Numerical and experimental studies of influence of trees on ground movements in expansive soils

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Doctor of Philosophy

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**Declaration**

This is to certify that except where due acknowledgement has been made, the work is that of the candidate alone, that the work has not been submitted previously, in whole or in part, to qualify for any other academic award, that the content of the thesis is the result of the work which has been carried out since the official commencement date of the approved research program, and that any editorial work, paid or unpaid, carried out by a third party is acknowledged.

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青玉案·佳期

八千里外观风月，世人言，南洋好，纵酒登高望辽阔。锦绣河山，却非故国，一曲东风破。
三十年前知命数，要提金榜沐兰泽，当游四海扬索波。今日佳期，已为人父，当于何人说。
Abstract

In Australia, distortions of residential buildings caused by tree roots are widely reported, particularly in areas of expansive soil. Often such damages are attributed to tree root absorption which changes the soil water content and causes shrinkage of soils. Australia has large areas of semi-arid climates, where the upper soil layers are naturally expansive. Expansive soil is predominantly clay soil which undergoes appreciable volume change following change in moisture content. This volume change occurs as shrinkage upon drying, and swelling upon wetting. Root water absorption in expansive soil is a very practical hydro-mechanical problem. Although the Australian Standard for Residential Slabs and Footings (AS2870, 2011) provides a guideline to estimate soil movement caused by tree root absorption, the standard method is mainly based on empirical estimation. The mechanism of the interaction between root water absorption and expansive soil is still not clear. The development of a root absorption model for predicting the effect of tree root on the footing and foundation design has become a crucial issue in recent years.

In this research, a numerical model has been developed for analysing root-water-soil interaction. The model is incorporated in an ABAQUS pore fluid diffusion and stress analysis. In this model, the factors influencing soil moisture content and movement including root absorption, rainfall and evaporation are integrated in flow equations. Root density and root distribution are analysed. Soil-water interaction analysis is based on a soil water characteristic curve. Stress-strain relationship in unsaturated soils is defined considering both mechanical and hydraulic behaviour.
As well as numerical modelling work, this research highlights the importance of experiments. The research encompasses field investigation and laboratory testing. A full-scale test site was established in an urban environment, in Glenroy Victoria and has been monitored since May 2011 on a monthly basis. In order to define the magnitude of tree root water uptake, the sap flow in tree trunk was measured every 30 minutes. To determine the influence of climate change, a Decagon weather station was installed on the research site to measure: (a) solar radiation, (b) rainfall, (c) temperature, (d) relative humidity and (e) wind speed and direction. A neutron moisture probe was used to measure the soil water content at the field site. Ground movements at a variety of locations depths were measured using a high precision laser level. A series of laboratory test were also performed on soil samples collected from the test site, which include soil suction measurement, soil consolidation tests, soil water characteristic curve tests and soil swelling and shrinkage tests.

Two case studies were conducted in this research. The first case study concerned a street tree, Golden raintree (koelreuteria paniculata) in Walkley Heights subdivision, South Australia. The second case study is related to the results of the field monitoring of a single, 2.5 m high eucalyptus ficifolia at the test site in Glenroy, Victoria. The soil suction and ground movement predicted by the numerical model are found to agree favourable with the measured data at the field sites. The parametric studies of the effect of climate, transpiration rate and elapsed time on tree root absorption and soil shrinkage settlement are also conducted.

A significant feature of this research is its inclusion and integration of a range of cross-disciplinary research areas, which include plant science, tree physiology, hydrology and geotechnics. The research has shown that the proposed numerical model is capable of modelling the climate-tree root-water-expansive soil interaction to a degree of sophistication that has not previously been so readily achievable. It is believed that the utilisation of such a model can lead to a higher level of understanding of the influence of trees on building footings over a range of climates and tree species and subsequently less risk in routine footing design.
Notation

$A_p$  tree root absorption rate
$T_p$  tree transpiration rate
$\sigma'$  effective stress
$\mu_a$  pore air pressure
$\mu_w$  pore water pressure
$\chi$  parameter related to the degree of saturation in effective stress equation
$\bar{\sigma}$  net stress
$s$  matric suction
$\delta_{ij}$  Kronecker’s delta
$\delta_{ij}^{\text{ave}}$  average stress
$S_{\text{ve}}^{\text{ef}}$  the effective degree of saturation
$S_{r}^{\text{ef}}$  residual degree of saturation
$\theta_r$  residual water content
$\theta_s$  saturated water content
$S_{ae}$  suction in air entry value
$S_{re}$  the residual suction
$\mu_d$  fitting parameter in Brooks and Corey SWCC equation
$\alpha_d$  fitting parameters in Fredlund and Xing SWCC equation
$\beta_d$  fitting parameters in Fredlund and Xing SWCC equation
$\gamma_d$  fitting parameters in Fredlund and Xing SWCC equation
$a_{dfw}$  fitting parameters in Van Genuchten equation
$m_{dfw}$  fitting parameters in Van Genuchten equation
$n_{dfw}$  fitting parameters in Van Genuchten equation
$D$  minimum horizontal distance
$H$  height of the tree
$I_{pt}$  instability index
$I_{ps}$ shrinkage index

$y_s$ characteristic surface movement

$\Delta u$ soil suction change averaged over the thickness of the soil layer

$H_s$ the value of depth of design suction change

$u_{eq}$ the equilibrium suction value

$u_{wp}$ wilting point suction

$H_i$ the influence depth of tree

$\theta$ volumetric water content

$K(\theta)$ hydraulic conductivity

$z$ gravity head

$\phi$ total head

$S(\theta)$ sink term

$z_m$ maximum depth of root zone

$r_m$ maximum root zone width

$\beta(r,z)$ root density

$G(\beta)$ root distribution parameter in Wollongong model

$F(T_p)$ transpiration parameter in Wollongong model

$E_i$ leaf evaporation rate

$LAI$ leaf area index

$q_i$ rainfall rate

$q_e$ evaporation rate

$LWP$ leaf water potential

$\rho_w$ water density

$R_i$ the net radiation flux to the area

$L_i$ the latent heat of evaporation

$L_iE_i$ the latent heat flux

$H_i$ the sensible-heat flux from the area

$T_s$ the temperature of the leaf surface
\( T_a \) the ambient air temperature
\( r_x \) the external heat-diffusion resistance
\( e_x \) the vapour pressure of the liquid surface within the leaf
\( e_a \) the ambient vapour pressure
\( r_e \) the external vapour-diffusion resistance
\( r_i \) the internal vapour-diffusion resistance, between the liquid and the leaf surface
\( Q_b \) blow canopy light
\( Q_a \) the above canopy light
\( V \) the velocity of sap flow
\( k \) the thermal diffusivity of wet wood
\( x \) the distance between the heat source (heater) and temperature sensors
\( v_1 \) the increases in temperature (from ambient) downstream
\( v_2 \) the increases in temperature (from ambient) upstream
\( \alpha(i) \) canopy reduce factor between 0 and 1
\( P_z \) empirical parameters in Vrugt root density equation
\( P_r \) empirical parameters in Vrugt root density equation
\( z^* \) empirical parameters in Vrugt root density equation
\( r^* \) empirical parameters in Vrugt root density equation
\( K_1 \) empirical parameter in root shape equation
\( K_2 \) empirical parameter in root shape equation
\( \psi \) total suction
\( \psi_o \) osmotic suction
\( S \) degree of situation
\( \alpha(s) \) reduction factor in soil aeration
\( k_s(e) \) the saturated coefficient of hydraulic conductivity
\( S_e \) effective degree of saturation
\( \varepsilon_v \) volumetric strain
\( \varepsilon_v^e \) volumetric strain in elastic region
\( \varepsilon_v^p \) volumetric strain in plastic region
\( \kappa \) swelling index
\( \lambda \) compression index
\( e \) void ratio
\( e_0 \) initial void ratio
\( m_0 \) initial mass of soil sample in SWCC test
\( V_0 \) initial volume of soil sample in SWCC test
\( w_i \) initial water content in SWCC test
\( m_w^0 \) initial water mass in SWCC test
\( V_w^0 \) initial water volume in SWCC test
\( m_s \) the mass of soil particles in SWCC test
\( V_s \) the volume of soil particles in SWCC test
\( V_w^1 \) new water volume in SWCC test
\( h \) soil sample ring height
\( D_{ring} \) soil sample ring diameter
\( J^{el} \) the elastic part of the volume ratio between the current and reference configurations
\( p_t^{el} \) the elastic tensile strength of the material
\( e^{el} \) the deviatoric part of the total elastic strain
\( e_{vol}^{el} \) logarithmic measure of the elastic volume change
\( p \) the equivalent pressure stress
\( q \) the Mises equivalent stress
\( r \) the third stress invariant
\( t \) the deviatoric stress measure
\( M \) a constant that defines the slope of the critical state line
\( \beta \) wet yield surface size
\( a_0 \) initial yield surface size
\( K \) flow stress ratio
\( \lambda_{vp} \) the slope of normal compression line for net stress change in consolidated soil
\( \lambda_{vs} \) the slope of normal compression line for suction change in consolidated soil
\( \kappa_{vp} \) the slope of unloading reloading line for net stress change in over consolidated soil
\( \kappa_{vs} \) the slope of unloading reloading line for suction change in over consolidated soil
\( D^{ep} \) a \( 6 \times 6 \) stress-stain matrix in the SFG model
\( R \) a row vector of six elements in the SFG model
\( W^{ep} \) a column vector of six elements in the SFG model
\( G \) a scalar
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Chapter 1 Introduction

1.1 General

In Australia, distortions of residential buildings caused by tree roots are widely reported, particularly in areas of expansive soil. Australia has large areas of semi-arid climates, where the upper soil layers are naturally expansive. Expansive soil is any soil composed predominantly of clay, which undergoes appreciable volume change in response to changes in soil water content. This volume change occurs as swelling upon wetting, and shrinkage upon drying (Li and Cameron, 2002).

Lightly loaded structures founded on expansive soils are frequently subjected to severe settlement due to the tree root activity. The problems are particularly significant in Australia as approximately 20% of the total land area is covered by expansive soils. In fact, the expansive soil problems are present in most capital cities and regional centres of Australia. Holland (1982) inspected over 500 cases of foundation failures in clay soil area in Melbourne metropolitan region and found that approximately 30% of these failures were directly attributed to tree drying settlement. Cameron (2000, 2001, and 2002) monitored shrinkage settlement and soil suction changes with depth and distance away from trees in various environments and proposed a simple model to account for the drying settlement caused by trees.
Australian Standard for Residential Slabs and Footings (AS 2870, 2010) attempts to provide guidelines for residential footing design for the effects of trees based on very limited data from site investigations of damaged houses (e.g. Cameron, 2001). An understanding of the influence of trees on building footings is needed over a range of climates and tree species. With the development of computer technology, finite element analysis has been used in simulating many complicated models in geotechnical problem. Over the past three decades, considerable research has been devoted to developing tree root water uptake models, and a number of approaches are now available (Feddes, 1978; Prasad, 1988; Li et al., 1999; Alonso and Lloret, 1995; Ojha and Rai, 1996, Fredlund and Hung, 2001, Kang et al., 2001; Blight, 2005; Fatahi et al., 2006; Indraratna, 2006).

The numerical modelling of root water uptake can generally be classified into two approaches (Hemmati and Gatmiri, 2008). The first category deals with water flow to a single root (radial flow) and is often referred to as a microscopic approach. The disadvantage of microscopic approach is that it requires detailed information of the geometry of root system, which is practically impossible to measure (Wu et al., 1999). The second category is a macroscopic approach, in which water extraction by roots is treated as a sink term distributed in the root zone. Most root water uptake models use the macroscopic approach. Some researchers classified the water uptake models into a third category or a hybrid approach which is similar to the second one, but also takes into account root density, root water potential and soil suction in the sink term.

Nevertheless, the current numerical models have some shortcomings. Firstly, the numerical models in the early stage are generally one dimensional, which is not applicable to huge trees with a spatial root distribution. Some studies directly adopt one dimensional empirical equation in two dimensional modelling, but could not get reasonable results. Secondly, in some models hydraulic and mechanical behaviour of unsaturated soil is not clearly described. These models could not provide reasonable results that satisfy the expansive soil conditions.
Finally, the input parameters used in most existing models were estimated, rather than measured from field experiments and/or laboratory tests.

1.2 Research methodology and objectives

This research project is mainly concerned with the behaviour of tree root-water-expansive soils interactions and their influence on ground movement and the performance of residential footings as well. It consists of: (1) development of a two-dimensional root absorption model; (2) field and laboratory experiments which provide data of root absorption and unsaturated soil model; (3) numerical implementations in finite element analysis (ABAQUS coupled pore flow and stress analysis) and FORTRAN coding; (4) case studies.

The specific objectives of the research are:
- To describe tree absorption behaviour by tree leaf transpiration, tree trunk sap flow, tree root density and root distribution.
- To describe the climate influence on root absorption by rainfall and infiltration.
- To describe both the hydraulic and mechanical behaviour of soil-water interactions in expansive soil.
- To develop a comprehensive finite element numerical model for analysis of climate-tree root-water-expansive soil interaction.
- To improve the footing design by considering the effect of tree roots in expansive soils.

1.3 Thesis arrangement

This thesis is divided into seven chapters, with Chapter 1 being this introduction. Chapter 2 provides a literature review of expansive soil problems, Australian Standard for residential footing and foundation design, root absorption behaviour
and influencing factors, hydraulic and mechanical behaviour of expansive soil and the development of numerical modelling in root water absorption.

Chapter 3 develops a main concept of two dimensional root absorption model and the influences on expansive soil by soil-water interactions.

Chapter 4 describes the field and laboratory experiments which provide parameters for numerical modelling. The methodologies and results of the experiments are described and discussed.

Chapter 5 introduces the implementation of the absorption model in ABAQUS coupled pore flow and stress analysis. The application of boundary and initial conditions by FORTRAN subroutines are introduced, the implementations of unsaturated soil behaviour are explained.

Chapter 6 explains the application of the proposed numerical model to two case studies relating to field measurement of soil suction and ground movement near a single golden raintree in Adelaide, South Australia, and a single eucalyptus ficifolia in Melbourne, Victoria. The validity of the proposed numerical model has been examined by the comparing the numerical results with the measured data.

Finally conclusions and recommendations for future research are presented in Chapter 7.
Chapter 2 Background and Literature Review

2.1 General

This chapter presents a literature review of the research related to tree root-water-expansive soil interactions. Australia has large areas of expansive soil and distortions of structures caused by tree roots are widely reported. In expansive soil, the water depletion caused by roots may significantly change the soil hydraulic and mechanical behaviour. The tree related desiccation, distortion of footing and pavements has intensified as a result of recent prolonged drought and widespread urban water restrictions and is required to be solved urgently.

The study of tree root-water-expansive soil interaction requires the integration of a range of cross-disciplinary research areas, which include plant science, tree physiology, hydrology and geotechnics. This chapter introduces the background of the research, followed by the literature review of botany characters of tree root absorption, and mechanical and hydraulic behaviour of expansive soil. It also reviews the studies in past several decades which attempted to provide guideline to root-water-expansive-soil interactions. With the advent of fast computer, recently there has been an increase of interest in modelling the complex processes involved in tree root induced ground movement. The numerical models of root absorption proposed by other researchers are also reviewed in this chapter.
2.2 Background

Trees are the most common type of plant on Earth which plays an important role in the eco-chain (Osman, 1996). Forest and woodlands are much loved features in the landscape and can generate a wide range of economic, environmental and social benefits. As trees are widely planted, the considerations of the influences by tree roots in engineering area are then evaluated.

Generally speaking, the influences of trees root on environment contain three aspects (Boettcher and Kalisz, 1990, Indraratna, 2006). To begin with, tree root growth may mechanically expand the soil environment. Secondly tree root system may reinforce the soil by increasing the shear stress. Finally, most researchers recognised (Holland, 1981; Cameron, 2001; Fredlund and Huang, 2001; Indraratna, 2006; Sorochan, 1991) tree root water absorption may cause moisture depletion from the soil. This variation may cause soil shrinking and water movement which significantly change the soil environment, especially in expansive soil area.

The investigation of root water expansive soil interaction is particularly significant in Australia as approximately 20% of the total land area is covered by moderately to highly expansive soils (Richards, 1984), as shown in Fig. 2.1. Approximately half of the surface area in Victoria was covered by moderate to highly expansive soils (McAndrew 1965). Six out of eight of Australia’s largest cities were significantly affected by clay foundation soils (Fityus et al., 2004).

In the expansive soil environment, highly reactive soils experience extensive volume changes related to swelling and shrinkage. When water content is increased, the soil swells and when it is decreased, the soil shrinks (Ferreira et al. 1999). According to Rogers et al. (1993), expansive soils could exert uplift pressures of as much as 250 kPa, which could do considerable damage to lightly-loaded wood-frame structures.
Holland (1981) inspected over 500 cases of foundation failures in clay soil area in Melbourne metropolitan region and found that approximately 30% of these failures were directly attributed to tree drying settlement. Almost all the failures in the quaternary basaltic clays were caused by edge heave or under flooring drying settlement in the clay beneath the structures. Cameron and Earl (1982) also notes three quarters of damage to light structures on expansive soils in Victoria resulted from local drying settlement caused by trees and shrubs planted too close to the structures. As Osman (2006) noted, understanding how trees absorbed and removed water could help the engineers identify the cause of the structural failure of light structures without jumping to conclusion that it was always due to the presence of trees.
2.3 Tree root absorption characters

2.3.1 Osmotic and passive absorption

The continuous absorption of water is essential for the growth and survival of trees because in their circulatory system they lose large amount of water daily. Unless the water is replaced immediately, tree may die from dehydration (Kramer, 1983). Biddle (1998) assumed 99% of the replaced water is gain by tree root absorption. Leaves and stem absorption is ignored as it is much less than root absorption. Thus, the magnitude of root water absorption is equal to number of water evaporation of tree.

\[ A_p = T_p \]  \hspace{1cm} (2.1)

where \( A_p \) is tree root absorption rate; \( T_p \) is the total transpiration rate of tree.

Absorption of water occurs along gradients of deceasing potential from the substrate to the roots (Renner, 1912). However, the gradient is produced differently in slowly and rapidly transpiring trees, resulting in two absorption mechanisms (Kramer and Boyer, 1995). In a moist environment, at night or on cloudy days, water in the xylem is often under osmotic pressure. At this time absorption is largely by the osmotic mechanism occurs in slowly transpiring process (Steudle and Frensch, 1989). Osmotic absorption is response to osmotic suction in soil (Zhou et al., 2011). While in hot and sunny days, the evaporation of leaves increases the demand for water in leaves, absorption increasingly occurs by the passive mechanism and water is regarded as moving through the plant in a continuous cohesive column, pulled by the matric suction forces. From the time perspective, passive absorption often lags a few hours behind transpiration during the day, indicating the occurrence of resistances to water flow and water capacitance in the absorption system (Running, 1980).
Minshall (1964) and Rufelt (1956) claimed osmotic absorption is important. Brouwer (1965) claimed that osmotic absorption could be operated as part of passive absorption. In this research, it is considered osmotic and passive absorptions are response to matric and osmotic suction in soil respectively (Zhou et al., 2011). They are both important.

2.3.2 Efficiency of root absorption

The efficiency of root systems in absorption of water depends on their depth and spread, their density, often expressed as root length density in centimetres of roots per cubic centimetre of soil, and their permeability or hydraulic conductivity (Hurd, 1974). Landsberg and Fowkes (1978) published a mathematical analysis of the effect of root system geometry and water potential in relation to water absorption. Meyer and Ritchie (1980) reported that resistance per unit of root length decreases toward root tips in sorghum, compensating for increased distances to the shoot. Stone (1975) found a similar situation in roots of red pine.

2.3.3 Factors influencing root absorption

Root absorption is significantly influenced by environmental factors, such as water content, concentration and composition of soil solution, soil aeration and soil temperature.

2.3.3.1 Water content

In a soil water characteristic, the soil water potential decreases sharply as the water content decreases and the hydraulic conductivity also decreases as water drains out of larger pores. Denmead and Shaw (1962) pointed out the availability of soil water to plants relative to need also decreases with increasing rate of transpiration. Dunham and Nye (1973) demonstrated the situation by direct sampling of thin layers of soil in the vicinity of a root mat. Hasegawa (1986) used
a technique somewhat similar to that of Dunham and Nye (1973) with soybean roots. The change in soil water in this procedure was measured by gamma ray attenuation.

2.3.3.2 Soil aeration

Aeration of soil not only reduces root growth but also reduces the absorption of water and minerals. The decrease in water absorption is caused mainly by an increase in the resistance to radial movement into roots (Kramer and Boyer, 1999). It sometimes also decreases the osmotic driving force (Everard and Drew, 1989). The reason is concluded as a decreased uptake of salt in soil aeration. Soil aeration is considered as a significant part in root absorption model. Feddes (1974) adopted the principle in calculating the absorption rate in a crop field. Feddes defined a “wilting point” and an “anaerobiosis point” follows the principle of aeration. In his assumption, “wilting point” is the critical point represents plant wilt. “Anaerobiosis point” is the critical point represents plant is in anaerobiosis conditions. Mckeen (1992) reported that soil suction at wilting point ranged from 1.55 to 3.1 MPa.

2.3.3.3 Plant age

It is easy to understand the root absorption is related to the plant age. To begin with, the transpiration rate is related to the total leave surfaces (Biddle, 1998). The augment of tree leave surfaces is accompany with the tree growing process. Thus, total potential of transpiration is increased with tree age; total root absorption is increased as well. Fiscus (1977) indicated there were also wide variations in permeability and hydraulic conductivity in association with the plant age.

2.3.3.4 Temperature

It was earliest observed by Hales in the 18\textsuperscript{th} century that cold soil reduces water absorption (Kramer, 1988). In low temperature, the water stress caused by chilling
roots reduces stomatal conductance and may cause both stomatal and nonstomatal reduction in photosynthesis. Musser et al. (1983) concluded that for most plants, the stomatal conductance, CO₂ uptake, water potential and leaf enlargement may significantly reduce when the temperature is below 5 to 7 °C.

When the temperature is high, it may increase the water diffusion in leaves, as well as the absorption. Bassirirad (1991) found when the root temperature increased from 15 to 25 °C, exudation from barley root systems increased.

2.4 *Hydraulic and mechanical behaviour of expansive soil*

2.4.1 Effective stress

The mechanical behaviour of soil (i.e., the volume change and shear strength behaviour) can be described in terms of the state of stress in the soil (Fuedlund and Rahardjo, 1993). The number of stress state variables required for the soil description depends upon the number of phases involved. The effective stress (\(\sigma - u_w\)) in saturated soil has often been regarded as a physical law. The validity of the effective stress as a stress state variable for saturated soils has been well explained and accepted (Bishop and Eldin, 1950; Laughton, 1955; Skempton, 1961).

Expansive soil is a typical unsaturated soil which has commonly been viewed as a three-phase system (Lambe and Whitman, 1979). Biot (1941) proposed the first theory of consolidation with air bubbles. It recognized the need for separating the effects of total stress and pore water pressure. Croney et al. (1958), Aitchison (1961) and Jennings (1961) developed effective stress equations which generally following this principle.

Bishop (1959) suggested a widely used effective stress equation with single constitutive variables (SCV) as:
\[
\sigma' = (\sigma - u_a) + \chi(u_a - u_w)
\]  
(2.2)

where \(\mu_a - \mu_w\) is matric suction. \(\mu_a\) and \(\mu_w\) are pore air pressure and pore water pressure respectively. \(\chi\) is a parameter related to the degree of saturation of the soil which determined by experimental data. Donald (1961) and Blight (1961) provided experimental data on cohesionless silt and compacted soils. Khalili and Khabbaz (1998) attempted to use Brooks and Corey SWCC equation (1964) to define parameter \(\chi\).

Although considerable contributions have been provided in single-valued effective stress equation, many researchers (Jennings and Burland, 1962; Morgenstern, 1979; Houlsby, 1997; Sheng, 2008; Gens 2010) stated that the parameter \(\chi\) is not a unique solution in volume and shear stress change. Burland (1964) suggested the unsaturated soil use dual constitutive variables (DCV). Matyas and Radhakrishna (1968) suggested the dual constitutive variables as:

\[
\begin{cases}
\overline{\sigma} = \sigma - u_a \\
 s = u_a - u_w
\end{cases}
\]  
(2.3)

where: \(\overline{\sigma}\) is the net stress and \(s\) is matric suction. Fredlund et al. (1978) gave both theoretical and experimental supports for above dual constitutive variables (DCV). Houlsby (1997) improved Bishop effective stress equation using dual constitutive variables:

\[
\sigma'_{ij} = \overline{\sigma}_{ij} + S_r s \delta_{ij}
\]  
(2.4)

where \(s\) is matric suction, \(S_r\) is the degree of saturation, \(\delta_{ij}\) is Kronecker’s delta, \(\sigma'_{ij}\) is average stress.
2.4.2 Soil suction profile

All engineering structures on expansive soils are subjected to variations of suction at the soil surface (Lytton 1997). The soil suction theory was mainly developed to describe the relation to the soil-water-plant system. The importance of soil suction in explaining the mechanical behaviour of unsaturated soils relative to engineering problem was introduced at the Road Research Laboratory in England (Croney and Coleman, 1948). The generally accepted definition of soil suction is that given in “Moisture equilibria and moisture changes in soils beneath covered areas” (Aitchison, 1965).

Soil suction is commonly referred to as the free energy state of soil water. The free energy of the soil water can be measured in terms of the partial vapour pressure of the soil water (Richards, 1965). In expansive soil, as noted by Aitchison and Woodburn (1969) cited in Osman et al. (2006) in Fig. 2.2, soil suction profile was a representation of a state of physical balance between various processes operating to add or to subtract water at any part of the profile. According to Fityus et al. (1998) and Masia et al. (2004), soil suction changes were greatest at the soil surface where soil moisture fluctuations occurred and reduced with depth to zero at the bottom, the depth of soil suction variation was
called an active zone. According to Cameron (2005), the active zone in south Adelaide is around 7 meters.

According to Australian standard (AS2870-2011), in most situations according to McKeen (2001), a range of 3pF (98kPa) to 5pF (9800kPa) were reasonable for wet and dry boundaries for suction at the soil surface. For clay soil, the soil water content at wilting point is around 39% and 54% for the field capacity (Raes 2002).

### 2.4.3 Soil water characteristic curve

Soil water characteristic curve (SWCC) is defined as the relationship between the degree of saturation (or volumetric water content) and soil suction (Campbell and Mulla, 1990). It is usually obtained by drying or wetting a soil sample under constant stress while monitoring the water discharge and displacement in the soil. The curve is also called the soil water characteristic curve or the soil water relation curve. SWCC is associated with water content and soil suction. It is a fundamental geotechnical properties in expansive soil in determining the shear stress, constitutive model and hydraulic conductivity (Mualem, 1976; Fredlund et al., 1994; wheeler, 1996).

Soil water characteristic curve represents the fundamental hydraulic behaviour of unsaturated soil. Over the last three decades, a number of equations have been proposed to define the SWCC curve. A summary is given in Table 2.1. These equations may help to calibrate data in a SWCC experiment or be directly used in unsaturated soil modelling.

