Diversity Techniques for Broadband Wireless Communications: Performance Enhancement and Analysis

A thesis submitted in fulfillment of the requirements for the degree of
Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged.

Arun K. Gurung
August 2010
This dissertation is dedicated to my mom Hari, dad Dal Bahadur, wife Saiphin and daughter Divya.
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been corrected/amended accordingly. I believe that the thesis is in a better shape and format now. I would like to sincerely thank both of them for their time and patience for going through it.

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Publications and Awards

Below are the publications and the awards in conjunction with the author’s PhD candidacy:

Refereed Conference Publications (Principal Author)


**Joint Refereed Conference/Journal Publications**


3. Fawaz S. Al-Qahtani, **Arun K. Gurung**, Salam A. Zummo, Seedahmed S. Mahmoud, and Zahir M. Hussain, “Performance Evaluation of Generalized Se-


Keywords

Diversity Techniques, Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Multiple Input Multiple Output (MIMO), Relay Networks, Cooperative System, Performance Analysis.
Preface

The demand for higher data-speed in wireless networks calls for innovative efficient communications technologies which are not only spectrally-efficient but also energy-efficient and resilient to channel impairments. Diversity techniques have been shown to be effective to overcome the channel fading and exploit broadcast nature of transmission to provide reliable and better links.

The thesis focus is primarily on the performance analysis/enhancement of some of the diversity techniques such as OFDM, MIMO (Multiple Input Multiple Output), and Relay Networks. These diversity techniques are already part of many current and future broadband wireless technologies such as WiFi (IEEE802.11n), WiMAX (IEEE802.16), LTE, UWB, etc. A new scheme for peak-to-average-power-ratio (PAPR) reduction has been proposed for OFDM signals. The impact of channel estimation error on the performance of MIMO systems forms the next contribution. The last section on relay networks covers performance analysis for various settings including multiple antenna nodes and different fading scenarios.

I hope that this work will foster further research in wireless communications and related technologies.

Arun K. Gurung

Melbourne

August 2010
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Acronyms

SNR  Signal-to-noise ratio
OFDM  Orthogonal Frequency Division Multiplexing
MIMO  Multiple Input Multiple Output
BER  Bit Error Rate
SER  Symbol Error Rate
PSD  Power Spectral Density
PDF  Probability Density Function
CDF  Cumulative Distribution Function
MGF  Moment Generating Function
PAPR  Peak to Average Power Ratio
dB  Decibel
Abstract

As the use of multimedia applications grows, there is an increasing appetite for higher data rate and ubiquitous (anywhere, anytime) access to such services. The challenge is come up with innovative technologies which are not just spectrally-efficient but also energy-efficient (‘Greener technology’). In order to achieve better channel utilization and higher capacity (spectral-efficiency), the newer technologies need to overcome the restrictions imposed by channels as well as the interference from other users/technologies operating in the same geographical location. The diversity techniques have been proven to be effective for such purpose, and are the focus of this thesis. The diversity techniques can be broadly categorized into three types: Space, Time, and Frequency. In this thesis, we are primarily concerned with the frequency and the space techniques.

Frequency diversity is a well-known diversity technique to combat frequency-selective channel fading. Multi-carrier technology where multiple narrow band carriers (subcarriers) are multiplexed and transmitted brings some quite useful properties. When the subcarriers are chosen to be orthogonal to each other, it is called orthogonal frequency division multiplexing (OFDM). OFDM signals are more immune to channel selectivity, delay spread, etc. Channel estimation is easier and the use of FFT/IFFT makes the implementation cheaper. Despite above mentioned advantages, there are few inherent problems. As several subcarriers are multiplexed, when added coherently gives high peak-to-average power ratio (PAPR). High PAPR demands large dynamic range in the transmitted chain such as digital to analog converter (DAC) and power amplifier (PA). Unless pre-processed, the transmitted signal gets distorted due to quantization errors and inter-modulation. In the ini-
Abstract

In the initial stage of PhD candidature, the author focused on PAPR reduction techniques. A simple modification on conventional iterative clipping and filtering (ICF) technique was proposed which has less computational complexity. The power savings achievable from clipping and filtering method was considered. Furthermore the ICF is compared with another distortion-less PAPR reduction technique called Selective Mapping (SLM) based on power savings. Finally, impact of clipping and filtering on the channel estimation was analyzed.

Due to multi-path propagation in wireless radio channel the receiver collects multiple copies of the (delayed) transmitted signal. The inter-symbol-interference (ISI) degrades the signal quality. Space diversity seeks to exploit the multi-path characteristics to improve the performance. The simplest form of the space diversity is the receive diversity where two or more antennas with sufficient spacing collect independent copies of the transmitted signal, which contributes to better signal reception. Post-processing techniques for receive diversity such as maximal-ratio-combining (MRC) or selection-combining (SC), have been known for years now; also called single-input-multiple-output (SIMO) system.

Later the researchers tried to implement MISO (multiple-input-single-output) where multiple-antenna transmitters (as in Base station) communicates with single-antenna receivers (as in smaller mobile phones). Today, we can see many communications systems with fully functional MIMO (multiple-input-multiple-output) capability. Design and performance analysis of MIMO system attracted significant research activities since last decade. In this thesis new analytical expressions of useful metrics such as spectral efficiency, capacity, and error rates were presented for adaptive SIMO systems with channel estimation error. Beamforming (steering signal towards desired receiver) is another useful technique in MIMO systems to further improve the system performance. MRT (Maximal Ratio Transmission) or MIMO-MRC is such system where the transmitter, based on channel feedback from the receiver, uses weighting factors to steer the transmitted signal. Closed form expressions for symbol error rates were derived for MRT system with channel estimation error. The results were extended to evaluate closed form expressions of error rates for Rectangular QAM. Same was derived for MRC system over Nakagami-q fading applicable for earth-satellite links. Antenna correlation was considered in another contribution
Relay and Cooperative networks represent another form of spatial diversity and have recently attracted significant research attention. These networks rely on intermediate nodes called "relays" to establish communication between the source and the destination. In addition to coverage extension, the relay networks have shown to offer cooperative diversity when there is a direct link or multiple relays. The first contribution is to analyze a dual-hop amplify-forward relay networks with dissimilar fading scenarios. Next error rates of Rectangular QAM for decode-forward selection relay system are derived. Multiple antenna at relay is included to analyze the benefits of dual spatial diversity over Rayleigh and Nakagami fading channels. Antenna selection is a cost-effective way to exploit the antenna diversity. General Order Antenna Selection (GOAS), based on Ordered Statistics, is used to evaluate signal statistics for a MIMO relay network.

A total of 18 (9 as principal author) refereed papers have been published as research contributions during the candidature. Author hopes that these contributions will inspire further advanced research on developing a truly wireless broadband communications system.

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Chapter 1

Introduction

1.1 Wireless Communications

Unlike in wireline communications, signal transmission/reception in wireless systems is achieved through the inherent broadcast nature of electromagnetic waves. In other words, the electromagnetic waves are not guided to the receivers through some medium such as copper, optical fiber, etc in wireless communications. Therefore the signal transmitted through the antenna will reach to the receiver often following different paths, faded and delayed differently. This phenomenon is known as multi-path fading in wireless communications, and dictates the quality of the received signal. The channel variation is normally higher due to the surrounding reflectors, scatterers, even the relative movement of the transmitter/receiver. The time-varying nature of the channel makes it more difficult to estimate, and requires constant monitoring of the channel variation. Depending on the severity of multipath fading, the channels can be categorized as frequency-selective or flat. If the coherence bandwidth of the channel is larger than the signal bandwidth, the channel is regarded as flat-fading and normally sought after. The need for higher data rate is normally satisfied with larger signal bandwidth, which means the channel coherence bandwidth is no longer larger than the signal bandwidth. This gives rise to frequency-selective channels which normally add more system complexity.

How to keep the system simpler and yet combat channel impairments? This is the task of modern communications engineers/scientists. Diversity has been always a effective way to mitigate such impairments. Some diversity techniques attempt to add redundancy to the system. For example, send the signal twice (called delay
diversity), or send in two different carries (called frequency diversity). Other diversity techniques seek to exploit the very broadcast nature of wireless channels. Use of multiple antennas is one such example. It will reduce the likelihood of receiving severely faded signal at all the antennas, therefore the transmitted signal can still be recovered from the antenna with better signal reception. And recently there is another form of spatial diversity - using relays. The relays basically relay the signal from the transmitter to the receiver. Originally, it was conceived for coverage extension. Now, it has been shown that a cooperation among multiple nodes in a network is possible and leads to what is referred as "cooperative diversity". The cooperative diversity not only improves the reception (reliability) but possibly increase the link rate (capacity) as well.

However, there are new issues to be dealt before one can achieve the full benefits of these diversity techniques. Some of such issues are described next.

1.2 Thesis Objectives

The focus of the thesis is on three different diversity techniques: OFDM (Orthogonal Frequency Division Multiplexing), MIMO (Multiple-Input Multiple-Output), and Relay/Cooperative Networks.

OFDM is a modulation technique where the high-rate data is divided into low-rate streams and modulated over orthogonal 'subcarriers'. Since the signal bandwidth is split into several narrow-band channels, each channel undergoes flat fading even the whole bandwidth is larger than the coherence bandwidth. The channel estimation and demodulation is a lot simpler as the receiver needs to work only on per-subcarrier basis. And the severely faded channels can be tracked and excluded for the signal transmission. The OFDM systems are therefore more resilient to channel impairments, yet offer broadband data rates. However there are some inherent issues in OFDM to be addressed. One issue is high Peak-to-Average-Power-Ratio (PAPR) which demands large dynamic range for blocks such as Digital-to-Analog-Converter (DAC) and Power Amplifier (PA) in the transmitter chain [1]. If not properly resolved, the transmitted OFDM signals suffer from the quantization errors and inter-modulation. Power back-offs in the PA may help on some part, but efficiency of PA drops and the transmitter eventually needs more power to operate.
Chapter 1. Introduction

There has been a big number of publications addressing this problem, mostly trying to reduce the PAPR. A short literature review on PAPR reduction techniques will be provided in next chapter.

Next, an analytical treatment of MIMO systems mostly with channel estimation error is given. Access to analytical closed-form expressions of performance metrics avoids lengthy simulation runs and very expensive measurement trials. The previous published works assumed perfect channel availability, which is generally not true in practical systems. Some works involved the inclusion of general order QAM (Quadrature Amplitude Modulation) to evaluate error rates. Antenna correlation (depending on the spacing) among the antennas is another issue to be considered in practical MIMO systems, and many cases dictate the suitability of use of multiple antennas [2]. For example, uncorrelated antennas may be desirable for spatial multiplexing i.e. to transmit independent stream of data between the transmitter and the receiver thereby increasing the spectral efficiency. But again uncorrelated antennas (large spacing among antennas) may not be available. In such cases, other strategies such as beamforming, precoding, etc will be more beneficial. Brief background on current knowledge status of MIMO system will be given in Chapter 2.

Author extended results from previous section to relay networks. Still an immature research area, relay/cooperative networks offer several topics to be identified and explored. The relay/cooperative networks utilize other wireless terminals to relay the information from the source to the destination, and often offer an additional spatial diversity to the system. Most of the published results revolve around the information theoretic aspects. Recently, more efficient and practical issues are being given emphasis. Our work mainly covered the performance analysis of dual-hop cooperative networks with multiple-antenna nodes.

1.2.1 Research Questions

The PhD thesis attempted to answer the following research questions:

1. Most of the PAPR reduction techniques add huge system complexity and delay. Can the PAPR reduction techniques be more efficient and practical?

2. How much power savings can be achieved with the PAPR reduction techniques?
And how they compare to each other on net power savings?

3. Do PAPR reduction techniques affect other communications aspects such as pilots intended for channel estimation at the receivers?

PAPR reduction techniques are expected to have minimum adverse effects to other such aspects of transmission system.

4. How does the MIMO systems perform with imperfect channel estimation?

5. How does the antenna correlation influence the performance of MIMO systems?

6. What performance can be achieved with multiple antennas in Relay/Cooperative networks? Is there any influence of fading parameters such as Nakagami-m metric on system performance?

1.2.2 Research Aims/Objectives

The specific research objectives arising from aforementioned questions have been addressed in the thesis. These can be summarized as follows:

1. Proposing a new simpler practical PAPR reduction technique

2. Comparing PAPR reduction techniques based on net power savings. Analyzing the impact of the PAPR reduction technique on pilot-aided channel estimation in OFDM systems.


4. Investigating the performance analysis of relay/cooperative networks under different settings such as fading scenarios and multiple antennas.

5. Extending analysis to more practical relay/cooperative systems.

1.3 Original Contributions

The thesis describes several novel contributions on diversity techniques for wireless broadband communications made during the candidature.

The key contributions of this dissertation are:
1. A new One-Iteration-Clipping-Filtering (OICF) scheme for PAPR reduction was proposed, which reduces the system complexity with similar performance as the conventional Iterative Clipping Filtering (ICF).

2. Net power savings analysis has been carried out for ICF taking account of a practical digital signal processor (DSP) parameters and practical power amplifier model. The analysis was extended to Selected Mapping (SLM), another popular PAPR reduction technique. These two techniques are compared based on net power savings.

3. Closed-form expressions for symbol error rate (SER) for MIMO-MRC system with channel estimation error were derived. The analysis was extended to Rectangular QAM which generalizes the constellation size/type.

4. A mathematical analysis for adaptive General Selection Combiner (GSC) with imperfect channel estimation has been carried out. Expressions of SNR statistics and performance metrics were obtained.

5. The impact of antenna correlation on the performance of MRC system has been analyzed.

6. End to end system performance of dual-hop amplify-forward relay system has been analyzed subject to different fading scenarios (Rice and Nakagami).

7. Performance of a dual-hop amplify-forward relay system with a multi-antenna relay was investigated for Rayleigh and Nakagami fading channels.

8. General Order Statistics (GOS) theorem was applied to the antenna selection in MIMO relay/cooperative system, and the SNR statistics were derived.

1.4 Thesis Organization

The thesis consists of six chapters, the major contributions are given in three chapters namely chapters 3 to 5.

The dissertation is organized as follows:

Chapter 2: Literature Review
Chapter 1. Introduction

A literature review on the diversity techniques within the scope of this thesis is presented in the chapter. A brief background on the relevant diversity techniques is provided, which will help readers to follow and understand the key results in the following chapters. The description is also basis for remaining chapters, therefore all the important references are mentioned.

Chapter 3: Clipping-Filtering in OFDM Systems

One of the diversity techniques explored in this work - OFDM systems form the content of this chapter. A brief background of OFDM, PAPR, and PAPR reduction techniques are given. Thereafter, conventional clipping-filtering is introduced before proposing our new scheme. Next analysis of power savings due to ICF follows. Two different techniques ICF and SLM are compared based on net power savings. Finally, a short research on the impact of Clipping-Filtering on pilot-assisted channel estimation is given.

Chapter 4: Performance Analysis of Multiantenna Systems

This chapter covers the performance analysis of multiantenna systems. First, the analysis for MIMO-MRC system with channel estimation error is described. The closed-form expressions of SER are derived. The analysis is extended to Rectangular QAM which generalizes the constellation size/type. Similar analysis was done for MRC system over Nakagami-q (Hoyt) channels useful for Earth-Satellite links. Next, an adaptive GSC (Generalized Selection Combiner) systems with channel estimation error is considered for analytical treatment. Antenna correlation is taken into account to analyze MRC system and performance metrics such as error rate are derived.

Chapter 5: Analysis of Relay/Cooperative Networks

This chapter includes the performance analysis of relay and cooperative networks. The key contribution is the SNR statistics analysis for various scenarios. Once the SNR statistics are known, it is often very easy to derive other performance metrics such as capacity, error rate, outage, etc which are of more importance in system design point of view. All the cases assumed dual-hop amplify-forward relay network. The first contribution was for dissimilar fading scenario (Rice and Nakagami). Multi-antenna relay is considered next over Rayleigh fading scenario. The analysis is extended to more general fading - Nakagami. Thereafter GOS (General Order Selec-
tion) theorem is applied to MIMO relay, where a general order antenna, unlike the best one, is chosen for transmission/reception. The derived analytical expressions are verified by computer simulations which confirm accuracy of the analysis.

Chapter 6: Conclusions and Future Work

This chapter summarizes the main research outcomes of the thesis and presents possible future research directions.
Chapter 2

Literature Review

2.1 Introduction

In this chapter, an overview of current research literature is provided on the diversity techniques for broadband wireless communications. The review is given for specific publications made during the candidature and does not offer general overview on the diversity techniques. Several books on diversity techniques (OFDM, MIMO, Relay) [1]- [2] have appeared and will be helpful to interested readers.

In the following sections, literature review on OFDM PAPR, MIMO and Relay Networks is given in sequence including some important and relevant references.

2.1.1 Orthogonal Frequency Division Multiplexing (OFDM)

OFDM is a modulation technique which offers quite a few interesting features to mitigate frequency-selective channel impairments [1]. Huge bandwidth savings is possible due to the orthogonality among subcarriers as shown in Figure 2.1 (Please refer to next page). The high-data rate is divided into several low-data rate streams which modulate orthogonal subcarriers. The narrow band signals are multiplexed together and sent through the channel (baseband representation). At the receiver, the signal is de-multiplexed in reverse order creating low-data rate streams which form the original high-data rate signal. Implementation has been cheaper because of the FFT/IFFT processing in the baseband signal processing unit. Another advantage of OFDM system is the efficient channel estimation/equalization as the broadband frequency-selective channel is split into several flat-fading channels due to narrow-band subcarriers. Service providers can use granularity (due to several narrow band
subcarriers) available to offer variety of data rate depending on the service types (e.g. data, voice, video, etc) and Quality of Service (e.g. reliability, priority, etc).

Discrete-time OFDM signal can be written as,

\[ x_n = x_n^{(NT/JN)} = \frac{1}{\sqrt{N}} \sum_{k=-N/2}^{N/2-1} X_{\langle k+N \rangle} \times \exp \left( \frac{j2\pi nk}{JN} \right), \]

\[ n = 0, 1, ..., JN - 1 \]  

(2.1)

where \( \langle k+N \rangle \) denotes \((k+N)\) modulo \(N\); \( X = \{X_0, X_1, ..., X_{N-1}\} \) represents input vector (of mapped symbols); \( N \) = number of subcarriers ; \( T \) = OFDM data symbol period ; \( \Delta f = 1/T \), frequency spacing for orthogonality ; \( J \) is oversampling factor. \( J=1 \) gives discrete-time signal sampled at Nyquist rate, whereas \( J=4 \) provides sufficient samples to capture continuous-domain signal peaks [1]. The oversampled signal can be obtained by \((J-1)N\) zero-padding in the middle of the original input vector and taking IFFT of it. The zero-padded input vector looks like

\[ X = \{X_0, ..., X_{N/2-1}, 0, 0, 0, 0, ..., X_{N/2}, ..., X_{N-1}\} \]  

(2.2)

Despite many desirable features in previous paragraphs, OFDM system does
come with some drawbacks. One of the major issues is the high peak-to-average-power-ratio (PAPR). Due to the addition of several narrow-band signals, the peak of resultant OFDM signals may be very high compared to the individual subcarriers as shown in Figure 2.2. 64 subcarriers are modulated by QPSK and added together to form OFDM signal which has peaks of up to 50 compared to just 4 of the individual subcarriers.

![Figure 2.2: High Peak-to-Average Power Ratio of OFDM Signals, N=64, QPSK](image)

Why is high PAPR an issue? The signals in the transmitter side are normally fed through a power amplifier before sent out to the channel. Looking at the characteristics of a typical power amplifier as shown in Figure 2.3, one can easily find out the answer which is explained below.

The power amplifiers are set to operate at certain back-off to produce least intermodulation distortion and yet is the most efficient. High PAPR signals call for high dynamic range in amplifiers, i.e. larger linear region. Due to regulatory requirements, transmitted signals also need to comply with spectral mask in their allocated bands. High PAPR signals if amplified with lower back-off (to achieve high efficiency) produce large distortion and will exceed the spectral mask which is not acceptable. On the other side, if a larger back-off is used in the amplifier, the efficiency will drop.
drastically requiring more input power. For portable devices, this would mean draining out of battery power too quickly. Therefore, the high PAPR in OFDM signals need to be addressed (reduced) to design efficient and practical devices.

2.1.2 Peak-to-Average Power Ratio (PAPR)

The PAPR of discrete-time OFDM signal is written as

$$\text{PAPR}_x \triangleq \frac{\max_{0 \leq n \leq JN-1} |x_n|^2}{E\{|x_n|^2\}} \quad (2.3)$$

where $E\{\cdot\}$ denotes expectation operator. PAPR is best described by its statistical parameter, complementary cumulative distribution function (CCDF). CCDF measures the probability of signal PAPR exceeding certain threshold $\gamma$, i.e. $\Pr[\text{PAPR}_x > \gamma]$. An approximated PAPR CCDF expression based on level crossing rate can be given as [4],

$$\Pr(\text{PAPR}_x > \gamma) \approx 1 - \exp\left(-N e^{-\gamma \sqrt{\frac{\pi}{3}}\gamma}\right) \quad (2.4)$$
and is accurate for a relatively high $\gamma$ and large number of subcarriers $N \geq 64$.

Another recent approach to evaluate PAPR distribution has been Extreme Value Theory. Based on this approach, T. Jiang [5] has shown new PAPR distribution for practical systems (e.g. WLAN, DAB I) with equal and unequal power allocations to subcarriers. New distribution is shown to be in closer resemblance to simulation curves than previous distribution (like (2.4)). In this thesis, the above distribution for analysis is followed, as done by [6]. Note that empirical results could be readily obtained through computer simulations, therefore the need of explicit numerical expression for PAPR distribution might not be necessary in many situations.

Following the same steps described in [6], $\gamma$ as a function of probability level $p$ can be given as,

$$\gamma(p) = \frac{-1}{2} W \left(-\frac{6 \ln(1-p)^2}{\pi N^2}\right)$$

(2.5)

where W is Lambert’s W-function defined by the inverse of $f(W) = We^W$ [7]. Table 2.1 gives some $\gamma$ values as a function of N and p.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$N = 64$</th>
<th>$N = 128$</th>
<th>$N = 256$</th>
<th>$N = 512$</th>
<th>$N = 1024$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-2}$</td>
<td>9.97</td>
<td>10.3</td>
<td>10.6</td>
<td>10.8</td>
<td>11.1</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>10.9</td>
<td>11.2</td>
<td>11.4</td>
<td>11.6</td>
<td>11.8</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>11.7</td>
<td>11.9</td>
<td>12.1</td>
<td>12.3</td>
<td>12.5</td>
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<tr>
<td>$10^{-5}$</td>
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<td>12.8</td>
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<td>$10^{-6}$</td>
<td>12.9</td>
<td>13.1</td>
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<td>13.4</td>
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</tr>
</tbody>
</table>

As can be seen from the table, the threshold PAPR strongly depends on probability of clipping and number of subcarriers. In order to accurately assess the gain in power efficiency, we need to set a common benchmark which in our case is bit-error-rate (BER). BER depends on the clipping probability of a linear amplifier (ideal). Besides a function of probability of clipping, system BER also depends on constellation size [8]. Higher the constellation size, more the errors with the same probability of clipping and the number of subcarriers. To simplify power analysis, a reasonable probability of clipping of $10^{-5}$ is assumed as a reference.
2.1.3 PAPR Reduction Techniques

There are several PAPR reduction techniques published in literature. Excellent overview of popular peak to average power (PAPR) reduction techniques is given in [9]. Some of them are as follows:

1. Clipping and Filtering

This is probably the most intuitive and simplest technique first appeared in [10]. The idea is to hard-clip the OFDM signal before sending to the channel. Filtering will be needed to restrict the out-of-band radiation within the regulatory spectral-mask. Repeated clipping-filtering [11] will go on iterative manner until target PAPR threshold is achieved.

Our contribution on OFDM PAPR revolves around this technique.

2. Selective Mapping (SLM)

Selective mapping is based on the idea of generating multiple copies of the original signal through some set of codes. The copy with lowest PAPR is chosen for transmission [12]. The side-information (index of the transmitted signal) is needed at the receiver to recover the original signal back for demodulation. As the number of subcarriers increases, larger the set of codes required to obtain a decent PAPR (5-6dB). High-computational complexity and need to transmit side-information have been criticized in the original SLM. Many efficient variants have emerged recently.

3. Phase Transmit Sequence (PTS)

Phase Transmit Sequence [13] is similar to the SLM, but creates several disjoint (smaller) signals from the original OFDM signal. All the disjoint signals are phase-rotated individually in such a way that results lower PAPR OFDM signal. Phase optimization is key to this technique and requires a good amount of computational complexity.

4. Coding

Several coding schemes, simplest being block coding [14], have been proposed which
have inherent low PAPR property. Some significant codes are Golay complementary
pairs and Reed-Muller codes. Interleaving (Bit & Symbol) can also possibly result
lower PAPR. To create good codes for a range of set of OFDM specifications (number
of subcarriers, modulation types, pilots, etc) along with efficient coding-rate is still
an open research issue.

5. Tone Reservation/Injection
With FFT/IFFT based OFDM, the number of subcarriers are always multiple of
base 2, and many OFDM based standards do not use all the subcarriers. Some (like
at the edge and in the middle) subcarriers are not used, others are assigned for spe-
cial purposes such as signalling or pilots. These subcarriers (also called tones) can
be used for the purpose of controlling PAPR of the signal \[15\]. The effectiveness of
this technique depends on number of tones available and their positions in OFDM
spectrum.

6. Active Constellation Extension (ACE)
Similar to the Tone Reservation/Injection method, ACE tends to modify the con-
stellation shape/size which alters the PAPR of the OFDM signals \[16\]. Again the
optimization required will add to the computation complexity. More signal power
may be needed in any case of extending the constellation or inserting dummy con-
stellation points.

Criteria for choosing suitable PAPR reduction methods are \[9\]:
- PAPR reduction capability and flexibility
- Power increase in transmit signal
- BER increase at receiver
- Loss in data rate
- Computational complexity
- Other consideration: hardware implementation, D/A converter, transmit filter, etc

There have been introduced ‘combo’ techniques which basically use more than
one compatible above schemes to obtain higher PAPR reduction with less complex-
ity/impairments in the system.
Moreover, the techniques can be extended to OFDM based system such as MIMO-OFDM. Traditional approaches can still be applicable to MIMO-OFDM, but there is a growing trend to look for schemes which can exploit the multiple antenna configuration, for example Cross Antenna Rotation and Inversion (CARI) [17], Space Frequency Permutation and Inversion (SFPI) [18].

2.2 Multiple Input Multiple Output - Multiantenna Systems

In this section, an overview of multiantenna systems relevant to this thesis is discussed. Multi-antenna (MIMO, Multi-Input Multi-Output) systems are now the state-of-the-art technology and part of many current and future broadband communication standards. MIMO systems exploit the spatial diversity at wireless terminals, where the signal copies transmitted from or received at multiple antennas can be used to improve the overall signal-to-noise ratio (SNR). Excellent analytical treatment of digital communication over fading channels and multi-antenna terminals can be found in [2].

2.2.1 MIMO-MRC (Maximal Ratio Transmission, MRT)

Antenna diversity is a widely used technique to increase the reliability and the capacity of wireless systems [2]. Multiple-antenna systems are proven to meet the demand of high data-rate applications over wireless channels as shown by its inclusion in many broadband next generation communications technologies. Of many variants of multiple-antenna systems, MIMO-MRC (Multiple-Input Multiple-Output Maximum Ratio Combining) or MRT (Maximum Ratio Transmission) is one where the forward channel information is known at the transmitter. The channel information, known as Channel State Information (CSI), is used to beamform (or direct) the transmitted signal to the desired receiver [19]. The estimated channel at the receiver is routed back to the transmitter error-free with zero-delay. The block diagram of MRT system is shown in Figure 4.1. In practical scenarios the assumption of the perfect channel information is not true.
Figure 2.4: Block Diagram of MIMO-MRC (MRT) System

The effect of imperfect channel estimation on system performance has always drawn a significant research attention. The MRT system with channel estimation error has been studied in [20] [21]. Following similar steps in [22], the authors have derived the probability density function and the moment generating function of MRT output Signal-to-Noise-Ratio (SNR) with channel estimation error. Recently, exact closed-form expressions of average symbol error rates for $M$-PSK and $M$-QAM signalling with perfect estimation were derived in [23]. In this thesis, similar expressions were derived but with imperfect channel estimation. The results were extended to Rectangular QAM which generalizes the expressions and is valid for any rectangular 2-Dimensional constellation.

### 2.2.2 Rectangular QAM for MRC over Hoyt Fading Channel

MRC is the optimum combining scheme among receive diversity techniques, and utilizes all the received branch signals for the demodulation. Performance analysis of MRC over various settings such as fading channels, modulation types, antenna correlation, etc has been hot research topics for years.

Among many popular wireless channel fading models, Nakagami-$q$ fading has
recently gained research attention [24]-[28]. The Nakagami-\(q\) fading is more severe than Rayleigh fading, and is used to model channels such as Earth-Satellite communications links with strong ionospheric scintillation. It spans from one-sided Gaussian fading (\(q=0\)) to Rayleigh fading (\(q=1\)). On the other hand, recent work on the integration of the product of two Gaussian \(Q\)-functions by Beaulieu [29] has stimulated research on rectangular Quadrature Amplitude Modulation (QAM) [30–32]. The Beaulieu’s results have been used to derive average symbol error probability (ASEP) for various scenarios for example - single-input single-output (SISO) Nakagami-\(m\) [30], SISO Nakagami-\(q\) [31] and Nakagami-\(m\) fading on multi-channel receiver in [32]. The final symbol error probability (SEP) expressions are given in terms of multivariate Lauricella hypergeometric functions which can be easily computed with its converging series representation.

We have extended the above analysis on Rectangular QAM to [28], and obtained close-form expressions of SER. The expressions specialize to previous results for Square QAM.

### 2.2.3 Antenna Correlation in MRC

Antenna correlation is one key factor influencing the performance of practical diversity systems. We consider a real scenario which may arise as antenna elements at the receiver are closely spaced or the received single envelopes can exhibit a degree of correlation.

The work published in 2004 by Mallik \textit{et al.} [33] derived probability density function (PDF) for \(L\) equal-correlated MRC diversity branches over Rayleigh fading channels and the channel capacity for different adaptive scenarios. The work is motivated by the result in [33] to obtain the exact closed-expression for ergodic Shannon capacity, an upper bound and its approximated ergodic capacity at high and low SNR region. The approximation expressions provide us with alternative analytical solution in case that the exact capacity is impossible to obtain in a closed-form. Next, the capacity statistics of correlated MRC were obtained; these statistics are valid for arbitrary number of receive antennas and are expressed as a function of the degree of correlation among receive antennas. The ergodic-approximation expressions and capacity statistics including moment generating function (MGF),
cumulative distribution function (CDF), and probability density function (PDF) are derived. Furthermore the effect of correlation among multi-channels on system performance is shown in terms of symbol error rate for $M$-PSK signalling. This study was undertaken jointly, and the derivation of error rates and validation of accuracy of the derived statistics were my contribution.

2.2.4 Generalized Selection Combiner (GSC)

A hybrid combination of SC (Selection Combining) and MRC, is called generalized selection combining (GSC) as shown in Figure 2.5. In the GSC, a fixed subset of size $L_B$ of a large number of available diversity channels of size $L$ is chosen and then combined using MRC. As mentioned before, the MRC combines the branch signals such that the instantaneous output signal-to-noise ratio (SNR) is maximized [34–36]. System performance is often analyzed with the assumption of perfect channel estimate. However, in practice the branch signal-to-noise ratio (SNR) estimates are corrupted with channel impairments making it difficult to achieve perfect estimation. Normally, a diversity branch SNR estimate can be obtained either from a pilot signal or data signals (by applying a clairvoyant estimator) [37]. For example, if a pilot signal is inserted to estimate the channel, a Gaussian error may arise due to the large frequency separation or time dispersion. Therefore, it is important to include estimation errors in system performance analysis. Previous works on the analysis of imperfect channel estimation with and without diversity can be found in [37]- [41].
Chapter 2. Literature Review

The paper in [41] considers the channel estimation error of the GSC per branch SNR as being complex Gaussian and derives the probability density function (PDF) of the output combiner.

The pioneering work of Shannon [42] has established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a channel. Spectral efficiency of adaptive transmission techniques has received extensive interest in the last decade. In [43], the capacity of a single user flat fading channel with perfect channel information at the transmitter and the receiver is derived for various adaptation policies, namely, 1) optimal rate and power adaptation (opra), 2) optimal rate adaptation and constant power (orm), and 3) channel inversion with fixed rate (cifr). The first scheme requires channel information at the transmitter and receiver, whereas the second scheme is more practical since the transmission power remains constant. The last scheme is a suboptimal transmission adaptation scheme, in which the channel side information is used to maintain a constant received power by inverting the channel fading [43]. In [44], the general theory developed in [43] is applied to achieve closed form expressions for the capacity of Rayleigh fading channel under different adaptive transmission and diversity combining techniques, also this work has been extended to many fading scenarios environments (here within [33,45]). Similar analysis was undertaken here but with Gaussian channel estimation error. This study was undertaken jointly, and the validation and discussion of accuracy of the derived statistics were my contribution.

2.3 Relay Networks

Relays are shown to significantly improve the link quality, thereby increasing the coverage area of many infrastructure or ad-hoc networks alike. As shown in Figure 2.6, the relay receives signals from the source and relay them towards the destination based on some protocols such as amplify-and-forward (AF), decode-and-forward (DF), etc. Some noteworthy papers on relay networks include [46]- [48] and the references therein.
2.3.1 Performance Analysis under Mixed Fading Scenario

Symmetric fading channels are intuitive choice for many of the researchers for the performance analysis of the relay networks (dual-hop, multi-hop, and multi-branch). Recently, there is an increased research attention on the performance of dual-hop relay networks where fading channels of two hops are asymmetric some recent works include [49]-[52]. Dissimilar or asymmetric Nakagami-\(m\) fading was assumed in [49] for decode-forward relay, and in [50] for amplify-forward case. Duong et al. [51] presented the symbol error probability of dual-hop Rayleigh-Rican fading channels where the source employs orthogonal space-time block codes (OSTBCs) and the relay operates in AF mode. Himal et al. [52] considered a dual-hop system with asymmetric fading channels which resembles practical scenarios of macro/micro cellular networks. The channel between the base station (source) and the relay has line-of-sight, whereas the channel between the relay to the mobile unit (destination) might not necessarily be same. The authors have derived exact expressions of the outage probability and the average bit error probability (ABEP) in terms of infinite series.

