A FUNDAMENTAL STUDY ON SELF-LOCKING
GEAR AND ITS APPLICATION IN SEAT
HEIGHT ADJUSTER

A thesis submitted in fulfilment of the requirements for
the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis/project is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Jiaxing Zhan

01/07/2016
Abstract

The self-locking gear has great potential application in controlling the position stability of gearbox, which is a critical requirement in some precision machineries and instruments. However, the design procedure, load capacity, and dynamic behaviours of self-locking gear has not been investigated and systemized, which is a great barrier for the potential users. This research focuses on addressing these issues by developing new mathematical models and finite element method. Moreover, in this PhD project, we attempt to investigate the design method of the self-locking gear in seat height adjuster, which may eliminate the back-driving thereby improving the occupant driving comfort, enhance the driving safety, and reduce the volume of gearbox.

A comprehensive literature review of spur gear, helical gear and epicyclical gear is presented in chapter 2. Chapter 3 gives an introduction of the analytical models of gear geometry, gear load capacity, and dynamics. Chapter 4 elaborates the procedure of finite element analysis. Chapter 5 presents the effects of different parameters on self-locking gear pairs. The dynamic performance of self-locking gears are presented in the Chapter 6. Chapter 7 depicts the mathematical models of seat cushion structure and plastic gears used to design the self-locking gear in seat height adjusters. Chapter 8 draws the conclusions and recommends the potential further

In summary, this thesis developed a new finite element method to investigate the static and dynamic characteristics of self-locking gear. The effects of torque, tip radius, centre to centre distance error and misalignment axes on self-locking gear
pairs were quantified. The non-linear dynamic behaviour of self-locking gear system subjected to the variation ratio of contact length were clarified. The design method of self-locking gear system in seat height adjuster subject to the automotive seat structure and plastic gears were developed.

The obtained results in this thesis provide significant knowledge for predicting the static and dynamic performance of self-locking gear pairs, optimizing their design parameters, and diagnosing possible design errors in self-locking gear pair design. The work detailed in this thesis resulted in five peer-reviewed journal publications.
Publications

Peer Reviewed Journal Articles


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Those who intend to walk a hundred miles often stop at ninety. To pursue a PhD is a long journey runs the gamut from happiness to bitterness. It is a process. A discovery. It is a process of self-discovery. A journey brings us face to face with ourselves. Thanks for those who travelled this journey with me together.

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CHAPTER 1
INTRODUCTION

1.1. Thesis Introduction

Gears have been in use for over 3000 years and they are an important element in all machinery used nowadays. There are more than ten types of gear system existing in the modern industry. Their design processes have been well developed and systematized in countless references [1-4]. To reduce the design cycle, simplify the development procedure and increase the design precision, plenty of codes and software were developed in the past decades. The generation of gear system could be realized in few minutes due to the advance of computer technology. Accompanied with the progress of gear design, the gear manufacturing also have been well developed. Techniques such as powder-metallurgy and plastic injection moulding improved the manufacturing precision and efficiency in gear industry.

Although the conventional gear system has been well studied, it is not without limits. In most of gear drives, the gear pair can be driven reversely. In another word, when the torque of output side is larger than input side, the driven gear is going to become the driving gear. This kind of reversible performance is unacceptable in many gear system because it could produce detrimental effects to other components such as torsional vibration and mechanical failure at the transmission input side. Traditionally, the worm-gear drives can be used to prevent the backdriving in gear transmission. However, their application has some disadvantages: relatively low speed, insufficient mesh efficiency, poor thermal performance, etc. [5, 6].
The concept of self-locking gears was initiated by Munster and Tzarev in 1968 [7]. However, few literatures presented the further research in the latter decades. Although Kapelevich reintroduced the self-locking gear design in his book published in 2002, the self-locking gears are not prevalently used in the industry. Part of the reason for this could be summarized by three aspects:

- No ready-made formulas provide the systematic design procedure compared with conventional gear design.
- The manufacture of self-locking gears is inconvenient due to their high pressure angle and high helical angle.
- Few works has been done to investigate the load capacity, dynamic characteristic and optimization methods of the self-locking gears.

In the automotive industry, gears are not only used in the power-train system but also utilized in window-lift system, fuel pumps system, adjusters, ABS, clutches, parking system and steering systems. In the automotive seat system, most of gear drives are used in the seat adjustment mechanisms. For example, Figure 1.1 shows a typical vertical seat height adjuster coupled with the seat pan structure. Figure 1.2 shows the main components in seat height adjuster.
There are three major functions of seat height adjuster used in automotive seat structure.

- Adjust the height of seat cushion to create a good driving vision for occupant.
- Absorb crash energy to avoid the submarine of seat structure.
- Lock the position of seat cushion both in static and vibration conditions - Downward movement of seat cushion is unacceptable.

Typically, the gearbox of seat height adjuster includes three stages as shown in Fig.1.3. As the power output stage, Stage 1 is coupled with the seat pan structure. Stage 2 is a planetary gear system used to reduce the speed of electric motor. Stage 1 is a power supply stage which using worm-helical gear drives to increase the motor torque and decrease the speed of power motor.

![Figure 1.3. Schematic gearbox of seat height adjuster.](image)

One of the important issues with the current gearbox of seat height adjuster is the stable locking of seat structure. During the adjustment, the sudden power loss or system vibration could reduce the driving torque. As a result, the gearbox system
could be driven by the outside load such as weight of seat structure and occupant. Different types of brake or clutch system have to be used to prevent this phenomenon.

Another important issue with contemporary seat height adjuster designed with spur gears is the safety problem. To reduce the weight and sound level of gear transmission, most of the helical gears are made with plastic material. However, when the external load is transferred into worm-helical system, the impact energy could lead to the potential failure of plastic helical gears. If the self-locking gear could be used in the gearbox of seat height adjuster, the seat height structure and complete seat structure could be simplified and the weight could be reduced about 5%. Also, the self-locking gears have numerous possible applications in precise transmissions, lifting device and other gear system where the reverse drag or inertial drive is not acceptable. Therefore, it is necessary to identify the static and dynamic behaviours of self-locking gears, and then incorporate this knowledge into the development of gear transmission system.

1.2. Aims and scope of PhD project
The main scientific objectives and research activities of this PhD project are as follows.

- Develop a finite element method to characterize the static and dynamic performance of self-locking gear.
- Analyse the load capacity of the self-locking gear pairs subjected to the helical angle, coefficient of friction, profile modifications and mechanical errors.
• Analyse the non-linear dynamic behaviour of self-locking gears subjected to the variation ratio of contact length.

• Determine the design method of self-locking gear design in seat height adjuster subject to the automotive seat structure.

1.3. Structure of the PhD thesis
A comprehensive literature review of spur gear, helical gear and epicyclical gear is presented in chapter 2. Chapter 3 gives an introduction of the analytical models of gear geometry, gear load capacity, and dynamics. Chapter 4 elaborates the procedure of finite element analysis. Chapter 5 presents the effects of different parameters on self-locking gear pairs. The dynamic performance of self-locking gears are presented in the Chapter 6. Chapter 7 depicts the mathematical models of seat cushion structure and plastic gears used to design the self-locking gear in seat height adjusters. Chapter 8 draws the conclusions and recommends the potential further work.
CHAPTER 2
LITERATURE REVIEW

2.1. Overview

Research related to self-locking gear system is extensive, and hence the primary aim of this literature review is to place the scope of the material covered in this thesis within the general body of knowledge. The second aim is to identify lacking areas in the field of knowledge and establish the importance of the research conducted. Through the review on the field of knowledge relating to the key subjects of this research, the current literature can be categorized as the study of the following,

Self-locking

Gear strength

Gear dynamics

In these three categories, I aim to identify lacking areas in the field of knowledge and to establish the importance of the research conducted.

2.2. Self-locking

Preventing the backdriving and controlling the position stability of gearbox are critical tasks in some precision machineries and instruments [9-12]. Consequently, many self-locking mechanisms have been developed to stabilize the rotation accuracy in gearing system [13-15]. However, these assistant devices could enlarge the volume of gearbox, increase the complexity of transmission system and introduce more errors between gear pairs. Therefore, alternative methods are
required to achieve design requirement without adding any extra electric or mechanical device.

Currently, the most common device to prevent the backdriving is the worm gear with a low lead angle and crossed axis shafts' arrangement [6, 16-18]. However, in this section, the review is focussed on publications related to the self-locking capacity and performance in parallel axis gear system.

### 2.2.1. Self-locking gear pairs

The self-locking system were originally used in textile industry reported from ref [7]. Then the application of self-locking gear pairs with high driving efficiency was proved in 1969. In 1995, Oledzki listed the classification of self-locking drives. The mathematical formulas of the self-locking kinematic pair was established and the dynamic behaviour of four different drives was studied.

In 2003, the criteria of the self-locking was analysed by Timofeev in the ref [19]. Kapelevich also reported the self-locking gear design in his direct gear design method in 2005[5]. Then in 2009, the self-locking gear pairs were used in a continuous variable valve lift (CVVL) to increase the efficiency and provide anti-backdriving capacity in the engine system(shows in Fig.2.1). In 2010, Kapelevich experimentally confirmed the self-locking condition by using a pair of self-locking helical gear prototype (shows in Fig.2.2).

### 2.2.2. Self-locking in planetary gear system

Muller fundamentally discussed the self-locking criteria in planetary transmissions [20]. He pointed out that the conditions of self-locking in planetary gear system are not widely understood. Therefore, the self-locking property of planetary
transmissions has been used only seldom because of the tooth-friction losses cannot be accurately predicted.

Figure 2.1. CVVL system designed with self-locking gear pair [21].
From 2000 onwards, the knowledge related to self-locking phenomenon of planetary gear system was extended in many literatures [23-35]. The emphasis was placed on two types of planetary gears, namely involute and cycloid. However, in their study, the self-locking of planetary gears was reckoned as an issue which should be avoided in whole gearbox. They investigated the relationship between transmission ratio and efficiency, and discussed the self-locking conditions in planetary gear system.

2.3. Gear load capacity of current gear pairs

The literature review in this section is focussed on the study of current international gear standard and the journal papers from 2000 onwards. The study of gear standards involved a review into ISO6336. To review the published papers, emphasis is placed on the calculation model of contact stress and bending stress. At the meantime, the study will touch on the optimization of gear strength by modifying the teeth profile and using new type of gear profiles.
2.3.1. Calculation methods in ISO standard

ISO6336 was first published in 1997, which is one of the most widely used international standards to predict the gear load capacity.

Part 2 is used to assess the surface durability of cylindrical gears on the basis of Hertzian pressure (shows in Fig.2.3) [3]. Considering the design, tangential speed and manufacturing accuracy, some empirical factors needed to be considered. Therefore, the contact stress is calculated with the equations:

$$ \sigma_H = Z_B \sigma_{H0} \sqrt{K_A K_v K_{H\beta} K_{H\alpha}} \tag{1} $$

$$ \sigma_{H0} = Z_H Z_E Z_t Z_{t\beta} \frac{F_t}{d_1 b} \frac{u + 1}{u} \tag{2} $$

where

$\sigma_{H0}$ represents nominal contact stress at the pitch point.

$Z_B$ represents tooth contact factor.

$K_A$ and $K_v$ are application factor and dynamic factor.

$K_{H\beta}$ and $K_{H\alpha}$ are load factor.

$Z_H$, $Z_E$, $Z_t$, $Z_{t\beta}$ represents zone factor, elasticity factor, contact ratio factor, and helix angle factor.

$F_t$ is the nominal tangential load, the transverse load tangential to the reference cylinder.

$b$ represents the gear width.
\( d_1 \) is reference pinion diameter.

\( u \) is gear ratio.

**Figure 2.3. Contact between two spheres.**

Part 3 is to compute the root strength of gear by rating the maximum stress at the tooth root [4]. Lewis considered gear tooth as a cantilever beam (shows in Fig.2.4) with static normal force applied at the gear tip. On the basis of this theory, the bending stress can be calculated as

\[
\sigma_F = \sigma_{F0}K_aK_vK_{F\beta}K_{F\alpha}
\]

(3)

\[
\sigma_{F0} = \frac{F_t}{b m_n}Y_F Y_S Y_{\beta} Y_B Y_{DT}
\]

\(<\text{for}\)

\( K_{F\beta} \) and \( K_{F\alpha} \) are the face load factor and transverse load factor.
\( F_t \) represents the nominal tangential load.