In table 2, $S_{\theta}^{\text{ref}}$ is the effective degree of saturation for the reference SWCC, $S_{\theta}^{\text{ref}}$ is the residual degree of saturation and $S_{\theta}^{\text{res}} = \frac{\theta_r}{\theta_s}$, with $\theta_r$ being the residual water content and $\theta_s$ the saturated water content. $S_{\infty}$ is suction in air entry value and $S_{r}$ is the residual suction. $\mu_d$, $S_{\theta}^{\text{res}}$, $a_{d/u}$, $m_{d/u}$, $n_{d/u}$, $\alpha_d$, $\beta_d$, $\gamma_d$ are fitting parameters.
Table 2.1 Selected SWCC Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Fitting parameters</th>
<th>Number of parameters in boom clay</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_e^{ref} = \begin{cases} 1 &amp; S \leq S_{ae} \ \left( \frac{S_{ae}}{S} \right)^{\mu_d} &amp; S &gt; S_{ae} \end{cases}$</td>
<td>$\mu_d$</td>
<td>$S_{ae} = 0.25 \text{MPa}$</td>
<td>Brooks and Corey (1964)</td>
</tr>
<tr>
<td>$S_r^{ref} = S_r^{res} + S_e^{ref} (1 - S_r^{res})$</td>
<td></td>
<td>$S_r^{res} = 0.0$</td>
<td>Van Genuchten (1980)</td>
</tr>
<tr>
<td>$S_e^{ref} = 1 + \left( \frac{S}{a_d^{i/w}} \right)^{m_d^{i/w}} e^{-n_d^{i/w}}$</td>
<td>$S_r^{res}, a_d^{i/w}, m_d^{i/w}, n_d^{i/w}$</td>
<td>$a_d = 4 \text{MPa}$, $m_d = 0.7$, $n_d = 1.0$</td>
<td></td>
</tr>
<tr>
<td>$S_r^{ref} = \frac{1 - \left[ \ln(1 + \frac{S}{S_{re}}) / \ln(1 + \frac{10^6}{S_{re}}) \right]}{\ln \left[ 2.71828 + \frac{S}{\alpha_d^2} \beta_d \gamma_d \right]}$</td>
<td>$\alpha_d, \beta_d, \gamma_d$</td>
<td>$\alpha_d = 2.0 \text{MPa}$, $\beta_d = 0.8$, $\gamma_d = 1.8$, $S_{re} = 10^5 \text{KPa}$</td>
<td>Fredlund and Xing (1994)</td>
</tr>
</tbody>
</table>

2.5 Guiding principles

In the past several decades, many researchers contributed guiding principles which attempt to illustrate the behaviour of root absorption through tree root distribution, proximity of distance, soil suction profiles and so on. Some of the principles are cited in Australian standard.
2.5.1 Concept of proximity

The definition of proximity, according to Cameron (2001), can be expressed by a ratio of minimum horizontal distance, $D$, between the base of the tree and the building parameter, to the height of the tree, $H$. The ratio $D:H$ could be expressed as a single or a group of trees. The proximity concept attempted to define a secure distance for building design considering the height of tree in a $D:H$ ratio in different environments (Fityus, 2004).

Bozozuk (1962) monitored the shrinkage settlement for a row of Gum tree in Canada. The effect of trees was limited at a $D/H$ ratio less than 0.5. Biddle (1983) studied a range of tree spaces and suggested a secure distance of $D/H$ value up to 1.5 in UK. Jaksa et al. (2002) reported the eucalypt root effect in clay, with a $D/H$ ratio of 0.5-1 for a single tree and 0.8-1.1 for a row of trees. Cameron (2001) concluded the highly affected zone of eucalypt within $D/H<0.7$ for a single tree and $D/H<1.5$ for a row trees in Adelaide. Blight (2005) suggested the influence of a root mass reaches a ratio $D/H=1$ of a single tree or $D/H=1.5$ for a row of trees. In South Australia, as a worst-case default value, the minimum distance of half the tree height is used as a conservative estimate (AS2870-2011). The method is a botanically naïve method which judging the affected zone area simply by the height of tree. Nevertheless, the simple method has been successfully working in South Australia for 15 years.

2.5.2 Site classification and suction change method

Australian Standard (AS2870-2011) provides a site classification method which is based on the expected ground surface movement and the depth to which this movement extends (see Table 2.2).
Table 2.2 Classification based on site reactivity

<table>
<thead>
<tr>
<th>Class</th>
<th>Foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Most sand and rock sites with little or no ground movement from moisture changes</td>
</tr>
<tr>
<td>S</td>
<td>Slightly reactive clay sites, which may experience only slight ground movement from moisture changes</td>
</tr>
<tr>
<td>M</td>
<td>Moderately reactive clay or silt sites, which may experience moderate ground movement from moisture changes</td>
</tr>
<tr>
<td>H1</td>
<td>Highly reactive clay sites, which may experience high ground movement from moisture changes</td>
</tr>
<tr>
<td>H2</td>
<td>Highly reactive clay sites, which may experience very high ground movement from moisture changes</td>
</tr>
<tr>
<td>E</td>
<td>Extremely reactive sites, which may experience extreme ground movement from moisture changes</td>
</tr>
</tbody>
</table>

In soil classification, an instability index \( I_{pt} \) is defined as the percent vertical strain per unit change in suction. According to Cameron (1989), instability index \( I_{pt} \) could be estimated by shrinkage index \( I_{ss} \), which can be measured from a swelling and shrinkage test. The characteristic surface movement \( y_s \) (mm) is calculated as:

\[
y_s = \sum_{n=1}^{N} (\Delta u \cdot h \cdot I_{pt})_n
\]  

(2.5)

where \( y_s \) is characteristic surface movement (mm), \( I_{pt} \) is instability index, in \%/pF. \( \Delta u \) is soil suction change averaged over the thickness of the layer under consideration (pF). \( h \) is thickness of soil layer (mm). \( n \) is number of soil layers. Australian standard assume the suction profile change linearly with depth. \( H_s \) is the value of depth of design suction change. Australia Standard (AS2870-2011) provides data for \( \Delta u \) and \( H_s \) as shown in Fig. 2.3. \( H_s \) is considered to be influenced by in a climatic index. The distribution of \( H_s \) in Victoria is shown in Fig. 2.4.
Fig. 2.3 Definition of depth of design and suction change (AS2870-2011)

<table>
<thead>
<tr>
<th>Location</th>
<th>Change in suction at the soil surface (Δu) pF</th>
<th>Depth of design soil suction change (Hs) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adelaide</td>
<td>1.2</td>
<td>4.0</td>
</tr>
<tr>
<td>Albury/Wodonga</td>
<td>1.2</td>
<td>3.0</td>
</tr>
<tr>
<td>Brisbane/Ipswich</td>
<td>1.2</td>
<td>1.5–2.3</td>
</tr>
<tr>
<td>Gosford</td>
<td>1.2</td>
<td>1.5–1.8</td>
</tr>
<tr>
<td>Hobart</td>
<td>1.2</td>
<td>2.3–3.0</td>
</tr>
<tr>
<td>Hunter Valley</td>
<td>1.2</td>
<td>1.8–3.0</td>
</tr>
<tr>
<td>Launceston</td>
<td>1.2</td>
<td>2.3–3.0</td>
</tr>
<tr>
<td>Melbourne</td>
<td>1.2</td>
<td>1.8–2.3</td>
</tr>
<tr>
<td>Newcastle</td>
<td>1.2</td>
<td>1.5–1.8</td>
</tr>
<tr>
<td>Perth</td>
<td>1.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Sydney</td>
<td>1.2</td>
<td>1.5–1.8</td>
</tr>
<tr>
<td>Toowoomba</td>
<td>1.2</td>
<td>1.8–2.3</td>
</tr>
</tbody>
</table>

Fig. 2.4 Distribution of $H_s$ in a climatic index in Victoria
Four parameters are used in the Australian Standard (AS2870-2011) to take into account the effect the tree on residential footing design, the equilibrium suction value $u_{eq}$, wilting point suction $u_{wp}$, the influence depth of tree $H_t$ and influence of tree absorption in suction $\Delta u_{tree}$. According to Richards and Chan (1971), equilibrium suction value $u_{eq}$ is the suction under the centre of a large paved area in the same environment as a constant number. Feddes (1974) suggested tree roots may stop to absorb water from soil when soil suction reaches a wilting point. Cameron (2001) also defined wilting point suction $u_{wp}$ as a constant number in different depth. The influence of tree absorption in suction is defined as $\Delta u_{tree}$:

$$\Delta u_{tree} = \log_{10}\left(\frac{u_{wp}}{u_{eq}}\right)$$  \hspace{1cm} (2.6)

The implementations of the parameters are shown in Fig. 2.5. Cameron (2001) estimated the soil surface movement by considering site classification and tree absorption. He provided data for this model in Adelaide with $H_s = 4m$, $H_t = 6m$, $\Delta u = 1.2 \log MPa$, $\Delta u_{tree} = 0.4 \log MPa$.

![Fig. 2.5 Root absorption model in the Australian standard (AS 2870-2011)](image-url)
2.5.3 *Foundation and Footings Society of Victoria Method*

![Diagram of tree root influenced areas](image)

**Fig. 2.6** Definition of tree root influenced areas in FFSV method (AS 2870-2011)

The Foundation and Footings Society of Victoria (FFSV) method was developed by Foundation and Footings Society of Victoria based on the input of arboriculturists, engineers and geologist in Victorian building industry. The FFSV method proposes a grading of trees with respect to the effect to their roots on nearby structures and suggested how their influence be reduced. As shown in **Fig. 2.6**, the method divided the tree influenced area into three areas. In this method, root water uptake is estimated related to various factors including: tree species, tree heath, stage of growth, total leaf area, height, root, trunk and branch mass, soil type, climate and water suction capacity. The detail is listed in Table 2.3. The definition of areas according to the tree effect score are shown in Table 2.4.
### Table 2.3 Tree effect score in FFSV method

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Option</th>
<th>Option score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree characteristics</td>
<td>Canopy</td>
<td>Dense</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Med Dense</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sparse</td>
</tr>
<tr>
<td>Height</td>
<td>Tall = &gt;15 m</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Med = 8 – 15 m</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Small = &lt;8 m</td>
<td>1</td>
</tr>
<tr>
<td>Stage of growth</td>
<td>Growing</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Mature</td>
<td>1</td>
</tr>
<tr>
<td>Drought resistance</td>
<td>Resistant</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Not resistant</td>
<td>0</td>
</tr>
<tr>
<td>Ground and site conditions</td>
<td>Depth of fill</td>
<td>≥1 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;1 m</td>
</tr>
<tr>
<td>Adverse conditions</td>
<td>Yes</td>
<td>1–2</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>0</td>
</tr>
<tr>
<td>Soil profile reactivity</td>
<td>High/Extreme reactivity</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Moderate reactivity</td>
<td>1</td>
</tr>
</tbody>
</table>

Total tree effect score (sum characteristic scores above)

### Table 2.4 The definition of influenced areas according to tree effect score in FFSV method

<table>
<thead>
<tr>
<th>Tree effect score</th>
<th>Tree effect</th>
<th>Climate zones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Distances (TD1 or TD2), m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;6</td>
<td>Low</td>
<td>1</td>
</tr>
<tr>
<td>6-9</td>
<td>Moderate</td>
<td>3</td>
</tr>
<tr>
<td>9-12</td>
<td>High</td>
<td>5</td>
</tr>
<tr>
<td>12-15</td>
<td>Very High</td>
<td>7</td>
</tr>
<tr>
<td>≥15</td>
<td>Extreme</td>
<td>9</td>
</tr>
</tbody>
</table>
2.6 Numerical modelling review

With the development of computer technology, more attempts of numerical modelling are contributed to model the complex processes involved vegetation induced ground movement (Fityus, 2004).

The numerical modelling of root water uptake can generally be classified into two approaches. The first category deals with water flow to a single root (radial flow) and is often referred to as a microscopic approach (Gardner, 1960; MolZ et al., 1968; Hillel et al., 1975). These models are developed in early stage. The second category is a macroscopic approach, in which water extraction by roots is treated as a sink term distributed in the root zone. Most root water uptake models use the macroscopic approach in recent two decades (for example, Feddes, 1978; Prasad, 1988; Li et al., 1999; Alonso and Lloret, 1995; Ojha and Rai, 1996, Fredlund and Hung, 2001, Kang et al., 2001; Blight, 2005; Fatahi et al., 2006; Indraratna, 2006 ).

In this analysis, the second method is adopted.

When describing the water extraction in root zone, it is generally an agreement to utilising a Richards flowing equation with a sink term. Celia et al. (1990) suggested a one dimensional equation:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ K \left( \frac{\partial \phi}{\partial z} + 1 \right) \right] - S(\theta) \quad (2.7)
\]

Whisler et al. (1968), Molz (1981) and Clausnitzer and Hopmans (1994) describing the equation as:

\[
\frac{\partial \theta}{\partial t} = \nabla [K \nabla (\phi - z)] - S(\theta) \quad (2.8)
\]

where \( \theta \) is the volumetric water content, \( K \) is the hydraulic conductivity of the soil (cm/sec) \( z \) donates the gravity head, \( \phi \) is the total head (cm) and \( S(\theta) \) is sink term.
Indraratna et al. (2006) suggested an equation using soil suction $\psi$ instead of total head $\phi$ which is more practical in modelling:

$$\frac{\partial \theta}{\partial t} = \nabla (K \nabla \Psi) - K(\theta) \frac{\partial \theta}{\partial z} - S(\theta) \tag{2.9}$$

In most root absorption models, the definition of hydraulic conductivity is following a soil water characteristic curve equation (Brooks and Corey, 1964; Van Genuchten, 1980). In defining the sink term $S(\theta)$, researchers gave different approaches in past several decades.

For one dimensional model, Feddes (1978) suggested:

$$S(\theta) = \frac{T_p}{z_r} \tag{2.10}$$

where $T_p$ is transpiration rate, $Z_r$ is the maximum depth of root zone. Prasad (1988) suggested another one dimensional linear model which assuming root density is proportional with depth:

$$S(\theta) = \frac{2T_p}{z_r} (1 - \frac{z}{z_r}) \tag{2.11}$$

Kang et al. (2001) developed an exponential equation for crop root extraction as:

$$S(\theta) = T_p \frac{1.8e^{-1.8z/z_r}}{(1 - e^{-1.8})z_r} \tag{2.12}$$

Ojha and Rai (1996) provided a model:

$$S(\theta) = \left[ \frac{T_p}{z_r} (\beta + 1)(1 - \frac{z}{z_r})^\beta \right] \tag{2.13}$$
where $\beta$ is a fitting parameter.

For two dimensional models, Vrugt et al. (2001) gave a model:

$$S(\theta) = \frac{\pi R^2 \beta(r, z) T_p}{2\pi \int_0^{z_m} \int_0^{r_m} r \beta(r, z) dr dz}$$

(2.14)

with the root density $\beta(r, z)$ and the maximum root zone width $r_m$ and depth $z_m$.

This equation is highly related to root density and root distribution. Indraratna et al. (2006) suggested using the following equations to present the sink term

$$S(\theta) = G(\beta) \cdot F(T_p)$$

(2.15)

$$G(\beta) = \frac{\tanh(k_z \beta(r, z, t))}{\int_{v(t)} \tanh(k_z \beta(r, z, t))}$$

(2.16)

$$\beta(r, z, t) = \beta_{\text{max}}(t) e^{-k_z |z| + k_t |t|}$$

(2.17)

$$F(T_p) = \frac{T_p (1 + k_4 (z_r - z))}{\int_{v(t)} G(\beta)(1 + k_4 (z_r - z)) dV}$$

(2.18)

where $G(\beta)$ is root distribution parameter, $F(T_p)$ is transpiration parameter and $\beta(r, z, t)$ is root length density.

2.7 Conclusion

This chapter presented the research background. The behaviour of root absorption and the hydraulic and mechanical behaviours of unsaturated soils were discussed. The existing numerical models of root water uptake and the design guidelines
recommended by Australian Standard for residential slab and footing design (AS2870, 2011) were reviewed and discussed as well.

A sophisticated numerical model should take into account not only soil evaporation, rainfall infiltration, tree root water take rate and root extent but also change in soil suction, and the soil deformation and footing behaviour. The main drawback of the current numerical models is that they have not been validated by the use of the high quality field data. The input physical parameters used in these models were generally estimated, rather than based on the laboratory tests and/or the field measurements. In addition, most numerical models focused on hydraulic behaviour of soil and little attention has been directed to the stress-strain behaviours of unsaturated soil.

Although a great deal of research has been conducted over the last 30 years, the profession’s understanding of the effect of trees on foundation and footing is still limited. It is therefore necessary that both theoretical and experimental research be conducted to gain insight into the tree root-water- expansive soil interaction.
Chapter 3 Concept development

3.1 General

Expansive or reactive soil is predominantly clay soil which undergoes appreciable volume change in response to changes in soil moisture content. Buildings constructed on expansive soils may be subjected to severe movement arising from non-uniform soil moisture changes, with consequent cracking and damage due to distortion (Li and Cameron 2002). These moisture changes are often induced by seasonal changes in rainfall and evaporation.

In addition to seasonal climate variation, there are many other environmental factors affecting expansive soil volume change, for example, tree-root drying, leaking underground water services and garden over-watering etc. In Australia, damage to residential buildings due to tree-root drying has been widely reported, particularly in areas of expansive soil. Often such damages are attributed to tree root absorption which changes the water content and causes shrinkage of soils.

In a tree root-water-soil interaction model, root absorption, rainfall infiltration and soil evaporation are considered to be the main factors contributing to the variation of soil water content. Expansive soils change in volume to varying degrees, in response to changes in water contents according to their shrink and swelling behaviour. They increase in volume upon wetting and decrease in volume if dried.
out. At the same time, suction force produced in root absorption causes water movement towards root zone area. Thus, re-distribution of soil suction occurs and significantly influences the ground movements.

This chapter presents the concept and theory for solving a tree root-moisture flow-expansive soil interaction problem. Firstly the methodology is described. The tree root water uptake model is then presented. The influence of absorption on the moisture flow-soil interaction is also discussed.

3.2 Methodology

As reviewed in Chapter 2, a root water absorption model involves knowledge in botany, hydraulics and geotechnics. In this thesis, the author reviews the works in past two decades and proposes a new methodology for modelling root water absorption. In this process, the author provides reasonable assumptions and solves several technical difficulties.

Firstly, although Biddle (1998) reported that the root absorption rate was equal to the transpiration rate in tree leaves, the transpiration of a plant was hard to define. In early models, transpirations were simply assumed to be constant according to the references in botany. Some researchers (Knox, 1995; Cameron, 2001) suggested that the transpiration be determined by leaf water potential ($LWP$), but the relationship between $LWP$ and transpiration was not clearly defined. Indraratna et al. (2006) defined transpiration in an indirect method with two empirical parameters. However, these parameters were difficult to determine. In this thesis, two methods are proposed to determine a transpiration rate. In the first method, the author selects the leaf diffusion model by Linacre (1963) and assumes the transpiration is equal to a leaf evaporation rate $E_t$ multiplied by the leaf area index ($LAI$). This assumption is according to the physical meaning of transpiration. In the second method, transpiration is obtained by an experimental approach in which the total sap flow in tree trunk is measured using a Heat Ratio Method. The details of experimental instrument and field monitor are given in Chapter 4.
Secondly, rainfall and soil evaporation may significantly change the water content in soil surface. According to the Australian Standard (AS2870-2011), the change in total suction at the soil surface ($\Delta u$) can be taken as 1.2 pF. Unfortunately, the climate effect was ignored in nearly all existing numerical models. For example, a dry only boundary condition was assumed by Indraratna et al. (2006) which totally ignored the influence of rainfall. A relatively sophisticated boundary condition which accounts both infiltration and evaporation is used in this research. In the proposed numerical model, the rainfall rate $q_i$ and the evaporation rate $q_e$ are used to represent the surface boundary conditions. The rainfall rate $q_i$ is reduced by the leaf canopy when rainfall is close to tree trunk. The rainfall rate $q_i$ and the evaporation rate $q_e$ can be obtained from the nearest Bureau of Meteorology station or measured from a weather station at the site.

Thirdly, although the definitions of flow equation and sink terms were provided in most two dimensional numerical models, the description of root distribution and root length density was not clear. In this thesis, the author gives a deduction of a two dimensional flow equation and improves Vrugt’s (2001) model by defining sink terms. In the new model, the author gives new root density definitions and implements two parameters $K_1$ and $K_2$ to define root shapes.

Finally, when modelling the soil-water interactions, most numerical models (Feddes, 1976; presad, 1988; Li, 1996; Vrugt et al., 2001) focused on the hydraulic behaviour of expansive soil. The author reviews the studies and demonstrates the reaction effect of soil suction to an absorption flow. On the other hand, in most numerical models, very little work was done in describing the mechanical behaviour of soil. Indraratna’s (2006) applied an effective stress equation to represent the stress variables in expansive soil. Nevertheless, the constitutive modelling was not clear. In this thesis, the author provides details of effective stress equation and constitutive modelling to simulate the soil movement.

In addition, in this modelling, the author highlights the significance of field and laboratory experiments. In defining transpiration, climate change and expansive
soil behaviour, large amount of parameters are obtained by the author from the experiments. The experiments improve the accuracy of the modelling in a significant manner. The experiments are discussed in Chapter 4.

The modelling methodology is illustrated in Fig. 3.1. It can be divided into five steps. The details are discussed in the following sections.

Fig.3.1 The proposed atmosphere-tree-soil interaction model
3.3 Transpiration rate

3.3.1 Transpiration and absorption

Tree root absorption is generally considered to be originated by water exhaustion in tree leaves. Transpiration is defined as the evaporation of water into the atmosphere from plants leaves. Transpiration is a passive process largely controlled by the humidity of the atmospheric and the soil suction. According to Biddle (1998), the loss of water from the leaves of a tree sets up a negative potential or suction through the tree, which provides the pulling power to drive water from the soil to the leaves through the plant’s root and xylem. Therefore the loss of water from the leaves of a tree through transpiration is the driving force behind all of the water movement involved in soil drying. The leaf water potential (LWP) is one measure of the potential of the tree water uptake, which is similar in the concept to soil suction (Cameron, 2002). A negative potential or suction is needed to drive the water through the plant and the atmosphere through humidity provides the pulling power to move water from the soil through the plant’s xylem foliage (Knox et, 1995). According to this principle, the tree root absorption rate can be assumed equal to the transpiration rate in tree leaves. Thus, to investigate the magnitude of root water absorption, one can measure the transpiration rate in tree leaves.

As Biddle (1998) summarised, more than 99% of plant water uptake is lost in the form of transpiration from leaves. Hence, it is generally assumed that the magnitude of tree root absorption is equal to the total transpiration in tree leaves. In this study, it has been assumed that the transpiration rate $T_p$ (mm/day) is proportional to the leaf area:

$$T_p = \frac{E_r \cdot LAI}{\rho_w} \quad (3.1)$$
where \( E_t \) is water evaporation rate in tree leaves. \( LAI \) is the leaf area index (m²/m²) defined as the single-side leaf area per unit ground surface area. \( \rho_w \) is the water density (kg/m³).

### 3.3.2 Leaf diffusion model

Leaf diffusion model determines the water evaporation rate \( E_t \) in tree leaves. Linacre (1963) proposed a leaf diffusion model illustrated in Fig. 3.2. In this model, heat flows only through an atmospheric resistance, whereas water-vapour diffuses through tissue inside the leaf surface, as well as through the layer of air outside. Thus the following equations apply to a single uniform area in the heat storage:

\[
R_t = L_t E_t + H_t \tag{3.2}
\]

\[
H_t = \frac{T_s - T_a}{r_s} \tag{3.3}
\]

\[
L_t E_t = \frac{e_s - e_a}{r_t + r_e} \tag{3.4}
\]

where:

- \( E_t \) = the water evaporation rate (g/cm²·sec)
- \( R_t \) = the net radiation flux to the area (cal/cm²·sec)
- \( L_t \) = the latent heat of evaporation (cal/g)
- \( L_t E_t \) = the latent heat flux (cal/cm²·sec), which is proportional to the water evaporation rate
- \( H_t \) = the sensible-heat flux from the area (cal/cm²·sec)
- \( T_s \) = the temperature of the leaf surface (°C)
- \( T_a \) = the ambient air temperature (°C)
\( r_x \) = the external heat-diffusion resistance (°C cm\(^2\) sec/cal)

\( e_x \) = the vapour pressure of the liquid surface within the leaf (mmHg)

\( e_a \) = the ambient vapour pressure (mmHg)

\( r_e \) = the external vapour-diffusion resistance (mmHg cm\(^2\) sec/cal)

\( r_i \) = the internal vapour-diffusion resistance, between the liquid and the leaf surface (mmHg cm\(^2\) sec/cal)

\( e_i - e_a \)

---

**Fig. 3.2** Model of the diffusion resistances assumed to exist about a leaf (Linacre, 1963)

The external vapour-diffusion resistance \( r_e \) can be related to the external heat-diffusion resistance \( r_x \) since both resistances concern diffusion through the same physical barrier.

\[
r_e = a \cdot r_x \tag{3.5}
\]

where \( a \) is the psychrometric constant, which can be can taken as 0.5 mmHg/°C

Since the internal liquid and the leaf surface can be assumed to have the same temperature due to the small distance between each other, \( e_i \) may be related to \( T_s \):
\[ e = f(T_s) \]  \hspace{1cm} (3.6)

Combining Eqs. 3.2 – 3.6 yields

\[ E_i = \frac{f((R_i - L_i E_i) r_i + T_a) - e_a}{L_i (r_i + a \cdot r_i)} \]  \hspace{1cm} (3.7)

### 3.3.3 Leaf area index

LAI is the leaf area index (m²/m²) defined as the single-side leaf area per unit ground surface area. Direct measurement of LAI is time consuming, labour-intensive and usually destructive. Alternatively, indirect methods can be used to measure LAI non-destructively. Gap fraction analysis is an indirect method that uses the relationship between the canopy leaf area and the fraction of the direct solar beam that penetrates the canopy to estimate LAI (Welles and Cohen, 1996).

According to Marshall and Waring (1986), the leaf area index can be estimated using the Beer-Lambert law:

\[ LAI = \frac{-\ln(Q_b / Q_a)}{k} \]  \hspace{1cm} (3.8)

where \( Q_b \) is the blow canopy light, \( Q_a \) is the above canopy light, and \( k \) is an extinction coefficient. The theory of gap fraction analysis led to the development of a number of commercial meters (e.g. Decagon AccuPAR LP-80, Li-Cor LAI-2000 and Delta-T Devices SunScan). These potable meters can be used in the field to estimate LAI by measuring incoming light above and below the tree canopy (Wilhelm et al., 2000). Combining Eq. 3.1, Eq. 3.7 and Eq. 3.8, transpiration rate could be calculated.
3.4 Tree trunk Sap flow

The importance of transpiration in studying soil-plant-water relations has resulted in numerous measurements by several methods, including gravimetric and volumetric measurements, measurement of water vapour, and the velocity of sap flow (Kramer, 1995). The most common approach is the sap flow methods that use heat as a tracer for sap movement because it provides direct and continuous measurements of whole plant water use in situ with high time resolution and little disruption to the sap stream (Smith and Allan, 1996). In a tree water circle system, because of leaf water potential, water comes from tree roots to tree leaves through xylem. Thus, the absorption rate (or transpiration rate) is equal to the total transfer rate of sap flow in tree truck.

