Optimal Design Techniques applied to Small Islanded Energy Systems

A Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

Mitchell Lennard

School of Engineering
College of Science Engineering and Health
RMIT University

October 2016
Declaration of Authorship

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the Thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Mitchell Lennard

October 2016
The Thesis explores the optimisation of Small Islanded Energy Systems (SIES). This work defines a small energy systems as one that is designed to meet Electrical and Hot Water loads in the range of 5 to 250 kWhrs per day. The process of optimisation is defined as the process of establishing the design configuration (size of generating arrays, storage and generator size) and operating rules such that the long term Cost of Energy (CoE) together with system CO₂ emissions are minimised.

The early sections of the thesis reviews existing techniques for energy system optimisation and explores advantages and disadvantages of different techniques. Through this process a question emerged regarding the ability of existing techniques to address the impact of consecutive days of incident solar energy on system designs. A concept is developed that this question of consecutive incident energy days could be addressed by a two stage design optimisation where the first stage is based around optimising the system design for the Modal day of incident energy and then iterating that solution based on the probability of occurrence of a particular incident energy on the days following the baseline Modal day. A review of suitable mathematical techniques to execute the suggested approach is conducted and a technique from the discipline of Operations Research (OR) referred to as Stochastic Programming with Recourse (SPwR) is nominated as a likely possible methodology.

The ability of this combination of a two stage optimisation, based around a baseline Modal day of incident energy executed using Stochastic Programming with Recourse together with Probability Distribution Functions of incident solar energy to produce optimal solutions is explored using a range of increasingly complex systems. The work concludes that the technique can be used to optimise the design of small energy systems to meet both electrical and hot water loads. The technique developed allows pre-processing of weather data accommodates load variation with weather and allows consideration of a range of commercial issues associated with grid connection of SIES.
Acknowledgements

Firstly thanks to Professor Xinghuo Yu for accepting me as a student and patiently guiding me through the early days of this work.

Thanks to Dr Roger Hadgraft for showing me that there is more than one way to approach research and teaching.

Thanks to Dr Abhijit Date for stepping in to guide me through to the bulk of the work, for his thoughtful questions and for showing me how to work in an academic environment.

Thanks to Lousie Lennard for her exceptional editing and attention to detail.
# Contents

Acknowledgements vii

1 Introduction 1
   1.1 Commercial Advantages of Small Islanded Energy Systems (SIES) 4
   1.2 Environmental Advantages of Small Islanded Energy Systems (SIES) 8
   1.3 The Role of Design Optimisation in the Development of SIES 8
   1.4 Research Questions 11
   1.5 Document Outline 11

2 Literature Review 13
   2.1 Introduction to Literature Review 13
   2.2 Approaches to Energy System Optimisation 14
      2.2.1 General Optimisation with Energy Systems as an Example 14
      2.2.2 Energy System Specific Optimisation 19
      2.2.3 Building on the Existing Approaches 23
   2.3 Mathematical Techniques to Accommodate the Run of Days Methodology 28
      2.3.1 Stochastic Programming Approaches 30
      2.3.2 Recourse Models and Problem Formulation 31
      2.3.3 Stochastic Programming and Recourse Models in Energy System Analysis 34
   2.4 Literature Review Summary 36

3 Stochastic Resource Optimisation - The Basic Methodology 37
   3.1 Basic Method - Introduction 37
   3.2 Basic Sample System and Form of the Objective Function 39
   3.3 Incident Energy as a Discrete Probability Distribution 40
      3.3.1 Other Issues Related to the use of Incident Energy Distributions 41
   3.4 General Form of the Recourse Model 42
   3.5 Stochastic Equation Forms for the Sample System One Day Post Modal 45
      3.5.1 Load Constraints for the Sample System 46
         Modal day 46
         non Modal day 47
      3.5.2 Battery Constraints for Sample System 48
   3.6 Stochastic Equation Forms for the Sample System- Two Days Post Modal 49
      3.6.1 Multi-Stage Recourse 49
   3.7 The Concept of Multi-Day Scenario Convolution 51
   3.8 Specific Example and Technical Models 53
      3.8.1 Decision Parameters 53
      3.8.2 Input Parameters 53
      3.8.3 Input Variables 54
      3.8.4 System Relationships 55
4 Expanded Models and Energy Mix Optimisation

4.1 Expanded Models - Introduction ............................................. 73
4.2 Expanded System Description .............................................. 74
  4.2.1 Small Solar Hot Water Collectors ..................................... 74
  4.2.2 Small Hot Water Storage Tanks ....................................... 77
4.3 Objective Function for the Expanded Form ................................. 80
4.4 Load Constraints for the Expanded System ................................ 81
  4.4.1 Modal Day Load Constraints ......................................... 82
  4.4.2 Non Modal Day Load Constraints ..................................... 83
  4.4.3 Storage Constraints for Sample System .............................. 84
4.5 Specific Example and Technical Models ................................... 85
  4.5.1 Decision Parameters .................................................. 85
  4.5.2 Input Parameters ..................................................... 85
  4.5.3 Input Variables ..................................................... 86
  4.5.4 System Relationships ................................................ 87
4.6 Worked Example ................................................................ 90
  4.6.1 Cost Function and the f matrix ....................................... 90
  4.6.2 Load Constraints and $A_{eq}$ ......................................... 91
  4.6.3 Storage Constraints and the $A$ Matrix .............................. 93
  4.6.4 Simulation Results ..................................................... 94
4.7 Energy Mix Optimisation - Introduction ................................... 95
  4.7.1 Existing Techniques and Approaches ................................. 95
4.8 Energy Flow Model ............................................................. 98
  4.8.1 Expanded Objective Function forms for Electric Hot Water Generation 98
  4.8.2 Load Constraints for the Expanded System ......................... 100
     Modal Day Load Constraints .............................................. 100
     Non Modal Day Load Constraints ....................................... 102
     Storage Constraints ..................................................... 103
  4.8.3 Energy Mix Optimisation - Worked Examples ....................... 104
  4.8.4 Cost Function and the f matrix ....................................... 105
  4.8.5 Load Constraints and $A_{eq}$ ......................................... 106
  4.8.6 Storage Constraints and the $A$ Matrix .............................. 108
  4.8.7 Simulation Results ..................................................... 109
4.9 Conclusion ................................................................. 111

5 Features of the Technique ........................................... 113
5.1 Introduction .......................................................... 113
5.2 Incorporating Weather Data ......................................... 114
  5.2.1 Multiple Years of Weather Data .............................. 114
  5.2.2 Filtering a Single Year of Weather Data ..................... 116
5.3 Incorporating Load Requirements .................................. 118
  5.3.1 Incorporating Load Increases associated with Low Incident Energy Days 119
5.4 Incorporating Load Increases associated with High Incident Energy days .......... 122
  5.4.1 Example of Loads Assessment in Greater than Modal days .......... 125
5.5 Assessment of the Ability to Export Energy and the Viability of Grid Connection .... 127
  5.5.1 Basic Substitution Grid Energy for ICE Energy ............. 127
  5.5.2 Energy Storage and Controlled Release ...................... 128
5.6 Multi-Objective Optimisation ....................................... 134
  5.6.1 The nature of the System Architecture and the Use of the Modal day .... 137
  5.6.2 Using the Technique to Support MOO Style Analysis .......... 138
5.7 Addressing Power Requirements ................................... 140
5.8 Ability to Assess Alternative Technologies - Hydrogen Systems as an Example ...... 143
  5.8.1 Electrolyzer Relationships ................................... 145
  5.8.2 Storage Tank Relationships .................................. 146
  5.8.3 Fuel Cell Relationships ...................................... 146
  5.8.4 Water Usage .................................................. 147
  5.8.5 Optimisation relationships .................................. 147
5.9 Summary ............................................................. 150

6 Contiguous Example .................................................. 151
6.1 Introduction .......................................................... 151
6.2 Statement of Requirements .......................................... 151
  6.2.1 Basic System Design ......................................... 151
  6.2.2 Weather Data .................................................. 152
  6.2.3 Load Data ..................................................... 153
6.3 First Pass Analysis .................................................. 155
  6.3.1 The Scenario Tree ............................................. 158
  6.3.2 Input Variables and Assumptions ................................ 161
  6.3.3 Cost Function and the f Matrix ................................ 165
  6.3.4 Load Constraints and the Aeq, beq Matrix .................... 166
  6.3.5 Storage Constraints and the A, b Matrix ....................... 172
6.4 First Pass Analysis Results ......................................... 175
  6.4.1 Impact of using a Larger Weather Record ....................... 176
6.5 Excess Energy and Grid Connection Trade Studies ..................... 178
  6.5.1 The Special Case of School Load profiles ...................... 180
  6.5.2 School Case Excess Energy ................................... 181
  6.5.3 Replacing ICE Energy with Grid Connection ................. 183
6.6 CO₂ Pollution Reduction ............................................ 187
6.7 The Impact of Geography ........................................... 189
6.8 Conclusion to Chapter 6 ............................................ 191
# 7 Conclusion

7.1 Introduction ......................................................... 193
7.2 Literature Review and the Basic Methodology ........................ 195
7.3 Adding System Complexity ........................................... 197
7.4 Exploring the Features of the Technique ............................. 199
7.5 Opportunities for Further Investigation ............................ 204
7.6 Summary ............................................................... 205

# 8 AppendixA

8.1 Introduction ............................................................ 207
8.2 A Generic Scenario Tree .............................................. 207
8.3 Incident Solar Energy, The Probability of that Energy Occurring and the concept of an Optimum Solution ................................. 209
8.4 How to find the Worse Case Solution by Scenario Tree Simplification .......................................................... 210
    8.4.1 The Case of Storing Energy on Greater than Modal days ....... 212

Bibliography ................................................................. 215
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AEMC</td>
<td>Australian Energy Market Commission</td>
</tr>
<tr>
<td>AUD</td>
<td>Australian Dollars</td>
</tr>
<tr>
<td>BOM</td>
<td>Bureau of Meteorology</td>
</tr>
<tr>
<td>CoE</td>
<td>Cost of Energy</td>
</tr>
<tr>
<td>CDD</td>
<td>Cooling Degree Days</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CHP</td>
<td>Combined Heat and Power</td>
</tr>
<tr>
<td>CPC</td>
<td>Concentrating Parabolic Concentrators</td>
</tr>
<tr>
<td>DE</td>
<td>Distributed Energy</td>
</tr>
<tr>
<td>DER-CAM</td>
<td>Distributed Energy Resources Customer Adoption Model</td>
</tr>
<tr>
<td>DHW</td>
<td>Domestic Hot Water</td>
</tr>
<tr>
<td>DoD</td>
<td>Depth of Discharge</td>
</tr>
<tr>
<td>DR</td>
<td>Demand Response</td>
</tr>
<tr>
<td>EBSS</td>
<td>Electrochemical Battery Storage System</td>
</tr>
<tr>
<td>EA</td>
<td>Evolutionary Algorithm</td>
</tr>
<tr>
<td>EAC</td>
<td>Equivalent Annual Cost</td>
</tr>
<tr>
<td>ES</td>
<td>Evolutionary Strategy</td>
</tr>
<tr>
<td>EV</td>
<td>Electric Vehicle</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GAMS</td>
<td>General Algebraic Modelling System</td>
</tr>
<tr>
<td>GHG</td>
<td>Green House Gas</td>
</tr>
<tr>
<td>HDD</td>
<td>Heating Degree Days</td>
</tr>
<tr>
<td>HES</td>
<td>Hybrid Energy System</td>
</tr>
<tr>
<td>HOMER</td>
<td>Hybrid Optimisation of Multiple Energy Resources</td>
</tr>
<tr>
<td>HVAC</td>
<td>Heating, Ventilation and Cooling</td>
</tr>
<tr>
<td>HW</td>
<td>Hot Water</td>
</tr>
<tr>
<td>HWL</td>
<td>Hot Water Load</td>
</tr>
<tr>
<td>ICE</td>
<td>Internal Combustion Engine</td>
</tr>
<tr>
<td>kWh</td>
<td>kilo Watt hours</td>
</tr>
<tr>
<td>LCA</td>
<td>Life Cycle Analysis</td>
</tr>
<tr>
<td>LCoE</td>
<td>Levalised Cost of Energy</td>
</tr>
<tr>
<td>LV</td>
<td>Low Voltage</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed Integer Linear Programming</td>
</tr>
<tr>
<td>MPC</td>
<td>Model Predictive Control</td>
</tr>
<tr>
<td>MOO</td>
<td>Multi Objective Optimisation</td>
</tr>
<tr>
<td>MOEA</td>
<td>Multi Objective Evolutionary Optimisation</td>
</tr>
<tr>
<td>OR</td>
<td>Operations Research</td>
</tr>
<tr>
<td>PEM</td>
<td>Point Estimation Method</td>
</tr>
<tr>
<td>PR</td>
<td>Performance Ratio</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimisation</td>
</tr>
<tr>
<td>PV</td>
<td>Photo Voltaic</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
</tr>
<tr>
<td>RES</td>
<td>Renewable Energy System</td>
</tr>
<tr>
<td>RO</td>
<td>Robust Optimisation</td>
</tr>
<tr>
<td>SAGSHP</td>
<td>Solar Assisted Ground Source Heat Pump</td>
</tr>
<tr>
<td>SA</td>
<td>Simulated Annealing</td>
</tr>
<tr>
<td>SIES</td>
<td>Small Islanded Energy System</td>
</tr>
<tr>
<td>SPEA</td>
<td>Strength Pareto Evolutionary Algorithm</td>
</tr>
<tr>
<td>SOO</td>
<td>Single Objective Optimisation</td>
</tr>
<tr>
<td>SLP</td>
<td>Stochastic Linear Programming</td>
</tr>
<tr>
<td>SPwR</td>
<td>Stochastic Programming with Recourse</td>
</tr>
</tbody>
</table>
List of Symbols

\[ C^T \cdot x = C_1 \cdot x_1 + \ldots + C_n \cdot x_n \]

\[ \min f = C^T \cdot x \] is the basis form of the characteristic equation used in this study

\[ Ax < b \] is the constraint equation for the decision variable \( x \)

\[ L \leq x \leq U \] is the basic form of the limits for the decision variable \( x \)

\( C^T \) is the cost coefficient vector

\( x \) is the decision variable vector

\( A \) is the constraint coefficient matrix

\( b \) is the constraint coefficient vector

\( L \) is the decision variable lower bound and

\( U \) is the decision variable lower bound

\( \text{Ir}_\text{rad}(n) \) is the incident radiation energy on day \( n \)

\( \text{Ir}_\text{rad}(n+1) \) is the incident radiation energy on the next day \( n + 1 \)

\( x_1 \) is the size of PV array \( (m^2) \)

\( x_3 \) is the battery size (kWh)

\( x_7 \) is the size of Hot water array \( (m^2) \)

\( x_8 \) is the hot water tank size (kWh)

\( x_2 \) is the ICE run-time for modal day (hours per day)

\( y_{2i} \) is the ICE run-time for the post modal day (hours per day)

The form of the constraint equation used in this study

\[ \min C_{\text{elec}} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i y_{2i} + d \]

\( \sum_i P_i y_{2i} \) is the arithmetic sum of each second stage scenario generator runtime
as factored by the probability of that scenario

\[ i = 1, 2, \ldots, n \ldots \] is the number of energy range days less than modal.

\[ P_{EL(0)} = e_0 x_1 + f x_2 \ldots \] is the modal day electrical constraint equation

\[ e_0 \ldots \] is the incident energy (factored by PV array efficiency) for the modal day and,

\[ f \ldots \] is the size of the system generator in kWh

\[ Q_{HWL(0)} = \mu_0 x_7 + \sigma x_2 \ldots \] is the modal day hot water constraint equation

\[ \mu_0 \ldots \] is the incident energy (factored by HW array efficiency) for the modal day and,

\[ \sigma \ldots \] is the heat output of the ICE in kWh per hour of running.

\[ e_i \ldots \] is the incident energy (factored by PV array efficiency) for the non-modal day(s)

\[ g \ldots \] is a factor that determines the percentage of the \( P_{load(0)} \) that must be stored.

\[ \mu_i \ldots \] is the incident energy (factored by HW array efficiency) for the non modal day

\[ \gamma \ldots \] is the percentage of the \( Q_{HWL(0)} \) that must be stored to support the Modal day load.
For

Lou, Scott and Sophie who always support my plans, regardless of how odd those plans must seem.
Chapter 1

Introduction

The study of Distributed Energy (DE) Systems encompasses a range of technologies that focus on the generation and distribution of electrical power (and also potentially heat energy) at a very local level, either for an individual building or a small cluster of buildings. Distributed Energy Systems provide an alternative to the existing system of industrial scale, remote power generation, connected to users via complex, monolithic distribution systems [1], [2], [3].

Microgrids is a terminology now in use to describe the combination of distributed energy system technologies with active load prediction, monitoring and control. Lawrence Berkeley National Laboratory Microgrid research group describe a Microgrid as follows:

“A Microgrid is a semi-autonomous grouping of generating sources and end-use sinks that are placed and operated for the benefit of its members, which may be one utility ‘customer’, a grouping of several sites, or dispersed sites that nonetheless operate in a coordinated fashion” [4].

Islanded Microgrid refers to a system that is not connected to a larger energy distribution networks or ‘grid’. This study is all about the design optimisation of "Urban Islanded Microgrids" which are Islanded Microgrids assumed to be operating in an environment where existing energy grid supply is possible.

One of the key advantages of Microgrids (and Distributed Energy Systems) is that they provide a technically and economically efficient architecture for the incorporation of renewable generation technologies into the electrical power supply system. A study by the Australian Government research organisation CSIRO into the benefits of Distributed Energy Systems [5] found the following:
• Results from the Energy System Model show that distributed energy has a future role to play in a carbon constrained future,

• In general Distributed Generation appears to be an effective early action greenhouse gas mitigation option for Australia when it is considered within a portfolio of other mitigation options,

• Sensitivity Analysis indicated that the more rapidly Distributed Generation technologies can move down the cost curve (i.e. technological breakthroughs, imported learning) the more competitive these options are to other alternatives Significant cost benefits resulting from the deployment of distributed energy solutions can be found in reduced water consumption and pollutant emissions,

• Islanded operation of distribution networks is in principal highly effective in realizing the full value from embedded generation. However the technical and commercial barriers remain formidable and will require substantial work to address; and,

• Before distributed energy achieves wide scale uptake … Technology and market development needs to be focused on reducing costs and improving reliability.

This thesis study is going to explore the optimisation of a form of Urban Islanded Microgrids that will be referred to from this point as Small Islanded Energy Systems (SIES). Figure 1 shows the general assumed form of the SIES.
In this study a **Small** energy system can be taken to mean anything from 5 to 250 KWhrs/day of electrical load. There is nothing significant about this ‘reference design’ other than it uses Internal Combustion Engine (ICE) driven generators to provide energy availability and assumes solar incident energy generated electricity and hot water with associated storage. This arrangement is chosen as it encompasses the main ‘renewable’ generation technologies presently used in small systems. It will be shown later in the study that alternate technologies can be assessed using the techniques developed.
1.1 Commercial Advantages of Small Islanded Energy Systems (SIES)

A specific advantage of the SIES architectures being examined by this study is that they allow the rapid and cost effective incorporation of new technological advances. Utility scale renewable systems lock into a particular technology and then, due to the financial construct, may be limited to the initial design technology for 25 years or more. The thesis study approach, being a combination of small, technologically agnostic designs means that as a new technology becomes available it can be specified, tested and rolled into the solution offering in a rapid (months) time-scale. SIES installed today will be viable upgrade targets across the course of their life. For instance manufacturer’s modelling and experience suggests that the batteries in the simple Figure 1.1 system will have a life of approximately 8 to 10 years (relative to the 25 year design life of the base system) which means every 8 years the opportunity exists to ‘roll-in’ new storage technologies. This ability to constantly exploit renewable technology advances, in an environment where new technology advances are occurring every year in commercial viable time frames, is a significant advantage of SIES over utility scale grids.

From a commercial perspective, this thesis study is attracted to and is focussed on Small Islanded Energy Systems (SIES) because they are seen as a way to deliver the benefits of renewable technologies in a commercially manageable and more importantly realistically achievable, low risk manner for investors.

The application of renewable technologies at the utility scale is by nature subject to the commercial complexity and rigour associated with large infrastructure development. Utility scale Business Plans involve someone (the public, private investors or a combination) taking on significant long term commercial risk and as a consequence such systems are difficult to bring to market. In contrast SIES entail many small commercial arrangements and consequently a myriad of small, manageable commercial risks. This concept of risk distribution as an enabler of commercial rollout is further enhanced when it is understood that the SIES can be, by design, a portable investment.

This concept of barrier to entry removal by utilising highly distributed risk is best illustrated by the roll out of ‘rooftop’ solar PV in Australia. According to the Australian Photovoltaic 2016 estimate [6] there is presently approximately 4 GW of installed solar PV in distributed form contributing to

the Australian energy supply. The investment in this size solar PV generation capacity is difficult to imagine as feasible if it had to be implemented as 4 x 1000MW or even 40 x 100MW utility scale facilities. The investment risk for the installed 4 GW is held by hundreds of thousands of small investors. Distributed risk supports simpler, faster and more targeted investment decisions.

Beyond the advantage of distributed commercial risk, SIES also make viable a complex and varied range of funding mechanisms that are not possible for utility scale Business Plans. Large utility scale Business Plans require ‘traditional’ investment models which are subject to limitations in the funding mechanisms and pool of investors available. SIES, because they require a large number of small packages of funding, are able to utilise a far more diverse range of investment/funding methodologies.

SIES have traditionally been used in remote areas (where there is no available grid connection) or in ‘end of grid’ situations [7]. Further there are many examples of Distributed Energy (DE) systems that can operate in either connected or Islanded modes (where connection decisions are based on optimum tariff analysis). In such systems the monolithic grid provides supply security.

The thesis study is focussed specifically on Islanded operation, even when grid connection is possible. The driving reasons for the decision to focus on Islanded operation are commercial. There are technical advantages to Islanded operation but these are ultimately realised as commercial issues. If a SIES solution is used to exploit the CO₂ reduction potential of renewable generation and storage then that solution must be commercially viable. This study defines commercial viability as being the ability to deliver energy at a cost equivalent to existing energy supplies, while at the same time providing supply security equivalent to or better than the monolithic utility grid.

The main reason that Islanded systems have a commercial advantage over optional grid connection systems is that Islanded systems do not carry the cost burden of the distribution networks. The recent Australian Energy Market Commission (AMEC) report into the cost of electricity in Australia suggests [8] states that:
"The distribution network component is a large part of the total retail electricity price paid by customers, contributing 37 per cent to the aggregated national retail electricity price in 2012/13. It is estimated that increases in the distribution network component will account for 81 per cent of the national increase in retail residential electricity prices from 2012/13 to 2014/15..... while the ‘wholesale’ component (simplistically the cost of generation) also contributes approximately 37 percent of the kWh price it is only estimated to increase by 1 per cent to 2015.”

While Australia is an extreme case (due to the small numbers of loads distributed over large distances) what the AEMC work suggests is that network cost will continue to grow as a percentage of the total retail Cost of Energy. (COE).

Another major cost advantage in the SIES architecture is the use of Combined Heat and Power (CHP) technologies. Combined heat and power technologies have traditionally been utilised in larger commercial systems but there is a suspicion, that will be explored by the techniques developed in this study, that small (less than 100kW) CHP system may be commercially attractive. The recovery of heat energy means that in many applications the SIES heat energy will be able to displace gas heating energy consumption and probably allow the users to disconnect from both electrical and gas reticulation networks. Consequently the SIES configuration produces commercial benefit from both gas consumption reduction, electrical network connection savings and gas network connection savings. The commercial equation for the SIES (and consequently the focus of this thesis study) now becomes one of the total cost of delivered energy rather than just the simple cost of electricity. It is noted that these commercial advantages are realised in addition to reductions in CO₂ emissions inherent in the use of SIES [9]

While it is possible for an urban SIES to be a product supplied and operated by a large commercial entity (e.g. the existing vertically integrated power supply oligopolies) it is also possible for an urban SIES to be ‘owned, operated and controlled’ by the end users of the energy. In between these two extremes are a wide range of potential commercial / ownership constructs.
The potential advantages of this flexibility may or may not ever be fully exploited, and the advantages may not even be obvious today. Part of the rationale for studying SIES is that the potential for energy generation and supply flexibility should lead to innovation in commercial structures and with innovation will come efficiency. The existing energy supply market in Australia is (as a result of the monolithic grid architecture) a quasi market controlled by a regulator that attempts to create competition where no natural competition would otherwise exist. The move toward vertical integration within suppliers (one company owning generation, distribution and retail assets) within Australia is evidence of the architectural construct of the grid driving monopolistic behaviour. This is common in many parts of the world.

It is not possible to clearly predict what the impact of Urban SIES economics will be, but it is reasonable to expect that the lowering of entry barriers and the ability for local independent ownership and control should lead to a range of market constructs and the potential for the benefits that flow from the subsequent competitive marketplace.
1.2 Environmental Advantages of Small Islanded Energy Systems (SIES)

The primary environmental advantage that flows from the use of Small Islanded Energy Systems (SIES) is that they provide a method that replaces Hydrocarbon based stationary energy supply with solar energy based energy. This directly reduces the \( CO_2 \) / Green House Gas (CHG) emissions associated with meeting a customers energy requirements.

The second environmental advantage comes from the use of small Combined Heat and Power (CHP) to meet energy needs when incident solar energy is not sufficient. CHP machines allow what would be otherwise waste heat produced by burning hydrocarbon fuels to produce electrical energy to be captured and used to meet consumer energy requirements. Hence each kg of hydrocarbon fuel burned produces more total usable energy, again reducing the GHG impact of customers energy needs.

The final environmental advantage is that the SIES being examined use very little water to produce meet the customers energy needs relative to traditional thermal power generation.

1.3 The Role of Design Optimisation in the Development of SIES

The preceding discussion regarding the economic potential for SIES is largely speculative and aspirational. It is included in order to set the scene and to remind readers that the design optimisation and energy system design optimisation specifically is primarily required for commercial reasons. The high level aspirational commercial aims noted need to be focussed into a specific question in order to provide a guiding framework for this thesis study.

This study assumes the test of commercial viability to be the ability to design a system that can provide energy at a price lower than competing sources without external subsidy. Further supply continuity must be equal to or exceed current energy system performance levels.

The key to achieving commercial viability is the ability to design systems *optimised* for a given load
1.3. The Role of Design Optimisation in the Development of SIES

requirement in a given geographical location and to be able to choose between (and analyse) a range of technical options. This thesis study is focussed on Design Optimisation of SIES. The following definition applies throughout this study:

**Design Optimisation of Small Islanded Energy Systems is defined as the process of establishing the design configuration (size of generating arrays, storage and generator size) and operating rules such that the long term Cost of Energy (COE) together with the system CO$_2$ emissions are minimised**

This thesis study aims to investigate optimisation techniques for the design of Small Islanded Energy Systems. The general or high level aim of the study is to explore techniques that could ultimately be used as the basis of a design tool. That final tool would need to have the following characteristics:

- Support design optimisation focussed around greenhouse gas reduction, technical efficiency and system life cycle costs minimisation,
- Allows demonstration of system reliability and availability (which is viewed by the CSIRO as a key issue in Distributed Energy acceptance by customers),
- Allows geographic variations in both climate and weather to be incorporated into the design solutions adopted,
- Allows technological variations to be assessed with a specific focus on being able to identify those technology research areas that have the most benefit in terms of economic and performance outcomes for the system(s); and
- Can evolve as lessons are learned from in service systems.

In addition to these initial requirements over the course of the study another key requirement regarding processing transparency was identified. During the initial literature review and across the course of the study one of the concerns that emerged regarding existing techniques regarded how complexities in those techniques prevented a complete understanding of exactly why some techniques
produced certain results, and why changing inputs or assumptions impacted outputs. This added another high level requirement for transparency. It became an aim that any technique developed should be sufficiently transparent to allow tool users to understand how the technique is working.
1.4 Research Questions

The above high level aims need to be further limited to a series of ‘Research Questions’ to meet the academic criteria for the thesis study. The following three request questions / topics where formulated during the initial literature review and agreed in the first year if the thesis study:

- Examine the form and of performance of specific optimisation objective functions when applied to SIES
- Assess and develop new forms of Resource and Technology models used in the optimisation of SIES
- Investigate the most effective optimisation techniques to be used to process the objective functions and resource and technology models examined.

At the same time, for the purposes of addressing the three academic questions Optimisation was agreed to be defined as:

*the ability to establish the size (capacity of batteries, capacity of hot water storage, capacity of PV array etc) and running time of the ICE such that the Cost of Energy (CoE) and CO₂ emissions are minimized over a 25-year system life.*

These research questions form the basis of the study that is described by this dissertation.

1.5 Document Outline

Chapter 2 is the Literature Review. This review explores existing work from two perspectives. There is an investigation of existing Small Energy System (the review was not limited to Islanded systems) and Combined Heat and Power (CHP) design optimisation tools and techniques. There is a further investigation where the research work is predominantly a discussion of mathematical optimisation techniques and energy systems are used as an example application. The Literature Review reaches a conclusion and recommends a two step optimisation process. This two step process involves an initial optimisation based around the ‘Modal Day’ of incident solar energy followed by a second
stage optimisation that is informed by the probability of non-modal incident solar energy. Having identified a candidate general approach, the Literature Review then concludes that the mathematical approach (developed in the area of Operations Research) known generically as ‘Stochastic Programming with General Recourse’ is a good candidate technique to use to mechanise the general approach.

The two step optimisation concept based around the use of a first stage Modal incident energy day and the use of Stochastic Programming to implement the SIES optimisation concept represent new and novel development elements of this study.

Chapter 3 demonstrates how the general approach and mathematical process identified in the Literature Review can be utilised to optimise a simple Solar PV/Battery Storage/Generator system. This chapter develops forms of objective functions, constraint functions and models of representing the performance of technologies that form the examined system. This chapter executes an optimisation then develops a verification technique that shows that the technique developed does produce a close to optimum solution.

Chapter 4 builds on the basic technique by adding hot water energy and the ability to analyse energy storage optimisation. Further the ability to trade off different means to generate hot water is added into the technique. The technique maintains the form of the equations developed in Chapter 3 and hence ensures that the Chapter 3 validation remains. The techniques developed in Chapters 3 and 4 represent new and novel development elements of this study.

Chapter 5 explores the features and utility of techniques developed. The chapter explores how the inherent features in the developed technique can be used to create a design tool and explores the features available in that tool. The chapter outlines how these features add to the capabilities in existing tools and how the techniques developed allow new styles of questions to be answered. This chapter is a summary of the new and novel functionality and capability supported by the developed techniques.

Chapter 6 is a "Contiguous Example" that applies the developed technique to a real world example and uses this example to explore aspects of the technique and its application as a design tool.
Chapter 2

Literature Review

2.1 Introduction to Literature Review

The first stage of this study involves investigating a range of areas related to existing approaches to design optimisation in general and specifically to the optimisation of the style of Small Energy Systems (the review was not limited to Islanded systems) defined in the introduction. In this study the literature review was broken into two high-level tasks:

- A review of Energy System Optimisation Methodologies and,
- A review of Optimisation Mathematical techniques

While literature from two separate areas of work has been investigated it should be noted that there is a linkage between a given energy system optimisation methodology and the mathematical techniques used to support that methodology. Particular mathematical techniques are reported as an element of discussions of energy system optimisation. In recognition of this interrelationship the literature review is structured around the following sequential questions:

- What is the existing body of work addressing energy system optimisation?,
- What mathematical techniques are being used by this body of work?,
- What deficiencies (gaps) exist in the existing optimisation methodologies and what new techniques or approaches may address these gaps?,
- What mathematical techniques are available to support possible new approaches?.
2.2 Approaches to Energy System Optimisation

An initial review of existing work addressing the modelling of energy systems, and energy system optimisation showed that the work could be allocated to one of two categories:

**Work focused on optimisation techniques where the energy systems are an example application:** This category of work was found to be useful in exploring the relationship between objective function formulation and mathematical technique. This work is focussed on expanding mathematical techniques.

**Work focussed on the optimisation of energy systems specifically:** This body of work is focussed on investigating the detailed modelling of energy systems and tends to focus on defining the optimisation problem.

Both categories of work inform this study.

2.2.1 General Optimisation with Energy Systems as an Example

Assessment of energy system sizing is addressed by a range of papers where the focus was found to be the exploration of mathematical techniques with the use of Microgrid optimisation as an example. There are a large number of these papers and in 2012 they were reviewed by Erdinc and Uzunoglu [10]. This review categorises studies according to the mathematical approach they adopt. Key mathematical categories looked at included Genetic Algorithms (GA), Particle Swarm Optimisation (PSO), Simulated Annealing (SA), design space based approaches, simulation approaches, Evolutionary Algorithms (EA), and stochastic/probabilistic approaches. It is noted that this review paper takes a generic view of the nature of Hybrid Energy Systems (note many papers in this area of study use the terminology Hybrid Energy Systems to describe systems with multiple generating sources, usually including wind, hydro or incident solar energy conversion technologies). When such papers are examined in detail is it possible to gain a view of the effectiveness of any particular approach as a design support methodology.
2.2. Approaches to Energy System Optimisation

In this Literature Review these style of papers have been examined with the aim of identifying how the mathematical approach adopted is driven by:

- The formulation of the base objective function
- The nature of the technology models used
- The nature of the system constraints applied and,
- The treatment of load data and weather requirements.

Evolutionary strategies are found to suit Hybrid Energy System problems where the technology models are non-linear or otherwise complex. In [11] Logenthiran provides an example of this class of paper. This paper establishes a simple objective function of the form

\[ \text{min} C_T = \sum_{i=1}^{N} C_{DER_i} + OMC \]  

\[ (1) \]

where
- \( C_T \) is the total cost of the distributed energy resources,
- \( C_{DER_i} \) is the capital cost of the Distributed Energy Resources (i.e. the system equipment) and
- \( OMC \) is the ongoing Operational and Maintenance Costs

This simple objective function is subject to simple constraints of the form

\[ LPSP < LPSP_{\text{max}} \]  

\[ (2) \]

where
- \( LPSP \) is the Probability of the Loss of Power Supply.
Complexity is introduced by the system component modelling which introduces non-linear relationships to model a wind plant and a photo-voltaic plant. This combination of simple objective functions and constraints together with complex plant models is a problem definition that suites the Evolutionary Strategy (ES) approach explored. The focus of Logenthiran’s work is on processing efficiency (and convergence) as much as it is on the energy system design issues. The ES approach explored does support the analysis of different system configurations (by amending the form and content of the ES ‘chromosome’). Further because the ES concept allows the systematic review of a large number of scenarios this approach can look at the performance of systems over an entire year (by simulating on a day by day basis).

This approach of using Genetic Algorithms is explored in depth by Bustos [12] who introduces two objective functions (expected energy not supplied and levelised cost of energy) together with a wide range of generating sources. This paper also introduces the concept of using Weibull distributions to estimate solar radiation and wind speed. This paper shows the scalability of using a Genetic Algorithm approach but is limited to analysis of systems over a 24 hour period.

While not directly addressing the question of system design optimisation, a number of papers, exemplified by Tina in [13], [14], examine the probabilistic estimation of incident energy and incident wind speed and then uses these estimates, together with complex but idealised technology models to analyse the performance of systems. These approaches produce ‘long term average performance’ and represent a method to try an incorporate longer term (yearly) incident energy variability into design. These approaches could form the basis of an optimisation technique that could address the aims of this study. Use of such approaches in an optimisation analysis is shown in [15]. In this work probability estimation of wind and solar incident energy is combined with complex technology models and objective functions for the minimisation of annual average power loss and maximisation of power stability and network security indices. The combination of the probabilistic estimation of generator output with complex technical models, load requirements and system constraints results in a question that is best suited to some form of Genetic Algorithm (GA) approach. In [15] a form of Particle Swarm Optimisation (PSO) is adopted. This approach is suitable for the form of the problem developed, especially given the nature of the objective function, but it can be seen that it would
not support an assessment of consecutive day performance, since the difference between consecutive days is hidden in the probability distribution created. As a consequence the answer produced is shown as an ‘average’ of system performance, in this case 4 days representing each annual season.

Another class of papers looks at the concept of ‘control strategies’ as an approach to energy system optimisation, In[16] Wang provides an example of such approaches. Wang suggests a simulation style strategy, that systematically checks all possible combination of variables, (for a given incident energy density, wind speed and load requirement) and optimises net present cost and ‘renewable market penetration’. This approach is possible due to the use of a pre-determined ‘control strategy’. In the context of this style of system the control strategy functions like a series of constraints; the control strategy constraints are calculated in an iterative process then become input conditions to the second stage of the GA analytical optimisation. The use of a pre-determined control strategy greatly reduces the number of possible system configuration options, making the GA analytical approach possible. The approach would support a form of sensitivity analysis where it would be possible to change the control rules and examine the impact on the result. The approach is limited by the manner of its incorporation of incident radiated energy, wind speed and load all of which must be pre-declared for a given time increment.

Another form of control strategy optimisation approach is outlined in [17]. In this approach the impact of a particular design change, (load control capability) on the optimal system configuration is examined. The approach uses a Weibull probability distribution function to produce wind speed data and simple averages to provide ‘light intensity’ data. The control strategy is used in conjunction with a simple energy balance constraint to support a simple analytical optimisation. The approach is suitable for the style of system architecture trade study being explored and provides a potential example of how a control law style (‘what if’) approach may be able to be used address the question of consecutive days. The approach is still limited to a single 24 hour period and as a result is only able to address ‘seasonal typical days’

Another class of papers examine the concepts of Robust Optimisation (RO) as applied to energy systems. As energy systems have inputs that are often uncertain (e.g. incident solar energy, wind
speed, and the connection time of electric vehicles) they are suited to use as exemplars of Robust Optimisation techniques. Battistelli [18] provides an example of this class of approach. This paper examines grid connected wind generators with battery storage provided by Electric Vehicles (EV). The approach adopted creates simple objective functions, with power balance and power flow constraints. These objective constraints are then amended by inclusion of terms representing ‘auxiliary variables’ and ‘deviation’ of certain parameters, (in this case power transferred in and out of the garage). The deviation parameters accommodate the uncertainty in the time EV’s spend connected to the system and the auxiliary parameters allow related variations in generation and load parameters. Importantly, this approach does not address the probability of a particular deviation occurring but rather establishes an optimum solution assuming that the uncertainty variable can occur anywhere within the range set by the deviation constraint. As such this approach looks like a form a sensitivity analysis. The paper reaches conclusions about those parameters that are sensitive to EV connection time and those that are not. In this way this approach does allow an insight into design decision making.

A further variation in technique that attempts to address the uncertainty inherent in incident solar energy is described by Cabral [19] who uses a Markov style analysis to estimate the incident solar energy at any time using some limited historical data. The Markov approach is used to estimate a daily ‘clearness index’ that is then used to estimate PV generation. A similar approach is used to estimate the state of the storage battery charge. These two inputs are then used analytically to optimise an objective function that addresses Loss of Power Supply Probability (LPSP). Importantly this work provides test evidence that allows a statistical review of the Markov estimation approach, demonstrating that it is useful in estimating and accommodating the uncertainty in incident solar energy.

The approach of using Markov estimation techniques to provide estimates of PV output and battery storage as input estimates to a stepwise analysis of supply continuity is further explored in [20]. Here the approach is expanded to illustrate how it can be utilised to support sensitivity analysis. In this work the sensitivity of the optimised solution to changes in financial inflation rate was reviewed. The approach to conducting sensitivity analysis would be applicable to a range of parameters and applicable to other analysis / operating rule style optimisations.
The final paper reviewed in this group was by Baziär[21]. Here a number of the concepts previously discussed are combined into a Theta-PSO based optimisation that addresses total operating cost of a hybrid solar, wind and storage energy system. The paper proposes a simple capital cost, running cost style objective function that could be simply expanded to accommodate a range of architectures. The constraints are related to both energy generation balance and a novel approach to battery charge rate limits. The method incorporates stochastic uncertainty using a double sided Point Estimate Method (PEM). The methodology solves the optimisation for the mean value of each random variable and then twice again for points above and below the random variable mean. The basic optimisation process used is Particle Swarm Optimisation (PSO). The paper proposes some sub process modifications to the PSO approach. The approach is limited to a 24 hour period and assumes the random variables in question are normally distributed.

The common aspect of all the work described in this section is that they primarily use the investigation of energy systems as a way to illustrate particular optimisation mathematical constructs. This does not mean that the insights provided are not useful, but what is illustrated is that most approaches use both objective function constructs and technology representations that are either simplified, or in some cases made more complex in order to suit the mathematical technique being investigated. The techniques mostly investigate a single 24 hour period. In some cases this is a limit of the technique adopted and in some cases reflects conventions associated with the analysis of energy systems. Those techniques that use ‘operational rule’ style analytical approaches could be used to address the consecutive day question.

### 2.2.2 Energy System Specific Optimisation

A small number of examples of optimisation papers that are focussed on energy system optimisation as the central question, rather than as a mathematical exemplar, have been identified.