\( b \) represents gear width.

\( m_n \) represents normal module.

\( Y_F, Y_S, Y_\beta, Y_B, Y_{DT} \) are form factor, stress correction factor, helical angle factor, rim thickness factor, deep tooth factor.

Figure 2.4. Gear tooth resisting bending load.

### 2.3.2. Contact stress

The international standard ISO6336 provides significant information to determine the gear load capacity. Based on this guidance, many new approaches have emerged since the 21st century. These new methodologies offered considerable contribution to enhance the computational precision in calculating the performance of gear pairs under load.
Pedrero et al. invented a new calculation method to evaluate the contact stress [36]. The load sharing ratio was represented in Fig.2.5. By using the same model, he also investigated the pitting of spur and helical gears in vacuum system[37]. In 2013, his non-uniform model was utilized to investigate the load capacity of helical and spur gears with high transverse contact ratio [38]. Similarly, the load sharing ratio was also being calculated as shown in Fig.2.6. Results showed the weakness of ISO standard due to the calculation method by using even load distribution.

Figure 2.5. Load sharing ratio of spur gears (contact ratio between 1 and 2). (From Pedrero et al. [36])
Hwang et al. presented a stress analysis of a pair of rotating gears [39]. The change of contact stress was compared with the results calculated by using the equation of AGMA. The similar work also have been done in ref [40] by considering the influence of friction. By using the Lagrange multiplier algorithm, the contact stress of gear pairs with different helical angle and static coefficients of friction were analysed by finite element method. Also, there are some novel numerical model were proposed to enhance the calculation accuracy of contact stress in static contact condition [41, 42].

The contact stress is also a significant factor to determine the service life of gears. Glodez et al. established computational model to compute the bending fatigue of gear teeth root [43]. It is successfully used to determine the service life of real spur gear. After two year, he presented another numerical model to simulate contacting surface fatigue mechanism[44].
2.3.3. Bending stress and its optimization

Pedrero et al. used a non-uniform model to evaluate the load capacity of gears [45]. Later on, Sanchez et al. a similar model was proposed to calculate the bending stress [46]. It was a study of the effect of location on the value of root bending stress.

Li studied root bending stress with mechanical errors, manufacturing errors (shows in Fig.2.7) and profile modifications. It was found that mechanical errors, manufacturing errors and profile modifications exerted great effects on root bending stress by comparing with FEM results (shows in Fig.2.8-Fig.2.15) [47, 48]. Later on, Li extended the research to study the effects of addendum coefficient on gear bending stress [49]. It was presented that the increase of addendum coefficient may increase the number of contact ratio thereby reducing the root bending stress.

Litvin et al. introduced a method to decrease the bending stress by using larger pressure angles for driving sides and smaller pressure angles on coast sides (shows in Fig.2.16) [50]. As a further extension of ref [50], Kumar et al. developed a new asymmetric rack cutter to generate the gear pair with asymmetric involute profiles and root fillet [51]. They developed a optimized bending stress on both gears by using this kind rack cutter, which optimize the performance of asymmetric gears.

Later on, Pedersen found the asymmetric gear profile could significantly reduce the bending stress [52]. Then the advantages of asymmetric gears were quantified by non-dimensional teeth [53]. The results showed that the load capacity was 28% higher compared with the standard gear with 20 degree pressure angle (shows in Fig.2.17).
Figure 2.7. Assembly errors: (a) theoretical position of gear shafts, (b) gear shafts with manufacturing errors and (c) coordinate systems with misalignment errors ‘‘e2’’ and ‘‘e3’’. (From Li [47])
Figure 2.8. Root bending stress with ME. (From Li [47])

Figure 2.9. Effect of MEmax on bending stress and influence factor. (From Li [47])
Figure 2.10. Effect of e3 on root bending stress.

(From Li [47])

Figure 2.11. Effect of misalignment error e3 on root strength and influence factor.

(From Li [47])

Figure 2.12. Effect of e2 on root bending stress.

(From Li [47])
Figure 2.13. Effect of misalignment error e2 on bending strength and influence factor. (From Li [47])

Figure 2.14. Effect of crowing on root bending strength. (From Li [47])
Figure 2.15. Effect of lead crowning on root strength and influence factor. (From Li [47])

Figure 2.16. Representation of driving profile (a), and coast profile (b).

(From Litvin et al. [50])
2.4. Gear dynamics of gear system

A considerable amount of literature on gear dynamic performance has been published to date. Ozguven and Houser [54] reviewed the literature relating to the mathematical models used in gear dynamics, from 1915 and up to 1986. They classified the models in five groups. Houser gives a good summary of gear noise in Gear Handbook with many references [1]. He concluded that in most applications, the term gear noise should more aptly be called transmission noise since the sound that produces from the meshing of gears is, in reality, transmitted via bearings and shafts, and finally radiated as a structure-bone noise, as depicted in Fig.2.18. Smith presented significant guidance to tackle the issues of noise in gear transmissions[55]. He also summarized that the complete vibration radiation path starts from the combination of mechanical errors, design errors and tooth deflections to produce the transmission errors (T.E.) (shows in Fig.2.19).
Figure 2.18. Gear noise transmission paths. (From Ref [57])
In this section, the various research approaches, developed from the year 1990, which used to investigate and predict the dynamic performance of gears are summarised. This includes the study of mesh stiffness, which is reckoned as the primary source of noise and vibration in gear transmission system. Following this, the influences of transmission error on the dynamic performance of gear system are summarised. Finally, this section also involves the effects of tooth profile modification on the identification and optimization of gear dynamic responses.

2.5. Conclusion
As a novel gear designed with anti-backdriving capacity, the self-locking gear has been of great interest to researchers and engineers to date. As Kapelevich[5] presented, the self-locking gear pairs could be designed using direct gear design method and its anti-backdriving capacity has been experimentally approved. This means the self-locking gear is applicable to many types of transmission mechanisms when the backdriving is not permitted. It is however important to note that due to the high pressure angle and large helical angle, there are not many studies that present the load capacity and dynamic characteristics of self-locking gears.

The author concludes that there is a gap in the literature in regards to:

1. Contact stress and bending stress of self-locking gear tooth
3. The effects of vibration and lubrication on the self-locking capacity.
The investigation of load capacity and dynamic characteristics of self-locking gear can provide significant knowledge for the potential users and fill the gap between the self-locking gear and conventional gear for the international standards. Furthermore, the study into the effects of vibration and lubrication on the self-locking gear pairs can investigate the design reliability of self-locking gear.
CHAPTER 3
THEORETICAL MODELLING OF SELF-LOCKING GEAR PAIRS

3.1. Design of Self-locking gear

3.1.1. Generation of tooth flank

For the conventional involute gear, the tooth surfaces can be generated by the screw motion of a straight line, as shown in Fig.3.1. The tooth flank of a left-hand helical gear and the unit normal to the surface can be determined as follows [56, 57]

\[
\mathbf{n}_1(\rho_1, \theta_1) = \begin{bmatrix}
    r_{b1} \cos(\theta_1 + \mu_1) + \rho_1 \sin(\theta_1 + \mu_1) \\
    -r_{b1} \cos(\theta_1 + \mu_1) + \rho_1 \sin(\theta_1 + \mu_1) \\
    -\rho_1 \sin \theta_1 + p_1 \theta_1
\end{bmatrix}
\]

(4)

\[
\mathbf{n}_1(\theta_1) = \begin{bmatrix}
    -\sin \lambda_{b1} \sin(\theta_1 + \mu_1) \\
    -\sin \lambda_{b1} \cos(\theta_1 + \mu_1) \\
    \cos \lambda_{b1}
\end{bmatrix}
\]

(5)

Similarly, the tooth flank of a left-hand helical gear and the unit normal to the surface can be represented as follows.

\[
\mathbf{n}_2(\rho_2, \theta_2) = \begin{bmatrix}
    r_{b2} \cos(\theta_2 - \mu_2) + \rho_2 \sin(\theta_2 - \mu_2) \\
    -r_{b2} \cos(\theta_2 - \mu_2) + \rho_2 \sin(\theta_2 - \mu_2) \\
    \rho_2 \sin \lambda_{b2} - p_2 \theta_2
\end{bmatrix}
\]

(6)

\[
\mathbf{n}_2(\theta_2) = \begin{bmatrix}
    \sin \lambda_{b2} \sin(\theta_2 - \mu_2) \\
    \sin \lambda_{b2} \cos(\theta_2 - \mu_2) \\
    -\cos \lambda_{b2}
\end{bmatrix}
\]

(7)

where \(r_{bi} (i = 1, 2)\) are the radii of base circle; \(\mu_i (i = 1, 2)\) are the start angles of the helicoids surfaces; \(p_i (i = 1, 2)\) are the pitches at the base circles; \(\lambda_{bi} (i = 1, 2)\) are the lead angles of helicoid surfaces at the base circles. The determination of \(\theta_i\)
(i = 1, 2) can be found in Fig. 3.1. The rest parameters in Equations (18)-(22) are calculated as follows.

\[ r_{bi} = r_{pi} \cos \alpha_{ti}, i = 1,2 \]  
\[ r_{pi} = \frac{z_{mi}}{2 \cos \beta_{i}} \]  
\[ \lambda_{bi} = \tan^{-1}\left(\frac{r_{pi}}{\tan \beta_{i} r_{bi}}\right), i = 1,2 \]  
\[ \mu_{i} = \frac{\pi}{2z_{i}} \mp [\tan \alpha_{ti} - \alpha_{ti}], i = 1,2 \]  
\[ p_{i} = r_{bi} \tan \beta_{i}, i = 1,2 \]
3.1.2. **Condition of self-locking**

The geometrical profile of self-locking gear can be generated by using direct gear design method without considering the standardized parameters utilized in conventional gear design. However, the realization of self-locking gear pairs needs some special design conditions as presented in ref. [22].

In order to make gears self-locking, the pitch point P should be located off the active portion of the contact line B1-B2 (shows in Fig.3.2). Another condition of self-locking is to have a sufficient friction angle, \( \gamma \), to deflect the force \( F' \) beyond the centre of the pinion \( O_1 \). It creates the resisting self-locking moment (torque) \( T'_1 = F'H'_1 \), \( H'_1 \) is a lever of the force \( F' \). This condition can be presented as

\[
H'_{1\text{min}} > 0 \tag{13}
\]

or

\[
\gamma > \arctan\left[\frac{1}{(1+u)\tan \alpha_W - \tan \alpha_{a_2}}\right] \tag{14}
\]

where

\[
u = \frac{n_2}{n_1},
\]

\[
\alpha_{a_2} = \arccos \frac{d_{b_2}}{d_{a_2}}
\]
3.2. Calculation of gear load capacity self-locking gears

To estimate the load capacity of spur and helical involute gear teeth, ISO Standard 6336 and AGMA Standard 2101-C95 established a common base for rating various types of gears for differing application. The analytical method presented in AGMA standard will be used to compare with the proposed finite element method.

3.2.1. Contact stress calculation

Based on the Hertzian theory, the contact stress equation of AGMA standard is given as:

$$\sigma_H = Z_E \sqrt{F_t K_a K_b K_n \frac{K_H}{2r p W Z_I}}$$  \hspace{1cm} (15)

$Z_E$ is the Elastic coefficient can be written as:

$$Z_E = \left[ \frac{1}{\pi \left( \frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g} \right)} \right]^{1/2}$$  \hspace{1cm} (16)
where \( v \) and \( E \) are the Poisson's ratio and Young's modulus, respectively. Suffix P represents the pinion and \( G \) stands for gear as shown in Fig.3.3. \( F_t \) is the transmitted tangential load applied on gear teeth. \( K_o \) is the overload factor. \( K_v \) is the dynamic factor. \( K_s \) is the size factor. \( K_H \) is the load distribution factor. \( r_p \) is the pitch radius of pinion and \( W \) is the face width.