![Fig. 3.3 ICT HRM30 sap flow sensor used in this research](image)

In this research, the SFM sap flow meters are used to obtain transpiration rate by measuring tree trunk sap flow (Fig. 3.3). The SFM is the second generation HRM (Heat Ratio Method) sensor from ICT International which is based on the HRM principle. Heat Ratio Method (HRM) is an improvement of the Compensation Heat Pulse Method (CHPM) by allowing very slow and reverse rate of sap flow to
be measured (Burgess et al. 2001). Both sap velocity \( V_s \) and volumetric water flow in xylem tissue can be measured using a short pulse of heat as a tracer.

In this method, by measuring the ratio of heat transported to two symmetrically placed temperature sensors, the magnitude and direction of heat pulse velocity can be calculated as:

\[
V_h = \frac{k}{x} \ln \left( \frac{v_1}{v_2} \right) \times 3600
\]  

(3.9)

where \( V_h \) is the heat pulse velocity (cm/h), \( k \) is the thermal diffusivity of wet wood (cm\(^2\)/s) \( x \), is the distance between the heat source (heater) and temperature sensors (cm), and \( v_1 \) and \( v_2 \) are the increases in temperature (from ambient) at equidistant points downstream and upstream, respectively, \( x \) cm from the heater. Installing sensors in xylem tissue causes substantial mechanical damage at tree trunk. Swanson and Whitfield (1981) used a finite-difference numerical model to produce a simple algebraic equation for wound correction. The model calculates three coefficients \( (a, b \text{ and } c, \text{ for varying wound widths}) \) to calculate corrected heat pulse velocity \( (V_c) \) measured with the CHPM:

\[
V_c = a + bV_h + cV_h^2
\]  

(3.10)

Finally, the sap velocity is calculated (Barrett et al., 1995):

\[
V_s = \frac{V_c \rho_h (c_w + m_c)}{\rho_s c_s}
\]  

(3.11)

where: \( \rho_h \) is the basic density of wood (dry weight/green volume), \( c_w \) and \( c_s \) are specific heat capacity of the wood matrix (1200 J kg\(^{-1}\) °C\(^{-1}\) at 20 °C (Becker and Edwards 1999)) and sap water, (4182 J kg\(^{-1}\) °C\(^{-1}\) at 20 °C (Lide 1992)), respectively, \( m_c \) is water content of sapwood and \( \rho_s \) is the density of water.
The data from SFM sap flow meters are analysed using the software SFT (Sap Flow Tool) provided by the instrument suppliers, ICT international Ltd. Fig. 3.4(a) shows the working interface of the software for data collection and result calculation. Fig. 3.4(b) presents the characteristic of the sap flow. The details of the sap flow measurement and the data analysis are introduced in Chapter 4.

Fig. 3.4 Main user interface and typical results of Sap Flow Tool software (ICT International Pty Ltd, 2010)
3.5 Infiltration and evaporation

Rainfall infiltration and soil evaporation represent the climate effects. Climate effects may influence the soil-tree root interaction in two aspects. Firstly, soil evaporation and rainfall infiltration may change the soil water content and lead to soil movement. According to Osman (2005), seasonal changes in rainfall were typically the principal cause of the change of soil water content. This led to downward movement during summer and upward movement during winter (Sorensen and Tasker, 1976; Freeman et al. 1994; Smith 1993; O’Malley and Cameron 2002). The consequent rising and settling of ground surface occurred in the dry and wet seasons resulting in seasonal subsidence and seasonal recovery respectively (Biddle 1998; Terzaghi et al. 1996).

On the other hand, according to the Australian standard (AS2870, 2011), climate has a major effect with tree root growing. In wet climates, more soil water is available and therefore trees rarely need to extend their roots to any great distance from the trunk other than for their stability. In temperate climates, there is sufficient water available other than in droughts. During drought times, the roots may have considerable extension to new territory. In a semi-arid or arid climate region, potential evapo-transpiration is much larger than precipitation. Trees struggle unless close to a permanent water source or extend their roots to great depths (AS2870, 2011).

This section discusses the influences of infiltration and evaporation on the soil water content and the top surface boundary conditions. Root distribution and root density will be discussed in the next section.

One of the limitations of the existing tree root – soil interaction models is that the climate influence was not include in the numerical analyses for simplicity. For instance, a dry only boundary condition was assumed by Indraratna et al. (2006) which totally ignored the influence of rainfall. Considering the climate model in the Australian standard, in this analysis a new evaporation and infiltration model with leaf canopy parameter is given.
Firstly, the direction of soil evaporation and rainfall infiltration are generally considered to be vertically. A relatively sophisticated boundary condition which accounts both infiltration and evaporation is used in this research:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial (q_i - q_e)}{\partial z}$$  

(3.12)

where $\theta$ is soil water content, $q_i$ and $q_e$ are rainfall rate and evaporation rate (cm/sec) respectively. The total water flow $q$ equals $q_i - q_e$. $z$ is the vertical coordinate. The infiltration and evaporation data can be obtained from a weather station at the site or from the nearest Bureau of Meteorology station as shown in Fig. 3.5. The details of the measurement from an on-site weather station at the test site are presented in Chapter 4.

**Fig. 3.5** Monthly rainfall and evaporation at Melbourne airport (Australian Government Bureau of Meteorology, [http://www.bom.gov.au/](http://www.bom.gov.au/))
Secondly, according to the Australian standard (AS2870-2011), leaf canopy effect is estimated in a rainfall infiltration process. The condition is shown in Fig. 3.6. The standard indicates that in the area of leaf canopy, because of the obstruction of tree leaves, rainfall sometimes could not directly drip to the soil surface. The standard sets a drip line at the edge of leaf canopy and assumes if the rainfall could not pass the leaf area and drip on to soil surface directly, it may flows to the edge of leaf canopy and falls to the land following the drip line. According to the assumption of Australian standard, in this analysis the author gives a new infiltration equation. When assuming the distance between tree trunk to drip line is $L_0$, rainfall infiltration $q_i$ is:

$$q_i = \begin{cases} 
q_r \cdot (1 - \alpha(i)) & (r < L_0) \\
q_r + q_r L_0 \alpha(i) & (r = L_0) \\
q_r & (r > L_0)
\end{cases} \tag{3.13}$$

where $q_i$ is infiltration rate, $q_r$ is rainfall rate, $\alpha(r)$ is a canopy reduce factor between 0 and 1. $r$ is the distance to tree trunk. When defining the canopy reduce factor, Australian standard gives the following discussions to canopy:

* **DENSE** for canopy that shows little background light

* **MED. DENSE** for canopy that shows approximately 50% background light

* **SPARE** for canopy that shows a high degree of background light
According to the definition by Australian standard, for DENSE canopy, $\alpha(r)$ is defined approximately between 0.6 and 0.9. For MED. DENSE canopy, $\alpha(r)$ is defined between 0.4 and 0.6. For SPARE canopy, $\alpha(r)$ is defined below 0.4.

Practically, $\alpha(r)$ can be obtained by a simple experiment using two rain gauges. One gauge is placed under leaf canopy and close to tree, another is placed beyond the drip line. After a rain event, the water volumes collected in gauges $V_c$ and $V_f$ are measured and then the canopy reduce factor $\alpha(r)$ can be calculated as:

$$\alpha(r) = 1 - \frac{V_c}{V_f}$$  \hspace{1cm} (3.14)

Combing Eq. 3.12 to Eq. 3.14, a rainfall and evaporation boundary in soil surface is established.

### 3.6 Root water uptake model

As introduced in Chapter 2, when establishing a root water uptake model, the numerical modelling of root water uptake can generally be classified into two
approaches. The first category deals with water flow to a single root (radial flow) and is often referred to as a microscopic approach. The second category is a macroscopic approach, in which water extraction by roots is treated as a sink term distributed in the root zone.

A macroscopic approach is adopted in this research, in which water uptake by the roots is treated as a volumetric sink term in a water flow equation. This section discusses the flow equation in both one and two dimensional models.

### 3.6.1 Sink term

Before the introduction of root water up take model, the definition of sink term and its relationship with transpiration is given. Sink term is the water uptake rate in a single root element. The relationship between sink term and transpiration can be written as:

$$T_p = \int V S(\theta) dV$$  \hspace{1cm} (3.15)

where $S(\theta)$ is the sink term and $V$ is the volume of the root zone. According to Eq. 3.15, the sink term is closely related with root distribution, and it is generally considered to be proportional with root density.

### 3.6.2 One dimensional flow equation

For one dimensional condition, root absorption flow is assumed to be vertically. As reviewed in Chapter 2 (Prasad, 1988), for vegetations like crop, shrub and grass, the influence of water up take by roots is supposed to be varied with depth.

Richard (1931) has shown that Darcy’s law can also be applied to the flow of water through an unsaturated soil. In vertical direction:
\[ q = -K(\theta) \frac{\partial \phi}{\partial z} \]  
\hfill (3.16)

where \( q \) is the flow rate, \( \theta \) is the volumetric water content, \( K(\theta) \) is the hydraulic conductivity of the soil (cm/sec) and \( \phi \) is the total head (cm).

Representing the root water uptake as a sink term \( S(\theta) \), the time rate of change of water content can be written as:

\[ \frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} + S(\theta) = 0 \]  
\hfill (3.17)

Philip (1969) stated the relationship between the total head \( \phi \) and water potential \( \psi \) (or soil suction):

\[ \phi = \psi - z \]  
\hfill (3.18)

Feddes (1976) combined Eq. 3.16, Eq. 3.17 and Eq. 3.18 gives the one-dimensional flow equation including a root water extraction term as:

\[ \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( K(\theta) \frac{\partial \psi}{\partial \theta} \cdot \frac{\partial \theta}{\partial z} \right) - \frac{\partial K(\theta)}{\partial z} - S(\theta) \]  
\hfill (3.19)

### 3.6.3 One dimensional root density

Obviously, when solving Eq. 3.19, one needs to define \( S(\theta) \). As introduced in Section 3.5.1, Sink term is closely related with root shapes and root length density. In one dimensional condition, root is linear so that the discussion of root shape is skipped. Vrugt (2001) suggested a parameter \( \beta(z) \) to represent the spatial root density (or water uptake) distribution with depth:
\[ \beta(z) = \left[ 1 - \frac{z}{z_{\text{max}}} \right] e^{-\frac{p_z}{z_{\text{max}}}z} \quad (z \geq 0) \]  

(3.20)

where \( \beta(z) \) denotes the spatial root density distribution with depth, \( z_{\text{max}} \) is the maximum rooting depth (L), \( p_z (-) \) and \( z^* \) (L) are empirical parameters. Eq. 3.18 allows defining the maximum root density in any depth. Nevertheless, in most conditions, plants have the maximum root density at the soil surface.

Considering the root density distribution in one dimensional condition, the relationship between sink term and transpiration (Eq. 3.15) is represented as:

\[ S(\theta) = \beta(z)T_p \int_{z=0}^{z_{\text{max}}} \beta(z)dz \]  

(3.21)

Prasad (1988) investigated a number of experimental data and suggested a model where root density is linear decreased by the root depth. In Eq. 3.18, when \( p_z = 0.01 \) and \( z^* = 1 \), Eq. 3.17 is simplified to the condition proposed by Prasad:

\[ \beta(z) = 1 - \frac{z}{z_{\text{max}}} \]  

(3.22)

where \( z_{\text{max}} \) is the maximum root depth, Combining Eq. 3.21 and Eq. 3.22:

\[ S(\theta) = \frac{2T_p}{z_{\text{max}}} \left( 1 - \frac{z}{z_{\text{max}}} \right) \]  

(3.23)

Eq. 3.23 is widely used in many one dimensional root absorption model.
3.6.4 Two dimensional flow equation

Two dimensional model is used to describe the absorption of large tree with root in long span and depth. In two dimensional, Darcy’s low is written as:

\[ q = -K(\theta) \nabla \cdot \phi \]  

(3.24)

where \( q \) is the flow rate, \( \theta \) is the volumetric water content, \( K(\theta) \) is the hydraulic conductivity of the soil (cm/sec) and \( \phi \) is the total head (cm). When analyse a root element, water comes from all directions and a root absorption rate \( S(\theta) \) is added at the element, thus:

\[ \frac{\partial \theta}{\partial t} + \frac{\partial q_s}{\partial r} + \frac{\partial q_s}{\partial z} + S(\theta) = 0 \]  

(3.25)

Eq. 3.23 is coming from a Laplace equation, combining Eq. 3.24 and Eq. 3.25:

\[ \frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial r} (K(\theta) \frac{\partial \phi}{\partial r}) - \frac{\partial}{\partial z} (K(\theta) \frac{\partial \phi}{\partial z}) - S(\theta) \]

\[ \frac{\partial \theta}{\partial t} = -\nabla \cdot V - S(r, z, t) \]  

(3.26)

Similarly, as shown in Eq. 3.18:

\[ \phi = \psi - z \]

where \( \phi \) is total potential or hydraulic head, \( z \) coordinate contributes the gravitational effect. \( \psi \) is soil suction. Combines Eq. 3.18 and Eq. 3.26:

\[ \frac{\partial \theta}{\partial t} = \nabla (K \nabla \psi) - \frac{\partial K(\theta)}{\partial z} - S(r, z, t) \]  

(3.27)
Eq 3.27 is a typical equation representing root absorption in two dimensional conditions.

3.6.5 Two dimensional root density and root distribution

In a two dimensional condition, root density is considered to change in both horizontal and vertical directions (Vrugt, 2001):

\[
\beta(r, z) = \left(1 - \frac{z}{z_{\text{max}}}ight) \left(1 - \frac{r}{r_{\text{max}}}ight) e^{-\left(\frac{r^*}{r_{\text{max}}} - 1\right)} e^{-\left(\frac{z^*}{z_{\text{max}}} - 1\right)}
\]

(3.28)

Where \(\beta(r, z)\) describes the spatial distribution of potential root water uptake with root depth \(z\) and along the radial distances \(r\). The \(z_{\text{max}}\) is the maximum root depth, \(r_{\text{max}}\) is the maximum root length in the radial direction, \(P_z, P_r, z^*\) and \(r^*\) are empirical parameters which are included to account for non-symmetrical root water uptake with depth and along the radial direction, and to allow for maximum root water uptake at any coordinate in root system. Vrugt (2001) gave a list of empirical parameters and the corresponding two dimensional root density distribution as shown in Table 3.1 and Fig. 3.7.

In a two dimensional condition, Eq. 3.21 is expanded to:

\[
S(\theta) = \frac{\beta(r, z)T_p}{\int \beta(r, z)dv} = \frac{\pi r_{\text{max}}^2 \beta(r, z)T_p}{2\pi \int_0^{z_{\text{max}}} \int_0^{r_{\text{max}}} r \beta(r, z) dr dz}
\]

(3.29)

Considering the research in botany (Smith, 1996), in natural conditions, the author suggests the distributions of tree root shapes could be various as shown in Fig. 3.8(a).
**Table 3.1** A list of empirical parameters in two dimensional root density model

(Vrugt, 2001)

<table>
<thead>
<tr>
<th>Figure</th>
<th>$z_m$</th>
<th>$r_m$</th>
<th>$z^*$</th>
<th>$r^*$</th>
<th>$P_z$</th>
<th>$P_r$</th>
<th>$z_{\text{max}}$</th>
<th>$r_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 3.7(a)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Fig. 3.7(b)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.20</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>Fig. 3.7(c)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>4.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Fig. 3.7(d)</td>
<td>1.00</td>
<td>2.00</td>
<td>0.20</td>
<td>1.00</td>
<td>5.00</td>
<td>2.00</td>
<td>0.20</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Fig. 3.7 Two dimensional root density distribution corresponding with Table 3.1

Vrugt (2001)
To conclude all these conditions in Fig. 3.8(b), an inflection point $P(x_0, y_0)$ is pointed to simulate the root geometry. To define the coordinate of $x_0$ and $y_0$, two parameters $K_1$ and $K_2$ are implemented, where $0 \leq K_1 \leq 1$ and $0 \leq K_2 \leq 1$. It is assumed:

$$x_0 = K_1 \cdot r_{\text{max}}$$ \hspace{1cm} (3.30)

$$y_0 = K_2 \cdot Z_{\text{max}}$$ \hspace{1cm} (3.31)

Where $r_{\text{max}}$ and $z_{\text{max}}$ is the maximum width and depth of tree root zone. When $K_1$ and $K_2$ gradually increase from 0 to 1, $P(x_0, y_0)$ is moving accordingly to simulate all the conditions in Fig. 3.8(b). For some special shapes, when $K_1 = 1$, $K_2 = 0$ or
\( K_1 = 0 \), \( K_2 = 1 \), the root shapes is coniform; When \( K_1 = 1 \) and \( K_2 = 1 \), the root shapes is cucumiform. In this assumption, Eq. 3.29 is converted to:

\[
S(\theta) = \frac{\pi r_{\max}^2 \beta(r, z) T_p}{2\pi} \int_0^{z_{\max}^2} r_1^2 \beta(r, z) dz + \int_{z_{\max}^2}^{z_{\max}} r_2^2 \beta(r, z) dz
\]  
\[\tag{3.32}\]

\[
r_1 = \frac{(1 - K_1)r_{\max}}{K_2 z_{\max}} (K_2 z_{\max} - z) + K_1 r_{\max} \quad (0 \leq z \leq K_2 z_{\max})
\]  
\[\tag{3.33}\]

\[
r_2 = \frac{K_1 r_{\max}}{(1 - K_2) z_{\max}} (z_{\max} - z) \quad (K_2 z_{\max} \leq z \leq z_{\max})
\]  
\[\tag{3.34}\]

Eq. 3.32, Eq. 3.33 and Eq. 3.34 give the equation of sink term considering root density and root distribution. As transpiration rate is obtained in a Leaf area index equation or a sap flow measurement, a sink term is calculated. Applied sink term in a flow equation Eq. 3.27, root absorption is simulated.

### 3.7 Root water and soil interaction

#### 3.7.1 General

As discussed in previous sections, root absorption is generally considered as a pore water flow in soil. The direct effect of absorption is a change in water content in a soil profile. In unsaturated soil, soil behaviour is closely related with the water content. The variation of water content may change soil suction, effective stress and cause soil movement. On the other hand, unsaturated soil has reaction to the absorption flow by reducing the sink term and changing the hydraulic conductivity. As shown in Fig. 3.9:
3.7.2 Soil suction

Soil suction is commonly referred to as the free energy state of soil water (Edlefson and Anderson, 1943). The free energy of the soil water can be measured in terms of the partial vapour pressure of the soil water (Richards, 1965). When discuss the component of soil suction, the total suction in expansive soil is represented as:

$$\psi = (u_a - u_w) + \psi_s$$  \hspace{1cm} (3.35)

Where $\psi$ is total suction, $u_a - u_w$ is matric suction, $\psi_s$ is osmotic suction (solute suction), $u_a$ and $u_w$ are pore air pressure and pore water pressure respectively. Soil suction is a significant stress variable related with effective stress and volumetric water content. Soil suction profile is a fundamental boundary condition in an expensive soil model.

According to Kramer and Boyer (1995), tree root absorption could be divided to two categories as passive absorption and osmotic absorption. Active or osmotic absorption occurs in slowly transpiring plants where the roots behave as osmometers whereas passive absorption occurs in rapidly transpiring trees where
water is pulled in through the roots, which act merely as absorbing surfaces. The two types of absorption are response with the matric suction and osmotic suction in an expansive soil. In most time, osmotic suction is generally concerned very minimal.

In this study, considering both active and osmotic absorption, total suction is used in the numerical analysis and the initial suction distributions used in the numerical model are based on the data measured at the filed site.

3.7.3 Soil water characteristic curve

As reviewed in Chapter 2. Soil water characteristic curve (SWCC) is defined as the relationship between the degree of saturation (or volumetric water content) and soil suction (Campbell and Mulla, 1990). The curve determines the hydraulic behaviour of unsaturated soil. Obviously, soil water characteristic curve needs to be considered in the root absorption model. When root absorption changes water content, soil suction will change following a soil water characteristic curve.

Soil water characteristic curve is obtained by laboratory experiment by the use of a SWCC device. The detail of the experiment is discussed in Chapter 4. The experimental data are analysed using Van Genuchten (1980) equation:

\[ S_e^{ref} = S_e^{res} + S_e^{ref} (1 - S_e^{res}) \]  \hspace{1cm} (3.36)

\[ S_e^{ref} = \left(1 + \left( \frac{S}{a_{dlw}} \right)^{n_{dlw}} \right)^{-n_{dlw}} \]  \hspace{1cm} (3.37)

where \( S_e^{ref} \) is the effective degree of saturation for the reference SWCC, \( S_e^{ref} \) is the residual degree of saturation and \( S_e^{res} = \theta_e / \theta_s \), with \( \theta_s \) being the residual
water content and $\theta_s$ the saturated water content. $a_{d/w}, m_{d/w}, n_{d/w}$ are fitting parameters.

The definition of characteristic points in a SWCC was introduced by Fredlund and Rahardjo (1993), in this study, the characteristic points are obtained by an equation suggested by Sheng (2008):

$$dS_r = -\lambda_{ws} \frac{ds}{s}$$

$$\lambda_{ws} = \begin{cases} 
0 & (s < s_{sa}) \\
\kappa_{ws} & (s_{sa} \leq s < s_{ae}) \\
\lambda_{ws} & (s_{ae} \leq s < s_{re}) \\
\kappa_{ws} & (s \geq s_{re}) 
\end{cases} \quad (3.38)$$

where $S_r$ is degree of situation, $s_{sa}$ is saturation suction, $s_{ae}$ is suction in air entry value and $s_{re}$ is residual suction. When a SWCC is obtained, the author determined the slope $\kappa_{ws}$ and $\lambda_{ws}$ on the curve, then the characteristic points $s_{sa}$, $s_{ae}$ and $s_{re}$ are obtained.

Soil water characteristic curve has considerably difference in different soils. In some numerical models, typical curve in text book or curve from other soil is adopted in the models. The adoption may cause significant difference when the water content is low and soil suction is high. The inaccuracy of suction may over several thousands kPa. The suction distribution and soil movement calculated in these conditions are unacceptable. In this analysis, it is highlighted the soil water characteristic curve used in a model must be measured by the soil sample in the field for modelling.
In this analysis, the author did several laboratory measurements of soil water characteristic curve and obtained data for both silty clay and clay in Victoria and South Australia. The samples came from the same field for numerical modelling. Fig. 3.10 shows a SWCC for South Australian clay as an example. After applied Eq. 3.38, three characteristic points were determined on the curve: \( s_{na} = 20 \text{KPa} \), \( s_{ae} = 250 \text{KPa} \) and \( s_{re} = 12000 \text{KPa} \). The details of laboratory measurement are presented in Chapter 4.

### 3.7.4 Expansive soil reaction to sink term

According to Feddes (1976), root water absorption is influenced by the soil reaction. In fact, when the water content is low, actual absorption \( S(x, y, z, t) \) in soil element is lower than the potential value \( S(\theta) \) because soil aeration and plant wilting may be restricted by the effect of soil suction, so that water uptake rate in root zone \( S(x, y, z, t) \) is not reaching the sink term value \( S(\theta) \). Feddes suggested
to use the “wilting point” and “anaerobiosis point”, for conditions wetter than anaerobiosis point or drier than wilting point, tree root water uptake is zero. In the area between wilting point and anaerobiosis point which called “soil aeration” area, tree root may have water uptake.

In this analysis, considering Sheng’s SWCC equation Eq.3.38, the author suggest the “wilting point” and “anaerobiosis point can be given in this SWCC equation. In the main drying curve, suction in “anaerobiosis point” is equal to $s_{sa}$ and suction in “wilting point” is equal to $s_{re}$. Air entry value $s_{ae}$ is the same. Thus, a new reduction factor $\alpha(s)$ associated with soil suction is suggested as:

$$
S(x, y, z, t) = \alpha(s) \cdot S(\theta)
$$

$$
\alpha(s) = \begin{cases} 
0 & (s \geq s_{re}) \\
\frac{s - s_{re}}{s_{ae} - s_{re}} & (s_{ae} < s \leq s_{re}) \\
1 & (s_{sa} < s \leq s_{ae}) \\
0 & (s < s_{sa}) 
\end{cases} 
$$

The graph to define reduction factor $\alpha(s)$ is shown in Fig. 3.11. Comparing with the equation suggested by Feddes, the new equation uses soil suction instead of a water head and is closely related with SWCC equation. The assumption is more reasonable. In this analysis, the variables $s_{sa}, s_{ae}$ and $s_{re}$ in Eq. 3.39 is directly obtained by a SWCC.

![Fig.3.11 The reduction factor $\alpha(s)$](image-url)
3.7.5 Expansive soil reaction to hydraulic conductivity

The definition of Hydraulic conductivity is given together with Darcy’s law. It influences the velocity of water flow. Hydraulic conductivity $K(\theta)$ is described by Brooks and Corey (1964) as:

$$K(\theta) = k_s(e)S_e^{ref} \left( \frac{2+3\lambda_{ws}}{\lambda_{ws}} \right)$$  \hspace{1cm} (3.40)

where $k_s(e)$ is the saturated coefficient of hydraulic conductivity estimated based on Kozeney–Carman equation (Mitchell, 1976), $S_e^{ref}$ is the effective degree of saturation and $\lambda_{ws}$ is the slope of the soil water characteristic curve. Considering Eq. 3.38, obviously, hydraulic conductivity is closely related with soil water characteristic curve. $S_e^{ref}$ and $\lambda_{ws}$ can be obtained by a SWCC equation Eq. 3.37 and Eq. 3.38. Thus, Hydraulic conductivity changes following the variation of soil suction.