An early example of such work is is provided by Lopez [22] who outlines a method for Multi Objective Optimisation (MOO) of a PV, Wind, Diesel, Hydrogen,Battery System Hybrid Energy system. The optimisation addresses total life cycle cost, un-met load and \( CO_2 \) emissions. This paper
applies the known technique of Multi-Objective Evolutionary Algorithm (MOEA) modified by using the concept of a Strength Pareto Evolutionary Algorithm (SPEA). The approach adopted is to use two linked Evolutionary Algorithms (EA). The first EA (the Main Algorithm) that optimises the configuration (size of each element) followed by a second algorithm that optimises 12 ‘control variables’ (running times, state of charge, hydrogen tank discharge etc.). The approach is computationally intensive but does allow the ability to incorporate complex constraints and system interactions (such as battery life vs. charge / discharge cycles). The approach supports whole year analysis but still uses limited weather and load variability. The main concern with this approach is the time taken to compute solutions that are still only a ‘Pareto Frontier’. The advantages of this approach is that it does examine a full year of data and it does support the use of sensitivity analysis during design investigation.

Zhang[23] describes the Multi Objective Optimisation (MOO) analysis of a small Combined Heat and Power (CHP) energy system. The analytical approach is developed to examine not only the trade-off between the cost of energy and the contribution of the systems operation to CO₂ emissions, but also allows an examination of different generating (electricity and heat) technologies. A contribution of this paper is the use of ‘nested’ objective functions that allows existing Mixed Integer Linear Programming (MILP) tools (in this case the CPLEX solver supported by the GAMS tool-set [24]. The method of MOO had previously been explored by the same author in [25]. The approach adopted illustrates the concept of tailoring the structure of constraints, in this case generator ramp limits, energy demand constraints, CHP life constraints and thermal storage constraints to support the specific analysis questions (in this case what mix of generating technologies is optimal). This paper introduces a second concept of incorporating total life cycle CO₂ emission, rather than just operating emissions. This is achieved by amending the cost objective functions with fixed indices (for CO₂ and for SO₂ emissions noting that gas fired CHP system produce SO₂). These indices are developed using a pre-existing Life Cycle Analysis (LCA) tool.

Where the focus is on ‘operational optimisation’ (establishing the best operating strategy for a given set of load requirements and cost constraints) rather than design / configuration optimisation then the ‘Model Predictive Control’ (MPC) approaches, as explored by Parisio [26] are useful. This
paper combines an MPC analysis that allows uncertainty in Renewable Energy System (RES), time varying load and time varying grid energy inputs using a control system like feedback together with an overarching Mixed Integer Linear Programming (MILP) analysis. The approach does support an analysis with uncertainty but only over a limited (24 hr) time frame. Another feature of this work is the use of ‘auxiliary variables’ that are used to simply the cost function.

One major stream of work in the area is the use of complex simulation as the basis of the optimisation method. The two key contributions in this area are the Hybrid 2 Simulation model [27] and the much used HOMER (Hybrid Optimisation of Multiple Energy Resources) optimisation tool. [28]. Hybrid 2 and HOMER both use the concept of time step by step simulation of a defined energy system performance. Hybrid 2 is predominantly a simulation that could be used to support optimisation. HOMER uses a simpler form of the Hybrid 2 style simulation and then adds an optimisation process layer and sensitivity analysis layer which provided designers with data that can be used to identify optimal system configurations and operational rules. The HOMER and Hybrid 2 simulations have the following key characteristics:

- they use time series simulations that, for each time step, model the load requirement, the generation capacity, distribute energy to loads or storage using pre-defined rules sets (constraints) then produce metrics to analyse how effectively system aims have been addressed

- they accommodate incident energy using a time step by time step estimate that could be historic data or an estimate based on historic data and probability distributions

- they accommodate load requirements using a time step by time step estimate

- they use the concept of the time step load requirement as the basis of all modelling. The rule sets and constraints are all developed to look at how that load requirement will be meet in that time period, whether or not energy needs to be ‘imported’ and what best to do with any energy that is generated (by renewable resources) but not required to meet that time step load

- Each uses detailed ‘models’ of the behaviour and performance of system components. These models could be modified without impacting on the overall structure of the analysis tool
Chapter 2. Literature Review

The HOMER tool [28] uses the time step simulation to then conduct an optimisation by investigating a range of configurations in pre-defined steps (e.g. increasing PV size in 5 kW steps). Configurations are then ranked according to pre-defined criteria (e.g. levelised cost of energy, fuel use, CO$_2$ emissions). The sensitivity analysis is the same process of investigating the changes in yearly performance that result from changes in ‘sensitivity parameters’ rather than physical configuration. It would be possible to use the Hybrid 2 simulation as the basis of a similar optimisation approach as HOMER.

What both tools illustrate is that because they view optimisation as a design task (i.e. a task to establish system configuration and operation), rather than a mathematical rigorous, continuous optimisation, they can use a stepwise approximation /simulation methodology and achieve a suitable result. A designer could use the functionality in either of these to ‘manually’ amend design configurations to close in on an approximate optimal solution.

The DER-CAM (Distributed Energy Resources Customer Adoption Model) tool-set [29][30] represents a comprehensive optimisation tool that is not simulation based but uses an analytical mathematical approach. The DER-CAM tool set has evolved over time and includes two main versions: an Investment and Planning tool set and an Operations tool set. The present capability of DER-CAM is exemplified by papers such as [31][32]. DER-CAM is primarily a tool to assess energy generation and energy balance in CHP systems with the aim of establishing the optimal configuration and operation in order to minimise energy costs and CO$_2$ emissions. The DER-CAM energy flow model is often depicted as in Figure 1.
2.2. Approaches to Energy System Optimisation

While DER-CAM mathematical constructs may become complex [31][32], [33] the basic approach initially adopted and subsequently maintained is to structure all objective functions as a simple linear relationship with the following form

\[
\min f = C^T \ast x = C_1 \ast x_1 + \ldots + C_n \ast x_n \tag{3}
\]

such that

\[Ax < b \text{ and,}
\]
\[L \leq x \leq U \text{ where}
\]

\(C^T\) is the cost coefficient vector
\(x\) is the decision variable vector
\(A\) is the constraint coefficient matrix \(b\) is the constraint coefficient vector
$L$ is the decision variable lower bound and $U$ is the decision variable lower bound

By maintaining this structure and capturing the energy system characteristics and external variables as a series of constraints and the energy system configuration as decision variables DER-CAM can use a Mixed Integer Linear Programming (MILP) solution. In the DER-CAM case this solution is mechanised using the CPLEX solver in the General Algebraic Modelling System (GAMS) tool. The basic DER-CAM structure limits the tool to examining energy flows in a discrete series of pre-defined time steps with the input energy and customer load defined for each of those time steps.

The potential for the basic DER-CAM structure to deal with uncertainty is explored in [32], which addresses uncertainty in Electric Vehicle (EV) connections and [34] which treats imported energy costs in a similar manner.

2.2.3 Building on the Existing Approaches

The optimisation of small energy systems (which are defined in the introduction as being smaller than 250 kWhrs / day) involves both the analysis of the energy system configuration during the design phase and then the optimisation of system operation once it starts to produce energy. The initial design phase requires decisions to be made regarding the configuration of the system specifically the size of components that generate and store the ‘renewable energy’ (e.g. solar Photo Voltaic electricity and solar heat hot water). On any given day the quantum of incident solar energy determines the ability to generate energy and the load requirement is determined in part by both the temperature of the day and the period of daylight. Hence for a given geographic location the likely weather profile (atmospheric conditions) is one of the primary inputs into the design process.

For the style of small energy systems being examined in this thesis study, energy that has not been generated from the incident solar radiation needs to be provided by either the extant energy grid, or in non-grid connected systems some form of ‘dispatchable’ generating source (note in this context dispatchable refers to a form of energy that can be generated and dispatched in a controlled manner, independent of external environmental conditions). In the case of the systems being examined this
dispatchable generation is most often implemented as an Internal Combustion Engine (ICE) coupled to an electrical generation. It is noted that the ICE solution is only one of many, for instance a hydrogen fuel cell could be used. For the purpose of this initial methodology discussion the dispatchable source will be considered to be an ICE ‘generating set’.

The analytical tools examined accommodate aspects of the optimisation question that this study seeks to address, either for initial configuration or identification of optimal operating rules. The more comprehensive tools such as HOMER or DERCAM go a long way toward supporting the optimisation questions but gaps remain. For the style of small systems with energy storage being examined for this thesis the most obvious analytical process relates directly to the question of how weather variation over a ‘run of days’ impacts on the validity of the optimal solution. HOMER allows for a daily/monthly/yearly weather profile to be used in the simulation but, for a given location, this will always be an estimate. DERCAM and HOMER would both support a designer who wished to conduct a sensitivity analysis with weather as the sensitivity variable but none of the existing tools are structured directly around the concept of weather variability as a ‘run of days’ question. For the systems being examined electrical storage is a significant cost driver and this leads to the following design questions that are not readily addressed by existing analytical approaches reviewed:

- how does the run of days weather profile impact the load requirements?
- how does the run of days weather profile impact the base storage solution?
- how does the run of days weather profile impact the trade off between electrical or hot water energy storage?
- how does the probability of a particular run of days weather profile impact upon the optimal solution?

Once it is recognised that the weather of a given day together with the run of days weather profile drives the ‘optimum’ design solution it becomes apparent that initial design decisions regarding the configuration of the ‘fixed’ system components (these are capital investment decisions) requires a consideration of potential weather variability. A design that is optimal for a given class or classes of days (which is how both DERCAM and HOMER are structured) may need to be further amended to
consider / accommodate the impact of weather in subsequent consecutive days.

The issue is best explained by the simple example of a Small Islanded Energy System comprised of a photovoltaic (PV) array, a diesel engine generator and a storage battery. In such a system, for any given day, design optimisation involves increasing the size of the PV array and battery (increased capital cost) while reducing the size (capital cost) and running time (daily cost) of the diesel engine generator. There will be an ‘optimal’ solution (size of PV array, battery, generator and generator running time) that produces the Lowest Cost of Energy (LCOE). That optimal solution will only be valid for days with the same cumulative incident solar energy and electrical load requirement (where the load has partial weather dependency). On days with different incident energy characteristics the solution will not be optimal.

If the day \((n + 1)\) has less incident energy than the analysed day \((n)\) then there are two possible changes to the design solution. One is to add more (than is necessary for day \(n\)) PV and storage capacity (increase in capital cost) or add more generator running time (running cost) on day \((n + 1)\) to compensate for the lower PV generation. This trade-off between capital cost and running cost represents a further optimisation question (beyond the day \(n\) optimisation) that is not addressed if the analysis is limited to 24 hours. This problem is further exposed if day \((n + 2)\) again has less incident energy than day \((n)\). Further the optimal design solution will be impacted by not only the weather profile of the run of days but the probability that this profile occurs. Hence, it is possible to optimise for a particular run of days, but if the probability of that run of days is low then the solution developed will not be optimal over multiple years. This problem in how to accommodate the probability of a run of days is identified in [34], [35]. The HOMER tool could support analysis for one run of day profile but could not examine the impact of the probability of a particular run of days. The DER-CAM work previously summarised [32], [34], does point to the potential of including the probability of particular variable state occurring.

For the form of SIES being examined the main variable of concern is the incident solar energy as this determines the amount of energy that can be generated (either as PV electricity or as hot water). The size of PV arrays, electrical storage, hot water arrays and water storage is optimised, for a
given load profile, based on the quantum of incident solar energy. The basic concept for the analysis proposed is to conduct a two stage optimisation. The first stage optimisation will be for the Modal day of incident solar energy for a given location over a 12 month period. Using the Modal day as the starting point means that, by definition, the outcome of the first stage optimisation is valid for the greatest number of days in a given year. The second stage optimisation examines changes in the baseline solution based on the incident solar energy on Non-Modal days, and the probability of those days occurring. This basic process flow is shown in figure 2.2
A further consideration is which days should be examined. There are two basic situations:

**Case A** The incident energy on day \( (n + 1) \) is greater than the Modal day \( (n) \) (i.e. \( I_{rad(n+1)} > I_{rad(n)} \))
In this case the solar array will be larger than is necessary to provide the load and the battery will not be large enough to store all of the excess energy. This situation represents ‘lost’ generation capacity but does not add to the overall Levelised Cost of Energy (LCOE). If all days where either Modal days or in this case then there would be no advantage, from a LCOE perspective, to modify the modal optimal parameters.

**Case B**The incident energy on day \((n + 1)\) is less than the Modal Day \((n)\) i.e. \(I_{rad(n+1)} < I_{rad(n)}\).

In this case the Modal day arrays size and storage will not be sufficiently large. The options available in this circumstance are to either:

- make up the shortfall in energy by importing non solar energy or
- increase the size of the array and the storage such that the energy shortfall on day \(n+1\) is captured and stored on day \(n\)

Hence the optimisation question, in words, is stated as follows:

*How to minimise the increase in capital cost (relative to the Modal day optimum configuration) versus the increased cost of imported (or non renewable generated) energy for the days \(I_{rad(n+1)} < I_{rad(n)}\) given that the a Modal day optimum exists and is known?*

This basic concept informs the next section of Literature Review into mathematical techniques that may support answering this question.
2.3 Mathematical Techniques to Accommodate the Run of Days Methodology

Having identified a capability gap in existing ‘optimisation’ techniques for energy systems and having proposed a basic methodology to address these gaps (as shown in Figure 2.2) it is now necessary to look for mathematical techniques that best suit the particular problem formulation. Searching for mathematical techniques requires a more formal, structured description of the question being addressed.

Optimisation within a design context involves establishing the best or optimal configuration of constituent elements within the design in order to achieve a pre-defined design goal. In the context of this study, the system architectures are established as a baseline to meet particular design requirements (e.g. an ICE driven generator to ensure availability). Then optimisation involves establishing the size of constituent elements (PV array size, battery size etc.) in order to achieve a pre-defined goal (cost of energy or CO₂ emissions or both). Mathematically such problems are often described as follows:

"Choose the design variable $x$ in accordance with the rules outlined in an objective function $f(x)$ taking into account particular constraints $g_i(x)$ and $h_j(x)$." This is often expressed as follows:

Optimise:

$$f(x) \ldots \ldots \ldots .(4)$$

$$g_i(x) \leq 0 (i=1,\ldots, I) \ldots \ldots \ldots \ldots .(5)$$

$$h_j(x) = 0 (j=1,\ldots, J) \ldots \ldots \ldots \ldots \ldots .(6)$$

In which case $g_i(x)$ are referred to as the inequality constraints and $h_j(x)$ are the equality constraints.
2.3. Mathematical Techniques to Accommodate the Run of Days Methodology

How a particular form of ‘uncertainty’ fits within this general construct has a significant impact on the selection of mathematical processes adopted to accommodate that uncertainty. Note, that as described previously, the key uncertainties that this study wishes to accommodate are incident energy and weather (which drives uncertainty in energy load requirement).

A structured way to categorise uncertainty is provided by Beyer [36] (which is based on early ideas in [37]). This categorises uncertainty by proposing a model where a system generates the required outputs $f$, which depend on environmental inputs $\alpha$, as controlled by system design parameters $x$, that is

\[ f = f(x, \alpha) \]..............(7)

In this structure uncertainties are categorised as follows:

- **Type A** uncertainty in the environmental inputs $\alpha$
- **Type B** uncertainty in the design variables $x$ e.g. then $f = (x + \delta, \alpha)$
- **Type C** uncertainty in the system output, which is a result of approximation errors in models or measurement errors and is expressed as the actual output $\tilde{f}$ as a random function of $f$ e.g. $\tilde{f} = \tilde{f}[(x + \delta, \alpha)]$
- **Type D** feasibility uncertainties, which in this construct are uncertainty with the constraints $g(x)$ and $h(x)$

For this study, identification of uncertainties as ‘type A’ assists in establishing the best mathematical approach to follow.

Mathematics that accounts for uncertainty in optimisation problems has been explored by both the engineering design field and by the Operations Research (OR) branch of mathematics. In the design context the search for an optimal design that ‘accommodates’ the range of possible uncertainties is
often referred to as Robust Design Optimisation (RO). The general aim of all Robust Design Optimisation is find an optimal solution that is the least sensitive, or most likely to be the least sensitive, to changes in the uncertainty variables. In the operations research area such approaches are generally referred to as Robust Optimisation. [38].

2.3.1 Stochastic Programming Approaches

Following the review summarised in the above section, and noting that the problem being addressed generally can be categorised as a ‘Type A’ uncertainty it was concluded that the techniques generically referred to as Stochastic Programming best suit the energy system problem being examined.

Early work on the theory and validation of the concept that stochastic program relationships can be expressed as an equivalent deterministic program was first proposed by Wets in [39]. A more recent outline of the basic concept of Stochastic Programming to accommodate Optimisation in presence of uncertainty is contained in [40]. In this tutorial style paper Higle outlines a number key issues that relate directly to this study. It is noted in the introduction that “When some of the data elements in a Linear Program (LP) are most appropriately described using random variables a Stochastic Linear Program (SLP) results”. It is further noted in the introduction that the recourse problems addressed by the paper best represent the class of problem where random variables exist in the linear program variables. Where problem formulations introduce probabilistic constraints, then the techniques generically described as Probabilistic Programming are best used [41].

Higle provides an introductory rationale regarding issues related to sensitivity analysis, an argument that is also outlined by King [42] Sensitivity Analysis is described as a ‘postoptimality investigation’ whereby following development of a basic problem formulation discrete coefficients in the objective function can be changed, new results calculated and the impact of those changes, and hence some insight into the most appropriate ‘design’ can be understood. (note that this is the approach adopted by the HOMER tool [28]). Mathematically sensitivity is a test of ‘robustness’ however it does not create robustness in the basic problem formulation. Sensitivity analysis provides a methodology to examine which uncertain variables, and the degree of uncertainty in those variables, that may have an impact on the solution. Both Higle and King [40], [42] suggest that if a sensitivity analysis shows
that a given formulation of an objective function is not robust (i.e. highly sensitive to changes in uncertain variables) then it is desirable for the model formulation and objective function to be structured and analysed to accommodate that uncertainty.

In building a case for Stochastic Programming Higle and King explore the additions to deterministic optimisation. 'What If' analysis is described as a deterministic solution is established and then sample values for uncertain parameters are tested to explore what impact they have. Worst Case analysis is described as a deterministic solution is established then the worst outcome possible for a given uncertain parameter is examined and used to drive the design conclusion. Examples of both approaches have been seen in the existing examples of energy system analysis e.g. [13], [21]. Both represent sophisticated examples of this underlying concept. Both authors explain the deficiencies associated with these approaches. As a consequence of these two studies this thesis study asserts the validity of these conclusions without further explanation. One issue raised is that of 'dependant random variables'. King offers, by way of a thought exercise, the notion that if all random variables are processed as their mean values then a deterministic optimisation problem is created, but the interdependencies between random variables are then hidden.

It was noted earlier that in Operations Research (OR), the general conclusion is that where random variables exist in the linear program variables then Recourse Models are the most appropriate.

### 2.3.2 Recourse Models and Problem Formulation

Generally Recourse problems are described as having some form of temporal information structure or scenarios. Traditionally recourse problems are described as having known variables at some time that allows an ‘initial’ decision to be made and then the ability for the solution to be amended at a later date, when new information becomes available, to allow a more optimal solution to be adopted. Hence recourse problems are always described as having at least two ‘stages’. The stages in the problem are often represented by scenario trees.

The general form of Stochastic Linear Program (SLP) expressed as a ‘two stage’ problem is [40]
\[ \text{Min} C_1^T x_a + E_w Q(x_a, w) \] ............(8)

such that
\[ Ax_a = b \]
\[ x_a \geq 0 \]

Where

\[ Q(x_a, w) = \text{mind}(w)^T y_i \] ............(9)

such that

\[ T(w)x_a + W(w)y_i = h(w) \] ............(10)

\[ y_i \geq 0 \]

The first linear relationship aims to minimise the first stage known costs \( C_1^T x_a \), plus the expected (recourse) costs \( Q(x_a, w) \) over all possible scenarios, assuming that the first stage constraints \( Ax_a = b \) are met.

The second linear relationships introduces a new set of variables, that can be used to minimise the cost for each second stage random scenario \( w \). The constraint
links the first stage variables (chosen by the first linear program) and the amended equivalents of those variables impacted by the new second stage scenarios.

The temporal ‘stage’ structure means that the first stage variables \( x_a \) are chosen independently of any future scenario(s) (non anticipative property) and each decision variable \( y_i \) depends upon the particular scenario that occurs. Hence the approach produces an optimum solution for the base scenario (\( x_a \)) and a series of supplemental solutions (\( y_i \)) one of which will be optimal given the occurrence of the random event \( w \).

The consequence of this recourse model approach is not an absolute optimum solution but rather a series of solutions that are the ‘least worse’ that can accommodate the likely uncertainty.

In order to process the two linear relationships it is possible, provided that \( w \) is a discrete random variable to create a Deterministic Equivalent relationship [39], [43] which has the following form:

\[
\text{Min} C_1^T x_a + \sum_{i=1}^{N} P_i d_i^T y_i \hspace{1cm} (11)
\]

such that
\[
Ax_a = b \\
T_i x_a + W_i y_i = h_i \hspace{1cm} i = 1, ..., N \\
x_a \geq 0 \\
y_i \geq 0 \hspace{1cm} i = 1, ..., N
\]
Where $N$ is the number of scenarios and $P_i$ is the probability of scenario $i$ occurrence. Some observations regarding this deterministic form, relative to the energy system question being studied are:

- there is only one first stage $x_a$ decision and there are $i$ second stage $y$ decisions.
- the first stage $x_a$ decision cannot anticipate any one second stage scenario and must be feasible for each scenario, which means $x_a$ is ‘optimal’ for all analysed scenarios.
- The $T$ and $W$ matrices are repeated for each scenario. So as the number of scenario increase the size of the problem grows but not the structure of the LP.

Finally it should be noted that the above statement of the Deterministic Equivalent, when developed by Wets in 1974 [39], was contingent upon a series of assumption and underlying theorems that need to be considered as the methodology is applied.

### 2.3.3 Stochastic Programming and Recourse Models in Energy System Analysis

Examples of the use of Stochastic Programming with recourse models being applied in Energy System analysis is limited, however there are a wide range of examples in other areas of investigation. A great deal of Operations Research (OR) work is focussed on supply chain and transport ‘reference problems’ and as a consequence work examining the use of Stochastic Methods often examines transport and supply chain problems. In [44] the authors use a multi-stage recourse approach with the probability of scenario events set at either one or zero (warehouse exists or does not exist). In [45] the approach described models uncertainty in customer demands as a multi-stage scenario tree however this problem is then assessed by examining fixed states of each scenario rather than looking at the probability of a given scenario. In [46] a multi-stage scenario structure in production manpower planning using a form of Deterministic Equivalent structure. All three papers provide an insight into the form of structures that may be available to assess energy system problems.

Many works address issues involved in trading markets optimisation and strategy using Stochastic Programming. Pousinh [47] provides an example of this application that happens to be focussed on trading wind energy, (noting that the wind energy aspect is not the primary aspect of the study).
2.3. Mathematical Techniques to Accommodate the Run of Days Methodology

While this paper used a two stage model, it does not utilise the Deterministic Equivalent but rather explores upper and lower bounds of potential energy prices.

A number of papers use Stochastic Programming concepts to investigate day ahead planning for Hydro Power production. Papers such as [48], [49], address the complex question of managing Hydro Energy System water resources using stochastic programming concepts. The approaches use temporal stage scenario diagrams that are unique to the Hydro water problem formulation. These papers do not provided a direct read across to the problem that this thesis study addresses, however, they do provide insight into how these techniques can be used for multi-day problems if the scenario tree and objective function are structured to support that.

One example of Stochastic multi-stage analysis of energy systems is outlined by Zhou [50]. In this paper uncertainty in both demand and supply is addressed as an element of the optimisation process. Rather than use a Deterministic Equivalent approach this paper processes the first stage problem using an Genetic Algorithm (GA) and then deals with uncertainty in the second stage by way of developing likely uncertainty variable values via Monte-Carlo simulation. The approach is interesting in that it applies known mathematical techniques in a novel two stage process. The use of the GA approach in the first stage makes it difficult to address the Multi-Day problem outlined in Figure 1.
2.4 Literature Review Summary

At the completion of the initial literature review study the following has been concluded:

- For Small Islanded Energy Systems (as defined in the introduction) the incident energy that occurs on consecutive days (the run of days) has an impact on the suitability of any design and could be considered in any optimisation technique developed.

- One potential approach to address the run of days question is to use a two stage process (see Figure 2.2) where the system design is optimised for the incident energy that occurs on the Modal incident energy day and then iterated based on the likely incident energy of the following days.

- A candidate mathematical approach suited to addressing the outlined ‘run of days’ question is based on work by Wets and Higle and is generally referred to as ‘Stochastic Programming with Recourse (SPwR)”
Chapter 3

Stochastic Resource Optimisation - The Basic Methodology

3.1 Basic Method - Introduction

The Literature Review outlined a proposed form of analysis that addressed uncertainty in incident energy and weather, and consequently renewable generation capability and load requirements for a given geographic location, together with the concept of a multi day energy balance. This basic approach is shown as Figure 2.2 in the Literature Review. The Literature Review suggested that such an analysis could be supported using Stochastic Programming with Recourse (SPwR). In this chapter these concepts are tested using a simple energy system example.

The exploration of the proposed basic method is laid out in a series of discrete steps to ensure all aspects of the method can be clearly explored and so as expansion of the method to more complex systems can be reviewed in a structured manner. The following aspects of the method are outlined in this chapter:

- Form of the Objective Function
- Incident Energy as a Discrete Probability Distribution
- General form of the Recourse Model
- The Concept of Multi-Day Scenario Convolution
• Specific Example and Technical Models

• Results and Discussion
3.2 Basic Sample System and Form of the Objective Function

The application of the method described above is illustrated using the the basic PV / Battery / Generator system shown in Figure 3.1

![Diagram of the basic PV / Battery / Generator system](image)

Figure 3.1 - Simple System being examined.

Note that while this example involves an Islanded system this is not a necessary condition. Even in such a simple system the Internal Combustion Engine (ICE) could be replaced with a grid interconnection and the same optimisation questions regarding the incident radiated energy on days $n + i_{(i=1,2,3,...)}$ would exist.

For the simple system the optimisation question is summarised by the following objective function:

$$min C_{elec} = C_{pv} + C_{ice} + C_{batt} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldotted{...}
where

\[ C_{\text{elec}} = \text{the cost of electricity} \]

\[ C_{\text{pv}} = \text{the cost of the PV generated electricity} \]

\[ C_{\text{ice}} = \text{the cost of the generator generated electricity} \]

\[ C_{\text{batt}} = \text{the cost of the battery storage} \]

Note: Throughout this thesis the nomenclature \( \min C_{\text{elec}} \) should be read to mean, "\( \text{minimise the cost of electricity which is defined by the relationship} \) ="
3.3 Incident Energy as a Discrete Probability Distribution

The basic concept for the approach outlined in the Literature Review is to develop a methodology that allows design tools to address the impact of variations in consecutive days of incident Solar Energy and variations in load requirements on a day by day basis. Using general recourse models and deterministic equivalent structures to analyse energy system performance becomes a larger (but not more complex) problem as the number of second and third stage scenarios grow. In the analysis being studied the scenarios are based upon the incident solar radiation. Table 1 shows a years worth of recorded incident solar radiation data for Melbourne Airport, Melbourne Australia, as recorded by the Australian Bureau of Meteorology (www.bom.gov.au).

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>9.1</td>
<td>6.6</td>
<td>7.3</td>
<td>2.8</td>
<td>2</td>
<td>0.8</td>
<td>1</td>
<td>2.8</td>
<td>4.2</td>
<td>4.4</td>
<td>7.3</td>
<td>8.1</td>
</tr>
<tr>
<td>2nd</td>
<td>8.8</td>
<td>7.7</td>
<td>7.4</td>
<td>2.1</td>
<td>3</td>
<td>1.7</td>
<td>2.4</td>
<td>1.8</td>
<td>4.3</td>
<td>3.5</td>
<td>6.9</td>
<td>8.5</td>
</tr>
<tr>
<td>3rd</td>
<td>9.3</td>
<td>5.6</td>
<td>7.4</td>
<td>2.4</td>
<td>2.8</td>
<td>1.3</td>
<td>2.5</td>
<td>1.2</td>
<td>4.3</td>
<td>4.3</td>
<td>5</td>
<td>3.9</td>
</tr>
<tr>
<td>4th</td>
<td>9.3</td>
<td>7.2</td>
<td>7.2</td>
<td>5.2</td>
<td>2.1</td>
<td>1.1</td>
<td>1.3</td>
<td>2</td>
<td>3.4</td>
<td>6.2</td>
<td>6.8</td>
<td>0.9</td>
</tr>
<tr>
<td>5th</td>
<td>8.9</td>
<td>8.2</td>
<td>7.1</td>
<td>4.3</td>
<td>1.6</td>
<td>1.4</td>
<td>2.1</td>
<td>1.6</td>
<td>0.7</td>
<td>3.4</td>
<td>7.7</td>
<td>1.7</td>
</tr>
<tr>
<td>6th</td>
<td>8.8</td>
<td>7.9</td>
<td>6.6</td>
<td>4.7</td>
<td>3.5</td>
<td>1.5</td>
<td>2</td>
<td>2.8</td>
<td>0.9</td>
<td>3.6</td>
<td>7.6</td>
<td>6.4</td>
</tr>
<tr>
<td>7th</td>
<td>9.1</td>
<td>6.8</td>
<td>4.3</td>
<td>3.1</td>
<td>2.8</td>
<td>1.7</td>
<td>1.5</td>
<td>1.8</td>
<td>3.1</td>
<td>5.6</td>
<td>2.8</td>
<td>9.1</td>
</tr>
<tr>
<td>8th</td>
<td>7</td>
<td>8.2</td>
<td>5.2</td>
<td>4</td>
<td>2.9</td>
<td>2.3</td>
<td>2</td>
<td>2.6</td>
<td>4.2</td>
<td>5.6</td>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>9th</td>
<td>7.7</td>
<td>7</td>
<td>5.8</td>
<td>4.2</td>
<td>3.2</td>
<td>2.3</td>
<td>2.6</td>
<td>0.8</td>
<td>3.3</td>
<td>5.2</td>
<td>3.1</td>
<td>0.7</td>
</tr>
<tr>
<td>10th</td>
<td>9.2</td>
<td>6.4</td>
<td>3.6</td>
<td>4.6</td>
<td>3.1</td>
<td>1.9</td>
<td>2.6</td>
<td>3.4</td>
<td>3.5</td>
<td>0.9</td>
<td>4.3</td>
<td>6.6</td>
</tr>
<tr>
<td>11th</td>
<td>8.3</td>
<td>8</td>
<td>5.6</td>
<td>0.9</td>
<td>3</td>
<td>0.8</td>
<td>2.2</td>
<td>3.4</td>
<td>3.9</td>
<td>6.3</td>
<td>3.5</td>
<td>4.9</td>
</tr>
<tr>
<td>12th</td>
<td>4.9</td>
<td>6.8</td>
<td>6.3</td>
<td>4.3</td>
<td>1.8</td>
<td>0.7</td>
<td>2.3</td>
<td>3</td>
<td>3.8</td>
<td>6.8</td>
<td>1</td>
<td>8.9</td>
</tr>
<tr>
<td>13th</td>
<td>1.6</td>
<td>7.9</td>
<td>4.7</td>
<td>1</td>
<td>1.2</td>
<td>0.8</td>
<td>0.8</td>
<td>3.5</td>
<td>3.2</td>
<td>2.9</td>
<td>1.6</td>
<td>5.1</td>
</tr>
<tr>
<td>14th</td>
<td>7.9</td>
<td>5.2</td>
<td>4.9</td>
<td>2.8</td>
<td>2.1</td>
<td>1.4</td>
<td>0.7</td>
<td>1.7</td>
<td>3.1</td>
<td>4.3</td>
<td>3.2</td>
<td>7.6</td>
</tr>
<tr>
<td>15th</td>
<td>9</td>
<td>7.8</td>
<td>5.2</td>
<td>1.5</td>
<td>1.9</td>
<td>1.2</td>
<td>1.9</td>
<td>3.2</td>
<td>4.9</td>
<td>6.7</td>
<td>5.4</td>
<td>7.9</td>
</tr>
<tr>
<td>16th</td>
<td>9.1</td>
<td>6.9</td>
<td>2.8</td>
<td>4.1</td>
<td>1.4</td>
<td>1.9</td>
<td>1.9</td>
<td>3.8</td>
<td>1</td>
<td>4.7</td>
<td>3.7</td>
<td>7.3</td>
</tr>
<tr>
<td>17th</td>
<td>8.2</td>
<td>7.5</td>
<td>2.1</td>
<td>2.8</td>
<td>2</td>
<td>0.8</td>
<td>1.8</td>
<td>3.1</td>
<td>3.3</td>
<td>5.4</td>
<td>7.9</td>
<td>8.4</td>
</tr>
<tr>
<td>18th</td>
<td>5.1</td>
<td>7.6</td>
<td>3</td>
<td>1.8</td>
<td>1.7</td>
<td>1.7</td>
<td>2</td>
<td>2.5</td>
<td>2.7</td>
<td>7.1</td>
<td>8.1</td>
<td>8.3</td>
</tr>
<tr>
<td>19th</td>
<td>7.7</td>
<td>1.2</td>
<td>6.1</td>
<td>3.3</td>
<td>2.4</td>
<td>2.3</td>
<td>2.3</td>
<td>3</td>
<td>2.9</td>
<td>7.2</td>
<td>6.8</td>
<td>8.1</td>
</tr>
<tr>
<td>20th</td>
<td>8.9</td>
<td>7.6</td>
<td>5.4</td>
<td>3.2</td>
<td>0.8</td>
<td>2.3</td>
<td>0.8</td>
<td>3.1</td>
<td>3.3</td>
<td>4.8</td>
<td>1.7</td>
<td>7.2</td>
</tr>
<tr>
<td>21st</td>
<td>8.4</td>
<td>4.7</td>
<td>2.6</td>
<td>3.8</td>
<td>2.6</td>
<td>2.3</td>
<td>2.7</td>
<td>3</td>
<td>3.2</td>
<td>4.2</td>
<td>1.2</td>
<td>6.1</td>
</tr>
<tr>
<td>22nd</td>
<td>7.7</td>
<td>6.8</td>
<td>1.9</td>
<td>1.5</td>
<td>1.9</td>
<td>2.3</td>
<td>2.3</td>
<td>2.8</td>
<td>4.8</td>
<td>1</td>
<td>5.8</td>
<td>1</td>
</tr>
<tr>
<td>23rd</td>
<td>8.6</td>
<td>7.5</td>
<td>4.6</td>
<td>2.8</td>
<td>2.4</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
<td>3.6</td>
<td>2.3</td>
<td>3.4</td>
<td>2.7</td>
</tr>
<tr>
<td>24th</td>
<td>8.8</td>
<td>6.8</td>
<td>4.1</td>
<td>2.5</td>
<td>2.5</td>
<td>1.5</td>
<td>1.7</td>
<td>2.3</td>
<td>4.1</td>
<td>4.1</td>
<td>2.7</td>
<td>9.4</td>
</tr>
<tr>
<td>25th</td>
<td>3.3</td>
<td>5.7</td>
<td>5.1</td>
<td>3.4</td>
<td>2.1</td>
<td>2.1</td>
<td>2.4</td>
<td>2.6</td>
<td>5.5</td>
<td>3.6</td>
<td>7.6</td>
<td>8.8</td>
</tr>
<tr>
<td>26th</td>
<td>4.2</td>
<td>3.4</td>
<td>5.4</td>
<td>0.9</td>
<td>2.3</td>
<td>2.6</td>
<td>3.7</td>
<td>4.8</td>
<td>3.1</td>
<td>7.3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>27th</td>
<td>6.5</td>
<td>2</td>
<td>4.1</td>
<td>3.3</td>
<td>1.5</td>
<td>1.6</td>
<td>2.4</td>
<td>3.5</td>
<td>5.6</td>
<td>4.2</td>
<td>6.7</td>
<td>7.5</td>
</tr>
<tr>
<td>28th</td>
<td>7.9</td>
<td>3.8</td>
<td>2.9</td>
<td>1.8</td>
<td>2.4</td>
<td>2.2</td>
<td>1.4</td>
<td>3.1</td>
<td>4</td>
<td>2.1</td>
<td>0.6</td>
<td>8.1</td>
</tr>
<tr>
<td>29th</td>
<td>5.5</td>
<td>2.4</td>
<td>2.8</td>
<td>2.1</td>
<td>1.3</td>
<td>2.9</td>
<td>2.6</td>
<td>5.7</td>
<td>4.3</td>
<td>2.7</td>
<td>9.6</td>
<td></td>
</tr>
<tr>
<td>30th</td>
<td>8.6</td>
<td>2.9</td>
<td>2.8</td>
<td>0.7</td>
<td>1.9</td>
<td>2.7</td>
<td>2.8</td>
<td>3.9</td>
<td>7.5</td>
<td>9.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31st</td>
<td>3.8</td>
<td>2.5</td>
<td>1.7</td>
<td>2.9</td>
<td>4.5</td>
<td>5.1</td>
<td>9.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 Australian Bureau of Meteorology (BOM) Yearly Incident Energy data (kWhr/day) for Melbourne Airport 2013.

Summarising Incident Energy Data
Table 3.1 illustrates how incident solar energy varies across the course of a year, in this case Melbourne Airport during 2103. It would be possible to assume that each particular incident energy level (to one decimal place) as listed in Table 3.1 was a discrete scenario. This would result in an increase in the number of scenarios examined (note that it is the nature of the stochastic approach that examines a ‘baseline case’ and then a range of ‘probable’ events) and a subsequent large (but not necessarily complex) deterministic equivalent problem. Given that the aim of the study is to produce a design tool the question becomes “what is the minimum number of discrete scenarios, expressed as bands of incident energy ranges, that have a material impact on the chosen design.”. Embedded within this statement is the notion that the energy systems are designed in discrete steps (the designer can have 10 PV panels or 12 panels or 13 panels, options are for have a 400 ltr water tank or a 450 ltr tank etc.). Consequently it is possible to be able to establish the minimum incident energy width (range) and hence the number of first stage scenarios that have a real world impact on a given design solution.

For this section of the study the width of the incident energy intervals is not crucial since the structure of the objective functions and constraint functions is independent of the total number of scenarios. Consequently to support this preliminary exploration of the method the following simple discrete incident energy distribution will be used.
3.3. Incident Energy as a Discrete Probability Distribution

Figure 3.2 Sample Incident Energy Distribution - Melbourne Airport 2013 (source BOM)

3.3.1 Other Issues Related to the use of Incident Energy Distributions

The structure proposed, whereby incident energy is treated as a discrete probability distribution introduces some additional features that are advantageous to the overall aims of the study;

- the size of the intervals can be scaled up and down to balance the scale of the analysis against the advantages from a design solution perspective.
- many years of incident energy data can be combined e.g Melbourne Airport data for 10 years could be ‘pre-processed’ into a distribution with no change to the basic solution approach.
- patterns in the daily distribution of incident solar energy, that have the potential to skew the optimisation solution can be identified, removed from the distribution and then a new answer calculated, producing a style of second pass sensitivity analysis.

All of these issues will be explored in later sections of this study.
3.4 General Form of the Recourse Model

The general form of a Stochastic Programming general recourse optimisation problem was stated previously as follows:

\[
\text{Min} C^T x_a + E_w Q(x_a, w) \ldots \ldots (2)
\]

such that

\[
Ax_a = b \\
x_a \geq 0
\]

Where \(E_w\) stands for the 'expectation' or probability and \(Q\) is referred to as the 'correction cost' or 'recourse function' which can be defined as

\[
Q(x_a, w) = \text{mind}(w)^T y_i
\]

such that

\[
T(w)x_a + W(w)y_i = h(w) \\
y_i \geq 0
\]

Where

\(x = \) the first stage variables
\(y = \) the second stage variables
\(A\) is the first stage constraint matrix and represents decisions that must be made before the values of the uncertainty parameters are known
\((w)y_i\) is the vector of second stage control decisions that represent the recourse actions that can be taken after the uncertain parameters are observed
\(T(w)\) and \(B(w)\) are random variable matrices that allow second stage costs to be established.
The first linear relationship aims to minimise the first stage known costs $C^T_1 x_a$, plus the expected (recourse) costs $Q(x_a, w)$ over all possible scenarios, assuming that the first stage constraints $Ax_a = b$ are met.

The second linear relationship introduces a new set of variables, that can be used to minimise the cost for each second stage random scenario $w$. The constraint $T(w)x_a + W(w)y_i = h(w)$ links the first stage variables (chosen by the first linear program) and the amended equivalents of those variables impacted by the new second stage scenarios.

In the energy system example the Modal day optimal solution is the "First Stage term".

$E_w Q(x_a, w)$ is the the expected value of the second stage problem where second stage variable are calculated for each $w$. In the energy system example the Less than Modal Incident Energy days are addressed by the expected value terms

$w_a$ is a discrete random variable where the probability of its value $P_w = P(w_a = w)$. In the energy system example this discrete random variable is incident energy, (which is broken into discrete bands as an pre-condition of the analysis.