Here, the analysis in [52] was extended to relay channels subject to a line-of-sight (Rician) and a Nakagami-\(m\) fading. Inclusion of Nakagami-\(m\) distribution generalizes the analysis as the Rayleigh fading is its special case. The final expressions specialize to the previously published work. Furthermore, approximation is presented which matches very well with exact result at high SNR region.
2.3.2 Rectangular QAM in Selection Relay System

The spatial diversity in relay system can be thought of relays collaborating together to form a virtual antenna array. The performance depends on the number of relaying nodes and the processing operation at both relays and destination. One popular relaying scheme, decode-and-forward (DF), where relays decode the received signals from the source and retransmit them toward the destination, was analyzed in [53]-[58] and references therein. In fixed DF case, the relays always respond to all received signals by decoding, re-encoding, and transmitting them to the destination. The relays forward only the correct decoded messages by applying cyclic redundancy check (CRC) at the relay nodes in adaptive DF relay networks [54]. In [58], the performance of fixed DF cooperative networks with relay selection over independent but not necessarily identically distributed (i.n.i.d) Nakagami-$m$ is presented. The authors derived the closed-form expression of SEP for $M$-ary phase-shift keying (M-PSK). We extend the results to the Rectangular QAM [29], [32] as it generalizes the error analysis. This study was undertaken jointly, and the validation of the derived error rate expressions was my contribution.

2.3.3 Dual Hop System with a Multiantenna Relay

As mentioned earlier, the cooperative diversity in wireless networks [46] [59] has shown tremendous potential to achieve next generation wireless communication networks. These intelligent wireless networks will consist of a number of wireless nodes cooperating with each other to relay information from one end to other, on ad-hoc basis in many cases. The very broadcasting nature (and therefore multi-path phenomenon) and the spatial diversity of wireless terminals can be exploited to achieve low cost reliable wireless networks.

Initial studies on cooperative networks included signal antenna terminals. Recently there is growing research attention towards inclusion of the multi-antenna terminals in cooperative networks [60]-[63]. Here, we consider a dual-hop relay system with amplify-and-forward (AF) transmission system with a multi-antenna terminal as shown in Figure 2.7. The intermediate node $R$ receives the signal from the source $S$, and forwards the amplified signal towards the destination $D$. There may or may not be the presence of direct link between $S$ and $D$. The end nodes
are single-antenna terminals whereas the relay node is equipped with $N$ multiple antennas. The scenario is applicable in many infrastructure wireless networks where the base stations can employ multiple antennas whereas the end-terminals have single antennas due to space constraints. The multi-antenna relay network has been considered in [60] with Maximal Ratio Combining (MRC) and Transmit Beamforming (TB), and it is shown that given a certain amount of available antennas in the network, wired cooperation (i.e. all the antennas belong to one terminal) outperforms wireless cooperation (i.e. each antenna belongs to different terminals). Very recently [61]- [63] have analyzed single-antenna relay and multi-antenna end-nodes with TB or MRC/TAS(Transmit Antenna Selection). Performance analysis was carried out in terms of outage probability and error rates. Here the case similar to [60] was considered, with a single multi-antenna relay. The MRC or Selection Combining (SC) with TAS is used to exploit the antenna diversity. In the TAS scheme, a single transmit antenna, which maximizes the effective received SNR, is selected for the signal transmission at any time [64]. TAS requires less channel feedback than TB and is less complex, and therefore may be better choice in some practical scenarios.

We extend above analysis to Nakagami-$m$ fading channels. The new expressions specialize to the previous results, and their accuracy was confirmed through Monte-Carlo simulation.
2.3.4 General Order Antenna Selection (GOAS) in Multiantenna Relay System

As stated earlier, antenna selection (traditionally SC at receiver) offers a cost-effective alternative to beamforming. The best antenna which maximizes the received signal-to-noise-ratio (SNR) is chosen for communication. Antenna selection was recently used in the context of two-hop [65], and multi-hop [66] MIMO relay networks. The idea of antenna selection comes from well-known theory of Ordered Statistics. The fundamental General Order Statistics (GOS) has been applied recently for system analysis in [67]- [69] where not only the highest but in general \( n^{th} \) order statistics are of interest. As [68] noted that \( n^{th} \) statistics is often required in signal detection/estimation. Another scenario where \( n^{th} \) statistics may be useful is in the evaluation of performance loss when the receiver/transmitter make error in selecting the best antenna. [69].

We apply GOS to a multi-antenna relay in dual-hop amplify-forward system, where \( n^{th} \) order antenna is chosen for both receiving and transmission. A fixed-gain is assumed at the relay, which offers less complicated alternative to the variable-gain scheme [70]. The end-nodes are single-antenna terminals, and the channels undergo i.i.d (identical and independently distributed) Rayleigh fading. Specifically, following SNR statistics were derived: 1) CDF (cumulative distribution function (CDF), 2) PDF (Probability Density Function), 3) Moment Generating Function (MGF), and 4) General Moments. The derived expressions reduce to previous results for special cases, and were validated through Monte-Carlo simulation. The statistics are then used to analyze system behavior in terms of outage probability and error rate.

2.3.5 GOAS in MIMO Relay System

We extend the results in previous section to a case where all the participating nodes are equipped with multiple antenna. Specifically, the results in [65] [71] are extended to include GOAS in MIMO dual-hop amplify-forward system, where \( n_1^{th} \) and \( n_2^{th} \) order link (antenna-pair) are chosen at transmitter and receiver respectively. All the channels and the links are subject to independently and identically distributed (i.i.d.) Rayleigh fading. We make following contributions to the current state-of-the-art knowledge: 1) Unlike in [65] [71], a fixed-gain amplification was assumed at the
relay. The fixed-gain relaying offers less-complicated alternative to the variable-gain scheme [70]. 2) General Order SNR statistics such as CDF (cumulative distribution function (CDF), PDF (Probability Density Function), MGF (Moment Generating Function), and General Moments, 3) Apply SNR statistics to investigate the presence of the direct link, and the multiple relays. From these results, special case e.g. conventional antenna selection, where the best antenna is chosen at both ends of the link, can be obtained. The analysis is validated through Monte-Carlo simulation.

2.4 Chapter Summary

A brief literature survey was conducted in this chapter. The literature review covered all the aspects of relevant systems considered in this thesis. Three different diversity techniques’ shortcomings and research gaps have been identified. The relevant research references are duly cited to show the current knowledge status on these topics. In next three chapters, we explain the main research outcomes of the thesis.
Chapter 3

Clipping-Filtering in OFDM Systems

3.1 Introduction

This chapter covers the contributions on OFDM PAPR reduction technique - clipping and filtering. First, the net power savings was analyzed achieved through conventional iterative clipping and filtering (ICF). Next ICF is compared with distortionless technique SLM based on net power savings achieved for same PAPR reduction. Thereafter, a new scheme One-Iteration-Clipping-Filtering (OICF) is proposed which is computational efficient with very similar performance to ICF. At the end, the impact of clipping-filtering on pilot-assisted channel estimation is shown in OFDM systems. The chapter summary lists the main contributions of this section.

3.2 Power Savings Analysis for ICF

The ultimate goal of PAPR reduction is to increase the efficiency of power amplifier and thus reduce power consumption or save input power. In following sections, the net power savings achievable due to the ICF scheme is given taking account of computational power spent for it.
3.2.1 Iterative Clipping and Filtering (ICF)

The signal peaks are clipped to a certain threshold. The clipped OFDM signal can be represented as,

\[
x_{n,c} = \begin{cases} 
    x_n, & |x_n| \leq A_{\text{max}} \\
    A_{\text{max}} \exp(j\psi_n), & |x_n| > A_{\text{max}} 
\end{cases}
\]  \hspace{1cm} (3.1)

\(\psi_n = \arg[x_n]\) represents phase of \(x_n\). The phase of signal is preserved whenever signal exceeds clipping threshold \(A_{\text{max}}\). The clipping severity is quantified in terms of clipping ratio \(\zeta\) given as the ratio of the threshold to the average signal power \(P_{\text{in}}\) (that of before clipping),

\[
\zeta = \frac{A_{\text{max}}^2}{P_{\text{in}}} \hspace{1cm} (3.2)
\]

The clipping reduces the signal power, the signal PAPR after clipping process, therefore, is always greater than \(\zeta\).

The out-of-band radiation which interferes adjacent channels is more severe issue than in-band distortion. Digital filtering such as mentioned in [11] can be used to remove out-of-band radiation. The clipped signal is converted into frequency domain by the forward Fourier transform; only first \(N/2\) and last \(N/2\) components are taken and all other samples are set to zero; finally the inverse Fourier transform of vector gives the filtered time-domain signal. Peaks regrow after the filter, increasing the signal PAPR. In iterative clipping and filtering (ICF) scheme, the regrown pulses are clipped and filtered in an iterative fashion until target PAPR is obtained [11].

The system model under consideration is shown in Figure 3.1. The digital clipping and filtering blocks reside before the digital-to-analog converter and the power amplifier. Pulse shaping is not considered so does cyclic prefix. Note that no change in receiver block is necessary for this scheme (unless performance improvement technique like described in [72] is used).

3.2.2 Power Amplifier and Efficiency

A linear power amplifier (PA) model (or ideally pre-distorted non-linear amplifier) is assumed as shown in the Figure 3.2. Input signal is amplified linearly until a certain level and beyond that clipped to the output saturation level \(P_{\text{sat}}\). To achieve the
Chapter 3. Clipping-Filtering in OFDM Systems

Figure 3.1: OFDM Transmitter Block Diagram: (a) Original, and (b) with ICF

Highest efficiency of the PA, the maximum output power should reach $P_{sat}$ [74]. Back-offs are often necessary to avoid excessive spectral leakage and in-band distortion. Back-offs are specified in terms of output back-off (OBO) or input back-off (IBO). OBO is the ratio of $P_{sat}$ and average output power $P_{out}$, whereas IBO refers to the ratio of input power corresponding to output saturation level $P_{max}$ and average input power $P_{in}$. In a linear amplifier, $IBO = OBO$. With the maximum input power satisfying

$$\max_{0 \leq n \leq N-1} |x_n|^2 = P_{max}$$

(3.3)

one can write $IBO = OBO = PAPR$.

The efficiency of a PA is defined as,

$$\eta = \frac{P_{out}}{P_{dc}}$$

(3.4)

$P_{dc}$ being a constant amount of power consumed by the amplifier regardless of the input power. Class A PAs are stated as one of the most linear amplifiers with maximum efficiency of 50% and its efficiency given by $\eta = 0.5/OBO$ [3]. Therefore, the efficiency in terms of input signal PAPR becomes,

$$\eta = \frac{0.5}{PAPR_x}$$

(3.5)

Following example ($N = 256$ and $p = 10^{-5}$) shows that how inefficient would be the PA in the case of original OFDM signals. From the Table 2.1, the input signal
PAPR is 12.7dB (=18.62) at which the amplifier has efficiency of only $0.5/18.62 = 2.68\%$. Such a low efficiency, for instance, would drain the battery power very quickly. To prolong the battery life or save the system power, the signal PAPR needs to be reduced. Every 3dB PAPR reduction results doubling the amplifier efficiency.

Combining equations (7) and (8), we get

$$P_{dc} = 2P_{out} \cdot \text{PAPR}_x$$  \hspace{1cm} (3.6)

or equivalently

$$P_{out} = \frac{0.5P_{dc}}{\text{PAPR}_x}$$  \hspace{1cm} (3.7)

Consider two cases to observe the impact of PAPR reduction on the power trade-offs analysis.

1) When the average output power $P_{out}$ is fixed, and

2) When supply power $P_{dc}$ is fixed.

For the first case when output power should be restricted by some regulatory limits, the assumption is that the PA amplifier can re-bias itself according to input PAPR change to offer maximum achievable efficiency. Take expectation of both sides in
equations (3.6) and (3.7), average power trade-offs can be written as,

\[ E[P_{dc}] = 2P_{out} \cdot E[P_{APRx}] \]  
\[ (3.8) \]

and,

\[ E[P_{out}] = \frac{0.5P_{dc}}{E[P_{APRx}]} \]  
\[ (3.9) \]

The PAPR reduction translates into either power savings when \( P_{out} \) is fixed or increased transmitted output power when \( P_{dc} \) is fixed.

### 3.2.3 Power Costs Analysis

The computational power needed for the clipping and filtering scheme and the net power savings were evaluated due to PAPR reduction achieved from it. Following similar steps in [6] and consider fixed-point DSP for all power computations. Table 3.1 gives a summary of parameters used here.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current/Processor cycle/Second</td>
<td>0.33 mA/MHz</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>1.26 V</td>
</tr>
<tr>
<td>Processor frequency</td>
<td>200 MHz</td>
</tr>
<tr>
<td>Cycles/256-point FFT</td>
<td>4786</td>
</tr>
<tr>
<td>Cycles/Radix 2 FFT core</td>
<td>5</td>
</tr>
<tr>
<td>Overhead cycles/FFT</td>
<td>306</td>
</tr>
<tr>
<td>Cycles/N-point FFT</td>
<td>( 306 + \frac{5N}{2} \log_2\left(\frac{N}{2}\right) )</td>
</tr>
<tr>
<td>Multiplications/Cycle</td>
<td>2</td>
</tr>
<tr>
<td>Additions/Cycle</td>
<td>4</td>
</tr>
<tr>
<td>Cycles/Complex Multiplications</td>
<td>3</td>
</tr>
</tbody>
</table>

The energy consumption per cycle and per N-point FFT/IFFT respectively are

\[ \text{Energy/cycle} = 0.33 \frac{\text{mA}.\text{sec}}{\text{Mcycle}} \cdot 1.26 \text{V} = 415.8 \frac{\text{pW}.\text{sec}}{\text{cycle}} \]  
\[ (3.10) \]

\[ \text{Energy/N-point} = 415.8 \cdot \left[306 + \frac{N}{2} \log_2\left(\frac{N}{2}\right)\right] \text{nJ} \]
\[ = \left[127.2 + 1.04N\log_2\left(\frac{N}{2}\right)\right] \text{nJ} \]  
\[ (3.11) \]
Chapter 3. Clipping-Filtering in OFDM Systems

Referring to Figure 3.1, it is obvious that the FFT/IFFT and clipping blocks incur the most of computational cost. The Table 3.2 shows the main steps of digital clipping. Note that the clipping threshold is 6dB. From the Figure 3.3, the probability of signal peaks exceeding PAPR of 6dB is almost 1, so the number of complex multiplications (line 3) required is equal to JN. Besides, assumption is each ‘loop’ and ‘if’ statements incur one cycle, therefore totalling 2JN cycles per clipping iteration. Finally, another JN cycles is taken as overhead cost. The table 3.3 summarizes of computational costs of ICF.

![Figure 3.3: Cumulative Complementary Distribution Functions (CCDF) after PAPR Reduction](image)

Using (3.10) and (3.11), and Table 3.3, total computational overhead can be given.

Table 3.2: Steps for Digital Clipping

```
loop n = 1:JN
if x_n >= threshold
    x_n = threshold * exp(jx_n)
else
    x_n = x_n
end if
end loop
```
Table 3.3: Operations necessary for ICF

<table>
<thead>
<tr>
<th>Operation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>K, iterations</td>
<td>3</td>
</tr>
<tr>
<td>Oversampling Factor, J</td>
<td>5</td>
</tr>
<tr>
<td>FFT/IFFT</td>
<td>$2K+1$</td>
</tr>
<tr>
<td>Multiplications</td>
<td>0</td>
</tr>
<tr>
<td>Additions</td>
<td>0</td>
</tr>
<tr>
<td>Complex Multiplications/iteration</td>
<td>$JN$</td>
</tr>
<tr>
<td>Overhead cycles/iteration</td>
<td>$3JN$</td>
</tr>
</tbody>
</table>

Table 3.4: Computation Cost for ICF (3 iterations)

<table>
<thead>
<tr>
<th>J</th>
<th>FFT</th>
<th>Others</th>
<th>Total Cycles</th>
<th>Powercost, µW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31666</td>
<td>4608</td>
<td>36274</td>
<td>15.09</td>
</tr>
<tr>
<td>2</td>
<td>71986</td>
<td>9216</td>
<td>81202</td>
<td>33.78</td>
</tr>
<tr>
<td>3</td>
<td>115690</td>
<td>13824</td>
<td>129510</td>
<td>53.87</td>
</tr>
<tr>
<td>4</td>
<td>161586</td>
<td>18432</td>
<td>180018</td>
<td>74.88</td>
</tr>
<tr>
<td>5</td>
<td>209120</td>
<td>23040</td>
<td>232160</td>
<td>96.57</td>
</tr>
</tbody>
</table>

as,

$$\text{Total cycles} = 7 \cdot \left[ 306 + 5 \frac{JN}{2} \log_2 \frac{JN}{2} \right] + 3 \cdot (3JN + 3JN) \quad (3.12)$$

From (3.10) and (3.12), the computational cost in Watts can be computed for different oversampling factor as shown in Table 3.4. For comparison purposes, it is assumed that both power amplifier and DSP work for the same amount of time so that the computational cost can be written in Watts instead of Joule. Note that the major component of computational cost comes from FFT/IFFT operations (e.g., for $J=5$, FFT/IFFT takes around 90% of total cost). It can be seen that oversampling factor has huge impact on processing cost. As mentioned in earlier section, $J \geq 4$ to capture and clip all the continuous signal peaks. However, the costs are in $\mu$W which is very small compared to the power savings achieved through PAPR reduction as shown below.

The net power savings becomes,

$$P_{netsavings} = \begin{cases} 
2P_{out} \cdot (\text{PAPR}_{bef} - \text{PAPR}_{aft}) - \\
7 \cdot \left[ 306 + 5 \frac{JN}{2} \log_2 \frac{JN}{2} \right] + 3 \cdot (3JN + 3JN) \\
\times 415.8 \text{ pW sec per cycle}
\end{cases} \quad (3.13)$$
With \( J = 5 \), 6dB clipping threshold, \( 10^{-5} \) clipping probability, the PAPR reduction gain is 12.15 (linear scale). The savings for \( P_{\text{out}} \) of nominal 120mW would be 2.92W, which is 68.54% of power required before PAPR reduction (PAPR\(_{\text{bef}}\) = 12.5dB(17.78) gives \( P_{\text{dc, bef}} \) = 4.26W). Net savings would be \( 2.92W - 96\mu W \approx 2.92W \). With less conservative clipping probability of \( 10^{-4} \), the net savings comes to 2.31W, 64.16% of original power consumption.

The dependence of net power savings on the number of subcarriers and modulation size would be another area to explore. This is due to the fact that system degradation from clipping method depends on those parameters. Similarly, the power savings analysis presented here could be extended to improve clipping and filtering methods such as Iterative Cancelation of Clipping Noise [72], SCF [75] and OICF [76].

In summary, the section presented the power savings due to PAPR reduction from clipping and filtering method. The savings takes account of improvement in power amplifier efficiency gained from PAPR reduction. The net power savings are shown to be in the order of Watts with this simplest PAPR reduction technique. For large clipping ratio (\( \geq 6\)dB), clipping and filtering method provides excellent trade-offs. Note that actual power savings would vary depending on the system requirements and processor model.

### 3.3 Power Savings Comparison of ICF and SLM

In this section, earlier analysis is extended to compare two different techniques based on net power savings for similar PAPR reduction.

#### 3.3.1 Power Amplifier Model

Here a realistic power amplifier model is assumed unlike in the previous section. According to [77], the efficiency depends on the PAPR and increases monotonically as the PAPR decreases. The specific relationship depends on the class of the power amplifier and on its particular design. The theoretical efficiency upper limits for class A and B power amplifiers were first described in [78], and later produced in an empirical formula by [77] as,
Chapter 3. Clipping-Filtering in OFDM Systems

\[ \eta = G \exp \left( -g \text{PAPR(dB)} \right) \]  

(3.14)

where the values of $G$ and $g$ are given in Table 3.5. The efficiency curves for class A and B amplifiers are shown in Figure 3.4 computed using (6) and Table 3.5. Note that (6) doesn’t fit to the original curves [78] accurately at the lower region of PAPR range (<4dB).

<table>
<thead>
<tr>
<th>Class</th>
<th>$G$ [%]</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>58.7</td>
<td>0.1247</td>
</tr>
<tr>
<td>B</td>
<td>90.7</td>
<td>0.1202</td>
</tr>
</tbody>
</table>

Table 3.5: Parameters for Amplifier Efficiency [77]

![Figure 3.4](image)

Figure 3.4: Theoretical Efficiency Upper Limits for Class A and B Amplifiers

Now from (5) and (6), the power savings after PAPR reduction can be written as,

\[ P_{\text{savings}} = P_{\text{dc, before}} - P_{\text{dc, after}} \]

(3.15)

\[ = \frac{P_{\text{out}}}{G} \left[ \exp \left( g \text{PAPR}_{\text{before}} \right) - \exp \left( g \text{PAPR}_{\text{after}} \right) \right] \]
3.3.2 Selective Mapping (SLM)

The popular distortion-less technique selective mapping for PAPR reduction was first described in [12]. For $x_n$, $n = 1 : N$ be identically and independently two dimensionally Gaussian distributed, the PAPR of the OFDM frame exceeding a certain threshold $\gamma$ is

$$P(\text{PAPR} > \gamma) = 1 - \left(1 - e^{-\gamma}\right)^N, \gamma > 0$$  \hspace{1cm} (3.16)

Assuming $U$ statistically independent OFDM frames representing the same information, and selecting the frame with the lowest PAPR for transmission, the probability that $\text{PAPR}_{\text{low}}$ exceeds $\gamma$ is given by

$$P(\text{PAPR}_{\text{low}} > \gamma) = \left(P(\text{PAPR} > \gamma)\right)^U = \left(1 - \left(1 - e^{-\gamma}\right)^\alpha N\right)^U$$  \hspace{1cm} (3.17)

where $\alpha = 2.8$ for oversampled signals [1]. One possible way to generate such signals as described in [12] is: Define distinct $U$ phase vectors, $\Phi^u \in \{0, 2\pi\}, u = 1 : U, n = 1 : N$ with which original OFDM block is multiplied to generate $U$ distinct data set, i.e.

$$X^u = X.B^u = X.e^{j\Phi^u}$$  \hspace{1cm} (3.18)

All new $U$ data set is transformed into time-domain, and the signal with the smallest PAPR is chosen for the transmission. The phase vector used has to be transmitted as side information along with the data signal. This allows receiver to recover the original information. Among different phase vectors such as random, complementary Golay, Walsh-Hadamard, Shapiro-Rudin and Orthogonal Spreading Variable Factor (OSVF), random phase vector shows the best performance [79]. The block diagram of SLM scheme is shown in Figure 3.5.

3.3.3 Costs Analysis

The ICF and SLM methods are analyzed in terms of computational cost and delay for similar PAPR reduction (refer Figure 3.3).
As shown in Figure 3.5, the SLM method employs $UJN$ complex multiplications before $UJN$-point IFFT operations. Afterwards, selection of the signal set with lowest PAPR is done. This involves a $JN$-length maximum value search to find the PAPR of each set and a $U$-length minimum index search to find the lowest PAPR set. Summary of computational cost is shown in Table 3.6. Additional overhead due to storage and fetching of phase vectors is assumed to be $JN$ cycles.

The total cycles incurred in two cases can be computed as follows:

\[
\text{Cycles}_{ICF} = \left(2K + 1\right) \left[306 + 5 \left(\frac{JN}{2}\right) \log_2 \left(\frac{JN}{2}\right)\right] + K \left(3JN + 3JN\right) + K \left(\frac{JN}{2} + 6\right) \tag{3.19}
\]

\[
\text{Cycles}_{SLM} = U \left[306 + 5 \left(\frac{JN}{2}\right) \log_2 \left(\frac{JN}{2}\right)\right] + 3UJN + U \left(\frac{JN}{2} + 6\right) + \left(17 + \frac{U}{2}\right) + JN \tag{3.20}
\]

Similarly, the computational delay can be expressed as,
\[ \text{Delay}_{\text{ICF}} = \text{Cycles}_{\text{ICF}} \]  

\[ \text{Delay}_{\text{SLM}} = \left[ 306 + 5 \left( \frac{\text{JN}}{2} \right) \log_2 \left( \frac{\text{JN}}{2} \right) \right] + 3 \text{JN} \]  

\[ + \left( \frac{\text{JN}}{2} + 6 \right) + \left( 17 + \frac{\text{U}}{2} \right) + \left( \frac{\text{JN}}{\text{U}} \right) \]

Table 3.6: Computational Cost of ICF and SLM

<table>
<thead>
<tr>
<th>Operations</th>
<th>ICF</th>
<th>SLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iterations</td>
<td>K</td>
<td>-</td>
</tr>
<tr>
<td>Oversampling Factor</td>
<td>J</td>
<td>J</td>
</tr>
<tr>
<td>Size of FFT/IFFTs</td>
<td>JN</td>
<td>JN</td>
</tr>
<tr>
<td>Total No. of FFT/IFFTs</td>
<td>2K+1</td>
<td>U</td>
</tr>
<tr>
<td>Multiplications</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Additions</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Complex Multiplications</td>
<td>KxJN</td>
<td>UxJN</td>
</tr>
<tr>
<td>JN-length max-value search</td>
<td>K</td>
<td>U</td>
</tr>
<tr>
<td>U-length min-index search</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Overhead cycles</td>
<td>Kx3JN</td>
<td>JN</td>
</tr>
</tbody>
</table>

Finally, for the comparison purposes it is assumed as in [6] that the DSP and the power amplifier work same amount of time. This way they can be compared in terms of power instead of energy. Then net power savings becomes,

\[
P_{\text{net}} = P_{\text{savings}} - \text{Cycles}_{\text{XXX}} \cdot 415.8 \times 10^{-12} \]

\[
= \frac{P_{\text{out}}}{G} \left[ \exp \left( gPAPR_{\text{before}} \right) - \exp \left( gPAPR_{\text{after}} \right) \right] - \text{Cycles}_{\text{XXX}} \cdot 415.8 \times 10^{-12} \quad \text{W}
\]

where XXX = ICF or SLM. Computer simulation for each method was conducted to produce similar CCDF curves shown in Figure 3.3. BPSK symbols modulate 64-subcarriers. The Nyquist OFDM signal is oversampled by a factor of \( J = 4 \). The number of random phase vectors required for SLM is \( U = 16 \). The Table 3.7 illustrates the computational cost incurred in each case. The Tables 3.8 and 3.9 show the net power savings for both cases. \( PAPR_{\text{bef}} = 10.75\text{dB} \) and \( PAPR_{\text{aft}} = 7\text{dB} \) at the probability of \( 10^{-3} \). The nominal output power (in ISM band) of \( P_{\text{out}} = 250\text{mW} \).
Chapter 3. Clipping-Filtering in OFDM Systems

is assumed.

First of all, the computation cost of SLM is more than twice that of ICF method. However, the SLM computation delay is one fifth of ICF. This is obvious because of parallel use of IFFT blocks in SLM, which on the other hand makes it more computational intensive technique. The IFFT/FFT operations incur much of the total computation overhead in both cases. Surprisingly 87% and 84% of the total cost come from IFFT/FFT blocks in ICF and SLM respectively. Therefore, computation reduction is considered in improved versions like in [75] of ICF method. Looking at the power consumptions, the SLM obviously needs twice the power consumed by the ICF, but they are still in µW range. Note that the savings due to the PAPR reduction is in mW. Therefore the power consumed by these techniques are negligible compared to the power savings achieved.

Following scenario gives an insight of how much percentage power savings could be achieved in case of Class A amplifier. The DC power required before PAPR reduction is \( P_{dc,bef} = 1.63 W \). After the PAPR reduction, the savings is 0.608W, 38% of \( P_{dc,bef} \) which is indeed a significant amount especially for low-power mobile terminals. Another interesting observation is about the power amplifier efficiency improvement. Higher efficiency improvement (Class B, 14.19%) does not necessarily translate into higher savings (364mW < 608mW Class A). The amplifier design/model plays important role on actual power savings (in our case, parameter G).

<table>
<thead>
<tr>
<th>Table 3.7: Comparison of ICF and SLM (K=3,J=4,N=64 and U=16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>ICF</td>
</tr>
<tr>
<td>SLM</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3.8: Power Savings Comparison of ICF and SLM, Class A Amplifier, ( \eta ) improvement 9.16%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>ICF</td>
</tr>
<tr>
<td>SLM</td>
</tr>
</tbody>
</table>

In summary, two popular peak reduction techniques Clipping and Selected Mapping in their conventional versions are analyzed and compared based on the power.
Table 3.9: Power Savings Comparison of ICF and SLM, Class B Amplifier, $\eta$ improvement 14.19%

<table>
<thead>
<tr>
<th>Name</th>
<th>Cost Savings</th>
<th>NetSavings</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICF</td>
<td>16\mu W</td>
<td>364mW</td>
</tr>
<tr>
<td>SLM</td>
<td>38\mu W</td>
<td>364mW</td>
</tr>
</tbody>
</table>

savings. The power savings are shown to be much higher than the power consumed by both methods. The choice of better method depends solely on the implementation complexity of any particular technique. The clipping technique is simpler but induce distortion, while selected mapping is distortion-less but has implementation complexity (due to the high number of IFFT blocks required). In addition, higher efficiency improvement doesn’t always provide higher power savings. Actual savings depends on the design/model of the amplifier.

### 3.4 One-Iteration-Clipping-Filtering (OICF)

Clipping amplitude [10] is stated as the simplest technique for PAPR reduction. As the occurrence of very high peaks is rare [4], the clipping method can produce peak reduction at small cost of performance degradation. Conventional error correction codes can offset such small degradation [10]. Clipping is a non-linear operation, therefore distorts the OFDM signal. The out-of-band radiation is reduced by filtering. However, filtering makes peaks to regrow. Iterative clipping and filtering (ICF) [11] of 2K+1 IFFT/FFT operations, where K is the number of iterations, is necessary to obtain the desired clipped signal. [80] proposed an efficient and fast algorithm for ICF. In [75] target clipped signal was produced through one iteration (of 3 IFFT/FFT operations) with some additional processing (two vector subtractions). They assumed that the clipped peaks as series of parabolic pulses, which is true for large clipping threshold. The processing overhead might still be considerable due to the oversampling (by a factor $\geq 4$) of original OFDM data block.

Here, a different approach was taken to produce the desired clipped signal through one iteration of clipping and filtering (OICF) with almost no additional processing. The OICF scheme employs a scaling of original clipping threshold. An empirical expression has been derived based on re-growth of clipped-filtered pulses, which
relates the original clipping threshold to new scaled one. The simulation results show that the performance of OICF is comparable to conventional method for large clipping threshold.

\[
y_{in}(t) = |x(t)|e^{j\psi(t)},
\]
\[
y_{out}(t) = G(|x(t)|)e^{j(\psi(t)+\Psi(|x(t)|))}
\]

Mathematical modeling of non-linear power amplifiers is often very complex [82]. A common approach is to assume the non-linearity as memoryless, thus frequency-selective. In that case, the continuous-time input and output signal can be given as,

\[
y_{in}(t) = |x(t)|e^{j\psi(t)},
\]
\[
y_{out}(t) = G(|x(t)|)e^{j(\psi(t)+\Psi(|x(t)|))}
\]

G and \(\Psi\) are known as the AM/AM and AM/PM conversions respectively. In this paper, a solid-state power amplifier (SSPA) model is used and given as,

\[
y_{out}(t) = G(|x(t)|)e^{j\psi(t)} = \frac{g_0|x(t)|}{\left(1 + \left(\frac{|x(t)|}{A_{sat}}\right)^{2p}\right)^{1/2p}}e^{j\psi(t)},
\]
and \(\Psi(|x(t)|) = 0\)
\( g_0 \) is the amplifier gain, \( A_{\text{sat}} \) is the input saturation level, and the AM/PM conversion is assumed to be negligibly small (set to zero here). Parameter \( p \) controls the AM/AM sharpness of the saturation region. The SSPA becomes Soft Limiter (SL) when \( p \to \infty \), whereas \( p = 2 \) or 3 seems appropriate for practical amplifiers (Figure 3.7, normalized axes).

\[
\text{IBO} = \frac{A_{\text{sat}}^2}{P_{\text{in}}}
\]

(3.27)

**3.4.1 Signal Clipping and Peak-Regrowth**

As stated before, clipping causes in-band distortion and out-of-band radiation. The out-of-band radiation leaks to adjacent channels. Filtering is necessary to reshape the spectrum and attenuate out-of-band radiation to a sufficiently small value. The
filtering scheme mentioned in [11] has been used here. The clipped N-subcarrier OFDM signal is converted into frequency domain; only first N/2 and last N/2 components are selected and all others are set to zero; taking inverse Fourier transform gives the filtered time-domain signal. However, after filtering, peaks are regrown which will increase the PAPR of the signal above the clipping threshold. The regrown pulses are clipped again and then filtered iteratively until desired PAPR is obtained [11].

3.4.2 Proposed Method

Following [81], for large clipping threshold the clipped pulse can be assumed as parabolic. In such case the clipped \( f(t) \) and clipped-filtered \( \hat{f}(t) \) (\( f_c \), cut-off frequency of ideal low pass filter; \( \tau \), the pulse duration, \( 2\pi f_c \tau / 2 \ll 1 \) for \( f \leq f_c \); \( b \), is a constant) pulse can be shown respectively as [81],

\[
f(t) = -\frac{1}{2}bt^2 + \frac{1}{8}b\tau^2, \quad -\frac{\tau}{2} \leq t < \frac{\tau}{2} \quad (3.28)
\]

\[
\hat{f}(t) = \frac{b\tau^3 f_c}{6} \text{sinc} 2\pi f_c t \quad (3.29)
\]

The simulation results for clipped and its regrown pulse is shown in Figure 3.8. Few observations can be made from the equations (3.28)(3.29) and Figure 3.8:

(a) pulses reach their peaks at same sampling time,
(b) pulses have same direction within pulse duration,
(c) mainlobe of \( \hat{f}(t) \) is wider than that of \( f(t) \),
(d) peak of \( \hat{f}(t) \) is less than that of \( f(t) \), and
(e) when \( N \) and \( A \) are large, filtered pulse sidelobes decay quickly and have negligible effects on adjacent peaks. (Readers are encouraged to refer [75] [81] for mathematical derivations)

Based on the above observations, Figure 3.9 was obtained showing the relationship between target and clipped-filtered PAPR. The proposed scheme One-Iteration-Clipping-Filtering (OICF) can be inferred from the graph. The graph shown is for 256 subcarriers, normalized QPSK symbols, oversampled by a factor of 5. Similar curves can be obtained for different number of subcarriers and mapping levels. For clipping ratios greater than 6dB, the scheme performs comparable to conventional
approach as will be shown shortly. However, for smaller clipping thresholds, performance degradation is large, and scheme is not suitable. This fact can be put as follows: For smaller clipping threshold, clipped pulses are close to each other. After filtering, the side lobes of pulses (refer to Figure 3.8) start to interfere adjacent pulses, thus increase the error floor.

The solid line is for target PAPR, the dashed line is for clipped signal (average power of clipped signal decreases, therefore PAPR will be higher than expected; however the solid and dashed lines merge together for higher values of clipping ratios), and finally dot-dashed line is for clipped and filtered signal (as filtering further reduces signal average power, and causes peak-regrowth, PAPR is higher than those of previous signals).

