An optimisation approach for assigning resources to defensive tasks during wildfires

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research programme; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and ethics procedures and guidelines have been followed.

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August 2015
Abstract

All over the world, wildfires have a big economic, social and environmental impact. It is expected that climate change will result in more frequent, large, catastrophic wildfires. Responding to these large wildfires is a difficult task with high stakes. Incident management teams (IMTs) managing the response to large, escaped wildfires operate in high-pressure environments where they must make complex, time-critical decisions under fast moving, changing conditions.

Past research on providing decision support to IMTs focused on modelling initial attack, fire line construction, pre-incident deployment and longer-term planning. However, on days of extreme fire weather, when large fires are burning in hot, dry and windy conditions, fire suppression may be both ineffective and unsafe. The aim of this thesis is to address the problem of assigning resources to alternative tasks besides direct fire suppression.

A description of the wildfire resource assignment problem is presented. A mixed-integer programming model is formulated to capture features that are unique to the problem of protecting assets during wildfires. The formulated model generalises the team orienteering problem with time windows, allowing for mixed vehicle types, interchangeable and complementary vehicle capabilities, and travel times which are determined by vehicle specific speed and road network information. The protection requirements of locations are defined in terms of vehicle capabilities.

Two approaches are presented to deal with the dynamic nature of wildfire planning: a dynamic rerouting approach and a two-stage stochastic programming approach. The rerouting approach is appropriate when disruptions are unexpected. The aim is to reassign vehicle in a manner that minimises changes to current vehicle assignment. The stochastic approach uses likelihood estimates for fire spread scenarios. Initial vehicle assignments are made in the first stage with the opportunity for adjustments in the second stage based on observed fire-weather outcomes.

The proposed approaches resulted in a set of complementary models for wildfire resource assignment. They can, among other, account for mixed vehicle capabilities, handle unexpected changes and incorporate fire spread scenario likelihoods. The models are computationally feasible and have the potential to provide real-time decision support to IMTs.
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<td>AIIMS</td>
<td>Australasian Inter-service Incident Management System</td>
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<tr>
<td>COPTW</td>
<td>Cooperative orienteering problem with time windows</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>HP</td>
<td>Heavy pumper</td>
</tr>
<tr>
<td>HT</td>
<td>Heavy tanker</td>
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<td>IMT</td>
<td>Incident management team</td>
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<td>LT</td>
<td>Light tanker</td>
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<td>MIP</td>
<td>Mixed integer programming</td>
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<td>MP</td>
<td>Medium pumper</td>
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<tr>
<td>MT</td>
<td>Medium tanker</td>
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<td>OP</td>
<td>Orienteering problem</td>
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<td>SVRPTW</td>
<td>Selective vehicle routing problem with time windows</td>
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<td>TFS</td>
<td>Tasmania Fire Service</td>
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<td>TOPTW</td>
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<td>Unmanned aerial vehicle</td>
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CHAPTER 1

Introduction

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1.1 Background

Taming fire has played a pivotal role in human development. Fire was used regularly for cooking as far back as 400 000 years ago, with some suggesting cooked food may already have appeared as early as 1.9 million years ago (Bowman et al., 2009). Cooking opened up a whole new range of edible food, changing and broadening the variety food that is available to us, impacting human evolution and steering the development of culture (Aiello and Wheeler, 1995; Laland et al., 2001). A form of fire that mankind is still striving to tame though, is the wildfire.

A wildfire is a fire burning out of control in a wilderness area. Wildfires are often referred to by names such as forest fires, veldfires or grass fires depending on the type of vegetation that is being burned. Wildfires can be a massive, destructive force, with the severity and impact of wildfires expected to increase. Bowman et al. (2009) noted that there has been a marked

\footnote{In Australia wildfires are called bushfires. The name bushfire is used to refer to wildfires burning in any of the vegetation types including grass, scrub, bush and forested areas.}
increase in the incidence of large, uncontrolled fires on all vegetated continents in the decade preceding their study, it is expected that global climate change will continue to increase the risk of extreme fire events (Bowman et al., 2009; Williams et al., 2009).

There are high economic, social and environmental costs associated with wildfires. These costs are often due to, among other factors, the destruction of forest plantations, property and infrastructure. Smoke and haze resulting from wildfires are known to cause adverse health effects (Bowman et al., 2009). In an effort to mitigate the incidence and effect of fire, governments commit large amounts of funding to fire planning, fire fighting resources and staffing. Wildfire is also a challenge to conservation efforts, causing negative impacts on some natural environments, while other environments depend on fire regimes to maintain biodiversity (Cowling, 1992). Wildfires are often necessary to maintain habitat quality and to stimulate forest and pasture regeneration (Bowman et al., 2009).

Worldwide large, catastrophic wildfires have had a big impact on landscapes, ecology and people. This is especially true in Australia where some of the world's largest and most intense bushfires occur. The following are examples of these fires. The 1851 Black Thursday fires caused the loss of 12 lives, one million sheep and thousands of cattle. Approximately 50,000 km$^2$ burned during the Black Thursday fires. During the fires of Black Friday in 1939 in Victoria, 71 were killed and 20,000 km$^2$ of land burned. Following the Black Friday fires, a Royal Commission was established to investigate the cause of the fire and to make recommendations on how future disasters may be prevented. The commission proposed the establishment of a regime of supervised burning which is still practised today. Ash Wednesday fires in 1983 burned 392,000 hectares of grass land and 76 people lost their lives. More recently in 2009, the Black Saturday bushfires in Victoria resulted in 173 fatalities, burning 4,500 km$^2$, more than 2,000 houses were destroyed, and numerous other structures were lost. The combined value of the damage caused by the 2009 Black Saturday fires is estimated to be over AU$4 billion (Teague et al., 2010).

1.2 Fire management and incident control

In fire management, decisions are made at three levels, namely strategic, tactical and operational. Strategic decisions are long-term decisions with typical planning horizons of years and decades, examples of which are facility location, budget allocation, resource mix determination, staffing levels and fleet acquisition. Tactical decisions are typically mid-term decisions made over seasons and days. Examples of tactical decisions are determining the seasonal and daily suppression resource deployment, spatial allocation of fuel treatment and fire prevention planning. Operational decisions are those decisions taken during a fire to mitigate the impact of the wildfire incident. Operational decisions are made on very short time scales, typically measured in hours and minutes. Fire suppression tactics and the dispatch of fire suppression crews are examples of operational decisions. In this thesis the focus is the use of operations research methods to aid in operational decisionmaking.
The initial attack is the first suppression actions taken on a wildfire (Martell, 2007). Most forest fire management agencies attempt to initiate suppression action while fires are small. Fast, aggressive initial attack could lead to containing a fire with little cost. Dispatchers must decide what resources to dispatch to each fire. Initial attack dispatching decisions must be resolved quickly and with limited information. An escaped fire is a fire which cannot be contained by initial attack resources. A campaign fire is a fire that requires substantial firefighting resources and possibly several days or weeks to suppress. A crown fire is a forest fire that jumps from crown to crown ahead of the ground fire, often advancing at great speed.

1.3 Fire management in Australia

Australasian Fire and Emergency Service Authorities Council (AFAC) is the body representing urban, rural and land management agencies within Australia and New Zealand, having a responsibly for the protection of life and property from fire and other emergencies (Australasian Fire Authorities Council, 2011). The respective fire agencies of each state are members of AFAC (AFAC, 2012). AFAC was established in 1993 by its members.

Due to the use of common emergency management doctrine and equipment, the techniques and principles used by fire management agencies share many similarities across jurisdictions. This allows national, and often international, cooperation and resource sharing among agencies when the capacity of local fire agencies to deal with a wildfire has been exceeded. With this universal applicability in mind, case studies from Tasmania are used to demonstrate the models that are developed in later chapters of this thesis.

In the state of Tasmania the Tasmania Fire Services (TFS) has the responsibility of managing fires in the state. The TFS fight both urban and wildfires and consists of more than 230 brigades with 250 career fire fighters and about 4800 volunteers (Tasmania Fire Service, 2012). The TFS undertakes emergency response, call handling and dispatch, fire investigation, training, community fire education, building safety, fire equipment sales and service, building and maintaining TFS vehicles, maintaining a state-wide communications network and fire alarm monitoring (Tasmania Fire Service, 2012). The TFS has mutual aid arrangements with Forestry Tasmania and the Parks and Wildlife Service to combat wildfires effectively.

1.4 Incident management systems

A number of guidelines have been drawn up by governments to deal with disaster response. These guidelines are often called Incident Management Systems (or Incident Command Systems in the United States). An Incident Management System aims to provide guidelines for dealing with large-scale events that require the response and cooperation of multiple agencies. The system sets out protocols for coordinating the agencies and sharing information between
the agencies. An *incident* is an unplanned event that requires emergency intervention. Examples of incidents are wildfires, urban fires, earthquakes and riots. The Australian government developed the Australasian Inter-service Incident Management System (AIIMS) based on the United State’s National Inter-Agency Incident Management System (Australasian Fire Authorities Council, 2011). AIIMS provides common terminology and structure, sets guidelines for communication between organisations at all levels of incidents and establishes the chain of command.

The AIIMS is guided by a number of core principles. The principle of management by objectives is applied, that is when objectives are communicated to all personnel so they know and understand the direction being taken during the operation. At any time, each incident can have only one set of objectives and one incident action plan for achieving it (Australasian Fire Authorities Council, 2011). Management structures should be scalable, enabling smooth and logical escalation and de-escalation as required by an incident. Ideally, up to five reporting groups or individuals are supervised by one person (Australasian Fire Authorities Council, 2011).

### 1.4.1 Incident classification

Incidents are classified according to the severity and extent of the impact of the incident (Australasian Fire Authorities Council, 2011). An incident which can be resolved by using local or initial response resources is called a *level 1* incident. The response to a level 1 incident is usually management by a single incident controller, taking on all the duties required to manage the incident. The major function is operations, that is to resolve the incident. The operations function can usually be carried out by the incident controller while planning and logistics are generally undertaken concurrently by the incident controller.

*Level 2* incidents are larger and more complex. These incidents require more resources and have higher risks associated. Resources need to be deployed beyond the initial response, or sectorisation of the incident, or the establishment of functional sections due to the levels of complexity, or a combination of the above.

*Level 3* incidents are characterised by degrees of complexity that may require the establishment of divisions for effective management of the situation. The incidents will usually involve delegation of all functions by the incident controller. The 2009 Black Saturday bushfires is an example of a level 3 incident.

### 1.4.2 Fire danger ratings

*Fire danger* is “a general term used to express an assessment of both fixed and variable factors of the fire environment that determine the ease of ignition, rate of spread, difficulty of control and fire impact” (Merrill and Alexander, 1987). A fire danger index is a quantitative measure of the fire danger, or one or more aspects of the fire danger. Typical factors that are considered when
calculating a fire danger index are dryness, based on rainfall and evaporation, and meteorological variables for windspeed, temperature and humidity. A number of different fire danger rating systems are used worldwide. The three most well known rating system are the United States National Fired Danger Rating System (NFDRS), the Canadian Forest Fire Weather Index (FWI) System and the McArthur Fire Danger Meters for forest and grasslands used in Australia (Dowdy et al., 2009, p.3).

The Australian fire danger rating system is based on the McArthur Forest Fire Danger Index, originally designed to be a scale from 1 to 100, but they are often used with values outside of the original intended design which lead to ratings exceeding 100. A rating of 1 means that fires will not burn, or will burn very slowly and can be easily controlled. A rating of 100 implies that fires will burn so fast and hot that it will be virtually impossible to control the fires (McArthur, 1973).

The danger rating is calculated using the temperature, the wind speed, the relative humidity and the drought factor. The drought factor represents the availability of fuel and is given as a number between 0 and 10, reflecting the influence of recent temperatures and rainfall on fuel availability (Dowdy et al., 2009, p.4).

Fire danger indices and ratings are important inputs in a number of fire management decisions. The fire danger index can aid decisions on initial attack preparedness of fire fighting authorities. The fire danger rating may be used as a guideline for policies, such as whether to impose fire bans. Public warnings and advice in Australia are issued based on five categories of fire danger ratings. The fire danger categories and their associated public warnings are shown in Figure 1.1 with explanations of the different categories. The information issued to the public is given in Table 1.1.

![Figure 1.1](image)

**Figure 1.1:** In Australia a fire danger rating system is used with 6 categories to warn and inform the public about the potential threat that a bushfire would pose if ignited on the given day.

Fire warnings are issued at a rating of 24 in Tasmania and above 50 in other states (Dowdy et al., 2009, p.3). Ratings above 50 occur in Tasmania about three times a year. The decision to stay and protect a home or not is left to home owners on fire danger ratings below 100. For
Chapter 1. Introduction

Fire danger index | Fire danger rating | TFS recommended action
--- | --- | ---
0–11 | Low-moderate | Know where to get more information and monitor the situation for any changes.
12–24 | High | Know where to get more information and monitor the situation for any changes.
25–49 | Very high | Only stay home if your home is well prepared and you can actively defend it.
50–74 | Severe | Leaving is the safest option for survival. Only stay if your home is well prepared and you can actively defend it.
75–99 | Extreme | Leaving is the safest option for your survival.
100 or higher | Catastrophic | Leaving is the only safe option for your survival – regardless of any plan to stay and defend.

| Table 1.1: The recommended actions issued by the TFS for a given fire danger rating. |

catastrophic ratings, i.e. ratings above 100, TFS advises that people move away from danger areas and that they should not attempt to protect their property regardless of any preparation they might have made.

1.4.3 The incident management team

In many jurisdictions, including Australia, Canada and the United States, incident management teams (IMTs) are responsible for coordinating, planning and managing wildfire response-related activities (Australasian Fire Authorities Council, 2011; ICS Canada, 2012; US Department of Homeland Security, 2008).

The size of an IMT depends on the severity of the incident. Incident management teams always have an incident controller, who is responsible for the management of all incident control activities, including control, planning, operations and logistics (Australasian Fire Authorities Council, 2011). The Australasian Inter-service Incident Management System (AIIMS) states that only one incident controller may manage an incident at any one time (Australasian Fire Authorities Council, 2011). The overall responsibility belongs to the incident controller. The incident controller’s responsibilities are grouped into four functional areas, namely control, planning, operations and logistics. In large and complex incidents the incident controller may elect to delegate some or all of the functions, creating an IMT. The incident controller is in charge of the IMT.

Planning involves the collection, analysis and dissemination of information and the development of plans to resolve the incident. The planning officer is responsible for all planning activities. The operations officer coordinates the tasking and application of resources to achieve the resolution of an incident. The logistics officer deals with the acquisition and provision of human and physical resources, facilities, services and materials to support the achievement of incident objectives. An overview of an IMT is shown in Figure 1.2. Each officer in the IMT may appoint
Figure 1.2: An Incident Management Team (IMT) consists of four core members, the incident controller, the operations officer, the logistics officer and the planning officer. In some cases, in addition to the four members, a safety advisor and liaison officer is appointed. During very large incident each officer may have a team to support him in his duties.
a team to assist him if the severity of the event requires it.

The incident controller may elect to appoint additional personnel to assist him or her in his or her responsibilities. The incident controller is responsible for maintaining the safety and welfare of crews and supporting personnel and may choose to appoint a safety advisor to oversee the occupational health and safety at the incident. Other potential roles that may be filled by persons working directly for the incident controller, as suggested by AIIMS, are intra-organisational liaison, specialist advising, community liaison and oversight of safety and performance.

The Victorian State Emergency control centre, where large emergencies are managed by Victorian IMTs, is shown in Figures 1.3(a)-(c).

1.4.4 The objectives of the incident management team

The first objective of any fire fighting operation is to save lives, as stated in the AFAC position on bushfires (Australasian Fire Authorities Council, 2010):

“5.1 The protection of people is always the highest priority

In all cases, the protection of people should be the first and highest priority for fire agencies and others while controlling bushfires.”

Once adequate consideration has been given to the protection of people, the IMT may prioritise other objectives. AFAC highlights the following priorities:

“5.2 Fire agencies should give priority to informing and protecting people, and protecting the assets communities value

If a bushfire cannot be controlled, efforts should be directed to anticipating the bushfire’s progress and safeguarding people threatened by it. Fire agencies should, subject to the availability of resources, focus on:

1. gathering and providing information and issuing warnings to enable those at risk to protect themselves;
2. protecting vulnerable people, in child care centres, schools, group homes, aged care facilities, hospitals, prisons, caravan parks and camping grounds, fire refuges and other places where vulnerable people gather; and if appropriate, recommending evacuation;
3. protecting assets the community has identified as valuable, and that will assist the community recover after the fire;
4. stopping building-to-building fire spread in built-up areas; and
5. protecting less valuable or more isolated assets.”
1.4. Incident management systems

Figure 1.3: The Victoria state emergency centre.

(a) Mapping section

(b) Fire behaviour analyst

(c) State map in the main control room.
Chapter 1. Introduction

The priorities may be summarised by their order of importance as: (I) protecting people, (II) defending valuable assets, (III) preventing building to building ignitions, (IV) defending other ‘non-valuable’ assets, and (V) containing or extinguishing the fire. These are the priorities that The Tasmania Fire Service use when facing decisions (Killalea, 2015). In most cases containing or extinguishing the fire takes care of all five of the priorities. But when the fire, or part of the fire, is not containable, then the first four goals need to be taken into account more explicitly. In the case where a fire is burning out of control with no hope of containment, then goal five is ignored and all resources are applied to satisfying the first four goals. It may happen that certain parts of the fire may be contained by suppression activities while other fronts are burning out of control. In this case a mix of resources may be applied for active suppression, defending assets and advising or protecting people.

To aid the IMT in their efforts, in some Australian jurisdictions the fire services prepare community protection plans (D. Killalea, Tasmania Fire Service, personal communication, 2014). These plans, among other things, identify community assets together with information pertinent to protecting those assets. The protection plans contain GPS coordinates, access information, number and type of resources required to protect the assets and importance of the assets to the community. Some examples of community assets are communication towers, hotels, historically significant buildings, schools, bridges, factories and hospitals. Extracts from a Community Protection Plan prepared by the Tasmanian Fire Service are shown in Figures 1.4 and 1.5.

1.5 Thesis scope and objectives

IMTs dealing with large escaped wildfires operate in high pressure environments where they must make complex, time-critical decisions under dynamic, changing conditions. The tasks that the IMT has to perform include assessing the merits of the available information, devising strategies for containing the fire, minimising the impact of the fire, managing firefighting crews and other resources, issuing warnings to the public and evacuating people. Factors affecting decisions include weather conditions, fire-spread predictions, fuel state, assets under threat, the value of assets, and the location of vulnerable people. A strong need for decision support tools has been identified in the literature (McLennan et al., 2006; Omodei et al., 2005a,b). Challenges and difficulties faced include IMTs becoming overwhelmed with the volume of information, dealing with parameter uncertainty and experiencing biases in human decisionmaking. In this context, application of operations research and supply chain logistics tools such as assignment, routing and scheduling models could lead to enhanced management of large fires (Martell, 2007).

On days of extreme fire weather, when large fires are burning in hot, dry and windy conditions, fire suppression may be both ineffective and unsafe. In these circumstances, fire agency resources may be better utilised by assigning them to “defensive” tasks such as asset protection, protecting vulnerable people in place, evacuating communities, collecting information and issuing warnings (CSIRO, 2009).
1.5. Thesis scope and objectives

Introduction

Command Intent: This is a Community Bushfire Response Plan. Its purpose is to inform operational decision-making when establishing protection priorities in response to bushfires affecting this area, particularly when fires are burning out of control.

It reflects the Tasmania Fire Service’s six operational priorities and its intent is to identify community protection priorities with particular reference to potential locations of vulnerable groups of people, as well as assets that are important for the community’s recovery.

It also includes hazards, water sources and other features of operational significance.

Scope: This plan is primarily for use by Incident Management Teams. It may also be used by TFS Brigades and other fire-fighting resources. It is supported by a Community Bushfire Protection Plan for community members.

Remember:

- CREW LEADERS:
  - Information in this plan may be modified by the IMT, in response to prevailing conditions. Incident Action Plans (IAPs) therefore take precedence over this plan.

