Heat Transfer Enhancement in Microchannels with Liquid-Liquid Slug Flow

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis/project is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

P.M. Thilaksiri Bandara
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The use of microchannels for fluid conduits has significant advantages in a variety of applications. Some of these applications include heat exchangers, micro-reactors, lab-on-a-chip, micro-electronics cooling, and micro-electro-mechanical systems (MEMS). The specific interest of this thesis is their applications in microelectronic cooling. At typical single-phase coolant flow through these channels, the flow is laminar which restricts the heat transfer performance.

Two-phase flow without phase change can radically increase the heat and mass transfer rates in microchannels due to the internal recirculation of the fluids. These two-phase flows are either made up of gas-liquid or immiscible liquid-liquid slug flows. There have been extensive studies on gas-liquid slug flow. Liquid-liquid slug flow has gained more attention recently due to the better thermal properties of liquids compared to gases. However, there is little information available on liquid-liquid slug flow, particularly their ability to enhance heat transfer performance and the pumping power requirements. Thus, this research presents the hydrodynamics and thermal characteristics of liquid-liquid slug flows in micro/mini channels.

The research project involves both experimental and numerical investigations and the study is categorised into three main components; namely slug flow formation and visualization, pressure drop and heat transfer studies. The modelling and experiments were carried out for a wide range of flow parameters, and the effect of those parameters on heat transfer and pressure drop are presented.

Numerical modelling was carried out for both circular and non-circular microchannels. Heat transfer in microchannels under constant temperature wall boundary conditions with liquid-liquid two-phase slug flow has been numerically simulated and validated with volume of fluid (VOF) method. The gradient adaptation method available in ANSYS Fluent was used for grid refinement around the two-phase interfaces avoiding the high computational times when having refined mesh elements throughout the computational domain.

Simulation of flow and heat transfer in circular channels was carried out in 2D axisymmetric space for a wide range of flow parameters such as slug lengths, film thicknesses, contact angles, and capillary numbers ($Ca$). Contact angle is a
defining factor for the interface shape of the droplets while \( Ca \) controls the liquid film thickness. Depending on the \( Ca \), sliding slug flow (without a film) or Taylor flow was observed. The upper limit of \( Ca \) for sliding slug flow was found to depend on the other flow parameters. The increase of slug length leads to decreased average Nusselt numbers showing that longer slugs tend to behave like single phase flows.

Fluid flow and heat transfer in square cross sectional microchannels with one adiabatic wall were simulated. Simulations were carried out for a range of flow rate ratios and different contact angles and droplet formation with a cross-junction is included in the modelling. Significant increase of \( Nu \) was observed and flows with shorter slug lengths gave higher heat transfer rates. This result is similar to that of the circular channels due to the higher intensity of recirculations in shorter slugs.

Three experimental setups were designed for flow visualisation, pressure measurement and heat transfer studies. A T-junction was fabricated from polydimethylsiloxane (PDMS) for the slug flow formation and flow was visualised within the PDMS channels with the high speed imaging in conjunction with microscopy. The silicon oil was selected as carrier fluid while water was the secondary phase fluid. Slug length, plug lengths, plug velocity and film thickness were determined from the captured images. The measured thickness values showed a good agreement with the predicted film thickness from the available correlations.

A U-tube manometer was used to measure the pressure drop across the channel due to the liquid-liquid slug flow. The pressure drop was measured for a range of flow combinations. The single phase pressure drop values were in good agreement with the theory. A significant increase of pressure drop was observed with slug flow. However, these measured pressure drop values showed a significant deviation from the predicted values with available correlations.

The heat transfer experiments were carried out in a stainless steel tube. The channel was heated with Joule heating and the channel wall was considered to have constant heat flux boundary condition. The experimental setup was validated with the single phase flow heat transfer results. For Reynolds numbers lower than 30, a slightly lower \( Nu \) values were obtained as compared to theory, most likely due to the scaling effects. A significant increase of \( Nu \), up to 400% was obtained with slug flow. The slug lengths, void fraction and film thickness showed a significant effect on heat transfer. A correlation based on slug length and volume fraction was developed to calculate the Nusselt number in liquid-liquid slug flow.

Both experimental and numerical studies showed a significant increase of heat transfer rates in liquid-liquid slug flow and the findings of this research can be used in designing systems where liquid-liquid slug flow micro-mini channels plays a key role.
# Table of Contents

Acknowledgments iii

List of Figures ix

List of Tables xv

List of Symbols xvi

## Chapter 1

**Introduction** 1

1.1 Two-phase slug flow . . . . . . . . . . . . . . . . . . . . . . . . . . 2

1.2 Flow fundamentals . . . . . . . . . . . . . . . . . . . . . . . . . . 4

1.2.1 Single phase flow . . . . . . . . . . . . . . . . . . . . . . . . . 4

1.2.1.1 Hydrodynamics . . . . . . . . . . . . . . . . . . . . . 4

1.2.1.2 Heat transfer . . . . . . . . . . . . . . . . . . . . . . 6

1.2.2 Two-phase slug flow . . . . . . . . . . . . . . . . . . . . . . . 7

## Chapter 2

**Literature Review** 11

2.1 Single phase flow heat transfer . . . . . . . . . . . . . . . . . . . . 11

2.2 Non-boiling two-phase slug flow . . . . . . . . . . . . . . . . . . . . 14

2.2.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . 14

2.2.2 Film thickness . . . . . . . . . . . . . . . . . . . . . . . . . . 15

2.2.3 Pressure drop . . . . . . . . . . . . . . . . . . . . . . . . . . 21

2.2.4 Void fraction . . . . . . . . . . . . . . . . . . . . . . . . . . . 25

2.2.5 Wall wettability and contact angle . . . . . . . . . . . . . . . 27

2.3 Two-phase flow heat transfer . . . . . . . . . . . . . . . . . . . . . . 29

2.3.1 Gas-liquid two-phase flow heat transfer . . . . . . . . . . . . . . 30

2.3.2 Liquid-liquid two-phase flow heat transfer . . . . . . . . . . . . . . 32

2.3.2.1 Hydrodynamics . . . . . . . . . . . . . . . . . . . . . 32

2.3.2.2 Heat transfer . . . . . . . . . . . . . . . . . . . . . . 33
2.3.3 Nusselt number correlations ............................................. 34
2.4 Summary ................................................................................ 45
2.4.1 Objectives and Outline ......................................................... 46

Chapter 3
Numerical study of slug flow heat transfer in circular microchannels 48
3.1 Introduction ........................................................................... 48
3.2 Problem formulation and solution procedure ................................. 50
  3.2.1 Interface tracking in two-phase flow ................................. 50
  3.2.2 Governing equations ....................................................... 51
  3.2.3 Differencing schemes ....................................................... 52
  3.2.4 Numerical Model and Boundary conditions ....................... 52
  3.2.5 Grid generation and validation study ............................... 55
3.3 Results and discussion ............................................................ 57
  3.3.1 Flow development .......................................................... 57
  3.3.2 Pressure drop ............................................................... 60
  3.3.3 Heat transfer ................................................................ 61
    3.3.3.1 Effect of temperature dependent properties on heat transfer ......................................................... 62
    3.3.3.2 Nusselt number variation in a unit cell ...................... 64
    3.3.3.3 Effect of contact angle on heat transfer ...................... 65
    3.3.3.4 Effect of film thickness and capillary number on heat transfer ......................................................... 69
    3.3.3.5 Effect of slug length on heat transfer ....................... 70
3.4 Summary ................................................................................. 73

Chapter 4
Numerical study of slug flow heat transfer in square microchannels 74
4.1 Introduction ........................................................................... 74
  4.1.1 Slug flow in non-circular microchannels ............................ 75
4.2 Problem formulation and solution procedure ............................. 76
  4.2.1 Differencing schemes ....................................................... 76
  4.2.2 Numerical Model and Boundary conditions ....................... 77
  4.2.3 Grid generation, independence and validation study ............ 79
    4.2.3.1 Can 2D modelling predict the accurate physics ...... 83
4.3 Results and discussion ............................................................ 85
  4.3.1 Flow development ......................................................... 85
  4.3.2 Flow field in slug flow .................................................... 86
  4.3.3 Heat Transfer .............................................................. 89
    4.3.3.1 Temperature evolution in slug flow ....................... 90
    4.3.3.2 Nusselt number variation along the channel wall .... 93

vii
List of Figures

1.1 Schematic diagram of a two-phase flow, (a) Taylor flow which has a thin liquid film, (b) sliding slug flow which does not have a liquid film. ......................................................... 3
1.2 The velocity boundary layer development for a laminar flow in a circular channel. ................................................................. 5
1.3 Thermal boundary layer development in a circular channel. ...... 6
1.4 Variation of channel wall temperature and bulk fluid temperature in channels under constant wall heat flux boundary condition. . . . 8
1.5 Variation of channel wall temperature and bulk mean fluid temperature at constant temperature wall boundary condition. ........ 9

2.1 Variation of single phase flow Nusselt number with Reynolds number \(Re\) for some studies in the literature [1] ................................. 13
2.2 Schematic diagram of a two-phase flow unit cell of slug and bubble. 17
2.3 Wall pressure distribution of a unit cell in gas-liquid Taylor flow in the axial direction for \(Ca = 0.01\) and \(Re = 100\). Dynamic holdup of the liquid (the fraction of channel length occupied by the liquid) in the channel is 0.65 and dimensionless length is 10 [2]. ............. 23
2.4 Variation of pressure drop with Reynolds number in a circular microchannels of diameter 800\(\mu\)m with a length of 50\(D\) based on different pressure drop correlations from the literature (equation 2.9, equation 2.10 and correlation in [3]). The data points are generated using the values and relationships shown in the Table 2.3 for a liquid-liquid two-phase flow with water as primary fluid and silicon as secondary fluid. ................................. 24
2.5 Schematic of the progression in the wetting behavior of bubbles at increasing \(Ca\) showing advancing and receding contact angles and film thickness such that (a) static contact angles, (b) dynamic contact angles, (c) de-wetting initiation and (d) complete detachment of formation of liquid film. ................................. 28
2.6 Effect of contact angle on slug formation in gas-liquid two-phase flow in a T-junction (Using CFD) [4]. ................................. 29
2.7 Variation of Nusselt number (nondimensionalized with respect to single phase) with Reynolds number and void fraction in a circular microchannels of diameter 800µm based on different correlations from the literature [5, 6, 7, 8]. The parameters (ratio of flow rates and slug lengths) are selected arbitrarily for a liquid-liquid two-phase flow with water as primary fluid and silicon oil as secondary fluid. The data points are generated using the values and relationships shown in the Table 2.3.

3.1 Schematic diagram of two-phase flow model

3.2 Mesh elements used in the simulations

3.3 Variation of velocity, oil volume fraction, and axial temperature using 1 µm, 2 µm and 4 µm mesh elements, (a) Velocity distribution at a plane of 9D away from the inlet (before the oil slugs), (b) Velocity distribution at a plane of 15D away from the inlet (in between the oil slugs), (c) Oil volume fraction distribution along the radial axis in the mid of a oil slug, (d) Temperature variation along the axis.

3.4 Droplet detachment from channel wall for the flow with Ca = 0.0044 given in Case 1.

3.5 Mesh around droplet showing refinement and presence of thin liquid film around the droplet.

3.6 Contours of oil volme fraction obtained from two different base mesh element sizes for the case with Ca = 0.001 (Case 4), (a) with 2 µm base mesh elements and (b) 0.5 µm base mesh elements.

3.7 Contours of oil volume fraction with different flow conditions, (a) Contour of volume fraction with 140° contact angle, Ca = 0.0044, (b) Contour of volume fraction with 120° contact angle, Ca = 0.0044, (c) Contour of volume fraction with 160° contact angle, Ca = 0.001, (d) Contour of volume fraction with 160° contact angle, Ca = 0.002.

3.8 Pressure variation across a droplet along the axial direction for case 1.

3.9 Interfacial pressure drop across a droplet as obtained from simulations and equation 2.7 with different Ca.

3.10 Local Nusselt number variation along the channel (case 1).

3.11 Local Nusselt number as a function of axial distance along channel for case 1 using constant fluid properties and temperature dependent fluid properties.

3.12 Nusselt number variation in a unit cell, flow field and temperature contours around a plug, (a) local Nusselt number variation in a unit cell (case 1), (b) flow field around and oil plug, and (c) temperature variation around and oil plug (case 1).
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.13</td>
<td>Average Nusselt number variation with different contact angles.</td>
<td>67</td>
</tr>
<tr>
<td>3.14</td>
<td>Average Nusselt number variation with different (a) Capillary numbers, (b) film thickness values.</td>
<td>69</td>
</tr>
<tr>
<td>3.15</td>
<td>The droplet shape for different film thickness values.</td>
<td>70</td>
</tr>
<tr>
<td>3.16</td>
<td>Average Nusselt number variation with slug length.</td>
<td>71</td>
</tr>
<tr>
<td>3.17</td>
<td>Average Nusselt numbers calculated from different correlations.</td>
<td>72</td>
</tr>
<tr>
<td>4.1</td>
<td>Schematic diagram of a two-phase flow model with boundary conditions, (a) isometric view of the set up, (b) front view of the set up, (c) side view of the set up, and (d) wall boundary conditions of the heating section.</td>
<td>77</td>
</tr>
<tr>
<td>4.2</td>
<td>Velocity distribution in a water slug and volume fraction of oil along a line, (a) velocity along a line in the mid plane perpendicular to symmetry plane (b) velocity along a line in the symmetry plane, (c) volume fraction of oil along a line in the mid plane perpendicular to symmetry plane (in the front of an oil plug), and (d) volume fraction of oil along a line in the mid plane perpendicular to symmetry plane (in the back part of an oil plug).</td>
<td>80</td>
</tr>
<tr>
<td>4.3</td>
<td>Average Nusselt number comparison between two-phase flow experiments [9], two-phase simulation, single phase flow experiments [9], single phase flow simulations and single phase flow theoretical for different total flow rate flows.</td>
<td>81</td>
</tr>
<tr>
<td>4.4</td>
<td>Velocity distribution at the middle plane of a longer water slug, (a) in radial direction and (b) in the diagonal direction.</td>
<td>82</td>
</tr>
<tr>
<td>4.5</td>
<td>Schematic diagram of 2D numerical model including boundary conditions.</td>
<td>83</td>
</tr>
<tr>
<td>4.6</td>
<td>Contours of oil volume fraction at different fixed Courant number for the total flow rate of 130 ( \mu l/\text{min} ) with 2 ( \mu m ) base mesh elements and IR thermography image from experiments [9] for the same flow rate.</td>
<td>84</td>
</tr>
<tr>
<td>4.7</td>
<td>Local Nusselt number variation in a unit cell in 2D and 3D simulations (water flow rate: 100 ( \mu l/\text{min} ), oil flow rate: 30 ( \mu l/\text{min} )).</td>
<td>85</td>
</tr>
<tr>
<td>4.8</td>
<td>Contours of volume fraction of oil in the mid plane parallel to glass wall with 4 ( \mu m ) mesh elements (water flow rate: 100 ( \mu l/\text{min} ), oil flow rate: (a) 30 ( \mu l/\text{min} ), (b) 10 ( \mu l/\text{min} )).</td>
<td>86</td>
</tr>
<tr>
<td>4.9</td>
<td>Contours of volume fraction of oil in the mid plane parallel to glass wall (water flow rate: 100 ( \mu l/\text{min} ), oil flow rate: 30 ( \mu l/\text{min} )) at different time steps with starting mesh size = 2( \mu m ), (a) ( t = 0 ), (b) ( t = 4.587 \text{ms} ) and (c) ( t = 9.179 \text{ms} ).</td>
<td>87</td>
</tr>
</tbody>
</table>
4.10 Contours of volume fraction of oil in the mid plane parallel to glass wall at $Ca = 0.0044$ (water flow rate: 100 µl/min, oil flow rate: 30 µl/min) with different contact angles, (a) $\theta = 130^\circ$ and (b) $\theta = 160^\circ$, (c) IR thermography image from experiments [9] for the same flow rate. ............................ 88

4.11 Relative velocity vectors in a water slug (water flow rate: µl/min, oil flow rate: 30 µl/min) ................................. 88

4.12 Relative velocity profile in the centre of a water slug (water flow rate: 100 µl/min, oil flow rate: 30 µl/min) ................................. 89

4.13 The local temperature and velocity field distribution in a water slug (water flow rate: 100 µl/min, oil flow rate: 30 µl/min). ................................. 90

4.14 The local temperature distribution in leading and trailing water slugs (water flow rate: 100 µl/min, oil flow rate: (a) 30 µl/min, (b) 20 µl/min and (c) 10 µl/min). ................................. 91

4.15 Evolution of temperature contours in slug flow in a square cross sectional microchannel with one adiabatic wall at different time steps, (a) at 0.25 ms with no oil plugs, (b) at 7.5 ms with 3 oil plugs in cold region, (c) at 10 ms with first few plugs entered to the heated section, (d) at 13.5 ms oil plugs more than half-way through the heated section, and (e) 22.5 ms with full of oil plugs in the heated section ................................. 92

4.16 Bulk temperature variation along the channel with different flow conditions in the heated section. ................................. 94

4.17 Local Nusselt number variation along the channel wall for slug flow (water flow rate: 100 µl/min in, oil flow rate: 30 µl/min) and single phase flow. ................................. 95

4.18 Local Nusselt number variation along the channel wall for slug flow (water flow rate: 100 µl/min, oil flow rate: 30 µl/min) with different contact angles. ................................. 96

4.19 Average $Nu$ for slug flow and single phase flow $Nu$ variation with $Re$. ................................. 97

4.20 Average Nusselt number variation in slug flow with magnitude of relative velocity. ................................. 98

4.21 Local Nusselt number variation in a unit cell with different base mesh elements sizes (water flow rate: 100 µl/min, oil flow rate: 30 µl/min). ................................. 99

5.1 Experimental setup for droplet formations and visualisation ................................. 102

5.2 Detailed illustration of PDMS T-junction ................................. 103

5.3 Experimental set up for pressure drop study ................................. 105

5.4 Experimental set up for heat transfer study ................................. 107

5.5 SEM images of the inside of steel tube ................................. 108
5.6 Thermal images taken from Flir IR camera of the heated steel tube with power supply wires connected by (a) silver soldering and (b) lead soldering. ................................................................. 109
5.7 Slug flow with 250 µl/min constant oil flow rate. .............................. 116
5.8 Slug flow with 500 µl/min constant oil flow rate. .............................. 117
5.9 Slug flow with 1000 µl/min constant oil flow rate. ............................. 118
5.10 Slug flow with 2000 µl/min constant oil flow rate. ............................ 119
5.11 Slug flow with 500 µl/min constant water flow rate. ................. 120
5.12 Variation of slug length (Ls), plug length (Lp) and unit cell length (LUC) with flow rate ratio, Qp/Qs with 1000 µl/min constant oil flow rate. ................................................................. 122
5.13 Variation of slug length (Ls), plug length (Lp) and unit cell length (LUC) with flow rate ratio, Qp/Qs for 500 µl/min constant water flow rate. ......................................................... 123
5.14 Variation of slug/plug lengths with Qp/Qs for different primary phase flow rates, (a) slug length (Ls) and (b) plug length (Lp). ........... 124
5.15 Liquid film around a water plug/droplet for the case of oil flow rate 1000 µl/min and water flow rate 500 µl/min .............................. 124
5.16 Liquid film around a water plug/droplet for the case of oil flow rate 3000 µl/min and water flow rate 500 µl/min .............................. 125
5.17 Liquid film around a water plug/droplet for the case of oil flow rate 2000 µl/min and water flow rate 1000 µl/min .............................. 125
5.18 Thickness of liquid film with Ca ................................................... 126
5.19 Variation of pressure drop with Re for single phase flow ................ 127
5.20 Variation of pressure drop with Re for slug flow ......................... 128
5.21 Variation of outer wall, inner wall, and bulk fluid temperatures along the channel at steady state ................................. 130
5.22 Local Nusselt number along the channel wall for different water flow rates ................................................................. 131
5.23 Comparison of single phase flow Nusselt numbers obtained from present study with experimental data from the literature. ......... 132
5.24 Local Nusselt number variation along the channel in slug flow for some selected flow rate combinations. ......................... 133
5.25 Comparison of experimental Nu with the theoretical Nu ........... 134
5.26 Variation of Nu* (Nusselt number normalised with single phase flow Nu) with slug length, where the water flow rate is constant value of 500 µl/min ........................................ 135
5.27 Variation of Nu* with volume fraction (φ) of water ......................... 136
5.28 Variation of Nu with volume fraction of water for different oil flow rate flows ................................................................. 137
5.29 Variation of Nu* and film thickness with Ca ................................. 138
5.30 Comparison of experimental and numerical Nu values ............. 140
5.3.1 Comparison of experimental $Nu$ and predicted $Nu$ values from equation 5.21 .......................... 141
## List of Tables

