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http://dx.doi.org/10.1364/OE.19.016182
Dipole emitters in fiber: interface effects, collection efficiency and optimization

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Abstract: Single photon emitters coupled to optical fibers are becoming important as sources of non-classical light and nano-scale sensors. At present it is not possible to efficiently interface single photon emitters with the optical fiber platform, and there are particular challenges associated with the need to ensure highly efficient collection and delivery of emitted photons. To model single particle emission, we have considered the coupling of a dipole to an optical fiber mode as a function of orientation and position with respect to the core-cladding interface. Our model shows that it is possible to significantly enhance the collection efficiency into the guided modes as a result of modifications to the dipole emission pattern and power resulting from the surrounding fiber environment. For certain geometries the fiber-dipole coupling can result in a factor of 2.6 increase in the power emitted by the dipole.

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OCIS codes: (060.0060) Fiber optics and optical communications; (270.0270) Quantum optics.

References and links
1. Introduction

There is a considerable impetus to improve the collection of single photon emission for quantum key distribution and quantum metrological applications [1]. However collection of emission from nano-scale objects is problematic, and in most practical settings, sensitivity is limited by photon collection efficiency. Here we explore a fiber-based approach to photon collection from dipole emitters. Some examples of the broad range of systems that are applicable to such analysis include fluorescence capture into optical fibers, crystal defect emission patterns and quantum dot radiation. In addition to single photon emission, our results also guide the exploration of the limits of sensing. The ultimate limit is single atom-scale sensors interacting with atomic objects of interest. Although there are many methods for signal transduction, optical readout provides one of the most convenient and robust methods.

The emission capture of fluorescence into the guided modes of optical fibers has been modelled previously [2], although this did not take into account the effects of the interface on the emitters and assumed an isotropic and homogeneous distribution of emitters. Both the spontaneous emission rate and radiation pattern [3–7] of dipoles are known to change near an interface between two materials. The spontaneous emission rate and radiation pattern changes have been modelled for the case of a dielectric cylinder [4, 5]. Measurements and modelling of the effect of a planar interface [6] and a gold nanoparticle [7] on dipole emission have been performed.
Emission capture into a tapered fiber from an emitter in a photonic crystal cavity has been demonstrated [8, 9], showing theoretical and experimental collection efficiencies of 84% and an estimated 70% respectively. Here we examine the coupling between a dipole and an optical fiber. Although this is a general problem, for concreteness we illustrate our theory by focussing on diamond color centers coupled to optical fibers. With minor modifications, our results can be translated to any nanoscale emitter coupled to any fiber mode.

Diamond color centers, defects in the diamond lattice, have been identified as an important platform for solid-state quantum information processing [10]. In particular, diamond can host a variety of room temperature, photostable bright single photon sources [1]. Photon collection from color centers in bulk diamond is limited in efficiency by the large refractive index contrast of the diamond/air interface, as seen previously in semiconductor LEDs [11]. Such poor efficiencies (~4–5% [11]) limit the application of these sources to tasks such as quantum key distribution and quantum metrology. Incorporation of these color centers into photonic structures is important, in order to fabricate devices with these quantum properties. Access to a hybrid diamond-glass material [12] could allow the en-masse fabrication of such a device in the form of drawn optical fiber.

Advances in this area have previously been achieved by coupling the color centers to a photonic structure in order to modify the emission pattern for practical collection. Diamond nanopillars [13], microrings [14] and solid immersion lenses [15–17] with embedded color centers have been fabricated from bulk diamond. Color center emission has been coupled to non-diamond cavity structures such as microdisk resonators [18], photonic crystals [19], or microspheres [20] by nanopositioning the diamond nanocrystal onto the surface of the structure and coupling evanescently to the cavity. Color centers have been coupled to fiber structures by growing [21], or nanopositioning [22–24], diamond nanocrystals onto the endfaces of fibers. For a review of diamond photonics see [25].