In this study, emphasis is given to calculate the contact stress of external involute spur gears. Therefore, the geometry factor \( Z_I \) can be calculated as:

\[
Z_I = \frac{\cos \varphi \sin \varphi_t}{2m_N} \frac{m_r}{m_r+1}
\]  

(17)

where \( \varphi_t \) is the transverse pressure angle. \( m_N \) is the load-sharing ratio (\( m_N = 1 \) for spur gears). \( m_r \) is defined as the gear ratio

\[
m_r = \frac{Z_G}{Z_P}
\]  

(18)

Where \( Z_G \) and \( Z_P \) are the teeth number of gear and pinion, respectively.

**3.2.2. Bending stress of gear root**

Lewis equation is derived from the basic beam bending stress equation, which forms the basis of the AGMA bending stress equation used nowadays. The final equation of bending stress, considering the same empirical factors \( (K_o, K_v, K_s, K_H) \) used in contact stress equation, can be defined as:

\[
\sigma_b = F_tK_oK_vK_s\frac{1}{Wm_t}\frac{K_HK_B}{Y_f}
\]  

(19)

Similarly, \( F_t \) is the transmitted load and \( W \) is the face width. \( K_B \) is the rim thickness factor. \( Y_f \) is the geometry factor for bending stress.
3.3. Dynamic model of self-locking gear pair system

3.3.1. Dynamic model with one degree of freedom (1 DOF)

Fig. 3.4 shows a typical dynamic model of gear pair system with one degree of freedom (1 DOF). The gear transmission is modelled as a pair of discs, connected along the line of action by a spring and a damper.

The model takes into account the influences of the static transmission error which is simulated by a displacement excitation $e(t)$ at the mesh. This transmission error arises from several sources, such as tooth deflection under load, non-uniform tooth spacing, tooth profile errors caused by machining errors as well as pitting, scuffing of teeth flanks. The meshing stiffness $K(t)$ is expressed as a time-varying function. The gear pair is assumed to operate under high torque condition with zero backlash. Effects friction forces at the meshing interface are neglected on the basis that in
particular, the coefficient of friction is low. Furthermore, the viscous damping coefficient of the gear mesh $C$ is assumed to be constant.

Figure 3.4. 1 DOF dynamic model for a gear pair system.

The system equation of this classical model with respect to the angular rotations $\varphi_p$ and $\varphi_g$ of the pinion and gear, respectively, can be written as [58]:

$$J_p\ddot{\varphi}_p + r_p C(\dot{r}_p \varphi_p + \dot{r}_g \varphi_g + e(t)) + K_t(r_p \varphi_p + r_g \varphi_g + e(t)) = T_p$$

(20)

$$J_g\ddot{\varphi}_g + r_g C(\dot{r}_p \varphi_p + \dot{r}_g \varphi_g + e(t)) + K_t(r_p \varphi_p + r_g \varphi_g + e(t)) = -T_g$$

(21)

where $J_p$ and $J_g$ represent the rotary inertia for pinion and gear, respectively. The angular velocity is represented by $\dot{\varphi}_p$ and $\dot{\varphi}_g$ while $\ddot{\varphi}_p$ and $\ddot{\varphi}_g$ are the angular acceleration. $T_p$ and $T_g$ denote the external torques acting in the system. $r_p$ and $r_g$ represent the base radii of the gear pair. The quantity $2e(t)$ is defined as the total gear backlash.

By introducing the composite coordinate

$$q = r_p \varphi_p + r_g \varphi_g,$$  

(22)
equations (37) and (38) yield a single differential equation in the following form:

\[ m_{red} \ddot{q} + K(t)q + C\dot{q} = F(t) - K(t)e(t) - Ce(t), \]  

\[(23)\]

where: \( m_{red} = \frac{J_pJ_g}{J_p + J_g} \), and \( F(t) = m_{red} \left( \frac{T_{p}r_p}{J_p} + \frac{T_{g}r_g}{J_g} \right) \).

### 3.3.2. Estimation of transmission error

In practice, the transmission error can be measured experimentally and at different operating conditions (different loads). The transmission error is usually expressed as:

\[ TE = \varphi_p - \varphi_g \]  

\[(24)\]

Where \( \varphi_p \) and \( \varphi_g \) represent the angular rotations of the pinion and gear, respectively.

### 3.3.3. Analytical model of the time varying meshing stiffness

According to method of Cai [59], the normalised linear stiffness with respect to the mean value can be expressed as follow:

\[ K(t) = \frac{1}{0.85\varepsilon} \left[ -1.8 t^2 + 1.8 \frac{\varepsilon t}{t_z} + 0.55 \right] \]  

\[(25)\]

where \( t_z \) is the meshing period or normal pitch length of the gear pair, \( \varepsilon \) is the contact ratio and \( t \) is the instant considered.

According to ISO 6336, for spur gears with \( \varepsilon \geq 1.2 \) and helical gears with \( \beta \leq 30^\circ \) the mesh stiffness is:

\[ c_y = c'(0.75\varepsilon + 0.25); \]  

\[(26)\]

where the maximum stiffness \( c' \) can be obtained from

\[ c' = c'_{th}C_M C_RC_B \cos \beta; \]  

\[(27)\]
where \( c'_{th} \) is the theoretical single stiffness, \( C_M \) is the correction factor which accounts for the difference between the measured values and theoretical values, \( C_B \) is the gear blank factor that accounts for the flexibility of gear rims and webs, \( C_B \) is the basic rack factor accounts for the deviation of the actual basic rack profile of gear, from the standard basic rack profile, \( \beta \) is the helical angle at reference pitch diameter. On the other hand, \( c'_{th} \) can be calculated as:

\[
c'_{th} = \frac{1}{q'}
\]  (28)

where \( q' \) is the minimum value for the flexibility of a pair of teeth;

\[
q' = C_1 + \frac{C_2}{z_{n1}} + \frac{C_3}{z_{n2}} + C_4 x_1 + \frac{C_5 x_1}{z_{n1}} + C_6 x_2 + \frac{C_7 x_2}{z_{n2}} + C_8 x_1^2 + C_9 x_2^2
\]  (29)

The values of \( C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9 \) are given in Table 1.

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>( C_2 )</td>
<td>( C_3 )</td>
<td>( C_4 )</td>
<td>( C_5 )</td>
<td>( C_6 )</td>
<td>( C_7 )</td>
<td>( C_8 )</td>
<td>( C_9 )</td>
</tr>
<tr>
<td>0.04723</td>
<td>0.15551</td>
<td>0.25791</td>
<td>-0.00635</td>
<td>-0.11654</td>
<td>-0.00193</td>
<td>-0.24188</td>
<td>0.00529</td>
<td>0.00182</td>
</tr>
</tbody>
</table>

Then the linear-translation tooth meshing stiffness along the line of contact, \( K_t \), can be calculated by

\[
K_t = k(t) \ c_r \ B
\]  (30)

where \( B \) represents the tooth width of gear pairs.
CHAPTER 4
FINITE ELEMENT MODELLING OF
SELF-LOCKING GEAR PAIRS

4.1. 3D Modelling
The creation of involute gear is an essential task which determines the accuracy of final results. For this reason, a lot of efforts have been made to programme the generation of involute curve [60, 61]. However, it is inconvenient for an inexperienced user and cumbersome to modify the design parameters. Therefore, in this study, a three dimensional gear pair is initially generated in SOLIDWORKS 2013. The involute gear profile was created using Equation Driven Curve which is a standard command in SOLIDWORKS.

4.2. Accuracy of 3D models

4.2.1. Calculation and measurement over pins
To examine the accuracy level of 3D model, the over pins measurement method was used. As shown in Figure 4.1, in standard ISO/TR 10064-2: 1996, the formulae of measurement over pins are expressed as follows [62].

For external gears with an even number of teeth;

\[
M_d = \frac{m_n z \cos \alpha_1}{\cos \beta \cos a_{M_t}} + D_M
\]  

(31)

For external gears with an odd number of teeth;

\[
M_d = \frac{m_n z \cos \alpha_1}{\cos \beta \cos a_{M_t}} \cos \left(\frac{90}{z}\right) + D_M
\]  

(32)

and
\[ \text{inv } \alpha_{Mt} = \text{inv } \alpha_t + \frac{D_M}{m_n z \cos \alpha_n} + \frac{2 \tan \alpha_n \chi}{z} - \frac{\pi}{2 z} \]  

(33)

For internal gears with an even number of teeth;

\[ M_d = \frac{m_n z \cos \alpha_t}{\cos \beta \cos \alpha_{Mt}} - D_M \]  

(34)

For internal gears with an odd number of teeth;

\[ M_d = \frac{m_n z \cos \alpha_t}{\cos \beta \cos \alpha_{Mt}} \cos \left( \frac{\pi}{z} \right) - D_M \]  

(35)

and

\[ \text{inv } \alpha_{Mt} = \text{inv } \alpha_t - \frac{D_M}{m_n z \cos \alpha_n} - \frac{2 \tan \alpha_n \chi}{z} + \frac{\pi}{2 z} \]  

(36)

where

- \( M_d \) is dimension over pins, mm.
- \( M_n \) is normal module, mm.
- \( Z \) is number of teeth.
- \( \alpha_t \) is transverse pressure angle, degree.
- \( \alpha_n \) is normal pressure angle, degree.
- \( \beta \) is helix angle, degree.
- \( \alpha_{Mt} \) is pressure angle in transverse plane, degree.
- \( D_M \) is diameter of pin used for measurement, mm.
- \( \chi \) is profile shift coefficient.
Traditionally, most of the parameters in the function can be computed by using hand calculation or Excel except the $\alpha_{Mt}$. Engineers and scholars need to calculate this number with the help of involute function table. It is inconvenient and timing-consuming to look up the number in the table. In the meantime, the final data is not accurate enough for the gear system with the requirement of high precise level.
Figure 4.1. Measurement over two pins: (a) external gears with even number of teeth; (b) external gears with odd number of teeth; (c) internal gears with even number of teeth; (d) internal gears with odd number of teeth [61].

4.2.2. Calculation of $\alpha_{Mt}$

In order to avoid the drawbacks within conventional method, MATLAB is introduced to enhance the precision of calculation. Considering the involute equation is a typical nonlinear function $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$, the solution of the equation is to compute a vector $x^*$ such that

$$f(x^*) = 0$$

(37)

The convergence of this kind of function has been investigated in many books [63-65]. Three representative iterative calculation methods and one typical matrix calculation method are described as follows.
4.2.2.1 Matrix method

In order to compute the pressure angle in transverse plane, the value of $\alpha_{Mt}$ was chosen in a particular range, which varied between 0 and $\pi/2$. Then the $\alpha_{Mt}$ is segmented and the matrix of $\alpha_{Mt}$ can be written as

$$A = [a_1, a_2, \cdots, a_n]$$

(38)

Then, the value of involute function $\text{inv}\alpha_{Mt}$ is represented as

$$B = [b_1, b_2, \cdots, b_n]$$

(39)

where $b_n = \tan(a_n) - a_n, \quad n=0, 1, \ldots$

We assume the value of $\text{inv}\alpha_{Mt}$ as $\text{inv}$ because it can be directly calculated with input data. By running some MATLAB codes, we compare the value of matrix $B$ and $\text{inv}$ and then get the exact matching value of $\alpha_n$ ($n=0, 1\ldots$) in matrix $A$. Finally, we can obtain the $\alpha_{Mt}$ which could make $\text{inv}\alpha_{Mt} \approx \text{inv}$.

4.2.2.2 Newton-Raphson method

With one variable, the Newton-Raphson method is implemented as follows:

Given a function $f$ defined over the reals $x$, and its derivative $f'$, a first guess $x_0$ is inputted as a root of the function $f$. Provided the function satisfies all the assumptions made in the derivation of the formula, a better approximation $x_1$ is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

(40)

Geometrically, $(x_1, 0)$ is the intersection with the x-axis of a line tangent to $f$ at $(x_0, f(x_0))$. 
The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$  \hspace{1cm} (41)$$

until a sufficiently accurate value is reached [66]. Figure 4.2 shows the algorithm used to compute the root.
4.2.2.3 Secant method

Newton’s method requires evaluations of first order derivatives at each step [67]. In order to avoid this, the secant method is introduced instead, as follows:

\[
x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)
\]

(42)
As shown in Figure 4.3, two values $x_0$ and $x_1$ need to be initialized. Then the $x_2$ is computed. We continue this process, solving for $x_3$, $x_4$, etc., until the value reaches a sufficiently high level of precision.