3.7.6 Soil movement

The behaviour of unsaturated soil determines the complexity of constitutive model in unsaturated soil which is considerably different with a saturated soil. Sheng (2008) suggested a reasonable constitutive model (SFG model) using a dual constitutive variable (DCV). Components in constitutive model in unsaturated soil are shown in Fig. 3.12. In this model the degree of saturation $S_r$ is contented in the constitutive equation metrix as an independent parameter. In this figure, $\sigma$ is main net stress, $S$ is soil suction, $\sigma$ and $S$ are stress variables. $\varepsilon_v$ is the volumetric strain, $S_r$ is the degree of saturation. The relation between $\sigma$ to $\varepsilon_v$ is a mechanical behaviour, the effect between $S$ and $S_r$ is a hydraulic behaviour. The mechanical and hydraulic behaviour have interactions. The relationship between stress, strain and Degree of saturation is a $7 \times 7$ matrix.
Fig. 3.12 Constitutive model proposed by Sheng (2008)

Nevertheless, in Numerical modelling, for most finite element analysis software, a constitutive model is limited to define the relationship only between stress and strain variables in a $6 \times 6$ matrix. The programs could not add a degree of saturation as a parameter in constitutive equation. In this analysis, to satisfy the numerical modelling definition, an effective stress method using single constitutive variable (SCV) and a Cam-clay model are selected. The principle is shown in Fig. 3.13.
The concept of effective stress in expansive soil was reviewed in Chapter 2. In this analysis, Stress variable $\sigma$ and $S$ is combined in an effective stress equation (Bishop, 1959) as:

$$\sigma' = (\sigma - \mu_a) + \chi(\mu_a - \mu_w)$$  \hspace{1cm} (3.41)

In this equation, $\mu_a - \mu_w$ is matric suction. $\mu_a$ is pore air pressure, $\mu_w$ is pore water pressure. $\chi$ is a parameter related to the degree of saturation of the soil which determined by experimental data. Considering the SWCC equation given by Brooks and Corey (1964), Khalili and Khabbaz (1998) attempted to define parameter $\chi$ by suggesting $u_a = 0.55$, thus:

$$\chi = \begin{cases} 
\left( \frac{s_{ae}}{s} \right)^{0.55} & (s \geq s_{ae}) \\
1 & (s < s_{ae})
\end{cases}$$  \hspace{1cm} (3.42)

where $s_{ae}$ is soil suction in air entry value. When defining the stress and strain relationship, assuming soil is homogenous and isotropic:

$$
\left\{ \begin{array}{l}
\frac{d\varepsilon_v^e}{\nu} = \frac{\kappa}{\nu} \frac{dp'}{p'} \\
\frac{d\varepsilon_v^p}{\nu} = \frac{\lambda - \kappa}{\nu} \frac{dp'}{p'}
\end{array} \right.
$$  \hspace{1cm} (3.43)

where $p'$ is the effective stress, $\varepsilon_v^e$ is the volumetric strain in elastic region. $\varepsilon_v^p$ is the volumetric strain in plastic region. $\kappa$ is the logarithmic bulk modulus of the material defined for the porous elastic material behaviour; $\lambda$ is the logarithmic hardening constant defined for the clay plasticity material behaviour. $\nu$ is specific
57

volume. \( \kappa \) and \( \lambda \) are the elastic slope and plastic slope in a \( e - \ln p' \) curve as shown in Fig. 3.14. Practically, \( \kappa \) and \( \lambda \) is obtained by a soil consolidation test. The discussion of the consolidation test is presented in Chapter 4. The implementation of the constitutive model in numerical modelling is introduced in Chapter 5.

![Graph showing the relationship between \( e \), voids ratio, \( \ln p' \), elastic slope, and plastic slope.](image)

**Fig. 3.14** \( e - \ln p' \) curve in constitutive model (ABAQUS Manual, 2005)

### 3.8 Conclusion

This Chapter describes a concept of root water absorption model considering the factors including transpiration, tree sap flow, climate change, root density, root distribution and root water soil interaction.

In this study, firstly, tree root absorption is assumed equal to the transpiration in tree leaves. A transpiration equation considering leaf diffusion and leaf area index is given. Alternatively, the transpiration rate can be obtained by measuring the sap flow of a tree. Secondly, the seasonal rainfall infiltration and evaporation from soil surface are taken as the boundary conditions. Thirdly, the implication of root
water uptake is considered as a flow equation adding a sink term. In this thesis, a new two dimensional sink term equation considering root length density is given. Soil aeration factor is also taken into account in this process. Finally, in root water and soil interaction, soil water characteristic (SWCC) is provided to describe the relationship between volumetric water content and soil suction. Soil suction and effective stress equation is utilized to describe stress variable in expansive soil. Finally, unsaturated soil constitutive model is applied and a soil movement is estimated. The whole methodology of the proposed root absorption model is illustrated in Fig. 3.15.
Fig. 3.15 The root water absorption modelling concept
Chapter 4 Field and Laboratory Experiments

4.1 General

Although the shrinkage settlement caused by tree root drying and its effects on buildings, pavement and other structures have long been a topic of interest and concern to practising engineers and academics, attempts to model them are few and relatively crude. This is because the physical processes involved are extremely complex. A rigorous theoretical or numerical model requires not only a good understanding of tree physiology, tree water demands in an urban environment, expansive unsaturated soil behaviour, structural response and tree root-water-soil interaction but also a large quantity of the measured field data which can be used to formulate and calibrate the numerical model.

As part of a long term study of the effects of climate and trees on the behaviour of expansive unsaturated soils and performance of the residential buildings, a field site was established in early 2011 at Glenroy, in a northern suburb of Melbourne. The Glenroy site was selected for this study because it is typical of many existing and new residential housing estates in Melbourne and is representative of the regional geological sequence. The site has been extensively instrumented to allow relative humidity, solar radiation, wind direction and speed, rainfall, sap flow of trees, soil moisture conditions and ground movements to be closely monitored. A
series of laboratory tests were also performed on soil specimens collected at the field site. The primary objective of the Glenroy field study is to collect high quality field data that can be used to evaluate and develop numerical model for soil drying by trees. A secondary aim is to develop an improved understanding of the physical processes that drive tree root-expansive soil interaction. With this information, it is hoped that more reliable and sophisticated models which take into account soil evaporation, rainfall infiltration, tree root water take rate, soil suction, root extent, the soil deformation and footing behaviour can be developed.

In this chapter, the site selection, establishment, instrumentation and field monitoring are described. The field data and the laboratory testing data are presented and discussed.

4.2 Field experiments

Although the effects of tree on soil desiccation have been of geotechnical interest for well over half a century, a review of the past 60 years of research into the desiccation of clay soil by trees, bushes and grass indicated that most of recorded research was not either preplanned or sustained (Blight 2005). The measured data available to formulate and calibrate models is scarce. For example, Indraratna et al. (2006) modelled a field study published by Biddle (1983) and showed the numerical results agreed reasonably well with the field measurements. However it must be pointed out that some 20 physical and empirical parameters used in their model were estimated as Biddle’s work contains no data on climate conditions and soil characteristics (Fityus et al. 2007).

After reviewing past research, it was decided to embark on a sustained research project in which high quality field data can be collected.
4.2.1 Description of the Experimental Site

Finding a suitable site for this study took nearly 12 months. The major concerns at the planning stage were the need to ensure site access for a period of at least five years and protection of instrumentation in an urban setting. The site was selected on the basis of the following criteria:

(a) an urban environment
(b) a highly reactive site in the basaltic clay area
(c) site access and security
(d) proximity to RMIT University

The site chosen for field instrumentation is located in Glenroy East, approximately 13 km north of Melbourne CBD and some 500 m north of the Northern Golf Club (Fig. 4.1). It lies within the City of Moreland council boundary.

Fig. 4.1 Map showing site location
Melbourne has a mild, temperate climate with warm to hot summers, mild autumns, cool to cold winters and cool springs. The climate at the Glenroy site is characterised by pronounced seasonal variations with a mean minimum temperature $^1$ of 5.2°C in the coldest month (July) and a mean maximum temperature $^2$ of 26.3°C in the hottest month (January). The annual average precipitation is around 588 mm and annual average sunshine is about 2,373 hours. The geography of the site is needed also to evaluate evapotranspiration. The site has average latitude of 37° 42’ 6.6” S and average longitude of 144° 56’ 0.9” E. The elevation above sea level is about 78 m.

The general plan of the test site is shown in Fig. 4.2. The site is flat, approximately 43 m long and 17 m wide. The house on the site is approximately 40 year old and is of single storey full masonry construction.

![Fig. 4.2 The general plan of the test site](image)

A 2.5 m high eucalyptus ficifolia was bought from a local nursery and planted at the centre of the front yard. The initial soil suction profile was measured. The initial height and stem diameter of eucalyptus ficifolia and root distribution were also recorded. Eucalyptus ficifolia was chosen for this study because it is widely used as a street tree and in home gardens. The use of the site for a period of at least five year was negotiated with the property owner.

---

$^1$ The long-term average daily minimum air temperature observed during a calendar month and over all years of record (1939-2011).

$^2$ The average daily maximum air temperature, for each month and as an annual statistic, calculated over all years of record (1939-2011).
The geotechnical profile for the soil found at the test site is given in Table 4.1. The average soil profile can be described as 0.12 m of sandy silt top soil underlain by high plasticity silty clay to a depth of approximately 2.5 m, then highly to extremely weathered basalt with high strength basalt encountered below 2.9 m. The site classification for reactivity following the Australian Standard for Residential Slab and Footings (AS2870, 2010) was Class H1 (i.e., highly reactive). Site classification is based on $y_s$, the predicted design site surface movement, over the life of the house, which is based on design soil suction change profiles for different climatic regions of Australia.

Table 4.1 Description of Typical Soil Profile

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 – 0.12</td>
<td>Sandy Silt (ML), dark grey, moist</td>
</tr>
<tr>
<td>0.12 – 1.00</td>
<td>Silty Clay (CH), dark grey with pale grey mottling, stiff/moist</td>
</tr>
<tr>
<td>1.00 – 1.65</td>
<td>Silty Clay (CH), becoming pale grey, some fissuring, very stiff/moist</td>
</tr>
<tr>
<td>1.65 – 2.50</td>
<td>Silty Clay (CH), becoming friable, moist</td>
</tr>
<tr>
<td>2.50 - 2.9</td>
<td>Quaternary Basaltic Clay, highly weathered, pale brown, trace calcareous material</td>
</tr>
<tr>
<td>2.9</td>
<td>Auger refusal on high strength basalt</td>
</tr>
</tbody>
</table>

A plan of the instrumentation layout at the front yard of the property is showed in Fig. 4.3 and a view of the instrumented site is presented in Fig. 4.4. The instrumentation installed at the site includes:

- Automatic weather station
- HRM sap flow meters
- Neutron moisture probe (soli moisture contents)
- Surface movement probes
- Sub-surface movement probes
- ECHO 10HS Soil Moisture Sensor

Laboratory tests that were conducted during the initial site investigation and are being continued on soil sample taken from the Glenroy site at various time to complement the filed data include:
• Conventional soil classification (PL, LL and LS)
• Soil shrink-swell tests
• Gravimetric water content measurements
• Soil suction measurements using Dewpoint Potentiometer (WP4)
• Soil suction measurements using Wescor Hygrometer
• Soil suction measurements using filter paper method
• Soil-Water Characteristic Curve (SWCC)
• Triaxial tests
• Consolidation tests

Fig. 4.3 The instrumentation layout at the front yard of the experimental site
4.2.2 Automatic weather station

A Decagon automatic weather station (Fig. 4.5) was installed at the site and has been operating since 12 May 2011. To avoid obstructions of nearby buildings and trees, the weather station was mounted on a steel post of 3.2 m, which is fixed to the garage wall.

As shown in Fig. 4.6, the sensors fitted to the Decagon weather station measure:

1) Solar radiation
2) Rainfall
3) Temperature
4) Relative humidity
5) Wind speed and direction

The weather station is powered by 5 x AAA rechargeable batteries which last up to six months. 1 MB memory of Em50 data logger can store approximately 3400 readings (about two months of data storage). The weather station data are downloaded to a laptop computer monthly.
The temperature and rainfall data from the weather station site are presented in Fig. 4.7 and Fig. 4.8 respectively. The temperature is shown as maximum and minimum daily temperature plotted against date. Over the period between 12 May and 8 December 2011, 475 mm of rain was recorded. Fig. 4.9 presents daily evapotranspiration (mm/day) which was determined based on the recorded weather station data and the FAO-56 Penman-Monteith equation.

As discussed in Chapter 3, the effect of climate on the tree-soil-footing interaction can be represented as an appropriate surface boundary condition which accounts both infiltration and evaporation. The surface boundary condition used in the numerical model developed in this study is defined by the following equation:

\[
\frac{\partial \theta}{\partial t} = - \frac{\partial (q_i - q_e)}{\partial z}
\]  

(4.1)

where \( \theta \) is soil water content, \( q = q_i - q_e \) is the total water flow, \( q_i \) and \( q_e \) are rainfall rate and evaporation rate respectively. \( z \) is the vertical coordinate.
Fig. 4.6 The decagon weather station

Fig. 4.7 Maximum and minimum daily temperature at Glenroy site
Fig. 4.8 Rainfall data at Glenroy site

Fig. 4.9 Daily evapotranspiration data at Glenroy site
4.2.3 Sap flow measurement

In this project, sap flow in xylem tissue is measured by using a HRN30 sensor from ICT International, which is based on the Heat Ratio Method (HRM). The concept of HRM method has been introduced in Chapter 3. As shown in Fig. 4.10, two probes (downstream and upstream) measure the ratio of the increase in temperature, following the release of a pulse of heat, at points equidistant downstream and upstream from a line heater. The magnitude and direction of water flux are then calculated. Finally, the device calculates the sap flow by recording the ratio of heat transported to two symmetrically placed temperature sensors. The sap flow measurement cycle which is automatically performed by the microprocessor of HRM30 sensor is outlined in Fig. 4.11. As introduced in Chapter 3, sap velocity $V_s$ is calculated as:

$$V_s = \frac{V_c \rho_b (c_w + m_c c_s)}{\rho_s c_s}$$  \hspace{1cm} (4.2)

Fig. 4.10 Schematic diagram showing HRM30 sensor installed in a stem (ICT manual, 2010)
The sap flow measurement cycle (ICT manual, 2010)

1. Measurement cycle initiated
   (time interval set for this project = 30 minutes)

2. Acquire an 80 s average of the initial
temperature of each individual thermocouple

3. Initial sapwood temperature measured and averaged
   (mV outputs from the TC’s are measured and converted to temperature in °C using
   the Smart Interface)

4. Heat pulse released
   (An exact power input in Joules is supplied via the
   sensors microprocessor)

5. Increases in sapwood temperature measured, compared and averaged
   (Between 60 to 100 s, begin summing Δtemperature for thermocouples down and
   upstream from the heater to obtain the average ratio of downstream temperature
   increase to upstream temperature increase)

6. Mathematical calculations performed on downstream/upstream temperature
   ratios
   (the natural log of the ratio of temperature increase is used to calculate heat pulse
   velocity by multiplying it by thermal diffusivity (k), dividing by the distance (x)
   between either one of the probes and the heater and converting this value to cm hr⁻¹
   by multiplying by 3600)

7. Measurements are stored in the logger memory as either raw
   heat pulse velocity, or corrected sap velocity (Vs)

8. Estimating the area of sap wood and heart wood, calculate
   transpiration or Sap flow by sap velocity (Vs) multiply
   cross sectional area.

Fig. 4.11 The sap flow measurement cycle (ICT manual, 2010)
where: $V_c$ is corrected heat plus velocity, $\rho_b$ is the basic density of wood (dry weight/green volume), $c_m$ and $c_s$ are specific heat capacity of the wood matrix and sap water respectively, $m_w$ is water content of sapwood and $\rho_s$ is the density of water.

Volumetric sap flow (transpiration) can readily be derived as the product of sap velocity ($V_s$) and cross sectional area of conducting sapwood:

$$T_p = V_s \cdot S_{sap}$$  \hspace{1cm} (4.3)

where $T_p$ is transpiration rate (or total sap flow), $V_s$ is sap velocity and $S_{sap}$ is sap wood cross-sectional area. According to Goldstein et al. (1988), the gross wood cross-sectional area is calculated from its under-bark radius. Heartwood area is discounted by staining the sapwood or by observing the dark colour often associated with heartwood.

In this research, two eucalyptus trees were bought from the local nursery, one was planted at the field site, the other was cut into pieces (Figure 4.12a) and used to determine the wood properties such as the bark thickness, the radius of heartwood and sapwood, weight and volume of a fresh sapwood sample, and weight of an oven-dried sapwood sample etc. The measured heartwood radius $r_1$ and xylem radius $r_2$ are 1.89 mm and 5.56mm respectively (Figure 4.12b), with bark thickness = 2 mm and stem diameter $D_{tree} = 33.39 \text{mm}$. Thus the sap wood cross-sectional area, $S_{sap}$, can be calculated as follows:

$$S_{sap} = \pi \left( \frac{D_{tree}}{2} - d \right)^2 \left( \frac{r_2^2 - r_1^2}{r_2^2} \right) = 600.08 \text{mm}^2 \hspace{1cm} (4.4)$$

The data from HRM30 probes were analysed using the software SFT (Sap Flow Tool) provided by the instrument suppliers, ICT international Ltd. The measured
sap flow rate and cumulated sap volume of eucalyptus ficifolia tree between 12 May and 8 December 2011 are plotted in Fig. 4.13.

(a) Cross section of cut eucalyptus  
(b) Wood dimensions

**Fig. 4.12** Cross-section of tree trunk

**Fig. 4.13** The measured sap flow rate (cm$^3$/h) and cumulated sap volume (cm$^3$)
From Fig. 4.14, it can be seen that this tree was transpiring around 0.2 L per day in winter (between June and early September). Once the warm weather occurred, transpiration increased to 0.8 – 1 L per day (during October and early December). In the numerical modelling, the daily sap flow presented in Figure 4.14 are used to calculate the tree transpiration, $T_p$.

Fig. 4.15 shows the diurnal variation of sap flow rates during representative clear days in November 2011. It can be seen that diurnal courses of sap flow exhibited a bell shape curve, flow rates began to rise from nearly zero after sunrise, reached a maximum around 12:00 noon, then decreased gradually to nearly zero until midnight. From Fig. 4.16, it can be seen that the sap flow also closely correlated with changes in solar radiation.
Fig. 4.15 Diurnal variation of sap flow rate during representative clear days in November 2011

Fig. 4.16 Relationship between sap flow and solar radiation
4.2.4 Ground movement monitoring

As the nearest Lands Department Benchmark (LDBM) was located approximately 150 m from the research site and was not easily accessed from the site. A temporary benchmark (a deep survey datum) was established at the site. A construction of temporary benchmark is shown in Fig. 4.17(a).

As shown in Fig. 4.17(a), a galvanised steel rod of 25 mm diameter was anchored in concrete at the bottom. A sleeve made from a polyvinyl chloride (PVC) pipe of 100 mm diameter was placed over the stainless steel rod to isolate the rod from soil movements occurring above the bed rock. The annulus between the hole and the PVC sleeve was backfilled with a ten-percent bentonite grout, which provides a low permeability backfill so as to minimise downward migration of water along the borehole. At the surface, a locking cap was installed to protect the benchmark from disturbance.

![Diagram of benchmark and sub-surface movement probes](image)

(a) Benchmark (seated into the bedrock )
(b) Sub-surface movement Probe (0.5 m, 1 m, 2 m)

Fig 4.17 Details of the benchmark and sub-surface movement probes

Three sub-surface movement probes were installed in 120 mm diameter holes which were either manually or mechanically bored to depths of 0.5m, 1.0 m and 2.0 m. Fig. 4.17(b) shows a diagram of the design of a typical sub-surface
movement probe. It consists of a 25 mm diameter galvanised steel rod with a 65 mm diameter steel base plate enclosed within a 90 mm diameter PVC tube. The base plate is seated directly in contact with the soil in the bottom of the hole and the annular space between the boring wall and the PVC pipe was filled with a ten-percent bentonite grout.

Nine surface movement probes are installed at various distances from the eucalyptus tree on the test site so that the effect of tree root drying on ground movement could be monitored. The layout is illustrated in Fig. 4.3. The surface movement probes consists of a 170 mm long by 30 mm diameter galvanised steel rod embedded into a 150 mm diameter by100 mm high concrete pad.

Ground movements are monitored on a monthly basis by using a fully automatic, self-levelling Spectra Precision laser level capable of 0.01 mm resolution. All ground levels are measured against the benchmark. Ground movement results are reported to a precision of 0.1mm relative to the levels at installation. The measured soil movements with time at various locations are plotted in Fig. 4.18. From Fig. 4.18, it can be seen that soil movement decreased with depth and were cyclical with climate although the general tread was settlement. It is interesting to note that the maximum shrinking settlement occurred at approximately 1 m away from eucalyptus, not at 0.5 m - a distance closer to tree trunk. The measured data will be compared with the results predicted by the numerical model in Chapter 6.
4.2.5 Neutron moisture meter

The in situ soil moisture content at the site are measured by using a CPN 503DR neutron moisture meter (NMM) on a monthly base (Fig. 4.19). Six aluminium access tubes of 50 mm external diameter and 2.0 mm wall thickness are installed at different distances from the tree to monitor the moisture patterns of the surrounding soil (Fig. 4.20). All access tubes are sealed at the bottom and fitted with a screw cap at the top end to prevent the ingress of rain and debris. The layout of the access tubes is shown in Fig. 4.3.
The NMM consists of a source of fast (high-energy) neutrons, a thermal neutron detector, and the associated electronic equipment necessary to power the detector and to display the results. Soil water content is estimated by lowering the NMM probe into the ground through the access tube, and counting the number of thermalised neutrons that find their way back to the detector (Li and Ren, 2010). The main advantage of the neutron method is that repeated measurement of soil moisture can be made in the access tubes at any interval. However, correlation of neutron moisture meter data (i.e. neutron counts) with soil moisture contents is a very difficult task. The calibration relationship is influenced by the strength of the
neutron source, the size and type of the neutron detector, the position of the detector relative to the source, the position of the detector relative to the ground surface (or water table), the size and composition of the access tube, the physical and chemical properties of the soil, the dry density and the water content of the soil (Li et al. 2003). The problem is more complicated for expansive soils due to the fact that the soil volume and bulk density change as in situ soil moisture changes. To get a satisfactory calibration for expansive soils at the test site, a theoretical calibration which is based the multigroup neutron diffusion theory and finite element method was conducted. The complete presentation of the numerical model for neutron moisture probe calibration is beyond scope of this thesis. The details of the finite element model and numerical analyses are described in Li et al. (2003).

In order to apply the numerical analysis, it is necessary to first know the elemental composition of the soil. A total of 196 chemical analyses of the Glenroy soil at different depths were carried out. A total of seven theoretical calibration equations have been developed for the Glenroy soil at different depths. The calibration equation generally takes the form:

\[ m_v = A + B \times (CR) \]  

(4.5)

where \( A, B = \) calibration constants (depend on the chemical composition of the soil), \( CR = \) count rate.

Figures 4.21 -4.24 show the variation of the NMM moisture content profiles with time at four different locations. From Figures 4.21 to 4.24, a number of observations can be made. These include:

- The soil moisture changes mainly confined within top 1 m.
- The effect of tree root drying is evident at a distance of 0.5 and 1 m, where soil moisture decreased while at a distance larger than 2 m, the moisture of the top soil increased or decreased with the climate changes.
The values of measured moisture content are generally much lower above a depth of about 0.3m. This can be attributed to the equipment constraints. The neutron moisture meter measures the average moisture content in a ‘sphere’ of soil of variable radius between 0.1m and 0.3m (decreasing with increasing moisture content) (Li et al. 2003). For reading taken at depths of less than 0.3m, some of this volume extends above the ground surface where there is only air and, hence, the values will be low.

![Figure 4.21 NMM moisture content profiles at 0.5 m away from tree](image)

Fig. 4.21 NMM moisture content profiles at 0.5 m away from tree
Fig. 4.22 NMM moisture content profiles at 1 m away from tree

Fig. 4.23 NMM moisture content profiles at 2 m away from tree
4.3 Laboratory experiments

In order to get a better understanding of the field measured data and obtain soil properties and parameters required for the numerical modelling, a series of laboratory experiments were also performed on soil samples collected from the test site, which include soil suction measurement, soil consolidation tests, soil water characteristic curve tests, and soil swelling and shrinkage tests.

4.3.1 Soil suction and moisture content tests

In this study, a series of soil suction and moisture content tests were conducted on soil samples taken from the field site during the initial site investigation and in September 2011. All samples were sealed in steel tubes and labelled on extraction from the borehole, and then transported to RMIT geotechnical laboratory. While the gravimetric water content of soil was determined using the oven drying method in accordance with AS 1289.2.1.1 (1992), the soil suction was measured
using three different methods/equipments, i.e., thermocouple psychrometer (Wescor HR-33T), dewpoint potentiometer (WP4) and the filter paper method. Suction tests were mainly conducted in accordance with AS1289.2.2.1 (1998) using a Wescor HR-33T as shown in Fig. 4.25. The total suction of a soil was determined by measurement of the dewpoint temperature of a thermocouple in a small air space in equilibrium with the soil. The equipment was regularly cleaned and calibrated. Calibration was carried out in accordance with the manufacturer’s instructions by placing stacks of filter paper discs in the sample cup and saturating them with salt solution of various concentrations to achieve the desired suction. This equipment is accurate over a range of 3.2 pF to 4.8 pF ($u(pF) = \log_{10} u(MPa) + 4.01$).

Fig. 4.25 Wescor HR-33T used in this research

Fig. 4.26 Soil suction and moisture profiles
Fig. 4.26 shows the vertical soil suction and moisture profiles during the course of field experiment. It should be noted that each reported suction value, expressed in pF units, was taken as the average value of at least three sub-samples.

A WP4-T dewpoint potentiometer, as shown in Fig. 4.27, was also used for suction measurements. The results compared very well with those measured using Wescor HR-33T. WP4 uses the chilled-mirror dewpoint technique. It measures the total suction of a sample by determining the relative humidity of the air above the sample in a closed chamber. Once the sample comes into equilibrium with the vapour in WP4’s sealed chamber, the instrument finds relative humidity using the chilled mirror method. A tiny mirror in the chamber is chilled until dew just starts to form on it. At the dewpoint, the WP4 measures mirror and sample temperature with 0.001°C accuracy.

Dewpoint hygrometers measure dewpoint temperature of air in vapor equilibrium with a soil sample and sample temperature to determine relative humidity. The relative humidity of air in vapour equilibrium with the sample is related to water potential by the Kelvin equation:

\[
S = \frac{RT}{M_w} \ln(r_h)
\]

(4.6)
where $S$ is soil suction (MPa), $R$ is the universal gas constant (8.3145 J/mol K), $T$ is temperature (K), $M_w$ is the molecular mass of water (18.02 g/mol), and $r_h$ is relative humidity.

WP4 measures total soil suction from 0 to -300 MPa with an accuracy of ±0.1 MPa from 0 to -10 MPa and ±1% from -10 to -300 MPa. It was calibrated using a 0.5 Molal/kg solution of potassium chloride as per the guidelines provided by the manufacturer. As mentioned before, Wescor HR-33T is only applicable for suction ranging from pF 3.2 to pF 4.8. Therefore WP4 and the filter paper method were used to measure final suction of a sample after shrinkage and swelling tests respectively.

The maximum suction allowed for the SWCC testes was limited to the air entry value of the HAEV ceramic disc (i.e., 1500 kPa). However, for expansive clays, the actual SWCC might extend well beyond the 1500 kPa limit. To obtain a complete SWCC, WP4 was also used to measure the final suction of a sample after it was removed out of the SWCC device and dried in air or a vacuum desiccator for weeks. It should be pointed out that in this research all suction tests were conducted in a contact temperature room at 23±2 °C and with a relative humidity maintained between 40% and 60%.

### 4.3.2 Soil consolidation tests

Soil consolidation test provides data for the determination of the one-dimensional consolidation properties of soil. The elastic modulus and plastic modulus used in the numerical model were obtained from laboratory consolidation testing experiment. The implementation of the constitutive modelling has been introduced in Chapter 5.