- ON THE FIREGROUND:
  - Fire crew safety is paramount. Use appropriate PPE, follow all applicable SOPs and apply LACES (Lookouts, Awareness, Communication, Escape Routes, Safety Zones).
  - Provide frequent Situation Reports. These will inform critical community warnings and information.
  - If you have completed your assigned task, have lost communications and it is safe to do so, move to the next protection priority that is identified in the IAP. If there is no IAP, consider the Public Safety Priorities and Community Assets listed in this plan to identify the next protection priority.

Area Overview

Area:

- Region: NORTHERN
- District: SOUTH ESK
- Council: BREAK O,DAY

Brigades:

- Primary: BINALONG BAY
- Support: ST HELENS, SCAMANDER

Radio:

- VHF (TFS): 15
- UHF (Ops): 12, 13 (Chat)
- VHF (Air Ops): 39
- VHF (PWS, FT): 54, 76

References:

- TasTowns: Page 123
- MapBook: Page 112
- 1:25k Topo: 6043, 6044

Contents:

- Area Profile: Pages 2-3
- Area List: Pages 4-8
- Area Maps: Pages 9-12
- Glossary: Pages 13-14

LEGEND (AREA MAP):

INDEX MAP:

Figure 1.4: An extract from a Community Bushfire Response Plan showing the location of important features such as water sources, nearby safer place, the location of vulnerable people and community assets.
<table>
<thead>
<tr>
<th>Map Reference</th>
<th>Description</th>
<th>Location/Manager</th>
<th>Comments</th>
<th>OPS USE ONLY</th>
</tr>
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<tbody>
<tr>
<td></td>
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<tr>
<td></td>
<td><strong>PUBLIC SAFETY PRIORITIES:</strong></td>
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</tr>
<tr>
<td>1</td>
<td>Nearby Safer Place</td>
<td></td>
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<tr>
<td></td>
<td>Boat Harbour Point</td>
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<td>2</td>
<td>Nearby Safer Place</td>
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<tr>
<td></td>
<td>Binalong Bay Beach</td>
<td></td>
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<tr>
<td>3</td>
<td>Nearby Safer Place</td>
<td></td>
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<tr>
<td></td>
<td>Round Hill Point</td>
<td></td>
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<td>4</td>
<td>Nearby Safer Place</td>
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<td></td>
<td>Taylors Beach (South)</td>
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<td>5</td>
<td>Nearby Safer Place</td>
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<td></td>
<td>Taylors Beach (North)</td>
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<tr>
<td>6</td>
<td>Nearby Safer Place</td>
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<td></td>
<td>Honeymoon Point</td>
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</tbody>
</table>

**FIRE GROUND INSTRUCTIONS:**
If there is direct threat to life, evacuate to nearby a safer place if possible/safe.

<table>
<thead>
<tr>
<th>Resources Type</th>
<th>Assignment</th>
<th>Priority</th>
</tr>
</thead>
</table>

**Figure 1.5:** An extract from a Community Bushfire Response Plan showing the location of important features such as water sources, nearby safer place, the location of vulnerable people and community assets.
The aim of this thesis is to address the problem of assigning resources to these defensive tasks. A number of modelling approaches are proposed in this thesis to contribute towards the development of decision support tools to aid IMTs at an operational (i.e. real-time) level. The underlying question guiding this research is: What is the best use of the available wildfire resources considering a large wildfire burning out of control and impacting people and assets? With this in mind, the following four objectives are pursued:

I Provide an efficient mixed-integer programming formulation for problem of assigning vehicles during wildfires to protect valuable assets.

II Formulate a model to assist IMTs assign wildfire vehicles to cooperative tasks.

III Propose a method for rerouting vehicles to account for changes in wildfire conditions which may cause disruption to existing vehicle routing plans.

IV Formulate a stochastic/probabilistic wildfire vehicle routing model which incorporates knowledge about expected future fire-spread scenarios.

1.6 Thesis organisation

Chapter 2 contains a review of the literature relevant to this thesis. The literature review is done in three parts. The first section focuses on past research on decision support in wildfire incident management to describe the context for this work; a brief review of wildfire-spread modelling is conducted in the second section. The problem of assigning wildfire resources, which is the focus of this thesis, has features in common with a number of variations on a well known vehicle routing problem, namely the orienteering problem. In the third section, vehicle routing literature is explored, highlighting problems in the literature that are the most similar to the wildfire resource assignment problem.

An efficient two-index mixed-integer programming formulation of the team orienteering problem with time windows (TOPTW) is presented in Chapter 3. The new formulation eliminates symmetry from the traditional TOPTW formulation. A new class of the team orienteering problem, the cooperative orienteering problem with time windows (COPTW) is introduced. The COPTW is motivated by the problem of assigning wildfire resources to defensive tasks and is a generalisation of the TOPTW that requires multiple vehicles to cooperatively provide service to a customer. Finally a genetic algorithm is presented for the COPTW. The algorithm is tested on generated benchmark instances, showing some promise to finding approximate solutions within minutes.

In Chapter 4, the problem of assigning resources to asset protection activities when large wildfires are burning out of control and fire suppression is not a viable option, is considered in more detail. A mixed-integer programming model assigning resources to asset protection with the aim of maximising the total protected asset value is formulated. The model allows for mixed vehicle
types with interchangeable capabilities and vehicle travel times determined by vehicle specific speeds and road network information. The protection requirements of locations are defined in terms of the vehicles’ capabilities.

In Chapter 5, a multi-objective mixed-integer linear programming approach is developed to assist in the rerouting of wildfire response vehicles once a disruption has occurred. The model maximises the total value of assets protected while minimising changes to the original vehicle assignments. A number of methods for quantifying these changes are proposed. The model is demonstrated using a realistic fire scenario impacting South Hobart, Tasmania, Australia. Computational testing shows that realistic-sized problems can be solved within a reasonable time using a commercial solver. Optimal solutions were found within seconds for test instances with 30 locations and narrow time windows. This MIP rerouting approach could thus be used in real-time for small assignment problems.

A two-stage stochastic programming approach is proposed for the problem of assigning resources to tasks in Chapter 6. The approach takes uncertainty in the modelling parameters into account. Initial vehicle assignments are made in the first stage with the opportunity for adjustments in the second stage based on observed fire-weather outcomes. The two-stage stochastic for wildfire vehicle routing is demonstrated using a case study with three second-stage scenarios.

The thesis closes in Chapter 7 with a summary of the work, an appraisal of the contributions of the thesis, as well as a discussion on possibilities for future work.
This chapter contains a review of the literature relevant to this thesis. The literature review is presented in three parts. The first section focuses on past research on decision support in wildfire incident management to establish the context of this work. Decision support tools are often dependent on wildfire simulations and forecasts, and this work is no exception. Bearing this in mind, a brief review of wildfire spread modelling is presented in the second section. The problem of assigning wildfire resources, which is the focus of this thesis, has features in common with a number of variations on the well-known vehicle routing problem. In the third section, vehicle routing literature is explored, highlighting problems in the literature that are most similar to the wildfire resource assignment problem.
2.1 Optimisation in wildfire and emergency incident management

Due to the complexity of decisionmaking, modelling, simulation and mathematical tools have become an integral part of the decisionmaking process. Designing fire regimes, allocating fire fighting resources and fighting fires on the ground are all done with the help of these tools. The development of decision support tools to aid fire management were reviewed by Martell (1982), Martell et al. (1998) and most recently by Minas et al. (2012).

Published research to date largely focuses on long-term planning, with considerably fewer studies concerned with the short-term IMT-level decisionmaking (Minas et al., 2012). Models developed to support short-term wildfire decisionmaking are concerned with the dispatch of resources to fires and with fire-line construction (Donovan and Rideout, 2003; Haight and Fried, 2007; Lee et al., 2013; Pappis and Rachaniotis, 2010a). Alternative tasks, besides direct fire suppression, that fire agency resources can perform were not so well studied.

Mees et al. (1994) considered the problem of optimally assigning resources to fireline construction in order to minimise the total expected cost plus loss. Their model calculates the probability of fire containment as a function of resource allocation, taking into account uncertainty in both the flame length of the fire and in the width of the fireline that is produced. Martin-Fernández et al. (2002) developed a model that employs discrete simulation algorithms and Bayesian optimisation methods for real-time dispatch of firefighting resources to wildfires. The model was applied in a case study in Northern Spain to demonstrate its ability to handle real-time weather changes and chaotic fire behaviour. An integer programming model to determine the optimal mix of firefighting resources to dispatch to a given fire to achieve containment with minimal resultant costs and damages was described by Donovan and Rideout (2003). Hu and Ntaimo (2009) presented a stochastic mixed-integer programming model for initial attack. Pappis and Rachaniotis (2010a) considered the non-linear dynamics of fire suppression, that is the observation that small delays in dispatch of initial attack resources can result in dramatic fire-loss increases. They constructed a non-linear programming model which utilises the concept of ‘deteriorating jobs’, that is jobs processed later in the sequence require more time. Pappis and Rachaniotis (2010a) scheduled a single firefighting resource for when there are several existing fires to be controlled. The model was later extended to allow scheduling of multiple firefighting resources (Pappis and Rachaniotis, 2010b). Ntaimo et al. (2012) presented a two-stage stochastic integer programming model for initial attack with the goal of containing as many fires as possible while minimising rental and travel cost and the expected future maintenance cost. The model allows for multiple types of firefighting resources and defines a standard response based on fireline production rates. A statistical sampling approach is employed to handle the large number of possible scenarios of fire occurrence and fire behaviour. Haight and Fried (2007) presented a scenario-optimisation integer programming model for initial attack resource deployment based on the classical maximal covering location model. The model’s objective is to minimise the expected number of fires that do not receive a standard response subject to resource availability.
constraints, where a standard response is defined as the required number of resources that can reach the fire within a maximum response time. The optimisation model was later expanded by Lee et al. (2013), allowing the sharing of resources between multiple planning units and adding resource types that differ from one another with respect to response time, fireline production, cost and basing constraints. Homchaudhuri et al. (2013) presented a genetic algorithm-based approach to efficiently allocate resources to fireline construction and optimum fireline building that minimises the total damage caused by wildfires.

Optimisation models have been successfully applied to a wide variety of emergency and disaster operations (Altay and Green, 2006; Caunhye et al., 2012). Urban search and rescue share some similarities with wildfire response (Chen and Miller-Hooks, 2012; Fiedrich et al., 2000). Chen and Miller-Hooks (2012) investigated optimal deployment of urban search and rescue teams to disaster sites by means of multi-stage stochastic programming. Urban search and rescue has a decaying probability of success, whereas wildfires have very strict time windows during which any action may have success. Hence, urban search and rescue models cannot be directly applied to wildfire asset protection.

2.2 Fire spread and behaviour models

Fire spread and behaviour models play an important role in decision support tools for both incident management and long-term planning. This section gives a brief overview of fire modelling literature and the fire spread models currently being used, with a focus on wildfire management in Australia.

Fire spread and behaviour models have been reviewed by Perry (1998), Pastor et al. (2003) and more recently in the review series by Sullivan (2009a,b,c).

Perry (1998) divides fire models broadly into two groups: those concerned with the quantification of fire behaviour through the prediction of parameters such as rate of spread and fire line intensity and those concerned with predicting the final shape, or spatial extent, of an event. They are known respectively as fire behaviour and fire spread models. Models do not always fall clearly into one of the two categories as the fire spread is dependent on the fire behaviour and a fire behaviour model may also predict fire spread.

In his review series, Sullivan groups fire spread or behaviour models into three categories depending on the underlying method used in the modelling approach, i.e. physical, empirical or simulation. Physical models are based on the fundamental chemistry and physics, or physics alone (Sullivan, 2009a). Empirical models are based on the statistical analysis of observed and measured data (Sullivan, 2009b). Simulation models are implementations of empirical and semi-empirical models, their primary function being to convert these one-dimensional models to two dimensions and then simulate the propagation of a fire perimeter across a landscape (Sullivan, 2009c).
Typical inputs into fire models are wind speed, wind direction, terrain slope, fuel type, fuel load and fuel moisture. The estimation of the two parameters fire intensity and rate of spread are of particular interest in fire modelling. Incident managers want to know how the fire will spread, the rate at which it will spread and how the shape of the developing perimeter will change over time. Knowing the intensity of the fire is important as this influences the effectiveness of suppression activities.

Two popular models being developed in Australia are Phoenix and Australis. Phoenix is a fire spread model being developed in Victoria at Melbourne University (Sullivan, 2009c; Tolhurst et al., 2008). Two basic fire behaviour models underpin Phoenix, the CSIRO southern grassland fire spread model and the McArthur Mk5 forest fire behaviour model. Changes were made to these models to account for the dynamic nature of the interaction between fire and its environment (Tolhurst et al., 2008). The landscape is divided into uniform square cells, each of which has 31 attributes. The data is stored in a personal geodatabase (MS-Access). Grid size may be specified by the user – 5 meter grids have been used, but grid sizes of 100 meters or 200 meters are common. Huygen’s fire spread algorithms are used to model the fire spread. Output characterises the fire in each cell across the landscape in terms of the origin of the source fire, the size of the fire at the time of impact, fireline intensity, flame height, time to impact the cell from ignition and ember density falling in the cell.

Considerable and ongoing attention has been given to fire spread modelling. It is reasonable to expect that continued improvement in speed and accuracy of fire spread simulation will lead to improved real-time fire spread forecasts available to IMTs. With wildfire decision support tools often reliant on fire spread predictions, these improvements in fire spread forecasting will in turn lead to better decision support tools. In this project, wildfire simulations output from Phoenix has been used. These simulated fire spreads were provided by the Tasmania Fire Service, taking local fuel and fire weather conditions into account.

2.3 Vehicle routing problems

Vehicle routing problems (VRPs) are concerned with the distribution of goods between depots and final users or customers (Toth and Vigo, 2002). Perhaps the simplest VRP is the capacitated VRP (CVRP). In the CVRP all customers correspond to deliveries. Each vehicle has a capacity restriction and the aim is to find the least cost routes which meet the demands of all the customers. The CVRP generalises the well-known travelling salesman problem (TSP).

Routing problems with profits is a class of vehicle routing problems in which not all customers have to be served (Feillet et al., 2005). A profit is associated with each customer and is gained when the customer is visited. Depending on the problem, the gained profit is either maximised, or a certain minimum profit must be gained (Archetti et al., 2013). Feillet et al. (2005) carried out a survey of routing problems with profits.
2.3. Vehicle routing problems

2.3.1 The orienteering problem

The orienteering problem (OP) is in the class of routing problems with profits. The OP borrows its name from the sport orienteering. Orienteering is a family of sports in which competitors have to navigate a route at speed from point to point in unfamiliar terrain by aid of a compass and a map. Various modes of travel are used depending on the type of orienteering; some of the popular orienteering modes are running, skiing and mountain biking. There are a number of variations on the type of orienteering events. The orienteering event most similar to the OP is the orienteering event known as the score event. In the score event competitors set off from a starting point and visit as many controls as possible within a time limit before returning to the same starting point. Each control has a score associated with it and competitors receive a penalty for each minute that they spend on the course beyond the allotted time. The winner is the competitor with the highest score. The OP is also known as the selective travelling salesman problem, the maximum collection problem and the bank robber problem in the operations research literature (Vansteenwegen et al., 2011).

2.3.2 Variations of the orienteering problem

The OP itself may be considered as a variation on the well-known travelling salesman problem. The OP is analogous to the salesman having limited time to complete his journey, but knows the expected sales in each city. The salesman aims to maximise his sales by visiting a subset of the towns.

A number of variations on the OP may be found in the literature (Vansteenwegen et al., 2011). The special case where the start and end vertices of the OP coincide, is called the tour orienteering problem. In the team orienteering problem a team of orienteerers work together to collect the scores. According to Montemanni et al. (2011) the team orienteering problem was first studied by Butt and Cavalier (1994) and Chao et al. (1996).

In the orienteering problem with time windows, a time window is associated with each location. The score at a location may only be collected during the time window. In the team orienteering problem with time windows (TOPTW), a team is available to collect the scores. The selective vehicle routing problem with time windows (SVRPTW) generalises the TOPTW by adding a capacity constraint to each vehicle. Team orienteering problems and the SVRP fall within the class of routing problems with profits. The TOPTW is discussed in more detail in the following section.

2.3.3 The team orienteering problem with time windows

The TOPWT is a generalisation of the team orienteering problem. In the TOPTW a time window is associated with each vertex. A vertex must be visited within its associated time window in order to collect the reward associated with that vertex. Kantor and Rosenwein
Chapter 2. Literature Review

(1992) was the first to solve the (single) orienteering problem with time windows, while the TOPTW was first studied by Vansteenwegen et al. (2009) and Montemanni and Gambardella (2009).

The TOPTW is in the class of vehicle routing problems with profits and is closely related to the selective vehicle routing problem with time windows (SVRPTW) studied by Gueguen and Dejax (1999). The SVRPTW generalises the TOPTW by adding a capacity constraint to each vehicle (Feillet et al., 2005). Vehicle routing problems with profits have been reviewed by Archetti et al. (2014b) and orienteering problems by Vansteenwegen et al. (2011).

The split delivery capacitated team orienteering problem allows multiple vehicles to service a single location (Archetti et al., 2014a), but differs from the COPTW in the sense that it does not require the service times at a single location to coincide.

2.3.4 Vehicle routing and assignment problems with cooperative service requirements

Kingston and Schumacher (2005) developed a mixed-integer linear programming model for the assignment of unmanned aerial vehicles to tasks such as surveillance. The formulation allowed for cooperative delivery of service and time windows. The problem was not formulated as a traditional vehicle routing problem, but calculated the travelling time of vehicles in a separate path planning process. Weinstein and Schumacher (2007) formulated the unmanned aerial vehicle (UAV) scheduling problem in a similar manner to the traditional vehicle routing problem with time windows. They included timing constraints, requiring certain locations to be serviced at the same time. They also considered soft time windows, applying a penalty function for service times occurring outside of the time windows. The manpower allocation problem with job teaming constraints is closely related to the vehicle routing problem with time windows, and requires cooperative service delivery (Dohn et al., 2009).

2.3.5 Benchmark instances and solution methods

Benchmark data sets used to test algorithms designed for the TOPTW are derived from datasets created by Solomon (1987) and Cordeau et al. (1997). Righini and Salani (2009) designed 58 instances based on the data sets of Solomon (1987) for the vehicle routing problem with time windows and the ten multi-depot vehicle routing problems of Cordeau et al. (1997). Montemanni and Gambardella (2009) created 27 instances based on Solomon’s sets and 10 based on Cordeau et al. for the OPTW. All the Solomon instances have 100 possible visits, and those of Cordeau et al. between 48 and 288. Montemanni and Gambardella (2009) designed TOPTW instances where the aforementioned instances are considered with two, three and four tours. No optimal solutions are available for these test instances. A repository of orienteering problem benchmark instances may be found online at http://www.mech.kuleuven.be/en/cib/op/ (KU Leuven, 2015).
A variety of solution approaches have been proposed for OPs. Boussier et al. (2006) proposed an exact algorithm for team orienteering problems, demonstrating its implementation for the team orienteering problem and the SVRPTW. Gueguen and Dejax (1999) and Butt and Ryan (1999) proposed exact algorithms for the SVRPTW. Both these approaches are neatly summarised by Feillet et al. (2005).


### 2.3.6 Vehicle rerouting

The problem of replanning and rerouting vehicles to deal with disruptions falls in the class of dynamic vehicle routing problems. Considerable attention has been given to dynamic vehicle routing problems which is evident from the review by Pillac et al. (2013). However, comparatively fewer studies have considered rerouting in orienteering problems. Minis et al. (2011) considered the rerouting of delivery vehicles when breakdowns occur, modelling the problem as a dynamic team orienteering problem with time windows. Mamasis et al. (2012) formulated a rerouting team orienteering problem and provided a heuristic solution method. Murray and Karwan (2010) developed a mixed-integer programming model for rerouting unmanned aerial vehicles when “pop-up” events occur. Pop-up events are new tasks or changes which were not initially planned for. Murray and Karwan (2010) provided a modelling framework which can model a wide variety of vehicle routing problems, including the TOPTW. A follow-up paper by the same authors presented a branch-and-bound method for solving their unmanned aerial vehicle assignment model (Murray and Karwan, 2013).