Figure 4.3. Flow chart of Secant method

4.2.2.4 Bisection method
The bisection method is also applicable when solve the equation \( f(x) = 0 \), where \( f \) is a continuous function on the closed interval \([x_0, x_1]\) such that \( f(x_0)f(x_1) < 0 \), then \( f \) must have at least one root in the interval \([x_0, x_1]\). At each step the method divides the interval in two by computing the midpoint

\[
x_2 = \frac{x_0 + x_1}{2}
\] (43)

of the interval and the value of the function \( f(x_2) \) at that point. If \( f(x_2) = 0 \), then \( x_2 \) is the root and the algorithm terminates. If \( f(x_0) \) and \( f(x_2) \) are opposite signs, then the method sets \( x_2 \) as the new value for \( x_1 \), and if \( f(x_1) \) and \( f(x_2) \) are opposite signs then the method sets \( x_2 \) as the new \( x_0 \). Convergence is finished when precision level is reached\[68\]. Figure 4.4 shows the flow chart of bisection method utilized in MATLAB.
4.2.2.5 Calculation experiments and analysis

The accuracy of test gear is level 10 and the quality reference standard is ANSI/AGMA 1328-1. The gear was manufactured by molding process and parameters are listed in Table 2. Using equation 6, the inv is equal to 0.016993274579186.

Using the matrix method, the matrix was defined as $A = [0, 0.00005, 0.0001, 0.00015, \ldots, 90]$ which has a reasonable segment that could not create a computer crash. The calculation result of $\alpha_{Mt}$ was equal to 20.86139332.
With different tolerance $\epsilon$, we analyzed other three calculation methods. All results are shown in Table 3.

The numbers computed by Newton’s method and Secant method are more accurate than other two methods and the convergence speed of them is faster as well. When the tolerance $\epsilon=1.0e^{-8}$, the accuracy level of bisection method became acceptable. However, with the decrease of tolerance, system needs more number of times for converging. Matrix method is not recommended to use for this kind of calculation, because the precise level is lower. Also without an appropriate setup, the system was easy to crash.

Table 2 Parameters of gear model

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Pitch Diameter (mm)</td>
<td>38.5918</td>
</tr>
<tr>
<td>Number of Teeth</td>
<td>48</td>
</tr>
<tr>
<td>Module</td>
<td>0.8</td>
</tr>
<tr>
<td>Normal Pressure Angle (Degree)</td>
<td>14.5</td>
</tr>
<tr>
<td>Lead Angle (Degree)</td>
<td>5.714088</td>
</tr>
<tr>
<td>Lead Direction</td>
<td>RIGHT</td>
</tr>
<tr>
<td>Normal Tooth Thickness (mm)</td>
<td>1.607</td>
</tr>
</tbody>
</table>
Table 3 Result of different methods

| Tolerance $\varepsilon$=1.0e-4 | Newton’s Method 20.86229173 | Bisection Method 20.859375 | Secant Method 20.86229173 |
| Tolerance $\varepsilon$=1.0e-5 | Times of convergence 5 | 5 | 6 |
| Tolerance $\varepsilon$=1.0e-6 | Times of convergence 5 | 5 | 7 |
| Tolerance $\varepsilon$=1.0e-7 | Times of convergence 5 | 14 | 7 |
| Tolerance $\varepsilon$=1.0e-8 | Times of convergence 6 | 22 | 7 |

4.2.3. Elements selection

As ANSYS Workbench provides direct, associative, bidirectional interfaces with SOLIDWORKS, the 3D model are subsequently imported into ANSYS14.5 Workbench without losing any accuracy. There are two types of 3D brick elements in the ANSYS system, the 8-node element (brick-SOLID 185) and the 20-node element (brick-SOLID 186)(see Fig.4.1). SOLID 185 is a lower-order version of SOLID 186 that exhibits linear...
displacement behaviour whereas SOLID 186 is a higher-order version of SOLID 185 that exhibits quadratic displacement behaviour.

Figure 4.5. Different types of 3D elements: (a) SOLID 185; (b) SOLID 186

In this study, the finite element model of gear pair is created by using solid element SOLID 186, which is defined by 20 nodes having three degree freedom per node: translations in the nodal x, y and z directions. As a higher order solid element could exhibit quadratic displacement behaviour, SOLID 186 is very suitable for determining the stress stiffening, creep, large deflection, and strain capabilities[69]. In other words, SOLID 186 could increase the computation accuracy because the simulation of gear load capacity is a complex non-linear contact problem. However, it should be noted that the use of SOLID 186 may produce a very large model that requires enormous computer time for solution. Therefore, the number of elements must be controlled in a reasonable range. The selection of SOLID 186 in ANSYS Workbench consists of two steps: First, the model must be discretized using brick element. Second, the midside nodes of element must be kept as shown in Fig.4.5.
Figure 4.6. Selection method of SOLID 186.

4.3. Procedure of finite element modelling

Then the approach for finite element analysis of gear load capacity and dynamic performance can be accomplished by following key steps:

Step 1: Analysis system (see Fig.4.7)

ANSYS Workbench Platform has variable Analysis Systems for different applications. In this method, the Transient Structural Analysis (ANSYS) is utilized for the finite element analysis.
Step 2: Definition of contact type (see Fig.4.8)

Contact regions between pinion and gear are defined as frictionless and the contact element types are CONTA 174 and TARGE 170. The contact normal stiffness factor and the penetration tolerance value are 1 and 0.1, respectively.
Figure 4.8. Definition of initial contact: (a) basic contact setup; (b) advanced setup to increase the possibility of convergence.

Figure 4.9. Definition of boundary conditions for gear pair.

Step 3: Increase the possibilities of convergence (see Fig. 4.8)

Most convergence failures occurred in non-linear contact problem is mainly due to the initial gap of 3D model and the oscillation between two contact parts. Therefore, in order to establish the initial contact between pinion and gear, the interface
treatment is defined as Adjust to Touch. Moreover, Augmented Lagrange is used as
the formulation to reduce the sensitivity to the contact stiffness.

Step 4: Definition of boundary conditions (see Fig.4.9)

Pinion and driven gear hubs are connected with Body-Ground Revolute joints. That is
to say, each gear has only one rotational degree of freedom. One joint is assigned
with a very low rotational velocity while the other one has a drive moment.

![Figure 4.10. Quasi-static analysis settings.](image)

Step 5: Analysis settings (see Fig.4.10)

Inertial effects of gear pairs needs to be deactivated by turning off the Time
Integration function when the gear load capacity was calculated. The Large
Deflection effect must be turned on. A system damping value of 0.1 is used for quasi-
static analysis. If necessary, the Auto Time Stepping can be activated to accelerate
the computation process.
Step 6: Results of solution

The contact stress on tooth surfaces and the equivalent stress on tooth root can be obtained using stress analysis function. The relative rotation of pinion and gear can be obtained using a joint probe.

4.4. Numerical examples

In this section, we will compare the numerical result of quasi-static FEM with AGMA standard in order to verify our proposed methodology. Then the effects of tip radius will be investigated. A pair of involute spur gear is illustrated in Fig.4.11. All elements have the shapes of a prism or hexahedron. The total number of elements is 31740 and the total number of nodes is 143938. To reduce the computational time, five teeth are modelled on each gear body but only three teeth pairs are fine meshed with small elements for computation (see Fig.4.12). The relevant data for the gear pair are listed in the Table 4. The pinion rotates with a constant rotational velocity (0.55 rad/s) while a 276Nm torque is assigned for the driven gear. Gears start to contact with the first teeth pair and end up at the third pair (Fig.4.13). For the sake of comparison, the effects of the tooth tip relief on gear tooth contact are analysed in detail. The contact stress and bending stress are extracted from the second contact tooth on pinion.
Figure 4.11. Illustration of a spur gear pair, with torque and rotational velocity applied on hubs.
Figure 4.12. Finite element model of meshing pair: (a) general mesh of gear pair; (b) enlarged fine mesh of three teeth pairs.

Table 4 Design parameters and material properties of gear pair

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of teeth</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Module (mm)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Circular tooth thickness (mm)</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>Reference pitch diameter (mm)</td>
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<td></td>
</tr>
<tr>
<td>Outside diameter (mm)</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Root diameter (mm)</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>Root Fillet (mm)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Pressure Angle (degree)</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Centre Distance (mm)</td>
<td>138</td>
<td></td>
</tr>
</tbody>
</table>
55

Density (Kg/m³) 2770
Young’s modulus (Mpa) 69000
Poisson’s ratio 0.33

Figure 4.13. Contact process of gear pairs: (a) contact starts from the first tooth pair and ends up at the third pair; (b) the tooth of second gear pair on pinion is used to investigate the effects of tip relief on contact stress and bending stress.

4.4.1. Comparisons of quasi-static FEM results with AGMA standard

Fig.4.14 shows the comparison of contact stress between quasi-static FEM and AGMA standard. The contact stress cycles calculated by quasi-static FEM agree with AGMA standard and ref [70] very well. The maximum contact stress computed by quasi-static FEM is about 650Mpa while the number calculated by using AGMA equations is about 666.7Mpa. The difference between finite element method and AGMA standard is only 2.5%.
Regarding the comparison of bending stress, the results of FEM are more severe than that of AGMA standard as presented in Fig. 4.15. This is because the calculation equations of AGMA standard are based on a uniform beam theory. However, the profile of gear teeth is an involute curve that is more complex than a uniform beam. It is obvious that this theory is not appropriate to examine the bending stress of gear teeth. From the Fig. 4.15, it can be seen that the maximum bending stress is about 158 Mpa using analytical method while the maximum compressive stress is about 280 Mpa using FEM. It is also interesting to find that the maximum stress on the tensile area of gear tooth is 16.7% lower than that of compressive area. This is mainly due to the axial load applied on gear teeth.

Although the time-vary contact stress and bending stress obtained by quasi-static FEM demonstrate the similar tendency as AGMA standard in approach and recess process, the one tooth contact region is reduced as shown in Fig. 4.14 and Fig. 4.15. This is mainly due to the deformation and deflection of gear teeth. As a result, the incoming teeth pair started to mesh earlier than theoretical start of contact. Likewise, the outgoing teeth left contact later than the end of contact. It also should be noted that the current AGMA standard equations are only suitable to examine the tensile stress on the root of gear teeth. It is however required that the compressive stress at every contact point to be known beforehand which is critical for the bending fatigue of current gear design. Therefore, it is recommended to use the new FEM presented in this paper to calculate the exact values of time-vary load capacity.
4.4.2. Effect of tip radius on gear load capacity

Fig.4.16 shows the contact stress is about 476 Mpa when the tooth tip is not modified. With the increase of tip radius (shown in Fig.4.17), the contact stress is decreasing constantly when two teeth start to contact. It is about 190 Mpa when the tip radius is 1.00mm. It is clearly seen that the use of tip relief increase the one tooth
contact region. This is mainly due to the use of tip relief decrease the gear contact ratio.

The tip radius has very little effect on the equivalent bending stress at gear root as presented in Fig.4.18, Fig.4.19, and Fig.4.20. However, it is interesting to find that the compressive stress during recess process is increased due to the tooth modification on tip area. The potential reason of this phenomenon is because the use of tip relief leads to the change of operating pressure angle between teeth pair. As a result, the axial load on gear teeth is increased, which leads to the increase of compressive stress during recess process.

Figure 4.16. Contact stress cycles with different tip radius.
Figure 4.17. Contact stress distribution when gear teeth start to contact: (a) 0.00mm tip radius; (b) 0.50mm tip radius; (c) 0.75mm tip radius; (d) 1.00mm tip radius.
Figure 4.18. Equivalent stress on gear root with different tip radius: (a) tension; (b) compression.