We consider a new configuration for the capture of dipole emission, by either embedding the emitters within the core of a glass optical fiber [12] or coating them onto the core surface. As demonstrated in this work, these architectures enable the performance of devices based on dipoles to be enhanced. Because the treatment we present here is applicable to a range of types of dipoles, we do not specify a precise emitter or wavelength. However, many of the most practical room-temperature single photon sources have their main emission features in the red to near infra-red band (e.g. negatively charged nitrogen vacancy, silicon-vacancy, NE8, silicon-nickel and chromium-related color centers) [1], hence this band is used as the starting point for our fiber designs. Due to our recent demonstration of a versatile new nanodiamond-glass hybrid material [12] based on tellurite [26], a soft glass, we will focus on a tellurite core design in this work. This modelling work can be employed to exploit the potential of fiber devices made using this material by guiding the fiber design and can be used for the optimization of a broad range of emitters and fiber materials.

Here we derive an equation to describe the nanocrystal emission, in a fashion similar to [4], and use this to investigate the emitted power coupled into the fiber. This begins in Section 2, with the derivation of the equations that describe this capture, by representing the diamond emission as a summation of guided fiber modes and radiation modes and calculating the power in each of these modes. In Section 3 we explore the effect of varying core diameter, index contrast, and dipole position and orientation on the emission captured into the fiber’s guided modes. High capture efficiencies and an enhancement of emission power from the high index contrast fiber are observed, which bodes well for future work towards highly efficient single photon devices based on microstructured optical fiber designs.
2. Theory

We model the diamond color center as a dipole with dipole moment $p_0$ and position $r_0 = [r_0, \theta_0, z_0]$ oscillating with a frequency $\omega$. Here we do not consider the linewidth of the dipole emission and assume that the emission is at a single wavelength. Since the typical dimensions of the diamond nanocrystals to be embedded in the tellurite [12] is 40nm and most single photon diamond center emission is at a wavelength of $\sim 650–750$ nm [1], it is reasonable to assume that the refractive index of the background medium (in this case tellurite glass, $n = 2.02$) is not significantly perturbed by the introduction of low concentrations (ideally isolated) of diamond nanocrystals ($n \approx 2.4$) and so we treat the optical properties of the material as isotropic. The fiber is modelled as an infinitely long dielectric cylinder of radius $r_{co}$ and refractive index $n_{co}$ surrounded by an infinite region of refractive index $n_{cl}$. This geometry (shown in Fig. 1) is a reasonable approximation for the guided modes of step-index optical fibers, nanowires, or some classes of microstructured optical fibers [27].

We expand the total field emitted by the dipole into two main components: the guided modes of the fiber and radiation modes, which is defined to be any part of the total field that cannot be expressed in terms of guided modes, as per Ref. [28]. Guided modes have discrete propagation constants ($kn_{cl} < \beta_j \leq kn_{co}$) and are solutions to the wave equation in the step-index fiber, while radiation modes span a continuum of propagation constants ($0 \leq \beta_{rad} < kn_{cl}$).

The total electric field emitted by the dipole can be written as:

$$E = \sum_j a_j(z)e_je^{-i\beta_jz} + a_{-j}(z)e_{-j}e^{i\beta_{-j}z} + \text{radiation modes}$$

where $\beta$ is the mode propagation constant, $e$ is the mode electric field expression, and $a$ is the mode coefficient, for the $j$th mode of the fiber (with $-j$ denoting a backward propagating mode). The magnetic field is expanded similarly. We can then solve for the mode coefficients, $a$, to find the power coupled into each mode from the dipole’s emission. The $e_j$ and $\beta_j$ can be calculated by solving Maxwell’s equations. For the guided modes, this gives the expected step-index mode field expressions [28]. To calculate the radiation modes, we separate the field into two terms, one representing the field in the absence of the waveguide (‘free-space’) and a second term that includes scattering from the waveguide. This separation appears in the cladding field as a Hankel function expression for boundary condition matching due to the presence of the fiber. Inside the fiber, as with the guided modes, a $J$ bessel function fully satisfies the boundary condition. The radiation modes are then of the form [28]:

$$e_z = \begin{cases} 
  a_v J_v(U) f_v(\theta) \\
  c_v J_v(Q) + c'_v H_v^{(1)}(Q) \end{cases} f_v(\theta) \\
  0 \leq r < r_{co} \\
  r_{co} \leq r < \infty \quad (2)
$$

$$h_z = \begin{cases} 
  b_v J_v(U) g_v(\theta) \\
  d_v J_v(Q) + d'_v H_v^{(1)}(Q) \end{cases} g_v(\theta) \\
  0 \leq r < r_{co} \\
  r_{co} \leq r < \infty \quad (3)$$
\[ f_\nu(\theta) = \begin{cases} 
\cos(\nu \theta) & \text{even modes} \\
\sin(\nu \theta) & \text{odd modes}
\end{cases} \] (4)

\[ g_\nu(\theta) = \begin{cases} 
-\sin(\nu \theta) & \text{even modes} \\
\cos(\nu \theta) & \text{odd modes}
\end{cases} \] (5)

where \( \nu \) is an azimuthal mode index, \( a_\nu, b_\nu, c_\nu', c_\nu'', d_\nu' \) and \( d_\nu'' \) are constants (see Appendix) found by applying the continuity conditions at the core-cladding interface, \( Q \) is used instead of \( \beta \) and is defined as

\[ Q = r_{\text{co}}(k_0^2 n_{\text{cl}}^2 - \beta^2)^{1/2}, \] (6)

and the superscripts \( f \) and \( s \) denote the ‘free-space’ and scattering terms, respectively. The four other components of the electric and magnetic field, \( e_r, e_\theta, h_r \) and \( h_\theta \), can be expressed in terms of the derivatives of \( e_z \) and \( h_z \) (see Appendix).

A collection of dipoles with dipole moments \( p_j \), frequencies \( \omega_j \) and positions \( r_j \) contribute to a polarisation density given by:

\[ P(r) = \sum_j p_j e^{i\omega_j t} \delta(r-r_j) \] (7)

For the case of a single dipole, this becomes \( P(r) = p_0 \exp(i\omega t) \delta(r-r_0) \). The current density associated with this dipole is:

\[ J = \frac{\partial P}{\partial t} = i\omega p_0 e^{i\omega t} \delta(r-r_0) \] (8)

We use the reciprocity theorem [28]

\[ \frac{\partial}{\partial z} \int_{A_w} F \cdot \hat{z} \, dA = \int_{A_w} \nabla \cdot F \, dA \] (9)

with \( F = E \times \bar{H}^* + E^* \times H \). In this equation \( E \) and \( H \) are the total fields as defined above in Eq. (1) with an associated current density \( J \) as defined in Eq. (8). Here the barred variables (\( \bar{E}, \bar{H} \) and associated current density \( \bar{J} \)) correspond to a system with no dipole and a single mode, such that \( \bar{E} \) and \( \bar{H} \) consist of any one mode from the expansion in Eq. (1), to give:

\[ \bar{E} = e_j e^{i\beta_j z}, \quad \bar{H} = h_j e^{i\beta_j z}, \quad \bar{J} = 0 \] (10)

We then use Eq. (9) to solve for the mode coefficients, \( a_j \), by substituting for these values. By assuming a non-absorbing waveguide (tellurite has a material loss of less than 1 dBm$^{-1}$ from 600–1400 nm [26]) and using the orthogonality of the modes, shown in Eqs. (13) and (14), we find the mode coefficients to be:

\[ a_j = \frac{i\omega}{4N_j} e^{-i\omega t} e^{i\beta_j z_0} \left[ e_j^*(r_0) \cdot p_0(r_0) \right] \] (11)

where \( \omega \) is the frequency of the dipole and \( N_j \) is a normalisation term given by:

\[ N_j = \frac{1}{2} \int_{A_w} (e \times h^*) \cdot \hat{z} \, dA \] (12)