Now from the graph, one can deduce a relation between the target and clipped-filtered PAPR curves. The solid and the dot-dashed lines are extended to establish a point (X2,Y2). With point (X1,Y1), the slopes m1 and m2 for solid and dot-dashed lines are defined respectively as m1 = Y2/X2 = 1 and m2 = (Y2-Y1)/(X2-X1) = (Y2-Y1)/Y2 (because X2=Y2 and X1=0). For equal PAPR, one can derive following
Figure 3.9: PAPR vs Clipping Ratio for clipped and clipped-filtered signals

expression between two curves,

\[ x_2 = \frac{x_1 - Y_1}{m_2} = \frac{x_1 - Y_1}{\frac{Y_2 - Y_1}{Y_2}} \]  

(3.30)

where \( x_1 \) is the original clipping threshold and \( x_2 \) is the scaled one.

In our case, \((X_1,Y_1) = (0,4.4)\) and \((X_2,Y_2) = (8.5,8.5)\). The scaled clipping threshold can be computed from the above equation for a target PAPR. For instance in order to get 6dB PAPR signal, conventional approach is to clip original signal at 6dB. But with this scheme, the signal is clipped at \( \sim 3.5 \)dB (From equation (3.30)). This in turn gives us the desired clipped signal in one iteration. In summary, the steps for this scheme are:

1) For particular mapping and number of subcarriers, get the PAPR vs CR graph as shown in Figure 3.9 through computer simulation,
2) Establish points \((X_1,Y_1)\) and \((X_2,Y_2)\), and derive equation (3.30), and
3) Compute the scaled clipping threshold and proceed for clipping and filtering.
3.4.3 Results and Discussion

The performance comparison between traditional ICF and OICF is made based on CCDF, error performance and power spectral density. No cyclic prefix is used with
Chapter 3. Clipping-Filtering in OFDM Systems

the assumption that it doesn’t affect the results. For ICF, three iterations (K=3) of clipping and filtering were considered. 4000 OFDM symbols were considered for each simulation run. Both amplifier back-off and clipping ratio are set to 6dB. Amplifier knee sharpness parameter p is 3. Additive Gaussian noise channel is assumed for all cases.

1) CCDF Curve: From Figure 3.10, it can be easily seen that the proposed method can achieve same CCDF curve as one from the ICF technique for 6dB. Note that ICF required 3 iterations. CCDF curve for unclipped OFDM signal is also shown as a reference and to underline the effectiveness of clipping method.

2) BER Curve: BER curves of ICF and OICF methods are very close to each other (less than 1dB degradation at $10^{-5}$ probability) as shown in Figure 3.11. In addition, BER curve for the case of without clipping is also shown. Clipping degrades performance by a small value for large clipping thresholds (> 6dB). The leftmost curve is obtained when amplifier is linear for the whole range of amplitude values. Curves for smaller clipping threshold is not shown for the reason stated earlier (i.e. this scheme is not suitable because it raises error floor). Note that the performance degradation of deeper clipping is large, and might outweigh the actual benefits obtained [10] [82].

3) Power Spectral Density (PSD): Figure 3.12 shows the one-sided power spectral density obtained for two methods. [11] mentioned that the PSD of OFDM symbols depends strongly on the precise nature of transitions at symbol boundaries. But in this work, no pulse shaping was considered for brevity. PSD was obtained using Matlab function 'pwelch' with 64-sample windows, 63-sample overlap, the default FFT length, and normalized frequency. From the Figure (3.12), one can observe that the out-of-band radiation for ICF is 3dB lower than without clipping, and that for OICF is approximately 3dB lower than that of ICF.

3.5 Impact of CF on Channel Estimation

Accurate channel estimation at the receiver is crucial for reliable information transfer. In addition to channel impairments, the transmitter added non-linearities contribute to channel estimation errors. The non-linearities distorts the pilots, used for channel estimation, even before they are transmitted to the channel. One such non-linearity
Chapter 3. Clipping-Filtering in OFDM Systems

deliberately introduced in the transmitter is due to clipping. Although the purpose of clipping OFDM signals is to reduce the PAPR, it generates in-band distortion. In-band distortion degrades the system performance. The degradation is exaggerated due to the distorted pilot-symbols.

The effect of clipping on pilot-assisted channel estimation was investigated in [83]. However, the authors avoided the basic components of the technique, i.e. oversampling and filtering. Oversampling of the digital signal is necessary to take account of all the peaks that would appear in its continuous form. Filtering, on the other hand, removes the out-of-band radiation which otherwise interferes the adjacent channels. Ideal low pass filtering reshapes the spectrum of clipped-filtered signal back to that of the original input signal. The system under consideration is shown in Figure 3.13.

3.5.1 Clipping and Filtering

Clipped and filtered signal can be expressed as [84],

$$x_{cf}^e = \alpha x_n + d_n$$  \hspace{1cm} (3.31)
Figure 3.13: OFDM System with Clipper and Filter, a) Transmitter, and b) Receiver.

which is derived based on Bussgang theory for complex Gaussian input signals fed to memory-less non-linearity. \( d_n \) is the distortion term uncorrelated with \( x_n \), assumed i.i.d random variables with zero means. Then the constant \( \alpha \) can be shown as,

\[
\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi} \gamma}{2} \text{erfc}(\gamma) \tag{3.32}
\]

And the signal power after clipping is [85],

\[
\epsilon_{cl} = (1 - e^{-\gamma^2}) \epsilon_x \tag{3.33}
\]

Filtering is necessary after the clipping to remove the out-of-band radiation. The filtering further reduces the signal power, and in [85] an empirical approximation is derived for \( \epsilon_{cl} \) and out-of-band power \( \epsilon_{cl, out} \),

\[
\frac{\epsilon_{cl, out}}{\epsilon_{cl}} \simeq 0.085 e^{-\sqrt{2} \gamma^2} \tag{3.34}
\]

After some manipulations, following expressions are obtained for the signal power
Figure 3.14: CCDF curves for Clipped and Clipped-Filtered OFDM Signals, $N=256$, QPSK and $\gamma=3$ dB. Note that $\xi = \gamma^2$

and the distortion after clipping and filtering,

$$\epsilon_{cl-fl} = (1 - e^{-\gamma^2})(1 - 0.085e^{-\sqrt{2}\gamma^2})\epsilon_x$$  \hspace{1cm} (3.35)

$$\sigma\epsilon_{d}^2 = \left((1 - e^{-\gamma^2})(1 - 0.085e^{-\sqrt{2}\gamma^2}) - \alpha^2\right)\epsilon_x$$  \hspace{1cm} (3.36)

$\epsilon_x$ is the unclipped signal power, one can note that the clipping distortion becomes less in clipping-filtering than in clipping.

### 3.5.2 Channel Estimation and Signal Detection

The analysis presented in [83] was followed for channel estimation and signal detection. The channel estimation takes account of both transmitter-added distortion and channel impairments.

First, we begin with the frequency domain representation of clipped and filtered signal,

$$X_{cf}^k = \alpha X_k + D_k, \quad k = 0, 1, 2, \ldots, N - 1$$  \hspace{1cm} (3.37)

Assuming perfect timing and synchronization, the received signal after discarding
cyclic prefix and FFT transformation is,

\[ Y_{cf}^k = H_k(\alpha X_k + D_k) + V_k \]  \hspace{1cm} (3.38)

\( H_k \) is N-point FFT of the time domain channel response \( h = \{ h_0, h_2, ..., h_{L-1} \} \), where \( L \) is the number of multipaths. Channel response is independent of the input signal and the clipping-filtering process, so is the additive white Gaussian noise \( V_k \). \( V_k \) and \( H_k \) are independent to each other as well. As the number of subcarriers increases, \( D_k \) approaches complex Gaussian distribution with zero means.

With the Least Square (LS) channel estimation, the signal to noise ratio of the detected signal on \( s^{th} \) subcarrier is \[ \lambda(g) \approx \frac{\alpha^2 g^2 \epsilon_s}{\sqrt{K_p} \left( \sigma_h^2 + \sigma_d^2 + \sigma_n^2 \right)} \]  \hspace{1cm} (3.39)

where \( \sigma_h^2 = \text{E}\{ \| h \|^2 \} \), \( \sigma_n^2 = \text{E}\{ |v_n|^2 \} = \text{E}\{ |V(k)|^2 \} \), \( \sigma_d^2 \) is given by (3.36), \( g \) is the channel gain in the \( s^{th} \) subcarrier and data signal power \( \epsilon_s \) may be equal or unequal to the pilot signal power \( \epsilon_p \).

Now the BER for clipped-filtered OFDM signals modulated with QPSK symbols under Rayleigh fading channel is,

\[ P_{QPSK} = \frac{2}{\sigma_h^2} \int_0^\infty g e^{-g^2/\sigma_h^2} Q(\sqrt{\lambda(g)}) \, dg \]  \hspace{1cm} (3.40)

Note that when oversampling is employed, the clipping distortion deviates per subcarrier basis and is not exactly white. Therefore the BER should be evaluated per subcarrier basis before computing total average BER of the system. However, for lower-order modulation such as BPSK, QPSK case, the results may not have significant impact [86].

### 3.5.3 Simulation Results and Discussion

In this section, some simulation results are shown to analyze the effect of oversampling-clipping-filtering on the performance of pilot-assisted OFDM systems. Consider a typical OFDM system, where \( N=256 \), \( L=6 \) and normalized QPSK modulated sub-
carriers transmitted over frequency selective Rayleigh fading channel and AWGN environment. Each OFDM block consists of $K=16$ QPSK pilot symbols, and are placed in equal spaced positions of the OFDM tones. The pilot symbols carry twice the power of that by data symbols. The $L$ path Rayleigh fading channel was produced in each symbol block independently and keep invariant during the symbol period. The channel response has exponential delay profile. The oversampling factor is 4. The amplifier is assumed to have sufficient back-offs so that the signals are linearly amplified before transmitted to the channel. All the figures are shown for three clipping ratios $\gamma = 100, 1, 0.5$.

Figure 3.15 shows the BER for the clipped and the clipped-filtered OFDM signals in additive Gaussian noise channel. Clipping causes quite significant degradation to the system performance. The clipping-filtering however seems to improve the performance, because the filtering has removed some of the the clipping distortion. The reduced distortion in the signal improves both channel estimation and signal detection. This level of improvement might not be achieved for fading channels as seen in next few figures.

![Figure 3.15: BER for Clipped and Clipped-Filtered OFDM Signals in AWGN Channel.](image)

Figure 3.16 compares the performance of the clipping and the clipping-filtering
in Rayleigh fading, degradation is again less in the clipping-filtering case than the
clipping only one. SNR improvement of up to 3dB is possible at BER of \(8 \times 10^{-2}\).
The effect of the fading itself degrades the system much higher than the clipping
or clipping-filtering (compared to AWGN channel), and the observation is similar
to [87]. More severe clipping produces more distortion, and more of the distortion
can be removed through the filtering. Therefore the SNR gain achieved in the case
of severe clipping is more than that of less-severe clipping (referring to \(\gamma = 0.5\) and
\(\gamma = 1\) curves) case.

![Figure 3.16: BER for Clipped and Clipped-Filtered OFDM Signals in Rayleigh Fading Channel](image)

To further improve the channel estimation, a simply strategy of re-inserting the
original pilots in clipped-filtered signal can be employed. It can be done after the
filter (ref. to Figure 3.13). This may slightly change the PAPR characteristics of the
transmitted signals. When the number of pilot tones is small compared to the number
of data tones, the change in signal PAPR will be insignificant. Figure 3.17 shows
that at clipping level of \(\gamma = 1\) and without pilot re-insertion, the BER floor starts
at \(10^{-3}\). However the re-inserted pilots improve the system performance keeping the
curve low and parallel to unclipped (\(\gamma = 100\)) case. Approximately 5dB of SNR gain
is achievable at BER of $10^{-3}$ and the clipping level $\gamma = 1$.

![Figure 3.17: BER for Clipped-Filtered OFDM Signals with Pilot Re-insertion in Rayleigh Fading Channel](image)

In summary, the effect of clipping-filtering on channel estimation is less severe than of clipping only. The filtering removes some of the clipping distortion from the signal band. The less distorted pilot symbols assist to achieve better channel estimation. At the clipping level $\gamma = 1$, SNR improvement of up to 3dB is possible at BER of $8 \times 10^{-2}$ over Rayleigh fading channel. The channel estimation can be improved further with pilot re-insertion strategy in clipping-filtering (it is not possible in clipping, unless overhead of converting clipped signal into frequency domain and back to time-domain is acceptable). The degradation due to the channel fading is far more severe than the clipping-filtering process.

### 3.6 Chapter Summary

In this chapter, few new contributions were made in terms of analyzing the clipping-filtering scheme as well as proposing a new modified one iteration clipping-filtering scheme which is more efficient requiring less computational overhead. Specifically,

1. Power savings analysis was conducted for ICF and SLM taking account of prac-
tical power amplifiers’ parameters and computational overhead. The results showed that net power savings are in order of milli-watts whereas the overheads were only just micro-watts. Moreover, the net power savings justifies the need for PAPR reduction due to lower back-off in the amplifier. The choice between the ICF and the SLM depends on other factors such as implementation complexity, space constraints, introduced distortion, bandwidth requirements, etc.

2) New scheme OICF was introduced, which is more efficient than conventional ICF and with similar performance evaluated in terms of BER and PSD.

3) The impact of clipping-filtering on pilot-assisted OFDM signal was analyzed. The results showed the clipping-filtering has less impact than the clipping-only scheme. Pilot-reinsertion is possible in clipping-filtering thereby reducing the degradation.
Chapter 4

Performance Analysis of Multiantenna Systems

4.1 Introduction

This chapter describes the performance analysis carried out for various multi-antenna systems, mostly for MRC systems with channel estimation error. The primary focus is the derivation of closed-form expressions which then can be used to evaluate the system performance. In addition to channel estimation error, the impact of antenna correlation on the system performance was also analyzed. The chapter summary at the end summarizes the main contributions of this chapter.

4.2 SER of MIMO-MRC with Channel Estimation Error

The system shown again in Figure 4.1 shows the ideal MIMO-MRC (also known as MRT) system where perfect channel estimation and zero-delay feedback are assumed.

The notations used are: All the matrices and the vectors are denoted by boldfaced capital letters and boldfaced small letters. The determinant, conjugate transpose, transpose, conjugate of \( X \) are denoted by \( \det(X) \), \( X^H \), \( X^T \) and \( X^* \) respectively. The small letters with subscripts \( x_{ij} \) denote the \( (i,j)^{th} \) element of the matrix \( X \). The average and the absolute value of \( x \) are denoted by \( E\{x\} \) and \( |x| \) respectively. And \( I_n \) denotes the \( n \times n \) identity matrix.

Consider a \((N_r,N_t)\) MRT system. A slow frequency flat Rayleigh fading channel is assumed and characterized by an \( N_r \times N_t \) matrix \( H = [h_{ij}] \) of i.i.d Complex Gaussian Random Variables (CGRVs) with zero mean and unit variance, i.e.,
$E\{|h_{ij}|^2\} = 1$, where $h_{ij}$ denotes the channel gain from the $j^{th}$ transmit antenna to the $i^{th}$ receive antenna. The $N_r \times 1$ received signal vector $\mathbf{r}$ can be written as,

$$\mathbf{r} = \mathbf{Hv}_t s + \mathbf{n}$$ \hspace{1cm} (4.1)

where $s$ is the transmitted data symbol with $E\{|s|^2\} = E_s$, $\mathbf{v}_t$ denotes the $N_t \times 1$ normalized transmit weighting vector and $\mathbf{n}$ is an $N_r \times 1$ additive white Gaussian noise vector with zero-mean and covariance $E\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2 \mathbf{I}_{N_r}$. The transmit weighting vector $\mathbf{v}_t$ is calculated at the receiver and sent back to the transmitter. The coherently combined received signal is

$$\hat{s} = \mathbf{u}^H \mathbf{H}^H (\mathbf{H} s + \mathbf{n})$$ \hspace{1cm} (4.2)

When the perfect CSI available at the receiver is fed back to the transmitter error-free with zero-delay, the SNR is maximized when the weighting vector is selected to
be the eigenvector $u$ of the largest eigenvalue $\lambda_{\text{max}}$ of the Wishart matrix $H^H H$, i.e.

$$
\gamma_{\text{mrt}} = \lambda_{\text{max}} \frac{E_s}{\sigma_n^2}
$$

(4.3)

Assuming the pilot symbol assisted channel estimate $\hat{H}$ differs from the actual channel $H$ by an independent complex Gaussian error $\Delta H$ which is an $N_r \times N_t$ matrix of i.i.d. CGRVs with zero-mean and variance $\sigma_e^2$, i.e., $\hat{H} = H + \Delta H$. Thus, the channel estimate $\hat{H}$ is an $N_r \times N_t$ matrix of i.i.d CGRVs with zero-mean and variance $\sigma_e^2$. The channel entries $\hat{h}_{ij}$ and $h_{ij}$ are jointly complex Gaussian distributed with normalized correlation coefficient $\frac{1}{\sqrt{1+\sigma_e^2}}$. The actual channel matrix $H$ can be written in terms of $\hat{H}$ as [89],

$$
H = \rho \hat{H} + \tilde{H},
$$

where $\rho = \frac{1}{\sigma_e^2}$, $\hat{h}_{ij}$’s are i.i.d. CGRVs with zero-mean and variance $\sigma_e^2$ and $E\{\hat{h}_{ij}\hat{h}_{lk}^*\} = 0$, $E\{\hat{h}_{ij}\hat{n}_{lk}^*\} = 0$ for any $i, j, k, l$. Then the effective output SNR becomes [20],

$$
\gamma_{\text{mrt}} = \frac{\lambda_{\text{max}}}{(1+\sigma_e^2)[\sigma_e^2 + (1+\sigma_e^2)\sigma_n^2/E_s]}
$$

(4.4)

$$
= \frac{G \lambda_{\text{max}}}{(1+\sigma_e^2)}
$$

where $G = \sigma_e^2 + (1+\sigma_e^2)/\gamma_o$, $\gamma_o = \frac{E_s}{\sigma_n^2}$ is the average SNR per receive antenna.

The statistical properties of largest eigenvalue of a complex central Wishart matrix investigated by Khatri in [90] where cumulative distribution function (cdf) of $\lambda_{\text{max}}$ is given as,

$$
F_{\text{mrt}}(\gamma) = Pr(\gamma_{\text{mrt}} \leq \gamma) = K_{mn} \det[S(g\gamma)]
$$

(4.5)

where,

$$
m = \min\{N_r, N_t\}, n = \max\{N_r, N_t\}, \text{ and } K_{mn} = \prod_{k=1}^{n} (n-k)! (m-k)! \text{ and the } m \times n \text{ Hankel matrix } S(g\gamma) = [s_{ij}] = (n-m+i+j-2)! - \Gamma(n-m+i+j-1, g\gamma)
$$

with $\Gamma(l, \mu) = \int_{\mu}^{\infty} e^{-x} x^{l-1} dx$.

The moment generation function of the instantaneous MRT output SNR can be
written as [20],

\[ \phi_{\text{mrt}}(v) = E\{e^{-(v\gamma)}\} \]
\[ = K_{mn} \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)k} l! d_{kl} \left( \frac{G}{v + kG} \right)^{l+1} \]  (4.6)

The coefficients \( d_{kl} \) can be obtained by curve fitting on the plot of \( \frac{df}{d\lambda} \text{det}[S(\lambda)] \) versus \( \lambda \). A simple numerical algorithm to compute \( d_{kl} \) is given in [21]. Symbolic software packages such as MATHEMATICA and MAPLE provide a direct implementation of the algorithm. (4.6) can be also written in another representation as below,

\[ \phi_{\text{mrt}}(v) = K_{mn} \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)k} l! d_{kl} \left( \frac{1}{k + Fv} \right)^{l+1} \]  (4.7)

where \( F = 1/G \). Note that (4.7) is exactly in same form as (9) in [23], which makes our analysis easier. For perfect channel estimation, \( F = \gamma_o = \frac{E_s}{\sigma_n^2} \). Now, the closed form expressions of average symbol error rate for coherent M-PSK and M-QAM signalling in MRT system with channel estimation error can be easily computed as shown below.

4.2.1 Average Symbol Error Rates Derivation

The average symbol error rate for coherent M-PSK can be given as [91],

\[ P_{s}^{\text{MPSK}} = \frac{1}{\pi} \int_{0}^{\pi/2} \phi_{\text{mrt}} \left( \frac{g_{\text{MPSK}}}{\sin^2 \theta} \right) d\theta \]
\[ = \frac{1}{\pi} \left[ \int_{0}^{\pi/2} I_1 \phi_{\text{mrt}} \left( \frac{g_{\text{MPSK}}}{\sin^2 \theta} \right) d\theta \right] \]
\[ + \frac{1}{\pi} \left[ \int_{\pi/2}^{\pi} I_2 \phi_{\text{mrt}} \left( \frac{g_{\text{MPSK}}}{\sin^2 \theta} \right) d\theta \right] \]  (4.8)

where \( g_{\text{MPSK}} = \sin^2(\pi/M) \). Following the similar steps as in [23], making a change of variable \( t = \cos^2 \theta \) for \( I_1 \), and \( t = \frac{\cos^2 \theta}{\cos^2(\pi/M)} \) for \( I_2 \) [73, eqn. 3.197-3] and [73, eqn.
where $B(z, w) = \frac{\Gamma(z)\Gamma(w)}{\Gamma(z+w)}$, $\Gamma(.)$ is Euler’s gamma function. $2F_1(a, b; c; z)$ and $F_1(a, b, b'; c; x, y)$ are the Gauss and the Appell hypergeometric functions respectively.

Similarly, the average symbol error rate for coherent square $M$-QAM is given as [91],

\[
P_s^{MQAM} = \frac{4q}{\pi} \int_0^{\pi/2} \phi_{nrt} \left( \frac{g_{MQAM}}{\sin^2 \theta} \right) d\theta
-
\frac{4q^2}{\pi} \int_0^{\pi/4} \phi_{nrt} \left( \frac{g_{MQAM}}{\sin^2 \theta} \right) d\theta
\]  

(4.10)

where $q = 1 - \frac{1}{\sqrt{M}}$ and $g_{MQAM} = \frac{3}{2(M-1)}$. Making a change of variable $t = 1 - \tan^2 \theta$ in $I_4$ and with [73, eqn. 3.221], following is obtained,
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\[ I_4 = \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)} c_{kl} \int_0^1 (1-t)^{l+1/2} (1-0.5t)^{-1} (1-\beta' t)^{-(l+1)} d\theta \]

\[ = \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)} c_{kl} B\left(l + \frac{3}{2}, 1\right) F_1\left(1, 1, l + 1, l + \frac{5}{2}, \frac{1}{2}, \beta'\right) \]  

(4.11)

Noting that \( I_3 \) is similar to \( I_1 \), the final expression of average symbol error rate of \( M \)-QAM becomes,

\[ P_{s}^{M\text{QAM}} = \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)} \left( b_{kl} B\left(l + \frac{1}{2}, l + \frac{3}{2}\right) - c_{kl} B\left(l + \frac{3}{2}, 1\right) F_1\left(1, 1, l + 1, l + \frac{5}{2}, \frac{1}{2}, \beta'\right) \right) \]  

(4.12)

where

\[ \beta = \frac{k}{g_{M\text{QAM}} F + k} \]

\[ \beta' = \frac{g_{M\text{QAM}} F + k}{2g_{M\text{QAM}} F + k} \]

\[ b_{kl} = \frac{2qK_{mn}d_{kl}!}{\pi (g_{M\text{QAM}} F + k)^{l+1}} \]

\[ c_{kl} = \frac{q^2K_{mn}d_{kl}!}{\pi (2g_{M\text{QAM}} F + k)^{l+1}} \]

The above expression (4.12) is in a simpler form than that derived in [23] where two additional Gauss hypergeometric functions are required. Note that the above expressions (4.9) and (4.12) specializes to (13) and (21) in [23] respectively for the perfect channel estimation at the receiver. The closed form expressions of average symbol error rate for other signalling schemes can be obtained in similar fashion.
4.2.2 Numerical Results and Discussion

In this section, numerical results are provided to show the impact of Gaussian estimation error on the performance of MRT system over i.i.d Rayleigh fading channels. All the results were calculated on Mathematica (which has built-in hypergeometric functions) using equations (4.9) and (4.12). Figure 4.2 shows the degradation of average symbol error rate of QPSK signalling for 2x2 MRT system for estimation errors with variance of $\sigma_e^2 = 0, 0.1, 0.5$. Clearly, the impact of estimation error is huge, a degradation of 6dB in SNR occurred at $10^{-3}$ of error rate. Higher modulation suffers more degradation as shown in Figure 4.3. Estimation error of just $\sigma_e^2 = 0.5$ results an error floor for 16QAM modulation of 2x2 MRT system.

Next the impact of estimation error is shown on the arrangement of diversity for a fixed cost. A total of six antennas is assumed at SNR=6dB. Figure 4.4 shows a system where the number of receive antennas is greater than that of transmit antennas. Evenly distributed antenna system (3x3 in this case) performs better than others. However as the estimation error increases, the performance degradation of evenly distributed antenna system becomes more acute (higher slope), therefore is
more sensitive to errors. This observation agrees with [20]. When there are more transmit antennas than receive antennas, similar observations can be made (refer to Figure 4.5). And the effect of estimation errors remains same for same number of total antennas, for example 1x5 and 5x1 have same performance degradation. The higher the diversity order, less sensitive is the system as shown in Figure 4.6 where $\sigma_e^2 = 0.1$ is considered. At the error rate of $10^{-3}$, the 3x3 system suffers only 1.5dB degradation compared to 6dB in 2x2 system. It essentially shows the trade-offs between the cost and the performance.

In this contribution, exact closed-form expressions of average symbol error rate were obtained for coherent $M$-ary MRT system over i.i.d Rayleigh fading assuming Gaussian channel estimation errors. The expressions are in terms of hypergeometric functions which are readily available as built-in functions in software tools like Mathematica. They are valid for any $M$-ary levels and arbitrary number of antennas. The results showed that the channel estimation errors significantly degrades the system performance. Evenly distributed antenna systems offer better performance.
Figure 4.4: Imperfect Channel Estimate on Diversity Order ($N_r > N_t$) of QPSK MRT system in Rayleigh Fading Channel

Figure 4.5: Imperfect Channel Estimate on Diversity Order ($N_t > N_r$) of QPSK MRT system in Rayleigh Fading Channel
4.3 SER for Rectangular QAM of MIMO-MRC with Channel Estimation Error

In this section, extension of earlier analysis [92]) is carried out to a Rectangular QAM signalling. The results are based on the recent work by Beaulieu [29] on closed-form expression for integral of two Gaussian Q functions. The derived expression reduces to those in [23, 29, 32, 92] to confirm our analysis.

Let the signal belong to an arbitrary $I \times J$ rectangular QAM constellation. Average SEP of arbitrary rectangular QAM over a fading channel is given by [29] [32],

$$P_s^{MRT} = 2q(I)I_1(g^{R}_{QAM}(I, J; \xi)) + 2q(J)J_1(g^{R}_{QAM}(I, J; \xi))$$

$$- 4q(I)q(J)I_2(g^{R}_{QAM}(I, J; \xi), g^{R}_{QAM}(I, J; \xi))$$  \hspace{1cm} (4.13)

where, $q(x) \triangleq 1 - x^{-1}; Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{t^2}{2}} dt; \xi \triangleq d^2_q/d^2_i$ as the squared quadrature to in-phase decision distances ratio, $g^{R}_{QAM}(I, J; \xi) \triangleq 3/[((I^2 - 1) + (J^2 - 1)\xi], R$ stands for rectangular; and
\[ I_1(A) \triangleq \int_0^\infty Q(\sqrt{2A\gamma})p_\gamma(\gamma)d\gamma, \quad \forall A > 0 \]  
\[ (4.14) \]

\[ I_2(A_1, A_2) \triangleq \int_0^\infty Q(\sqrt{2A_1\gamma})Q(\sqrt{2A_2\gamma})p_\gamma(\gamma)d\gamma, \quad \forall A_1, A_2 > 0 \]  
\[ (4.15) \]

### 4.3.1 Average SEP Analysis

Following similar steps as in [32] and re-writing the integral (4.14) in terms of the moment generating function as below,

\[
I_1(A) = 1 \pi \int_0^{\pi/2} M_{MRT}(A \sin^2 \varphi) d\varphi \\
= K_{mn} \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)k} K_{mnl}(l)!d_{kl} \int_0^{\pi/2} \left( \frac{1}{k + FA} \right)^{l+1} \\
(4.16)
\]

A change of variable \( t = \cos^2(\varphi) \) in (4.34) results,

\[
I_1(A) = \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)k} \frac{K_{mnl}(l)!d_{kl}}{2\pi(k + FA)^{l+1}} \int_0^1 t^{-\frac{1}{2}}(1 - t)^{l+\frac{1}{2}}(1 - \alpha t)^{(l+1)} dt \\
(4.17)
\]

which can be written in terms of Gaussian hypergeometric series as [93, eqn. 3.197-3],

\[
I_1(A) = \sum_{k=1}^{m} \sum_{l=n-m}^{(n+m-2k)k} \frac{K_{mnl}(l)!d_{kl}}{2\pi(k + FA)^{l+1}} B\left(\frac{1}{2}, l + \frac{3}{2} \right) \\
2F_1\left(l + 1, \frac{1}{2}; l + 2; \frac{1}{1 + \frac{FA}{\alpha}} \right) \\
(4.18)
\]

where \( B(z, w) = \frac{\Gamma(z)\Gamma(w)}{\Gamma(z+w)} \), \( \Gamma(.) \) is Euler’s Gamma function. In similar fashion, the integral (4.15) can be expressed as [32],
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\[ I_2(A_1, A_2) = \frac{1}{2\pi} \sum_{\kappa \in \Omega_2(2)} \int_0^{\tan^{-1}\left(\frac{\sqrt{A_1^2 + A_2^2}}{A_1}\right)} M_{\text{MRT}}\left(\frac{A_{\kappa_2}}{\sin^2 \phi}\right) d\phi \]  

(4.19)

where \( \kappa \in (\kappa_1, \kappa_2) \) is a permutation belonging to \( \Omega_2(2) = \{(1, 2), (2, 1)\} \). With a change of variable \( t = 1 - \frac{A_{\kappa_2}^2}{A_{\kappa_1}^2} \tan^2(\phi) \) and some manipulations, above becomes,

\[ I_2(A_1, A_2) = \sum_{k=1}^m \frac{(n+m-2k)^k}{4\pi (k + F(A_1 + A_2))^{l+1}} \sum_{l=n-m} \int_0^1 \frac{(1 - t)^{l+\frac{1}{2}} (1 - \beta t)^{-1}}{(1 - \delta t)^{l+1}} dt \]  

(4.20)

which can be written in terms of hypergeometric series as [93, eqn. 3.211],

\[ I_2(A_1, A_2) = \sum_{k=1}^m \sum_{l=n-m} \frac{(n+m-2k)^k}{4\pi (k + F(A_1 + A_2))^{l+1}} \sum_{\kappa \in \Omega_2(2)} B\left(l + \frac{3}{2}, 1\right) \frac{1}{1 + \frac{F(A_{\kappa_2})}{k}} \frac{1}{1 + \frac{F(A_{\kappa_1} + A_{\kappa_2})}{k}} \]  

(4.21)

Now substituting (4.18) and (4.21) in (4.13) with values of \( A, A_1 \) and \( A_2 \), the SEP for rectangular QAM signalling can be obtained for MIMO-MRC systems with imperfect channel estimation in (4.26).

When the in-phase and quadrature decision distances \( d_I \) and \( d_J \) are equal, as induced by a unit value for the parameter \( \xi \), then \( g_{\text{QAM}}^R(I, J; \xi) = g_{\text{QAM}}^R(I, J) \triangleq g_{\text{QAM}}(\sqrt{M}, \frac{I^2 + J^2}{2}) \triangleq 3/[(I^2 - 1) + (J^2 - 1)] \). In this case, (4.26) specializes to (4.27).

Below are two cases highlighting two special cases where new expression reduces to previous results.

**A. Square QAM**

For square \( M \)-ary QAM with \( \xi = 1 \), \( I = J = \sqrt{M} \) implying that \( g_{\text{QAM}}^R(\sqrt{M}, \sqrt{M}; \xi) = g_{\text{QAM}}^S(3/2(M - 1)) \), \( S \) stands for square; \( q(I) = q(J) = q(\sqrt{M}) = q \). Taking this into account in (4.27) yields (4.28) since \( A_1 = A_2 \), which is exactly as in [92, eqn. 19].

When the channel estimation is perfect, \( F = \gamma_o \), then (4.28) specializes to [23, eqn. ...
B. SISO Channel

For a single-input single-output system, \( N_r = N_t = 1 \) implying \( m = n = 1, k = 1, \ell = 0, K_{mn} = 1 \) and \( d_{10} = 1 \). Making use of [93, eqn. 9.121.24],

\[
2F_1\left(\frac{1}{2}; 1; \frac{1}{1 + F_{QAM}^R(I, J; \xi)}\right) = 2 \left(1 + F_{QAM}^R(I, J; \xi)\right) \left[1 - \sqrt{1 + F_{QAM}^R(I, J; \xi)}\right]
\]

(4.22)

and using [94, eqn. 18],

\[
F_1\left(1, 1, 1, 1; \frac{5}{2}; x, y\right) = \frac{3}{x - y} \left\{\sqrt{1 - \frac{1 - y}{y} \sin^{-1} \sqrt{y}} - \sqrt{1 - \frac{1 - x}{x} \sin^{-1} \sqrt{x}}\right\}
\]

(4.23)

along with the identity \( \tan(\sin^{-1}(\sqrt{x})) = \sqrt{\frac{x}{1-x}} \), the Appell hypergeometric function in (4.26) reduces to (4.24) which in turn with close observation can be written as (4.25) [32].

\[
F_1\left(1, 1, 1, \frac{5}{2}; 1 + (1 + \xi) F_{QAM}^R(I, J; \xi)\right) = \\
-3(1 + \xi) \left(1 + (1 + \xi) F_{QAM}^R(I, J; \xi)\right) \left[\frac{(1 + \xi - \xi^\kappa_2^{-1}) F_{QAM}^R(I, J; \xi)}{1 + \xi^\kappa_2^{-1} F_{QAM}^R(I, J; \xi)}\right] \\
\times \tan^{-1}\left(\sqrt{\frac{1 + \xi F_{QAM}^R(I, J; \xi)}{(1 + \xi - \xi^\kappa_2^{-1}) F_{QAM}^R(I, J; \xi)}} - \xi^\kappa_2^{-1} \tan^{-1}\left(\xi^\kappa_2^{-n_1}\right)\right)
\]

(4.24)

Inserting (4.23) and (4.24) into (4.26) and noticing that \( \sum_{\kappa \in \Omega(2)} \tan^{-1}\left(\xi^\kappa_{2-n_1}\right) = \pi/2 \) as \( \tan^{-1}(x^{-1}) = \frac{\pi}{2} - \tan^{-1}(x) \) for \( x > 0 \) [92] and \( B\left(\frac{1}{2}, \frac{3}{2}\right) = \pi/2 \), final expression of SEP is obtained for rectangular QAM signalling in MIMO-MRC system with channel estimation error in (4.29). At perfect channel estimation, (4.29) becomes [29, eqn. 12].