Figure 4.19. Equivalent stress (tension) distribution on tooth root: (a) 0.00mm tip radius; (b) 0.50mm tip radius; (c) 0.75mm tip radius; (d) 1.00mm tip radius.
4.4.3. Conclusion and discussion

In this section, the proposed FEM for analysing the time-varying load capacity of gears has been developed using ANSYS software. According to the analytical and numerical investigations, the following conclusions may be drawn:

AGMA equations give acceptable accuracy to determine the contact stress on gear teeth. On the other hand, AGMA standard does not give good results for the
equivalent bending stress at gear root. Thus, the traditional method for determining the gear load capacity based on the AGMA equations has some limitations.

The proposed quasi-static FEM can demonstrate the time-varying load capacity of gears in a complete mesh cycle. The calculation results are more accurate than those obtained from the AGMA equations due to the consideration of gear deformation and deflection.

Designers should pay attention to the tip relief of gear teeth. The increase of tip radius can reduce the contact stress when teeth pair starts to contact. Conversely, the increase of tip radius can increase the compressive stress at gear root during the recess process.

The quasi-static FEM proposed in this section can provide a new approach for determining the time-varying load capacity of gear system. Effects of manufacturing errors and assembly errors can also be investigated using this new method.
Presented in this chapter are the results of an investigation into the effect of torque, tip radius, centre to centre distance error (CTCDE), and misaligned axes (MA) on the load capacity and transmission error of self-locking gear pair. According to the design method presented in section 3.1, a pair of self-locking gear was designed as shown in Fig. 5.1. The design parameters are illustrated in Table 5.

**Gear: 11 Teeth**

**Pinion: 6 Teeth**

Figure 5.1. Self-locking gear pair
Table 5 Parameters of self-locking gear pair

<table>
<thead>
<tr>
<th></th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>Normal module (mm)</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Normal pressure angle</td>
<td>63°</td>
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</tr>
<tr>
<td>Helix angle on the pitch diameter</td>
<td>75°</td>
<td></td>
</tr>
<tr>
<td>Transverse pressure angle</td>
<td>82.5°</td>
<td></td>
</tr>
<tr>
<td>Transverse contact ratio</td>
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<td></td>
</tr>
<tr>
<td>Axial contact ratio</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Tooth width (mm)</td>
<td>9.76</td>
<td></td>
</tr>
<tr>
<td>Centre distance (mm)</td>
<td>49.26</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus (Mpa)</td>
<td>200000</td>
<td></td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

5.1. Effect of torque

Geared devices can change the speed of a power source, creating a mechanical advantage, through their gear ratio, and thus may be considered a simple machine. Moreover, as a rotating machine part having cut teeth, the main function of gear part is to mesh with another toothed part to transmit torque constantly. Therefore, in this section, the effect of torque on gear contact stress and transmission errors are investigated.

Fig.5.2 summarizes the Von Mise stress of self-locking gears for the case of different torque loaded on gear pairs when the tip radius is 0mm. Here, all stress graphs,
regardless of torque, show a similar variation: the peak value occurs when one pair of gear teeth is in meshing. The peak value of gear stress increased from 3700MPa to about 16000MPa when the running torque increases from 27.6KNmm to 138KNmm.

Fig.5.3 summarizes the transmission error of self-locking gears for the case of different torque loaded on gear pairs. The curve of transmission error tends to be straight line when the torque is 27.6KNmm. Other curves show a similar tendency. The peak value of transmission error in whole mesh cycle increase from about 2 degrees to about 8 degrees when the running torque increases from 27.6KNmm to 138KNmm.
5.2. Effect of tip radius

Transmission error occurs when the driven gear is often momentarily ahead or behind its theoretical position in respect to the constant speed position. As presented in section 2.4, the transmission error is the main source of noise and vibration in gear system. To compensate for transmission error it is a well-established practice to apply small profile corrections to the gear teeth-often termed tip reliefs. In this section, the effect of tip radius on gear stress and transmission error are investigated. Five sorts of amount of tip relief are studied.
Fig. 5.4 presented the Von Mise stress of self-locking gear pair corresponding to the change of tip radius when a 27.6 KNmm was applied in the system. As expected, all curves present similar variation. The peak value increased from 10000MPa to about 30000MPa. However, it is interesting to find that the stress curves are similar when the tip radius are 0mm and 0.3mm. When the tip radius is larger than 0.3mm, the gear stress was increased due to the increase of tip radius.

Figure 5.4. Comparison of the Von Mise stress corresponding to the change of tip radius

Fig. 5.5 summarizes the transmission error of self-locking gears for the case of different tip radius on gear pairs. It can be seen that the tip radius lightly shifts the peak value of transmission error from below 6 degrees to about 6.5 degrees. However, it is interesting to find that the increase of tip radius has minor effect on the transmission error when the amount increases from 0.3mm to 0.6mm.
Figure 5.5. Comparison of the transmission error corresponding to the change of tip radius

5.3. Effect of CTCDE

Gear pairs may operate at a not-standard centre to centre distance to adjust space constraints, to create backlash and to accommodate for anticipated deflections under load as well as geometry changes due to thermal expansion. Since the centre to centre distance is a machined dimension and maintained within predetermined tolerances, the actual dimension may vary in different manufacturing conditions. Moreover, due to the assembly deviation, the desired centre to centre distance is practically not possible to accurately achieve or maintain. Therefore, it is significant to investigate the effect of CTCDE on gear system.

Fig. 5.6 summarized the effect of CTCDE on Von Mise stress on gear teeth. All curves present similar variation and magnitude. Similarly, as shown in Fig. 5.7, the effect of CTCED on transmission error of self-locking gear pair is minor as well.
70.3. Mechanical errors, such as tooth profile deviation, MA, system runout, or the mounting position tolerance, are unavoidable in gear systems. Among them, MA may shift the load distribution on gear teeth that result in the increase of contact and bending stresses, and the increase of transmission error of gear system. In this section, the effect of MA is evaluated.

Figure 5.6. Comparison of the Von Mise stress corresponding to the change of CTCDE

Figure 5.7. Comparison of the transmission error corresponding to the change of tip radius

5.4. Effect of MA

Mechanical errors, such as tooth profile deviation, MA, system runout, or the mounting position tolerance, are unavoidable in gear systems. Among them, MA may shift the load distribution on gear teeth that result in the increase of contact and bending stresses, and the increase of transmission error of gear system. In this section, the effect of MA is evaluated.
It can be clearly seen that the increase of gear misalignment can greatly increase the stress of gear teeth of self-locking gear pairs (shows in Fig.5.8), especially on one teeth pair meshing condition. Fig.5.9 summarized the effect of MA on transmission error of gear pairs. It can be seen that the general tendency of MA curves are similar. However, the increase of MA increase the general magnitude of transmission error which means there are more errors were introduced in the system.

Figure 5.8.Comparison of the Von Mise stress corresponding to the change of MA

Figure 5.9.Comparison of the transmission error corresponding to the change of MA
CHAPTER 6
DYNAMIC ANALYSIS OF SELF-LOCKING GEARS

There is little knowledge in literature about the dynamic performance of self-locking gears, especially in the study of dynamic responses related to the time-varying contact length. It is therefore interesting to investigate the dynamic performance of self-locking gear in relation to the contact length for the sake of filling the potential user’s knowledge gap. As an important excitation of noise and vibration, the time-varying contact length of gear pairs has been studied in many aspects. Coy [71] developed a mathematical model for surface fatigue life of gear, pinion, and entire meshing gear train. He presented that the contact length of helical gear was a critical factor to compute the maximum contact stress when two gears mating together. Maatar and Velex [72] found that the contact lengths could be expressed as Fourier series. The numerical expansions correlated very well with the results of numerical time-step simulations based on contact length discretization. Based on this Fourier series, Velex and Sainsot [73] analytically studied the tooth friction excitations caused by time-varying contact lengths. Kar and Mohanty [74] developed an algorithm to find the contact length of a single tooth to determine the time-varying frictional force and torque in helical gear system. They expanded this algorithm to investigate the bearing forces produced by variation of contact length [75]. Li et al. [76] analyzed the meshing error of every contact point along the time-varying contact lines in meshing period. It was used to predict the excitation produced by dynamic transmission error under sliding friction. He [77] divided the contact length
into different contact zones to investigate the meshing stiffness of helical gears. Jiang et al. [78] proposed a method to calculate the friction excitations and contact stiffness based on the time-varying length of contact line in helical gears. The analytical model provided a new method for the study of excitation characteristics in helical gears with tooth spalling defect.

In this chapter, instead of focusing on the detailed computation of time-varying contact length and gear mesh stiffness, the variation ratio of contact length (VRCL) in self-locking pair corresponding to transverse contact ratio and axial contact ratio is investigated using an analytical approach. As key dynamic responses, the tooth root stress, bearing force, and axial acceleration of three self-locking gear pairs with different VRCL are investigated by using transient dynamic finite element analysis (FEA). Our attempt is to find the relationship between the VRCL and these three significant dynamic responses thereby demonstrating the dynamic performance of self-locking gear pairs.

6.1. **Contact ratio and VRCL of self-locking gear pair**

Considering the material of gear pairs and the lubrication condition, most of the cases, the sliding friction coefficient $f$ is between 0.1 and 0.3. In the worst case, it requires the transverse operating pressure angle equal to $\alpha_w = 75^\circ - 85^\circ$. As a result, the transverse contact ratio $\varepsilon_\alpha$ is less than 1. Thus, the axial contact ratio needs to be selected properly to compensate the total contact ratio for the sake of achieving a smooth meshing.

The transverse contact ratio in a pair of self-locking gears is given as
$$\varepsilon_\alpha = \frac{n_1}{2\pi} \times [\tan\alpha_{a1} + \mu \tan\alpha_{a2} - (1 + \mu)\tan\alpha_w]$$

(55)

The axial contact ratio due to face advance is

$$\varepsilon_\beta = \frac{b \tan\beta}{p_t}$$

(56)

Figure 6.1. Meshing of self-locking gear showing the contact length

6.2. Variation ratio of contact length

The length of contact lines varies during the different phase when two gears engaging with each other[79]. For conventional helical gear pairs, in order to study the variation of contact length, the average contact length $L$, the minimum contact
length $L_{\text{min}}$, and the maximum contact length $L_{\text{max}}$ can be calculated by using the following relations

$$L = \frac{\varepsilon_a \varepsilon_{\beta} P_{bx}}{\cos b}$$  \hspace{1cm} (56)$$

$$L_{\text{max}} = \frac{[\varepsilon_a \varepsilon_{\beta} - \varepsilon'_a \varepsilon'_\beta + \min(\varepsilon'_a, \varepsilon'_\beta)] P_{bx}}{\cos \beta_b}$$  \hspace{1cm} (57)$$

$$L_{\text{min}} = \frac{(\varepsilon_a \varepsilon_{\beta} - \varepsilon'_a \varepsilon'_\beta) P_{bx}}{\cos \beta_b}$$  \hspace{1cm} \forall \varepsilon'_a + \varepsilon'_\beta \leq 1 \hspace{1cm} (58)$$

$$L_{\text{min}} = \frac{(\varepsilon_a \varepsilon_{\beta} - \varepsilon'_a \varepsilon'_\beta + \varepsilon'_a + \varepsilon'_\beta - 1) P_{bx}}{\cos \beta_b}$$  \hspace{1cm} \forall \varepsilon'_a + \varepsilon'_\beta \geq 1 \hspace{1cm} (59)$$

However, since the contact length of self-locking gear pairs along transverse direction is constantly smaller than the transverse base pitch as shown in Fig.6.1, equations of $L_{\text{min}}$ and $L_{\text{max}}$ must be modified. It can be expressed as following equations:

$$L'_{\text{max}} = \frac{[\varepsilon_a \varepsilon_{\beta} - \varepsilon'_a \varepsilon'_\beta + \min(\varepsilon'_a, \varepsilon'_\beta)] P_{bx}}{\cos \beta_b}$$  \hspace{1cm} (60)$$

$$L'_{\text{min}} = \frac{(\varepsilon_a \varepsilon_{\beta} - \varepsilon'_a \varepsilon'_\beta) P_{bx}}{\cos \beta_b}$$  \hspace{1cm} \forall \varepsilon'_a + \varepsilon'_\beta \leq 1 \hspace{1cm} (61)$$

$$L'_{\text{min}} = \frac{(\varepsilon_a \varepsilon_{\beta} - \varepsilon'_a \varepsilon'_\beta + \varepsilon'_a + \varepsilon'_\beta - 1) P_{bx}}{\cos \beta_b}$$  \hspace{1cm} \forall \varepsilon'_a + \varepsilon'_\beta \geq 1 \hspace{1cm} (62)$$

Based on the Equations (56), (60), (61), and (62), the VRCL of self-lock gear can be defined as

$$\lambda_{\text{min}} = \frac{L'_{\text{min}}}{L}$$  \hspace{1cm} (63)$$
\[
\lambda_{max} = \frac{L'_{\text{max}}}{L}
\]  

(64)

The difference between \(\lambda_{min}\) and \(\lambda_{max}\) can be calculated by following equation.

\[
\lambda_{\text{diff}} = \lambda_{max} - \lambda_{min}
\]  

(65)

Figure 6.2. VRCL corresponding to contact ratio.