The vehicle rerouting studies by Minis et al. (2011) and Mamasis et al. (2012) did not consider the cooperative delivery of service, which is often required in wildfire response services. Our approach is similar to that of Murray and Karwan (2010). However, we do not consider an infinite resource decision variable for infeasible assignments. Murray and Karwan (2010) have multiple nodes, each representing a task to be performed at a single location. In contrast, we consider only a single node for each location, thus reducing the number of integer decision variables required to describe the model. Besides this, we consider continuous time as opposed to discrete time units. These differences allow us to provide an efficient formulation for the wildfire asset protection rerouting problem.
2.3.7 Stochastic programming

In this section published research related to stochastic planning for the team orienteering problem with time windows and wildfire resource management is reviewed. The paper by Higle (2005) is a good brief introduction to stochastic programming.

As mentioned earlier in this chapter, Haight and Fried (2007) and Ntaimo et al. (2012) developed a two-stage programming approach to wildfire resource deployment. However, in these models resources are being assigned to initial attack activities, which is fundamentally different from routing vehicles for defensive tasks.

Barbarosoğlu and Arda (2004) proposed a two-stage stochastic programming model to plan the transportation of vital first-aid commodities to disaster-affected areas during emergency response. A multi-commodity, multi-modal network flow formulation was developed to describe the flow of material over an urban transportation network. Resource mobilisation is treated in a random manner, and the resource requirements are represented as random variables. Random arc capacities and supply amounts arise from the vulnerability of the transportation system. A finite sample of scenarios for capacity, supply and demand triplet represents the randomness.

Evers et al. (2014) considered a number of extensions to the orienteering problem (OP) to model uncertainty that is present when planning UAVs reconnaissance missions. Assigning a single UAV was considered. The travel and recording times are uncertain and the information about each target can only be obtained within a predefined time window. They also considered the appearance of new targets during the flight, which should be visited immediately if possible. They developed a heuristic approach that is used to re-plan the tour each time before leaving a target. The approach balances two objectives: the expected profits of foreseen targets, and the expected percentage of time-sensitive targets reached on time.

Allahviranloo et al. (2014) proposed a new generalisation of the selective VRP to account for different optimisation strategies under uncertain demand levels. Developing parallel genetic algorithms and a classic genetic algorithm.

There has been limited research published with regards to modelling uncertainty in the TOP. According to Allahviranloo et al. (2014), past formulations of the selective VRP have all been deterministic. In a review of VRPs with profits, Archetti et al. (2014b) mention only one variation of the TOP that considers uncertainty, referring to the paper by Ke et al. (2013).

2.4 Chapter summary

This chapter provides an overview of literature related to topics pursued in this thesis.

Optimisation literature in wildfire and emergency incident management is reviewed, demonstrating that past research focused on providing decision support for and modelling initial attack, fire line construction, and longer-term planning with regards to incident management, for instance
determining resource levels. The defensive tasks that wildfire resources can perform were not so well studied.

Vehicle routing literature is reviewed focusing on the orienteering problem with time windows and its variations. The team orienteering problem with time windows is of special interest, because it is used as the basis of the resource assignment models presented later in this thesis.
An efficient two-index mixed-integer programming formulation of the *team orienteering problem with time windows* (TOPTW) is presented in this chapter. The new formulation eliminates symmetry from the traditional TOPTW formulation. The traditional MIP formulation of the TOPTW is a three-index formulation which assigns vehicles to specific routes. However, the vehicles in the TOPTW are identical and the vehicle routes of a solution can be permuted without any change to the solution. It is thus possible to eliminate this symmetry by dropping
the third vehicle related index, formulating the TOPTW as a two-index vehicle flow MIP instead. Test results on standard benchmark instances illustrate the computational improvement achieved by using the new formulation.

Further, a new class of the team orienteering problem, the *cooperative orienteering problem with time windows* (COPTW), is defined. The COPTW is a generalisation of the TOPTW that requires multiple vehicles to collect the reward from a location. The COPTW is demonstrated with the aid of a wildfire scenario in South Hobart, Tasmania, Australia. Computational testing suggests that it is feasible to apply the COPTW to realistic problems.

Finally, a genetic algorithm is presented for the COPTW. The algorithm is tested on generated benchmark instances, showing some promise to finding approximate solutions within minutes.

### 3.1 The TOPTW formulation

Traditionally the TOPTW is presented as a mixed-integer program utilising a three-index vehicle flow formulation (Vansteenwegen et al., 2009, 2011; Montemanni and Gambardella, 2009). The following notation is used to formulate the TOPTW. Consider the connected weighted graph $G$ of order $N$ with vertex set $V$ and arc set $E$. Each arc $e_{ij} \in E$ has an associated weight $w(e_{ij}) = t_{ij}$. The arc weight is analogous to the travelling time (or cost) between vertices. Each vertex has an associated reward $s_i$. Each vehicle starts at a depot located at vertex $v_1$ and ends at a depot located at vertex $v_N$. Let $a_i$ be the service duration associated with vertex $v_i$. Each vertex $v_i$ has an associated time window, the time window’s opening time is $o_i$ and its closing time is $c_i$. The number of members in the orienteering team is given by $P$. The following decision variables are used. The binary variable $Y_{ip}$ equals 1 if vertex $v_i$ is visited in path $p$. The binary decision variable $X_{ijp}$ equals 1 if in path $p$ a visit to vertex $v_i$ is followed by a visit to vertex $v_j$. Finally, $S_{ip}$ is the time that service starts at vertex $v_i$ in path $p$. The problem can be formulated as a mixed-integer programming problem:

$$\text{Max } \sum_{p=1}^{P} \sum_{i=2}^{N-1} s_i Y_{ip}$$  \tag{3.1}$$

subject to

$$\sum_{p=1}^{P} \sum_{j=2}^{N} X_{1jp} = \sum_{p=1}^{P} \sum_{i=1}^{N-1} X_{iNp} = P;$$  \tag{3.2}$$

$$\sum_{i=1}^{N-1} X_{ikp} = \sum_{j=2}^{N} X_{kjp} = Y_{kp} \quad \forall k = 2, \ldots, N-1, \ p = 1, \ldots, P;$$  \tag{3.3}$$

$$S_{ip} + t_{ij} + a_i - S_{jp} \leq M(1 - X_{ijp}) \quad \forall i, j = 1, \ldots, N, \ p = 1, \ldots, P;$$  \tag{3.4}$$

$$\sum_{p=1}^{P} Y_{kp} \leq 1 \quad \forall k = 2, \ldots, N-1;$$  \tag{3.5}$$
3.2 An efficient TOPTW formulation

\[ \sum_{i=2}^{N-1} Y_i a_i + \sum_{i=1}^{N-1} \sum_{j=2}^{N} t_{ij} X_{ijp} \leq T_{\text{max}} \quad \forall \ p = 1, \ldots, P; \]  
\[ a_i \leq S_{ip} \quad \forall p = 1, \ldots, P; \]  
\[ S_{ip} \leq c_i \quad \forall i = 1, \ldots, N, p = 1, \ldots, P; \]  
\[ X_{ijp}, Y_{ip} \in \{0, 1\} \quad \forall i, j = 1, \ldots, N, \ p = 1, \ldots, P. \]  

The objective function (3.1) maximises the total collect reward. Constraints (3.2) ensure all paths start at vertex \( v_1 \) and end at vertex \( v_N \). The connectivity and timeline of paths are determined by constraints (3.3) and (3.4). Constraints (3.5) ensure that at most one vehicle visits a vertex. The time limit is enforced on each path by constraint (3.6). The start of each service time is restricted to the time windows by constraints (3.7) and (3.8). Constraints (3.9) enforce the binary conditions on the decision variables.

3.2 An efficient TOPTW formulation

An efficient two-index mixed-integer programming formulation of the TOPTW is presented next. In the efficient formulation, a number of symmetries are eliminated and infeasible arcs are removed in a pre-processing step. The symmetries are eliminated by not labelling individual vehicle, but instead using an approach similar to commodity flow formulations. Eliminating these symmetries reduces the number of solutions that need to be considered when searching the solution space.

Similar notation is used as in Section 3.1 for the TOPTW formulation with exception to the following decision variables. The decision variable \( Y_i = 1 \) if vertex \( v_i \) is serviced during the time window, otherwise \( Y_i = 0 \). The binary decision variable \( X_{ij} \) indicates whether or not a vehicle is travelling from vertex \( v_i \) to vertex \( v_j \). Finally, \( S_i \) is the start time of service at vertex \( v_i \).

The next step is to eliminate those arcs that are infeasible due to the time window constraints. Consider two vertices \( v_i \) and \( v_j \), chosen such that the earliest possible departure from vertex \( v_i \) results in an arrival at vertex \( v_j \), which is later than the closing time of vertex \( v_j \). Since no feasible solution will contain the arc \( e_{ij} \), it is possible to ignore this arc. Let \( E \) be the index set excluding the infeasible arcs, that is \((i, j) \in E \) if, and only if, \( e_{ij} \in E \) and \( a_i + a_i + t_{ij} \leq c_j \).

Two sets \( \delta^- (i) \) and \( \delta^+ (i) \) are defined to simplify the model notation: \( \delta^- (i) \) is the index set of vertices adjacent to vertex \( v_i \), that is \( j \in \delta^- (i) \) if \((j, i) \in E \), and \( \delta^+ (i) \) is the index set of vertices adjacent from vertex \( v_i \), that is \( j \in \delta^+ (i) \) if \((i, j) \in E \).

Based on the notation introduced above, the TOPTW may be formulated as a mixed-integer program:

\[ \text{Maximise} \sum_{i=2}^{N-1} s_i Y_i \]  

(3.10)
Chapter 3. An Efficient Formulation of the TOPTW and the COPTW

subject to

\[ \sum_{j \in \delta^+(1)} X_{1j} = \sum_{i \in \delta^-(N)} X_{iN} = P; \]  
\[ \sum_{i \in \delta^-(k)} X_{ik} = \sum_{j \in \delta^+(k)} X_{kj} = Y_k \quad \forall \; k = 2, \ldots, N - 1; \]  
\[ S_i + t_{ij} + a_i - S_j \leq M(1 - X_{ij}) \quad \forall \; (i, j) \in E; \]  
\[ \sum_{i=2}^{N-1} Y_i a_i + \sum_{(i,j) \in E} t_{ij} X_{ij} \leq T_{max} P; \]  
\[ o_i \leq S_i \quad \forall \; i = 1, \ldots, N; \]  
\[ S_i \leq c_i \quad \forall \; i = 1, \ldots, N; \]  
\[ X_{ij} \in \{0, 1\} \quad \forall \; (i, j) \in E; \]  
\[ Y_i \in \{0, 1\} \quad \forall \; i = 2, \ldots, N - 1. \]

The objective function (3.10) is to maximise the total collected reward. Constraint (3.11) ensures that each vehicle starts at vertex \( v_1 \) and ends at the vertex \( v_N \). The vehicle flow from and to each vertex is balanced by constraints (3.12). Constraints (3.13) ensure that service at a vertex may only start after service at a previously visited vertex has been completed and sufficient time for travel has been allowed, with \( M \) representing a large constant. Setting \( M \) as \( \max(c_i) + \max(t_{ij}) + \max(a_i) - \min(o_i) \) is sufficiently large as it will never bound the left-hand side of that constraint. Constraint (3.14) reduces the solution space to improve the performance of the solver. The start of service times at vertices are limited to their respective time windows by constraints (3.15) and (3.16). The binary conditions are enforced on the decision variable in constraints (3.17) and (3.18).

Distinct routes are guaranteed by constraints (3.11) and (3.12). Constraints (3.11) require \( P \) vehicles to depart the source and arrive at the sink. At most one vehicle can travel along an edge as enforced by constraints (3.12). Subtours are prevented by constraint (3.13) as long as all travel times are greater than zero. A solution is thus guaranteed to contain \( P \) distinct, continuous paths.

3.2.1 Comparison of solution times

The computational performance of the above formulation is demonstrated next. This new formulation is compared with the traditional mixed-integer programming formulation of the TOPTW as presented in Vansteenwegen et al. (2011). Computational testing was carried out on a single node of a computer cluster. The node had two Intel Xeon E5-2670 processors and 64GB of RAM. CPLEX 12.6 was used to solve the problem instances and performance was measured in CPU time. The solver’s parallel optimisation mode was set to deterministic, while all the remaining CPLEX solver parameters were left at their default values. Problem instances were
3.3. The cooperative orienteering problem with time windows

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<th>Old 1</th>
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<td>4.2</td>
<td>101.2</td>
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Table 3.1: The average solution time of each truncated benchmark set. Problem instances which could not be solved within the three-hour time limit were excluded from the comparisons, the entries belonging to these sets are indicated in boldface.

created using truncated versions of the well-known TOPTW benchmark instances. The full set of TOPTW benchmark instances were described by Vansteenwegen et al. (2011) and are available from the author’s website at http://www.mech.kuleuven.be/cib/op. These benchmarks were truncated to the first $N - 1$ locations, and the initial depot is repeated to act as the final depot. The graph representation of each problem thus has $N$ vertices, a vertex for each of the $N - 2$ locations and two vertices representing the initial and final depots. The results of the computational testing are shown in Table 3.1 with the solution time restricted to three hours (10 800 seconds). The comparison demonstrates that the new formulation significantly reduces the solution time for most of the problem instances.

Solution times for the old and new formulation for problems in the set c100 limited to a size of 35 and $P = 3$ are shown in Table 3.2. The results demonstrate how dramatic the improvement in solution time can be. For example, the problem instance c102 was solved in under two minutes using the new formulation, whereas the old formulation required close to three hours before an optimal solution was found.

3.3 The cooperative orienteering problem with time windows

Next, we define a class of the team orienteering problem – the cooperative orienteering problem with time windows (COPTW). The COPTW, requiring multiple vehicles to cooperatively collect
Table 3.2: A detailed comparison of the solution times of the old and new formulations of the TOPTW for the problems in set c100. The problems were solved for \( P = 3 \) and \( N = 35 \).

<table>
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<th>Problem</th>
<th>Solution time</th>
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<tbody>
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<td></td>
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</tr>
<tr>
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<td>c104</td>
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<td>c109</td>
<td>807.1</td>
</tr>
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</table>

the reward from a location, is a generalisation of the team orienteering problem with time windows. We introduce a two-index vehicle flow formulation for the COPTW based on the efficient TOPTW formulation.

Consider the connected weighted graph \( G \) of order \( N \) with vertex set \( V \) and arc set \( E \). Each arc \( e_{ij} \in E \) has an associated weight \( w(e_{ij}) = t_{ij} \). The arc weight is analogous to the travelling time (or cost) between vertices. Each vertex has an associated reward \( s_i \) and resource requirement \( r_i \). Each vehicle starts at a depot located at vertex \( v_1 \) and ends at a depot located at vertex \( v_N \). There is no value associated with the initial and final vertices, therefore \( s_1 = 0 \) and \( s_N = 0 \). Let \( a_i \) be the service duration associated with vertex \( v_i \). Each vertex \( v_i \) has an associated time window, the time window’s opening time is \( o_i \) and its closing time is \( c_i \). The vehicle fleet consists of \( P \) identical vehicles.

The following decision variables are defined. The decision variable \( Y_i = 1 \) if vertex \( v_i \) is serviced, otherwise \( Y_i = 0 \). The non-negative integer decision variable \( X_{ij} \) is the number of vehicles travelling from vertex \( v_i \) to vertex \( v_j \). The binary variable \( Z_{ij} = 1 \) if \( X_{ij} > 0 \), otherwise \( Z_{ij} = 0 \).

A vertex is considered serviced if \( r_i \) vehicles visit the vertex, all arriving at or before the start of service, \( S_i \). The vehicles then cooperatively provide the service for a duration of \( a_i \).

The next step is to eliminate those arcs that are infeasible due to the time window constraints. Consider two vertices \( v_i \) and \( v_j \), chosen such that the earliest possible departure from vertex \( v_i \) results in an arrival at vertex \( v_j \), which is later than the closing time of vertex \( v_j \). Since no feasible solution will contain the arc \( e_{ij} \), it is possible to ignore this arc. Let \( \mathcal{E} \) be the index set excluding the infeasible arcs, that is \((i, j) \in \mathcal{E} \) if, and only if, \( e_{ij} \in E \) and \( o_i + a_i + t_{ij} \leq c_j \).

Two sets \( \delta^- (i) \) and \( \delta^+ (i) \) are defined to simplify the model notation: \( \delta^- (i) \) is the index set of vertices adjacent to vertex \( v_i \), that is \( j \in \delta^- (i) \) if \((j, i) \in \mathcal{E} \), and \( \delta^+ (i) \) is the index set of vertices adjacent from vertex \( v_i \), that is \( j \in \delta^+ (i) \) if \((i, j) \in \mathcal{E} \).
Based on the notation introduced above, the COPTW may be formulated as a mixed-integer program:

\[
\begin{align*}
\text{Maximise} & \quad \sum_{i=2}^{N-1} s_i Y_i \\
\text{subject to} & \quad \sum_{j \in \delta^+(1)} X_{1j} = \sum_{i \in \delta^-(N)} X_{iN} = P; \\
& \quad \sum_{i \in \delta^-(k)} X_{ik} = \sum_{j \in \delta^+(k)} X_{kj} \quad \forall \ k = 2, \ldots, N - 1; \\
& \quad r_k Y_k = \sum_{j \in \delta^+(k)} X_{kj} \quad \forall \ k = 2, \ldots, N - 1; \\
& \quad X_{ij} \leq PZ_{ij} \quad \forall \ (i, j) \in \mathcal{E}; \\
& \quad S_i + t_{ij} + a_i - S_j \leq M(1 - Z_{ij}) \quad \forall \ (i, j) \in \mathcal{E}; \\
& \quad o_i \leq S_i \quad \forall \ i = 1, \ldots, N; \\
& \quad S_i \leq c_i \quad \forall \ i = 1, \ldots, N; \\
& \quad X_{ij} \in \{0, 1, 2, \ldots, P\} \quad \forall \ (i, j) \in \mathcal{E}; \\
& \quad Y_i, Z_{ij} \in \{0, 1\} \quad \forall \ (i, j) \in \mathcal{E}.
\end{align*}
\]

The objective function (3.19) is to maximise the total collected reward. Constraint (3.20) ensures that each vehicle starts at vertex \( v_1 \) and ends at the vertex \( v_N \). The vehicle flow to and from each vertex is balanced by constraints (3.21). Constraints (3.22) ensure that the required number of vehicles arrive at a serviced location. Constraints (3.23) and (3.24) ensure that service at a vertex may only start after service at a previously visited vertex has been completed and sufficient time for travel has been allowed, with \( M \) representing a large constant.

The assignment \( M = \max(C_i) + \max(t_{ij}) + \max(a_i) - \min(O_i) \) is a sufficiently large value for \( M \). The start of service times at vertices are limited to their respective time windows by constraints (3.25) and (3.26). The integer and binary conditions are enforced on the decision variables by constraints (3.27) and (3.28) respectively.

### 3.4 Case study - Wildfire asset protection in Tasmania

The working of the COPTW is demonstrated in this section with the aid of a hypothetical wildfire scenario. The Tasmanian Fire Service’s community protection plans identify various community assets such as communication towers, hotels and historically significant buildings. A number of these community assets located in South Hobart are shown in Figure 3.1. For the scenarios presented here, a simple fire spread radiating outwards at a rate of 3km/h in a circular fashion from a single point of origin is considered. The fire front is indicated as the dark-grey shaded area on the map in Figure 3.1. Further, it is assumed that each asset requires 30 minutes
of protection at the time of impact. Travel times between assets were calculated using Google Maps’ Distance Matrix service. Random values were generated for the protection requirement and value of each asset.

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</table>

Figure 3.1: Community assets located in South Hobart. The table contains the parameters associated with each location.

