take forward the information of difference between the first round and second round including what he did and how it felt, for instance, to over correct. Another familiar example is driving a car in which the body of the vehicle gradually becomes assimilated as the extended body of the driver while in the driving context. A similar spatial appropriation occurs for the digital design modeller, or any similarly engaged activity in which the translation to particular abstractions, symbols or procedures rapidly become invisible to conscious thought as the operator progressively “occupies” the operational space. The space we occupy while reading a book is not too different as an analogy, the only difference being the extent of the contribution of the visual as opposed to language to the visualisation. However a better analogy is the creative act of writing the book in which there is a constant exchange self projection into the space, spatially engaged exploration and pulling back to a more transcendent viewpoint to analyse and redefine intention.

Geir Kaufmann in his book Imagery Language and Cognition, writing at a time when information processing theory had become the dominant paradigm for thought, underwritten by symbol processing and hence linguistics, examines in some detail the early history of the imagist position. The theory that thought and words are based on mental images is closely aligned with the British Empiricist philosophical tradition. Images are in these terms briefer, less intense copies of sensations. Kaufmann believes that imagist theory is a more balanced position than either the extreme linguistic Symbolist position (language the necessary for vehicle for thought) or the Conceptualist position (thought, an abstract operation of which language and images are purely products). The Imagist stance gives an important function to language – images can be cashed in for language and language has a function in linking, representing the abstract and communicating – while in the extreme linguistic and Conceptualist position, images are dispensed with completely. He quotes Paivo in linking imagery to the concrete aspects of a situation, task–focus and as a dynamic system that promotes flexibility and speed in the mediating process, while the verbal has a more static, labelling process. Visual imagery is

391 Ibid., 201.
393 Ibid., 18.
394 Ibid.,37.
also characterised as a visual–spatial representational system, specialized for parallel processing of information, while the verbal processes are of an auditory–motor nature and are specialised for sequencing. Piaget’s research with young children and the challenge of ‘classes’ (some of my cars are red) appears to support the idea that language may constitute a necessary condition for the attainment of logical structures but he maintains that there are operational structures that transcend natural language.

Kaufmann concludes: “The principal thesis that we shall try to defend here is the following. Visual imagery is particularly suited for the execution of transformational activity needed in tasks with a high degree of novelty.”

Kaufmann also explores the process of invention and discovery – activities that transcend the boundaries of previous experience and the ‘flash of inspiration’ idea. This is something that I shall revisit in chapter 6.

An example of the need for exploration, or constructive activity for novel problem solving is The Hat Rack Problem, Kaufmann and Raaheim, 1973, in which a group of students are given a longer stick, a shorter stick and a C–clamp and asked to create a hat rack while a control group are given pencil and paper and asked to solve the same problem. This illustrated the value of embodied space encompassing what Poincaré would term Geometric and all aspects of Representative Space (visual, tactile, motor) in order to find spatial design solutions. An example of the same phenomenon taken from the design modelling examples is the work for the Temple Sagrada Familia, for which although there is both a high level of fidelity and flexibility in the digital modelling for the church and large scale modelled assemblies are created with good opportunity to navigate the space virtually during design, the value of physical scale models sitting in space and around which the group navigates have been found to hold much greater value for understanding the complex novel spatial relationships and for collective design decision making despite the exigencies of scale compared to the unconstrained scale of the virtual models.

Figure 87 The relationship of the value of linguistically–based thought, image–based thought and overt exploratory activity in space in relationship to the novelty of the task (after Kaufmann)

Not only did a much higher percentage of the first group find the solution to this challenge in the time available they also did so in a much shorter time than the successful members of control group working in relative abstraction on the problem and relying more heavily on a geometrical understanding of the space. (The main trick was the discovery that the longer stick was longer that the floor to ceiling height and could be wedged between them, something that could have been identified geometrically but was clearly more apparent while exploring the physical space).
Figure 88a: working with the scale plaster models prototyped from instances of the digital system at the Temple Sagrada Familia (continued next page)
To step back from visualisation and mental imagery and return to the relationship between geometrical and visual space, Lorenzo Magnani writes that the geometry that “precisely and naturally fits the actual considerations of the visual field” is non–Euclidean, two dimensional, elliptical geometry, a thesis already supported by Thomas Reid (1764), many years before the discovery of the non–Euclidean geometries.

“What we call protogeometry consists largely of a collection of somaesthetic, tactile and kinaesthetic data, where action in the external world mediated by the body is central”. For a phenomenologist such as Angell, Magnani notes,

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398 Ibid., 178.

399 Angell, R.B., ‘The geometry of invisibles’, in Nous, 8 (1974), this is indispensable in explaining the cognitive “origins” and developments of geometrical idealities. In order to recognise perception as a structured “intentional constitution” of external objects, the objective space we usually subjectively experience has to be subjected to transcendental reduction. In this way we will see that space and geometrical idealities, like the Euclidean ones, are “constituted” objective properties of these transcendental objects.

In this light of Magnani’s proposition, I conducted a number of small observational experiments, of passing through doors.

As Norbert Schulz would have it, a door is a convention with which we are very familiar, that now works at both the syntactic and semantic levels. We read it, no doubt on

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400 Norberg-Schulz, C. Meaning in Western Architecture, Rizzoli, 1993. (For a discussion of semantics and syntactics.)
many occasions unconsciously, as we approach it. I was less interested on this occasion in architectural readings than in the spatial relationship of the body to the door and the extent to which three metrical dimensions (related to three mutually perpendicular planes but embodied as resistance to gravity, eyes forward, and the distinctions between our left and right sides) are invoked analytically in approaching, opening and passing through the door. To this end I approached different doors on different trajectories with as little conscious attention as I could manage to bring to the task. On each occasion I noticed that wherever I had stopped in relation to the door, it was always within comfortably reach of the door handle, without outstretching the upper arm, when I opened the door, it always cleared my stationary body but most intriguingly, it would do so with a tolerance of less than 5 millimetres. It seemed that the first two conditions would not be simple to achieve through geometrical calculation, but the third would require an extraordinary level of precision in mapping the current position of the body in space to the ideal projected space in front of the door, even disregarding the complexity of motor responses to achieve it. While this was a very inconclusive experimental approach compared, for instance, to Piaget’s painstaking observations of activities at different stages of early development, it was the level of precision in relatively unconscious spatial navigation of such a casual nature (compared for example to an expert honing their hand eye coordination for an international tennis competition) that was most provocative. Which combination of sensory inputs, feedback against successive images, and process of unconscious calculation could arrive at such an outcome? The philosopher Gilbert Ryle has written in support of engaged thought, of which he gives the example of perfecting the golf swing, as a higher form of activity than the passive, contemplative, reflection traditionally associated with thought and immortalised by Auguste Rodin in his bronze: *Le Penseur.* Clearly passing through doors is not a high level, analytical activity, as at least equal precision and dexterity is observed in equivalent activities by a range of ambulant animals. But perhaps it is possible to intuit from these observations something about the origins, the cost, the value and the limitations of the overlay of the three dimensions as a dominant spatial conception.

Ernst Mach, too, sees visual perception as the clue to a more primitive ‘geometry’ of space:

“The localities, the distances…of visual space differ only in quality, not in quantity. What we term visual measurement is ultimately the upshot of primitive physical and metrical experiences”. “The other properties of visual space also are adapted to biological conditions. The biological needs would not be satisfied with the pure relations of geometric space”. “Visual space, therefore, which ordinarily is well filled with objects, thus affords the best means of localization”. He emphasizes the role that the perception of multiple objects in space plays in our ability...
He, like Poincaré, distinguishes between perceptual and conceptual (geometrical) space but in making the distinction, he treats them more as constituents of an undivided whole, both necessary, neither sufficient for a) our biological needs and spatial being in the world and b) the construction of geometry itself.

Mach writes, “It is erroneous to assert that the straight line is recognised as the shortest line by mere visualization…this is something we can reproduce with perfect accuracy and precision in imagination.” 407 Mach is clear that visualization and reasoning are each insufficient in themselves for the construction of geometry but “no sharp division can be drawn between the instinctive, the technical and the scientific acquisition of geometric notions”. 408

Architects are sometimes accused of ocular–centricity, designing places and artefacts to be seen and photographed rather than experienced in a fully embodied way by engaging aurality, feelings of air movement and temperature, olfactory sensibility. Whether or not this is a valid observation, experience suggests that visualization, the projection of imagined space, makes reference not only to visual recall but also more broadly sourced sensation. The representation and communication of architecture engages visual space and visualization ahead of any other sensory contribution to spatial understanding. But to what extent can the visual be divorced from other sensory and conceptual inputs to spatial understanding, interaction and production?

405 Ibid.
406 Ibid., 35.
407 Ibid., 62.
408 Ibid., 69.
**Tactile and motor space**

Visual Space is one facet of that which Poincaré terms representative space. Tactile Space he considers more complicated still than Visual Space, while Motor Space (spatial perception as the aggregate of the sensations from the muscles) would have as many dimensions as we have muscles. In this last, it is sense of direction of each movement as an integral part of sensation that he considers the key to this facet of spatial perception. Sense of direction is very complicated, the result of habit which in turn is the result of many experiments. He returns to this theme in ‘Science and Method’ where he develops the idea of restricted space (the immediate space of the stationary body), extended space (space created by the relative relocation of the body) and the great space (an imaginary space in which the universe may be lodged). Why should all these spaces have three dimensions, he muses, and conceives, by way of response, of a theoretical tri-partite inner distribution board which, incidentally, lays no claim to be analogous to the neural system. He then describes how a sequence of parries, alarms and responses may be orchestrated in relation to a relative space of perception that alters with our movement. We may contrast this to Bateson’s marksmen whose visual, tactile and motor perceptions are trained to combine within the same Poincaréian immediate space without translational movement of the body.

The characteristics of Representative Space “in its triple form – visual, tactile and motor” (Poincaré is conspicuously and strangely silent on the topic of aural spatial perception) differ essentially, in his analysis, from Geometrical Space. It is neither homogeneous nor isotropic nor can we categorically claim it is three dimensional, so Poincaré argues in Science and Hypothesis. “Thus we do not represent to ourselves external bodies in geometrical space, but we reason about these bodies as if they were situated in geometrical space”.

“None of our sensations, if isolated, could have brought us to the concept of space; we are brought to it solely by studying the laws by which those sensations succeed one another… Whether an object changes its state or only its position, this is always translated for us in the same manner, by a modification in an aggregate of impressions. How, then, have we been able to distinguish them?” It is only through our own opportunity for movement that we can distinguish between the movement and change of state in other bodies Poincaré concludes. Thus for Poincaré it is not single sensation but an “aggregate of impressions” in combination with our own possibility of movement that brings us our concept of space. This very generalised conclusion is nevertheless prescient in relation to the later experiments of Adelbert Ames which demonstrate that of all the contributions to visual perception, those internalised rules relating to our own movement are

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412 Ibid., 57.

413 Ibid., 58.

414 Adelbert Ames Jr. (1880-1955), American scientist who contributed to physics, physiology, ophthalmology, psychology and philosophy, pioneer in the study of psychological optics. He is best known for constructing illusions of visual perception such as the Ames room and the Ames window.
most privileged and yield the information that is prioritised above all other in the overall processing of the visual field.

Such a prioritisation once more becomes ambiguous in relation to virtual design modelling. Virtual navigation implies the visual perception of movement, albeit without necessarily leaving the chair. However, it is clear from observations in practice that virtual reality simulation of movement, while convincing in a passive medium such as cinema, is not sufficiently authentic or full-bodied to yield the same spatial appreciation and situational awareness in relation to a design simulation – an instance or series of instances of the architectural model – even at the more sophisticated end of the virtual reality spectrum. This is despite the fact that, in theory, the virtual simulation permits a more holistic level of simulated space habitation than an overview of scaled-down physical models or the necessarily limited extent of the 1:1 physical prototype. We are satisfied with virtual representation for passive engagement – watching a movie or interacting in a game but it does not furnish the same opportunity for critical analytical engagement with unfamiliar or novel spatial organisation of the propositional world in the process of design. In this discussion of visual space that encompasses both visual perception and visualisation, I have strayed into the territory of representation of the designed space, as opposed to the design (or system model) space, although specifically in relation to the iterative review process during design activity as a subset of the progressive definition and refinement of the design space. For Poincaré, visual space is definitively differentiated from homogeneous, geometrical space; for Mach they are two necessary facets of holistic spatial knowledge. Ames prioritizes the aspects of visual perception through a catalogue of detailed psychological experiments. It is difficult to draw from this the significance of visualization (external or in imagination) of the system model. The examples point to an experiential model that is close to sequential event–based experience, possible change sequences as a series of events rather than the multidimensional space of the model envisaged as a contiguous spatial phenomenon.

**Auditory space: Strawson’s Individuals and the No–Space world**

If visual perception is so fundamental to the construction of space, what is the role of the perception of sound? Daniel Kish describes in deep sonar hues his experience as a blind person of navigating space at quite fine resolution by taking soundings from a click of the tongue. It is not fully clear from his writing whether he gains a visualization or auralization as his internal representation of the space to himself but it gives him the details of his surroundings including the position and architecture of doorframes.\(^{415}\)

There is a curious absence of reflection on the aural contribution to spatial understanding in Poincaré’s writing and in philosophy generally before the 20th century. This contrasts with the longstanding classical association between Pythagorean harmonic proportions in music and spatial proportions in Renaissance and post–Renaissance art and architecture.\(^{416}\)


Vitruvius gives quite a detailed description of harmonics in the fifth of his Ten Books of Architecture as a prelude to describing the design of sounding vessels as amplifiers in the Roman theatre. But he notes the need for a good knowledge of Greek as there is no Latin reference on the topic available at the time of writing in the first century B.C.\(^\text{417}\)

Curiously sound is treated in philosophy as intrinsically temporal while the visual relates to *space*, which when abbreviated to three dimensions is independent of any temporal dimension. The curious aspect is that this means that either light and sound are treated very differently — light produces the static image while a static sonic image cannot be perceived (sound is difference sensed over time rather than over a spatial field) or space is treated completely independently of light and perception, as a completely abstract geometrical construct in the Cartesian, Newtonian and modern empiricist tradition. But the static image is another ideal conception. It is idealised from the unstable reality of visual perception in which movement and change are intrinsic. In experience, it is a reconstruction of common attributes among a temporal sequence of impressions. With reference to Kant’s two forms of sensibility or intuition, namely Space and Time, and the Kantian contention that all representations are in inner sense, of which Time is the form, but only some representations are in outer sense, of which Space is the form, Strawson inquires whether a scheme of objective particulars could exist that dispenses with outer sense. He posits a non–spatial world without bodies as the basic particulars of the system.\(^\text{418}\)

Strawson’s line of argument is designed to test the robustness of his central non–solipsistic thesis that we operate with the scheme of a single unified spatiotemporal system, in which objects and episodes exist outside our perceptual experience. We think of this world as containing particular things of which some are independent of ourselves, and its history as made up of particular episodes in which we may or may not have a part. Everything in the world and every episode can be related one to another in spatial and temporal terms, for instance the same object can exist in one location at a particular time and be part of a different episode at a different location at a different time. Strawson tests this through the remote contingency of a purely auditory world.

He pursues the seemingly tenuous argument that while the objects of hearing possess, in their own right, direction and distance characteristics, these are the result of combining auditory sense–experience with tactual, kinesthetic and, usually, visual sense–experience and that in the total absence of these last three, intrinsic spatial characteristics such as ‘to the left of’, ‘above’ ‘further’ have no significance.\(^\text{419}\) This does not entirely accord with Kish’s description of his own experience in which although the possibility of movement undoubtedly contributed to his sense of space, he can click and hear a detailed portrayal


\(^{419}\) Ibid., 65.
of his surroundings from a series of static reference points. Strawson's purely auditory world is difficult to project in imagination, and to relate to Poincaré's speculations on the shifting understanding of location and direction in relation to the body and its movement. In order to establish whether objective particulars are possible in such a 'non–spatial' world, or in the 'absence of Space', Strawson must establish whether the re–identification of particulars is possible – the process by which we know material objects in the spatial world. He strikes a problem with the logical possibility of the continuing existence of sound that is not perceived. "in the auditory as in the ordinary world, the possibility of reidentification of particulars depends on the idea of a dimension in which unperceived particulars may be housed, which they may be thought of as occupying" and this is essentially spatial. He turns this around to argue that all independently reidentifiable particulars must at least be intrinsically spatial things, occupiers of space. Sound particulars, not being of this character, are not independently reidentifiable. This contradicts the hypothesis of the non–spatial world and supports the existence of the unified spatiotemporal world.

The weakness of his argument in this area seems to be the assumption, never robustly supported, that the essential dimensions of sound: timbre, pitch and loudness, are temporal but fundamentally non spatial. He arrives at the conclusion that sounds are not independently reidentifiable (due to being non–spatial) and that thus a purely auditory, non–spatial world of objective particulars is subject to contradiction. I would contest that the dimensions of sound support a more subtle refutation of Kant's division between inner and outer sense based on the physical reciprocity between temporal and spatial dimension inherent in pitch and timbre.

5.8 Perceptual and representational space

In the 1940s, Piaget presents the idea of topology as a more fundamental concept than Cartesian space. In his introduction to the Child's Conception of Space, he challenges both what he defines as Kant's Space "as an a priori structure of 'sensibility'"and Poincaré's Kantian ascription of the formation of spatial concepts to sensory impressions. He does this on the grounds that our derivation of co–ordinate systems from embodied knowledge of vertical–horizontal axes in physical experience are quite a late and complex connection, only fully developed in the child by the age of eight or nine. By contrast, he presents evidence that the mapping or representation of the physical world by non–metrical, non–axial proximities and semantic relationships occurs at a much earlier age. We may take issue with Piaget's interpretation of Kant's pure intuition, which is Kant's name for the form of sense–intuition not the matter of sense–intuition, knowledge, which is not grounded in sensation at all. But it is hardly important to Piaget's principle objective; to dispel the adult 'misconception'
regarding the relationship of spatial perception to spatial representation (that spatial representations are derived directly from perceptual knowledge). It is his contention, that during the development of representational space, representational activity is projected back on to perceptual activity. In other words, as we evolve and learn new spatial representations, (axial coordinate systematic understandings of space being an example), they effectively become assimilated in our knowledge of space in ways that are no longer separable from perceptual knowledge.

This realisation is also thought-provoking for architects, whose traditional representational conventions of descriptive and projective geometry, considered through this lens of space in relationship to development and learning, have the potential to influence, perhaps straight jacket, the space of their perception, and hence, through re-representation, their conceptions. Theoretical concepts of morphology have affected some form of release in architecture, and for this we have to go back to Antoni Gaudí’s work in the early twentieth century, not merely more recent digital enactments, but these too can become formalist conventions once we move away from their deeper implications.

This capacity to “learn” space through an amalgam of sensory feedback and representational overlay, also allows the space of our perceptions to change. Nothing illustrates this more clearly than the demonstrations in perception by Adelbert Ames. Gregory Bateson\textsuperscript{423} writes a detailed description of his visit to Ames’ laboratory and experience of experiments in which all aspects of his visual perception were challenged through small tricks of relative parallax and though unexpected combinations of visual and tactile sensation. After having had his faith in his own image formation profoundly shaken, Bateson afterwards had difficulty crossing the street, so uncertain was he about his perception of the location of the oncoming cars. The best known of these experiments is the Ames room, which produces the visual illusion that people standing at either end of the room are dramatically different in size.\textsuperscript{424}

Gregory Bateson provides a long description of interacting with another of the Ames experiments, the trapezoidal room. When inspected objectively from above this was a box of strange trapezoidal shape but when viewed through a peephole in the side of the box using a pair of prismatic glasses, its interior space appeared perfectly rectangular by virtue of the position and shape of windows painted onto the inside of the box. When asked to hit first the right hand end wall with a stick protruding into the box, then swing it round to hit the left hand end, the exercise appeared simple but would be prevented each time by the stick hitting the back wall. Bateson describes how, even after 50 attempts he could not overcome his visual perception to make the right correction pulling back the stick and always hit the back wall but, in the process, he improved, the stick swung further, and most interestingly, the room became more visually trapezoidal in doing so.\textsuperscript{425}

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There is nothing given about our spatial perception from the sensory inputs we receive, although, they seem self-evident, and, although experiments such as these can support scientific hypothesis about the order of priority given to particular senses and perceived relationships that stimulate them, but Husserl writes that there is something universal and unchanging about geometrical knowledge. "The Pythagorean theorem, [indeed] all of geometry, exists only once, no matter how often or even in what language it may be expressed." Writing down (such as Euclid’s Elements)

Husserl argues, “effects a transformation of the original mode of being of the meaning–structure ... It becomes sedimented, so to speak. But the reader can make it self-evident again, can reactivate the self-evidence.”  

Geometrical space and perceptual space, then, are not far distant from one another. Ramsay and Richtmyer write in their Introduction to Hyperbolic Geometry: “The notions of geometry go, in a sense, beyond the notions of analysis, in that they are things that we ‘visualize.’ ... The ability to visualize is a human ability that should be encouraged rather than suppressed in the teaching of mathematics. Our impression from teaching talented young students is that they can visualize the hyperbolic plane, in a sense. From that point of view the main models, those of Beltrami, Klein and Poincaré, are unsatisfactory for intuitive geometrical visualization.”

In his plaster models for the Sagrada Família church, Antoni Gaudí, familiarises us with a family of hyperbolic surfaces, brought together in composition to create a new formal freedom of expression in ecclesiastical architecture, to admit and modulate light and facilitate its construction through its underlying (non-obvious) logic. Similarly the use of hyperbolic surfaces in mid-twentieth century concrete shell structures gives visceral pleasure through the taut impression of structural economy and elegant curvature. Unburdened with the mathematician’s duty to the rigorous generality of axioms and analysis, there is no difficulty in assimilating an intuitive and concrete sense of hyperbolic space, at least in a geometrically local sense. Daina Taimina’s crocheted hyperbolic surfaces have also been very successful in this respect.

What of Riemannian geometrical space? Can this too be visualised? Jeffrey Weeks has given his best shot at broad enfranchisement through visual access to such space in his book *The Shape of Space*. Apery provides exquisite computer renders of the Real Projective plane but we are at once seeing them immersed in three dimensions, pictured in two, just as we understand the Klein bottle as a self-intersecting surface and generally must grapple with intellect rather than visual imagination to picture it embedded more gracefully in four dimensions. There are other pitfalls in the quest for visual access such as the use of such terms as *visibility condition* introduced in Riemannian geometry by Eberlain and O’Neill that states that one point is visible from another if there is a geodesic joining them. Visualisation is not the same as visual metaphor even when buried deep in the discipline of mathematics.

Poincaré stated that “Euclidean geometry is, and will remain, the most convenient [geometry] for two reasons: firstly,

427  Ibid., 361.
429  Dr Daina Taimina, Latvian mathematician, Adjunct Associate Professor, Cornell University (2007-2010).
because it is the simplest, and it is not so only because of our mental habits or because of the kind of direct intuition that we have of Euclidean space: it is the simplest in itself, just as a polynomial of the first degree is simpler than a polynomial of the second degree; and secondly, because it sufficiently agrees with the properties of natural solids, those bodies which we can compare and measure by means of our senses.” This second reason is more troubling from the author of the idea that geometries are mere conventions. Would the various developments in geometry of the last two centuries have occurred at all, had other or better ways than Euclidean geometry of mapping aspects of the world of our experiences, observations and measurements not been needed? As Mandelbrot wrote in 1976, “Many important spatial patterns of Nature are either irregular or fragmented to such an extreme degree that Euclid … is hardly of any help in describing their form”.433

We can visualise fractals, but perhaps, not at all the scales at which the fractal exists all at once. We can understand a planar Euclidean representation of an object at a glance and all at once, but it does not give us all the geometrical facets at all scales of the object in the world that it represents. Should we not be able to visualise large topological architectural models with many variables (dimensions) through visualising change. After all we do not so much see objects as construct them from what we see. What we see is change and difference. Many of the ‘objects’ change not only through their, or our, changing position and viewpoint but change their form between every sighting. Faces are a prime example. Poincaré explores this with regard to how our three embodied dimensions move with us, using the vocabulary of fencing.434 Thom writes, “this recognition of the same object in the infinite multiplicity of its manifestations is, in itself, a problem”.435 Returning now to Piaget’s interpretation, is geometrical space not a vital part of our perceptual object making? Conversely can we not immerse ourselves in geometrically constructed spaces, models that defy a simple, seen all–at–once Euclidean encapsulation and reconstruct the space perceptually?

Recapitulation: Chapter 5

In the preceding chapter, ‘Inside the model space: bifurcations and holes’, I explored the non–homogenous nature of ‘design space’ in parametric geometrical models through some examples of different types of discontinuity and continuity. The aim of this chapter has been to address the question of what contemporary architectural modellers can learn about computational geometrical system model space from the philosophy and psychology of mathematics. Does it, for instance, help answer the question: in what sense the system model space is really space at all and how human engagement


in such a space is differentiated from human spatial engagement in general. What does it offer in regard to what space is and the place of geometry in the way that we perceive, understand and represent it?

One answer is that space, especially in respect of geometrical and mathematical considerations has been profoundly shaped by both modern Cartesian geometry and Immanuel Kant’s Aesthetic in the Critique of Pure Reason, which in each case, places the human at the centre. Descartes’ Geometry emphasizes the active engagement in constructing geometry, virtuosity in the art of invention; Kant’s, the form of intuition, which precedes both cognition and the deployment of logic. In Chapter 6, I will illustrate how this provided a basic ground for philosophical mathematical thought into the early twentieth century. It was a fundamental tenet against which ideas were tested and the refutation of which became a major point of division and definition. Modern philosophy furnished a new paradigm for spatial and temporal understanding, one in which human sensibilities and actions have a much greater impact in shaping external reality.

Another answer is that the dichotomy of mathematical space and external reality is contentious and has been continually contested in philosophical history. Thus the understanding of the external space is subject to variation in the conditions for visual perception and also the assimilated conventions of mathematical space. This places virtual model space, however abstract, within a relativistic continuum of spatial knowledge rather than segregated from the embodied knowledge of external reality by its geometrical nature.