Fig.6.2 and Fig.6.3 depict the change law of VRCL determined by different transverse contact ratio and axial contact ratio, which perform in self-locking gear pairs. As can be seen from the graphs, the change law of variation ratio has following features:

(1) \(L\) is a fixed value when the \(\epsilon_\alpha\) or \(\epsilon_\beta\) is an integer.

(2) When \(\epsilon_\alpha = \epsilon_\beta\), \(\lambda_{min}\) reaches at minimum value and \(\lambda_{max}\) reaches at maximum value.
(3) When neither $\varepsilon_\alpha$ nor $\varepsilon_\beta$ is integer, the $L$ becomes a time-varying length. It should be noted that the $L$ could not be calculated as the average of $L'_{\max}$ and $L'_{\min}$.

Figure 6.3. Three-dimensional change of VRCL corresponding to contact ratio.

6.3. Finite element modelling of self-locking gear pairs

The design parameters are shown in chapter 5. The finite element model of gear pair is created by using tetrahedral element as shown in Fig.5.1. In order to increase the computation accuracy and decrease the running time of simulation, only the elements on gear teeth are refined by using small elements. The total number of elements is 123708 and the total number of nodes is 213079.

The boundary conditions are defined as follows. The revolute joints are applied in the inner hole of both gears. The pinion has an angular velocity as a function of time shown in Fig.6.4. At the meantime, a torque relates the function's time domain is applied on the gear shown in Fig.6.5.
Influences of VRCL on dynamic performance

As discussed in the Section 6.1, the transverse contact ratio of self-locking gear pair is consistently smaller than one. Therefore, in order to compensate the total contact ratio, a proper axial contact ratio has to be designed in the self-locking gear system.

The effects of transverse contact ratio and axial contact ratio on the VRCL have been presented in Section 6.2. The influences of the VRCL on the transient meshing performance of self-locking gears are studied in the following sections.
Based on the design reported in Section 3.1, three different gear pairs are used for the study. The transverse contact ratio of these pairs is all fixed at 0.5. The gear width is adjusted to create different axial contact ratio. Typically, in helical gears, the overall contact ratio is generally between 2 and 3. As a benchmarking, the first gear pair has an axial contact ratio of 2. The axial contact ratio of second pair is 2.25 and the third is 2.5. Therefore, the VRCL of gear pairs can be calculated and the results are summarized in Table 6.

Table 6. Calculation results of VRCL in three gear pairs.

<table>
<thead>
<tr>
<th>Gear Pair Number</th>
<th>Axial contact ratio</th>
<th>$\lambda_{\text{min}}$</th>
<th>$\lambda_{\text{max}}$</th>
<th>$\lambda_{\text{Diff}}$</th>
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<tr>
<td>1</td>
<td>2</td>
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<td>0</td>
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<td>2.25</td>
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<td>1.11</td>
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<tr>
<td>3</td>
<td>2.5</td>
<td>0.8</td>
<td>1.2</td>
<td>0.4</td>
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</table>

6.5. Results

6.5.1. Gear root stress analysis

In order to investigate the effects of VRCL, two fixed nodes on pinion and gear are selected as the position to extract simulation data. The equivalent stress of gear root corresponding to different VRCL is summarized in Fig.6.6. Considerable peak values, in the rotating period of pinion and gear, occurred when the gear pairs are just in meshing. Generally speaking, the deviation of stress on pinion root is small. The peak value decreases from approximately 90Mpa to about 80Mpa. On the other hand, the root stress of gear decreases consistently from 160Mpa to just above 120Mpa. All in
all, although increasing the axial contact ratio makes the pinion and gear width thicker, only stresses on gear root present an obvious decrease during two gears in meshing.

(a1) Pinion

(a2) Gear
(b1) Pinion

Equivalent Stress (Mpa) vs Time (s)

(b2) Gear

Equivalent Stress (Mpa) vs Time (s)

(c1) Pinion

Equivalent Stress (Mpa) vs Time (s)
Figure 6.6. Comparison of the tooth root stress corresponding to the change of VRCL in three gear pairs: (a1) and (a2), $\lambda_{\text{max}}=1$, $\lambda_{\text{min}}=1$; (b1) and (b2), $\lambda_{\text{max}}=1.11$, $\lambda_{\text{min}}=0.89$; (c1) and (c2), $\lambda_{\text{max}}=1.2$, $\lambda_{\text{min}}=0.8$.

6.5.2. Bearing force analysis

Fig.6.7 shows the bearing force on pinions and gears obtained by FEM of three gear pairs. Note that the data labels has been used to better visualize the peak force values of three gear pairs. Also, it should be noted that although the magnitude of bearing force produced between pinion and gear is identical, the force direction are totally reversed. It is obvious that the bearing force increases dramatically with the increase of $\lambda_{\text{max}}$ and the decrease of $\lambda_{\text{min}}$. 
6.5.3. Axial acceleration analysis

The axial acceleration curves are shown in Fig. 6.8. The abscissa axis refers to time and the vertical axis is the amplitude of the axial acceleration. Two peak values could be observed in the time interval of about 0.1 second on pinions. The maximum acceleration of first pinion is just over 4000mm/s$^2$, while the other two designs show larger increase in their rotating period. On the other hand, the maximum acceleration produced on first gear is around 10000mm/s$^2$, while the acceleration of
second gear increases to about 14000mm/s$^2$ and the third soars to about two times as large as the first one.
Figure 6.8. Comparison of the axial acceleration corresponding to the change of VRCL in three gear pairs: (a1) and (a2), \( \lambda_{\text{max}}=1, \lambda_{\text{min}}=1 \); (b1) and (b2), \( \lambda_{\text{max}}=1.11, \lambda_{\text{min}}=0.89 \); (c1) and (c2), \( \lambda_{\text{max}}=1.2, \lambda_{\text{min}}=0.8 \).

### 6.6. Discussion

The change of VRCL causes the change of the dynamic performance of three gear pairs. However, the dynamic responses corresponding to the change of VRCL show different influences on three models. It was expected that the increase of \( \lambda_{\text{Diff}} \) could lead to the increase of gear root stress. However, it is interesting to find that the equivalent stress on both gears demonstrates negative correlation corresponding to the increase of \( \lambda_{\text{Diff}} \) (shown in Fig. 6.9). In other words, no negative effect is found due to the increase of \( \lambda_{\text{Diff}} \) by considering the equivalent stress only. Part of the reason is that the element stress is reduced due to the gear pair width becoming thicker. Apart from this, it should be noted that in gear system design, it is expected that the teeth stress of pinion is smaller than gear due to the rotational speed differences[80]. As shown in the Fig. 6.10, all these gear pairs have this desired feature. The third pair shows the best performance in respect to design requirement of gear stress.

For the bearing force, the peak value increases dramatically with the increase of \( \lambda_{\text{Diff}} \) (that of minimum variation ratio \( \lambda_{\text{min}} \) decreases from 1, 0.89 to 0.8 and that of maximum variation ratio \( \lambda_{\text{max}} \) increases from 1, 1.11 to 1.2). In other words, when the axial contact ratio is not an integral multiple of base axial pitch, the bearing force demonstrates strong positive correlation with the increase of \( \lambda_{\text{Diff}} \) as shown in Fig. 6.11.
The increase of $\lambda_{Diff}$ is observed to increases the maximum acceleration of driven gear in axial direction, which is expected to result in considerable increase of the instability of the self-locking gear pairs. However, the maximum axial acceleration of pinion demonstrates different trend compared with driven gear. The obtained results show an obvious decrease in the third gear pair, which is an interesting finding in this study.

It is suggested that the use of self-locking design needs to be considered comprehensively. At the same time, in order to balance the different design requirements, some proper trade off needs to be considered. The detailed discussion to achieve an optimized trade off will be discussed in our future work. Also, studies using different gear pairs (gear pairs with different transverse contact ratio and axial contact ratio), and different setup of boundary conditions will further expand the results of this research.
Figure 6.9. Comparison of the maximum tooth stress generated on pinion and gear corresponding to the increase of $\lambda_{\text{Diff}}$.

Figure 6.10. Comparison of the maximum bearing force generated in three gear pairs corresponding to the increase of $\lambda_{\text{Diff}}$. 
Figure 6.11. Comparison of maximum axial acceleration generated on pinions and gears corresponding to the increase of $\lambda_{\text{Diff}}$.

6.7. Summary
This chapter presented a comprehensive study about the dynamic performance of self-locking gear pairs corresponding to the change of VRCL. The main objective of this research was to provide important knowledge to the potential user of self-locking gears. For this purpose the tooth root stress, bearing force, and axial acceleration of three self-locking gear pairs are investigated by using transient dynamic finite element analysis (FEA). According to obtained FEA results, the increase of $\lambda_{\text{Diff}}$ reduced the tooth root stress but increased the bearing force significantly. The axial acceleration of these three gear pairs was increased due to the increase of $\lambda_{\text{Diff}}$. However, the growth of acceleration on pinion and driven gear demonstrated different trend which is an interesting finding in this thesis. The obtained analysis results can be used as criteria for proper selection and application of self-locking gears.
CHAPTER 7
SELF-LOCKING GEAR DESIGN IN SEAT HEIGHT ADJUSTERS

As mentioned in the section 1.1, the stability of seat cushion both in static and vibration conditions is significant due to the safety consideration. Therefore, priority must be given to the strength of self-locking gear in the design stage. As shown in Fig.1.1 and Fig.1.3, there are three stages in seat height adjusters system. In this chapter, the methodology to clarify the torque range of self-locking gear pairs in seat height adjusters are presented.

7.1. Effect of seat structure

Many car makers have added the height adjustment devices (power actuator is used in luxury cars and manual pump is used in most of the compact cars) in their seat structure design. On the one hand, it improves the sitting comfort and flexibility of the seat system, but on the other hand, the adjustable height seat system creates more design complexity. In the first place, the seat height adjustment affects the motor vehicle driver's eyellipse, which is used to facilitate design and evaluation of vision in motor vehicles. In the second place, the seat height adjustment affects the H-point position which has major ramifications in the overall vehicle design.

The objective of this section is to study the influence of seat pan structure design on the seat adjustment ranges and the dynamic wrist torque variation of the height adjustment wheel (HAW), which are two significant factors impact the torque range of self-locking gear. In this section, we present a mathematical model of the
dynamics of seat pan movement and evaluate the wrist torques required for driving
the HAW and the adjustment ranges of H-point in horizontal direction and vertical
direction.

The nomenclature used in this section are listed as follows:

1 - Seat track
2 - Support link
3 - Bracket
4 - Geared link
5 - Driving pinion
6 - Sector of Geared link

\( \theta_1, \theta_2, \theta_3, \theta_4, \theta_5 \) – Horizontal angles of parts with ground

a\(_1\) – Angle from teeth boundary to down stop position

a\(_2\) – Angle from design position to down-stop position

a\(_3\) – Angle from design position to up-stop position

a\(_4\) – Adjustable angle of linkage (Min = -a\(_2\), Max = a\(_3\))

V – Vertical height of H-point

\( \phi \) – Eye vision angle

F – Occupant load on cushion
7.1.1. Description of seat pan structure

A seat pan structure was originally designed to support the occupant of a vehicle during driving. In modern product, a seat pan structure functions to create more flexibility by adding multi-way adjustment devices into the seating system. In this section we describe the fundamental operating principle of a geared four-bar linkage of automotive seat pan, which consists of the driving pinion, geared-link, support link, bracket, and seat track. Fig.7.1 shows the 3D layout and partial definitions of the geared-four bar linkage system. As shown in Fig.7.1, the system is based on the seat track, which is mounted on the vehicle body. The support link and the geared-link are pivoted to the seat track. The bracket is connected to the geared-link and support link with tube. The driving pinion is mounted on the bracket. When drivers adjust the HAW, the pinion is rotated about the axis P. The geared-link has a rocker
motion around the centre of axis D. Then the motion is transmitted to the whole four-bar linkage. Eventually, the seat cushion could be lifted up and down to adjust the position of H-point, thereby adjusting the eye vision of the occupant.