As outlined in the previous chapter, provided that $w_a$ is a discrete random variable then (2) can be shown to expand to the Deterministic Equivalent [39]:

$$\text{Min} C^T_1 x_a + \sum_{i=1}^{N} P_i d^T_i y_i, \ldots, (3)$$

such that

$Ax_a = b$

$T_i x_a + W_i y_i = h_i, \ldots, i = 1, \ldots, N$

$x_a \geq 0$
$y_i \geq 0 \quad \forall i = 1, \ldots, N$

Where $N$ is the number of scenarios and $P_i$ is the probability of scenario $i$ occurrence.
3.5 Stochastic Equation Forms for the Sample System One Day Post Modal

A common method for representing recourse Stochastic Programming problems is to use scenario trees (Figure 3.3). This is outlined by Casey in [51]. If for the days $I_{rad_{m(n+1)}} > I_{rad_{m(n)}}$ we quantify the possible range of incident energy into discrete intervals $i = 1...n$ then the situation can be represented by the following scenario tree.

As the technique being explored is aimed at being a design tool it is necessary to declare those variables which represent ‘initial design decisions’ and those variables that depend on system running.
conditions. In this example the variables:

\( x_1 \) Size of PV array \((m^2)\)

\( x_3 \) Battery Size \((\text{kWh})\)

are initial design decisions and the parameter

\( x_2 \) ICE Run-Time (hours per day)

is able to be varied at a later date once the probability and quantum of the reduced incident energy days is understood. In order to support the standard recourse problem form a new variable is declared

\( y_{2i} \) ICE Run-Time (hours per day) for the post modal day

Transferring these defined variables back into the deterministic form shown as equation (3) gives:

\[
\min C_{elec} = ax_1 + bx_2 + cx_3 + \sum_i P_i y_{2i} + d \ldots \ldots \ldots \ldots (4)
\]

\( \sum_i P_i y_{2i} \) is the arithmetic sum of each second stage scenario generator runtime as factored by the probability of that scenario, where \( i = 1, 2, \ldots, n \) number of energy range days less than modal and

\( a, b \) and \( c \) are cost scaling factors for the PV capital cost, generator running cost and battery capital cost. While \( d \) is a factor covering all non variable fixed capital costs, which in this case includes the generator capital cost.

This can then be optimised using the following constraints:
3.5.1 Load Constraints for the Sample System

Modal day

The constraint is set such that on the Modal day the system is just able to supply all loads from the PV system, with no ICE runtime required. So:

\[ P_{\text{load}(0)} = e_0 x_1 + f x_2 \ldots \ldots \ldots \ldots \ldots (5) \]

where

- \( e_0 \) is the incident energy (factored by PV array efficiency) for the Modal day and,
- \( f \) is the size of the system generator in kWh

NOTE:

- \( e_0 \) is a factor that includes a scalable performance metric for the PV array that would be determined by review of literature or by test. This is discussed in more details in later chapters

- \( f \) is chosen by the designer as a fixed element. In later more complex examples the reason to chose a fixed machine size (rather than make this a further variable) will be shown to be a ‘constraint’ associated with the ability to generate heat in a suitable time period.

non Modal day

On the non Modal days the total load can be supplied by PV generation, ICE running or from extra energy stored on the Modal day. This leads to the following constraint:
Chapter 3. Stochastic Resource Optimisation - The Basic Methodology

\[ P_{\text{load}(i)} = e_i x_1 + f y_{2i} + \text{energy stored in the Modal day} \]

\[ P_{\text{load}(i)} = e_i x_1 + f y_{2i} + (x_3 - g P_{\text{load}(0)}) \]

\[ P_{\text{load}(i)} + g P_{\text{load}(0)} = e_i x_1 + f y_{2i} + x_3 ...........(6) \]

where

- \( e_i \) is the incident energy (factored by PV array efficiency) for the non-Modal day(s) and,
- \( g \) is a factor that determines the percentage of the \( P_{\text{load}(0)} \) that must be stored. This is a simple ratio style factor that accounts for the notion that the total incident energy for a day is not evenly distributed across 24 hours but is available for direct load support for only a few hours of any given day.

### 3.5.2 Battery Constraints for Sample System

From the Modal day, as an absolute minimum the battery must store at least a specific fraction \( g \) of the total daily load:

\[ x_3 \geq P_{\text{Load}(0)} x g \]

Substituting (5)

\[ 0 \geq e_0 g x_1 + f g x_2 - x_3 .................(7) \]

Note that this ‘inequality constraint’ is expressed in this particular form as the intent is to be able to use the simple LINPROG routine in Matlab that requires the following form:

\[ \text{Min} f^T x \]

such that
\[ A \boldsymbol{x} \leq b \]
\[ A_{eq} \boldsymbol{x} = b_{eq} \]
\[ l_b \leq \boldsymbol{x} \leq u_b \]
3.6 Stochastic Equation Forms for the Sample System- Two Days Post Modal

The above discussion provides a method to accommodate the idea that the incident energy on day (n+1) (i.e. the day following the modal day) is less than the modal day (n). This is expressed mathematically as $I_{rad}^{m(n+1)} < I_{rad}^{m(n)}$.

The next design driver is the concept that there could be two or more consecutive days where the incident energy is less than the modal day. This is expressed mathematically as;

$$I_{rad}^{m(n+j)} < I_{rad}^{m(n)},\ ...\ j = 2, 3, \ldots k$$

Note that this is the critical design case since it is the total run of days less than modal that is important to the design approach, once any consecutive day has incident energy equal to or greater than the modal day $I_{rad}^{m(n+j)} > I_{rad}^{m(n)}$ then the system will generate more energy than the loads require, the energy capability of the system will "reset" and the optimisation that was valid for the modal day is valid again. There are two ways to deal with this critical design case.
3.6. Stochastic Equation Forms for the Sample System—Two Days Post Modal

3.6.1 Multi-Stage Recourse

The scenario tree for this approach is illustrated below

![Scenario Tree Diagram]

This scenario tree can be processed using a range of approaches [52]. Mathematically robust approaches are suggested in [51] and [40]. Another technique used is to work out the optimum solution and cost for each individual branch and then statistically combine the results then use statistical measure (such as the variance) to establish the most likely solution [45]. These approaches are mathematically viable but complex and result in very large processing requirements as the number of non Modal incident energy bands increases.

It is again emphasised that simplicity of processing is a requirement since this work is intended to be able to be used as a design tool and transparency is an important aspect of ensuring design tools are understood by users. The primary aim is for users to be clearly able to see the impact of input changes of outputs and this may not necessarily be the case if complex statistical methods are employed.
3.7 The Concept of Multi-Day Scenario Convolution

In the approach being proposed, while all days being analysed have less incident energy than the Modal day, the incident radiated energy on any day can be viewed as statistically independent from the preceding or following day. This suggests that simple convolution can be used to combine the three level scenario tree shown in Figure 4 back to into a two level scenario structure as shown in Figure 3. While the ‘derived’ scenario structure will have more nodes than the original $I_{rad}^{m(n+1)} < I_{rad}^{m(n)}$ scenario it will now be a single level structure able to be solved using the sample simple mathematical approach. The following scenario tree represents the situation $I_{rad}^{m(n+j)} < I_{rad}^{m(n)}$, $j = 2, 3, \ldots, k$.

![Multi-Day Convoluted Scenario Tree](image)

Figure 3.5: Multi-Day Convoluted Scenario Tree

where:

$$e_z = e_i + e_j$$
3.7. The Concept of Multi-Day Scenario Convolution

\[ P_{e2z} = P_{e2i} \cdot P_{e3j} \]

\[ P_{Load(z)} = P_{load(i)} + P_{Load(j)} \] and the resultant objective function becomes the same form as the \( Irad_{rn(n+1)} \) case

\[ \min C_{elec} = ax_1 + bx_2 + cx_3 + \sum_z P_z y_{2z} \]

with the resultant constraints being of the same form as previously discussed.

This simple convolution approach is possible because of the nature of the formulation of the original stochastic objective function and the underlying assumptions that the energy load in the less than incident energy days can only be meet by additional energy stored on the modal day, by sub-modal day generator running time or a combination of both. Once this assumption is made then it is possible to combine strings of consecutive less than Modal days together provided that the total incident energy and total load for those days is aggregated.

This approach will be explored in later chapters and in Appendix A.
3.8 Specific Example and Technical Models

Earlier equation 4 provided the basic form of the objective function:

\[
\min C_{elec} = ax_1 + bx_2 + cx_3 + \sum_i P_i by_{2i} + d \cdots \cdots \cdots \cdots \cdots (4)
\]

In optimisation approaches in general and specifically when using stochastic programming with recourse parameters are described as follows:

3.8.1 Decision Parameters

In this example the decision parameters are

- \(x_1\) Size of PV array (\(m^2\))
- \(x_2\) ICE Run-Time (hours per day)
- \(x_3\) Battery Size (kWh)

note: this is true if the generator size is not a variable, which is the case in the process being developed.

3.8.2 Input Parameters

The 'Input Parameters' are used to establish the cost factors (\(a, b, c\) and \(d\)) in the objective function. These are in this example defined as follows:

- \(C_{batt/n}\) Battery cost per kWh
- \(C_{pv/n}\) PV cost per \(m^2\)
- \(ICE_{run/n}\) Cost per hour (i.e. fuel cost)
3.8. Specific Example and Technical Models

\[ ICE_{\text{cap}} = \text{Capital cost (engine plus generator)} \]
\[ ICE_{\text{gen}} = \text{ICE Generator Size (kW)} \]

\[ SL = \text{System Life (years)} \]

3.8.3 Input Variables

The "input variables" are declared by the designer and are factors that are variable from a design perspective but are fixed in advance of the analysis.

\[ P_{\text{load}} = \text{Electrical load (kWh/day)} \]

\[ I_{rad_{m}} = \text{Total Incident Radiation for Modal day (kWh/m}^2\text{)} \]

\[ \mu_{pv} = \text{PV Panel efficiency} \]

\[ t_{pvhrs} = \text{assumed hours of sunshine where array produces hourly load.} \]

\[ DoD_{\text{spec}} = \text{Depth of Discharge (DoD) required to meet life estimate (manufactures estimate)} \]

\[ B_{L} = \text{Battery Life} \]

\[ r = \text{assumed interest rate} \]

Note that the analysis approach assumes that the designer can vary the input parameters and input variables as a way to conduct a sensitivity style analysis. It would be possible to incorporate some of these variables into the optimisation but other important capabilities of the optimisation approach may be lost. Examples of how these variables can be captured in the optimisation, in the form of constraints, is explored in later chapters.

For this first example the following parameters are used:
Table 3.2 Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{min}}$</td>
<td>Battery cost per kWh</td>
<td>A$1000</td>
<td>Based on LG Lithium 5 kWh battery</td>
</tr>
<tr>
<td>$C_{\text{PV}}$</td>
<td>PV cost per m²</td>
<td>A$250</td>
<td>Based on Winaico WST260</td>
</tr>
<tr>
<td>$C_{\text{ICE}}$</td>
<td>ICE Cost per hour (i.e., fuel cost)</td>
<td>A$2.37</td>
<td>Based on Paguro P6000</td>
</tr>
<tr>
<td>$P_{\text{ICE}}$</td>
<td>ICE Generator Size (kW)</td>
<td>5 kW</td>
<td>Driven by Paguro P6000 6.5 kW machine</td>
</tr>
<tr>
<td>$\eta_{\text{PV}}$</td>
<td>PV Panel efficiency</td>
<td>0.14</td>
<td>Based on Winaico WST260</td>
</tr>
<tr>
<td>$SL$</td>
<td>System Life (years)</td>
<td>20</td>
<td>aligns with panel performance</td>
</tr>
<tr>
<td>$t_{\text{hours}}$</td>
<td>assumed hours of sunshine where array produces hourly load</td>
<td>6</td>
<td>an initial low estimate, conservative.</td>
</tr>
<tr>
<td>$\text{DoD}_{\text{spec}}$</td>
<td>DoD required to meet life estimate (manufacturers estimate)</td>
<td>80</td>
<td>Manufactures Specification</td>
</tr>
<tr>
<td>$BL$</td>
<td>Battery Life</td>
<td>10 years</td>
<td>Manufactures Specification</td>
</tr>
</tbody>
</table>

3.8.4 System Relationships

The factors a, b, c are used to translate the equipment characteristics shown in Figure 3.2 into the daily equivalent costs that match the form of the objective function equation [3]. Daily cost are used as the loads, and incident energy and probability of incident energy are all analysed on a per day basis. Costs for long-life capital purchases are calculated using Equivalent Annual Cost which is then reduced to a daily cost as follows:

\[
EAC = \frac{\text{InitialCost}}{A_{SL,r}}\quad (8)
\]

where $A_{SL,r}$ is the Annuity Rate
where $SL$ is the System Life in years and $r$ is the assumed long term interest rate.

It was decided the the cost comparison would be done on an COE per annum on a present year baseline. This is a highly conservative approach as it allows the COE for the CHP system to be compared with the COE from the competing grid energy, in baseline dollars. Across the course of the project the COE for the CHP system stays at the baseline dollar amount, so effectively drops each year, whereas the cost of grid energy notionally rises with inflation.

The ‘Annuity Rate’ is a way to establish the total cost of the capital investment (today dollar capital cost plus interest charges assuming a payback equal to the system life) the distribute this equally across the system life years. (keeping in mind this is all in baseline dollars). This approach is used in both the Hybrid 2 [27] and DERCAM [29] model as way to distribute capital costs.

The second construct is where there is a component life that is less than the System Life. In this case the baseline cost is factored up by the ratio. Again this is conservative since the mid life capital purchase will be paid in later year dollars but accounted for in today dollars.

\[ a = \frac{C_{pv}/n}{A_{SL,r}} \times (SL/PL) \] ........(10) (This is an annualised cost per m²)

\[ b = 365x ICE_{run}/n \] ...(10) is the yearly fuel cost which assumes a defined generator / ICE combination

\[ c = \frac{C_{batt}/n}{A_{SL,r}} \times (SL/BL) \] ......(11) (This is an annualised cost per kWh)

Note: The battery life concept is simplified because the analysis assumes the battery size is made ideal for the Modal day (see constraint equations) Moving away from the Modal day changes (for some forms of battery chemistry) the battery life, and hence the total battery cost which is ignored at this point in the development of the basic relationships
the factors e, f and g are ..........

\[ e_0 \] is the incident energy (factored by PV array efficiency) for the modal day,
\[ e_0 = I_{rad_m} x \mu_{pv} \] .....(12)

Note \( \mu_{pv} \) is a scaling factor that combines the specified efficiency of a given PV array together with a second factor that account for the variation in performance of the panel across the course of the day. This will be left as a simple scaling factor in this example and explored in a more representative fully detailed example in later chapters.

\( f \) is the size of the system generator in kWh .....(13)

\( g \) is a factor that determines the percentage of the \( P_{load(0)} \) that must be stored, that is the proportion of the load that occurs on the Modal day when the PV cannot generate energy ( i.e.the time after the sun has gone down).
\[ g = [(24 - t_{phhrs})/24] / (DaD_{spec}) \] .....(14)

\[ d = ICE_{cap} / (A_{SL,r}) \times (SL/MachineLife) \] .....(15) annualised capital cost of the ICE

### 3.8.5 Specific Weather Scenarios

Figure 3.2 shows the incident solar energy daily probability distribution for Melbourne Airport in 2013. The Modal day incident radiant energy for this example was \( 2.8 MJ/m^2 \). This means that there are three classes of days with less incident energy than the modal day. This is represented by the following scenario tree:
Figure 3.6 Example Single Stage Scenario Tree
Chapter 3. Stochastic Resource Optimisation - The Basic Methodology

3.9 Worked Example

3.9.1 Cost Function and the f matrix

Using the baseline equipment data in table 3.2 and the the scenario tree in Figure 3.6 is formatted into the defined equations as follows:

assuming an interest rate of 6% and a design life of 20 years

\[ A_{SL,r} = \frac{1 - 1/(1 + r)^{SL}}{r} \]
\[ A_{SL,r} = \frac{1 - 1/(1 + 0.6)^{20}}{0.6} \]
\[ A_{SL,r} = 11.5 \]

assuming \( C_{pv/n} = \$250/m^2 \) then

\[ a = \frac{C_{pv/n}}{(A_{SL,r})} \times \frac{SL}{PL} \]
\[ a = 250/11.5 \times (20/20) \]
\[ a = 21.74 \]

for the Paguro machine there is 0.35 L/kWh quoted fuel burn. It is 6.5 kW machine therefore one hour running = 2.275 L. @ $1.20 per L = $2.73/hr

\[ b = 365 \times ICE_{run/n} \]
\[ b = 365 \times 2.73 \]
\[ b = 997 \]

assuming a $1000 per kWhr battery and a 10 year battery life

\[ c = \frac{C_{batt/n}}{(A_{SL,r})} \times \frac{SL}{BL} \]
\[ c = 1000/11.5 \times (20/10) \]
\[ c = 174 \]
Based on the $11,000 cost of the Paguro machine

\[ d = \frac{ICE_{\text{cap}}}{A_{\text{SL,r}}} \times (SL/Machine\ Life) \]
\[ d = \frac{11000}{11.5} \times \frac{20}{20} \]
\[ d = 956 \]

Based on the detail in the scenario tree

\[ P_{e1} = 0.03385 \]
\[ P_{e1} = 0.17494 \]
\[ P_{e1} = 0.22739 \]

Substituting back into the cost function, equation (4) results in the following:

\[ \min C_{\text{elec}} = ax_1 + bx_2 + cx_3 + \sum_i P_i y_{2i} + d \]
\[ \min C_{\text{elec}} = 21.74x_1 + 997x_2 + 174x_3 + 0.03385 * 997y_{21} + 0.0.17494 * 997y_{22} + 0.22739 * 997y_{21} + d \]
\[ \min C_{\text{elec}} = 21.74x_1 + 997x_2 + 174x_3 + 33.74845y_{21} + 174.415y_{22} + 226.797y_{21} + d \]

This results in an \( f \) matrix as follows:

\[
\begin{bmatrix}
21.74 \\
997 \\
174 \\
33.748 \\
174.415 \\
226.797
\end{bmatrix}
\]

**3.9.2 Load Constraints and the \( A_{eq} \) Matrix**

To compete the load constraints:

\[ f = ICE_{\text{gen}} \] which in this case is equal to 5 kW
assuming $t_{pvhrs}$, which is the total hours that the PV can supply load directly without storage, is 6 hours, and assuming a Battery Depth of Discharge (DOD) limit of 80% (typical of modern design Li Ion batteries) then:

$$g = \left[\frac{(24 - t_{pvhrs})}{24}\right]/(DOD_{spec})$$

$$g = \left[\frac{(24 - 6)}{24}\right]/(0.8)$$

$$g = 0.9375$$

The PV conversion factors are defined as assuming a total conversion efficiency ($\mu_{pv}$) of 0.14:

$$e_0 = I_{rad} \times \mu_{pv}$$

$$e_0 = 2.8 \times 0.14 = 0.392$$

$$e_1 = 0.4 \times 0.14 = 0.056$$

$$e_0 = 1.4 \times 0.14 = 0.196$$

$$e_0 = 2.4 \times 0.14 = 0.336$$

Substituting back into equations (5) and (6) results in the following equality constraints:

$$P_{load(0)} = e_0 x_1 + f x_2$$

$$P_{load(0)} = 0.392 x_1 + 5 x_2$$

and

$$P_{load(1)} + g P_{load(0)} = 0.056 x_1 + 5 y_{21} + x_3$$

$$P_{load(2)} + g P_{load(0)} = 0.196 x_1 + 5 y_{22} + x_3$$

$$P_{load(3)} + g P_{load(0)} = 0.366 x_1 + 5 y_{23} + x_3$$

This results in $A_{eq}$ being 4 x 6 matrix.
3.9. Worked Example

\[ A_{eq} = \begin{pmatrix}
0.392 & 5 & 0 & 0 & 0 & 0 \\
0.056 & 0 & 1 & 5 & 0 & 0 \\
0.196 & 0 & 1 & 0 & 5 & 0 \\
0.336 & 0 & 1 & 0 & 0 & 5
\end{pmatrix} \]

and the \( b_{eq} \) matrix is defined as:

\[ b_{eq} = \begin{pmatrix}
P_{load(0)} \\
P_{load(1)} + gP_{load(0)} \\
P_{load(2)} + gP_{load(0)} \\
P_{load(3)} + gP_{load(0)}
\end{pmatrix} \]

3.9.3 Storage Constraints and the \( A \) Matrix

As this is a simple system there is a single storage constraint as shown in equation (6)

\[
0 \geq e_0 x_1 + f g x_2 - x_3 \\
0 \geq 0.392 \times 0.9375 x_1 + 5 \times 0.9375 x_2 - x_3 \\
0 \geq 0.3675 x_1 + 4.6875 x_2 - x_3
\]

This results in \( A \) being a 6 x 1 vector

\[ A = \begin{pmatrix}
0.3975 & 4.6875 & -1 & 0 & 0 & 0
\end{pmatrix} \]

and \( b = 0 \)

3.9.4 Simulation Results

The one day post modal solution defined above was processed using the LINPROG function in MATLAB and the load was set as equivalent on the Modal and non Modal days. The following results
were obtained.

Table 3.3 Simple System Results

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Load 20 kWh/day</th>
<th>Load 40 kWh/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = \text{PV in m}^2$</td>
<td>51</td>
<td>102</td>
</tr>
<tr>
<td>$X_2 = \text{Gen Run Time in hrs (Modal day)}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X_3 = \text{Batt Size in kWh}$</td>
<td>20.3</td>
<td>40.6</td>
</tr>
<tr>
<td>$Y_{21} = \text{Gen Run Time in hrs (Prob 0.034)}$</td>
<td>3.4</td>
<td>6.7</td>
</tr>
<tr>
<td>$Y_{22} = \text{Gen Run Time in hrs (Prob 0.175)}$</td>
<td>2</td>
<td>3.8</td>
</tr>
<tr>
<td>$Y_{23} = \text{Gen Run Time in hrs (Prob 0.228)}$</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The results in Table 3.3 show that the ‘optimal’ initial design decision regarding the size of the PV array varies when Non Modal days are factored into the solution using the Stochastic Programming methodology.

The results are interpreted as follows:

- The values for $x_1$ and $x_3$ are slightly greater than the minimum required to meet the model day constraints. This illustrates that these values have been amended by consideration of the non modal days.

- The values for $y_{21}$, $y_{22}$, and $y_{23}$ represent the generator running time that will result if those less than modal days occur (i.e. there is a probability of 0.034 that on any given day a system with a 51.02 m$^2$ PV array and a 20.3 kWh battery will need to run the generator for 3.37 hours
In the literature discussion it was noted that

"The consequence of this recourse model approach is not an absolute optimum solution but rather a series of solutions that are the 'least worse' that can accommodate the likely uncertainty.

This example illustrates this concept. The design solution chosen for the PV size and battery analysis may not be the minimum possible (hence cheapest) solution for most common (Modal day) but it does represent the least worse solution when the probability of non-modal days is considered.
3.10 Validation of Results

The approach developed needs to be validated. Validation of an optimisation technique is a non-trivial exercise and much published work reviewed did not conduct the validity of a particular approach but the performance (processing time) of one approach relative to another. Validation for the simple example needs to demonstrate that the stochastic optimisation methodology and the form of the equations developed produce a design solution (size of PV array, size of battery and ICE running time) that results in the lowest possible Cost of Energy (COE) over a given time period. If this can be shown to be the case then the other features of the approach developed can be exploited knowing that the basic optimisation is valid.

For the simple electrical load only system the Cost of Energy (electricity) over the year is determined by the amortised capital cost of the PV, battery and ICE generator and the running cost of the generator. In simple terms the larger the PV array and the battery the less running time is required in the diesel generator over a given year. Consequently there is a combination of component sizes and generator running time that represents the least cost of energy for a given year. The optimisation approach developed is trying to establish this least cost combination using a particular approach (Stochastic programming).

Another approach to identify the lowest cost configuration, is to ‘manually’ work out the cost of operation of a range of configurations. If enough configurations are examined eventually the least cost configuration will be identified. This approach is possible because the system being examined in this section is simple. This approach of sampling system combinations is not possible for the more complex systems examined later in this work but it can be used as a form of validation for the basic system. The following section summarises how this ‘manual analysis’ was conducted:

3.10.1 Validation Methodology

It is necessary to have some methodology to search through possible solution options is a systematic manner. The approach adopted for the verification study is shown in Fig 3.8
3.10. Validation of Results

It is important to note that in this process the ‘initial incident energy’ is not a variable but rather just a point of reference used to create a system configuration to be tested. Using ‘steps’ of initial incident energy days to design a candidate system configuration just provides a structured way to create candidate configurations that can then be assessed to establish a yearly cost.

Once the initial incident energy reference point is chosen the PV and battery size is chosen using the following relationships:

The PV array is sized assuming that on this day all the load must be generated by the PV array (i.e. no generator running). This constraint results in the following relationships:
\[ PV \text{Array size}_{m_2} = \text{P}_{\text{ref load}} \times (I_{\text{rad initialise}} \times \mu_{\text{pv}}) \ldots (16) \]

where
\[ P_{\text{ref load}} = \text{The reference load which is assumed to be constant for each day of the year.} \]
\[ I_{\text{rad initialise}} = \text{Total Incident Radiation for Modal day (kWh/m2)} \]
\[ \mu_{\text{pv}} = \text{PV Panel efficiency} \]

Once the PV Array size has been determined the required battery size is calculated using the same constraint that was used in the stochastic programming structure, i.e.

assuming \( t_{\text{pv hrs}} \), which is the total hours that the PV can supply load directly without storage, is 6 hours, and assuming a Battery Depth of Discharge (DOD) limit of 80\% then:

\[ g = \frac{[(24 - t_{\text{pv hrs}})/24]}{(\text{DoD spec})} \]
\[ g = \frac{[(24 - 6)/24]}{0.8} \]
\[ g = 0.9375 \]

and then:

\[ \text{Batt Size} = \text{P}_{\text{ref load}} \times g \ldots (17) \]

This approach is necessary to ensure that the ‘constraints’ applied in the stochastic case are equivalent to the verification case so as an ‘apples with apples’ comparison is being conducted.

Once the PV array size and battery size is chosen this ‘system configuration’ is then tested for each
day of the reference year (and site) which in this case is Melbourne Airport 2013. The incident energy for each day of 2013 at Melbourne Airport is shown in Table 3.1. For each day of the year the ICE run time is calculated using the energy balance relationship shown and assuming the same 5 kW generator as the stochastic example.

\[ \text{Generator\text{\_}W\text{\_}hr} = P_{\text{ref\_load}} - P_{\text{V\_this\_day}} - \text{StoredEnergy\_from\_previous\_day} \ldots (18) \]

The total generator running time for the year is the sum of each individual days running. This then gives a total running hours for the year.

The total cost for the year is then calculated using the same relationships that where used in the Stochastic example and the baseline cost data outlined in Table 3.2.

\[ \text{Yearly\_Cost} = \left( \frac{250 \times \text{PV\_Array\_size}}{\text{SL}} \right) + \left( \frac{1000 \times \text{Batt\_Size}}{\text{SL}} \right) + \left( \frac{\text{Total\_runtime\_hrs} \times 2.37}{\text{SL}} \right) \ldots (19) \]

where SL is the system life. This is a simplified relationship but is suitable for the purpose of verification so long as the cost of the system established by the stochastic programming approach is calculated the same way.

The results of the verification approach are shown in the following graph. Note that the horizontal axis uses the design reference incident energy as this energy maps to a PV /battery size using equations (17) and (18).
The results are interpreted as follows. In Figure 3.10 the red line is the calculated annual cost of energy for a range of different system configurations. Those configurations are established by designing a system for a given day of incident energy, which is shown on the X-axis. The ‘Y’ marker is the annual yearly cost (in AUD) for the system configuration chosen using the stochastic programming methodology.

The verification demonstrates the following:

- The high yearly cost of the systems where the analysis commenced by considering low incident energy days occurs since these days produce designs with excessive capital investment in PV and batteries.

- As the reference incident energy days increase (as we move right along the x-axis) the resultant designs have smaller and smaller PV and battery capacity which means across the year the amount of generator running time increases and hence yearly costs slowly increase.
3.10. Validation of Results

- The two impacts noted above balance out and achieve a minimum cost solution around the Modal incident energy day .......this validates the first assumption of the stochastic methodology which was to use the Modal day for the first stage assumption.

- in both load cases (20 kWh/day and 40 kWh/day) the stochastic method has produced a design solution that is not quite the minimum possible but is a compromise minimum that takes into account the probability of poor energy days.

In summary this ‘manual searching for a minimum’ approach can be used for a simple system. It shows that the stochastic technique is producing the correct minimum and hence as long as the form of the stochastic equations stays consistent as system complexity increases then the approach should remain valid.

This is an important finding because:

- as the systems being modelled become more complex the ‘manual’ search approach would become too complex to be practical

- the form of the equations being used have advantages when viewed as a design tool and its essential that the equations support finding a minimum as well as facilitating the required design processes.
3.11 Observations from the Simple System Stochastic Solution

A number of capabilities not present in existing modelling approaches (i.e. HOMER and DERCAM) have been accommodated during the development of the simple stochastic model. These are summarised as follows:

3.11.1 Incorporating Weather Variability

The form of the stochastic equations adopted allow the ‘pre-processing’ of weather data since the probable variation in incident solar energy (which is the key environmental variable for renewable microgrids) is incorporated as the combination of a simple discrete input energy and a probability of occurrence of that energy. In this example the weather data is taken from a single year and has been divided into 10 discrete levels. This is an arbitrary breakdown. It would be possible to break down the data into smaller and smaller intervals which would result in a ‘closer’ to optimal solution. Using smaller energy intervals in this way does not impact on the simple form of the solution. Halving the energy intervals would result in the example case producing a 6 node ‘less than modal’ scenario tree (relative to the three node tree examined) which would be solved in the same way, with no change to either the form of the objective function or constraints.

Similarly it would be possible to use multi-years of historical data (for a particular geographic location) to produce a more robust prediction for future years. The method used to develop the incident radiation probability distribution is independent of the form of the stochastic equations. This is important since the quality of data available for different geographic locations varies greatly. Hence being able to separate weather data processing from the optimisation technique and to be able to pre-process weather using statistical techniques independent of the optimisation technique is a useful characteristic of this method.

Of particular interest is the simple manner that the technique provides to explore the impact of multiple consecutive less than Modal days has on the optimal solution. In the data set used for this example there is a cluster of strongly less than Modal days for the months June, July and August. The results reported included these days into the overall yearly probability distribution. Having the
ability to identify consecutive day incident energy patterns, and review the impact on the probabilities that flow to the stochastic equations, allows decisions to be taken about what weather patterns to use in the design optimisation.

The ability to post-process weather data and examine the impacts of consecutive day weather variation is considered a key advantage of the technique described.

This capability of the approach is explored in detail in Chapters 4 and 5

3.11.2 Incorporating Load Data

Energy load data is incorporated as discrete daily totals in the constraint equations. As with the weather data this allows ‘pre-processing’ of the load requirements which in turn keeps the form of the optimisation solution simple. In this example the concept of a daily load profile (load vs. incident energy vs. time) is greatly simplified. It is possible using the form of the equations presented to incorporate the variation in load profiles by adjustment of the pre-calculated scaling factors in the constraint relationships. The form of the constraints also allows variation of load profile in each non-Modal day to be incorporated while leaving the core processing technique unchanged.

This capability of the approach is explored in detail in Chapter 4 and 5

3.11.3 Incorporating System Complexity

The form of the solution presented is able to scale up to incorporate increased system complexity. Adding system components (e.g., Hot water generation and hot water storage) will increase the terms in the objective function without increasing the complexity of the mathematical technique required to find a solution. The relationships between hot water and electrical storage, and hot water and electrical load will be expressed as an increased in the range of constraint relationships. While this will lead to larger solution matrices the core technique, and the ability to use simple solvers will not be made more complex. Likewise the ability to accommodate weather variation using the stochastic method is not impacted by the increase in system complexity.
This capability of the approach is explored in detail in Chapter 4

3.11.4 Basis of a Transparent Design Tool

A further advantage of the technique outlined is that the structure of the objective function and constraint equation provides the basis of an easy to use design tool where the impact of assumptions on the final result is clear to the user. The structure of the solution would allow designers to conduct transparent sensitivity analysis using simple equation solvers.

3.11.5 Transparent commercial risk

The method proposed provides an opportunity to not only assess the variable costs (ICE run time or imported energy) associated with a given design solution but the probability that those costs will be incurred. This is a measure of potential commercial risk and is a unique capability of the approach being adopted.
3.12 Conclusion

This chapter has shown using a very simple system how the Stochastic programming approach identified in the literature can be applied to energy system analysis.

It is suggested in this chapter that the form of the equations used in the solution provide some advantages when assessing more complex systems. In the following chapters these concepts are further examined and the capability of the technique when used as a design tool is demonstrated.
Chapter 4

Expanded Models and Energy Mix

Optimisation

4.1 Expanded Models - Introduction

In Chapter 3 a form of characteristic equations and constraints were developed for a highly simplified energy system such that that system could be analysed using Stochastic Programming with Recourse (SPwR). The aim was to optimise the design from a system life-cycle cost perspective while allowing consideration the impacts of consecutive day(s) of incident solar energy. The characteristic equations and constraint equations were established to allow the use of the simple LINPROG solver in Matlab.

In this chapter a more complex energy system model is addressed. The aims of this chapter are to:

- Develop characteristic equations and constraint equations for the more complex system and confirm that the Chapter 3 basic techniques can be ‘scaled-up’.
- Explore how the trade-off between different energy storage mechanisms can be addressed
- Explore how different load types can be accommodated
4.2 Expanded System Description

The basic architecture of the energy system being explored in this chapter is shown as figure 4.1

This system adds to the basic system shown in figure 3.1 as follows:

- Additional hot water reticulation and load
- Hot water generation from solar collectors or ICE (Combined Heat and Power)
- Ability to convert and store electrical energy as Hot Water (HW)

Note that while this example involves an Islanded (not grid connected) system this is not a necessary condition. Even in such a simple system the Internal Combustion Engine (ICE) could be replaced with a grid interconnection and the same optimisation questions regarding the incident radiated energy on consecutive days would still exist. The same model form could be used to optimise incident solar energy generation and storage cost against the cost of energy imported from an external grid.
In Chapter 3 simple relationships/models were used for both the PV generation and the battery storage. The hot water components could potentially have more complex models due to the impact that differential water temperatures have on thermal energy system performance and the temporal aspect of water heating and cooling. The following sections discuss appropriate ways to model hot water components such that they can be accommodated within the basic mathematical forms developed in Chapter 3.

### 4.2.1 Small Solar Hot Water Collectors

In Chapter 3 the energy generated by the PV array was expressed as a percentage of the total incident energy, that percentage being related to the base conversion efficiency of the PV panel and an allowance for the loss of conversion efficiency across the course of the day. For solar hot water collectors the basic relationship for the amount of energy that can be generated is described by the Hottel-Whillier-Bliss equation which is described in Smith in [53]

\[
Q_u = F_r A_c [((\tau \alpha)_c I - U_L(T_{f,i} - T_a)] \ldots \ldots (1)
\]

where

- \( Q_u \) = the useful heat collected per unit area of array
- \( I \) = incident solar energy per \( m^2 \)
- \((\tau \alpha)_c\) = the transmittance / absorptance product
- \( F_r \) = the heat removal factor (impacted by flow rate)
- \( U_L \) = the heat loss co-efficient and
- \((T_{f,i} - T_a)\) = is the difference between the working fluid and the ambient temperature.

While the \( F_r A_c (\tau \alpha)_c \) term can be viewed as a design parameter (analogous to the PV array efficiency) the \( U_L(T_{f,i} - T_a) \) term does not match the form of the constraint equations developed in the previous chapter as it suggests that the ability of the array to generate heat is influenced by the rate of
temperature rise of the storage (effectively the $T_{f,i}$ term). The rate of temperature rise of the storage tank is in turn a function of its capacity, the starting temperature and system flow rates. This inter-relationship is further complicated when thermal stratification within the storage tank is considered [54].

The nature of the thesis study is that it is addressing the size of equipment purchased by looking at total energy balances over a given day (with given incident solar energy). The total energy nature of the modelling ignores the short term (minutes, hours) energy state of a given component (e.g. instantaneous state of battery charge). Consequently it is acceptable to model the hot water system in terms of the total energy gathered and stored in a given day, assuming a final system ‘end state’. The end state of interest in the case of the hot water system is the final tank steady state temperature (assuming no stratification) as this determines the energy that is stored, and available to be used, when the incident solar energy is no longer present.

Note this concept of the total quantum of stored energy required is already seen in the simplified Chapter 3 P.V. equations in that the total daily load requirement is distributed evenly between hours where incident solar energy is present and then not present. This even distribution of load is not a realistic assumption (i.e. the greatest electrical load may occur late in the day when incident energy is reduced) and is made at this time while the basic form of the equations is being developed. A modification to the basic equation forms to deal with “non symmetrical” load distribution is addressed in later chapters.

For the purpose this section of the study the steady state ‘working temperature’ of the tank will be stated as a fixed design attribute. This will allow equation (a) to be used as an estimate of collector performance. Further setting a steady state system temperature as a fixed design attribute allows the heat load and (and transfer rates) to be established.

This concept of ‘total energy in a given day’ is explored in [55] by Hossain. This summary paper of small solar hot water collectors introduces modelling that uses collector efficiency that ignores the short term temporal considerations that occur before steady state temperatures are reached. Consequently this approach matches the form of the base equations from Chapter 3. Hossain suggests
4.2. Expanded System Description

Defining collector efficiency as “the ratio of useful energy gain $Q_u$ to the incident energy of a particular time period.”

$$Q_u = \frac{\int Q_u \, dt}{\int I \, dt}$$

And the instantaneous thermal efficiency $\eta$

$$\eta = \frac{Q_u}{A}$$

What this suggests and as is shown in [55] is that the efficiency of a given system can be either calculated (modelled) or established by experiment. This means that for this thesis study a particular collector /tank/operating temperature configuration can be declared, the efficiency established then the cost vs size vs energy harvesting and storage can be optimised against the other technologies. Configuration impacts on efficiency (and the cost of those efficiency impacts) can also be explored using the basic form of the equations. This is demonstrated in later chapters.

The concept of estimating or measuring a general efficiency is addressed in a range of studies. Notably work on Compound Parabolic Concentrators (CPC) tends to illustrate how “averaged daily efficiencies” can be established. It is noted that it is the nature of the CPC as a technology that tends to emphasise the performance of the array over a given day. The original discussion of CPC arrays is described by Winston in [56]. Estimates of efficiency of CPC designs are shown in [57] with the same author reporting on three years of test data in [58]. Much later in [59] and [60] Oommen reviewed the original results and subsequently the designs were improved. Advanced design concepts using the same approaches in evacuated solar hot water collecting tubes [61] also produces an analytical estimate of efficiency of the form required to match the Chapter 2 equations. For the reminder of this study it will be assumed that this average daily efficiency is a known characteristic of the array being sized. This will require that an array design (with area as the variable) and a target operating temperature are know or declared.
4.2.2 Small Hot Water Storage Tanks

From first principles the amount of energy that can be stored in a hot water tank is a function of the working steady state temperature, the tank volume and the time available to heat the tank. Small domestic hot water (DHW) storage tanks are also subject to stratification (different discrete thermal layers of hot water). Recently the complexity of establishing the temperature within tanks has led to studies using Computational Fluid Dynamics (CFD) approaches, this is illustrated by [62], and [63]. While such approaches provide a detailed prediction of the temperature of the tank at any given time they are not suitable for the form of equations being developed in this study.

In [64] Buzas uses a series of equations that model the temperature states in tanks and coupled hot water collectors based on energy balance concepts. The relationships in this paper are further simplified by ignoring losses, which occur on a different time scale to the heating. Buzas’s work provides the following simple relationship for a Domestic Hot Water (DHW) tank with a heating coil.

![Figure 4.2 - Relationships in Simple DHW Tank (from Buzas)](image)

Buzas establishes the basic energy balance relationship:
\[
\frac{d(\rho_1 c_p_1 V T_s)}{d_t} = F_1 \rho_1 c_p_1 (T_d - T_s) + F_c \rho_2 c_p_2 (T_c - T_1) \tag{2}
\]

where

\(F_x\) = the relevant flow rate

\(\rho_x\) = the relevant fluid density

\(c_x\) = the energy storage capacity of the working fluid per unit volume.

What is important about equation (2) is that it shows that, if no water is removed from the storage tank that a steady state temperature will be reached when \(T_c = T_1 = T_s\). Further this steady state temperature will be approached exponentially.