This chapter covers a broad range of spatial topics. While this has been a philosophical exploration of space, it has emphasized physical and metrical aspects. I acknowledge that these do not define or encompass the domain. But it is the role of geometry both as an essential attribute of space and as the representational spatial medium in computer–based architectural geometrical system modelling that is the core topic.

While there have been cursory excursions into the temporal, and even tacit acknowledgement of the unified nature of our spatiotemporal framework, the geometrical representation of Time, per se, presents particular challenges, with the usual attribution of a sole dimension. The linear representation of time brings it closer to number than geometry. We now have accessible opportunities for representing time within digital modelling. As the model contains innumerable spatial instances, we can view a sequence of changes over time. However, the linear nature of time implies that we can only view one very particular sequence of change, a highly selective one–dimensional path through a multi dimensional design space. Thus time itself is not a particularly useful framework for the exploration of all the spatial dimensions of the model.

Kant neatly marries the perceptual and representational aspects of space by ascribing the mathematical rules of space to human sensibility, to pure intuition, which is innate and not derived from experience or logic. Space and time are not actual things or qualities of things but are what make existence possible for spatiotemporal things. Space and Time have no independent

existence. We cannot imagine the absence of space (I have summarised Strawson’s investigative attempt). The Kantian answer to the question of what space is has influenced all subsequent thinking on the topic, concurrent or reactive.

What is the role of geometry in our perception and representation of space under Kant’s description? Geometry too, for Kant, is pure a priori intuition, “a science that determines the properties of space”.\textsuperscript{437} So, Kantian space does have properties although it is not a thing and these properties are determined by geometry.

The design space of the architectural computational geometrical system model, although it can be represented, as a map of geometrical relations is a space that generally cannot be represented in Cartesian 3–space. Hence I have questioned whether it falls within the scope of Kantian pure intuition or not and have found that the level of technical philosophical understanding necessary to answer this, takes the question and its resolution outside the scope of this thesis. However greater philosophical thinkers of the Kantian school have applied their thinking to this and we have seen the apparently contradictory positions taken by Poincaré in different volumes on the question of the necessity implicit in a priori synthetic judgment and the development of new geometries and mathematical structures for framing them. I will give further consideration to the conflict between Kant’s Pure Intuition and the position of the logicists and the implication for architectural model space in the chapter that follows. I will also examine the cognitive parallels between the processes of design and mathematical discovery and draw comparisons between positions taken on aesthetics in mathematics and in design. I will give some space to computational programming heuristics and their influence in shaping the design space of the model. Finally, I will also test the specific spatial descriptive issues encountered in the modelling examples in Chapter 4 in the light of some of the discussion of the philosophy of space in Chapters 5 and 6.

\textsuperscript{437} Kant, I. \textit{Critique of Pure Reason}. Translated by Kemp Smith, N. London and Basingstoke: Macmillan and co Ltd, 1970 (First edition 1929) (First Published (German) 1781), 70.
CHAPTER 6

INTUITION AND LOGIC IN DESIGN SPACE
Introduction

The previous chapter considered the confluence of intuition, space and geometry, in particular in Kant’s Aesthetic. Kant’s Pure Intuition became the new prevalent spatial paradigm for philosophical mathematical thought in the nineteenth century. As any new paradigm, it stood as the next bulwark against which to test the strength of the models that followed. I have already touched on the conflict that arose between Intuition and the logicist approach to mathematics, the program led by Hilbert to bring the whole of mathematics into a consistent logical structure.

Logic from the Greek λογική logikē is the study of reasoning, argument or inference. Thus, it corresponds closely to the analytic in Kant’s terminology, particularly in the cases of deductive reasoning – drawing conclusions from definitions and axioms. The argument in this thesis rests mainly on inductive reasoning – drawing general conclusions from specific examples. Logic may be informal – that is arguments using natural language, seeking fallacies in the Platonic tradition. But the more usual meaning is formal logic, descended from Aristotle; that is, systems of purely abstract rules and their inference. This last meaning is the language of computation.

In this chapter I will pick up the thread of the Logic–Intuition debate. I will consider the challenges posed in the computational design system model in negotiating the translation of intentions into an informal logic that can then be expressed in formal logic for computation and the implications of the invisibility of the logical model. The last part of the chapter is a comparative examination of aesthetics in mathematics and in architecture, which is seen as critical to both spatial perception in models and to the creative processes of design and mathematical discovery.

6.1 Logicism versus Intuition

Bertrand Russell famously thought that his own logicism conflicted with Kant’s philosophy of mathematics although none of his own writing, at least up until 1912, refuted the claim that mathematics is synthetic (the apparently obvious conflict, as strict logicism would imply that it is analytic). Russell held two doctrines simultaneously known as standard and conditional logicism. Mathematical theories for which there appeared to be no alternative (i.e. arithmetic) were to be reduced to logic in the standard sense; those for which there were several legitimate alternatives (e.g. geometries) were to be reduced to logic only in the conditional sense.438

This last sometimes known as “If–thenism” focuses not on whether one or other conflicting set of axioms is true but on the truth of the implications from a particular set of axioms. In other words, the mathematician’s job is to study inference of certain given truths.

So the conflict between Russell and Kant turns on a more subtle point of interpretation than whether mathematics is synthetic or analytic. One interpretation is that the role of intuition for Kant would be restricted to the mathematical context (the basic axioms of an internally consistent geometry, for example). Once chosen, theorems could result from the axioms by formal logical deduction. But if the role

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of intuition is not limited in this way and intuition, not formal logic, directs the whole of geometric reasoning then even the result of a geometric construction for a proof would be determined not by the definition of the context but by spatial intuition.\(^{439}\) It is this aspect of the Kantian view that mathematical reasoning is not strictly formal, but always uses intuition, (the a priori knowledge of space and time), that Russell refutes unequivocally, based on progress in symbolic logic by the early twentieth century.\(^{440}\)

A concrete objection to a priori intuitions providing methods of reasoning that could not be substituted by formal logic, was the necessity of the figure, real or imagined to all geometrical proofs.\(^{441}\) In a sense, the same question has hung over architectural system modelling. To what extent can a real or imagined figurative engagement with an architectural system model be sacrificed in order to embrace the greater sophistication of relationships within the representation? The figurative sampling of the model remains at some level central to its value in communicating but is it always equally central to designers’ own designerly ways of knowing the space in which they work?

Russell for a while endorsed the idealist tradition in which sensibility had a dominating role, but the foundational research of Weierstrass, Dedekind, Cantor and their followers led, in Russell’s writing, to “the collapse of the idea that analysis must rely on the imagination\(^{442}\) in order to deal with the basic space–time notions of infinity and continuity”.\(^{443}\)

Poincaré also picked up on the distinction between intuition in the choice of axiomatic context and intuition in the process of inference. For him there are fresh appeals to intuition in the mathematician’s ongoing work.

Before moving to the arguments of the intuitionists regarding formal mathematical propositions and empiricism, I will just consider, for a moment, whether a similar debate could ever be conducted in architectural design. Of course, once a context is set in terms of physical and intellectual site, program, in theory it would be possible to generate architecture on a purely algorithmic basis – follow and develop formal recipes using axioms, theorems, symbols and rules. Paul Coates has written in support of the idea that it is much more exciting to harness computation to generate form in response to emergent processes – that is not to know exactly what it is (in the sense of output shape information) that it is being generated but simply to initiate the parallel processes by which it comes into being.\(^{444}\) These would not be deductions as in the mathematical argument, but syntheses nevertheless constructed using logic based on logical formulations that have already proven themselves in terms of what ever it is that they represent, such that it is no


\(^{441}\) Ibid., 456–7.

\(^{442}\) This is the imagist imagination, implicitly pictorial.

\(^{443}\) Coffa, Russell and Kant, 254.

longer necessary to pay ongoing detailed attention to these referents. This would be architecture that had resolved itself purely into game play with symbols.

Do any of the examples in the dissertation come close to this? The fact that the emphasis in this research is on geometry and mathematics in architecture undoubtedly gives the examples a bias towards applications of computational geometry for building system models rather than the bottom-up parallel process examples that Coates espouses. But the search for examples under the five themes in Chapter Three does spread the net wider by including Chaos, complexity and emergence, tiling and optimisation.

Let’s consider some of the examples. Minifie Nixon’s Australian Wildlife health Centre features both a top-down geometrical description of a very novel use of surface (the Costa minimal surface) and a bottom-up description for generating the patterning in the bi coloured masonry set out (the cellular automata). But ultimately both have their meaning in the context of the overall building design, in terms of spatial composition and visual readings of the architecture. It is the intriguing figure of the Costa surface in the region of its extrema as an embodied phenomenon rather than its algebraic description, or the details of the proof of its existence as a previously undiscovered minimal surface that introduces it into the sphere of the design for the building. This artistic potency has a wholly figurative basis. Similarly it is the marriage of the concept of the cellular automata to its imagined representation in the brickwork that brings this into the architecture. The cellular automata are used as emergent visual pattern generators that can be manipulated through the rules of the automaton. The figurative and phenomenal are fundamental to all aspects of the conceiving, imagining, designing and realising the building regardless of its exposition of geometry or use of generative computing. For Gehry and Partners’ Disney Concert Hall, the architecture is born in paper and card. A highly analytical, mathematical process is developed to rationalise or idealise this to constrained geometrical descriptions, in order to re translate it into descriptions for construction materials and components that will compose these shapes, albeit never conforming precisely to the idealised geometries. As in the Minifie Nixon example, the modelling process, for all that it harnesses abstract mathematical analytical spatial concepts, never departs completely from a directly referential relationship in the representation of the generative physical models or the representation of the constructible physical realisation of the building. The figurative is critical in conception, imagination, realisation and repeated aesthetic performance analysis throughout the process. The only departure is in the intensive experimental workshopping to develop the representational tools and methods to do this. Gaudí found procedures for the realisation of certain challenging geometries in gypsum plaster as the model testing ground for his architecture. The translation of these Boolean subtractions of doubly curved surfaces to a computational setting, while it has used computational geometry and generic analytical description of shape and functions, has had as its emphasis, a visual, as well as metrical, dynamic form-finding activity. This activity is fitting geometrically described geometrical assemblies to Gaudí’s restored plaster
assemblies where these are available, and more recently to the more fragmentary evidence in images. Once again the departure from the figurative is only in order to develop or refine the technique of representation. Figurative is used here in the sense of ‘representing by use of a figure or likeness’ but could also have the meaning of ‘metaphorically’ where the metaphor is clearly visual or phenomenal. The point is clear. Mathematics can engage in argument about the validity of a purely formal or logical basis for what is within mathematics. Architecture can only do this where it is the architecture of things, which are in their own right symbolic, such as software, but never entirely in relation to the architecture of buildings which have shape and other phenomenal qualities that will not entirely relinquish imagery.

One of the best examples to examine in the preceding chapters to test the concept of a purely formal architecture (formal in the sense of strict logical form, symbol manipulation without regard for meaning) might be Biothing’s Invisibles. This is emergent architecture, and I believe in its phenomenal nature, sense that it could exist physically, when I see it in animation. It could be generated without any direct representational content i.e. the components of the algorithm do not necessarily represent anything (that already has real world qualities and constraints) it just makes stuff happen. However the means to translate this from algorithms generating changing imagery (animation), into algorithms that generate the physical, endlessly morphing, biological phenomenon that our imagination leaps to make it represent is not yet known.

6.2 Intuitionism, formal and informal

Luitzen Egbertus Jan Brouwer (1881–1966) developed the philosophy of mathematics called Intuitionism, which while it gives individual intuition sway over the independent validity of logic in mathematics, is, in a sense, antithetical to Kant’s Pure Intuition with regard to Space and Geometry. It does, however, maintain Time as an a priori notion. It is based on the idea that mathematics is a creation of mind that is not universal: the truth of a mathematical statement is conceived via a mental process that proves it to be true and the communication between mathematicians allows the same mental process to be shared in different minds. This view topples certain logical constructs used in mathematical proofs, notably the principle of the excluded middle (A ∨ ¬A). Proof that a statement is not not true is no longer proof that it is true. There are propositions for which there exists no proof of the statement and no proof of its negation at this time. Time is important in intuitionism and statements or their negations could become provable in time. Intuitionism is not a limitation on classical reasoning; it contradicts classical reasoning in important ways, which makes the Intuitionist view and the Classical view alternative viewpoints.

Not only does Brouwer’s philosophy make mathematics a product of the free mind but it also removes mathematics from a play with symbols according to fixed rules. It does this by separating mathematics from mathematical language as a language–less activity of the mind. 445

Brouwer wrote that:

“An a priori character was so consistently ascribed to the laws of theoretical logic that until recently these laws, including the principle of the excluded middle, were applied without reservation even in the mathematics of infinite systems and we did not allow ourselves to be disturbed by the consideration that the results obtained in this way are in general not open, either practically or theoretically, to any empirical corroboration. On this basis extensive incorrect theories were constructed, especially in the last half century… an incorrect theory, even if it cannot be inhibited by any contradiction that would refute it, is nonetheless incorrect, just as a criminal policy is nonetheless criminal even if it cannot be inhibited by any court that would curb it.”

Brouwer’s work has meaning principally when viewed in reaction to mathematical formalism. Oblique reference has been made to David Hilbert’s program for a complete and consistent axiomatization of all of mathematics. This was to be constructed from the single assumption that the “finitary arithmetic”, a subsystem of the arithmetic of positive integers, was itself consistent. This is generally regarded as the basis of formalism in mathematics, a way of thinking that locates mathematics primarily as series of games played according to rules that govern inference, in which strings of symbols called axioms are moved around generating new strings according to the rules. Proof of validity of a theorem rests on whether it can be constructed strictly within the rules of engagement. Formalism is subject to the criticism that the mathematical ideas that occupy mathematicians are actually far removed from the string manipulation at its heart. But formalism does not dictate which axiom systems should be studied; in fact none is more meaningful than any other from a formalist viewpoint. This is very different from, for instance, Hardy’s view of mathematics in which some theorems are much deeper than others, a value system that, nevertheless, does not necessarily rest on their usefulness in application in any given time or context. The grip of formalism in

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447 David Hilbert (1862–1943). Hilbert put forward his list of 23 unsolved problems in mathematics at the International Congress of Mathematics in 1900 which was considered one of the most deeply considered compilations of unsolved problems and highly influential in twentieth century mathematics. ‘Hilbert’s Program’ (1920) proposed an explicit research program in metamathematics to establish the logical foundation of mathematics by showing that a) all mathematics follows from a correctly chosen finite system of axioms and that b) some such axiom system is provably consistent.

448 Finitary signifies an operation that takes a finite number of inputs to give an output. (Many operations in mathematics take an infinite number of inputs, such as the integral of a function in calculus.)

449 An axiom is a proposition whose truth is taken for granted, a starting point for deducing and inferring other dependent truths. In formalism, it can be thought as the opening state of the game.
mathematics has been pervasive and became doctrinaire during the twentieth century, sometimes obscuring mathematical thought based on a closer allegiance with an external reality/ies outside the formalist system.

There is a useful analogy to design in Brouwer’s thinking. Design, too, can be played as a formal game with ordering or compositional rules. *Possible Palladian Villas* was an obvious, formative, post-digital example of this but the rules can become very sophisticated and, ‘non standard’ in the individualist artistic sense once electronic computation is enlisted. (This analogy to Brouwer’s thinking itself, highlights what Douglas Hofstadter refers to as *analogical awareness*, a crucial side of human intelligence, something that representing knowledge in a logical formalism is apt to miss. He opposes it with *deductive awareness* of the domain that is represented, which is strong within logical formalisms.) Minifie Nixon’s application of cellular automata and the Costa surface in the Australian Wildlife Health Centre (ref Chapter 3) is a good example of appropriation of a principle for generating rules. The cellular automaton is highly adaptable; it has rules that can be endlessly tweaked to produce a suitable aesthetic outcome. But this example also serves to demonstrate the way the rules must also be tested back in the domain of what is physically constructible. In this case the 2-D variants on the Game of Life correspond well to a masonry set out of comparatively regular blocks in the wall, despite the adaptation to the non-rectilinear planning of the centre. In this sense it is independently (empirically) verifiable outside the game and the rules of the game, in a way that is analogous to that called for by Brouwer in the games of mathematical formalism.

Douglas Hofstadter writes: “fantasy and fact intermingle very closely in our minds, and this is because thinking involves the manufacture and manipulation of complex descriptions, which need in no way be tied down to real events or things.”

Hofstadter is principally concerned in this reference with getting to the heart of *formal systems*. He uses an example invented by the American logician Emi Post in the 1920s, which he calls the MU Puzzle. The main point about a formal system is that you must *not* do anything *outside the rules* (the restriction of formality).

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452 The puzzle is: “can you produce ‘MU’?” You are given the string ‘MI’ and some rules with which to change one string to another. You can use any applicable rule at any time – there are no rules about which rule you should use where several are applicable. The formal system the *MIU system* uses only these three letters – the only strings of the MIU system use only the letters of the alphabet ‘M’, ‘I’ and ‘U’. Rules: (1) If you possess a string whose last letter is ‘I’, you can add on a ‘U’ at the end, e.g. MI gives MIU. (2) Supposing you have ‘Mx’, you can add ‘Mxx’ to your collection of strings e.g. MI produces MII, MUI produced MUIUI etc.; (3) If ‘III’ occurs in one of the strings, you can replace ‘III’ with ‘U’ e.g. MI gives MU; (4) If ‘UU’ occurs inside one of your strings, you can drop it e.g. From UUU you get U. Starting from the string MI, play to try to make MU.

Thought and formal systems diverge in this particular. Hofstadter says, “...the strange flavour of AI work is that people try to put together long sets of rules in strict formalisms which tell inflexible machines how to be flexible...” based on the understanding that intelligence is characterised by the ability to: - respond very flexibly to situations, - take advantage of fortuitous situations, - make sense of ambiguous or contradictory messages etc. This leads to just plain rules; metarules to modify just plain rules; then metarules to modify the metarules, and so on. Hofstadter also writes: “Ever since Pascal and Leibniz, people have dreamt of machines that could perform intellectual tasks” but “The once exciting phrase ‘Giant Electronic Brain’ remains only as a sort of ‘camp’ cliché, a ridiculous vestige of the era of Flash Gordon and Buck Rogers”. This tendency to become blasé quickly is encapsulated in a ‘Tesler’s Theorem’ about progress in AI: “once some mental function is programmed, people soon cease to consider it as an essential ingredient of ‘real thinking’. The ineluctable core of intelligence is always in that next thing which hasn’t yet been programmed.”

Taking a step back to consider the limits of logical computability, Gödel’s Theorem is paraphrased as all consistent axiomatic formulations of number theory include undecidable propositions. In other words, there are questions for which it is impossible to reach a simple yes or no answer within a finite period of time. Alan Turing proved in 1936 that, given a description of a program on a Turing machine, to decide whether the program finishes running or continues to run, and will, thereby, run forever is undecidable. This is known as the Halting Problem. Hofstadter refers to these and related matters as “Strange Loopiness” and asks, rhetorically, why not ban, not only paradoxes, but all self-reference and its causes [from the game]. He finds, as might be anticipated, that the cost of such a stipulation is too high, both in terms of what can be performed and in terms of the interest and quirkiness of mathematics. In the section on Chaos, Complexity and Emergence in Chapter 3, I have touched on the architectural opportunities of recursion and in particular generative recursive systems using computation. The presence of self-reference in natural systems is so fundamental that it is hardly imaginable that self-referential systems could be totally excluded from the formal systems conceived for computation.

Another twentieth century deviation from mainstream formalist mathematical thought was Abraham Robinson’s (1918–1974) discovery and development of nonstandard analysis, a rigorous theory of infinitesimals. Robinson spoke for the first time on Model Theory and Non–standard arithmetic in 1959 at an international symposium in Warsaw devoted to the discussion of infinitistic methods in the foundation of mathematics.

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454 Ibid., 26.
455 Ibid., 600.
456 Ibid., 601.
457 Gödel’s Theorem: To every w-consistent recursive class k of formulæ there correspond recursive class-signs r, such that neither vGenr nor Neg(vGenr) belongs to Flg(k) (where v is the free variable of r).
458 Hofstadter, Gödel, Escher, Bach, 21.
In 1961 he published his first paper on the topic of Non-standard analysis.\textsuperscript{460} Robinson revisited the argument of Gottfried Leibnitz that the theory of infinitesimals implied the introduction of ideal numbers that might be infinitely small or infinitely large compared with the real numbers but which were to possess the same properties as the \textit{reals}.

While neither Leibnitz nor his followers were able to give rational development of a system of this sort (in fact it was the ethereality of the idea of infinitesimals that gave detractors like Berkeley most grounds for criticism of the early development of calculus), Robinson was able to show that the ideas can be vindicated and lead to novel and useful new approaches to analysis, which he called \textit{nonstandard analysis}. Nonstandard analysis leads to simplification of many propositions and proofs, for instance the definition of when a real function is continuous. The theory is based on analysis of the relationship between mathematical languages and mathematical structures which is the basis of model theory. Model theory\textsuperscript{461} is the branch of logic studying mathematical structures by considering only first-order sentences, true of those structures and the sets, which are definable in those structures by first order formulas.\textsuperscript{462}

Robinson was someone of whom the philosopher John Kaplan at UCLA later said “he talked philosophy the way philosophers did.”\textsuperscript{463} Robinson himself made a claim for the importance of playfulness in mathematics and was credited with a remarkable ability to understand the potential applications of his work through his own very broad mathematical knowledge. He was also said to be able to communicate beauty in mathematics.

I have noted already Frédéric Migayrou’s adoption of the term \textit{non standard} for an architectural lineage he identifies from the early twentieth century modernism exemplified by Henry van der Velde’s stand for the upholding of the individuality of artists (in design) against Muthesius’ call for standardisation (in design) at the Werkbund meeting in 1914. Migayrou’s definition of non-standard also encompasses what he calls “the constituent logic of a new architectural singularity… how a singularity organises itself within a dynamic system”.\textsuperscript{464}

It is never quite clear whether this is expressly a mathematical

\begin{itemize}
  \item including quantifiers, for instance “for every object (in the domain of discourse)…”.
  \item Second order logic is more powerful including additional quantifiers such as “for every property of objects (in the domain of discourse)” or “for every set of objects”. In the foundation of mathematics, first order logic has become the standard formal logic of axiomatic systems.
  \item Ibid., 19.
\end{itemize}
singularity\textsuperscript{466} with reference to functions of morphology, for instance, or another name for a particular instance of any variable geometrical description as a “generator of infinite forms” in architecture. \textsuperscript{467}

One more important twentieth century mathematical and philosophical thinker who highlighted the potential monoculturalism of mathematical formalism and the cost of disregarding what it swept aside was Imre Lakatos.\textsuperscript{468} His book *Proofs and refutations* is based on the first three chapters of his Cambridge doctoral thesis *Essays in the logic of mathematical discovery*.\textsuperscript{469} It is largely a fictional dialogue within a mathematics class. The students are attempting to prove the Euler characteristic in algebraic topology, the theorem about polyhedra $V-E+F=2$ where $V$ is the number of vertices, $F$, the number of faces and $E$, the number of edges in a polyhedron. The class dialogue represents that actual historical series of proofs offered for the conjecture by mathematicians, only to be repeatedly refuted by counterexamples. Lakatos attempted to show that no theorem of informal mathematics is final or perfect. It stands until a counterexample is found, at which point the theorem is adjusted, possibly qualifying the domain of its validity. Thus mathematical knowledge builds and adapts through proof and refutation. Lakatos: “Teacher: I admit that the traditional name “proof” for this thought-experiment may rightly be considered a bit misleading. I do not think that it establishes the truth of the conjecture.”

\textsuperscript{466} Singularity theory in mathematics is the study of the failure of manifold structures – singularities are exemplified by the double points where a piece of string representing a one dimensional manifold crosses itself when you drop it on the ground. Some singularities in a function are stable, others not so. For a much better layperson’s explanation of Morse’s theorem and singularity theory, read Chapter 3 of Casti, J.L. *Five Golden Rules Great Theories of 20th-century Mathematics - and Why they Matter*. New York, Chichester, Brisbane, Toronto, Singapore: John Wiley and Sons, Inc., 1996.

\textsuperscript{467} Migayrou, *Future City experiment and utopia in architecture*, 17.

\textsuperscript{468} Imre Lakatos (1922-1974) Hungarian philosopher of mathematics and science.


\textsuperscript{470} Ibid.,9.

\textsuperscript{471} Ibid.,8.
“Alpha (first student): Imagine a solid bounded by a pair of nested cubes – a pair of cubes, one of which is inside, but does not touch the other. This hollow cube falsifies your first lemma, because on removing a face from the inner cube, the polyhedron will not be stretchable on to a plane. Nor will it help to remove a face from the outer cube instead. Besides, for each cube V-E+F = 2, so that for the hollow cube V-E+F = 4.

Teacher: Good Show! Let us call it counter-example 1. 472

This counter example is of historic interest (first noticed by Lhuilier (1812-13), but claimed as an earlier discovery by Gergonne, Lhuilier’s editor). 473 Also, interestingly, the subtraction of a cube from the interior of a solid cube, where the two do not intersect and there is no passage from the internal hollowed out internal space to the exterior space is a proposition that has until recently posed a serious challenge to certain three dimensional modelling software, as it relies on a particular definition of solids.

But the central point here is Lakatos’ counterpoint to the formalist school of mathematics. In his introduction to Proofs and Refutations, he notes that there are problems, which fall outside the range of the [Hilbert’s] metamathematical abstraction. 474 Lakatos writes that Formalism disconnects the history of mathematics from the philosophy of mathematics, since, according to the formalist concept of mathematics, there is no history of mathematics proper. He cites Russell’s remark that Boole’s Laws of Thought (1854) was the first book ever written on mathematics. The nineteenth and twentieth century formalists take Descartes’ modernist clean slate to a new extreme. But, according to Lakatos they leave open a small back door for fallen angels: “Newton had to wait four centuries until Peano, Russell, and Quine helped him into heaven by formalising the Calculus.” Paraphrasing Kant, Lakatos writes: “the history of mathematics, lacking the guidance of philosophy, has become blind, while the philosophy of mathematics, turning its back on the most intriguing phenomena in the history of mathematics, has become empty.