Figure 4.7 illustrates the effect of channel estimation (denoted by \( \sigma^2 \)) on average
\[ F_1 \left( 1, 1, 1, \frac{5}{2}; \frac{1}{1 + \xi^{\kappa_2 - \kappa_1}}, \frac{1 + \xi^{\kappa_2 - \kappa_1} F g_{\text{QAM}}^R(I, J; \xi)}{1 + (1 + \xi) F g_{\text{QAM}}^R(I, J; \xi)} \right) = \] (4.25)

\[ 3 \left[ 1 + (1 + \xi) F g_{\text{QAM}}^R(I, J; \xi) \right] \left( \frac{\tan^{-1} \frac{\xi^{\kappa_2 - \kappa_1}}{2}}{\sqrt{\xi + \frac{1}{\sqrt{\xi}}}} \right)^{-1} \left[ \frac{\xi^{\kappa_2 - \kappa_1}}{2} \sqrt{1 + \xi^{\kappa_2 - \kappa_1} F g_{\text{QAM}}^R(I, J; \xi)} \right] \left( 1 + \frac{\xi^{\kappa_2 - \kappa_1} F g_{\text{QAM}}^R(I, J; \xi)}{\xi^{\kappa_2 - \kappa_1} F g_{\text{QAM}}^R(I, J; \xi)} \right) \]

\[ P_s^{\text{MRT}} = \sum_{k=1}^{m} \sum_{l=m-n}^{(n-m-2k)} k_{mn} d_{kl} B \left( \frac{1}{2}, l + \frac{3}{2} \right) \left( \frac{q(I) q(J)}{\pi (k + F g_{\text{QAM}}^R(I, J; \xi))^{l+1}} \right) \left( \frac{2 F_1(l + 1, \frac{1}{2}; l + 2, \frac{k + F g_{\text{QAM}}^R(I, J; \xi)}{k})}{k + F g_{\text{QAM}}^R(I, J; \xi)} \right) \]

\[ \sum_{k=1}^{m} \sum_{l=m-n}^{(n-m-2k)} \frac{q(I) q(J) k_{mn} d_{kl}}{\pi (k + F g_{\text{QAM}}^R(I, J; \xi))^{l+1}} \left( F_1 \left( 1, 1, l + 1, l + \frac{5}{2}; \frac{1}{1 + \xi^{\kappa_2 - \kappa_1}}, \frac{1 + \xi^{\kappa_2 - \kappa_1} F g_{\text{QAM}}^R(I, J; \xi)}{1 + (1 + \xi) F g_{\text{QAM}}^R(I, J; \xi)} \right) \right) \] (4.26)

\[ P_s^{\text{MRT}} = \sum_{k=1}^{m} \sum_{l=m-n}^{(n-m-2k)} \frac{(q(I) + q(J)) k_{mn} d_{kl}}{\pi (k + F g_{\text{QAM}}^R(I, J; \xi))^{l+1}} \left( 2 F_1 \left( l + 1, \frac{1}{2}, l + 2, \frac{k}{k + F g_{\text{QAM}}^R(I, J; \xi)} \right) \right) \]

\[ - \sum_{k=1}^{m} \sum_{l=m-n}^{(n-m-2k)} \frac{q(I) q(J) k_{mn} d_{kl}}{2 \pi (k + 2 F g_{\text{QAM}}^R(I, J; \xi))^{l+1}} \left( F_1 \left( 1, 1, l + 1, l + \frac{5}{2}; \frac{1}{1 + \xi^{\kappa_2 - \kappa_1}}, \frac{1 + \xi^{\kappa_2 - \kappa_1} F g_{\text{QAM}}^R(I, J; \xi)}{1 + (1 + \xi) F g_{\text{QAM}}^R(I, J; \xi)} \right) \right) \] (4.27)

SEP of 8x4 QAM modulations for various \( \xi \) values. The MRT system consists of 2x2 antennas and i.i.d Rayleigh faded channels. The average SNR per symbol for an 8x4 QAM is calculated as [29] \( 10 \log_{10}(E_T/\sigma_n^2) \), where \( E_T \) is the average total energy per symbol and given by \( E_T = (21 + 5\xi) \sigma_n^2 d^2, \sigma_n^2 \) being average channel fade. In the case of \( \xi = 1 \) i.e. the in-phase and quadrature-phase decision distances are equal, the
\[ P_{S_{\text{SQAM}}} = \sum_{k=1}^{m} \sum_{l=n-m}^{m-2k} \left[ \frac{2qK_{mn}!d_{kl}B \left( \frac{1}{2}, l + \frac{3}{2} \right)}{\pi(k + Fg_{QAM}^{R})^{l+1}} - 2F_{1} \left( l + 1, \frac{1}{2}, l + 2; \frac{k}{k + Fg_{QAM}^{R}} \right) \right] \]

\[ q^{2}K_{mn}!d_{kl}B \left( l + \frac{3}{2}, 1 \right) \sum_{\kappa \in \Omega(2)} F_{1} \left( 1, 1, l + 1, l + \frac{5}{2}; \frac{k + FA_{\kappa_{2}}}{k + 2Fg_{QAM}^{R}} \right) \]

\[ P_{S_{\text{SISO}}} = q(I) \left[ 1 - \sqrt{\frac{g_{QAM}^{R}(I, J; \xi)}{1 + Fg_{QAM}^{R}(I, J; \xi)}} \right] + q(J) \left[ 1 - \sqrt{\frac{g_{QAM}^{R}(I, J; \xi\xi)}{1 + Fg_{QAM}^{R}(I, J; \xi\xi)}} \right] \]

\[ -q(I)q(J) \times \left[ 1 - \frac{2}{\pi} \sum_{\kappa \in \Omega(2)} \left[ \frac{\xi^{\kappa_{2}-1}Fg_{QAM}^{R}(I, J; \xi)}{1 + \xi^{\kappa_{2}-1}Fg_{QAM}^{R}(I, J; \xi)} \right] \right] \]

\[ \tan^{-1} \left( \frac{\xi^{\kappa_{2}-\kappa_{1}}}{\xi^{\kappa_{2}-1}Fg_{QAM}^{R}(I, J; \xi)} \right) \]

Figure 4.7: Average Symbol Error Probability for 8x4 Rectangular QAM in 2x2 MRT System over Rayleigh Fading with Channel Estimation Error
in-phase signal has 21/5 times the average quadrature signal energy. When \( \xi = 21/5 \), both in-phase and quadrature-phase signals have equal energies. The ASEP is higher at moderate to high SNR region. And finally, \( \xi = (21/5)^2 \) represents a case where the quadrature signal energy is 21/5 times that of in-phase signal. For the perfect channel estimation, the error probability is shown to have similar trends as observed in [29] for all three cases. The diversity gain is evident. Yet the latter case is the worst of all incurring almost 5dB of SNR loss at large SNR values. The Gaussian channel estimation error at the receiver aggravates the loss. Even for error of \( \sigma^2_e = 0.01 \), the error rate floor starts at 25dB, the worst being the latter case.

In summary, exact closed-form expression was derived for rectangular QAM signalling in MIMO-MRC system with channel estimation error. The expression are valid for i.i.d slow Rayleigh fading channels and arbitrary number of receive and transmit antennas. The derived expression is shown to reduce to previous results. The numerical results for 8x4 QAM shows the strong influence of decision distances’ ratio on the symbol error probability. The Gaussian channel estimation error at the receiver incur further performance loss.

\section*{4.4 SER for Rectangular QAM in MRC over Nakagami-\( q \) (Hoyt) Fading}

The analysis is extended to a spatial diversity system in independent but not-necessarily (ind) Nakagami-\( q \) fading channels. The signals received at \( L \) branches are combined using Maximal Ratio Combining (MRC), this diversity technique is optimum but requires the channel information known at the receiver. The derived final expressions are in general form and shown to reduce to previous published results for special cases.

\subsection*{4.4.1 System Model}

The probability distribution function (pdf) of Nakagami-\( q \) faded random variable \( \alpha \) with mean-square value \( \Omega \triangleq E\{\alpha^2\} \) is given by [2],

\begin{equation}
 p_\alpha(\alpha) = \frac{(1 + q^2)\alpha}{q\Omega} \exp\left(-\frac{(1 + q^2)\alpha^2}{4q^2\Omega}\right) I_0\left(\frac{(1 - q^2)\alpha^2}{4q^2\Omega}\right), \alpha \geq 0. \quad (4.30)
\end{equation}
where \( q \in [0, 1] \) is the fading severity parameter and \( I_0(.) \) is the zeroth-order modified Bessel function of the first kind. Let \( \gamma \) be the instantaneous symbol signal-to-noise-ratio (SNR) of a flat-fading SISO channel defined by \( \gamma \triangleq \frac{\alpha^2 E_s}{N_o} \). Then the moment generating function (mgf) of \( \gamma \) is [2],

\[
\Phi_\gamma(s) = \left[ \left( 1 + \frac{2s\bar{\gamma}}{1 + q^2} \right) \left( 1 + \frac{2s\bar{\gamma}q^2}{1 + q^2} \right) \right]^{-\frac{1}{2}} \tag{4.31}
\]

where \( \bar{\gamma} = \Omega E_s/N_o \) is the average SNR per symbol.

Let the transmitted symbol be flat-faded and received over \( L \) independent branches. The instantaneous SNR at the receiver employing maximal ratio combining (MRC) can be given by,

\[
\gamma_{MRC} \triangleq \sum_{i=1}^{L} \gamma_i = \sum_{i=1}^{L} \frac{\alpha_i^2 E_s}{N_o} \tag{4.32}
\]

where \( \alpha_i, i = 1, 2, \ldots, L \) is the fading amplitude of the \( i \)th branch Nakagami-\( q \) fading channel with fading parameter \( q_i \) and the mean-square value \( \Omega_i = E\{\alpha_i^2\} \). As the received symbols are independently faded, the moment generating function of MRC output SNR \( \gamma_{MRC} \) can be written as,

\[
\Phi_{\gamma_{MRC}}(s) = \prod_{i=1}^{L} \left[ \left( 1 + \frac{2s\bar{\gamma}_i}{1 + q_i^2} \right) \left( 1 + \frac{2s\bar{\gamma}_i q_i^2}{1 + q_i^2} \right) \right]^{-\frac{1}{2}} \tag{4.33}
\]

### 4.4.2 Average SEP Derivation

Here, error probability is derived for multi-channel receiver employing MRC scheme over independent but non-identical Nakagami-\( q \) fading channels. As illustrated in [2] and by several other authors [31]-[94], the moment generating function (mgf) approach offers more convenient way to evaluate error probability than the conventional probability density function (pdf) one. Here same mgf method was followed to reach the final expressions.

Re-writing the integral \( I_1(A) \) as below [32],

\[
I_1(A) = \frac{1}{\pi} \int_{0}^{\pi/2} \Phi_{\gamma_{MRC}} \left( \frac{A}{\sin^2 \theta} \right) d\theta \tag{4.34}
\]
A change of variable $t = \cos^2(\theta)$ in (4.34) results,

$$I_1(A) = \frac{1}{2\pi} \int_{0}^{1} t^{-\frac{1}{2}} (1-t)^{-\frac{1}{2}} \Phi_{\gamma_{\text{MRC}}} \left( \frac{A}{1-t} \right) dt$$

$$= \frac{1}{2\pi} \int_{0}^{1} t^{-\frac{1}{2}} (1-t)^{-\frac{1}{2}} \prod_{i=1}^{L} \left( 1 + \frac{2A\gamma_i}{(1-t)(1+q_i^2)} \right)$$

$$\left( 1 + \frac{2A\gamma_i q_i^2}{(1-t)(1+q_i^2)} \right)^{-\frac{1}{2}} dt$$

$$= \frac{\Phi_{\gamma_{\text{MRC}}}(A)}{2\pi} \int_{0}^{1} t^{-\frac{1}{2}} (1-t)^{L-\frac{1}{2}}$$

$$\times \prod_{i=1}^{L} (1-\alpha_i t)^{-\frac{1}{2}} (1-\beta_i t)^{-\frac{1}{2}} dt \quad (4.35)$$

where,

$$\alpha_i = \frac{1}{1 + \frac{2A\gamma_i}{1+q_i^2}}$$

$$\beta_i = \frac{1}{1 + \frac{2A\gamma_i q_i^2}{1+q_i^2}}$$

The above integral can be expressed in terms of Lauricella multivariate hypergeometric function $F^{(n)}_D$ which has a series and an Euler integral representation as shown below [95]

$$F^{(n)}_D (a, \{b_i\}_{i=1}^{n}; c; \{x_i\}_{i=1}^{n}) = \sum_{i_1, \ldots, i_n=0}^{\infty} \frac{(a)_{i_1+\ldots+i_n} (b_1)_{i_1} \ldots (b_n)_{i_n}}{(c)_{i_1+\ldots+i_n}} \frac{x_1^{i_1}}{i_1!} \ldots \frac{x_n^{i_n}}{i_n!}$$

$$= \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_{0}^{1} t^{a-1} (1-t)^{c-a-1} \prod_{i=1}^{n} (1-x_i t)^{-b_i} dt,$$

$$\max\{|x_1|, \ldots, |x_n|\} \leq 1, \Re(c) > \Re(a) > 0 \quad (4.36)$$

where $(x)_y = \Gamma(x+y)/\Gamma(y)$ is the Pochhammer symbol for $y \geq 0$, $\Gamma(.)$ is Euler gamma function, and $\Re(.)$ denotes real part of the argument. Note that $F^{(1)}_D$ and $F^{(2)}_D$ reduce to the Gauss hypergeometric function $2\, F_1(a, b; c; x)$ and the Appell hypergeometric function $F_1(a, b, b'; c; x, y)$ respectively, and are readily available as built-in functions in standard numerical software.

Now, the first integral (4.35) can be written as,
\[ I_1(A) = \frac{\Gamma(L + 1/2)}{2\sqrt{\pi} \Gamma(L + 1)} \Phi_{\gamma_{\text{MRC}}}(A) F_D^{(2L)} \left( \frac{1}{2} \left[ \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2} \right]; L + 1; \{\alpha_i\}_{i=1}^L, \{\beta_i\}_{i=1}^L \right) \] (4.37)

Note that for \( n \geq 2 \), there is no direct implementation of \( F_D^{(n)} \) in standard numerical software, therefore one need to use its series representation for computation.

As shown in [27], only first few terms of the series are required. Therefore the expressions in Lauricella functions are much less computation intensive than the direct numerical integration in the final expressions.

Similarly the integral \( I_2(A_1, A_2) \) can be expressed as [32],

\[ I_2(A_1, A_2) = \frac{1}{2\pi} \sum_{\kappa \in \Delta_2(2)} \int_0^{\tan^{-1}\left( \sqrt{\frac{A_1^2}{A_2^2}} \right)} \Phi_{\gamma_{\text{MRC}}}(\frac{A}{\sin^2\theta}) d\theta \] (4.38)

where \( \kappa \in (\kappa_1, \kappa_2) \) is a permutation belonging to \( \Delta_2(2) = \{(1, 2), (2, 1)\} \). With a change of variable \( t = 1 - \frac{A_{\kappa_2}}{A_{\kappa_1}} \tan^2\theta \), (4.38) becomes,

\[ I_2(A_1, A_2) = \frac{1}{4\pi} \left( \sqrt{\frac{A_1}{A_2}} + \sqrt{\frac{A_2}{A_1}} \right) \sum_{\kappa \in \Delta_2(2)} \int_0^1 (1 - t)^{-\frac{1}{2}} \times (1 - \delta t)^{-1} \Phi_{\gamma_{\text{MRC}}}(A_{\kappa_2} + \frac{A_{\kappa_1}}{(1 - t^2)}) dt \] (4.39)

where \( \delta = \frac{A_{\kappa_2}}{A_{\kappa_1}} \). Inserting (4.32) in the above integral and after some rearrangements, one obtains

\[ I_2(A_1, A_2) = \frac{\Phi_{\gamma_{\text{MRC}}}(A_1 + A_2)}{4\pi} \left( \sqrt{\frac{A_1}{A_2}} + \sqrt{\frac{A_2}{A_1}} \right) \sum_{\kappa \in \Delta_2(2)} \int_0^1 (1 - t)^{-\frac{1}{2}} \times \prod_{i=1}^L (1 - \eta_i t)^{-\frac{1}{2}} (1 - \zeta_i t)^{-\frac{1}{2}} (1 - \delta t)^{-1} dt \] (4.40)

where,

\[ \eta_i = \frac{1 + \frac{2A_{\kappa_2} \gamma_i}{1+q_i}}{1 + \frac{2(A_1+A_2) \gamma_i}{1+q_i}} \]
\[ \zeta_i = 1 + \frac{2 \Delta_i^2 \Omega_i^2}{1 + q_i} \]

which can be written as,

\[
I_2(A_1, A_2) = \frac{\Phi_{\text{MRC}}(A_1 + A_2)}{2\pi(2L + 1) \left( \sqrt{\frac{A_1}{A_2}} + \sqrt{\frac{A_2}{A_1}} \right)} \sum_{\kappa \in \Delta_2(2)} F_D^{(2L+1)} \left( 1, \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}, 1; L + \frac{3}{2}; \{\eta_i\}_{i=1}^L, \{\zeta_i\}_{i=1}^L, \delta \right)
\]

Substituting (4.37) and (4.41) in (4.13), the final expression of ASEP is obtained for a general order rectangular QAM signalling in MRC over ind Nakagami-\(q\) fading channel as shown in (4.43).

Below are given few special cases.

**A. IID MRC**

If the branch fadings are independently and identically Nakagami-\(q\) distributed (iid), i.e. \(q_i = q, \Omega_i = \Omega, i = 1, 2, ..., L\), one can use following reduction formula [28, A.1]

\[
F_D^{(n)}(a, \{b_i\}_{i=1}^n; c; \{x_i\}_{i=1}^n) = F_D^{(n-m+1)} \left( a, \sum_{i=1}^m b_i, \{b_i\}_{i=m+1}^n; c; x^*, \{x_i\}_{i=m+1}^n \right)
\]

for \(x_1 = x_2 = ... = x_m = x^*, m \leq n\).

**B. SISO**

When there is no diversity at the receiver i.e. \(L = 1\), (4.43) becomes (4.45) which is equivalent to that derived in [31].

**C. Square QAM**

For square QAM, the in-phase and the quadrature decision distances \(d_I\) and \(d_J\) are equal, as induced by a unit value for the parameter \(\xi\), then \(g_{\text{QAM}}^R(I, J; \xi) = g_{\text{QAM}}^R(I, J) = 3/[(I^2-1)+(J^2-1)]\). Then \(I = J = \sqrt{M}\) implies that \(g_{\text{QAM}}^R(\sqrt{M}, \sqrt{M}; \xi) = g_{\text{QAM}}^S = 3/(2(M - 1))\), S stands for square; \(q(I) = q(J) = q(\sqrt{M}) = q_{\text{SISO}}\). Taking these into account and since \(A_1 = A_2\), (4.43) becomes (4.46) which is identical to [28, eqn. 16].
\[ P_{e}^{\text{MRC}} = \frac{\Gamma (L + \frac{1}{2}) q (I) \Phi_{\gamma_{\text{MRC}}} (A) \Gamma (L + 1)}{\sqrt{\pi} \Gamma (L + 1)} F_{D}^{(2L)} \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}, L + 1; \right) \]

\[ = \frac{\Gamma (L + \frac{1}{2}) q (J) \Phi_{\gamma_{\text{MRC}}} (A \xi) \Gamma (L + 1)}{\sqrt{\pi} \Gamma (L + 1)} \]

\[ F_{D}^{(2L+1)} \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}, L + 1; \right) \left( \frac{1}{1 + \frac{2 A_{\gamma_{2}} \gamma_{2} q_{2}^{2}}{1 + q_{2}^{2}}} \right)_{i=1}^{L} \left( \frac{1}{1 + \frac{2 A_{\gamma_{2}} \gamma_{2} q_{2}^{2}}{1 + q_{2}^{2}}} \right)_{i=1}^{L} \]

\[ P_{e}^{\text{HDMRC}} = \frac{\Gamma (L + \frac{1}{2}) q (I) \Phi_{\gamma_{\text{MRC}}} (A) \Gamma (L + 1)}{\sqrt{\pi} \Gamma (L + 1)} F_{1} \left( \frac{1}{2}, \frac{L}{2}, \frac{L}{2}, L + 1; \right) \left( \frac{1}{1 + \frac{2 A_{\gamma} \gamma q^{2}}{1 + q^{2}}} \right)_{i=1}^{L} \]

\[ + \frac{\Gamma (L + \frac{1}{2}) q (J) \Phi_{\gamma_{\text{MRC}}} (A \xi) \Gamma (L + 1)}{\sqrt{\pi} \Gamma (L + 1)} F_{1} \left( \frac{1}{2}, \frac{L}{2}, \frac{L}{2}, L + 1; \right) \left( \frac{1}{1 + \frac{2 A_{\gamma} \gamma q^{2}}{1 + q^{2}}} \right)_{i=1}^{L} \]

\[ - \frac{2 q (I) q (J) \Phi_{\gamma_{\text{MRC}}} (A_{1} + A_{2}) \Gamma (L + 1)}{\pi (2L + 1) \left( \sqrt{\frac{A_{2}}{A_{1}}} + \sqrt{\frac{A_{1}}{A_{2}}} \right)} \sum_{\kappa \in \Delta_{2}(2)} F_{D}^{(2L+1)} \left( \frac{1}{2}, \frac{L}{2}, \frac{L}{2}, L + 1; \right) \left( \frac{1}{1 + \frac{2 A_{\gamma} \gamma q^{2}}{1 + q^{2}}} \right)_{i=1}^{L} \left( \frac{1}{1 + \frac{2 A_{\gamma} \gamma q^{2}}{1 + q^{2}}} \right)_{i=1}^{L} \]

\[ \times \sum_{\kappa \in \Delta_{2}(2)} \frac{2 q (I) q (J) \Phi_{\gamma_{\text{MRC}}} (A_{1} + A_{2}) \Gamma (L + 1)}{\pi (2L + 1) \left( \sqrt{\frac{A_{2}}{A_{1}}} + \sqrt{\frac{A_{1}}{A_{2}}} \right)} \sum_{\kappa \in \Delta_{2}(2)} F_{D}^{(3)} \left( \frac{1}{2}, \frac{L}{2}, \frac{L}{2}, L + 1; \right) \left( \frac{1}{1 + \frac{2 A_{\gamma} \gamma q^{2}}{1 + q^{2}}} \right)_{i=1}^{L} \left( \frac{1}{1 + \frac{2 A_{\gamma} \gamma q^{2}}{1 + q^{2}}} \right)_{i=1}^{L} \]

D. Rayleigh Channel

Rayleigh fading is a special case of Nakagami-\( q \) when \( q = 1 \), and is the least severe.

From (4.36), the following identity can be used,

\[ F_{D}^{(2L)} \left( a, \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}; \{ x_{i} \}_{i=1}^{2L} \right) = F_{D}^{(L)} \left( a, 1, 1, \ldots, 1; \{ x_{i} \}_{i=1}^{L} \right) \]

in (4.43) which becomes equivalent to [32, eqn. 19] with \( m = 1 \).

In summary, exact closed-form expression of average symbol error probability was derived for a general order rectangular QAM in Maximal Ratio Combiner over ind Nakagami-\( q \) fading. The integral of the product of two Q-functions by [29]
\[ P_{e}^{\text{SISO}} = 2\{q(I) + q(J)\} \Phi_{MRC}(A) F_{1}\left(\begin{array}{lll}1 & 1 & 1 \\
 & 2 & 1 \\
 & 2 & 2 \end{array}; 1; 1 + \frac{2A_{1}\gamma}{1 + q_{1}}, 1 + \frac{1 + 2A_{1}q_{1}}{1 + q_{1}} \right) \]
\[ - \frac{2q(I)q(J) \Phi_{MRC}(A_{1} + A_{2})}{3\pi \left(\sqrt{\frac{A_{1}}{A_{2}}} + \sqrt{\frac{A_{2}}{A_{1}}}\right)} \times \left[ F_{D}^{(3)}\left(1, 1, 2; 2; \frac{1}{2} + \frac{1 + 2A_{1}\gamma}{1 + q_{1}}, 1 + \frac{2A_{1}q_{1}}{1 + q_{1}}, A_{1} + A_{2}\right) \right. \]
\[ \left. + F_{D}^{(3)}\left(1, 1, 2; 2; \frac{1}{2} + \frac{2A_{2}\gamma}{1 + q_{2}}, 1 + \frac{2A_{2}q_{2}}{1 + q_{2}}, A_{2}\right) \right] \]
\[ P_{e}^{\text{SQAM}} = \frac{2q_{I}q_{J}(L + \frac{1}{2}) \Phi_{MRC}(g_{\text{SQAM}}^{S})}{\sqrt{\pi} \Gamma(L + 1)} F_{D}^{(2L)}\left(\begin{array}{lll}1 & 1 & \frac{1}{2} \\
 & 2 & \frac{1}{2} \end{array}; \frac{1}{2} - L + 1; \right. \]
\[ \left. \left\{ \frac{1}{1 + \frac{2g_{\text{SQAM}}^{S}}{1 + q_{i}}}, i = 1 \right\}^{L}, \right\}^{L} \left\{ \frac{1}{1 + \frac{2g_{\text{SQAM}}^{S}q_{i}}{1 + q_{i}}}, i = 1 \right\}^{L} \right) \frac{2q_{I}q_{J} \Phi_{MRC}(2g_{\text{SQAM}}^{S})}{\pi (2L + 1)} \]
\[ F_{D}^{(2L+1)}\left(1, 1, \frac{1}{2}; \frac{1}{2}, \frac{1}{2} + \frac{1 + 2g_{\text{SQAM}}^{S}}{1 + q_{i}}, 1 + \frac{2g_{\text{SQAM}}^{S}q_{i}}{1 + q_{i}} \right) \left\{ \frac{1 + \frac{2g_{\text{SQAM}}^{S}}{1 + q_{i}}}{1 + \frac{4g_{\text{SQAM}}^{S}q_{i}}{1 + q_{i}}}, i = 1 \right\}^{L} \]
\[ \left(1 + \frac{2g_{\text{SQAM}}^{S}q_{i}}{1 + q_{i}} \right)^{L} \left(1 + \frac{1}{2} \right) \]

formed the basis of analysis, and the final expressions are expressed in terms of multivariate Lauricella hypergeometric functions which can be efficiently computed using its converging series representation.

### 4.5 Performance Analysis of MRC in Correlated Rayleigh Fading

This study was undertaken jointly, and my main contribution included the derivation of error rates and validation of accuracy of the derived statistics. Here, antenna correlation is taken account for the performance analysis of MRC system over Rayleigh fading channels. Consider an \(L\)-branch diversity receiver in slow fading channels.
Assuming perfect timing and no inter-symbol interference (ISI), the received signal on the \( l \)th branch due to the transmission of a symbol \( s \) can be expressed as

\[
    r_l = g_l s + n_l, \quad l = 1 \ldots L,
\]

where \( g_l \) is a zero-mean complex Gaussian distributed channel gain, \( n_l \) is the complex additive white Gaussian noise (AWGN) sample with a variance of \( N_0/2 \). The AWGN is assumed to be independent from channel to channel and independent of the fading amplitude \( g_l \), and \( s \) is the data symbol taken from a normalized unit-energy signal set with an average power \( E_s \). The instantaneous SNR per symbol of the \( l \)th channel is defined as

\[
    \gamma_l = \frac{g^2_l E_s}{N_l} = \frac{g^2_l \bar{\gamma}}{\gamma_l},
\]

where \( \gamma \) is average SNR. When MRC is used, instantaneous SNR \( \gamma \) is given by

\[
    \gamma = \sum_{l=1}^{L} g^2_l \gamma = \sum_{l=1}^{L} \gamma_l.
\]

When the diversity branches are correlated, the analysis proceeds in a similar fashion to the independent fading. So far in the literature, two correlation models have been proposed namely constant (equal) correlation and exponential correlation, each with its own advantages and disadvantage. The desired MGF of combiner SNR can be obtained as follows:

\[
    \phi_{\gamma_s}(s) = \det(I_L + s\bar{\gamma} \Gamma)^{-1} = \prod_{l=1}^{L} \frac{1}{1 + s\gamma_l},
\]

where \( \bar{\gamma} \) is the branch covariance matrix and \( I_L \) is the \( L \times L \) identity matrix. Consider \( \Gamma \) as a matrix of constant correlation coefficient \( \rho \in (0, 1) \), i.e. \( \Gamma(\rho) \) can be expressed as

\[
    \Gamma(\rho) = \begin{pmatrix}
        1 & \text{if } i = j \\
        \rho & \text{otherwise}
    \end{pmatrix}_{L \times L}.
\]

It is easily seen that \( \Gamma \) has exactly two distinct eigenvalues, namely \( a = 1 - \rho \) with multiplicity \( L-1 \) and \( b = 1 + (L-1)\rho \). Thus, the MGF is

\[
    \phi_{\gamma_s}(s) = \frac{1}{(1 + (1-\rho))^{L-1}(1 + (1+(L-1)\rho))}.\]

Upon applying partial fraction, taking the first with respect \( s \) and evaluating the result at \( s = 0 \), and after further manipulation the PDF can be expressed as

\[
    p_{\lambda}(\lambda) = \frac{(1 + (L-1)\rho)^{L-1}}{\bar{\gamma}((1 + (L-1)\rho)\lambda)^{L-1} \rho^{L-1}} \exp\left(\frac{-\gamma}{(1 + (L-1)\rho)}\right) - \sum_{k=1}^{L-1} \frac{((\bar{\gamma}(1-\rho)((1 + (L-1)\rho))^{L-k} \gamma^{k-1})}{\bar{\gamma}}(1-\rho)\lambda)^{L-k} \rho^{L-k}(k-1)! \exp\left(\frac{-\gamma}{(1 - \rho)}\right).
\]

### 4.5.1 Ergodic Capacity and Bounds

The channel capacity is the most important parameter in communications. It gives a quantitative measure of information that can be transmitted over a channel to the
receiver. In this section, the ergodic capacity of correlated MRC Rayleigh fading channels and its approximation was derived.