In the context of this thesis study (as outlined in the previous chapter) there is an assumption that a quantum of energy is used while the incident energy exists and then a further quantum of energy has to be stored. While these two things happen simultaneously, and so the temperature in the tank at any time is variable, the amount of energy able to be stored in the steady state condition is a function of the storage temperature and the volume for the tank. The steady state energy storage capacity of the tank is related to the specific heat of the storage fluid, so assuming hot water the following relationship applies:

\[
Q_{halt} = [4.184(T_s - T_a) \times 2.778E - 07] \times x_8...(kWh)\tag{3}
\]

where

\(x_8\) = the size of the tank in litres
4.3 Objective Function for the Expanded Form

For the simple system the optimisation question is summarised by the following objective function:

\[
\min C_{\text{elec}} = C_{\text{pv}} + C_{\text{ice}} + C_{\text{batt}} + C_{\text{hwa}} + C_{\text{tank}} \ldots \tag{4}
\]

where
\[
C_{\text{elec}} = \text{the cost of electricity}
\]
\[
C_{\text{pv}} = \text{the cost of the PV generated electricity}
\]
\[
C_{\text{ice}} = \text{the cost of the generator generated electricity and Hot Water}
\]
\[
C_{\text{batt}} = \text{the cost of the battery storage}
\]
\[
C_{\text{hwa}} = \text{the cost of the solar hot water generation}
\]
\[
C_{\text{tank}} = \text{the cost of the hot water storage tank}
\]
4.4 Load Constraints for the Expanded System

As was outlined in Chapter 3 a common method for representing recourse Stochastic Programming problems is to use scenario trees [51]. If for the the days $I_{rad_{m(n+1)}}, I_{rad_{m(n)}}$ we quantify the possible range of incident energy into discrete intervals $i = 1...n$ then the situation can be represented by the following scenario tree

![Scenario Tree Diagram]

Expanding on the approach developed in Chapter 3 it is necessary to declare those variables which represent ‘initial design decisions’ and those variables that depend on system running conditions. for the expanded system the initial design variables are:

- $x_1$ Size of PV array ($m^2$)
- $x_3$ Battery Size (kWh)
- $x_7$ Size of Hot Water collector ($m^2$)
- $x_8$ Hot Water Tank Size (kWh)
are initial design decisions and the parameter

\[ x_2 \text{ ICE Run-Time (hours per day)} \]

is able to be varied at a later date once the probability and quantum of the reduced incident energy days is understood. In order to support the standard recourse problem form a new variable is declared

\[ y_{2i} \text{ ICE Run-Time (hours per day) for the post modal day} \]

This results in a new objective function:

\[
\min C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum P_i y_{2i} + d \ldots \ldots \ldots (4)
\]

\[ \sum P_i y_{2i} \] is the arithmetic sum of each second stage scenario generator runtime as factored by the probability of that scenario, where \( i = 1, 2, \ldots, n \) is the number of energy range days less than modal.

This can then be optimised using the following constraints:

### 4.4.1 Modal Day Load Constraints

From Chapter 3 it was shown:

\[
P_{EL(0)} = e_0 x_1 + f x_2 \ldots \ldots (5)
\]
where

\( e_0 \) is the incident energy (factored by PV array efficiency) for the modal day and,

\( f \) is the size of the system generator in kWh

Using a similar structure for the hot water system results in:

\[
Q_{HWL(0)} = \mu_0 x_7 + \sigma x_2 \ldots \ldots (6)
\]

where

\( \mu_0 \) is the incident energy (factored by HW array efficiency) for the modal day and,

\( \sigma \) is the heat output of the ICE in kWh per hour of running.

### 4.4.2 Non Modal Day Load Constraints

From Chapter 3 it was shown:

\[
P_{EL(i)} = e_i x_1 + f y_{2i} + \text{energy stored in the Modal day}
\]

\[
P_{EL(i)} = e_i x_1 + f y_{2i} + (x_3 - g P_{EL(0)})
\]

\[
P_{EL(i)} + g P_{EL(0)} = e_i x_1 + f y_{2i} + x_3 \ldots \ldots (7)
\]

where

\( e_i \) is the incident energy (factored by PV array efficiency) for the non-modal day(s) and,

\( g \) is a factor that determines the percentage of the \( P_{load(0)} \) that must be stored.
Extending this concept to the Hot Water system results in:

\[ Q_{HWL(i)} = \mu_i x_7 + \sigma y_2 + (\text{additional HW energy stored in the Modal day}) \]

\[ Q_{HWL(i)} = \mu_i x_7 + \sigma y_2 + (x_8 - \gamma Q_{HWL(0)}) \]

\[ Q_{HWL(i)} + \gamma Q_{HWL(0)} = \mu_i x_7 + \sigma y_2 + x_8 \]

(8)

where

\( \mu_i \) is the incident energy (factored by HW array efficiency) for the non modal day and,

\( \gamma \) is the percentage of the \( Q_{HWL(0)} \) that must be stored to support the Modal day load.

4.4.3 Storage Constraints for Sample System

From the Modal day, as an absolute minimum, the battery must store at least a specific fraction \( g \) of the total daily load:

\[ x_3 \geq P_{EL(0)} \times g \]

Substituting (e)

\[ 0 \geq c_0 g x_1 + f g x_2 - x_3 \]

(9)
The first storage constraint is analogous to the battery size constraint in that, at the minimum system temperature:

\[ x_8 \geq Q_{HWL(0)} \times \gamma \]
\[ x_8 \geq \gamma (\mu_0 x_7 + \sigma x_2) \]

\[ 0 \geq \gamma \mu_0 x_7 + \gamma \sigma x_2 - x_8 \ldots \ldots (10) \]
4.5 Specific Example and Technical Models

Earlier equation 4 provided the basic form of the objective function:

\[ \text{min} C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i y_{2i} + d \ldots (4) \]

4.5.1 Decision Parameters

In this example the decision parameters are:

- \( x_1 \) Size of PV array (\( m^2 \))
- \( x_3 \) Battery Size (kWh)
- \( x_7 \) Size of hot water collector (\( m^2 \))
- \( x_8 \) Hot Water Tank Size (kWh)

These are initial design decisions and the parameter

\( x_2 \) ICE Run-Time (hours per day)

Can be varied during operation

Note: this is true if the generator size is not a variable

4.5.2 Input Parameters

The input parameters are used to establish the cost factors (\( a, b, c, m, n \) and \( d \)) in the objective function. These are in this example defined as follows:

\( C_{batt}/n = \) Battery cost per kWh
\( C_{pv}/n = \) PV cost per \( m^2 \)
\( ICE_{run}/n = \) Cost per hour (i.e. fuel cost)
4.5. Specific Example and Technical Models

\[ ICE_{cap} = \text{Capital cost (diesel engine plus generator)} \]

\[ ICE_{gen} = \text{ICE Generator Size (kW)} \]

\[ C_{hw tank/n} = \text{Hot Water tank cost per kWh} \]

\[ C_{hwa/n} = \text{Hot Water array cost per } m^2 \]

4.5.3 Input Variables

The ‘input variables’ are declared by the designer and are factors that are variable from a design perspective but are chosen and fixed in advance of the optimisation analysis.

\[ P_{EL} = \text{Electrical load (kWh/day)} \]

\[ I_{radm} = \text{Total Incident Radiation for Modal day (kWh/m}^2\text{)} \]

\[ SL = \text{System Life (years)} \]

\[ t_{pvhrs} = \text{assumed hours of sunshine where array produces hourly load.} \]

\[ t_{hwahrs} = \text{assumed hours of sunshine where the HW array produces hourly load.} \]

\[ Q_{HWL} = \text{Hot Water load (kWhr/day)} \]

\[ DoD_{spec} = \text{DoD required to meet life estimate (manufacturer’s estimate)} \]

\[ B_{L} = \text{Battery Life} \]

\[ r = \text{assumed interest rate} \]

For this first example the following parameters are used:
Figure 4.4 System Parameters for Figure 4.1 system

### 4.5.4 System Relationships

The factors a, b, c, m and n are used to translate the equipment characteristics shown in Figure 4.2 into the daily equivalent costs that match the form of the objective function equation (4). Daily costs are used because the loads, and incident solar energy and probability of incident energy are all analysed on a per day basis. Costs for long-life capital purchases are calculated using Equivalent Annual
Cost these factors were calculated for the electrical system in Chapter 3.

The Hot Water equivalents are calculated as follows:

The cost scaling factor for the tank

\[ n = C_{\text{hwtank}}/(A_{SL,r}) \times (SL/TL) \] (annualised cost per kWh of the tank)

Establishing \( C_{\text{hwtank}} \) requires a translation and an assumption of storage temperature. Setting the storage temp at 90°C is necessary.

Using the specific heat of water as \( C_p = 11.66 \times 10^{-4} \text{ kWhr/kg.C} \) (based on \( C_p = 4.184 \text{ kJ/kg.C} \)) at 90°C stored energy is 0.1050 kWhr/kg. So let's assume a 400 L tank (400Kg) is 1200 AUD then

Cost of tank per kWhr = 1200/(0.105 x 400)

Cost of tank per kWhr = 28.6 AUD/kWhr @ 90°C

to therefore assuming the tank life is equivalent to the system life

\[ n = 28.6/11.5 = 2.484 \]

\( m \) is the scaling factor for the Hot Water CPC array

\[ m = C_{\text{hwa}}/(A_{SL,r}) \times (SL/AL) \]

Assuming a simple array cost of 50 AUD per \( m^2 \) gives:

\[ m = 4.35 \]
the factors $\mu, \sigma, \gamma$, are established as follows

$\mu_0$ is the incident energy (factored by HW array efficiency) for the modal day,

The concept of the efficiency of the solar hot water collectors is related to temperature differentials and state operating conditions but there are some rough ranges. Ultimately this figure is best established by test of the arrays being examined. For this discussion a figure of 0.5 for the CPC design was identified by Oommen in [60] and is used in this sample calculation. Note that in this thesis study the absolute accuracy of the figure is not crucial and in a real use of this approach this value would be established by test for each array type.

$\sigma$ is the heat output of the ICE in kWh per hour of running. Again such figures need to be established by test as the ability to recover the heat is heavily design dependant but using the rough rule of thumb that in a small ICE twice as much heat energy as shaft (torque) energy is created in this case the 5kW shaft power would create 10 kWh of heat for every hour of running, hence

$\sigma \approx 10$

$\gamma$ is the percentage of the $Q_{HWL(0)}$ that must be stored to support the Modal day load. In line with the analysis in Chapter 3 for the PV array it is assumed as a starting point that 6 hours of incident energy is available. As there are no depth of discharge allowances for the water tank (unlike the battery) $\gamma$ becomes a simple ratio. Hence the 6 hours assumption requires 18 hours of stored heat energy, this gives the value:

$\gamma \approx 0.75$
4.6 Worked Example

4.6.1 Cost Function and the f matrix

The same weather scenario (Melbourne Airport 2013) is utilised in this example. The Modal day incident radiant energy for this example was $2.8 MJ/m^2$. From figure 3.2 there are three classes of days with less incident energy than the modal day;

$P_{e21} = 0.03385$ $I_{e21} = 0.4$ kWhr/day

$P_{e22} = 0.17494$ $I_{e22} = 1.4$ kWhr/day

$P_{e23} = 0.22739$ $I_{e23} = 2.4$ kWhr/day

Re-using the parameters calculated in chapter 3 together with the hot water related factors calculated in the previous section results in the following:

$$\text{min}C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i y_{2i} + d...................(4)$$

$$\text{min}C_{elec} = 21.74x_1 + 997x_2 + 174x_3 + 4.35x_7 + 2.484x_8 + 33.748y_{21} + 174.415y_{22} + 226.979y_{23}$$

This results in an $f$ matrix as follows:

$$f_x = \begin{bmatrix}
21.74 \\
997 \\
174 \\
4.53 \\
2.48 \\
33.748 \\
174.415 \\
226.797
\end{bmatrix}$$
4.6.2 Load Constraints and $A_{eq}$

\[ P_{EL(0)} = e_0 x_1 + f x_2 \]........(5)

\[ P_{EL(i)} + g P_{EL(0)} = e_i x_1 + f y_{2i} + x_3 \]........(7)

\[ Q_{H WL(0)} = \mu_0 x_7 + \sigma x_2 \]........(6)

\[ Q_{H WL(i)} + \gamma Q_{H WL(0)} = \mu_i x_7 + \sigma y_{2i} + x_8 \]........(8)

Re-using the parameters calculated in Chapter 3

\[ f = 5 \]

\[ g = 0.09375 \]

\[ e_0 = 0.392 \]

\[ e_1 = 0.056 \]

\[ e_2 = 0.196 \]

\[ e_3 = 0.336 \]

and based on the parameters discussion in the previous sections

\[ \sigma = 10 \]

\[ \gamma = 0.75 \]

\[ \mu_0 = 0.5 \times 2.8 = 1.4 \]

\[ \mu_1 = 0.5 \times 0.4 = 0.2 \]

\[ \mu_2 = 0.5 \times 1.4 = 0.7 \]

\[ \mu_3 = 0.5 \times 2.4 = 1.2 \]

This results in:

\[ P_{load(0)} = 0.392 x_1 + 5 x_2 \]

\[ P_{load(1)} + g P_{load(0)} = 0.056 x_1 + 5 y_{21} + x_3 \]
4.6. Worked Example

\[ P_{\text{load}(2)} + gP_{\text{load}(0)} = 0.196x_1 + 5y_{22} + x_3 \]
\[ P_{\text{load}(3)} + gP_{\text{load}(0)} = 0.366x_1 + 5y_{23} + x_3 \]

and

\[ Q_{\text{HW}(0)} = 1.4x_7 + 10x_2 \]
\[ Q_{\text{HW}(1)} + \gamma Q_{\text{HW}(0)} = 0.2x_7 + 10y_{21} + x_8 \]
\[ Q_{\text{HW}(2)} + \gamma Q_{\text{HW}(0)} = 0.7x_7 + 10y_{22} + x_8 \]
\[ Q_{\text{HW}(3)} + \gamma Q_{\text{HW}(0)} = 1.2x_7 + 10y_{23} + x_8 \]

This results in \( A_{eq} \) being an 8 x 8 matrix:

\[
A_{eq} = \begin{bmatrix}
0.392 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.056 & 0 & 1 & 0 & 0 & 5 & 0 & 0 \\
0.196 & 0 & 1 & 0 & 0 & 0 & 5 & 0 \\
0.336 & 0 & 1 & 0 & 0 & 0 & 0 & 5 \\
0 & 10 & 0 & 1.4 & 0 & 0 & 0 & 5 \\
0 & 0 & 0 & 0.2 & 1 & 10 & 0 & 0 \\
0 & 0 & 0 & 0.7 & 1 & 0 & 10 & 0 \\
0 & 0 & 0 & 1.2 & 1 & 0 & 0 & 10 \\
\end{bmatrix}
\]

\( b_{eq} \) matrix is defined as:

\[
b_{eq} = \begin{bmatrix}
P_{\text{load}(0)} \\
P_{\text{load}(1)} + gP_{\text{load}(0)} \\
P_{\text{load}(2)} + gP_{\text{load}(0)} \\
P_{\text{load}(3)} + gP_{\text{load}(0)} \\
Q_{\text{HW}(0)} \\
Q_{\text{HW}(1)} + \gamma Q_{\text{HW}(0)} \\
Q_{\text{HW}(2)} + \gamma Q_{\text{HW}(0)} \\
Q_{\text{HW}(3)} + \gamma Q_{\text{HW}(0)} \\
\end{bmatrix}
\]
4.6.3 Storage Constraints and the $A$ Matrix

As identified earlier in section 4.4.3 there are two storage constraint equations:

\[
0 \geq c_0g x_1 + f g x_2 - x_3 \quad \cdots \quad (9)
\]
\[
0 \geq \gamma \mu_0 x_7 + \gamma \sigma x_2 - x_8 \quad \cdots \quad (10)
\]

Substituting in the variables gives

\[
0 \geq 0.3675 x_1 + 4.6875 x_2 - x_3
\]
\[
0 \geq 1.05 x_7 + 7.5 x_2 - x_8
\]

This results in $A$ being a $8 \times 2$ vector

\[
A = \begin{bmatrix}
0.3975 & 4.6875 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 7.5 & 0 & 1.05 & -1 & 0 & 0 & 0
\end{bmatrix}
\]

and $b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
4.6.4 Simulation Results

Results were developed for a 20 kWhr electrical load coincident with a 40 kWhr heat load and a 30 kWhr electrical load /80 kWhr hot water load requirement and the results are shown in Table 4.2

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Load 20 kWhr/day electrical 40 kWh/day thermal</th>
<th>Load 30 kWhr/day electrical 80 kWhr/day thermal</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1 = PV in m²</td>
<td>51</td>
<td>78</td>
</tr>
<tr>
<td>X2 = Generator Run Time in hrs (Modal day)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>X3 = Batt Size in kWh</td>
<td>18.8</td>
<td>31.3</td>
</tr>
<tr>
<td>X7 = Hot Water array in m²</td>
<td>28.6</td>
<td>57</td>
</tr>
<tr>
<td>X8 = Hot Water Tank in kWh</td>
<td>30</td>
<td>69</td>
</tr>
<tr>
<td>Y21 = Gen Run Time in hrs (Prob 0.034)</td>
<td>3.4</td>
<td>4</td>
</tr>
<tr>
<td>Y22 = Gen Run Time in hrs (Prob 0.175)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Y23 = Gen Run Time in hrs (Prob 0.228)</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 4.2 Simple Analysis Results

What can be seen in these results is that the electrical system and hot water system having been designed for the Modal day and having no interconnection other than the ICE running time have been optimised separately. In the following section more interaction between the two systems is modelled.
4.7 Energy Mix Optimisation - Introduction

One issue identified during the literature review was the inability of existing techniques to support an examination of the trade-off possible with thermal loads and thermal storage. In many small energy system applications (residential, commercial and light industrial / agricultural) a proportion of the total energy load is thermal in nature. This could be either heating or cooling.

In the previous sections the relationship between the thermal load and the electrical load was limited to impacts on the ICE / generator run time. It is noted that the previous simple examples would allow a thermal load (such as hot water heating or air-conditioning) to be accounted for as an electrical load but there was no ability to address the optimisation of meeting thermal loads. One of the aims of the method being developed is to support the optimisation of thermal generation in a transparent manner.

In the following sections additions are made to the energy system model to support the optimisation of generation to meet thermal loads. The aims of this section are to:

- Review existing techniques to address optimisation of thermal load generation.
- Develop amendments to the existing equations to support the optimisation of thermal load related generation
- Explore how different load types can be accommodated using the base concepts developed.

4.7.1 Existing Techniques and Approaches.

A number of recent articles addressing the optimisation of thermal storage were identified. Typical of these investigations is [65] which utilises the DER-CAM tool-set. In this paper DeForest explores the concept of storing thermal energy, in this case chilled water, as a technique to reduce the cost of imported electrical energy. In this case DER-CAM is used as the basis of scenario testing rather than a complete optimisation tool. The paper introduces the concept of Cooling Degree Days (CDD) as a way estimate cooling load demand using a combination of temperature and humidity. Such a concept has potential application for this study.
In [66] Geidl et al introduce the mathematical concept of an energy hub. The authors describe the energy hub as a model that contains either direct energy connections, energy conversion or storage. The advantage of the concept proposed is that it is highly scalable since multiple energy conversion elements, and connections between conversion elements can be added. The technique provides a mechanism that would allow the trade off between generation for different thermal loads to be accommodated through the use of the coupling matrix introduced by the technique. The discussion focuses on dispatch optimisation since this question best suits the ‘power flow’ nature of the approach. While the approach outlined provides insights into how such problems can be addressed it results in a non-linear, inequality constrained multi-objective optimisation problem which quickly becomes complex (non-convex) to solve as real world constraints are considered.

Many papers were identified in the area of Combined Heat and Power (CHP) optimisation. These papers tend to look at storage sizing and optimisation. A significant advance was made by Rasoul Asaee as outlined in [67]. In this paper two discrete hot water generating sources, together with storage, are considered as is the electrical output from the CHP machine. The modelling approach is a combination of simulation with scenario testing (similar to HOMER [28]) and as a consequence does not allow investigation of run of days weather patterns however it does provide some insight into how measures of energy conversion device efficiency can be used to address cost optimisation, especially where operational cost is the primary concern. Merkel [68] utilises a similar approach that addresses multiple heat generation sources using a similar technique but adds a secondary analysis step that corrects the theoretical assessment with real system measures.

Chicco [69] adopts an energy vector approach to explore the trade-off between energy inputs required to meet specific thermal and electrical loads when using a CHP machine. The basis of the analysis involves applying simple cost and efficiency measures to establish if the cooling load should be meet by electrical direct electrical inputs or by electrical and hot water outputs from a CHP machine. This is similar to the question being asked in this study but focussed on inputs rather than loads. The same authors expand on the energy vector approach in [70] where energy outputs are mapped to energy inputs using a simple transfer matrix that captures efficiencies.
In [70] a given energy input stream, primarily fuel of the CHP machine and electrical power are split into a range of energy transformation devices that then combine to meet the load requirements. Interestingly by adopting this concept of energy branches the approach develops a range of ‘intermediate’ variables that build to form the overall transfer matrix structure. The structure created allows each transformation device to be "switched" on or off in the analysis and variations in performance can be dealt with by changes in the ‘efficiency’ parameters used. The solution is generated using a non-linear optimisation tool and then sensitivity style scenario analysis is conducted. The following section uses some of the concepts of energy path accounting from these authors in the context of the base methodology developed in previous chapters.
4.8 Energy Flow Model

The first iteration of the base model is to look at the way the hot water thermal load can be generated. In the reference Combined Heat and Power (CHP) system (see Figure 4.1) there are three possible ways to create hot water to meet the thermal load:

- Hot water generation using the concentrating solar array
- Hot water generation using the CHP machine
- Hot water generation using the electric heating element.

This system configuration allows for the generation of hot water on greater than Modal incident energy days $I_{rad_{m(n+1)}} > I_{rad_{m(n)}}$ using the electric hot water heater which utilises excess PV capacity and allow the use of Hot Water Storage (HWS) as a second way to store PV generated energy.

This architecture opens up the possibility of optimising, (establishing the optimum way to generate hot water) from a cost perspective the generation of hot water. Figure 4.1 shows the general energy flows that are analysed for in the following sections, using the energy vector concept as is suggested by Chicco and Mancarella in [70].
4.8.1 Expanded Objective Function forms for Electric Hot Water Generation

The aim of the approach being adopted is to expand the equations to cover the new analysis requirement while maintaining the form of the base equations. The existing objective function for the base system is shown as equation (d)

\[
min C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i by_2i + d \ldots \ldots \ldots \ldots (4)
\]

In line with the ‘split’ energy flows shown in figure 4.1 (and as utilised in [70].) it is necessary to
split the existing variables. The aim is to keep the basic form of the objective function and address the new energy flows in the constraints. To allow more complex constraint equation the following variables will be used:

\[ x_9 \text{ PV array size derived from the core electrical load requirement} \]
\[ x_{10} \text{ PV array size to support part of the hot water load requirement} \]

As a consequence \( x_1 \) is now defined as

\[ x_1 = x_9 + x_{10} \]

In a similar fashion

\[ x_{11} \text{ Generator running time for electrical requirement} \]
\[ x_{12} \text{ Generator running time for water heating requirement} \]

As a consequence \( x_2 \) is now defined as

\[ x_2 = x_{11} + x_{12} \]

Note: The ICE machine runs once and creates both hot water and electricity simultaneously. Two variables are created to allow consistency in the form of the equations and to support the ability to address the possibility of not exploiting or storing some heat or electrical energy associated with ICE running.

the final expansion required is as follows:

\[ x_{13} \text{ Battery capacity for electrical requirement} \]
\[ x_{14} \text{ Battery capacity for heating requirement} \]
As a consequence \( x_3 \) is now defined as

\[
x_3 = x_{13} + x_{14}
\]

The final expansion is required in the second stage terms so the description between first and second stages is consistent. The following variables are created:

\( y_{2ai} \) ICE second stage run time to meet the \( ith \) non Modal day electrical load \( y_{2bi} \) ICE second stage run time to meet the \( ith \) non Modal day electrical hot water load

As a consequence \( y_{2i} \) is now defined as

\[
y_{2i} = y_{2ai} + y_{2bi}
\]

It is noted that \( x_9 \) thru \( x_{13} \) and \( y_{2ai}, y_{2bi} \) do not need to be captured in the cost function (as cost are captured by \( x_1 \) thru \( x_3 \) and \( y_{2i} \)) but they will need to be included in the LINPROG objective function with zero cost parameters in the \( C^T \) matrix.

## 4.8.2 Load Constraints for the Expanded System

### Modal Day Load Constraints

Previously it was shown in section 4.4.1:

\[
P_{EL(0)} = e_0 x_1 + f x_2
\]

where

\( e_0 \) is the incident energy (factored by PV array efficiency) for the Modal day and,

\( f \) is the size of the system generator in kWh

and

\[
Q_{HWL(0)} = \mu_0 x_7 + \sigma x_2
\]
where

\( \mu_0 \) is the incident energy (factored by HW array efficiency) for the modal day and,  
\( \sigma \) is the heat output of the ICE in kWh per hour of running.

These two constraints are given the same form and rely on the performance of the solar energy arrays being able to be characterised by a single ‘efficiency’ parameter. These equations are modified to represent the split energy flows as follows:

**Electrical Modal Energy Load Constraint**

Keeping in mind that in these equations the electrical loads are a requirement that is meet using the PV array and storage partitioned for provision electrical loads only (excluding heating)

\[
P_{EL(0)} = e_0x_9 + f_{11} \ldots \ldots \ldots \ldots \ldots (11)
\]

**Hot Water Modal Energy Load Constraint**

Unlike the electrical Modal day load provision the Hot Water load can be provided by both hot water primary sources (the HW array and the ICE) and the electric hot water heater as a secondary source. This results in the following constraint equation:

\[
Q_{HWL(0)} = \mu_0x_7 + \sigma x_2 + \text{electrical heating component}
\]

\[
Q_{HWL(0)} = \mu_0x_7 + \sigma x_2 + \alpha(e_{0}x_{10} + f_{12}) \ldots \ldots (12)
\]

where
\( \alpha = \) the efficiency of the electrical hot water heater (hot water energy output for a given electrical energy input.

**Non Modal Day Load Constraints**

**Electrical Non-Modal Energy Load Constraint**

In section 4.4.2 it was shown:

\[
P_{EL}(i) = e_i x_1 + f y_{2i} + \text{energy stored in the Modal day}
\]

\[
P_{EL}(i) = e_i x_1 + f y_{2i} + (x_3 - g P_{EL(0)})
\]

\[
P_{EL}(i) + g P_{EL(0)} = e_i x_1 + f y_{2i} + x_3
\]

This equation is modified to account for the split energy flows as follows:

\[
P_{EL(i)} + g P_{EL(0)} = e_i x_9 + f y_{2ai} + x_{13} \ldots (13)
\]

**Hot Water Non-Modal Energy Load Constraint**

As for the Modal day constraints the non-Modal Hot Water constraints need to account for the ability to heat water using electrical energy either generated on the non Modal day or generated on the Modal day and stored. This results in the following.

\[
Q_{HWL(i)} = \mu_i x_7 + \sigma y_{2i}
\]

+ (battery stored electrical Hot Water heating)

+ (stored Hot Water)

+ (electrical direct Hot Water heating)
4.8. Energy Flow Model

The following relationships apply:

battery stored electrical Hot Water heating = $\alpha x_{14}$

stored Hot Water = $(x_8 - \gamma \cdot Q_{HWL(0)})$

electrical direct Hot Water heating = $\alpha f y_{2bi}$

expressed in the standard form

$$Q_{HWL(i)} + \gamma Q_{HWL(0)} = \mu_i x_7 + \sigma y_{2i} + x_8 + \alpha x_{14} + \alpha f y_{2bi} \ldots \ldots (14)$$

Storage Constraints

Electrical Storage Constraint

From the Modal day, as an absolute minimum, the battery must store at least a specific fraction $g$ of the total daily load:

$$x_3 \geq P_{Load(0)} \cdot g$$

or

$$0 \geq e_0 g x_1 + f g x_2 - x_3 \ldots \ldots (15)$$

this statement still holds true if the expanded forms of the array and storage size are also defined

Hot Water Load Constraints for the Expanded System

Equation (10) in section 4.4.3 provides the storage constraint for the hot water tank. This constraint remains suitable for this new analysis because the relationship between $x_8$ and $x_{10}$ and $x_{14}$ is defined
\[ 0 \geq \gamma \mu_0 x_7 + \gamma \sigma x_2 - x_8 \ldots \ldots (16) \]

In order for all variables to be accommodated and converge to a feasible solution within the LINPROG function the following relationships are defined:

\[
\begin{align*}
0 & \geq -x_1 + x_9 + x_{10} \\
0 & \geq -x_2 + x_{11} + x_{12} \\
0 & \geq -x_3 + x_{13} + x_{14} \\
0 & \geq -y_{2i} + y_{2ai} + y_{2bi}
\end{align*}
\]

4.8.3 Energy Mix Optimisation - Worked Examples

As the expanded energy mix equations take the same form as the basic hot water system equations the worked example forms can be developed by simple expansion of the hot water system relationships.

The additional factor added is \( \alpha \) which is the base energy efficiency of the electric hot water heater. For this example this will be set at 0.8, which is typical for resistive heaters.

4.8.4 Cost Function and the \( f \) matrix

This worked example expands on the simple Hot Water example used in section 3.6. As in this previous example the base weather data used is for Melbourne Airport 2013 and the system base parameters are the same.

The base cost function remains the same as in section 3.6.
4.8. Energy Flow Model

\[ \min C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i y_{2i} + d \] ..................(4)

As determined previously using the parameters calculated in section 3.6 the following cost function is used

\[ \min C_{elec} = 21.74x_1 + 997x_2 + 174x_3 + 4.35x_7 + 2.484x_8 + 33.748y_{21} + 174.415y_{22} + 226.979y_{23} \]

Note that in this case the \( x_9 \), thru \( x_{14} \) terms and the second stage terms still need to be captured in the \( f \) matrix (while not shown in the base cost equation) to allow them to be used within the constraint equations to map energy flows.

This results in an 20 x 1 \( f \) matrix as follows:
4.8.5 Load Constraints and $A_{eq}$

As outlined in the previous sections the constraint equations that match the form of the developed methodology are defined as follows:

\[ P_{EL(0)} = e_0 x_9 + f x_{11} \] \hspace{1cm} (11)

\[ Q_{HWL(0)} = \mu_0 x_7 + \sigma x_{12} + \alpha (e_0 x_{10} + f x_{12}) \] \hspace{1cm} (12)
4.8. Energy Flow Model

\[ P_{EL(i)} + gP_{EL(0)} = e_ix_9 + fy_{2i} + x_{13} \]......................(13)

\[ Q_{HWL(i)} + \gamma Q_{HWL(0)} = \mu_ix_7 + \sigma y_{2i} + x_8 + \alpha x_{14} = \alpha fy_{2i} \]...........(14)

All of the parameters for these equations (with the exception of \( \alpha \)) have previously been developed in either section 3.6. Hence substituting the parameters from section 3.6 back into these equations provides the following:

\[ P_{load(0)} = 0.392x_9 + 5x_{11} \]
\[ P_{load(1)} + gP_{load(0)} = 0.056x_9 + 5y_{2a1} + x_{13} \]
\[ P_{load(2)} + gP_{load(0)} = 0.196x_9 + 5y_{2a2} + x_{13} \]
\[ P_{load(3)} + gP_{load(0)} = 0.366x_9 + 5y_{2a3} + x_{13} \]

and

\[ Q_{HW(0)} = 1.4x_7 + 10x_2 + 0.3136x_{10} + 4x_{12} \]
\[ Q_{HW(1)} + \gamma Q_{HW(0)} = 0.2x_7 + x_8 + 0.84x_{14} + 10y_{2a1} + 4y_{2b1} \]
\[ Q_{HW(1)} + \gamma Q_{HW(0)} = 0.7x_7 + x_8 + 0.84x_{14} + 10y_{2a2} + 4y_{2b2} \]
\[ Q_{HW(1)} + \gamma Q_{HW(0)} = 1.2x_7 + x_8 + 0.84x_{14} + 10y_{2a3} + 4y_{2b3} \]

This results in \( A_{eq} \) being 8 x 20 matrix:

\[
A_{eq} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0.392 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.056 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.196 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.366 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 & 0 & 1.4 & 0 & 0 & 0.3136 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2 & 1 & 0 & 0 & 0 & 0 & 0 & 0.8 & 10 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.7 & 1 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 1.2 & 1 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 4
\end{bmatrix}
\]
and the $b_{eq}$ matrix is defined as:

$$b_{eq} = \begin{bmatrix}
P_{load(0)} \\
P_{load(1)} + gP_{load(0)} \\
P_{load(2)} + gP_{load(0)} \\
P_{load(3)} + gP_{load(0)} \\
Q_{HW(0)} \\
Q_{HW(1)} + \gamma Q_{HW(0)} \\
Q_{HW(2)} + \gamma Q_{HW(0)} \\
Q_{HW(3)} + \gamma Q_{HW(0)}
\end{bmatrix}$$

### 4.8.6 Storage Constraints and the $A$ Matrix

The storage inequality relationships are shown as equations (15) and (16). Substituting in the values developed in section 3.6 results in the following:

$$
0 \geq 0.037x_1 + 4.69x_2 - x_3 \\
0 \geq 1.05x_7 + 7.5x_2 - x_8 \\
0 \geq -x_1 + x_9 + x_{10} \\
0 \geq -x_2 + x_{11} + x_{12} \\
0 \geq -x_3 + x_{13} + x_{14} \\
0 \geq -y_{21} + y_{2a1} + y_{2b1} \\
0 \geq -y_{22} + y_{2a2} + y_{2b2} \\
0 \geq -y_{23} + y_{2a3} + y_{2b3}
$$

This results in $A$ being a $8 \times 20$ matrix.
4.8. Energy Flow Model

\[
A = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0.365 & 4.688 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 7.5 & 1.05 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

and \(b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\)

4.8.7 Simulation Results

Results were developed for a 20 kWhr electrical load coincident with a 40 kWhr heat load and a second scenario a 30kWhr electrical load coincident with a 80 kWhr heat load requirement. The results are shown in Table 4.3
Table 4.3 Complex System Analysis Results

Comparing these results with those shown in Table 4.2 it can be seen that in the complex analysis the optimised solution is biased toward electrical HW heating rather than array heating. The results show how it is possible to address the energy generation balance question for CHP systems, originally raised in [70] using the basic technique developed.
4.9 Conclusion

In this chapter the following has been achieved:

- Hot water generation and storage has been added to the basic equations in support of the optimisation of CHP systems
- Hot water loads have been incorporated
- The ability to optimise between hot water generating and storage derived from electricity, solar hot water and ICE hot water has been added into the basic form of the technique.

At the end of this chapter the form of the technique necessary to optimise the design of CHP style Small Islanded Energy Systems has been demonstrated by exploring how Hot Water loads can be accommodated via a range of analytical approaches.
Chapter 5

Features of the Technique

5.1 Introduction

In the previous chapters the basic technique has been established and expanded. In this chapter a range of features that have been incorporated into the basic technique will be explored.

This chapter addresses the following functionality:

- Ability to incorporate long term weather data,
- Ability to vary load with daily weather data,
- Ability to support assessment of grid interconnection
- Ability to support Multi-Objective Optimisation like processes
- Ability to support power analysis
- Ability to support new technologies
Chapter 5. Features of the Technique

5.2 Incorporating Weather Data

In the previous chapters the weather data assessed was for a single year (2013 Tullamarine Melbourne). As was shown in chapter 3 the weather characteristics of a particular site are processed by the analysis in the following way:

- develop a day by day record of total incident solar energy (in kWh) for a particular site for a particular year
- develop a discrete probability distribution for daily incident energy; the width of the band of daily incident energy determining the fidelity of the eventual solution,
- Identify the Modal day of incident energy and the day(s) of less than modal incident energy,
- For the modal day and the less than modal days calculate the total energy gathered by each system (this was the \( e_i \) factor for PV arrays) using a single conversion factor (this was the \( \mu_{pv} \) factor for PV arrays)

It should be remembered that the conversion factors (e.g. \( \mu_{pv} \)) are not a crude efficiency (such as the data sheet specification for a particular PV array) but rather a factor that incorporates both crude efficiency further factored to account for inefficiencies that occur across any given day. These conversion factors can be established by either analysis or test.

This concept of ‘total energy gathered’ on a given incident solar energy day is the primary way that weather data is incorporated in the solution. This approach to incorporating weather data allows a number of ‘pre-processing’ techniques which are described in the following sections.

5.2.1 Multiple Years of Weather Data

The technique looks at the Modal incident energy day, for a given time period, and the related less than modal days for the time period. The time period does not need to be a year and the larger the sample of days notionally the more accurate (in terms of the ability of the analysis to predict long term,
5.2. Incorporating Weather Data

20 year, system behaviour) the analysis will be. Analysis of multiple years of weather data should remove the potential for an atypical incident energy year to impact the analysis.

For the Melbourne Airport weather station, the daily record of incident solar energy between the 1st January 2000 and 31st December 2015 was analysed and the following results produced:

**Modal Incident Solar Energy:** $= 1.84 \text{ kWh} / \text{m}^2$

<table>
<thead>
<tr>
<th>Distribution Range kWh/day</th>
<th>Days Count</th>
<th>Probability of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.4</td>
<td>24</td>
<td>0.004</td>
</tr>
<tr>
<td>0.41 to 0.8</td>
<td>94</td>
<td>0.0180</td>
</tr>
<tr>
<td>0.81 to 1.2</td>
<td>288</td>
<td>0.4928</td>
</tr>
<tr>
<td>1.21 to 1.6</td>
<td>325</td>
<td>0.556</td>
</tr>
<tr>
<td>1.61 to 2.0</td>
<td>486</td>
<td>0.0832</td>
</tr>
<tr>
<td>2.1 to 2.5</td>
<td>658</td>
<td>0.1126</td>
</tr>
</tbody>
</table>

Table 5.1 2000-2015 Melbourne Airport Weather Summary
Table 5.2 - Design Solution 2000-2015 vs 2013 Only

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Load 30 kWh/day electrical 80 kWh/day thermal - 2013</th>
<th>Load 30 kWh/day electrical 80 kWh/day thermal - 2000-2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ = PV in m²</td>
<td>76</td>
<td>116</td>
</tr>
<tr>
<td>$X_2$ = Generator Run Time in hrs (Modal day)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X_3$ = Battery Size in kWh</td>
<td>31</td>
<td>89</td>
</tr>
<tr>
<td>$X_7$ = Hot Water array in m²</td>
<td>57</td>
<td>88</td>
</tr>
<tr>
<td>$X_T$ = Hot Water Tank in kWh</td>
<td>69</td>
<td>185</td>
</tr>
<tr>
<td>$Y_{21}$ = Gen Run Time in hrs</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$Y_{22}$ = Gen Run Time in hrs</td>
<td>2</td>
<td>2.4</td>
</tr>
<tr>
<td>$Y_{23}$ = Gen Run Time in hrs</td>
<td>0</td>
<td>1.1</td>
</tr>
<tr>
<td>$Y_{24}$ = Gen Run Time in hrs</td>
<td>n/a</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The results summarised in Table 5.2 show the significant impact that the specific weather data used can have on the analysis result. If a system was designed for Melbourne Airport using the 2013 data alone then it may not be the optimum configuration over the medium term. The ability of the technique proposed to support the assessment of optimum design solutions using a large sample of days, without changing the basic technique is capability not seen in other analysis approaches reviewed. In this example 15 years of data between 2000 and 2015 has been considered. The significant difference between the 15 year based result and the 2013 based result suggests that either 2013 was an atypical year or that in the 15 year sample there was a number of low incident energy years. In the context of this work the exact details of the Melbourne Airport weather data is not crucial. What is important is the ability to assess the impact of weather variation that is possible because the technique was specifically designed to support such investigations during system design.
This ability to easily incorporate large weather data sets, which should produce a more accurate long term estimate of system performance is a key feature that differentiates this technique form others used to optimise the design of SIES.

5.2.2 Filtering a Single Year of Weather Data

Weather data pre-processing can also be used to look at the impact that specific patterns of weather may have on the calculated design solution. Table 5.3 below is shaded to show the days of less than Modal energy for the Melbourne Airport 2013 reference site.