‘Formalism’ is a bulwark of logical positivist philosophy.” 475

Lakatos offers a “overdue challenge” to this stronghold of dogmatist epistemology but through the modest “aim of elaborating the point that informal quasi-empirical, mathematics does not form through a monotonous increase in the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations.” 476

And this is closer to the iterative cycle of proposition and testing of design than the formalist dogma. But in order to determine how much closer, let us now consider descriptions mathematics and to its growth, and all problems relating to the situational logic of mathematical problem-solving.

472 Ibid., 13.

473 Ibid., 11. Simon Antoine Jean L’Huilier (or L’Huillier) (1750–1840), Swiss mathematician of French Hugenot descent. He is known for his work in mathematical analysis and topology, and in particular the generalization of Euler’s formula for planar graphs. Joseph Diaz Gergonne (1771—1859) was a French mathematician and logician.

474 Ibid., 1. Among these are all problems relating to informal

475 Lakatos, Proofs and Refutations, 2. (Introduction)

476 Ibid., 5.
of the experience of mathematical discovery itself from others in the field who also question the most dogmatic formalist stand on what is valid in mathematics. This will show better the extent to which mathematical and design thought processes can be seen to align. To adapt Poincaré’s statement, architectural computational geometrical system modelling uses “off the shelf” proven geometrical relationships but can make use and reconfigure them only by fresh recourse to intuition.

### 6.3 Minds of mathematicians and computational designers

In Jacques Hadamard’s book *The Mathematician’s Mind*, Henri Poincaré, the French mathematician and philosopher, whose writing plays a central role in this argument, appears once more as an important protagonist. Poincaré provides an example of a problem that had occupied him consciously for a fortnight without success before one night drinking black coffee, “Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination.” He then went away on a geological expedition for some days and in a moment of certainty in the instant he boarded a bus, “the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define Fuchsian functions were identical with those of non–Euclidean geometry ... It seems, in such cases, that one is present at his own unconscious work, made partially perceptible to the over–excited consciousness...”

Poincaré rejects the purely logical view of mathematics, however evident logical deduction may be in the finished product. It cannot account for the productive combinatorial process in which a great deal of choice is involved in selecting which deductions to pursue and how they are to be made. It was his contention that, were the rules of inference actually employed rigorously in this process, they would quickly lead to contradictions and paradoxes. This process as described, most associated in Hadamard’s book with those of rare talent – Mozart, Gauss, Helmholz, for example – is a form of unconscious thought in which many combinations of ideas are shaken together (the etymological root of ‘cognito’) until a chance fit occurs and the result is thrown close to the surface of ‘fringe’ consciousness. Here we see that the terms ‘synthetic’ and ‘figurative’ are, in certain creative circumstances in particular, just as central to mathematical exploration as to the exploration of the design space and its definition in the architectural design process.

This idea of mathematical clarity arising out of crowded, murky and disorderly space is a familiar one. John Stillwell wrote that “Rigor and precision are necessary for communication of mathematics to the public but they are only the last stage in the mathematician’s own thought. New ideas generally emerge from confusion and obscurity, so they cannot be grasped precisely until they have first been grasped vaguely and even inconsistently.”

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478 Stillwell, J. *Numbers and Geometry*. Edited by Axler, S., Gehring, F. W., and K.A. Ribet, Undergraduate Texts in
This intuitive search within a very ill defined space – the outcomes only acquiring their logical analytical structure in the proofs, which come much later – makes mathematical discovery appear indistinguishable from design process in all but its aims. Thus, although mathematics is ultimately a search for truth or at least consistency, and design a search for form (whether of space or object) consistent with a large number of explicit and implicit criteria, the cognitive journeys are not necessarily very dissimilar. In computational geometrical system modelling, the searching within obscurity may be for the form of the computational design model system – the form of the design space. Directed mechanized computation, then, underpins the targeted search within this (in and of itself unstable) design space for the form of the designed space.

In contrast to Hadamard’s examples of high level creative mathematical thought, Papert posits that an innate sense of mathematical aesthetics is accessible to those without a high level of mathematical skill – a democratised notion of sensitivity to mathematical aesthetics. He frames this as a radical viewpoint outside mainstream culture. While he attributes to Poincaré the view that the distinguishing characteristic of a mathematical mind is aesthetic, not logical, he nevertheless, accuses him of enshrining theoretically the prevalent view that appreciation of mathematical beauty is accessible only to the elite. Of Piaget’s work and other theories of the psychology of mathematical development, he writes that they ignore the aesthetic entirely. He claims that mathematical education over-emphasizes the logical facets and neglects an aesthetic projection that, were it given full rein, might dispel the elitist ‘innate’ perception about mathematical thought promulgated by Poincaré.

Papert gives small simple algebraic example that he has observed giving pleasure to non–mathematical students. As they attempt from first principles to prove that the square root of 2 is irrational, they start with the square root of 2 = p/q where p and q represent natural numbers with the intention of moving to reductio ad absurdum (showing that the square root of two cannot in fact be given by the ratio of two natural numbers). From this they are able to proceed to

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479 While it may be fair to say that a similar pattern is found by ethnographers of science, and there is some kind of simile in many creative pursuits, the cursory exploration of select history of the philosophy of mathematics has revealed that this portrayal of mathematical discovery and even the description of mathematics as discovery is far from uncontested ground. The attention that Lachterman draws to the classical tradition, as witnessed in Euclid’s Elements, of referring to geometrical constructions as ‘pre-given’ and their almost passive role in the proof of the existence of certain geometrical entities counters this modern, active and human-centred portrayal of the process of mathematical discovery. More recently, the strictly formalist tendency in mathematics with its roots in Hilbert’s aim to bring the whole of mathematics within a coherent logical framework also has no place for intuition by any definition, or mathematical discovery as anything less than a purely rational, analytical, derivative process. Design has also made periodic claims on logic and consistency in its methods but the description of design as an intuitive search within a very ill defined space is, arguably, less contentious than the same description applied to mathematics.


481 Ibid., 105–6.
Whether or not they are able to continue to the proof from this point, this result alone gives them happiness and satisfaction that he considers to have only aesthetic basis. They have eliminated a square root and a ratio to arrive at this simplified expression. It seems to me that to reach this point and certainly from this point forwards, it is necessary to harness a certain measure of logical deduction: $p^2$ must be an even number which is contradictory for the squares of odd numbers etc. However, accepting his case for the importance of the aesthetic component (even as a reward to the participants) this is clearly an aesthetic that has no closely direct relationship to the senses and no close relationship to ‘artistic questions’ and it is a value not directly governed by proof. It appeals to a different sensibility, some deep recognition of order, pattern, and simplicity. Papert’s idea of aesthetics could also be a product of his time, reflecting the reductivist tendencies of twentieth century modernism.

Does Papert’s idea of broad enfranchisement through the aesthetics of mathematics appear at face value as plausible as the idea of a meaningful appreciation of architectural aesthetics by those without formal education in the architecture or first hand in depth architectural design experience? Many would see architecture as valueless without the precondition that it communicates aesthetically with or at least provokes those inhabiting, animating and experiencing it.

Paralleling Papert’s broadening of the idea of mathematical aesthetics, discussion of the value of the figurative in mathematics extends more broadly than just in high level mathematical discovery – Poston and Stewart, for instance, while they are very clear on the reality that Catastrophe theory is not a qualitative theory (it is the detailed mathematical application that is useful in the physical sciences), nevertheless, use physical catastrophe machines using cardboard, elastic bands, pins and pencils to maintain the link between theory and practice and strengthen the role of physical intuition.

While of hyperbolic geometry, Ramsay and Richtmyer write: “the notions of geometry go, in a sense beyond the notions of analysis, in that they are things that we ‘visualize’. Although we must keep in mind the limitations of diagrams, and so on, the ability to visualize is a human ability that should be encouraged rather than suppressed, in the teaching of mathematicians. Our impression from teaching talented young students is that they can visualize the hyperbolic plane, in a sense. From that point of view the main models, those of Beltrami, Klein, and Poincaré, are unsatisfactory for intuitive geometrical visualization.”

Architecture not only shares with mathematics the value of the ‘large generalization, limited by a happy particularity’ as the basis of its aesthetic success, but mathematics, it seems, in certain fields at least, shares with architectural design the value of visualization, at least in imagination, if not in external figurative or synthetic representation, as the basis of its aesthetic success.

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482 Ibid., 113.


6.4 The invisibility of the model

The computational design model space is potentially invisible, seen only through its instances, or the manifestations of particular trajectories through the space. It is these traces that are seen, not the model itself, which must be understood through more abstract, linguistic, mathematical, diagrammatic, and perhaps logical means. There may be no transcendent, objective view of the whole. In relinquishing the primacy of the object, the value of the image also fades. Whether we speak of the image as the projection of the object into two dimensions in the scientific tradition of Desargues or of Bachelard’s poetic image that collects and creates resonance in imagination, the system model realized through design computation on a programmable machine confounds the ocular-centricity of the designer. The image that has been central and all-powerful in design thinking, in the field of both the outer and the inner eye gives place to less immediate ways to know the model space in computational design. Donald Schön gives us three types of seeing for designing: literal visual apprehension, appreciative judgments of quality, and apprehension of spatial gestalts. The first and the last, at least, are compromised as we move into model spaces that can be experienced as having as many spatial dimensions as they have variables or degrees of freedom. The second, ‘appreciative judgments of quality’ are adaptable to aesthetic frameworks predicated more on ‘deep’ pattern recognition through more logical, intellectual and less sensorially-led appreciation. These are neither visible nor readily visualizable spaces. With reference to Nigel Cross’ warnings about failure to recognize the distinct nature of design in relation to science we must now ponder how this computational design space is assimilated into design’s own distinct “things to know, ways of knowing them, and ways of finding out about them.”

First, I will briefly compare two contrasting portrayals of the image pertinent to the consideration of the role of vision and visualization in the perception of space and design thought.

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487 In the early days of the Joint Center for Urban Studies at Harvard and MIT, and the Centre for Land Use and Built Form studies LUBFs at Cambridge, despite a culture of scientific analytical methodology in bringing computation into design problem-solving that had no time for the consideration of ‘appearances’ in architectural design, Lionel March has written that his primary interest and motivation was aesthetic. (March , L. ‘Modern Movement to Vitruvius: Themes of Education and Research.’ *Royal Institute of British Architects Journal*, 81 (1972): 101–9) (Keller, S.B., ‘Systems Aesthetics: Architectural Theory at the University of Cambridge, 1960–75’, in Architecture, Landscape Architecture, and Urban Planning (Cambridge: Harvard University, 2005).172.) For one reflection on the nature of aesthetics in mathematics, see: Hardy, G.H. *A Mathematician’s Apology*. Cambridge: University Press, 1940.


489 Ibid., 221–
6.5 Projection and the image

Robin Evans writes that “what connects thinking to imagination, imagination to drawing, drawing to building, and buildings to our eyes is projection in one guise or another”. He is exploring not only the optical dimension of architecture but visualisation in its most cerebral and mysterious sense. His treatment, however, is explicitly instrumental, and geometrical. It gives a sense of the continuity and flow as well as the physics and geometry of incident and imagined light but in ‘Newtonian’ space. This is in sharp contrast to Gaston Bachelard’s phenomenological consideration of the ‘poetic image’ in imagination. It is interesting to contrast the emphasis on geometry as the means of representation in Evans’s descriptive statement with the more subjective treatment of the image as a given phenomenon that mysteriously collects and creates resonance in Bachelard’s writing – a somehow more abstract concept of visualization as a Fourier carrier and transmitter. I have resorted to metaphor from physical science while Bachelard himself rejects the shallow inference of metaphor and deliberately sidesteps analytical ownership and explanation through the sciences, particularly those concerned with psychology and physiology. Evans is concerned with one aspect of geometry – projection – and its entwinement with the art of architectural and painterly space making.

Geometry need not in itself be viewed exclusively as the preserve of reason and rationality in space making. The significance of Evans’s work for this chapter is his linking of the space of external reality to the space of ‘imagination’ through a single means of geometrical representation. In other words, he conceives both vision and visualisation, geometrically. The projective geometry defines also the view of and our visual experience of the built architecture. Symmetrically, by the inverse geometrical route, we reconstruct the internal image and construct the imagined image. This is the empirical Newtonian understanding of the nature of light and its interaction with the object world and the Newtonian understanding of space in which the action of seeing and imagining inhabit a continuum with their observed physics. By contrast, Bachelard’s image is a more mythical synthesis that operates outside the space that can be described primarily through its geometrical construction.

6.6 Phenomenology

The comparison of the treatment of the image by the architectural historian Robin Evans and its treatment in the earlier writing of the phenomenologist Gaston Bachelard serves to make the distinction between a view of space in which the conceptual mechanics of a system of construction (geometry) has a central position and one in which subjective human experience is central. Designers must operate in the extremes of both rational scientific, and phenomenological space in order to address both the operation of making and the deep experience and resonance in the subject. This thesis dwells consistently in the realm of the design space.
represented by the system model rather than the designed space. The subject in this investigation is the operator, the maker, the modeller. For this reason it takes as a framework the Kantian lineage in order to specifically investigate the non–homogeneous nature of system model space and its implications for the maker, whilst largely marginalising other philosophical pathways through space, notably phenomenology. This is not to dismiss the significance of a different investigation, for example, to discover whether or not the deep engagement in constructing complex design (model) spaces geometrically and algorithmically poses a distraction from phenomenological aspects of spatial design. It is merely to state a particular emphasis on the form rather than the matter of intuitions (to adopt the Kantian terminology).

So, returning to the central theme of this study, what can the contemporary architectural system modeller learn from the philosophy of mathematics that will help them to understand and navigate space of relatively unlimited dimensionality?

This is really a question concerning aesthetics. This means aesthetics in the sense of the sensory apprehension of the world and aesthetics in the sense of space, and, possibly, of time. Architectural design, the design of spatial organisation and artefact, always returns at some point to the figurative and concrete, and this problem of the unlimited dimensionality and non homogeneity of the model space in relation to its representation of a more concrete and ultimately physically constructible reality was already understatedly present in the work of Descartes, and even in Greek mathematics. I have already shown ways in which aesthetics are central to both design and mathematical thought. But do the natures of these aesthetic sensibilities constitute genuine affinity between mathematics and design in the philosophy underpinning the spatial thinking? Moving on from the aesthetics of mind in the activities of mathematical discovery and architectural design, the next part of the chapter is a comparative examination of aesthetics in mathematics and in architecture, which is seen as critical to spatial perception in models.

6.7 Architectural and Mathematical aesthetics

During the 1960s and 1970s, at the same time that mainstream postmodernism mounted its first vociferous reaction to modernist’s aesthetic, reductionist functional dogma, discourse on aesthetics was banished from certain architectural quarters. The early Cambridge–based LUBFS work described in chapter 2 exemplifies this position. ‘Appearances’ were superficial and irrelevant; design should focus on deeper mathematical formal models, formulated through rigorous analysis. This manifesto fell prey to substantive criticism, including internal scholarly criticism, on grounds that can be generalised to the dual problems that design criteria are not consistently and meaningfully quantifiable and that analysis, per se, yields no synthetic formal direction. Architecture’s alignment with the arts was its perceived weakness and its post war realignment with empirical and mathematical sciences was a key to public funding and civic seriousness. Inevitably this seriousness spawned novel aesthetic outcomes. The Smithson’s Smithdon School in Hunstanton, Norfolk (1949–54) has been accorded the beginning of British Brutalism while
its stripped back minimalism was motivated, ironically, by a puritanical drive away from expression.\(^{492}\) It was also the era in Britain of Cedric Price’s Fun Palace for Joan Littlewood, the theatre director and innovator, and Peter Cook’s Plug–in City, two examples of time–based proposals for infinitely changing and adapting architectures that also, incidentally, provided the progenitors of the subsequent hi–tech aesthetic for projects such as Rogers and Piano’s Centre Pompidou. Robert Maxwell’s ‘eye for a fine building’ had until this time been the ultimate arbiter of aesthetic success and architecture had rules of composition and proportion applicable in an unlimited litany of ways and conforming to different temporal aesthetic systems. In the Cambridge work, Sean Keller has identified a transition from aesthetic systems to systems aesthetic, which, he has written, at its most extreme, was, for Lionel March and colleagues, ‘completely immaterial’ i.e. pure mathematics.

This calls into question the aesthetic of mathematics and its ‘immateriality’.

Certainly the statement of the roving and brilliant mathematician Paul Erdős leaves mathematical aesthetics as an ineffable quality: “Why are numbers beautiful? It is like asking why is Beethoven’s Ninth Symphony beautiful? If you can’t see why, someone can’t tell you. I know numbers are beautiful. If they aren’t beautiful, nothing is.”\(^{493}\)

Hardy has more refined definitions of mathematical beauty. “A chess problem is genuine mathematics, but in some way ‘trivial’ mathematics”.\(^{494}\) … “the beauty of a mathematical theorem depends a great deal on its seriousness, as even in poetry the beauty of a line may depend to some extent on the significance of the ideas which it contains”.\(^{495}\) Some theorems are not serious in his definition by virtue of their high degree of speciality in the enunciations and proofs, their inability to be generalized. He quotes Whitehead, “The certainty of mathematics depends on its complete abstract generality … it is the large generalization, limited by a happy particularity which is the fruitful conception”.\(^{496}\)

Hardy gives us a hierarchical strata of ideas “the idea of an irrational is deeper than the idea of an integer; Pythagoras’s theorem is for that reason deeper than Euclid’s.”\(^{497}\) He is referring to Euclid’s theorem that there is an infinity of prime numbers. This was proved by reducio ad absurdum – that is, by demonstrating that the opposite proposition is absurd. “Euclid’s theorem is very important but not very deep.”\(^{498}\) … A mathematical proof should resemble a simple and clear–cut constellation, not a scattered cluster in the Milky Way”.\(^{499}\)


\(^{495}\) Ibid., 90.


\(^{497}\) Hardy, A Mathematician’s Apology, 110.

\(^{498}\) Ibid., 111.

\(^{499}\) Ibid., 113.
However, giving transcendent status to mathematical ordering, including geometry, is considered by Hardy’s critic Wittgenstein untenable and anachronistic or at least impervious to the modern history of philosophy in philosophical terms:

“The talk of mathematicians becomes absurd when they leave mathematics, for example, Hardy’s description of mathematics as not being a creation of our minds. He conceived philosophy as a decoration, an atmosphere, around the hard realities of mathematics and science. These disciplines, on the one hand, and philosophy on the other, are thought of as being like the necessities and decoration of a room. Hardy is thinking of philosophical opinions. I conceive of philosophy as an activity of clearing up thought.”

The words ‘deep’ and ‘deeper’ are also present in the justification for the transition from formal to mathematical systems in the architectural and planning work of the LUBFS centre and their contemporaries. The idea that a ‘mathematical’ basis for aesthetics is somehow deeper and carries more meaning, or meaning at more levels than a ‘formal’ basis for aesthetics which is more closely aligned with figurative, concrete and synthetic thinking, is a powerful idea at this time. As has been noted above, in 1960s and 70s architecture, it is strangely anachronistic logical positivist position in relation to the earlier philosophical writing of Quine and Popper.

The potentially abstract nature of mathematical aesthetics is highlighted in relation to the form of proofs. In geometry, this also relates to the philosophical opposition between synthetic representation (constructing the geometry figuratively or descriptively) and analytic representation – a higher level of algebraic abstraction. “Geometers are concerned with the attractiveness of a proof. Some proofs using coordinates are long and turgid, and can be replaced by a short synthetic proof. But the true geometer tries to be conscious of both methods of approach to a problem, and a solution by one method often illuminates the other.”

It is generally considered poor mathematical practice to prove a theorem for one class when the same applies to a much broader category, for instance a property of certain

composition, colour, texture and materiality”. While ‘mathematical’ in this context replaces the other, more philosophical, meaning of ‘form’ as the underlying ordering of a system in opposition to matter. I have used the words ‘figurative, concrete and synthetic’ in conjunction with ‘formal’, as opposed to ‘mathematical’ aesthetics, cautiously skirting around direct reference to the space of sensory perception. The senses present two antonymic difficulties here. The first is the contested ground between representational, sensory and perceptual space with respect to ‘external reality’. In particular it is difficult to divorce sensory experience (as opposed to sensory input data) from representational information in our perceptions. The second is the stimulation of sensory experience from what would conventionally be thought of as intellectual or cognitive activity. As this has been recorded most with respect to deep exploratory mathematical thought, it is difficult to divorce the sensory component from mathematical aesthetics.

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501 I am using ‘formal’ in its most common sense in architecture to mean an emphasis on object and spatial “shape,

502 Pedoe, Geometry and the Liberal Arts, 175.
types of quadratic equation when the same truth applies to all quadratic equations. Once again this is beauty and elegance measured by generality and universality.

In architectural design the aesthetic drive to generality is never absent either. This is not merely associated with programmatic efficiency or with a particular minimalist aesthetic. The act of composition is to bring disparate elements into a cogent overall pattern. A particular architect’s signature style is a function of being able to direct the satisfaction of many different programs, sites, and clients, within particular material, organisational or formal languages. However it is the well judged break with symmetry as accorded to Froebel by March, the rule breaking or exploration of exception within the system that makes the connection with the exceptional human pattern recognition heuristic acknowledged by Steadman and which is the ultimate aesthetic achievement. Whitehead’s ‘large generalization, limited by a happy particularity’ summarises value and success in architectural design as aptly as in mathematics.

6.8 Mapping intentions

In computer science parlance, the relationship between design intentions and those intentions expressed as a map of geometrical relations in a parametric design model schema is non–obvious. Similarly, the relationship between the design schema as a diagram or map of the model and its relationships and parameters and the actual ranges of the parameter values is also non–obvious. In other words these things are not easily seen, they are not open to view; not plain, manifest, clear, palpable and certainly not unmistakable. There is a more positive connotation for non–obvious than simply hidden; it is the essential quality of a patentable idea, in other words, its inventiveness.

In addressing the question of ‘designerly ways of knowing’ more complex digital geometrical model spaces, we have already considered some aspects of seeing what is not easily seen in terms of the relationship of perceptual and geometric space in the previous section. Leaving aside, for the moment, the question of whether and how we can see or visualize the geometrically constructed model space, let us turn to whether and how we can palpate that which is not palpable.


507 Palpate comes from Latin palpare – ‘to feel, touch gently’ and it is used in this sense to denote examination by touch (particularly of a part of the body for medical purposes) but palpable is also used to mean ‘so intense as to be almost touched or felt’ as in a palpable sense of loss or a palpable absence. Thus, when we move ‘blindly’ around a design space unable to discern it visibly as a shape or field, despite the fact that we are working with an ostensibly visual medium (computer graphics...
One way is to explore it by steps until encountering the boundaries. Test it through incremental change until it fails. This runs counter to the constructionist and fiercely optimistic, near utopian character of architectural design and discourse. If the graph of dependencies is large, the viable range of each variable may also depend on the values of many others. For an exercise that is sculpturally and aesthetically-led, the search must be driven by swags of designerly intuition about the regions of change in which to explore. Where it is driven by quantifiable outputs in relation to inputs (best natural lighting levels for least glass area), it exploits the indifference of the computer itself to the tedium of searching and comparison in order to find better and optimal solutions. This provides some, albeit partial, empirical knowledge of the complex boundaries, navigable expanses and holes in the space of the model that feed back into its intuitive spatial exploration and manipulation.

This answer to the question of how designers are to know these model spaces is intertwined with the reasons why they might wish to. These models are systems, generally constructed from components, which are familiar geometrical objects. By relating these objects in various ways (without being specific about whether these are individual successive and specialised and visualisation), seeking out boundaries and limits, this seems very akin to literal touch in the examination of the body to find unseen internal organs and interfaces. But of course there is nothing to touch (unless we literally work with a haptic or force-fed interface programmed for the task – but at a literal level it is generally text or slider bars with an image of a three dimensional instance as an output) so it is not palpable. Yet it is literally palpable in the second sense of so intense an engagement as to be almost felt or touched. So in the second sense we can palpate it.

The model has a shape of its own. The singularities and bifurcations in the shape of this space to which Cache, Deleuze and Migayrou have alluded, are not metaphorical, they are palpable, yet visualisable only through visualisation of change. This is not new space – we can find similar spaces in the models of biological and embryological systems and processes. We see their mathematical treatment in the writing of Thom. They are newer to design. In science their complexity is constrained by that which can usefully be computed for analysis. In design they may be purely synthetic and may grow to any order of complexity until constrained by their own brittleness to serve no more.

and relationships, or relationships from a more generalised pattern, generated by a recursive function perhaps), within a few generations of relations, the system has become complex.

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The bounds of the model space are not known, nor are the possible outputs or exactly how they relate to one another in the space. The model should be a rich Ali Baba cave of possible and perhaps novel architecture, which can be pulled from the space, pre-constrained to the needs of the project. In reality, depending on the original solution definition, there could be nothing in the dark cave — no possible outcomes that meet the criteria (the script that will not run) or it might generate a set of geometrical outcomes that make no sense at all as design solutions (the script that failed to describe the intention or map the constraints as understood from the original schema description — in which case the fault might lie in the script or in a fallacy in the original description). There may be fruitful regions within the space, shelves of jewels, but these may lead to impossible ‘holes’, ‘wells’ and we will not know if there are further even richer seams in the space just beyond (bifurcations and geometrical discontinuities). The model space may be more of a Pandora’s box than an Ali Baba cave. But ‘hope’, of course, dwelt amongst the evils inside Pandora’s box. This is different from some other explorations of unknown territory because the multidimensional nature of the space frustrates the deployment of sensory understandings of its form. It is not significantly different from the unseen process of reconciling innumerable different parameters in design thought and conversation that is part of every design but its particular treacherous nature is in explicitly linking design intent and trade-offs to geometrical or shape knowledge in a modelling environment of intrinsically more than three dimensions. Its explicitly geometrical characterisation lacks the semantic fluidity of the loose mix of symbolic and spatial representation possible in thought, conversation or even in sketching. It is a model for the design process but a partial model nevertheless of the design product with integrated geometrical description of its (possible) shape(s).