An optimal solution utilising five vehicles is shown in Figure 3.2(a). Note the convergence of paths at vertices requiring multiple vehicles. The locations are visited from left to right as the fire spreads across the landscape. In this scenario not all assets can be saved with only five vehicles. Twenty vehicles would be required to provide adequate protection to all of the assets. The roads used by the vehicles are highlighted in Figure 3.2(b).

3.5 Computational study

In this section the solution time of the COPTW is demonstrated. The aim of the testing was to obtain an indication of the size of problems that can be solved within a reasonable time using MIP solution approaches. Computational testing was done on randomly generated problem instances. These instances were generated using the existing TOPTW benchmarks by adding a column for the resource requirements. The resource requirement values were generated by drawing randomly from the set \{1,2,3\}.

The results of the computational testing are summarised in Table 3.3. The same hardware is used as before and is described in §3.2.1. The solution times are again measured in CPU time and the parallel optimisation mode of CPLEX was set to deterministic. All the remaining parameters were left at their default values.

The results indicate that an optimal solution may be found for the majority of problems with 20
Figure 3.2: Applying the COPTW to a wildfire asset protection scenario. The problem parameters are contained in the table in Figure 3.1. An optimal solution is shown for five vehicles, a total reward of 240 is collected by servicing locations 3, 7, 9, 11, 12, 14, 17, 20, 21 and 22. (a) The solution in graph representation. (b) The roads utilised by vehicles in this solution.
<table>
<thead>
<tr>
<th>Set</th>
<th>Size</th>
<th>Solution time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P = 3$</td>
</tr>
<tr>
<td>c100</td>
<td>15</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td><strong>3 615.0</strong></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td><strong>6 005.6</strong></td>
</tr>
<tr>
<td>c200</td>
<td>20</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>48.2</td>
</tr>
<tr>
<td>pr1-10</td>
<td>15</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>119.0</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td><strong>8268.8</strong></td>
</tr>
<tr>
<td>pr11-20</td>
<td>10</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td><strong>7 848.9</strong></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td><strong>10 803.5</strong></td>
</tr>
<tr>
<td>r100</td>
<td>10</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td><strong>3 291.8</strong></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td><strong>7 130.5</strong></td>
</tr>
<tr>
<td>r200</td>
<td>20</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>3 231.3</td>
</tr>
<tr>
<td>rc100</td>
<td>10</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td><strong>3 027.2</strong></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td><strong>7 976.8</strong></td>
</tr>
<tr>
<td>rc200</td>
<td>20</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>206.5</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td><strong>8 151.1</strong></td>
</tr>
</tbody>
</table>

**Table 3.3:** The average solution time of each benchmark set. The problems were truncated to the indicated sizes. Sets containing problems which could not be solved within the three-hour time limit are indicated in boldface.
or fewer locations. In wildfire asset protection the time windows are determined by the anticipated time to impact, therefore the time windows are correlated to their spatial positions. New problem instances were generated to investigate the effect of this correlation on computational performance. These new problem instances were generated with uniform spatially distributed vertices. Ten problem instances with a 100 vertices each were generated. The vertices of each instance are uniformly distributed in a 80km by 80km square region. The travel time is directly proportional to the distance between vertices. The opening time of each time window is correlated to the $x$-coordinate of its vertex. This was done to capture the spatially correlated property of time windows in wildfire scenarios. A parameter $w$ is used to determine the length of the time window of each vertex ($C_i = O_i + w$). The smaller problem instances (25, 50 and 75 vertices) are subsets of the ten 100-vertex instances that were generated.

The solution times of these problems are summarised in Table 3.4. The solution times were limited to three hours (10 800 seconds). Small problems, those with 25 vertices, were solved within a couple of seconds, while the solution times of the larger problems were highly dependent on the problems’ parameters. In the instances considered in Table 3.4, increasing either the length of the time windows or the number of vehicles resulted in increased computation time. In some cases it may be possible to easily solve problems with a 100 locations.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$w$</th>
<th>$P \to 3$</th>
<th>$6$</th>
<th>$3$</th>
<th>$6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.7</td>
<td>2.4</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>4.8</td>
<td>91.5</td>
<td>1828.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>28.9</td>
<td>121.9</td>
<td><strong>5382.3</strong></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>57.6</td>
<td><strong>2216.3</strong></td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: The solution times in seconds for a number of test instances with each $N$ uniformly distributed vertices. Here $P$ is the number of vehicles and $w$ is the length of the time windows. A dash indicates that none of the problems could be solved within the three-hour time limit and bold face entries are sets in which solutions were found for all problems except one.

The various branch and price methods proposed by Boussier et al. (2006), Gueguen and Dejax (1999) and Butt and Ryan (1999) for team orienteering problems use a column generation approach. These approaches employ a set formulation approach, where the set of all feasible routes is the set being considered. The aim is then to choose routes from the set in order to maximise the reward. In the COPTW, however, vehicles must collect rewards cooperatively and it is not possible to calculate the reward of each route independently. Therefore, these branch and price methods cannot be applied directly to solve the COPTW.

### 3.6 A genetic algorithm for the COPTW

The genetic algorithm (GA) presented here is based on the genetic algorithms proposed by Tasgetiren and Smith (2000) and Tasgetiren (2001) for the orienteering problem (without time windows). These algorithms are modified here to allow for multiple vehicles, to account for the
cooperative element of the COPTW and to take time windows into consideration.

### 3.6.1 Representation of solutions

Each solution is presented as an array listing the order in which vertices are visited for each path. For example, the array \([1, 2, 4, 10, 1, 3, 10, 1, 5, 3, 10]\) represents a solution for a problem with ten vertices, utilising three vehicles. The first vehicle visits vertices 1, 2, 4 and 10, the second vehicle visits vertices 1, 3 and 10, and the third vehicle visits vertices 1, 5, 3 and 10. Given the path for each vehicle, it is possible to calculate the service times of vertices and the objective value associated with the solution.

### 3.6.2 Initialisation

The GA is initialised by generating \(N_{GA}\) solutions. For each solution, a random ordering of the vertices is selected. This defines the order in which vertices will be visited by vehicles in each path. Then, for each vehicle, a vertex is selected with probability \(p\) and inserted into the vehicle’s path. Finally the vehicle paths are concatenated to form the entire solution.

### 3.6.3 Crossover

The following crossover steps are repeated until \(\lambda N_{GA}\) offspring solutions have been created, where \(\lambda\) is the crossover rate and \(N_{GA}\) is the population size. Two parents are selected from the parent population using deterministic binary tournament selection with replacement. A pair of solutions are selected at random from the entire parent population; the fittest of the two is kept as the first parent. The step is repeated to find a second parent. The two parents are the crossed over using the following method. The vehicle path which will be cut is randomly selected. Then, for each parent, the position in the path where the cut will occur is selected at random. The offspring solution is constructed by taking the part of the solution from the first parent before the cut, and the part of the solution after the cut from the second parent. For example, let the first parent be given by \(P_1 = [1, 2, 4, 10, 1, 3, 10, 1, 5, 3, 10]\) and the second parent by \(P_2 = [1, 3, 9, 10, 1, 7, 8, 10, 1, 3, 10]\), and assume that the solutions will be cut and crossed in the second path at the vertices indicated in bold. Cutting \(P_1\) before the third vertex of the second path and \(P_2\) before the second vertex of the second path, results in the offspring \([1, 2, 4, 10, 1, 3, 7, 8, 10, 1, 3, 10]\). This crossover may result in duplicate vertices occurring in a path, the first occurring vertex of each duplicate pair is removed from the path to fix this.

### 3.6.4 Mutation

A proportion \(\mu\) of the offspring solutions and parent solutions are selected for mutation. Two mutations are considered, each occurring with equal probability; a vertex is either removed from
3.6. A genetic algorithm for the COPTW

A path in a solution, or a vertex is inserted into a path in a solution. If the path is already empty, then only inserting a vertex into the path is considered. The chosen mutation (insertion or deletion) is attempted \( \sigma \) times, and the solution with the highest fitness of all the mutation attempts is taken as the final mutation result. Each selected solution is replaced by its mutated version.

3.6.5 Calculating fitness

A solution’s fitness is a function of its objective value. A solution \( s \) has a fitness given by

\[
f(s) = \begin{cases} 
\text{ObjVal} - N_i & \text{if the overall time limit is not exceeded}, \\
(\text{ObjVal} - N_i) \times 0.9 & \text{if the overall time limit is exceeded}, 
\end{cases}
\]

where \( N_i \) is the number of visits to vertices by vehicles which do not contribute to the objective value.

3.6.6 Selecting individuals and stopping criteria

Deterministic binary tournament selection without replacement is used to select individuals for the next generation. The entire current parent generation and the newly created offspring solutions are eligible for selection. The selection is carried out until \( N_{GA} \) solutions have been selected.

One iteration of the above crossover, mutation and selection steps is considered one generation. The fitness of each generation is determined, keeping record of the solution found with the highest fitness. The genetic algorithm runs until the best solution found remains unchanged for \( \beta \) generations.

3.6.7 Testing and results

The algorithm was implemented in Matlab and tested using the full-size version of the test instance described in §3.5. The parameters used for the genetic algorithm are summarised in Table 3.5. These parameter values were found to give good solutions through trial and error. The best solution found by CPLEX after an hour was used to provide a comparison of the quality of the solutions provided by the genetic algorithm.

The computational results of the genetic algorithm are summarised in Tables 3.6, 3.7, and 3.8. The genetic algorithm ran on average under 4 minutes. The genetic algorithm performed very well compared to CPLEX for problem sets pr1-10 and pr11-20. The genetic algorithm performs poorly for set c200, giving solutions with objective values less than, but on average within 30% of, the best solution found by CPLEX. Excluding the set c200, the GA on average gave solutions within -15% and 60% of the best solution found by CPLEX.
Chapter 3. An Efficient Formulation of the TOPTW and the COPTW

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>$N_{GA}$</td>
<td>100</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>$\lambda$</td>
<td>.7</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>$\mu$</td>
<td>.3</td>
</tr>
<tr>
<td>Local search iterations</td>
<td>$\sigma$</td>
<td>10</td>
</tr>
<tr>
<td>Initialisation density</td>
<td>$p$</td>
<td>.1</td>
</tr>
<tr>
<td>Stopping criteria (generations)</td>
<td>$\beta$</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.5: The parameters used in the computational analysis of the genetic algorithm.

<table>
<thead>
<tr>
<th>Set</th>
<th>Objective value</th>
<th>% improvement</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c100</td>
<td>0.04</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>c200</td>
<td>-0.19</td>
<td>197</td>
<td></td>
</tr>
<tr>
<td>pr1-10</td>
<td>0.46</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>pr11-20</td>
<td>0.58</td>
<td>202</td>
<td></td>
</tr>
<tr>
<td>r100</td>
<td>0.14</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>r200</td>
<td>-0.07</td>
<td>189</td>
<td></td>
</tr>
<tr>
<td>rc100</td>
<td>0.11</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>rc200</td>
<td>0.15</td>
<td>175</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.6: Genetic algorithm performance compared to the best solution found by CPLEX after 1 hour for $P = 3$.

<table>
<thead>
<tr>
<th>Set</th>
<th>Objective value</th>
<th>% improvement</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c100</td>
<td>-0.05</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>c200</td>
<td>-0.29</td>
<td>201</td>
<td></td>
</tr>
<tr>
<td>pr1-10</td>
<td>0.31</td>
<td>245</td>
<td></td>
</tr>
<tr>
<td>pr11-20</td>
<td>0.50</td>
<td>213</td>
<td></td>
</tr>
<tr>
<td>r100</td>
<td>0.01</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td>r200</td>
<td>-0.08</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>rc100</td>
<td>0.03</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>rc200</td>
<td>0.03</td>
<td>242</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7: Genetic algorithm performance compared to the best solution found by CPLEX after 1 hour for $P = 4$.

<table>
<thead>
<tr>
<th>Set</th>
<th>Objective value</th>
<th>% improvement</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c100</td>
<td>0.00</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>c200</td>
<td>-0.29</td>
<td>306</td>
<td></td>
</tr>
<tr>
<td>pr1-10</td>
<td>0.40</td>
<td>273</td>
<td></td>
</tr>
<tr>
<td>pr11-20</td>
<td>0.60</td>
<td>283</td>
<td></td>
</tr>
<tr>
<td>r100</td>
<td>0.07</td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>r200</td>
<td>-0.14</td>
<td>313</td>
<td></td>
</tr>
<tr>
<td>rc100</td>
<td>-0.02</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>rc200</td>
<td>-0.15</td>
<td>280</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.8: Genetic algorithm performance compared to the best solution found by CPLEX after 1 hour for $P = 5$. 

3.6.8 Algorithm analysis

The algorithm showed some promise towards providing approximate solutions to the COPTW. However, in some case the solutions found were far from optimal. This may be attributed to the role played by the crossover and mutation steps in the algorithm presented here. Traditionally the aim of the crossover step is to improve the fitness of the population by combining the properties of two solution. The mutation step introduces variation in the solution population, increasing the size of the solution space that is explored. However, these roles are reversed in the algorithm presented here. The mutation step, with the combined local search algorithm, generally improves the fitness of the solution population, and the crossover step provided variation. Often in the process of crossing solutions an infeasible offspring solution was created, this property reduced the effectiveness of the genetic algorithm.

3.7 Chapter summary

In this chapter, a new efficient mixed-integer programming formulation for the TOPTW is introduced. The new formulation leads to improved solution times for all problems that were tested. The mixed-integer programming formulation was then modified to account for the resource requirements of locations resulting in the COPTW. The COPTW is motivated by a problem arising in the management of resources during wildfires.

The COPTW provides a first step towards the development of decision support tools for asset protection during wildfires. The computational results demonstrate that it would be possible to apply the model to realistic problems. Problems may arise, however, which are difficult to solve, due to either the underlying properties of the problem, or simply due to a large number of locations. In these cases the development of fast heuristic approaches or efficient exact methods may be very useful.

A genetic algorithm is presented in this chapter. The algorithm was tested on generated benchmark problems. These benchmarks are based on the well-known benchmarks for the TOPTW. Although the algorithm showed some promised towards providing approximate solutions, in some cases the solutions were far from optimal. Infeasibility is often introduced in the crossover steps, reducing the effectiveness of the algorithm.

In the next chapter, modifications to the COPTW is considered to account for specific wildfire management cases. For example, fire agencies often have a mix of different resource types at their disposal. In this case, vehicle capabilities have to be matched to appropriate roles and tasks.
Chapter 3. An Efficient Formulation of the TOPTW and the COPTW
In this chapter, we consider the problem of assigning resources to asset protection activities when large wildfires are burning out of control and fire suppression is not a viable option. We formulated a mixed-integer programming model, assigning resources to asset protection with the aim of maximising the total saved asset value. The model allows for mixed vehicle types with interchangeable capabilities and vehicle travel times determined by vehicle-specific speeds and road network information. The protection requirements of locations are defined in terms of the vehicles’ capabilities.
A description of the wildfire asset protection problem is provided in the next section. Similarities between the wildfire asset protection problem and the team orienteering problem are discussed. A mixed-integer programming formulation of the problem is presented and explained in §4.2. This is followed by a discussion in §4.3 of the model parameters and how different conditions and scenarios could be parameterised. The model’s functionality is then demonstrated on a hypothetical wildfire scenario in South Hobart, Tasmania, Australia. The model’s computational performance is evaluated in §4.4. The chapter concludes with a discussion of the results and of possible future research directions.

4.1 The wildfire asset protection problem

When escaped wildfires impact communities and infrastructure, it is often possible to carry out a number of activities to protect the assets being threatened. Wetting down structures, clearing gutters of combustible material and putting out spot fires are a few examples. The responding fire services need to decide how best to assign the available resources to these asset protection tasks at various locations.

To aid the IMT in their efforts, the fire services prepare community protection plans in some Australian jurisdictions (D. Killalea, Tasmania Fire Service, personal communication, 2014). These plans, among other things, identify various community assets together with information pertinent to protecting those assets. The protection plans contain GPS coordinates, access information, number and type of resources required to protect the assets and the importance of the assets to the community. Some examples of community assets are communication towers, hotels, historically significant buildings, schools, bridges, factories and hospitals.

4.1.1 Asset value and protection requirements

Each asset under threat is assigned a protection priority by the IMT, either explicitly or implicitly. For modelling purposes, this protection priority is translated into a value, and the aim is to protect the maximum total value of assets with the available limited resources. The value of locations may be expressed as a monetary amount or a relative value. Often values would be perceived values rather than calculated values. Communities are often consulted to aid in assessing the value of an asset. A factor that may be considered is the contributions of an asset to the recovery of a community after a wildfire, for example, communication infrastructure is key in coordinating relief efforts. Values can be determined using existing operations research techniques such as analytic hierarchy process.

The community protection plans identify the protection requirements for each asset. For an asset to be protected, the resources with the required capabilities must arrive in time and remain at the asset for a sufficient period of time, called the service duration, to carry out the necessary protection tasks. The number and type of resources required to provide an adequate
level of protection to an asset will depend on several factors. Examples are the accessibility of the location, whether four-wheel drive vehicles are required, the availability of reticulated water, and the type of protection activities required to protect the asset.

4.1.2 Resources

The typical resource units being assigned are fire trucks, commonly referred to as tankers. Besides tankers, various types of vehicles may be available to the IMT in dealing with a fire threatening a community. As an example, the vehicles and resources of the Tasmania Fire Service are shown in Table 4.1. The average travelling speeds of each vehicle type are summarised in Table 4.2.

4.1.3 Time windows

The advancing fire fronts impose time constraints on protection activities. These time constraints can be translated into time windows during which asset protection tasks must commence in order to be successful. The time that a resource starts working on a task is called the service start time. The time windows are determined by the anticipated time to impact, which is the time remaining before the asset is impacted by the fire. The time to impact may be estimated using fire spread modelling. Extensive research had been carried out in the modelling and prediction of fire spread, which was summarised in a series of reviews undertaken by Sullivan (2009a,b,c).

4.1.4 Related problems

The problem of assigning tasks to resources during large escaped wildfires as described above has features in common with the team orienteering problem with time windows (TOPTW). In the TOPTW, a team of orienteerers have a limited time to collect rewards from various locations. The reward at each location is only available for a period of time specified by the location’s time window.

Drawing the analogy to the TOPTW, fire tankers may be seen as members of an orienteering team. The assets requiring protection are equivalent to control points, each with an associated time window and value. However, in the wildfire asset protection problem, multiple resources are often required to protect a single asset, whereas the TOPTW requires the visit of only a single team member to claim a reward from a location. The cooperative orienteering problem with time windows (COPTW) addresses this shortcoming.

The COPTW generalises the TOPTW to allow multiple resources to converge on a single location and cooperatively collect the associated reward. In this chapter, we further extend the cooperative orienteering problem to allow for mixed resource types with different interchangeable
<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Abbreviation</th>
<th>Typical crew size</th>
<th>Capabilities and limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy pumper</td>
<td>HP</td>
<td>4</td>
<td>Asset protection; limited to formed roads, both sealed and unsealed; limited to reticulated water</td>
</tr>
<tr>
<td>Medium pumper</td>
<td>MP</td>
<td>4</td>
<td>Asset protection; limited to formed roads, both sealed and unsealed; limited to reticulated water</td>
</tr>
<tr>
<td>Heavy tanker</td>
<td>HT</td>
<td>4</td>
<td>Suppression and asset protection; roads (formed/unformed and 4WD vehicular tracks)</td>
</tr>
<tr>
<td>Medium tanker</td>
<td>MT</td>
<td>4</td>
<td>Suppression and asset protection; roads (formed/unformed and 4WD vehicular tracks)</td>
</tr>
<tr>
<td>Light tanker</td>
<td>LT</td>
<td>2</td>
<td>Suppression and asset protection; roads (formed/unformed and 4WD vehicular tracks)</td>
</tr>
<tr>
<td>Hydraulic ladder</td>
<td>Aer</td>
<td>2</td>
<td>Asset protection; limited to formed roads (sealed, unsealed) and reticulated water (e.g. snorkel)</td>
</tr>
<tr>
<td>Transportation vehicle</td>
<td>Trans</td>
<td>&lt; 10</td>
<td>Information gathering, firefighter transport (e.g. troop carrier)</td>
</tr>
<tr>
<td>Miscellaneous vehicle</td>
<td>Misc</td>
<td>-</td>
<td>Miscellaneous; limited to formed roads (e.g. canteen vehicle)</td>
</tr>
<tr>
<td>Dozer</td>
<td>-</td>
<td>-</td>
<td>Fire break construction</td>
</tr>
<tr>
<td>Excavator</td>
<td>-</td>
<td>-</td>
<td>Fire break construction</td>
</tr>
</tbody>
</table>

Table 4.1: Vehicle types, their abbreviation, typical crew capacity and roles that the vehicle can perform.
4.2 Model formulation

The resource units will be referred to as vehicles. Let $Q$ be the set of vehicle types. There is a total of $p_q$ vehicles of each type $q \in Q$ available for assignment. The value of the asset at location $i$ is $v_i$. Let $a_i$ be the service duration associated with location $i$, that is the duration vehicles are required stay at the location to protect the asset. Each asset has an associated time window specifying the time during which protection activities must commence in order to be successful. The earliest time that protection activities may commence is $o_i$, also called the time window’s opening time. The latest time that protection activities may commence is $c_i$, called the time window’s closing time. We assume that service has to be provided cooperatively by the required resources, and that the sequence in which service is delivered at each asset is not important.