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>9.1</td>
<td>6.6</td>
<td>7.3</td>
<td>2.8</td>
<td>2</td>
<td>0.8</td>
<td>1</td>
<td>2.8</td>
<td>4.2</td>
<td>4.4</td>
<td>7.3</td>
<td>8.1</td>
</tr>
<tr>
<td>2nd</td>
<td>8.8</td>
<td>7.7</td>
<td>7.4</td>
<td>2.1</td>
<td>3.7</td>
<td>1.7</td>
<td>2.4</td>
<td>1.8</td>
<td>4.3</td>
<td>3.5</td>
<td>6.9</td>
<td>8.5</td>
</tr>
<tr>
<td>3rd</td>
<td>9.3</td>
<td>5.6</td>
<td>7.4</td>
<td>2.4</td>
<td>2.8</td>
<td>1.3</td>
<td>2.5</td>
<td>1.2</td>
<td>4.3</td>
<td>4.3</td>
<td>5</td>
<td>3.9</td>
</tr>
<tr>
<td>4th</td>
<td>9.3</td>
<td>7.2</td>
<td>7.2</td>
<td>5.2</td>
<td>2.1</td>
<td>1.1</td>
<td>1.3</td>
<td>2</td>
<td>3.4</td>
<td>6.2</td>
<td>6.8</td>
<td>0.9</td>
</tr>
<tr>
<td>5th</td>
<td>8.9</td>
<td>8.2</td>
<td>7.1</td>
<td>4.3</td>
<td>1.6</td>
<td>1.4</td>
<td>2.1</td>
<td>1.6</td>
<td>0.7</td>
<td>3.4</td>
<td>7.7</td>
<td>1.7</td>
</tr>
<tr>
<td>6th</td>
<td>8.8</td>
<td>7.9</td>
<td>6.6</td>
<td>4.7</td>
<td>3.5</td>
<td>1.5</td>
<td>2</td>
<td>2.8</td>
<td>0.9</td>
<td>3.6</td>
<td>7.6</td>
<td>6.4</td>
</tr>
<tr>
<td>7th</td>
<td>9.1</td>
<td>6.8</td>
<td>4.3</td>
<td>3.1</td>
<td>2.8</td>
<td>1.7</td>
<td>1.5</td>
<td>1.8</td>
<td>3.1</td>
<td>5.6</td>
<td>2.8</td>
<td>9.1</td>
</tr>
<tr>
<td>8th</td>
<td>7</td>
<td>8.2</td>
<td>5.2</td>
<td>4</td>
<td>2.9</td>
<td>2</td>
<td>2.6</td>
<td>4.2</td>
<td>5.6</td>
<td>1</td>
<td>3.4</td>
<td>7.1</td>
</tr>
<tr>
<td>9th</td>
<td>7.7</td>
<td>7</td>
<td>5.8</td>
<td>4.2</td>
<td>3.2</td>
<td>2.3</td>
<td>2.6</td>
<td>0.8</td>
<td>3.3</td>
<td>5.2</td>
<td>3.1</td>
<td>0.7</td>
</tr>
<tr>
<td>10th</td>
<td>9.2</td>
<td>6.4</td>
<td>3.6</td>
<td>4.6</td>
<td>3.1</td>
<td>1.9</td>
<td>2.6</td>
<td>3.4</td>
<td>3.5</td>
<td>0.9</td>
<td>4.3</td>
<td>6.6</td>
</tr>
<tr>
<td>11th</td>
<td>8.3</td>
<td>8</td>
<td>5.6</td>
<td>0.9</td>
<td>3</td>
<td>0.8</td>
<td>2.2</td>
<td>3.4</td>
<td>3.9</td>
<td>6.3</td>
<td>3.5</td>
<td>4.9</td>
</tr>
<tr>
<td>12th</td>
<td>4.9</td>
<td>6.8</td>
<td>6.3</td>
<td>4.3</td>
<td>1.8</td>
<td>0.7</td>
<td>2.3</td>
<td>3</td>
<td>3.8</td>
<td>6.8</td>
<td>1</td>
<td>8.9</td>
</tr>
<tr>
<td>13th</td>
<td>1.6</td>
<td>7.9</td>
<td>4.7</td>
<td>1.1</td>
<td>2.2</td>
<td>0.8</td>
<td>0.8</td>
<td>3.5</td>
<td>3.2</td>
<td>2.9</td>
<td>1.5</td>
<td>5.1</td>
</tr>
<tr>
<td>14th</td>
<td>7.9</td>
<td>5.2</td>
<td>4.9</td>
<td>2.8</td>
<td>2</td>
<td>1.4</td>
<td>0.7</td>
<td>1.7</td>
<td>3.1</td>
<td>4.3</td>
<td>3.2</td>
<td>7.6</td>
</tr>
<tr>
<td>15th</td>
<td>9</td>
<td>7.8</td>
<td>5.2</td>
<td>1.5</td>
<td>1.9</td>
<td>1.2</td>
<td>1.9</td>
<td>3.2</td>
<td>4.9</td>
<td>6.7</td>
<td>5.4</td>
<td>7.9</td>
</tr>
<tr>
<td>16th</td>
<td>9.1</td>
<td>6.9</td>
<td>2.8</td>
<td>4.1</td>
<td>1.4</td>
<td>1.9</td>
<td>1.9</td>
<td>3.8</td>
<td>1</td>
<td>4.7</td>
<td>3.7</td>
<td>7.3</td>
</tr>
<tr>
<td>17th</td>
<td>8.2</td>
<td>7.5</td>
<td>2.1</td>
<td>2.8</td>
<td>2</td>
<td>0.8</td>
<td>1.8</td>
<td>3.1</td>
<td>3.3</td>
<td>5.4</td>
<td>7.9</td>
<td>8.4</td>
</tr>
<tr>
<td>18th</td>
<td>5.1</td>
<td>7.6</td>
<td>3</td>
<td>1.8</td>
<td>1.7</td>
<td>1.7</td>
<td>2</td>
<td>2.5</td>
<td>2.7</td>
<td>7.1</td>
<td>8.1</td>
<td>8.3</td>
</tr>
<tr>
<td>19th</td>
<td>7.7</td>
<td>1.2</td>
<td>6.1</td>
<td>3.3</td>
<td>2.4</td>
<td>2.3</td>
<td>3</td>
<td>3.2</td>
<td>2.9</td>
<td>7.2</td>
<td>6.8</td>
<td>8.1</td>
</tr>
<tr>
<td>20th</td>
<td>8.9</td>
<td>7.6</td>
<td>5.4</td>
<td>3.2</td>
<td>0.8</td>
<td>2.3</td>
<td>0.8</td>
<td>3.1</td>
<td>3.3</td>
<td>4.8</td>
<td>1.7</td>
<td>7.2</td>
</tr>
<tr>
<td>21st</td>
<td>8.4</td>
<td>4.7</td>
<td>2.6</td>
<td>3.8</td>
<td>2.6</td>
<td>2.3</td>
<td>2.7</td>
<td>3.2</td>
<td>4.2</td>
<td>1.2</td>
<td>6.1</td>
<td>1.4</td>
</tr>
<tr>
<td>22nd</td>
<td>7.7</td>
<td>6.8</td>
<td>1.9</td>
<td>1.5</td>
<td>1.9</td>
<td>2.3</td>
<td>2.3</td>
<td>2.8</td>
<td>4.8</td>
<td>1</td>
<td>5.8</td>
<td>1</td>
</tr>
<tr>
<td>23rd</td>
<td>8.6</td>
<td>7.5</td>
<td>4.6</td>
<td>2.8</td>
<td>2.4</td>
<td>2.3</td>
<td>1.4</td>
<td>1.5</td>
<td>3.6</td>
<td>2.3</td>
<td>3.4</td>
<td>2.7</td>
</tr>
<tr>
<td>24th</td>
<td>8.8</td>
<td>6.8</td>
<td>4.1</td>
<td>2.5</td>
<td>2.5</td>
<td>1.5</td>
<td>1.7</td>
<td>2.3</td>
<td>4.1</td>
<td>4.1</td>
<td>2.7</td>
<td>9.4</td>
</tr>
<tr>
<td>25th</td>
<td>3.3</td>
<td>5.7</td>
<td>5.1</td>
<td>3.4</td>
<td>2.1</td>
<td>2.1</td>
<td>2.4</td>
<td>2.6</td>
<td>5.5</td>
<td>3.6</td>
<td>7.6</td>
<td>8.8</td>
</tr>
<tr>
<td>26th</td>
<td>4.2</td>
<td>3.4</td>
<td>5.4</td>
<td>2</td>
<td>0.9</td>
<td>2.3</td>
<td>3.6</td>
<td>7</td>
<td>4.8</td>
<td>3.1</td>
<td>7.3</td>
<td>6</td>
</tr>
<tr>
<td>27th</td>
<td>6.5</td>
<td>2</td>
<td>4.1</td>
<td>3.3</td>
<td>1.5</td>
<td>1.6</td>
<td>2.4</td>
<td>3.5</td>
<td>5.6</td>
<td>4.2</td>
<td>6.7</td>
<td>7.5</td>
</tr>
<tr>
<td>28th</td>
<td>7.9</td>
<td>3.8</td>
<td>2.9</td>
<td>1.8</td>
<td>2.4</td>
<td>2</td>
<td>1.4</td>
<td>3.1</td>
<td>4</td>
<td>2.1</td>
<td>0.6</td>
<td>8.1</td>
</tr>
<tr>
<td>29th</td>
<td>5.5</td>
<td>2.4</td>
<td>2.8</td>
<td>2.1</td>
<td>1.3</td>
<td>2.9</td>
<td>2.6</td>
<td>5.7</td>
<td>4.3</td>
<td>2.7</td>
<td>9.6</td>
<td>9.6</td>
</tr>
<tr>
<td>30th</td>
<td>8.6</td>
<td>2.9</td>
<td>2.8</td>
<td>0.7</td>
<td>1.9</td>
<td>2.7</td>
<td>2.8</td>
<td>2.2</td>
<td>3.9</td>
<td>7.5</td>
<td>9.6</td>
<td>9.6</td>
</tr>
<tr>
<td>31st</td>
<td>3.8</td>
<td>2.5</td>
<td>1.7</td>
<td>2.9</td>
<td>4.5</td>
<td>5.1</td>
<td>9.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.3 2013 Incident Energy Melbourne Airport - Days less than Modal

What the data in table 5.3 shows is that there are three months of the year (June, July, August) where most days are below the Modal day design point. In order to assess the impact of this run of days the data was re-analysed with these three months removed. The following results were returned:

**Modal Incident Energy:** $= 2.8 \text{ kWh} / m^2$

<table>
<thead>
<tr>
<th>Distribution Range kWh / day</th>
<th>Days Count</th>
<th>Probability of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.8</td>
<td>4</td>
<td>0.01</td>
</tr>
<tr>
<td>0.9 to 1.8</td>
<td>28</td>
<td>0.102</td>
</tr>
<tr>
<td>1.9 to 2.8</td>
<td>35</td>
<td>0.128</td>
</tr>
<tr>
<td>2.9 to 3.8</td>
<td>49</td>
<td>0.179</td>
</tr>
<tr>
<td>3.9 to 4.8</td>
<td>36</td>
<td>0.131</td>
</tr>
</tbody>
</table>

Table 5.4 - 2013 Melbourne Airport Weather Data - June, July August removed

The data in the above table highlights a few issues:

- The original Modal day calculation is not effected by removing the predominantly less than days, which re-enforces the importance of using this as the design starting point,
- As the Modal day energy level remains unchanged the baseline design will be equivalent to the design in section 3.6,
- If an analysis was conducted on this reduced data set the only change would be the estimate of the number of running hours for the ICE.
This result highlights an issue regarding the basic design assumptions that are inherent in the reference system being used in this study. The reference system included an ICE machine as a supply availability guarantee, on the assumption of a non-grid connected system. Hence the capital cost of the ICE machine could be viewed as a fixed ‘insurance cost’ directly related to the decision to be non-grid connected. What the above results show is that the system cost increase that comes from the inclusion of the ICE is limited to the capital cost and the inclusion of the ICE is not driving additional running costs that would not otherwise be included. This is worthwhile to remember when looking at the use of grid supplied energy (in place of ICE energy) to meet the system shortfall over the less than Modal days. This also further demonstrates how the technique is producing a minimum cost optimum.
5.3 Incorporating Load Requirements

The second capability that has been built into the technique is the ability to pre-process the system load requirements. For small scale energy systems the load requirements are related to the weather conditions (which also impacts renewable generation). The simple relationships can assumed to be as follows:

- Low incident solar energy days usually result from cloud cover which (in temperate climates especially) usually correlates with lower temperatures and increase heating load demand.

- Low incident energy days result from reduced daylight hours which are winter days that correlate with lower temperatures, increased heating demand and increase lighting demands.

- Higher incident energy days tend to be associated with higher temperatures and longer daylight hours, which may reduce heating and lighting loads but increase cooling loads.

There are prior examples of energy system optimisation techniques that incorporate the variation of loads that can occur. In [71] Milan models small CHP system using the DER-CAM tool set. In this approach loads are based on sample survey data and are embedded within the DER-CAM non-linear equations. The complexity of the approach results in the use of specific sample days, which for the comparative work being undertaken is suitable but may not be applicable if used in a design context.

A similar and typical approach is adopted by Merkel in [68] where real world data on energy use is gathered and used within a time step simulation of a CHP system. In this case there is no attempt to correlate load demand with climatic conditions as there is no consideration of non CHP generation in the study. A more sophisticated approach is used by Reda [72] in exploring control strategies for Solar Assisted Ground Source Heat Pumps (SAGSHP). This approach uses a yearly simulation model that includes a Domestic Hot Water (DHW) demand model based on survey data together with an integrated building performance model that uses daily incident energy data and daily temperature data to create and hour by hour estimate of load requirement. This is a comprehensive approach that produces highly accurate estimates. The approach is difficult to translate into a generic design tool as it requires highly detailed knowledge of the building being assessed. In addition optimisation is by trialing of defined system configurations. Despite that this work does show the potential
way in which incident energy and temperature data can be used to establish correlated load patterns.

There is significant body of work that explores the relationship between building energy requirements and climate. Typical of the advanced studies that are available is the work by Fikuru [73]. In this paper existing techniques to estimate load requirements based on weather variables, and the ability to generate predictive models is explored. Hong [74] adopts a simulation approach to the same problem while exploring a way to predict High Volume Air Conditioning (HVAC) loads for given cities (cities in this case representing different geographic locations and hence different relationships between incident energy, temperature and humidity. In an earlier study Kuffmann [75] introduces the simplified notion of Cooling Degree Days (CDD) and Heating Degree Days (HDD) as a mechanism to estimate how the total energy requirement for a given building varies with climate over a given sample period, in this case a year.

For this study what is important is not the benefit of any particular technique but rather the understanding that there are a range of techniques available to link weather with load requirements, and ultimately this means techniques are available to correlate incident solar energy with load requirements.

The following sections explore how the technique developed in this study can accommodate these relationships.

5.3.1 Incorporating Load Increases associated with Low Incident Energy Days

The technique developed uses a constraint equation form that was developed in order to allow a simple linkage between the required load and the incident energy day. In Chapter 4 it was shown:

\[ P_{EL(0)} = e_0 x_1........(4) \]

where \( e_0 \) is the incident energy (factored by PV array efficiency) for the Modal day and,
Using a similar structure for the hot water system results in:

\[ Q_{HWL(0)} = \mu_0 x_7 \ldots \ldots (5) \]

where \( \mu_0 \) is the incident energy (factored by HW array efficiency) for the Modal day

\[ P_{EL(i)} + gP_{EL(0)} = e_i x_1 + f y_{2i} + x_3 \ldots \ldots (6) \]

where

- \( e_i \) is the incident energy (factored by PV array efficiency) for the non-Modal day(s) and,
- \( g \) is a factor that determines the percentage of the \( P_{load(0)} \) that must be stored.

\[ Q_{HWL(i)} + \gamma Q_{HWL(0)} = \mu_i x_7 + \sigma y_{2i} + x_8 \ldots \ldots (7) \]

\( \mu_i \) is the incident energy (factored by HW array efficiency) for the non modal day and,

\( \gamma \) is the percentage of the \( Q_{HWL(0)} \) that must be stored to support the Modal day load.

As the method already addresses less than Modal days it is possible to pre-process the available weather data and then incorporate load variations. This is possible since the load for each particular band of incident energy is a simple left hand term in the constraint equation that correlates to that incident energy band. As a consequence, if it is possible to correlate load requirement with incident energy bands (which is possible via the related temperature and humidity data) then that correlation can be simple captured in each constraint relationship.

Table 5.5 was created by correlating the 2013 Melbourne Airport daily weather data for incident energy and temperature maximums and minimums. The table also shows assumed energy requirements that relate to these temperatures. Assumed energy consumption vs temperature data is used in this case since no measured or reported data of this type was found for Melbourne. For the purposes of this study what is important is that it is possible to conduct the analysis using the techniques developed, assuming that input data can be sourced or measured as required.
5.3. Incorporating Load Requirements

Table 5.5 - Load Profile Derived from 2013 Lees than Modal day vs Temperature Profile

Using the Table 5.5 Load Profile the simplifies analysis from section 3.6 was repeated and the results are shown below in Table 5.6

<table>
<thead>
<tr>
<th>Incident radiation range kWh/m²</th>
<th>Average High Temp (°C)</th>
<th>Average low temp (°C)</th>
<th>Electrical Load Assumption (kWh/day)</th>
<th>Hot Water Load Assumption (kWh/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8 Modal</td>
<td>18.1</td>
<td>10.2</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>0 to 1.0</td>
<td>16</td>
<td>2.9</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>1.1 to 2.0</td>
<td>18.0</td>
<td>5.7</td>
<td>18</td>
<td>30</td>
</tr>
<tr>
<td>2.1 to 3.0</td>
<td>18.5</td>
<td>8.6</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 5.6 - Simple Optimisation with Table 5.5 Loads Variable with Incident Energy

What is shown in Table 5.6 is the effect of increasing load requirements as incident energy drops. Compared with the simple section 3.6 results, where loads are considered to be constant, the table 5.6 results show that the arrays and storage are larger than they need to be to meet the Modal day requirements. This is occurring as the marginal cost of increasing the non-ICE generating and storage elements of the system, to meet the increased non-modal day loads, is cheaper than increasing the ICE run time.
Chapter 5. Features of the Technique

The main point illustrated by this section is that, as a result of the basic form of the constraint equations developed, it is easy to pre-process load vs incident energy correlations (for less than incident energy days) and establish the impact these correlations have on the optimal solution.
5.4 Incorporating Load Increases associated with High Incident Energy days

As was shown in the previous section the ability to explore the impact of load changes that occur as a result of weather variations is possible for lower than incident energy days. Exploring weather / load interactions and the impact on design solutions is not as simple for greater than Modal days since the basic methodology does not assess greater than Modal incident energy days.

The concept of not exploring the days of greater than modal incident energy was summarised in section 2.2.3 of the literature review as follows:

The incident energy on day \((n+1)\) is greater than the Modal Day \((n)\) i.e. \(I_{rad_{m(n+1)}} > I_{rad_{m(n)}}\) In this case the solar array will be larger than is necessary to provide the load and the battery will not be large enough to store all of the excess energy. This situation represents 'lost' generation capacity but does not add to the overall levelised Cost of Energy (COE). If all days where either modal days or in this case then there would be no advantage, from a COE perspective, to modify the modal optimal parameters.

This assumption was demonstrated to be correct in section 3.10 where it was shown that the designs suggested by the technique did produce the lowest cost of energy across the sample year. Inherent in both the assumption that it is not necessary to analyse \(I_{rad_{m(n+1)}} > I_{rad_{m(n)}}\) days, and also inbuilt within the section 3.10 proof was an unexplored assumption that the load on those days was equal to the load on the Modal day. As long as the load on \(I_{rad_{m(n+1)}} > I_{rad_{m(n)}}\) days is equal to or less than the Modal day then the technique will produce the minimum Cost of Energy (COE) solution. The issue of what happens if the energy load requirement on the greater than Modal day is greater than the Modal day load has so far not been considered.

For the style of small energy systems being considered, the most likely scenario is that there will be days where incident energy is greater than the Modal day and on these days the temperature and humidity may also be greater than Modal. On these days the electrical load requirement may be significantly higher than on the modal day due to the addition of air-conditioning loads. It is noted that cooling energy requirements may also lead to additional hot water requirements for higher capacity
High Volume Air Conditioning (HVAC) systems as such system use hot water absorption chillers to generate cold water for cooling. For the purpose of exploring how the base methodology can be modified to support an analysis of increasing loads on $I_{rad_{m(n+1)}} > I_{rad_{m(n)}}$ this section will focus on the impacts of increasing electrical loads only (which is the most likely scenario for small energy systems due to the dominance of compressor air refrigeration is small systems.)

The first option for conducting this analysis would be to include the requirement in the constraint equations. A constraint of the following form, that aligns with the existing methodology, was examined:

$$P_{EL(a)} = e_a x_1 + f y_{2a} \ldots \ldots (1)$$

where

$P_{EL(a)} =$ the electrical load requirement for the day ‘a’ (where a is a greater than Modal incident energy day)

$e_a =$ PV energy co-efficient for the day ‘a’

$y_{2a} =$ the ICE running time on day ‘a’

While this constraint has the form of the existing constraints it will not be able to be processed in such a way that a single optimal solution can be identified. A problem exists since this an equality expression, there is a unique solution, and no ability for the possibility of a solution that produces excess energy generation on greater than modal days as a result of optimisation and Modal or less than modal days. The most appropriate form of the constraint would be:

$$P_{EL(a)} \leq e_a x_1 + f y_{2a} \ldots \ldots (1a)$$
and while this can be processed mathematically it will not necessarily produce an optimal solution since the constraint can be satisfied by increasing $x_1$ with zero solutions for $y_{2a}$.

An alternative approach is to use a two step optimisation that develops the optimal solution using the baseline 'less than Modal' technique and then runs a post optimisation check to see if there are any greater than Modal day loads that cannot be met by the basic design. This approach can also be used to establish the quantum of energy that the system has available for export if the system is configured to do so. The following is the basic process that can be utilised. This approach allows the energy flows in the greater than Modal days (including increases in loads on greater than Modal days) to be analysed while maintaining the advantages of the base process.

Figure 5.1 Expanded Process Flows

The process used to review the performance of the design for greater than Modal days could be
similar to that used in section 3.10 as the verification methodology. In this approach the energy balance and potential energy deficit for each day was analysed in order to look at ICE run time (fuel burn) and hence variable operating costs. In section 3.1 of this study it was asserted that discrete energy bands could be used because from a design perspective possible solutions are only possible in discrete steps (size of PV panels, size of tanks, size of batteries). Since the designer only has the option of discrete solutions there is little point in searching mathematically for a single point optimum, especially if such a process results in a loss of design transparency, which is the risk when complex processing techniques are employed. As a consequence in this study the greater than Modal day question is addressed as follows:
5.4. Incorporating Load Increases associated with High Incident Energy days

The following section outlines an example of how load increases in greater than Modal days can be assessed.

Figure 5.2 Greater than Modal Day Load Assessment

The following section outlines an example of how load increases in greater than Modal days can be assessed.
5.4.1 Example of Loads Assessment in Greater than Modal days

The process outlined in figure 5.2 was completed for the Melbourne Airport weather data reference year of 2013. In summary the following processing approach was followed:

- Correlate the Minimum and Maximum temperatures for each day within the assessment bands (of incident solar energy),
- Average the temperatures occurring in the given days,
- Assume an electrical load associated with these temperature ranges
- Using the $51 m^2$ PV array area from example 3.8 work out the total PV generated energy for the day ranges in question
- Calculate the energy excess of deficit for the stated load.

It is noted that in this example, the correlation between temperature range and required power has been assumed as no actual data is available. This is suitable for illustrating the process. Using the above process the following results were achieved.

<table>
<thead>
<tr>
<th>Average Incident Energy (kWh/m²)</th>
<th>Low Average Temp °C</th>
<th>High Average Temp °C</th>
<th>Assumed Electrical Load (kWh)</th>
<th>Energy Deficit (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>9.2</td>
<td>19.6</td>
<td>20</td>
<td>4.3</td>
</tr>
<tr>
<td>4.4</td>
<td>11.3</td>
<td>22.4</td>
<td>25</td>
<td>6.4</td>
</tr>
<tr>
<td>5.4</td>
<td>11.8</td>
<td>24.2</td>
<td>30</td>
<td>8.5</td>
</tr>
<tr>
<td>6.4</td>
<td>12.6</td>
<td>26.2</td>
<td>30</td>
<td>15.7</td>
</tr>
<tr>
<td>7.4</td>
<td>12.6</td>
<td>27.2</td>
<td>35</td>
<td>17.8</td>
</tr>
<tr>
<td>8.4</td>
<td>12.3</td>
<td>30.1</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>9.4</td>
<td>9.6</td>
<td>26.1</td>
<td>30</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 5.7 Greater than Modal Day Energy Assessment

The results in table 5.7 show that there are no bands of incident energy days (since all ‘deficits’ are positive) where there is a deficit of energy generated by the optimised design. Without actual
load vs. temperature data there are no detailed conclusions that can be drawn. The results illustrate the following concepts:

- The Modal day design does produce excess energy for the greater than Modal days, even when loads are increased above the modal day load. This validates the initial assumption that was made at the start of the technique design that only less than modal incident energy days need to be included in the main optimisation.

- The technique supports the simple post analysis process outlined in figure 5.2.

- The technique supports a simple sensitivity analysis to investigate how much additional load could be added to the base system design before the optimisation LCOE would be exceeded.

It is further noted that the technique used does not need to be broken down into incident energy bands. It would be possible using the same approach to analyse a data set (incident solar energy, temperature, load requirement) and establish whether or not the Modal day optimisation produces sufficient energy to meet the load requirement on any given day.

Crucially what this section shows, is that a comprehensive analysis of the impact of load variations that occur with temperature (and humidity if required) can be conducted, in a simple transparent manner using the developed technique, without losing the basic advantages of the technique.
5.5 Assessment of the Ability to Export Energy and the Viability of Grid Connection

The reference system for this study has so far been described as a non grid-connected or ‘Islanded system’. While a non-grid system has been the basis of developing the technique being described it is possible to use the developed methodology to review aspects of grid connected systems. Following are some ways that the technique can be extended to explore questions associated with grid connection.

5.5.1 Basic Substitution Grid Energy for ICE Energy

The most basic question that can be addressed is whether or not the ICE machine used in the design can economically replaced with grid imported energy. Put another way the question is is ’What penalty cost is associated with using the ICE machine instead of using grid energy as a backup (supply security) source of energy and a way to make up any energy shortfall?’ There are only two substitutions required to assess how the system design, and hence how the Levalised Cost of Energy (LCOE) is impacted by using grid energy.

The first substitution required is to the cost function:

\[
\min C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i y_{2i} + d \ldots \ldots \quad (4)
\]

The \(bx_2\) term can be replaced by a direct substitution where \(b\) is the cost of imported energy per kWh and \(x_2\) is the required import energy expressed in kWh.

Similarly \(y_{2i}\) is the amount of import energy required on the less than Modal days being examined.
The second substitution is in the two constraint equations:

\[ P_{EL(i)} + g P_{EL(0)} = e_1 x_1 + f y_{2i} + x_3 \ldots \ldots \ldots (5) \]
\[ 0 \geq e_0 g x_1 + f g x_2 - x_3 \ldots \ldots \ldots \ldots \ldots \ldots (6) \]

As these are energy balance equations the terms \( f y_{2i} \) and \( f x_2 \) can be replaced directly by imported energy expressed in kWh.

So the technique allows the comparison of the LCOE that results for a given cost of either ICE energy or grid energy. The technique does not replicate the ability of other approaches, such as DER-CAM [29], to optimise the mix of imported versus self generated energy in any give time period. This is because the developed technique is primarily intended to be a design decision support tool and once a capital investment has been made, and a system deployed, then the only question becomes the potential to reduce ICE running costs by importing energy or alternatively to decide at the time of design that it is likely that importing grid energy is cheaper than provisioning for ICE energy. Operational rules, once a design is implemented, are a simpler question that does not require optimisation since the cost of importing energy v.s. the cost of ICE running are two known costs.

There is a further variation that is required to be considered, which is how to assess the CO\(_2\) impact of grid v.s ICE energy and this is addressed later in this chapter.

### 5.5.2 Energy Storage and Controlled Release

Price, government support and ease of integration has seen highly distributed Photo-Voltaic (PV) generation become a significant form of Renewable Energy in the electrical grids in many developed
countries [76] [77]. The widespread integration of PV generation into Low-Voltage (LV) networks produces power system quality issues. The primary LV circuit technical issue is that injected real power may raise circuit voltage above specification levels [78] [79]. Power system managers and regulators address this power quality risk by limiting the amount of PV an individual distributed generator can install [80] and by requiring that individual PV generating systems automatically disconnect if the circuit voltage exceeds a given limit [81]. The consequence of these quality management approaches is that PV system size is often not optimal from user economic and CO₂ reduction perspectives. This issue is amplified in climates with a wide yearly variation in incident solar radiation. In these circumstances high summer solar radiation limits PV system size such that little or no power is generated in winter months. [82]. The problems associated with Low Voltage (LV) distribution power quality as a result of excessive real power injection from distributed Photo-Voltaic (PV) interconnection have been extensively reviewed [83] [84] [85]. One possible solution is to limit in real time, the total real power injected by a pre-defined ‘cluster’ of closely (in circuit terms) connected generating sources. [86] [87]. The control mechanisms suggested involve monitoring bus voltage and then disconnecting PV generating capacity or reducing injected real power using “droop based” active power curtailment, in real time, in response to specific voltage limits being exceeded. Note that in the LV systems being discussed active power is a more appropriate control parameter than the reactive power techniques used in higher voltage transmission systems [88]. The approaches suggested result in an increase in the size of allowable PV systems that can be connected but still result in an un-defined economic loss for PV system owners due to lost energy supply income (assuming a grid interconnection and feed in tariff) from the PV capital investment.

To address this issue one possible technique is for real power that is being generated by the PV arrays, but cannot be feed into the network (because the LV feeder voltage has exceeded the allowable limit) to be stored and then injected into the network in a controlled manner when this can be done without impacting LV circuit quality. The concept of using Electrochemical Battery Storage Systems (EBSS) to time shift energy input to the network has been studied recently [77] [89]. The main emphasis of these studies has been to ‘reduce’ peak network loads (an economic benefit to the Network operator) [15] or to gain economic advantage by supplying energy when there is most economic advantage to the User/Generator (UG)[90] [90]. Using EBSS with the aim of allowing increased distributed
PV nameplate capacity does not require any new technical understanding but it does present new economic optimisation questions that can be addressed by the technique being developed. Further the necessity for central control of highly distributed resources is not a new concept (as it the basis of the very notion of the ‘Smart Grid’) [3] but the commercial considerations and design approaches discussed by the analysis in this chapter are unique to the concept of co-operative operation of highly distributed generators in a pre-existing market environment.

The original design rules outlined in this work have not considered the potential for income from excess energy that is generated on greater than Modal days. The amount of energy imported on a given day will vary depending on the incident energy for that day and the preceding days. On the Modal day the system, by design, does not import any energy. On days with incident energy greater than the Modal day the opportunity exists to: export energy in real time (as it is generated), store energy for later export, or provide the Demand Response (DR) equivalent contribution.

Starting from the base of the Modal day optimised, non-exporting system the three options above represent additional income with no additional cost. This is due to the assumption that the cost of providing the grid interconnection and the cost of the monitoring and control (that is required to allow the large PV array necessary to meet the Modal day requirement) has already been included in the base system.

The amount of ‘excess energy’ available on greater than Modal days was outlined in the proof in section 2.10 and can be described as follows:

On any given day \(k\) where \(I_{rad_{m(n+1)}} > I_{rad_{m(n)}}\) then the excess energy available for export is established by:

\[
P_{exp(k)} = c_{k}x_{1} - P_{load(k)} (2)
\]

The total yearly quantum of energy available is the arithmetic sum of the energy available on each
of the days of excess incident energy. The energy available on any given day can be either dispatched immediately or stored for later dispatch. If the energy is to be stored for later dispatch there will need to be an increase in the size of the battery as follows:

Using equations (2) and (3) the excess energy inherent in the baseline optimised design and the delta cost of batteries to allow controlled dispatch of that excess energy can be established.

Beyond the value of energy that may be generated by the baseline system it would be possible to increase the size of the PV array, the size of the storage or both if the income from the exported energy justified this investment. The following basic relationship summarises the key decision criteria: it is beneficial to invest if:

\[
I_{exp} + S_{ICE} \geq a x_{1 \Delta} + c x_{3 \Delta} \ldots \ldots \ldots (4)
\]

where

\( I_{exp} \) = the income from exported energy
\( S_{ICE} \) = the reduction in ICE running cost as PV array size increases
\( x_{1 \Delta} \) = increase in PV array size to provide additional export capacity
\( x_{3 \Delta} \) = increase in battery size to provide additional export capacity

Note that these relationships specifically relate to electrical energy but the hot water relationships would be identical. Hot Water is not addressed in this analysis as there are fewer market mechanisms for trading hot water.

The design produced by utilising equations (i), (j) and (k) remain an ‘optimum’ configuration for
the following reasons:

- The starting point is the Modal day optimum configuration established using the rules in chapter 3
- If the configuration remains unchanged and the only energy exported is the excess energy then this reduces the long term Levelised Cost of Energy (LCOE) hence the solution optimum is maintained.
- additional capital investment in either PV array capacity or battery capacity can be limited only to those investments that result in a positive rate of return, hence the LCOE optimum is maintained.

There are a wide range of potential commercial arrangements that could be entered into by the owners of small energy systems that are connected to the grid. It is not the role of this study to explore all of these potential schemes but rather to put in place a framework that allows any potential scheme to be analysed. What is shown is the technique is well suited to supporting analysis of commercial options because:

- The technique produces a defined minimum lowest cost of operation system configuration which can be then iterated to address potential export /import commercial agreement.
- The technique produces a probability estimate of the amount of energy that needs to be generated by an ICE or imported. This supports an analysis of commercial risk.
- The technique does not have import / export rules and does not seek to trade off import energy vs self generation in the basic form, hence it contains no pre-determined commercial rules. Rather the technique optimises the cost of ‘renewable’ generation and storage vs ‘non-renewable’ ( which is modelled as ICE energy contribution.)The resultant analysis structure allows the simple substitution of any source of less than modal day top up energy and any commercial costs without change to the basic technique form.

The ability of the technique to optimise the mix of non-renewable generation against renewable generation (generation dependant upon incident solar energy) in a simple manner and then be used as
a basis to explore further variations is a key advantage over existing techniques. The optimisation technique, because it starts from an assumption of self contained (no energy import, islanded) operation uses simple energy balance constraints and includes no further commercial constraints. As a result the simple optimisation produces a renewable v.s. non renewable optimum that can then be used as the basis to explore any commercial proposal to interact with energy sources external to the defined system boundary.
5.6 Multi-Objective Optimisation

So far the analysis presented has been a Single Objective Optimisation (SOO). The single objective, the subject of the objective function, has been the Cost of Energy (COE). There is a substantial body of work addressing issues associated with Distributed Energy Systems using the techniques of Multi Objective Optimisation (MOO). In this section the aims of MOO use in the analysis of DER style systems is discussed and the way that these ideas can be accommodated within the bounds of the developed technique are examined.

In very simple terms Multi Objective Optimisation describes a process whereby multiple outcomes or objectives are weighed up against each other and the ‘optimised’ solution is a ‘balanced’ outcome between all of, what are normally assumed to be, competing objectives. Analysis of Distributed Energy Resource (DER) Systems suggest MOO style questions since cost reduction and CO$_2$ reduction are often two desirable outcomes. DER optimisation often wants to address the balance between energy cost and CO$_2$ emissions.

Alarcon-Rodriguez et al. [91] have produced a summary of both the issues associated with MOO as applied to DER systems and the range of approaches used by different researchers. As noted in this paper the fundamental difference between SOO approaches and MOO approaches is that the SOO approach aims to identify a single optimum solution (i.e. in the case of this study so far the lowest cost of energy over a 20 year period). MOO techniques on the other hand cannot every produce a single answer but rather aim to identify what is often referred to as the ‘Pareto Front’
Figure 5.3 (extracted from [91]) illustrates how the two objective Pareto solution space is generally described. The nature of the solution set leads to the further concept of dominance being used in MOO problems. Dominance is used to assist in ‘choosing’ a single solution within the possible set of solutions described within the Pareto set. With reference to figure 4.3 solution ‘a’ is said to dominate solution ‘b’ if:

- ‘a’ is no worse than ‘b’ in all objectives and,
- ‘a’ is better than ‘b’ in at least one objective

This concept of ‘better’ and ‘worse’ solutions is a common thread that runs through all MOO approaches and is crucial in this study.

[91] provides a useful classification of MOO methods that will be useful in exploring the capabilities of the developed technique in addressing multi objective issues. The following classification of MOO techniques are suggested:
5.6. Multi-Objective Optimisation

- **‘a priori’** - where multi-objectives are aggregated into a single objective function, often using weights or value factors, and then processed as a SOO.

- **‘a posteriori’** - where multi-objectives are processed using complex objective functions to produce a solution set that is as close to a Pareto front (i.e., single line for a two-objective problem). Once this front is available, it can then be used to explore options and allow a conclusion about the desired solution to be reached.

A range of MOO techniques have been used by researchers to explore issues related to DER systems. Many authors use MOO approaches to look at the trade off between energy sources and technologies in CHP systems. In [92], DiZang analyses alternative energy and technology sources in a CHP system and includes estimates of total life cycle analysis using an ‘a priori’ style approach where multiple SOO objective functions are processed, using value weighting as constraints. The multiple SOO results are then post-processed to identify a solution set. Mallikarjun [93] addresses a complex architectural level optimisation of a CHP style system using three objectives (levelised cost of energy, aggregated emissions and reliability) using a goal programming methodology that requires significant pre-categorisation of the quality or value of specific attributes. A very simple version of the ‘a priori’ approach is used by Widodo in [94] to explore country level energy system designs. Here, a range of SOO analysis (lowest cost of energy, lowest CO2 emission) and then the results are compared manually to illustrate impacts of various system configurations.

Many authors, exemplified by [95], [96], [97] and [22], analyse small energy systems using variations of Multi-Objective Evolutionary Algorithms (MOEA). While they are different in detail, each approach utilises a two-stage process that uses a constrained SOO objective function, processes that function using genetic (GA) or evolutionary (EA) style methods to create a Pareto set. This set is further refined toward a smaller solution set using a second stage EA approach. All approaches have different advantages, related to the particular problem being addressed but all require that significant value judgements and weights are added to each parameter being addressed or included within the objective functions.

A very useful approach is developed by H. Ren et al in [98]. In this example, which addresses a
Chapter 5. Features of the Technique

small CHP system an exclusively economic objective function is processed then a separate environmental objective function is processed (two SOO style objective functions). Once this is completed there is a third ‘trade off’ objective function developed (with inputs from the two SOO processes) and this objective function is then analysed using Multi Objective Linear programming (MOLP) tool that produces a Pareto solution set. These sets are then graphically displayed and used to conduct a series of ‘what if’ style reviews. The approach is a great example of the advantages of the ‘a posteriori’ approach in that it is very subjective (few embedded value judgements or weighting), impacts of weights are transparent and easily communicated and information is conserved throughout.

The applicability and potential to conducted Multi-Objective analysis and optimisation using the technique developed in this thesis has to be reviewed within the context of the original purpose and focus of the technique. The only reason to develop, and hence to be able to model and optimise, the small energy systems that are the subject of this study is to provide a mechanism and architecture to support the use of ‘renewable’ (non fossil fuel) generating systems. The primary energy source being utilised is incident solar energy. The aim of the entire program is to reduce CO$_2$ emissions related to stationary energy supply. In this context the ‘optimum’ CO$_2$ emission is zero. With this in mind the following is observed regarding the rationale and potential for the use of Multi-Objective Optimisation for this study.

5.6.1 The nature of the System Architecture and the Use of the Modal day

The system design or reference architecture that has been used by this study is introduced in Chapter 3 (figure 3.1) and Chapter 4 (figure 4.1). This architecture aims to introduce renewable generation into small energy systems and includes an Internal Combustion Engine (ICE) driving a electrical generator. The ICE is primarily included in the design as a means to ensure supply security. The necessity to include the ICE for security reasons recognises that the probability exists that there will be a run of consecutive days such that the renewable resources may be insufficient to meet the load needs. Once the decision to include the ICE has been taken, and the capital expense added into the project, then the optimisation includes the ability of the ICE to deliver energy. The system is carrying the capital cost of the ICE as a form of supply security insurance.
Having included the ICE into the design the optimisation technique sets a constraint that requires all energy needs on the Modal day must be meet by the renewable sources. Another way to view this constraint is to say that:

\[ CO_2 \text{ emissions for Modal days and days of greater than Modal incident energy must be zero} \]

This is already a value judgement about the behaviour of the system with respect to \( CO_2 \) emissions. It may be that if this constraint was removed that a lower Levelised Cost of Energy (LCOE) could be achieved. This constraint is, in the terminology of Multi Objective Optimisation (MOO), an ‘\textit{a priori}’ approach. This approach limits the Pareto set being explored by the optimisation technique.

It is noted that the validation process used in section 3.10 produced the Table 3.3 results which are representing a MOO style Perato front since each ‘discrete design reference incident energy point’ directly relates to a specific level of \( CO_2 \) emissions.

5.6.2 Using the Technique to Support MOO Style Analysis

As noted above for the style of systems being explored in this work the primary ‘objectives’ associated with the exploration of design options are:

- the cost of energy produced
- the \( CO_2 \) emissions that result from energy production

Given the nature of the reference design the two objectives are in direct competition. A reduction in \( CO_2 \) emissions may lead to an increase in operating costs. Hence on the surface this looks like a task for Multi Objective Optimisation (MOO). Given the limited nature of this trade-off the existing technique supports, without significant modification, MOO like analysis. This can be done in two ways:
• use variations in the constraint associated with allowable ICE run time and build an understanding of how this impacts system LCOE

• assign a price to $CO_2$ emission and add this into the ICE running cost in the objective function

Both approaches are easily supported within the existing technique. What is inherent in both approaches is that any MOO ultimately requires a value judgement by the designer to reach a conclusion.

In the first approach the LINPROG process can be used to place bounds on the allowable values of the $x_2$ parameter. This would produce a range of values of $x_1$ (PV array size) and $x_3$ battery size. Using this information a form of Pareto front could be developed. Once this information is created then the designer would need to make a value judgement to answer the question what increase in LCOE will be accepted for a given reduction in $CO_2$ emissions. This is a value judgement, and as long as the technique can produce the information to support this judgement then it has provided MOO style functionality.

In the second approach the cost of $CO_2$ emissions can be captured in the cost objective function term $bx_2$ where the cost factor "b" was originally the cost of running the ICE generator per kWh, essentially the fuel cost. To support a MOO style analysis the ‘b’ term could be re-defined as follows:

$$ b = \text{ICE cost of fuel per kWh} + \text{ICE cost of } CO_2 \text{ pollution per kWh} $$

again this approach involves the application of a value judgement regarding the cost that should be assigned to $CO_2$ pollution.