While each model may represent many design solutions, it is nevertheless a highly defined and constrained design domain. It may be one particular path amongst many approaches with limited opportunities to backtrack or move laterally within the same model. For this reason, the large-scale high-resolution computational model, while it is an exploratory tool, will likely have its construction deferred to a design stage when more is known, decisions have been taken. The earliest stage of design may be best served by a rapid set of diverse alternative propositions or solutions that cannot necessarily be represented coherently in a single model and throughout the design process an array of evaluative models or processes may be deployed outside the principal representation of the proposal that provide feedback on the performance of the design that will influence its overall direction.

6.9 Tools

The computational tools already move beyond standard mathematical categories, relationships, notation and axioms to build more specific frameworks that sit somewhere between geometry and the higher level object structure of things in the physical world. Familiar examples of this are ‘layers’ adopted from the traditional practice of drawing on layers of transparent paper – tracing information from other drawings. Within more sophisticated “product management” paradigm software like Dassault CATIA®, ‘files’, another concept adopted from the paper world, can be linked in
a hierarchy that permits referencing between objects and attributes sorted in different “documents”. Thus “Parts” can be grouped within a “Product”, which not only permits their collective viewing and manipulation within a shared (visual) context but also permits the geometry in one Part to be driven by that in another. Moreover the structure of relations between them can be replicated alone with the geometry with new high–level dependencies in a new context. Catalogues facilitate reuse of content and relations in the model at any level. Although the specific application of the CATIA Parts, Catalogues, Products, Assemblies is not prescribed – they are much closer to receptacles of objects in the physical world than to abstract set theoretical notions in which an element of a set may also be a set and the intersection of two sets may be the empty set. Geometrical Sets and Ordered Sets provide literal set building tools. Their intersection, union and other set theoretical functions all have meaning in the modelling software context. The same object can belong to many sets, two or more sets can sit within a set. The set definition can be extremely hybrid in terms of the objects chosen to belong to it and the relationships defined between them. Even in the software terminology, they are “hybridbodies” subject to “hybridshapedesign”. The set is any group of objects with a collective significance to the modeller.

Software as a possible world is a collection of concepts driven by the software architects’ adoption of conventions, metaphors, as well as their acquired understanding of a generalised workflow, in some cases predicated primarily on earlier paradigms e.g. paper, ink, parallel motions, or machining, component, assembly. In this sense it is never the world of pure geometry or mathematics, but has also an overlay of artefacts, conventions and methods for applying them.

The history of architectural computational modelling has been strongly influenced by object-oriented programming – a paradigm that uses datafields, methods, together with their interactions. An object is a discrete bundle of functions and procedures, all relating to a particular real–world concept such as a bank account holder or a hockey player, or, in architectural modelling, pre–defined geometrical objects and associated attributes. Programmers are equally vociferous for and against the advantages of object–orientation for keeping programs simple. Dr Alan Kay coined the term object–oriented in an allegedly off–the–cuff exchange in 1967. As a biologist and mathematician, he has described his idea of objects being like biological cells or individual computers, only able to communicate between one another by messaging. He has also written of an ambition at that time ‘to get rid of data’, his inspiration for the programming architecture coming out of sketchpad, ARPA.net (refer to Chapter 2) and the Burroughs B5000. Driven by the aim for computing to interface more effectively with real world applications, the vision for objects was, at base, mathematical, high level but generic in the sense of versatile and ubiquitous. “I wanted quite a bit more than functions. I made up a term ‘genericity’ for dealing with generic behaviours in a quasi–algebraic form...OOP [object–oriented programming] to me means only messaging, local retention and protection and hiding

Nevertheless its implementation within modelling software has undoubtedly reinforced the atomised world of discrete, albeit related, objects located in empty but clearly oriented and metric Cartesian space. Other types of world or space must be built against this background.

6.10 Artificial (design?) intelligence

In section 6.2, I have already included (1) Douglas Hofstadter’s summary of the objective of Artificial Intelligence research as putting together long sets of rules in strict formalisms to tell inflexible machines how to be flexible and (2) the truism of Tesler’s theorem that the essential ingredients of intelligence are the diminishing list of things, which have not yet been programmed. In general, I have not fore grounded the topic of artificial or computational ‘intelligence’ or even computation in general. That is because I have focused my attention on system models constructed as edifices of geometrical relations, in other words, deterministic systems in which the action of construction is, more or less, with the modeller and what is computed is the implication and representation of their geometrical moves and instructions. These are top-down models with a direct lineage from geometrical and algebraic thinking. Some would argue that this is a very limited use of computation – and that there is a much more exciting, more programmatic, less mathematical approach to finding form or setting the computer to make things. Of recent publications, Paul Coates, Programming Architecture, is the greatest advocate of harnessing parallel processing of emergent outcomes in architecture. He gives sequence of examples of experiments in this vein since the 1960s. Early in the book he presents a direct comparison between generating a Voronoi boundary net between points using a very short attract/repel code in logo that causes ordinary points to move in relation to the target Voronoi points to cell boundaries, and generating the same Voronoi net using a computational geometrical approach, programmed in BASIC where the code runs to several pages. Marvyn Minski is one of the leading names in Artificial Intelligence as co-founder with John McCarthy of the Massachusetts Institute of Technology’s (MIT) AI laboratory in 1959. He worked with Seymour Papert on the first Logo “turtle.” Paul Coates writes: “the early pioneers

513 Ibid.


515 Marvyn Minski (1927-) American cognitive scientist in the field of artificial intelligence.

516 John McCarthy (1927-) American computer scientist and cognitive scientist who received the Turing prize in 1971 for his contributions in the field of artificial intelligence.

517 Seymour Papert (1928-) South African MIT mathematician, computer scientist and educator, leading researcher into the impact of new technology in learning.

518 Logo is a computer programming language created at the Bolt, Beranek and Newman research firm, adapted from Lisp. It was created in 1967 for educational use by Wally Feurzeig and Seymour Papert. The name comes from the Greek Logos meaning word. Logo’s best known character is the turtle, an on-screen cursor that can be triangular or turtle–shaped adapted from a robot also called the turtle. The turtle can be given movement and drawing instructions to programmatically introduce line graphics. Turtle graphics were added by Seymour Papert in the late 1960s to support the
of artificial intelligence at the…MIT began to think about the epistemological importance of their new machine – the computer - almost as soon as it was invented. It was realised that the computer allowed a new way of thinking about knowledge… The original motivation of the pioneers could perhaps be summarised as: the need to encourage abstract thought rather than learn the standard procedures.”519 Coates replaces the word theory in a quotation that he attributes to Minski by the word design: “you have to distinguish between writing a program that helps you to test your design or analyse your results, and writing a program that is your design”.520 Much contemporary work in computational design research focuses on closing the loop between these two types of program or model so that the outcomes of the program that is the design can be rapidly tested by the program that tests the design, and the programs that test the design can rapidly inform changes in the program that is the design. Alan Kay coined the term Object Oriented Programming (OOP) while working on the computer program Smalltalk at the Learning Research Group (LRG) of Xerox PARC. Smalltalk was an object-oriented, dynamically–typed, reflective programming language developed for educational use, and particularly constructionist learning models. Its designers were working in the spirit of human-computer symbiosis in the 1970s that had grown out of the Artificial Intelligence research in the 1960s. Kay was also at MIT and influenced by Papert’s focus on human learning.

The meaning of intelligence is very illusive. The first test of intelligence in a computer was Turing’s 1950 “Turing test” or imitation game. Could a computer convincingly play the part of a human in a bilateral natural language conversation conducted by text?521

Terry Winograd’s doctoral research at MIT 1968-72 worked with a subset of the same problem – could a computer understand questions in English about the situation before it, respond in English, respond to requests to act, in this case to manipulate blocks of different shapes, sizes and colours on a table, break down the request into operations it could perform, understand what it had done and why and describe it in English? The program he developed was to interpret semantics in language using syntactic clues and break down a request into executable procedures. Others had worked with the visual problem of how a computer with a TV feed could come to understand a pile of blocks on a table where some were in front, some stacked on top of others. Winograd’s program was notable as an approach to constructing intelligence because it could not be broken down into cleanly separable procedures. The operations of parsing the English sentence, representing this in its own internal system, reasoning about the world represented inside itself, answering questions etc. were all inextricably intertwined. There were different procedures but it was as though they were all knotted together, unable to be prised apart. It would interpret a sentence and break it down into a set of instructions in the

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520 Ibid.

language PLANNER, encapsulating, in a user-transparent way, a tree of goals, sub-goals and sub-sub-goals. If a goal could not be reached along one branch of the tree, it would backtrack and try another path. It had no capacity for pattern recognition or eliminating repetition – if asked to pick up the green pyramid and put it down, over, and over and over again, it would do this. Despite being a computer program, it had no capacity for numerical processing and only knew the numbers up to ten. Winograd’s program was called SHRDLU after ‘ETAOIN SHRDLU’ the top ten letters of the alphabet according to their frequency of use in English, and the old code used by linotype operators to mark typos in a newspaper columns. Winograd’s approach was based on a model of language as a way of activating a response or procedures within the hearer. Winograd wrote:

“The different possibilities for the meaning of ‘the’ are procedures which check various facts about the context, then prescribe actions such as “Look for a unique object in the data base which fits this description”, or “assert that the object being described is unique as far as the speaker is concerned.” The program incorporates a variety of heuristics for deciding what part of the context is relevant.”

Thus although Winograd’s work is dealing exclusively with symbolic representation, the use of words in natural language, even the simplest and most common is only a signpost, open to multiple interpretation. Derrida’s concept of difference is brought to mind – the problem of a spectrum of subtly different context-specific ‘the’s and other articles with which to refine the meaning. Douglas Hofstadter wrote that “writing a program which can fully handle the top five words of English – “the”, “of”, “and”, “a”, and “to” – would be equivalent to solving the entire problem of AI, and hence tantamount to knowing what intelligence and consciousness are.”

Hofstadter has also written of a fleeting image of what he perceived thought might be, which was, for him, stimulated by the output sentences of a sentence-writing program he wrote. He was inspired by the natural tendency of human readers to imbue every word they read with its full flavour and nuances, even knowing that an artificial symbol processor has arranged them. He had “a sense that real thought was composed of much longer, much more complicated trains of symbols in the brain – many trains moving simultaneously down many parallel and crisscrossing tracks, their cars being pushed and pulled, attached and detached, switched from track to track by a myriad neutral shunting engines…”

What an intriguing hybrid of the imagist and linguistic positions on thought this represents.

What does such formalisation of linguistics have to do with design modelling? An interesting development in this regard was the adoption of an analogous approach to shape. George Stiny and James Gip published a paper in 1971 about the development of a formal generative Shape grammar. They wrote, “Different rule types consistent with

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524 Ibid., 623.
the idea of shape grammars are possible and can define classes of grammars analogous to the different classes of phrase structure grammars.”

525 Their aim at this time was to identify grammars (combinations of shape or geometrical primitives and rule sets) that would generate successful two-dimensional works of art. What is clear from the work is that there are a multitude of readings of a given geometrical figure in terms of the primitives or ingredients of which it is composed. As humans we are subject to Gestalt psychological behaviour, which means that we are apt to see certain shapes ahead of others even if both are present of overlapping in a figure. Computers do not have the same evolutionary background and will compose and decompose shapes according the components and rule sets with which they are programmed. This is significant for the design of Computer Aided Design software. It is also significant if you would like the computer to perform tasks based on shape recognition. George Stiny has continued to investigate Shape Grammars throughout his career.

526 Shape grammars can be viewed in the lineage of centuries of pattern exploration through decorative tiling, which similarly play with overlapping alternative pattern readings. It introduces comparable heuristics but with fewer constraints into the field of artificial intelligence. Paul Coates extends this grammatical concept to “design grammars” (of higher level CAD functions). He makes reference to Chomsky and writes, “Generating Automatically Defined Functions (ADFs) could be seen as a method of isolating useful sub-clauses in the evolving language.”

'Design intelligence’ was coined in 2002 and in relation to neither image- nor language-based thought nor its artificial imitation. Michael Speaks has used the term to make reference to an obscure sense of self organising aspects of the information exchange within, and between, design practices and the world and the potential to design better more connected and informed systems. He argues that contemporary architectural practise as a body becomes more powerful “to the degree that it transforms the chatter of little truths into design intelligence”.


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526 Die Gestalt is the German word for shape. The Gestalt effect describes the capacity of people to see whole forms and shapes rather than just the component edges, lines, curves and patches of which they are made up in the visual field. It is an aspect of pattern recognition.


Interest in artificial intelligence as it manifest, did grow out of an interest in human intelligence including decision-making. Herbert Simon the highly influential political and social scientist worked on the subject of human decision-making and in the 1950s he moved into systems research and computer simulation of problem solving and worked with collaborator Alan Newell. This work had an important influence on the early research in artificial intelligence. He wrote: “The human being striving for rationality and restricted within the limits of his knowledge has developed some working procedures that partially overcome these difficulties. These procedures consist in assuming that he can isolate from the rest of the world a closed system containing a limited number of variables and a limited range of consequences.”

This brings the topic back to the architectural computational geometrical model, which is a highly constrained system for facilitating design decision-making. By using the apparent 'self-organising' potential of parallel computation it can sometimes represent a tacit decision making system in its own right.

6.11 Bringing the philosophy back to the architectural modelling

In this chapter and the preceding one, I have selectively reviewed the modern history, philosophy and psychology of mathematics and space. How does this philosophical and psychological framing of space and logic in relation to geometry contribute to a new understanding for constructing and navigating the types of modelling space outlined in the case studies in Chapter 4?

Case study 1: The Crestaria or stepping pediment of the Sagrada Familia Passion Façade

The steps in the pediment increase in depth and height towards the apex of the assembly, viewed in front elevation. The cross section conforms to the same pattern in each rank, although the width and height of the steps vary. The edges of the successive ranks of steps conform to a series of smooth curves in plan. Each relationship was easy to comprehend projected into plan, front elevation and cross section. It was also simple to operate as a seemingly infinitely variable geometrical system in two dimensions, whether in plan, front elevation or cross section. However in combining the algorithms for each of the three, the overall system not only became much more constrained but also much more unpredictable as to which combinations of variables would result in a viable outcome. A viable outcome is a three-dimensional surface or solid model. It also became difficult to predict how changing a variable would alter the whole. It would work very flexibly as a system for generating points (vertices of the model shape) but being able to communicate

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Spina.


532 Alan Newell (1927- ) computer scientist and cognitive psychologist at the RAND Foundation and Carnegie Mellon University, contributor to Information Processing Language (1956) and two of the earliest AI programs, the Logic Theory Machine (1956) and General Problem Solver (1957) (with Herbert Simon).

the detailed surface or solid construction joining those dots in a way that allowed them to be computed in every case was a harder goal to reach. So perhaps this particular case must be seen as a *programming* problem rather than a mathematical challenge as such. Each part of the ‘form’ of the model could be simply described in analytical geometry – the quadratic (or substitute function) defining the step growth, the parabola (or substitute function) defining the curve shapes in plan, the translational position and rotation of the lines defining the pitch lines of the steps in front elevation. For a geometer it would be simple to combine these architecturally intuitive parts of the description in a notationally economical and coherent way. But the challenge lies in the conditionals needed to adequately describe how to construct the possible variants that this schema will generate. This is a question for programmatic intelligence in the model, which is to say logic and linguistics rather than geometry.

**Case study 2: the interlocking bone-like columns of the colonnade of the upper Passion Façade of the Sagrada Família Passion Façade**

In case study two, the parametric variation of the interlocking columns constructed from intersecting doubly ruled surfaces is a slightly different case. The intersection a hyperboloid of revolution of one sheet and a particular line parallel to the central axis of the hyperboloid yields two points. One of these two points lies on two lines in the hyperboloid surface that link it to two other given (variable) points on the surface, which were used for the construction of the original line to intersect. Sometimes the upper intersection point lies on the lines in the surface, sometimes the lower intersection point lies on the two lines in the surface but never both. Whether it is the upper or lower point alters in an unpredictable way depending the parameters determining the exact shape of the surface, the inclination of the column, its slenderness ratio, the two given points chosen (these are the triple intersection points for the hyperboloid surface and two neighbouring hyperbolic paraboloid ‘branch’ surfaces.) But it is critical that the correct point is chosen for the construction of the hyperbolic paraboloid branches for the column to maintain its integrity. The problem of predicting which of the two points is the right point for any given set of conditions seems to be out of reach geometrically.

I see this as much more of a ‘problem for Descartes’. The conditions that the construction must meet were *given* linguistically and programmatically (at least from my point of view as the modeller working within a design team to create a variable computational system representation.) The challenges were geometrical and finding a logical structure for a model that would respond, that could be operated to explore the design variations and their implications. As in Case Study One, it was easy to comprehend the synthetic implications of simple components of the geometry but once it went beyond second order or included too many variables, imagination failed and being able to retranslate the geometrical conditions into program conditions was again a much more difficult goal. However, there could be a simpler solution – to formulate a test and solution that in pseudocode could be written: If a condition is met that indicates the line lies in the surface, continue, else substitute point b for point a. This would be a way of bypassing all the
geometrical complexity of the model space but maintaining the model integrity under a much wider range of variables.

In both these examples the geometry itself is an important component of the brief because, although still under construction, and subject to detailed design and description, the Sagrada Familia church is the historical design of Gaudí, deploying a particular heuristic that moved from Gothic revival geometries in the original proposals for the church and use of freeform in his own concurrent work to hyperbolic geometries. He developed a very detailed codex for applying the surfaces in the architecture and a very highly developed plaster modelling technique for developing the design. The aim of the contemporary modelling is both interrogative, pragmatic in meeting fabrication and construction constraints, and ultimately definitive in its use to interpret his intentions where these have been modelled explicitly previously.

Case study 3: Student research project using mathematical surfaces

The third modelling case study takes an approach, which, as I have noted, also coincidently interested Le Corbusier at the time of the design of the Philips Pavilion. That approach is to take the surface representations of certain mathematical functions and explore their use for thin shell construction. This student research project grew into learning empirically to edit, redesign and parametricise the mathematical function for a very specific design brief with its own geometrical relations and constraints. This was strongly within a constructionist learning model\(^\text{534}\) – as most project based design learning is. In this case it was simultaneously about design and learning to manipulate functions through simultaneously making graphical artefacts. The surface had marionette qualities and the first exercise was to learn which strings to pull (and how) within the function in order to affect the surface shape in different ways. This would be unimaginable without the responsive graphical representation in the computer and is a very startling manifestation of Seymour Papert’s vision for the possible role of computers in learning, which he espoused, for instance, in his book *Mindstorms.*\(^\text{535}\) At the other end of the spectrum, this project, while it showed how much progress could be made by the relatively mathematically unschooled, given the right tools and motivation, ultimately also reinforced the modernist emphasis on virtuosity in mathematical problem construction and supported Poincaré’s elitist view regarding an aesthetical sense of the mathematical. When the students were stuck in terms of persuading a surface to conform both to their aesthetic bidding and meet one of the critical conditions for acting as a bridge, arching, they turned to an astrophysicist and expert mathematician who with their list of performance criteria before him could parametricise the function immediately for their needs.

\(^{534}\) “Constructionism holds that learning can happen most effectively when people are also active in making tangible objects in the real world. In this sense, constructionism is connected with experiential learning and builds on some of the ideas of Jean Piaget.” (It is inspired by the Constructivist theory that individual learners construct mental models to understand the world around them.) http://en.wikipedia.org/wiki/Constructionism_(learning_theory) Last accessed 13th February 2011.

Recapitulation: Chapter 6

In transition from object to system modelling the questions of derivation and deduction come to the fore. The architectural object was ever the result of a series of moves, generally geometrically defined dependencies, more or less apparent in the outcome. System modelling permits the designer to make these relations explicit in the model and continue to vary the model within its constraints or decision framework, which has already been formalised beyond the sketch. Systems can be generative, emergent or highly controlled and deterministic. They are procedural and adhere to an imposed logic. The questions seem to be:

a. Whether this logic can ever be intact and complete in the sense sought in mathematics by Hilbert, Russell, Whitehead and others, the formalist approach to mathematics. There has been an ideology within design that has continued to pursue the perfect logical model, the model in which, for example, the components will always conform perfectly to changes in the spatial and structural configuration.

b. What the place of intuition is. This turns on what intuition is and whether to accept Kant’s definition of space and the spatial patterns called mathematics as intuition (something we know when we look at it) or whether, as Russell contended latterly, these are logical relations that go beyond intuition and can be constructed without recourse to it.

In common usage, aesthetics takes on highly intuitive connotations – an innate sense of beauty and order. It is clear that in mathematics, as in architecture, there are different aesthetic standards, some highly reductivist, some to do with “depth” of inference. Kant’s Aesthetic concerns Space and Time. Aesthetics provides a key to ordering, discovery and sense making in both mathematics and in architectural design.

A review of literature on aesthetics in mathematical form, proofs, lay appreciation, and expert mathematical discovery uncovers surprising levels of similarity with the nature of the design process. Whitehead’s ‘large generalization limited by the happy particularity’ is not only central to mathematics but an underlying characteristic of successful architectural design in application to traditional aesthetics. The geometrical bases of both the generality and the particularity, or in Gregory Bateson’s words ‘the pattern that connects’ are very variable. Similarly visualization and figurative thinking appear as central in some quarters to abstract mathematical understanding and discovery as to architectural design and representation.

While the dream and the unconscious mind no doubt continue to play a crucial role in providing the goals and methods for both design and mathematical discovery, system design has also taken on an increasing role in representation, problem-solving and discovery. Computer programming heuristics should not be overlooked in their influence on the way that designers approach design. In this respect it is interesting to consider the pioneers in the pursuit of machine ‘intelligence’ and the relatively untapped potential that still exists for those prepared to tailor the representational and problem-solving tools to their own design challenges.

Spatial design presents an interesting challenge. Its goals
and outcomes are generally predicated on realisation in the external physical world. They have a presence that privileges a conceptualist, imagist model of thought. In bringing the philosophy back to the case studies in the section above it is clear that it is the close relationship of the abstract formulae to their graphical manifestation that allows the design students in Case Study 3 to interact with the abstract notation so successfully. This is a constructionist paradigm – the mental model in combination with the actual act of making and change.

Similarly it is an image-based understanding of the relationships between the parts and shapes of Gaudi’s drawing of the Passion Façade of the Sagrada Familia that allows it to be constructed as a computational geometric model. Yet the geometry in each case is so complex in its relational entirety within the models that to mitigate issues in three dimensions in the exploration of the effects of varying the numerous variables, it is actually a symbolist procedural heuristic that is called for, not a holistic geometrical understanding of the dimensions of the model space.
DISCUSSION
The journey so far...  

In the introduction to the dissertation, I open with the idea that, for the last half century, architecture has been slowly adapting its representational practices from the conception of objects of sensory engagement to the construction of systems of formally described relationships. Since the production and nature of architecture are inextricably entwined with its modes of representation, this poses a profound shift in the active role of geometry in architecture. The architectural model space is changed. This happens under the influence of four centuries of developments in mathematics and mathematical philosophical thought. The assimilation of these developments has been accelerated through the use of electronic computation for architectural representation, but also, as acknowledged in Chapter 6, channelled, geometrically and spatially by prior developments in computer science. The central question is: what can selective historical reflection on the history, philosophy and psychology of mathematics offer the architect grappling with the potential geometrical complexity of contemporary virtual system model space? 

In breaking down this question, I started in the second chapter, with a brief review of the historical relationship between architectural modelling, geometry and mathematics. This reflected on the reciprocal contribution of practical problem solving in architectural representation to the development of projective and descriptive geometry. It also outlined the relative distance placed between active, living, mathematics and architecture by mathematics’ journey from geometry to more abstract analysis as its foundation. 

Yet, in contrast, the growth of topology and set theory are the bridge to the computational system modelling, including, particularly in the last two decades, its use to represent architectural design. In the shift to dynamic analogue modelling and to early computational system modelling in architecture, I contrasted two groups of pioneers. Ivan Sutherland and colleagues at MIT sought to adapt the interface with the machine to design thinking, in Sutherland’s case introducing drawing as a programming interface. The LUBFS

536 It is interesting to reflect on what a profound influence the birth and development of Computer Aided Design has had. By effectively imprinting existing processes and techniques from the drawing board, albeit with huge productivity gains, the dominance of top-down geometrical approaches has been very marked. Some authors would argue that there are much more interesting ways to harness computing, exploiting the potential of emergent programming approaches (see: Coates, P. Programming Architecture, London: Routledge, 2010, 21.) Coates distinguishes between the use of computational geometry, (programmed in Basic in his example), and simple parallel process system programming (using NetLogo). He illustrates his point by giving the code for developing a Voronoi net between points, using each method. The Netlogo code is a few lines instructing small points to progressively distance themselves from their nearest large points, while the Basic code to calculate the geometry of the net runs to several pages. In researching Chaos, Complexity and Emergence as one of the five themes for example projects in Chapter 3, it was easy to identify examples of fascinating virtual architecture projects in this domain, but more difficult to identify built projects that already whole-heartedly embrace an emergent approach to design. This research has focused principally on system models using the more deterministic approach of computational geometry to construct the system.
group at Cambridge tried to mould design thinking to the greater potential design profundity possible, as they understood it, using mathematical representation and its manipulation, through symbol processing.