**A. Exact Ergodic Capacity**

The ergodic capacity of a channel is given by [96] [97]

\[ C = \frac{B}{\ln(2)} \int_0^\infty \ln(1 + \gamma) f_\gamma(\gamma) d\gamma \quad (4.50) \]

By using the change of variable \( x = 1 + \gamma \), and applying the integral equality

\[ \int_0^1 \ln(x) e^{-\mu x} = E_1(\mu) / \mu, \]

where \( E_1(x) = \int_0^1 e^{-xt} \frac{dt}{t} \) is exponential integral function of first order. The desired closed-form expression for the capacity per unit bandwidth (in bits/seconds/hertz) can be obtained after some mathematical manipulation as

\[ C = \left[ \frac{(1 + (L - 1)\rho)^{L-1}}{(L\rho)^{L-1}} \exp \left( \frac{1}{\gamma(1 + (L - 1)\rho)} \right) E_1 \left( \frac{-1}{\gamma(1 + (L - 1)\rho)} \right) \right. \]

\[ - \sum_{k=1}^{L-1} \frac{(\gamma(1 - \rho))^{L-k}}{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-1}} \frac{((1 + (L - 1)\rho))^{L-1}}{((L\rho)^{k-1}(k-1)!} \]

\[ \left( \frac{(\gamma(1 - \rho))^{m+j+1}}{(\gamma(1 - \rho))^{i+1}} E_1 \left( \frac{-1}{\gamma(1 - \rho)} \right) \right) \quad (4.51) \]

**B. Approximate Ergodic Capacity**

**Asymptotic Approximation**

The series representation of Exponential integral of first order function expressed as

\[ E_1(x) = -E - \ln(x) - \sum_{i=1}^{\infty} \frac{(-x)^i}{i,i!} \]

where \( E = 0.5772156659 \) is the Euler-Mascheroni constant, can be used to obtain asymptotic approximation capacity. Thus, the asymptotic approximation \( C^{\infty} \) per unit bandwidth (in bits/seconds/hertz) can be expressed as
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\[ C = \left[ \frac{(1 + (L - 1)\rho)^{L-1}}{(L\rho)^{L-1}} \exp \left( \frac{1}{\gamma(1 + (L - 1)\rho)} \right) \right] \left( -E - \ln \left( \frac{1}{\gamma(1 + (L - 1)\rho)} \right) \right) \]

\[ - \ln \left( \frac{-1}{\gamma(1 + (L - 1)\rho)} \right) + \left( \frac{-1}{\gamma(1 + (L - 1)\rho)} \right) \]

\[ - \sum_{k=1}^{L-1} \frac{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}}{\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}} \frac{1}{(k - 1)!} \left( \frac{\gamma(1 - \rho)}{\gamma(1 - \rho)} \right)^n \]

\[ \sum_{i=0}^{k-1} \binom{k - 1}{i} (-1)^{k-i-1} \sum_{j=0}^{i} \sum_{m=0}^{i-j} \frac{1}{(i - j)} \sum_{m=0}^{i-j} \frac{1}{(i - j - 1 - m)!} \left( \frac{\gamma(1 - \rho)}{\gamma(1 - \rho)} \right)^{m+j+1} \left( \frac{\gamma(1 - \rho)}{\gamma(1 - \rho)} \right)^{i+j} \]

\[ \times \left( -E - \ln \left( - \frac{1}{\gamma(1 - \rho)} \right) \right) \left( - \frac{1}{\gamma(1 - \rho)} \right) \]  

(4.52)

**Upper Bound**

The capacity expression can be upper bounded by applying Jensen’s inequality to (4.50) as follows:

\[ C_{UP}^{UB} = \ln \left( 1 + E[\gamma] \right), \]

one can evaluate \( C_{UP} \) using the pdf \( \gamma \) given in (4.49) and the identity \[88], \[ \int_0^\infty x^ne^{-\mu x}dx = n!\mu^{-n-1} \] for \( \text{Re}[\mu] > 0 \) and simplify the resulting expression to obtain the capacity (4.51) upper bound

\[ C_{UB}^{UP} = \ln \left[ 1 + \left( \frac{(1 + (L - 1)\rho)^{L-1}}{\gamma(1 + (L - 1)\rho))^{L-1}} \sum_{k=1}^{L-1} \frac{1}{(k - 1)!} \right] \times \left( \frac{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}}{\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}} \right) \]  

(4.53)

### 4.5.2 Capacity Statistics

The non-ergodic capacity of correlated MRC system is given in [bit/s/Hz] by [97]

\[ C = \log_2(1 + \gamma). \]  

(4.54)
Chapter 4. Performance Analysis of Multiantenna Systems

A. Moment Generating Function (MGF)

The MGF of capacity of correlated MRC system is given by

$$\Phi_C(\tau) = E[e^{\tau C}] = E\left[(1 + \gamma)^{\frac{\tau}{\tau_m}}\right]. \quad (4.55)$$

Expressing the expectation in an integral form over the distribution of $p_\gamma(\gamma)$ in (4.49), the MGF can be expressed as follows:

$$\Phi_C(\tau) = \left\{ \frac{(1 + (L - 1)\rho)^{L-1}}{\gamma(1 + (L - 1)\rho)\rho^{L-1}} \times \left( \frac{1}{\gamma(1 + (L - 1)\rho)} \right)^{-\left(\frac{\tau}{\tau_m} + 1\right)} \exp\left(\frac{1}{\gamma(1 + (L - 1)\rho)}\right) \Gamma\left(\frac{\tau}{\ln(2)} + 1, \left(\frac{1}{\gamma(1 + (L - 1)\rho)}\right)\right) \right. $$

$$- \sum_{k=1}^{L-1} \frac{\exp\left(\frac{1}{\gamma(1 - \rho)}\right)}{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-1}\rho^{L-k}(k - 1)!} \left[ \sum_{i=0}^{k-1} \binom{k-1}{i} (-1)^{k-i-1} \left(\frac{1}{\gamma(1 - \rho)}\right)^{-\left(\frac{\tau}{\tau_m} + i + 1\right)} \Gamma\left(\left(\frac{\tau}{\ln(2)} + i + 1\right), \left(\frac{1}{\gamma(1 - \rho)}\right)\right) \right] \right\} \quad (4.56)$$

The MGF can be expressed in another form by taking the use of $\Psi(a, b; z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$ [93]. Note that using alternative notation for $\Psi(a, b; z) = z^{-a}F_0(a, 1 + a - b; \cdots; -1/z)$ where $F_0(., .; .)$ is a generalized hypergeometric series, the MGF of $C$ can simply be written as

$$\Phi_C(\tau) = \left[ \frac{(1 + (L - 1)\rho)^{L-1}F_0(1, \frac{\tau}{\tau_m}; \beta)}{\gamma(1 + (L - 1)\rho)\rho^{L-1}} - \sum_{k=1}^{L-1} \frac{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}(k - 1)!}{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-1}} \right] \times \left[ \Gamma(k) \beta^{-k}F_0(k, -\frac{\tau}{\tau_m}; 1; -\frac{1}{\gamma}) \right] \frac{\rho^{L-k}(k - 1)!}{L} \right\} \quad (4.57)$$

where $\beta = -\gamma(1 + (L - 1)\rho)$. 
B. Complementary Cumulative Distribution Function (CCDF)

The CCDF of $C$ is defined as follows

$$F_C(C) = 1 - \text{Prob}(C \leq C) = \int_0^{2^C-1} p_{\gamma}(\gamma) d\gamma. \quad (4.58)$$

Averaging over the distribution of $\gamma$ in (4.49), it gives the exact expression for CCDF as

$$F_C(C) = 1 - \left[ \frac{\gamma(1 + (L - 1)\rho)L\rho^{L-1}\exp(2^C - 1) - 1}{\gamma(1 + (L - 1)\rho)L\rho^{L-1}} \right. \right.$$

$$\left. - \sum_{k=1}^{L-1} \frac{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}}{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-1}L\rho^{L-k}} \right] \times \left[ 1 - \exp \left( -\frac{2^C + 1}{\gamma(1 - \rho)} \left( \sum_{n=0}^{k-1} \frac{1}{n!} (2^C - 1)^n (\gamma(1 - \rho))^n \right) \right) \right]. \quad (4.59)$$

C. Probability Density Function (PDF)

The PDF of $C$ is defined as the derivative of $F_C(C)$ with respect to $C$. Taking the derivative of $F_C(C)$ in (4.59) results in

$$p_C(C) = 2^C \ln(2) \frac{(1 + (L - 1)\rho)^{L-1}}{\gamma(1 + (L - 1)\rho)L\rho^{L-1}} \times \exp \left( -\frac{2^C + 1}{\gamma(1 + (L - 1)\rho)} \right) \quad (4.60)$$

$$- \sum_{k=1}^{L-1} \frac{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}2^C - 1^{k-1}}{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-1}L\rho^{L-k}(k - 1)!} \exp \left( -\frac{2^C + 1}{\gamma(1 - \rho)} \right)$$

4.5.3 Symbol Error Rate

A closed form expression of SER is derived for $M$-PSK signalling. Similar expressions can be obtained for other modulation schemes. For a given instantaneous SNR $\gamma$ over an AWGN channel, the SER for $M$-ary signaling is given by $P_{e,\text{PSK}}(\gamma) = a_{\text{PSK}}Q(\sqrt{g_{\text{PSK}}\gamma})$ where $a_{\text{PSK}} = 2$, $g_{\text{PSK}} = 2\sin^2 \left( \frac{\pi}{M} \right)$. In order to obtain the SER in fading channel, $P_e$ is averaged over the pdf of $\gamma$. Thus, one can obtain SER expression for $M$-PSK, by exploiting the following identity $\int_0^\infty e^{bx} [1 - \Phi(\sqrt{a}x)] dx = \frac{1}{\sqrt{a}} \left[ \frac{\sqrt{a}}{\sqrt{\alpha}} - 1 \right]$ and $I(p, q, m) = \frac{\Gamma(m)}{\sqrt{q}} \int_0^\infty Q(\sqrt{px}) e^{-qx} x^{m-1} dx = \frac{1}{2\sqrt{\sigma}^{(m+1)}} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)} F_1 \left( m, \frac{1}{2}, m + 1; \frac{1}{s+1} \right)$, where $s = \frac{p}{2q}$. Therefore, the SER for $M$-PSK modulation can be expressed
Figure 4.8: Capacity of correlated MRC over a Rayleigh fading with various values of receive antennas and $\rho$, and fixed $\gamma = 15\text{dB}$.

\[
SER_{\text{PSK}} = a_{\text{PSK}} \left[ \frac{(1 + (L - 1)\rho)^{L-1} \left(1 - \frac{\sqrt{g_{\text{PSK}}}}{\sqrt{g_{\text{PSK}} + \sigma}}\right)}{\gamma(1 + (L - 1)\rho)L\rho^{L-1}} \right. \\
- \left. \sum_{k=1}^{L-1} \frac{(\gamma(1 - \rho)(1 + (L - 1)\rho))^{L-k}}{b^k} \left(1 + \frac{g_{\text{PSK}}}{2b}\right)^{-k} \right. \\
\left. \times \frac{\Gamma(k + \frac{3}{2})}{\Gamma(k + 1)} \left(\frac{g_{\text{PSK}}}{2\sqrt{\pi}g_{\text{PSK}} + \sigma}\right)^{\frac{1}{2}} \times {}_2F_1\left(1, k + \frac{1}{2}; k + 1; \frac{g_{\text{PSK}}}{2\sqrt{\pi}g_{\text{PSK}} + \sigma}\right) \right] (4.61)
\]

4.5.4 Numerical Results

In this section some numerical results are presented for the channel capacity, capacity statistics and symbol error rate as a function of average receiver SNR $\gamma$ in dB for a correlated MRC over slow Rayleigh fading. All curves provided are obtained using the closed-form expressions (4.51), (4.52), (4.53), (4.56), (4.57), (4.59), (4.60), and (4.61). Figures 4.8 and 4.9 show the channel capacity per unit bandwidth of correlated MRC over Rayleigh fading channels. Both figures plot the channel capacity per unit bandwidth against the correlation coefficient and they illustrate that the channel...
capacity decreases with increase of correlation coefficient $\rho$. Note that the impact of correlation is negligible for $\rho \leq 0.5$, therefore the antennas must be put apart at least half of the wavelength so that the channel fading can be considered independent. Also, the channel capacity increases with the increase of number of receive antennas and average SNR $\gamma$. Figure 4.10 depicts the exact closed-form expression capacity of (4.51) for $(L = 3)$ and $\rho = 0.3$ as well as the corresponding approximation given in (4.52). The curves are shown here along with exact result for comparison. It can be observed that the upper bound for and the high-SNR approximation for the correlated MRC match on each other for SNR $\geq 5$ dB which show a tight approximation of the exact average capacity. Furthermore, the approximations for low SNR region are two-fold: 1) the low-SNR approximation becomes tight for SNR values $< -5$ dB, whereas; 2) the expression of the low-SNR approximation II becomes tight to exact capacity between 0 and 10 dB as shown in Figure 4.10. Figure 4.11 depicts the PDF curves for different values of the correlation coefficient $\rho$, considering an average SNR per branch of $\gamma = 15$ dB and $(L = 3)$. It can be observed that the capacity distribution has a Gaussian-like shape even in the presence of correlation. As expected, the distribution of $C$ shifts slightly towards the left indicating a decreasing
value of its mean as the value of $\rho$ increases. Figure 4.12 depicts similar observations for the CCDF curves. Lastly, SER for BPSK is shown in Figure 4.13 for three correlated branch with each undergone slow Rayleigh fading. These expressions exactly resembles the SER results for uncorrelated MRC when $\rho = 0$ and a single carrier/SISO when $\rho = 1$.

To summarize, the impact of channel correlation was investigated analytically on the channel capacity statistics and SER performance of MRC over slow Rayleigh fading channels. The closed form expressions were obtained for the ergodic capacity, as well as its asymptotically tight approximation. The closed-form expressions for capacity statistics: moment generating function, probability density function and cumulative distribution function were derived. In addition, the exact analytical expression of SER for $M$-PSK modulation was obtained. The derived expressions are valid for arbitrary number of receivers. The results shows that the correlation between channels degrade the system performance.
Figure 4.11: Probability density function $P_C(C)$ for a correlated MRC with $L = 3$ at $\gamma = 15$ dB and different values of $\rho$.

Figure 4.12: Cumulative distribution function $F_C(C)$ for a correlated MRC with $L = 3$ at $\gamma = 15$ dB and different values of $\rho$. 
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4.6 Spectral Efficiency of GSC with Estimation Errors

This work was collaborated with Fawaz, and my main contribution included the validation and discussion of accuracy of the derived statistics. This section describes the performance analysis of GSC with channel estimation errors. The results in [44] were extended to obtain closed-form expressions for the single-user capacity of SCD system in the presence of Gaussian channel estimation errors.

4.6.1 System Model

Consider an $L$-branch diversity receiver in slow fading channels. Assuming perfect timing and inter-symbol interference (ISI) free transmission, the received signal on the $l$th branch due to the transmission of a symbol $s$ can be expressed as

$$r_l = g_l s + n_l, \quad l = 1 \ldots L,$$

where $g_l$ is a zero-mean complex Gaussian distributed channel gain, $n_l$ is the complex additive white Gaussian noise (AWGN) sample with a variance of $N_0/2$, and $s$ is the data symbol taken from a normalized unit-energy signal set with an average power.
Under channel estimation error, the PDF of the received instantaneous SNR $\gamma$ is given by [41]

$$p_{\gamma}(\gamma) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l D_{l-k}}{\Lambda_l} \left[ \gamma (1 - \rho^2) \right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \left( k - 1 \right) \left[ \frac{M \rho^2 \gamma}{\Lambda_l (1 - \rho^2) (l + M)} \right]^n \exp \left( -\frac{\gamma}{\Lambda_l} \right)$$

(4.63)

where $\mu$ denotes the coefficients of partial fractions, $\mu_l^0 = \left( \frac{L}{M} \right)$ for $l = 0$, and $(-1)^{M-k} \left( \frac{M}{M+1} \right) \left( \frac{M}{M+k} \right)^{M-1} \left( \frac{L}{M} \right)^{L-M}$ for $l \neq 0$. For $\rho_0^{M-k} = \sum_{i=0}^{L-M}$ $(-1)^{i+M-k-1} \left( \frac{M}{M+1} \right) \left( \frac{M}{M+i} \right)^{M-k}$ if $k < M < L$ and $(-1)^{M-l-1} \left( \frac{M}{M+1} \right) \left( \frac{L}{M} \right)^{L-M}$ for $l \neq 0$. $\Lambda_l = \gamma [(M + l(1 - \rho^2))/(l + M)]$ and $\rho$ denotes the correlation between the actual channel gains and their estimates and it can be expressed as:

$$\rho = \frac{\text{cov}(g_l, \hat{g}_l)}{\sqrt{\text{var}(g_l) \text{var}(\hat{g}_l)}} = \sqrt{1 - \epsilon^2}$$

(4.64)

The actual channel gain $g$ is related to the channel estimate $\hat{g}$ as follows

$$\hat{g}_l = \sqrt{1 - \epsilon^2} g_l + \epsilon z_l$$

(4.65)

Where $z_l$ is a complex Gaussian random variable independent of $\hat{g}$ with zero-mean and a unit variance and $\epsilon \in [0, 1]$ is a measure of the accuracy of the channel estimation. The true channel is scaled to keep the covariance of the estimated channel and the true channel to be the same. For $\epsilon = 0$, the estimated channel is fully correlated with the true channel (perfect channel estimation $\rho = 1$). Note that for perfect channel estimation, the pdf of (4.63) reduces to

$$p_{\gamma}(\gamma) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l D_{l-k} \gamma^{k-1} (l + M)}{M(k-1)! \gamma} \exp \left( -\frac{(l + M)\gamma}{M\gamma} \right)$$

(4.66)

In the following sections, the close-form expressions are derived for different adaptive schemes with GSC over Rayleigh fading channels. The analysis relied on the main results from [44].
4.6.2 Power and Rate Adaptation

Given an average transmit power constraint, the channel capacity $C_{opra}$ in (bits/seconds) of a fading channel \[43,44\] is given by

$$C_{opra} = \frac{B}{\ln 2} \int_{\gamma_0}^{\infty} \ln \left( \frac{\gamma}{\gamma_0} \right) p_{\gamma}(\gamma) d\gamma,$$  \hspace{1cm} (4.67)

where $B$ (in hertz) is the channel bandwidth and $\gamma_0$ is the optimum cutoff SNR satisfying the following condition

$$\int_{\gamma_0}^{\infty} \left( \frac{1}{\gamma_0} - \frac{1}{\gamma} \right) p_{\gamma}(\gamma) d\gamma = 1.$$ \hspace{1cm} (4.68)

To achieve the capacity in (4.67), the channel fading level must be tracked at both transmitter and receiver. The transmitter has to adapt its power and rate accordingly by allocating high power levels and transmission rates for good channel conditions (large $\gamma$). Since the transmission is suspended when $\gamma < \gamma_0$, this policy suffers from outage, whose probability $P_{out}$ is defined as the probability of no transmission and is given by

$$P_{out} = 1 - \int_{\gamma_0}^{\infty} p_{\gamma}(\gamma) d\gamma.$$ \hspace{1cm} (4.69)

To obtain the optimal cutoff SNR $\gamma_0$, in (4.69), we follow the following procedure. Let $x = \frac{\Lambda}{\gamma_0}$ and define $f_{GSC}(x)$. Now, differentiating the function $f_{GSC}(x)$ with respect to $x$ over the interval $(0, +\infty)$ resulting in

$$f'_{GSC}(x) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[ \frac{M \rho^2}{(1-\rho^2)(l+M)} \right] \left[ \frac{\Lambda_l}{\gamma_0} \right] \left( n + 1, \frac{\gamma_0}{\Lambda_l} \right) - \left( n, \frac{\gamma_0}{\Lambda_l} \right) = 1.$$ \hspace{1cm} (4.69)

To obtain the optimal cutoff SNR $\gamma_0$, in (4.69), we follow the following procedure. Let $x = \frac{\Lambda}{\gamma_0}$ and define $f_{GSC}(x)$. Now, differentiating the function $f_{GSC}(x)$ with respect to $x$ over the interval $(0, +\infty)$ resulting in

$$f'_{GSC}(x) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[ \frac{M \rho^2}{(1-\rho^2)(l+M)} \right] \left[ \frac{\Lambda_l}{\gamma_0} \right] \left( n + 1, x \right) \frac{1}{x^2}.$$ \hspace{1cm} (4.69)

Hence, $f'_{GSC}(x) < 0, \forall x > 0$, meaning that $f'_{MRC}$ is a strictly decreasing function of $x$. From (4.69) in terms of $x$, it can be observed that $\lim_{x \to 0} f_{GSC}(x) = +\infty$ and $\lim_{x \to +\infty} f_{GSC}(x) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \sum_{n=0}^{k-1} \frac{1}{n!} \left( k-1 \right)$ \hspace{1cm} (4.69)

Note however that $f_{GSC}(x)$ is a continuous function of $x$, which leads to a unique positive $\gamma_0$ such that $f_{GSC}(x) = 0$. One thereby can conclude that for each $\gamma > 0$ there is a unique $\gamma_0$ satisfying $f_{GSC}(x)$. Numerical results using MATLAB shows that
\[ C_{\text{o}pra} = \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \frac{\mu_l \lambda_{l,k}^{-1}}{\lambda_{l,k}} \left[ \gamma (1 - \rho^2) \right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2 \gamma}{(1 - \rho^2)(l + M)} \right]^n \]

\[ \int_{\gamma_0}^{\infty} \ln \left( \frac{\gamma}{\gamma_0} \right) \gamma^n \exp \left( -\frac{\gamma}{\lambda_l} \right) d\gamma \]

(4.70)

Evaluating \( I_1 \) by taking the help of the following identity \([44]\) given by \( J_n(\mu) = \int_1^\infty t^{n-1} \ln(t) e^{-\mu t} dt = \frac{\Gamma(s)}{\mu^s} \{ E_1(\mu) + \sum_{k=1}^{s-1} \frac{1}{k} P_k(\mu) \} \), where \( E_1(\mu) \) denotes the Exponential integral of the first order \([93]\) defined as \( E_1(x) = \int_1^\infty \frac{e^{-x t} t}{t - 1} dt \), \( x \geq 0 \) and \( P_k(\mu) \) denotes the poisson distribution \([93]\) given \( P_k(x) = \frac{\Gamma(k, x)}{k!} = e^{-x} x^{k-1} \frac{x^k}{k!} \). Upon substituting \( J_n(\mu) \) into (4.70), the following closed-form expression for capacity \( C_{\text{o}pra} \) per unit bandwidth (in bits/seconds/Hz) can be obtained as follows:

\[ C_{\text{o}pra} = \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \frac{\mu_l \lambda_{l,k}^{-1}}{\lambda_{l,k}} \left[ \gamma (1 - \rho^2) \right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2 \gamma}{(1 - \rho^2)(l + M)} \right]^n \]

\[ \times \left\{ E_1 \left( \frac{\gamma_0}{\gamma} \right) + \sum_{k=1}^{n+1} \frac{1}{k} P_k \left( \frac{\gamma_0}{\gamma} \right) \right\} \]

(4.71)

The above capacity expression allows to examine the limiting cases for \( (M = L, \text{and} \ M = 1) \) more conveniently for MRC and SC, respectively.

A. Asymptotic Approximation The asymptotic approximation was obtained \( C_{\text{o}pra} \) using the series representation of Exponential integral of first order function \([93]\) expressed as \( E_1(x) = -E - \ln(x) - \sum_{i=1}^{\infty} \frac{(-x)^i}{i!} \), where \( E = 0.5772156659 \) is the Euler-Mascheroni constant. Then, the asymptotic approximation \( C_{\text{o}pra} \) per unit bandwidth (in bits/seconds/hertz) can be shown to be

\[ C_{\text{o}pra} = \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \frac{\mu_l \lambda_{l,k}^{-1}}{\lambda_{l,k}} \left[ \gamma (1 - \rho^2) \right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2 \gamma}{(1 - \rho^2)(l + M)} \right]^n \]

\[ \left\{ -E - \ln \left( \frac{\gamma_0}{\gamma} \right) + \left( \frac{\gamma_0}{\gamma} \right) + \sum_{k=1}^{n+1} \frac{1}{k} P_k \left( \frac{\gamma_0}{\gamma} \right) \right\} \]

(4.72)

B. Upper Bound The capacity expression of \( C_{\text{o}pra} \) can be upper bounded by ap-
plying Jensen’s inequality to (4.67) as follows \( C_{UP}^{OPRA} = \ln \left( \mathbb{E}[\gamma] \right) \), \( C_{UP}^{OPRA} \) was evaluated using the pdf \( \gamma \) given in (4.63) and the identity [93], \( \int_0^\infty x^n e^{-\mu x} dx = n! \mu^{-n-1} \) for \( \text{Re}[\mu] > 0 \) and simplify the resulting expression to obtain the capacity (4.67) upper bound

\[
\frac{C_{UB}^{OPRA}}{B} = \ln \left( \sum_{l=0}^{L-M} \sum_{k=1}^D \frac{\mu_l^{D_l-k} \Lambda_l^{2-k} [\gamma(1 - \rho^2)]^{k-1} \sum_{n=0}^{k-1} \frac{(n+1)!}{n!} \left( \frac{k-1}{n} \right)}{(1 - \rho^2)(l + M)} \right) ^{n}.
\]

(4.73)

### 4.6.3 Constant Transmit Power

By adapting the transmission rate to the channel fading condition with a constant power, the channel capacity \( C_{ora} \) [42,96] is given by

\[
C_{ora} = \frac{B}{\ln 2} \int_0^\infty \ln (1 + \gamma) p_\gamma(\gamma) d\gamma.
\]

(4.74)

Substituting (4.63) into (4.74) results in

\[
\frac{C_{ora}}{B} = \sum_{l=0}^{L-M} \sum_{k=1}^D \frac{\mu_l^{D_l-k} \Lambda_l^{2-k} [\gamma(1 - \rho^2)]^{k-1} \sum_{n=0}^{k-1} \frac{(n+1)!}{n!} \left( \frac{k-1}{n} \right)}{(1 - \rho^2)(l + M)} \int_0^\infty \ln (1 + \gamma) \gamma^n \exp \left( -\frac{\gamma}{\Lambda_l} \right) d\gamma.
\]

(4.75)

The integral \( I_2 \) in (4.75) can be evaluated by taking use of equality [93], \( \int_0^\infty \ln (1 + y) y^{l-1} e^{-y} dy = (t - 1)! e^t \sum_{i=1}^l \frac{\Gamma(-t+i,x)}{x^i} \), where \( \Gamma(\cdot, \cdot) \) is the complementary incomplete Gamma function which can be related to the exponential integral function \( E_\ell(x) \) through [93], \( E_\ell(x) = x^{l-1} \Gamma(1 - l, x) \). It has been shown in [44] that the integral \( I_2 \) has closed form expression which is given by, \( \int_0^\infty \ln (1 + y) y^{l-1} e^{-y} dy = \frac{\Gamma(0)}{x^l} \left[ P_l(-x) E_1(x) + \sum_{i=1}^{l-1} \frac{P_{l-i}(x) P_{l-i}(-x)}{x^i} \right] \). Upon substituting this result into (4.75) yielding
closed-form expression for the capacity $C_{ora}$ per unit bandwidth (in bits/seconds/hertz)

$$C_{ora} = \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \mu_l^{D_l-k} \Lambda_l^{1-k} \sum_{n=0}^{k-1} \frac{k-1}{n} \left[ \frac{M \rho^2}{(1-\rho^2)(l+M)} \right]^n \times P_{n+1} \left( \frac{-1}{\Lambda_l} \right) E_1 \left( \frac{1}{\Lambda_l} \right) + \sum_{i=1}^{n} \frac{P_i \left( \frac{1}{\Lambda_l} \right) P_{n+1-i} \left( \frac{1}{\Lambda_l} \right)}{i} \right]$$

(4.76)

A. Asymptotic Approximation Following the same procedure in Section 4.6.2, the asymptotic approximation $C_{ora}^{\infty}$ per unit bandwidth (in bits/seconds/hertz) can be computed as

$$C_{ora}^{\infty} = \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \mu_l^{D_l-k} \Lambda_l^{1-k} \sum_{n=0}^{k-1} \frac{k-1}{n} \left[ \frac{M \rho^2}{(1-\rho^2)(l+M)} \right]^n \times P_{n+1} \left( \frac{-1}{\Lambda_l} \right) \left( -E - \ln \left( \frac{1}{\Lambda_l} \right) + \frac{1}{\Lambda_l} \right) + \sum_{i=1}^{n} \frac{P_i \left( \frac{1}{\Lambda_l} \right) P_{n+1-i} \left( \frac{1}{\Lambda_l} \right)}{i} \right]$$

(4.77)

B. Upper Bound The capacity $C_{ora}$ can be upper bounded by applying Jensen’s inequality to (4.67) as follows

$$\frac{C_{ora}^{UB}}{B} = \ln \left( 1 + \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \mu_l^{D_l-k} \Lambda_l^{1-k} \sum_{n=0}^{k-1} \frac{n+1}{n} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2}{(1-\rho^2)(l+M)} \right]^n \right)$$

(4.78)

C. Higher SNR Region The Shannon capacity can be approximated at high SNR region using the fact $\log_2(1 + \gamma) = \log_2(\gamma)$ as $\gamma \to \infty$ for $x > 0$ yields an asymptotically tight bounds for (4.76) in high SNR per unit bandwidth (in bits/seconds/hertz) as

$$C_{high} = \sum_{l=0}^{L-M} \sum_{k=1}^{D_l} \mu_l^{D_l-k} \Lambda_l^{1-k} \sum_{n=0}^{k-1} \frac{n+1}{n} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2 \gamma}{\Lambda_l(1-\rho^2)(l+M)} \right]^n \left[ \psi(n+1) - \ln \left( \frac{1}{\Lambda_l} \right) \right]$$

(4.79)

where $\psi(x)$ denotes Psi function defined as $\psi(x) = \frac{d}{dx} \ln \left( \Gamma(x) \right)$. For integer values of $x$, Psi can be represented as $\psi(x) = -E + \sum_{i=1}^{x-1} \frac{1}{i}$.

D. Lower SNR Region The approximate Shannon capacity can be derived in
low SNR region by the square capacity of the argument \((\gamma)\) as \(\log_2(1 + \gamma) \approx \sqrt{\gamma}\) [98]. Upon using this approximation along with definition of incomplete gamma function yields the approximated Shannon capacity at low SNR per unit bandwidth (in bits/seconds/hertz) as

\[
C_{\text{Low}} \approx L - M \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{D_{l-k} \kappa_{l-k}}{k-3/2} \left[ \gamma(1 - \rho^2) \right] \sum_{n=0}^{k-1} \frac{\Gamma(n+3/2)}{n!} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2 \gamma}{(1 - \rho^2)(l+M)} \right]^n
\]

\[(4.80)\]

E. Lower SNR Region II The Shannon capacity can be approximated as well in low SNR region by exploiting the fact \(\log_2(1 + \gamma) \approx \frac{1}{\ln(2)}(\gamma - \frac{1}{2} \gamma^2)\) gives the approximated Shannon capacity in low SNR region per unit bandwidth (in bits/seconds/hertz)

\[
C_{\text{Low}}^2 \approx \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{D_{l-k} \lambda_{l-k}^2}{\ln(2)n!} \left[ \gamma(1 - \rho^2) \right] \sum_{n=0}^{k-1} \left( \frac{k-1}{n} \right) \left[ \frac{M \rho^2 \gamma}{(1 - \rho^2)(l+M)} \right]^n \left[ \Gamma(n+2) - \frac{1}{2} \Gamma(n+3) \Lambda_f \right]
\]

\[(4.81)\]

4.6.4 Numerical Result

Some numerical results are given to illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of average receiver SNR \(\bar{\gamma}\) in dB for two different adaptation policies with (GSC) over slow Rayleigh fading with weight estimation errors. All curves provided are obtained using the closed-form expressions.

Figure 4.14 shows the comparison of the capacity per unit bandwidth for \(opa\), and \(ora\) policies for GSC \((L = 4, M = 3)\). The result indicates how the \(opa\) policy achieves the highest capacity for any average receive SNR, \(\bar{\gamma}\). From the same figure, it can be noticed that \(ora\) achieves less capacity than \(opa\). However, both \(opa\) and \(ora\) achieve the same result when there is no power adaptation implemented at the transmitter as in \(opa\). The results in Figure 4.14 is plotted for the case of fully estimated channel \((\rho^2 = 1)\). Figure 4.15 compares \(C_{\text{opa}}\) for different values of correlation between the channel and its estimate; namely, \(\rho^2 = 0.1, \rho^2 = 0.5\), and \(\rho^2 = 0.9\). It can be noticed that the highest \(C_{\text{opa}}\) that can be achieved is when \(\rho^2 = 1\) as shown in Figure 4.14 with perfect estimation. Furthermore, \(C_{\text{opa}}\) decreases
Figure 4.14: Capacity per unit bandwidth for a Rayleigh fading with GSC diversity \((L = 4, M = 3)\) for two adaptation schemes with perfect estimation \(\rho^2 = 1\).

Figure 4.15: Capacity per unit bandwidth for a Rayleigh fading with GSC diversity \((L = 4, M = 3)\) and various values of different \(\rho^2\) under power and rate adaptation.

when the value of \(\rho^2\) decreases where in this case the weight error increases. It can be observed from Figure 4.15 that there is almost a 3 dB difference in \(C_{opra}\).
between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Figure 4.16 shows the plot of $C_{ora}$ as well as its asymptotic approximation and upper bound as a function of the average received SNR $\gamma$ for $(L = 4, M = 3)$. As can be seen from the same figure that the $ora$ policy is more sensitive to the estimation error than the $opra$ policy by 1 dB difference between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Figure 4.17 depicts the exact closed-form expression capacity of (4.76) for $(L = 4, M = 3)$ and $\rho^2 = 0.9$ as well as the corresponding approximations given in (4.77)-(4.81). It can be observed that (4.78) and (4.79) correspond to each other for SNR $\geq 5$ dB which show a tight approximation of the exact average capacity. Furthermore, the approximations for low SNR region are two fold: 1) the expression in (4.80) becomes tight in SNRs $<-15$ dB, whereas; 2) the expression in (4.81) becomes tight to exact capacity between 0 and 10 dB as shown in Figure 4.17.

In conclusion, the closed-form expressions are derived for the channel capacity per unit bandwidth for two different adaptation policies including their approximations and upper bounds over a slow Rayleigh fading channel for GSC $(L, M)$ with estimation error. Furthermore, an upper bound as well as asymptotically tight approximations were obtained for $ora$ policy for the high and low SNR regions. The
results showed that \( \text{opra} \) outperforms \( \text{ora} \) by 1 dB difference between \( \rho^2 = 0.9 \) and \( \rho^2 = 0.1 \). Finally, it is worth to mention that the derived capacity expressions represent general formulas for GSC \( (L,M) \) with estimation error over slow Rayleigh fading channel from which those spacial limited cases of GSC (i.e, \( L = M \), MRC; \( M = 1 \), SC; \( \rho^2 = 1 \), perfect estimation; \( \rho^2 = 0 \) no diversity) can be derived.

4.7 Chapter Summary

The performance analysis of multiantenna systems with the channel estimation error and the antenna correlation was the main contribution in this chapter. Performance measures such as error rates, capacity, etc are derived in closed-form expressions. The error rates are also evaluated for rectangular QAM signalling. Following summarizes main contributions out of this chapter:

1) SER closed-form expressions were obtained for QPSK and Rectangular QAM for MRT system with channel estimation error. The results showed that evenly distributed antenna system, for a fixed number of antennas, is less susceptible to the estimation error.
2) The analysis was extended to MRC over independent but not identically distributed (i.n.d) Nakagami-$q$ fading. The obtained results specialize to previously published expressions.

3) Antenna correlation is considered to evaluate performance analysis in MRC system. Derivation of expressions for error rates forms my contribution in this joint work. The results showed that the antenna correlation degrades the system performance.

4) Finally, spectral efficiency was evaluated for GSC with Gaussian channel estimation errors. The validation of the derived capacity expressions was done as my contribution in this joint research.
Chapter 5

Analysis of Relay/Cooperative Networks

5.1 Introduction

Relay/Cooperative networks have been relatively new research topic and offer new dimension to wireless communications. Similar to spatial diversity obtained from multiple-antenna, relay/cooperative networks provides distributed spatial diversity and will be an integral part of future communications. Such communications will exploit the access to almost unlimited number of wireless nodes (both mobile subscriber and fixed base stations) in the networks to transfer information to one another. Immediate benefits would be coverage extension, diversity gain, reliability, low power, etc.

This chapter contains the research outcomes obtained during the last stage of PhD candidature. Mathematical analysis of relay/cooperative system is the main focus. First, a dual-hop relay system under mixed fading scenario is considered for performance analysis. Next evaluation of SER was performed for Rectangular QAM for selection decode-forward relay network. Afterwards, multi-antenna relay is included for the analysis. The analysis was carried out for Rayleigh fading, and then extended to Nakagami-\(m\) fading. General Order Antenna Selection (GOAS) was applied then. Finally GOAS was applied to fully MIMO relay system.
5.2 Amplify-Forward Relay in Mixed Nakagami-$m$ and Rician Fading Channels

A dual-hop relay, as shown in Figure 5.1, consists of the Source ($S$) sending signals towards the Destination ($D$) via the assistance of the Relay ($R$). The total transmission involves two time-slots. In AF relay systems, $R$ amplifies the received signal before forwarding to the $D$ node. Here, no direct link exists between $S$ and $D$ is assumed. The instantaneous end-to-end signal-to-noise-ratio (SNR) $\gamma_{eq}$ at $D$ can be shown as [48]

$$\gamma_{eq} = \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + c}$$

(5.1)

where $\gamma_{sr} = |h_{sr}|^2P_s/N_o$ and $\gamma_{rd} = |h_{rd}|^2P_d/N_o$. $P_s$ and $P_d$ are the transmit powers at $S$ and $R$, and $N_o$ is the power of additive white Gaussian noise, assumed equal at both $R$ and $D$. The $h_{sr}$ and $h_{rd}$ are the fading envelopes of the $S - R$ and $R - D$ links respectively. Finally $c$ is a constant, equal to either 1 or 0 according to the gain $G = 1/(h_{sr}^2 + N_o)$ or $G = 1/h_{sr}^2$.

Consider an asymmetric fading scenario where the $S - R$ link experiences Nakagami-$m$ fading, and the $R - D$ link is subject to Rician fading (line-of-sight) fading. Due to the symmetry of link SNRs in (5.1), the assumption is valid other way around as well. If a link experiences Nakagami-$m$ fading, the PDF and the CDF of $\gamma_{sr}$ are
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given by,

\[ p_{\gamma_{sr}}(\gamma) = \frac{m^m \gamma^{m-1}}{(m-1)!} \frac{e^{-\frac{m\gamma}{\gamma_{sr}}}}{\gamma_{sr}^m} \]  \hspace{1cm} (5.2)

\[ F_{\gamma_{sr}}(\gamma) = 1 - e^{-\frac{m\gamma}{\gamma_{sr}}} \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{m\gamma}{\gamma_{sr}} \right)^i \]  \hspace{1cm} (5.3)

where \( m = 1, 2, \ldots \) is fading severity parameter, \( \gamma_{sr} = E\{|h_{sr}|^2\} P_s/N_o \) is average SNR of S – R hop. Note that when \( m = 1 \), the distribution becomes Rayleigh. And the \( R – D \) SNR \( \gamma_{rd} \) is a noncentral-\( \chi^2 \) distributed, given by

\[ p_{\gamma_{rd}}(\gamma) = \frac{(K+1)e^{-K \gamma_{rd}}}{\bar{\gamma}_{rd}} e^{-\frac{(K+1)\gamma_{rd}}{\gamma_{rd}}} I_0 \left( 2\sqrt{\frac{K(K+1)}{\bar{\gamma}_{rd}}} \right) \]  \hspace{1cm} (5.4)

where \( K \) is the ratio of the powers of the line-of-sight component to the scattered components, \( \bar{\gamma}_{rd} = E\{|h_{rd}|^2\} P_r/N_o \) is the average SNR of \( R – D \) hop, and \( I_0(\cdot) \) is the zeroth order modified Bessel function of the first kind [73, pp 900].