What is important is that the basic optimisation technique developed can support either the first approach (an ‘a posteriori’ technique) or the second approach (an ‘a priori’ technique). This ability to support the key Multi-Objective question is such a simple manner, without amendment to the base equation form, is considered to be a key attribute of the developed technique.
5.7  Addressing Power Requirements

One of the significant potential disadvantages of the technique being explored is the loss of consideration of the impact of short term power variations on the optimisation solution.

Most of the key optimisation techniques presently used such as HOMER [28] and DERCAM [29] utilise short duration (per hour) analysis which looks at energy deficits. Variations on this approach is adopted by many authors, exemplified by Doroudchi. [99]. Underlying all of these methods is a deficit energy/power calculation that looks at instantaneous energy generation from the renewable, instantaneous state of storage and then meets any instantaneous energy shortfall by importing energy. Typical of the style of daily power requirement analysis is figure 4.4 that is taken from [100]

![Figure 5.4 Typical DER CAM Daily Power simulation](image)

The technique developed in this study does not utilise the approach of hour by hour deficit accounting and relies on a estimate of daily and consecutive day accounting. This approach means that the
Chapter 5. Features of the Technique

The technique cannot see short duration power gaps such as those depicted in Figure 5.4.

The reason for the difference between the technique developed and the alternative approach is the basic intent of the technique. The technique developed has been conceived specifically as a design support tool. It is assumed that the basic architecture has been chosen, that basic technologies have been chosen and then the developed design is optimised. It has been assumed that issues of short term power availability is/will be dealt with via initial design analysis and hence is not required to be captured within the optimisation. The second assumption, that has not previously been explained, is that the technique is based upon the system design methodology (that the optimisation technique has been developed to support), that uses a detailed load analysis rather than the more common ‘load following’ design methodology (as used by normal utility design.) The detailed load analysis approach is more commonly used in transport energy system design, specifically in aerospace and marine engineering. The detailed load analysis approach is further expanded by the assumption that the ICE machine does not operate in a load following mode but is operated in blocks of time to charge the storage (electrical or hot water) with load transients meet by the storage characteristics not by the ICE. This detailed load analysis approach is demonstrated in the Chapter 5 design example and is summarised as follows:

- total loads (energy users) are defined
- the occurrence of loads is categorised against time of operation
- loads are accounted for as 5 second, 5 minute and continuous
- average loads and peak (5 second) loads are summed for a given time epoch

This load analysis design approach allows peak power loads to be accommodated by design details. These details are then feed forward into the optimisation as constraints. An example would be a limit applied on the size of a battery and minimum state of charge allowable in a battery in order to support the peak load associated with the starting of a specific pump load.

As noted that sample architectures have been assumed not to use the ICE machines in the load following mode. This requires that particular technologies and architectures are adopted to meet the
5.7. Addressing Power Requirements

load analysis requirements. As an example while the system may require 15 kWh of battery capacity following a first pass design optimisation the technology of the storage is also important in the design, since while the most cost effective way to provide that 15 kWh may be to use Aqueous (NaCl electrolyte) energy storage batteries these batteries would not be able to provide the necessary short term transient power requirements. This may require the addition of some lithium ion batteries as an element of the total 15 kWh storage capacity. If this is the case the kWh cost of the batteries, from the perspective of the optimisation technique, will need to be higher and constraints on minimum state of charge (SOC) or total minimum battery capacity may be required.

In summary:

- in non-load following designs approaches considerable design (based on detailed transient load analysis) is required to ensure all aspects of system operation are understood and captured

- using such design process means power capacity is accounted for prior to optimisation and then feed into the optimisation as constraints.

- the optimisation technique developed is suited to this overall design approach as a clear distinction is drawn between sizing of relative elements and design attributes related to transient load requirements and supply availability

These issues are all explored in detail in the Chapter 6 example that illustrates the design process flow and how that impacts on the attributes required in the optimisation methodology.
5.8 Ability to Assess Alternative Technologies - Hydrogen Systems as an Example

The technique developed has been illustrated by the use of a ‘reference architecture’ (see figure 3.1). One of the original aims was to develop a technique to optimise the design of small energy systems and to ensure that technique could be technology agnostic or that the technique could be used to assess the commercial viability of new and emerging technologies. In this section the ability of the technique to assess new technologies is explored.

While the technique is developed to allow new technologies to be assessed there are some fundamental architectural aspects that underpin the optimisation technique. The technique has been developed to assess architectures with the following attributes:

- the system architectures are based around the primary source of energy generation being incident solar energy.
- The system architect requires a way to store energy to ensure supply when there is no incident energy or insufficient energy.
- The system architecture requires a way to provide energy availability in the event that there is a run of lower than normal incident energy days.

The reference system (figure 4.1) has used existing technologies configured in an architecture that is already used and known to work. There are a large number of technologies in a range of developmental states that could be used to meet the three aims of small energy system noted above. One technology, that is close to being commercially viable, but not in widespread use is small hydrogen energy systems. Sample hydrogen technologies will be used to show how the technique can be used to assess new technologies.

A significant body of work by Andrews and Shabani exists that explores the design and economic viability of small scale Hydrogen systems.\cite{101}, \cite{101}, \cite{102}. In this work Hydrogen conversion and generation technologies are used as both storage technologies (figure 5.5) and then further modified
to create CHP style functionality (figure 5.6).

Figure 5.5 Basic Hydrogen Solar Storage System (extract from[101])

Figure 5.6 Diagram of a hydrogen system for CHP.
Figure 5.6 Cooling System for Fuel Cell to Create CHP Like Functionality (extract from [101])

If such a system was to be utilised the following functional substitutions into the Figure 3.1 reference system could be made:

<table>
<thead>
<tr>
<th>Function</th>
<th>ICE / Battery Architecture</th>
<th>Hydrogen Architecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Energy Storage</td>
<td>Electrochemical Battery</td>
<td>Electrolyser / Hydrogen storage tank / Fuel cell</td>
</tr>
<tr>
<td>Energy Availability low incident energy day</td>
<td>ICE Generator and Hydrocarbon fuel storage</td>
<td>Hydrogen storage tank / Fuel cell</td>
</tr>
<tr>
<td>CHP Functionality</td>
<td>ICE Generator, Hydrocarbon fuel storage and Hot water tank</td>
<td>Hydrogen storage tank / Fuel cell / cooling system / hot water tank.</td>
</tr>
</tbody>
</table>

Table 5.8 Small Energy System Functional Allocation for Hydrogen Technologies

Once the functional allocations are understood the characterisation of the Hydrogen system elements can be completed in a way that can then be substituted into the basic technique equations.

While it is possible to map high level functions, as is done in Table 5.8, there are fundamental differences between the way the Hydrogen architecture can function and the ICE base reference architecture functionality.

The approach adopted for the ICE based architecture used the Modal day as a first stage reference and asserted that excess energy generated on greater than modal days had no economic benefit in the non grid connected system. This conclusion is based on the cost and short term nature of electrochemical battery storage. This assertion proved to be correct when checked in the section 2.10 validation process. This assertion is not valid for the Hydrogen system architecture due to the ability of hydrogen generated in times of high incident energy to be used months later during periods of consecutive days of low incident energy. This feature is recognised and exploited in the analysis
conducted by Andrews [102].

In order to understand options for analysing the Hydrogen system it is necessary to identify the key economic trade-off decisions and these can be informed the the following hydrogen system component characteristics.

### 5.8.1 Electrolyzer Relationships

The electrolyzer converts DC electricity to hydrogen (and oxygen). The relationship between the power that can be generated from a Photo Voltaic (PV) panel (which has a specific current v.s. voltage characteristic) when it is coupled to a electrolyzer (which also has a current v.s. voltage characteristic) is complex and is outlined in [103]. That said in order to support simple analysis there are simple performance metrics available. [104] suggest that a conservative estimate of a conversion rate of 50 kWh/kg (lower heating value conversion efficiency of 67 percent).

Electrolyzers are rated (and sold) on input power size (kW) and production rate in $Nm^3$ or kg per hour. As a consequence any design trade-off the addresses the size (and hence cost) of the electrolyzer needs to consider:

- the amount of electrical energy that needs to be stored
- the ‘excess’ PV energy available to generate hydrogen for storage
- the time for which this electrical excess is available.

### 5.8.2 Storage Tank Relationships

The storage tank relationships are simple depending only on the volume of the tank and the storage pressure. In [102] an estimate of 300 dollars to 500 dollars per kg is quoted for atmospheric pressure tanks. The key issue beyond cost is the space available in a given system for hydrogen storage tanks. Space available becomes a constraint that results in either less storage capacity or more cost if higher
pressure tanks (and processing equipment) is used.

5.8.3 Fuel Cell Relationships

In a similar fashion to the electrolyzer the fuel cell obeys a particular output current v.s. terminal voltage characteristic. In energy systems because the output voltage is usually a fixed characteristic commercial fuel cells are usually specified in power output and voltage. As an example of the manner in which devices are specified the ACTA Power 1000 stand alone Hydrogen energy system can deliver 2.5 kW at 48VDC. This device uses hydrogen at the rate of approximately 730NL / kWh of delivered electrical energy (note NL is a non-SI unit used to describe gas volumes at zero degrees C and 1 standard atmosphere).

Fuel cells generate heat which can potentially be captured to create a CHP style functionality. The potential to gather ‘waste’ heat is addressed in [103]. This paper suggests that a 500W device produced about 200W of recovered heat (assuming a 48VDC output voltage). Further the paper suggest an available heat output of about 22 percent of the hydrogen energy input.

While the exact performance may vary dramatically from system design to system design the important thing to note is that such relationships can be established.

5.8.4 Water Usage

The final relationship that should be considered when exploring the optimisation energy system is the cost of water. The input water required for a given system is highly dependant on the design of the system (as water can be captured and re-cycled) and a review has shown that water consumption is often not stated. For the purpose of this study it is just noted that the ability to cost water should be included.
5.8.5 Optimisation relationships

The following general relationships apply to the Hydrogen system optimisation:

- There is a minimum size of the fuel cell which is set by the load requirement that exists in time periods where there is no incident energy. The only reason to increase the size of the fuel cell would be if there was a load deficit (incident solar energy available - fuel cell capacity) that would need to be met via stored energy.

- The maximum storage volume (at standard pressure) is a physical constraint set by the system physical arrangement.

- The minimum PV array size is related to the size of storage and the ability to add to the storage on days of lesser incident energy, which is related to the size of the electrolyzer.

Given the above the question is can the technique developed, or at least the principles used, be utilised to address the optimisation of the energy system.

The design question can be summarised as:

"What is the minimum possible cost derived from PV array size, electrolyzer and tank size that allows the load requirements to be meet for the worst possible run of low incident energy days"

There is a fundamental difference between the Battery / ICE system and the hydrogen system. In the Battery / ICE system the ICE machine is available to make up any shortfall in Incident energy load if the battery stored energy is not sufficient. In the Hydrogen system this is not the case and the hydrogen system sizing must be varied at the time of initial design in order to meet the load requirement. Consequently the Hydrogen design question does not have the same 'first stage decision variable / second stage variable' form as generally assumed by the Stochastic programming with recourse problem. However it is possible to use the same basic concepts associated with the notion of the incident emergency probability to assess the hydrogen optimisation question. The concepts developed in this thesis can be adapted to assess the hydrogen system design question as follows:
Chapter 5. Features of the Technique

- Establish the size of the Fuel Cell based on the load requirement for the hours (on the Modal day) where there is no incident energy (i.e. the \( g \) term from Chapter 2) Establish the size of the storage required to provide fuel for the fuel cell for the hours (on the Modal day) where there is no incident energy. This also provides a size for the electrolyser.

- Establish the size of the PV array necessary to supply the load and also power the electrolyser while the incident energy is available.

These steps provide a system design that is optimised for the modal day. So as has been previously discussed this design will be close to the cost optimum over the year, since it is optimum for most days in the year. The second stage of the design process is to iterate this design based upon the probability of particular incident energy days. This will involve the following steps:

- Establish the less than modal day energy deficit as a deficit vs probability of occurrence distribution. Establish the largest storage tank necessary to meet the probable energy deficits. Develop a probability distribution for excess electrical energy for greater than modal days of incident energy using the modal day PV array size. This PDF then determines if the Modal PV array size is large enough. If not the PDF will allow the required increase in PV array size to be established.

The hydrogen system is chosen as an example as it illustrates how the technique developed can only be directly re-used if the first stage design then second stage operational variables exist inherently within the system design architecture. Despite the fact that the hydrogen system design question cannot be addressed directly by the developed technique the underlying concepts of the use of a first pass modal day design then the use of the probability distribution of incident energy to iterate that design can be re-applied.

A further design question that arises is the trade-off between importing grid energy and generating and storing hydrogen. If this was the design question being addressed then the existing technique would be useful as the entire hydrogen system would represent a storage cost parameter that is impacted by the probability of incident solar energy. The cost of hydrogen storage could be then optimised against the cost of imported energy.
Other technologies do drop simply into the structure of the existing technique, such as replacing the ICE generator with a Stirling Cycle machine.
5.9 Summary

The initial aims of this thesis study was to produce an analytical optimisation technique that supported a investigation into the impact of long term incident solar energy data, that supported the ability to assess the impact of load variation with weather changes, that supported investigation of grid interconnection, that supported Multi-Objective optimisation and that allowed a review of alternative technologies. This chapter confirms that the technique developed meets all of these requirements.
Chapter 6

Contiguous Example

6.1 Introduction

In the previous chapters the basic technique has been established and expanded and a range of features of the technique have been explored. In this chapter a single real world example is conducted as a way to further investigate the technique and its application.

The analysis is going to utilise as an example an imaginary Secondary School campus located Beaumaris Victoria. The size of the school has been estimated as has the energy usage, which has been based on a room by room analysis for a modern designed building. Such a school is currently in development and its has been used as an example since this shows how the technique can be used to form the basis of a design tool.

6.2 Statement of Requirements

The following are the assumed requirements for the energy system

6.2.1 Basic System Design

The design for the energy system for the school has the following basic attributes

- The primary source of electrical energy will be PV panels
• Electrical storage will utilise aqueous batteries for energy and Lithium ion batteries for power

• Heating will be provided by hot water reticulation with passive radiators

• Primary water heating will be via passive Hot water collectors

• Energy availability (electricity and will be ensured by a backup bio-diesel fuelled ICE machine, size based on potential load requirements

• Cooling will be provided by discrete ‘inverter style’ air conditioners in each room

This is the system architecture that is shown in chapter 4 figure 4.1.

6.2.2 Weather Data

The weather data used for the initial design is from the closest weather station at Moorabbin Airport Victoria (accessed from the Australian Bureau of Meteorology www.bom.gov.au). The data shown in figure 6.1 below is the incident solar energy for 2015 for the Moorabbin Airport weather station. The daily incident energy is divided up in 20 discrete bands of 0.5 kWhrs per day increments. The probability distribution that results is shown as Figure 6.1

![Figure 6.1 - 2015 Moorabbin Airport Incident Solar Energy.](image)

The preliminary analysis of this data shows that the Modal Day of incident solar energy occurs
within the band 2.1 to 2.5 kWhrs/day of incident energy. For the preliminary analysis the Modal Day incident energy will be assumed to be 2.25 kWhrs/day

Later in this example the impact of using 10 years of data for Moorabbin Airport weather station, relative to the 2015 snapshot, will be examined.

6.2.3 Load Data

A theoretical load analysis has been created for the ‘sample school’. The load analysis was developed as a bottom up analysis on a room by room basis. The school operating time was broken into three distinct time periods across the day:

- The "Day" period when students are in attendance and the majority of class rooms are occupied. Assumed to be 9 am until 4 pm.
- The "Evening" period when staff and some students are on site. Assumed to be 4 pm until 8 pm.
- The "Continuous" period which is for loads that operate constantly regardless of the attendance of staff and students

The load analysis of the school was based on an assumed class room configuration and use pattern as summarised below:

- 30 ‘standard’ classrooms with 26 occupied each hour of the day period
- 4 laboratories with 3 occupied each hour of the day period
- 2 kitchens with 1 occupied each hour of the day period
- 4 student common rooms occupied each hour of the day period
Chapter 6. Contiguous Example

- 10 staff rooms occupied constantly for the day and evening periods
- 6 offices occupied constantly for the day and evening time periods
- one each of sports hall and auditorium which where averaged to one large space operating across the day and evening periods
- one library operating constantly in the day period.

The continuous load is an allowance for items such as Energy System controls, IT servers, security lighting and security systems and storage refrigerators. The following table shows the summary load analysis.

<table>
<thead>
<tr>
<th>Load Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electrical</strong></td>
<td></td>
</tr>
<tr>
<td>Total Day (Amp-Hours)</td>
<td>3290</td>
</tr>
<tr>
<td>Total Evening (Amp-Hours)</td>
<td>541</td>
</tr>
<tr>
<td>Total Continuous (Amp-Hours)</td>
<td>934</td>
</tr>
<tr>
<td>Peak Current Day (Amperes)</td>
<td>411</td>
</tr>
<tr>
<td>Peak Current Evening (Amperes)</td>
<td>135</td>
</tr>
<tr>
<td>Peak Current all day (Amperes)</td>
<td>223</td>
</tr>
<tr>
<td><strong>Heating</strong></td>
<td></td>
</tr>
<tr>
<td>Daily Load 5 to 12 C (kWhrs)</td>
<td>3360</td>
</tr>
<tr>
<td>Daily Load 12 to 18 C (kWhrs)</td>
<td>2192</td>
</tr>
<tr>
<td><strong>Cooling</strong></td>
<td></td>
</tr>
<tr>
<td>Daily Load 20 to 30 C (kWhrs)</td>
<td>640</td>
</tr>
<tr>
<td>Daily Load 30 to 40 C (kWhrs)</td>
<td>1200</td>
</tr>
</tbody>
</table>

Table 6.1 - Summary Load Analysis for Hypothetical Beaumaris School

The load in this example is expressed as a total energy demand in the established time periods. This is a different approach that is used in many other approaches such as HOMER [28] or DER CAM [29] where hour by hour style analysis is used. This use of average loads is in line with the design concept of the energy system which is an energy storage system architecture rather than a load following system. Such a design approach is possible when a load analysis is constructed to define both the total
6.2. Statement of Requirements

energy load and the peak power consumption.
6.3 First Pass Analysis

The first pass analysis will use the equations for electrical / hot water systems developed in Chapter 4. Note that the system architectures is shown in Figure 4.1 in this first pass analysis all Hot Water generation will assumed to be created from the Hot Water arrays and the ICE, that is there is no electrical generation of hot water as was explored in section 4.1 In the first pass analysis the cooling load is assumed to be created by distributed electrical air-conditioning.

Referring back to Chapter 4 the following equations are applicable:

For the simple system the optimisation question is summarised by the following objective function:

\[ \min C_{elec} = C_{pv} + C_{ice} + C_{batt} + C_{hwa} + C_{tank} \]

where

- \( C_{elec} \) is the cost of electricity
- \( C_{pv} \) is the cost of the PV generated electricity
- \( C_{ice} \) is the cost of the ICE generator generated electricity and Hot Water
- \( C_{batt} \) is the cost of the battery storage
- \( C_{hwa} \) is the cost of the solar Hot Water generation
- \( C_{tank} \) is the cost of the Hot Water storage tank

The applicable objective function is:

\[ \min C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_{i} P_i y_{2i} + d \]

where for the expanded system the design variables are:
6.3. First Pass Analysis

$x_1$ Size of PV array ($m^2$)

$x_3$ Battery Size (kWh)

$x_7$ Size of Hot Water collector ($m^2$)

$x_8$ Tank Size (kWh)

$x_2$ ICE Run-Time (hours per day)

and

$\sum_i P_i y_{2i}$ is the arithmetic sum of each second stage scenario generator runtime as factored by the probability of that scenario, where $i = 1, 2, ..., n$ is the number of energy range days less than modal.

$y_{2i}$ ICE Run-Time (hours per day) for the post modal day

Modal Day Load Constraints

In Chapter 4 the Modal day constraints were defined as:

$$P_{EL(0)} = e_0 x_1 + f x_2$$

where

$e_0$ is the incident energy (factored by PV array efficiency) for the modal day and, $f$ is the size of the system generator in kWh

and

$$Q_{HWL(0)} = \mu_0 x_7 + \sigma x_2$$
where

$\mu_0$ is the incident energy (factored by HW array efficiency) for the Modal day and,

$\sigma$ is the heat output of the ICE in kWh per hour of running.

**Non Modal Day Load Constraints**

Similarly the Non Modal day constraints are defined as:

\[
P_{EL(i)} + gP_{EL(0)} = e_i x_1 + f y_2 + x_3
\]

where

$e_i$ is the incident energy (factored by PV array efficiency) for the non-Modal day(s) and,

$g$ is a factor that determines the percentage of the $P_{load(0)}$ that must be stored.

and

\[
Q_{HWL(i)} + \gamma Q_{HWL(0)} = \mu_i x_7 + \sigma y_2 + x_8
\]

where

$\mu_i$ is the incident energy (factored by HW array efficiency) for the non-Modal day and,

$\gamma$ is the percentage of the $Q_{HWL(0)}$ that must be stored to support the Modal day load.

**Storage Constraints**
Finally the storage constraints were established as

\[ 0 \geq e_0 g x_1 + f g x_2 - x_3 \]

and

\[ 0 \geq \gamma \mu x_7 + \gamma \sigma x_2 - x_8 \]

### 6.3.1 The Scenario Tree

Figure 6.1 shows the distribution of incident energy for the school site in 2015. The distribution was developed using discreet steps of 0.5 kWh / day of incident energy. Using this discrete distribution the Modal Incident solar energy day was established as 2.25 kWh/day. This leaves three discrete ranges of incident energy days (expressed as kWhr/day) that are less than the Modal energy day with the following probabilities of occurrence:

\[ I_{rad0} = 0.25, P_{I_{rad0}} = 0 \]
\[ I_{rad1} = 0.75, P_{I_{rad1}} = 0.04383 \]
\[ I_{rad2} = 1.25, P_{I_{rad2}} = 0.05205 \]
\[ I_{rad3} = 1.75, P_{I_{rad3}} = 0.10685 \]

If all three probability scenarios were to be analysed a scenario tree as shown in Figure 6.2 be required:
In section 3.7 it was asserted that provided each occurrence of an incident energy day was independent (which they are) then the multi-stage scenario tree can be collapsed back to simple two stage scenario tree (as shown in figure 6.3) using the following approach.

**Nomenclature**
In order to support the convolution process the following naming convention and generalised scenario tree has been used:
Figure 6.3 - Generic Equivalent Incident Energy Scenario Tree

The generic labels define the state of the incident energy day and the probability of the occurrence of that day, i.e:

$$e_{a1} = \{I_{rad_{a1}}, P(I_{rad_{a1}})\}$$

In the specific case of the scenario tree convolution shown in Figure 6.4 the following reduction can occur
where the specific figure 6.3 nodes $e_{b12}$ to $e_{b121}, e_{b122}, e_{b123}$ network is reduced to three new equivalent nodes $e_{b12a}, e_{b12b}, e_{b12c}$ where

\[ e_{b12a} = \{I_{radb12a}, P(I_{radb12a})\} \]

and

\[ I_{radb12a} = I_{radb12} + I_{radb121} \]
\[ P(I_{radb12a}) = P(I_{radb12}) \times P(I_{radb121}) \]

It can be seen that such a convolution based reduction will result in an equivalent two stage tree comprised of \(3 \times 3 \times 3 = 27\) new equivalent second stage terms.
While this is complex diagrammatically it is a simple process to mechanise using a spreadsheet.
The final convoluted stage two terms were calculated to be as follows:

<table>
<thead>
<tr>
<th>2nd Stage Equivalents</th>
<th>Incident Energy (kWh/day)</th>
<th>Probability of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>2.25</td>
<td>8.42004E-05</td>
</tr>
<tr>
<td>e2</td>
<td>2.75</td>
<td>9.99916E-05</td>
</tr>
<tr>
<td>e3</td>
<td>3.25</td>
<td>9.99916E-05</td>
</tr>
<tr>
<td>e4</td>
<td>2.75</td>
<td>9.99916E-05</td>
</tr>
<tr>
<td>e5</td>
<td>3.25</td>
<td>0.000118744</td>
</tr>
<tr>
<td>e6</td>
<td>3.75</td>
<td>0.000243762</td>
</tr>
<tr>
<td>e7</td>
<td>3.25</td>
<td>0.000205266</td>
</tr>
<tr>
<td>e8</td>
<td>3.75</td>
<td>0.000243762</td>
</tr>
<tr>
<td>e9</td>
<td>4.25</td>
<td>0.000500404</td>
</tr>
<tr>
<td>e11</td>
<td>2.75</td>
<td>9.99916E-05</td>
</tr>
<tr>
<td>e12</td>
<td>3.25</td>
<td>0.000118744</td>
</tr>
<tr>
<td>e13</td>
<td>3.75</td>
<td>0.000118744</td>
</tr>
<tr>
<td>e14</td>
<td>3.25</td>
<td>0.000118744</td>
</tr>
<tr>
<td>e15</td>
<td>3.75</td>
<td>0.000141014</td>
</tr>
<tr>
<td>e16</td>
<td>4.25</td>
<td>0.000289478</td>
</tr>
<tr>
<td>e17</td>
<td>3.75</td>
<td>0.000243762</td>
</tr>
<tr>
<td>e18</td>
<td>4.25</td>
<td>0.000289478</td>
</tr>
<tr>
<td>e19</td>
<td>4.75</td>
<td>0.000594251</td>
</tr>
<tr>
<td>e21</td>
<td>1.5</td>
<td>0.000205266</td>
</tr>
<tr>
<td>e22</td>
<td>3.75</td>
<td>0.000243762</td>
</tr>
<tr>
<td>e23</td>
<td>4.25</td>
<td>0.000243762</td>
</tr>
<tr>
<td>e24</td>
<td>3.75</td>
<td>0.000243762</td>
</tr>
<tr>
<td>e25</td>
<td>4.25</td>
<td>0.000289478</td>
</tr>
<tr>
<td>e26</td>
<td>4.75</td>
<td>0.000594251</td>
</tr>
<tr>
<td>e27</td>
<td>4.25</td>
<td>0.000500404</td>
</tr>
<tr>
<td>e28</td>
<td>4.75</td>
<td>0.000594251</td>
</tr>
<tr>
<td>e29</td>
<td>5.25</td>
<td>0.001219898</td>
</tr>
</tbody>
</table>

Table 6.2- Convoluted Equivalent Incident Energy Bands and Probabilities
IMPORTANT NOTE

Collapsing the three stage scenarios back to one as illustrated above is a ‘worse case’ assessment as it addresses all the probabilities of having three consecutive days of less than Modal. There are a number of other scenarios that involve one less than modal consecutive days and two less than Modal consecutive days. These scenarios are not included since they do not change the optimum solution. This is because the trade-off being examined is total likely increase in Modal optimum PV / battery size vs total likely ICE generator running time. Considering the single consecutive and double consecutive cases would not include as large a load that had to be covered by PV / battery size increase on the Modal day. The full explanation is shown in Appendix A.

The 27 equivalent second stage terms are then addressed in the analysis as follows:

6.3.2 Input Variables and Assumptions

The Input variables as outlined in section 4.5 are predominantly system cost and economic assumptions. For this example they are assumed to be as follows:
### Table 6.3- Input Variables and Assumptions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value (AUD)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{bat}$</td>
<td>Cost of Battery per KWh</td>
<td>$1000 Li Ion $500 NaCl</td>
<td>Two costs are used for a hybrid battery design. Lithium battery is used for power delivery and Salt-Water battery for energy storage.</td>
</tr>
<tr>
<td>$C_{PV}$</td>
<td>PV cost per m²</td>
<td>$250 / m²</td>
<td>Based on Winacoo WST style panels</td>
</tr>
<tr>
<td>$n_{PV}$</td>
<td>PV efficiency</td>
<td>0.14</td>
<td>Based on Winacoo WST style panels</td>
</tr>
<tr>
<td>$ICE_{cost}$</td>
<td>Cost of running ICE per hour</td>
<td>$14</td>
<td>See Note 1</td>
</tr>
<tr>
<td>$C_{HP/Tank}$</td>
<td>Cost of Hot Water tank per kWh</td>
<td>$28.6</td>
<td>See Note 2</td>
</tr>
<tr>
<td>$C_{HP/Array}$</td>
<td>Cost of Hot Water array per m²</td>
<td>$50</td>
<td>Established based on previous published data for CPC collectors</td>
</tr>
<tr>
<td>System Assumptions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DoD</td>
<td>Allowable Battery discharge</td>
<td>80%</td>
<td>This is related to the two battery technologies chosen together with a factor of safety related to energy availability</td>
</tr>
<tr>
<td>SL</td>
<td>System life</td>
<td>20 years</td>
<td>This is an economic assumption</td>
</tr>
<tr>
<td>BL</td>
<td>Battery life</td>
<td>10 years</td>
<td>See Note 3.</td>
</tr>
<tr>
<td>$r$</td>
<td>Assumed interest rate</td>
<td>6%</td>
<td></td>
</tr>
</tbody>
</table>

#### NOTES

1. The ICE machine basic size is selected outside the optimisation process. This is done because the machine is seen as a non discretionary capital investment since it is required to provide a supply availability guarantee. Reviewing the total load requirement on a given day is that the system needs to be able to deliver 4766 Amp-hours over an 8 hour period or 596 Amps = 142 kW per hour. This number drives the design requirement and there is little room for optimisation. The design choice for this example is to use a 190 kW Steyr multi fuel marine diesel engine (SE266E40) driving 200 kW AC three phase generator. The capital cost estimate is 30,000 AUD per machine. Fuel consumption is 20 L/hr or 14 AUD / hr cost.

2. The calculations in support of this estimate is contained in section 4. The calculation assumes
a 90 degree C storage temperature.

3. The battery life is a function of the number of charge-discharge cycles. As the reference design is not load following but rather assumes a energy storage design it is assumed that the batteries are subject to one cycle per day. This is an economically conservative assumption.

Cost Parameter Calculation

Equation (b) is the primary objective function. It includes cost scaling factors \((a, b, c, m, n)\) which all established as follows:

As previously discussed in Chapters 3 and 4 daily cost are used because the loads, and incident energy and probability of incident energy are all analysed on a per day basis. Costs for long-life capital purchases are calculated using Equivalent Annual Cost (EAC) which is then built into a per annum objective function.

\[
EAC = \frac{InitialCost}{A_{SL,r}}
\]

where \(A_{SL,r}\) is the annuity rate

\[
A_{SL,r} = \frac{1 - 1/(1 + r)^{SL}}{r}
\]

where \(SL\) is the system life in years and \(r\) is the assumed long term interest rate.

In the technique established cost comparison is based on an estimate of Levalised Cost of Energy (LCOE) per annum on a present year baseline. This is a highly conservative approach as it allows the LCOE for the Combined Heat and Power (CHP) system to be compared with the LCOE from the competing grid energy, in baseline dollars. Across the course of the project the LCOE for the CHP system stays at the baseline dollar amount, so effectively drops each year, whereas the cost of grid energy notionally rises with inflation.
The ‘Annuity Rate’ is a way to establish the total cost of the capital investment (today dollar capital cost plus interest charges assuming a payback equal to the system life) the distribute this equally across the system life years. This approach is used in both the Hybrid 2 [27] and DERCAM [29] model as way to distribute capital costs.

The second construct is where there is a component life that is less than the system life. in this case the baseline cost is factored up by the ratio of the component life to system life. Again this is conservative since the initial interest charge will be higher (as its costed over a longer period) and then the mid life capital purchase will be paid in later year dollars but accounted for in today dollars.

hence

\[ A_{SL,r} = \frac{[1 - 1/(1 + r)^{SL}]}{r} \]
\[ A_{SL,r} = \frac{[1 - 1/(1 + 0.6)^{20}]}{0.6} \]
\[ A_{SL,r} = 11.5 \]

\[ a = C_{pv/n}/(A_{SL,r}) \times (SL/PL) \] ....(ii)......(This is an annualised cost per m² of PV array)

hence

\[ a = C_{pv}/(A_{SL,r}) \times (SL/PL) \]
\[ a = 250/11.5 \times (20/20) \]
\[ a = 21.74 \]

\[ b = 365 \times ICE_{run} \] ....(iii) is the yearly fuel cost which assumes a defined generator / ICE combination

hence, remembering \( x_2 \) is ICE run time in hours per day

\[ b = 365 \times ICE_{run}/n \]
In the earlier chapters one battery type was considered. In this example the approaches illustrates how two battery types can be considered as this is seen as a mechanism to optimise higher cost (power) against lower cost (energy storage) batteries. In order to conduct this optimisation equation (b) is amended as follows:

\[ b = 365 \times 14 \]
\[ b = 5110 \]

\[ \sum c = c_a x_{3a} + c_a x_{3a} \]

In earlier chapters the value of \( c \) was established as follows:

\[ c = \frac{C_{\text{batt}}}{n} \times \left( \frac{A_{SL,r}}{SL/BL} \right) \]  

(This is an annualised cost per kWh)

hence \( c_a \) (lithium ion) and \( c_b \) (aqueous) can be established as follows:

\[ c_a = \frac{C_{\text{batta}}}{(A_{SL,r})} \times (SL/BL) \]
\[ c_a = \frac{1000}{11.5} \times (20/10) \]
\[ c_a = 174 \]

\[ c_b = \frac{C_{\text{battb}}}{(A_{SL,r})} \times (SL/BL) \]
\[ c_b = \frac{500}{11.5} \times (20/10) \]
\[ c_b = 87 \]

\[ n = \frac{C_{\text{hwtank}}}{(A_{SL,r})} \times (SL/TL) \]  

(annualised cost per kWh of the tank)

therefore assuming the tank life is equivalent to the system life

\[ n = \frac{28.6}{11.5} = 2.484 \]
$m$ is the scaling factor for the Hot Water CPC array

$$m = \frac{C_{hwa}}{A_{SL,r}} \times (SL/AL)$$

Assuming a simple HW array cost of 50 AUD dollars per $m^2$ gives:

$$m = 4.35$$

Using the above factors the objective functions and Matrices are calculated as follows:

### 6.3.3 Cost Function and the $f$ Matrix

As noted earlier the basic objective function is

$$\min C_{elec} = ax_1 + bx_2 + cx_3 + mx_7 + nx_8 + \sum_i P_i b y_{2i} + d$$

This is amended for two batteries to create:

$$\min C_{elec} = ax_1 + bx_2 + c_a x_{3a} + c_b x_{3b} + mx_7 + nx_8 + \sum_i P_i b y_{2i} + d$$

Substituting in the factors developed in the previous section gives:

$$\min C_{elec} = 21.74x_1 + 511.0x_2 + 174x_{3a} + 87x_{3b} + 4.35x_7 + 2.484x_8 + \sum_i P_i b y_{2i}$$

The second order terms scaling parameters are defined by $P_i \ast b$
Table 6.4- $P_i \times b$ Parameters for the Cost Function Matrix

This results in an $f$ matrix (33 x 1) as follows:

<table>
<thead>
<tr>
<th>2\textsuperscript{nd} stage Equivalent</th>
<th>Incident Energy kWhr</th>
<th>Probability</th>
<th>$P_i \times b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>2.25</td>
<td>8.42004E-05</td>
<td>0.430264299</td>
</tr>
<tr>
<td>e2</td>
<td>2.75</td>
<td>9.99916E-05</td>
<td>0.510957261</td>
</tr>
<tr>
<td>e3</td>
<td>3.25</td>
<td>9.99916E-05</td>
<td>0.510957261</td>
</tr>
<tr>
<td>e4</td>
<td>2.75</td>
<td>9.99916E-05</td>
<td>0.510957261</td>
</tr>
<tr>
<td>e5</td>
<td>3.25</td>
<td>0.000118744</td>
<td>0.606783606</td>
</tr>
<tr>
<td>e6</td>
<td>3.75</td>
<td>0.000243762</td>
<td>1.245625904</td>
</tr>
<tr>
<td>e7</td>
<td>3.25</td>
<td>0.000205266</td>
<td>1.048910343</td>
</tr>
<tr>
<td>e8</td>
<td>3.75</td>
<td>0.000243762</td>
<td>1.245625904</td>
</tr>
<tr>
<td>e9</td>
<td>4.25</td>
<td>0.000500404</td>
<td>2.557062974</td>
</tr>
<tr>
<td>e11</td>
<td>2.75</td>
<td>9.99916E-05</td>
<td>0.510957261</td>
</tr>
<tr>
<td>e12</td>
<td>3.25</td>
<td>0.000118744</td>
<td>0.606783606</td>
</tr>
<tr>
<td>e13</td>
<td>3.75</td>
<td>0.000118744</td>
<td>0.606783606</td>
</tr>
<tr>
<td>e14</td>
<td>3.25</td>
<td>0.000118744</td>
<td>0.606783606</td>
</tr>
<tr>
<td>e15</td>
<td>3.75</td>
<td>0.000141014</td>
<td>0.72058149</td>
</tr>
<tr>
<td>e16</td>
<td>4.25</td>
<td>0.000289478</td>
<td>1.479234047</td>
</tr>
<tr>
<td>e17</td>
<td>3.75</td>
<td>0.000243762</td>
<td>1.245625904</td>
</tr>
<tr>
<td>e18</td>
<td>4.25</td>
<td>0.000289478</td>
<td>1.479234047</td>
</tr>
<tr>
<td>e19</td>
<td>4.75</td>
<td>0.000594251</td>
<td>3.03662167</td>
</tr>
<tr>
<td>e21</td>
<td>1.5</td>
<td>0.000205266</td>
<td>1.048910343</td>
</tr>
<tr>
<td>e22</td>
<td>3.75</td>
<td>0.000243762</td>
<td>1.245625904</td>
</tr>
<tr>
<td>e23</td>
<td>4.25</td>
<td>0.000243762</td>
<td>1.245625904</td>
</tr>
<tr>
<td>e24</td>
<td>3.75</td>
<td>0.000243762</td>
<td>1.245625904</td>
</tr>
<tr>
<td>e25</td>
<td>4.25</td>
<td>0.000289478</td>
<td>1.479234047</td>
</tr>
<tr>
<td>e26</td>
<td>4.75</td>
<td>0.000594251</td>
<td>3.03662167</td>
</tr>
<tr>
<td>e27</td>
<td>4.25</td>
<td>0.000500404</td>
<td>2.557062974</td>
</tr>
<tr>
<td>e28</td>
<td>4.75</td>
<td>0.000594251</td>
<td>3.03662167</td>
</tr>
<tr>
<td>e29</td>
<td>5.25</td>
<td>0.001219898</td>
<td>6.233679644</td>
</tr>
</tbody>
</table>
6.3. First Pass Analysis

\[
x = \begin{bmatrix}
21.74 \\
5011 \\
174 \\
87 \\
4.53 \\
2.48
\end{bmatrix}
\]

\[t_x = P_{e1} \times b \]
\[P_{e2} \times b \]
\[P_{e3} \times b \]
\[.. \]
\[.. \]
\[P_{e29} \times b \]

6.3.4 Load Constraints and the \( A_{eq}, b_{eq} \) Matrix

\[ P_{EL(0)} = e_0 x_1 + f x_2 \]
\[ P_{EL(i)} + g P_{EL(0)} = e_i x_1 + f y_{ei} + x_{3a} + x_{3b} \]
\[ Q_{HWL(0)} = \mu_0 x_7 + \sigma x_2 \]
\[ Q_{HWL(i)} + \gamma Q_{HWL(0)} = \mu_i x_7 + \sigma y_{ei} + x_8 \]

The scaling factor \( f \) is defined in chapter 3 as the size of the generator in kW connected to the ICE machine. In this example the design, which is driven by the need to meet the load with ICE generation alone, is 1 x 200 kW machines. Hence \( f = 200 \)

The scaling factor \( g \) is defined in chapter 3 as a factor that determines the percentage of the total electrical load that must be stored. In effect this is a factor that considers what percentage of the load occurs while the incident solar energy is present. In this present example the load profiles is more
complex than the Chapter 3 and 4 examples hence $g$ is established as follows:

\[
\begin{array}{|c|c|}
\hline
\text{Load Parameter} & \text{Amp \_hours} \\
\hline
\text{Daytime Load} & 3290 \\
\text{2 hours of storage} & 822 \\
\text{daytime load} & \\
\text{Evening load} & 541 \\
\text{24 hour Load} & 934 \\
\text{Total Storage (A)} & 2297 \\
\text{Total Day Load (B)} & 3290 \\
G = (A) / (B) & 0.7 \\
\hline
\end{array}
\]

Table 6.5- Calculation of $g$ Factor

e_0$ and $e_i$ have in previous chapters been defined as the total incident solar energy on a given day factored by the ‘PV panel efficiency’. In practice it is known that the PV array does not ‘harvest’ all of the incident solar energy available. The factors that impact on PV array energy conversion performance have been known for many years and where explored from a theoretical perspective by Evans [105] and by Kurokawa [106]. Subsequent to these early works there has been a significant body of work looking at the impacts on PV array performance exemplified by Kirn [107]. Other work such as that by Yan [108] uses longitudinal experimental data. Sharma conducted a review of all the recent approaches [109]. In the work Sharma defines the concept of a solar array Performance Ratio which is defined as the ratio of the final yield (The total energy generated by the PV system for a defined period (day, month or year) divided by the rated output power of the installed PV system) to the reference yield (the ratio of total in plane solar insolation (Ht) ($kWh/m^2$) to the reference irradiance (G) ($1kW/m^2$). Sharma’s reference yield is equivalent to the figures for $e_0$ and $e_i$ used in earlier chapters where as in the more detailed analysis in this chapter, to be realistic, should be using a final yield style figure. Sharma reviews a number of papers all of which suggest a performance ratios of between 0.6 and 0.8 are achieved in real experimental systems. This range of performance accords with theory [106] and long term experiment [108]. For this analysis example the actual figure is not as important as the notion that the Rerformance Ratio (PR) depends on a wide range of parameters,
can be measured for a particular design and is compatible with the technique developed. The PR can be incorporated into the developed technique as follows:

Let

\[ e_i = I_{rad(i)} \times \eta_{pv} \times PR \]

where
- \( e_i \) is the final incident energy yield on day \( i \)
- \( I_{rad(i)} \) is the total incident energy on day \( i \)
- \( \eta_{pv} \) is the basic cell efficiency and
- \( PR \) is the Performance Ratio

Using this approach with a performance ratio of 0.7 the values for \( e_i \) have been established as follows:

\[ e_o = 2.25 \times 0.14 \times 0.7 = 0.22 \]

and the \( e_i \) values are as follows:
Table 6.6 $e_i$ Values for the Figure 6.4 Scenario Tree

<table>
<thead>
<tr>
<th>2nd Stage Equivalents</th>
<th>Energy kWh</th>
<th>$e_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>2.25</td>
<td>0.2205</td>
</tr>
<tr>
<td>e2</td>
<td>2.75</td>
<td>0.2695</td>
</tr>
<tr>
<td>e3</td>
<td>3.25</td>
<td>0.3185</td>
</tr>
<tr>
<td>e4</td>
<td>2.75</td>
<td>0.2695</td>
</tr>
<tr>
<td>e5</td>
<td>3.25</td>
<td>0.3185</td>
</tr>
<tr>
<td>e6</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e7</td>
<td>3.25</td>
<td>0.3185</td>
</tr>
<tr>
<td>e8</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e9</td>
<td>4.25</td>
<td>0.4165</td>
</tr>
<tr>
<td>e10</td>
<td>2.75</td>
<td>0.2695</td>
</tr>
<tr>
<td>e11</td>
<td>3.25</td>
<td>0.3185</td>
</tr>
<tr>
<td>e12</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e13</td>
<td>3.25</td>
<td>0.3185</td>
</tr>
<tr>
<td>e14</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e15</td>
<td>4.25</td>
<td>0.4165</td>
</tr>
<tr>
<td>e16</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e17</td>
<td>4.25</td>
<td>0.4165</td>
</tr>
<tr>
<td>e18</td>
<td>4.75</td>
<td>0.4655</td>
</tr>
<tr>
<td>e19</td>
<td>4.75</td>
<td>0.4655</td>
</tr>
<tr>
<td>e20</td>
<td>1.5</td>
<td>0.147</td>
</tr>
<tr>
<td>e21</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e22</td>
<td>4.25</td>
<td>0.4165</td>
</tr>
<tr>
<td>e23</td>
<td>3.75</td>
<td>0.3675</td>
</tr>
<tr>
<td>e24</td>
<td>4.25</td>
<td>0.4165</td>
</tr>
<tr>
<td>e25</td>
<td>4.75</td>
<td>0.4655</td>
</tr>
<tr>
<td>e26</td>
<td>4.75</td>
<td>0.4655</td>
</tr>
<tr>
<td>e27</td>
<td>4.25</td>
<td>0.4165</td>
</tr>
<tr>
<td>e28</td>
<td>4.75</td>
<td>0.4655</td>
</tr>
<tr>
<td>e29</td>
<td>5.25</td>
<td>0.5145</td>
</tr>
</tbody>
</table>

The above details result in the following

\[
P_{\text{load}(0)} = 0.222x_1 + 400x_2
\]
\[
P_{\text{load}(e_1)} + gP_{\text{load}(0)} = 0.221x_1 + 200y_{e1} + x_{3a} + x_{3b}
\]
\[
P_{\text{load}(e_2)} + gP_{\text{load}(0)} = 0.270x_1 + 200y_{e2} + x_{3a} + x_{3b}
\]
\[
P_{\text{load}(e_3)} + gP_{\text{load}(0)} = 0.319x_1 + 200y_{e3} + x_{3a} + x_{3b}
\]
\[
\]
\[
P_{\text{load}(e_{29})} + gP_{\text{load}(0)} = 0.515x_1 + 200y_{e29} + x_{3a} + x_{3b}
\]

The same approach is used to develop the Hot Water load equations.
As noted in Chapter 4 $\sigma$ is the heat output of the ICE in kWh per hour of running. Again such figures need to be establish by test as the ability to recover the heat is heavily design dependant but using the rough rule of thumb that in a small ICE twice as much heat energy as shaft (generator) energy is created in this case the $1 \times 200$ kW shaft power would create $800$ kW of heat for every hour of running, hence

$$\sigma = 400$$

$\gamma$ is the percentage of the $Q_{HWL(0)}$ that must be stored to support the Modal day load. In line with the analysis in Chapter 3 for the PV array it is assumed as a starting point that 6 hours of incident energy is available. As there are no depth of discharge allowances for the water tank, $\gamma$ becomes a simple ratio. The heat load occurs across $(8 + 4 = 12)$ hours and hence

$$\gamma = 0.5$$

$\mu_i$ is the incident energy (factored by HW array efficiency) for the incident energy on day $i$.