The guided modes satisfy the orthogonality condition
\[
\int_{A_w} (\mathbf{e}_j \times \mathbf{h}_k^*) \cdot \hat{z} \, dA = \int_{A_w} (\mathbf{e}_j^* \times h_k^*) \cdot \hat{z} \, dA = 0 \quad \text{for } j \neq k
\]  

and in a similar fashion for the radiation modes, taking into account the continuous nature of the propagation constant,

\[
\int_{A_w} [\mathbf{e}_j(Q) \times \mathbf{h}_k^*(Q')] \cdot \hat{z} \, dA = \int_{A_w} [\mathbf{e}_j^*(Q') \times h_k(Q)] \cdot \hat{z} \, dA = 0
\]

for all \( Q \) if \( j \neq k \), and for \( Q \neq Q' \) if \( j = k \) (14)

The power in each mode, \( P_j \), can then be calculated from these mode coefficients by \( P_j = |a_j|^2 N_j \). The total dipole emission power is also calculated from the mode coefficients by summing the power captured into all modes (all guided and radiation modes), i.e.

\[
P_{\text{total}} = \sum_j P_j + \sum_{\nu} \int P_{\nu}(Q) \, dQ
\]

Summation of the radiation power includes an integral due to the continuous nature of the allowed \( \beta \) values, where \( \beta \) has been re-defined in terms of \( Q \) as in Eq. (6). The total power emitted by the dipole must be calculated in this way, rather than as in a bulk material, because of the creation of a cavity due to the fiber core-cladding interface. This alters the dipole emission characteristics [3–7] and leads to constructive or destructive interference resonances as the core diameter changes. Such resonances lead to the enhancement in the dipole emission in some cases. By using the mode coefficients, we include the effects of the core and cladding on the total field, and hence on the total power. This allows the full effect of the fiber on the emission to be calculated, and enables the estimation of both the capacity of the fiber to capture the emitted photons and also the changes in the dipole emission due to the surrounding environment.

3. Results

The total power emitted by a radially-oriented dipole located in the center (labeled ‘center’) and at the core-cladding interface (labeled ‘interface’) of the tellurite fiber is shown in Fig. 2a. The surrounding environment clearly has a significant effect on the dipole emission behaviour, with both the total emission power and the emission pattern changing significantly when constructive or destructive interference occurs due to the core-cladding interface. This is consistent with previous reports [4, 5] that placing the dipole in the vicinity of the step-index waveguide can increase the emitted power above that of a dipole in a bulk (homogeneous) material. Here we have calculated this power change as a function of the fiber’s core diameter. This power has been normalized to the power emitted in bulk tellurite to highlight the effect of the waveguide.

Our results show that the total power oscillates above and below the bulk power as the varying core diameter changes the resonance condition of the dipole emission. It can be seen from Fig. 2a that the power emitted by a dipole on the core-cladding interface is approximately double that of a dipole located in the core center for the case of a radially oriented dipole. It can also be seen that, for a dipole located in the core center, as the core diameter increases, the emitted power tends towards the bulk power. In other words, the fiber begins to resemble the bulk tellurite at large core diameters, as expected. For the case of a dipole at the center, the power approaches zero for small core diameters rather than the cladding emission power. This can be understood by looking at the dipole environment. Regardless of core size, the dipole remains located in the core material and, as the core diameter reduces, the dipole approaches the core-cladding interface. In contrast, for a dipole located on the interface the surroundings approach an environment that consists only of the cladding material.
Simulations based on the equations derived in Section 2 show good agreement (see ‘center’ curve in Fig. 2a) with data obtained computationally using the finite-difference time domain (FDTD) method, shown as black dots in Fig. 2a. This confirms the validity of this model as the FDTD method directly calculates the electric and magnetic fields in a grid system using Maxwell’s equations and so is an independent method to the equations shown here. The FDTD package Meep [29] was used to perform these calculations. The boundaries of the computational space consisted of absorbing perfectly matched layers (PMLs) with a thickness of 1 distance unit for all calculations. The distance unit was defined as the free-space wavelength of 700 nm, to which all distances were normalized. Grid resolutions (the number of calculated points per distance unit) of 55, 55, and 30 were used for the 0.2, 0.42 and 0.6 μm core diameter calculations, respectively. The power was calculated by summing the integral of the Poynting vector normal to a sphere (with radius greater than the core radius and centered on the dipole emitter) at each time step in the calculation. Figure 2b shows convergence data for the FDTD method as a function of increasing grid resolution for the 0.42 μm core diameter case.