5.2.1 SNR Statistics

1. Cumulative Distribution Function (CDF)

The cumulative distribution function (CDF) statistics of received SNR at the destination can be obtained as,

\[ F_{\gamma_{eq}}(\gamma) = \Pr\left( \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + c} < \gamma \right) = 1 - \int_0^{\infty} C_{\gamma_{sr}} \left( \gamma + \frac{\gamma^2 + c^2}{w} \right) p_{\gamma_{rd}}(\gamma + w)dw \]

where \( C_{\gamma_{xx}}(\gamma) = 1 - F_{\gamma_{xx}}(\gamma) \) is complementary CDF of \( \gamma_{xx} \). Inserting (5.4) and (5.3) into above equation yields

\[ F_{\gamma_{eq}}(\gamma) = 1 - \frac{K+1}{\bar{\gamma}_{rd}} e^{-K \left[ \frac{\bar{\gamma}_{rd}+c}{\gamma_{sr}} \right]} \int_0^{\infty} e^{-\frac{(K+1)w}{\gamma_{rd}}} e^{-\frac{m(\gamma+c)w}{\gamma_{sr}w}} \]

\[ \times I_0 \left( 2\sqrt{\frac{K(K+1)}{\bar{\gamma}_{rd}}} \right) \sum_{i=0}^{m-1} \frac{1}{i!} \left( \frac{m\gamma}{\gamma_{sr}} \right)^i \left( 1 + \frac{c + \gamma}{w} \right)^i dw \]  \hspace{1cm} (5.5)

Using infinite-series representation of \( I_0(z) = \sum_{j=0}^{\infty} \frac{(\frac{z^2}{2})^{2j}}{(2j)!} \) [73, eqn. 8.44.1] and binomial expansion for the last term \([ \text{i.e. } (x + y)^n = \sum_{p=0}^{n} \binom{n}{p} x^{n-p} y^p \] , following is
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obtained,

$$F_{\gamma_{eq}}(\gamma) = 1 - e^{-K - \lambda \gamma} \sum_{j=0}^{\infty} K^j \left( \frac{K + 1}{\gamma_{rd}} \right)^{j+1} \times \sum_{l=0}^{\gamma} \sum_{p=0}^{\infty} \sum_{i=0}^{\infty} \gamma^{i+j-l}(\gamma + c)^{i-p} \frac{m^{i+p+1}}{i!l!j!} \times \int_{0}^{\infty} e^{-\frac{(K+1)w - m(\gamma + c)\gamma}{\gamma_{sr}w}} w^{i-p} dw \right)$$  (5.6)

where \(\lambda = \frac{(K+1)}{\gamma_{rd}} + \frac{m}{\gamma_{sr}}\). With the help of [73, eqn. 3.471.9] to evaluate \(I_1\), the CDF of end-end SNR for amplify-forward dual-hop relay under mixed Nakagami-\(m\) and Rician fading can be written as

$$F_{\gamma_{eq}}(\gamma) = 1 - 2e^{-K - \lambda \gamma} \sum_{j=0}^{\infty} K^j \left( \frac{K + 1}{\gamma_{rd}} \right)^{j+1} \sum_{l=0}^{\gamma} \sum_{p=0}^{\infty} \sum_{i=0}^{\infty} \gamma^{i+j-l}(\gamma + c)^{i-p} \frac{m^{i+p+1}}{i!l!j!} \times \left( \frac{m}{\gamma_{sr}} \right)^{\theta_1} \left( \frac{K + 1}{\gamma_{rd}} \right)^{\theta_2} \gamma^{2i+l-p+1} \frac{(\gamma + c)^{i+p+1}}{l!} \right) \times K_{l-p+1} \left( 2 \sqrt{\frac{m(K + 1)(\gamma + c)\gamma}{\gamma_{sr}\gamma_{rd}}} \right)$$  (5.7)

where \(\theta_1 = \frac{2i+l-p+1}{2}, \theta_2 = \frac{2i+l-p+1}{2}\), and \(K_l(.)\) is \(l\)-order modified Bessel function of second kind [73, eqn. 8.446]. For \(m=1\), (5.7) specializes to [52, eqn. 8] as expected.

Figures 5.2 and 5.3 plots the CDFs for two different scenarios. In the first case, the average SNR in \(R - D\) link (Rician fading) is twice that of in \(S - R\) link. The Rician factor \(K = 5dB\) and the Nakagami fading parameter \(m = 3\) are assumed. The analytical plots (continuous and dot curves) were drawn using (5.7) for \(c = 0\) and \(c = 1\). Obviously, higher link average SNR improves the system performance. The difference between the curves for \(c = 0\) and \(c = 1\) is small, and diminishes as threshold SNR \(\gamma\) increases. Therefore \(c = 0\), which aids mathematical analysis, can be used for system performance analysis without sacrificing much of accuracy.

Figure 5.3 confirms the observation for different settings where Rayleigh-Rayleigh fading \((K = -10dB\) and \(m = 1\)) is compared with less severe fading \((K = 10dB\) and \(m = 3\)). Both links have same average SNR in this case. Monte-carlo simulation results are also shown and validates the analytical results.
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Figure 5.2: Comparison of Cumulative Distribution Functions (CDFs) for $c = 0$ and $c = 1$, $K = 5\, dB$, $m = 3$, $\bullet$ indicates Monte-Carlo simulation points.

2. Probability Density Function (PDF)

The probability density function (PDF) can be obtained by taking derivative of (5.7) with respect to $\gamma$ as shown below

$$p_{\gamma_{eq}}(\gamma) = 2e^{-K-\lambda\gamma} \sum_{j=0}^{\infty} \frac{K^j}{j!} \sum_{l=0}^{j} \frac{j!}{l!} \sum_{i=0}^{m-1} \frac{i!}{p!} \left( \frac{m}{\gamma_{sr}} \right)^{\theta_1} \left( \frac{K + 1}{\gamma_{rd}} \right)^{\theta_2} \frac{\gamma^{i+j+1}}{i!} \left[ K_{I-p+1}(2\gamma\zeta) \right]$$

$$\times \left\{ \lambda + \frac{l-p-i-j}{\gamma} \right\} + 2\zeta K_{I-p}(2\gamma\zeta)$$

(5.8)

where $\zeta = \sqrt{\frac{m(K+1)}{\gamma_{sr}\gamma_{rd}}}$.

Note that $z\frac{dK_v(z)}{dz} + vK_v(z) + zK_{v-1}(z) = 0$ [88, eqn. 8.486-12] was used to arrive above expression. Two special cases can be deduced:

Rayleigh-Rician and Rayleigh-Rayleigh fading scenarios.

Case I: When $m = 1$, one gets Rayleigh-Rician fading scenario considered in [52]
where only CDF was derived. In this case, \( i = p = 0 \) thereby yielding

\[
p_{\gamma_{eq}}(\gamma) = 2e^{-K-\lambda\gamma} \sum_{j=0}^{\infty} \frac{K^j}{j!} \sum_{l=0}^{j} \binom{j}{l} \left( \frac{K + 1}{\bar{\gamma}_{rd}} \right)^{\frac{2j-l+1}{2}} \times \frac{\gamma^{j+1}}{\bar{\gamma}_{sr}^{j+1}} \left[ K_{l+1}(2\gamma\zeta) \left\{ \lambda + \frac{l-j}{\gamma} \right\} + 2\zeta K_{l}(2\gamma\zeta) \right]
\]

(5.9)

**Case II:** When \( m = 1 \) and \( K = 0 \), fading scenario becomes Rayleigh-Rayleigh case considered in [48]. It can be easily seen that the infinite sum reduces to a single term \((j = 0)\), thereby (5.9) specializes to [48, eqn. 19].

Figure 5.4 shows the comparison of PDF plots for four different settings. The first two cases include Rayleigh-Rayleigh \((K = -10dB, m = 1, \bar{\gamma}_{sr} = \bar{\gamma}_{rd} = 5dB)\) and Rayleigh-Rician \((K = 10dB, m = 1, \bar{\gamma}_{sr} = \bar{\gamma}_{rd} = 5dB)\) fadings. Next two cases were considered to observe the impact of Nakagami fading parameter \( m \) and unbalanced link SNR values on the end-end statistics. Higher \( m \) reduces the variance, however doubling the \( R-D \) SNR has a opposite effect. The analytical results are exactly in accordance with the simulation confirming accuracy of analysis.
3. Moment Generating Function (MGF)

The moment generating function (MGF) is an important statistics, which can be used to derive error probabilities efficiently for a varieties of modulation types. MGF is related to PDF by $M_{\gamma_{eq}}(s) = \int_0^\infty e^{-sy}p_{\gamma_{eq}}(y)dy$. Here, the MGF can be easily derived in the form of hypergeometric functions with the aid of [88, eqn. 6.621-3].

4. When the Direct Link Exists

When $S \rightarrow D$ link is not in deep fade and active, one can easily extend above analysis to offer cooperative diversity at the destination. Two most popular combining schemes are Maximal Ratio Combining (MRC) and Selection Combining (SC). The MRC requires continuous channel estimation of all branches ($S \rightarrow R \rightarrow D$ and $S \rightarrow D$), and provides optimum gain whereas SC is a sub-optimum scheme. Assuming independence between the relayed $S \rightarrow R \rightarrow D$ and the direct link ($S \rightarrow D$), and when MRC is employed at the destination, the MGF of the total SNR becomes, $M_{\gamma_{tot}}(\gamma) = M_{\gamma_{eq}}(\gamma).M_{\gamma_{sd}}(\gamma)$. Once the MGF is known, ASEP can be easily computed. Taking inverse Laplace transform of the MGF results the PDF.

If SC is used at the destination instead, one can write the CDF of the total SNR as $F_{\gamma_{tot}}(\gamma) = F_{\gamma_{eq}}(\gamma).F_{\gamma_{sd}}(\gamma)$. Similarly the PDF can be calculated as $f_{\gamma_{tot}}(\gamma) =$
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\[ F_{\gamma_{eq}}(\gamma) \cdot f_{\gamma_{rd}}(\gamma) + f_{\gamma_{eq}}(\gamma) \cdot F_{\gamma_{rd}}(\gamma). \]

5.2.2 Performance Analysis

In this section, the analysis presented in the previous section was used to compute two important measures - outage probability (OP) and average symbol error probability (ASEP).

A. Outage Probability (OP)

The outage probability quantifies the likelihood of the instantaneous end-to-end SNR falling below some predetermined threshold \( \gamma_{th} \), given by \( P_{out} = F_{\gamma_{eq}}(\gamma < \gamma_{th}) \).

A lower bound of outage probability can be obtained by simple approximation as shown below. This approximation takes account of the lower link SNR and independence of two links, and has been used in several papers such as in [52]. It simplifies the analysis and is tight at medium to high SNR values.

\[
P_{out} = \Pr \left[ \min(\gamma_{sr}, \gamma_{rd}) < \gamma \right] = 1 - C_{\gamma_{sr}}(\gamma)C_{\gamma_{rd}}(\gamma) = 1 - e^{-\frac{m\gamma}{\bar{\gamma}_{sr}}} Q_{1}\left(\sqrt{2K}, \sqrt{\frac{2(K+1)\gamma}{\bar{\gamma}_{rd}}}\right) \sum_{i=0}^{m-1} \frac{1}{i!} \left(\frac{m\gamma}{\bar{\gamma}_{sr}}\right)^i \tag{5.10}
\]

where \( Q_{1}(., .) \) is the first-order Marcum Q-function [2, eqn. 4.33] and has following series representation \( Q_{1}(\alpha, \beta) = e^{-\frac{\alpha^2 + \beta^2}{2}} \sum_{k=0}^{\infty} \left(\frac{\alpha}{\beta}\right)^k I_k(\alpha\beta) \) where \( I_k(x) \) is the \( k^{th} \)-order Bessel function of the first kind.

B. Average Symbol Error Probability (ASEP)

Average symbol error probability (ASEP) is an important system performance metric useful for system designers. The ASEP of communication link over fading channels is normally computed by averaging the conditional symbol error probability (SEP) in AWGN channel over the range of fading distribution. Specifically,

\[
P_s(e) = \int_{0}^{\infty} Q\left(\sqrt{\beta\gamma}\right) p_{\gamma_{eq}}(\gamma) d\gamma \tag{5.11}
\]

where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt \) is the Gaussian Q-function and essentially represents the conditional SEP over AWGN channel, and \( \beta \) is a constant specific to modulation type (BPSK, \( \beta = 2 \); QPSK, \( \beta = 1 \), etc). (5.11) can be written into following form [52]
which allows to directly use CDF (5.7) to compute ASEP,

\[ P_s(e) = \frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq}} \left( \frac{t^2}{\beta^2} \right) e^{-\frac{t^2}{\beta^2}} dt = \frac{1}{2\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq}} \left( \frac{t}{\beta} \right) e^{-\frac{1}{2}t - \frac{1}{2}t} dt \]  

Inserting (5.7) into (5.12) yields

\[ P_s(e) = 1 - \frac{e^{-K}}{\sqrt{2\pi}} \sum_{j=0}^\infty \frac{K^j}{j!^2} \sum_{l=0}^j \frac{(j)^{m-1} \sum_{i=0}^l \sum_{p=0}^{j-l}(i)}{l! m^{\theta_1}} (K + 1)^\theta_2 \]

\[ \times \int_0^\infty t^{i+j+\frac{1}{2}} e^{-\frac{2\lambda+\beta}{2\lambda}} t K_{p+l-i+1} \left( \frac{2t}{\beta} e^{\frac{m(K+1)}{\gamma_{sr} \gamma_{rd}}} \right) \]  

which using the following identity [73, eqn. 6.621.3],

\[ \int_0^\infty x^{\mu-1} e^{-\alpha x} K_{\nu}(\beta' x) dx = \frac{\sqrt{\pi}(2\beta')^{\nu} \Gamma(\mu + \nu) \Gamma(\mu - \nu)}{(\alpha + \beta')^{\mu+\nu} \Gamma(\mu + 1/2)} 2F1(\mu + \nu, \nu + 1/2; \mu + 1/2; \alpha - \beta') \text{Re}(\alpha + \beta') > 0 \]  

becomes (c=0),

\[ P_s(e) = \frac{1}{2} - \psi \sum_{j=0}^\infty (K \gamma_{sr})^j (K + 1)^{j+l} \sum_{l=0}^j \frac{(j)^{m-1} \sum_{i=0}^l \sum_{p=0}^{j-l}(i)}{l! m^{\theta_1}} (K + 1)^\theta_2 \]

\[ \times \left( i \right) \frac{2^{i-p}(p+\theta_5)}{i!} \frac{\Gamma(\theta_3) \Gamma(\theta_4)}{q^{\theta_5} \Gamma(\theta_5)} 2F1(\theta_3 + l - i + 1/2; \theta_5, \omega) \]  

where \( \psi = 2 \Gamma(2\beta' \gamma_{sr} \gamma_{rd}) \sqrt{2} \gamma_{sr} \gamma_{rd} \), \( \theta_3 = i + j + l - p + 1/2, \theta_4 = i + j - l - p + 1/2, \theta_5 = i + j + 2, \nu = (2\lambda + \beta) \gamma_{sr} \gamma_{rd} + 2 \sqrt{(K + 1) \gamma_{sr} \gamma_{rd}}, \omega = \nu - 4 \sqrt{(K + 1) \gamma_{sr} \gamma_{rd}}, \Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \) is the gamma function, and \( 2F1(a; b; c; x) \) is the Gauss hypergeometric function defined in [73, eqn. 9.111]. Using approximated CDF (5.10), it is easy to derive an expression for approximated ASEP (not shown here).

The final ASEP expression (5.15) specializes to [52, eqn. 14] for Rayleigh fading \( (m=1) \). Modified Bessel function in (5.7) and Gauss hypergeometric function in (5.15) are available as built-in functions in standard software packages such as MATLAB, MATHEMATICA, and therefore facilitates efficient computation of the
final expressions.

5.2.3 Numerical Results and Discussion

Some numerical results are presented based on the earlier analysis. The purpose is to understand the system behavior for different settings of severity parameters $m$ and $K$, and average link SNRs. The figures were drawn using (5.7), (5.10) and (5.15) in Matlab. The infinite summation was computed for first 20 terms ($j = 0, 1, 2, ..., 19$) which was found sufficient for the effective SNR range considered here. Please note that the terms $1/j!^2$ and $1/i!$ in the final expressions, which makes the contributions of higher terms negligible. Mathematical justification is possible however out of scope of this contribution.

Figure 5.5 shows the outage probability of dual-hop relay system with mixed Nakagami-$m$ and Rician fading channels. Equal average link SNRs are assumed. Consider $m = 1, 5$ for $K = 5dB$ and $\gamma_{th} = 10dB$ first. The gain with higher $m$ (less severe fading) is obvious and remains same (shown by equal slopes) irrespective of link SNRs at high SNR values. With higher $K = 10dB$ and lower threshold $\gamma_{th} = 5dB$, the gain becomes more prominent. The outage behavior remains similar to previous case at very high SNR region (not shown here). The approximate outage probability is also depicted and matches well with the exact one at high SNR region ($> 20dB$).

Figure 5.6 shows the ASEP for QPSK modulation ($\beta = 1$) for similar settings. For $m = 1$ which is Rayleigh fading case, $K = 6dB$ offers constant gain of 2.5dB over $K = 0dB$ in high SNR region. And with a fixed $K = 6dB$, higher $m$ provides only marginal (almost none for $m > 3$) gain as illustrated by $m = 2, 3$. Similar observation was taken for $K = 0dB$ and $m = 2, 3, 5$ (not shown here to avoid ambiguity in the figure). Another interesting observation is the diversity order of the system. The diversity order of the system is independent of $m$, and equal to one. This is similar to the observation in [52].

In summary, the system performance of dual-hop relay system over mixed fading scenarios has been considered. The analysis is carried over Nakagami-$m$ and Rician fading channels for amplify-and-forward scheme. The SNR statistics have been derived and validated through computer simulation. Furthermore, closed-form ex-
expressions for the OP and the ASEP are derived, which are in terms of infinite series. The diversity order of the system is one and does not depend on severity parameter.
m. The derived expressions are in more general forms than the previously published results.

5.3 Rectangular QAM in Selection Decode-and-Forward Relay Networks over Nakagami-m Fading Channels

This work was done as co-investigator, and my main contribution included the validation of the derived error rates expression. This section describes the extension of the previous work [58] on fixed selection DF relay networks to derive closed form expression of average SEP for rectangular QAM signals. Unlike [58] where the final SEP expression is in the integral form, the SEP expression are given in terms of finite sums which is simpler and more tractable. The final expression is in general form, expressions for Rayleigh fading and Square QAM cases can be derived as special cases.

Consider a wireless system where a source node $S$ transmits to a destination $D$ with assistance of $K$ relay nodes $\{R_1, R_2, \ldots, R_k\}$. The source $S$ transmits the signal (broadcasting phase) to $K$ relays $(S - R_k)$. Then, only the best relay is selected to forward the source’s signal to the destination $(R_k - D)$. Let us denote $h_{SR_k}$ and $h_{R_k D}$ as the channel coefficients of $S - R_k$ and $R_k - D$ links, respectively. The channels for all links are assumed i.n.i.d Nakagami-m fading. Moreover, the instantaneous signal-to-noise ratio (SNR) for $S - R_k$ and $R_k - D$ links are denoted as $\gamma_{SR_k} = \varpi |h_{SR_k}|^2$ and $\gamma_{R_k D} = \varpi |h_{R_k D}|^2$ respectively, where $\varpi$ is the average SNR. The effective channel powers $|h_{SR_k}|^2$ and $|h_{R_k D}|^2$ follow the gamma distribution with different fading parameters $1/\Omega_{SR_k}$, $1/\Omega_{R_k D}$ and fading severity parameters $m_{1k}$, $m_{2k}$, respectively.

The fixed DF relay protocol is considered, where the selected relay terminal is always active to assist the direct communication. With the fixed DF relaying operation, the dual-hop $S-R_k-D$ channel can be tightly approximated in the high SNR regime as follows [29] [32]:

$$\gamma_{eq_k} = \min\{\gamma_{SR_k}, \gamma_{R_k D}\}$$

(5.16)

For the relay SC scheme, the relay with largest equivalent received SNR is selected, and thus, the instantaneous SNR at the destination of the SC system with fixed DF
relays can be expressed as follows:

\[ \gamma_{sc} = \max_{k=1 \ldots K} \gamma_{eq_k} \]  

(5.17)

The MGF of \( \gamma_{sc} \) is defined as \( M_{\gamma_{sc}}(s) = E[e^{s\gamma_{sc}}] \) with \( E[\cdot] \) is the expectation operator, and can be written as [58, eqn. 16]

\[
M_{\gamma_{sc}}(s) = \sum_{k=1}^{K} \frac{A_k n_k^1!}{(\beta_{A_k} - s)^{n_k^1 + 1}} + \sum_{k=1}^{K} \frac{B_k n_k^2!}{(\beta_{B_k} - s)^{n_k^2 + 1}}
\]  

(5.18)

where \( \alpha_k = \frac{m_{1k}}{n_{SR_k}}, \beta_k = \frac{m_{2k}}{n_{RC_k}}, \beta A_k = \beta B_k = (\alpha_k + \beta_k + \sum_{t=1}^{f}(\alpha_m + \beta_m)), n_k^1 = m_{1k} - 1 + u + \sum_{t=1}^{f}(\gamma_m + \beta_m), \) and \( n_k^2 = m_{2k} - 1 + v + \sum_{t=1}^{f}(\gamma_m + \beta_m) \). Furthermore, \( A_k \) and \( B_k \) are calculated as follows:

\[
A_k = \frac{\alpha_k^{m_{1k}}}{\Gamma(m_{1k})} \sum_{u=0}^{m_{1k}-1} \frac{\beta_u}{u!} \sum_{t=0}^{K} \left( \frac{(-1)^t}{t!} \sum_{n_1=1}^{K} \cdots \sum_{n_t=1}^{K} \prod_{l=1}^{K} \left( \frac{\alpha_{n_l}}{\beta_{n_l}} \right) \right)
\]  

(5.19)

\[
B_k = \frac{\beta_k^{m_{2k}}}{\Gamma(m_{2k})} \sum_{v=0}^{m_{2k}-1} \frac{\alpha_v}{v!} \sum_{t=0}^{K} \left( \frac{(-1)^t}{t!} \sum_{n_1=1}^{K} \cdots \sum_{n_t=1}^{K} \prod_{l=1}^{K} \left( \frac{\beta_{n_l}}{\beta_{n_l}} \right) \right)
\]  

(5.20)

5.3.1 Average Symbol Error Probability

The average SEP in terms of MGF can be written as

\[
P_s = \frac{2q(I)\pi/2}{\pi} \int_{0}^{\pi/2} M_{\gamma_{sc}} \left( -\frac{A_1}{\sin^2 \phi} \right) d\phi + \frac{2q(J)\pi/2}{\pi} \int_{0}^{\pi/2} M_{\gamma_{sc}} \left( -\frac{A_2}{\sin^2 \phi} \right) d\phi
\]  

\[-\frac{2q(I)q(J)}{\pi} \sum_{\kappa \in P_2} \tan^{-1}(\sqrt{A_{\kappa_2}/A_{\kappa_1}}) \int_{0}^{\tan^{-1}(\sqrt{A_{\kappa_2}/A_{\kappa_1}})} M_{\gamma_{sc}} \left( -\frac{A_{\kappa_2}}{\sin^2 \phi} \right) d\phi
\]  

(5.21)
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\[ P_s = \frac{2q(I)}{\pi} \sum_{k=1}^{K} \left[ A_k n_1^k ! I_1(\beta_{A_k}, A_1, n_1^k) + B_k n_2^k ! I_1(\beta_{B_k}, A_1, n_2^k) \right] + \] (5.22)

\[ \frac{2q(J)}{\pi} \sum_{k=1}^{K} \left[ A_k n_1^k ! I_1(\beta_{A_k}, A_2, n_1^k) + B_k n_2^k ! I_1(\beta_{B_k}, A_2, n_2^k) \right] - \frac{2q(I)q(J)}{\pi} \]

\[ \sum_{\kappa \in P_2(2)} \sum_{k=1}^{K} \left[ A_k n_1^k ! I_2(\beta_{A_k}, A_{\kappa_1}, A_{\kappa_2}, n_1^k) + B_k n_2^k ! I_2(\beta_{B_k}, A_{\kappa_1}, A_{\kappa_2}, n_2^k) \right] \]

where \( A_1 = g(I, J, \zeta), \ A_2 = g(I, J, \zeta) \zeta \) and \( \kappa = (\kappa_1, \kappa_2) \) is a permutation to \( P_2(2) = \{(1, 2), (2, 1)\} \). By incorporating (5.18) in (5.21), it yields the average SEP of arbitrary rectangular QAM over i.i.d Nakagami-m fading channels as given by (5.22) at the top of the next page. Furthermore, the \( P_s \) can be obtained in closed form expression by evaluating the integrals \( I_1, I_2 \) in (5.22). By making change of variable \( t = \cos^2 \phi \), the integral \( I_1 \) can be obtained in closed form expression as

\[
\int_{0}^{\pi/2} \left( \frac{1}{\beta_{A_k} + \frac{A_1}{\sin^2(\phi)}} \right)^{n_1^k + 1} d\phi = \frac{1}{2} \left( \frac{\beta_{A_k} + A_1}{\beta_{A_k} + A_1} \right)^{-n_1^k - 1} \int_{0}^{1} t^{-1/2} (1-t)^{n_1^k + 1/2} dt \\
\times \left( 1 - \frac{\beta_{A_k}}{\beta_{A_k} + A_1} \right)^{-n_1^k - 1} t^{-1/2} \left( \frac{\beta_{A_k}}{\beta_{A_k} + A_1} \right) \\
= \frac{1}{2} 2F_1 \left( n_1^k + 1, \frac{1}{2}; n_1^k + \frac{3}{2}; \beta_{A_k} \right) \beta_{A_k} + A_1)_{n_1^k + 1} B \left( \frac{1}{2}, n_1^k + \frac{3}{2}; \right) \] (5.23)

where \( 2F_1(a; b; c; x) \) is Gauss’s hypergeometric function and \( B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \) denotes the Beta function [93]. For the integral \( I_2 \) in (5.22), we carry out change of variable \( t = 1 - \frac{A_{\kappa_2}}{A_{\kappa_1}} \tan^2 \phi, \ dt = -2 \tan \phi \sec^2 \phi d\phi, \sin^2 \phi = \frac{1-t}{2-t} \), then after further
manipulation, the integral \( I_2 \) can be written in terms of hypergeometric series

\[
I_2(a, A_{\kappa_1}, A_{\kappa_2}, n) = \int_0^\eta \left( \frac{\sin^2 \phi}{\sin^2 \phi + A_{\kappa_2}} \right)^{n+1} d\phi
\]

\[
= \frac{A_{\kappa_1}}{A_{\kappa_2}(a + 2b)^{n+1}} \int_0^1 (1 - t)^{n+\frac{1}{2}} \left( 1 - \frac{A_{\kappa_2} t}{a + 2A_{\kappa_2}} \right)^{-n-1}
\]

\[
\times \left( 1 - \frac{t}{2} \right)^{-1} dt
\]

\[
= \frac{A_{\kappa_1} F_1 \left( 1, 1, n + 1, n + \frac{5}{2}, 1; \frac{A_{\kappa_1}}{a + 2A_{\kappa_2}} \right)}{2A_{\kappa_2}(a + 2A_{\kappa_2})^{n+1} B(n + 1/2, 1)} \quad (5.24)
\]

where \( \eta = \tan^{-1} \left( \sqrt{\frac{A_{\kappa_2}}{A_{\kappa_1}}} \right) \) and \( F_1(a, b_1; b_2; c; x_1; x_2) \) is Appell’s hypergeometric function [93]. By incorporating the corresponding results of integral \( I_1 \) and \( I_2 \) into (5.22), it yields the final closed form expression of the average SEP of selection DF relay over (i.n.i.d) Nakagami-\( m \) fading channels for general rectangular QAM as given by (5.25) at the top of the next page. The SEP expression given in (5.25) involves finite summations of functions which can be efficiently evaluated using popular mathematical software such as MATLAB, MATHEMATICA and MAPLE. It is worthwhile to mention that when in-phase and quadrature distance between \( d_I \) and \( d_J \) are equal, i.e., \( \zeta = 1 \) then \( A_1 = A_2 = g(I, J, \zeta) = 3/[(I^2 - 1) + (J^2 - 1)] = A \) yielding a simplified expression for a particular case of (5.25).

Similar expression can be obtained for square \( M \)-ary QAM with \( d_I = d_J \) with \( I = J = \sqrt{M} \) implying that \( g(I = \sqrt{M}, J = \sqrt{M}, \zeta) = 3/(2M - 2) \).

### 5.3.2 Numerical Results and Discussion

Few numerical results are shown to illustrate mathematical analysis in previous section. The analytical results are validated with Monte-Carlo simulations. A general case of Nakagami-\( m \) fading channels is considered with similar/dissimilar fading parameters. A selection DF relay networks with three relays \( (K = 3) \) is assumed for two cases configured as: 1) Similar case: \( \{\Omega_{SR_k}\}_{k=1}^3 = \{\Omega_{RD}\}_{k=1}^3 = 3 \) and \( \{m_{1k}\}_{k=1}^3 = \{m_{2k}\}_{k=1}^3 = 2 \) and 2) Dissimilar case: \( \{\Omega_{SR_k}\}_{k=1}^3 = 1, 2, 3, \{\Omega_{RD}\}_{k=1}^3 = 3, 2, 1 \) and \( \{m_{1k}\}_{k=1}^3 = 1, 2, 3, \{m_{2k}\}_{k=1}^3 = 3, 2, 1 \). Figure 5.7 shows the SEP of \( 8 \times 4 \) QAM with \( \zeta = 21/5 \), computed using (5.25), versus average SNR.
\[ P_s = \frac{q(I)}{\pi} \sum_{k=1}^{K} A_k n_1^{k1} 2F_1 \left( n_1^k + 1, \frac{1}{2}; n_1^k + \frac{3}{2}; \frac{\beta_{A_k}}{\beta_{A_k} + 1} \right) \left( \frac{\beta_{A_k} + 1}{A_1} \right) n_1^k B \left( \frac{1}{2}, n_1^k + \frac{3}{2} \right)^{-1} \]

\[ + \frac{q(J)}{\pi} \sum_{k=1}^{K} B_k n_2^{k1} 2F_1 \left( n_2^k + 1, \frac{1}{2}; n_2^k + \frac{3}{2}; \frac{\beta_{B_k}}{\beta_{B_k} + A_2} \right) \left( \frac{\beta_{B_k} + A_2}{A_2} \right) n_1^k B \left( \frac{1}{2}, n_1^k + \frac{3}{2} \right)^{-1} \]

\[ + \frac{q(I)}{\pi} \sum_{k=1}^{K} A_k n_1^{k1} 2F_1 \left( n_1^k + 1, \frac{1}{2}; n_1^k + \frac{3}{2}; \frac{\beta_{A_k}}{\beta_{A_k} + A_1} \right) \left( \frac{\beta_{A_k} + A_1}{A_2} \right) n_2^k B \left( \frac{1}{2}, n_2^k + \frac{3}{2} \right)^{-1} \]

\[ + \frac{q(J)}{\pi} \sum_{k=1}^{K} B_k n_2^{k1} 2F_1 \left( n_2^k + 1, \frac{1}{2}; n_2^k + \frac{3}{2}; \frac{\beta_{B_k}}{\beta_{B_k} + A_2} \right) \left( \frac{\beta_{B_k} + A_2}{A_2} \right) n_2^k B \left( \frac{1}{2}, n_2^k + \frac{3}{2} \right)^{-1} \]

\[ - \frac{q(I)q(J)}{\pi} \sum_{\kappa \in \mathcal{P}_2(2)} \sum_{k=1}^{K} B_k^2 n_1^{k1} A_1 n_1 A_2 A_2 \left( \frac{A_2}{\beta_{A_k} + 2A_2} \right) \left( A_1 \left( \frac{\beta_{A_k} + 2A_2}{\beta_{A_k} + 1} \right) \right)^{-n_2^k - 1} \]

\[ \times F_1 \left( 1, 1, n_2^k + 1, n_2^k + \frac{5}{2}; \frac{1}{2}; \frac{A_2}{\beta_{B_k} + 2A_2} \right) \]

As can be observed from this figure, analytical results are in good agreement with those of Monte-Carlo simulations. Furthermore, to quantify the effect of in-phase and quadrature decision distance of QAM, i.e., \( \zeta \), on the SEP performance, numerical results of SEP with different values of \( \zeta = 1, \sqrt{30/7}, \) and \( 30/7 \) were shown in Figure 5.8. It is evident that the SEP performance improves with the decrease of \( \zeta \) i.e., Square QAM performs the best. At moderate-to-large SNR regime, the cases with \( \zeta = \sqrt{30/7} \) and \( \zeta = 30/7 \) lose 0.3 dB and 2 dB respectively relative to the the case of \( \zeta = 1 \). The closed-form expressions of SEP with arbitrary M-ary rectangular QAM for selection DF relay networks over i.n.i.d Nakagami-\( m \) fading channels are derived. The analytical expressions are in terms of finite summation and hypergeometric series which can be efficiently computed in standard software packages. Special cases
Figure 5.7: Symbol error probability of selection DF relay networks in Nakagami-
 fading channels versus SNR.