2.1 Non-dimensional groups associated with two-phase flow and heat transfer in microchannels ........................................ 14
2.2 Experimental studies of liquid film thickness in two-phase flow in milli/microchannels .................................................. 19
2.3 Different parameters and data values used for the Nusselt number calculations in Figure 2.7 ................................................. 39
2.4 Review of experimental and numerical two-phase flow heat transfer in mini/microchannels ............................................. 40

3.1 Material properties ................................................................. 53
3.2 Different test cases .................................................................. 54
3.3 Calculated film thickness (in micrometers) from different correlations and from present study .............................................. 57
3.4 Average Nusselt numbers for different test cases. Film thickness, $\delta$ is in micrometers and † denotes sliding slug flow .................. 66
3.5 Droplet shapes, fluid temperature, and average Nusselt numbers with different contact angles ............................................. 68

4.1 Different test cases ................................................................. 79

5.1 Material properties ................................................................. 101
5.2 Range of non-dimensional parameters used in pressure drop experiments ................................................................. 106
5.3 Range of non-dimensional parameters used in heat transfer experiments ................................................................. 111
5.4 Uncertainties associated with the measured and derived variables ................................................................. 114
5.5 Different oil and water flow rates ............................................. 115
# List of Symbols

## Roman symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{cond}}$</td>
<td>area of heat conduction</td>
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<td>$C_0$</td>
<td>two-phase distribution parameter</td>
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<td>$Ca$</td>
<td>Capillary number</td>
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<td>$C_f$</td>
<td>skin friction coefficient</td>
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<td>$Co$</td>
<td>Courant number</td>
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<td>current</td>
</tr>
</tbody>
</table>

xvi
$k$  thermal conductivity

$k_{eff}$  effective thermal conductivity

$k_{st}$  thermal conductivity of steel

$kn$  Knudsen number

$L$  length

$L^*$  non-dimensional length

$L_h$  heating length

$L_{hyd}$  hydrodynamically developing length

$L_p$  plug length

$L_s$  slug length

$L_{th}$  thermal entry length

$L_{UC}$  unit cell length

$\dot{m}$  mass flow rate

$n$  unit vector

$Nu$  Nusselt number

$Nu_{L^*}$  apparent slug Nusselt number

$Nu_s$  slug Nusselt number

$Nu_{sp}$  single phase Nusselt number

$Nu_{TP}$  two phase Nusselt number

$Nu_x$  local Nusselt number

$\bar{Nu}$  mean Nusselt number

$p$  pressure

$Pe$  Peclet number

$Pr$  Prandtl number

$Q$  flow rate

$q_{in}$  input energy
$q_{loss}$  energy loss

$q$  heat flux

$\bar{q}$  mean heat flux

$R$  radius

$Re$  Reynolds number

$Re_G$  gas phase Reynolds number

$Re_L$  liquid phase Reynolds number

$T$  temperature

$t$  time

$T_{in}$  inlet temperature

$T_m$  mean fluid temperature

$T_{out}$  outlet temperature

$T_{wi}$  inner wall temperature

$T_{wo}$  outer wall temperature

$U$  velocity

$U_{avg}$  average velocity

$U_B$  bubble velocity

$U_m$  mean velocity

$U_p$  primary phase velocity

$U_s$  secondary phase velocity

$U_{sg}$  gas superficial velocity

$U_{sl}$  liquid superficial velocity

$U_{TP}$  mixture velocity

$U_{ug}$  gas phase drift velocity

$V$  voltage

$We$  Weber number
$x$  flow quality

**Greek symbols**

- $\beta$  void fraction
- $\delta$  film thickness
- $\Delta P$  pressure drop
- $\Delta P_{\text{bub}}$  bubble pressure drop
- $\Delta P_{\text{slug}}$  slug pressure drop
- $\Delta P_{\text{tot}}$  total pressure drop
- $\Delta T$  temperature difference
- $\theta$  contact angle
- $\lambda$  mean free path
- $\mu$  dynamic viscosity
- $\mu_B$  dynamic viscosity of fluid at bulk fluid temperature
- $\mu_{\text{eff}}$  effective viscosity
- $\mu_o$  dynamic viscosity of oil
- $\mu_p$  dynamic viscosity of primary phase
- $\mu_s$  dynamic viscosity of secondary phase
- $\mu_W$  dynamic viscosity of fluid at wall temperature
- $\mu_w$  dynamic viscosity of water
- $\nu_s$  secondary phase fluid kinematic viscosity
- $\rho$  density
- $\rho_{\text{eff}}$  effective density
- $\rho_g$  gas density
- $\rho_l$  liquid density
- $\rho_p$  primary phase density
- $\rho_s$  secondary phase density
σ  interfacial tension
φ  volume fraction

**Acronyms**

*ETFE*  Ethylene Tetrafluoroethylene
*ID*    inner diameter
*LIF*   laser induced fluorescence
*MEMS*  micro electro mechanical systems
*OD*    outer diameter
*PDMS*  Polydimethylsiloxane
*PIV*   particle image velocimetry
*VOF*   volume of fluid
Introduction

The use of microchannels for fluid conduits has significant advantages in a variety of applications. Some of these applications include, heat exchangers, micro-reactors, lab-on-a-chip, micro-electronics, and micro electro mechanical systems (MEMS). In heat exchangers, the amount of heat that can be removed scales as the surface area of the cooling channels, so massively parallel microchannels have the potential to transfer large heat fluxes. Large heat flux removal has become particularly important with the increase in transistor density in microelectronics and with the rise in concentration solar radiation collectors. For example, traditional air cooling has become ineffective in the latest microelectronic systems due to the reduced equipment size, increased heat flux and increased resistance to air flow by compact packing of components in the systems. Therefore, micro-electronic cooling has gained significant interest over the past few decades. Cooling techniques such as falling film cooling, spray cooling, and heat pipes were introduced. However, these techniques proved not to be as effective as expected to cool chips [10], and other low-cost, efficient heat removal methods may be required.

The concept of heat removal by means of liquid flow in microchannels was first introduced by Tuckerman and Pease for electronic cooling. A heat removal rate of 0.79kW/cm² with single phase flow was demonstrated [11, 9]. While impressive, single-phase heat transfer is still limited to comparatively low Nusselt numbers and thus heat fluxes. Computer chips currently require cooling rates up to approximately 1 kW/cm² [12], meaning new solutions need to be found for the next generation of devices that maximise heat transfer rates with minimal pressure
drops.

The heat transfer rate for boiling flow in microchannels is much higher than that of its single-phase counterpart due to the large heat of vaporization [9, 13]. Mudawar and Bowers [14] have shown that flow boiling can dissipate heat at a rate of 10kW/cm², which is 10 times higher than that for single phase flow [9, 14]. Even though flow boiling has been shown to be effective for electronic cooling, it has the drawback of being difficult to control due to back flow and instabilities in the flow. These instability constraints may be overcome while maintaining high heat transfer rates by using a separate fluid phase such as gas or an immiscible liquid into a main continuous liquid - so called two phase flow without phase change.

1.1 Two-phase slug flow

The potential of two-phase flow to provide a high heat transfer rate compared to traditional single phase flow is due to two main reasons; internal recirculation within the liquid slugs which promotes the radial mixing of fluids, leading to a greater radial heat transfer rate, and the higher local fluid velocity in the secondary phase plug leading to a higher heat transfer coefficient (refer to [15]). Above a critical capillary number there exists a thin liquid film between the channel wall and the secondary phase fluid bubble, which has a significant effect on heat and mass transfer. A detailed explanation of this will be given in section 2.2.2. This type of flow was named as Taylor flow after the pioneering studies of Taylor in 1961 [16]. Figure 1.1a illustrates the main properties of Taylor flow including the liquid film between the droplets and the wall. However, these fluid droplets can flow without creating a thin film at low capillary numbers (Ca) by sliding along the channel wall (Figure 1.1b) due to the weak shear forces which can not overcome the adhesion forces. This type of flow is called sliding slug flow/slug flow, while some researchers use slug flow for both two-phase flows with and without a thin liquid film [8, 17].

Other than slug and Taylor flow, there are various other types of two-phase flow patterns such as dispersed bubbly flow, liquid ring flow, and liquid lump flow etc, which have been identified in flow visualization experiments [18, 19]. However, slug/Taylor flow is very easy to produce at non-boiling flow conditions, particularly
Figure 1.1: Schematic diagram of a two-phase flow, (a) Taylor flow which has a thin liquid film, (b) sliding slug flow which does not have a liquid film.

in microchannels where surface tension forces often dominate. Extensive research work has been carried out in the literature on two-phase slug flow in microchannels particularly concerning hydrodynamic characteristics such as velocity of bubbles, void fraction, liquid film thickness, pressure loss and mass transfer enhancement [20, 19, 21, 22, 23, 24]. In fact mass transfer in microreactors can be the limiting factor in reaction rates and Taylor flow has been shown to significantly increase in mass transfer for gas-liquid and liquid-liquid two-phase flows compared to single-phase liquid flow of the same carrying fluid in their review of experimental and modeling studies. [25, 26, 19]. It has been shown that the mass transfer increases through the interface and internal diffusion rates increase too as a function of Capillary number.

While there have been extensive studies on gas-liquid slug flow, there is little information available on liquid-liquid slug flow, particularly during heat transfer. Therefore, further studies and investigation are required determines the hydrodynamics and heat transfer characteristic of liquid-liquid slug flow.
1.2 Flow fundamentals

The channels with largest dimension of the cross section below 1 mm are defined as microchannels while the channels with dimensions between 1 mm - 5 mm are considered as mini-channel, and the above 5 mm are considered as conventional channels [27]. Fluid flow in microchannels is only different from macro/conventional channel flows if the molecular effects such as slip flow become important in small scale. The Knudsen number given in equation 1.1 is used to determine whether the flow can be considered continuum.

\[ Kn = \frac{\lambda}{L} \]  

(1.1)

where \( \lambda \) is the mean free path and \( L \) is the characteristics channel dimension. If \( Kn < 10^{-3} \) the flow is considered as a continuum, while flows with \( Kn \) bigger than \( 10^{-3} \) become non-continuous and consideration of special physical phenomena is required to analyse these types of flows [27]. The mean free path in a liquid is very small. As discussed in [27], \( Kn \) for a water flow in a 50 \( \mu \)m channel is \( 6 \times 10^{-6} \) which is within the range obeying continuum flow. Therefore, the fluid flow in the present study are analysed with continuum flow theories as the channel diameters are ranged from 100 \( \mu \)m - 1 mm.

1.2.1 Single phase flow

1.2.1.1 Hydrodynamics

Micro/mini channel flows are often laminar due to small characteristic dimensions of the channels. The Figure 1.2 shows the velocity boundary layer development in a circular channel for a case with fluid entering with a uniform velocity. The fluid particles near the channel wall come to a complete stop under the no-slip condition. Therefore, the flow velocity in the mid of the channel has to be increased to maintain constant mass flow rate in the channel. This creates a velocity gradient along the channel and the region where this velocity gradient is significant is called hydrodynamically developing region or entrance region. The region beyond the entrance region where the velocity profile fully developed and remain unchanged along the channel axis is called hydrodynamically fully developed region. The
Hydrodynamic entrance region, $L_{hyd}$

Velocity boundary Layer

Hydrodynamically fully developed region

Developing velocity profile

Fully developed velocity profile

Figure 1.2: The velocity boundary layer development for a laminar flow in a circular channel.

Length of this developing region is called entry length and it is given approximately by equation 1.2 for a laminar flow,

$$L_{hyd} = 0.05ReD$$  \hspace{1cm} (1.2)$$

where $D$ is the diameter of the channel and $Re$ is the Reynolds number which measures the relative effect of inertial and viscous forces.

The velocity distribution in fully developed laminar flow is given by equation 1.3.

$$U(r) = \frac{2Q}{\pi R^2} \left(1 - \frac{r^2}{R^2}\right)$$  \hspace{1cm} (1.3)$$

Here, $Q$ is the flow rate of fluid and $R$ is the radius of the channel. The maximum velocity in the channel is taken place at the centre of the channel when $r = 0$. The pressure drop over a length $L$ for laminar flow can be calculated from,

$$\Delta P = \frac{32\mu U_{avg} L}{D^2}$$  \hspace{1cm} (1.4)$$

where, $D$ is the diameter of the channel and $U_{avg} = Q/\pi R^2$ is the average flow velocity.
1.2.1.2 Heat transfer

Heat is the amount of energy that can be transferred from one system to another due to the temperature differences. Heat can be transferred in the forms conduction, convection and radiation while the form of heat transfer considered in this thesis is internal forced convection. Similar to hydrodynamic entry length, a thermal entry region as shown in Figure 1.3 is formed in channels when a cold fluid (at a temperature below the channel wall temperature) enters a hot channel or vice versa. The thermal entry length is calculated from equation 1.5.

\[ L_{th} = 0.05RePrD \]  

Here, \( Pr \) is the Prandtl number which defines the relative effect of momentum diffusivity over thermal diffusivity such that, \( Pr = \mu c_p / k \), where \( \mu, c_p, k \) are viscosity, specific heat and thermal conductivity of fluid respectively. The flow region beyond the thermal entry length is called thermally fully developed region where the non-dimensional temperature \( (T_w - T(r)) / (T_w - T_m) \) remains unchanged, where \( T_w, T(r), \) and \( T_m \) are wall temperature, fluid temperature at distance \( r \), and bulk mean fluid temperature respectively. The channel wall can be heated by imposing either constant wall temperature (\( T_w \)) or constant wall heat flux (\( q'' \)) conditions.
In this study both constant wall temperature and constant wall heat flux boundary conditions are imposed.

**Constant wall heat flux**
The temperature of the channel wall and the bulk mean fluid temperature are increased linearly as in fully developed region under the constant wall heat flux boundary condition as shown in Figure 1.4. $T_{in}$, $T_{out}$, $T_{wi}$, $T_{wo}$, $T_m$, $\Delta T$ are inlet temperature, outlets temperature, inside wall temperature, outside wall temperature, bulk fluid temperature and temperature difference between inside channel wall and bulk fluid respectively. The experimental studies in this thesis are carried out under uniform wall heat flux conditions via Joule heating and the basic theories related to conduction and convection are used to calculate the heat transfer rates as explained in Chapter 5.

**Constant wall temperature**
The difference between the channel wall temperature ($T_{wi}$) and the bulk fluid temperature ($T_m$) decays exponentially under constant wall temperature heating as show in Figure 1.5. The numerical studies are carried out with constant wall temperature boundary condition and further details are explained in Chapters 3 and 4.

### 1.2.2 Two-phase slug flow
Slug flow in micro/mini channels are analysed using continuum flow assumptions similar to single phase flow. As shown in Figure 1.1 slug flow is comprised of a train of secondary phase fluid plugs or droplets with a thin liquid film or without a film. In the case of Taylor flow, the fluid in the secondary phase plug and the carrier phase fluid in the film have slightly different velocities. If the secondary phase plug is long enough to have a uniform film thickness, the velocity in the region far from the rear and front of the film can be represented by an ideal annular flow velocity profile as discussed in [28, 29]. Thus the velocity can be calculate from equations 1.6 and 1.7.
Figure 1.4: Variation of channel wall temperature and bulk fluid temperature in channels under constant wall heat flux boundary condition.

for $0 < r < R_i$,

$$U(r)_s = 2U_T P \frac{\left[1 - \left(\frac{R_i}{R}\right)^2\right] + \left(\frac{\mu_g}{\mu_s}\right) \left(\frac{R_i}{R}\right)^2 \left[1 - \left(\frac{r}{R_i}\right)^2\right]}{\left[1 + \left(\frac{R_i}{R}\right)^4\right] \left[\frac{\mu_g}{\mu_s} - 1\right]}$$  \hspace{1cm} (1.6)
for \( R_i < r < R \)

\[
U(r)_p = 2U_{TP} \frac{\left[ 1 - \left( \frac{R_i}{R} \right)^2 \right]}{\left[ 1 + \left( \frac{R_i}{R} \right)^4 \right] \left[ \frac{\mu_p}{\mu_s} - 1 \right]}
\]

(1.7)

where, \( U_{TP}, R_i, \mu_p, \) and \( \mu_s \) are mixture velocity, plug radius, primary phase viscosity, and secondary phase viscosity respectively. The primary phase velocity profile in the middle of a long slug can be represented from equation 1.3.

Temperature variations of fluids in slug flow is different to single phase flow due to the continuous disruption of the thermal boundary layer from the sec-
ondary phase plugs. Recirculation within the primary and secondary phase fluids cause this continuous variation of thermal boundary layer. Temperature will continuously change until the slugs and plugs come to an stable value based on the wall temperature.

This chapter gave the background and some theory related to the fluid mechanics and heat transfer which is required in the later chapters.
This chapter will present a comprehensive review on heat transfer in microchannels including flow hydrodynamics. The chapter starts with a brief review on single phase flow heat transfer in micro/mini channel followed by the slug flow. Both numerical and experimental studies on the hydrodynamics and heat transfer of two-phase flow without phase change in small channels and tubes are reviewed under slug flow section. The studies on film thickness, pressure drop and void fraction are reviewed and shows that there is little agreement between measured and predicted pressure drop. Furthermore heat transfer rates are examined in the form of Nusselt number ($Nu$) correlations based on different flow parameters. Values are compared using a standard flow regimes for two-phase slug flow indicating huge variability (over 500%) in the Nu values obtained from reported correlations. Finally, the objectives of the present study are outlined based on the existing gaps in the literature.

2.1 Single phase flow heat transfer

Liquid flow heat transfer in conventional channels is well understood and reported in the literature. Similarly, single phase flow heat transfer in microchannels has also been studied extensively in the recent literature due to their high surface area to volume ratio. In 2004, Morini [30] reported a comprehensive review on the experimental work on hydrodynamics and heat transfer in microchannels. He reported the disagreement between experimentally determined friction factors and Nusselt
numbers in microchannels and conventional theory. These disagreements were explained due to reasons such as rarefraction and compressibility effect, viscous dissipation effects, property variation effects, channel surface roughness effects etc., Hetsroni et al. [31] showed the discrepancies between experimental and modelling results when a one dimensional model is used with assumptions such as uniform heat flux and constant heat transfer coefficients. However, they reported fairly good agreement between available experimental data and their modelling in the cases modelled by considering real geometry of the microchannels, temperature dependent physical properties, axial conduction in fluid and wall, and non-adiabatic thermal boundary conditions at the inlet and outlet of the heat sink.