4.2.1 Depots

Initially the vehicles are located at one of $m$ depots at locations $1, \ldots, m$. The depot may be a vehicle storage area, a fire station or a staging area. For brevity these locations will be referred to as depots. There are $stock_{iq}$ vehicles of type $q$ stationed at depot $i$. The assets are located at locations $m+1, \ldots, n-1$. Note that location $n$ is a dummy location representing the sink in the model formulation. Further $a_i = 0$ for all $i = 1, \ldots, m$. The departure of vehicles from a depot may be delayed by specifying $o_i > 0$.
4.2.2 Asset protection requirements

Let $U$ be the set of vehicle capabilities. Each vehicle type $q$ has an associated capability vector $\text{cap}_q$. The protection requirement for each location is defined in terms of the capabilities required to protect the assets at that location.

The protection requirement of an asset $i$ is given by the protection vector $r_i$, specifying the amount of each capability required. An asset is considered protected if the combined capabilities of the vehicles assigned to the asset meets or exceeds the capabilities required. Furthermore, the vehicles must arrive before or at the start of service time $S_i$ and stay for the service duration $a_i$.

For example, one way of satisfying the protection vector $r_i = (2, 3)$ is by combining the following three vehicles: one vehicle with $\text{cap}_1 = (2, 1)$ and two vehicles with $\text{cap}_2 = (0, 1)$.

4.2.3 Travel time

The time it takes for a vehicle to travel between two locations will depend on the vehicle type and the roads being used. Further, certain roads may only be accessible by some vehicle types, for example roads accessible only by four-wheel drive vehicles. As a result, each vehicle type will often have a unique travel time between two locations. The travel time from location $i$ to location $j$ is denoted by $t_{ijq}$ for each vehicle type $q$.

4.2.4 Preprocessing

We eliminate the paths that are not feasible due to the time window constraints. This preprocessing approach is equivalent to the approach in the previous chapter. Let $L$ be the set of all possible location pairs. For vehicles of type $q$, consider two locations $i$ and $j$ chosen such that the earliest possible departure from location $i$ results in an arrival at location $j$ which is later than the closing time of location $j$. Since no feasible solution will require vehicles of type $q$ to travel from $i$ to $j$, it is possible to ignore this route. Let $E_q$ be the index set including only feasible routes, that is $(i,j) \in E_q$ if and only if $(i,j) \in L$ and $o_i + a_i + t_{ijq} \leq c_j$.

Two sets $\delta^-(k)$ and $\delta^+(k)$ are defined to simplify the model notation: $\delta^-(k)$ is the index set of locations adjacent to location $k$, that is $i \in \delta^-(k)$ if $(i,k) \in E_q$, and $\delta^+(k)$ is the index set of locations adjacent from location $k$, that is $j \in \delta^+(k)$ if $(k,j) \in E_q$.

4.2.5 The mixed-integer programming model formulation

The following decision variables are used in the model formulation:

$X_{ijq}$ is an integer decision variable indicating the number of vehicles of type $q$ travelling from location $i$ to location $j$;
4.3 Model demonstration

\( Y_i = 1 \) if asset \( i \) is protected, otherwise \( Y_i = 0 \);
\( Z_{ijq} = 1 \) if a vehicle of type \( q \) is travelling from location \( i \) to location \( j \), otherwise \( Z_{ijq} = 0 \);
and
\( S_i \) for all \( i \in 1, \ldots, n \) is the start time of service at location \( i \).

Based on the notation introduced above, the problem being considered may be formulated as a mixed-integer programming problem:

\[
\text{Maximise } \sum_{i=m+1}^{n-1} v_i Y_i \quad (4.1)
\]

subject to

\[
\sum_{j \in \delta_i^+ (k)} X_{kjq} = \text{stock}_{kq} \quad \forall \ k = 1, \ldots, m, \ q \in Q; \quad (4.2)
\]

\[
\sum_{i \in \delta_i^- (k)} X_{ikq} = \sum_{j \in \delta_i^+ (k)} X_{kjq} \quad \forall \ k = m + 1, \ldots, n - 1, \ q \in Q; \quad (4.3)
\]

\[
\sum_{q \in Q} \sum_{i \in \delta_i^- (k)} X_{ikq} \text{cap}_{qu} \geq r_{ku} Y_k \quad \forall \ u \in U, k = m + 1, \ldots, n - 1; \quad (4.4)
\]

\[
X_{ijq} \leq p_q Z_{ijq} \quad \forall \ (i, j) \in E_q, \ q \in Q; \quad (4.5)
\]

\[
S_i + t_{ijq} + a_i - S_j \leq M(1 - Z_{ijq}) \quad \forall \ (i, j) \in E_q, \ q \in Q; \quad (4.6)
\]

\[
o_i \leq S_i \quad \forall \ i = 1, \ldots, n - 1; \quad (4.7)
\]

\[
S_i \leq c_i \quad \forall \ i = 1, \ldots, n - 1; \quad (4.8)
\]

\[
X_{ijq} \in \{0, 1, 2, \ldots, p_q \}, Z_{ijq} \in \{0, 1\} \quad \forall \ (i, j) \in E_q, \ q \in Q; \quad (4.9)
\]

\[
Y_i \in \{0, 1\} \quad \forall \ i = m + 1, \ldots, n - 1. \quad (4.10)
\]

The objective function (4.1) maximises the total protected asset value. Constraints (4.2) define the starting position of vehicles as depots. The vehicle flow to and from each location is balanced by constraints (4.3). Constraints (4.4) enforce the condition that an asset is protected only if the vehicles assigned to the asset collectively meet the protection requirement. Constraints (4.5) and (4.6) ensure that service at a location may only start after protection activity at a previously visited location has been completed and sufficient time for travel has been allowed, with \( M \) representing a large constant. Setting \( M = \max(a_i) + \max(t_{ijq}) + \max(a_i) - \min(c_i) \) is sufficiently large for this purpose. The start of protection activities at locations are limited to their respective time windows by constraints (4.7) and (4.8). Constraints (4.9) and (4.10) enforce the integer and binary conditions on the appropriate decision variables.

4.3 Model demonstration

In this section we demonstrate how the model could be used in practice. The modelling approach’s flexibilities are discussed with regards to protection activities and interchanging or
combining resources to protect assets. Finally a case study is considered using assets located in South Hobart, Tasmania, Australia.

4.3.1 Time windows

There are two types of tasks considered in this study: active defence tasks and strategic defence tasks. Active defence tasks are those tasks that take place during the time that a fire is actively impacting the assets, either through direct flame contact or embers. Examples of active defence tasks are putting out spot fires near assets and wetting down structures. The duration of active tasks depends on the intensity of the fire, the structure being threatened and the fuel surrounding the asset, but typically is between fifteen minutes and six hours. To ensure active protection activities commence at the time of impact, the time window’s opening time is equal to its closing time, i.e. time windows represent a single point in time.

Strategic defence tasks are preparatory tasks that can be carried out before a fire impacts an asset. Examples of strategic defence tasks include: clearing fuel around a structure, wetting down the roof, setting up a sprinkler system and applying fire retardant expansion foam to a structure. The time windows associated with strategic defence tasks start some time before the anticipated time of impact and close near the time of impact, depending on the activity.

4.3.2 Interchanging resources

The model allows for combining and substituting resources to meet a given location’s protection requirements. Although a myriad of possibilities of interchanging and combining resources to meet protection requirements exist, three cases are discussed next as an illustration.

Possibly the simplest case is when there is no overlap in the capability of vehicle types. Consider the following example: two vehicles with capabilities \( \text{cap}_1 = (1, 0) \) and \( \text{cap}_2 = (0, 1) \), respectively, are not substitutable.

The second case is where vehicles can perform the same task, but some vehicles can provide more of a required capability than others. In this case, the vehicle capability vectors are a scalar multiple of each other. As an example, assigning a vehicle with \( \text{cap}_3 = (2, 4) \) to protect an asset, is the same as assigning two vehicles with \( \text{cap}_4 = (1, 2) \).

The third case is where one vehicle can perform the role of another, but not vice versa. For example, tankers can replace pumpers, but since pumpers do not carry their own water supply, they cannot always replace tankers. Pumpers can only operate where a water source is available; tankers on the other hand, do have their own water supply and are not limited by the availability of water. To illustrate how this would be handled in the model, consider the vehicle capabilities contained in Table 4.3.

Consider an asset \( i \), which has no water source, that has a protection requirement of \( r_i = (2, 2) \). The first entry indicates that the location requires tankers and the second entry indicates that
4.3. Model demonstration

<table>
<thead>
<tr>
<th>Vehicle type (q)</th>
<th>( cap_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light tanker (LT)</td>
<td>((1, 1))</td>
</tr>
<tr>
<td>Heavy tanker (HT)</td>
<td>((2, 2))</td>
</tr>
<tr>
<td>Medium pumper (MP)</td>
<td>((0, 4/3))</td>
</tr>
<tr>
<td>Heavy pumper (HP)</td>
<td>((0, 2))</td>
</tr>
</tbody>
</table>

Table 4.3: The capability vectors for each vehicle type to demonstrate how resource substitution may occur.

a heavy vehicle (or equivalent) is required. The protection requirement of asset \( i \) can be met by either one heavy tanker, since \( cap_{HT} = (2, 2) \geq (2, 2) = r_i \), or two light tankers, since \( 2 \cdot cap_{LT} = 2 \cdot (1, 1) \geq (2, 2) = r_i \). Note that no combinations of pumpers can satisfy the protection requirement.

Next, consider a location \( j \) that has a reticulated water source and a protection requirement of \( r_j = (0, 2.5) \). The protection requirement of location \( j \) may be met by two medium pumpers, since \( 2 \cdot cap_{MP} = 2 \cdot (0, 4/3) \geq (0, 2.5) = r_j \), a medium pumper and a heavy pumper \( cap_{MP} + cap_{HP} = (0, 4/3) + (0, 2) \geq (0, 2.5) = r_j \), or two heavy pumpers, \( 2 \cdot cap_{HP} = 2 \cdot (0, 2) \geq (0, 2.5) = r_j \). The protection requirement can also be met by the appropriate combination of tankers. For example, a heavy tanker and a light tanker would meet the protection requirement, since \( cap_{HT} + cap_{LT} (1, 1) + (2, 2) \geq (0, 2.5) = r_i \).

The entries in the capability vectors may be viewed as resources being delivered to a location by the vehicle. For example, the vehicle capacity vector could specify the number of people and litres of water each vehicle can deliver per minute. Each location’s protection requirement may specify how much of each resource (i.e. people and water) is required to protect the assets at that location.

4.3.3 Case study: South Hobart

In January 2013, several fires burned out of control near Hobart with devastating consequences. Among the losses were 203 residential buildings, approximately 662 km of commercial fencing and 10,000 head of livestock, mainly sheep. The estimated cost of the losses was in the order of AUD100 million, not taking into account the cost of emergency response and recovery operations and the longer-term economic impact (Hyde, 2013).

In our demonstration, fire stations located in Hobart and assets specified in the South Hobart protection plan are used. The location of these fire stations and assets are shown in Figure 4.1, which also contains the parameter values for these locations.

In our scenario we assume a simple fire spread, radiating outwards at a rate of 3 km/h from a single point of origin in a circular fashion, impacting South Hobart, is assumed. Each asset requires 30 minutes of active defence commencing at the time of impact. The travel times between assets have been calculated using Google Maps’ Distance Matrix service. We assume
that there are four vehicle types. The capability of each vehicle is shown in Table 4.3. The first entry indicates whether the vehicle requires a reticulated water source. The second entry indicates the vehicle’s size, or litres of water per minute it can provide. A total of 14 vehicles are available for assignment: 5 light tankers, 2 heavy tankers, 5 medium pumpers and 2 heavy pumpers. Each asset is randomly assigned a value of either 10, 20 or 30, each with equal probability.

The scenario is solved for two variations considering different starting locations of vehicles. In the first variations, all vehicles are located at a fire station on the eastern side of the Derwent river. The optimal assignment of vehicles is shown in Figures 4.2 and 4.3. In the second variation, vehicles are distributed among the various fire stations. An optimal solution is shown in Figures 4.4 and 4.5.

In the second variation, a total value of 270 is protected, compared to 240 in the first variation. The reduced protected value is due to the increased travel distances. For example, in the first variation the required resources can’t reach the three eastern assets 11, 12 and 14 in time. In the second variation resources are located closer to these assets and asset 12 is saved. High-value assets are prioritised over low-value assets, for example the southern assets 1 and 2 are unprotected in both cases. In this scenario, tankers, not being reliant on a water source, are the highest utilised resource with all the tankers assigned to protection activities while some pumpers remain unassigned. It is also interesting to note that some roads are heavily utilised, giving an indication of which roads are critical to keep open.

Optimal solutions for both of these problem instances could be found within 2 seconds using
4.4 Computational study

Computational testing was carried out on a single computer cluster node. The node has two Intel Xeon E5-2670 processors and 64GB of RAM. CPLEX 12.6 was used to solve the problem instances and performance was measured in elapsed time (wall-clock time). The solver’s parallel optimisation mode was set to deterministic while all the remaining CPLEX solver parameters were left at their default values.

Ten problem instances with 60 locations each were generated. The location of assets were randomly chosen with a uniform probability density function inside a 80km by 80km square. The travel times between locations were calculated by taking the direct distance between the locations and using a travel speed of 60km/h.

In the generated instance, the opening time of each time window is correlated to the $x$-coordinate of its location. The opening time is given by $o_i = 10x_i$, which translates to a fire spreading across the landscape at a rate of 10 km/h. It is assumed that all the time windows have the same length $w$, the closing time of each window is thus given by $c_i = o_i + w$. Location values are taken from the well-known orienteering problem benchmark instance r101 (Christofides, 1979), while each entry of the protection requirement vectors is randomly selected, with equal probability, from the set $\{1,2,3\}$. The smaller problem instances (30, 40 and 50 locations) are subsets of the 60-location instances.

Our first set of experiments consider only two entries in the vehicle capability vectors and four vehicle types. The rest of the parameters are set as summarised in Table 4.4. The results of these experiments are contained in Table 4.5(a). The problems generally become harder as the number of locations and vehicles increase. Problems of size 30 are generally quick to solve, while the solution time for larger problems typically depends on their properties. Larger time windows result in harder problems.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$q=1$</td>
<td>$q=2$</td>
</tr>
<tr>
<td>$cap_q (</td>
<td>U</td>
</tr>
<tr>
<td>$cap_q (</td>
<td>U</td>
</tr>
<tr>
<td>$p_q (p=6)$</td>
<td>2</td>
</tr>
<tr>
<td>$p_q (p=10)$</td>
<td>3</td>
</tr>
</tbody>
</table>

*Table 4.4: The parameter values used for computational testing.*

The second set of experiments considered three entries in the vehicle capability vectors. The rest of the parameters were set as summarised in Table 4.4. The results of these experiments
Figure 4.2: An optimal solution for the first variation described in the text. All of the vehicles are located at a fire station on the eastern side of the Derwent river (Map data ©2014 Google).

Figure 4.3: The solution in Figure 4.2 shown by vehicle type. The map has been cropped to the area highlighted by the rectangle in Figure 4.2 (Map data ©2014 Google).
**Figure 4.4:** An optimal solution for the second variation described in the text. The vehicles are located at various fire stations across Hobart (Map data © 2014 Google).

(a) Light tankers  
(b) Heavy tankers  
(c) Medium pumpers  
(d) Heavy pumpers

**Figure 4.5:** The solution presented in Figure 4.4 shown by vehicle type. The map has been cropped to the area highlighted by the rectangle in Figure 4.4 (Map data © 2014 Google).
are available in Table 4.5(b). Although all the problems of size 30 considered could be solved within a couple of minutes, increasing the number of elements in the vehicle capability vector increased the solution times.

<table>
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<tr>
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<tr>
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<td>20</td>
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<td>40</td>
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<tr>
<td>40</td>
<td>6 7 178 939</td>
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<td>50</td>
<td>12 48 5 178</td>
<td></td>
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<tr>
<td>60</td>
<td>31 2 34 113</td>
<td></td>
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\( (a) |U| = 2 \)

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<th>6</th>
<th>10</th>
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<tbody>
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<tr>
<td>30</td>
<td>3 2 34 113</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>40</td>
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<tr>
<td>60</td>
<td>34 2 673</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( (b) |U| = 3 \)

Table 4.5: The solution times for test instances in seconds. The number of unsolved problems after twenty minutes elapsed time (wall time) is indicated in parenthesis while a dash indicates that none of the problems could be solved within the twenty minute time limit.

These results indicate that problems containing 50 locations or more are very hard to solve with this integer programming approach.

### 4.5 Chapter summary

In this chapter we presented a mixed-integer programming approach to the problem of protecting assets during large escaped wildfires.

The mixed-integer programming model presented in this chapter generalises the COPTW by allowing mixed vehicle types, introducing a vector specifying the protection requirement for each location and allowing each vehicle type to have a unique travel time between two locations.

The working of the model was demonstrated using the locations of assets and fire stations in Hobart, Tasmania, Australia. Although parts of the data used to demonstrate the model was sourced from Tasmania Fire Service, the modelling approach is general and the model could be applied to other locations.

Testing of the asset protection model formulation demonstrated that it is computationally feasible to apply the model to real-life asset protection problems. However, as the problem size increases, the model becomes harder to solve. We also showed that the solution time depends on the properties of the problem. With this in mind, the development of methods to improve solution times could prove beneficial. Potential approaches may employ CPLEX as a heuristic solver to find a good enough solution or the development of meta-heuristic solution techniques. Further research would be required to answer the question of which heuristic methods are most suitable and when an approximate solution may be considered of acceptable quality. Heuristic techniques have already proved very useful for solving larger instances of the team orienteering problem with time windows within seconds. Techniques to solve the team orienteering problem often rely on the ability to independently generate paths for each vehicle, calculating the contribution of each individual vehicle to the overall objective function. The cooperative element of
our approach does not allow the independent calculation of the contribution of a vehicle to the objective function, which makes it impossible to apply these techniques directly to the wildfire asset protection problem. Therefore, further research is required to assess the effectiveness of heuristics in solving the problem described here.

In our model we assumed that vehicles will be engaged continuously at a location for the entire service duration to carry out the required protection activities. The service at a location can be split into separate tasks, with vehicles only required to be present at an asset for the appropriate tasks. Having one or more tasks at a location can be modelled by representing each asset with multiple nodes, a node for each protection task to be carried out. To collect the reward (or save the asset) service must be delivered to all the nodes belonging to a single asset, each node having an opening time, a closing time and a service duration. The travel time between nodes belonging to the same location would be zero. This approach may also be used to model the case where either an active or a strategic defence task can be carried out to protect an asset.