The power captured into the guided modes of the fiber is shown in Fig. 3 for a range of core diameters and two dipole positions (at the core center and on the core surface). The captured power has been normalized to the total power emitted by a dipole in bulk diamond, rather than bulk tellurite, to highlight the changes from the addition of the fiber structure and to more easily compare the captured fiber power to that obtained with the more traditional approach of collecting from a color center in a diamond crystal. Due to the large capture efficiency of the fiber and the increase in total power of the dipole, we find that the power captured into the guided modes of the fiber can be greater than the total emitted power of a dipole in bulk diamond. This is shown in Fig. 3a with a radially oriented dipole at the core-cladding interface. That is, for a specific dipole orientation and position, a fiber can collect ~102% of the power
emitted in total in a bulk diamond environment. More generally, for many core diameters and dipole positions, the power captured is greater than 20% of the power emitted in bulk diamond and it is typically higher for a radially oriented dipole (~50–60%). The discontinuities in the plots are due to the appearance of new guided modes as the core diameter is increased. Note that the radial and azimuthal orientations are equivalent for the case of a dipole at the center of the core.

Figure 4 shows the effect of the choice of the core material on the emission capture. The plot shows the captured power, in Figs. 4a and 4c, and the total power, in Figs. 4b and 4d, from the dipole at varying core diameters for three core materials: F2, SF57, (both commercially available silicate glasses) and tellurite (n = 1.61, 1.83 and 2.02 respectively at 700 nm wavelength). This shows that, as might be expected, strong capture is best achieved with a higher core refractive index. This effect is due to both the total emission power increasing and an increase in capture from the higher index, which can be seen by comparing the increase in total and guided power. As can be seen in Fig. 4, the peak total power increase from F2 to tellurite for a dipole at the interface is 2.2/1.39 = 1.58, whereas the peak guided power increase is 1.02/0.49 = 2.08,
showing that the increase in total power alone does not account for the guided power increase.

![Fig. 4. Power vs. core diameter for an air-clad step index fiber with three different core materials: F2, SF57 and tellurite (n = 1.61, 1.83 and 2.02 respectively at 700 nm). (a) Total power and (b) guided power for a dipole positioned at the core center. (c) Total power and (d) guided power for a dipole positioned on the cladding side of the core-cladding interface. The dipole is radially oriented and emitting at a wavelength of 700nm. Power is normalized to the total power emitted by the dipole in a bulk (homogeneous) diamond material. As might be expected, captured power increases as the core index is increased.]

The dipole position and orientation also has an effect on the capture of the dipole emission into the guided modes. Figure 5 shows the captured power as a function of dipole radial position for a radially oriented dipole. The discontinuity of the electric field at the core-cladding interface, visible at $r = r_{co}$, generates a large radial electric field for the guided modes on the core surface (the cladding side of the core-cladding interface). As the dipole coupling equation, Eq. (11), includes an $E \cdot p$ term, this large radial field leads to a large capture into the guided modes for a radially oriented dipole.