Following this historical context, the third chapter examined selected examples of recent architectural projects based on geometrical spatial system models for their designed space (aims and outcome) and/or their design space (means and process), organized into five geometrical themes. The new mode of representation that supports multi-dimensional relational models has led to a shift in the active role of geometry in architecture that has affected its engagement as problem solver for design resolution and geometry’s appropriation as idea. This is an aesthetic drive at two different levels and needs to be considered in this light. In other words, it has moved the aspirational territory of architectural modelling to find expression more aligned to contemporary models of process in the world: the stochastic, the changing, and variation within the ‘pattern that connects’. More generally, computation provides a way to map the design process into a graph of associations between geometrical objects, parameters and attributes that constructs a space of potentially fantastic complexity. This space, representational of the design as much as the designed, is itself a territory with real geometrical shape and properties that, as it grows, quickly defies all-at-once visual apprehension and recognition. With this in mind, in the fourth chapter I revealed some more specific spatial issues encountered in multidimensional computational geometrical system modelling. This is done through the examples of two parts of a large associative model I was engaged in constructing to resolve the detailed design of Gaudí’s west transept for the Sagrada Família church, and a third example from a student employing mathematical functions. To help to generalise these issues further, I also included a review of the design questions, which students in a course I have taught chose to tackle through computational geometrical system modelling. I reflected on the impact of system modelling on their spatial and design thinking, drawing on my observations and the students’ responses to a qualitative questionnaire. The students’ spatial and design thinking was, in most cases, altered by the change of media (modelling including text-based and spreadsheet interaction alongside graphical interaction), by the potential for generative modelling, and by the experience of constructing a very precise and explicit design domain rather than a largely implicit design object. Chapters Three and Four presented an overview of the geometrical nature of some recent architectural system modelling and framed a problem: the altered perception of the space of the model. The holistic Newtonian overview with its

Discussion

Direct (Euclidean) spatial mapping to what is represented is replaced by knowledge circumscribed by what can be done in the space – this is space determined by action.

Geometrical space still dominates the interpretation and explication of design intent into built artefact in the system model, but the active role of geometry is changed under the influence of machine-based logic. Computation has extended the geometrical vocabulary of architectural modelling and representation into geometrical model spaces that no longer map to perceptual space in the way that the simple static Euclidean space of the model once did, nor even in the way that sophisticated dynamic physical analogue models, such as those of the architect Gaudí, and twentieth century shell builders such as Candela and Isler did. The mapping between representational space and perceptual space in architectural design modelling is altered. With this change comes a shift in the way that the designer’s innate spatial capacities are tapped in practice, in particular, the reliance on the visual and visualizable. This raises questions about the nature and role of intuition in the design process. How does the perceptual space of the designer, so fundamental to concrete architectural expression, remain engaged when designing in architectural models that have adopted language, concepts and ideas from post seventeenth century mathematics and philosophy of mathematics, which, within those disciplines themselves, have been acknowledged as distancing perceptual and representational space? This adoption is occurring both actively through appropriation and ideation, such as the adaption of the Weaire Phelan packing model to generate the geometry of the structure of the Beijing Watercube, and passively, through the modelling opportunities of computation, such as the iterative optimization to size the structural members of the Beijing airport roof or the models to be able to progressively manipulate a large number of relations to achieve the best fit to Gaudi’s drawn design intentions for the western transept, detailed in Chapter 4.

Having exposed some examples of the geometrical challenges of modelling systems of, effectively, unlimited dimensionality, and framed the problem of altered space

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538 For instance Judith Wechsler questions whether in a computer age, intuition can play the same role as it did for the physicists, chemists and engineers of the first decade of the twentieth century. (1988 preface to reissue of Wechsler, J. On Aesthetics in Science. Edited by Loeb, A.L. 2 ed. 1 vols, Design Science Collection. Boston, Basel: Birkhauser, 1978.) Ramsay and Richtmyer make the distinction between a model in mathematics, which serves to prove the consistency of a system, and a model that ‘looks like’ something. The Poincare half disk as a representation of the hyperbolic plane is an example of the former. Although analysis in mathematics now precedes geometry, (whereas from ancient to modern times the properties of the real number system were derived from geometry, today they are derived from set theory), nevertheless this does not mean that geometry should now be treated as a branch of analysis. “The notions of geometry go, in a sense, beyond the notions of analysis in that they are things that we visualise...” Ramsay, A. and R.D. Richtmyer. Introduction to Hyperbolic Geometry. Edited by Ewing, J.H., F.W. Gehring and P.R. Halmos, Universitext. New York: Springer-Verlag, 1995.

539 Note that geometrical space known or determined primarily through action rather than formal analytical knowledge is a kind of reversal or place swapping with sensory space where movement and change have been shown to be the primary authorities over perception. Geometrical space, traditionally
in architectural modelling, in Chapter 5, I looked to locate this through a selective history of the relationship between ‘space’, (the oft stated principal object of architecture), and geometry in a philosophical sense. Between the seventeenth and the early twentieth centuries, there were changes of epic proportion in the geometrical understanding of space within mathematics. This was led by the increasing power of analytic representations, discoveries leading to definitions of much more generalised frames of reference – starting from such work as Descartes’ construction of a problem. Later, in the nineteenth century, the power of analysis led to the growth of Hilbert’s and others’ ambition to bring the whole of mathematics within a framework of logic, based on the minimum required axioms. This emphasis on analysis led to questioning within mathematics about the continuing role of synthetic geometry, the value of the visual and visualizable, the cognitive nature of original ideation for mathematical problems and whether intuition could be dispensed with from the foundations of mathematics; in other words, fundamental questions about the relationship between representational space and perceptual space in mathematics.

I traced the mathematical and philosophical shift from ancient to modern thought, which commenced in earnest with the work of Descartes in the early seventeenth century. Genuine seeds of modernism, in the Cartesian sense, (outlined by Lachterman as a new meaning for ‘construction’), were evident in architecture at least as early as the late nineteenth century. But Frédéric Migayrou’s juxtaposition of the standard and non standard in architecture is one of many sources that outlines the counter forces to this trend, sometimes generalised to the normative influence of industrial production, focussing architectural and related design production in the direction of the more static, object-centric and reductivist application of Classical Euclidean geometry from the early twentieth century onwards. I explored the implications of Kant’s form and matter of Pure Intuition and the framework and the conflict it presented for philosophers and mathematicians in the nineteenth and early twentieth centuries with the growth of formalism and logicism in mathematics. I also explored the relationship between ‘geometrical space’ and ‘sensory space’, named by Poincaré Geometric and Representative space, respectively. More contemporary significant, in the light of subsequent developments in psychology, is the question of the altered relationship between representational and perceptual space in architectural modelling. The principal aim of Chapters Five and Six was to find out what selective reflection on the modern history, philosophy and psychology of mathematics offer the architect grappling with the potential geometrical complexity of contemporary virtual system model space.

The struggle between logic and intuition in the twentieth century history of mathematics, with its particular implication for the synthetic, for the concrete, for the image in a world potentially dissolving into symbol manipulation, is explored in greater depth in Chapter 6. Here I included reflections on the apparent kinship between design and mathematical discovery as creative cognitive processes. I also compared the meaning of aesthetics in relation to various aspects of mathematics and design.
To recap, I have illustrated, through selected examples, the implications of the altered active role of geometry within system models in architectural representation. Regardless of whether the system is constructed through a series of synthetic geometrical moves or a more abstract analytical process, the outcome is a space that is geometrical in definition and representation but which commonly lies outside the immediate reach of visualisation. It is only perceptually accessible through action, change or virtual movement.

What has been learnt?

Why revisit the vexed philosophical question of the relationship between representational and perceptual space in reference to computational geometrical architectural system modelling? What is revealed?

First, there is a parallel between the static product or object model in architecture and the Greek Euclidean construction of geometry, with the implication in each case that the act of construction is to reveal what already is and prove its veracity or holism. In his analysis of the transition to modernity, David Lachterman identifies a lineage between Descartes’ construction of a problem, Leibniz’s construction of an equation and Kant’s construction of a concept as active interventions that bring something into being. The construction of the system model in architecture is much closer to Descartes’ construction of a problem than the Classical Greek construction of geometry as proof. Lachterman presents Descartes’ construction as a generative activity, partially complete, always open to extension and variation. Unlike Euclidean construction, Descartes’ activity is without the implication of the pre-existence of the geometry, and proof has little status as a motivation. The nature of the architectural system model, unlike traditional object representation, is no longer the proof that a number of given aspects of the design synthesize into a coherent proposal but a representation of the design problem – the model represents a well-defined question rather than an answer. It is the problem rather than the solution that is refined by reforming the model. Similarly, the designers perceive themselves to be within a problem space or design space in the model. Whereas in the static object model (physical model or drawings) the model is perceived and the subjects project themselves in imagination into the potential designed space represented by that model. From this point they can react and inform the design in a feedback loop that is analogous to Gregory Bateson’s marksman with the shotgun. In the system model, the designer is more immanent within the model itself. Bateson’s rifleman with his adjustable sights is a better analogy. The model is a representation, but potentially a representation of the design deliberations, it is a space of action, a design domain, within which it is possible to act and move. Conversely, the space can only be known through action, movement and change.

Second, geometry is highly developed as a formal system of representation and is often equated with a studied system.[mathematics not as proofs of theorems but “as transposition of mathematical intelligibility and certainly from the algebraic to the geometrical domain, or from the interior forum of the mind to the external forum of space and body.”]

of notation and a particular body of knowledge. Thus it is revealing to cast it, for instance, in the light of Kantian *synthetic a priori intuition*; geometry as a fundamental of our subjective knowledge of external reality and of our understanding of spatial relationships. Geometry, in this modernist, humanist light, becomes inseparable from space itself. In Kantian terms, neither one is a thing; rather they are sensibilities by which spatiotemporal things can be known. Geometry in the architectural system model is both the medium of action and simultaneously the medium of knowledge. It is not merely an overlay on the space, the structure by which the model is constructed. It is *of* the space. Geometrical spatial thinking is integral to framing the design problem or proposition, it is not a translation from another spatial paradigm.

Third, in the words of Mach, for instance, geometrical space and sensory space, similarly, and by implication, are less than cleanly separable. They are bound together in the processes of both perception and in our conceptual projection of phenomena. Space is neither the void in which everything exists nor is it an emptiness ‘contained within’; although, as Mach has pointed out, we often act as though it were. Space is inescapably hybrid. Piaget has conceptual space reprojected on perceptual space such that perceptual space is permanently altered. There is, in Piaget’s construction, no return to the innocence of the pre-metrical perceptual space of the young child. Brouwers arguments against formalism reintroduce an empirical component to mathematical evaluation. Lakatos similarly emphasizes the place of the empirical and the value of refutation. In asserting the scientific necessity of possible falsification in mathematics and geometry, they open the door to the influence of perceptual space, even the space of the image, real or imaginary, on geometrical space. So there is an implied reciprocity. Knowledge from perceptual space, images we can create or imagine, or other sensory phenomena can alter or reinforce irrevocably the “rules” of geometrical space, just as metrical understanding permanently alters the developing child’s perceptual space in Piaget’s analysis. The spatial hybridity and the reciprocal influence of the conceptual and perceptual knowledge is tacitly familiar to designers who work in a world of partially known and projected future proposals. They move between hierarchies of representations, imagined, figurative, highly abstract and concrete prototypes. The architectural system model generally encompasses all these. While it has figurative representations and prototypes as outputs, the model is essentially a more abstract, but highly explicit, map of intention with regard to mutual spatial relationships and constraints. To summarise, the architectural system model, however geometrical in its construction is never a purely geometrical space, rather it represents an intense interaction between representational and perceptual or more phenomenal aspects of space.

Fourth, the notion of *three dimensions* has been labelled a *convention*, highly useful, very convenient for description
and representation, and significantly embodied. It need not, however, in all situations be considered a fundamental of all activity space. Poincaré introduces this idea when he writes Science and Hypothesis and in effective refutation of Kant’s Aesthetic. This is an essential part of the puzzle of spatial knowledge. But Poincaré also accords to three dimensional space very special status. He wrote:

“Now, Euclidean geometry is, and will remain, the most convenient: 1st because it is the simplest, and it is not so only because of our mental habits or because of the kind of direct intuition that we have of Euclidean space; it is the simplest in itself, just as a polynomial of the first degree is simpler than a polynomial of the second degree; 2nd, because it sufficiently agrees with the properties of natural solids, those bodies which we can compare and measure by means of our senses.”

So convincing is the metrical overlay on the space of existence, in Piaget’s terms, and our respect for space in relation to the body – what lies ahead, behind, to each side, above and below (referenced via the sensations attributed to gravity), that the convention often extends to considering physical space as ‘three dimensional’. This is powerfully explicit within computer modelling programs, where subliminal awareness of the force of gravity is supplanted by the orientation of the base coordinate system, the first geometrical object or construct in every virtual space modelling accessed graphically on a computer. However, systems in general are not constrained to three dimensions. There are inevitably more variables that

three in most. Architects working in system model spaces need to become as comfortable with that reality as the engineers and scientists solving the curve in six dimensional space of the Apollo 11 mission’s path in time from Cape Canaveral to the Sea of Tranquillity, obeying Newton’s laws of motion, starting with the velocity determined by the rockets and landing with zero velocity to avoid a crash. Multidimensional spaces cannot always be fully or adequately represented graphically even following Euler’s graph theoretical lead. They may need to be known and understood in other ways. We know the world in the way that we do, (we identify, or as Strawson would have it, reidentify objects) through movement and change. In the complexity of the spatiotemporal reality neither movement nor change is adequately represented by one further temporal dimension. Even in a purely intuitive context (meaning: in the context of things that we cannot help but know), there is a place for multiple additional dimensions. Three dimensions is a static hypothesis that is bound up in the empty Western scientific model of space and idealisation of the object. It is nevertheless a very powerful, useful convention and likely inescapable for designers dealing with idea of physical solids and their representation.

Fifth, lastly and awkwardly, intuition is a difficult topic to research. Like intelligence for those programming for it, it


slips from the grasp. In design no less than in other spheres it both attaches to common sensibilities of designers and singles out differences between individuals. If moving to system modelling is a fundamental spatial cognitive shift for designers, is it possible to learn effectively to marry up understanding of spatial propositions through imagined and constructed imagery with the more challenging and, in some ways, more abstract, in some ways, more worldly, multidimensionality of the model space, where possibilities are represented through action and change? This question opens up another research seam in relation to the topic of the human propensity for learning and adaption. Perhaps it is more productive to leave behind the word and concept of intuition, and its relation to learning and adaptation, let Kemp-Smith have it for his translation of Kant, knowing without analytical thought or empirical knowledge, and concentrate in architectural modelling instead on imagination. How can the abstract complexity of computational geometrical systems be assimilated in the architectural imagination?

The space of the dynamical system model presents certain representational difficulties, especially in the context of the conventions of static two and three-dimensional representations. The model space represents a constrained but potentially infinite domain of ‘real world’ spatial possibilities but the representation of the model space itself is problematic. However, we can learn to move and act in this model space, a fundamental tenet of space as we know, or impose, it.

CONCLUSION
This dissertation has provided a purely inductive journey, offering evidence through very selective examples in architecture, architectural modelling and in the philosophy of mathematics. The foray back in time to the immediate philosophical forbears of contemporary mathematical space has provided a lens that has altered my own encounters with geometrically-constructed virtual architectural system model spaces using computation. How this will play out in active engagement with the model in design, is, as yet, barely tested.

The journey started with some questions about the tendency of the model space to break out and exhibit unpredictable and not always desirable behaviour. How could it be so fickle when it was simply an edifice of geometry, carefully authored and, where possible, hemmed in by succinct algebraic definitions? Perhaps it was not ‘mathematical enough’, or it was too ineptly constructed? Surely the logic of mathematics would guarantee spaces of integrity and predictability, spaces of unlimited flexibility and, above all, controllable spaces. Where was the realisation of Descartes’ modernist promise? At this stage, for me, at least, the question of what Descartes’ virtuosity can deliver to this space remains an open one.

The dissertation has many sub-conclusions. Each chapter recapitulation offers a summation of these. The journey has raised many more productive questions than conclusions. Some of these questions have spun out to contribute to problem definitions for other collaborative research projects or publications in the course of the research. Some will lead to future research.

The opposition between the reverse engineering to interpret and model the hybrid geometry in Gaudí’s Sagrada Familia Passion Façade and algebraic sculpting to design the bridge in Steven Swain and engineering partner’s student research student research project has significant mileage to be explored in a generic way in the design studio. The opposition between programming as an exercise in formal expression – indebted to linguistics- and the overtly spatial and mathematical nature of computational geometry offers a similarly rich vein of potential exploration. Finally, although many, it seems, have attempted to teach mathematics to architecture students and reported on their aims, methods and outcomes, there is little material reporting the outcome of architect-mathematician collaborations in design practice. The place of expert geometers in architecture is amply demonstrated by such examples as Chris William’s work on the British Museum roof, the digital computational version of twentieth century structural art, and the success of the members of the Institute of Geometry led by Prof Helmut Pottmann at TU Wien:


546 http://www.dmg.tuwien.ac.at/pottmann/
and the spin-off specialist consultancy to the design industry and software solution builders, Evolute\textsuperscript{TM}.\textsuperscript{547} The powerful cocktail of design and mathematical understanding is seen in the work of the great structural artists and shell builders of the twentieth century, using in many cases, analogue computing through very precise physical modelling and measurement.\textsuperscript{548} With powerful digital computation in the mix, the question that leads forth from here is how the open-ended solution-led thinking of architects, perlocutionary approaches of computer scientists and problem-solving capacity of mathematicians can come together to represent architecture in ways that match the elegance and conviction of the structural artists but expand the design conversation beyond purely structural art: systems that build on multiple philosophies.

\textsuperscript{547} http://www.evolute.at/ last accessed 15\textsuperscript{th} February 2011.

\textsuperscript{548} Nordenson, G., ed., Seven Structural Engineers: The Felix Candela Lectures (New York: The Museum of Modern Art, 2008): 87-102. For Heinz Isler’s chapter: ‘Shell Structures: Candela in America and What We Did in Europe’. This records the level of precision required, including controlling the temperature in order to take measurements from the models.


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All Figures were photographed, drawn or diagrammed by the author unless listed.

**Figures 7, 9, 10, 12, 16, 17, 20, 25, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 45, 46, 49, 50** contain content adapted from images previously published in Burry, J. and M. Burry The New Mathematics of Architecture. 1 ed. London: Thames and Hudson, 2010.

**Figure 5.** Photographs by Patricio López and Antonio Trogu respectively from Flickr under Creative Commons.

**Figure 6.** Steadman, P., ‘Graph-theoretic Representation of Architectural Arrangement’, in Architectural Research and Teaching (1973). (digital .pdf)

**Figure 8.** [http://www.sial.rmit.edu.au/Projects/Paramorph_II.php](http://www.sial.rmit.edu.au/Projects/Paramorph_II.php)

**Figure 11.** Escher and Droste Effect [http://escherdroste.math.leidenuniv.nl/](http://escherdroste.math.leidenuniv.nl/)

Another Escher Grid [http://www.tiac.net/~sw/2005/05/escher_grid_2/](http://www.tiac.net/~sw/2005/05/escher_grid_2/)


**Figure 14.** Apery, R. Models of the Real Projective Plane: Vieweg, 1987. (photographed from book)

**Figure 15.** Erik de Neve [http://www.usefuldreams.org/spereev.htm](http://www.usefuldreams.org/spereev.htm) last accessed January 29th 2010

**Figure 22.** Photographs: Rupert Truman from Architects Journal April 1st 1992

**Figures 23, 35, 48.** Mark Burry


**Figure 27.** 2G. International Architecture Review. N. 6. 1998 / II. Ushida Findlay. (digital scan)

**Figure 36.** Williams, C.J.K., ‘The Analytical and Numerical Definition of the Geometry of the British Museum Great Court Roof’, in Mathematics and Design 2001.


**Figure 44.** Steven Hyde, Cristophe Oguey, and Stuart Ramsden, [http://mesoscale.anu.edu.au/](http://mesoscale.anu.edu.au/) Last Accessed 20th January 2010.


**Figures 77- 84 inc.** Steven Swain, architect and former student, used with formal consent.

**Figure 85.** (Stillwell 1998, (fig 2.27) p.67)

**Figure 90.** [http://olewnick.blogspot.com/search?q=Ames](http://olewnick.blogspot.com/search?q=Ames) last accessed 20th January 2010
APPENDIX I

SELECTED ANNOTATED
BIBLIOGRAPHY
*An asterix indicates that the entry is referenced in the dissertation

**Architecture and Geometry**


Cecil Balmond 1943- Sri Lankan-born British engineer, Arup Fellow, worked at Arup for 40 years until 2010, Head of the Advanced Geometry Unit. Co-author Christian Brensing, British author. In *Informal*, a series of geometrically complex architectural projects undertaken at Arup in collaboration with leading architects, for instance Toyo Ito’s Serpentine Gallery and UN Studio’s Arnhem Central have their genesis and underlying ideas exposed through juxtaposed text, sketch and diagrams within a powerful graphical regime. The geometrical ideas, their relation to the world, schematisation are relevant to the design model thinking investigated in this research.


William Blackwell writes from the position that geometry and architecture are symbiotic, that architecture is built geometry and that rational simplicity, not complexity often makes things work [in architecture]. He reviews architecture through geometrical phenomena: planar geometry, regular polygons, equilateral triangle and hexagon, square, octagon and progression of fourths, pentagon and decagon etc. This book serves as a post Renaissance benchmark of the application of Platonic geometries in architecture.


Book written and published during the PhD research period by this author, as lead author and co-author Professor Mark Burry, Innovation Professor at RMIT University, Australia, who wrote a number of project descriptions for those projects where he had had an involvement or association with the designers. The book investigates the update of ideas and technique from mathematics (particularly post-calculus mathematics) in the period since the 1990s when powerful and affordable graphical computing became widely available to the profession in a creative capacity. The research undertaken for the PhD contributed to the chapter introductory essays in the book and the project research for the book by this author also contributed case study material for the PhD.


Mark Burry (1957-) is an architect and Professor of Innovation at RMIT University, Australia. He is a scholar of the late work of the Catalan architecture Antoni Gaudí (1851- 1926) and has worked on the continuing description for construction of the Sagrada Família church in Barcelona since 1979. His research included finding novel computational approaches to modelling the intersecting second order doubly ruled surfaces, which are the geometrical basis of much of Gaudí’s design for the church from 1990 onwards.

Bernard Cache (1958- ) is an architect, furniture designer and co-founder of Paris-based practice *Objectile* with Patrick Beauce. Cache is guest professor at the Berlage Institute. He is the author of two books. He is interested in the intersection of the mathematical, philosophical and digital and has developed geometrical modelling software. This, his first book is the development of his thesis, presents an architectural development of the Deleuzian concept of the “fold” in a broad philosophical and artistic framing of the architectural domain. His work is significant as an early post-digital architectural practitioner and architectural theorist with a unique interest in the underlying mathematics of his propositions. This book explores the idea of singularities, the ambiguous relationship between singular points and continua, and in particular the point of inflection.


Michele Emmer (1945- ), Dipartimento di Matematica, Sapienza Università di Roma, is an Italian professor of mathematics and film maker in the subject area of art and mathematics. He is also author of *Visual Mind II and Mathematics and Culture*. Mathland, targeted at a readership in the arts, and full of examples for art, architecture and film, was an invaluable compendium for my early research covering the subjects of what mathematics is, the geometrical revolution in the nineteenth century leading to non-Euclidean
geometry, Turing’s universal machine and its relationship to mathematics, topology and its significance and celebration in the arts. It was an important early source of references.


Nigel Hiscock, former principal lecturer in Architecture at Oxford Brookes University, England has written a number of books on the topic of mediaeval church architecture including *The Symbol at your Door: Number and Geometry in Religious Architecture of the Greek and Latin Middle Ages* (2007). The Wise Master Builder investigates the use of Platonic geometry in the planning of mediaeval abbeys, consistent with the Ottonian Revival of the tenth century, when monastic reform, a revival of learning, especially Christian Platonism emerged. This formed part of the early body of contextual reading around the topic of geometry and mathematics in architecture. Mary Weitzel Gibbons’ review in *Church History*, 2002, highlights that this is a very thoroughly researched body of circumstantial evidence to bring to bear on a controversial subject.


George Hersey (1927-2007), former Emeritus Professor of Art History at Yale University. Also first author of *Possible Palladian villas (plus a few instructively impossible ones)*. General background reading on the history (or benchmarks) of geometry in architecture.


Alicia Imperiale, architect, is Assistant Professor at Tyler School of Art, Temple University. She also wrote the book *New Flatness: surface Tension in Digital Architecture* (Birkhauser, 2000) This book contributed to general reading on the contemporary geometrical interests in architecture.


Kandinsky (1866 – 1944) Russian painter and art theorist, one of the first modernist abstract painters, writes of the two elements of the work of art as the *inner* and the *outer* – rather like Kant’s *inner* and *outer* sense in relation to time and space. Kandinsky writes, “The essentially immutable means [of artistic expression] are: Music – sound and time; Literature – words and time; Architecture – line and extension; Sculpture – extension and space; Painting – color and space.” (Kandinsky, *Complete Writings* Vol. 1, 89) It is intriguing that he awards *space* to sculpture and painting but only *line and extension* to architecture – solidity ahead of space.

Jean-Michel Kantor (1946-) French researcher at the Centre national de la recherche scientifique (CNRS) in mathematics, history and philosophy of mathematics and mathematical literature. Refereed journal article recounting the history of Euler’s Seven Bridges of Konigsberg, the early history of topology and graph theory, knots and non orientable surfaces, published in Nexus Network Journal, the first refereed journal in architecture and mathematics.


John Maeda (1966-), Japanese American graphic designer, computer scientist, university professor and author, software engineering graduate of MIT, PhD in Art and Design from Tsukuba University, Japan, President of the Rhode Island School of Design. In 2006, he published his eighth book The Laws of Simplicity, MIT Press. Maeda brings a highly visual approach to concepts of pattern, publishing and computation at the cusp of design and computer science.


Professor Achim Menges is Director of the Institute for Computational Design at Stuttgart University and architect partner in the network practice Ocean North. His research encompasses evolutionary computation, algorithmic design, biomimetic engineering and computer-aided manufacturing.


Richard Padovan (1935-) architect, former lecturer at the University of Bath, UK, is also the author of a subsequent book: Towards Universality: Le Corbusier, Mies and De Stijl, 2002. This is a very general wide-ranging review of the place of proportion across several fields over the last 2,500 years. Background reading in the areas of the history of geometry in architecture.


Dan Pedoe (1910-1998), English-born mathematician and geometer, who has written a number of other books on geometry, the last of which was Japanese Temple Geometry Problems: Sangoku. This book is predicated on a general history of, principally, Euclidean geometry but forges the connection of the origin of geometry to


Greg Missingham (1946-) is Associate Professor of architecture at University of Melbourne, Australia and has twice been a review panel member for progress reviews at RMIT Graduate Research Conferences for this research.
surveying and building. It provided some general leads in the early stage of the research and is referenced in the history of the relationship between architecture and geometry in Chapter 2.