While there is no new physics for flow in microchannels, some of the physics and phenomena that are not important in macroscale play an important role. Some of these scaling effects are viscous heating, conjugate heat transfer, entrance effects and axial conduction. Morini [32] performed analytical modelling for 2D laminar fully developed flow with taking the scaling effects into account. He reported that the entrance effect is significant at moderate or high Reynolds number flows. Celata [33] conducted single phase flow heat transfer experiments in different diameter tubes (ranging from 120 to 528 µm) and reported the evidence for thermal behaviour in developing flow (higher local Nusselt numbers close to inlet and reducing values towards developed region) at large diameter channels while this was not significant in smaller diameter tubes. However, their finding showed an overall decrease in Nusselt numbers from the respected value for conventional tubes for all Reynolds numbers up to transition to turbulent flow. They attribute this to a heat loss term which has similar effects as axial wall conduction as axial conduction was not considered significant in their experiments. Furthermore, they explained that convective heat transfer term is not the mechanism in micro tubes as in conventional tubes, but is counterbalanced by a dissipation term that was not accounted for in their experiments.

A comprehensive review on all experimental and analytical work on single phase flow heat transfer including the scaling effects was published by Rosa et al. [34] in 2008. Despite the reported discrepancies between the microchannel heat transfer results with conventional theories, some researchers found a good agreement with conventional theories. Lelea et al. [35] found a good agreement between
their experimental results with their numerical results, analytical results. They conducted experiments for three different size stainless steel circular microchannels from 125.4-500 μm diameters with water as the working fluid. However, they have not reported data for low Re flows as their Reynolds numbers start at 95. Experimental studies by Schilder et al. [36] also claims that their single phase flow heat transfer results agreed well with the conventional theories. The recent experimental studies by Choo and Kim [1] which conducted on heat transfer in stainless steel microchannels (140 to 506 μm inner diameter) reported the lower Nusselt numbers than the theoretically expected value of 4.36 under the constant wall heat flux boundary condition when the Reynolds number is less than 400 as shown in Figure 2.1.

![Figure 2.1: Variation of single phase flow Nusselt number with Reynolds number (Re) for some studies in the literature [1]](image)

The classical theories of single phase fluid flow and heat transfer can be used for predicting the corresponding values in other types of fluids (gas/liquid) besides water in microchannels. A wide range of experimental and numerical studies have been carried out on single phase flow heat transfer in microchannels with different working fluids other than water as discussed in [37]. Some gas flows heat transfer and fluid flow results shown deviation from classical theories while some agreed well with theories as shown in [37]. Despite the amount of research work in the area, there are still unexplained physics related heat transfer in microchannels
especially in low $Re$ flows as they give low $Nu$ values.

## 2.2 Non-boiling two-phase slug flow

### 2.2.1 Introduction

The earliest studies of non-boiling two-phase flow with slugs of a discontinuous phase in a continuous liquid were reported by Fairbrother and Stubbs [38], Taylor [16] and Bretherton [20]. Effects which are not important in macro sized channels become dominant in small channels, such as the surface tension force. Multi-phase flow in microchannels takes different forms, such as suspended droplets, channel-spanning slugs and annular flow based on the relative magnitude of the different forces such as viscous, interfacial, and inertial [39]. Due to the relative forces associated with the small scales in microchannels, gravitational forces are almost always negligible. This type of flow is generally dictated by surface tension and viscosity. This means the three important non-dimensional parameters for slug flow in microchannels are Reynolds number, capillary number and Weber number which are given in Table 2.1. The symbols, $U$, $D$, $\rho$, $\mu$, and $\sigma$ represent two-phase mixture velocity, diameter of channel, density of primary fluid, dynamic viscosity of primary fluid and interfacial tension. These non-dimensional groups define the nature of the flow and heat transfer rates. Controlled hydrodynamics of slug flow allow to increase heat and mass transfer rates in microchannels. Some of those important slug flow parameters are discussed in details below.

### Table 2.1: Non-dimensional groups associated with two-phase flow and heat transfer in microchannels

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Formula</th>
<th>Physical Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds Number</td>
<td>$Re$</td>
<td>$\frac{\rho U}{\mu}$</td>
<td>Inertial force, Viscous force</td>
</tr>
<tr>
<td>Capillary number</td>
<td>$Ca$</td>
<td>$\frac{\mu U}{\sigma}$</td>
<td>Viscous force, Surface tension force</td>
</tr>
<tr>
<td>Weber number</td>
<td>$We$</td>
<td>$\frac{\rho U^2 D}{\sigma}$</td>
<td>Inertial force, Surface tension force</td>
</tr>
</tbody>
</table>
2.2.2 Film thickness

Taylor bubbles/droplets may be separated from the channel wall by a thin liquid film of the primary liquid phase as shown in Figure 1.1. Shear stress on the bubbles from the liquid film is much smaller than the shear stress on the wall from the liquid film. As a result of this, the bubbles or droplets usually flow with a slightly higher velocity than the continuous fluid velocity. The knowledge of film thickness is important for practical applications that involve heat and mass transfer from the channel wall to the liquid and pressure drop [19, 40]. Therefore, it is necessary to understand the underlying physics of liquid film thickness and to have proper measurement techniques.

The film thickness can be measured experimentally using direct and indirect methods. In the direct method, the film thickness is measured using high quality images, either free from the optical distortion caused by curved channel walls [41], or corrected for the optical distortion [42, 43]. In indirect methods, bubble velocity is measured experimentally and then the film thickness is calculated based on the velocity. This requires knowledge of the velocity profile in the liquid film. Some researchers considered the film as being stagnant such as Suo and Griffith [22] and Warnier et al. [44]. Suo and Griffith introduced a relationship between film thickness and flow velocities as shown in equation 2.1, where $\delta$ is the film thickness, $U_{TP}$ and $U_B$ are two-phase mixture velocity and bubble velocity respectively.

$$\frac{\delta}{D} = \frac{1}{2} \left( 1 - \sqrt{\frac{U_{TP}}{U_B}} \right)$$  \hspace{1cm} (2.1)

Many experimental and numerical studies have been carried out on liquid film thickness in two-phase flow. Bretherton [20] suggested an analytical expression for the liquid film thickness based on lubrication theory where $R$ is the radius of channel:

$$\frac{\delta}{R} = 1.34Ca^{2/3}$$  \hspace{1cm} (2.2)

Bretherton’s expression agreed with the experimentally measured film thickness for a range of capillary numbers from $10^{-3}$ to $10^{-2}$, but the measured film thickness was significantly larger than the theoretical values for $Ca$ below $10^{-3}$. Ratulowski and Chang [45] explained the discrepancy between experimental values and analytical
values for low capillary numbers by taking the effect of trace impurities (Marangoni effect) into account.

Irandoust and Andersson [46] proposed an empirical correlation as shown in equation 2.3 to predict the film thickness based on their experimental results over a wide range of conditions ($9.5 \times 10^{-4} < Ca < 1.90, 0.42 < Re < 860$, and $1 < Re/Ca < 140,000$).

$$\frac{\delta}{D} = 0.18 \left[ 1 - exp(-3.08Ca^{0.54}) \right] \quad (2.3)$$

Aussillous and Quere [41] developed an expression for the film thickness based on scaling arguments and Taylor’s experimental data [47] as shown in equation 2.4. It was found that the liquid film thickness agreed well with Taylor’s data for small $Ca$ and flow with negligible inertia ($Re < 1$).

$$\frac{\delta}{D} = \frac{1}{2} \left( \frac{1.34Ca^{2/3}}{1 + 3.35Ca^{2/3}} \right) \quad (2.4)$$

In reality, inertial effects can have an effect on the film thickness even for low $Re$. Heil [48] studied numerically the effect of inertial force on the liquid film thickness for capillary and Reynolds numbers in the ranges $0.05 < Ca < 5$ and $0 < Re < 280$, and showed that film thickness depends on both Reynolds number and capillary number. Kreutzer et al. [2] investigated the film thickness experimentally and numerically and showed similar trend to Heil. This indicates that inertia can be significant even when $Re \ll 1$ which agrees with the work of de Ryck [49] who showed that the ratio of $Re/Ca$ plays an important role in the inertia to film thickness relationship but no specific correlation was presented.

Recently Leung et al. [50] measured the film thickness over a wide range of capillary and Reynolds numbers ($0.001 < Ca < 0.18$ and $10 < Re < 1100$). Three different liquid phases (Water, Water/Ethylene Glycol, and Ethylene Glycol) were used in their experiments and the film thickness for Ethylene glycol and gas two-phase flow had a good agreement with equation 2.4. While their results compared favourably to equation (2.3) for the Water/Ethylene Glycol system there were unexplained discrepancies for the other liquid combinations.

There are other parameters that affect the film thickness. Using a high resolu-
Figure 2.2: Schematic diagram of a two-phase flow unit cell of slug and bubble.

A laser focus displacement meter was used to measure the film thickness in a microtube directly by Han and Shikazono [43, 42], indicating that the liquid slug length ($L_s$ in Figure 2.2) has a weak effect on film thickness. However, the bubble length ($L_b$) has a considerable effect on the film thickness. Short bubbles with length $L_b < 2D$ had a thicker film than longer bubbles. They also derived an empirical correlation for the dimensionless liquid film thickness in circular channels based on capillary number, Reynolds number, and Weber number at high inertia (refer to [42] for more details) as shown in equation 2.5.

$$
\left[ \frac{\delta}{D_h} \right]_{\text{steady}} = \frac{0.670Ca^{2/3}}{1 + 3.13Ca^{2/3} + 0.504Ca^{2/3}Re^{0.589} - 0.352We^{0.629}}
$$

where, $D_h$ is the hydraulic diameter, and $Ca < 0.3$ and $Re < 2000$.

The film thickness for low $Ca$ flows in microchannels can be very small; on the order of microns or less, which are dimensions that start to push the limits of standard measurement techniques. It has been shown that it is possible to measure micro and nanofilms of fluids using fluorescence microscopy [51]. The fluorescence molecules absorb incident light and emit light of a different wavelength [52]. This technique improves the spatial and temporal range and resolution compared to other optical methods. The temporal resolution is largely determined by the exposure time but is generally too large to capture the transient nature of most two-phase flows in microchannels [51]. Thus, the capabilities of the techniques are restricted by the sensitivity of the camera. Recently Grad et al. [53] demonstrated the use of optical ring resonators as time-resolved refractive index sensors embed-
ded in microfluidic channels that enables sensing thin liquid films. This technique allows measuring thin films in the range of 250 to 400 nanometers in oil-water two-phase flow systems.

Optical white light microscopy can also be used to measure the liquid film, although for thicker films (down to about 5 µm). Howard and Walsh extended the range of applicability of correlation (equation 2.5) in [42] for film thickness prediction from a $Ca$ of 0.3 up to 1.9 for gas-liquid two-phase flow with a wide selection of liquids for carrier phase. Furthermore, they revealed a coupling between equations whereby prediction of bubble velocity is dependent on film thickness and prediction of film thickness on bubble velocity. Consequently, they presented an iterative approach for predicting these parameters. Mac Giolla Eain et al. [54] measured the film thickness for liquid-liquid slug flow with four different carrier oil/water combinations. They studied the effects of aqueous slug length and carrier phase fluid properties on the magnitude of film thickness and showed the variation of film thickness with upper and lower threshold values of aqueous slug length. Similar to Han and Shikazono correlation given in equation 2.5, the mean slug velocity and the capillary forces exert the greatest influence on the magnitude of liquid film. An overview of some of the recent experimental methods used to measure liquid film thickness are summarised in Table 2.2, indicating that there is still a need for higher resolution and faster measurement techniques to measure the very thin film.

The convective heat transfer rate is intrinsically linked to the thermal boundary layer thickness, which in turn depends on the fluid mechanics of the film (if there is one) between the slug and the wall. Thus the understanding of the mechanics of the transient film thickness is essential for understanding heat and mass transfer enhancement. However, in microchannels it is very difficult to measure the film thickness which can be less than 1 µm. Therefore, new techniques need to be developed that can measure the transient nature of these films.
Table 2.2: Experimental studies of liquid film thickness in two-phase flow in milli/microchannels

<table>
<thead>
<tr>
<th>Authors</th>
<th>Flow condition</th>
<th>Geometry</th>
<th>Measuring technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bretherton [20]</td>
<td>$10^{-4} \leq Ca \leq 10^{-2}$</td>
<td>1 mm diameter tube</td>
<td>Volumetry</td>
</tr>
<tr>
<td>Suo and Griffith [22]</td>
<td>$7 \times 10^{-6} \leq Ca \leq 2 \times 10^{-4}$</td>
<td>0.5 - 0.8 mm diameter tubes</td>
<td>Conductimetric techniques</td>
</tr>
<tr>
<td>Irandoust and Andersson [46]</td>
<td>$9.5 \times 10^{-4} &lt; Ca &lt; 1.90$ and $0.42 &lt; Re &lt; 860$</td>
<td>1 – 2mm diameter tubes</td>
<td>Light Absorption</td>
</tr>
<tr>
<td>Takamasa and Kobayashi [55]</td>
<td>$8 &lt; Re &lt; 3000$</td>
<td>10 - 30 mm circular tubes</td>
<td>Laser focus displacement meter</td>
</tr>
<tr>
<td>Aussillous and Quere [41]</td>
<td>$10^{-3} \leq Ca \leq 1.4$</td>
<td>0.4- 1.5 mm tubes</td>
<td>Video recording</td>
</tr>
<tr>
<td>Hazuku et al. [56]</td>
<td>Gas velocity, 0.44 m/s and Liquid velocity 0.070-1.1 m/s</td>
<td>Fluorocarbon tube of 25 μm-2 mm internal diameter</td>
<td>Laser focus displacement meter</td>
</tr>
<tr>
<td>Han and Shikazono [42]</td>
<td>$0 &lt; Ca &lt; 0.2$ and $0 &lt; Re &lt; 2000$</td>
<td>0.3, 0.5, 0.7, 1.0, 1.3 mm diameter circular tubes</td>
<td>Laser focus displacement meter</td>
</tr>
<tr>
<td>Han and Shikazono [43]</td>
<td>$0 &lt; Ca &lt; 0.2$ and $0 &lt; Re &lt; 2000$</td>
<td>0.3, 0.5, and 1.0 mm hydraulic diameter square tubes</td>
<td>Laser focus displacement meter</td>
</tr>
<tr>
<td>Grad et al. [53]</td>
<td>$0.002 &lt; Ca &lt; 0.8$</td>
<td>tubes ranging from 100 to 200 μm</td>
<td>Optical resonators</td>
</tr>
<tr>
<td>Authors (continued)</td>
<td>Flow condition (continued)</td>
<td>Geometry (continued)</td>
<td>Measuring technique (continued)</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------------</td>
<td>----------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>Howard and Walsh [57]</td>
<td>0.0059 &lt; $Ca$ &lt; 1.823 and 0.72 &lt; $Re$ &lt; 122.98</td>
<td>FEP tubing with an internal diameter 1.58±0.05 mm</td>
<td>Optical microscopy</td>
</tr>
<tr>
<td>Mac Giolla Eain et al. [54]</td>
<td>0.002 &lt; $Ca$ &lt; 0.119, 14.46 &lt; $Re$ &lt; 100.96 and 0.047 &lt; $We$ &lt; 0.697</td>
<td>FEP tubing with an internal diameter 1.59 mm</td>
<td>Optical microscopy</td>
</tr>
</tbody>
</table>
2.2.3 Pressure drop

The pressure drop for two-phase flow in microchannels is significant in terms of system design, parasitic energy loss, pump sizing and flow stability. Flow instabilities of parallel channels two-phase flow systems are mainly caused by the pressure drop fluctuations. These effects mean it is important to quantitatively analyse pressure drop in two-phase flow systems using experimental, empirical and semi analytical methods.

Various methods have been developed to determine pressure drop values in Taylor flow with none seemingly able to predict the full range of conditions associated with even laminar flow in microchannels. The model developed by Lockhart and Martinelli [58] and the homogenous flow model [59] have been used as a basis for theoretical correlations using experimental results. However, Liu et al. [23] reported the incompatibility of the above methods for slug-annular flow, annular flow and Taylor flow at relatively low Reynolds numbers due to the flow-regime-independence of those methods. Consequently, an analytical model for the pressure drop in two-phase flow was developed based on the unit cell concept as shown in Figure 2.2 [2, 60]. The total pressure drop, $\Delta P_{tot}$ in a unit cell comprises of two components: frictional pressure drop in the liquid slug and the pressure drop over the bubble. It is assumed that frictional loss in the liquid film is negligible compared to slug frictional loss, however this analysis neglects cases where the slug touches the wall as $Ca \to 0$. The frictional loss in the film region could have a considerable impact if the bubble is really long.

$$\Delta P_{tot} = \Delta P_{slug} + \Delta P_{bub}$$  \hspace{1cm} (2.6)

Here $\Delta P_{slug}$ is the pressure drop in the liquid slug which is the combination of the pressure drop of fully developed laminar flow and the pressure drop caused by internal recirculation. The pressure drop across a bubble (interfacial pressure drop) in a circular tube, $\Delta P_{bub}$ is calculated based on the analytical solution introduced by Bretherton [20], as shown in equation 2.7. The theory assumes a single gas bubble and takes into account the Laplace pressure from surface tension and the curvature. Viscous forces were assumed to be negligible so it applies when $Ca <$
\[ \Delta P_{\text{bub}} = 7.16 (3Ca)^{2/3} \frac{\sigma}{D} \] (2.7)

Kreutzer et al. [2] introduced a friction factor for Taylor flow as given in equation 2.8 considering the frictional loss from both liquid slug or length \( L_s \), and bubble in the unit cell for inertia dominated flows \( (Re = \mathcal{O}(100)) \) and lower capillary numbers \( (Ca = \mathcal{O}(0.01)) \).

\[ f = \frac{16}{Re} \left[ 1 + a \frac{D}{L_s} \left( \frac{Re}{Ca} \right)^{1/3} \right] \] (2.8)

The value of the \( a \) was found to be 0.07 and 0.17 numerically and experimentally respectively. Hence the pressure drop in Taylor flow can be calculated using equation (2.9).

\[ \frac{\Delta P}{L} = \frac{16}{Re} \left[ 1 + a \frac{D}{L_s} \left( \frac{Re}{Ca} \right)^{1/3} \right] \frac{4}{D} \left( \frac{1}{2} \rho U^2 \right) \varepsilon_l \] (2.9)

where \( L \) is the length of the channel and \( \varepsilon_l \) is the volume fraction of primary fluid phase. Kreutzer et al. [2] numerically showed that the pressure drop over a bubble increases with increasing Reynolds number and decreasing capillary number. The pressure difference over the frontal transition region is higher than that at the rear transition region, and a large oscillation takes place at the rear as shown in Figure 2.3 due to the presence of increased recirculation around the gas-liquid interface. They selected a unit cell with a gas bubble and two adjacent half liquid slugs. However, there are only limited microfluidics cases in this inertia dominated regime.