The tests were carried in accordance with AS 1289.6.6.1 (1998). The sample was loaded step by step to a pressure of about 1920 kPa and then was unloaded in 4-6
steps. The deformation of the sample was measured for each load/unload stage. The loading was applied following the normal compression line (NCL) and the unloading is following the unloading-reloading line (URL). The objective of the test is to obtain the log elastic modulus \( \kappa \) in URL and log plastic modulus \( \lambda \) in NCL for constitutive modelling. The consolidation apparatus used in this research are shown in Fig. 4.28. During the tests, loading and unloading were applied though the lever. When applying a load, the vertical displacement was recorded. The test followed the equation:

\[
\begin{align*}
\frac{d\varepsilon_v^e}{v} &= \frac{\kappa}{p'} dp' \\
\frac{d\varepsilon_v^p}{v} &= \frac{\lambda - \kappa}{p'} dp'
\end{align*}
\]  

(4.7)

where \( v \) is the specific volume, \( \varepsilon_v^e \) is the volumetric strain in elastic region. \( \varepsilon_v^p \) is the volumetric strain in plastic region. \( \kappa \) is the logarithmic elastic modulus; \( \lambda \) is
the logarithmic hardening constant. In expansive soil, the relationship between strain and void ratio is:

\[ \varepsilon_v = \frac{e_0 - e}{1 + e} \]  

(4.8)

where \( \varepsilon_v \) is the volumetric strain, \( e \) is void ratio, \( e_0 \) is initial void ratio. The consolidation tests were conducted on two types of soil samples (silty clay and clay) collected from the Glenroy field site (Zhou et al. 2012).

**Sample 1: Silty clay at the Glenroy test site, Victoria \( \lambda = 0.09, \kappa = 0.02 \)**

**Table 4.2** Uploading data in silty clay consolidation test

<table>
<thead>
<tr>
<th>Uploading (kg)</th>
<th>Stress (kPa)</th>
<th>Displacement (mm)</th>
<th>Strain</th>
<th>( e_0 )</th>
<th>( e )</th>
<th>( 1+e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>13.81</td>
<td>-0.861</td>
<td>-0.04305</td>
<td>0.7251</td>
<td>0.650834</td>
<td>1.650834</td>
</tr>
<tr>
<td>0.4</td>
<td>27.62</td>
<td>-1.113</td>
<td>-0.05565</td>
<td>0.7251</td>
<td>0.629098</td>
<td>1.629098</td>
</tr>
<tr>
<td>0.8</td>
<td>55.25</td>
<td>-1.421</td>
<td>-0.07105</td>
<td>0.7251</td>
<td>0.602532</td>
<td>1.602532</td>
</tr>
<tr>
<td>1.8</td>
<td>124.31</td>
<td>-1.891</td>
<td>-0.09455</td>
<td>0.7251</td>
<td>0.561992</td>
<td>1.561992</td>
</tr>
<tr>
<td>3.8</td>
<td>262.43</td>
<td>-2.574</td>
<td>-0.1287</td>
<td>0.7251</td>
<td>0.50308</td>
<td>1.50308</td>
</tr>
<tr>
<td>7.8</td>
<td>538.67</td>
<td>-3.453</td>
<td>-0.17265</td>
<td>0.7251</td>
<td>0.427261</td>
<td>1.427261</td>
</tr>
<tr>
<td>11.8</td>
<td>814.92</td>
<td>-3.845</td>
<td>-0.19225</td>
<td>0.7251</td>
<td>0.39345</td>
<td>1.39345</td>
</tr>
<tr>
<td>15.8</td>
<td>1091.16</td>
<td>-4.159</td>
<td>-0.20795</td>
<td>0.7251</td>
<td>0.366365</td>
<td>1.366365</td>
</tr>
<tr>
<td>19.8</td>
<td>1367.40</td>
<td>-4.429</td>
<td>-0.22145</td>
<td>0.7251</td>
<td>0.343077</td>
<td>1.343077</td>
</tr>
<tr>
<td>23.8</td>
<td>1643.65</td>
<td>-4.623</td>
<td>-0.23115</td>
<td>0.7251</td>
<td>0.326343</td>
<td>1.326343</td>
</tr>
<tr>
<td>27.8</td>
<td>1919.89</td>
<td>-4.871</td>
<td>-0.24355</td>
<td>0.7251</td>
<td>0.304952</td>
<td>1.304952</td>
</tr>
</tbody>
</table>

**Table 4.3** unloading data in silty clay consolidation test

<table>
<thead>
<tr>
<th>Uploading (kg)</th>
<th>Stress (kPa)</th>
<th>Displacement (mm)</th>
<th>Strain</th>
<th>( e_0 )</th>
<th>( e )</th>
<th>( 1+e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.8</td>
<td>1919.89</td>
<td>-4.871</td>
<td>-0.24355</td>
<td>0.7251</td>
<td>0.304952</td>
<td>1.304952</td>
</tr>
<tr>
<td>18.8</td>
<td>1298.34</td>
<td>-4.782</td>
<td>-0.2391</td>
<td>0.7251</td>
<td>0.312629</td>
<td>1.312629</td>
</tr>
<tr>
<td>8.8</td>
<td>607.73</td>
<td>-4.586</td>
<td>-0.2293</td>
<td>0.7251</td>
<td>0.329535</td>
<td>1.329535</td>
</tr>
<tr>
<td>0.2</td>
<td>13.81</td>
<td>-3.386</td>
<td>-0.1693</td>
<td>0.7251</td>
<td>0.433041</td>
<td>1.433041</td>
</tr>
</tbody>
</table>
Fig. 4.29 Normal compression and Unloading-reloading lines for silty clay

**Sample 2: Clay at the Glenroy test site, Victoria** $\lambda = 0.12 \quad \kappa = 0.025$

Table 4.4 Uploading data in clay consolidation test

<table>
<thead>
<tr>
<th>Unloading (kg)</th>
<th>Stress (kPa)</th>
<th>Displacement (mm)</th>
<th>Strain</th>
<th>$e_0$</th>
<th>$e$</th>
<th>$1+e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>13.81</td>
<td>-0.943</td>
<td>-0.04715</td>
<td>0.8094</td>
<td>0.724087</td>
<td>1.724087</td>
</tr>
<tr>
<td>0.4</td>
<td>27.62</td>
<td>-1.197</td>
<td>-0.05985</td>
<td>0.8094</td>
<td>0.701107</td>
<td>1.701107</td>
</tr>
<tr>
<td>0.8</td>
<td>55.25</td>
<td>-1.487</td>
<td>-0.07435</td>
<td>0.8094</td>
<td>0.674871</td>
<td>1.674871</td>
</tr>
<tr>
<td>1.8</td>
<td>124.31</td>
<td>-2.021</td>
<td>-0.10105</td>
<td>0.8094</td>
<td>0.62656</td>
<td>1.62656</td>
</tr>
<tr>
<td>3.8</td>
<td>262.43</td>
<td>-2.658</td>
<td>-0.1329</td>
<td>0.8094</td>
<td>0.568931</td>
<td>1.568931</td>
</tr>
<tr>
<td>7.8</td>
<td>538.67</td>
<td>-3.402</td>
<td>-0.1701</td>
<td>0.8094</td>
<td>0.501621</td>
<td>1.501621</td>
</tr>
<tr>
<td>15.8</td>
<td>1091.16</td>
<td>-4.204</td>
<td>-0.2102</td>
<td>0.8094</td>
<td>0.429064</td>
<td>1.429064</td>
</tr>
<tr>
<td>23.8</td>
<td>1643.65</td>
<td>-4.704</td>
<td>-0.2352</td>
<td>0.8094</td>
<td>0.383829</td>
<td>1.383829</td>
</tr>
</tbody>
</table>

Table 4.5 Unloading data in clay consolidation test

<table>
<thead>
<tr>
<th>Uploading (kg)</th>
<th>Stress (kPa)</th>
<th>Displacement (mm)</th>
<th>Strain</th>
<th>$e_0$</th>
<th>$e$</th>
<th>$1+e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.8</td>
<td>1643.65</td>
<td>-4.704</td>
<td>-0.2352</td>
<td>0.8094</td>
<td>0.383829</td>
<td>1.383829</td>
</tr>
<tr>
<td>13.8</td>
<td>953.04</td>
<td>-4.59</td>
<td>-0.2295</td>
<td>0.8094</td>
<td>0.394143</td>
<td>1.394143</td>
</tr>
<tr>
<td>8.8</td>
<td>607.73</td>
<td>-4.451</td>
<td>-0.22255</td>
<td>0.8094</td>
<td>0.406718</td>
<td>1.406718</td>
</tr>
<tr>
<td>0.2</td>
<td>13.81</td>
<td>-3.549</td>
<td>-0.17745</td>
<td>0.8094</td>
<td>0.488322</td>
<td>1.488322</td>
</tr>
</tbody>
</table>
4.3.3 Soil water characteristic curve test

In this research, soil water characteristic curve was measured using a Fredlund SWCC Device (Figure 4.31). The Fredlund SWCC Device is a simple unsaturated soil testing apparatus with great flexibility for applying soil suctions while following various stress paths. The device can be used to obtain the complete soil water characteristic curve and allows to control matric suctions from near zero values up to 1500 kPa. The pore-air pressure, at the top of the specimen is controlled through the use of dual pressure regulators and precise gauges.

The stainless steel cell is constructed with simple knobs and screws for fast soil specimen setups. The device includes a pressure panel with dual gauges and regulators for increased precision in the low-pressure range. The apparatus also includes the necessary plumbing and valves for periodic flushing and measuring of diffused air. During a test, the soil specimen can either be subjected to a token vertical load, or a load similar to the overburden pressure in the field. The in situ overburden pressure can be applied with this apparatus by simply adding dead
weights to the weight plate. Fig. 4.31 shows the components of the device. The components and their effects in experiment are listed in Table 4.6.

![Fig. 4.31 Component of Fredlund SWCC Device](image)

**Table 4.6 Components and their effects in a SWCC device**

<table>
<thead>
<tr>
<th>Number</th>
<th>Component</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>High air pressure supplier</td>
<td>Provide air pressure</td>
</tr>
<tr>
<td>(2)</td>
<td>By-pass valve control</td>
<td>Control the magnitude of air pressure</td>
</tr>
<tr>
<td>(3)</td>
<td>Pressure booster</td>
<td>Boost air pressure</td>
</tr>
<tr>
<td>(4)</td>
<td>Water volume change indicator</td>
<td>Measure the volume of water discharged</td>
</tr>
<tr>
<td>(5)</td>
<td>Loading frame</td>
<td>Apply vertical load (Optional)</td>
</tr>
<tr>
<td>(6)</td>
<td>Stainless cell</td>
<td>Container of soil sample</td>
</tr>
<tr>
<td>(7)</td>
<td>Dial gage</td>
<td>Measure the soil vertical displacement</td>
</tr>
<tr>
<td>(8)</td>
<td>Syringe</td>
<td>Flush air bubble</td>
</tr>
</tbody>
</table>
Before the test, the initial mass \( m_0 \) and the initial volume \( V_0 \) of specimen are recorded. Initial water content \( w_i \) is measured and initial void ratio \( e_0 \) is calculated. Thus, initial water mass, \( m_w^0 \), initial water volume \( V_w^0 \), the mass of soil particles \( m_s \) and the volume of soil particles, \( V_s \), are obtained. During the experiment, the mass of soil particles \( m_s \) and the volume of soil particles \( V_s \) are two constant numbers.

In the test, in order to measure a soil water characteristic curve, the Fredlund SWCC device monitors both water discharge \( \Delta m_w \) using a water volume change indicator (component 4) and vertical displacement \( \Delta L \) (\( \Delta V = \Delta L \times \text{cross-section area} \)) using a digit dial gauge with a resolution of 1 \( \mu \text{m} \) (component 7). The new water mass \( m_w^1 \) is obtained:

\[
m_w^1 = m_w^0 - \Delta m_w
\]  

(4.8)

The new water volume \( V_w^1 \) is calculated:

\[
V_w^1 = V_w^0 - \Delta V
\]  

(4.9)

Then the new water content is:

\[
w_i = \frac{m_w^1}{m_s}
\]  

(4.10)

The new void ratio is:

\[
e_i = \frac{V_w^1}{V_s}
\]  

(4.11)

The Degree of saturation is calculated as:
\[ S_r = \frac{e_i}{w_i G_r} \]  

(4.12)

After the new water content, \( w_1 \), new void ratio \( e_i \) and degree of saturation \( S_r \) is obtained, it is able to draw soil water character curves: \( w \) vs \( \ln s \) and \( S_r \) vs \( \ln s \), and \( e \) vs \( \ln s \) curve (the relationship between void ratio and suction).

Before testing, the ‘undisturbed’ soil sample was first saturated with distilled water in a container for about two weeks. The soil sample was carefully trimmed to size and weighted. The initial water content was measured by the oven dry method. Then the specimen was placed into the SWCC cell and a pressure of about 10 kPa was applied gradually to the specimen and kept constant through the test. During the test the vertical displacement and water discharge were recorded. The details are provided in Table 4.8.

The maximum matric suction allowed for the SWCC testes is limited to 1500 kPa, the air entry value of the HAEV ceramic stone. In this study, the suction range above 1500 kPa (i.e., total suction) was obtained by using either a Dewpoint Potentiometer (WP4) or the filter paper method. Van Genuchten equation (1980) was used in this study as it has been proven to be the best fitting equation for SWCC of unsaturated soils.

Considering the requirement of numerical modelling, in this research, SWCC tests were carried out on three different type soils, two (silty clay and clay) from Glenroy test site, Victoria and one (extremely expansive clay) from Walkley Heights, South Australian. The results are shown below.
<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut soil sample</td>
<td></td>
</tr>
<tr>
<td>Saturate the sample in a container</td>
<td>filled with distilled water</td>
</tr>
<tr>
<td>Weight the sample</td>
<td></td>
</tr>
<tr>
<td>Place sample into the SWCC cell</td>
<td></td>
</tr>
</tbody>
</table>
**Sample 1: Glenroy silty clay, Victoria**

**Table 4.8** Initial condition of SWCC test (Glenroy silty clay, Victoria)

<table>
<thead>
<tr>
<th>Initial parameters</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring height $h$</td>
<td>$20 \ mm$</td>
</tr>
<tr>
<td>Ring diameter $D_{ring}$</td>
<td>$45 \ mm$</td>
</tr>
<tr>
<td>Mass of specimen $m_0$</td>
<td>$61.18 \ g$</td>
</tr>
<tr>
<td>Initial water content $w_i$</td>
<td>$27.35%$</td>
</tr>
<tr>
<td>Mass of soil particles $m_s$</td>
<td>$48.03 \ g$</td>
</tr>
<tr>
<td>Volume of soil particles $V_s$</td>
<td>$18429.09 \ mm^3$</td>
</tr>
<tr>
<td>Mass of water $m_w^0$</td>
<td>$13.14 \ g$</td>
</tr>
<tr>
<td>Volume of water $V_w^0$</td>
<td>$13140 \ mm^3$</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>$0.7251$</td>
</tr>
</tbody>
</table>
Table 4.9 Water discharge and calculation of water content (Glenroy silty clay, Victoria)

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Reading on tube (mm)</th>
<th>water discharge by reading</th>
<th>water discharged (ml)</th>
<th>weight of water, Ww (g)</th>
<th>water content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>84</td>
<td>0</td>
<td>0</td>
<td>13.14</td>
<td>27.36324</td>
</tr>
<tr>
<td>10</td>
<td>92</td>
<td>8</td>
<td>0.56368</td>
<td>12.58</td>
<td>26.18978</td>
</tr>
<tr>
<td>20</td>
<td>115</td>
<td>31</td>
<td>2.18426</td>
<td>10.96</td>
<td>22.81609</td>
</tr>
<tr>
<td>50</td>
<td>143</td>
<td>59</td>
<td>4.15714</td>
<td>8.99</td>
<td>18.70899</td>
</tr>
<tr>
<td>100</td>
<td>165</td>
<td>81</td>
<td>5.70726</td>
<td>7.44</td>
<td>15.48199</td>
</tr>
<tr>
<td>200</td>
<td>173</td>
<td>89</td>
<td>6.27094</td>
<td>6.87</td>
<td>14.30853</td>
</tr>
<tr>
<td>400</td>
<td>184</td>
<td>100</td>
<td>7.046</td>
<td>6.10</td>
<td>12.69502</td>
</tr>
<tr>
<td>800</td>
<td>196</td>
<td>112</td>
<td>7.89152</td>
<td>5.25</td>
<td>10.93484</td>
</tr>
<tr>
<td>1000</td>
<td>200</td>
<td>116</td>
<td>8.17336</td>
<td>4.97</td>
<td>10.34811</td>
</tr>
</tbody>
</table>

Table 4.10 Volume change and calculation of void ratio and Degree of saturation (Glenroy silty clay, Victoria)

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Disp. Reading (mm)</th>
<th>Disp. volume (mm3)</th>
<th>Total Volume (mm3)</th>
<th>e = (V-Vij)/Vs assuming A=Const</th>
<th>Sij (%)</th>
<th>ln (s)</th>
<th>Van</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>31792.5</td>
<td>0.725126</td>
<td>100</td>
<td>0</td>
<td>0.991271</td>
</tr>
<tr>
<td>10</td>
<td>0.09</td>
<td>143.0663</td>
<td>31649.43</td>
<td>0.717363</td>
<td>96.74732</td>
<td>2.302585</td>
<td>0.900692</td>
</tr>
<tr>
<td>20</td>
<td>0.12</td>
<td>190.755</td>
<td>31601.75</td>
<td>0.714775</td>
<td>84.58975</td>
<td>2.995732</td>
<td>0.861321</td>
</tr>
<tr>
<td>50</td>
<td>0.16</td>
<td>254.34</td>
<td>31538.16</td>
<td>0.711325</td>
<td>69.69928</td>
<td>3.912023</td>
<td>0.790417</td>
</tr>
<tr>
<td>100</td>
<td>0.21</td>
<td>333.8213</td>
<td>31458.68</td>
<td>0.707012</td>
<td>58.02909</td>
<td>4.60517</td>
<td>0.721473</td>
</tr>
<tr>
<td>200</td>
<td>0.22</td>
<td>349.7175</td>
<td>31442.78</td>
<td>0.706149</td>
<td>53.69628</td>
<td>5.298317</td>
<td>0.640617</td>
</tr>
<tr>
<td>400</td>
<td>0.32</td>
<td>508.68</td>
<td>31283.82</td>
<td>0.697524</td>
<td>48.23034</td>
<td>5.991465</td>
<td>0.551478</td>
</tr>
<tr>
<td>800</td>
<td>0.3</td>
<td>476.8875</td>
<td>31315.61</td>
<td>0.699249</td>
<td>41.44064</td>
<td>6.684612</td>
<td>0.459574</td>
</tr>
<tr>
<td>1000</td>
<td>0.27</td>
<td>429.1988</td>
<td>31363.3</td>
<td>0.701837</td>
<td>39.07247</td>
<td>6.907755</td>
<td>0.430422</td>
</tr>
</tbody>
</table>
Fig. 4.32  $e - \ln s$ curve (Glenroy silty clay, Victoria)

Fig. 4.33 Volumetric water content versus soil suction for Glenroy silty clay
Fig. 4.34 Degree of saturation versus soil suction for Glenroy silty clay (Van Genuchten equation: $a_d = 1600 \, MPa, m_d = 1.3, n_d = 0.6$)

**Sample 2: Glenroy clay, Victoria**

Table 4.11 Initial condition of SWCC test (Glenroy clay, Victoria)

<table>
<thead>
<tr>
<th>Initial parameters</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring height $h$</td>
<td>20 mm</td>
</tr>
<tr>
<td>Ring diameter $D_{ring}$</td>
<td>45 mm</td>
</tr>
<tr>
<td>Mass of specimen $m_0$</td>
<td>62.18g</td>
</tr>
<tr>
<td>Initial water content $w_i$</td>
<td>31.67%</td>
</tr>
<tr>
<td>Mass of soil particles $m_s$</td>
<td>47.22g</td>
</tr>
<tr>
<td>Volume of soil particles $V_s$</td>
<td>17283.87 mm$^3$</td>
</tr>
<tr>
<td>Mass of water $m_w^0$</td>
<td>14.96g</td>
</tr>
<tr>
<td>Volume of water $V_w^0$</td>
<td>14960 mm$^3$</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.8094</td>
</tr>
</tbody>
</table>
Table 4.12 Water discharge and calculation of water content (Glenroy clay, Victoria)

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Reading on tube (mm)</th>
<th>water discharge by reading</th>
<th>water discharged (ml)</th>
<th>weight of water, Ww (g)</th>
<th>water content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>123</td>
<td>0</td>
<td>0</td>
<td>14.96</td>
<td>31.67668</td>
</tr>
<tr>
<td>20</td>
<td>140</td>
<td>17</td>
<td>1.19782</td>
<td>13.76</td>
<td>29.14009</td>
</tr>
<tr>
<td>50</td>
<td>148</td>
<td>25</td>
<td>1.7615</td>
<td>13.20</td>
<td>27.9464</td>
</tr>
<tr>
<td>100</td>
<td>160</td>
<td>37</td>
<td>2.60702</td>
<td>12.35</td>
<td>26.15587</td>
</tr>
<tr>
<td>200</td>
<td>176</td>
<td>53</td>
<td>3.73438</td>
<td>11.22</td>
<td>23.7685</td>
</tr>
<tr>
<td>400</td>
<td>194</td>
<td>71</td>
<td>5.00266</td>
<td>9.96</td>
<td>21.0827</td>
</tr>
<tr>
<td>800</td>
<td>230</td>
<td>107</td>
<td>7.53922</td>
<td>7.42</td>
<td>15.7111</td>
</tr>
<tr>
<td>1000</td>
<td>245</td>
<td>122</td>
<td>8.59612</td>
<td>6.36</td>
<td>13.47294</td>
</tr>
</tbody>
</table>

Table 4.13 Volume change and calculation of void ratio and Degree of saturation (Glenroy clay, Victoria)

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Disp. Reading (mm)</th>
<th>Disp. volume (mm3)</th>
<th>Total Volume (mm3)</th>
<th>e = (V - V_s)/V_s assuming A=Const</th>
<th>S_s (%)</th>
<th>In (s)</th>
<th>Van</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>31792.5</td>
<td>0.809432</td>
<td>100</td>
<td>0</td>
<td>0.999752</td>
</tr>
<tr>
<td>20</td>
<td>0.51</td>
<td>810.7088</td>
<td>30981.79</td>
<td>0.792526</td>
<td>97.4368</td>
<td>2.995732</td>
<td>0.971611</td>
</tr>
<tr>
<td>50</td>
<td>0.6</td>
<td>953.775</td>
<td>30838.73</td>
<td>0.784249</td>
<td>94.4317</td>
<td>3.912023</td>
<td>0.937632</td>
</tr>
<tr>
<td>100</td>
<td>0.7</td>
<td>1112.738</td>
<td>30679.76</td>
<td>0.775052</td>
<td>89.43022</td>
<td>4.60517</td>
<td>0.889876</td>
</tr>
<tr>
<td>200</td>
<td>0.76</td>
<td>1208.115</td>
<td>30584.39</td>
<td>0.769534</td>
<td>81.85025</td>
<td>5.298317</td>
<td>0.8132</td>
</tr>
<tr>
<td>400</td>
<td>0.84</td>
<td>1335.285</td>
<td>30457.22</td>
<td>0.762176</td>
<td>73.30218</td>
<td>5.991465</td>
<td>0.701843</td>
</tr>
<tr>
<td>800</td>
<td>1</td>
<td>1589.625</td>
<td>30202.88</td>
<td>0.74746</td>
<td>55.70118</td>
<td>6.684612</td>
<td>0.561359</td>
</tr>
<tr>
<td>1000</td>
<td>1.26</td>
<td>2002.928</td>
<td>29789.57</td>
<td>0.723548</td>
<td>49.34476</td>
<td>6.907755</td>
<td>0.512982</td>
</tr>
</tbody>
</table>
Fig. 4.35 $e - \ln s$ curve (Glenroy clay, Victoria)

Fig. 4.36 Volumetric water content versus soil suction for Glenroy clay
Fig. 4.37 Degree of saturation *versus* soil suction for Glenroy clay (Van Genuchten equation: $a_d = 900 MPa$, $m_d = 0.9$, $n_d = 0.9$)

**Sample 3: Walkley Heights clay, South Australia**

**Table 4.14**  Initial condition of SWCC test (Walkley Heights clay, South Australia)

<table>
<thead>
<tr>
<th>Initial parameters</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring height $h$</td>
<td>20 mm</td>
</tr>
<tr>
<td>Ring diameter $D_{ring}$</td>
<td>45 mm</td>
</tr>
<tr>
<td>Mass of specimen $m_0$</td>
<td>59.89g</td>
</tr>
<tr>
<td>Initial water content $w_i$</td>
<td>30.85%</td>
</tr>
<tr>
<td>Mass of soil particles $m_s$</td>
<td>45.76g</td>
</tr>
<tr>
<td>Volume of soil particles $V_t$</td>
<td>17489.51 mm$^3$</td>
</tr>
<tr>
<td>Mass of water $m_w^0$</td>
<td>14.12g</td>
</tr>
<tr>
<td>Volume of water $V_w^0$</td>
<td>14120 mm$^3$</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.8178</td>
</tr>
</tbody>
</table>
**Table 4.15** Water discharge and calculation of water content (Walkley Heights clay, South Australia)

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Reading on tube (mm)</th>
<th>water discharge by reading</th>
<th>water discharged (ml)</th>
<th>weight of water, Ww (g)</th>
<th>water content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>72</td>
<td>0</td>
<td>0</td>
<td>14.12</td>
<td>30.86053</td>
</tr>
<tr>
<td>10</td>
<td>78</td>
<td>6</td>
<td>0.42276</td>
<td>13.70</td>
<td>29.9368</td>
</tr>
<tr>
<td>50</td>
<td>84</td>
<td>12</td>
<td>0.84552</td>
<td>13.28</td>
<td>29.01306</td>
</tr>
<tr>
<td>100</td>
<td>92</td>
<td>20</td>
<td>1.4092</td>
<td>12.71</td>
<td>27.78141</td>
</tr>
<tr>
<td>150</td>
<td>100</td>
<td>28</td>
<td>1.97288</td>
<td>12.15</td>
<td>26.54976</td>
</tr>
<tr>
<td>200</td>
<td>106</td>
<td>34</td>
<td>2.39564</td>
<td>11.73</td>
<td>25.62603</td>
</tr>
<tr>
<td>300</td>
<td>114</td>
<td>42</td>
<td>2.95932</td>
<td>11.16</td>
<td>24.39438</td>
</tr>
<tr>
<td>400</td>
<td>120</td>
<td>48</td>
<td>3.38208</td>
<td>10.74</td>
<td>23.47064</td>
</tr>
<tr>
<td>800</td>
<td>132</td>
<td>60</td>
<td>4.2276</td>
<td>9.90</td>
<td>21.62317</td>
</tr>
<tr>
<td>1000</td>
<td>146</td>
<td>74</td>
<td>5.21404</td>
<td>8.91</td>
<td>19.46778</td>
</tr>
</tbody>
</table>

**Table 4.16** Volume change and calculation of void ratio and Degree of saturation (Walkley Heights clay, South Australia)

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>Disp. Reading (mm)</th>
<th>Disp. volume (mm3)</th>
<th>Total Volume (mm3)</th>
<th>e = (V - Vv)/Vv assuming A=Const</th>
<th>Sr (%)</th>
<th>ln (s)</th>
<th>Van</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>31792.5</td>
<td>0.817804</td>
<td>100</td>
<td>0</td>
<td>0.999686</td>
</tr>
<tr>
<td>10</td>
<td>0.06</td>
<td>95.3775</td>
<td>31697.12</td>
<td>0.812351</td>
<td>97.65796</td>
<td>2.302585</td>
<td>0.987625</td>
</tr>
<tr>
<td>50</td>
<td>0.08</td>
<td>127.17</td>
<td>31665.33</td>
<td>0.810533</td>
<td>94.85686</td>
<td>3.912023</td>
<td>0.956454</td>
</tr>
<tr>
<td>100</td>
<td>0.17</td>
<td>270.2363</td>
<td>31522.26</td>
<td>0.802353</td>
<td>91.75607</td>
<td>4.60517</td>
<td>0.926368</td>
</tr>
<tr>
<td>150</td>
<td>0.23</td>
<td>365.6138</td>
<td>31426.89</td>
<td>0.796899</td>
<td>88.28827</td>
<td>5.010635</td>
<td>0.900735</td>
</tr>
<tr>
<td>200</td>
<td>0.27</td>
<td>429.1988</td>
<td>31363.3</td>
<td>0.793264</td>
<td>85.60704</td>
<td>5.298317</td>
<td>0.877939</td>
</tr>
<tr>
<td>300</td>
<td>0.32</td>
<td>508.68</td>
<td>31283.82</td>
<td>0.788719</td>
<td>81.96211</td>
<td>5.703782</td>
<td>0.838165</td>
</tr>
<tr>
<td>400</td>
<td>0.37</td>
<td>588.1613</td>
<td>31204.34</td>
<td>0.784175</td>
<td>79.31547</td>
<td>5.991465</td>
<td>0.803893</td>
</tr>
<tr>
<td>800</td>
<td>0.46</td>
<td>731.2275</td>
<td>31061.27</td>
<td>0.775995</td>
<td>73.8425</td>
<td>6.684612</td>
<td>0.699232</td>
</tr>
<tr>
<td>1000</td>
<td>0.52</td>
<td>826.605</td>
<td>30965.9</td>
<td>0.770541</td>
<td>66.95244</td>
<td>6.907755</td>
<td>0.659178</td>
</tr>
</tbody>
</table>
Fig. 4.38 $e - \ln s$ curve (Walkley Heights clay, South Australia)

Fig. 4.39 Volumetric water content versus soil suction for Walkley Heights clay
Fig. 4.40 Degree of saturation versus soil suction for Walkley Heights clay (Van Genuchten equation: $a_d = 3000 \text{MPa}, m_d = 0.8, n_d = 1.2$)

4.3.4 Swelling and shrinkage test

As reviewed in Chapter 2, the Australian Standard (AS2870-2011) suggested an empirical equation to calculate the expansive soil surface movement:

$$ y_s = \sum_{n=1}^{N} (\Delta u \cdot h \cdot I_{pi})_n $$

(4.13)

where $y_s$ is characteristic surface movement, $I_{pi}$ is instability index, in % picofarads (pF). $\Delta u$ is soil suction change averaged over the thickness of the layer under consideration. $h$ is thickness of soil layer. $n$ is number of soil layers. According to Cameron (1989) and his later research (Cameron 2001, 2002), the instability index could be measured as a shrinkage and swelling index $I_{ss}$. 