4. Conclusion

We have presented a full-vectorial model describing the emission of a dipole in the vicinity of a step-index optical fiber. We show that the total power of the dipole changes depending on its orientation and position with respect to the fiber and that the emitted power can be $\sim 2.6$ times that of a dipole in a bulk material with the same refractive index. Interestingly, we have found that due to changes in the emitted power from interface effects, the emission power collected by the fiber can be larger than the total power emitted by a center in bulk diamond. This shows that a high index contrast fiber could perform well for single photon collection, due to both efficient capture and increased emission power. This model can be used to guide the fabrication of a single photon source which, because of the large capture efficiency obtainable, could be used in applications such as quantum key distribution or quantum metrology. The fabrication
Fig. 5. Plots for a tellurite core fiber in air cladding with a radially oriented dipole emitting at a wavelength of 700 nm. (a) Power captured into the fiber guided modes vs. dipole radial position for a fiber with a core diameter of 0.22 μm. (b) Power captured vs. core diameter, showing the core diameter at which the previous plot was calculated. Power is normalized to the total power emitted by the dipole in a bulk (homogeneous) diamond material. The discontinuity at the core-cladding interface is due to the discontinuity of the radial field.

of such a device presents many practical challenges, such as reducing the background photon count (from both the pump source and material fluorescence) and preventing the excitation of multiple color centers. However, these results show that such a device design could be a candidate for an efficient single photon source.

Appendix

The radiation mode expressions for the radial and azimuthal fields can be found using [28]

\[
e_r = \frac{i}{k^2 n^2 - \beta^2} \left[ \beta \frac{\partial e_z}{\partial r} + \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} \frac{k}{r} \frac{\partial h_z}{\partial \theta} \right] \tag{16}
\]

\[
e_\theta = \frac{i}{k^2 n^2 - \beta^2} \left[ \beta \frac{\partial e_z}{\partial \theta} - \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} \frac{k}{r} \frac{\partial h_z}{\partial r} \right] \tag{17}
\]

Substituting the expressions for \(e_z\) and \(h_z\), Eqs. (2) and (3), into the above equations, Eqs. (16) and (17), gives:

For \(0 \leq r < r_{co}\)

\[
e_r = \frac{f_n(\theta)}{k^2 n_{core}^2 - \beta^2} \frac{iU}{2r_{co}} \left\{ \beta a_n [J_{n-1}(UR) - J_{n+1}(UR)] \right. \\
- \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} k b_n [J_{n-1}(UR) + J_{n+1}(UR)] \} \tag{18}
\]

\[
e_\theta = \frac{g_n(\phi)}{k^2 n_{core}^2 - \beta^2} \frac{iU}{2r_{co}} \left\{ \beta a_n [J_{n-1}(UR) + J_{n+1}(UR)] \right. \\
- \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} k b_n [J_{n-1}(UR) - J_{n+1}(UR)] \} \tag{19}
\]
For $r_{co} \leq r < \infty$

\[
e_r = \frac{f_v(\theta)}{k^2 n_{clad}^2 - \beta^2} \frac{iQ}{2r_{co}} \left( \beta \left\{ c_{v}^f [J_{v-1}(QR) - J_{v+1}(QR)] + c_{v}^t \left[ H_{v-1}^{(1)}(QR) - H_{v+1}^{(1)}(QR) \right] \right\} - k \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} \left\{ d_{v}^f [J_{v-1}(QR) + J_{v+1}(QR)] + d_{v}^t \left[ H_{v-1}^{(1)}(QR) + H_{v+1}^{(1)}(QR) \right] \right\} \right) (20)
\]

\[
e_\theta = \frac{g_v(\phi)}{k^2 n_{clad}^2 - \beta^2} \frac{iQ}{2r_{co}} \left( \beta \left\{ c_{v}^f [J_{v-1}(QR) + J_{v+1}(QR)] + c_{v}^t \left[ H_{v-1}^{(1)}(QR) + H_{v+1}^{(1)}(QR) \right] \right\} - k \left( \frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} \left\{ d_{v}^f [J_{v-1}(QR) - J_{v+1}(QR)] + d_{v}^t \left[ H_{v-1}^{(1)}(QR) - H_{v+1}^{(1)}(QR) \right] \right\} \right) (21)
\]

where $U = r_{co}(k^2 n_{co}^2 - \beta^2)^{1/2}$, $Q = r_{co}(k^2 n_{cl}^2 - \beta^2)^{1/2}$, $V^2 = U^2 - Q^2$, and $R = r/r_{co}$. The radiation mode coefficients (in Ref. [28] Table 25-3, page 525) appearing in Eqs. (2) and (3) are shown here in Table 1 for completeness. These coefficients are calculated by applying the boundary conditions to the electric and magnetic field expressions at the fiber core-cladding interface, i.e. $e_z$, $e_\theta$, $h_z$, and $h_\theta$ are continuous at the interface.
Table 1. Coefficients Appearing in Eqs. (2) and (3) for ‘TM-like’ (ITM) and ‘TE-like’ (ITE) Radiation Modes