Alberto Pérez Gómez (1949- ) is an architectural historian and phenomenologist. He graduated in engineering and architecture in Mexico City and completed his PhD at Essex University in the UK. This is a dense and scholarly account that covers many of the same topics as Robin Evans’s *The Projective Cast*, interestingly, suggesting a zeitgeist, as it was published only two years later. From page 377, Perez Gomez and Pelletier do consider ‘digital space’ with some pessimism about its true capacity for the extension of spatial representation as opposed to its role as an instrument of control.


This is a contemporary encyclopaedia of descriptive geometry applied to computational design modelling presented in a highly consistent graphical format as a ‘how-to’ manual. It draws on many years of formal practical teaching of descriptive geometry to architects at the Technical University in Vienna by Professor Pottmann and his colleagues in the mathematics department and the experience of developing parametric geometrical modelling software for construction design during the 1990s and 2000s. It is published by Bentley Institute Press, which is the publishing outlet of Bentley Systems.


Mutsumo Sasaki (1946-) Japanese structural engineer who has been involved with award winning innovative architecture projects. He has famously worked with architects Toyo Ito and Isosaki and this book reviews their joint projects and records conversations between these creative’s and interviews with Sasaki. Sasaki has been responsible for bringing evolutionary structural optimisation techniques into real built projects – (a technique pioneered by Professor Y.M. Xie, RMIT university). He is articulate in discussing underlying architectural and metaphorical ideas. His claim to an early interest in Riemannian geometry is intriguing.


Dr Denis Shelden is a Founder and Chief Technology Officer of Gehry Technologies and Associate Professor of the Practice in Design and Computation at MIT University. He worked at Gehry and Partners during the 1990s as a computational design pioneer and this is his PhD
dissertation which offers deep insights into computational
descriptions of surfaces and the underlying spatial and
mathematical meanings and syntax as well as the process of
bridging between physical phenomenal space, geometrical
space and the constraints of physical construction space.

Stevens, G. *The Reasoning Architect, Mathematics and Science

Dr Garry Stevens is an Australian architect, sociologist and
information technologist and former research associate at
University of Sydney’s Department of Architecture and
Design Science. The Reasoning Architect as its sub title
suggests is a highly accessible, encyclopaedic review of
mathematics and science in design.

*Treib, M. *Space Calculated in Seconds: the Philips Pavilion, Le
Corbusier, Edgard Varese* Princeton: Princeton University Press,
1996.

Professor Marc Treib (1943- ), Emeritus Professor
Department of Architecture, University of California.
Treib’s detailed account of the design of the pavilion,
retold largely from the letters of the protagonists, portrays
a process at times as frenetic, rhythmic and arrhythmic as
the Poème électronique itself, and it is an engaging read. It
spares non of the human interest around the uncertainty of
working with the temperament of the great Le Corbusier
or the uncomfortable claims made by Iannis Xenakis.
It is also rich in photographs and detail of the physical
prototypes. The succession of mathematical schemas is
implied but less deeply examined.

*Tomlow, J. *The model: Antoni Gaudí’s Hanging Model and
Its reconstruction – New Light On the Design of the Church of
the Colonia Güell*. Vol. 34, IL. Stuttgart: Institutut für leichte
Flächentragwerke, 1989.

Prof Dr Jos Tomlow (1951- ) Roemond, Holland studied
engineering at Delft, co-founder of the Gaudi Research
Delft Group headed by Jan Molema. Tomlow received a
PhD from Stuttgart University in 1986 for a thesis based
on his part in the work of reconstructing Antoni Gaudi’s
hanging model for the Colonia Güell church. Other
significant participants in that project were pioneering
computational designers and experts on difficult building
Rainer Graefe and Arnold Walz.

Weinstock, M., ‘Morphogenesis and Mathematics of
Emergence’, in *Emergence: Morphogenetic Design Strategies*,

Michael Weinstock is an architect and, since 1989,
academic at the Graduate School of the Architectural
Association School of Architecture. He is an advocate for
the architecture of emergence – and in this piece reviews the
mathematical basis of the processes that produce emergent
forms and behaviours, in nature and in computational
environments. He argues for a more mathematical approach
in architecture in the light of biological, physical and
chemical precedents and outlines the potential architectural
consequences of such an approach.

Chris Williams is a structural engineer and academic at the University of Bath, UK. His application of programming and dynamic relaxation techniques to the detailed design of the shape and distribution of nodes and triangular facets across the complex, asymmetrical dome built over the British Museum Great court by Foster + Partners, Waagner Biro and Buro Happold (reported in this paper) is well known and a formative example that has influenced the application of computational techniques to comparable problems of structure, form finding, and fabrication and contributed to raising formal architectural aspirations.


Rudolf Wittkower (1901-1971), German art historian who taught at London University and Columbia University where he was Chairman of the department of Art history and Archaeology. This book is foundational to the study of architecture and geometry. It shaped subsequent understanding of Renaissance architecture, in particular the theory of harmonic proportion and the incorporation of musical ratios into the architecture of Andrea Palladio. It enters the subject through the significance of the centralised church, Alberti’s pursuit of perfection through the circle, seen as the emulation of the round forms of nature.

**Architecture history and theory**


David Billington (1927- ), American engineer, Gordon Y.S. Wu Professor of Engineering at Princeton University. His research interests include the design and rehabilitation of bridges. He has written nine other books on structural engineering topics, or the work of leading structural engineers. I consulted this book to find out more about the history of physical analogue structural and form-finding modelling particularly in the work of Heinz Isler.


Sigfried Giedion (1888-1968), Swiss historian and architectural critic. This is a particular cultural overview of modern or contemporary architecture – which is seen as a struggle between rectilinear rationalism and the organic. Le Corbusier is seen as the protagonist most successful in reconciling these two very different realms. Historically architecture is given three space conceptions – the first in ancient Egypt, Sumer, Greece it is the interplay of
volumes. The second, seen in Roman architecture, is the preoccupation with interior space and the vaulting problem. The third dates from the beginning of the twentieth century with an optical revolution that abolished the single viewpoint. “Organiscists” such as Alvar Aalto are seen as returning to the vaulting problem.


Michael Ulrich Hensel (1965-), German-born architect whose research is concerned with the performance potential of new building materials in architecture. In common with co-authors Achim Menges and Michael Weinstock he is interested in exploring a biological paradigm for design.


Chris Hight, Associate Professor at the School of Architecture, Rice University. Space Reader is an edited collection of original and republished writings on Space. It was useful to read An introduction to Unwelt (Jakob von Uexkull), (the spatial world and place of the bee, for instance) in response to other reading by Malpas reviewing Heidegger and his counter position to Uexkull. The adoption of the term Heterogeneous Space in this volume is intriguingly different to my own adoption of the term to try and encompass the careful mathematically encapsulated translations between physical and virtual spaces implied, for instance, in the work of Shelden.


Santiago Huerta, Escuela T.S. de Arquitectura, Departamento de Estructuras, Universidad Politécnica de Madrid. Spanish engineer and construction historian – this paper is interesting not only for the reflections on the structures in Gaudí’s work but also references to the vaulting work of Gustavino, Catalan engineer who worked in Massachusetts and for the detailed history and references to the modern discovery of the significance of the catenary curve in structures.


Charles Jencks (1939-), American architectural theorist, landscape architect and designer, and George Baird (1939-), architect. This book marks the official birth of “post modernism” in architecture by denying the blank modernist architectural slate and claiming that architecture always come with its meaning. Semiotics applies in built form as much as in language. Just as in Jencks’ subsequent works Modern Movements in Architecture and the Language of Post-Modern Architecture there is a tension implied by the different possible simultaneous meanings taken.

Kojin Karatani (1941-), educated in economics and English literature in Tokyo, regular visiting Professor at Columbia University, retired from Kinki University, Osaka in 2006. He has written in Japanese and English, his three English works being *Origins of Modern Japanese Literature, Architecture as Metaphor, and Transcritique: On Kant and Marx*. He introduced the concept of 'the will to architecture'. I read this book early in the research, curious about the metaphorical relationship between architecture and number implied in the title.


Greg Lynn (1964-), American architect, early adopter of computer aided design software and film software for novel formal architectural experiments. This book focuses on the use of animation and motion graphics software for design generation.


A collection of essays including 'Blobs, or Why Tectonics is Square and Topology is Groovy' republished from ANY magazine. Lynn makes the case for morphology within defined topological descriptions to produce multiple possible formal outcomes.


Sarah Menin, architecture graduate researcher of the parallels between the creative works of Aalto and Sibelius, lecturer in the School of Architecture, Planning and Landscape at Newcastle University. This is an edited anthology of the conception, creation, perception and interpretation of Place as a construct of the mind. It is divided into two parts: Mind and Matter, with appropriate contributors to each. Mind is concerned with relationship and philosophy, matter with mediation and intervention, including built architecture. I read this to extend the consideration of Place, after consulting Malpas...
and in consideration of the place of the modeler within the virtual architectural model.


Frédéric Migayrou, French philosopher, art and architectural critic and curator. This is the essay translated from the original written in French as the opening piece in the non-standard architecture exhibition catalogue, 2003.


Professor Michael J. Ostwald is Dean of the School of Architecture and the Built Environment University of Newcastle, Australia. This book investigates the claims of similarity between the architecture of the Baroque period in the 1600s and its revival in the 1800s and what has been called The New Baroque in contemporary architecture. It is another view on a group of buildings of formal complexity, using computational design processes.


The most comprehensive source available on the influence of complexity science in architecture in the 1980s and 1990s.


Maria Luisa Palumbo, Scientific Director of the Master of Digital Architecture at the National Institute of Architecture (Italy), reflects on the way technology has turned architecture back to the body as a model of “sensitivity, flexibility, intelligence and communicative capacity”. She speculates on the future directions in biotechnology, in a world of architecture that “blends the digital and genetic”, organic and inorganic, real and virtual. This is a book of its time that expresses the speculative excitement around the opportunities of digital update in architecture for generative and creative practice.


Wolfgang Pehnt (1931- ) German architectural historian and critic wrote this book on architecture of the Expressionist movement through Europe. Expressionism is a significant stream in the history of modernist architecture. I was investigating the influence of Hugo Häring and Hans Sharoun and also Catalan Expressionism and considering whether Expressionists took up mathematical surface description – not a very fruitful avenue of enquiry.

Dimity Reed, Australian architect, former Professor of Urban Design and local councilor wrote this monograph on the design and realization of the National Museum of Australia in Canberra. The title refers both to complex intertwined cultural histories and its representation through the tangled thread of the architecture.


Colin Rowe (1920 -1999) British born, American naturalised architectural historian, critic, theoretician and Professor of Architecture (at Cornell) and Fred Koetter (1938 – ) American architect and author (Koetter, Kim and Associates, Boston, Massachusetts.) Collage City is a seminal critique of the modernist vision for the city, based on historical analysis of urban space and city plans. It was immortalised by the image of Le Corbusier's Unite d'Habitacion sailing like a great ocean liner into the urban space of Giogio Vasari's Uffizi in Florence.


A book of deeply scholarly and engaging essays of which the title essay explores the geometrical and proportional similarities between Le Corbusier's Villa at Garches and Andrea Palladio's Villa Malcontenta. Rowe's work reinforces the powerful geometrical connections between the Renaissance and 20th century modernist architecture in adopting the International Style.


Translated from the German *Die Geschichte der Bauingenieurkunst* this book by Hans Straub is introduced with the topic of the historical symbiosis of arts and sciences and the central position of mathematics in the liberal arts of the Renaissance. It has been republished including by the MIT Press in 1964.


Robert Charles Venturi (1925 -) American architect, writer and teacher, winner of the 1991 Pritzker Prize in Architecture compiled this "manifesto" from course lectures at the University of Pennsylvania to demonstrate through a wide variety of examples ways of understanding architectural composition and complexity leading to richness and visual stimulus. He draws on examples from different periods of history, well known and less well known to make a case for the complex holism in architecture in contrast to the prevalent diagrammatic minimalism and functionalism of late modernism.
## Architecture/Science


Giuseppa Di Cristina, Faculty of architecture, University of Rome “La Sapienza”.

This is an extended edition of Architectural Design edited and introduced by Di Cristina calling on architectural practitioners and architectural theoreticians and academics to compile an account of the contemporary influence of science and technology in architecture.


Murray Gell-Mann (1929-) American physicist and Nobel laureate. This is his account of the connection of particle physics to just about everything else.


James Gleick (1954- ) author, journalist and biographer writing about the cultural significance of science and technology. This book is an un-put-downable read about weather, turbulence (its resistance to computation) and the dangerous birth of Chaos theory in the 1960s, through the life and work of meteorologist Edward Norton Lorenz, reference to Thomas Kuhn’s, science history theory of how paradigm shifts occur and Werner Heisenberg’s two questions for God on his deathbed: “Why relativity?” and “Why turbulence?”

Christopher Alexander (1936-) English born architect and influential architectural theoretician and writer, Emeritus Professor at University of California, Masters in mathematics and Bachelors degree and PhD in Architecture. This is Alexander’s case for the application of complexity science in architectural design as a way of harnessing the comparatively self-correcting systems of nature.


D’Arcy Wentworth Thompson (1869-1948), Scottish mathematical biologist. On Growth and Form addresses a possible overemphasis on evolution in determining the physical form of organisms through a detailed study of examples of the role of physical laws and mechanics in influencing natural form. This book has been influential in many fields including in architecture and structural design and optimization.

General source reading on morphogenesis in biology in order to better understand the relationship between the science and the appropriation of ideas from natural morphogenesis in architecture. Not very accessible


General source reading on morphogenesis in biology in order to better understand the relationship between the science and the appropriation of ideas from natural morphogenesis in architecture.


Illustrated to show the simultaneous additive and subtractive activity of bone growth and the influence of physical factors.


Antoni Picon, Professor of Architectural History and Technology at Harvard Graduate School of Design, trained in engineering, architecture and history of art and Alexandra Ponte. This is an edited work about a “similarity of operation between science and architecture that makes the relationship productive” and “a new type of connection between architecture and science for which the computer is central.” It marries history and contemporary preoccupation with science in architecture. Architectural examination of the natural sciences seems to lead back to the nineteenth century. Martin Bressani’s writing about Violet le Duc was particularly influential for this research.


Eugène Emmanuel Viollet-Le-Duc (1814-1879), French architect famous for his restorations of mediaeval buildings and known as the first architectural theorist of modern architecture. This book provides a powerfully anatomical and detailed analysis of architecture that reflects violet-Le-Duc’s keen interest, knowledge and library of books on biology.

**Cognition**


William Ross Ashby (1909-1972), English Psychiatrist. This book provides an introduction to systems thinking from the natural world to computation delivered with dry clarity and very accessible.

This is a book about adaptation and the learning component in it, adaptation in the context of system. Other key words: stability, dynamic systems, the Homeostat, ultrastability, Iterated and serial systems. It is a speculative and qualitative work.


Gregory Bateson (1904-1980) British anthropologist, social scientist and linguist. A holistic view of mind and spirit (animus), starting with “the pattern which connects” similarities understood through qualitative similarity, homology etc. Second order connections, or “phylogenetic homology”. Introduces through this ordering; first, second, third, explained metaphorically, the idea of levels of meta-meaning. Minds contain no things, only ideas about things.


Mark H. Bickhard, Professor of Cognitive Robotics and Philosophy of Knowledge, Departments of Philosophy and Psychology, Lehigh University. This is a critique detailing the limitations of the essential symbolic encoding understanding of representation for the further development of Artificial Intelligence and Cognitive Science.


Gabriela Goldschmidt, Faculty of Architecture and Town Planning, Technion, Israel Institute of Technology. This is a paper defining sketching as form of visual reasoning as opposed to only an irrational, generative process.


Jacques Salomon Hadamard (1865-1963), important French mathematician who contributed to number theory, complex function theory, differential geometry, partial differential geometry and the study and observation of dynamical systems. This is a pre-computer and very unfashionable (in the 1940s period of Behaviourism) review of introspection as a method of mathematical and scientific (and even musical) discovery.


Julian Jaynes (1920 -1997), American psychologist. This is an intriguing thesis about possible mind changes since ancient times. At the centre is the proposition that the Ancient Egyptians, characters in Epic of Gilead, early figures in the Old Testament were not fully conscious in
the modern sense despite their highly developed language but were responding to imperative voices, potentially generated by the opposite side of the brain in the manner of modern schizophrenia.


Daniel Kish, President of World Access for the Blind, himself blind since an early age, writing in New Scientist about early childhood memories of using echo-soundings to understand and navigate space.


George Lakoff (1941-), Professor of Linguistics at the University of California Berkeley and Rafael Núñez, PhD, Associate Professor at the Department of Cognitive Science at the University of California. Mathematics as a product of the human brain is shaped by the cognitive structures of the brain, related to other aspects of spatial and linguistic function and constructed on a series of foundational metaphors.


Geir Kaufmann (1943-), Norwegian Professor of Psychology in a Department of Leadership and Organizational Management. This book compares the Symbolist and Conceptionist theories of thought drawing conclusions on the importance of imagist thought for innovative thought around concrete and synthetic problems.


Seymour Papert (1928-), South African MIT mathematician, computer scientist and educator, leading researcher into the impact of new technology in learning. In this contributed chapter he argues for aesthetics rather than procedure as central to mathematical thought and constructive learning and challenges Poincaré’s elitist view of sensitivity to the mathematical aesthetic.


Roger Penrose (1931-), English mathematical physicist, Emeritus Rouse Ball Professor of Mathematics at the Mathematical Institute, Oxford University.


Jean Piaget (1896-1980) and Bärbel Inhelder (1913-1997), Swiss developmental psychologists. This is the only volume in the series referenced in the dissertation. It reports on investigations of the development of spatial concepts in young children, particularly the growth of projective and Euclidean concepts out of the more basic topological understanding.

One aspect of Piaget’s quest to “trace the development of the operations which give rise to number and continuous quantities, to space, time, speed, etc., operations which, in these essential fields, lead from intuitive and egocentric pre-logic to rational coordination that is both deductive and inductive.”


Introduced as a sequel to the Child’s Conception of Space. In this volume the authors deal with the measurement and metrical geometry that has bearing on the complex issue of spatial intuition.


John Searle (1932-), American philosopher. This is an early series of essays by Searle amplifying John L. Austin’s (1911-1960, British philosopher of language) linguistic theory of *Speech Acts*. Austin divides Speech Acts into *locutionary* (the literal utterance and its most literal meaning), *illocutionary* (the multiple meanings e.g. “Is there any tea?” also implying that the utterer would like some tea) and *perlocutionary* (causing another to act). Searle’s work focuses particularly on illocutionary – the performative nature of speech and inference in language. This work in linguistics can be seen as counterpoint to the close symbol processing analogy to natural language.


A short book of non-technical essays for a general audience that cover the mind-body problem, the mind-brain problem, the question of whether computers can think (strong AI), cognitive science as an inversion of strong AI, Free Will.


Alan Newell (1927-1992) American researcher in computer science and cognitive psychology contributed to the Information Processing Language (1956) and two of the earliest AI programs the *Logic theory machine* (1956) and the *General Problem Solver* (1957), the second of these with Herbert Simon (1916 – 2001), American political scientist, economist, sociologist and psychologist. They were awarded the ACM’s A.M. Turing Award together in 1975 for their basic contributions to artificial intelligence and the psychology of human cognition. In this book they propose an architecture as a model of human cognition, which is seen primarily as symbol manipulation, performing serial operations, solving problems through searching through a problem space with an explicit representation of goals. (This representation is for the purpose of researching problem solving).

Nicholas Swindale (1951- ), British PhD in neurobiology, Professor in the Department of Opthalmology and Visual Sciences at University of British Columbia. The organization of topographic maps in the cerebral cortex may be understood through considering mathematical topology – in particular direction and movement for navigation understood visually may be connected to the topological organization of structures in the cortex.


Ernst von Glasersfeld (1917-2010), philosopher and Emeritus Professor of Psychology at the University of Georgia, Adjunct Professor at MIT (Amhurst). He was a proponent of Radical Constructivism, a theory that he developed by elaborating on Giambattista Vico, Jean Piaget's theory of genetic epistemology, Bishop Berkeley's theory of perception, James Joyce's Finnigan's Wake and other texts. I focused in this book on Chapter 1 – Growing up Constructivist, Chapter 8 - the Cybernetic Connection, Chapter 9 - Units Plurality and Number. The first deals with the solipsism of Constructivism – we only know what we personally experience. The eighth introduces the basic concepts of cybernetics – self-regulation and control, autonomy and communication- then deals with one specific area – Piaget's theory of cognition and constructivist epistemology (cognitive adaptation in the human mind (148)). The ninth acknowledges that “Maths presents a problem for the constructivist model: a host of results that are ‘objective’ in the sense of unquestionable.” Then it goes on to examine through history and philosophy, whether it is possible to know what characteristics have to be abstracted in order to form the concept of number in experience.


This is an edited collection of essays. In his introduction von Glasersfeld defines Radical constructivism as a way of knowing rather than a theory of knowledge or epistemology. The fact that we do agree and can communicate does not prove that what we experience has objective reality. With reference to Jean Piaget’s “adaptive function” definition of knowledge, von Glasersfeld advocates active and affirmative environments for learning mathematics. Paul Steedman writes on the idea that there is no more certainty in mathematics than in the empirical sciences, citing Popper.


Judith Wechsler, Art historian who has engaged in interdisciplinary studies including art and science, National Endowment for the Humanities Professor,
Tufts University, 1989-. This book was written during her time at MIT. In the preface to the first edition, Wechsler notes that this is a book that came out of a course run from 1972-78 to encourage students to see scientific models as human creations affected by traditions, styles and sensibilities. In her preface to the 1988 edition she is more expansive on the challenging changes in the patterns of aesthetics around phenomena over time and the way these run parallel to developments in the visual arts. I focused on Seymour Papert’s essay on The Mathematical Unconscious (105-119), which I have referenced in the section of aesthetics in Chapter 6.

**Design and Computation**


Christopher Alexander (1936-) English born architect and influential architectural theoretician and writer, Emeritus Professor at University of California, Masters in mathematics and Bachelors degree and PhD in Architecture. This is his first book developed from his PhD work. It is historically significant as a very early, published attempt to find a way to program architectural design synthesis using a computer. It is referenced more than once in the dissertation.


Alexander and Ishikawa’s popular book of over 200 spatial patterns that the author's believe encapsulate the basis of meaningful human interaction at the scale of settlement, city, institution, home and room. The highly structured, systematic ordering of these *design patterns* with their interrelated language, syntax, and grammar has been adopted as a useful programming model in fields other than architecture, such as software engineering and computer science.


This is another lengthy sequel to The Synthesis of Form with didactic overtones. It deals with similar argument to *A Pattern Language* – a culturally ubiquitous *Fen Shui* delivered in an unequivocal dogmatic (in the literal sense of ‘based on absolute truth’) style.


Erik K. Antonsson is now Research Director at an Aerospace Company in Los Angeles, former Professor of Mechanical Engineering at California Institute of Technology (-2009) where he ran the Engineering Design Research Laboratory. Jonathan Cagan, Professor, Mechanical Engineering, Carnegie Mellon University. This is a valuable reflective collection combining contributions from architecture and engineering on the history, and the very possibility of formal design science – scientific theories of design synthesis.

Co-authors: Peter Felicetti – Australian practicing structural engineer with architectural background specialising in complex architecture, Jiwu Tang, Research Fellow, Innovative Structures Group, RMIT University, Professor Mark Burry, Director of the Design Research Institute, RMIT University, Professor Y. M. Xie, Head of Civil, Structural and Chemical Engineering, RMIT University. This journal paper reports on collaborative research to apply Evolutionary Structural Optimisation to find the constraint and loading conditions for evolving Gaudí’s structure for the Passion Façade of the Sagrada Familia Church.


Steven Anson Coons (-1979), a professor of mechanical engineering at the Massachusetts Institute of Technology and a pioneer in computer graphical methods. This forward thinking manifesto for the hypothetical design of a computer-aided design system to include: graphical input of information, linked to natural language, computation for analysis linked directly to synthetic input, the capacity for extending the system or library of tools – being able to design the design within the system, multiple designers conversing with the system simultaneously and able to view each others interventions, has been referenced extensively in Chapter 2 of the dissertation.


Professor Richard Coyne, Architectural Computing at the University of Edinburgh wrote this book while at University of Sydney, where he completed his PhD. Coyne wrote several books on the implications of information technology in design. This one emphasizes the pragmatic aspects of context and community.


Stylianos Dritsas and Mirco Becker, formally of architects Kohn Pedersen Fox Associates. This is a joint reflection on the development of computational design from analogue origins supported and punctuated by a series of computational design explorations for projects in which the authors have been engaged.


Charles M. Eastman, Professor in the Colleges of Architecture and Computer Science, Georgia institute of Technology. This book is a comprehensive review of the history, and current state of digital building representation for design interaction and construction.
at the end of the twentieth century. It identifies concepts, methods, standards and remaining research gaps at that time.


James Glymph, former Principal at Frank O. Gehry and CEO of Gehry Technologies, the company formed in 2002 to introduce AEC project solutions. Dennis Shelden Chief Technology officer at Gehry Technologies and Professor of Architectural Practice at MIT, Cristiano Ceccato, Associate at Zaha Hadid architects and former member of the founding team at Gehry Technologies, Judith Mussel, founder of XP& Architecture, formerly Associate at Gehry Partners, Hans Schober Schlaich Bergerman & Partners, Glass industry professional. This paper reports on the work for the Jerusalem Museum of Tolerance project where in contrast to other free-form glass roofs using either triangular facets or curved (formed) glass planes, the free from roof is achieved using quadrilateral panels. This was read as a useful comparison with Foster + Partners Smithsonian roof. This is a development of the Gehry technique of translational surfaces applied to achieving the freeform surface with the quad panels in the surface.