An important aspect of wildfire asset protection, namely vehicle and crew safety, can be considered in the current modelling framework. For example, time windows can be restricted to periods when it will be safe to carry out asset protection activities and roads that are unsafe to travel on can be excluded from the model. Modelling some aspects of crew safety may require model extensions. For example, routes which may be safe at certain periods and unsafe at others, can be modelled using an approach similar to that of the TOPTW with arc time windows. By adding time windows to the arcs and time dependent travel times, the model would be able account for routes which are only safe to use at certain times and take into account road conditions that change over time.

Changes in weather, vehicle breakdowns or road closures may result in disruptions to asset protection assignment plans. In the next chapter we develop a model to aid in reallocation of emergency resources in response to changes in conditions.
Incident managers assigning wildfire response vehicles to provide protection to community assets may experience disruptions to their plans arising from factors such as changes in weather, vehicle breakdowns or road closures. For example, during the February 7th 2009 Black Saturday wildfires, a number of fires burned out of control in south-eastern Australia and resulted in the loss of 173 lives and of several thousand homes. A major contributing factor to the devastation caused by these fires was a change in wind direction, from north-westerly to south-easterly, that occurred late in the day. This wind change had a dramatic effect on fire orientation and the resultant impact. A point in case being the Kilmore East fire, this was initially a long narrow fire band, but following the wind change the flank of the fire became a 55 km wide fire-front that burned through a number of townships with tragic consequences (Cruz et al., 2012). Here we are concerned with the development of a modelling approach to aid in reallocation of emergency resources in response to such changes in conditions.
In this chapter a multi-objective mixed-integer linear programming model is developed to assist in the rerouting of wildfire response vehicles once a disruption has occurred. The model maximises the total value of assets protected while minimising changes to the original vehicle assignments. A number of potential disruption measures are proposed. The model is demonstrated using a realistic fire scenario impacting South Hobart, Tasmania, Australia. Computational testing shows that realistic-sized problems can be solved within a reasonable time using a commercial solver.

5.1 Rerouting vehicles during wildfires

The wind change that occurred during the 2009 Black Saturday fires is one example of the myriad of disruptions that typically occur during wildfire response, requiring replanning and updating of any existing plans. Some of the disruptions that may occur during wildfires are discussed in more detail next.

The weather - temperature, relative humidity, wind speed and wind direction - affects the direction, intensity and speed at which a wildfire spreads. As such, fire spread predictions are heavily reliant on weather forecasts. A change in wind speed can delay or hasten the fire’s anticipated time to impact and hence the required timing of asset protection activities. A change in the direction or extent of fire spread may result in additional assets being impacted, or conversely, assets previously requiring protection may no longer be under threat. A fire burning with a higher intensity than originally predicted may require that additional resources be assigned to the asset in order to provide adequate protection.

Vehicle breakdowns are a common source of disruption in vehicle routing problems. The problem of vehicle breakdowns is exacerbated in the wildfire context by the extreme conditions in which the emergency vehicles must operate. If a vehicle breaks down, it may be necessary to reroute other vehicles to perform the tasks of the broken vehicle. In the worst case, due to a lack of resources, it may no longer be possible to adequately protect an asset which previously had sufficient resources assigned to it.

Modified road conditions can result in changes to travel times between locations. For example, roads may become inaccessible due to falling trees, or congested as a result of increased traffic flow as residents evacuate their homes. Increased traffic times may be experienced because of this. Similarly, reduced visibility due to smoke can force responding vehicles to slow down in order to travel safely. Conversely, travel times could be reduced due to road access restrictions whereby roads are closed to the public with access only permitted to emergency vehicles.

Fire trucks and other fire-fighting appliances are often reliant on a functioning reticulated water supply. A loss in reticulated water may change the vehicle type required or increase the number of vehicles needed to protect an asset. Additionally, the turn-around time of vehicles between assets may increase, due to the need to source a static water supply prior to moving onto the
next protection priority.

The types of disruptions described above by no means represent an exhaustive list. A wildfire rerouting model should also be flexible enough to be able to handle less common and unforeseen disruptions. Furthermore, given the time-critical nature of wildfire response, it is important that asset protection plans are updated and implemented as quickly as possible following a disruption.

There is an incentive for minimising deviations from the original vehicle assignment when rerouting vehicles. Vehicles are dispatched with information about the routes to use, how to protect specific assets and in some cases with specialised equipment. There is also a burden placed on the IMT in terms of communicating and managing the updated plans. Having updated plans as close as possible to the original pre-disruption plans would result in a reduced burden on the IMT and improved efficiency in the process of updating plans. With this in mind, we investigate three methods for measuring the deviations from the original plans. In preparing a revised asset protection plan, the primary aim is to maximise the value of protected assets. The secondary aim is to minimise deviation from the original plans as represented by the chosen deviation measure. In the next two sections we introduce our approach. In section 5.2 we present a modified version of the model proposed in Chapter 4 that we are going to use to maximise protected value. In section 5.3 we introduce the deviation measures we use to minimise deviation from the pre-disruption vehicle assignment plan.

5.2 Vehicle assignment to maximise value protected

The model formulated here extends the mixed integer programming model presented in Chapter 4. In the previous formulation, vehicles are grouped by vehicle type using integer vehicle flow decision variables. The rerouting formulation explicitly keeps track of each vehicle, using binary decision variables to describe each vehicle’s individual path. The additional information provided by the binary vehicle flow formulation enable us to model disruptions affecting specific vehicles. Further, we introduce a number of parameters and decision variables to track changes to the routes of vehicles or the vehicle-asset assignments with the goal of measuring deviations from the original pre-disruption vehicle assignments. The following notation is used in the model formulation:

Sets:

- $\mathcal{U}$ is the set of vehicle capabilities.
- $\mathcal{P}$ is the set of available vehicles.
- $\mathcal{E}_p$ is the set of feasible routes for vehicle $p$.
- $\delta^+_p(k)$ is the set of locations that can be reached directly from location $k$. That is, for each $p$ in $\mathcal{P}$, $\delta^+_p(k) = \{i \mid (i, j) \in \mathcal{E}_p, j = k\}$.
- $\delta^-_p(k)$ is the set of locations from where location $k$ can be reached. That is, for each $p$ in $\mathcal{P}$, $\delta^-_p(k) = \{j \mid (i, j) \in \mathcal{E}_p, i = k\}$. 
Parameters:
- \( a_i \) is the service duration associated with location \( i \).
- \( \text{cap}_p \) the capability vector associated with vehicle \( p \).
- \( c_i \) the latest time that protection activities may commence.
- \( m \) is the number of depots.
- \( n \) is the number of nodes in the graph representation of the problem.
- \( o_i \) is the earliest time that protection activities may commence.
- \( r_i \) the protection requirement of asset \( i \).
- \( \text{start}_{ip} \) 1 if vehicle \( p \) is at depot \( i \), 0 otherwise.
- \( t_{ijp} \) the travel time from location \( i \) to location \( j \) of vehicle \( p \).
- \( v_i \) is the value of the asset at location \( i \).
- \( x_{ijp} \) 1 if vehicle \( p \) was assigned to travel from location \( i \) to location \( j \) in the pre-disruption assignments, 0 otherwise.

Variables:
- \( S_i \) is the time at which service commences at location \( i \).
- \( X_{ijp} \) 1 if vehicle \( p \) travels from location \( i \) to location \( j \), 0 otherwise.
- \( Y_i \) 1 if location \( i \) is serviced, 0 otherwise.
- \( Z_p \) 0 if all assignments of vehicle \( p \) remains unchanged in the updated plan, otherwise 1.
- \( Z_p^* \) 1 if an asset has been added to or removed from a vehicle’s path, 0 otherwise.
- \( Z_{ip}^+ \) 1 if location \( i \) is assigned to vehicle \( p \) in the updated plans, but not the pre-disruption plans, 0 otherwise.
- \( Z_{ip}^- \) 1 if location \( i \) is assigned to vehicle \( p \) in the pre-disruption plans, but not the updated plans, 0 otherwise.

Based on the notation above, the problem of rerouting vehicles to minimise deviation may be formulated as a bi-objective mixed-integer programming problem. The primary objective is to maximise the total protected value and the secondary objective is to minimise deviation from the original pre-disruption plans, that is

\[
\text{Maximise} \sum_{i=m+1}^{n-1} v_i Y_i, \quad \text{Minimise} \ f_2, \quad (5.1)
\]

where \( f_2 \) is the chosen deviation measure.

The assignment of vehicles are subject to the following constraints:

\[ a_i + a_i + t_{ijp} \leq c_j. \]
5.2. Vehicle assignment to maximise value protected

\[ \sum_{j \in \delta^+_p(i)} X_{ijp} = \text{start}_{ip} \quad \forall \ i = 1, \ldots, m, \ p \in \mathcal{P}; \]  
\[ \sum_{p \in \mathcal{P}} \sum_{i \in \delta^-_p(n)} X_{inp} = |\mathcal{P}|; \]  
\[ \sum_{i \in \delta^-_p(k)} X_{ikp} = \sum_{j \in \delta^+_p(k)} X_{kjp} \quad \forall \ k = m + 1, \ldots, n - 1, \ p \in \mathcal{P}; \]  
\[ \sum_{p \in \mathcal{P}} \sum_{i \in \delta^-_p(k)} X_{ikp} \text{cap}_p \geq r_k Y_k \quad \forall \ k = m + 1, \ldots, n - 1; \]  
\[ S_i + t_{ijp} + a_i - S_j \leq M_1(1 - X_{ijp}) \quad \forall \ (i,j) \in \mathcal{E}_p, \ p \in \mathcal{P}; \]  
\[ o_i - S_i \leq M_2(1 - Y_i) \quad \forall \ i = 1, \ldots, n; \]  
\[ S_i - c_i \leq M_3(1 - Y_i) \quad \forall \ i = 1, \ldots, n; \]  
\[ Y_i \in \{0, 1\} \quad \forall \ i = m + 1, \ldots, n - 1; \]  
\[ X_{ijp} \in \{0, 1\} \quad \forall \ p \in \mathcal{P}, (i,j) \in \mathcal{E}_p; \]  
\[ S_i \in \mathbb{R} \quad \forall \ i = 1, \ldots, N. \]  

Constraints (5.2) enforce the starting position of vehicles. Constraint (5.3) ensures vehicles end at the sink node. The vehicle flow to and from each location is balanced by constraints (5.4). Constraints (5.5) enforce the condition that an asset is protected only if the vehicles assigned to the asset collectively meet the protection requirement. Constraints (5.6) ensure that service at a location may only start after protection activity at a previously visited location has been completed and sufficient time for travel has been allowed, with \( M_1 \) representing a large constant. Setting \( M_1 = \max(o_i) + \max(t_{ijp}) + \max(a_i) - \min(c_i) \) is sufficiently large for this purpose.

The start of protection activities at locations are limited to their respective time windows by constraints (5.7) and (5.8). Note that visits to locations are allowed outside of the time windows if they do not contribute to the protected value. This is to allow for original vehicle assignments which may have become infeasible after the disruption due to changes in time windows. These infeasible routes need to be accounted for in order to measure the disruption that occurs when vehicles are rerouted. Setting \( M_2 = M_3 = \max(c_i) \) is sufficiently large for this purpose.

Constraints (5.9), (5.10) and (5.11) define the domains of the binary and continuous decision variables respectively.

Note that the initial locations do not have a service duration associated with them, therefore
\[ a_i = 0 \] for each \( i = 1, \ldots, m \). Additionally, for each initial location \( S_i = 0 \) if vehicles are allowed to depart immediately, otherwise, \( S_i = q_i \) where \( q_i \) is the time that is required before the vehicle at location \( i \) is ready for departure.
5.3 Measuring deviation

Recall that the parameter $x_{ijp}$ represents the vehicle assignment before any disruption occurred, $x_{ijp} = 1$ if vehicle $p$ travelled from $i$ to $j$ in the initial assignment, otherwise $x_{ijp} = 0$ and $\mathcal{P}$ is the set of available vehicles for reassignment.

5.3.1 Minimising the number of vehicles with updated routes

In our first proposed deviation measure, the aim is to minimise the number of vehicles that are affected by planning changes. Let $Z_p = 0$ if the route associated with vehicle $p$ remains unchanged in the updated plan, otherwise $Z_p = 1$. Since $X_{ijp}$ represents the new assignment of vehicles, the constraints

$$X_{ijp} - x_{ijp} \leq Z_p \text{ and } X_{ijp} - x_{ijp} \geq -Z_p \forall (i,j) \in \mathcal{E}_p, \ p \in \mathcal{P}.$$  \hspace{1cm} (5.12)

enforce the appropriate values of $Z_p$. In this case the deviation minimising objective is given by

$$\text{Minimise } \sum_{p \in \mathcal{P}} Z_p.$$  \hspace{1cm} (5.13)

The complete rerouting model is described by equations (5.1)–(5.11),(5.12) and (5.13).

5.3.2 Minimising the number of vehicles with updated routes, allowing changes in servicing sequence

Since a vehicle already has all the information and tools required to provide protection to the assets assigned to it, it may be desirable to allow vehicles to change the sequence in which they provide protection to assets. This will allow greater flexibility in updating vehicle assignments.

Hence in our second deviation measure, the aim is to minimise the number of disrupted vehicles, allowing for changes in route sequence. Let $Z^*_p = 0$ if vehicle $p$ is protecting the same assets, but potentially in a different sequence, 1 otherwise. Note that $\sum_{i=1}^{n-1} x_{ijp} = 1$ if vehicle $p$ is assigned to asset $i$ in the updated plans. The parameter $\sum_{i=1}^{n-1} x_{ijp} = 1$ if vehicle $p$ was assigned to asset $i$ before any disruptions. Therefore the appropriate value of $Z^*_p$ is enforced by the constraints

$$\sum_{i=1}^{n-1} X_{ijp} - \sum_{i=1}^{n-1} x_{ijp} \leq Z^*_p \text{ and } \sum_{i=1}^{n-1} X_{ijp} - \sum_{i=1}^{n-1} x_{ijp} \geq -Z^*_p \forall j = 1, \ldots, n, \ p \in \mathcal{P}. $$  \hspace{1cm} (5.14)

The deviation minimising objective is given by

$$\text{Minimise } \sum_{p \in \mathcal{P}} Z^*_p.$$  \hspace{1cm} (5.15)
The complete model is described by equations (5.1)–(5.11), (5.14) and (5.15).

5.3.3 A general deviation measure

A limitation of the above proposed deviation measures is that multiple changes to a single vehicle’s path are counted as a single change. In some case it may be desirable to make a couple of minor changes to multiple vehicles to deal with a disruption instead of making major changes to the assignment of a single vehicle. In such a case the main concern would be to maintain vehicle-asset pair assignment while allowing partial changes to a vehicle path.

Let $Z_{ip}^+ = 1$ if location $i$ is assigned to vehicle $p$ in the updated plans, but not in the pre-disruption plans, 0 otherwise. Let $Z_{ip}^- = 1$ if location $i$ is assigned to vehicle $p$ in the pre-disruption plans, but not in the updated plans, 0 otherwise. These values are enforced by the constraints

$$\sum_{i=1}^{n-1} x_{ijp} - \sum_{i=1}^{n-1} \sum_{j=m+1}^{n} x_{ijp} = Z_{ip}^+ - Z_{ip}^- \quad \forall j \in m+1, \ldots, n, \ p \in \mathcal{P};$$

(5.16)

The cost of reassigning a vehicle may be dependent on a variety of factors related to the specific asset and vehicle in question. The following parameters allow an IMT flexibility when considering various assignment changes, allowing the IMT to weigh each change in the objective function to reflect the cost or the IMT’s own management priorities. Let $cost_{ip}^+$ be the cost of adding asset $i$ to vehicle $p$’s assignment, and and let $cost_{ip}^-$ be the cost of removing asset $i$ from vehicle $p$’s assignment. The deviation minimising objective is given by

$$\text{Minimise} \sum_{p \in \mathcal{P}} \sum_{i=1}^{N} cost_{ip}^+ Z_{ip}^+ + cost_{ip}^- Z_{ip}^-.$$  

(5.17)

The complete model is described by equations (5.1)–(5.11), (5.16) and (5.17). Note that if $cost_{ip}^+ = cost_{ip}^- = 1$, then the number of asset-vehicle reassignments are minimised. Setting $cost_{ip}^+ = 0$ (or $cost_{ip}^-$) would mean that adding (or removing) an asset-vehicle assignment pair from the assignment plans will not contribute to the deviation measure.

5.4 Model implementation and demonstration

We use the algorithm proposed by Özlen and Azizoğlu (2009) to determine all non-dominated solutions to our bi-objective integer programming problem. The problem is implemented as a single objective MIP, maximising the objective $f_1 - \epsilon f_2$, where $f_1$ is the primary objective to be maximised, $f_2$ the secondary objective to be minimised, and $\epsilon$ a scaling factor which enforces the lexicographical ordering on the objective functions. A constraint specifying an upper bound on the secondary objective is iteratively decreased until all non-dominated solutions have been
found. In the case of the first deviation measure, the constraint \( \sum_{p \in P} Z_p \leq \bar{P} \) is added, where \( \bar{P} \) is the upper bound that is decreased in each iteration.

Next, we demonstrate the model with the aid of a wildfire scenario in which an unforeseen change in weather occurs. In the scenario, a wildfire ignites south of Hobart, Tasmania, Australia. A fire spread forecast is generated taking into account the current, expected weather conditions. The location of the fire, together with the initial fire spread prediction, is shown in Figure 5.1.

Realistic fire spread data, provided by Tasmania Fire Service, is used in this demonstration.