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The Shrinkage swell test is to perform both core shrinkage and swell test at same time by using two samples but both cut from same core sample. The shrinkage swell index $I_{ss}$ can be calculated after the combination of both tests results. The equation is:

$$I_{ss} = \frac{(\varepsilon_{sw}) + \varepsilon_{sh}}{1.8}$$

(4.14)

where $I_{ss}$ is the shrinkage and swelling index. $\varepsilon_{sw}$ is the magnitude of the swelling strain and $\varepsilon_{sh}$ is the magnitude of the total shrinkage strain to the oven dry condition.

In a shrinkage test (Fig. 4.41), both ends of a sample were trimmed very carefully and then positioned with a drawing pin at the centre as a reference point so the reading can be taken with high accuracy. The sample was left in the air to dry out during the day and was wrapped with a plastic membrane during night. The reason for doing that was to avoid the large crack caused by drying out too quick. The changes in length, weight and diameter of shrinkage sample were recorded regularly.

Fig. 4.41 the shrinkage test
Fig. 4.42 the swelling test

Fig. 4.43 the swelling curve of soil samples

Fig. 4.44 the shrinkage curve of soil samples
In a swelling test (Fig. 4.43), the sample was setting into a consolidation ring with flat top and bottom surfaces. The ring was put in a consolidation cell is fulfilled with distilled water. Then the consolidation cell was placed into a loading device where a 25 kPa seating load is applied. The reading of changing in axial strain versus time was record by computer program. The test was continuing until the difference in the movement of the sample is less than 5% between the last two readings or two week time.

In this analysis, two different samples were tested the swelling and shrinkage behaviour. They were silty clay and clay in Glenroy field, the swelling and shrinkage curve of the samples are plotted in Fig 4.43 and Fig. 4.44. As shown in the figures, For silty clay in Glenroy field, the shrinkage stain $\varepsilon_{sh} = 7.98\%$, the swelling strain $\varepsilon_{sw} = 1.35\%$ the swelling and shrinkage index:

$$I_{ss} = \frac{(\varepsilon_{sw}) + \varepsilon_{sh}}{2} = \frac{1.35}{2} + \frac{7.98}{1.8} = 4.80\% / pF$$

(4.15)

For clay in Glenroy field, the shrinkage stain $\varepsilon_{sh} = 5.01\%$, the swelling strain $\varepsilon_{sw} = 1.15\%$ the swelling and shrinkage index:

$$I_{ss} = \frac{(\varepsilon_{sw}) + \varepsilon_{sh}}{2} = \frac{1.15}{2} + \frac{5.01}{1.8} = 3.10\% / pF$$

(4.16)

4.4 Calibration of volume change behaviour in constitutive modelling by the experimental results

The soil parameters obtained form the laboratory experiments are summarised in Table 4.17. The results of the laboratory experiments were used to compare with the volume change behaviour predicted by different constitutive models for
unsaturated soil. This work has significant meaning in the investigation of the mechanical and hydraulic behaviour of expansive soil.

As shown in Table 4.17, firstly, in the soil consolidation test, the logarithmic bulk modulus $\kappa$ and the logarithmic hardening constant $\lambda$ were obtained. Secondly, the procedure of a soil water characteristic test is fundamentally a drying process. Because the soil samples for these two tests are the same, by applying $\lambda$ and $\kappa$ obtained in the consolidation test into the drying process in the soil water characteristic test by using different constitutive modelling, the theoretical results of drying curve can be worked out and compared with the experimental drying curve.

<table>
<thead>
<tr>
<th>Soil sample</th>
<th>$\lambda$</th>
<th>$\kappa$</th>
<th>$e_0$</th>
<th>$s_{ra}$ (kPa)</th>
<th>$s_{ae}$ (kPa)</th>
<th>$s_{re}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenroy silty clay</td>
<td>0.09</td>
<td>0.02</td>
<td>0.725</td>
<td>25</td>
<td>60</td>
<td>10000</td>
</tr>
<tr>
<td>Glenroy clay</td>
<td>0.12</td>
<td>0.025</td>
<td>0.8094</td>
<td>25</td>
<td>200</td>
<td>15700</td>
</tr>
<tr>
<td>Walkley Height clay</td>
<td>N/A</td>
<td>N/A</td>
<td>0.8174</td>
<td>20</td>
<td>250</td>
<td>12000</td>
</tr>
</tbody>
</table>

4.4.1 Effective stress model

The effective stress equation for unsaturated soils can be written as:

$$\sigma' = (\sigma - \mu_a) + \chi(\mu_a - \mu_w)$$  \hspace{1cm} (4.17)

where $\mu_a - \mu_w$ is matric suction. $\mu_a$ and $\mu_w$ are pore air pressure and pore water pressure respectively. $\chi$ is a parameter related to the degree of saturation of the soil. In the soil water characteristic curve test, the soil sample was fully saturated before put into the cell, i.e. pore air pressure $\mu_a = 0$. Thus the matric suction is equal to the total suction and Eq. 4.17 is simplified to:
\[ \sigma' = \sigma + \chi \cdot s \]  \hspace{1cm} (4.18)

As discussed in Chapter 3, Khalili and Khabbaz (1998) attempted to use Brooks and Corey SWCC equation (1964) to define parameter \( \chi \) by suggesting \( u_d = 0.55 \):

\[
\chi = \begin{cases} 
\left( \frac{s_{ae}}{s} \right)^{0.55} & (s \geq s_{ae}) \\
1 & (s < s_{ae})
\end{cases}
\]  \hspace{1cm} (4.19)

where \( s_{ae} \) is soil suction in air entry value. Combining Eq. 4.18 and Eq. 4.19:

\[
\begin{cases} 
\sigma' = \sigma + s & (s \leq s_{ae}) \\
\sigma' = \sigma + \left( \frac{s_{ae}}{s} \right)^{0.55} s & (s > s_{ae})
\end{cases}
\]  \hspace{1cm} (4.20)

In drying process, according to the Com-clay model, the constitutive equation is:

\[ d\varepsilon_v = \frac{\dot{\lambda}}{\nu} \cdot \frac{dp'}{p'} \]  \hspace{1cm} (4.21)

Where \( \varepsilon_v \) is the volumetric strain, \( \nu \) is the specific volume. In the SWCC tests conducted in this research, \( \sigma \) is a constant value of 11 kPa. Thus \( dp' = ds \). The constitutive equation can written as:

\[
\begin{cases} 
\frac{d\varepsilon_v}{ds} = \frac{\dot{\lambda}}{1+e} \cdot \frac{ds}{11+s} & (s \leq s_{ae}) \\
\frac{d\varepsilon_v}{ds} = \frac{\dot{\lambda}}{1+e} \cdot \frac{ds}{11 + \left( \frac{s_{ae}}{s} \right)^{0.55}} & (s > s_{ae})
\end{cases}
\]  \hspace{1cm} (4.22)
4.4.2 The SFG model

Sheng (2008) proposed the SFG model:

\[
d\varepsilon_v = \frac{\lambda_{vp}}{v} \frac{d\bar{p}}{\bar{p} + s} + \frac{\lambda_{vs}}{v} \frac{ds}{\bar{p} + s}
\]  
(4.23)

where \( \lambda_{vp} \) and \( \lambda_{vs} \) are the slopes of normal compression line (NCL). \( v \) is the specific volume. The slope \( \lambda_{vs} \) is defined as:

\[
\lambda_{vs} = \begin{cases} 
\frac{\lambda_{vp}}{s + 1} & (s < s_{sa}) \\
\frac{\lambda_{vp}}{s + 1} + \frac{1}{s_{sa}} & (s \geq s_{sa})
\end{cases}
\]  
(4.24)

where \( s_{sa} \) is the saturated suction. In the soil water characteristic curve test, \( \bar{p} \) is a constant value 11 kPa, and \( dp = 0 \). Eq. 4.23 is simplified to:

\[
d\varepsilon_v = \frac{\lambda_{vs}}{v} \frac{ds}{\bar{p} + s}
\]  
(4.25)

Combining Eq. 4.24 and Eq. 4.25, in this condition:

\[
\begin{cases} 
\varepsilon_v = \frac{\lambda}{1 + e} \frac{ds}{11 + s} & (s < s_{sa}) \\
\varepsilon_v = \frac{\lambda}{1 + e} \left( \frac{s_{sa} + 1}{s + 1} \right) \frac{ds}{11 + s} & (s \geq s_{sa})
\end{cases}
\]  
(4.26)

The relationship between void ratio \( e \) and the volumetric strain \( \varepsilon_v \) is:
\[ e = \frac{e_0 - \varepsilon_v}{1 + \varepsilon_v} \]  

(4.27)

where, \( e_0 \) is the initial void ratio. Comparing Eq. 4.22 and Eq. 4.26, the difference between the two constitutive models are clear.

Fig. 4.32 and Fig. 4.35 show the experimental drying curve in the SWCC test. As discussed in this section, combining Eq. 4.22 and Eq. 4.27 the theoretical drying curve \((e - \ln s)\) curves based on the effective stress model can be plotted. Similarly, according to Eq. 4.26 and Eq. 4.27, the theoretical drying \((e - \ln s)\) curve calculated by the SFG model can be plotted. The drying curves for Glenroy silty clay and Glenroy clay are shown in Fig. 4.45 and Fig. 4.46 respectively.

The theoretical calculations prove the constitutive models in unsaturated soil. At the same time, it also checked the accuracies of the data in laboratory experiments. According to the fitting results, the experimental and theoretical drying curves are matched with each other.
Fig. 4.45 Experimental and theoretical drying curve in the SWCC test (Glenroy silty clay)

Fig. 4.46 Experimental and theoretical drying curve in the SWCC test (Glenroy clay)
4.5 Conclusion

In order to study the effects of tree on residential footings, a full scale experimental site was set up at Glenroy, Victoria. The sap flow of tree, \textit{in situ} soil moisture variation and ground movement have been monitored on a monthly basis. The field experiments were complemented by a series of laboratory tests, which included test for soil suction, moisture content, shrink-swell index, SWCC and compression parameters. In this chapter, the results of both field and laboratory tests have been discussed in detail.

The experiments described in this chapter can not only provide physical parameters and soil properties for the numerical model discussed in Chapter three, but also be used to calibrate and validate the numerical model. The numerical implementation of the proposed numerical model will be presented in the following chapter and the back-analysis of field experiment will be given in Chapter 6.
Chapter 5 Numerical implementation

5.1 General

In the last several decades, the analysis and research of geotechnical engineering were mainly dependent on accumulated experiences and experimental data. The Australian Standard (AS2870-2011) for residential slabs and footings design also use extensive empirical criteria. Experimental and statistical data has advantage of facticity while it has the disadvantage of non-determinacy. The development of geotechnical engineering requires a new scientific method to solve practical considerations. Fortunately, by the development of computer technology, especially the finite element analysis techniques, geotechnical problems can be solved by numerical modelling.

Numerical modelling can provide visual and detailed results of a practical model. Nevertheless, the accuracy of the model is highly dependent on the input parameters and boundary conditions. In this thesis, Chapters 3 and 4 provide the details of the root absorption and unsaturated soil theories and experimental results. This chapter focuses on the numerical implementation of the concepts developed in Chapter 3 and the application of the parameters measured from the field work and laboratory testing.
5.2 ABAQUS pore fluid diffusion and stress analysis

Tree root absorption is essentially a plant science and hydrology problem including partially saturated flow which occurs when the wetting liquid is absorbed into or expelled from the medium by capillary action.

In this thesis, soil-water interaction caused by tree root absorption is simulated by the ABAQUS coupled pore fluid diffusion and stress analysis. According to the ABAQUS manual (2005), coupled pore fluid diffusion and stress analysis is used to model single phase, partially or fully saturated fluid flow through porous media. It can solve either linear or nonlinear problem in both transient and steady-state conditions. It requires the use of pore pressure elements with associated pore fluid flow properties defined. It can be performed in terms of either total pore pressure or excess pore pressure by including or excluding the pore fluid weight.

ABAQUS is a powerful finite element analysis software. It has its own coding called INP file with a very strict format. In a root absorption model, considering the format in pore fluid diffusion and stress analysis, the methodology of INP file coding is shown in Fig. 5.1. The INP file is also attached with notes. In this model the mesh consists of 4-noded linear plane strain quadrilateral elements.

In this chapter, considering the format of ABAQUS coding in Fig. 5.1, the discussions are divided into following sections: definition of absorption water flow, soil mechanical behaviour, soil hydraulic behaviour, initial condition and finally an example. The introductions in this chapter give a complete structure of how the root absorption model is implemented in numerical modelling in ABAQUS.
Define nodes and elements

Node

Element

Element type (CPE4P)

Mechanical behaviour

Density

Elasticity

Clay Plasticity (Cam-clay model)

Expansive soil behaviour

Soil suction

SWCC

Hydraulic conductivity

Void ratio

Boundary conditions

Geostatic boundary condition $U_1,U_2,U_3$ and gravity load

Initial conditions

Initial saturation

Initial stress

Initial Void ratio

Apply root absorption

Pore water flow by absorption

Variation of effective stress

Result

Result

Fig. 5.1 Methodology of ABAQUS INP files coding
• COUPLED DIFFUSION/STRESS ANALYSIS (modified from ABAQUS Manu)

• *HEADING

• *************************************

• *Node

  *Data lines to define node

• *Element

  *Data lines to define element

• The mesh consists of 4-noded linear strain quadrilateral elements (CPE4P)

• *************************************

• *MATERIAL, NAME=soil

  *Data lines to define mechanical properties of the solid material

• *DENSITY

  *Data lines to define density

• *POROUS ELASTIC, SHEAR= POISSON

  *Data lines to define $\kappa$, $\nu$, and $p_{el}$

• *CLAY PLASTICITY, HARDENING=EXponential, INTERCEPT=$e_i$

  *Data lines to define $\lambda$, $M$, $a_0$, $\beta$, $k$

• *PERMEABILITY, TYPE=SATURATION

  *Data lines to define the dependence of permeability on saturation,

• *SORPTION, TYPE=ABsorption

  *Data lines to define absorption behaviour

  *Define SWCC, Pore water pressure and $S$

• *SORPTION, TYPE=EXsorption

  *Data lines to define eXsorption behavior
• **MOISTURE SWELLING**
  - Data lines to define moisture swelling strain as a function of saturation
  - Define swelling strain and $S$ in partially saturated flow

• **BOUNDARY**
  - Define Geostatic boundary conditions

• **INITIAL CONDITIONS, TYPE=STRESS, GEOSTATIC,**
  - User define initial stress
  - Data lines to specify initial stresses

• **INITIAL CONDITIONS, TYPE=PORE PRESSURE,**
  - User define soil suction (pore water pressure)
  - Data lines to define initial values of pore fluid pressures

• **INITIAL CONDITIONS, TYPE=RATIO**
  - Data lines to define initial values of the void ratio

• **INITIAL CONDITIONS, TYPE=SATURATION**
  - Data lines to define initial saturation

• **STEP 1**
  - **GEOSTATIC**

• **CLOAD and/or DLOAD and/or TEMPERATURE and/or FIELD**
  - Data lines to specify effective stress

• **FLOW and/or SFLOW and/or DFLOW and/or DSFLOW**
  - Data lines to specify pore fluid flow
5.3 Definition of water flow in ABAQUS

5.3.1 Definition of absorption flow

As introduced in early chapters, Biddle (1998) found that more than 99% of plant water uptake was lost in the form of transpiration from leaves. Hence, it is generally assumed the magnitude of tree root absorption is equal to total transpiration in tree leaves. The transpiration rate $T_p$ is the first variable in numerical modelling.

In field experiment, the magnitude of $T_p$ can be obtained by measuring either the sap flow rate (Fig. 4.14) or leave area index (LAI) of a tree. In numerical modelling, $T_p$ is set as a function of time $t$. After $T_p$ is implemented, the magnitude of sink term $S(\theta)$ is defined by the equation:

$$S(\theta) = \frac{\beta(r, z)T_p}{\int \beta(r, z)dv} = \frac{\pi r_{max}^2 \beta(r, z)T_p}{2\pi \int_0^{z_{max}} \int_0^{r_{max}} r\beta(r, z)drdz}$$

(5.1)

where $z_{max}$ is the maximum root depth, $r_{max}$ is the maximum root length, $\beta(r, z)$ is root length density. $r$ and $z$ are horizontal and vertical coordinates. When defining Eq. 5.1 in numerical modelling, the geometry shape of root zone is considered and $z_{max}$ and $r_{max}$ are determined. The geometry of root zone can be drawn directly in ABAQUS part definition following the real condition. After root
zone is defined, root length density $\beta(r,z)$ can be counteracted between numerator and denominator. Once the magnitude of sink term $S(\theta)$ is calculated by Eq. 5.1 and the reduce factor $\alpha(s)$ is obtained from the following equation, $S(x,y,z,t)$ can be determined.

$$S(x,y,z,t) = \alpha(s) \cdot S(\theta)$$

$$\alpha(s) = \begin{cases} 
0 & (s \geq s_{re}) \\
\frac{s - s_{re}}{s_{ae} - s_{re}} & (s_{ae} < s \leq s_{re}) \\
1 & (s_{ia} < s \leq s_{ae}) \\
0 & (s < s_{ia})
\end{cases}$$ \hspace{1cm} (5.2)

The definition of the root geometry and the calculation from $T_p$ to $S(x,y,z,t)$ is processed by FORTRAN programming.

Finally, root absorption $S(x,y,z,t)$ is set as a negative distributed flow. ABAQUS employs Laplace equation and Darcy’s law as fundamental equations for continuous water flow. As shown in Fig. 5.2, in one element of soil:

![Fig. 5.2 Element in soil](image)
In \( x \) direction: \[ q_x d_y d_z - \left( q_x + \frac{\partial q_x}{\partial x} \right) d_y d_z = -\frac{\partial q_x}{\partial x} d_y d_z \]

In \( y \) direction: \[ -\frac{\partial q_y}{\partial y} d_x d_z \]

In \( z \) direction: \[ -\frac{\partial q_z}{\partial z} d_x d_y \]

\[ \frac{\partial \theta}{\partial t} d_x d_y d_z = \left( -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} \right) d_x d_y d_z \]

\[ \frac{\partial \theta}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0 \quad (5.3) \]

where: \( \theta \) is the water content. \( \text{Eq. 5.3} \) describes the flow without root sorption by applying the Laplace equation. When the sink term is defined as a negative flow in the root element, \( \text{Eq. 5.3} \) is modified to:

\[ \frac{\partial \theta}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} + S(\theta) = 0 \quad (5.4) \]

As introduced in Chapter 3, after combining with the Darcy’s law

\[ \frac{\partial \theta}{\partial t} = -\nabla \nabla - S(x, y, z, t) \quad (5.5) \]

And according to Philip (1969):

\[ \phi = \psi - z \quad (5.6) \]

where \( \phi \) is total potential or hydraulic head, \( z \) coordinate contributes the gravitational effect. \( \psi \) is soil suction. Combining \( \text{Eq. 5.5} \) and \( \text{Eq. 5.6} \). The flow equation with sink term is adopted to simulate the water uptake as:

\[ \frac{\partial \theta}{\partial t} = \nabla (K \nabla \psi) - \frac{\partial K}{\partial z} - S(x, y, z, t) \quad (5.7) \]
According to the derivations in this paragraph, in numerical modelling as soon as defining the sink term \( S(x, y, z, t) \) as a negative flow in the root element, by applying Laplace equation and Darcy’ law, flow equation Eq. 5.7 is then implemented and the root absorption is simulated.

### 5.3.2 Definition of evaporation and rainfall

As discussed in Chapter 3, soil evaporation and rainfall infiltration are considered as the climate boundary condition in the root absorption model. Fundamentally, evaporation and rainfall are defined as a surface flow in numerical modelling. When defining the rainfall infiltration rate as \( q_i \) and the soil evaporation rate as \( q_e \), the surface flow \( q_s \) is defined as \( q_e - q_i \). As discussed before, \( q_i \) under the tree canopy is influenced by leaf canopy because the rainfall is obstructed by leaves. In a semi-arid area (e.g. South Australia), evaporation rate is much larger than rainfall.

In this research, the rainfall infiltration rate \( q_i \) and the soil evaporation rate \( q_e \) are directly obtained from the weather station on field site or the Bureau of Meteorology. The climate model has been discussed in Chapter 3. In numerical modelling the rainfall and evaporation rate are defined as function of time \( t \) in a FORTRAN subroutine DFLOW.

### 5.3.3 FORTRAN subroutine DFLOW

In ABAQUS coupled pore fluid diffusion and stress analysis, to specify complicated conditions in a model, user subroutines in FORTRAN code are implemented. Subroutine DFLOW (distributed flow) is used to define non-uniform pore fluid in soil environment.
According to ABAQUS manual, subroutine DFLOW is used to define the variation of the seepage magnitude as a function of position, time, pore pressure, etc. The subroutine will be called at each flow integration point for each element-based or surface-based non-uniform flow definition in the analysis and ignores any amplitude references that may appear with the associated non-uniform flow definition.

Substantially, in this analysis, the root absorption, rainfall and evaporation are defined as non-uniform flow. In ABAQUS modelling, the definitions described in Section 5.2.1 and 5.2.2 are achieved by the DFLOW subroutine. The methodology of programming is shown in Fig. 5.3. The format of subroutine is shown below.

```
SUBROUTINE DFLOW (FLOW,U,KSTEP,KINC,TIME,NOEL,NPT, COORDS,1 JLTYL,SNAMЕ)

C
INCLUDE 'ABA_PARAM.INC'
C
DIMENSION TIME(2),COORDS(3)
CHARACTER*80 SNAME

(User coding to define FLOW)

RETURN
END
```

### 5.4 Definition of soil mechanical behaviour in ABAQUS

Soil is a three phase property contains soil particles, air and water. The mechanical behaviour is complicated. In this analysis, a porous elastic model is applied to describe the soil behaviour in the elastic region. A critical state plasticity model for clay soil (Cam-clay model, Roscoe 1968) is applied in the plastic region. In addition, in unsaturated soil the effective stress equation is also influenced by the soil hydraulic behaviour. The definitions of soil hydraulic behaviour are discussed in the next section.
Fig. 5.3 Methodology of Fortran programming of water flow

Define root zone $r_{\text{max}}$ and $z_{\text{max}}$

If $P(x, y)$ in root zone

Yes

Input $T_p$

Define $S(\theta)$ by $T_p$

Define $S(x, y, z)$

Considering soil reaction $\alpha(s)$

$\alpha(s)$

$F_1 = S(x, y, z)$

$F_1 = 0$

No

If $P(x, y)$ at surface

Yes

Define $q_s = q_e - q_i$

$S(x, y, z)$

$F_2 = q_s$

$F_2 = 0$

No

FLOW = $F_1 + F_2$
5.4.1 Porous material elastic model

In elastic region, soil is considered as a nonlinear, isotropic elastic property, which the pressure stress varies as an exponential function of the volumetric strain. According to ABAQUS manual (2005), a porous elastic soil model is valid for small elastic strains, allows a zero or nonzero elastic tensile stress limit and can have properties that depend on temperature and other field variables.

The elastic part of the volumetric behaviour of porous material is modelled by assuming that the elastic part of the change in volume of the soil is proportional to the logarithm of the pressure stress \( e - \ln p' \) curve, as shown in Fig. 5.4.