Figure 7.1. Passengear car seat structure

However, there is a limited range of angles for the linkage system. Fig.2 shows a two-dimensional schematic illustrating the operating principle of the geared four-bar linkage. The original design position of the linkage system is constrained by $\theta_1, \theta_2, \theta_3, \theta_4$ in static condition. When the linkage system was driven by occupant, the movement range of the geared-link could not exceed the up-stop position and down-stop position. $a_2$ is the angle from the design position to down-stop position. $a_3$ is the angle from the design position to up-stop position. $a_1$ is the angle from teeth boundary to down stop position.

7.1.2. **Force analysis of geared four-bar linkage**

In order to analyse force acting on the linkage system, angles between different parts need to be calculated firstly. Considering in actual engineering design, $L_1, L_2, L_3, L_4, \theta_3, \theta_4, a_1, a_2, a_3$ and $\angle PCB$ will be defined as original design parameters, the operation torque of driving pinion can be expressed as:
\[ T = \frac{Z_T}{Z_{MC}} \times F \times L_4 \times (\cos(\Theta_2) + \left( \frac{\cos(L_{csx}(\sin(\Theta_2) - \tan(\Theta_1) \times \cos(\Theta_2))}{L_3 \times (\tan(\Theta_1) \times \cos(\Theta_3) + \sin(\Theta_3))} \right)) \]  

(66)

### 7.1.3. Path of the H-point

Because the path of load point S is a simple planar question, the geared four-bar linkage can be simplified as a typical four-bar linkage, which is shown in Fig.3. The point S is a point on the coupler BC. The basic four links has the following vector relationship:

\[ \overrightarrow{L_1} + \overrightarrow{L_4} = \overrightarrow{L_2} + \overrightarrow{L_3} \]  

(67)

Projecting the (23) along axis X and axis Y, the equation becomes

\[
\begin{align*}
(L_1 \times \cos(\Theta_4) + L_4 \times \cos(\Theta_2) &= L_2 \times \cos(\Theta_1) + L_3 \times \cos(\Theta_3) \quad (68) \\
(L_1 \times \sin(\Theta_4) + L_2 \times \sin(\Theta_1) &= L_4 \times \sin(\Theta_2) + L_3 \times \sin(\Theta_3) \quad (69)
\end{align*}
\]

Then, the coordinates of the point A, point B, point C, and point D are determined by the expressions

\[
\begin{bmatrix}
A_x \\
B_x \\
C_x \\
D_x
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & L_1 \\
0 & \cos(\Theta_1) & 0 & 0 & L_2 \\
0 & \cos(\Theta_1) & \cos(\Theta_3) & 0 & L_3 \\
\cos(\Theta_4) & 0 & 0 & 0 & L_4
\end{bmatrix}
\]

(70)

\[
\begin{bmatrix}
A_y \\
B_y \\
C_y \\
D_y
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & L_1 \\
0 & \sin(\Theta_1) & 0 & 0 & L_2 \\
-\sin(\Theta_4) & 0 & 0 & \sin(\Theta_2) & L_3 \\
-\sin(\Theta_4) & 0 & 0 & 0 & L_4
\end{bmatrix}
\]

(71)

Finally, the position of the point S moving along the desired path is given by the following equations:
\[
\begin{bmatrix}
S_x \\
S_y
\end{bmatrix} = \begin{bmatrix}
0 & -\sin(\theta_4) \\
\cos(\theta_1) & 0 \\
\cos(\theta_3) & 0 \\
0 & \sin(\theta_2) \\
-\cos(\theta_5) & \sin(\theta_3)
\end{bmatrix}^T \begin{bmatrix}
L_1 \\
L_2 \\
L_3 \\
L_4 \\
L_5
\end{bmatrix}
\] (72)

Evaluation of seat structure

Using the equations (66) and (72), the evaluation method described above was used to study the seat adjustment ranges and dynamic wrist torque of HAW. According to the definition of vehicle classification in Euro market, cars can be divided into nine segments. In this study, one A-segment car seat, two B-segment car seats, two C-segment car seats, two M-segment car seats and one J-segment car seats were selected for analysis and assessment. To obtain design data, the pan structure of these seats were scanned by using a small coordinate measurement machine (CMM). The CMM had a Quill-Mounted M8 motorized probe head. The system was controlled by a DC240C controller and the data was collected by PC-DMIS CAD, which is a commercial CMM assistant software. The parameters of these eight seat pan structures were listed in the table 7. The seat pan structures were selected from eight vehicle manufacturers. The pan structures were all designed with geared four-bar linkage system and they are the up to date platform products of each vehicle company. In this section, the reader will note that the seat pan structures are distinguished using the letters A through H. There is no name on seat pan structure because no permission was granted by the vehicle manufacturers.

The seats were designed variably by different automotive seat companies. The accurate load point and H-point of every seat pan structure is impossible to be obtained for this research. Therefore, in order to compare the wrist operation
torque and lift distance fairly, the eight seat cushions were analysed with similar set-up as follows:

- The friction and clearance in the linkage system were not taken into account.
- The length of \(L_5\) is 151.3275mm and \(\theta_5\) is 7.595°.
- The teeth number of gear \(Z_T\) is 8.
- The load on point \(H\) is 1000N (95th percentile male).
- The position of load point is identical with \(H\)-point in this study.

Table 7 Design parameters of geared four-bar linkage

<table>
<thead>
<tr>
<th>Cushion</th>
<th>A-segment</th>
<th>B-segment</th>
<th>C-segment</th>
<th>MPV</th>
<th>SUV</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD((L_1))</td>
<td>291</td>
<td>291</td>
<td>295.3</td>
<td>278</td>
<td>309.1</td>
</tr>
<tr>
<td>AB((L_2))</td>
<td>80</td>
<td>80</td>
<td>78</td>
<td>77</td>
<td>90</td>
</tr>
<tr>
<td>BC((L_3))</td>
<td>316</td>
<td>316</td>
<td>318.4</td>
<td>292.5</td>
<td>329.9</td>
</tr>
<tr>
<td>CD((L_4))</td>
<td>97</td>
<td>97</td>
<td>95</td>
<td>87</td>
<td>100</td>
</tr>
<tr>
<td>(a_1)</td>
<td>19</td>
<td>21</td>
<td>21</td>
<td>31</td>
<td>19</td>
</tr>
<tr>
<td>(a_2)</td>
<td>17.5</td>
<td>19</td>
<td>26</td>
<td>14.5</td>
<td>11</td>
</tr>
<tr>
<td>(a_3)</td>
<td>29.5</td>
<td>33.5</td>
<td>27</td>
<td>29.5</td>
<td>38.5</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>39.941</td>
<td>41.846</td>
<td>43.897</td>
<td>38.686</td>
<td>40.037</td>
</tr>
<tr>
<td>(\theta_2)</td>
<td>27.62</td>
<td>29.8</td>
<td>33.4</td>
<td>31.21</td>
<td>27</td>
</tr>
<tr>
<td>(\theta_4)</td>
<td>4</td>
<td>5.501</td>
<td>7.296</td>
<td>3.774</td>
<td>6.688</td>
</tr>
<tr>
<td>(\angle PCB)</td>
<td>8.9</td>
<td>10.2</td>
<td>13.6</td>
<td>14.3</td>
<td>3</td>
</tr>
<tr>
<td>(Z_{MC})</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>64</td>
<td>64</td>
</tr>
</tbody>
</table>
7.1.4. Result

Fig. 7.2 presents the operation torque from lowest position to highest position when people adjusted the seat pan structure. The smallest torque at lowest position is 7Nm (Pan G) and the largest is about 10.5Nm produced by Pan H. The operation torque of Pan G, decreases from 7Nm to about 3Nm, is consistent lower than other seat pan structures. It was interesting to note that all wrist operation torque curves performed parabolic trajectory except Pan H. The torque curve of Pan H dropped sharply from 10.5Nm to approximately 4 Nm, which performed a smaller operation torque.

![Figure 7.2. Comparison of operation torque](image)

From the Fig. 7.3, readers could find that Pan D gave a narrow distance (about 50mm) for occupant to adjust while Pan F brought 85.3mm adjustable range at vertical direction, which is the widest among these eight designs. At the horizontal direction, shown in Fig. 7.4, there is about 73.5mm displacement of Pan H, whereas the design of Pan B only gave 44.1mm space for the adjustment of customer.
7.2. Effect of plastic gears

Plastic gears are usually used in the third stage of automotive seat height adjusters as shown in the Fig.1.3. As the plastic gears may withstand some load produced by crash test, the accurate calculation of ultimate shear strength could influence the
safety performance of seat adjuster. Moreover, plastic gears not only lead to a reduction in weight, noise, and cost, but also bring numerous benefits, such as self-lubrication and chemical resistance. However, compared to the wide utilization of plastic gears in technical areas, the basic knowledge of the plastic gears does not satisfy the increasing requirements from engineers and scientists. Especially, when it comes to the ultimate static shear strength when plastic gears mating with steel worm. In this section a method is presented to calculate the maximum static shear strength of plastic gears when mating with a steel worm. Shear mechanisms of plastic helical gears was analysed. Maximum static shear strength of gears was tested by using different plastic material. The test results were compared with theoretical calculation.

The nomenclature used in this section are listed as follows:

\[ R_r = \text{root radius} \]

\[ R_o = \text{outer radius} \]

\[ R_b = \text{base radius} \]

\[ l = \text{length of contact area} \]

\[ f = \text{tooth width} \]

\[ T_s = \text{shear torque} \]

\[ F_{\text{max}} = \text{max shear force} \]

\[ \Sigma = \text{surface} \]
C = curve

LC = centre to centre distance

LS = distance from shear area to gear centre

B = lead angle

a = pressure angle

S = area of surface

n= number of teeth in (full) contact

t= tooth thickness (mm)

d= pitch diameter

τ= shear strength

σy= yield strength at design temperature (Mpa)

S= 1.3-1.5 (safety factor needs to be applied if the material to be used is ether ZYTEL nylon resin or DELRIN 100)

7.2.1. Meshing mechanism of a helical gear and a worm

The mesh between helical gear and worm is different compared with worm-worm wheel pairs and helical-helical gear pairs. Fig.7.5 shows a helical gear meshed with a worm. In the normal plane (shown in Fig.7.6), the mesh is identical with the mating of rack and gear. In order to create a smooth running and increase the strength of helical gear, the teeth width (f) of helical gear is normally larger than the length (l) of contact area (A).
7.2.2. Calculation of plastic helical gear static shear strength

4.2.2.1 Calculation principles of DuPont

From the design principles of DuPont, it indicates that in some applications, the gear strength may be limited by the shear strength of the loaded teeth, as given by the equation:

\[ T_s = d \times F_{\text{max}} \]  \hspace{1cm} (73)
where

\[ F_{max} = n f t \tau. \]  

(74)

The shear strength can be calculated by the equation:

\[ \tau = \frac{\sigma_y}{(1.7 S)} \text{ (Mpa).} \]  

(75)

Figure 7.7. Shear area of DuPont’s principles

Fig. 7.7 shows the shear model of the plastic gear. From the theory of DuPont, it indicates that the failure area will be happened at the shadow area B, which is a rectangular area. The length of the rectangle is tooth width f and the width is tooth thickness t.

4.2.2.2 Proposed calculation method

The equation of DuPont is an accurate method to calculate the shear strength of gear-gear pairs. But for worm-helical gear pairs, it is not a proper equation. Fig. 7.8 shows when plastic gears were sheared by a steel worm, the failure area on each tooth is a surface Σ, which intersected by the worm outer diameter cylinder.
surface $\Sigma_1$ of worm, two involute surfaces of gear tooth $\Sigma_2$ and $\Sigma_3$, and the gear outer diameter cylinder surface $\Sigma_4$.