In an attempt to take into account liquid/liquid two phase pressure drop, Jovanovic et al. [17] developed a model to analyse the influence of the film velocity on the slug flow pressure drop considering a constant thickness moving film between the secondary phase liquid droplet and the capillary wall in liquid-liquid two-phase flow. The total pressure drop per unit cell was broken into three components which are frictional losses in the secondary phase, frictional losses in the primary phase, and the interfacial pressure drop. Thus the pressure drop over a unit length is given
Figure 2.3: Wall pressure distribution of a unit cell in gas-liquid Taylor flow in the axial direction for $Ca = 0.01$ and $Re = 100$. Dynamic holdup of the liquid (the fraction of channel length occupied by the liquid) in the channel is 0.65 and dimensionless length is 10 [2].

as in equation 2.10.

$$\Delta P = \frac{4U_s \beta}{(R^2 - (R - \delta)^2)/\mu_p + (0.5(R - \delta)^2)/\mu_s} + \frac{8U_{TP}(1-\beta)\mu_p}{R^2} + \frac{7.16}{L_{UC}} (3Ca)^{2/3} \frac{\sigma}{\rho}$$ (2.10)

where, $\beta$, $\mu_p$, $\mu_s$, $U_{TP}, U_s$, and $L_{UC}$ are void fraction, viscosity of primary phase, viscosity of secondary phase, mixture velocity, secondary phase velocity and unit cell length respectively.

Most recently Eain et al. [3] pointed out the lack of data or consistent correlations for Taylor flow pressure drop. They commented on the wide range of values calculated for identical flow conditions, citing the lack of taking into account the flow physics when developing the models for the discrepancy, particularly
Figure 2.4: Variation of pressure drop with Reynolds number in a circular microchannels of diameter 800µm with a length of 50D based on different pressure drop correlations from the literature (equation 2.9, equation 2.10 and correlation in [3]). The data points are generated using the values and relationships shown in the Table 2.3 for a liquid-liquid two-phase flow with water as primary fluid and silicon as secondary fluid.

liquid/liquid flows. They developed a correlation for the skin friction coefficient, $C_f$, where the pressure drop is $\Delta P = 0.5(C_f)\rho U^2$. $C_f$ is in terms of the non-dimensional length, the capillary number and the Reynolds number, and is given by equation 2.11

$$C_f = 14.486 \left[ (L_s)^{0.65} \times (Ca_p)^{-0.616} \times (Re_p)^{-1.05} \right]$$

(2.11)

where, $p$ and $s$ denote the primary and secondary phase respectively. While it fits
their data quite well there is still a long way to go.

Figure 2.4 shows the variation of pressure drop with Reynolds number for a liquid-liquid two-phase flow system in a 800µm diameter microchannel, calculated using pressure drop correlations discussed above. There is a very large discrepancy in the values which is larger than the uncertainty in the correlations. It should be noted that each study related to different correlations has different uncertainties in pressure drop measurements (refer to original work). Compared to gas-liquid two phase flow, liquid-liquid two phase flow research is still relatively immature leaving to gaps in some of the fundamental physics.

Heat and mass transfer enhancement almost always comes with the penalty of increased pressure drop which is detrimental to a system as the pumping energy is higher. Thus any study of heat and mass transfer increase must have data on the pressure drop increase to ensure a balance is reached between the increase in heat and mass transfer rate and the increased pumping power.

2.2.4 Void fraction

The percentage of the flow domain occupied by the secondary phase (bubble or droplet) in two-phase flow is known as the void fraction. Void fraction is the key parameter of determining other flow parameters in slug flow, namely two-phase density and slug and droplet/bubble velocities. It is also very important in terms of heat transfer, flow pattern transition and pressure drop as the two phases invariably have different viscosity and thermal conductivity. The void fraction is the volume of space occupied by the each phases such that, \( \beta = Q_s/(Q_p + Q_s) \). However, determining the void fraction based on the inlet flow conditions can be misleading, due to the slippage between the two fluid phases [61] in gas-liquid two-phase flows.

It can be challenging to measure the void fraction in micro scale flow systems, and most of the experimental measurements in the literature have been done using flow visualization or conductivity techniques [39]. In conductivity measurements, the impedance between two electrodes in contact with fluid is measured, as the bulk impedance depends upon the relative concentrations of two phases [62, 63]. The void fraction for air-water two-phase flow in microchannels has been measured
successfully using this technique [64, 65, 66, 59, 18, 67].

The experimental results obtained for void fractions in air/liquid flows by Bao et al. [65] showed a good agreement with Lockhart and Martinelli [58] and CISE [68] correlations which were extrapolated from macro-scale models. Triplett et al. [59] found that their experimental data of void fraction in bubbly and slug flow agreed well with the prediction from the homogeneous flow model. Data obtained by Serizawa et al. [18] was in good agreement with the Armand correlation [69] unlike the Kawahara et al. [67] results. In the experimental studies by Godbole et al. [70] in an upward vertical two-phase flow system, the void fraction correlations were compared with the existing correlations in the literature, and best void fraction correlations were highlighted for four different ranges of void fraction. A flow pattern independent drift flux model based void fraction correlation was developed in the most recent experimental studies by Bhagwat and Ghajar [71]. A comprehensive list of correlations for void fraction can be found in [70]. They also showed that the Rouhani and Axelsson correlation [72] predicted the highest number of data points within ±10 and ±15% agreement with the experimental results. This correlation is given in the following equation.

\[
\beta = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}
\]  

Here, \( U_{sg} \) and \( U_{sl} \) are gas and liquid superficial velocities respectively and \( u_{ug} = 1.18 \left[ g\sigma (\rho_l - \rho_g)/\rho_l^2 \right]^{0.25} \) is the gas phase drift velocity. \( g, \rho \) and \( \sigma \) are gravity, density and surface tension while \( l \) and \( g \) stand for liquid and gas phases. Two-phase distribution parameter, \( C_0 \) can be calculated from

\[
C_0 = \begin{cases} 
1 + 0.2(1 - x)(gD\rho_l^2/G^2)^{0.25} & \text{if } 0 < \beta \leq 0.25, \\
1 + 0.2(1 - x) & \text{if } \beta > 0.25.
\end{cases}
\]  

\( G, D \) and \( x \) in the above equation are mass flux, tube diameter and flow quality respectively.

These experimental studies have become a strong foundation for two-phase flow studies. Void fraction is an important parameter in the description of slug/Taylor flow and should be taken into account when describing above conditions. Again
most of the studies have been carried out for gas-liquid two-phase flow with a noticeable few on liquid-liquid flow.

The void fraction is particularly relevant to heat transfer when there is a marked difference between the thermal conductivity of the two phases. For example liquid/gas systems or aqueous/liquid liquid systems.

### 2.2.5 Wall wettability and contact angle

Surface wettability plays a key role in droplet formation in microchannels and in the droplet shape with two-phase flow, when the bubble or droplet touches the wall (sliding slug). The interface shape of a sliding slug is a function of the viscous deformation force, the surface tension force and contact angle. The first two forces are taken into account with the Capillary number but contact angle is not considered except in a couple of studies [73].

Wetting is categorised into four regimes which are, completely wetting (when $\theta = 0^0$ and liquid spreads completely over the solid also called super-hydrophilic surfaces), wetting (when $0^0 < \theta < 90^0$ and liquid spreads partially over the solid), non wetting (when $90^0 < \theta \geq 150^0$ and liquid spread partially over the solid), and super-hydrophobic (when $\theta > 150^0$ [74, 75]). Experimental observations reveal that most material systems typically exhibit contact angle hysteresis [76] i.e., the contact angle depends on which directions the interface is moving. This is important in sliding slug flow as the front and rear of the bubble will have different contact angles when the slug is attached to the wall.

A slug (diameter larger than the channel diameter) at rest for a sufficient amount of time for the liquid film to be drained, will be wetting the wall. If that slug is then accelerated (increased Capillary number) the slug will eventually detach from the wall. The effect of capillary number on contact angle for a bubble in a small channel is illustrated in Figure 2.5. Initially the interface will have a given static angle, $\theta$, at zero velocity (Figure 2.5a). With the external pressure force the bubble will move and the front and back contact angles start to change as a function of $Ca$. This will create a receding ($\theta_{rec}$) and an advancing ($\theta_{adv}$) angle as shown in Figure 2.5b. With the increment of $Ca$, the viscous force become significant, the dynamic contact angle increases and the slug separates from the wall.
wall as shown in Figure 2.5c. A continuous phase films forms between the wall and the slug interface and grows until the discrete phase is completely separated from the wall as show in the Figure 2.5d. This separating liquid film may be very thin (<1 µm depending on the channel diameter and capillary number) and sometimes it is hard to determine whether it exists or not. Therefore, an apparent contact angle ($\theta_{\text{aprnt}}$) is introduced as shown in the Figure 2.5d.

![Figure 2.5: Schematic of the progression in the wetting behavior of bubbles at increasing $Ca$ showing advancing and receding contact angles and film thickness such that (a) static contact angles, (b) dynamic contact angles, (c) de-wetting initiation and (d) complete detachment of formation of liquid film.](image)

The experimental and numerical studies carried out by Rosengarten et al. [77] demonstrate the droplet behavior in a sudden contraction. Furthermore, they highlight the role the contact angle plays when the liquid film surrounding the bubble becomes very thin and can be influenced by long range of van der Waal forces. The contact angle in two-phase slug flow microchannels is important because it affects the shape of the slug and the slug velocities in the channel particularly at low $Ca$. Santos and Kawaji [4] reported the effect of the contact angle on flow formation as shown in Figure 2.6. This effect is significant in microchannels, where surface tension becomes significant due to the smaller diameters. The contact angle affects
the length and speed of the slug which in turn will influence the heat transfer.

Figure 2.6: Effect of contact angle on slug formation in gas-liquid two-phase flow in a T-junction (Using CFD) [4].

Experimental measurements of contact angle in two-phase flow can be used for selecting the appropriate dynamic contact angle in modelling. There are few experimental studies on measurement of contact angle in two-phase slug flows in the literature [78, 79]. However, a static contact angle was measured in these experiments while the contact angle was measured from the photos taken during the flow visualization in some experiments [80].

2.3 Two-phase flow heat transfer

There has been considerable research on the relationship between the heat transfer rate and the hydrodynamics of two-phase flow, both experimentally and numerically. Various correlations in terms of the flow parameters have been developed, however, as will be shown, the agreement in the values from the heat transfer correlations is far from good.
2.3.1 Gas-liquid two-phase flow heat transfer

Oliver and Wright [5] experimentally investigated the effect of void fraction in two-phase flow heat transfer for both Newtonian and non-Newtonian fluids for a constant wall temperature. However, they could not control the slug length due to the experimental limitations. The follow up work by Oliver and Young Hoon [81] reported the effect of slug length on heat transfer. A number of researchers extended this work and suggested expressions for the Nusselt number based on parameters such as channel diameter, heating length, and slug length [21] (refer to section 2.3.3). Walsh et al. [8] also studied the effect of slug length on heat transfer and they identified an entrance region of about one slug length for which initially high values of the Nusselt number relaxed towards a constant asymptotic value similar to single phase flow. Numerical investigations of heat transfer in slug flow with constant wall heat flux have been carried out revealing that the overall heat transfer coefficient is largely controlled by the value for the slug region and the fraction of the wall occupied by the slug [82]. Importantly this is different to the controlling variables for the pressure drop.

In recent experimental work by Leung et al. [21], the dependency of the hydrodynamic characteristics (mixture velocity, homogeneous void fraction and liquid film thickness) on heat transfer rate for gas-liquid Taylor flow was investigated. They extended these experiments in their followup work and analyzed the heat transfer characteristics for three different fluids as the liquid phase. They also used a wide range of capillary numbers ($0.001 < Ca < 0.190$) for the experiments in order to study the underlying control mechanism in two-phase flow heat transfer. They concluded that the size of the recirculation zone and the recirculation efficiency, which are important parameters for Taylor flow heat transfer, are strong functions of capillary number [50]. Further more, they revealed that the liquid film thickness around the gas bubble significantly influences heat transfer performance. In their most recent work Leung et al. [83] studied the gravitational effect on Taylor flow in horizontal microchannels with gas-liquid, and showed that there is only a minor effect of gravity on heat transfer rate.

Recently, Howard et al. experimentally examined the effect of Prandtl and capillary numbers on heat transfer performance in gas-liquid two-phase flow microchannels [84]. They showed a 600% enhancement in heat transfer rates over
conventional Poiseuille flow which was applicable for Nusselt number over inverse Graetz number \((Gr = D Pe/x, Pe = UD/\kappa)\) where, \(x\) is the length, and \(\kappa\) is the thermal diffusivity) ranges from \(10^{-4}\) to 1 and slug length to channel diameter ratio from 0.88 to 32.

The effect of channel diameter on heat transfer (145, 190, 303, and 506 \(\mu m\) channels) and pressure drop was studied experimentally by Choo and Kim [1] for a gas-liquid two-phase flow in stainless steel microchannels. They studied the heat transfer characteristics by keeping the water flow rate constant and varying the air flow rate. They showed that the Nusselt number increased with increasing gas flow rates for the larger diameter channels, due to the presence of turbulent mixing in the liquid film. However, they used different types of flow regimes other than slug flow in heat transfer experiments due to the high Reynolds number flows. Experimental studies by Majumder et al. [85] showed a 1.2 - 1.6 times heat transfer enhancement with gas-liquid Taylor flow in square mini channel. They observed the temporal fluctuations of the fluid temperature using embedded-thermocouples but no fluctuations in the wall temperature due to the relatively large thermal mass of the wall. In the recent experimental studies by Lim et al. [86], the pressure drop variation was studied under adiabatic and heat transfer conditions revealing a significant decrease of pressure drop under the heat transfer condition due to the change in fluid viscosity. They also presented an optimal heat transfer condition for the two-phase flow, is to keep the bubble diameter close to the channel diameter and the void fraction to be around 10%. They showed a 176% maximum thermal performance enhancement under these conditions.

The geometry of the channel cross section also plays an important role in hydrodynamics and heat transfer performance. In microchannels particularly due to different fabrication methods, the cross section of the microchannels may be circular, square, rectangular, triangular, elliptical and trapezoidal etc. [87] which affects the film thickness and uniformity.

While gas-liquid two-phase flow makes a large impact on heat removal, drawbacks and limitations of the technique were identified. A drawback of the introduction of gas bubbles into the liquid flow is the decrease of the flow-averaged values of the thermodynamic properties of the gas-liquid medium due to the low thermal conductivity and heat capacity of the gas compared to those of the liquid
The introduction of immiscible liquid droplets with higher heat capacity and conductivity has the potential to overcome these problems.

2.3.2 Liquid-liquid two-phase flow heat transfer

Liquid-liquid two phase flow offers a higher potential for heat transfer enhancement due to the inherently higher thermal conductivity of liquids compared to gases. This can become marked for very high thermal conductivity liquids such as metals and nanofluids. Characteristics such as flow field and pressure drop of liquid-liquid two-phase flow in microchannels are still not well understood, and limited research work has been carried out [17, 88, 89]. However, various investigations have been carried out on viscous oil-water flows in small and conventional horizontal/vertical pipes [88]. The interest of studying on oil-water two-phase flow in microchannels was driven by applications of microfluidic devices with precisely controlled droplet size and polydispersity in creating emulsions commonly used the chemical, textile, food and other industries [89].

2.3.2.1 Hydrodynamics

While there has been significant research work in the literature on applications of liquid-liquid two-phase flow, generally related to lab-on-a-chip and individual droplet reactors. However, this area will not be covered extensively in this review. A few recent examples include Zhao [90], Howard and Walsh [57] and Rosenfeld et al. [91]. Dreyfus et al. revealed the controllability of flow patterns by the wetting properties of the fluid, in their droplet formation experiments [88, 92, 93]. However, injection methods such as that used by Foroughi and Kawaji may not be stable as in a T-junction or other flow focusing devices. Kashid and Agar [94] investigated the effect of various operating conditions on the flow patterns, slug size, interfacial area, and pressure drop in a Y-junction microchannel. Pressure drop, flow patterns and wettability of oil-water two-phase flow in microchannels were studied by Salim et al. [89]. They carried out experiments using quartz and glass microchannels initially saturated with oil and microchannels initially saturated with water. They visualized different flow patterns in two different channels when the channel was initially saturated with water. While they had droplet,
slugs, and stratified flows for the quartz microchannels, slugs, semi-stratified, and stratified flows were observed in glass channels. This indicates the importance of surface forces in two-phase flow. Foroughi and Kawaji [92] conducted experiments on two liquids in microchannels in order to form droplets or plugs, and observed different types of flow patterns and the associated pressure drop. Recent studies by Jovanovic et al. [17] developed a model for pressure drop analysis including studies on other hydrodynamics characteristics such as slug length and film thickness.

Nanofluids can increase the heat transfer performance in two-phase slug flow due to their larger thermal conductivity. Temperature dependence on droplet formation with nanofluids has been studied in few studies [95, 96]. They have showed the decrease of surface and interfacial tension of nanofluids with increasing temperature. The viscosity of nanofluids was reported to be higher than the base fluids and to decrease with increasing temperature. Furthermore, they reported larger droplets with nanofluids compared to its base fluid.

2.3.2.2 Heat transfer

While wall to liquid heat transfer during slug/Taylor flow in microchannels has been studies numerically, there has only been few experimental studies to date [9, 97]. Heat transfer enhancement in mineral oil carrying water droplets in microtubes was numerically investigated by Urbrant et al. [98] and they revealed improvement of Nusselt number caused by the flow interruption in the carrier flow and the internal circulation within the droplets [9]. The effect of droplet size on heat transfer also was investigated, showing increased droplet size yields more efficient heat transfer and higher values of Nusselt number. Numerical simulations of immiscible fluids with water as the carrying fluid were investigated by Fischer et al. [99]. Their simulations also included nano-particles in the secondary phase fluid and included Marangoni and colloidal effects. They showed that the use of a second suspended liquid (with or without nano-particles) is an efficient way to improve the heat transfer without unacceptably high pressure losses. They also reported a 400% increase in the Nusselt number relative to single-phase flow in the case of slug-train coflow. Experimental work carried out by Asthana et al. [9] investigated the heat transfer enhancement in microchannels with liquid-liquid two-phase flow. A serpentine microchannel was utilized for their experiments and heat transfer
between single-phase liquid and segmented liquid-liquid flow was compared. The comparison was performed using the measurements of local temperature, and velocity and pressure drop at various flow rates. Using Laser Induced Fluorescence (LIF) for measuring temperature and micro-PIV (Particle Image Velocimetry) for velocity measurements, they demonstrated a four-fold Nusselt number improvement in slug flow compared to that with pure water. Recent experimental studies carried out by Eain et al. [97] investigated the Taylor flow heat transfer in a stainless steel tube with a three different oil/water combinations. They reported up to 600% increase of $N_u$ over single phase flow due to the introduction of second immiscible phase with the shorted slug and longest plug length. A significant effect of film thickness on heat transfer was also found in their experiments. The increasing plug lengths and decreasing slug lengths cause for increased heat transfer rates while the increasing film thickness found to reduce Nusselt number. IR thermography was used to measure the wall temperature in their experiments.

### 2.3.3 Nusselt number correlations

Heat transfer in a two-phase flow system depends on factors such as flow conditions (particularly near the walls), geometry of the channel, and properties of the fluids. Thus, a model developed based on the average flow properties of the two-phases is not sufficient to describe the underlying physics. It is necessary to carefully consider the near-wall and bulk fluid properties. Consequently, theoretical models have been developed based on different flow conditions and flow parameters to explain the heat transfer behavior in two-phase flow. Che et al. [100, 101, 102] developed analytical and numerical models for mass and heat transfer of plug flow in cylindrical capillaries. These works predicted one order of magnitude improvement for the heat transfer coefficient using plug flow without phase change.