This is a review of the accomplishments of Structural engineering in the 20th Century with projection of the expectations and needs for the 21st. This includes acknowledgement of the impact of digital computation in relieving the engineer of much of the tedium of detailed analysis, freeing them up to concentrate on the more creative interests of exploring alternatives, accounting for uncertainties and integrating all the components of the system.


Mark Goulthorpe, principal of the dECOi atelier and MIT Professor of Architectural design, Professor Mark Burry, RMIT University and Grant Dunlop, architect. This is now a source of information and reflection regarding the design and development of the Aegis Hyposurface.

George Hersey (1927-2007), former Emeritus Professor of Art history at Yale University and Richard Freedman, Microsoft Corporation, designer of the program to generate the villa plans and facades. This book follows the lead of George Stiny and Bill Mitchell who showed that a parametric generative grammar or shape grammar could be used to generate Palladian plans. While it is clear that Palladio's villas embody geometrical rules, it is more challenging to unravel exactly what these rules are and how consistently they should be applied.


Douglas Hofstadter (1945-), American academic, graduate of mathematics, PhD in Physics, researcher in consciousness, son of Robert Hofstadter, Nobel laureate physicist. A book exploring consciousness, artificial intelligence, self-referential systems, self-representational systems, paradox, through several media including reframed and diverting Classical dialogue, games, axioms and theorems and straight theoretical and historical account.


Sawako Kaijima, computational designer at Adams Kara Taylor (AKT) in London, BA major: media design, Keio University, Masters of Architecture, MIT (2005) and Michalatos Pangiotis, also at AKT, architecture graduate, programmer and specialist in real time motion analysis. This paper examines the inhibition represented by rigid models of practice, particularly in being able to leverage computational representation. It explores the role of the computational designer as mediator (between disciplines) and mitigator of practice. Influenced by Karitani’s writing, they present design practice as contingency planning, transformed through execution, aiming to realize the design as a problematic field that is enriched by contingency (as opposed to as an idea that is diluted, polluted or corrupted by contingency).


This paper is an attack on the complexification (literally, a technical term in mathematics for extending multiplication to include multiplication by complex numbers) of the geometrical description of design proposals through ill-advised application of digital tools, countered by a manifesto supported by examples of fine tuning a system to respond to a specific and
appropriately simplified design problem. It is a play on the true meaning of *complex* with regard to systems built on simple components and rules.


Dr Sean Keller, PhD, assistant professor at Illinois Institute of Technology. This journal article is written summarizing the content of Keller's PhD dissertation regarding the history of the Land Use and Built Form Studies (LUBFs) Centre framed ironically in the light of the near closure of the Architecture Department at Cambridge in 2004/2005 by the University of Cambridge in response to the poor performance of its research when measures of scientific research were applied.


A very detailed account of the activities and critiques of the LUBFS centre at Cambridge University in the 1960s and '70s focusing on the construction of highly syntactical systems in the name of architecture. I have drawn on this work extensively for historical and cultural context for the early development of computational system modelling in architecture in Chapter 2 of this dissertation.


Dr Axel Kilian, PhD (MIT), Assistant Professor, Princeton University. Kilian frames his thesis, which describes a series of important design modelling experiments around the grail of bidirectionality in modelling constraints – escaping from the classic tree-structured graph of relations in models to be able to intervene from both ends, rather than having a rigid hierarchy from inputs to outputs. He has a number of useful demonstrators including the digital version of the analogue 3D hanging model which allows live editing of both loading and shape inputs. This is about two-way mappings between design representations and overcoming the problem that most reverse mappings are non-deterministic (the output infers one of many inputs). This is a highly visual publication.


Dr Branko Kolarevic, Chair in integrated Design, Professor of Architecture, University of Calgary and Ali M. Malkawi, Professor of Architecture; founder and Director, T.C. Chan Centre for Building Simulation and Energy Studies, PennDesign. Edited collection of short essays by architectural technologists and leading provocateurs from architectural practice based on a symposium on the title topic. It is another useful reminder that a performance involves a protagonist or protagonists; it is not a passive concept or measure in relation to architecture and its design.

Greg Lynn (1964- ), owner of Greg Lynn FORM office, Professor of Architecture at the University of Applied Arts, Vienna, Studio Professor at UCLA School of the Arts and Architecture, Davenport visiting Professor at the Yale School of Architecture, Fellow of United States Artists. An account of morphogenesis and mutation in architecture told through the work of the practice, Greg Lynn’s writing and invited essays from science, from fiction, and from science fiction authors (J. G. Ballard and Bruce Sterling). This was read to support the research of architecture underpinned by ideas from topology.

Maloney, J. and B. Dave, ’Mixed reality at the sketch design stage’, in *Proceedings of the 15th International Conference on Computer Aided Architectural Design Research in Asia / Hong Kong* (Chinese University of Hong Kong, 2010).

Jules Maloney, Senior Lecturer at Victoria University of Wellington, formerly University of Melbourne; Associate Professor Bharat Dave, University of Melbourne. This paper reports on research into novel visual systems for linking contextual representations to design representations, such as simulated energy use or solar incidence to building orientation, and also real time on-site live site video with sketch design interventions.

March, L., 1934-. *Architectonics of humanism: essays on number in architecture / Lionel March*. Chichester, West Sussex :: Academy Editions,, c1998.

Lionel March former Director of the Centre for Land Use and built Form studies at Cambridge University, Rector and Vice-Provost of the Royal college of Art, London, Chair of Architecture and Urban Design UCLA, also author of *Geometry and Environment, Urban Space and Structures*. This book is a thoughtful and critical account (reinterpretation) of the relationship between architectural discourse and mathematics from the Renaissance to the Twentieth Century.


One of the first papers I read in order to try to understand the cultural context of the mathematics architecture nexus in the overlapping period of modern architecture and digital computation.

*March, L., ‘Some elementary models of built forms (Working paper 56 of the centre for Land Use and Built Form studies)’, (University of Cambridge, 1971).


An outline summary of mathematical applications in architectural and urban design over forty years, structured around projects that the author has been
involved in – principles such as the economy of perimeter block development compared to solid towers, illustrated by Fresnel squares of different widths and the same area, the interaction of block size and road size in city planning, reference to Shape grammars and the distinction between computation and computing. March’s references include: Pappus, Polya, Froebel, Whitehead, Stiny and amongst city planners, Cerda and in recent decades, McCormac.


An edited book divided into three parts: Description, Prediction and Evaluation. The contributors include: Lionel March, Michael Derbyshire, Philip Steadman, Dean Hawkes and Richard Stibbs, Robin Forrest, Paul Richens, Philip Tabor, Tom Willoughby, Patricia Apps. Description includes Boolean description of a class of forms (March), Graph theoretic representation (Steadman), Computer description etc. Prediction includes environmental performance, surface luminance, environmental impact of motorways, route patterns etc. Evaluation means the problem of evaluating design – balancing architectural objectives.


This is a comprehensive volume introducing ideas from geometry and their application in design. I read this early in the research as part of the literature search for works on mathematics and architecture.


Thomas Maver, Professor of CAD at the University of Strathclyde, Director of Abacus, director of the Graduate School in the Department of Architecture and Building Science, researcher in the field of Computer Aided Design for over 40 years. In this paper he supports his case for the use of the computer to make more explicit certain numerical constraints, properties, performance criteria for buildings within the design process – exposing the cost benefit of design decisions. This is couched as criticism of over emphasis on formal aspects of design at the expense of clarity of communication. He supports this with examples of CAAD applications from the previous 25 years.


William Mitchell (1944-2010) former Dean of MIT’s school of Architecture and Planning and Director of the Media Lab’s Smart Cities research group. This book is a
formal treatment of the logic of architectural design and structure of design thought. It considers the language of architectural form and its specification using formal grammars, based on timeless principles.


Dr Paul Nicholas, Adjunct Professor at Kunstakademiet, Arkitektskole, Copenhagen, Senior Architectural Designer at edaw I AECOM, PhD from RMIT, SIAL where he was embedded with Arup in Melbourne and Professor Mark Burry, RMIT University. This is a very clear paper about the complexity of integrating design and analysis using digital tools in design. It frames an example around lighting noting the specificity of interaction to the design stage, the specific project and the need for both the design and analysis frameworks to be active simultaneously – rather than one passive, one reactive. This is useful paper for framing the integrated system model in design as a system that is much more extensive than the geometry model, or even the digital representations in their collective entirety but includes design and performance interpretation.


Les A Piegl, Professor of Computer Science and Engineering at the University of South Florida, researcher in geometric computing including Computer Aided Design and CAD/CAM, computer graphics and software engineering. Wayne Tiller, Geoware Inc., Tyler, Texas. A comprehensive and mathematical account of non-uniform rational b-spline curves and surfaces targeting a visual audience. Deemed mathematical rather than computational by the programming community, difficult to use as a manual given the quality of the images for the uninitiated, but universally hailed as comprehensive and containing the code needed as a foundation to implementing and manipulating the curves and surfaces.


Seymour Papert, (1928-) South African MIT mathematician, computer scientist and educator. This book mediates Constructivist and Piagetian theories of learning with computer-based technology in education. He proposes a computer based learning environment called the Microworld, designed to complement the natural knowledge building mechanisms of children and result in improved quality of knowledge gained through learning activities.


Ingeborg Rocker, Assistant Professor at the Harvard Graduate School of Design. This article maps the changing codes and constraints shaping architecture, following the transition of calculus into computation and the introduction of computers into architecture to pose questions about the effect of recoding architecture.

Prof. Dr. Kristina Shea, (1971-), American graduate of mechanical engineering, Professor for Applications of Virtual Product Development, TU Munich, (formally with Arup, London and based at University of Cambridge Engineering Department, UK), Dr Robert Aish, British graduate of industrial and PhD in Human Computer Interaction, Autodesk’s Building Solutions researcher, former Director of Research at Bentley Systems and creator of Bentley Systems Generative Components parametric software for the AEC sector, Co-founder of the Smart Geometry Group; Marina Gourtovaia, Research Associate, software development, University of Cambridge, formerly Russian polymer chemist. This paper is about the integration of eifForm, a generative application using structural shape annealing, combining grammatical parametric shape generation, performance evaluation, including structural analysis, stochastic optimization for exploration of discrete structural forms related to behaviour, spatial and cost performance with Custom Object, the alpha forerunner of Bentley System’s Generative Components graph-based parametric geometrical software. The Case study is a series of related roof truss models. It usefully elucidates the specific aspects of the complexity of system modelling integrating design and analysis.


Philip Steadman, Professor of Urban and Built Form Studies at the Bartlett Faculty of the Built Environment, London. Referenced in Chapter 2 as a well-known and important pioneering application of enumeration – in this case finding all the solutions to the linkages between a certain number of rooms and re-representing the graphs as rectangular room packings within a rectangular boundary.


Ivan Edward Sutherland (1938- ), American computer scientist and Internet pioneer, received the Turing Award in 1988 for Sketchpad. Referenced in Chapter 2 as the quintessential and earliest example of successfully interfacing visual design thinking with computer programming. Far from being a translation of traditional approaches to representation, it demonstrates most of the fundamental offerings of computation – that are only now becoming current in some design disciplines.

Hugh Whitehead, British architect, founder and leader of the Specialist Modelling Group (SMG) at Foster + Partners; Brady Peters PhD Fellow at the Centre for Information Technology and Architecture (CITA) and an architectural researcher with JJW Arkitekter and with Grontmij/CarlBro Engineers. His research focuses on acoustic performance of complex surfaces. He was formerly a leading computational designer in the SMG at Foster + Partners.


Dr Megan Yakeley, (now Website consultant). This PhD dissertation is an in depth study of design pedagogy starting with the premise that design education confounds product and process. It investigates the way in which the introduction of computer programming in a particular framework, can contribute to the development of a personal design process. Yakeley refers to a transition from Behaviourist to Constructionist model of learning.

**Design Theory and Education**


Nigel Cross, Emeritus Professor of Design Studies, researcher in design cognition, applying protocol analysis. In this paper Cross takes up the argument for design as a third area of education and contrasts it with the other two: sciences and humanities. In this he is picking up an argument from Bruce Archer published in the first issue of Design Studies. In this paper, Cross develops both the criteria which design must satisfy to be acceptable as part of a general education – a less instrumental orientation than traditional design pedagogy, – and the distinguishing features of design as a way of knowing (the constructivist construction, as opposed to the Classical body of knowledge). This article offers up one particular potential set of benchmarks for evaluating the designerly value of activities, heuristics and frameworks.


Peter Downton, a professor of architecture at RMIT University. This is a useful general reference to design research. It makes the distinctions between research about design, research for design and research by design, examines the nature of new knowledge that is generated in each case and the relationship between knowing and knowledge in design and design research.
Appendix I | Selected annotated bibliography


This unpublished PhD Chapter was very useful in exploring the definition and application of models. The PhD dissertation is developed in a sonata format form which I have borrowed the term ‘recapitulation’ to describe the section at the end of each chapter which both summarises what has just gone before adds new material and interpretation.


Linda Groat, Professor at the University of Michigan, A. Alfred Taubman college of Architecture and Urban Design and David Wang Associate Professor of Architecture present this as a compendium of approaches to architectural research, looking closely at practice, communication and organisational issues as well as the traditional architectural science modes of research in academia. This was background reading early in the doctorate.


Tom Kvan is currently Professor and Dean of the Faculty of Architecture, Building and Planning at University of Melbourne and wrote this paper while at the University of Hong Kong. This paper unpacks terms computer-supported collaborative design and computer-supported co-operative work used apparently interchangeably in journal titles and keywords by exposing the difference between collaboration and co-operative workflow in this context. This is important as the emphasis of much research into computational design modeling is done in the name of supporting collaboration or cooperation across design teams, model systems engaging team members as well as geometry.


Ruffina Karatne is currently Senior Sustainability Design Manager at Leigh and Orange Ltd and authored this paper while at the University of Hong Kong. This research revisits the role of the model in the design conversation – specifically, how digital modeling and rapid prototyping change this cycle. It adds to ‘models of’ and ‘models for’, ‘models with’ and reports on the work of architectural students.


Professor Bryan Lawson, Dean of the Faculty of Architectural Studies, University of Sheffield, UK. This is the Fourth Edition and significantly edited and extended since the title was published in 1985. Lawson has now written a companion volume, titled What Designers Know. It is a very useful book for drawing clear distinctions between design thinking and other ways of thinking, such as scientific enquiry.

Horst Willhelm Jacob Rittel (1930-1990) Professor of the Science of Design, University of California, Berkeley and Melvin M. Webber (1920-2006), urban designer and pioneer in thinking about cities of the future. They are best known for coining the term “Wicked Problem” in relation to social policy meaning a problem that is difficult or impossible to solve because of incomplete, contradictory and changing requirements, often hard to recognize. The level of interdependency means that trying to solve one aspect can simply reveal a new problem. (This is relevant because computational design system models in trying to represent aspects of design deliberations can quickly take on this level of interdependency and intractability despite the constrained domain of design solutions inherent in the model.)


Department of Urban Studies and Planning, MIT, Cambridge MA

Donald Alan Schön (1930-1997) Bostonian Professor of Urban Studies at Massachusetts Institute of Technology interested in the uptake of technological change (or not) in social systems and the power of ‘generative metaphor’ – the influence of the descriptors attached to social situations on actions wrote this article with Grant Wiggins, design educator.


The oldest cited published source for the neat distinction between ‘models of’ and ‘models for’, that is, models that represent something in order to illustrate or communicate it and those that represent something in order to have an active input to an iterative process.

**Geometry**


This is a paper about Disordered systems; Networks; Random networks; Critical phenomena; Scaling; and the world-wide web. I read this as an effort to contextualise topology, networks and topological holes in a physical world context.


Roger Apéry (1916-1994) French mathematician born to French mother and Greek father, professor at the University of Caen. This is beautiful book showing many excellent computer graphic images of representations of the real projective plane. The introductory chapter gives some history of the real projective plane.

Tomaso Aste and Dennis Weaire, professor at Trinity College, Dublin, PhD supervisor of Robert Phelan and coauthor of the most efficient geometrical packing solution the *Weaire Phelan model*. Their answer to Kelvin’s Conjecture has been made famous in architecture by its use in the structural design of the space frame and cladding bubble units of the Beijing Olympic Watercube (PTW architects/Arup/CSCCE/CCDI). This is a fascinating and very readable history and survey of packing.


Garrett Birkhoff’s paper about the significance of Group Theory in the context of Évariste Galois’s (1811-1832) work, also known as the theory of symmetry or the theory of ambiguity. It is an interesting paper that assesses the extent of attribution of Group theory to Galois and largely accessible to non mathematicians.


Borrowed as background reading after reading Dennis Shelden’s Thesis – I found it rather technical.


Marcel Berger (1927-) French Mathematician, differential geometer, former director of the Institut des Hautes Études Scientifiques (IHES), France. This is an 826 page book providing an overview of the vast subject of Riemannian Geometry. It starts with the author’s apology for its inability to be comprehensive, exclusion of proofs etc. Much of it is too technical for non-mathematical readership but it starts with a three chapter introduction to the concepts and tools of Riemannian geometry, following the work of Gauss and Riemann written in a very accessible way. The breadth of applications throughout the book is also fascinating even to the non-technical reader.


Algebraic curves are graphs of polynomials in two variables. Bix’s book starts with the outline history from highly developed Greek geometry, the subsequent development of algebra and the analytic geometry of Fermat and Descartes in the 1600s. It gives important definitions and explanations of the tools for working with algebraic curves.


General background reading – partially non-technical.


John Horton Conway (1937-), Professor of Mathematics at Princeton University, inventor of the cellular automaton or game of life. A book written to explain the relationship of transfinite numbers and mathematical games through the medium of game playing. Transfinite is Cantor’s term for numbers that are larger than all finite number numbers but not in some strict sense infinite, for instance w + 1 where (1, 2, 3, …w) is the set of finite ordinal numbers.


A book for the curious rather than necessarily the mathematically equipped about all the many different kinds of number there are.


Celso Costa is the Brazilian mathematician who discovered the Costa Surface in 1982. It is only the fourth embedded minimal surface that can be formed by puncturing a compact surface after the plane, catenoid and helicoid. Most of the mathematics in this paper was beyond my grasp but it was interesting to try and contextualize this discovery.


Richard Courant (1888-1972), German mathematician David Hilbert’s assistant in Göttingen, Professor at New York University from 1936, known for the finite element method before this was later rediscovered by engineers. Herbert Robbins (1915-2001), American mathematician and statistician, known for his work in Topology. This book was written as a popular mathematics reference and has remained in print.


Ludwig Danzer (1928-) German mathematician and discrete geometrician who worked extensively on aperiodic tilings including collaborations with Branko Grunbaum and Geoffrey Shephard and was one of the first mathematicians to seriously
study challenging mathematical problems on aperiodicity in response to the 1984 discovery of quasicrystals. This paper is only partially accessible to the non-technical reader.


Philip J. Davis (1923-) American applied mathematician especially of numerical analysis and approximation theory and historian of mathematics and Ruben Hersch (1927- ), American mathematician and academic, write on the practice and social impact of mathematics. Example: Chapter 5 Selected Topics in Mathematics: Group theory, The Prime Number Theorem, Non-Euclidean Geometry, Non-Cantorian Set Theory, Nonstandard Analysis, Fourier Analysis. A very engaging read for knowledge rather than criticism or argument. This is at a good level for the non-technical reader to engage with the mathematical concepts – challenging but not impenetrable.


A further reference to Ludwig Danzer and his work on packing theory.


A history and translation of Desargues Rough Draft on Conics.


Prof. Dr. Gerd Fischer, German mathematician at the Mathematisches Institut der Universität Düsseldorf. This is a beautiful catalogue of collections of mathematical plaster models from the nineteenth and early twentieth century used for geometrical demonstrations. They include models of very complex surfaces. The first volume contains 132 photographs of mathematical plaster or string models of surfaces from various collections, grouped according to their geometrical derivations. The second volume is the commentary on the models. It is authored by mathematics professors from German Universities and edited by Professor Gerd Fischer.


Nat Friedman, American mathematician and sculptor, formally a member of the Department of Mathematics at the University of Albany, State University of New York 1968-2000 who in recent years has let mathematics influence his sculpture and has also found
ways to present knot theory to young and non-technical audiences. This paper provides a very clear accessible introduction to the knot theory that underpins Friedman's expressive work.


This paper provided a very graphic real work understanding of a topological rather than a geometrical or topographical mapping of a situation and its implications. It was a usefully concrete way to grasp the concept of topological holes.


Karl Friedrich Gauss (1777 - 1855), brilliant German mathematician who contributed to many fields, including these papers on curved surfaces that both summarize existing knowledge and investigate new theorems about curved surfaces. The abstracts are accessible and it is instructive, even as a non-mathematician to read, in translation, Gauss's account of breaking down these problems.


Ivor Grattan-Guinness (1941-), Emeritus Professor of the history of mathematics and logic. This is an account of Whitehead's contribution to the foundations of pure and applied mathematics, particularly between 1890 and 1920. He was an algebraist and increasingly, under the influence of his student Bertrand Russell, a logician.


G.H. Hardy, was recognised by David Hilbert as ‘England’s top mathematician’. (Hardy saw himself as 5th pure mathematician in the world at the height of his career). This small volume is a beautifully concise and slightly sad, in the sense of pathos, reflection on the value of mathematics and author's contribution to it written at the age of sixty years. C P Snow's foreword is very compelling in this edition. Hardy's major collaborators were Littlewood (at this time during the war 2nd Lieutenant working in Ballistics in the royal Artillery) and Ramanuyen, Brahmin a prodigy from Madras who died very young in England from Tuberculosis. This was one of the first books I read during the course of this research and it had a profound influence on my thinking about mathematical aesthetics and its highly abstract basis for Classically grounded mathematicians such as Hardy.

Sadly inpenetrable to such a non-technical reader.


A biography of Paul Erdös (1913-1996), Hungarian mathematician and inveterate traveller, who is said to have published more papers than any other mathematician in history with hundreds of collaborators.


David Hilbert (1862-1943), German mathematician (biography footnoted in Chapter 6) and S. Cohn-Vossen. This is a wonderfully thorough and graphically illustrated book describing surface geometry, written for both a mathematical and a broad readership.


Steven Hyde is Professor in the Department of Applied Mathematics at Australian National University researching the relevance of low-dimensional geometry and topology in complex physical, geological and biological systems, this includes novel 2D hyperbolic geometrical representations of crystal structures.


Peter Janich (1942-), professor of philosophy at the University of Marburg, founded methodological culturalism. This book considers not only whether space is three dimensional, but in what ways it is three dimensional, what ‘purely spatial’ means, whether geometry is purely formal, what dimension is and other questions of this ilk.


Johannes Kepler (1571-1630), German mathematician, astronomer and astrologer, best known for his laws of planetary motion. This book is divided into five chapters on first, regular polygons; second, the congruence of figures; third, the origin of harmonic proportions in music; fourth, harmonic configurations in astrology; and the last, the harmony of the motion of the planets including his “third law” of planetary motion.


Kenneth R. Meyer, Professor Emeritus of mathematics, University of Cincinnati, interested in qualitative theory.
of differential equations and dynamic systems, specifically Hamiltonian systems and celestial mechanics. This is a paper for geometers but it is possible to gain some small insights into Jacobi Elliptic functions for the non-mathematician.


Filippo Morabito’s paper is too technical for me, even in English, but from the abstract I can gather qualitatively what this is about as an exercise.


Heinz-Otto Peitgen et al presenting fractal geometry using only elementary mathematics. This was background reading for the second theme in Chapter 3 of the dissertation.


Tim Poston (1945-), English mathematician, best known for his work on catastrophe theory, has coauthored a list of books on that topic. Ian Stewart (1945-), a professor of mathematics at the University of Warwick, is widely known as an author of popular science and has received the Christopher Zeeman Medal for his work to promote mathematics. While these authors strongly argue against a purely qualitative application of catastrophe theory without understanding the mathematics behind it, nevertheless, instructions for constructing your own basic catastrophe mechanism from pins and string in the early chapters enfranchise the non-mathematician. “A proper understanding of catastrophe theory involves a feeling for the geometry of space of many dimensions, backed by suitable algebraic and analytic techniques.”


Vera Martha Winitzky de Spinadel (1929-), Argentinian mathematician, professor of mathematics at the University of Buenos Aires, director of the Research Center of Mathematics & Design. This book is a well illustrated but rigorous examination of relationships that span natural formations, maths and art and seem to appeal to an innate sense of beauty.

John Stillwell (1942– ), Australian mathematician on the faculties of the University of San Francisco and Monash Universities. This is a book about Numbers, Geometry, Arithmetic, Geometry, Coordinates, rational points, Trigonometry, Finite Arithmetic, Complex Numbers, Conic Sections and Elementary Functions.


This is an undergraduate mathematics textbook that includes mathematical exercises as learning aids in each chapter. It is divided into themes dealt with historically. They are too numerous to list exhaustively but examples are Pythagorus theorem, Greek geometry, Greek number theory, Number theory in Asia, Polynomial Equations, Analytic geometry, Projective Geometry, Number theory revival (Fermat), Elliptic functions, Mechanics, Complex numbers and Curves, Complex numbers and Algebra, Group theory, Hypercomplex numbers, Topology, Set theory. It was valuable to read this in parallel to Morris Kline’s History of Mathematics.


Jeffrey Weeks, American mathematician, MacArthur scholarship recipient, his Princeton PhD was supervised by William Thurston. Weeks brings the question of the shape of space to readers from all disciplinary backgrounds. Is the flatland analogy of the universe a plane, curved back on itself, a sphere, a donut? This book offers a very comprehensible way to visualise the hypersphere. While the maths is elementary – it includes exercises at first year university level, it also holds some potential surprises for the more mathematically literate.


Now out-of-print but available in libraries.

**Mathematics and Space**


Edwin Abbott Abbott (1838 – 1926), schoolmaster and theologian. A satirical and allegorical novelette featuring the first literary mention of the four-dimensional cube or hypercube – a drama played out on a flat land with an infinitesimal third dimension experienced as brightness, but a flat land that is anecdotally more like a closed surface, such as a sphere. Abbott introduces his reader to a social hierarchy of shape and the heretical nature of a third or fourth dimension in a two-dimensional world.