![Fig. 5.4 Porous elastic volumetric behaviour (AB AQUS manual, 2005)](image)

The equation is:

\[
\frac{\kappa}{(1 + e_0)} \ln \left( \frac{p_0 + p_{el}}{p + p_{el}} \right) = J^{el} - 1
\]

(5.8)

Where \( \kappa \) is the logarithmic bulk modulus, \( e_0 \) is the initial void ratio, \( p \) is the equivalent pressure stress, \( p_0 \) is the initial value of the equivalent pressure stress;
\( J^{el} \) is the elastic part of the volume ratio between the current and reference configurations:

\[
J^{el} = \frac{1 + e}{1 + e_0}
\]  
(5.9)

and \( p_t^{el} \) is the elastic tensile strength of the material, in most conditions, \( p_t^{el} \) is 0.

The deviatoric elastic behaviour of soil can be defined in two ways. Firstly, by defining the shear modulus \( G \), the deviatoric stress \( S \) is then related to the deviatoric part of the total elastic strain \( e^{el} \), the equation is:

\[
S = 2Ge^{el}
\]  
(5.10)

In this case the shear behaviour is not affected by compaction of the material. The application of the definition in ABAQUS CAE is shown in Fig. 5.5.

![Fig. 5.5 Definition of porous elastic model by shear modulus](image-url)
Secondly, by defining Poisson’s ratio $\nu$, the instantaneous shear modulus is then defined from the instantaneous bulk modulus and Poisson’s ratio as:

$$G = \frac{3(1-2\nu)(1+e_0)}{2(1+\nu)\kappa}(p + p_i^{el}) \exp(\varepsilon_{vol}^{el})$$  \hspace{1cm} (5.11)

Where $\varepsilon_{vol}^{el} = \ln J^{el}$ is the logarithmic measure of the elastic volume change. In this condition:

$$dS = 2Gde^{el}$$ \hspace{1cm} (5.12)

The application of the definition by poison’s ratio in ABAQUS CAE is shown in Fig. 5.6. In this analysis, elastic model is defined by the bulk modulus and Poisson’s ratio.

![Fig. 5.6 Definition of porous elastic model by poison’s ratio](Image)
5.4.2 Critical state (clay) plasticity model

The critical state clay plasticity model utilised in this analysis is an extension of the Cambridge model (Roscoe, 1968). The model describes the inelastic behaviour of the material by a yield function that depends on the three stress invariants, an associated flow assumption to define the plastic strain rate, and a strain hardening theory that changes the size of the yield surface according to the inelastic volumetric strain. The model requires that the elastic part of the deformation be defined by using the linear elastic material model or the porous elastic material model within the same material definition; and allows for the hardening law to be defined by a piecewise linear form or by an exponential form.

In ABAQUS modelling, the Cam-clay model, the yield surface is:

\[
\frac{1}{\beta^2} \left( \frac{p}{a} - 1 \right)^2 + \left( \frac{t}{Ma} \right)^2 - 1 = 0
\]  \hspace{1cm} (5.13)

\[
t = \frac{1}{2} q \left[ 1 + \frac{1}{K} - (1 - \frac{1}{K})(\frac{r}{q})^3 \right]
\]  \hspace{1cm} (5.14)

Where:

- \( p \) is the equivalent pressure stress.
- \( q \) is the Mises equivalent stress.
- \( r \) is the third stress invariant.
- \( t \) is the deviatoric stress measure.
- \( M \) is a constant that defines the slope of the critical state line.
- \( \beta \) is a constant that is equal to 1.0 on the dry side of the critical state line but may be different from 1.0 on the wet side of critical state line.
- \( a_0 \) is a hardening parameter that defines the initial size of the yield surface.
- \( K \) is the ratio of the flow stress in triaxial tension to the flow stress in triaxial compression.
The clay yield surfaces in \( p - t \) plane are shown in Fig. 5.7:

![Fig. 5.7 Clay yield surfaces in \( p - t \) plane (ABAQUS manual, 2005)](image)

Considering the conjunction with the porous elastic material model, the hardening law of the model can have an exponential form. The law is written in terms of some of the porous elasticity parameters. The size of yield surface at any time is determined by the initial value of the hardening parameter \( a_0 \), and the amount of inelastic volume change that occurs according to the equation:

\[
a = a_0 \exp \left[ (1 + e_0) \frac{1 - J^{pl}}{\lambda - \kappa J^{pl}} \right]
\]  

(5.15)

where \( J^{pl} \) is the inelastic volume change (that part of \( J \), the ratio of current volume to initial volume, attributable to inelastic deformation); \( \kappa \) is the logarithmic bulk modulus of the material defined for the porous elastic material behaviour; \( \lambda \) is the logarithmic hardening constant defined for the clay plasticity material behaviour; and \( e_0 \) is the user-defined initial void ratio.

In this analysis, the definitions of parameters for the Cam-clay model are coming from experimental results. As discussed in Chapter 4, the log elastic modulus \( \kappa \) and the log plasticity bulk modulus \( \lambda \) are defined in a soil consolidation test. The
triaxial compression tests allow the calibration of the yield parameters $M$ and $\beta$. $M$ is the ratio of the shear stress $q$, to the pressure stress $p$, at critical state and can be obtained from the stress values when the material has become perfectly plastic (critical state). $\beta$ represents the curvature of the cap part of the yield surface and can be calibrated from a number of triaxial tests at high confining pressures (on the wet side of critical state). $\beta$ must be between 0.0 and 1.0. To calibrate the parameter $K$, which controls the yield dependence on the third stress invariant, experimental results obtained from a true triaxial (cubical) test are necessary. These results are generally not available. The value of $K$ is generally between 0.8 and 1.0. In an ABAQUS CAE model, the implementation of Cam-clay model is shown in Fig. 5.8.

![Fig. 5.8 Implementation of Cam-clay model in ABAQUS](image)

In this thesis, three different types of soils are analysed, which are Glenroy silty clay, Glenroy clay, and South Australia clay. Considering the consolidation tests result Fig. 4.22 and Fig. 4.23 in Chapter 4 and the investigation of Cameron in
Walkley Heights (Malley and Cameron, 2005), the definitions of the mechanical behaviour of the three soils are shown below.

**Table 5.1** Mechanical behaviour of Glenroy silty clay

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log elastic modulus $\kappa$</td>
<td>0.02</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.09</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.12</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.7251</td>
</tr>
<tr>
<td>Poison’s ratio</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 5.2** Mechanical behaviour of Glenroy clay

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log elastic modulus $\kappa$</td>
<td>0.025</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.12</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.12</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.8094</td>
</tr>
<tr>
<td>Poison’s ratio</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 5.3**: Mechanical behaviour of Walkley Heights clay

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log elastic modulus $\kappa$</td>
<td>0.02</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.1</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.36</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.8176</td>
</tr>
<tr>
<td>Poison’s ratio</td>
<td>0.25</td>
</tr>
</tbody>
</table>
5.5 Definitions of soil hydraulic behaviour in ABAQUS

This section introduces the definition of soil hydraulic behaviour in numerical modelling. In this paragraph, the implementations of soil suction profile, soil water characteristic, hydraulic conductivity, swelling behaviour and effective stress in ABAQUS are introduced.

5.5.1 Definition of soil suction profile

As introduced in Chapters 2 and 3, soil suction is a significant variable which represents the suction stress and associates with volumetric water content. In expansive soil, as the pore air pressure $\mu_a$ and the osmotic suction $\psi_s$ are generally very small, ABAQUS considers soil suction is mainly represented by the pore water pressure.

In ABAQUS coupled pore fluid diffusion and stress analysis, a specific subroutine UPOREP is implemented to define the pore water pressure. According to ABAQUS manual (2005), user subroutine UPOREP allows the specification of the pore water pressure values of a porous medium. It can be used to define pore water pressure values as functions of nodal coordinates and node number and will be called to define initial fluid pore pressure values at all nodes of a coupled pore fluid diffusion and stress analysis whenever user-defined initial pore pressure conditions are specified. The format of subroutine is below:

```c
SUBROUTINE UPOREP(UW0,COORDS,NODE)
C
INCLUDE 'ABA_PARAM.INC'
C
DIMENSION COORDS(3)
C
user coding to define UW0

RETURN
END
```
In ABAQUS modelling, Soil suction is assumed equal to pore water pressure. Pore water pressure is defined following the experimental data as a function of \( y \) coordinate so that the initial suction profile is implemented. If the soil is divided in different soil layers, the definition of pore water pressure in each layer could be specified separately. The measurement of the soil suction profile has been discussed in Chapter 4. According to the experimental results in Glenroy field and the measurement by Cameron (2005), soil suction profile is implemented. In ABAQUS modelling, the pore water pressure needs to be defined in negative, the unit is \( kPa \). The definitions are shown below:

**Table 5.4** Soil suction profile 1: Glenroy suburb, Victoria \( H_s = 2.5m \)

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>UW0 (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-200</td>
</tr>
<tr>
<td>0.5</td>
<td>-400</td>
</tr>
<tr>
<td>1</td>
<td>-600</td>
</tr>
<tr>
<td>1.5</td>
<td>-1000</td>
</tr>
<tr>
<td>2</td>
<td>-1400</td>
</tr>
<tr>
<td>2.5</td>
<td>-1600</td>
</tr>
<tr>
<td><strong>Below 2.5</strong></td>
<td><strong>-1600</strong></td>
</tr>
</tbody>
</table>

**Table 5.5** Soil suction profile 2: Walkley Heights, SA (Malley and Cameron, 2005), \( H_s = 4m \)

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>UW0 (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-200</td>
</tr>
<tr>
<td>0.5</td>
<td>-400</td>
</tr>
<tr>
<td>1</td>
<td>-600</td>
</tr>
<tr>
<td>1.5</td>
<td>-800</td>
</tr>
<tr>
<td>2</td>
<td>-1000</td>
</tr>
<tr>
<td>2.5</td>
<td>-1200</td>
</tr>
<tr>
<td>3</td>
<td>-1400</td>
</tr>
<tr>
<td>3.5</td>
<td>-1500</td>
</tr>
<tr>
<td>4</td>
<td>-1600</td>
</tr>
<tr>
<td><strong>Below 4</strong></td>
<td><strong>-1600</strong></td>
</tr>
</tbody>
</table>
5.5.2 Definition of soil water characteristic curve

ABAQUS allows defining soil water characteristic curve in the “sorption” option. According to ABAQUS manual (2005), sorption defines a porous material’s absorption/exsorption behaviour under partially saturated flow conditions. It is used in the analysis of coupled wetting liquid flow and porous medium stress. ABAQUS highlights the effect of pore water pressure in partially saturated soil while ignores the effect of pore air pressure. As it notes “A porous medium becomes partially saturated when the total pore liquid pressure $u_w$ is negative. Negative values of $u_w$ represent capillary effects in the medium.”

In ABAQUS modelling, soil water characteristic curve is defined as a relationship between pore water pressure and degree of saturation. This is because, in unsaturated soil, ABAQUS considers soil suction is equal to the pore water pressure. In sorption definition, absorption and exsorption behaviour can be defined separately or can be considered in the same curve. In ABAQUS modelling, SWCC can be defined in either tabular form or an analytical form. In this analysis, tabular form is selected (Fig. 5.9).

The definitions of SWCC for different soils are following the Fredlund SWCC device results (Fig. 4.34, Fig. 4.37, Fig. 4.40) in Chapter 4. The input data in ABAQUS definition are shown in Tables 5.6 to 5.8. In ABAQUS modelling, the pore water pressure needs to be defined in negative, the unit is kPa.
Fig. 5.9 Definition of SWCC in ABAQUS

Table 5.6 Definition of SWCC in ABAQUS (Glenroy silty clay)

<table>
<thead>
<tr>
<th>Pore water pressure (kPa)</th>
<th>Degree of saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>1</td>
</tr>
<tr>
<td>-10</td>
<td>0.9674</td>
</tr>
<tr>
<td>-20</td>
<td>0.8458</td>
</tr>
<tr>
<td>-50</td>
<td>0.6969</td>
</tr>
<tr>
<td>-100</td>
<td>0.5802</td>
</tr>
<tr>
<td>-200</td>
<td>0.5369</td>
</tr>
<tr>
<td>-400</td>
<td>0.4823</td>
</tr>
<tr>
<td>-800</td>
<td>0.4144</td>
</tr>
<tr>
<td>-1000</td>
<td>0.3907</td>
</tr>
<tr>
<td>-32860</td>
<td>0.0974</td>
</tr>
</tbody>
</table>
Table 5.7 Definition of SWCC in ABAQUS (Walkley Heights clay)

<table>
<thead>
<tr>
<th>Pore water pressure (kPa)</th>
<th>Degree of saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>1</td>
</tr>
<tr>
<td>-10</td>
<td>0.9765</td>
</tr>
<tr>
<td>-50</td>
<td>0.9485</td>
</tr>
<tr>
<td>-100</td>
<td>0.9175</td>
</tr>
<tr>
<td>-150</td>
<td>0.8828</td>
</tr>
<tr>
<td>-200</td>
<td>0.8560</td>
</tr>
<tr>
<td>-400</td>
<td>0.7931</td>
</tr>
<tr>
<td>-800</td>
<td>0.7384</td>
</tr>
<tr>
<td>-1000</td>
<td>0.6695</td>
</tr>
<tr>
<td>-15700</td>
<td>0.1504</td>
</tr>
</tbody>
</table>

Table 5.8 Definition of SWCC in ABAQUS (Glenroy clay)

<table>
<thead>
<tr>
<th>Pore water pressure (kPa)</th>
<th>Degree of saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>1</td>
</tr>
<tr>
<td>-20</td>
<td>0.9743</td>
</tr>
<tr>
<td>-50</td>
<td>0.9443</td>
</tr>
<tr>
<td>-100</td>
<td>0.8943</td>
</tr>
<tr>
<td>-200</td>
<td>0.8185</td>
</tr>
<tr>
<td>-400</td>
<td>0.7330</td>
</tr>
<tr>
<td>-800</td>
<td>0.5570</td>
</tr>
<tr>
<td>-1000</td>
<td>0.4934</td>
</tr>
<tr>
<td>-15000</td>
<td>0.0916</td>
</tr>
</tbody>
</table>
5.5.3 Definition of swelling and shrinkage behaviour

ABAQUS provides a moisture swelling option to define the saturation-driven volumetric swelling of the solid skeleton of a porous medium in partially saturated flow conditions. Generally the moisture swelling model assumes that the volumetric swelling of the porous medium’s solid skeleton is a function of the saturation of the wetting liquid in partially saturated flow condition. The swelling behaviour is assumed to be reversible. It can be either isotropic or anisotropic. Similarly, Moisture swelling data can be defined in either tabular form or an analytical form as shown in Fig. 5.10. The definitions of free swelling behaviour are listed in Table 5.9.

Table 5.9 Soil drying behaviour

<table>
<thead>
<tr>
<th>Soil</th>
<th>$e - \ln(s)$ curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenroy silty clay</td>
<td>Fig. 4.32</td>
</tr>
<tr>
<td>Walkley Heights clay</td>
<td>4%-9% per log MPa (Cameron 2005)</td>
</tr>
<tr>
<td>Glenroy clay</td>
<td>Fig. 4.35</td>
</tr>
</tbody>
</table>

Fig. 5.10 Definition of drying curve
5.5.4 Definition of hydraulic conductivity

Hydraulic conductivity (or permeability ratio) is defined associated with Darcy’s law. It refers to the velocity of water flow in soil. There are many factors that may influence the number of hydraulic conductivity. As discussed in Chapter 3, hydraulic conductivity is set as a function of the degree of saturation. In ABAQUS coupled pore fluid diffusion and stress analysis, when defining the material, it provides an option to define hydraulic conductivity associated with degree of saturation as shown in Fig. 5.11. In this analysis, the definition of hydraulic conductivity is following the research by Biddles (1998).

![Fig. 5.11 Definition of hydraulic conductivity](image)

5.5.5 Application of the effective stress

As discussed in chapter 3, when describing effective stress in expansive soil, Bishop (1959) equation is:
\[
\sigma' = (\sigma - \mu_a) + \chi(\mu_a - \mu_w)
\]  \hspace{1cm} (5.16)

where \(\sigma'\) is the effective stress, \(\sigma\) is total stress, \(\mu_a\) is the pore air pressure, \(\mu_w\) is the pore water pressure, \(\mu_a - \mu_w\) is the matric suction. Substantially, effective stress equation represents the difference in stress (and strain) between saturated and unsaturated soil conditions associated with water content.

The relationship between stress variable and water content is established in Eq. 5.16. In this equation, matric suction (or soil suction) and the parameter \(\chi\) is related to the volumetric water content, \(\mu_a\) is assumed to be very minimum, the total stress \(\sigma\) is generally caused by the soil gravity. After implemented these parameters, the effective stress is calculated. When simulating the soil movement, the effective stress \(\sigma'\) is applied in the constitutive model, volumetric strain is then calculated.

The application of the effective stress is done by the FORTRAN subroutine DLOAD, which is used to define distributed load in elements in ABAQUS. The format of programming is also listed below. The methodology of programming is shown in Fig. 5.12.

```fortran
SUBROUTINE DLOAD(F,KSTEP,KINC,TIME,NOEL,NPT,LAYER, KSP, COORDS,JLTYP,SNAME)
C INCLUDE 'ABA_PARAM.INC'
C DIMENSION TIME(2), COORDS (3)
CHARACTER*80 SNAME

user coding to define F

RETURN
END
```
5.6 Definition of initial conditions

5.6.1 General

In numerical modelling, initial condition means the condition before the numerical analysis. In a root absorption model, it is the condition before the root absorption start to affect the expansive soil. Generally, the initial conditions in root absorption model are divided into initial saturation, initial ratio and initial stress distribution.
5.6.2 Initial saturation

ABAQUS has a compulsory rule to define initial saturation before the analysis. Initial saturation indicates the water content distribution in a soil environment. In saturated soil environment, initial saturation is a value of 1.

In expansive soil condition, as discussed in Chapter 3, initial saturation is closely related to the soil suction and soil water characteristic curve. In the root absorption model, soil suction is defined as pore water pressure implemented by the FORTRAN subroutine USPORP. At the same time, soil water characteristic curve (SWCC) is defined in sorption option. After the application of SWCC, initial saturation will follow the sorption curve and reach the new value matched with the pore water pressure. Thus, initial saturation is adjusted to the real condition. Fig. 5.13 shows the initial saturation in a 10 × 10 meter clay soil model in South Australia.

Fig. 5.13 Initial saturation in ABAQUS modelling, South Australian model

5.6.2 Initial void ratio

As shown in the mechanical behaviour, in both porous elastic model and clay plastic model, initial void ratio $e_0$ is closely related to the hardening law, and is
involved in the definition of the soil water characteristic curve. In this analysis, initial void ratio $e_0$ is defined following the SWCC experimental data which has been measured in Chapter 4.

During the absorption process, the variation of the void ratio is associated with effective stress. In constitutive modelling, as $\lambda$ and $\kappa$ is obtained in a soil consolidation test. Void ratio $e$ changes with effective stress $p'$ following an $e - \ln p'$ curve.

### 5.6.3 Initial stress distribution

Most geotechnical problems begin from a geostatic state, which is a steady-state equilibrium configuration of the undisturbed soil under geostatic loading and usually includes both horizontal and vertical components. It is important to establish the initial stress distribution correctly so that the problem begins from an equilibrium state. Considering the effective stress equation in unsaturated soil, effective stress in initial condition is:

$$\sigma' = \rho gh - \mu_a + \chi (\mu_a - \mu_w)$$  \hspace{1cm} (5.19)

where $\rho$ is soil density, $h$ is soil depth, $\mu_a$ is pore air pressure, $\mu_w$ is pore water pressure, $\mu_a - \mu_w$ is matric suction. Initial stress has the equal value in both horizontal and vertical components.

The application of initial stress in ABAQUS is done by the FORTRAN subroutine SIGINI. The methodology of programming is similar to the effective stress as shown in Fig. 5.14. The state of initial stress in a 10 x 10 meter clay model in South Australia is shown in Fig. 5.15.
SUBROUTINE SIGINI(SIGMA, COORDS, NTENS, NCRDS, NOEL, NPT, LAYER, KSPT, LREBAR, NAMES)
C
INCLUDE 'ABA_PARAM.INC'
C
DIMENSION SIGMA(NTENS), COORDS(NCRDS)
CHARACTER NAMES(2)*80

user coding to define SIGMA(NTENS)

RETURN
END

Fig. 5.14 Application of initial stress in FORTRAN programming
5.7 A simple example

Before the case studies, the author introduces a simple model utilizing the root absorption concept in this thesis. As shown in Fig. 5.16, the model is 10 meters in depth and 20 meters in width. Tree root zone is assumed to be 3 meters wide and 1.5 meters deep. The transpiration rate is simply set as a constant number 2L/day. The expansive soil behaviour in this model are assumed to be the same with South Australian. The suction profile is setting in a wet season. The climate influence is ignored. The list of the parameters is shown in Table 5.10.
After setting the initial parameters, as discussed in early sections, sink terms, flow equations, initial stress and effective stress are calculated by FORTRAN subroutines, after an ABAQUS coupled pore flow and stress analysis in a 30 weeks period, the degree of saturation and suction distribution are worked out as shown in Fig. 5.17. Soil movement is shown in Fig. 5.18.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transpiration $T_p$</td>
<td>2 L/day</td>
</tr>
<tr>
<td>Maximum root depth $z_{max}$</td>
<td>1.5m</td>
</tr>
<tr>
<td>Maximum root width $r_{max}$</td>
<td>3 m</td>
</tr>
<tr>
<td>Root density $\beta(\theta)$</td>
<td>$\beta(z) = 1 - \frac{z}{z_{max}}$</td>
</tr>
<tr>
<td>Hydraulic conductivity $K(\theta)$</td>
<td>5.8E-7 m/s</td>
</tr>
<tr>
<td>Surface flow $q_s$</td>
<td>N/A</td>
</tr>
<tr>
<td>Suction profile</td>
<td></td>
</tr>
<tr>
<td>Log elastic modulus $K$</td>
<td>0.02</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.1</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.36</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.8176</td>
</tr>
<tr>
<td>Soil water characteristic curve</td>
<td></td>
</tr>
<tr>
<td>$e - \ln s$ curve</td>
<td>4% per log MPa</td>
</tr>
<tr>
<td>Depth of design suction change $H_s$</td>
<td>4 m</td>
</tr>
<tr>
<td>Saturated suction</td>
<td>20 kPa</td>
</tr>
<tr>
<td>Air entry value</td>
<td>250 kPa</td>
</tr>
<tr>
<td>Residual suction</td>
<td>12000 kPa</td>
</tr>
</tbody>
</table>

Table 5.10 List of the input parameters
Fig. 5.17 Root absorption with (a) degree of saturation (%) and (b) suction distribution in pF (Suction in pF = log (suction in kPa) + 1.01)

Fig. 5.18 Soil movement after root absorption
5.8 Conclusion

By defining soil properties, flow equation, constitutive equation, and inputting initial and boundary conditions, a root absorption model is implemented in the ABAQUS coupled pore flow and stress analysis. The finite element model can now be used to predict the soil suction distribution and soil movement due to tree root absorption.

The details of the laboratory measurement of the hydraulic and mechanical behaviour of Glenroy soil (silty clay and clay) and Walkley Heights clay have been given in Chapter 4. This chapter has explained in details how the measured parameters are implemented into ABAQUS to define the flow equation, soil suction, SWCC and constitutive equations etc. The corresponding FORTRAN subroutines used in ABAQUS modelling and an application example are provided as well.

In next chapter, two case studies will be carried out using the model and procedure described in this chapter. Also presented is the comparison between numerical result and field measured data.
Chapter 6 Case Studies

6.1 General

With the advent of faster computers and more sophisticated method of numerical analysis, there have been many recent attempts to model the influence trees on soil shrinkage settlement. However, most models are simply and have not been validated using high quality field data.

As discussed in previous chapters, a root water absorption model needs more than 20 input parameters. Obviously the accuracy and reliability of the results predicted by numerical models depends directly upon the quality of physical and soil property parameters. Most of these parameters can only be obtained from either the field measurement or laboratory tests. Therefore case studies and validation are essential before a model for root water absorption and soil desiccation can find practical application. In this study, two different case studies have been carried out.

The first case study concerns a street tree, Golden raintree (koelreuteria paniculata) in Walkley Heights subdivision, South Australia. A finite element analysis based on the proposed root water absorption is used to predict the profile of soil suction and associated ground movements. The FEM results are then compared with the field measurements by Cameron (2005). The second case study is based on the field monitoring of a Eucalyptus ficifolia at Glenroy test site. The parameters used in this case study are obtained from filed measurements and laboratory tests, and
the details are given in Chapter 4. And the implementations of these parameters in FE modelling are discussed in Chapter 5.

The parametric studies are also conducted to evaluate the effect of climate, transpiration rate and elapsed time on tree root absorption and soil shrinkage settlement. In this chapter, the results of case studies and parametric studies are presented and discussed.

6.2 Case Study One - Golden raintree at Walkley Heights, South Australia

6.2.1 Soil and site description

The field site for case study one is located in the north eastern corner of Walkley Heights subdivision, approximately 17 km north east of the city of Adelaide, South Australia. The site was selected for the study because it is typical of many new and existing residential housing estates in Adelaide and has a deep clay soil profile. The site was part of a long term research program led by Dr Cameron to study the influence of trees on soil water content, dwellings and pavements in an urban environment (Malley and Cameron, 2005). The project was funded by Local Government Association and City of Salisbury Council. The locations of the site, tree and soil boreholes are illustrated in Fig. 6.1. The tree planted at the front of house was a 3.1 m high golden raintree (koelreuteria paniculata). The site has been monitored by Dr Cameron since August 2001.

As shown in Fig. 6.1, four boreholes were drilled to approximately 5.5 m deep at a distance of 1 meter, 4.5 meters, 4.5 meters (below the pavement) and 49 meter to the tree trunk to measure the soil suction and surface movement. A house floor is located 3 meters north of the tree. The geotechnical profile for the soil found at position BH1 is shown in Fig. 6.2. The soil profile across the site is relatively uniform and the average of soil profile can be described as 100 mm of sandy clay topsoil underlain by low plasticity clay (CI) to 1 m then high plasticity clay (CH)
to a depth of approximately 4.5 m. The shrink-swell tests were conducted in accordance with AS1289 and the results are presented in Fig. 6.2. The shrink-swell indices were low near the surface (2.7%/pF for top 0.5 m) and quite high (6 – 9.5 %/pF) below 2.2 m. It should be noted that 4%/pF would be regarded as a highly reactive soil, 6%/pF very highly reactive and 8%/pF, an extremely reactive soil. Fig. 6.2 also shows profiles of Atterberg limits and soil free swell.

Fig. 6.1 Location of the site, tree and boreholes at Walkley Heights, Adelaide, SA

![Soil profile at BH1](image)

Fig. 6.2 Soil profile at BH1
The site classification for reactivity (based the predicted surface movement, $y_s$, for the site) following Australian Standard for Residential Slabs and Footings AS2870 (2011) is E (ie. extremely reactive with $y_s > 75$ mm). This is consistent with data obtained at Northfield, a suburb adjacent to Walkley Heights, during geotechnical investigation for housing (Bayetto, 1993).