The heat transfer coefficient on channel walls can be non-dimensionalized into local Nusselt number ($N_{ux}$) which is defined as

$$N_{ux} = \frac{q D_h}{k(T_w - T_m)} = \frac{h D_h}{k} \tag{2.14}$$

where $q$, $T_w$ and $T_m$ are heat flux, wall temperature, and mean flow temperature respectively. The mean Nusselt number can be calculated by integrating the Nusselt...
number over the non dimensional duct length.

$$Nu = \frac{1}{L^*} \int_0^{L^*} Nu_x dx = \frac{\bar{q} D_h}{k \Delta T} \quad \text{(2.15)}$$

where $L^*$ is the length of the channel non-dimensionalized with respect to $D_h$ and $\bar{q}$ is the mean heat flux. The value, $k$ is thermal conductivity of the primary phase.

Experimental work carried out by different research groups has clearly shown a significant increase in two-phase flow heat transfer relative to that for single phase flow. Theoretical models have also been developed to correlate experimental data. Oliver and Wright [5] introduced a model, which is a modification to Gratz-Leveque solution for thermally developing laminar flow, to explain their experimental results that reportedly showed 2.5 times heat transfer enhancement. The model was in the form of a Nusselt number, $Nu$, given by

$$Nu = 1.615 \left( Re Pr \frac{D}{L} \right)^{1/3} \left( \frac{1.2}{(1-\beta)^{0.36}} - \frac{0.2}{1-\beta} \right) \left( \frac{\mu_B}{\mu_W} \right)^{0.14} \quad \text{(2.16)}$$

where $\mu_B$ and $\mu_W$ are the liquid viscosities at bulk fluid temperature and the heat transfer boundary temperature respectively. $L$ is the heated length of the channel. However, there was no quantitative explanation on effects of slug length on heat transfer due to the experimental limitation of controlling of slug lengths. A modification of the Graetz-Leveque solution considering the fraction of the cross section occupied by liquid (in gas-liquid flow) for a thermally developing laminar flow proposed by Hughmark [6] is

$$Nu \sqrt{1-\beta} = 1.75 \left( Re Pr \frac{D}{L} \right)^{1/3} \left( \frac{\mu_B}{\mu_W} \right)^{0.14}. \quad \text{(2.17)}$$

However, they also have not considered the slug length into account in their correlation.

A Nusselt number correlation for the slug region ($Nu_s$) based on the slug length was proposed by Kreutzer et al. [7] to represent their results of two-dimensional CFD simulations. A wide range of conditions ($1 < L_s/D < 16$, $7 < Pr < 700$, $10 < Re < 400$) for two-phase flow in a 1 mm diameter tube were considered in
their simulations, with their correlation given as

$$Nu_s = 20 \left[ 1 + 0.003 \left( \frac{L_s}{RePrD} \right)^{-0.7} \right]. \quad (2.18)$$

As discussed in section 2.3.1, Walsh et al. [8] developed a correlation for the Nusselt number based on slug length which is given as

$$Nu = (1 - \beta) \left[ Nu_{sp} + 25 \left( \frac{L_s}{D} \right)^{-0.5} \right], \quad (2.19)$$

where $Nu_{sp}$ is the fully developed liquid-only (single phase) Nusselt number for constant heat flux conditions. In this study they used a wide range of values for $Ca$ and $Re$ numbers ($6.6 \times 10^{-4} < Ca < 8.8 \times 10^{-3}$ and $56.4 < Re < 1127$) and identified that the effective heat transfer area is the part of the wall covered by the liquid slugs but not by the gas bubbles.

Following a similar method to [8], Leung et al. [21] argued that characteristic length in heat transfer calculations should be slug length, rather than the total length from the entrance, and correlated their experimental results to an apparent slug Nusselt number as a function of the dimensionless slug length as given in equation 2.20.

$$Nu_{L*} = 4.364 + \frac{0.29}{L_s^* + 0.15L_s^{*1/3}} \quad (2.20)$$

Apparent slug Nusselt number, $Nu_{L*}$ is related to Nusselt number as

$$Nu_{L*} = Nu \frac{L_{UC}}{L_s} \quad (2.21)$$

where $L_{UC}$ is the length of unit cell. $L_s^* = \frac{L_s}{RePrD}$ is the dimensionless slug length.

Unfortunately, there is little agreement in the literature in terms of heat transfer as these correlations give widely varying values of the Nusselt number for identical conditions. In order to demonstrate the variation, the Nusselt number is plotted as a function of the Reynolds number and void fraction for liquid-liquid two-phase flow in a circular microchannel is shown in Figure 2.7 (a) and (b), using the correlations given above. A circular channels of diameter 800$\mu$m with heating length, $L = 50D$ is used to illustrate the variation in predicted Nusselt number for the arbi-
Figure 2.7: Variation of Nusselt number (nondimensionalized with respect to single phase) with Reynolds number and void fraction in a circular microchannels of diameter 800 µm based on different correlations from the literature [5, 6, 7, 8]. The parameters (ratio of flow rates and slug lengths) are selected arbitrarily for a liquid-liquid two-phase flow with water as primary fluid and silicon oil as secondary fluid. The data points are generated using the values and relationships shown in the Table 2.3.

The flow rate of primary phase (water) is varied from 1206 - 2413 µl/min, with secondary phase (silicon oil) flow rates of 302 - 2715 µl/min. The slug length to hydraulic diameter ratio, $L_s/D = 3$ and unit cell length to diameter ratio, $L_{UC}/D = 5$ are maintained constant for both Nusselt number against Reynolds number and Nusselt number against void fraction calculations. Homogeneous void fraction is calculated using $\beta = U_s/(U_p + U_s)$ where $U_p$ is the velocity of primary fluid and $U_s$
is the velocity of secondary fluid. Reynolds and capillary numbers are calculated based on the mean two-phase velocity, $U_{TP} = U_p + U_s$. The Nusselt number for fully-developed single-phase flow is 4.36 for a circular channel flow with constant heat flux boundary conditions. As Figure 2.7 shows most correlations show a significant increase in Nusselt number relative to single-phase flow, however there is a massive difference in the values. These differences could be due to; differences in boundary conditions used in each study, accuracies related to temperature measurements, surface effects that were not taken into account, or the parameters are beyond the limits for which the correlations were developed. Additionally, there are more interrelated parameters that affect heat transfer rates than the current correlations taken into account, while each correlation was developed for a specific set of conditions.

A summary of the key heat transfer studies in two-phase flow is given in Table 2.4.
Table 2.3: Different parameters and data values used for the Nusselt number calculations in Figure 2.7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculation method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water flow rates</td>
<td>1206 - 2413 µl/min</td>
</tr>
<tr>
<td>Silicon oil flow rate</td>
<td>302 - 2715 µl/min</td>
</tr>
<tr>
<td>Mixture velocity</td>
<td>$U_{TP} = U_p + U_s$</td>
</tr>
<tr>
<td>Homogeneous void fraction</td>
<td>$\beta = \frac{U_s}{U_p + U_s}$</td>
</tr>
<tr>
<td>Capillary number</td>
<td>$Ca = \frac{\mu_p U_{TP}}{\sigma}$</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>$Re = \frac{\rho_p U_{TP} D}{\mu_p}$</td>
</tr>
<tr>
<td>Single phase Nusselt number</td>
<td>4.36 for constant heat flux wall</td>
</tr>
<tr>
<td>Viscosity of water</td>
<td>$\mu_p = 0.001002$ Pa.s</td>
</tr>
<tr>
<td>Kinematic viscosity of silicon oil</td>
<td>$\nu_s = 1.5$ cSt</td>
</tr>
<tr>
<td>Interfacial tension of water/silicon oil</td>
<td>$\sigma = 0.036$ N/m</td>
</tr>
<tr>
<td>Prandtl Number</td>
<td>$Pr = \frac{C_p \mu_p}{k}$</td>
</tr>
<tr>
<td>Slug length to diameter ratio</td>
<td>$L_s/D = 3$</td>
</tr>
<tr>
<td>Unit cell length to diameter ratio</td>
<td>$L_{UC}/D = 5$</td>
</tr>
</tbody>
</table>
Table 2.4: Review of experimental and numerical two-phase flow heat transfer in mini/microchannels

<table>
<thead>
<tr>
<th>Authors</th>
<th>Mode of study</th>
<th>Geometry</th>
<th>Working Fluids</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prothero and Burton [103]</td>
<td>experimental</td>
<td>4 mm diameter circular tubes</td>
<td>air and water</td>
<td>( \frac{Nu}{Nu_{sp}} = 2 )</td>
</tr>
<tr>
<td>Oliver and Wright [5]</td>
<td>experimental</td>
<td>6.4 mm diameter circular channels</td>
<td>gas-liquid (Newtonian and non-Newtonian liquids)</td>
<td>( 10 \leq Re \leq 1000, \frac{Nu}{Nu_{sp}} = 2.5 )</td>
</tr>
<tr>
<td>Oliver and Young Hoon [81]</td>
<td>experimental</td>
<td>6.4 mm diameter circular channels</td>
<td>gas-liquid (Newtonian and non-Newtonian liquids)</td>
<td>( 10 \leq Re \leq 1000, \frac{Nu}{Nu_{sp}} = 2.5 )</td>
</tr>
<tr>
<td>Bao et al. [104]</td>
<td>Experimental</td>
<td>1.95 mm diameter circular channel</td>
<td>air and water</td>
<td>( 70 \leq Re_G \leq 11110 ) and ( 150 \leq Re_L \leq 1400 ), constant wall heat flux, increased heat transfer at higher gas and liquid velocities</td>
</tr>
<tr>
<td>Fukagata et al. [105]</td>
<td>Numerical</td>
<td>20 ( \mu )m diameter circular channel</td>
<td>gas-liquid</td>
<td>( \frac{Nu}{Nu_{sp}} \simeq 2 )</td>
</tr>
<tr>
<td>Lakehal et al. [106]</td>
<td>Numerical</td>
<td>1 mm diameter circular channel</td>
<td>400 ( \leq Re \leq 3000 ), gas-liquid</td>
<td>( \frac{Nu}{Nu_{sp}} = 3 - 4 )</td>
</tr>
<tr>
<td>Urbrant et al. [98]</td>
<td>Numerical</td>
<td>100 ( \mu )m diameter channel</td>
<td>liquid-liquid (mineral oil and water)</td>
<td>( Ca \leq 0.15 ) and ( Re \leq 0.1 ), effect of slug size ( a/D = 0.4, 0.5, 0.6, ) and ( 0.7 ) on heat transfer was studied. Higher Nusselt number for larger droplets of mineral oil</td>
</tr>
<tr>
<td>Authors (continued)</td>
<td>Mode of study (continued)</td>
<td>Geometry (continued)</td>
<td>Working Fluids</td>
<td>Remarks (continued)</td>
</tr>
<tr>
<td>---------------------</td>
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<td>---------------------</td>
</tr>
<tr>
<td>Hetsroni et al. [107]</td>
<td>Experimental</td>
<td>130 $\mu$m hydraulic diameter triangular cross-sectional channels</td>
<td>air and water</td>
<td>$4.7 \leq Re_G \leq 270$ and $4 \leq Re_L \leq 56$, increases the heat transfer coefficient with increasing liquid velocity and decreases with increasing air velocity</td>
</tr>
<tr>
<td>Walsh et al. [8]</td>
<td>Experimental</td>
<td>1.5 mm diameter stainless steel channel</td>
<td>gas-liquid</td>
<td>$6.6 \times 10^{-4} \leq Ca \leq 8.0 \times 10^{-3}$ and $56.4 \leq Re \leq 1127$, constant wall heat flux, Nusselt number increases two-fold when the slug length decreased from 15D to 2D</td>
</tr>
<tr>
<td>He et al. [82]</td>
<td>Numerical</td>
<td>1.5 mm diameter gas-liquid channel</td>
<td></td>
<td>$32 \leq Re \leq 720$, overall heat transfer rate depends upon the mean thermal resistance in the film and slug region, as well as the heat transfer coefficient at the interface between the two fluids</td>
</tr>
<tr>
<td>Authors (continued)</td>
<td>Mode of study</td>
<td>Geometry (continued)</td>
<td>Working Fluids (continued)</td>
<td>Remarks (continued)</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>---------------</td>
<td>-----------------------</td>
<td>----------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Gupta et al. [108]</td>
<td>Numerical</td>
<td>0.5 mm diameter channel</td>
<td>gas-liquid</td>
<td>$Re = 280, Ca = 0.006$, $\frac{Nu}{Nu_{sp}} = 2.5$ for both constant wall heat flux and constant wall temperature cases, Nusselt number increase with decreasing homogenous void fraction</td>
</tr>
<tr>
<td>Leung et al. [21]</td>
<td>Experimental</td>
<td>2 mm diameter channel</td>
<td>gas-liquid (water and Nitrogen)</td>
<td>$200 \leq Re \leq 1100$, constant wall heat flux, $\frac{Nu}{Nu_{sp}} = 3.2$</td>
</tr>
<tr>
<td>Fischer et al. [99]</td>
<td>Numerical</td>
<td>100-1000 $\mu$m diameter channel</td>
<td>liquid-liquid (Water, 5cSt silicone oil, and PAO, with $Al_2O_3$ nanoparticles)</td>
<td>$0.01 \leq Re \leq 100$, strong effect of interfacial tension on the radial heat transfer</td>
</tr>
<tr>
<td>Howard et al. [84]</td>
<td>Experimental</td>
<td>1.5 mm internal diameter with 0.25 mm thickness channel</td>
<td>gas-liquid (water, Ethylene-Glycol and water mixture, PD5 oil, Pure Ethylene Glycol, AS100 Silicon oil, and Air)</td>
<td>$0.002 \leq Ca \leq 0.677$, 600% enhancement in heat transfer rates over conventional Poiseuille flow</td>
</tr>
<tr>
<td>Asthana et al. [9]</td>
<td>Experimental</td>
<td>rectangular silicon microchannel</td>
<td>liquid-liquid (water and mineral oil)</td>
<td>$\frac{Nu}{Nu_{sp}} = 4$</td>
</tr>
<tr>
<td>Authors (continued)</td>
<td>Mode of study (continued)</td>
<td>Geometry (continued)</td>
<td>Working Fluids (continued)</td>
<td>Remarks (continued)</td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------------------</td>
<td>----------------------</td>
<td>-----------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Choo and Kim [1]</td>
<td>Experimental</td>
<td>140, 222, 334, and 506 μm internal diameter stainless steel microchannels</td>
<td>gas-liquid (air and water)</td>
<td>gas superficial velocity, 1.24 - 40.1 m/s, liquid superficial velocity, 0.57 - 2.13 m/s, and wall heat flux, 0.34 - 0.95 MW/m², Effect of channel diameter on the Nusselt number and pressure drop size of the recirculation zone and the recirculation efficiency are important parameters for heat transfer enhancement</td>
</tr>
<tr>
<td>Leung et al. [50]</td>
<td>Experimental</td>
<td>2 mm diameter channel</td>
<td>gas-liquid (water, water-Ethylene Glycol, pure Ethylene Glycol and Nitrogen)</td>
<td>minor effects of gravity on heat transfer</td>
</tr>
<tr>
<td>Leung et al. [83]</td>
<td>Experimental</td>
<td>1.12 mm, 1.69 mm, and 2.12 mm diameter channels</td>
<td>gas-liquid (Ethylene Glycol and Nitrogen)</td>
<td></td>
</tr>
<tr>
<td>Lim et al. [86]</td>
<td>Experimental</td>
<td>300 μm and 500 μm diameter circular channels</td>
<td>gas-liquid (de-ionized water and Nitrogen)</td>
<td>Flow visualization and heat transfer experiments carried out. 176% enhancement in heat transfer was gained for bubbly flow</td>
</tr>
<tr>
<td>Majumder et al. [85]</td>
<td>Experimental</td>
<td>3.3 mm × 3.3 mm × 350 mm square channel</td>
<td>air and water</td>
<td>1.2 to 1.6 times heat transfer improvement</td>
</tr>
<tr>
<td>Authors (continued)</td>
<td>Mode of study (continued)</td>
<td>Geometry (continued)</td>
<td>Working Fluids (continued)</td>
<td>Remarks (continued)</td>
</tr>
<tr>
<td>--------------------</td>
<td>---------------------------</td>
<td>----------------------</td>
<td>---------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Eain et al. [97]</td>
<td>Experimental</td>
<td>1.5 mm circular stainless steel channel</td>
<td>Three different combinations (Pd5, Dodecane and AR20 silicon oil as primary fluids and water as secondary fluid)</td>
<td>upto 600 % increase of ( Nu ), effect of slug length and film thickness on heat transfer</td>
</tr>
</tbody>
</table>
2.4 Summary

Experimental and numerical studies on the hydrodynamics and heat transfer of slug flow in microchannels have been reviewed. Two-phase slug flow significantly increases heat flux relative to single phase flow with little consensus on the amount of enhancement. The studies revealed that using two immiscible liquids rather than gas-liquid gives further enhancement due to the higher heat capacity and thermal conductivity of liquids relative to gases. However, due to the larger viscosity of the liquids, the pump power of the system will be increased. Therefore, it is important to have a quantitative comparison between the energy consumption and the heat transfer enhancement in practical systems. Whilst the hydrodynamics of slug/Taylor flow are generally well understood there remain some gaps in the understanding of heat transfer process.

Slug flow formation is an important stage in two-phase flow applications and proper techniques need to be studied experimentally in order to have controlled slug and lengths and velocities. The liquid film thickness in two-phase flow has been studied using both experimental and numerical methods. However, there are difficulties in capturing the liquid film thickness in numerical simulations due to the need for high grid resolution near the wall which is computationally costly. Therefore, it is necessary to develop modeling techniques that can capture the liquid film thickness that affects heat transfer, mass transfer, and pressure drop in microchannels. There are no significant experimental or numerical studies on liquid film thickness measurement in liquid-liquid two-phase flow and thus there are opportunities to develop that area.

A flow regime that has had little attention has been highlighted. At low Ca the slug can slide along the channel wall without a liquid film and thus the shape and the heat transfer depends on the contact angle.

Even though comprehensive studies have been conducted in the literature on two-phase flow pressure drop in microchannels, most of these experimental and numerical works are on gas-liquid two-phase flow with very little on liquid-liquid two-phase flow. Thus the analysis of pressure drop in liquid-liquid two-phase flow will have significant interest among researchers correlations are needed in designing microfluidic systems.
There are discrepancies of over 500% between $Nu$ correlation results, even though a number of theoretical models have been developed to explain the relationship between the heat transfer and hydrodynamic parameters. New theoretical models are required to explain the relationship between heat transfer and parameters such as film thickness, pressure drop, void fraction, and contact angle etc in order to help resolve these discrepancies. Thus in this thesis some of the knowledge and gaps are filled via the following research objectives.

2.4.1 Objectives and Outline

This research presents the hydrodynamics and thermal characteristics of liquid-liquid slug flows in micro/mini channels. Two different liquid combinations were used in this study. Water was selected for the primary phase fluid while light mineral oil was used for the secondary phase in numerical studies. However, low viscous silicon oil was used for the primary phase while water was used for the secondary phase in experimental study. The objective of the research to study the factors affecting liquid-liquid slug flow heat transfer in micro/mini channels. Therefore, following research tasks were identified to reach the research objective.