Figure 5.8: Symbol error probability of selection DF relay networks in Nakagami-
fading channels versus SNR.

can be easily derived, square QAM for example.
5.4 Amplify and Forward Cooperative Transmission with a Multi Antenna Relay

Consider a case where the source and the destination nodes are single-antenna terminals whereas the relaying node is equipped with $N$ multiple antennas. The scenario can be found in many infrastructure wireless networks. The relay $R$ processes the received signal from the source by Maximal Ratio Combining (MRC). The MRC is an optimal combining scheme, but requires complete channel state information. The output signal is amplified and forwarded to the destination. Only one antenna at the relay is chosen for the transmission for $R \rightarrow D$ link, called Transmit Antenna Selection (TAS). The antenna is selected based on the largest channel gain between the relay antennas and the destination antenna. Only an index of antenna is necessary for the feedback, as such TAS is a bandwidth efficient scheme. Mathematically, the received signal vector at $R$ can be written as,

$$y_r = h_1 x + n_r$$

(5.26)

where $x$ is a modulated signal with normalized energy, $n_r$ is the additive white Gaussian noise vector with $E[n_r n_r^H] = I_n N_0$, $I_n$ is an identity matrix of size $N$ and $N_0$ is the noise power at the relay antennas. In MRC, the $y_r$ is multiplied by
some vector $w^H$ to obtain the received signal as,

$$y_r = w^H y_r = w^H h_1 + w^H n_r$$

(5.27)

This signal gets amplified by $G$ at $R$ and transmitted through the best antenna to the $D$. The signal at $D$, therefore is,

$$y_d = h_{2,b} G y_r x + n_d = h_{2,b} G w^H h_1 x + h_{2,b} G w^H n_r + n_d$$

(5.28)

where $h_{2,b}$ is the channel gain between the best antenna at the relay and the destination, $n_d$ is the AWGN noise at the destination with power $N_0$. Now, the end-end equivalent system SNR becomes,

$$\gamma_{eq} = \frac{|h_{2,b}|^2 G^2 |w^H h_1|^2}{|h_{2,b}|^2 G^2 |w^H n_r|^2 + N_0} = \frac{|h_{2,b}|^2 |w^H h_1|^2}{N_0 + \frac{|h_{2,b}|^2}{G^2 N_0}}$$

(5.29)

There are several ways to choose gain $G$. One way is to choose a constant gain, which introduces a fixed amplification irrespective of the $S \rightarrow R$ instantaneous link quality. This scheme is simpler and more practical than CSI-assisted gains. Let $G^2 = 1/(CN_0)$, then

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\bar{\gamma}_1 + \bar{\gamma}_2}$$

(5.30)

where the first hop SNR $\gamma_1 = \frac{1}{N_0} |w^H h_1|^2 = \bar{\gamma}_1 |w^H h_1|^2$ and the second hop SNR $\gamma_2 = \frac{1}{N_0} |h_{2,b}|^2 = \bar{\gamma}_2 |h_{2,b}|^2$. The $\gamma_1$ can be maximized when $w = \frac{h_1}{\|h_1\|_F}$, then $\gamma_1 = \bar{\gamma}_1 |h_1|_F^2$. $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are average link SNRs. $h_1$ is the $S \rightarrow R$ channel gain vector. $\| . \|_F^2$ stands for Forbenius Norm.

For independent identically distributed (IID) Rayleigh fading, the probability density function (PDF) of $\gamma_1$ is well known central Chi-square distribution with $2N$ degrees of freedom given by, $f_{\gamma_1}(\gamma_1) = \frac{\gamma_1^{N-1}}{(N-1)\bar{\gamma}_1^N} e^{-\gamma_1/\bar{\gamma}_1}$ When the 2nd hop link with the largest channel gain is chosen, i.e., $\gamma_2 = \max_{1 \leq i \leq N} \bar{\gamma}_2 |h_{2,b}|^2$, according to the ordered statistics the cumulative density function (CDF) of $\gamma_{2,b}$ can be written as, $F_{\gamma_2}(\gamma_2) = (1 - e^{-\gamma_2/\bar{\gamma}_2})^N$. 
5.4.1 Outage Analysis without Direct Link

In this section, the closed form expressions for the end-end system SNR in terms of CDF, PDF and MGF. These metrics facilitate to obtain the performance analysis parameters such as outage probability, error rates, etc.

A. MRC/TAS SNR Statistics

The CDF of system SNR $\gamma_{eq}$ can be written as,

$$F_{\gamma_{eq}}(\gamma) = \Pr\left(\frac{\gamma_1 \gamma_2}{\gamma_2 + C} < \gamma\right)$$

$$= \int_0^\infty \Pr\left(\gamma_2 < \frac{C\gamma}{\gamma_1 - \gamma}\right) f_{\gamma_1}(\gamma_1) d\gamma_1$$

$$= 1 - \int_{\gamma}^\infty \Pr\left(\gamma_2 > \frac{C\gamma}{\gamma_1 - \gamma}\right) f_{\gamma_1}(\gamma_1) d\gamma_1$$

$$= 1 - \int_{\gamma}^\infty \left[1 - \left[1 - e^{-\frac{C\gamma_1}{\gamma_1 - \gamma}\gamma_2}\right]^N\right] f_{\gamma_1}(\gamma_1) d\gamma_1$$

With the aid of binomial theorem for the expansion of inner integrand and after some intermediate steps, following is obtained,

$$F_{\gamma_{eq}}(\gamma) = 1 + \sum_{i=1}^{N} \binom{N}{i} \frac{(-1)^i}{(N-1)!\gamma_1^N}$$

$$\times \int_{\gamma}^\infty \gamma_1^{N-1} \exp\left(-\frac{\gamma_1}{\gamma_1 - \gamma_2}\right) d\gamma_1$$

To solve integral $I_1$, let $x = \gamma_1 - \gamma$. Then,

$$I_1 = e^{\frac{-\gamma_1}{\gamma_2}} \sum_{j=0}^{N-1} \binom{N-1}{j} \gamma_1^{N-1-j}$$

$$\times \int_0^\infty x^j \exp\left(-\frac{x}{\gamma_1 - \gamma_2}\right) dx$$

(5.31)

Using [88, eqn. 3.471.9], integral $I_2$ can be written as,

$$I_2 = 2 \left(\frac{C_i\gamma_1}{\gamma_2}\right)^{\frac{j+1}{2}} K_{j+1}\left(2 \sqrt{\frac{C_i\gamma_1}{\gamma_1\gamma_2}}\right)$$

(5.32)
where $K_l(.)$ is $l$-order modified Bessel function of second kind [88, eqn. 8.446]. The final expression for the CDF of end-end SNR of MRC/TAS relay system can be given by,

$$F_{\gamma_{eq}}^{\text{MRC}}(\gamma) = 1 + 2 \sum_{i=1}^{N} \binom{N}{i} (-1)^i e^{-\frac{\gamma}{\bar{\gamma}_1}} \frac{N!}{(N-1)!} \sum_{j=0}^{N-1} \binom{N-1}{j}$$

$$\times \gamma^{\frac{2N-j-1}{2}} \left( \frac{Ci\bar{\gamma}_1}{\gamma_2} \right)^{\frac{i+j+1}{2}} K_{j+1} \left( 2 \sqrt{\frac{Ci\gamma}{\bar{\gamma}_1\bar{\gamma}_2}} \right)$$ (5.33)

When $N = 1$ indicating single antenna relay node, (5.33) specializes to [70, eqn. 9]. The CDF plots are shown in Figure 5.10 for various settings. The multi-antenna obviously improves the performance which becomes more pronounced when $\bar{\gamma}_2$ is larger than $\bar{\gamma}_1$. The PDF of $\gamma_{eq}$ can be obtained by taking derivative of (5.34) with respect to $\gamma$. With the help of an identity $z \frac{d}{dz} K_v(z) + vK_v(z) + zK_{v-1}(z) = 0$ [88, eqn.
8.486-12], the PDF can be shown as,

\[
f_{\gamma_{eq}}^{MRC}(\gamma) = 2 \sum_{i=1}^{N} \binom{N}{i} (-1)^i e^{-\frac{\gamma}{\gamma_1}} \sum_{j=0}^{N-1} \binom{N-1}{j} \gamma j \times \gamma^{2N-2j-1} \binom{C_i}{\gamma}^{j+1} e^{\frac{C_i \gamma}{\gamma_1 \gamma_2}} \left[ K_{j+1} \left( 2 \sqrt{\frac{C_i \gamma}{\gamma_1 \gamma_2}} \right) \right]
\]

(5.34)

Again when \(N = 1\), (5.34) becomes [70, eqn. 10].

The Moment Generating Function (MGF) can be easily derived from the PDF. Since the MGF \(M_{\gamma_{eq}}^{MRC}(s) = E[e^{-s \gamma}]\), the final expression can be obtained using [88, eqn. 6.641-3] in terms of Whittaker function [88, eqn. 9.220]. Error rates for general modulation types can be efficiently obtained using MGF based approach [2].

B. SC/TAS SNR Statistics

When the SC scheme is employed at \(R\) for \(S \rightarrow R\) link, \(\gamma_1 = \frac{1}{N_0} |h_{1,b}|^2 = \bar{\gamma}_1 |h_{1,b}|^2\) in (5.30), where \(|h_{1,b}| = \max_{1 \leq i \leq N} |h_{1,i}|\). Following similar steps as in previous subsection, it is easy to show the CDF and the PDF of \(\gamma_{eq}\) for SC/TAS relay system as follows,

\[
F_{\gamma_{eq}}^{SC}(\gamma) = 1 + 2 \frac{N}{\bar{\gamma}_1} \sum_{i=1}^{N} \binom{N}{i} \sum_{j=0}^{N-1} \binom{N-1}{j} \left( \frac{-1}{e^{\frac{\gamma}{\gamma_1}}} \right)^{i+j} \times \sqrt{\frac{C_i \gamma}{(j+1) \gamma_1}} K_1 \left( 2 \sqrt{\frac{C_i \gamma}{\gamma_1 \gamma_2}} \right) \]

(5.35)

\[
f_{\gamma_{eq}}^{SC}(\gamma) = -2 \frac{N}{\bar{\gamma}_1} \sum_{i=1}^{N} \binom{N}{i} \sum_{j=0}^{N-1} \binom{N-1}{j} \left( \frac{-1}{e^{\frac{\gamma}{\gamma_1}}} \right)^{i+j} \times \left[ \frac{(j+1)}{\gamma_1} K_1 \left( 2 \sqrt{\frac{C_i (j+1) \gamma}{\gamma_1 \gamma_2}} \right) + \sqrt{\frac{C_i (j+1)}{\gamma_1 \gamma_2}} \right] K_0 \left( 2 \sqrt{\frac{C_i (j+1) \gamma}{\gamma_1 \gamma_2}} \right)
\]

(5.36)

which, as expected, becomes equivalent to (5.33) and (5.34) respectively for the case
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Figure 5.11: Probability Density Function, Comparison between SC/TAS and MRC/TAS schemes, $\bar{\gamma}_1 = 5\text{dB}, N = 5$, ‘o’ indicates monte-carlo simulation points.

of single-antenna relay. The PDFs of MRC/TAS and SC/TAS schemes are compared in Figure 5.11 for $N = 5$. The SC/TAS scheme is less optimal than MRC/TAS one, but less complex SC/TAS may outweigh the advantage of MRC/TAS in practical scenarios.

C. Outage Probability

The Outage Probability (OP) quantifies the likelihood of the instantaneous end-to-end SNR falling below some predetermined threshold $\gamma_{th}$, and given as $P_{out} = F_{\gamma_{eq}}(\gamma_{th})$. The OPs for MRC/TAS and SC/TAS schemes can be obtained directly from (5.33) and (5.35), i.e. $P_{out}^{MRC}(\gamma) = F_{\gamma_{eq}}^{MRC}(\gamma_{th})$ and $P_{out}^{SC}(\gamma) = F_{\gamma_{eq}}^{SC}(\gamma_{th})$.

5.4.2 Outage Analysis with Direct Link

A. MRC at Destination

When there is a direct link from $S \rightarrow D$, and the signals from direct and relayed links are combined coherently, i.e. $\gamma_{tot} = \gamma_{sd} + \gamma_{eq}$. The OP for this case can be written as [99, eqn. 6-44],

$$P_{out}^{MRC}(\gamma_{th}) = \Pr [(\gamma_{sd} + \gamma_{eq}) < \gamma_{th}] = \int_0^{\gamma_{th}} F_{\gamma_{sd}}(\gamma_{th} - \gamma) f_{\gamma_{eq}}(\gamma) d\gamma$$  (5.37)
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Note that the simplified representation (instead of joint PDF) comes from the fact that two instantaneous SNRs are independent. Using (5.34) and the direct link CDF, \(1 - e^{-\bar{\gamma}_{sd}}\), \(P^{\text{MRC-D}}_{\text{out}}(\gamma_{th})\) can be obtained. However, to author’s best knowledge there is no closed-form solution for the definite integrals involved. Numerical integration can be employed to solve them.

B. SC at Destination When the largest signal is selected between the relayed and the direct signals, we can derive a closed form expression for the OP. The output SNR \(\gamma_{tot} = \max\{\gamma_{sd}, \gamma_{eq}\}\). Therefore,

\[
P^{\text{SC-D}}_{\text{out}}(\gamma_{th}) = F^{\text{SC}}_{\gamma_{tot}}(\gamma_{th}) = F_{\gamma_{sd}}(\gamma_{th})F_{\gamma_{eq}}(\gamma_{th})
\]

\[
= [1 - e^{-\bar{\gamma}_{sd}}]F_{\gamma_{eq}}(\gamma_{th})
\]

The PDF can be obtained by taking derivative of (5.38), i.e. \(f^{\text{SC-D}}_{\gamma_{tot}}(\gamma) = \frac{d}{d\gamma} F^{\text{SC-D}}_{\gamma_{tot}}(\gamma) = f_{\gamma_{eq}}(\gamma)F_{\gamma_{sd}}(\gamma) + F_{\gamma_{eq}}(\gamma)f_{\gamma_{sd}}(\gamma)\).

5.4.3 Numerical Results and Discussion

Some numerical scenarios are given to analyze the outage behaviour of the dual-hop amplify-forward cooperative network with a multi-antenna relay based on the analysis presented in previous sections. All the analytical results were confirmed through the Monte-Carlo simulation. As depicted, analytical and simulation results exactly match with each other. 10^6 samples were generated to produce each plot.

The modified bessel function \(K_l(\cdot)\) in the final expressions is provided as a built-in function in Matlab. The gain \(G\) is fixed through the constant \(C = \frac{\bar{\gamma}_1}{e^{\bar{\gamma}_1}E_1(1/\bar{\gamma}_1)}\) [70, eqn. 16], where \(E_1\) is Euler integral.

Figure 5.12 shows the comparison of outage probability of MRC/TAS and SC/TAS schemes when there is no direct link between \(S\) and \(D\). The average SNRs of both links are assumed equal, and the threshold is \(\gamma_{th} = 0\) dB. The MRC/TAS scheme offers a definite SNR gain which improves as the number of antenna \(N\) increases. At \(10^{-4}\), MRC/TAS has just 1dB gain over SC/TAS for \(N = 2\) whereas the gain reaches up to 3dB for \(N = 5\). The diversity order (defined as negative slope of OP, i.e. \(-\frac{d}{d\bar{\gamma}} \log_{10}(P_{\text{out}})\) as \(\bar{\gamma} \to \infty, \bar{\gamma}\) is in dB) is equal to \(N\) as can be seen from the
The diversity order analysis can be carried out as shown recently in [100]. Figure 5.13 shows the similar settings but with the direct link. The average $S \rightarrow D$ link SNR is equal to those in other links. The outage behaviour looks similar to that without direct link, but the slope has become steeper because of the diversity order being $N + 1$ now.

The SNR gain/loss due to unbalanced links with/out direct link is shown in Figure 5.14. The number of antennas is $N = 3$, threshold $\gamma_{th} = 0$ dB and the $S \rightarrow R$ and $R \rightarrow D$ links are balanced, i.e. $\bar{\gamma}_1 = \bar{\gamma}_2$. When the direct link average SNR is half that of other links, SNR gain of almost 3 dB at $10^{-4}$ is obvious from the figure. Stronger direct link between $S$ and $D$ improves the gain, for example further up to 5 dB for $\bar{\gamma}_{sd} = 2\bar{\gamma}_1$. Note that the MRC at destination will improve the performance further (not shown here). In summary, the performance of a fixed gain dual-hop amplify-forward cooperative transmission system was analyzed in terms of outage probability. The intermediate Relay node is equipped with multiple-antenna whereas the end nodes (Source and Destination) are single-antenna terminals. The end-end SNR statistics such as CDF and PDF have been derived in closed-form for MRC/TAS.
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Figure 5.13: Comparison of Outage Probability of MRC/TAS and SC/TAS Schemes with Direct Link, SC at Destination, $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_{sd}$. $N = 5$, $\gamma_{th} = 5\, \text{dB}$

Figure 5.14: Comparison of Outage Probability of SC/TAS Scheme with and without Direct Link, SC at Destination, $\bar{\gamma}_1 = \bar{\gamma}_2$.

$\gamma_{sd} = 2\bar{\gamma}_1$, $\bar{\gamma}_1$, $0.5\bar{\gamma}_1$
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and SC/TAS schemes. It is shown that the higher second hop SNR improves the performance. When there is a direct link between the end nodes, the relayed and the direct signals can be combined to reduce the system outage. Closed-form expression of outage probability for cooperative mode is derived when the SC is used at the destination. The diversity order of system equals to $N$ for relaying mode (absence of direct link), whereas $N+1$ for cooperative mode. Analytical results exactly matches with Monte-Carlo simulation.

5.5 SNR Statistics of Dual-Hop Amplify and Forward Cooperative Transmission with Multi Antenna Relay over Nakagami-$m$ Fading

This contribution covers the extension of earlier analysis to Nakagami-$m$ fading. For independent identically distributed (IID) Nakagami-$m$ fading, the $\gamma_1$ is well known gamma distribution with the probability density function (PDF) given by,

$$f_{\gamma_1}(\gamma_1) = \frac{\gamma_1^{m_1N-1}}{\Gamma(m_1N)} \left( \frac{m_1}{\gamma_1} \right)^{m_1N} e^{-\frac{m_1\gamma_1}{\bar{\gamma}_1}}$$

(5.39)

When the 2nd hop link with the largest channel gain is chosen, i.e., $\gamma_2 = \max_{1 \leq i \leq N} |h_{2,i}|^2$, according to the ordered statistics the cumulative density function (CDF) of $\gamma_{2,b}$ can be written as,

$$F_{\gamma_2}(\gamma_2) = \left[ 1 - e^{-\frac{m_2\gamma_2}{\bar{\gamma}_2}} \sum_{j=0}^{m_2-1} \frac{1}{j!} \left( \frac{m_2\gamma_2}{\bar{\gamma}_2} \right)^j \right]^N$$

(5.40)

Note that above expression is valid for integer $m$. The cases where non-integer $m$ is necessary can be readily approximated from the ones with neighboring integer $m$ values.

5.5.1 SNR Analysis without Direct Link

The closed form expressions for the system SNR statistics in terms of CDF and PDF are presented here. These metrics are useful to obtain the link design parameters such as outage probability and error rates.
**A. MRC/TAS SNR Statistics**

Writing the CDF of system SNR $\gamma_{eq}$ as,

$$F_{\gamma_{eq}}^{MRC}(\gamma) = \Pr(\gamma < \frac{\gamma_1 \gamma_2}{\gamma_2 + C})$$

$$= 1 - \int_{\gamma}^{\infty} \left[ 1 - \left\{ 1 - e^{-\frac{m_2 C \gamma}{\gamma_2 (\gamma_1 - \gamma)}} \right\}^{N} \right] f_{\gamma_1}(\gamma_1) d\gamma_1$$

With the aid of binomial theorem for the expansion of inner integrand, one obtains,

$$F_{\gamma_{eq}}^{MRC}(\gamma) = 1 + \sum_{i=1}^{N} \binom{N}{i} (-1)^i \int_{\gamma}^{\infty} e^{-\frac{m_2 C \gamma}{\gamma_2 (\gamma_1 - \gamma)}} \left\{ \sum_{j=0}^{m_2-1} \frac{1}{j!} \left( \frac{m_2 C \gamma}{\gamma_2 (\gamma_1 - \gamma)} \right)^j \right\} f_{\gamma_1}(\gamma_1) d\gamma_1$$

To solve the power of multinomial $M_1$, we refer to [67] [101] where it is shown that,

$$\left\{ \sum_{j=0}^{m_2-1} \frac{1}{j!} \left( \frac{m_2 C \gamma}{\gamma_2 (\gamma_1 - \gamma)} \right)^j \right\}^i$$

$$= \prod_{j=1}^{m_2-1} \left[ \sum_{n_j=0}^{n_{j-1}} \binom{n_j}{n_j} \left( \frac{1}{j!} \right)^{n_j-n_j+1} \left( \frac{m_2 C \gamma}{\gamma_2 (\gamma_1 - \gamma)} \right)^{\Phi_1} \right]$$

\[\Omega(n_2)\]

where $\Phi_1 = \sum_{q=1}^{m_2-1} n_q$, $n_0 = i$, and $n_{m_2} = 0$. Using (5.39), the CDF becomes,

$$F_{\gamma_{eq}}^{MRC}(\gamma) = 1 + \sum_{i=1}^{N} \binom{N}{i} (-1)^i \frac{\Omega(m_2)}{\Gamma(m_1 N)} \frac{m_1 N}{\gamma_1} \left( \frac{m_2 C}{\gamma_2} \right)^{\Phi_1} \int_{\gamma}^{\infty} \gamma^{m_1 N-1} e^{-\frac{m_2 C \gamma}{\gamma_2 (\gamma_1 - \gamma)}} \left( \frac{m_1 N}{\gamma_1} \right)^{\Phi_1} d\gamma_1$$

Let $x = \gamma_1 - \gamma$. Then $I_1$ has a following form,

$$I_1 = \sum_{k=0}^{m_1 N-1} \binom{m_1 N - 1}{k} \gamma^{m_1 N-k-1} e^{-\frac{m_1}{\gamma_1}} \int_{0}^{\infty} e^{-\frac{m_2 C \gamma}{\gamma^2 x}} \frac{m_1 N}{\gamma_1} x^{k-\Phi_1} d\gamma_1$$

which can be written in a closed-form using [88, eqn. 3.471.9]. Therefore the final expression of the CDF for MRC/TAS scheme over Nakagami-$m$ fading channel...
becomes,

\[
F_{\gamma_{eq}}^{MRC}(\gamma) = 1 + 2 \sum_{i=1}^{N} \left( \begin{array}{c} N \\ i \end{array} \right) (-1)^i \Omega_j(m_2) e^{-m_1\gamma} \Gamma(m_1N) \\
\times \sum_{k=0}^{m_1N-1} \left( \begin{array}{c} m_1N - 1 \\ k \end{array} \right) \left( \begin{array}{c} m_1 \\ \gamma_1 \end{array} \right)^{2m_1N-k+\Phi_1-1} \\
\times \left( \frac{m_2C}{\bar{\gamma}_2} \right)^{k+\Phi_1+1} \gamma^{2m_1N-k+\Phi_1-1} e^{-m_1\gamma} \Gamma(m_1N-k+\Phi_1-1) \\
\times \left( \frac{m_1+\Phi_1+1}{\gamma_2} \right)^{j-k-1} 2^{j+1} K_{k-j+1}(2\sqrt{\lambda})
\] (5.43)

where \( K_l(\cdot) \) is \( l \)-order modified Bessel function of second kind [88, eqn. 8.446], and \( \lambda = \frac{m_1m_2C\gamma}{\bar{\gamma}_1\bar{\gamma}_2} \). Note that when \( m_2 = 1 \), \( \Phi_1 = 0 \) and \( \Omega(m_2) = 1 \). When \( N = 1 \) indicating single antenna relay node,

\[
F_{\gamma_{eq},N=1}^{MRC}(\gamma) = 1 - 2 e^{-m_1\gamma/\bar{\gamma}_1} \sum_{j=0}^{m_2-1} \sum_{k=0}^{m_1-1} \left( \begin{array}{c} m_1-1 \\ k \end{array} \right) \\
\times \left( \frac{m_1}{\bar{\gamma}_1} \right)^{2m_1+j-k-1} \left( \frac{m_2C}{\bar{\gamma}_2} \right)^{j+1} \\
\times \gamma^{2m_1+j-k-1} K_{k-j+1}(2\sqrt{\lambda})
\] (5.44)

which is an alternative representation to [103, eqn. 18], and further reduces to [70, eqn. 9] for \( m_1 = m_2 = 1 \). The CDF plots are shown in Figure 5.15 for various settings. The multi-antenna relay obviously improves the performance and becomes more pronounced for higher value of fading severity parameters \( (m_1 \text{ and } m_2) \) and when \( \bar{\gamma}_2 \) is larger than \( \bar{\gamma}_1 \).

The PDF of \( \gamma_{eq} \) can be obtained by taking derivative of (5.43) with respect to \( \gamma \). With the help of an identity \( \frac{\partial}{\partial z} K_v(z) + v K_v(z) + z K_{v-1}(z) = 0 \) [88, eqn. 8.486-12],
Figure 5.15: Cumulative Distribution Function, MRC/TAS scheme, $\gamma_1 = 5dB$, ‘o’ indicates monte-carlo simulation points.

the PDF of AF multi-antenna Relay with MRC/TAS scheme can be shown as,

$$f_{\gamma_{eq}}^{MRC}(\gamma) = 2^N \sum_{i=1}^{N} \binom{N}{i} \left( -1 \right)^i \frac{\Omega(m_2)e^{-\frac{m_2}{\gamma_1}}}{\Gamma(m_1N)} \sum_{k=0}^{m_1N-1} \left[ \begin{array}{c} K_k - \Phi_1 + 1 \\left( 2\sqrt{\lambda} \right) \left\{ \frac{m_1N - k + \Phi_1 - 1}{\gamma} \right\} \\
\frac{m_1m_2iC}{\gamma_1\gamma_2} \end{array} \right]$$

(5.45)

B. SC/TAS SNR Statistics When the SC scheme is employed at $R$ for $S \rightarrow R$ link, $\gamma_1 = \frac{1}{N_0}|h_{1,b}|^2 = \bar{\gamma}_1|h_{1,b}|^2$ in (5.30), where $|h_{1,b}| = \max_{1 \leq i \leq N} |h_{1,i}|$. Following similar steps as in previous subsection, the CDF of $\gamma_{eq}$ for SC/TAS multi-antenna
relay system is shown as follows,

\[
F_{\gamma_{eq}}^{SC}(s) = 1 + 2N \sum_{i=1}^{N} \binom{N}{i} \sum_{k=0}^{N-1} \binom{N-1}{k} \sum_{j=0}^{m_{1}-1} \Omega(m_{2}) \Omega(m_{1}) e^{-\frac{m_{1}(k+1)\gamma_{1}}{\gamma_{1}}} \left(\frac{m_{1}}{\gamma_{1}}\right)^{j+\Phi_{2}+1} \\
\times \left(\frac{m_{2}C}{\gamma_{2}}\right)^{\Phi_{1}} \left[ \sum_{p=0}^{\Phi_{2}} \left( \frac{j + \Phi_{2} - 1}{q} \right) \gamma_{q} \frac{\mu}{2} K_{q-\Phi_{1}+1}(2\sqrt{\nu}) \right]
\]

where \(\Omega(m_{1}) = \prod_{j=1}^{m_{1}-1} \left[ \sum_{n_{j}=0}^{n_{j}+1} \left( \frac{1}{j!} \right) n_{j}^{-n_{j}+1+1} \right]\), \(\Phi_{2} = \sum_{q=0}^{m_{1}-1} n_{q}, n_{0} = k, n_{m_{1}} = 0\), \(\mu = \frac{m_{2}C_{\gamma_{3}}}{m_{1}(k+1)\gamma_{2}}\), and \(\nu = \frac{m_{1}m_{2}(k+1)C_{\gamma_{3}}}{\gamma_{1}\gamma_{2}}\). The derivation of the PDF for SC/TAS scheme is similar to that for MRC/TAS.

### 5.5.2 SNR Analysis with Direct Link

The presence of direct link between \(S\) and \(D\) offers cooperative diversity and was taken into account to derive the CDF of the end-end SNR. cooperative selection (CS) is assumed at the destination. When the highest signal between the relayed and the direct signals is selected, we have the output SNR \(\gamma_{tot} = \max\{\gamma_{3}, \gamma_{eq}\}\), \(\gamma_{3}\) being the instantaneous SNR for \(S \rightarrow D\) link. Let \(m_{3}\) and \(\bar{\gamma}_{3}\) denote the fading severity parameter and average SNR for \(S \rightarrow D\) Nakagami-\(m\) faded link. Using (5.40) and (5.43), the CDF of dual-hop AF relay with MRC/TAS at multi-antenna relay and
cooperative selection (CS) at the destination can be written as,

\[
F_{\gamma_{tot}, MRC}^{SC}(\gamma) = -1 + F_{\gamma_1}(\gamma) + F_{\gamma_{eq}}^{MRC}(\gamma) - 2 \sum_{i=1}^{N} \frac{(-1)^i \Omega(m_2)e^{-\left(\frac{m_1}{\gamma_1} + \frac{m_3}{\gamma_3}\right)\gamma}}{\Gamma(m_1N)} \\
\times \left\{ \sum_{j=0}^{m_3-1} \left( \frac{m_3\gamma}{\gamma_3} \right)^j \right\} \sum_{k=0}^{m_1N-1} \binom{m_1N-1}{k} \\
\times \frac{m_1}{\hat{\gamma}_1} \frac{2^{m_1N-k+\Phi_1-1}}{i^{-k+\Phi_1+1}} \times K_{k-\Phi_1+1}(2\sqrt{\lambda})
\] (5.47)

The Figure 5.16 shows the CDFs for the case when a direct link S \(\rightarrow\) D is present. The plot is for \(N = 2\) and the identical fading parameter \(m_1 = m_2 = m_3 = 2\) with varying link SNRs. As expected, the presence of direct link always improves the system performance. The level of improvement depends on the value of direct link SNR compared to relayed link SNRs. When the direct link SNR \(\bar{\gamma}_3\) is greater
than relayed link SNRs ($\bar{\gamma}_1 = \bar{\gamma}_2$, balanced links), the improvement is significant (as seen from leftmost curves). However for smaller $\bar{\gamma}_3$, not much benefit (the rightmost curves) is achieved when $\gamma$ becomes higher.

The PDF can be obtained by taking derivative of (5.47) with respect to $\gamma$, i.e.

$$f_{\gamma_{\text{tot}},\text{MRC}}(\gamma) = \frac{d}{d\gamma} F_{\gamma_{\text{tot}},\text{MRC}}(\gamma) = f_{\gamma_{\text{eq}}}(\gamma) F_{\gamma_3}(\gamma) + F_{\gamma_{\text{eq}}}(\gamma) f_{\gamma_3}(\gamma).$$

Similarly, the SNR CDF for SC/TAS scheme at relay and CS at destination can be given as follows,

$$F_{\gamma_{\text{tot}},\text{SC}}(\gamma) = -1 + F_{\gamma_3}(\gamma) + F_{\gamma_{\text{eq}}}(\gamma) - 2N \sum_{i=1}^{N} \binom{N}{i} \times$$

$$\times \sum_{k=0}^{m-1} \left( \begin{array}{c} N - 1 \\ m \end{array} \right) \left\{ \sum_{l=0}^{m-3} \frac{1}{l!} \left( \frac{m_3}{\bar{\gamma}_3} \right)^l \right\} \times$$

$$\times \sum_{j=0}^{m_1-1} \frac{\Omega(m_2)\Omega(m_1)e^{-\left(\frac{2m_1\beta+1}{\bar{\gamma}_3}\right)}}{\left(-1\right)^{-l+k+1}j!} \times$$

$$\times \left( \frac{m_1}{\bar{\gamma}_1} \right)^{j+\Phi_1+1} \left( \frac{m_2C}{\bar{\gamma}_2} \right)^{\Phi_1} \left[ \sum_{p=0}^{j+\Phi_2} \binom{j+\Phi_2}{p} \times$$

$$\times \gamma^{j-p+\Phi_1+\Phi_2} \mu \frac{\bar{\gamma}_1}{2} K_{p-\Phi_1+1}(2\sqrt{\nu}) \right]$$

$$+ \frac{\bar{\gamma}_1}{m_1} \sum_{q=0}^{j+\Phi_2-1} \binom{j+\Phi_2-1}{q} \gamma^{j-q+\Phi_1+\Phi_2-1} \times \mu \frac{\bar{\gamma}_1}{2} \times K_{q-\Phi_1+1}(2\sqrt{\nu})$$

(5.48)

The derived SNR statistics can be used to evaluate the system performance metrics through standard techniques. For example, the Outage Probability is simply $P_{\text{out}} = F_{\gamma_{\text{tot}}}(\gamma_{th})$, where $\gamma_{th}$ is the outage threshold. The Error Rate can be calculated from $P_e = \frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{\text{tot}}}(\frac{t^2}{2})e^{-t^2/2}dt$, where $\beta$ is a constant specific to modulation type, $\beta = 1$ for QPSK for instance.

### 5.5.3 Numerical Results

All the analytical results were confirmed through the Monte-Carlo simulation. As shown, the analytical results are in exact accordance with the simulation results. $10^6$ samples were generated to produce each plot. The modified bessel function $K_i(.)$ in the final expressions is provided as a built-in function in Matlab. The gain $G$ is fixed through the constant $C = \frac{\bar{\gamma}_1}{e^{\bar{\gamma}_1} E_1(1/\bar{\gamma}_1)}$ [70, eqn. 16], where $E_1$ is Euler integral.
Figure 5.17: Cumulative Distribution Function, SC/TAS at Relay, with/out Direct Link, $\gamma_1 = \gamma_2 = 5$ dB; SC at Destination and $\gamma_3 = 2\gamma_1$ when the Direct Link is present. ‘$o$’ indicates monte-carlo simulation points.

Figure 5.17 shows the comparison of CDFs for the case where SC/TAS is employed at the Relay with/out the presence of direct link. The relayed link is balanced ($\gamma_1 = \gamma_2 = 5$ dB), whereas the direct link SNR $\gamma_3 = 2\gamma_1$. The number of antennas is $N = 3$, and the fading severity $m$ is varied from 1 to 3. The CDF curves show similar trends for both with/out the direct link for varying $m$. After a certain value of $\gamma$, the lower $m$ provides higher performance (shown by the rightmost solid lines). In other words, at higher threshold values (which corresponds to lower SNR regime), the higher $m$ (less fading) results in worse performance.

To understand the impact of $m$ on system outage behavior, Figure 5.18 is plotted. Both MRC/TAS and SC/TAS schemes with direct link are considered for two different values of $m$. The outage threshold $\gamma_{th} = 6$ dB and $m_1 = m_2 = m_3$. As expected, higher $m$ offers better system performance (except at low SNR regime). For example, in SC/TAS scheme at $10^{-5}$ outage probability, $m = 5$ offers 3 dB SNR gain over $m = 3$. The MRC/TAS scheme offers slightly better gain 3.5 dB for same settings. In addition, one can observe that MRC/TAS provides 2 dB gain than SC/TAS when $m = 2$, and becomes 2.5 dB when $m = 5$ at $10^{-5}$. 
In summary, new closed-form expressions of system SNR CDF were obtained for a multi-antenna cooperative system in IID Nakagami-$m$ fading channels. The spatial diversity at relay is exploited through the MRC/TAS or SC/TAS, and at destination through the CS. The expressions are valid for integer $m$, and specializes to previous results. The analysis is confirmed with Monte-Carlo simulation. The results clearly indicate the impact of severity parameter $m$, multi-antenna, and presence of direct link on the system behavior. The analysis can be extended to diversity-order calculation as recently shown in [100].

5.6 General Order Antenna Selection in Amplify Forward System with Multi-Antenna Relay

A dual-hop relay system, as shown in Figure 5.19 is considered here. The source and the destination nodes are equipped with single antenna whereas relaying node has $M$ antennas.