![Figure 5.1: The study area is located near Hobart, Tasmania, Australia. The perimeter of the initial expected fire spread is indicated by a dashed contour, the locations of fire stations are indicated by ⋄ and community assets are indicated by ●.](image)

5.4.1 Initial assignment

The fire, illustrated in Figure 5.2, is expected to impact 22 community assets listed in Table 5.2. Fire crews operating various fire-fighting vehicle types, i.e. medium/heavy pumpers and light/medium/heavy tankers are available to respond to the wildfire to provide protection to the community assets. There are 25 vehicles in total. Each vehicle is stationed at one of 10 fire stations listed in Table 5.3. The protection requirement descriptions are based on expert opinion (C. Collins, Tasmania Fire Service, personal communication, 2015). The capability vectors of vehicles and protection requirement vectors of locations derived from these protection requirement descriptions are shown in Table 5.2. The derived vehicle capability vectors are \( cap_{HP} = (1, 0, 0, 0, 0) \), \( cap_{MP} = (1, 0, 0, 0, 0) \), \( cap_{HT} = (0, 1, 1, 0, 0) \), \( cap_{MT} = (0, 0, 1, 0, 1) \) and \( cap_{LT} = (0, 0, 0, 1, 1) \). Driving times were taken from Google’s Distance Matrix Service API.
It is assumed that the service duration of each asset is 30 minutes. The value of assets has been picked, with equal probability, from the set \{10,20,30\}. In reality these values are determined by planning bodies before any wildfire commences or by IMTs during the management of a wildfire. Optimising the resource allocation results in the assignments shown in Table 5.4. All the assets can be protected with the 25 available vehicles in the initial assignment.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Protection requirement</th>
<th>Time to impact (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ionospheric Research Station</td>
<td>1×HT/MT &amp; 1×LT</td>
<td>(0, 0, 1, 1, 0)</td>
</tr>
<tr>
<td>2 Transend Communications Site</td>
<td>1×HT/MT</td>
<td>(0, 0, 1, 0, 0)</td>
</tr>
<tr>
<td>3 Communications Tower</td>
<td>1×HT/MT</td>
<td>(0, 0, 1, 0, 0)</td>
</tr>
<tr>
<td>4 Broughton Ave Reservoir</td>
<td>1×LT/MT</td>
<td>(0, 0, 0, 0, 1)</td>
</tr>
<tr>
<td>5 Plants of Tasmania Nursery</td>
<td>1×HT/MT</td>
<td>(0, 0, 0, 0, 1)</td>
</tr>
<tr>
<td>6 Island Bonsai Plant Nursery</td>
<td>1×HT/MT</td>
<td>(0, 0, 0, 0, 1)</td>
</tr>
<tr>
<td>7 Communications Exchange</td>
<td>1×HT/MT</td>
<td>(0, 0, 0, 0, 1)</td>
</tr>
<tr>
<td>8 Arts Centre</td>
<td>1×HP/MP</td>
<td>(1, 1, 0, 0, 0)</td>
</tr>
<tr>
<td>9 The Signalman’s Cottage</td>
<td>HP/MP &amp; MT/LT</td>
<td>(1, 1, 0, 0, 0)</td>
</tr>
<tr>
<td>10 Communications Towers (Bramble Street)</td>
<td>1×HT/MT</td>
<td>(0, 0, 1, 0, 0)</td>
</tr>
<tr>
<td>11 Mountain Lodge</td>
<td>1×HT/MT</td>
<td>(0, 0, 1, 0, 0)</td>
</tr>
<tr>
<td>12 Telstra Fern Tree Exchange</td>
<td>1×LT/MT</td>
<td>(0, 0, 0, 0, 1)</td>
</tr>
<tr>
<td>13 Communications Tower (Chimney Pot Hill)</td>
<td>1×HT/MT</td>
<td>(0, 0, 1, 0, 0)</td>
</tr>
<tr>
<td>14 St Raphaels Anglican Church</td>
<td>1×HT</td>
<td>(0, 1, 0, 0, 0)</td>
</tr>
<tr>
<td>15 Mt Nelson Store</td>
<td>1×HP/MP</td>
<td>(1, 0, 0, 0, 0)</td>
</tr>
<tr>
<td>16 Communications Tower (Hobart College)</td>
<td>1×HT/MT</td>
<td>(0, 0, 0, 0, 0)</td>
</tr>
<tr>
<td>17 Communications Tower (Chimney Pot Hill)</td>
<td>1×HT &amp; 1×LT/MT</td>
<td>(0, 1, 0, 0, 1)</td>
</tr>
<tr>
<td>18 Pump Station (Potable Water)</td>
<td>2×LT/MT</td>
<td>(0, 0, 0, 0, 2)</td>
</tr>
<tr>
<td>19 Communications Tower (Olinda Grove)</td>
<td>1×HT</td>
<td>(0, 0, 0, 0, 0)</td>
</tr>
<tr>
<td>20 Communications Tower (Tolmans Hill)</td>
<td>1×HT</td>
<td>(0, 0, 0, 0, 0)</td>
</tr>
<tr>
<td>21 HCC Mountain Park Depot</td>
<td>1×HT &amp; 1×LT/MT</td>
<td>(0, 0, 0, 0, 1)</td>
</tr>
<tr>
<td>22 Cascade Hotel</td>
<td>1×HP/MP</td>
<td>(1, 0, 0, 0, 1)</td>
</tr>
</tbody>
</table>

Table 5.2: Community assets, their protection requirements, and the time to impact in minutes. The time to impact is measured from the time the fire is ignited; ‘N/A’ indicates that the asset is not being impacted and does not require any protection. Two protection requirements are shown: the ‘described’ column shows the description in terms of number and type of trucks and the ‘vector’ column shows the parametrised vector representation of the protection requirement. The abbreviations have the following meanings: HP - heavy pumper, MP - medium pumper, HT - heavy tanker, MT - medium tanker, and LT - light tanker.

5.4.2 Disruption

A change in the forecasted weather is observed after planning for the initial expected weather conditions has been completed and vehicles have been dispatched. The changed, disrupted, fire spread perimeter is shown in Figure 5.3. The new fire spread prediction shares ignition conditions with the initial expected fire spread. The disrupted time to impact is shown in the last column of Table 5.2. The change in the weather conditions results in changes to the expected time of impact for most locations. A number of the assets are no longer being impacted, whereas locations not threatened previously are being impacted in the new conditions. It is assumed that the protection requirement of each asset is unchanged.
Table 5.3: Fire stations located near South Hobart and the number of vehicles of each type stationed at the fire stations.

<table>
<thead>
<tr>
<th>Brigade</th>
<th>HP</th>
<th>MP</th>
<th>HT</th>
<th>MT</th>
<th>LT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Fern Tree</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2 Hobart–Glenorchy</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>3 Hobart</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>4 Hobart–Clarence</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5 Hobart–Tarooma</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6 Hobart–Lenah Valley</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7 Hobart–MT Nelson</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>8 Kingston</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9 Summerleas</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10 Wellington</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Total: (=25) 4 1 3 6 11

Figure 5.2: The initial fire spread prediction. A number of community assets located in South Hobart are labelled 1 through 22. The time to impact and protection requirements for each asset are listed in Table 5.2.

Table 5.4: The initial vehicle assignment.

<table>
<thead>
<tr>
<th>Vehicle Assigned assets</th>
<th>Vehicle Assigned assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 4,22</td>
<td>14</td>
</tr>
<tr>
<td>2 12</td>
<td>15</td>
</tr>
<tr>
<td>3 16</td>
<td>-</td>
</tr>
<tr>
<td>4 15,18</td>
<td>17</td>
</tr>
<tr>
<td>5 17,22</td>
<td>18 1,18</td>
</tr>
<tr>
<td>6 17,20</td>
<td>19</td>
</tr>
<tr>
<td>7 16,18</td>
<td>20 21</td>
</tr>
<tr>
<td>8 14,21</td>
<td>21</td>
</tr>
<tr>
<td>9 2 22</td>
<td>18</td>
</tr>
<tr>
<td>10 5,19</td>
<td>23 19</td>
</tr>
<tr>
<td>11 1,3</td>
<td>24 4,22</td>
</tr>
<tr>
<td>12 11,13</td>
<td>25 17</td>
</tr>
<tr>
<td>13 6</td>
<td></td>
</tr>
</tbody>
</table>

5.4.3 Rerouting vehicles

The Pareto-optimal results when rerouting vehicles and minimising the number of changes to vehicle paths, i.e. considering secondary objective (5.13), are plotted in Figure 5.4. The plot shows that increasing the number of changes made to the vehicle assignments results in an increase of the total value of assets protected. If no vehicles are rerouted after the disruption occurs, then the total protected asset value is 65% of the highest possible value that can be protected. However, not all the vehicle routes have to be changed to reach this maximum protection level; rerouting nine vehicles is sufficient (shown in Table 5.5).

It took on average 5 minutes to generate a complete Pareto-frontier for each deviation measure. The problem was solved on a desktop computer with an Intel i7 processor and 8GB of RAM. The models were described in CMPL (Schleiff and Steglich, 2015) and solved using CPLEX 12.6.
5.5 Computational testing

Computational testing was carried out on a single computer cluster node. The node has two Intel Xeon E5-2670 processors and 64GB of RAM. The problem instances were solved using CPLEX 12.6 and performance was measured in elapsed time (wall clock time) and a time limit of 30 minutes (1800 seconds) was set.

The problem instances proposed in Chapter 4 are again considered. These problem instances range in size from 30 to 60 assets with 10 vehicles available for assignment. The length \( w \) of time windows in an instance are either all equal to 20 or 40 minutes.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Assigned assets</th>
<th>Vehicle</th>
<th>Assigned assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>–</td>
<td>14</td>
<td>10,21</td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>8,9,21</td>
<td>16</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>15,21</td>
<td>17</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>18</td>
<td>7,18</td>
</tr>
<tr>
<td>6</td>
<td>3,20</td>
<td>19</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>16</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>5,17,21</td>
<td>21</td>
<td>–</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>5,19</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>24</td>
<td>4.22</td>
</tr>
<tr>
<td>12</td>
<td>11,13</td>
<td>25</td>
<td>17</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: The changed vehicle assignment to account for the disruption. Vehicles that have been reassigned are indicated by boldface entries.

Figure 5.3: The updated fire spread prediction. A number of community assets located in South Hobart are labelled 1 through 22. The time to impact and protection requirements for each location are listed in Table 5.2.

Figure 5.4: The Pareto-optimal solutions for the weather change scenario when minimising the changes to vehicle paths.

(IBM Corporation, 2015). We investigate the computational behaviour of the model in the next section.
An initial, pre-disruption vehicle assignment is created by solving these problem instances in the absence of any disruption. Vehicles are then rerouted to account for the breakdown of a single vehicle. The breakdown vehicle is chosen at random. The third deviation measure is considered, minimising the number of vehicle-asset assignment changes and the time required to find a single optimal solution, allowing maximum deviation, is reported.

The average time required to find an optimal solution for each set of instances is shown in Table 5.6. Problems with 30 assets and small time windows yield optimal solutions in a matter of seconds. We expect that practical problems with 30 or less assets and small time windows would generally be quick to solve to optimality using the current approach. Solving problems of size 40 with small time windows may still be feasible in practice, but would result in an average solution time of 10 minutes. However, for larger time windows and more than 40 assets, alternative solution methods or heuristics will be required to provide solutions within a reasonable time frame.

<table>
<thead>
<tr>
<th>Size</th>
<th>$w = 20$</th>
<th>$w = 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>13</td>
<td>1166</td>
</tr>
<tr>
<td>40</td>
<td>660</td>
<td>–</td>
</tr>
<tr>
<td>50</td>
<td>1488</td>
<td>–</td>
</tr>
<tr>
<td>60</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 5.6: The solution times for test instances in seconds. A dash (–) indicates that none of the problems could be solved to optimality within the 30 minute time limit.

5.6 Chapter summary

In this chapter we presented a method for rerouting vehicles to adapt to changes during wildfire asset protection. A number of potential secondary objectives were considered to minimise changes made to the assignment of vehicles. These deviation measures provide a varying degree of flexibility when rerouting vehicles. The specific deviation measure chosen would depend on the priority and objectives of the IMT.

The rerouting framework presented here allows for the consideration of a variety of disruptions that can occur when assigning vehicles to asset protection activities. An unexpected weather disruption case study was presented using a wildfire scenario in South Hobart. Vehicle breakdowns can be considered by removing the vehicle from the set $\mathcal{P}$ and then reassigning the subset of available vehicles. If a vehicle cannot be rerouted, then its route can be fixed by adding constraints of the type $X_{ijp} = x_{ijp}$ for the appropriate edges and vehicle.

Although the discussion focused on asset protection, other tasks performed by wildfire resources can also be accommodated by the modelling approach presented here. For example, assisting with the evacuation of people, collecting information or aiding with direct suppression activities.
can be parameterised without any changes to the model by associating a value with the successful completion of the activity. A mandatory task at location $i$ can be considered by adding the constraint $Y_i = 1$.

Computational testing showed that realistic-sized problems can be solved within a reasonable time. Feasible solutions were found for the larger problems that were not solved to optimality. A possible area for further work is finding efficient solution methods to solve larger problems in a short time frame. Murray and Karwan (2013) used a branch and bound method to solve a similar rerouting model. Modifications to this approach could make it applicable to the problem presented here.

Changes and disruptions during wildfires are often difficult to forecast. For example, predicting when and where a truck will break down, or forecasting the exact nature of changes to the weather are inherently difficult. Updating plans are thus often reactionary by nature and rerouting vehicles may be the only option available. Our approach quickly provides a suit of solutions, allowing the IMT to choose the most appropriate assignment plan.

The rerouting method described in this chapter does deal with totally unpredictable cases. Sometimes there is some knowledge about the likelihood of a disruption occurring. For example, weather forecast scenarios may have some probability associated with them and planning with these probabilities in mind could lead to improved vehicle assignments. In the next chapter the wildfire asset protection problem is formulated as a two-stage stochastic programming model, with initial vehicle assignments made in the first stage with the opportunity for adjustments in the second stage based on observed fire-weather outcomes.
As discussed in the previous chapter, adapting to changes and reacting to new information is a common task performed by IMTs. Previously in this thesis, asset protection was modelled assuming perfect information and adopting a rerouting approach to deal with any changes that may occur. The rerouting approach is applicable to cases where disruptions are unexpected. In some cases, information about likely fire spread scenarios is available, and using this knowledge could further improve the initial assignment of resources. In this chapter, a two-stage stochastic programming approach is considered which takes knowledge about future outcomes into account and allows for the consideration of uncertainty in the modelling parameters. In the two-stage stochastic programming approach, initial vehicle assignments are made in the first stage, with the opportunity for adjustments in the second stage based on observed conditions and fire-weather outcomes.

There are a number of factors in wildfire planning which may lead to uncertainty in the modelling parameters. Possibly the the greatest source of uncertainty is the weather. The temperature,
relative humidity, wind speed and wind direction all affect the direction, intensity and speed at which a wildfire spreads. Any uncertainty in the weather forecast translates to uncertainty in the predicted fire spread. Parameters which may be uncertain include the time to impact, which is reflected in the time windows; protection requirements, which is a function of fire intensity; and travel times due to variability in road conditions. There have been recent efforts towards developing fire spread models to reflect the uncertainty involved in fire spread forecasting (French et al., 2013).

6.1 Modelling uncertainty

A finite number of second-stage scenarios are used to reflect the values that the parameters may adopt. Time \( t \) marks the separation between the first- and second-stage problems. We assume that at time of planning, the likelihoods of various scenarios that may occur after time \( t \) are known. All locations that are impacted before time \( t \) are considered to be part of the first stage problem, and locations impacted after time \( t \) are part of the second stage. It is expected that new information will arrive at time \( t \). The vehicle assignments are adapted, routing vehicles for the scenario which materialises at time \( t \). The aim is to assign resources in the first stage in such a manner as to provide the best opportunity to protect the maximum expected value of assets. The two-stage wildfire asset protection problem is illustrated in Figure 6.1. Each second-stage scenario is expected to occur with probability \( p_l \), where \( l \) is a scenario from the scenario set \( \mathcal{S} \).

![Figure 6.1: The fire front perimeters for the first-stage and second-stage scenarios. The solid curve indicates the extent of the expected fire spread for the first-stage problem. The dashed curve shows the fire perimeter for the second-stage scenario A, and the dotted curve shows the fire perimeter for the second-stage Scenario B. Dots indicate the locations of assets.](image)
6.1. Modelling uncertainty

6.1.1 Model notation

The notation used in the two-stage stage stochastic model formulation is presented next. Let $S$ represent the set of second-stage scenarios. Let $\mathcal{E}^L_q$ be the set of edges that are time feasible with respect to time windows, associated with vehicles of type $q$ and scenario $l$. The following sets for the various types of locations are defined: $\mathcal{A}_d$ is the set of depots, $\mathcal{A}_s$ is the set of staging locations, $\mathcal{A}^0_a$ is the set of first-stage locations, and $\mathcal{A}^l_a$ is the set of second-stage locations associated with scenario $l \in S$.

As in Chapters 4 and 5, $\delta^-(i)$ and $\delta^+(i)$ are adjacency sets for location $i$, that is $\delta^-(i)$ is the index set of vertices adjacent to vertex $v_i$, that is $j \in \delta^-(i)$ if $(j, i) \in \mathcal{E}$, and $\delta^+(i)$ is the index set of vertices adjacent from vertex $v_i$, that is $j \in \delta^+(i)$ if $(i, j) \in \mathcal{E}$.

The decision variables for service times of a location, vehicle flow between locations and whether a location is serviced are defined as before with an additional superscript to indicate which stage and scenario the variable belongs to. A description of the decision variables and a summary of the model notation may be found in Table 6.1.

The objective is to maximise the overall expected protected asset value, that is

$$\text{maximise} \sum_{i \in \mathcal{A}^0_a} v_i Y^0_i + \text{expected value of the second stage objectives} \sum_{i \in \mathcal{A}^0_a} p_1 v_i Y^1_i + p_2 \sum_{i \in \mathcal{A}^2_a} v_i Y^2_i + \cdots.$$ 

6.1.2 Staging locations

Once a vehicle has carried out all its assigned first-stage tasks, the vehicle travels to any of the staging locations. A staging location may be an asset or a separate staging area where vehicles wait for their second-stage instructions. Vehicles wait at the staging locations until they receive new instructions at time $t$. The following constraints apply to the staging locations. The number of vehicles departing from the depots must equal the number of vehicles arriving at the staging location,

$$\sum_{k \in \mathcal{A}_d} \sum_{j \in \delta^+_q(k)} X^0_{kj} = \sum_{k \in \mathcal{A}_s} \sum_{i \in \delta^-_q(k)} X^0_{ik} \quad \forall \ q \in \mathcal{Q}.$$ 

The number of vehicles arriving at each staging location must equal the number of vehicles departing from the staging location,

$$\sum_{i \in \delta^-_q(k)} X^0_{ikq} = \sum_{j \in \delta^+_q(k)} X^l_{kj} \quad \forall \ k \in \mathcal{A}_s, q \in \mathcal{Q}, \ l \in S.$$ 

Chapter 6. A Two-stage Stochastic Programming Approach

Sets:
\( \mathcal{U} \) is the set of vehicle capabilities.
\( \mathcal{Q} \) is the set of vehicle types.
\( \mathcal{E}_l^q \) is the set of feasible routes for vehicles of type \( q \) in scenario \( l \).
\( \mathcal{A}_0 \) is the set of first stage assets.
\( \mathcal{A}_l^l \) is the set of second stage assets for scenario \( l \in S \).
\( \mathcal{A}_d \) is the set of depots.
\( \mathcal{A}_s \) is the set of staging locations.

Parameters:
\( a_i \) is the service duration associated with location \( i \).
\( \text{cap}_p \) the capability vector associated with vehicle \( p \).
\( c_i \) the latest time that protection activities may commence.
\( m \) is the number of depots.
\( n \) the number of nodes in the graph representation of the problem.
\( o_i \) the earliest time that protection activities may commence.
\( p_q \) the number of vehicles of type \( q \).
\( r_i \) the protection requirement of asset \( i \).
\( \text{start}_{ip} \) 1 if vehicle \( p \) is at depot \( i \), 0 otherwise.
\( t_{ijp} \) the travel time from location \( i \) to location \( j \) of vehicle \( p \).
\( v_i \) is the value of the asset at location \( i \).

Variables:
First stage
\( S_i^0 \) is the time at which service commences at location \( i \).
\( X_{ijq}^0 \) is the number of vehicles of type \( q \) travelling from location \( i \) to location \( j \).
\( Y_i^0 \) 1 if location \( i \) is serviced, 0 otherwise.

Second Stage
\( S_i^l \) is the time at which service commences at location \( i \) in scenario \( l \).
\( X_{ijq}^l \) is the number of vehicles of type \( q \) travelling from location \( i \) to location \( j \) in scenario \( l \).
\( Y_i^l \) 1 if location \( i \) is serviced in scenario \( l \), 0 otherwise.