- Numerically study the effect of flow parameters such as slug length, film thickness, void fraction, contact angle on heat transfer in circular microchannels.
- Numerically study the effect of flow parameters such as slug length, film thickness, void fraction, contact angle on heat transfer in non-circular microchannels.
- Experimentally study slug flow formation and visualisation.
- Experimentally measure the liquid film thickness in liquid-liquid slug flow
- Experimentally measure the pressure drop associated with liquid-liquid slug flow.
- Experimentally study the heat transfer characteristic in liquid-liquid slug flow including effects of flow parameters on heat transfer.
- Develop new correlations to predict the heat transfer rates in liquid-liquid slug flow over a wide range of parameters.

To complete these research tasks, Chapter 3 presents a comprehensive study on numerical studies of liquid-liquid slug flow heat transfer in microchannels including the effect of flow parameters on heat transfer. Chapter 4 studies the numerical modelling of liquid-liquid slug flow heat transfer in square cross sectional microchannels. 3D simulations are carried in this chapter and the effect of flow parameters on heat transfer is discussed. Experimental studies on liquid-liquid slug flow heat transfer in mini channels are presented in Chapter 5. This includes setting up the experiments, uncertainty analysis, and results related to hydrodynamics and heat transfer in slug flow. Finally, in Chapter 6, conclusions and future research directions are given. The research outcomes from these research tasks will give a comprehensive understanding on hydrodynamics and thermal behaviour of liquid-liquid slug flow in micro/mini channels.
Chapter 3

Numerical study of slug flow heat transfer in circular microchannels

This chapter presents a comprehensive numerical study of two-phase slug flow heat transfer in microchannels. It will illustrate the effect of slug flow parameters on heat transfer in circular microchannels. Findings of the study will be important in designing microscale systems where two-phase slug flow plays a key role.

3.1 Introduction

Numerical modelling is a promising technique for understanding and analysing the effects of parameters in fluid and heat flow systems in designing and product development. This saves a lot of time where the systems are quite complex and complicated to build and test practically and can give detailed data impossible to obtain experimentally. Many microscale fluid and heat flow systems are comprised with circular microchannels. Therefore, it is necessary to understand and study the fluid flow and heat transfer behaviour in circular microchannels.

Despite of the fact that much research work has been carried out on two-phase slug flow in circular microchannels as discussed in Chapter 2, there are still research gaps in the area including the effect of boundary conditions such as moving contact line, the no-slip boundary conditions at the microscale [109] and heat transfer phenomena. Particularly, there are no significant research publications in the literature on variation of heat transfer for the transition from sliding slug flow
to Taylor flow. Very few experimental and numerical studies on heat transfer in microscale channels have been carried out, and they are mostly on gas-liquid two-phase flows. The thin liquid film around the secondary phase fluid plug as discussed in 2.2.2 has been a debated topic in the literature. Many theoretical correlations have been developed for the film thickness ($\delta$) based on capillary number [22, 20, 46, 41] and Gupta et al. [108] reported that the Bretherton’s correlation as in equation 2.2, is the most appropriate one for representing the thin film. Some experimental and numerical studies reported a thin liquid film [9, 22, 20, 46, 41, 108, 109] while some reported no thin liquid film around the secondary fluid [110, 108, 109]. The numerical simulations which could capture the thin liquid film required either an ultra fine mesh along the channel walls as discussed by Gupta et al. [108], or a special technique such as gradient adaptation as used in [109] to refine the mesh at the interface of two fluids. However, using ultra fine mesh elements in the simulation is computationally costly. Recent numerical studies by Talimi et al. [111] simulated a moving liquid slug, using a moving frame of reference simulation approach considering there is no liquid film. They used fixed interfaces for 3D liquid slug to simplify the simulation and compared the results with existing experimental results. They reported higher Nusselt numbers ($Nu$) compared to experimental $Nu$ values, which is not surprising as they neglected the thin film in their simulation.

Surface wettability plays a key role in determining the droplet shape in microchannels with two-phase flow, when the bubble or droplet is touching the wall (sliding slug). Consequently, the three-phase contact angle is an important parameter in this type of simulation. Furthermore, contact angle affects the shape of the slug and the slug velocities (if there is a film) in the channel particularly at low $Ca$. A numerical study by Santos and Kawaji [4] has shown the contact angle effect on flow formation. Rosengarten et al. [77] demonstrated the role the contact angle plays when the liquid film surrounding the bubble becomes very thin and showed the influence by long range of van der Waal forces. They discussed the grid independency when simulating moving contact lines along a surface as the grid size near the wall will affect how the contact line moves.

Slug length ($L_s$) is another important parameter in two-phase slug flow heat transfer. Numerical investigations of heat transfer in slug flow with constant wall heat flux have been carried out by He et al. [82] revealed that the overall heat
transfer coefficient is largely controlled by the value for the slug region and the fraction of the wall occupied by the slugs. Heat transfer enhancement in mineral oil carrying water droplets in micro-tubes was numerically investigated by Urbant et al. [98]. They investigated the effect of slug size on heat transfer showing increased droplet size yields more efficient heat transfer and higher values of Nusselt number.

In this chapter, new findings related to liquid-liquid slug flow heat transfer in circular cross sectional microchannels will be presented using numerical simulation of water and light-mineral oil slug flow.

3.2 Problem formulation and solution procedure

Modelling of circular cross sectional channels becomes computationally less expensive because they can be transformed into a two-dimensional (2D) model in axisymmetric space. Therefore, this chapter will present numerical modelling in 2D space where the computational time is reasonably small compared to full three-dimensional (3D) simulations.

3.2.1 Interface tracking in two-phase flow

Well known interface tracking methods, such as volume-of-fluid (VOF), level-set (LS), and phase-field are used for capturing the interfaces of two non-mixing fluids in two-phase flows. However, each methods has its own advantages and disadvantages for interface tracking in microscale systems. For example, the level-set method is a popular technique for tracking the complex interfaces. However, the level-set method has a deficiency in conserving the volume, where the VOF method is implicitly conserves volume. Thus VOF method has become popular in two-phase slug flow modelling where conservation of volume is important. Gupta et al. [108] compared the performance of VOF and LS methods implemented in ANSYS Fluent and TransAT, and reported both codes are capable of predicting the interface shape in Taylor flow regime. The volume of fluid with geo-reconstruction scheme in Fluent requires cells with aspect ratio close to one for accuracy. In the present study, mesh elements throughout the whole computational domain have an aspect ratio of 1. Thus VOF method is used for capturing the interface of water.
and light mineral oil, the fluids are considered as Newtonian and incompressible while there is no mass transfer at the interface. The interfacial tension is assumed as constant for the simulation. However, one case is simulated with temperature dependent interfacial tension in order to check the effect of temperature dependent interfacial tension on heat transfer.

3.2.2 Governing equations

The VOF method which was initially proposed by Hurt and Nichols [112] is one of the most convenient methods to predict the dynamics of the immiscible fluids, including surface tension and contact angle effects. In this method, different phases are not allowed to mix and single momentum and energy equations are solved throughout the domain and shared by all the phases. Therefore, the equations which are solved during the simulation are

Continuity:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{U} = 0, \]  

(3.1)

Momentum:

\[ \rho \left( \frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} \right) = -\nabla p + \nabla \cdot (\mu(\nabla \mathbf{U} + \nabla \mathbf{U}^T)) + \mathbf{F}, \]  

(3.2)

Energy:

\[ \frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\mathbf{U}(\rho E + p)) = \nabla \cdot (k \nabla T), \]  

(3.3)

and Volume fraction equation:

\[ \frac{\partial (\phi)}{\partial t} + \nabla \cdot (\phi \mathbf{U}_s) = 0. \]  

(3.4)

The velocity, \( \mathbf{U} \) is treated as mass averaged variable, such that \( U = [\phi \rho_s U_s + (1-\phi)\rho_p U_p]/\rho \). The subscript \( s \) and \( p \) denote secondary and primary phase and \( \rho \) is the density of mixture. The properties such as density and viscosity are calculated using the volume fraction of phases in each cell, such that \( \rho = \phi \rho_s + (1-\phi)\rho_p \) and \( \mu = \phi \mu_s + (1-\phi)\mu_p \). The \( \mathbf{F} \) in equation 3.2 is surface tension force which is calculated from the Continuum Surface Force (CSF) model proposed by Brackbill.
et al. [113] as in equation 3.5.

\[ F = \sigma \frac{\rho \kappa \mathbf{n}}{2(\rho_s + \rho_p)} \]  (3.5)

where, \( \kappa = - (\nabla \cdot \mathbf{n}) = \frac{1}{n} \left[ (n \cdot \nabla) | \mathbf{n} \mid - (\nabla \cdot \mathbf{n}) \right] \). The contact angle in conjunction with surface tension model of [113] is used in this simulation. The fluid contact angle (\( \theta_w \)) at the wall is considered as a dynamic boundary condition and, is used to adjust the surface normal in cells near the wall and thus to adjust the curvature and the interface shape near the wall. This is implemented using \( \mathbf{n} = n_w \cos \theta_w + t_w \sin \theta_w \), where, \( n_w \) and \( t_w \) are unit surface normal and tangential to the wall, respectively.

### 3.2.3 Differencing schemes

The solver in ANSYS Fluent 14.5 was used for solving the above equations using finite volumes in 2D axisymmetric space. An explicit geometric reconstruction scheme with a maximum Courant number \( (Co = \Delta t U/\Delta x) \) of 0.25 is used for solving the volume fraction equation. The geo-reconstruction scheme represents the interface between fluids using a piecewise-linear approach. A first order non-iterative fractional step is used for the time marching of the momentum and continuity equations in transient simulation. A variable time step \( (\Delta t) \) based on a fixed global Courant number of 0.25 is used for momentum, pressure and energy equations similar to the simulations of [108] and [109]. This Courant number was chosen to allow a stable solution at relatively short time steps. Very small time steps in the order of \( 10^{-7} \) are used, as the elements at the interfaces are around 0.5 or 1 \( \mu \text{m} \). However, this time steps are even smaller in the case of refining the base mesh elements into 1 \( \mu \text{m} \). The second order upwind scheme is used for the spatial discretization of momentum and energy equations.

### 3.2.4 Numerical Model and Boundary conditions

The numerical model consists of a 3000 \( \mu \text{m} \) (30\( D \)) long circular microchannel with 100 \( \mu \text{m} \) internal diameter. The microchannel has two sections with high and low temperature regions to have a hydrodynamically developed flow in the heated
section as shown in Figure 3.1. The channel wall is considered to be silicon to mimic the conditions of Asthana et al. [9]. Water and light mineral oil were used as primary and secondary fluid, respectively. Properties of the materials are shown in Table 3.1. The viscosity of the oil was 0.023 kg/m s and the interfacial tension with water is 0.04925 N/m used in the simulation.

<table>
<thead>
<tr>
<th>material</th>
<th>density [kg/m³]</th>
<th>specific heat [J/kg K]</th>
<th>thermal conductivity [W/m K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>998</td>
<td>4182</td>
<td>0.6</td>
</tr>
<tr>
<td>light mineral oil</td>
<td>838</td>
<td>1670</td>
<td>0.17</td>
</tr>
<tr>
<td>Silicon</td>
<td>2330</td>
<td>710</td>
<td>149</td>
</tr>
</tbody>
</table>

A length of $10D$ (bigger than the development length correspond to the highest $Re$, $5.5D$) is allocated for the slug flow to be developed with a wall temperature of $23^\circ C$ before entering the heating section. In this study the wall is considered to be heated with a uniform temperature boundary condition. Therefore, the heating wall ($20D$) is maintained as at constant temperature of $65.15^\circ C$ which is approximately the heated electronic chip temperature [9]. The inlet is set as a uniform velocity with different velocities for different cases while the outlet of the channel is set as a $0Pa$ pressure outlet. Different cases are studied with different flow velocities, different contact angles and different slug length for analysing their effects on heat transfer. Those cases are illustrated in Table 3.2.
Table 3.2: Different test cases

<table>
<thead>
<tr>
<th>Purpose of case</th>
<th>Case no</th>
<th>Inlet velocity [m/s]</th>
<th>Slug length [µ m]</th>
<th>Length of oil patches [µ m]</th>
<th>Contact angle [degrees]</th>
<th>Capillary number [Ca]</th>
<th>Reynolds number [Re]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study capillary number effect</td>
<td>1</td>
<td>0.220</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.183</td>
<td>750</td>
<td>110</td>
<td>160</td>
<td>0.0038</td>
<td>18.3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.098</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0020</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.049</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0010</td>
<td>4.9</td>
</tr>
<tr>
<td>Study slug length effect</td>
<td>5</td>
<td>0.220</td>
<td>350</td>
<td>115</td>
<td>160</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.220</td>
<td>500</td>
<td>115</td>
<td>160</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.220</td>
<td>750</td>
<td>115</td>
<td>160</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td>Study contact angle effect</td>
<td>8</td>
<td>0.220</td>
<td>280</td>
<td>115</td>
<td>175</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.220</td>
<td>280</td>
<td>115</td>
<td>150</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.220</td>
<td>280</td>
<td>115</td>
<td>140</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>0.220</td>
<td>280</td>
<td>115</td>
<td>120</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.220</td>
<td>280</td>
<td>115</td>
<td>90</td>
<td>0.0044</td>
<td>21.9</td>
</tr>
<tr>
<td>study film thickness effect</td>
<td>13</td>
<td>0.253</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0051</td>
<td>25.2</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>0.580</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0118</td>
<td>57.8</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.717</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0146</td>
<td>71.4</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>0.750</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0153</td>
<td>74.7</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>0.900</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0183</td>
<td>89.6</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>1.100</td>
<td>280</td>
<td>115</td>
<td>160</td>
<td>0.0224</td>
<td>109.6</td>
</tr>
</tbody>
</table>
3.2.5 Grid generation and validation study

Grid generation is the most important step in numerical modelling of fluid and heat flow as the number of grid elements in computational domain decides the accuracy of the results produced in simulations. Very fine mesh elements (in the order of $D/100$ or smaller) are required for capturing the thin liquid film of few hundreds nano meters in microchannel flows as discussed in the numerical simulation in the literature [108, 109]. Various types of mesh refinement methods such as, refinement of mesh elements along the channel close to the channel wall only, or refinement of mesh elements throughout the computational domain have been used [108]. A technique such as gradient adaptation based on the volume of fluid will make the model less computational expensive by refining the mesh elements at the interface of two phases while maintaining the base mesh elements elsewhere. Thus, the gradient adaptation technique was utilised in the present study as in [109]. Structured square shape elements of $2 \mu m \times 2 \mu m$ ($D/50$) as show in Figure 3.2 were used throughout the study. However, test cases were simulated with $1 \mu m \times 1 \mu m$ ($D/100$) and $4 \mu m \times 4 \mu m$ ($D/25$) for the grid independency test. It is not expected that a VOF simulation will be truly grid independent as discussed by
Figure 3.3: Variation of velocity, oil volume fraction, and axial temperature using 1 \( \mu \text{m} \), 2 \( \mu \text{m} \) and 4 \( \mu \text{m} \) mesh elements, (a) Velocity distribution at a plane of 9\( D \) away from the inlet (before the oil slugs), (b) Velocity distribution at a plane of 15\( D \) away from the inlet (in between the oil slugs), (c) Oil volume fraction distribution along the radial axis in the mid of a oil slug, (d) Temperature variation along the axis.

Rosengarten et al. [77]. However, it is shown here that there is no significant variation in results from refining the mesh. Figure 3.3 shows the variation of velocity along radius, volume fraction of oil along radius and temperature variation along the axis for three different mesh cases with 4 \( \mu \text{m} \) (coarse mesh), 2 \( \mu \text{m} \) and 1 \( \mu \text{m} \) (refined mesh). The data was selected when oil droplets are at a similar distance away from the inlet in each model.

According the figures, there is only slight variations in the temperature and velocity values for three different mesh elements sizes (maximum approximate values of 4% for temperature from 4 \( \mu \text{m} \) to 2 \( \mu \text{m} \) and 1.5% for velocity from 4 \( \mu \text{m} \) to 2 \( \mu \text{m} \)). However, the volume fraction across the channel at presence of oil plugs is an important parameter as it depicts the existence of thin film. As shown in Figure 3.3 (c), there is considerable variation of volume fraction values close to the
Table 3.3: Calculated film thickness (in micrometers) from different correlations and from present study

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Ca</th>
<th>0.0044</th>
<th>0.0038</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 1.34RCa^{2/3}, 10^{-2} &lt; Ca &lt; 10^{-1}$ [20]</td>
<td></td>
<td>1.82</td>
<td>1.61</td>
</tr>
<tr>
<td>$\delta = \frac{D}{2} \left( \frac{1.34Ca^{2/3}}{1+3.35Ca^{2/3}} \right), 10^{-3} &lt; Ca &lt; 1.4$ [41]</td>
<td></td>
<td>1.67</td>
<td>1.49</td>
</tr>
<tr>
<td>$\delta = \frac{0.670DCa^{2/3}}{1+3.13Ca^{2/3}+0.504Ca^{0.672}Re^{0.589}-0.352We^{0.629}}, Ca &lt; 0.3$ [42]</td>
<td></td>
<td>1.68</td>
<td>1.50</td>
</tr>
<tr>
<td>present study</td>
<td></td>
<td>1.8±0.5</td>
<td>1.5±0.5</td>
</tr>
</tbody>
</table>

channel wall between 4 $\mu$m to 2 $\mu$m where these values are almost equal at 2 $\mu$m and 1 $\mu$m cases. Therefore, 2 $\mu$m size mesh elements were selected throughout the study and the dynamic mesh adaptation method as in [109] was used for the simplicity and the accuracy of the results. The mesh is refined automatically during the calculations based on gradient of volume fraction, and the threshold values for refining and coarsening were 0.15 and 0.1, respectively similar to [109].

Gradient adaptation of volume of fluid is an efficient method to predict the accurate physics in two-phase flow systems with less computational time. However, it is necessary to validate the results obtained from the selected model. Film thicknesses measured in the numerical results are compared with the thickness values calculated from the correlations available in the literature. A comparison of the thickness values are shown in Table 3.3. According to the table, there is a good agreement between numerical film thickness and the calculated film thicknesses from different correlations. The maximum error is 8% which is given with the [41] correlation for $Ca = 0.0044$ case.

### 3.3 Results and discussion

#### 3.3.1 Flow development

In this study, the droplet formation was not simulated. Instead two initial oil patches and with an initial inlet velocity is used. The detachment of the droplets (if it happens under those flow conditions) from the wall with the flow progression can be shown as in Figure 3.4. As shown in Figure 3.4, droplet is detached from the wall after approximately 1 ms time and then maintains its shape. This is the
Figure 3.4: Droplet detachment from channel wall for the flow with $Ca = 0.0044$ given in Case 1.

classical Taylor flow with conditions shown in case number 1 in the Table 3.2. The thin liquid film formed here is approximately equal to the theoretical value of $\delta = 1.8\mu m$ calculated from equation 2.2. The size of the film is easily calculated from the refined mesh elements around the bubble as shown in Figure 3.5.

Liquid film formation is dependent on the $Ca$ of the two-phase flow. The smaller $Ca$ leads to very thin nano-meter scale films and difficulties are encountered in capturing it due to the limitations in using very fine mesh elements which need to be smaller than the thin film. The smallest $Ca$ number in the present study is 0.001 and the approximate film thickness calculated from equation 2.2 is 0.67 $\mu m$. The simulations showed this to be sliding slug flow with the 2 $\mu m$ base mesh elements which were used throughout the study. However, a test case was carried out with 0.5 $\mu m$ base mesh elements in order to check the existence of a measurable
Figure 3.5: Mesh around droplet showing refinement and presence of thin liquid film around the droplet.