The concept of the efficiency of the collectors is related to temperature differentials and state operating conditions but there are some rough ranges. Ultimately this figure is best established by test of the arrays being examined. For this discussion a value of 0.5 for the CPC design was identified by Ooommen in [60] and is used in this sample calculation. It is worth noting that unlike the flat panel PV arrays being explored in this example the CPC, by design, has a uniform ‘performance ratio’ [56], [57]. For the purpose of this example the performance ratio will be set as 1, remembering that if experimental results showed this was in error that the technique remains the same and the only result is further scaling of the $\mu_i$ value.

Based on the above

$$Q_{HW(0)} = 1.125x_7 + 400x_2$$
$$Q_{HW(e1) + 0.5Q_{HW(0)}} = 1.125x_7 + 400y_{e1} + x_8$$
\[ Q_{HW(e_2)} + 0.5Q_{HW(0)} = 1.375x^7 + 400y_{e2} + x_8 \]
\[ Q_{HW(e_3)} + 0.5Q_{HW(0)} = 1.625x^7 + 400y_{e3} + x_8 \]
\[ \cdot \]
\[ \cdot \]
\[ \cdot \]
\[ Q_{HW(e_{29})} + 0.5Q_{HW(0)} = 2.625x^7 + 400y_{e29} + x_8 \]

At this point a constraint is applied to drive a solution that does not require the use of the ICE on the Modal day. This is done in order to establish the upper bound of the PV array area. This can be implemented by setting the \( x_2 \) terms in the \( A_{eq} \) array to zero.

As illustrated above there are 6 first stage parameters and 27 second stage parameters. These result in 56 equality equations. Together this results in a 56 x 33 \( A_{eq} \) matrix with the following form:

\[
A_{eq} =
\]

While the 27 second stage terms create a large sparse \( A_{eq} \) matrix the nature of the structure of the problem allows the use of simple solvers such as Matlab LINPROG.
The $b_{eq}$ matrix is formed from the Left Hand terms, which in these energy balance equations relate to the system loads. The load data is shown in table 6.1. The technique is designed to allow different load data to be used for different days of incident energy. For the purpose of this example the following summary loads are being used.

**Daily Electrical Load**

Total Load one day = 4766 Amp hours = 1144 kWh

**Daily Heat Load**

Using the low temperature day requirement = 3360 kWh

Remembering that the convoluted scenario tree covers three successive days there is a requirement to account for 3 days of load, hence

\[ P_{EL(i)} + gP_{EL(0)} = (3 \times 1114) + (0.7 \times 1114) \]

\[ P_{EL(i)} + gP_{EL(0)} = 4232 \text{ and} \]

\[ Q_{HWL(i)} + \gamma Q_{HWL(0)} = (3 \times 3360) + (0.5 \times 3360) \]

\[ Q_{HWL(i)} + \gamma Q_{HWL(0)} = 11760 \]

The 56 x 33 $A_{eq}$ matrix results in a need for a 56 x 1 $beqmatrix$ of the following form:
6.3.5 Storage Constraints and the $A, b$ Matrix

The electrical storage constraints developed in chapter 4 are modified to support the two battery types as follows:

\[
0 \geq e_0 g x_1 + f g x_2 - x_{3a} - x_{3b}
\]

substituting the values developed earlier gives:

\[
0 \geq 0.154 x_1 + 140 x_2 - x_{3a} - x_{3b}
\]
and the Hot Water equation becomes

\[ 0 \geq \gamma \mu_0 x_7 + \gamma \sigma x_2 - x_8 \]

substituting the values developed earlier gives:

\[ 0 \geq 0.5625 x_7 + 200 x_2 - x_8 \]

The 27 second stage scenarios mean that there are 33 variables resulting in a 2 x 33 \( A \) matrix of the following form:

\[
A = \begin{bmatrix}
0.154 & 140 & -1 & -1 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 200 & 0 & 0 & 0.562 & -1 & 0 & 0 & \ldots & 0
\end{bmatrix}
\]

and \( b = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)

Battery Configuration

With the equations established as noted above the lithium ion battery (\( x_{3a} \)) will be set to zero by the optimisation as it is more expensive than the aqueous (\( x_{3b} \)) battery. There is a minimum size for the Lithium Ion battery which is a system design attribute related to the ability to deliver the peak current. Table 6.1 notes the daily maximum current as 1595 Amps.

The load analysis suggests a 500 Amp peak can occur for 5 minutes. Without an actual load measurement the peak is not fully able to be established. Prior to optimisation its possible to estimate the crude battery requirement at about 800 kWh (refer table 5.5). As a starting point it will be assumed that 20 percent of the battery (160 kWh) should be Lithium Ion. This limit could be expressed as
an equality constraint however the LINPROG tool makes allowance for upper and lower parameter values. Hence the Lithium Ion battery requirement can be set as a ‘lower bound’ limit in the LINPROG code and this will have the same result.
6.4 First Pass Analysis Results

The first pass analysis results are shown in table 6.7:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simple Answer</th>
<th>PV Size Limit 1</th>
<th>PV Size Limit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>5200.00</td>
<td>3900.00</td>
<td>2499.99</td>
</tr>
<tr>
<td>X2</td>
<td>0.01</td>
<td>1.43</td>
<td>2.97</td>
</tr>
<tr>
<td>X3</td>
<td>207.90</td>
<td>437.78</td>
<td>854.16</td>
</tr>
<tr>
<td>X4</td>
<td>560.15</td>
<td>418.27</td>
<td>873.80</td>
</tr>
<tr>
<td>X5</td>
<td>2986.67</td>
<td>2986.67</td>
<td>2986.67</td>
</tr>
<tr>
<td>X6</td>
<td>3729.92</td>
<td>3061.01</td>
<td>4064.29</td>
</tr>
<tr>
<td>Y1</td>
<td>15.41</td>
<td>12.11</td>
<td>11.21</td>
</tr>
<tr>
<td>Y2</td>
<td>13.56</td>
<td>11.48</td>
<td>8.97</td>
</tr>
<tr>
<td>Y3</td>
<td>11.70</td>
<td>9.80</td>
<td>7.34</td>
</tr>
<tr>
<td>Y4</td>
<td>14.14</td>
<td>11.48</td>
<td>8.97</td>
</tr>
<tr>
<td>Y5</td>
<td>12.87</td>
<td>10.67</td>
<td>8.54</td>
</tr>
<tr>
<td>Y6</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y7</td>
<td>12.87</td>
<td>10.67</td>
<td>8.54</td>
</tr>
<tr>
<td>Y8</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y9</td>
<td>10.33</td>
<td>8.76</td>
<td>7.31</td>
</tr>
<tr>
<td>Y10</td>
<td>14.14</td>
<td>11.62</td>
<td>9.15</td>
</tr>
<tr>
<td>Y11</td>
<td>12.87</td>
<td>10.67</td>
<td>8.54</td>
</tr>
<tr>
<td>Y12</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y13</td>
<td>12.87</td>
<td>10.67</td>
<td>8.54</td>
</tr>
<tr>
<td>Y14</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y15</td>
<td>10.33</td>
<td>8.76</td>
<td>7.31</td>
</tr>
<tr>
<td>Y16</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y17</td>
<td>10.33</td>
<td>8.76</td>
<td>7.31</td>
</tr>
<tr>
<td>Y18</td>
<td>9.07</td>
<td>7.80</td>
<td>6.70</td>
</tr>
<tr>
<td>Y19</td>
<td>17.30</td>
<td>14.01</td>
<td>10.68</td>
</tr>
<tr>
<td>Y20</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y21</td>
<td>10.33</td>
<td>8.76</td>
<td>7.31</td>
</tr>
<tr>
<td>Y22</td>
<td>11.60</td>
<td>9.71</td>
<td>7.93</td>
</tr>
<tr>
<td>Y23</td>
<td>10.33</td>
<td>8.76</td>
<td>7.31</td>
</tr>
<tr>
<td>Y24</td>
<td>9.07</td>
<td>7.80</td>
<td>6.70</td>
</tr>
<tr>
<td>Y25</td>
<td>10.33</td>
<td>8.76</td>
<td>7.31</td>
</tr>
<tr>
<td>Y26</td>
<td>9.07</td>
<td>7.80</td>
<td>6.70</td>
</tr>
<tr>
<td>Y27</td>
<td>9.07</td>
<td>7.80</td>
<td>6.70</td>
</tr>
<tr>
<td>Y28</td>
<td>7.80</td>
<td>6.85</td>
<td>6.09</td>
</tr>
</tbody>
</table>

Table 6.7 First Pass Analysis Results

The ‘Simple Answer’ results are the baseline configuration where the system is designed such that on the Modal day the system does not require the use of the ICE. This results in a very large PV array of 5200m². While a school site may have sufficient roof space and other opportunities to mount PV panels (e.g. outdoor area shade or car park shading ) it could occur that there is a limit to the
available space for PV Panels. The ability of the technique to explore the consequence of limiting PV size has been explored in this example. While the LINPROG solver allows upper and lower bounds to be set this approach was not used as this may not be possible if another solver was utilised. To allow a maximum PV array size to be defined the following changes to the base relationships were made:

- A third ‘inequality constraint’ of the form $X_1 \leq PV_{arealimit}$ was added
- In the $Aeq$ matrix the $P_{EL(0)} = e_0 x_1 + f x_2$ had the $f$ value set to no zero

The results can be seen in table 6.7 columns "PV Size Limit". It is noted that these results are not the minimum possible Levelised Cost of Energy (LCOE) for the system but rather the minimum LCOE configuration that is possible for the given PV array size limit. (note that the limits set where $3900m^2$ and $2500m^2$ and that the slightly higher sizes represent issues with the ability of the LINPROG solver to settle on an absolute solution.) The main point of this example is to illustrate the ability of the technique developed to be used as a design investigation tool by exploring how real world constraint can be simply incorporated without amending form of the basic technique.

### 6.4.1 Impact of using a Larger Weather Record

As was discussed in Chapter 5 it is possible (and desirable from a modelling perspective) to use a larger weather data sample when creating a solution. For this example a second analysis was created using 15 years of weather data (from 2000 to 2015). This larger sample of incident energy data was processed into the same discrete bands as used in Figure 6.1.
6.4. First Pass Analysis Results

Comparing the data in Figures 6.1 and 6.8 illustrates how using a larger weather data set results in a more symmetrical (skewed normal) probability distribution, which is what would be expected for a natural phenomenon such as incident solar energy.

The data in 6.8 was used as input for the example system analysis. The Modal day incident energy level remains as 2.25 kWh/day. This leaves three discrete ranges of incident energy days that are less than the modal energy day with the following probabilities of occurrence:

\[ I_{rad_0} = 0.25, P_{I_{rad_0}} = 0 \]
\[ I_{rad_1} = 0.75, P_{I_{rad_1}} = 0.044 \]
\[ I_{rad_2} = 1.25, P_{I_{rad_2}} = 0.076 \]
\[ I_{rad_3} = 1.75, P_{I_{rad_3}} = 0.087 \]

As there are three less than Modal bands the same ‘equivalent’ 27 second stage scenario tree, as
shown in figure 6.4 was used. Parameters for the scenario tree and to suit the technique equations were established as follows:

The second order terms scaling parameters are defined by $P_i \times b$

<table>
<thead>
<tr>
<th>2nd Stage Equivalents</th>
<th>Energy kWh</th>
<th>Probability</th>
<th>$P_i \times b$</th>
<th>$e_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>2.25</td>
<td>1.068E-04</td>
<td>0.55</td>
<td>0.2205</td>
</tr>
<tr>
<td>e2</td>
<td>2.75</td>
<td>1.857E-04</td>
<td>0.95</td>
<td>0.2695</td>
</tr>
<tr>
<td>e3</td>
<td>3.25</td>
<td>1.655E-04</td>
<td>0.85</td>
<td>0.3185</td>
</tr>
<tr>
<td>e4</td>
<td>2.75</td>
<td>1.655E-04</td>
<td>0.85</td>
<td>0.2695</td>
</tr>
<tr>
<td>e5</td>
<td>3.25</td>
<td>2.878E-04</td>
<td>1.47</td>
<td>0.3185</td>
</tr>
<tr>
<td>e6</td>
<td>3.75</td>
<td>3.277E-04</td>
<td>1.67</td>
<td>0.3675</td>
</tr>
<tr>
<td>e7</td>
<td>3.25</td>
<td>1.884E-04</td>
<td>0.96</td>
<td>0.3185</td>
</tr>
<tr>
<td>e8</td>
<td>3.75</td>
<td>3.277E-04</td>
<td>1.67</td>
<td>0.3675</td>
</tr>
<tr>
<td>e9</td>
<td>4.25</td>
<td>3.732E-04</td>
<td>1.91</td>
<td>0.4165</td>
</tr>
<tr>
<td>e11</td>
<td>2.75</td>
<td>1.655E-04</td>
<td>0.85</td>
<td>0.2695</td>
</tr>
<tr>
<td>e12</td>
<td>3.25</td>
<td>2.878E-04</td>
<td>1.47</td>
<td>0.3185</td>
</tr>
<tr>
<td>e13</td>
<td>3.75</td>
<td>2.564E-04</td>
<td>1.31</td>
<td>0.3675</td>
</tr>
<tr>
<td>e14</td>
<td>3.25</td>
<td>2.564E-04</td>
<td>1.31</td>
<td>0.3185</td>
</tr>
<tr>
<td>e15</td>
<td>3.75</td>
<td>4.459E-04</td>
<td>2.28</td>
<td>0.3675</td>
</tr>
<tr>
<td>e16</td>
<td>4.25</td>
<td>5.078E-04</td>
<td>2.59</td>
<td>0.4165</td>
</tr>
<tr>
<td>e17</td>
<td>3.75</td>
<td>2.920E-04</td>
<td>1.49</td>
<td>0.3675</td>
</tr>
<tr>
<td>e18</td>
<td>4.25</td>
<td>5.078E-04</td>
<td>2.59</td>
<td>0.4165</td>
</tr>
<tr>
<td>e19</td>
<td>4.75</td>
<td>5.783E-04</td>
<td>2.95</td>
<td>0.4655</td>
</tr>
<tr>
<td>e21</td>
<td>1.5</td>
<td>1.884E-04</td>
<td>0.96</td>
<td>0.147</td>
</tr>
<tr>
<td>e24</td>
<td>3.75</td>
<td>2.920E-04</td>
<td>1.49</td>
<td>0.3675</td>
</tr>
<tr>
<td>e25</td>
<td>4.25</td>
<td>5.078E-04</td>
<td>2.59</td>
<td>0.4165</td>
</tr>
<tr>
<td>e26</td>
<td>4.75</td>
<td>5.783E-04</td>
<td>2.95</td>
<td>0.4655</td>
</tr>
<tr>
<td>e27</td>
<td>4.25</td>
<td>3.325E-04</td>
<td>1.70</td>
<td>0.4165</td>
</tr>
<tr>
<td>e28</td>
<td>4.75</td>
<td>5.783E-04</td>
<td>2.95</td>
<td>0.4655</td>
</tr>
<tr>
<td>e29</td>
<td>5.25</td>
<td>6.585E-04</td>
<td>3.36</td>
<td>0.5145</td>
</tr>
</tbody>
</table>

Table 6.9- Extended Weather Data Analysis Scenario Parameters 2000-2015

Using these parameters the following results were returned.
Table 6.10 shows that the analysis has returned the same result for both sets of weather data. This is because the large data set and the 2015 data set returned the same Modal day and similar probabilities for non-modal days. What the example does illustrate is the ability of the technique to accommodate pre-processed weather data and to use multiple years of data in developing the solution.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2015 Answer</th>
<th>2000-2015 Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>5200.00</td>
<td>5200.00</td>
</tr>
<tr>
<td>X2</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>X3a</td>
<td>207.90</td>
<td>207.90</td>
</tr>
<tr>
<td>X3d</td>
<td>560.15</td>
<td>560.15</td>
</tr>
<tr>
<td>X7</td>
<td>2986.67</td>
<td>2986.67</td>
</tr>
<tr>
<td>X8</td>
<td>3729.92</td>
<td>3729.92</td>
</tr>
<tr>
<td>y1</td>
<td>15.41</td>
<td>15.41</td>
</tr>
<tr>
<td>y2</td>
<td>13.56</td>
<td>13.56</td>
</tr>
<tr>
<td>y3</td>
<td>11.70</td>
<td>11.70</td>
</tr>
<tr>
<td>y4</td>
<td>14.14</td>
<td>14.14</td>
</tr>
<tr>
<td>y5</td>
<td>12.87</td>
<td>12.87</td>
</tr>
<tr>
<td>y6</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y7</td>
<td>12.87</td>
<td>12.87</td>
</tr>
<tr>
<td>y8</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y9</td>
<td>10.33</td>
<td>10.33</td>
</tr>
<tr>
<td>y11</td>
<td>14.14</td>
<td>14.14</td>
</tr>
<tr>
<td>y12</td>
<td>12.87</td>
<td>12.87</td>
</tr>
<tr>
<td>y13</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y14</td>
<td>12.87</td>
<td>12.87</td>
</tr>
<tr>
<td>y15</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y16</td>
<td>10.33</td>
<td>10.33</td>
</tr>
<tr>
<td>y17</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y18</td>
<td>10.33</td>
<td>10.33</td>
</tr>
<tr>
<td>y19</td>
<td>9.07</td>
<td>9.07</td>
</tr>
<tr>
<td>y21</td>
<td>17.30</td>
<td>17.30</td>
</tr>
<tr>
<td>y22</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y23</td>
<td>10.33</td>
<td>10.33</td>
</tr>
<tr>
<td>y24</td>
<td>11.60</td>
<td>11.60</td>
</tr>
<tr>
<td>y25</td>
<td>10.33</td>
<td>10.33</td>
</tr>
<tr>
<td>y26</td>
<td>9.07</td>
<td>9.07</td>
</tr>
<tr>
<td>y27</td>
<td>10.33</td>
<td>10.33</td>
</tr>
<tr>
<td>y28</td>
<td>9.07</td>
<td>9.07</td>
</tr>
<tr>
<td>y29</td>
<td>7.80</td>
<td>7.80</td>
</tr>
</tbody>
</table>
6.5 Excess Energy and Grid Connection Trade Studies

The design concept that provided the framework in which the technique has been developed was in part commercial. The requirement was to look at techniques that best suited SIES (non-grid connected) energy systems. The approach was adopted in recognition that the design looks at the long term (20 to 25 years) investment question and the optimisation is focussed on the cost of energy over 25 years. In such an analysis any consideration of grid connection, while technically possible, is commercially invalid as it requires significant assumptions (that are difficult to validate) about the long term cost of grid energy. Hence the focus on SIES operation is driven by the desire of the work to produce a technique that can form the basis of a design tool. This is in contrast with tools such as DER-CAM [29] which was in part developed to support energy trading decisions. Further it was intended that a design that is feasible in an Islanded configuration can then be further investigated to explore the impacts of grid interconnection. In this section the ability of the technique to explore grid connection issues will be explored, using the school example data developed in earlier sections.

The design approach and technique has shown that, for a SIES, designed around the Modal day of incident solar energy and then iterated for less than Modal days produces the lowest cost energy but does result in a significant excess of generating capacity on greater than Modal days. There are two questions related to the investigation of grid interconnection:

- Can the excess energy generated on greater than Modal days be utilised?
- Can grid energy replace the ICE generator in the design in a cost effective manner?

Note that the following discussion is primarily about electrical energy as this is most easily transferred to other users using pre-existing electrical grid infrastructure. Hot Water can also be shared and the analytical approach would be the same as explored for electrical energy in this section.

The amount of ‘excess energy’ available on greater than Modal days was outlined in the proof in section 3.10 and can be described as follows:

On any given day $k$ where $I_{rad_{m(n+1)}} > I_{rad_{m(n)}}$ then the excess energy available for export (into a
Chapter 6. Contiguous Example

wider grid) is established by:

\[ P_{\text{exp}}(k) = e_k x_1 - P_{\text{load}}(k) \]

where

- \( P_{\text{exp}}(k) \)...... is the excess power available on day \( k \)
- \( e_k \)...... is the incident solar energy factor on day \( k \)
- \( P_{\text{load}}(k) \)...... is the load on day \( k \)

The total yearly quantum of energy available is the arithmetic sum of the energy available on each of the days of excess incident energy. Hence:

\[ P_{\text{exp(yr)}} = \sum_k P_{\text{exp}}(k) \times P_{\text{Prob}}(k) \times 365 \]

\[ P_{\text{exp( yr)}} = \sum_k (e_k x_1 - P_{\text{load}}(k)) \times P(k) \times 365 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1) \]

where

- \( P_{\text{Prob}}(k) \) ... is the probability of the day \( k \) occurring.

In section 5.4 the ability to accommodate increases in load on greater than modal days was explored. In this example this load increase is likely to occur as a result of the requirement for air-conditioning on some days of greater than Modal energy. In the example in section 5.4 (see table 5.7) weather data was processed to find the correlation between incident energy bands and minimum and maximum temperatures. In table T.1 two additional loads are provided for air-conditioning. For the purposes of this example it will be assumed that the 20 to 30 degree C temperature range correlates to the 6 to 8 kWhr / day incident energy bands and the 30 to 40 degree C temperature range correlates to the 8 to 10 kWhr / day incident energy bands. Based on these assumptions, using the extended 2000 to
6.5. Excess Energy and Grid Connection Trade Studies

2015 incident energy data set and the ‘simple answer’ PV array size of $5200m^2$ the following results where produced:

<table>
<thead>
<tr>
<th>Incident Energy Band (KWh/m²)</th>
<th>Excess Energy Available these days (kWh)</th>
<th>Total Excess energy per annum (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.51 to 3.0</td>
<td>257.4</td>
<td>7517.37</td>
</tr>
<tr>
<td>3.1 to 3.5</td>
<td>512.2</td>
<td>12561.55</td>
</tr>
<tr>
<td>3.51 to 4.0</td>
<td>767.0</td>
<td>17470.26</td>
</tr>
<tr>
<td>4.1 to 4.5</td>
<td>1021.8</td>
<td>19448.09</td>
</tr>
<tr>
<td>4.51 to 5.0</td>
<td>1276.6</td>
<td>23182.44</td>
</tr>
<tr>
<td>5.1 to 5.5</td>
<td>1531.4</td>
<td>26376.00</td>
</tr>
<tr>
<td>5.51 to 6.0</td>
<td>1786.2</td>
<td>27754.96</td>
</tr>
<tr>
<td>6.1 to 6.5</td>
<td>2041.0</td>
<td>32605.75</td>
</tr>
<tr>
<td>6.51 to 7.0</td>
<td>2295.8</td>
<td>33954.21</td>
</tr>
<tr>
<td>7.1 to 7.5</td>
<td>2550.6</td>
<td>38836.80</td>
</tr>
<tr>
<td>7.51 to 8.0</td>
<td>2805.4</td>
<td>42016.25</td>
</tr>
<tr>
<td>8.1 to 8.5</td>
<td>3060.2</td>
<td>37620.74</td>
</tr>
<tr>
<td>8.51 to 9.0</td>
<td>3315.0</td>
<td>44476.77</td>
</tr>
<tr>
<td>9.1 to 9.5</td>
<td>3569.8</td>
<td>37647.99</td>
</tr>
<tr>
<td>9.51 to 10.0</td>
<td>3824.6</td>
<td>6205.41</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>407,674.60</td>
</tr>
</tbody>
</table>

Table 6.11- ‘Excess’ Energy Generated by the 5200 $m^2$ Solar PV Array (2000-2015)

6.5.1 The Special Case of School Load profiles

The ability to identify excess energy generated is useful to support commercial analysis, “can we reduce the overall cost of energy to the user by gaining some commercial advantage from the excess energy generated?”. In the case of schools the excess energy calculation is more complex because of the unique usage pattern that schools present. In this section the wider ability if the technique to deal with these questions is explored.

In the simple case of the school the analysis (table 6.11) the design for an entire year whereas the school does not operate, or operates with significantly reduced load on weekends and during school holidays which can be up to 10 weeks per annum. A number of approaches to correcting for this effect are supported by the technique developed. In summary:
Remove Weekends from the Input Weather Data: This approach is simple to implement but does not match the philosophy of the technique since the notion of the probability of a given day of incident energy assumes that days are independent random events. The result improves the more total days of data are used to create the probability distributions. Removing weekends just reduces the size of the data set.

Add 2/7 of the total Daily Load to the Estimate of the Excess Energy: This approach is supported by the technique and provides a rough estimate but the underlying assumption is that all weekend days are Modal days or that there are an equal number greater than and less than Modal days distributed across the weekends.

Conduct a Separate Excess Analysis for Weekdays and Weekends: the approach used to assess the excess over 365 days can be broken into two separate analyses, one for the weekdays and one for the weekends, both assuming the same probability distribution but using different loads. This approach can be implemented using the technique and is consistent with the assumption that individual days incident solar energy is a random event. This approach is illustrated below.

The ‘no load’ weekend is not unique to schools and could also be a characteristic of a range of commercial buildings (compared with residential buildings). Another unique attribute of schools is that they are often closed for blocks of time, especially across summer months. Given that the summer months may have, on average, higher incident energy days means the design solution may be impacted by consideration of months when the school is not open. The technique allows this impact to be addressed as follows:

- Remove the closed weeks as a block from the incident energy data sets
- Reduce the analysis to the number of days the school operates
- Adjust the probability distribution for the reduced number of days.
This approach will not be explored further as the execution is trivial. The point to note is that the technique developed supports such approaches.

### 6.5.2 School Case Excess Energy

Previously the amount of excess energy was defined as:

\[
P_{\text{exp(yr)}} = \sum_{k} (e_{k}x_{1} - P_{\text{load(k)}}) * P(k) * 365
\]

For the school example this can be broken down into three elements:

\[
P_{\text{exp(yr)}} = \sum_{p} \text{ExcessEnergyWeekdays} + \sum_{q} \text{ExcessEnergyWeekends} + \sum_{r} \text{ExcessEnergysummerholidayclosed} \ldots (2)
\]

where

\[
\sum_{p} \text{ExcessEnergyWeekdays} = (e_{k}x_{1} - P_{\text{load(p)}}) * P(k) * (n_{p}/365)
\]

\[
\sum_{q} \text{ExcessEnergyWeekends} = (e_{k}x_{1} - P_{\text{load(q)}}) * P(k) * (n_{q}/365)
\]

\[
\sum_{r} \text{ExcessEnergysummerholidayclosed} = (e_{k}x_{1} - P_{\text{load(r)}}) * P(k) * (n_{r}/365)
\]

where

\(e_{k} \ldots\) is the incident energy factor on day \(k\)

\(P_{\text{load(p)}} \ldots\) is the load on weekdays

\(P_{\text{load(q)}} \ldots\) is the load on weekends

\(P_{\text{load(r)}} \ldots\) is the load on summer holiday closed days

\(\text{Prob}(k) \ldots\) is the probability of the day \(k\) occurring.
 Chapter 6. Contiguous Example

\( n_p \) is the number of working weekdays in a year

\( n_q \) is the number of working weekends in a year

\( n_r \) is the number of summer holiday days in a year

Using

\[
\begin{align*}
P_{\text{load}(p)} &= 1143 \\
P_{\text{load}(q)} &= 224 \\
P_{\text{load}(r)} &= 224 \\
n_p &= 230 \\
n_q &= 92 \\
n_r &= 42
\end{align*}
\]

The above relationships produced the following results:

<table>
<thead>
<tr>
<th>Incident Energy Band (kWh/m²)</th>
<th>Excess Energy Weekdays (kWh)</th>
<th>Excess Energy Weekday p/a Excess (kWh)</th>
<th>Excess Energy Weekend (kWh)</th>
<th>Excess Energy Weekend p/a Excess (kWh)</th>
<th>Excess Energy Summer close (kWh)</th>
<th>Summer close p/a Excess (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.51 to 3.0</td>
<td>258.4</td>
<td>4755.37</td>
<td>1177.4</td>
<td>8667.15</td>
<td>1177.4</td>
<td>3956.74</td>
</tr>
<tr>
<td>3.1 to 3.5</td>
<td>513.2</td>
<td>7930.95</td>
<td>1432.2</td>
<td>8853.24</td>
<td>1432.2</td>
<td>4041.70</td>
</tr>
<tr>
<td>3.51 to 4.0</td>
<td>768.0</td>
<td>11023.01</td>
<td>1687.0</td>
<td>9685.32</td>
<td>1687.0</td>
<td>4421.56</td>
</tr>
<tr>
<td>4.1 to 4.5</td>
<td>1022.8</td>
<td>12266.96</td>
<td>1941.8</td>
<td>9315.59</td>
<td>1941.8</td>
<td>4252.77</td>
</tr>
<tr>
<td>4.51 to 5.0</td>
<td>258.4</td>
<td>2956.87</td>
<td>1177.4</td>
<td>5389.19</td>
<td>1177.4</td>
<td>2460.28</td>
</tr>
<tr>
<td>5.1 to 5.5</td>
<td>1277.6</td>
<td>13865.97</td>
<td>2196.6</td>
<td>9536.00</td>
<td>2196.6</td>
<td>4353.39</td>
</tr>
<tr>
<td>5.51 to 6.0</td>
<td>1532.4</td>
<td>15004.37</td>
<td>2451.4</td>
<td>9601.07</td>
<td>2451.4</td>
<td>4383.10</td>
</tr>
<tr>
<td>6.1 to 6.5</td>
<td>1787.2</td>
<td>17991.17</td>
<td>2706.2</td>
<td>10896.98</td>
<td>2706.2</td>
<td>4974.71</td>
</tr>
<tr>
<td>6.51 to 7.0</td>
<td>2296.8</td>
<td>21405.12</td>
<td>3215.8</td>
<td>11987.91</td>
<td>3215.8</td>
<td>5472.74</td>
</tr>
<tr>
<td>7.1 to 7.5</td>
<td>2551.6</td>
<td>24482.10</td>
<td>3470.6</td>
<td>13319.89</td>
<td>3470.6</td>
<td>6080.82</td>
</tr>
<tr>
<td>7.51 to 8.0</td>
<td>2806.4</td>
<td>26485.43</td>
<td>3725.4</td>
<td>14063.40</td>
<td>3725.4</td>
<td>6420.25</td>
</tr>
<tr>
<td>8.1 to 8.5</td>
<td>3061.2</td>
<td>23713.96</td>
<td>3980.2</td>
<td>12333.24</td>
<td>3980.2</td>
<td>5630.39</td>
</tr>
<tr>
<td>8.51 to 9.0</td>
<td>3316.0</td>
<td>28034.91</td>
<td>4235.0</td>
<td>14321.82</td>
<td>4235.0</td>
<td>6538.22</td>
</tr>
<tr>
<td>9.1 to 9.5</td>
<td>3570.8</td>
<td>23730.04</td>
<td>4489.8</td>
<td>11934.93</td>
<td>4489.8</td>
<td>5448.56</td>
</tr>
<tr>
<td>9.51 to 10.0</td>
<td>3825.6</td>
<td>3911.28</td>
<td>4744.6</td>
<td>1940.35</td>
<td>4744.6</td>
<td>885.81</td>
</tr>
</tbody>
</table>

| Total(s)                      | 237557.51                   | 151846.08                            | 69321.04                   |
| Total                         | 458724.63                   |                                     |                             |

Table 6.12- ‘Excess’ Energy Generated by the 5200 m² Solar PV Array allowing for School use profile
6.5. Excess Energy and Grid Connection Trade Studies

Whether or not the total excess energy available is commercially significant is not required to be determined by this analysis. What is important to note is that the technique developed and explored in this work can establish a system configuration that results in the lowest Cost of Energy over the life of the system. A consequence of the approach adopted is that for many days the system generates energy that is greater than the load requirement. The technique supports analysis to determine the probable amount of excess energy available. Once the amount of excess energy has been established what is done with this energy, if anything, can be determined.

6.5.3 Replacing ICE Energy with Grid Connection

The ICE machine is provided in the design to provide back-up energy in the event of lack of sufficient incident solar energy. The ICE machine supplements both electrical and Hot Water generation. Another design decision required to be made by system designers is the question of whether or not to use the ICE machine or to use imported grid energy in place of the ICE machine.

Existing tools such as DER-CAM [29] and HOMER [28] by design support an optimisation of grid energy against system generated power. As previously discussed the decision to not include grid energy in the initial design brief for the technique developed by this work is a commercial decision not a technical one. It would be possible to substitute the value of grid energy directly into equation (a) in place of the $bx_2$ term. While this works technically, commercially it is difficult to do. This is because the whole approach being developed is designed to make initial design or capital purchase/investment decisions for a life cycle of 20 years or more. In this context it is not possible to effectively conduct the primary assessment with grid energy in the place of the $bx_2$ term as the cost of this energy will not be known and hence the optimisation will be compromised by the approximation applied to the value of grid energy.

The approach adopted by this work is to first use the known costs of the ICE machine as the reference point for the optimisation. Once this is completed it is then possible to investigate the yearly cost of operating the ICE machine and establish whether or not the existing price or future estimated
prices for grid power are more attractive than the design reference point using the ICE machine. Estimating the yearly cost of the ICE machine can be estimated using the following equation:

\[
C_{\text{ice}p/a} = bx_2 + \sum_i P_i b y_{2i} ............(3)
\]

In order for this cost to be compared with grid energy it is important to remember that the ICE machine delivers both hot water and electricity, both of which would need to be replaced by grid electricity. The amount of electrical energy that has to be purchased from the grid is defined as:

\[
P_{\text{importelectricity}p/a} = P_{\text{electricalreplacement}} + P_{\text{HWequivalentreplacement}} ...........(4)
\]

where

\[
P_{\text{electricalreplacement}} = 365 * f * P(\text{modal}) * x_2 + \sum_i (P_i * f * y_{2i} * 365) \text{ and}
\]

\[
P_{\text{HWequivalentreplacement}} = (365 * \sigma * P(\text{modal}) * x_2 + \sum_i (P_i * \sigma * y_{2i} * 365)) / \alpha
\]

where \(P(\text{modal})\) = the probability of the Modal day
\(\alpha\) = the efficiency of the electrical hot water heater
\(\sigma\) = the heat output of the ICE in kWh per hour of run time
\(f\) = the size of the generator in kW.

This analysis was completed for the sample school with the following assumptions:

- The Modal day, unlimited PV design is assumed so \(x_2 = 0\)
6.5. Excess Energy and Grid Connection Trade Studies

- $\alpha = 0.8$
- $f = 200$
- $\sigma = 400$

Using these assumptions the required grid replacement electricity was determined to be.