Table 6.1: The notation used to formulate the two-stage stochastic wildfire asset protection model.
6.1.3 Vehicle flow constraints

The starting position of vehicles are defined as before. The number of vehicles departing from a depot may not exceed the number of vehicles stationed at the depot

$$\sum_{j \in \delta^i_q(k)} X_{kjq}^0 \leq \text{start}_{kq} \quad \forall \ k \in A_d, \ q \in Q.$$ 

For each asset, the number of vehicles arriving at the asset must equal the number of vehicles departing from that asset,

$$\sum_{i \in \delta_q^-(k)} X_{ikq}^l = \sum_{j \in \delta_q^+(k)} X_{kjq}^l \quad \forall \ k \in A_a^l, \ q \in Q, \ l \in \{0, S\}.$$ 

Vehicle flow formulations require start and end nodes for vehicle paths. In this case the starting nodes are the vehicle depots. The end node is a dummy location which can be reached from any location. This reflects the fact that vehicles are not required to finish at a specific location, but simply travel back to their depot once all tasks have been completed. The travel time to the final sink node does not influence any of the protection assignments or vehicle flow constraints

$$\sum_{(i,j) \in E_q^l} X_{ijq}^l = \sum_{i \in \delta(n)} X_{inq}^l \quad l \in S, q \in Q.$$ 

6.1.4 Protection requirements

Protection requirements are defined in a similar fashion as in Chapter 4 and 5,

$$\sum_{q \in Q} \sum_{i \in \delta_q^-(k)} \text{cap}_{pq} X_{ikq}^l \geq r_{ku} Y_{k}^l \quad \forall \ u \in U, k \in A_u^l, \ l \in \{0, S\}.$$ 

6.1.5 Timing constraints

The start of service time at a staging location is \( t \), that is \( S_i = t \ \forall \ i \in A_s \). The following constraints restrict the start of service time to the respective time windows while considering travel time and service duration at previous locations,

$$X_{ijq}^l \leq p_q Z_{ijq}^l \quad \forall \ (i,j) \in E_q^l, \ q \in Q, \ l \in \{0, S\};$$

$$S_i^l + t_{ijq} + a_i - S_j^l \leq M(1 - Z_{ijq}^l) \quad \forall \ (i,j) \in E_q^l, \ q \in Q, \ l \in \{0, S\};$$

$$a_i \leq S_i^l \quad \forall \ i \in A_u^l, \ l \in \{0, S\};$$

$$S_i^l \leq c_i \quad \forall \ i \in A_u^l, \ l \in \{0, S\}. $$
6.1.6 Sign, binary and integer restrictions

The following constraints apply to the decision variables:

\[ X_{ijq}^l \in \{0, 1, 2, \ldots, p_q\} \quad \forall (i, j) \in \mathcal{E}_q^l, \; q \in \mathcal{Q}, \; l \in \{0, S\}; \]
\[ Z_{ijq}^l \in \{0, 1\} \quad \forall (i, j) \in \mathcal{E}_q^l, \; q \in \mathcal{Q}, \; l \in \{0, S\}; \]
\[ Y_i^l \in \{0, 1\} \quad \forall i \in \mathcal{A}_a^l, \; l \in \{0, S\}; \]
\[ S_i \in [0, t] \quad \forall i \in \mathcal{A}_a^0; \]
\[ S_i \in [t, T_{max}] \quad \forall i \in \mathcal{A}_a^l, \; l \in S. \]

Further, since vehicles may depart immediately from depots \( S_i = 0 \) and \( a_i = 0 \) for all \( i \in \mathcal{A}_d \). The earliest that vehicles may depart from staging locations is \( S_i = t \) \( \forall i \in \mathcal{A}_s \). The sink \( S_n = T_{max} \), where the smallest possible value for \( T_{max} \) is chosen without restricting the vehicle paths.

6.2 Model demonstration

The two-stage stochastic model is demonstrated in this section. The approach is implemented using data provided by Tasmania Fire Service. Three fire spread scenarios are considered shown in Figure 6.2. The problem is implemented in Matlab (The MathWorks Inc., 2012) and solved using the Cplex Class API for Matlab of CPLEX 12.6 (IBM Corporation, 2015).

In addition to community assets, we also consider nearby safer places, and vulnerable groups. The community assets and their protection requirements are described in Chapter 5 and Table 5.2. It is assumed that the locations of nearby safer places and vulnerable groups require any vehicle type to visit the location for a duration of 30 minutes before the time of impact. The available vehicles are described in Chapter 5, Table 5.3. The solutions are shown in Figures 6.3, 6.4, 6.5 and 6.6. It took 5 minutes to find an optimal solution, this includes preprocessing time and time to build the model, which is solved using CPLEX.
Figure 6.2: Three fire spread scenarios.
Figure 6.3: Assets protected in the first stage.

Figure 6.4: Assets protected in scenario 1 of the second stage.
Figure 6.5: Assets protected in scenario 2 of the second stage.

Figure 6.6: Assets protected in scenario 3 of the second stage.
6.3 Computational testing

In this section, computational testing is undertaken on a series of randomly generated test instances. The aim is to provide some indication of the size of practical problems that the model can solve. The model was implemented in Matlab and solved using CPLEX 12.6 and the CPLEX Matlab API. A desktop computer with an Intel i7 processor and 8GB of RAM was used.

The test problem instances were generated using the following method. Locations are uniformly distributed across a 80km by 40km landscape. Driving times are taken as the direct distance between locations at a driving speed of 60km/h. Initially the fire spreads at a rate of 10km/h from left to right across the landscape. The second stage starts after 4 hours, at which time the fire front has travelled 40km. Three scenarios are considered for the second stage with a fire spread rate of 10km/h, 15km/h and 20km/h respectively. The location service time is 30 minutes. The number of vehicles required to service each location is randomly selected from the integer range [1,6] and the value of each location is selected from the integer range [1,3].

The closing time of a location’s time window is determined by the time to impact. The opening time is determined by the time window length. Two cases are considered for the length of the time windows. In the first case, all locations have 20 minute time windows. The average solution times for the first case are listed in Table 6.2.

In the second case, half of the locations have time windows of 40 minutes, and half of the locations have time windows of zero length. The average solution times for the second case are listed in Table 6.3. The average solution time of 10 instances for each parameter combination is reported in Tables 6.2 and 6.3. The solver was given a time limit of 600 seconds (10 minutes). The relative optimality gap is reported for those instances to which an optimal solution is not found within the 600 second time limit.

<table>
<thead>
<tr>
<th>Locations per scenario</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>6 (0%)</td>
<td>264 (0.8%)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>14 (0%)</td>
<td>373 (0.8%)</td>
</tr>
</tbody>
</table>

Table 6.2: The average solution time in seconds and relative optimality gap percentage for the two-stage stochastic programming benchmark instances described in the text with 20 minutes time windows.

At least one feasible solution was found for all the problems considered. Best solutions were found within 11% of the optimal on average given 10 minutes of computation time for the test instances with 40 or less locations. The exception being problem instances with 40 locations per scenario for the second set of test instances.

These results indicate that optimal or near optimal solutions can be found within minutes for practical problems with 30 locations or less per scenario. Note that a problem with 30 locations
6.4. Chapter summary

A two-stage stochastic programming model with recourse is presented in this chapter. The model is demonstrated using three fire spread scenarios impacting South Hobart. Computational testing shows that the problem is tractable using this approach.

The two-stage stochastic approach determines optimal solutions for each of the second-stage scenarios that are considered. This has the benefit that vehicle assignments have been prepared in advance and are ready to be implemented when the second-stage scenario is realised. This approach gives IMTs the option of choosing the appropriate assignment from the list of possible second-stage assignments. If the realised fire spread scenario differs dramatically from any of the expected scenarios, then a new assignment plan can be generated, using either an approach similar to that of the asset protection approach in Chapter 4 or a rerouting approach similar to that of Chapter 5 to minimise deviation from the existing second-stage assignments.

### Table 6.3: The average solution time in seconds and relative optimality gap percentage for the two-stage stochastic programming benchmark instances described in the text; half of the locations have 40 minute time windows, and half have zero length time windows.

| $|A_s|$ | 20  | 30  | 40  |
|-----|-----|-----|-----|
| 30  | 63 (0%) | 601 (7.7%) | 601 (20%) |
| 60  | 221 (1%) | 601 (10.3%) | 602 (22.8 %) |

per scenario correlates with a practical problem with a total of between 60 and 120 assets, vulnerable people and nearby safer places.
CHAPTER 7

Conclusion

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This chapter concludes the thesis with a summary of the work contained therein, an appraisal of the contributions of the thesis and suggestions for further investigation.

7.1 Thesis summary

The first chapter provides the reader with an introduction and background to wildfire incident management. The problem of assigning resources to defensive tasks during wildfires is discussed and the scope and objectives of the study are described.

Chapter 2 provides an overview of literature related to topics pursued in this thesis. Optimisation literature in wildfire and emergency incident management is reviewed, demonstrating that past research focused on providing decision support and modelling initial attack, fire line construction and longer-term planning with regards to incident management, for example determining resource levels. The defensive tasks that wildfire resources can perform are were not studied as thoroughly. Further, vehicle routing literature is reviewed, focusing on the orienteering problem with time windows and its variations. The team orienteering problem with time windows is of special interest because it is used as the basis of the wildfire resource assignment models presented in this thesis.

In Chapter 3, a new, efficient mixed-integer programming formulation for the TOPTW is introduced in fulfilment of Objective I in §1.5. By using existing TOPTW benchmark instances, it is demonstrated that the new TOPTW formulation leads to dramatically improved solution times. A modified formulation for the TOPTW is then suggested. The modification accounts
for the cooperative delivery of service at locations, resulting in the COPTW – a feature of the wildfire resource assignment problem. This partly fulfils Objective II in §1.5.

In Chapter 4, a mixed-integer programming approach to the problem of assigning resources to defensive tasks during wildfires is presented in fulfilment of Objective II in §1.5. The model formulation captures a number of the characteristics of the wildfire resource assignment problem. The formulated model further generalises the TOPTW, introducing a vector specifying the protection requirement for each location and allowing each vehicle type to have a unique travel time between two locations. The working of the model is demonstrated using the locations of assets and fire stations in Hobart, Tasmania, Australia. Computational testing of the model demonstrates that it is computationally feasible to apply the model to real-life resource assignment problems.

In Chapter 5, a method for rerouting vehicles to adapt to changes is presented in fulfilment of Objective III in §1.5. A number of potential secondary objectives are considered to minimise changes made to the assignment of vehicles. These deviation measures provide a varying degree of flexibility when rerouting vehicles. The specific deviation measure chosen would depend on the priority and objectives of the IMT. The rerouting framework presented in Chapter 5 allows for the consideration of various disruptions that can occur when assigning vehicles to asset protection activities. An unexpected weather disruption case study is presented, using a wildfire scenario in South Hobart.

A two-stage stochastic programming formulation of the wildfire resource assignment problem is presented in Chapter 6. The model takes future fire spread scenarios into account, fulfilling the final research objective, Objective IV in §1.5. The resource assignment approaches proposed in Chapter 4 and Chapter 6 are used for initial assignments, or generating plans independently from any past vehicle assignment. The stochastic approach in Chapter 6 is applicable when there is likelihood estimates available for some of the parameters. The rerouting approach in Chapter 5 can be used to update initial vehicle assignments under changing conditions, minimising deviations from the existing vehicle assignment plans that were generated using either the deterministic or stochastic approaches.

7.2 Appraisal of the work contained in this thesis

There is no doubt that IMTs and fire crews battling wildfires across the world face a demanding job with potentially serious consequences. A suite of models is presented in this thesis with the aim of providing decision support to IMTs in assigning resources during wildfires.

Not only do these models have application to operational decisionmaking, but they can be used to inform strategic planning decisions such as vehicle fleet composition and home-basing of response vehicles. By simulating potential fire spread scenarios, it is possible to determine the resources required to provide adequate protection to community assets and identify locations
which would go unprotected. In addition, the models may be adapted for use in IMT training exercises, demonstrating various assignment scenarios and how it may be possible to deal with disruptions while minimising deviation from original plans.

It is demonstrated that the models are of practical use in the sense that they can be implemented by using existing software and hardware technology to produce solutions for problems within a reasonable time. The contribution to wildfire management literature is summarised by the following four points.

**Contribution 1** Characterisation of the wildfire resource assignment problem for defensive tasks

This is the first time that decision support modelling has focused on the defensive tasks, as opposed to fire suppression activities. A description of the wildfire resource assignment problem is presented, proposing practical mixed-integer programming formulations and demonstrating how these models may be implemented. The wildfire asset protection model developed in Chapter 4 has been published in the *Canadian Journal of Forest Research*, a flagship journal in forestry management (Van der Merwe et al., 2015a). This research is a step towards providing IMTs with tools that may be used in real-time to reduce the impact of wildfires on communities.

**Contribution 2** Rerouting and stochastic planning for wildfire resource assignment

Two approaches are presented to deal with the dynamic nature of wildfire planning. A dynamic rerouting approach and two-stage stochastic programming approach is presented. Which of these two approaches are to be adopted would depend on whether knowledge of future disruptions is available. A manuscript describing the rerouting approach has been submitted to the Annals of Operations Research for publication (Van der Merwe et al., 2015b). A manuscript on the stochastic approach is in preparation for submission.

There are a number of features which distinguishes the wildfire resource assignment problem from existing vehicle routing formulations. In the process of formulating the wildfire resource assignment models, the following key contributions with regards to the operations research literature were made.

**Contribution 3** An efficient formulation for the TOPTW

A mixed-integer programming formulation for the TOPTW was presented which eliminates symmetry from the traditional TOPTW formulation. Further, edges which are infeasible due to timing constraints has been removed from the graph representation, using a preprocessing step. The result is a formulation of the TOPTW which dramatically improved solution times compared to traditional TOPTW formulations.

**Contribution 4** Novel generalisations and application of the TOPTW

In order to capture features of the wildfire resource assignment problem, a number of generalisations were made to the TOPTW.
7.3 Suggestions for further investigation

In this section, three suggestions are made with respect to possible future research pertaining to implementing the approaches developed here and to further refinement of these approaches.

**Suggestion 1** *Solution approaches for the COPTW*

Due to the quick-moving nature of many wildfires, decisionmaking in wildfire response will always stand to benefit from faster solution times. Solution approaches for the TOPTW often rely on, or exploit, the ability to independently generate paths for each vehicle. For example, exact approaches using column generation techniques calculate the contribution of each individual vehicle to the overall objective function (Gueguen and Dejax, 1999). The cooperative element of the models formulated in this thesis do not allow for the independent calculation of the contribution of each vehicle to the overall objective value.

In the past, heuristic methods have proved very successful for finding approximate solutions to the TOPTW and could prove the same for cooperative orienteering problems. However, some elements that make these methods very successful, are not as easily applied to the models in this thesis. Some insertion heuristics measure the ‘slack’ in a route to check whether a location can be added without violating time window constraints (Vansteenwegen et al., 2009). The cooperative element of models in this thesis makes this type of ‘accounting’ more difficult, since a location may need to be inserted or removed from more than one route. Heuristic algorithms designed for the TOPTW may thus require further work to adapt them for cooperative orienteering problems, or it may be necessary to develop new algorithms based on fundamentally different ideas.

Algorithms for finding solutions to the wildfire asset protection problem could exploit the structure of real-world problems. Such as the spatially correlated time windows and the narrow time windows of active defence tasks.

**Suggestion 2** *Trade-off between fire suppression and defensive tasks*

Suppression may be effective on parts of a fire front. Any successful fire suppression which is undertaken will have an effect on the asset protection problem, potentially reducing the number of assets impacted, or delaying the time to impact. Since the resources used for defensive activities can also perform direct fire suppression activities, there exists an inherent trade-off between applying resources to fire suppression and defensive tasks. Integrating suppression and the resource assignment models developed in this thesis could lead to improved resource assignment.

**Suggestion 3** *Implementation*

The project was undertaken in cooperation with Tasmania Fire Service. The modelling was informed and supported by feedback from Tasmania Fire Service. The proposed methods were
positively received, and it was acknowledged that this work has the “potential to add considerable value to the management of fires and other natural hazard events” (Killalea, 2015). Further work, firstly, to ensure that all the nuances of wildfire incident management has been captured and, secondly, to develop decision support tools to implement the proposed approaches is a natural next step.


McArthur A (1973). Forest fire danger meter mk. 5. *Forest Research Institute, Forestry and Timber Bureau, Canberra*.


Merrill D and Alexander M (1987). *Glossary of forest fire management terms*. Canadian Committee on Forest Fire Management, NRCC.


The MathWorks Inc (2012). *MATLAB, Version 8.0.0.783 (R2010b)*. Natick, MA.


Tools, Methods and Software

This appendix contains additional information about the tools and methods used in this thesis. Methods pertaining to processing data, estimating drive times and drawing maps are presented in §A.1. A list of the software is presented in §A.2 and a description of computing hardware in §A.3. A descriptive list of the electronic files supplementing this thesis is provided in §A.4. The layout of the accompanying data files is explained §A.5.

A.1 Mapping and data processing

Travel times and driving directions were determined using Google Maps’ application programming interface (API). The Google Distance Matrix service computes travel distance and journey duration between multiple origins and destinations. A php script was used to request driving times from the Google Distance Matrix Service via the API. The script, FetchGoogleDriveTimes.php, is one of the supplementary files accompanying this thesis. Google Maps impose usage limitations on this service, a maximum of 25 origins and/or destinations per request with an upper bound of 100 elements. The number of elements is the number of origins multiplied by the number of destinations. The daily usage is limited to 2500 requests per day. Usage beyond these limits is a paid service. For these research purposes, the free usage limitations were sufficient. Driving directions were determined using Google Maps’ Directions Service.

Google maps uses the WGS 84 (EPSG:4326) coordinate system. Part of processing the data received from Tasmania Fire Service involved converting coordinates to the appropriate coordinate system. The data for fire spread scenarios, community protection plans and depot locations were provided in the form of GIS shapefiles.

The maps in Chapters 5 and 6 were created using Natural Earth data, which is public domain vector and raster map data and is available online from naturalearthdata.com.
A.2 Software packages

The following software packages were used during the preparation of this thesis.

**IBM ILOG CPLEX Optimization Studio (12.6)** Is a propriety optimisation software package, often simply referred to as CPLEX.

**CMPL** Is an open source mathematical programming language and a system for mathematical programming and optimisation of linear optimisation problems available from [https://projects.coin-or.org/Cmpl](https://projects.coin-or.org/Cmpl).

**Matlab R2012b** Is a proprietary computing environment and programming language. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

**QGIS** QuantumGIS (or QGIS) is a free, open source geographic information system (GIS) application. The application allows for the viewing, editing and analysis of GIS data.

A.3 Computing hardware

Implementation and computational testing of the models were carried out on either a desktop computer or a node high performance computer cluster called Trifid. The desktop computer has an Intel i7-3770 3.40 GHZ processor and 8GB of RAM. Each node of the cluster has two Intel Xeon E5-2670 processors and 64GB of RAM with 16 cores available. The cluster belongs to the V3Alliance.

A.4 Supplementary material

The following electronic files accompany this thesis. The files are also available online from [https://github.com/mvdmerwe/PhDThesisCode](https://github.com/mvdmerwe/PhDThesisCode).

<table>
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<th>Model formulations:</th>
</tr>
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<td>AP.cmpl</td>
</tr>
<tr>
<td>Asset protection model formulation in CMPL format.</td>
</tr>
<tr>
<td>COPTW.cmpl</td>
</tr>
<tr>
<td>Formulation for the cooperative orienteering problem with time windows in CMPL format.</td>
</tr>
<tr>
<td>Rerouting1.cmpl</td>
</tr>
<tr>
<td>Rerouting model formulation with secondary objectives and constraints presented in §5.3.1.</td>
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<tr>
<td>Rerouting2.cmpl</td>
</tr>
<tr>
<td>Rerouting model formulation with secondary objectives and constraints presented in §5.3.2.</td>
</tr>
</tbody>
</table>
Rerouting3.cmpl Rerouting model formulation with secondary objectives and constraints presented in §5.3.3.

SP.m Stochastic model formulation in Matlab’s m-file format.

TOPTW.cmpl Traditional formulation for the team orienteering problem with time windows in CMPL format.

TOPTWnew.cmpl New formulation for the team orienteering problem with time windows presented in Chapter 3.

Data:

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rerouting1.csv</td>
<td>Pre-disruption data used for the rerouting model demonstration in Chapter 5.</td>
</tr>
<tr>
<td>Rerouting2.csv</td>
<td>Post-disruption data used for the rerouting model demonstration in Chapter 5.</td>
</tr>
<tr>
<td>Stochastic.csv</td>
<td>Data used for the stochastic model demonstration in Chapter 6.</td>
</tr>
<tr>
<td>VehicleProperties.csv</td>
<td>The vehicle capabilities and starting position data file used in Chapters 5 and 6.</td>
</tr>
</tbody>
</table>

Script:

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FetchDriveTime.php</td>
<td>Takes a list of longitude latitude coordinates in csv format as input. The output is drive time and distance matrices which are constructed using the Google Distance Matrix Service.</td>
</tr>
<tr>
<td>PlotSolutionGoogleMap.m</td>
<td>Matlab script for creating maps with the routes of vehicles overlayed. For example see Figure 3.2(b).</td>
</tr>
</tbody>
</table>

A.5 Data structure

The data for the optimisation models are read from comma separated value (CSV) tables. The CSV files for the model demonstrations were created by exporting and processing the shape files provided by Tasmania Fire Service. The final data is saved as CSV files, each with the layout shown in Figures A.1 and A.2. Note that in this context $x_i$ and $y_i$ refer to the latitude and longitude coordinates of locations $i$. The definitions of the other symbols may be found in Table 6.1.
Figure A.1: The layout of the csv data files containing location-specific information.

Figure A.2: The layout of the csv data files containing vehicle-specific information.