(a) contour of volume fraction of oil with 2 micron base mesh elements

(b) contour of volume fraction of oil with 0.5 micron base mesh elements

Figure 3.6: Contours of oil volume fraction obtained from two different base mesh element sizes for the case with $Ca = 0.001$ (Case 4), (a) with 2 $\mu$m base mesh elements and (b) 0.5 $\mu$m base mesh elements.

film thickness for this particular case. The mesh elements near the interfaces are refined up to 0.125 $\mu$m with gradient adaptation method in this case. Figure 3.6 shows the contours of oil volume fraction obtained from two different base mesh elements. It can be clearly seen that there is no film even in the 0.5 micron base mesh elements case as in Figure 3.6 (b). Thus, these cases are considered as sliding slug flow. In reality, at very small film thicknesses, roughness and other forces such as electrostatic may break the film down allowing the slug to slide. If the flow is in or close to the sliding slug flow regime, then the contact angle is important. As an example, the contact angle in case number 11 created sliding slug flow with a deformed droplet shape as shown in Figure 3.7 (b). Figure 3.7 (a) shows the shape of droplets with 140°contact angle for case 10. The rear interface of the droplet is
Figure 3.7: Contours of oil volume fraction with different flow conditions, (a) Contour of volume fraction with 140° contact angle, $Ca = 0.0044$, (b) Contour of volume fraction with 120° contact angle, $Ca = 0.0044$, (c) Contour of volume fraction with 160° contact angle, $Ca = 0.001$, (d) Contour of volume fraction with 160° contact angle, $Ca = 0.002$.

less curved in this case compared to the cases with 160° contact angle as shown in Figure 3.7 (c) and (d). The effect of contact angle and $Ca$ on heat transfer will be discussed in the later of the section.

### 3.3.2 Pressure drop

The pressure drop in liquid-liquid two phase flow is due to frictional pressure drop and interfacial pressure drop as discussed in the literature review chapter. The pressure variation along the axial direction for the flow conditions given in case 1 is shown in Figure 3.8. A significant variation of pressure can be seen in the frontal and rear interfaces as shown in figure. The pressure gradients in the primary phase (pressure along the wall) and in the secondary phase (pressure along the axis) are slightly different due to the non uniformity of the film thickness along the oil plug.

Pressure drop across an oil droplet obtained from the simulations for different $Ca$ flows are illustrated in Figure 3.9. The pressure drop is calculated from the mass weighted average pressure values on the planes at front and rear of the droplet. The values obtained from simulations are compared with the pressure drop calculated from the Bretherton’s correlation given in equation 2.7. A good agreement between the values were observed with a maximum difference of 9.5%. It is believed this small variation is due to the uncertainties associated with the position of line created to get the static pressure along the two sides of the droplet.
3.3.3 Heat transfer

Heat transfer performance in the flow channel is measured with the Nusselt number as explained in 2.3.3, calculated from equation 2.14. The mean flow temperature $T_{mx}$ at a plane perpendicular to the axis is computed as a mass weighted average value and the wall temperature, $T_{wx}$ is uniform along the channel. Total surface heat flux on the heating wall gives the $q_x$ value for each point along the channel. The calculated Nusselt number along the channel for case number 1 is shown in Figure 3.10.

According to Figure 3.10, the Nusselt number of the water only flow asymptotes towards 3.67 which is the theoretical value for single phase flow in circular channels under constant wall temperature boundary condition. The figure shows significantly increased Nusselt numbers with slug flow. Thus, having a train of
droplets along the whole channel will lead to higher heat removal rates. There is a large spatial variation of the local $Nu$ in the slug region.

### 3.3.3.1 Effect of temperature dependent properties on heat transfer

The properties such as viscosity and interfacial tension of the working fluids in this study have a strong dependency on temperature as discussed in [114]. Therefore, one case (case 1) was simulated with temperature dependent properties in order to test the effect of temperature dependent properties on heat transfer. The following relationships were used during the simulations for the viscosity of oil (equation 3.6) and water (equation 3.7) and the interfacial tension (equation 3.8) between two
fluids. Here, $T$ is temperature in °C.

$$\mu_{oil} = 4.4367 + 45.11e^{-0.054(T-8.64)} \quad (3.6)$$

$$\mu_w = 0.0000002T^2 - 0.00003T + 0.0015 \quad (3.7)$$

$$\sigma = 51.83 - 0.103T \quad (3.8)$$

A slight decrease of film thickness was observed with the temperature dependent properties. The local $Nu$ variation in a unit cell was also compared as given in Figure 3.11. The effect on the local $Nu$ and average $Nu$ is relatively small with the overall difference less than 8%. This is within the uncertainty due to the effect of a finite mesh size. Thus, the simulations were carried out with the constant fluid properties. Temperature dependent properties become more important if there is significant deformation like for example during droplet formation at a T-junction.
3.3.3.2 Nusselt number variation in a unit cell

Figure 3.12 (a) shows the variation of Nusselt number in a unit cell (one oil and one water section). The minimum heat transfer occurs at the rear end of the thin liquid film which is the same as discussed in [109]. The differences in velocity between the water and oil slugs (slightly higher oil velocities), the water in the front of the oil droplet has to be accelerated. This acceleration together with low bulk mean temperature of the fluid in the front end of the film (as in Figure 3.12 (b) and (c)) gives higher heat transfer rates. The overall heat transfer rate in the water slug region is higher than that in the thin film region. However, this could be changed in the cases of sliding slug flow. Heat transfer in slug flow is dominated by internal circulation. The cold water in the middle of the channel is pushed to the wall through the back of the oil droplet while the heated water near the wall
Figure 3.12: Nusselt number variation in a unit cell, flow field and temperature contours around a plug, (a) local Nusselt number variation in a unit cell (case 1), (b) flow field around and oil plug, and (c) temperature variation around and oil plug (case 1).

is circulated towards the channel axis through the front of the next oil droplet. Therefore, there exists a significant temperature difference between the front and back of the oil droplet as shown in Figure 3.12 (c).

The liquid film plays an important role in recirculation as discussed previously. The temperature of the liquid draining to the trailing water slug is relatively high and leads to poor heat removal capacity [111]. However, this does not imply that recirculation within the sliding slug flow always leads to higher heat removal rates. This is explained in Table 3.4 which illustrates the average Nusselt number values for different flow conditions stated in Table 3.2. The actual slug length is different from the initial slug length due to stretching of the droplet during formation of the liquid film. A comprehensive analysis of film thickness on heat transfer will be discussed in the section 3.3.3.4.

3.3.3.3 Effect of contact angle on heat transfer

According to Figure 3.13, the highest average local Nusselt number for a unit cell is for case number 10 (among cases 1, 8, 9,10, 11, and 12) with 140° contact angle. In
Table 3.4: Average Nusselt numbers for different test cases. Film thickness, $\delta$ is in micrometers and † denotes sliding slug flow.

<table>
<thead>
<tr>
<th>Case</th>
<th>Flow parameters</th>
<th>$Nu_{avg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$Ca = 0.0044, L_s/D = 2.42, \theta = 160^\circ, \delta = 1.8$</td>
<td>10.6</td>
</tr>
<tr>
<td>Case 2</td>
<td>$Ca = 0.0038, L_s/D = 7.11, \theta = 160^\circ, \delta = 1.5$</td>
<td>7.2</td>
</tr>
<tr>
<td>Case 3</td>
<td>$Ca = 0.0020, L_s/D = 2.55, \theta = 160^\circ, \delta = 0$</td>
<td>6.4†</td>
</tr>
<tr>
<td>Case 4</td>
<td>$Ca = 0.0010, L_s/D = 2.52, \theta = 160^\circ, \delta = 0$</td>
<td>5.5†</td>
</tr>
<tr>
<td>Case 5</td>
<td>$Ca = 0.0044, L_s/D = 3.09, \theta = 160^\circ, \delta = 1.8$</td>
<td>10.3</td>
</tr>
<tr>
<td>Case 6</td>
<td>$Ca = 0.0044, L_s/D = 4.64, \theta = 160^\circ, \delta = 1.8$</td>
<td>9.2</td>
</tr>
<tr>
<td>Case 7</td>
<td>$Ca = 0.0044, L_s/D = 7.12, \theta = 160^\circ, \delta = 1.8$</td>
<td>7.9</td>
</tr>
<tr>
<td>Case 8</td>
<td>$Ca = 0.0044, L_s/D = 2.40, \theta = 175^\circ, \delta = 1.8$</td>
<td>10.4</td>
</tr>
<tr>
<td>Case 9</td>
<td>$Ca = 0.0044, L_s/D = 2.38, \theta = 150^\circ, \delta = 1.8$</td>
<td>10.5</td>
</tr>
<tr>
<td>Case 10</td>
<td>$Ca = 0.0044, L_s/D = 2.55, \theta = 140^\circ, \delta = 0$</td>
<td>10.7†</td>
</tr>
<tr>
<td>Case 11</td>
<td>$Ca = 0.0044, L_s/D = 2.67, \theta = 120^\circ, \delta = 0$</td>
<td>10.1†</td>
</tr>
<tr>
<td>Case 12</td>
<td>$Ca = 0.0044, L_s/D = 1.80, \theta = 90^\circ, \delta = 0$</td>
<td>6.3†</td>
</tr>
<tr>
<td>Case 13</td>
<td>$Ca = 0.0051, L_s/D = 2.38, \theta = 160^\circ, \delta = 2.0$</td>
<td>11.1</td>
</tr>
<tr>
<td>Case 14</td>
<td>$Ca = 0.0118, L_s/D = 2.38, \theta = 160^\circ, \delta = 3.5$</td>
<td>14.5</td>
</tr>
<tr>
<td>Case 15</td>
<td>$Ca = 0.0146, L_s/D = 2.15, \theta = 160^\circ, \delta = 6.0$</td>
<td>11.9</td>
</tr>
<tr>
<td>Case 16</td>
<td>$Ca = 0.0153, L_s/D = 2.08, \theta = 160^\circ, \delta = 6.5$</td>
<td>11.7</td>
</tr>
<tr>
<td>Case 17</td>
<td>$Ca = 0.0183, L_s/D = 2.02, \theta = 160^\circ, \delta = 8.0$</td>
<td>11.4</td>
</tr>
<tr>
<td>Case 18</td>
<td>$Ca = 0.0224, L_s/D = 1.75, \theta = 160^\circ, \delta = 11.3$</td>
<td>10.4</td>
</tr>
</tbody>
</table>

These flow conditions, the oil droplets maintain a higher interface curvature in the front compared to rear end as shown in Figure 3.7 (a). This flow is in the sliding slug flow condition and has strong recirculation within the water slugs as discussed previously, hence leading to higher heat transfer rates. However, non-existence of thin liquid film does not always lead to higher Nusselt numbers; specially at lower $Ca$ cases. It is a combined effect of both $Ca$ and the thin liquid film. As shown in the average $Nu$ table, lower flow velocities with no liquid film lead to decreased heat transfer rates. The variation of droplet shape, temperature and heat transfer rates with different contact angles are given in Table 3.5. The droplets with 90° and 120° contact angles maintain a deformed shape than the standard plugs as shown in Table 3.5. The droplet shape reaches to the standard plug shape with increasing contact angle.
Figure 3.13: Average Nusselt number variation with different contact angles.
Table 3.5: Droplet shapes, fluid temperature, and average Nusselt numbers with different contact angles

<table>
<thead>
<tr>
<th>Contact angle</th>
<th>Contour of oil volume fraction</th>
<th>Contour of temperature</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>![Image of 90° droplet]</td>
<td>![Image of 90° temperature]</td>
<td>6.3</td>
</tr>
<tr>
<td>120°</td>
<td>![Image of 120° droplet]</td>
<td>![Image of 120° temperature]</td>
<td>10.1</td>
</tr>
<tr>
<td>140°</td>
<td>![Image of 140° droplet]</td>
<td>![Image of 140° temperature]</td>
<td>10.7</td>
</tr>
<tr>
<td>150°</td>
<td>![Image of 150° droplet]</td>
<td>![Image of 150° temperature]</td>
<td>10.5</td>
</tr>
<tr>
<td>160°</td>
<td>![Image of 160° droplet]</td>
<td>![Image of 160° temperature]</td>
<td>10.6</td>
</tr>
<tr>
<td>175°</td>
<td>![Image of 175° droplet]</td>
<td>![Image of 175° temperature]</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Unit: K
**3.3.3.4 Effect of film thickness and capillary number on heat transfer**

Liquid film forms between the channel wall and the secondary fluid plug is strongly dependent on the flow capillary number. Therefore, it is not possible to study the independent effect of film thickness on heat transfer in slug flow. Therefore, different $Ca$ number flows were used for formation of slug flow with different film thickness values. Averaged Nusselt number increases gradually with increasing $Ca$ as in cases 1, 2, 3, 4, 9, and 10 as given in Figure 3.14 (a). However, average $Nu$ starts to drop at a certain $Ca$ number and average $Nu$ for $Ca = 0.0146$ is lower than the value for $Ca = 0.0118$. At $Ca = 0.0118$, there is an approximately four folds increase of Nusselt number compared to single phase flow. The variation of Nusselt number with different film thickness values is given in the Figure 3.14 (b). The average $Nu$ increases up to a certain film thickness value as the $Ca$ is also increasing in those cases. However, above a certain film thickness value, the heat transfer performance starts to decrease as shown in the Figure 3.14 (b). However, film thickness and $Ca$ are dependent on each other and they have a combined effect on heat transfer as explained in the previous section. The droplet shapes for different film thickness values are given in the Figure 3.15. At higher $Ca$ flows, the film thickness varies from the front to rear of the droplet which can be seen clearly in Figure 3.15 (b) to (f). The thickness was calculated as a mean value in these particular cases.
Figure 3.15: The droplet shape for different film thickness values.

3.3.3.5 Effect of slug length on heat transfer

The slug length also plays an important role in heat transfer in two-phase flows. As shown in Table 3.4, the heat transfer rates have decreased with increasing slug lengths, and the minimum average Nusselt number (considering case 1, 5, 6 and 7) is reported in the flow with slugs of length 750 µm as shown in Figure 3.16. This happens because the increased slug lengths tend to behave as single phase flow decreasing heat transfer as expected. Average Nusselt number for a unit cell length was compared with some of the correlations developed in the literature as given in following equations.

Figure 3.16: Average Nusselt number variation with slug length.

\[
Nu = \frac{1.75}{\sqrt{1 - \phi}} \left( \frac{Re Pr D}{L} \right)^{1/3} \left( \frac{\mu_B}{\mu_W} \right)^{0.14} \quad (3.9)
\]


\[
Nu = (1 - \phi) \left[ 4.364 + 25 \left( \frac{L_s}{D} \right)^{-0.5} \right] \quad (3.10)
\]


\[
Nu_{L^*} = 4.364 + \frac{0.29}{L_s^* + 0.15L_s^{1/3}} \quad (3.11)
\]

where \( L_s^* = \frac{L_s}{Re D Pr} \).
Figure 3.17: Average Nusselt numbers calculated from different correlations.

Even though the above correlations were developed for the gas-liquid two-phase flow, there was no limitation specified on the parameters. The Nusselt numbers calculated using the above correlation are shown together with the values from present study in Figure 3.17. The change in $Nu$ with slug length shows a similar trend to the correlations even though there are some differences in the values. These differences between the values (shown in Figure 3.17) could be due to the differences in boundary conditions used in each study, surface effects which are not taken into account or because the parameters are beyond the limits for which the correlation was developed. However, there are more interrelated parameters (such as void fraction and thermal properties of fluids, $Ca$ and film thickness) that affect the heat transfer rates than the current correlations taken into account.
Overall, liquid-liquid two-phase slug flow increase Nusselt number compared to its single phase counter part by approximately 400%. These findings can be applied practically in many applications involved cooling and heat transfer enhancement.

3.4 Summary

Heat transfer in microchannels under constant temperature wall boundary conditions with liquid-liquid two-phase slug flow has been numerically simulated and validated with VOF method, presented in this chapter. Two isolated oil slugs in water were considered for the simplicity of the simulations.

Maximum heat transfer rates were observed with the slug flow with \( Ca = 0.0118 \) and 160\(^\circ\)contact angle given in case 14. Contact angle has an important effect on heat transfer and highest heat transfer rates were observed in the sliding slug flow with a 140\(^\circ\)contact angle (among the cases which have same \( Ca \) while other parameters are also set to be constant) showing that without a liquid film, internal recirculations within the primary fluid slugs increases. It is necessary to use the right contact angle in simulations as it is a defining factor for the interface shape of the droplets in slug flow development. This shows the \( Nu \) is dependent on the type of slug flow, ie sliding slug or Taylor flow. Sliding slugs induce more recirculation and thus for the conditions simulated show the highest \( Nu \). This has not been reported previously.

The film thickness is a strong function of \( Ca \) and they have a combined effect on heat transfer. Increasing \( Ca \) increases the heat transfer up to certain value and then decreases beyond that point. A similar trend is exhibited with film thickness on heat transfer as the film thickness increases with increasing \( Ca \). Slug length is another important parameter for heat transfer in two-phase slug flows. The increase of slug length leads to decreased average Nusselt numbers showing that longer slugs tend to behave like single phase flow. Furthermore, there is little agreement in the literature in terms of heat transfer as these correlations give varying values of the Nusselt number for identical conditions.
Chapter 4

Numerical study of slug flow heat transfer in square microchannels

Geometry of the flow channel is an important factor for heat and mass transfer in microchannels. Square cross sectional microchannels have gained a significant interest in the recent history due to easy fabrication processes, and the easier optical access compared to circular microchannels. Thus, it is important to study heat transfer in square cross-sectional microchannels in order to have better understanding for design and product development. This chapter will present the numerical modelling of slug flow heat transfer in square cross sectional microchannels.

4.1 Introduction

Numerical studies of slug flow heat transfer have gained more attention recently due to advances in computational power enabling more accurate simulations of the physics involved. Most of these studies are on gas-liquid two-phase flow in circular microchannels [105, 115, 116, 117, 118, 119, 120, 109, 121]. There is a higher computational cost to simulate the flow and heat transfer in non-circular cross-sectional channels which requires 3D modelling. Few studies are available in the recent literature on non-circular channel two-phase flow heat transfer [111], and even there they simplified the model to avoid the computational cost. However, these simplifications lead to over estimation of heat transfer rates [121, 111]. Therefore, there is a lack of reliable simulation data on 3D modelling of slug flow
heat transfer.

The thin liquid film between the secondary phase fluid droplet and the channel wall as described previously has a significant effect on heat and mass transfer in slug flow. Both numerical and experimental studies in the literature reported the existence of the thin liquid film [9, 20, 73, 116, 109, 41, 46, 122, 22]. The film thickness for low $Ca$ flows in microchannels can be very small ($\delta \approx D/100$). Mesh elements smaller than the thickness of the film are required to capture this. This can be achieved either by having fine mesh elements along the channel wall as in [116] or refined mesh elements at the two-phase interface as in [109, 73]. However, having refined mesh elements, specially in 3D simulations is extremely computationally costly.

4.1.1 Slug flow in non-circular microchannels

Slug flow in non-circular microchannels has not been studied much both experimentally and numerically. In the experimental studies by Molla et al. [123] pressure drop shown to increase with increasing void fraction as well as with increasing slug length and slug velocities. They showed a nearly constant mean pressure gradient along the channel in spite of a significant variation of the void fraction along the channel. Luo and Wang [124] also studied the liquid velocity distribution in a gas-liquid slug flow in a rectangular microchannel. Hydrodynamics and mass transfer of slug flow in a rectangular microchannel was studied by Yue et al. [125] followed by the experiments in a square microchannel [126]. Yue et al. [126] showed that the measured velocity agreed well with the calculated velocity based on the approximately measured film thickness. They also showed a deviation of pressure values from the correlation in Kreutzer et al. for the shorter slugs.