David Joseph Bohm (1917-1992), British-born quantum physicist who completed his PhD at Berkeley in 1943 and immediately had his particle scattering calculations classified, denied access to his own work.


A book that proposes a non-atomised understanding of energy and matter in the universe.


Professor John L. Casti is a mathematics PhD, Professor of Operations Research and Systems Theory who has written both technical publications on mathematical modeling as well as popular science books such as this one. I found it enormously helpful in providing a non-technical view of the mathematical ideas underlying, for instance, Morse’s theorem or Brouwer’s fixed point theorem and their metaphorical and applied relationships to the physical world.


This book revisits, ten years on, the arguments for and against six key convictions about key questions – the origins of life, the genetic contribution to behaviour, acquisition of language, machine intelligence, existence of extra-terrestrial life and quantum reality, that were first addressed in Casti’s book Paradigms Lost. It is interesting to chart the changes over that period.


This book extends the scope of Five Golden Rules, exploring further mathematical theories.


Manuel Delanda (1952-), Mexican, New York based writer, artist and philosopher. In his own words this is a book ‘to present the work of Philosopher Gilles Deleuze to an audience of analytical philosophers of science, and scientists interested in philosophical questions.’ I focused on the chapter The Mathematics of the Virtual: Manifolds, Vector Fields and Transformation Groups. In exploring Deleuze’s world rather than his words, Delanda departs into some very eloquent summaries of mathematical concepts like symmetry.

Jacques Harthong, École nationale Superieure de Physique, and Georges Reeb (1920-1993), French mathematician, differential topologist etc. This paper is written in French and the name makes reference to Abraham Robinson’s Formalism ’64. Far from reigniting the antagonism between formalism and intuitionism, the ism that came into existence as Brouwer’s critique of formalism in mathematics, this paper, by clarifying the terms of the debate highlights the points in which the approaches are very close.


Benoit Mandelbrot (1924-2010), French American mathematician. Although he worked on a wide variety of mathematical problems including mathematical physics, he is best known as the father of fractal geometry. He gave a new collective name to geometrical phenomena that arose in mathematics in the period
1875 – 1922 from the work of such figures as Weierstrasse, Cantor, Peano, Lesbegue and Hausdorff (from whom comes the ‘Hausdorff dimension’ of fractals). Other fractals were earlier identified by Koch, Sierpinski and Besiociovitch. The word that Mandelbrot created for this purpose ‘fractal’ is from the Latin adjective fractus meaning ‘irregular or fragmented’. Mandelbrot made the case that fractal geometry was better for describing many of the irregular and fragmented patterns in nature than Euclidean geometry. The book is pitched at a broad audience, with passages for general readership and passages for a technical, mathematical or scientific readership. It is beautifully written. This is a revised and enlarged version of:


Translated from the original in French, dated 1975.


Roger Penrose (1931–), English mathematician who gave his name to a number of aperiodic tilings with very small numbers of tiles. This, as the name suggests, is a very fat book. It progresses from a popular approach to a more mathematically challenging one in its aim to be both comprehensive and inclusive.


René Thom (1923 – 2002), French mathematician, initially topologist and subsequently for his development of singularity theory to publish, what he is best known for catastrophe theory. This last had a poor reception in the popular science press where the mathematical foundations were poorly interpreted or understood. Erik Christopher Zeeman further developed the mathematical theory. Through its qualitative emphasis in bringing Catastrophe theory to the attention of the world, many sections of this book (translated from French) are accessible to the non-technical reader. Significantly, it links the mathematics to phenomena observed in natural systems such as embryology.

**Philosophy**


Alice Ambrose (1906-2001) American philosopher studied at Cambridge University with Wittgenstein for her second PhD, awarded in 1938; Ludwig Joseph Johann Wittgenstein (1989-1951), Viennese-born philosopher and professor of philosophy at Cambridge University. The final chapter or lecture: *Philosophy for mathematicians* sheds some clarity on the relationship of philosophy to mathematics and whether logic is the
foundation of mathematics or whether, as Wittgenstein argues, logic is simply part of mathematics.


Gaston Bachelard (1884-1962), French philosopher who contributed to poetics and the philosophy of science. Bachelard developed a theory of the influence of psychology in the history of science including the concept of an ‘epistemological rupture’ which influenced Thomas Kuhn’s development of the idea of a paradigm shift. This book, however, is a highly poetic study of the home and the psychology of space of domestic environment that informs all other aspects of life. It was first published in English in 1964. In this dissertation it is used it as an example of spatial representation and thought very free from geometry, metrics and empirical science.


Paul Benacerraf, American philosopher and professor of philosophy at Princeton University and Hilary Whitehall Putnam (1926-), American philosopher and mathematician. This book is an edited series of essays in philosophy of mathematics divided into four sections: the foundations of mathematics; the existence of mathematical objects; mathematical truth; and the concept of set. Notably, David Hilbert’s essay *On the Infinite* is published in this collection. This is referenced in Chapter 5 of the dissertation.


Jonathan F. Bennett (1930-), New Zealand born British philosopher who lectured at University of Oxford, Simon Fraser University, University of British Columbia, Syracuse. This text was recommended on Kant’s Analytic.

*Brouwer, L.D.J., ‘On the Significance of the principle of excluded middle in mathematics, especially in function theory’, in *Annual convention of the Deutsche Mathematiker-Vereinigung* (Marburg and der Lahn: 1923(b)).

Luitzen Egbertus Jan Brouwer (1881-1966), Dutch mathematician and philosopher who worked in topology, set theory, and complex analysis. He is best known for his writing on the significance of the excluded middle in mathematics and for Fixed Point theorem in topology. This is idea that for any continuous function \( f \) there is a point \( x_0 \) such that \( f(x_0) = x_0 \) (a point that remains fixed). This theorem is used across many fields of mathematics. The significance of the excluded middle in classical logic is that, in mathematics, there could be counter examples where ‘not not true’ is not the same as ‘true’. Brouwer pointed out that many assertions in formal mathematics were deemed proven while untestable empirically.

J Alberto Coffa explains the finer points of Russell’s refutation of Kant’s a priori intuition applied to not only the apprehension of geometry but to formally and logically constructed proofs in mathematics.

*Comte, A. The Positive Philosophy of Auguste Comte.*
Translated by Martineau, H. New York: Calvin Blanchard, 1855.

Auguste Comte (1798-1857), French philosopher. The Positive Philosophy aimed to define the laws of social evolution and is a founding text for social science. In relation to mathematics, it provides a social historical interpretation.


Gilles Deleuze (1925-1995), French philosopher whose writing is full of non-philosophical references, and neologisms. He conceived of philosophy as the production (rather than construction) of concepts. To the unschooled Deleuze's writing seems very like a performance or production built on references to a multitude of diverse sources. In this case interpretation of concepts from Leibniz provides the narrative. The book is full of resonance and although it makes frequent reference to geometry seems to construct, in itself, a space that is neither geometrical nor phenomenal.


Dr Simon Duffy, Postdoctoral Fellow, Department of Philosophy, University of Sydney, interested in modern and contemporary European philosophy. This is collection of essays examining the work of Gilles Deleuze exposing many connections between mathematics and philosophy. Duffy argues in his introductory essay that Abraham Robinson's axioms in the 1960s allow the pre-foundational proofs of the calculus to be verified, allowing the reintroduction of relations between mathematical and metaphysical developments of the calculus that had been marginalized in its rigorous algebraic foundation. Deleuze described himself as a pure metaphysician.

Paul Karl Feyerabend (1924-1994), Austrian born philosopher of science, professor of philosophy at the University of California, Berkeley. This is a collection of essays on the importance of relativism and its application in the interpretation of science no less than in other spheres.


A detailed and rigorous argument that science as practiced cannot be strictly constructed or described according to any clear methodology.


Edmund Husserl (1859-1938), philosopher and mathematician, considered father of phenomenology. For this thesis, I have studied and quoted from Appendix VI *The Origins of Geometry*. This focuses on the ontic (real) nature of geometry and the question of its pre-Euclidean origins rather than any relationship to philological, symbolic or logical derivations.


Nicholas Jolley, Professor of Philosophy, School of Humanities, University of California, Irvine. A book that tackles the difficulty of understanding Leibniz’s work from his scattered and unpublished writing.

*Kant, I. Critique of Pure Reason*. Translated by Kemp Smith, N. London and Basingstoke: Macmillan and co Ltd, 1970 First edition 1929 First Published (German) 1781.

Immanuel Kant (1724-1804), German philosopher, geographer and anthropologist. This was the first of three volumes, the others being the Critique of Practical Reason and the Critique of Judgement. The dissertation makes extensive reference to *The Aesthetic* and Kant’s concept of *Pure Intuition* in relation to Time, Space and Geometry in Chapter 5.


I have made reference to both translations of *The Aesthetic* where there are differences that seem to offer clarity to interpretation.


The book is divided into the Critique of Aesthetic Judgement and the Critique of Teleological Judgement and also offers an overview of the whole Critical project which highlights the opposition between the empirical scientific position of *causal determinism* and the *spontaneous causality* understood in relation to moral behaviour.


A cultural history, recording the changes in the people’s perception of past, present, future, speed, form, distance,
direction in response to technological change and the relationship to the Great War that followed.


Philip Kitcher (1947-), British philosophy professor specializing in the philosophy of science. Kitcher divides Kant’s views on the nature of mathematics into two sub theses: a metaphysical sub thesis and an epistemological sub thesis. Metaphysically, the truths of pure mathematics are necessary although they do not owe their truth to the nature of our concepts. Epistemologically, the truths of pure mathematics can be known independently of particular bits of experience, although one cannot come to know them through conceptual analysis alone. The arguments are philosophically technical but the paper is readable outside philosophy. Good to read in combination with Coffa’s Russell and Kant.


David Rapport Lachterman, writer in philosophy. Not a simple nor potentially critical read for a non-philosopher but a very rewarding task in trying to gain from Lachterman’s detailed analysis the nuanced distinctions between Classical Greek geometrical construction and Modernist geometrical construction, personified by René Descartes. This work is referenced extensively in Chapter 5 of the dissertation.


Sanford Kwinter, Canadian, New York-based writer and architectural theorist. To be read in combination with the subsequent writing Soft Systems which deals with the shift towards a biological model and more systematic thinking. This book focuses on the interest of time to architecture and design. Kwinter writes that this is the discipline’s greatest hope for systematic renewal in “a field, however, that is not always as attentive to the nuances of historical understanding as it is enthusiastic about what it charmingly imagines to be its own discoveries.” Kwinter argues that it is time that makes the virtual real and coexistent with that which has already emerged.


Imre Lakatos (1922-1974), Hungarian philosopher of mathematics and science. This book is based on the first three chapters of his Cambridge doctoral thesis Essays in the Logic of Mathematical Discovery. It is largely a fictional dialogue within a mathematics class. The students are attempting to prove the Euler characteristic in algebraic topology, the theorem about polyhedra $V-E+F=2$. The class dialogue represents the actual historical series of proofs offered for the conjecture by mathematicians, only to be repeatedly refuted by counterexamples. This is a revolutionary text in highlighting the same essential value of negation in mathematics as in the empirical sciences and thus undermining the positivist formalist dogma.

Gottfried Wilhelm Leibniz (1646-1716), German philosopher and mathematician. These are letters, some in response to other thinkers such as Bayle. They deal with the ideality of extension, motion, which in this sense are just like space and time, even as dealt with in mathematics.


Henri Lefebvre (1901-1991), French sociologist and philosopher. In this book he takes an eclectic look at space, its postmodern theorization, how we perceive, construct and reproduce space actually and mentally. He gives emphasis to what he calls "social space" and is in places dismissive of mathematically or scientifically constructed space.


Ernst Mach (1838-1916), Austrian physicist and philosopher. He was a professor of experimental physics for most of his career, contributing to knowledge of spark and ballistics shock waves and supersonic effects. He was also interested in psycho-physics and sensory perception. This book explores the difference between the sensory space of our perception and geometrical space.


Lorenzo Magnani (1952-) Italian professor of philosophy at the University of Pavia, primary interests: philosophy of science, logic and artificial intelligence. This is a study
of geometry and its roles in thought based on a particular interpretation of Kant's Transcendental philosophy (implicit and somatic) and linking the history (and pre-history) of geometry to Pierce's theory of abduction (guessing, or more technically, argument based on certain predicate and uncertain subject) and recent developments in cognitive science. Mary Domski's book review suggests that the book title could be misleading in its generality and that the treatment of Kant is implicit like the interpretation itself. It is a very interesting book, usefully framed by these cautions.


Jeff Malpas, New Zealand born professor of philosophy at University of Tasmania. This is a very interesting book for a general, and certainly a design and spatial science readership as well a philosophical readership. It provides a useful overview of the cultural and historic shaping of understanding of space and progressive exile of an understanding or acknowledgement of the importance of place. Malpas is a scholar of Heidegger to whom “place is integral to very structure and possibility of experience.”


This book is introduced by Malpas as the second in a trilogy. It is far more technical and philosophically specific than Place and Experience and less accessible to a general design readership except where already familiar with Heidegger’s work.


That oblivion of being that is central in modernism is accurately diagnosed by the poet Friedrich Hölderlin. He also proposes a way of thinking in the face of it. Thinking as an essential form of topology, in which way it is both poetic and able to recognise and understand the nihilistic character of modernity, depends on thinking in its proper place and our orientation within it. This paper deals with many of the same terms that are central in Lachterman, but in a very different way: ‘thesis’, ‘posit’, ‘gestell’, also ‘kehre’ or turning back – thinking is always a turning back to presence, to being, to questionability.


This is a paper about how space is implicated in world in Heidegger, which is not a matter of spatial containment but of active involvement. “Heidegger’s attempts to clarify the concept of world also lead inevitably to an increasingly spatialized analysis.”(p9) But he also works at severing the connection between the idea of spatiality and the particular Cartesian understanding of space as homogeneous extension. Projection is described as ‘a throw’. “In the end, space and
spatiality belong, as does the concept of world also, not so much to the domain of strict physical science nor even of a certain ‘empirist metaphysics', as to the domain of properly philosophical, and perhaps also poetic, critique and reflection.” (Malpas p29).


This paper is an introduction to the other Kant, the Kant who lectured for 40 years from 1756 to 1796 in physical geography in the summer term and anthropology, or human pre-history in the winter. “Empirical cognition of nature as the comprehensive object of outer sense (geography) has its counterpart in the empirical cognition of the human being, or rather its “soul” (Seele), as the comprehensive object of the inner sense (anthropology).” Kant drew on the most advanced biological theories of his time – ontogenic generation serves as the model for phyogenic generation. He presents predispositions and germs that contain the genetic enabling grounds and limiting conditions of the individual’s formation. Drawing on Leibniz’s infinitesimals and monads he maintains the unity of the human species and its natural historical origin in one biological entity “phylus”.


Mason is editor and translator of the letters between Leibniz and Arnauld into English. Antoine Arnauld (1612-1694), French Roman Catholic theologian, philosopher and mathematician, penetrating thinker, author of L’art de penser or Port-Royal Logic and his 32 volume Complete Works. Leibniz corresponded with him via the Count Ernst von Hessen-Rheinfels in 1686 (Arnauld was by this time in exile from France for his theological differences with the Jesuits) on the subject of his (Leibniz’s) Discourse on Metaphysics.


Leonard Nelson (1882-1927), German mathematician and philosopher at Göttingen, friend of David Hilbert. This was a recommended authoritative text for interpretation of Kant’s Transcendental Aesthetic. It is also a source for some clear definitions with respect to the nature and scope of philosophy. The dissertation makes multiple references to this book in Chapter 5.


Henri Poincaré (1882-1912), French mathematician, theoretical physicist, engineer and a philosopher of science. He has been variously described as the father of topology, the last polymath, having laid the foundations for chaos theory with his three-body problem. He gave the Poincaré conjecture: that every simply connected, closed 3-manifold is homeomorphic to the 3-sphere. (The 3-sphere is the three dimensional sphere in 4 dimensional space.) Science and Hypothesis highlights the difference
between verification and proof – verification leads only to the premises translated to another language. While experimentation plays a considerable role in the genesis of geometry this does not lead to the conclusion that geometry is an experimental science – it is too approximate and provisory for this. The object of geometry is the study of a particular group, which Poincaré infers we have a predisposition to understand (similar to Chomski’s linguistic theory of the predisposition for certain forms in language). In Chapters 5 and 6 of the dissertation, I have made reference to Poincaré’s overview of non-Euclidean geometry, his case for the convention of Euclidean geometry in the world of our experience, and his analysis of the relationship of geometry and space.


This book written later that Science and Hypothesis concludes firmly that logic is barren unless fertilized by intuition. As an example Cantorian logic is useful when applied to a real problem but overlooks the reality that there is no actual infinity – this forgetting leads to contradiction. Acknowledging the originality, profundity and truth of the work of Russell and Hilbert he states that they have nevertheless not destroyed the mathematical theory of Kant or solved the conflict between Kant and Leibniz. Certain truths are irreducible to logic in both the old and new sense of logic. In mathematics ‘exists’ means ‘exemption from contradiction’. Certain indemonstrable axioms of mathematics are nothing but disguised definitions and in defining an object we assert that the definition involves no contradiction. Mathematical discovery is the process in which the human mind borrows least from the external world, so studying the process of geometrical thought should lead to what is most essential in the human mind. His analysis of the relativity of space leads to somatic and deep evolved cognitive habits shaping the associations with perceived objects in ultimately amorphous space in which we have no direct intuition of magnitude. In this way he demolishes absolute space. Written for access of a general readership.


Graham Priest (1948-) is Boyce Gibson Professor of Philosophy at the University of Melbourne. The logic of Russell and Frege has mysteriously become known as ‘classical logic’ (although it was actually revolutionary with respect to the logic of Ancient Greece and the Roman Empire). This has been polished up by generations of logicians since. Many of the most interesting developments in logic in the last forty years have occurred in quite different areas: intuitionism, conditional logics, relevant logics, paraconsistent logics, free logics, fuzzy logic. These are designed to supplement or replace classical logic where it transgresses. They are known collectively as non-classical logic. Each logic, starting with classical logic itself, is introduced in the book in a chapter with a descriptive and historical introduction before moving to the formal constructions of that logic.

William van Orman Quine (1908-2000), American philosopher and logician in the analytic tradition and Joseph Ullian (1930-), professor of philosophy at Washington University in St Louis, Missouri. Written as a compact introduction to the study of rational belief and as an undergraduate textbook, it is only 145 pages long, divided into 10 chapters but nevertheless dense reading. It has been seen as an attack on positivism and a distinct view of observation language, free from language theory. It deals with the impossibility of proving every reasonable belief, the place of the hypothesis and of confirmation and refutation in giving value to hypotheses.


Peter Remnant and Jonathan Bennett are the editors and commentators on the translation of Leibniz’s rather one-sided enactment of a conversation with Locke in answer to Locke’s *An Essay Concerning Human Understanding* written just before his death in 1704 and not published until 1763. Philalethus as fictional spokesperson for Locke deals up arguments from Locke’s Essay in a fairly wooden way while Theophilus, fictional spokesperson for Leibniz has more license as the only real time participant in the conversation. This essay, too, deals with continuous quantity as the common logical genus of time and space, two very heterogeneous things.


The two ‘mistakes’ that Ross identifies in the work of Jules Henri Poincaré (1854-1912) are, first his assertion (in Science and Hypothesis) that if Kant were correct that geometrical axioms were synthetic a priori intuition then they would impose themselves with such force upon us that we could not imagine the contrary and consequently there would be no non-Euclidean geometry. Ross contests this interpretation of synthetic matter, or matters of fact, in which the predicate is not implied in the subject. Here the contrary matter is possible in logic without contradiction. Incidentally, Poincaré’s second mistake according to Ross is his understanding that there can be no scientific validation of ethics (loosely based on the argument that scientific propositions are in the indicative (what is) and ethics in the imperative (what ought to be).)


Gilbert Ryle, (1900-1976), English philosopher, was the Waynflete Professor of Metaphysical Philosophy and a Fellow of Magdalen College, Oxford. In *The Concept of Mind* (1949) he attacked the mind-body dualism of Descartes, which he referred to as the ghost in the machine. The corollary of this critique was the development of a deeply embodied theory of thinking. In this paper he also develops the idea of the thick description – an approach to observation that includes multiple levels of inference in any
human act or utterance. This was very influential on the subsequent work of anthropologists such as Clifford Geertz.


This paper also deals with the subject of embodied thought – the way a golfer perfects their stroke through endless repetition of the act rather than through analytical thought about the act and thick description: the difference between a twitch and a wink or a wink and a rehearsed burlesque, imitative, parody of a wink. This is relevant to consideration of embodied thought and levels of inference for the design modeler working within a virtual computational geometrical model space.


General reference.


This is a very difficult book to read from outside the domain but it was instructive to read the definitions and the types of definition that are relevant to “making a distinction between semantic and deductive logic” and why these distinctions collapse in relation to first order logic but may be relevant to second order logic. First order logic is a formal logical system used in mathematics, philosophy, linguistics, and computer science. It includes quantifiers and a domain of discourse over which the quantifiers range. Second (and higher) order logic is an extension of first order logic. As well as variables that range over individual elements in the domain of discourse, second order logic has variables that range over sets of individuals. This increases the expressive power of quite short expressions, for instance, in programming.


Sir Peter Frederick Strawson (1919-2006), English philosopher and Waynflete Professor of Metaphysical Philosophy at the University of Oxford and a fellow of Magdalen College from 1968-1987. Individuals is described at a metaphysical project as it tries to expose the place of basic particulars in our spatiotemporal conceptual framework by defining in a detailed way what humans think about reality. I have made reference to this book in Chapter 5 as the earliest philosophical text that I could find that makes claims about the nature of auditory space.


W.H. Walsh was a prolific publisher of philosophical texts from the late 1930s to the 1980s. This book was a recommended text for further insights into Kant’s Pure Intuition in the Transcendental Aesthetic in his Critique of Pure Reason. I read this whole book closely and it did
offer some alternative angles to Nelson, for instance the emphasis on Kant's division of thought into sensibility and understanding and the contrast with Leibniz's position that sensing is a confused form of thinking.


Alfred North Whitehead (1961-1947), English mathematician who became a philosopher. He was Bertrand Russell's PhD supervisor. The central idea in these lectures is that of event-based relativity of space and time rather than matter-base relativity of space and time. He advocates that the modern account of nature should be an account of what the mind knows of what nature does to the mind. He points out the inconsistencies in a matter-based account in which many of the constituents are not really there – for instance 'points' and Locke's 'secondary qualities', such as colour.


Whitehead is first a mathematician. He published The Axioms of Projective Geometry (1906), The Axioms of Descriptive Geometry (1907), and An Introduction to Mathematics (1911). In the years 1910, 1912, and 1913 respectively there appeared the three volumes of Principia Mathematica, of which Bertrand Russell was the co-author. This work is credited with demonstrating the usefulness of philosophy to the physical sciences. In this volume in the chapter On Material Concepts of the Material World he writes:

“The Material World is conceived as a set of relations and of entities which occur as forming the ‘fields’ of these relations. The Fundamental Relations of the material world are those relations in it, which are not defined in terms of other entities, but are merely particularized by hypotheses that they satisfy certain propositions.” Leibniz's theory of the relativity of space stands in opposition to the classical concept. Bertrand Russell who is the harshest critic of Leibniz's theory also went further than any of its supporters to give it mathematical precision.


Ludwig Wittgenstein (1889-1951), Austrian philosopher who held the professorship of philosophy at the University of Cambridge from 1939 to 1947. This is his early work written during his time as a soldier and prisoner of war in the First World War. It is written in declarative style, groups of supposedly self-evident statements and sub statements, without argument, to show the application of modern logic via language to metaphysics. It developed out of notes and correspondence including with Bertrand
Russell and developed as both a continuation of and reaction to Russell and Frege’s conceptions of logic. It has been important, as has the much later *Philosophical Investigations*, in linguistics for relating language and reality. It also explores the limits of science.


This work published posthumously retracted many of the ideas in *Tractatus*. It rejects the traditional view of language as words standing for objects combined in sentences to make meaning, arguing instead for a much more complex and versatile model of language and its role.

### Also consulted


Euclid. *The thirteen books of Euclid’s Elements vol III.*


Sanford Kwinter: Canadian writer and architectural theorist based in New York. Kwinter’s afterword, in (Benjamin Aranda and Chris Lasch, *Pamphlet Architecture 27: Tooling* (New York, 2005), 92.) was a very apt reminder drawing on Alfred North Whitehead that number as a thing has not always been with us and its immense power as a thing once it comes into existence. I have borrowed this book three times and had to return it before achieving a satisfactory reading.


APPENDIX II

RELATED PUBLICATIONS DURING THE DOCTORATE
Appendix II | Related publications during the doctorate

**Book**


**Book chapters**


**Refereed Journals**


**Refereed Conference papers**


Student Research Questionnaire

1. What discipline/subject area are you enrolled in?

2. What were your main reasons for selecting this subject?

3. Did you have any prior digital solid modelling experience?  
   (If so please detail software and modelling experience.)

4. Did you have any prior experience of Flexible (system) modelling?  
   (If so, please detail whether this was using proprietary software, or scripting with explicit modelling programs, programming or physical analogue modelling.)

5. Please try and describe how you found it different working in a flexible modelling environment from creating digital solid or surface models using explicit geometry.

6. a) How did your modelling experience in this class meet your expectations?  
   b) Did you find it easy? What were the positive aspects, in particular surprises  
   c) Did you find it hard? What were the principal sources of frustration?

7. Do you have any comments on the types of representation of the model that you could see using the software? (e.g. the tree, the graph of parents and children, the geometry window showing you the current instance of the model geometry, parameters and relations in an excel spreadsheet) and those you created (e.g. sketches, conceptual models)

8. Which aspects of the modelling software did you use and find useful? (e.g. advanced replication, controlling parameters and relationships within spreadsheets; scripting; being able to edit by interacting with either the 3D model or the tree; other things…).

General Comments

(These might address the question of modelling systems rather than objects; experiences of different software and scripting interfaces; challenges of conceptualizing the schema in order to facilitate the types of flexibility and variability sought in the model or any other comments.)