<table>
<thead>
<tr>
<th>Second Stage event</th>
<th>Electrical Load Grid Power (KWh) p/a</th>
<th>Heating Load Grid Power (KWh) p/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>120.11</td>
<td>300.27</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>183.81</td>
<td>459.52</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>141.32</td>
<td>353.30</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>170.79</td>
<td>426.99</td>
</tr>
<tr>
<td>$\theta_5$</td>
<td>270.36</td>
<td>675.89</td>
</tr>
<tr>
<td>$\theta_6$</td>
<td>277.49</td>
<td>693.71</td>
</tr>
<tr>
<td>$\theta_7$</td>
<td>177.02</td>
<td>442.56</td>
</tr>
<tr>
<td>$\theta_8$</td>
<td>277.49</td>
<td>693.71</td>
</tr>
<tr>
<td>$\theta_9$</td>
<td>281.39</td>
<td>703.48</td>
</tr>
<tr>
<td>$\theta_{11}$</td>
<td>170.79</td>
<td>426.99</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>270.36</td>
<td>675.89</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>217.13</td>
<td>542.84</td>
</tr>
<tr>
<td>$\theta_{14}$</td>
<td>240.91</td>
<td>602.27</td>
</tr>
<tr>
<td>$\theta_{15}$</td>
<td>377.63</td>
<td>944.06</td>
</tr>
<tr>
<td>$\theta_{16}$</td>
<td>382.94</td>
<td>957.35</td>
</tr>
<tr>
<td>$\theta_{17}$</td>
<td>247.26</td>
<td>618.15</td>
</tr>
<tr>
<td>$\theta_{18}$</td>
<td>382.94</td>
<td>957.35</td>
</tr>
<tr>
<td>$\theta_{19}$</td>
<td>382.88</td>
<td>957.20</td>
</tr>
<tr>
<td>$\theta_{21}$</td>
<td>237.96</td>
<td>594.89</td>
</tr>
<tr>
<td>$\theta_{22}$</td>
<td>277.49</td>
<td>693.71</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>220.19</td>
<td>550.47</td>
</tr>
<tr>
<td>$\theta_{24}$</td>
<td>247.26</td>
<td>618.15</td>
</tr>
<tr>
<td>$\theta_{25}$</td>
<td>382.94</td>
<td>957.35</td>
</tr>
<tr>
<td>$\theta_{26}$</td>
<td>382.88</td>
<td>957.20</td>
</tr>
<tr>
<td>$\theta_{27}$</td>
<td>250.74</td>
<td>626.85</td>
</tr>
<tr>
<td>$\theta_{28}$</td>
<td>38.29</td>
<td>95.72</td>
</tr>
<tr>
<td>$\theta_{29}$</td>
<td>374.95</td>
<td>937.37</td>
</tr>
<tr>
<td><strong>Total(s)</strong></td>
<td>6985.30</td>
<td>17463.24</td>
</tr>
<tr>
<td><strong>Total Input</strong></td>
<td><strong>24448.54</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.13- Grid Input Electricity required to replace the ICE machine for the school example

The interesting finding in this result is how little grid energy is required to replace the generator running time. This occurs because the system has been designed around the Modal day which in
this geographic location a day of lower incident energy (relative to the total spread of days). This, together with the high marginal cost of ICE running means that the optimal solution has ‘chosen’ very large arrays supported by a very few hours of ICE run time.

This analysis shows very clearly the impact of the concept of designing around SIES. In such systems, with the relatively low cost of solar PV arrays (for both electricity and hot water) the ICE machine is effectively an emergency back-up supply rather than a main generating component. Commercially the capital cost of the ICE machines is equivalent to a yearly insurance charge. Whether or not it is commercially sensible to replace the ICE machine with a grid connection depends on the cost of maintaining the connection when the required grid input energy will be very limited.

What also becomes commercially interesting is the value of the excess energy relative to the cost of input energy. Looking at the results in Tables 6.11 and 6.12 there is 458724 KWh of electrical energy available for export and only 2700 KWh of input energy required. Currently in many grid systems there is a significant price differential for export vs input energy but the significant amount of energy available for export may offset this differential and make the grid energy commercially viable.

The analysis that has been completed to look at input and export energy can also be used to explore the reduction in Greenhouse Gas (CHG) emissions that come from the design. This is explored in the next section.
6.6 CO₂ Pollution Reduction

The technique, provides a probabilistic estimate of ICE run time and therefore allows an estimate of the potential for CO₂ reduction, relative to grid energy, that the system can provide. Such estimates are important not only from an environmental design perspective but because in the future there may be a commercial value to CO₂ mitigation.

Using the previously outlined relationships the total CO₂ emissions per annum can be estimated as follows:

\[
CO₂ = (365 \times \lambda \times P\text{(modal)} \times x_2 + \sum_{i} (P_i \times \lambda \times y_{2i} \times 365))...............(5)
\]

where

- CO₂ = The annual CO₂ emissions in kg
- P\text{(modal)} = the probability of the Modal day
- \lambda = the CO₂ emissions in kg/hr running at 200kW shaft ( generating) output

The CO₂ emissions of stationary diesel engines is highly variable and not well researched. Generally the emphasis in Diesel engine development has been on CO and NOₓ emissions rather than CO₂ emission reduction. For CO₂ auditing purposes the US Energy Information Agency have recommend use of a figure of 2.35kg of CO₂ per L of fuel consumed[110].

In section 6.3.2 the fuel consumption of the Styer SE266E40 Marine Diesel engine was assumed to be 20 L /hr. Thus gives a simplistic estimate of 47 kg /hr of CO₂ emission. Substituting into equation (aa) and assuming again that \(x_2 = 0\) ( the 5200\(m^2\) PV array ) gives the following results:
<table>
<thead>
<tr>
<th>Second Stage event</th>
<th>CO₂ Emissions (kg/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>28.23</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>43.20</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>33.21</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>40.14</td>
</tr>
<tr>
<td>$\theta_5$</td>
<td>63.53</td>
</tr>
<tr>
<td>$\theta_6$</td>
<td>65.21</td>
</tr>
<tr>
<td>$\theta_7$</td>
<td>41.60</td>
</tr>
<tr>
<td>$\theta_8$</td>
<td>65.21</td>
</tr>
<tr>
<td>$\theta_9$</td>
<td>66.13</td>
</tr>
<tr>
<td>$\theta_{11}$</td>
<td>40.14</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>63.53</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>51.03</td>
</tr>
<tr>
<td>$\theta_{14}$</td>
<td>56.61</td>
</tr>
<tr>
<td>$\theta_{15}$</td>
<td>88.74</td>
</tr>
<tr>
<td>$\theta_{16}$</td>
<td>89.99</td>
</tr>
<tr>
<td>$\theta_{17}$</td>
<td>58.11</td>
</tr>
<tr>
<td>$\theta_{18}$</td>
<td>89.99</td>
</tr>
<tr>
<td>$\theta_{19}$</td>
<td>89.98</td>
</tr>
<tr>
<td>$\theta_{21}$</td>
<td>55.92</td>
</tr>
<tr>
<td>$\theta_{22}$</td>
<td>65.21</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>51.74</td>
</tr>
<tr>
<td>$\theta_{24}$</td>
<td>58.11</td>
</tr>
<tr>
<td>$\theta_{25}$</td>
<td>89.99</td>
</tr>
<tr>
<td>$\theta_{26}$</td>
<td>89.98</td>
</tr>
<tr>
<td>$\theta_{27}$</td>
<td>58.92</td>
</tr>
<tr>
<td>$\theta_{28}$</td>
<td>9.00</td>
</tr>
<tr>
<td>$\theta_{29}$</td>
<td>88.11</td>
</tr>
</tbody>
</table>

**Total(s)** 1641.54
In 2016 Victorian Electricity produces 1.26 kg of CO\textsubscript{2} per kWhr [111], at the consumers connection point. (Note that this is a worst case figure and it will reduce over the next decade as the brown coal contribution to Victorian energy reduces).

The total CO\textsubscript{2} emissions if all of the school load came from Victorian Grid electrical energy can be calculated as:

\[
\text{CO}_2 = \left( \frac{\text{Daily Electrical Load} + \text{Daily Heating Load}}{\alpha} \right) \times 365 \times 1.26
\]

\[
\text{CO}_2 = \left( \frac{1144 + 3360/0.8}{0.8} \right) \times 365 \times 1.26
\]

\[
\text{CO}_2 = 2457705 \text{ kg/annum }
\]

Hence in simplistic terms the reference design reduces the school’s CO\textsubscript{2} pollution footprint by over 99 percent. This figure assumes the 5200m\textsuperscript{2} array which is unrealistic but this example does illustrate how savings can be calculated.
6.7 The Impact of Geography

In this section the design for the school’s load requirement is re-assessed for the incident energy weather profile in a different geographic location. Broome on the North West coast of Western Australia has been chosen as it provides a significantly different weather profile than that for Moorabbin Airport (Victoria) which is used in the initial analysis, (see figure 6.1).

Analysing the Australian Bureau of Meteorology (BOM) 2015 data on incident solar energy for Broome Airport, using the same 0.5 kWh/day ‘bands’ of incident energy as used for the Moorabbin analysis produced the following result.

Examination of the probability distribution in figure 5.4 shows that the 7.1 to 7.5 kWh/day band has the most days (the highest probability of occurrence) however it is also clear that there are more days in total between 5.1 to 6.5 kWh than there are between 7.1 to 7.5. This means if the technique uses the 7.1 to 7.5 band as the modal day (first stage term) it will identify an optimum result. In addition if the 7.1 to 7.5 band is used as the first stage term at least 5 less than modal day scenarios will need to
be accounted for to ensure that the 4.5 to 7.0 kWhr days are considered by the optimisation. This will require a \((5^5)\) matrix analysis. While this is possible within the design of the technique it is complex.

An alternative approach is to amend the Probability Distribution Function (PDF) incident energy bands. Figure 6.5 show the PDF when the incident energy bands are set to 1.5 kWhr / day intervals.

![Figure 6.5 - 2015 Broome Airport Incident Solar Energy, 1.5 kWhr/m² Probability Distribution Function](image)

The data in the figure 6.5 PDF illustrates that the most common days lie in the 5 to 6.5 kWHr/day range and suggest that any design should use a day in this range as the Modal (stage 1). Using a modal day of 5.1 kWhr /day would provide the most conservative answer. Examining figure 6.4 and 6.5 together suggests that if a Modal day of 5.75 kWhr is used then an optimal design will be achieved. The consideration is that using a higher modal day will mean a smaller PV array will be chosen and the ICE will need to run on a few extra days, however since the ICE capital cost is always incurred, (as this is a design requirement to ensure absolute availability) this is a reasonable trade-off.

**Changes to the Load Calculation** Table 6.1 shows the assumed loads for the school when located in Beaumaris Victoria. As the weather in Broome is significantly different than Beaumaris there is a need to review the loads. The cooling load was originally not included in the Beaumaris Modal day
and less than Modal day assumptions. In Broome the cooling load can be assumed to exist every day and hence the new baseline daily electrical load is amended to $1144 + 1200 = 2344 \text{ kWhr/day}$. Similarly the heating hot water load will not be required, but there is always a Domestic Hot Water (DHW) load which will be set at $100 \text{ kWhr/day}$ (this is an assumption provided to support the example).

The figure 6.5 Broome weather data was processed producing the following results:

<table>
<thead>
<tr>
<th>2nd Stage Equivalents</th>
<th>Energy kWh</th>
<th>Probability</th>
<th>$P_i * b$</th>
<th>$e_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>6.75</td>
<td>2.05483E-05</td>
<td>0.105001861</td>
<td>0.6615</td>
</tr>
<tr>
<td>e2</td>
<td>8.25</td>
<td>2.67151E-05</td>
<td>0.13651392</td>
<td>0.8085</td>
</tr>
<tr>
<td>e3</td>
<td>9.25</td>
<td>2.67151E-05</td>
<td>0.13651392</td>
<td>0.9065</td>
</tr>
<tr>
<td>e4</td>
<td>8.25</td>
<td>2.67151E-05</td>
<td>0.13651392</td>
<td>0.8085</td>
</tr>
<tr>
<td>e5</td>
<td>9.75</td>
<td>3.47325E-05</td>
<td>0.177483049</td>
<td>0.9555</td>
</tr>
<tr>
<td>e6</td>
<td>10.75</td>
<td>4.54224E-05</td>
<td>0.232108553</td>
<td>1.0535</td>
</tr>
<tr>
<td>e7</td>
<td>9.25</td>
<td>3.49374E-05</td>
<td>0.178529999</td>
<td>0.9065</td>
</tr>
<tr>
<td>e8</td>
<td>10.75</td>
<td>4.54224E-05</td>
<td>0.232108553</td>
<td>1.0535</td>
</tr>
<tr>
<td>e9</td>
<td>11.75</td>
<td>5.94025E-05</td>
<td>0.303546625</td>
<td>1.1515</td>
</tr>
<tr>
<td>e11</td>
<td>8.25</td>
<td>2.67151E-05</td>
<td>0.13651392</td>
<td>0.8085</td>
</tr>
<tr>
<td>e12</td>
<td>9.75</td>
<td>3.47325E-05</td>
<td>0.177483049</td>
<td>0.9555</td>
</tr>
<tr>
<td>e13</td>
<td>10.75</td>
<td>3.47325E-05</td>
<td>0.177483049</td>
<td>1.0535</td>
</tr>
<tr>
<td>e14</td>
<td>9.75</td>
<td>3.47325E-05</td>
<td>0.177483049</td>
<td>0.9555</td>
</tr>
<tr>
<td>e15</td>
<td>11.25</td>
<td>4.51566E-05</td>
<td>0.230747403</td>
<td>1.1025</td>
</tr>
<tr>
<td>e16</td>
<td>12.25</td>
<td>5.90541E-05</td>
<td>0.301766541</td>
<td>1.2005</td>
</tr>
<tr>
<td>e17</td>
<td>10.75</td>
<td>4.54224E-05</td>
<td>0.232108553</td>
<td>1.0535</td>
</tr>
<tr>
<td>e18</td>
<td>12.25</td>
<td>5.90541E-05</td>
<td>0.301766541</td>
<td>1.2005</td>
</tr>
<tr>
<td>e19</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e20</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e21</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e22</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e23</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e24</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e25</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e26</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e27</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e28</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
<tr>
<td>e29</td>
<td>13.25</td>
<td>7.72297E-05</td>
<td>0.394643859</td>
<td>1.2985</td>
</tr>
</tbody>
</table>

Table 6.15 - 2015 Broome Equivalent Incident Energy Bands and Probabilities

The important thing about this example is that it illustrates how the technique supports and also encourages thought about the nature of the weather (incident solar energy) Probability Distribution.
Function profile and how that can drive the solution. By first examining the incident energy PDF for this case, and also giving consideration to the low requirement for heating water allows the technique to be tailored up front to simplify the optimisation.
6.8 Conclusion to Chapter 6

The contiguous example has illustrated the following issues:

- The technique is based on an energy storage approach rather than a load following approach. This approach is supported by the use of a load analysis.

- The technique supports the ‘pre-processing’ of weather (incident solar energy) data and the review / refinement of incident energy Probability Distributions to support the analysis. It is well suited to incorporating long term (multiple years) of weather data as a way to improve long term predication accuracy.

- Complex ‘scenario trees’ can be simplified, using a process of convolution’ to allow the basic form of the technique to be maintained while at the same time allowing multiple consecutive days of energy to be analysed.

- The technique is well suited to exploring issues associated with the generation of excess energy and is also able to be used to investigate grid interconnection issues. The technique can be used to examine the long term $CO_2$ emissions and supports a Multi-Objective Optimisation style analysis.
Chapter 7

Conclusion

7.1 Introduction

The work described by this thesis was driven by a number of commercial issues. Primary among these issues was an assertion that while Small Islanded Energy System (SIES) architectures provide an effective way to introduce ‘Renewable Energy’ generating technologies it was important that these systems could be designed in a commercially viable manner. There are two main issues that underpin the notion of a commercial viability:

- The need to minimise life cycle costs of energy by optimisation of capital investments and ongoing running costs
- The need to have predictable costs and predictable system performance

The commercial requirement for predictability informed the study in two ways. Firstly it created an emphasis on Small Islanded Energy Systems (SIES) or Non Grid connected systems. This is because the commercial predictability of a design that assumes grid connection is always dependent upon an estimate of the long term cost of grid energy. The only real way to have certainty and predictability of costs is not to be reliant upon external suppliers of energy.

The second consequence of the need for predictability is a focus on having a design / optimisation methodology that addresses the impact of long term weather variability on the performance and hence the initial design of the system.
In the literature review the existing methodologies for both initial design optimisation and operational cost optimisation of micro grid systems were investigated. The existing techniques were reviewed from the perspective of how well they supported an initial design optimisation process, specifically for smaller (10 to 250 kWhr/day) Combined Heat and Power (CHP) systems. The literature review suggested that existing approaches may be able to be improved by more directly addressing the notion that there is a probability distribution associated with what was described as the ‘run of days’ question. It was asserted that the incident energy experienced by the system on consecutive days of operation has a significant impact on the performance and hence the design optimisation of such systems.

Based on what was found in the literature review the key issues for the investigation were agreed to be:

- Examine the form and performance of specific optimisation objective functions when applied to Small Islanded Energy Systems (SIES)
- Assess and develop new forms of resource and technology models used in the optimisation. Investigate the most effective optimisation techniques to be used to process the objective functions and resource and technology models examined.

The issues have been examined within the commercial context outlined above and with a view to use the techniques developed as the basis of a design tool. Following is a summary of what has been developed in response to the aims and what has been learned throughout the investigation.
7.2 Literature Review and the Basic Methodology

The literature review suggested that a two step process of optimisation be used. In the first step a 'baseline design optimisation' is conducted. In the second step that baseline is then iterated based on an understanding of the probability of incident energy states after the baseline day. This is shown as figure 2.2 in Chapter 2. Associated with this idea was the nomination of the Modal day as the 'baseline day'. It was assumed that the use of the Modal day would result in the optimum solution since the baseline solution would be optimal for the most days in a given year. This assumption was tested in Chapter 3 and found to be valid.

In the literature review the notion of the use of a baseline Modal day followed by a second stage iteration based on the incident energy profile was re-stated as a simple optimisation question:

*How to minimise the increase in capital cost (relative to the Modal day optimum configuration) versus the increased cost of imported (or non-renewable generated) energy for the days $I_{rad,m(n+1)} < I_{rad,m(n)}$ given that the Modal day optimum exists and is known.*

The literature review then sought to identify a mathematical technique best suited to the two stage concept and the optimisation question. The review identified an approach that Wets [39] describes as "Stochastic Linear Programming with Fixed Recourse using the Equivalent Deterministic Approach." This mathematical approach was used throughout the remainder of this study as the basis of the techniques developed. Importantly it was established that using the Stochastic Programming with Recourse approach does not produce a single point 'optimal solution' but rather a series of solutions that taken together provides the likely 'least worse' solution.

Incident Energy as a Probability Distribution Function

Chapter 3 examines how to apply the identified mathematical technique to the analysis of a very simple (Battery, PV, ICE generator) system. This example first explores how incident solar energy data for a given geographic site can be described as a discrete Probability Distribution Function (PDF). The approach for processing weather data first introduced in Chapter 3 is adopted throughout the
remainder of the study. In Chapter 3 it is shown that the width if the discrete energy bands is able to be modified at the discretion of the designer without impacting the general validity of the technique. Further it is suggested that:

- the size of the incident solar energy intervals can be scaled up and down to balance the scale of the analysis against the advantages from a design solution perspective.
- many years of incident energy data can be combined e.g Melbourne Airport data for 10 years could be ‘pre-processed’ into a distribution with no change to the basic solution approach.
- patterns in the daily distribution of incident energy, that have the potential to skew the optimisation solution can be identified, removed from the distribution and then a new answer calculated, producing a style of second pass sensitivity analysis.

These assumptions regarding the versatility of the technique of using incident solar energy discrete Probability Distribution Functions is explored and shown to be valid in chapters 5 and 6. It is concluded that the ability to address incident energy data in this way, and especially the ability to process multi years of weather data into a single PDF is a unique and novel feature of the technique.

First Stage Variables and Decision Variables

Chapter 3 introduces the concept that for the style of Small Islanded Energy Systems (SIES) being explored the size of generating and storage components associated with using incident solar energy are defined as ‘first stage terms’. This aligns with the reality of such systems where PV array and battery sizes are a system design decision and a capital investment. The second stage term or ‘expected value term’ is defined as the ICE generator running time. Again this aligns with the reality of how the system operates once it is in service. Also at this stage of the project it was concluded that the ICE machine and generator size was a fixed design parameter that would not be subject to optimisation. This conclusion reflects the notion that the ICE/Generator is provided in the system primarily to ensure supply availability (when there is a deficit in incident solar energy). As a consequence the ICE machine and generator must be sized to meet the load requirements, assuming no or limited incident solar energy.
Multi-Stage Recourse - Run of Days

A fundamental requirement of the technique being developed is the ability to address consecutive days of less than Modal incident solarenergy, \( I_{rad(m+i)} \) < \( I_{rad(m)} \), \( i = 1, 2, 3, ..., n \). In Chapter 3 it was asserted that these consecutive days could be assessed using the notion of multi-stage recourse. Further it was suggested that convolution could be used to simplify the multi-stage optimisation problem. The use of convolution is explored in detail in the Chapter 6 example and in appendix 1. It was concluded in Chapter 3 that it was possible to use Convolution because the incident solar energy on consecutive days \((n + i)\) are independent from a probability perspective.

Validation of Results

Chapter 3 suggests a methodology and then applies that methodology to confirm that the technique produces an optimum solution. The conclusion reached is that the technique does produce a minimum Levalised Cost of Energy (LCOE) solution. Importantly the claim is not that the technique produces an absolute minimum solution, and this is not the purpose of the technique. As was noted earlier the mathematical technique being used is meant to address a stochastic problem by addressing the probability of second stage or ‘expected value’ events. As such the technique is not producing a single minimum but rather a design that results in the ‘least worse’ costs over an extended period of time. This approach suites the commercial nature of the problem.

7.3 Adding System Complexity

A goal of the project was to develop an optimisation technique that could both be scaled up (to address more complex systems) and address meeting loads with different energy sources without making changes to the basic approach. Both of these attributes were identified as weaknesses in
existing techniques. The ability to optimise energy source mixes (ICE Hot Water vs solar Hot Water vs electrically heated Hot Water) is not a focus of existing techniques identified in the literature, which tend to be designed around optimising either CHP hot water or different electrical sources. This limitation in existing techniques is addressed by Chicco and Mancarella in [70]. It is concluded in Chapter 3 that the technique is well suited to addressing the ‘energy vector’ analysis question outlined in [70]. This is seen as an important capability of the developed technique since in most SIES the heating and cooling loads are significant element of the total load. The second issue that is highlighted in Chapter 4 is the notion of using efficiency factors (simple scalar factors) to represent the performance of system elements.

Many existing approaches, typified by Logenthiran [11] use a mathematical model to describe the performance of items of equipment. Others [34], [70] use parametric scaling factors (often referred to as efficiency factors). In this study a version of the efficiency factor approach has been used. The use of the efficiency factor is explored initially in Chapter 4 and expanded upon in Chapter 6. Importantly incorporating the performance of items of equipment in this way allows the performance of a technology that is captured within the developed technique to be established by either test or by review of existing data outside the basic technique. Further it is possible to use the technique developed as a method to conduct a sensitivity analysis of the impact of increases in technology performance vs the cost associated with that increased performance.

The main conclusions from the work in chapter 4 are that:

- The technique developed in Chapter 3 can be expanded to include more components while maintaining its basic form and validity
- The technique allows analysis of the optimisation of Hot Water generation (electric vs passive solar vs ICE) and addresses the ‘energy vector’ question raised by previous analysis.
7.4 Exploring the Features of the Technique

Chapter 5 and Chapter 6 explore the use and features of the technique, primarily from the perspective of a design tool.

Pre-Processing Weather Data

One of the key features of the technique is the ability to ‘pre process’ weather (incident solar energy) data. When a system design is focussed on utilising incident solar energy and when predicting the performance of the system, given the variation of incident energy, is a focus the ability to easily explore the impact of incident energy variability is crucial. In Chapter 5 and Chapter 6 different ways to process incident energy data, and the impacts on the resultant designs is explored. In both chapters the influence of creating an incident energy PDF from a sample of many years is explored. This capability is important since:

- It can remove the influence of atypical years
- It allows the incorporation of longer term weather cycles (such as the Australian El Nino / El Nina influences)
- It allows long term patterns to feed into what is by definition a design for long term operation. This means that the design assumptions more closely match the probable environment.
- It allows years (or months) with abnormal or ‘outlying’ data to be excluded or for the impact of these anomalies to be investigated.

While the ability to pre-process weather data is able to be created using existing approaches (such as HOMER or DERCAM) the technique developed in this study is particularly suited to the long term analysis since it is built upon the notion of the Probability Distribution Function (PDF). Building the PDF using a large sample of weather data (15 years in both Chapter 5 and Chapter 6 examples) produces an optimisation impact since, as was shown in Chapter 4, the large data PDF will represent a closer approximation to the likely incident energy profile that will be encountered by the system in service.
Chapter 7. Conclusion

The second capability reviewed was the ability to look at incident energy patterns within a given year. Table 5.3 showed how simply by colouring a table patterns in incident energy to be revealed to the designer. In the example in section 5.2.2 the removal of certain days from the sample had no material impact on the analysis result. No specific conclusion is drawn from this example as it is possible that in other circumstances, with other patterns of incident energy, there may well have been a different result. What is important is the technique allows such reviews to be conducted.

The example in Chapter 6 of the analysis if the Broome incident solar energy data highlights a further important aspect of the use of Probability Distribution Functions (PDF). In this case the ability to explore the PDF allows identification of the most appropriate first stage design incident solar energy. Again what is important is not the particular answer but rather the utility of the technique.

Load Data Correlation

Techniques such as HOMER and DERCAM support the analysis of variable loads, or loads correlated with incident energy. The technique developed also allows a consideration of variation in load requirement by incorporating different electrical and heating loads for different days of incident energy. Further the technique supports a process where there can be a statistical correlation between load, temperature and incident energy. The technique allows this correlation to be another process that can be conducted outside the optimisation process. The technique well suits a process whereby larger load data sets could be processed such that the probability of a particular load can be correlated and processed with the corresponding probability of a day of incident solar energy.

The second issue explored is the investigation of load and ‘excess’ energy on greater than Modal days. This is a question that at first seems unique to this particular technique given that this technique is the first to be based around the notion of a Modal day. On further examination it is possible to see how all existing techniques, because they aim to produce an ‘optimum’ solution will, on certain days, produce ‘excess’ energy. The advantage of the developed technique is that the amount of excess energy can not only be predicted but also associated with a probability of occurrence, which is
important from a commercial perspective. The ability to calculate excess energy, or more importantly energy that is available to export to the grid is illustrated in Chapters 4 and 5.

Grid Connection Analysis

The project focus has been on Small Islanded Energy Systems for commercial reasons. This approach is also useful in assessing the co-ordination of small energy systems with the grid. There are four areas where the technique can support investigation of grid interconnection:

- the economic viability of substituting grid electrical energy for ICE energy
- the ability to accurately predict when energy is available for export
- the ability to model how excess energy can be exported in a controlled manner, which supports electrical connection of much larger PV arrays than would otherwise be the case.
- the ability to explore the sharing of hot water.

The benefits of the technique when exploring grid interconnection are primarily about commercial issues. Such issues are explored in chapters 5 and 6.

Multi-Objective Optimisation

For SIES of the nature being explored the primary use of Multi-Objective Optimisation (MOO) is to ‘optimise’ both the cost of energy with the $CO_2$ emissions from the system. The developed technique allows the balancing of $CO_2$ emissions and cost without the complexity and with significantly more transparency than existing MOO optimisation techniques. The ability to conduct an MOO process is illustrated in Chapter 5. The technique clearly illustrates that for the systems being discussed system constraints drive the potential MOO solutions. For instance in Chapter 6 the solution approaches the lowest possible $CO_2$ emissions are already identified by defining a constraint that the generator (ICE machine) should not be operated on the Modal day. Using this approach results in a very large PV array size. This large PV array size produces the lowest possible CoE but it may not be practical. In this circumstance the $CO_2$ minima and cost minima will be bounded significantly by the space
available for the installation of the PV array. Such constraints will exist in all designs and as a consequence it is asserted that the technique developed, and its ability to quickly assess multiple solutions with multiple constraints meets the same functional intent (while providing additional benefits) as an MOO scheme. This constraint based ‘sensitivity analysis’ in place of true MOO assessment is seen in other existing techniques, most notably in the HOMER system [28].

The Ability to Assess New Technologies and the Scalar Quality Factor

The technique allows any given technology to be assessed using three basic parameters:

- the capital cost of a given size element
- the ability to transform a given quantum of input incident solar energy into electricity or hot water
- the life, in cycles, that can be translated into years, of the technology.

This very simplistic characterisation of technologies has some disadvantages. For instance the life cycle ratio concept applied to some batteries will not provide as accurate an answer as other techniques that include battery ‘accumulated life’ estimates based on simulation of Depth of Discharge. The advantage of the simplistic incorporation of technology performance is that it provides a transparent mechanism to allow new (or competing versions of an existing technology) to be tested, characterised and then incorporated into the technique with no change to its basic form. This allows new technologies to be assessed for financial viability and provides a simple, way to compare different technologies within a design study. This is illustrated in Chapter 5. This ability to incorporate the assessment of new technologies quickly is an advantage of the technique.

Running (Operational) Costs as a Probability Distribution

For any system of this type, while the total life cycle cost of energy is a crucial factor, commercial decisions are based on a consideration of the mix of capital investment and operational costs. While many tools, exemplified by DER CAM [29] allow a day to day examination between running in Islanded mode and importing energy, in reality, once the initial design is completed and the capital
investment made, the operating costs, and trade off between importing energy (addressing energy deficiencies) with ICE machines is locked in and can no longer be ‘optimised’ (other than choosing to input energy on a given day if it is cheaper than running the generator.) The technique developed in this thesis reflects this investment reality and provides a mechanism to both predict the quantum of energy that will need to be generated by ICE machine runtime (consuming fuel) or importing grid energy across the system life as well as providing estimates of probabilities for this consumption. The notion of providing the probability of occurrence of a given running cost in a given time period is a crucial commercial attribute as it allows long term contractual arrangements to be agreed between suppliers and consumers. This capability to provide a robust estimate of operating cost addresses one of the key requirements of the original task and is a new and novel contribution to this area of study.
7.5 Opportunities for Further Investigation

This study has developed and validated a new approach to the optimisation of Small Islanded Energy Systems. There are a number of follow on tasks that could be conducted to further explore and enhance the technique. These are summarised as follows:

**Long Term Demonstration System** The time frame associated with this study have precluded the conduct of a long term demonstration of system performance. The technique makes system configuration ("first stage" decisions) and then assumes /asserts long term performance, in the form of an estimate of the probable use of the ICE machine to meet the load requirements over a period of years. The performance of a full scale system could be monitored over a 12 to 18 month period with the following aims:

- observe how closely the measured distribution of ICE machine running times matches the predicted distribution
- measure the actual long term energy generation from incident energy to allow ‘correction’ of the assumed generation ‘efficiency factors’
- measure the long term load performance to further develop the load analysis technique and to explore the creation of a load probability distribution.

It is noted that one of the advantages of the technique is that it supports a process of ongoing refinement (by correcting PDFs and efficiency factors) with real world performance. Long term trials are required since weather patterns vary from year to year and the greater the trial period the closer the system performance will be to the design PDF for incident energy.

**Medium term Generating Technology Efficiency factors**

The technique relies upon ‘holistic’ efficiency factors to predict the daily energy generation from a given day of total incident solar energy. While there is a large number of theoretical models for generating technologies available and some medium term studies the efficiency factor concept used in this technique should encompass a whole range of non-linear behaviours most notably the impact
of cloud cover dynamics, atmospheric transmission changes and the impact of non-tracking technologies. While the BOM style data for total incident solar energy on a given day accounts for the atmospheric variability and cloud cover, and while the day to day variability in input energy will be ‘smoothed’ out by use of the large sample PDF the conversion dynamics (represented by the ‘efficiency factor’) may not correctly account for long term weather behaviour. A real world performance measure of the generating technologies would ensure that a wide range of variability and dynamic performance is more correctly captured in the scalar ‘efficiency factor’ which will improve the techniques performance.

Use of Alternative Solvers

This study has been focussed on the notion of the Stochastic methodology and the structure of the optimisation function and constraint equations in order to ensure the aims of the study can be met. The study has attempted to create simple equation forms in order to ensure that the impact of different system attributes can be fully understood. This has been done in the belief that for the system designer it is particularly useful for the mechanics of the technique to be highly transparent. The consequence of this drive for simplicity is that the equations developed in this thesis have been able to be solved using simple techniques. In this study the LINPROG solver in Matlab has been used for convenience, and because solver performance was not a key restraint in understanding the validity and efficacy of the technique. Now that the high level structure of the technique has been established a further task exists to explore the most appropriate solver to be used. This becomes more important as larger scenario trees (that create large sparse equality matrices) are required to be processed.

7.6 Summary
For the style of Small Islanded Energy Systems (SIES) being addressed, the long term pattern of day to day variations in incident solar energy is a significant design and system performance driver. This study set out to explore ways to optimise the design of SIES with the aim to minimise the long term Levalised Cost of Energy (LCOE) and \( CO_2 \) emissions. The requirement was to develop a technique that would above all else produce highly predictable system performance estimates and hence highly predictable and accurate costs estimates.

Using the concepts of Stochastic Programming with Recourse together with the use of probability distribution functions to represent incident solar energy a totally new and novel technique has been developed that is specifically tailored to the needs of system designers and the commercial requirements of such systems.

The technique developed has been shown to have a number of attributes, outlined above, that taken together represent a new and useful assessment capability that adds to the existing techniques in use.
Chapter 8

Appendix A

8.1 Introduction

In Chapter 6 the weather data for the example was processed as a three stage scenario tree (see Figure 6.2). This three stage scenario tree was transformed using concepts of simple convolution into an 'equivalent' 27 state single stage scenario tree. This approach was adopted as it allowed the same basic form of the developed technique to be utilised as well as allowing use of the simple Matlab LINPROG solver.

The transformation from a three stage scenario tree to a single stage scenario tree as shown in Chapter 6 is a simplification of all the possible scenarios. This simplification has been utilised as it represents a set of worse case outcomes. This appendix explores why this simple transformation is suitable for the purpose of design optimisation.

8.2 A Generic Scenario Tree

In Chapter 6, Figure 6.3 Generic Equivalent Incident Energy Scenario Tree was used to represent the combination of three less than Modal days that where identified in the PDF shown as figure 6.1 "2015 Moorabbin Airport Incident Solar Energy". The concept shown in Figure 6.3 can be expressed as a more generic scenario tree as shown in Figure A.1 where \( n = 1, 2, \ldots n \) and \( k = 1, 2, \ldots k \):
Figure A.1 - Generic Scenario Tree

The first issue to be considered is that the number of serial scenarios \( n, k \) is not unlimited or ill defined. The number of \( n, k \) terms is within the control of the designer. The number of less than Modal incident energy bands is determined by the designer setting the ‘width’ of the incident energy bands within the PDF. The number of bands can be reduced to ease analysis and as the width of the band is reduced the answer will be more accurate. The manipulation of the incident solar energy PDF is shown in Chapter 6.7. As is noted in Chapter 3.3.1 a point is reached where reducing the band width (increasing the number of less than Modal day scenarios) has no impact since the system components (e.g. number of PV panels) can only be realized in a minimum discrete step size.

Returning to the example in Chapter 6 it can be seen that the suggested approach is to ‘collapse’ all the incident solar energy and the probability that this energy will be available to the system back into a new ‘equivalent’ day \( e_{eq} \) such that
8.3 Incident Solar Energy, The Probability of that Energy Occurring and the concept of an Optimum Solution

\[ e_{eq} = [I_{rad_{eq}}, P(I_{rad_{eq}})] \text{ where} \]

\( I_{rad_{eq}} \) is the equivalent total incident solar energy and

\( P(I_{rad_{eq}}) \) is the probability that this incident energy will be available to the system

and

\[ I_{rad_{eq}} = I_{rad_k} + I_{rad_{k+1}} \]
\[ P(I_{rad_{eq}}) = P(I_{rad_k}) \times P(I_{rad_{k+1}}) \]

The following sections explore the validity of this notion of an ‘equivalent day’

8.3 Incident Solar Energy, The Probability of that Energy Occurring and the concept of an Optimum Solution

In order to understand why the simplified contraction is suitable it is necessary to return to a discussion on the nature of the optimisation being executed by the developed technique, and why that optimisation approach is used in the first place. The optimisation technique developed aimed to meet the two step approach identified in the literature review and shown in figure A.2
Chapter 8. AppendixA

Figure A.2 - Basic Optimisation Process Flow (repeat of Figure 2.1)

When this concept of a Modal energy day design then iterated by a further analysis of incident energy days and the probability of those days was implemented using the Stochastic Programming with General Recourse approach the technique created became a special form of optimisation. The technique does not produce an absolute minimum cost single point solution but rather a solution that is over the long term (20 years) going to represent the likely minimum range of possible costs. This concept is illustrated in the following diagram which for simplicity is shown as a two variable problem.
Figure A.3 is attempting to portray the following aspects of the optimisation technique developed. The first stage modal day analysis would produce an optimised (lowest cost of energy) solution represented by the red dot. This solution is made up of an answer for the first stage (capital investment) variables. (i.e. how large a PV array and battery should the designer invest in). Consideration of the probability of lower than Modal incident energy days has two impacts on the solution.

- The optimum Modal day configuration is amended to address the potential range of variation in incident energy
- An additional cost is added (the $x_3$ parameter) which has an outer limit based on the potential range of variation in incident energy

So the technique produces two ‘outer boundaries’ of cost. One for the fixed capital (first stage investment) and one for the second stage (‘running’) costs. The system long term costs will lie on these outer boundaries if the ‘worst case’ incident energy scenario occurs. Any incident solar energy scenario that is not ‘worse case’ will lie within the established solution boundaries. The optimum
first stage solution, the red dot, will never change as it is absolutely optimal as it is the lowest cost for the most common day of the year.

Returning to the introduction the aims of this project are ultimately economic. It was outlined in the introduction that for SIES to be viable its economic performance has to be highly predictable. In commercial terms what the designer needs to know is what is the worst possible Cost of Energy outcome that results from the worst possible probability distribution of incident solar energy days. This concept of a worse possible rather than the traditional single point optimal solution is a commercial construct and it is what differentiates the technique that has been developed from those that already exist. Hence the aim of the technique should be to find that boundary cases in figure A3 knowing that a probability exists that the final operating cost of energy over the long term (20 years) will always fall some where within those boundaries.

8.4 How to find the Worse Case Solution by Scenario Tree Simplification

If the Probability Distribution function produces ‘k’ bands of incident energy less than modal (k = 1, 2, 3, ..., n) then there are the following possible run of consecutive days to consider in an analysis:

Scenario 1 = Modal day + next day (k=1) less energy + Modal day
Scenario 2 = Modal day + next day (k=1) less energy + next day (k=2) less energy + Modal day
........
........
Scenario n = Modal day + next day (k=1) less energy + next day (k=2) less energy + ................. + next day (k=n) less energy + Modal day

The basic approach for the technique developed is that the load requirement for days of less than Modal incident solar energy has to be meet either by energy captured and stored on the Modal day or by energy generated on the less than Modal day(s) by the ICE machine (or imported from the grid). The greater than energy deficit that aggregates on the less than Modal days then the greater
the energy that must be stored in batteries or generated by ICE, and hence the greater the cost.

If Scenario 1 or Scenario 2 is assessed then the solution produced would fall within the boundaries shown in figure A3. The capital investment decisions would sit between the red dot and the inner boundary and the operating cost and hence total cost would sit within the outer boundary.

If scenario n is processed then the ‘worse case scenario’ is identified. Scenario n identifies the capital investment boundary, and the running cost boundary and hence the likely worse case cost. Given the underlying commercial nature of the question being explored, and given that what the solution seeks is a robust way to estimate cost this worse case cost is what is required. If a system can be designed and provide the energy at this cost, and then there is more incident solar energy than predicted in a given time period then extra energy is produced not less. Projected costs have not been exceeded by needing to increase ICE run time or to import more grid energy than initially predicted.

The worse case scenario for the generic scenario tree shown as Figure A1 can be established by ‘collapsing’ by convolution every possible path from the $k^{th}$ nodes back to the three second stage terms to create a new series of equivalent second stage terms.

This convolution approach is valid because while the probability of the amount of incident solar energy is initially established on a day by day basis when it comes to the second stage processing decision (represented by a two stage scenario tree) all that matters is the total energy deficit that has to be provided for on the Modal day or by ICE machine runtime and the Probability that that energy deficit will be required to be met. The fact that the energy deficiency occurs over 1, 2 or k days is not important.

This approach to creating an equivalent scenario tree by collapsing a four stage tree to a two stage equivalent using simple convolution is illustrated in the Chapter 6 contiguous example.
8.4.1 The Case of Storing Energy on Greater than Modal days.

There is another possible scenario that has not been explored in the development of the technique: the energy deficit that occurs on less than Modal days can be met in three possible ways:

- The deficit met by using energy captured and stored on the Modal day
- The energy provided by the ICE machine run time of imported from the grid

or

- The deficit met by using energy captured on the day(s) before the Modal day and stored, assuming the days before the Modal have greater incident energy than the Modal.

This third case has not been explored in this study so far. The optimisation question that is posed by the third case can be expressed as follows:

Assuming that $I_{rad_{n-1}} > I_{rad_n} > I_{rad_{n+1}}$ and given that a first stage modal solution has been established what is the optimal mix of increase in storage size to exploit the excess energy on the day $n - 1$ vs ICE run time for the day $n + 1$?

The technique developed can address this question using the basic form of the objective function without amendment. This is because the basic cost function already allows a trade off between battery cost and running time. The excess energy available from day(s) could be captured as a series of inequality equations where the reference incident solar energy used is a series of energy x probability values (as per the existing $e_0$ structure.)

While this is possible, there is no particular benefit since this mathematical process would be a complex way to execute what is a simple marginal case analysis. This marginal case analysis can be executed by applying a third analysis step to the existing methodology using a similar technique to the ICE versus grid energy analysis detailed in chapter 5. This approach is shown in the flow diagram Figure A.4.
8.4. How to find the Worse Case Solution by Scenario Tree Simplification

The process outlined in Figure A4 looks at the marginal cost of energy storage only and not at the costs associated with increasing electrical generation size. (such as an increase in PV panel area). The marginal cost of increasing generation size is already factored into the basic optimisation. The greater than Modal (n-1) day already produces more energy but it occurs at a lower probability than the modal day. Hence from first principals the base process array size is always the lowest cost.
option.
Bibliography