The closed form expressions for the General Order Statistics in terms of CDF,
PDF, MGF and Moments of the system SNR are derived. First, following theorem from Order Statistics [102] is useful for our analysis.

**Theorem:** Let $\Gamma_1, \Gamma_2, \ldots, \Gamma_M$ be $M$ independent and identical random variables, and arranged in decreasing order denoted by $\Gamma^{(1)}, \Gamma^{(2)}, \ldots, \Gamma^{(M)}$ where $\Gamma^{(1)}$ corresponding to the highest order statistic (largest of the $\Gamma_i$'s). If $f(\gamma)$ and $F(\gamma)$ are the PDF and the CDF of $\Gamma$'s respectively, the PDF of $n^{th}$ order statistic $\Gamma^{(n)}$ is given by,

$$f_{\Gamma^{(n)}}(\gamma) = M \binom{M-1}{n-1} [F(\gamma)]^{M-n} [1 - F(\gamma)]^{n-1} f(\gamma)$$

(5.49)

When the both links are subject to i.i.d Rayleigh fading, it is easy to obtain the PDF and the CDF of $n^{th}$ order statistic as follows,

$$f_{\gamma^{(n)}}(\gamma_i) = M \binom{M-1}{n_i-1} \sum_{k_i=0}^{M-n_i} \binom{M-n_i}{k_i} \frac{(-)^{k_i}}{\bar{\gamma}_i^{k_i}} e^{- (n_i+k_i) \frac{\gamma_i}{\bar{\gamma}_i}}$$

(5.50)

and,

$$F_{\gamma^{(n)}}(\gamma_i) = M \binom{M-1}{n_i-1} \sum_{k_i=0}^{M-n_i} \binom{M-n_i}{k_i} \frac{(-)^{k_i}}{n_i + k_i} \left[ 1 - e^{- (n_i+k_i) \frac{\gamma_i}{\bar{\gamma}_i}} \right]$$

(5.51)
where \(i = 1, 2\) to indicate 1st or 2nd hop link. 1. CDF Starting with the CDF of SNR \(\gamma_{eq}\) as [103],

\[
F^{GOS}_{\gamma_{eq}}(\gamma) = \Pr \left( \gamma_{1} + C < \frac{\gamma_{2}}{\gamma_{1}} \right) = \int_{0}^{\gamma} \Pr \left( \frac{\gamma}{\gamma_{1}} < \frac{\gamma_{1} + C}{\gamma_{2}} \right) f_{\gamma_{2}}(\gamma_{2})d\gamma_{2}
\]

\[
= 1 - M^{2} \frac{(M-1)}{n_{1}-1} \frac{(M-1)}{n_{2}-1} \sum_{k_{1}=0}^{M-n_{1}} \sum_{k_{2}=0}^{M-n_{2}} \left( -1 \right)^{k_{1}+k_{2}} e^{-\left( n_{1}+k_{1} \right) \frac{C}{\gamma_{1}}} \cdot \left( \frac{M-n_{1}}{k_{1}} \right) \left( \frac{M-n_{2}}{k_{2}} \right)
\]

\[
\times \int_{0}^{\gamma} e^{-\left( n_{1}+k_{1} \right) \frac{C}{\gamma_{1}} \gamma_{2}} - \left( n_{2}+k_{2} \right) \frac{C}{\gamma_{2}} d\gamma_{2}
\]

\[
= 1 - 2M^{2} \frac{(M-1)}{n_{1}-1} \frac{(M-1)}{n_{2}-1} \sum_{k_{1}=0}^{M-n_{1}} \sum_{k_{2}=0}^{M-n_{2}} \left( M-n_{1} \right) \left( M-n_{1} \right)
\]

\[
\times \left( \frac{-1}{k_{1}+k_{2}} \right) e^{-\left( n_{1}+k_{1} \right) \frac{C}{\gamma_{1}}} \sqrt{\frac{C}{\gamma_{1}}} \frac{K_{1}(\lambda)}{(n_{1}+k_{1})(n_{2}+k_{2}) \gamma_{1} \gamma_{2}} (5.52)
\]

where [88, eqn. 3.324-1] was used to evaluate integral \(I_{1}\) in terms of first order modified Bessel of second kind \(K_{1}(.)\), \(\lambda = 2 \sqrt{\frac{(n_{1}+k_{1})(n_{2}+k_{2})C}{\gamma_{1} \gamma_{2}}}\). When \(M = 1\) indicating single-antenna relay,(5.52) specializes to [70, eqn. 9].

At high SNR region i.e. \(\bar{\gamma}_{1}, \bar{\gamma}_{2} \to \infty\) which in other words entails a small argument for bessel function, following approximation is valid,

\[
K_{\alpha}(x) \approx \frac{2^{\alpha-1} \Gamma(\alpha)}{x^{\alpha}}, \quad 0 < x << \sqrt{\alpha + 1} \quad (5.53)
\]

Therefore, the CDF at high SNR can be approximated by,

\[
F^{GOS}_{\gamma_{eq, appx}}(\gamma) = 1 - M^{2} \frac{(M-1)}{n_{1}-1} \frac{(M-1)}{n_{2}-1} \sum_{k_{1}=0}^{M-n_{1}} \sum_{k_{2}=0}^{M-n_{2}} \left( M-n_{1} \right) \left( M-n_{1} \right)
\]

\[
\times \left( \frac{-1}{k_{1}+k_{2}} \right) e^{-\left( n_{1}+k_{1} \right) \frac{C}{\gamma_{1}}} \sqrt{\frac{C}{\gamma_{1}}} K_{1}(\lambda) (5.54)
\]

Figure 5.20 shows the CDF plot for varying order statistics. The first and the second hop average SNRs are assumed equal, i.e. \(\bar{\gamma}_{1} = \bar{\gamma}_{2} = 5dB\), and the number of antennas at relay \(M = 4\). One can see that how the lower order statistics result a
loss in the system performance. The Monte-Carlo simulation results are also shown and validate the analysis.

3. PDF The PDF of $\gamma_{eq}$ can be obtained by taking derivative of (5.59) with respect to $\gamma$. With the help of an identity $z \frac{d}{dz} K_v(z) + v K_v(z) + z K_{v-1}(z) = 0$ [88, eqn. 8.486-12], the GOS PDF of dual-hop amplify-forward relay can be given by (5.50). As expected when there is single antenna at Relay i.e. $M = 1$, (5.60) becomes [70, eqn. 10]. Figure 5.21 illustrates PDF plot for varying order statistics. The first link SNR is half that of the second link SNR, i.e. $\bar{\gamma}_1 = 0.5 \bar{\gamma}_2 = 5dB$, and $M = 4$. One important observation here is the performance difference from the pairs $n_1 = 1, n_2 = 2$ and $n_1 = 2, n_2 = 1$. The second highest order statistics in the first link results higher performance loss than that in the second link.

$$f_{\gamma_{eq}}(\gamma) = 2M^2 \binom{M-1}{n_1-1} \binom{M-1}{n_2-1} \sum_{k_1=0}^{M-n_1} \sum_{k_2=0}^{M-n_2} \binom{M-n_1}{k_1} \binom{M-n_2}{k_2} \left[ \frac{C\gamma}{(n_1 + k_1)(n_2 + k_2)\bar{\gamma}_1\bar{\gamma}_2} \right]$$

$$\times \left[ \frac{n_1 + k_1}{\bar{\gamma}_1} K_1(\lambda) + \frac{\lambda}{2\gamma} K_0(\lambda) \right]$$

$$\times \left[ \frac{1}{\lambda} \right]$$

(5.55)
3. MGF Since MGF $M_{\gamma_{eq}}^{GOS}(s) = E[e^{-s\gamma}]$, 

$$M_{\gamma_{eq}}^{GOS}(s) = \int_0^\infty e^{-s\gamma} f_{\gamma_{eq}}^{GOS}(\gamma) d\gamma$$

Using (5.50), the final expression for MGF becomes,

$$M_{\gamma_{eq}}^{GOS}(s) = M^2 \binom{M-1}{n_1-1} \binom{M-1}{n_2-1} \sum_{k_1=0}^{M-n_1} \sum_{k_2=0}^{M-n_2} \binom{M-n_1}{k_1} \binom{M-n_2}{k_2} (-1)^{k_1+k_2} \exp \left( \frac{(n_1 + k_1)(n_2 + k_2)C}{2s_2(n_1 + k_1 + s_1)} \right)$$

$$\times \left[ W_{-1,1/2}(\sigma) \frac{W_{-1,0}(\sigma)}{(n_2 + k_2)(n_1 + k_1 + s_1)} + \sqrt{\frac{C(n_1 + n_2 + s_1)}{(n_1 + k_1)(n_2 + k_2)}} \right]$$

where $\sigma = \frac{(n_1 + k_1)(n_2 + k_2)}{s_2(n_1 + k_1 + s_1)}$. [88, eqn. 6.643-3] was used to arrive at the final expression, and $W_{.,.}(\cdot)$ is Whittaker function defined in [88, eqn. 9.22]. For $N = 1$, (5.56) is equivalent to [70, eqn. 12].

![Figure 5.21: SNR Probability Density Function of Multi-Antenna Relay with General Order Antenna Selection, ‘•’ indicates Monte-Carlo simulation points](image)

4. SNR Moments The SNR moments are useful statistics to evaluate important system performance measures such as the average SNR, Amount of Fading (AoF),
etc. The generalized moments can be given by,

$$
\mu_n = E[\gamma_{eq}^n] = \int_0^\infty \gamma^n f_{\gamma_{eq}}(\gamma) d\gamma
$$

which after integration by parts can be written as,

$$
\mu_n = n \int_0^\infty \gamma^{n-1} [1 - F_{\gamma_{eq}}(\gamma)] d\gamma
$$

Using (5.59) and [88, eqn. 6.643-3], the final expression of SNR moments is obtained as follows,

$$
\mu_n = M^2 \left( \frac{M-1}{n_1-1} \right) \left( \frac{M-1}{n_2-1} \right) \sum_{k_1=0}^{M-n_1} \sum_{k_2=0}^{M-n_2} \binom{M-n_1}{k_1} \binom{M-n_2}{k_2}
$$

$$
\times (-1)^{k_1+k_2} \frac{n(n+2)(n!)^2 e^{C}}{(n_1+k_1)(n_2+k_2)} \left( \frac{\bar{\gamma}_1}{n_1+k_1} \right)^{n_1+1}
$$

$$
\times W_{-(n+1),1/2} \left( \frac{(n_2+k_2)C}{\bar{\gamma}_2} \right)
$$

Note that first order statistics \((n = 1)\) is the average SNR \(\mu_1 = E[\gamma_{eq}]\), and the second order statistics \((n = 2)\) is the SNR variance \(\mu_2 = E[\gamma_{eq}^2]\). The AoF, which is a measure of fading severity, can be obtained by, \(\text{AoF} = \frac{E[\gamma_{eq}^2] - (E[\gamma_{eq}])^2}{(E[\gamma_{eq}])^2} = \frac{\mu_2}{\mu_1^2} - 1\).

### 5.6.1 Performance Analysis and Discussion

Numerical results are shown to understand the impact of order selection on the system performance in terms of outage probability and error rate. The modified bessel function \(K_l(.)\) and the Whittaker function \(W_{.,.}(.)\) in the final expressions were evaluated in MATLAB. The gain \(G\) is fixed through the constant \(C = \frac{\bar{\gamma}_1}{e^{\gamma_1} E_1(1/\gamma_1)}\) [70, eqn. 16], where \(E_1\) is Euler integral.

Figure 5.22 shows the outage probability \(P_{out} = F_{\gamma_{eq}}(\gamma_{th})\) where \(\gamma_{th}\) is the outage threshold. The links are symmetric, i.e. \(\bar{\gamma}_1 = \bar{\gamma}_2\), \(M = 4\) and \(\gamma_{th} = 5dB\). Just one order lower statistic results a loss of 2dB at \(10^{-3}\) outage depicted from \(n_1 = 1, n_2 = 1\) and \(n_1 = 1, n_2 = 2\). Furthermore, \(n_1 = 2, n_2 = 1\) indicating second highest order antenna selection in first link results more loss, almost 5dB at same outage rate. This observation was expected as the system behavior depends more on the first link quality in amplify-forward scheme.
The average bit error probability (ABEP) for DBPSK modulation is shown in Figure 5.23. ABEP for DBPSK is $P_e = 0.5M_{\text{eq}}\gamma(1)$. For other modulation schemes, ABEP can be similarly evaluated based on well-known MGF-based approach [2]. The error rates can be obtained using the CDF as well which appears to be method of choice for relay networks [61] [63]. Here focus revolves around the main idea of the paper, impact of order selection on performance analysis. From Figure 5.23, one can observe the diversity gain achievable from the multiple antenna at relay compared to a single-antenna terminal ($M = 1$). Nevertheless, the gain depends on the order selection. When the antenna selection is performed with the lower order statistics, the performance gain diminishes. At $10^{-4}$ error rate, loss of almost 5dB is possible when second best antenna are chosen for both hops.

Figure 5.24 depicts the average SNR of the system, which is the first moment in (5.57). One can observe the loss in average SNR when $n_1 = n_2 = 2$. When the second hop average SNR is doubled, there is slight improvement shown by right dash line, but not sufficient to compensate the loss due to the second order antenna selection. In this contribution, new general expressions of SNR statistics were obtained.

Figure 5.22: Outage Probability of Multi-Antenna Relay with General Order Antenna Selection, • indicates Monte-Carlo simulation points.
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Figure 5.23: Average Bit Error Rate for DBPSK of Multi-Antenna Relay with General Order Antenna Selection

Figure 5.24: Average End-End SNR of Multi-Antenna Relay with General Order Antenna Selection, $M = 4$
for general order antenna selection in a dual-hop amplify-forward fixed-gain relay network. The antenna selection is applied at multi-antenna relay for both hops. The SNR statistics such as CDF, PDF, MGF and General Moments were derived, and then used to analyze the system performance in terms of outage and error rate. The results indicate the usefulness of the derived expressions to accurately quantify the metrics such as performance loss due to selection of lower order antenna (instead of highest order or best antenna). The expressions specialize to earlier results, and were verified by computer simulation.

5.7 General Order SNR Statistics in MIMO Cooperative Relay Network

A dual-hop relay system, as shown in Figure 5.25, consists of the source $S$ sending signals towards the destination $D$ via the assistance of the relay $R$. In Amplify-Forward (AF) relay systems, $R$ amplifies the received signal before forwarding it to the $D$ node. The total transmission involves two time-slots. The source has $M_s$ antennas where the relay and the destination are equipped with $M_r$, and $M_d$ antennas.

Note that unlike in [65], assumption here is the same set of antennas at Relay for receiving and transmission. $H_1$ denotes $M_r \times M_s$ channel matrix for the first hop whereas $H_2$ is $M_d \times M_r$ matrix for the second hop, both matrix elements are i.i.d complex Gaussian random variables (CGRVs) with mean zero and variance 0.5 per dimension. The channel elements are ordered in the decreasing order of their absolute magnitudes. The $n_1^{th}$ and $n_2^{th}$ best links (Tx/Rx antenna pairs) are then selected for transmission in $S \rightarrow R$ and $R \rightarrow D$ hops respectively, therefore corresponds to the channel gains $h_{n_1}$ and $h_{n_2}$. 
The CDF of SNR $\gamma_{eq}$ can be given by [103],

$$F_{GOS}^{\gamma_{eq}}(\gamma) = 1 - 2M_{sr}M_{rd}\left(\frac{M_{sr} - 1}{n_1 - 1}\right)\left(\frac{M_{rd} - 1}{n_2 - 1}\right)\sum_{k_1=0}^{M_{sr}-n_1} \times \sum_{k_2=0}^{M_{rd}-n_2} \left(\frac{M_{sr} - n_1}{k_1}\right)\left(\frac{M_{rd} - n_2}{k_2}\right) \times \left(-1\right)^{k_1+k_2}e^{\left(n_1+k_1\right)\frac{-C_{\gamma}}{\gamma_1\gamma_2}} K_1(\lambda) \times \frac{C_{\gamma}}{\gamma_1\gamma_2} \sqrt{(n_1+k_1)(n_2+k_2)\gamma_1\gamma_2}$$

(5.59)

where $K_1(.)$ is first order modified Bessel of second kind, $\lambda = 2\sqrt{\frac{n_1+k_1(n_2+k_2)C_{\gamma}}{\gamma_1\gamma_2}}$, $M_{sr} = M_sM_r$, and $M_{rd} = M_rM_d$. When $M_s = M_r = M_d = 1$ indicating single-antenna nodes,(5.59) specializes to [70, eqn. 9]. Furthermore when $n_1 = n_2 = 1$ (highest order statistics i.e. best Tx/Rx antennas pair in both hops), the CDF can be obtained for antenna selection in MIMO dual-hop amplify-forward system with a fixed-gain relay. Note that [65] [71] obtained results for an ideal-gain relay.

Figure 5.26 shows the CDF plot for varying order statistics. The first and the second hop average SNRs are assumed equal, i.e. $\bar{\gamma}_1 = \bar{\gamma}_2 = 5dB$, and the number of antennas at relay $M = 4$. One can see that how the lower order statistics result a loss in the system performance (the CDF curves moving towards right). The Monte-
Carlo simulation results validate the analysis.

**B. PDF** The PDF of $\gamma_{eq}$ can be obtained by taking derivative of (5.59) with respect to $\gamma$. With the help of an identity $z \frac{d}{dz} K_v(z) + v K_v(z) + z K_{v-1}(z) = 0$ [88, eqn. 8.486-12], the GOS PDF can be given by (5.50). As expected for single-antenna nodes, (5.60) becomes [70, eqn. 10]. Figure 5.27 illustrates the behaviour of PDF depending on the antenna order.

$$\begin{align*}
\int_{\gamma_{eq}}^{GOS} \gamma &= 2M_{sr}M_{rd} \left( \begin{array}{c} M_{sr} - 1 \\ n_1 - 1 \end{array} \right) \left( \begin{array}{c} M_{rd} - 1 \\ n_2 - 1 \end{array} \right) \sum_{k_1=0}^{M_{sr} - n_1} \sum_{k_2=0}^{M_{rd} - n_2} \left( \begin{array}{c} M_{sr} - n_1 \\ k_1 \end{array} \right) \\
&\times \left( \begin{array}{c} M_{rd} - n_2 \\ k_2 \end{array} \right) (-1)^{k_1+k_2} \frac{C_\gamma}{(n_1 + k_1)(n_2 + k_2)\bar{\gamma}_1\bar{\gamma}_2} \\
&\times \left[ \frac{n_1 + k_1}{\bar{\gamma}_1} K_1(\lambda) + \frac{\lambda}{2\bar{\gamma}} K_0(\lambda) \right] 
\end{align*}
$$

(5.60)

![Figure 5.26: Cumulative Distribution Function for General Order Antenna Selection in MIMO Relay, '*' indicates Monte-Carlo simulation points](image-url)
Chapter 5. Analysis of Relay/Cooperative Networks

C. MGF Since MGF $M_{\gamma_{eq}}^{GOS}(s) = E[e^{-s\gamma}]$,

$$M_{\gamma_{eq}}^{GOS}(s) = M_{sr} M_{rd} \left( \frac{M_{sr} - 1}{n_1 - 1} \right) \left( \frac{M_{rd} - 1}{n_2 - 1} \right) \sum_{k_1=0}^{M_{sr} - n_1} \sum_{k_2=0}^{M_{rd} - n_2} \left( \frac{M_{sr} - n_1}{k_1} \right)$$

$$\left( \frac{M_{rd} - n_2}{k_2} \right)(-1)^{k_1+k_2} \exp \left( \frac{(n_1 + k_1)(n_2 + k_2)C}{2\gamma_2(n_1 + k_1 + s\gamma_1)} \right) \right) \times \left[ \frac{W_{-1,1/2}(\sigma)}{(n_2 + k_2)(n_1 + k_1 + s\gamma_1)} + \frac{C(n_1 + n_2 + s\gamma_1)^{-1}}{(n_1 + k_1)(n_2 + k_2)\gamma_2} W_{-1/2,0}(\sigma) \right]$$

where $\sigma = \frac{(n_1+k_1)(n_2+k_2)C}{\gamma_2(n_1+k_1+s\gamma_1)}$. [88, eqn. 6.643-3] was used to arrive at the final expression, and $W_{\cdot}(\cdot)$ is Whittaker function defined in [88, eqn. 9.22]. When $M_s = M_r = M_d = 1$, (5.61) is equivalent to [70, eqn. 12].
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D. General Moments

The generalized moments \( \mu_n = E[\gamma_{eq}^n] \) can be given by,

\[
\mu_n = M_{sr}M_{rd}\left(\frac{M_{sr} - 1}{n_1 - 1}\right)\left(\frac{M_{rd} - 1}{n_2 - 1}\right) \sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \left(\frac{M_{sr} - n_1}{k_1}\right) \\
\times \left(\frac{M_{rd} - n_2}{k_2}\right)\left(-1\right)^{k_1+k_2} \frac{n(n+2)(n!)^2 e^{\frac{C(n_2+k_2)}{2\gamma_2}}}{(n_1+k_1)(n_2+k_2)} \\
\times \left(\frac{\bar{\gamma}_1}{n_1+k_1}\right)^{n+1} W^{-(n+1),1/2}\left(\frac{(n_2+k_2)}{\bar{\gamma}_2}\right)
\]

(5.62)

Note that first order statistics \( (n = 1) \) is the average SNR \( \mu_1 = E[\gamma_{eq}] \), and the second order statistics \( (n = 2) \) is the SNR variance \( \mu_2 = E[\gamma_{eq}^2] \). The AoF, which is a measure of fading severity, can be obtained by, \( AoF = \frac{E[\gamma_{eq}^2] - (E[\gamma_{eq}])^2}{(E[\gamma_{eq}])^2} = \frac{\mu_2}{\mu_1^2} - 1 \).

Figure 5.28 shows the average end to end SNR for different antenna configurations given equal number of total antennas. Evenly distributed antenna configuration seems to produce better performance.

![Figure 5.28: Average End-end SNR for General Order Antenna Selection in MIMO Relay for same total number of antennas](image-url)
5.7.1 SNR Statistics with Direct Link

SNR statistics are presented for GOAS in MIMO Relay taking account of the direct link between S and D. The relayed and the direct links are assumed independent of each other, and the received signals at D can be processed either using Selection-Combining (SC) or Maximal-Ratio-Combining (MRC). Note that the antenna selection at the source for both the relayed and the direct is not possible at the same time unless there are separate antenna sets one each for the relayed and the direct links. The motivation is to compare the performance gain when the direct link exists.

A. SC at Destination

When the largest signal is selected between the relayed and the direct signals, i.e. selection combining (SC) at the destination, we can obtain a closed-form expression for the CDF \( F_{\gamma_{\text{tot}}}^{\text{SC}}(\gamma) \). Since output SNR \( \gamma_{\text{tot}} = \max\{\gamma_{sd}, \gamma_{eq}\} \), we can write
\[
F_{\gamma_{\text{tot}}}^{\text{SC}}(\gamma) = F_{\gamma_{sd}}^{\text{GOS}}(\gamma) F_{\gamma_{eq}}^{\text{GOS}}(\gamma).
\]

The PDF \( f_{\gamma_{\text{tot}}}^{\text{SC}}(\gamma) \) can be obtained as
\[
f_{\gamma_{\text{tot}}}^{\text{SC}}(\gamma) = \frac{d}{d\gamma} F_{\gamma_{\text{tot}}}^{\text{SC}}(\gamma) = f_{\gamma_{sd}}^{\text{GOS}}(\gamma) F_{\gamma_{eq}}^{\text{GOS}}(\gamma) + F_{\gamma_{eq}}^{\text{GOS}}(\gamma) f_{\gamma_{sd}}^{\text{GOS}}(\gamma).
\]

B. MRC at Destination

When the signals from the direct and the relayed links are combined coherently at the destination (MRC), i.e. \( \gamma_{\text{tot}} = \gamma_{sd} + \gamma_{eq} \), it is difficult to obtain a closed-form expression. The conventional approach is based on MGFs. When two independent random variables (RVs) are coherently combined (added), the MGF of the resultant RV is equal to the product of MGFs of the individual RVs, i.e.
\[
M_{\gamma_{\text{tot}}}^{\text{MRC}}(s) = M_{\gamma_{eq}}^{\text{GOS}}(s) M_{\gamma_{sd}}^{\text{GOS}}(s),
\]
where
\[
M_{\gamma_{sd}}^{\text{GOS}}(s) = M_{sd}^{\text{sd}}(\frac{M_{sd}-1}{n_{3}-1}) \sum_{k_{3}=0}^{M_{sd}-n_{3}} \left( \frac{M_{sd}-n_{3}}{k_{3}} \right) (-1)^{k_{3}} n_{3}^{k_{3}+k_{3}+s_{2}},
\]
with \( n_{3} = M_{s} M_{d} \). The inverse Laplace transform of the MGF gives the PDF.

5.7.2 SNR Statistics with Multiple Relays

The analysis presented in previous section can be extended to the scenario where there are multiple relays to relay the signal from the source to the destination. Assumption is that the multi-antenna relays have same number of antenna \( M_{r} \) and subject to similar fading scenario (i.i.d. Rayleigh fading among antennas). The highest relayed signal is selected (Selection Relaying, SR) for demodulation at the destination, i.e. \( \gamma_{\text{tot}} = \max\{\gamma_{eq1}, ..., \gamma_{eq}, ..., \gamma_{eqN}\} \). The SNR CDF \( F_{\gamma_{\text{tot}}}^{\text{SR}}(\gamma) \) can then
Figure 5.29: A MIMO Dual-Hop System with Multiple Relays

be given as,

\[ F_{\gamma_{\text{tot}}}^{SR} (\gamma) = [F_{\gamma_{\text{eq}}}^{GOS} (\gamma)]^N \]

\[ = \sum_{i=0}^{N} \binom{N}{i} \Omega \left[ \sum_{k_1=0}^{M_{sr}-n_1} \sum_{k_2=0}^{M_{rd}-n_2} \binom{M_{sr} - n_1}{k_1} \binom{M_{rd} - n_2}{k_2} \right. \]

\[ \times \left. (-1)^{k_1+k_2+1} e^{\left(\frac{C\gamma}{n_1+k_1}\right)} K_1(\lambda) \right] ^i \]

(5.63)

where \( N \) is the number of relays and \( \Omega = [2M_{sr}M_{rd}\binom{M_{sr}-1}{n_1-1}\binom{M_{rd}-1}{n_2-1}]^i \). The above expression is applicable when the relayed signals are independent to each other. Same set of ordered antennas (i.e. \( n_1 \) and \( n_2 \) are same for all the relayed signals) is selected in all the relays to facilitate simpler expression.

One scenario of practical interest could be when the best link is selected for all relayed
signals, i.e. $n_1 = n_2 = 1$. Above expression becomes,

$$F_{\gamma_{\text{tot}}}^{SR}(\gamma) = \sum_{i=0}^{N} \left( \begin{array}{c} N \\ i \end{array} \right) \left[ 2M_{sr}M_{rd} \sum_{k_1=0}^{M_{sr}-1} \sum_{k_2=0}^{M_{rd}-1} \binom{M_{sr}-1}{k_1} \binom{M_{rd}-1}{k_2} \right] \times (-1)^{k_1+k_2+1} \frac{C\gamma}{e^{(k_1+1)\gamma_1}} \sqrt{\frac{C\gamma}{e^{(k_1+1)(k_2+1)\gamma_1\gamma_2}}} K_1(\lambda) \right]^i$$

(5.64)

The MRC of all the relayed signals provides the optimum performance, but would call for more complicated derivations. Above analysis can be further applied to the case which takes account of the presence of both the direct link and the multiple relays, and again for the selection combining at the destination the derivation is straightforward.

### 5.7.3 Performance Analysis

The impact of order selection on the system performance is analyzed in terms of outage probability and error rate. The modified Bessel function $K_1(\cdot)$ and the Whittaker function $W_{\cdot \cdot}(\cdot)$ in the final expressions were evaluated in MATLAB. The gain $G$ is fixed through the constant $C = \frac{\gamma_1}{e^{1/\gamma_1} E_1(1/\gamma_1)}$ [70, eqn. 16], where $E_1$ is Euler integral.

Figures 5.30-5.32 show the outage probability for different configurations. Evenly distributed antenna configuration performs better and presence of direct offers additional diversity gain (Figure 5.30). The impact of antenna order selection can be accurately quantified according to Figure 5.31. From Figure 5.32, in case of availability of multiple relay nodes more the relays better the performance is.

New general expressions of SNR statistics are obtained for general order antenna selection in a dual-hop amplify-forward fixed-gain relay network. The antenna selection is applied at multi-antenna relay for both hops. The SNR statistics such as CDF, PDF, MGF and General Moments were derived, and then used to analyze the system performance in terms of outage and error rate. The results indicate the usefulness of the derived expressions to accurately quantify the metrics such as performance loss due to selection of lower order antenna (instead of highest order or best antenna). The expressions specialize to earlier results, and were verified by computer simulation.
Figure 5.30: Comparison of Outage Probability for Best Antenna Selection in MIMO Relay for same total number of antennas, with Direct Link; '*' showing for $M_s = M_r = M_d = 2$ and '+' for $M_s = 1, M_r = 4, M_d = 1$

Figure 5.31: Impact of General Order Antenna Selection on Outage Probability in MIMO Relay with Direct Link
Chapter 5. Analysis of Relay/Cooperative Networks

5.8 Chapter Summary

The chapter described the performance analysis of relay/cooperative networks and their performance analysis under various settings. Several possible research extensions are possible based on the results presented here, and the author will look forward to conduct some of them in near future. In summary, following key contributions were observed:

1) Dual-hop amplify-forward relay system was analyzed under mixed fading scenario - Rician and Nakagami-\(m\) fading. Outage probability and SER expression were derived.

2) SER expressions were derived for Rectangular QAM in selection decode-forward relay system. Only the best relay is selected for the signal transmission. The obtained expression generalizes the SER expression for rectangular 2D signal constellation. The validation of the derived expression was my contribution in this joint work.

3) Multi-antenna relay was considered next. Outage behaviour was analyzed for MRC/TAS and SC/TAS at the relay. MRC/TAS performs better than SC/TAS.
scheme. The analysis was extended to Nakagami-$m$ fading.

4) General Order Antenna Selection (GOAS) was applied to multi-antenna relay system to obtain general SNR statistics valid for any order selection at the relay. The analysis was extended to full MIMO relay system. The derived results were verified with computer simulation.
Chapter 6

Conclusions and Future Directions

Performance Enhancement and Analysis of diversity techniques for broadband wireless communications have been the research focus of this thesis. All the diversity techniques are integral part of current and future broadband technologies such as IEEE802.11n, WiMAX, LTE, UWB, etc. These diversity techniques will enable future wireless communications systems to be more resilient to channel impairments, more reliable, offer broadband data rates and ubiquitous access.

Below are a summary of key findings and outcomes of this thesis.

6.1 Summary of Results

In Chapter 3, a new scheme One-Iteration-Clipping-Filtering (OICF) was proposed for PAPR reduction in OFDM systems. This scheme has an advantage of reduced system complexity with almost similar performance with respect to the conventional iterative method. The peak-regrowth after clipping-filtering process has been the key idea behind this scheme. The target PAPR can be obtained by clipping the signal at scaled-threshold at first place, thereby avoiding any further iterations of clipping and filtering. Power savings analysis is carried out for ICF taking account of a practical digital signal processor parameters and power amplifier model. The ultimate target of PAPR reduction is to improve the power efficiency (reduce power consumption) at given spectral mask. Our analysis quantifies the actual power savings due to the PAPR reduction. The analysis is applied to another distortion-less scheme Selected Mapping (SLM). Two schemes are compared for identical PAPR reduction based on net power savings. Although the SLM scheme’s computational cost is almost twice
that of ICF, the net power savings is of both schemes are almost equal. The choice between two schemes will depend on other factors such as space constraint, system complexity, etc. Finally, impact of clipping-filtering on the channel estimation was considered. Pilot-aided channel estimation use some pilot subcarriers. The clipping-filtering has less severe impact on clipping-only process. Pilot re-insertion is also possible which further improves the performance.

The Chapter 4 covered the performance analysis on multi-antenna systems with channel estimation error. First the closed-form expressions of SER were derived for MIMO-MRC with estimation error. The analysis is extended to Rectangular QAM modulation. Similar SER expressions are derived for MRC system over Nakagami-q (Hoyt) channels. Then adaptive GSC system is considered for analysis. SNR statistics such as PDF, CDF and MGF are first obtained, and then used to derive spectral efficiency, error rates, etc. In all the cases, the estimation error is assumed as a Gaussian distributed. Estimation error as expected decreases the system performance. The impact of antenna correlation on MRC performance is analyzed as the final contribution in the chapter. Higher antenna correlation degrades the system performance.

Next Chapter 5 describes the research results on relay and cooperative networks. Primary outcome is the SNR statistics for various fading scenarios and multi-antenna terminals. Relay network under consideration includes a dual-hop system with amplify-forward transmission scheme. The first contribution deals with a relay system subject to dissimilar fading (Rice and Nakagami). Next a system with a multi-antenna relay and single-antenna source and destination was considered. The relay receives the signal from the source and processes it using either MRC or SC. Then the signal is amplified and forwarded to the destination through the best antenna. The scheme using MRC at relay offers better performance than that using SC. The analysis is extended to the case where the fading is assumed Nakagami fading which generalizes the previous results.

General order statistics (GOS) theorem is applied to antenna selection in the multi-antenna relay to derive a general expression for SNR statistics. Unlike in conventional antenna selection where the best antenna is selected, \( n^{\text{th}} \) order antenna is selected for the transmission/reception, called as General Order Antenna Selection.
Chapter 6. Conclusions and Future Directions

(GOAS). GOAS is applied to MIMO relay where all the participating nodes are equipped with multiple antennas, and $n^{th}$ order antenna is selected at each node. The presented analysis can easily be extended to a case where a direct link exists or multiple relay nodes are present. In both cases, system performance improves due to the cooperation diversity.

6.2 Future Directions

There are several areas in this thesis that can lead to further research. Some of them are listed below:

- The proposed scheme OICF can be further improved if the threshold scaling can be made adaptive. Moreover, use of coding or iterative noise canceler may improve the performance further. Furthermore, concurrent use of several PAPR reduction schemes might be beneficial for OFDM systems with large number of subcarriers and higher modulation size.

- In MIMO systems, imperfect channel estimation for the performance analysis was considered. Other practical issues such as feedback delay, power allocation, synchronization, etc can be considered together which will provide valuable insight on the performance achievable of such practical systems. Rectangular QAM has been studied for few systems. Arbitrary modulation constellation shape/size may be another area to explore.

- Finally, relay/cooperative networks are relatively new areas of communications and there are heaps of work to be done. Multi-hop, multi-branch, multi-antenna terminals, resource allocation, spectral efficiency, coding for cooperative networks, synchronization, scheduling, channel estimation, etc are topics of interest. Many of them are already being explored or considered for investigation.
Bibliography


VITA

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