Unlike most numerical models previously published for soil drying by trees, the climate conditions and soil characteristics were both taken into account based on the measured data instead of the assumed data in this research. The SWCC test was conducted on the clay samples collected from the Walkley Heights site by the author at RMIT geotechnical lab. The results of SWCC test are presented in Chapter 4, Section 4.3.3. The data of SWCC used for the case study and details of ABAQUS implementations are discussed in Chapter 5.

The climate of Adelaide, South Australia, is semi-arid with a relative dry period from October to April and a wet period from May to September. Generally, in a semi-arid or arid climate region, potential evapo-transpiration is much larger than precipitation. Fig. 6.3 shows the infiltration and evaporation data obtained from the Bureau of Meteorology station nearest to the field site. These data are needed to evaluate the surface boundary conditions in the proposed numerical modelling so that the effect of climate can also be taken into account.

![Fig. 6.3 Monthly rainfall and evaporation at Walkley Heights, Adelaide, SA](image)
6.2.2 Numerical modelling

A soil-tree root system, as shown in Fig. 6.4, was analysed using the numerical model discussed in previous chapter. According to the soil profile at BH1 (Fig. 6.2), the soil was divided into three different layers as shown in Fig. 6.4. The boundary and initial conditions of the model are schematically illustrated in Fig. 6.4. Based on the tree height, stem diameter and canopy coverage, the length and depth of root zone were estimated to be 2 meters and 1 meter respectively. The transpiration rate of golden raintree was taken as 1 - 5 L/day (5L per day during summer and 1.0 L/day for winter).

The finite element model, 20 meters in length and 10 meters in depth, consisted of four hundred and fifty 4-noded linear plane strain quadrilateral elements (CPE4P). The parameters input in the finite element model are listed in Table 6.1.

![Fig. 6.4 Geometry and boundary conditions of the FE model for case study one](image-url)
Table 6.1 Parameters applied in the finite element model for case study one

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transpiration $T_p$</td>
<td>1-5 L/day</td>
</tr>
<tr>
<td>Maximum root depth $z_{\text{max}}$</td>
<td>1 m</td>
</tr>
<tr>
<td>Maximum root width $r_{\text{max}}$</td>
<td>2 m</td>
</tr>
<tr>
<td>Root density $\beta(\theta)$</td>
<td>$\beta(z) = 1 - \frac{z}{z_{\text{max}}}$</td>
</tr>
<tr>
<td>Hydraulic conductivity $K(\theta)$</td>
<td>$5.79 \times 10^{-8}$ m/s</td>
</tr>
<tr>
<td>Surface flow $q_s$</td>
<td>Fig. 6.3</td>
</tr>
<tr>
<td>Suction profile</td>
<td></td>
</tr>
<tr>
<td>Log elastic modulus $\kappa$</td>
<td>0.02</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.1</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.36</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.82</td>
</tr>
<tr>
<td>Soil water characteristic curve</td>
<td></td>
</tr>
<tr>
<td>Depth of design suction change $H_s$</td>
<td>4 m</td>
</tr>
<tr>
<td>Saturated suction $S_{sa}$</td>
<td>20 kPa</td>
</tr>
<tr>
<td>Air entry value $S_{ae}$</td>
<td>250 kPa</td>
</tr>
<tr>
<td>Residual suction $S_{re}$</td>
<td>12000 kPa</td>
</tr>
<tr>
<td>$e - \ln s$ curve (layer 1: 0 -1 m)</td>
<td>4% per log MPa</td>
</tr>
<tr>
<td>$e - \ln s$ curve (layer 2: 1 4.5 m)</td>
<td>6% per log MPa</td>
</tr>
<tr>
<td>$e - \ln s$ curve (layer 3: 4.5 -10 m)</td>
<td>9.5% per log MPa</td>
</tr>
</tbody>
</table>
6.2.3 Results of the finite element analysis

Using the numerical model outlined in Chapter 5, the back-analysis of soil suction distribution and soil movement under influence of Golden raintree and climate was conducted. The time-dependent FE analysis was started from 22 August 2001 when the initial soil suction profile was measured, at the end of the wet season. The distributions of soil suction and degree of saturation near Golden raintree after 30 weeks (end of first dry season) and 100 weeks (end of second wet season) are plotted in Fig. 6.5 and Fig. 6.6 respectively.

![Diagram of soil suction distribution](image)

**Fig. 6.5** Calculated soil suction distribution near golden raintree (pF)

(suction in pF = log (suction in kPa) + 1.01)
Fig. 6.6 Calculated degree of saturation distribution near golden raintree (%)

Figures 6.7 and 6.8 show the calculated soil suction profiles at BH1 (approximately 1 m away from tree) and BH2 (4.5 m away from tree). The maximum change in soil suction occurred at the top surface. The climate influence is evident as soil suction was increased during the dry season and reduced during the wet season. The depth of seasonal moisture variation, $H_s$, is taken as 4m at Adelaide (AS2870, 2011). The soil suction change became very small at about 4m but then appears to remain at a constant value to a depth of 5.5m. This is consistent with the expectation that a tree is able to extend the active depth, beyond the depth of seasonally induced moisture change. Also shown in Fig. 6.7 and Fig. 6.8 are the measured soil suctions during the field investigations. It can be seen that the field values of soil suction generally followed the trend predicted by the finite element model.
Fig. 6.7 Soil suction profiles at BH1 (1 m away from tree) – comparison of results predicted by the numerical model with the measures results.

Fig. 6.8 Soil suction profiles at BH2 (4.5 m away from tree) – comparison of results predicted by the numerical model with the measures results.
The contour plot of the calculated vertical displacement and the deformed shape of the whole model are presented in Figs. 6.9 and 6.10 respectively. Provided the profile of suction variations are known and soil shrink-swell indices are measured, the vertical shrinkage settlement or heave of an expansive soil profile can be estimated using the simply equation recommended by AS2870 (2011). Fig. 6.11 shows the calculated (by the FE model) and estimated (by using AS2870 method) surface movement as a function of time at the location of BH1 and BH2. Also shown in Fig. 6.11(b) are the measured data by level survey. Although the general trends are similar, the soil movement estimated by one dimensional shrink-swell approach was approximately 60% larger than that predicted by the finite element model. It should be pointed out that movement predictions by shrink-swell approach usually fall within ±25% of the actual movement (Cameron 1989). Although the measured movement at BH1 generally followed the trend predicted by the numerical model, the correspondence between observed and predicted movement was not strong, which could be attributed to differences in soil profiles.
(a) After 30 weeks (April 2002)

Fig. 6.10 The deformed shape of soil profile (deflection magnified by 25)

(b) After 100 weeks (September 2003)

Fig. 6.11 Ground surface movement with time - comparison of results predicted by the numerical model with the results estimated by AS2870 method
6.3 Case study two - *Eucalyptus ficifolia* at Glenroy, Victoria

6.3.1 Soil and site description

The second case study is related to the results of the field monitoring of a single, 2.5 m high *Eucalyptus ficifolia* at the test site in Glenroy, Victoria. The details of the experimental site have been described in Chapter 4. The site is some 44 m by 18 m and has a single storey solid masonry dwelling at the middle of the site. This site contains approximately 3 m layer of expansive soil underlain by basaltic bed rock. The geotechnical profile for the soil found at the site is shown in Fig. 6.12. The soil profile across the site is relatively uniform. Monitoring of this site began in May of 2011 and is to continue for a period of at least five years. The results of the field moisture content and soil movement measured near *Eucalyptus ficifolia* were used to calibrate and validate the proposed finite element model. The soil parameters used in the finite element analysis were obtained from a series of laboratory tests.

![Soil profile at Glenroy site](image-url)
6.3.2 Results of numerical modelling and analysis

Melbourne has a mild, temperate climate. Therefore the depth seasonal moisture variation is much small compared to Adelaide which has semi-arid climate. According to the Australian Standard for Residential Slabs and Footings (AS2870, 2011), the depth of seasonal soil suction change, $H_s$, at the research site should be taken as 2.3 m. Considering that the tree planted is relative young and the shallow bedrock is present at this site, the finite element model, as shown in Fig. 6.13, is 10 meters in length and 5 meters in depth. The root geometry of eucalyptus ficifolia was measured before planting. The horizontal distance, $r$, and the depth $d$, of root water uptake zone are both taken as 0.5 meters. According to the soil profile description shown in Fig. 6.12, the soil profile was divided into two layers. The drying curves of the different soil layers were obtained from the SWCC tests. Table 6.2 shows the physical parameters used for the second case study, which were based on the field data. Soil properties and parameters used in the finite element analysis for silty clay and clay soil are given in Table 6.3 and Table 6.4 respectively. These parameters were obtained from laboratory tests.

![Geometry and boundary conditions of the FE model for case study two](image)

Fig. 6.13 Geometry and boundary conditions of the FE model for case study two
Table 6.2 Parameters used in the finite element model for case study two

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value/reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transpiration $T_p$</td>
<td>Fig. 4.14</td>
</tr>
<tr>
<td>Maximum root depth $z_{max}$</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Maximum root width $r_{max}$</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Root density $\beta(\theta)$</td>
<td>$\beta(z) = 1 - \frac{z}{z_{max}}$</td>
</tr>
<tr>
<td>Hydraulic conductivity $K(\theta)$</td>
<td>5.8E-7 m/s</td>
</tr>
<tr>
<td>Surface flow $q_s$</td>
<td>Fig. 4.10</td>
</tr>
<tr>
<td>Suction profile</td>
<td>Table 5.4</td>
</tr>
<tr>
<td>Depth of design suction change $H_s$</td>
<td>2.3 m</td>
</tr>
</tbody>
</table>

Table 6.3 Soil properties and parameters adopted in FE analysis for silty clay

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value/reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log elastic modulus $\kappa$</td>
<td>0.02</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.09</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.12</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.7251</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>$e - \ln s$ curve</td>
<td>Fig.4.32</td>
</tr>
<tr>
<td>Saturated suction $S_{sa}$</td>
<td>25 kPa</td>
</tr>
<tr>
<td>Air entry value $S_{ae}$</td>
<td>60 kPa</td>
</tr>
<tr>
<td>Residual suction $S_{re}$</td>
<td>10000 kPa</td>
</tr>
<tr>
<td>Soil water characteristic curve</td>
<td>Table 5.6</td>
</tr>
</tbody>
</table>
Table 6.4 Soil properties and parameters adopted in FE analysis for clay

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value/reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log elastic modulus $\kappa$</td>
<td>0.025</td>
</tr>
<tr>
<td>Log hardening constant $\lambda$</td>
<td>0.12</td>
</tr>
<tr>
<td>Stress Ratio $M$</td>
<td>1.12</td>
</tr>
<tr>
<td>Initial yield surface size $a_0$</td>
<td>1</td>
</tr>
<tr>
<td>Wet yield surface size</td>
<td>1</td>
</tr>
<tr>
<td>Flow stress ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>Initial void ratio $e_0$</td>
<td>0.8094</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>$e – \ln S$ curve</td>
<td>Fig. 4.35</td>
</tr>
<tr>
<td>Saturated suction $S_{ra}$</td>
<td>25 kPa</td>
</tr>
<tr>
<td>Air entry value $S_{ae}$</td>
<td>200 kPa</td>
</tr>
<tr>
<td>Residual suction $S_{re}$</td>
<td>15000 kPa</td>
</tr>
<tr>
<td>Soil water characteristic curve</td>
<td>Table 5.8</td>
</tr>
</tbody>
</table>

The soil moisture variation and ground movement induced by root absorption of eucalyptus were simulated by using the finite element model for a period of 100 days (between May 2011 and Sep 2011). The distribution of soil suction and degree of saturation predicted by the numerical model are shown in Figs. 6.14 and 6.15 respectively. From Fig. 6.16, it can be seen that the effect of the root absorption on soil moisture flow is evident.

![Contour plot of calculated soil suction distribution after 100 days (pF)
(suction in pF = log (suction in kPa) + 1.01)](image.png)
As shown in Chapter 4, the volumetric water content at the test site was measured by a neutron moisture probe. To compare the field results with the suction distribution predicted by the finite element model, the field suction profiles were estimated based on the SWCC of Glenroy soil. Van Genuchten equation (1980) was used to convert the volumetric water content to soil suction. For silty clay at Glenroy site, $a_{d/w} = 1600 \text{MPa}$, $m_{d/w} = 1.3$, $n_{d/w} = 0.6$, initial void ratio $e_0 = 0.7251$. Soil suction $s$ can be calculated as follows:

$$s = 1600 \left[ (2.1513 \cdot V_w /100)^{-0.77} - 1 \right]^{0.67} \tag{6.1}$$

where $V_w$ is the measured volumetric water content.
For clay below 1.65 m, $a_{d/w} = 900\text{MPa}, m_{d/w} = 0.9$, $n_{d/w} = 0.9$, initial void ratio $e_0 = 0.8094$. The soil suction can be estimated using the following equation

$$s = 900\left[(1.9259 \cdot V_w / 100)^{-1.1} - 1\right]^{1}$$

(6.2)

The calculated soil suctions at various locations and depths by the finite element model are given in Table 6.5. From Fig. 6.17, it can be seen that soil suction profiles predicted by the numerical model compare reasonably well with those estimated using SWCC and Van Genuchten equation.

Table 6.5 Soil suction profiles predicted by the numerical model

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Initial Suction (pF)</th>
<th>Suction d = 0.5m (pF)</th>
<th>Suction d = 1m (pF)</th>
<th>Suction d = 2m (pF)</th>
<th>Suction d = 3m (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.31</td>
<td>3.51</td>
<td>3.56</td>
<td>3.63</td>
<td>3.68</td>
</tr>
<tr>
<td>0.5</td>
<td>3.61</td>
<td>3.48</td>
<td>3.5</td>
<td>3.59</td>
<td>3.72</td>
</tr>
<tr>
<td>1</td>
<td>3.78</td>
<td>3.58</td>
<td>3.63</td>
<td>3.7</td>
<td>3.81</td>
</tr>
<tr>
<td>1.5</td>
<td>4.01</td>
<td>3.85</td>
<td>3.88</td>
<td>3.91</td>
<td>3.94</td>
</tr>
<tr>
<td>2</td>
<td>4.15</td>
<td>3.97</td>
<td>3.99</td>
<td>4.05</td>
<td>4.09</td>
</tr>
<tr>
<td>2.5</td>
<td>4.21</td>
<td>4.08</td>
<td>4.12</td>
<td>4.13</td>
<td>4.13</td>
</tr>
<tr>
<td>3</td>
<td>4.21</td>
<td>4.19</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
</tr>
<tr>
<td>4</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
</tr>
<tr>
<td>5</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
<td>4.21</td>
</tr>
</tbody>
</table>

Fig. 6.17 Comparison between the soil suction profiles estimated using SWCC and Van Genuchten equation and predicted by the numerical model.
Fig. 6.18 shows the contour plot of the calculated vertical soil movement after 100 days. The ground movements predicted by the numerical model at various locations and depths are plotted against the measured data in Fig. 6.19 and Fig. 6.20. It can be seen that the numerical results agree reasonably well with the measured data at the field site.

As shown in Fig. 6.19, the observed and predicted soil settlements close to the tree were both increasing with time. This can be attributed to the tree root water absorption effect. From Fig. 6.19, it can be seen that the observed maximum surface settlement is 6 mm and the predicted maximum soil movement is approximately 7.2 mm, both occurred at a distance of about 1 m from the tree trunk. The observed movements are smaller than predicted results although they followed the trend predicted by the numerical model. A number of factors might cause this discrepancy. For example, the actual root density distribution may be different with that assumed in the numerical modelling and the influence of climate on ground movement may be underestimated in the finite element analysis.

Fig. 6.18 Contour plot of the calculated vertical displacement after 100 days (mm)
(a) The measured surface movements  
(b) The calculated surface movements

Fig. 6.19 Comparison between the measured surface movements and the results of finite element analysis

(a) The measured movements at various depths  
(b) The calculated movements

Fig. 6.20  Comparison between the measured soil movements at various depths and the results of finite element analysis
6.4 Parametric study and discussion

In the above sections, two case studies were discussed in detail. Further study has been carried out to evaluate the relative importance of various parameters on tree root-water-soil interaction analysis.

6.4.1 Elapsed time

To assess the effect of elapsed time on the changes of soil moisture and ground movement, the surface settlement and soil suction profiles versus times are plotted in Fig 6.21 and Fig. 6.22. It can be seen that tree root absorption has a continuous affect to the soil behaviours. Both the soil settlement and soil suction are increased with the elapsed time. Nevertheless, there are no linear relationship between the soil movement and time or between the soil suction and time. It is interesting to note that the effect of tree root on soil settlement is more significant in first several months.

![Fig. 6.21 Calculated surface movements versus times](image)
6.4.2 Transpiration rate

The tree growing factors and shrinkage settlement induced by the tree root drying are closely related to the transpiration rate of a tree. A number of different transpiration rates (0.6L/day, 1.5L/day, 5L/day and 10L/day) were used in the numerical analyses to assess this effect of the transpiration rate on the ground movement and soil suction change. The results are presented in Figures 6.23 and 6.24. As shown in Fig. 6.23, when the transpiration rate is taken as 0.6L/day, the maximum settlement is only 7.2 mm. When the transpiration rate is increased to 10L/day, the effect of root absorption becomes dominant and the maximum soil settlement is increased to 30 mm. Increasing transpiration rate from 0.6L/day to 10L/day leads to an increase of soil suction at the surface from 3.51 pF to 3.86 pF (Fig. 6.24).
Fig. 6.23 Effect of tranpiration rate on soil settlement near a tree

Fig. 6.24 Effect of tranpiration rate on soil suction variations
6.4.3 Climate effect

Climate has a major effect on the extent of tree root growth and tree-water-soil interaction as well. However nearly all existing numerical models ignore the influence of climate on the changes of in situ soil moisture and ground movement. The same finite element model used for case study two was re-run with a “no water in-flow” (i.e. ignore rainfall and evaporation) condition and the results are plotted in Fig. 6.24. Compared with Fig. 6.21 (rainfall and evaporation taken into account in boundary conditions), it can be seen that the influence area of root absorption was decreased from 6.2 m to 4.7 m. The maximum settlement is almost unchanged but it occurred just under the tree (distance = 0) when the climate effect is ignored. It is interesting note that when the effect of climate was taken into account (Fig. 6.21), the maximum settlement occurred at about 1 m from the tree trunk, the same distance observed during the field investigation.

In addition to transpiration rate and climate, the tendency of soil settlement is also related to the soil layers. When the soil foundation is assumed as homogenous and has just one soil layer, as shown in the example in Chapter 5, the settlement of soil simply decreases with the depth to the tree root. When a soil profile is divided into a number of different soil layers, the situation becomes complicated. The different mechanical and hydraulic behaviours of different soil layers may lead to irregular results of the soil settlement. In particularly, the soil drying behaviour may influence the soil settlement significantly. In case study one, the drying curve of the soil below 4.5 m is 9.5%/pF. This soil is more sensitive with the variation of water content. This is the reason why the influenced depth of root absorption in this particular case is over 5.5 m in both numerical results and the real condition.
6.5 Conclusions

In this chapter, two case studies and back-analysis have been presented, one for a single, 2.5 m high eucalyptus ficifolia at a carefully monitored field site in Melbourne (temperate climate), the other for a single, 3.1 m high golden raintree (koelreuteria paniculata) in Adelaide (semi-arid climate). The results of the back-analysis agreed well with the measured data at the field sites.

From the results of the parametric studies, the following observations can be made:

- As the tree transpiration rate increase, *in situ* soil suction and soil shrinkage settlement all increase.

- Considering the effect of climate reduces the influence area of tree root absorption and leads to a more realistic ground deformation pattern.

- The influence area of root absorption, *in situ* soil suction, and soil settlement all increase with time and growth of the tree.
Chapter 7 Conclusions and Recommendations

7.1 Conclusions

In this thesis, a finite element model has been developed for analysing and modelling the interactions between tree root and expansive soils. The model was incorporated in the finite element software ABAQUS.

The numerical model developed in this research is more rational than all existing numerical models for root water absorption and soil desiccation in that the effect of climate is taken into account and SWCC is incorporated into the constitutive model for tree root-water-soil interaction. Movements of the soil foundation are generated by the initial soil suction conditions and subsequent changes in the boundary conditions due to the climate and tree root activities. The tree root water uptake is treated as a volumetric sink term in a flow equation and is estimated based on the measured sap flow rate or leave area index (LAI) of a tree. A new three dimensional sink term equation considering complex root geometry shape is given. The hydraulic and mechanical behaviour of expansive soil are taken as a function of soil suction, soil water characteristic curve and effective stress. To implement root absorption and unsaturated soil model into ABAQUS, five FORTRAN subroutines were written and tested.
As well as numerical modelling work, the research encompassed field experiments and laboratory testing. In order to collect high quality field data that can be used to evaluate and develop numerical model for soil drying by trees, a field site was established at Glenroy, in a northern suburb of Melbourne. The site has been extensively instrumented to allow relative humidity, solar radiation, wind direction and speed, rainfall, sap flow of trees, soil moisture conditions and ground movements to be closely monitored. The field experiments were complemented by a series of laboratory tests, which included test for soil index properties, soil suction, moisture content, shrink-swell index, SWCC and compression parameters.

Two case studies have also been carried out. The validity of the proposed numerical model has been verified by the back-analysis of the soil moisture variation and ground movements induced by root absorption of a eucalyptus ficifolia at a highly reactive site in Melbourne (temperate climate) and a golden raintree (koelreuteria paniculata) at a extremely reactive site in Adelaide (semi-arid climate). Good agreement is obtained between the numerical analyses and field measurements.

Overall, the research has shown that the proposed numerical model is capable of modelling the climate-tree root-moisture flow-expansive soil interaction to a degree of sophistication that has not previously been so readily achievable. It is believed that utilisation of such a model and the results of the field experiments can lead to a higher level understanding of the impact of trees on soil profile and houses and subsequently less risk in the design of footings for drying of the foundation by trees.

7.2 Recommendations

It is recommended that further research be carried out in the following areas:
7.2.1 More case studies

The numerical model developed in this study allowed for a detailed back analysis of field measured soil suction and ground movement near a tree. More case studies of different tree species, in different climates with different tree height and root distributions are required. It is believed that current design footing methods recommended by AS2870 (2011) could be improved and a greater insight into the influence of trees on residential footings and foundations could be achieved by a large number of the case studies.

7.2.2 Tree growing factors

The tree growing factors in this thesis is not specifically demonstrated. The author affiliates this problem into the definition of transpiration rate. When tree is growing, it requires more water to live and the transpiration rate is increased. The variation of transpiration rate could be measured be a sap flow device. To take the tree growing factors into account, real transpiration rate can be implemented into the numerical model as a function of time.

Tree growing factors actually influence the root absorption model in two ways: i.e., the vibration of total transpiration; the redistribution of root area. When a tree is growing, the root spreads in the soil and the root zone area is extended.

The extension of tree root zone is very hard to measure; Indraratna et al. (2006) suggested to use the following empirical formula from Borg and Grims (1986) to consider crop root growing:

\[ z_{\text{max}} (t) = z_{\text{max}, f} \{0.50 + 0.5 \sin\{3.03(t/t_f) - 1.47\}\} \]  
\[ r_{\text{max}} (t) = r_{\text{max}, f} \{0.50 + 0.5 \sin\{3.03(t/t_f) - 1.47\}\} \]
where $z_{\text{max}}(t)$ is the maximum depth of root zone at time $t$, $r_{\text{max}}(t)$ is the maximum lateral distance of root zone at time $t$, $z_{\text{max},f}$ is the maximum possible root zone depth, $r_{\text{max},f}$ is the maximum possible lateral distance and $t_f$ is the time that root growth stops and the root zone reaches its maximum.

Nevertheless, in this equation, the maximum possible depth and lateral distance is difficult to define as the fitting parameters for tree growth are not a constant. The author found it was very hard to apply this equation in the numerical analysis. Therefore a more reasonable tree growing model needs to be developed.

### 7.2.3 Constitutive modelling

As reviewed in Chapter two, in the current constitutive modelling, the descriptions of stress variables can be divided into single constitutive variables (SCV) and dual constitutive variables (DCV). In this thesis, the author uses the Bishop (1959) effective stress equation with single constitutive variables (SCV). Although the effective stress principle has been widely adopted in unsaturated soil models for decades, the argument on the definition of $\chi$ is continued. Some researchers argued the parameter $\chi$ in different conditions are not a constant number.

According to the theoretical and experimental justification by Fredlund and Morgenstern (1977), two independent stress state tensors are used in constitutive modelling for unsaturated soil as:

$$
\begin{align*}
\bar{\sigma} &= \sigma - u_a \\
\bar{u} &= u_a - u_w
\end{align*}
$$

(7.3)

where $\bar{\sigma}$ is the mean net stress, $\sigma$ is total stress and $s$ is matric suction.
The constitutive equations with dual constitutive variables (DCV) seem more reasonable. As described in chapter four, the author also used experimental data to calibrate the SFG model developed by Sheng et al. (2008), the result is quite reasonable. Nevertheless, the application of this model in numerical modelling has difficulty.

According to Sheng et al. (2008) an incremental stress-strain relationship is derived for the model:

\[
\begin{pmatrix}
\frac{d\sigma}{d\theta} \\
\frac{d\varepsilon}{ds}
\end{pmatrix} = \begin{pmatrix}
D^{ep} W^{ep} \\
R & G
\end{pmatrix}
\begin{pmatrix}
d\varepsilon \\
ds
\end{pmatrix}
\] (7.4)

Where \( D^{ep} \) is a 6×6 matrix, \( R \) is a row vector of six elements, \( W^{ep} \) is a column vector of six elements, and \( G \) is a scalar. \( \theta \) is degree of saturation. Obviously, this model contains the degree of saturation \( \theta \) and soil suction \( s \) as a part of constitutive equation.

As presented in Chapter three, in most finite element analysis softwares, it is generally defines the relationship between stress \( \sigma \) and strain \( \varepsilon \) by giving different 6×6 matrix \( D^{ep} \). Unfortunately, when the degree of saturation \( \theta \) and soil suction \( s \) are integrated in a constitutive model, no commercial finite element software provides options to define a 7×7 stress-strain relationship. It is a big challenge for all researchers who do numerical modelling in unsaturated soil to implement a dual constitutive variables constitutive model which contains a hydraulic line (7×7 matrix) in numerical modelling.

7.2.4 The interaction between tree root, expansive soil and footing

This thesis focused on the interactions between root water absorption and expansive soil. The effects of tree root water uptake and soil desiccation on the
performance of a reinforced concrete footing on expansive soils have not been included in this study.

Trees have been found to be a major cause of damage to residential buildings on expansive soils. Current design guides and common practice have limited tree near a house. There has now arisen the requirement that engineers should directly design the footing and superstructure to accommodate the effects of nearby trees. Further research is urgently required in this area.

The numerical model proposed in this research can be used to study the influence of trees on residential footings. It has been demonstrated that soil suction change within foundation soil due the effect of tree root drying can be predicted by the proposed numerical model. A residential footing can be modelled with ABAQUS BEAM and SLAB elements. Once the soil suction distribution is determined, the foundation movement induced by change in soil suction, and the deformation and stresses of the footing can be calculated.
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