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>ITM modes</th>
<th>ITE modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_v$</td>
<td>$-\frac{4}{\pi} \frac{\beta}{kn_c} \frac{\nu}{\nu_0^2} \frac{1}{Q_s} J\nu(\nu H^{(1)}_\nu(Q)M_v}$</td>
<td>$\frac{4}{\pi} \frac{\beta}{kn_c} \frac{\nu}{\nu_0^2} \frac{1}{Q_s} J\nu(\nu H^{(1)}_\nu(Q)M_v)$</td>
</tr>
<tr>
<td>$b_v$</td>
<td>$-\frac{4}{\pi} \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{G_c}{r_0^2}$</td>
<td>$-\frac{4}{\pi} \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{G_c}{r_0^2}$</td>
</tr>
<tr>
<td>$c_{\nu}^t$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$c_{\nu}^r$</td>
<td>$-\frac{4}{\pi} \frac{\beta}{kn_c} \frac{\nu}{\nu_0^2} \frac{1}{Q_s} \frac{1}{J^2(\nu H^{(1)}_\nu(Q)M_v)}$</td>
<td>$2i \frac{Q}{\mu_0} J\nu(\nu H^{(1)}_\nu(Q)M_v)$</td>
</tr>
<tr>
<td>$d_{\nu}^t$</td>
<td>$2i \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{G_c}{r_0^2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$d_{\nu}^r$</td>
<td>$-2i \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{G_c}{r_0^2} \frac{1}{J^2(\nu H^{(1)}_\nu(Q)M_v)}$</td>
<td>$-\frac{4}{\pi} \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{G_c}{r_0^2}$</td>
</tr>
<tr>
<td>$N_j$</td>
<td>$\frac{2\pi k_c}{Q_s} \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{1}{Q_s} \frac{1}{J^2(\nu H^{(1)}_\nu(Q)M_v)}$</td>
<td>$2\pi k_c \left( \frac{\nu_0}{\mu_0} \right) \frac{1}{Q_s} \frac{1}{Q_s} \frac{1}{J^2(\nu H^{(1)}_\nu(Q)M_v)}$</td>
</tr>
</tbody>
</table>

*These correspond to the pure TM and TE modes in the 'free-space' terms that are subsequently altered by the perturbation of the waveguide. From [28].

\[ F_v = \frac{J\nu(U)}{U J\nu(U)} - \frac{H^{(1)}_\nu(Q)}{Q H^{(1)}_\nu(Q)} \] (22)

\[ G_v = \frac{J\nu(U)}{U J\nu(U)} - \frac{n_c^2 H^{(1)}_\nu(Q)}{n_c^2 Q H^{(1)}_\nu(Q)} \] (23)

\[ A_v = M_v \frac{2i n_c^2}{\pi} \frac{F_v}{Q^2 J\nu(Q)H^{(1)}_\nu(Q)} \] (24)

\[ B_v = M_v \frac{2i G_v}{\pi} \frac{G_v}{Q^2 J\nu(Q)H^{(1)}_\nu(Q)} \] (25)

\[ M_v = \left( \frac{vB}{kn_c} \right)^2 \left( \frac{V}{U Q} \right)^4 F_v G_v \] (26)
Acknowledgments

A.D.G. acknowledges the support of an ARC Queen Elizabeth II Fellowship (DP0880466). T.M.M. acknowledges the support of an ARC Federation Fellowship (FF0883189). The authors acknowledge Stephen Warren-Smith for preceding fluorescence capture modelling.