![Diagram](image)

**Figure 7.8.** Shear area of worm and helical gear

The surface $\Sigma_1$ can be represented by the following equation:

$$y^2 + (z - LC)^2 = Ra_1^2.$$  \hfill (76)

As shown in Fig.4, the gear tooth surface $\Sigma_2$ and $\Sigma_3$ are involute surfaces, and the equation can be represented in the coordinate system as follows

$$x = Rb_2 \cdot (\cos a + a \sin a).$$  \hfill (77)

$$z = Rb_2 \cdot (\sin a + a \cos a).$$

The equation of surface $\Sigma_4$ can be written as

$$x^2 + z^2 = R_o^2.$$  \hfill (78)

As the $C_1$ and $C_3$ are the intersection lines of surface $\Sigma_1$ and surface $\Sigma_2$, then the equations can be represented as follows

$$\begin{cases}
  y^2 + (z - LC)^2 = R_{o_1}^2 & (C1: x>0, C3: x<0), \\
  x^2 + z^2 = R_{b_2}^2(1 + a^2)
\end{cases}$$  \hfill (79)
Similarly, the equations of C2 and C4 can be written as

\[
\begin{align*}
\begin{cases}
y^2 + (z - LC)^2 = R_{o1}^2 \\
x^2 + z^2 = R_{o2}^2
\end{cases}
\end{align*}
\tag{80}
\]

(C2: \(y>0\), C4: \(y<0\)).

According to the surface area equation, the area of surface \(\Sigma\) satisfies

\[
S = \iint_R \sqrt{\left(\frac{\partial x}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial y}\right)^2 + 1} \, dA,
\tag{81}
\]

Where

\[
\frac{\partial z}{\partial x} = 0, \quad \frac{\partial z}{\partial y} = \frac{y}{LC-z}
\]

Then, the equation can be written as:

\[
S = \iint_R \sqrt{\left(\frac{x}{LC-z}\right)^2 + 1} \, dA.
\tag{82}
\]

Finally, the equation of surface area on one tooth can be represented as follows

\[
S = \int_{c2}^{c4} \int_{c3}^{c1} \sqrt{\left(\frac{x}{LC-z}\right)^2 + 1} \, dx \, dy.
\tag{83}
\]

The total shear area of gear is determined by the number of teeth in full contact. So the whole shear area of gear is

\[
ST = \sum_{i=0}^n S_i.
\tag{84}
\]

Consequently, the proposed equation of gear max shear torque can be represented as follows

\[
T_s = ST \times \tau \times LS.
\tag{85}
\]
7.2.3. Gear static shear strength test

As shown in the Fig.7.9, the fixture supplies a fixed centre to centre distance between worm and helical gear. The screw fitted with gear internal geometry. The shear ability of gear was obtained by applying torque on screw.

![Test setup](image)

Figure 7.9. Test setup

Two pairs of worm and gear were used for the test as shown in Table 8. The PA66 GF43% and PA66 CF40% were used as the material of plastic gears. The material property and moulding parameters are shown in Table 9 and Table 10 respectively.
Table 8. Parameters of worms and gears

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th></th>
<th>B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Worm</td>
<td>Gear</td>
<td>Worm</td>
<td>Gear</td>
</tr>
<tr>
<td>Reference Pitch Diameter (mm)</td>
<td>7.3034</td>
<td>32.3489</td>
<td>8.035</td>
<td>38.5918</td>
</tr>
<tr>
<td>Normal Tooth Thickness (mm)</td>
<td>0.8</td>
<td>1.4</td>
<td>0.947</td>
<td>1.6</td>
</tr>
<tr>
<td>Outer Diameter (mm)</td>
<td>9.15</td>
<td>33.55</td>
<td>9.75</td>
<td>40.68</td>
</tr>
<tr>
<td>Root Diameter (mm)</td>
<td>6</td>
<td>30.25</td>
<td>6</td>
<td>36.54</td>
</tr>
<tr>
<td>Tooth width (mm)</td>
<td>N/A</td>
<td>8</td>
<td>N/A</td>
<td>8</td>
</tr>
<tr>
<td>Number of Teeth</td>
<td>1</td>
<td>46</td>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td>Module</td>
<td>0.7</td>
<td></td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Pressure Angle (Degree)</td>
<td>14.5</td>
<td></td>
<td>14.5</td>
<td></td>
</tr>
<tr>
<td>Axial Pitch (mm)</td>
<td>2.209</td>
<td></td>
<td>2.526</td>
<td></td>
</tr>
<tr>
<td>Lead Angle (Degree)</td>
<td>5.5°</td>
<td></td>
<td>5.7°</td>
<td></td>
</tr>
<tr>
<td>Lead Direction</td>
<td>Right Hand</td>
<td></td>
<td>Right Hand</td>
<td></td>
</tr>
<tr>
<td>Shaft Angle (Driver and Driven) (Degree)</td>
<td>90°</td>
<td></td>
<td>90°</td>
<td></td>
</tr>
<tr>
<td>C to C distance (mm)</td>
<td>20</td>
<td></td>
<td>23.58</td>
<td></td>
</tr>
<tr>
<td>Teeth In Full Contact</td>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Three combinations with different design parameters (shown in Table 8) and material (shown in Table 9) were used for testing, which were A1, A2 and B1. Each sort of test was replicated three times.
Table 11. Comparison of test and theoretical calculation (Dry as mold, 23°C)

<table>
<thead>
<tr>
<th>Test (Nm)</th>
<th>A1</th>
<th>A2</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>25.0</td>
<td>35.5</td>
<td>50.1</td>
</tr>
<tr>
<td>#2</td>
<td>25.0</td>
<td>35.5</td>
<td>51.3</td>
</tr>
<tr>
<td>#3</td>
<td>27.0</td>
<td>34.0</td>
<td>49.5</td>
</tr>
<tr>
<td>Average</td>
<td>25.7</td>
<td>35.0</td>
<td>50.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Theoretical Calculation (Nm)</th>
<th>DuPont</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>34.3</td>
<td>26.7</td>
</tr>
<tr>
<td></td>
<td>43.4</td>
<td>33.8</td>
</tr>
<tr>
<td></td>
<td>70.0</td>
<td>51.4</td>
</tr>
</tbody>
</table>

From the results shown in Table 11, the deviations between test and proposed calculation are less than 4%, while the deviations between DuPont and experimental test are more than 20%. The deviation between B1 and DuPont even reaches about 30%, which could not be accepted in engineering design.

7.3. Summary

In this chapter, a mathematical model was presented to evaluate the seat adjustment range and the adjustment torque of HAW. The vehicle designers can use this model to determine the parameters of adjustable seat pan structure to ensure the drivers of varying size can comfortably reach their preferred driving position. The model can also be used to estimate whether the adjustment torque of HAW exceeds the strength demands for different populations related to anthropometry. This proposed mathematical model can be used to clarify the maximum load in self-
locking planetary gear system. Gear teeth load capacity and dynamic behaviours under this load can be investigated.

In this chapter, a mathematical model to calculate the maximum shear stress of plastic gear was also presented. Comparing the mathematical model and experimental test, it can be seen that the proposed calculation method is suitable for gear shear strength calculation with different geometry and material. The equation of DuPont is not a proper method to calculate the shear strength of worm and helical gear pairs. Compared with the calculation of DuPont, the proposed calculation is quite match with the actual test result and the deviation is within 2 Nm. The accuracy of calculation is increased. This model can be used to clarify the maximum allowable load applied in worm and helical gear system. As a result, the minimum load in the self-locking planetary gear system can be clarified. Gear teeth load capacity and dynamic behaviours under this load can be quantified.
The self-locking gear has great potential application in controlling the position stability of gearbox, which is a critical requirement in some precision machineries and instruments. However, the design procedure, load capacity, and dynamic behaviours of self-locking gear has not been investigated and systemized, which is a great barrier for the potential users. This research focuses on addressing these issues by developing new mathematical models and finite element method. Moreover, in this PhD project, we attempt to integrate the self-locking gear into seat height adjuster to eliminate the back-driving thereby improving the occupant driving comfort, enhance the driving safety, and reduce the volume of gearbox. The obtained results provide significant knowledge for predicting the static and dynamic performance of self-locking gear pairs, optimizing their design parameters, and diagnosing possible design errors in self-locking gear pair design.

Overall, this thesis achieved four main areas of contribution:

1. The development of a finite element method to investigate the static and dynamic characteristics of self-locking gear.
2. The analysis of the load capacity of the self-locking gear pairs subjected to the torque, tip radius, centre to centre distance error and misalignment axes.
3. The analysis of the non-linear dynamic behaviour of self-locking gear system subjected to the variation ratio of contact length.
8.1. Outcomes of new developed finite element method

By using ANSYS Workbench, a general CAD-FEM integrated approach for determining the static and dynamic performance of self-locking gears were developed. The effectiveness of the proposed methodology was validated by using a pair of involute gear. It is also verified that the distribution of stress within the body of a gear tooth are not satisfied with Saint-Venant’s principle due to the complex geometry of gear. Therefore, the use of mathematical expressions developed in the AGMA and ISO standard is not capable of returning accurate value for the gear stress analysis. In short, this finite element approach can provide a reliable pre-assessment of the strength capacity of self-locking gear pairs.

8.2. Outcomes of quasi-static analysis of self-locking gears

By using the proposed numerical method, the stress distribution and the transmission errors of self-locking gears corresponding to the change of rotation angle were quantified. The torque applied on the gear pairs has great influence on the Von Mise stress and transmission errors of self-locking gears. Therefore, it is recommended that the self-locking gear pairs should be run in relative lower load condition. The use of tip radius increase the gear stress and transmission error in self-locking gear pairs. Therefore, this kind of profile modification is no need for self-locking gear, which is cost-effective. The effects of center to center distance error and misalignment axes are insignificant. Therefore, self-locking gear are not sensitive to the mechanical errors.
8.3. Outcomes of dynamic analysis of self-locking gears

Countless papers have investigated the dynamic performance of conventional gear by studying the time-varying contact length and time-varying mesh stiffness between gear pairs. However, these factors are variable with the change of design parameters. In this thesis, instead of focusing on the detailed computation of time-varying contact length and gear mesh stiffness, the variation ratio of contact length (VRCL) in self-locking pair corresponding to transverse contact ratio and axial contact ratio is investigated using an analytical approach. As key dynamic responses, the tooth root stress, bearing force, and axial acceleration of three self-locking gear pairs with different VRCL are investigated by using transient dynamic finite element analysis. The dynamic performance of self-locking gear pairs were clarified by quantified the relationship between the VRCL and these three significant dynamic responses.

8.4. Design method of self-locking gear design in seat height adjuster

This research assumes that the self-locking gear is going to be used in a seat height adjuster system. The self-locking gear parameters used in this research is limited by the torque range of seat height adjuster, because the seat height adjuster is coupled with the automotive seat cushion. In this thesis, two mathematical models were developed for investigating the effects of seat structure and plastic gears. These models were proven as an effective way to determining the torque range generated on self-locking system.

8.5. Recommendations for future work

The following areas are noted as being worthy of extending the present work.
(1) Although the proposed numerical approach has relative higher calculation accuracy compared with current analytical methods, the computational efficiency could be improved through optimizing the finite element model of gear pairs in the future research.

(2) Effect of gear tooth flank microgeometry (root relief, lead crowning, and involute crowning) on gear load capacity could be investigated in the future work.

(3) Chapter 6 showed the dynamic performance of self-locking gear pairs corresponding to the change of VRCL. However, highly detailed gear stiffness were not developed in this thesis. Therefore, as one of the most important parameters to investigate the dynamic performance of self-locking gear system, the development of time-varying meshing stiffness is set as the first challenge to be resolved by future work.

(4) Further experiments should be conducted to validate the load capacity and dynamic performance of self-locking gear system. The selection of proper lubrication condition on self-locking gear pair is seen as the second challenge for further research.


[77] S. He. Effect of sliding friction on spur and helical gear dynamics and vibro-acoustics: The Ohio State University; 2008.