However, very little research work has been carried out on heat transfer in square or rectangular microchannels. Betz and Atinger [13] performed experiments for a gas-liquid slug flow in a square microchannel with constant heat flux wall boundary conditions. They reported a 140% increase in the Nusselt number relative to liquid only flow. Experimental work carried out by Asthana et al. [9] investigated the heat transfer enhancement in a square serpentine microchannel with liquid-liquid two-phase flow under the constant temperature wall boundary
conditions. They reported a four-fold $Nu$ increase in water-light mineral oil two-phase slug flow compared to that with water only flow. Experimental studies by [85] showed a 1.2 - 1.6 times heat transfer enhancement with gas-liquid Taylor flow in square mini channel. They observed the temporal fluctuations of the fluid temperature using embedded-thermocouples but no fluctuations in the wall temperature due to the relatively large thermal mass of the wall. Recent numerical studies on slug flow heat transfer in a square microchannel by Talimi et al [111] reported higher $Nu$ values than the experimental values presented in [13] when they mimicked the same boundary conditions. This was due to no thin film in their moving frame of reference simulation approach with fixed interface for 3D liquid slugs. However, these interfaces are dynamic in reality and that need to be considered in simulations for the accurate results.

4.2 Problem formulation and solution procedure

Volume-of-fluid (VOF) is used for the interface capturing same as in 2D axisymmetric simulations. Therefore, the governing equation of the problem will be same as in the circular microchannels (equations 3.1, 3.2, 3.3, and 3.4). However, the each variable in the computational domain will have three components in x, y, and z directions as the problem becomes 3D in nature. This makes modelling computationally expensive and a detailed study on applicability of 2D modelling will be discussed in more details in section 4.2.3.1.

4.2.1 Differencing schemes

The solver in ANSYS Fluent 14.5 was used for solving the above equations using finite volumes. An explicit geometric reconstruction scheme with a maximum Courant number ($Co = \Delta t U/\Delta x$) of 0.25 is used for solving volume fraction equation. The geo-reconstruction scheme represents the interface between fluids using a piecewise-linear approach. A first-order non-iterative fractional step is used for the time marching of the momentum and continuity equations in transient simulation. A variable time step ($\Delta t$) based on a fixed global Courant number of 0.25 is used for momentum, pressure and energy equations similar to the simulations
of [116],[109], and [73]. However, this fixed Courant number needs to be defined carefully in 2D planar simulations for obtaining the right droplet shape. This will be explained in detail in section 4.2.3.1. Very small time steps in the order of $10^{-7}$ or less are used, as the elements in the interfaces between 0.5 and 1 microns.

![Diagram](image)

Figure 4.1: Schematic diagram of a two-phase flow model with boundary conditions, (a) isometric view of the set up, (b) front view of the set up, (c) side view of the set up, and (d) wall boundary conditions of the heating section.

### 4.2.2 Numerical Model and Boundary conditions

The numerical model consists of square microchannels with hydraulic diameters of 100 $\mu$m for both inlet and outlet channels. The microchannel geometry is chosen to be the same as that used by Asthana et al. [9] in their experiments in order to make a direct comparison. The flow focusing cross junction was also included in the model. The out flow microchannel has two main sections with
high and low temperature regions to introduce a temperature gradient and to have a hydrodynamically developed flow in the heated section [9]. A length of 18D from the flow forming junction is used for the slug flow to be developed with a wall temperature of 23°C before entering the heating section. The heating wall (20D) is set as a constant temperature of 65.15°C which is approximately the temperature of working electronic chips [9]. Symmetric boundary conditions are used as in schematic diagram of the model shown in Figure 4.1. The channel wall is considered to be silicon and one wall is made up with borosilicate glass which is considered as adiabatic wall in the simulations. Water and light mineral oil were used as primary and secondary fluid respectively similar to circular channel modelling and the water is introduced from inlet \( p \) and oil is introduced from inlet \( s \). The inlet is set as a uniform velocity inlet with different velocities corresponding to different flow rates of water and oil as in the experiments while the outlet of the channel is set as a 0 Pa pressure outlet. The other dimensions given in the Figure 4.1 are set to be \( H = 100 \mu m, L_1 = 200 \mu m, L_4 = 200 \mu m, W_1 = 50 \mu m, \) and \( W_2 = 100 \mu m. \)

Properties of the materials are given in Table 3.1. Different cases are studied with different flow velocities and different contact angles for analysing their effects on heat transfer including flow formation. However, this flow formation cross junction is not considered in the refined mesh cases due to the computational cost. Instead two 3D rectangular oil droplets are patched at the beginning of the channel and allowed to flow with the mixture velocity \( U_{TP} = U_p + U_s \). The size of the droplets and the distance between two consecutive droplets are taken from the flow formation results with the coarse mesh which have a good agreement with experimental results in [9]. Different cases as shown in Table 4.1 are studies with different flow velocities similar to [9] and with different contact angles. However, the appropriate contact angle for this modelling was selected by comparing the numerical volume fraction contours with the experimental results by Ashthana et al. [9].
Table 4.1: Different test cases

<table>
<thead>
<tr>
<th>water flow rate</th>
<th>oil flow rate</th>
<th>contact angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>µl / min</td>
<td>µl / min</td>
<td>degrees</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>160</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>160</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>160</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>130</td>
</tr>
</tbody>
</table>

4.2.3 Grid generation, independence and validation study

Very fine mesh elements are required for capturing the thin liquid film thickness that can be in the order of a few hundreds of nano-meters thick in microchannel flows as discussed in the previous chapter. However, due to the limited computational resources this become exceedingly long simulations. Therefore, structured cubical shape elements of 4 µm x 4µm x 4µm (D_h/25) were used for studying heat transfer including flow formation. It is not expected that a VOF simulation will be truly grid independent as discussed by Rosengarten et al. [77]. However, a grid independency study was performed with 2 µm, 4 µm and 8 µm cubical mesh elements. Figure 4.2 shows the variation of volume fraction of oil and fluid velocities for different mesh sizes. Figure 4.2 (a) presents the velocity profile between two oil slugs in the mid plane perpendicular to symmetry plane, while Figure 4.2 (b) shows the velocity profile in the symmetry plane at the same position. Figure 4.2 (c) and (d) present the volume fraction of oil in the mid plane perpendicular to the symmetry plane at two different positions across the oil plug as shown in each figure. Figures 4.2 (a) and (b) show that the velocity variation between 4 and 2 µm mesh elements cases is very small (maximum error of 3.5%), while there is a significant difference between 4 and 8 µm mesh elements cases. Similarly in volume fraction graphs also show a significant difference in 4 and 8 µm mesh elements cases while the difference is minimal in the 4 and 2 µm mesh elements cases. Considering the large computational time, the 4 µm mesh elements were selected throughout the simulation as there was only a little difference compared to 2 µm mesh elements case. The mesh refinement was carried out with gradient adaptation of volume of fluid as in the axisymmetric simulations. The original mesh elements are refined automatically at the two-fluid interface during the simulations based on gradient
of volume fraction, with the threshold values for refining and coarsening of 0.15 and 0.1.

It is necessary to perform a validation study for the selected mesh elements throughout the simulations. A good agreement was observed between the average $Nu$ calculated from the 4 $\mu$m mesh elements cases and the results obtained in
experiments by Ashthana et al. [9] as shown in Figure 4.3. For the total flow rate of 110 \( \mu l/min \) case, there is a considerable difference between the experimental and numerical two-phase \( Nu \) values. However, the single phase \( Nu \) value obtained from the simulations has a very good agreement with the theoretical Nusselt number for this particular boundary conditions.

To further validate the simulations, the slug velocity is compared with the theoretical velocity. At higher primary to secondary fluid flow ratios, the slug length (distance between two consecutive oil droplets) was increased leading the fluid in between the two oil droplets to behave as single phase flow [73]. Therefore, the velocity profile across the cross section in the middle of the slug should approach that of single phase Poiseuille flow as discussed in [111]. The velocity profile for a single phase flow in a rectangular microchannel can be calculated from equations...
Figure 4.4: Velocity distribution at the middle plane of a longer water slug, (a) in radial direction and (b) in the diagonal direction.

For the lower flow rate case (water: 100 µl/min and oil: 10 µl/min) in the present study, the slug length is approximately 8D. Therefore, the velocity profile of the cross section at the middle of a water slug is calculated using the analytical

\begin{equation}
\begin{split}
    u(y, z) &= -\frac{16c_1a^2}{\pi^2} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n^3} (-1)^{(n-1)/2} \times \\
    &\quad \left[ 1 - \frac{\cosh(n\pi z)/2Y}{\cosh(n\pi Z)/2Y} \right] \cos \left( \frac{n\pi y}{2Y} \right), \\
    u_m &= -\frac{c_1a^2}{3} \left[ 1 - \frac{192}{\pi^5} \left( \frac{Y}{Z} \right) \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n^5} \tanh \left( \frac{n\pi Z}{2Y} \right) \right]
\end{split}
\end{equation}

(4.1)
solution in order to compare and validate the simulation results. The Figure 4.4 (a) and (b) show the radial velocity along the z axis and the velocity along the diagonal axis respectively. The velocity values calculated in simulation with 4 µm mesh elements have an excellent agreement with the analytical values obtained from equations 4.1 and 4.2 with 50 terms in the series. The mean velocity, \( U_m \) is calculated by dividing the total flow rate by the cross sectional area of the channel.

4.2.3.1 Can 2D modelling predict the accurate physics

A test case was carried out on 2D simulations in order to check the applicability for modelling two-phase flow and heat transfer in non-circular microchannels. A mid plane was selected as shown in Figure 4.5 with given boundary conditions for the model. The lengths of the channel was selected similar to be 3D geometry. The modelling was carried out similar to [73] except the 2D space is planar instead of axisymmetric.

![Figure 4.5: Schematic diagram of 2D numerical model including boundary conditions](image)

The simulation was run with a variable time step based on a fixed global Courant number similar to axisymmetric simulations. However, the fixed Courant number which was used for axisymmetric simulations was not suitable for 2D planar case. Therefore, the simulation was run with few different fixed Courant numbers in order to find the most suitable value for this simulations. Figure 4.6 shows the contours of oil volume fraction at different fixed Courant numbers used in the simulations. Based on the contours plots, 0.13 was selected as the fixed Courant number for 2D planar simulations comparing the shape of the droplet with the experimental droplet shape for this particular boundary conditions.

The local Nusselt number on the heating wall was calculated for a unit cell in order to compare with 3D results. Figure 4.7 shows the comparison of local \( Nu \)
Figure 4.6: Contours of oil volume fraction at different fixed Courant number for the total flow rate of 130 µl/min with 2 µm base mesh elements and IR thermography image from experiments [9] for the same flow rate.

from 2D and 3D simulations for the case of total flow rate of 130 µl/min. There is a significant difference between Nusselt number values. The average $Nu$ value calculated from 2D local $Nu$ values was 5.03, where it was 7.62 and 7.9 from 3D modelling and experiments for this particular flow rate and $Ca$. The 2D simulation predicts low heat transfer rates due to the missing walls in the sides and the lower intensity of recirculations as it is only in one plane. However, this recirculations are radially distributed in 3D modelling where it improves heat transfer performance. Therefore, it is necessary to perform 3D modelling in order to predict accurate
Figure 4.7: Local Nusselt number variation in a unit cell in 2D and 3D simulations (water flow rate: 100 µl/min, oil flow rate: 30 µl/min).

4.3 Results and discussion

4.3.1 Flow development

Slug flow formation is a very important step in heat transfer studies due to the effect of size of droplets/slugs and the liquid film on the heat removal process. Flow formation was carried out with the flow focusing cross junction for the models with 4 µm cubical structured mesh elements. Three different water/oil flow rate ratio were used as in [9]. Figure 4.8 shows the contours of oil volume fraction at different flow rate ratio. The results clearly showed that the slug length is increased with the increasing ratio of primary to secondary fluid velocity as discussed in [128] for the
gas-liquid Taylor flow system. The droplet/plug length decreases with increasing primary to secondary fluid flow ratio same as in experimental observation [9]. However, the thin liquid film between the droplet and the channel wall could not be captured with these simulations due to the insufficient refinement of the mesh elements near the wall region. This film formation was observed with the refined mesh elements (2 μm structured cubical elements) as shown in Figure 4.9. A comparison of heat transfer rates with two-different base mesh sizes (2 μm and 4 μm) is presented in the heat transfer section.

![Figure 4.8: Contours of volume fraction of oil in the mid plane parallel to glass wall with 4 μm mesh elements (water flow rate: 100 μl/min, oil flow rate: (a) 30 μl/min, (b) 10 μl/min ).](image)

The liquid film thickness is mainly dependent on the $Ca$. However, the contact angle also plays a key role in liquid film formation together with $Ca$ if the droplet wets the wall, as shown in Figure 4.10. The figure shows how the smaller contact angle tends to maintain a larger area of oil droplet attaching to the channel wall. Thus, the contact angle has a significant effect on heat transfer, specially during sliding slug flow, and this will be discussed under the heat transfer section.

### 4.3.2 Flow field in slug flow

The typical relative flow field in the slug is shown in Figure 4.11. These flow fields are calculated in a frame of reference relative to the moving oil plug. As shown in Figure 4.11, the fluid flow is blocked by the rear interface of the oil droplet/plug, changing the flow direction towards the wall when the fluid moves forward in the
Figure 4.9: Contours of volume fraction of oil in the mid plane parallel to glass wall (water flow rate: 100 µl/min, oil flow rate: 30 µl/min) at different time steps with starting mesh size = 2µ m, (a) t = 0, (b) t = 4.587 ms and (c) t = 9.179 ms.

primary phase fluid. This fluid is slower along the wall due to the frictional forces in the opposite direction to the flow. Similarly, the fluid moving backward is blocked by the front interface of the next oil plug and direct the fluid towards the centre of the channel, resulting in recirculation.

The relative velocity at the central section of the water slug has a parabolic profile as show in Figure 4.12 with zero transverse component as explained in the
Figure 4.10: Contours of volume fraction of oil in the mid plane parallel to glass wall at $Ca = 0.0044$ (water flow rate: $100 \mu l/min$, oil flow rate: $30 \mu l/min$) with different contact angles, (a) $\theta = 130^\circ$ and (b) $\theta = 160^\circ$, (c) IR thermography image from experiments [9] for the same flow rate.

Figure 4.11: Relative velocity vectors in a water slug (water flow rate: $\mu l/min$, oil flow rate: $30 \mu l/min$)
experiments by Ashthana et al. [9]. The negative values represent the relative velocity directed backward. Negative velocity component is increased with decreasing water slug length, indicating stronger flow recirculation in the primary fluid. This will lead to higher heat and mass transfer rates. However, the liquid film thickness is also a key factor on the intensity of recirculations, and hence the heat transfer as discussed in the previous chapter.

4.3.3 Heat Transfer

Temperature measurement is a challenging task in microchannels as it needs to have very small temperature measuring probes or some non invasive methods such as light induced fluorescence (LIF) [9]. In numerical studies, a mass weighted average value is calculated when the bulk mean fluid temperature is required in a local cross sectional plane along the channel.

Temperature contours in the channel for the case of total flow rate 130 µl/min, are shown in Figure 4.13. The figure shows the high temperature zones at the back of the slug while lower temperature zones in the front of the water slug
due to the recirculation. The cold water in the centre of the channel is pushed towards the wall in the front of the slug while the heated water near the wall is circulated towards the channel axis from the back of the water slug. This causes lower convective heat transfer rates at the back of the water slugs due to the lower temperature differences of the wall and bulk fluid temperatures. The temperature distribution around an oil plug is shown in Figure 4.14. It shows a significant temperature difference in the primary fluid between the front and back of the oil droplets similar to the experimental observations [9] and the numerical observations in [73] and [99]. These differences are due to the internal recirculations. These temperature differences lead to a large variation of heat transfer rates between the front and back the oil droplets. Comparison of two temperature contours for different flow rates in Figure 4.14 indicates the temperature difference is smaller for the higher total flow rate case as explained in experimental observations.

4.3.3.1 Temperature evolution in slug flow

The temperature evolution in a typical slug flow in a square channel with constant temperature boundary conditions is shown in Figure 4.15. In this channel, three walls are transferring heat while the glass wall is assumed as adiabatic mimicking the experiments. The first figure in each pair shows the temperature in the mid plane parallel to the glass wall while the second figure shows the temperature contours in the mid plane perpendicular to the glass wall. Thermal boundary
Figure 4.14: The local temperature distribution in leading and trailing water slugs (water flow rate: 100 µl / min, oil flow rate: (a) 30 µl/min, (b) 20 µl/min and (c) 10 µl/min.

layer is formed as shown in Figure 4.15(a). Thermal boundary layer deforms with introduction of droplets in the heated section. The oil plugs moving in the heated section will absorb heat from the wall while distorting the thermal boundary layer as in Figure 4.15 (b), (c), and (d). The heated fluid near the wall is pushed towards the central section of the water slugs and oil plugs due to the recirculations while the cold fluid in the middle is circulated towards the wall from the leading edge of the slugs and plugs. However, there will be a temperature gradient in the upper
Figure 4.15: Evolution of temperature contours in slug flow in a square cross sectional microchannel with one adiabatic wall at different time steps. (a) at 0.25 ms with no oil plugs, (b) at 7.5 ms with 3 oil plugs in cold region, (c) at 10 ms with first few plugs entered to the heated section, (d) at 13.5 ms oil plugs more than half-way through the heated section, and (e) 22.5 ms with full of oil plugs in the heated section.

and lower halves in the planes perpendicular to the glass wall as observed in [129]. This leads for heat transfer from fluid in the upper half to the fluid in the lower half. The temperature of the fluid in the glass wall side is gradually increased due to this heat transfer within the fluid together with convection as shown in Figure 4.15(d). The difference between the channel wall temperature and the bulk
fluid temperature decreases when the slugs move forward. It should be noted that temperature contours presented in Figure 4.15 are not shown up to the end of the channel.

Bulk temperature variation along the channel in different flow conditions is illustrated in the Figure 4.16. The fluid temperature increases along the channel until it reaches equilibrium with the wall temperature. The graph clearly shows that the bulk temperature with slug flow rises more rapidly compared to the single phase flow with the same flow conditions. Slug flow with higher total flow rate reaches to the temperature equilibrium quicker than the lower total flow rate flow as explained in [9].

4.3.3.2 Nusselt number variation along the channel wall

The wall heat flux together with bulk mean fluid temperature are used to calculate the local $Nu$ along the channel wall using equation 2.14. Calculated local $Nu$ values for the slug flow case with total flow rate 130 $\mu$l/min are illustrated in Figure 4.17. The presence of oil plugs gives significantly higher $Nu$ values compared to those for single phase flow, indicating higher heat transfer rates. Heat transfer rates are higher throughout the channel with slug flow unlike in single phase flow $Nu$ which asymptotes to a constant value of 3.02 for this particular boundary conditions limiting the heat transfer performance.

Flow parameters such as slug length, capillary number and contact angle affect heat transfer in microchannels. Decrease of slug lengths (distance between two oil plugs) leads to higher heat transfer performance as explained in Figure 4.12 due to more intense recirculation within the water slugs. The contact angle also plays a key role in heat transfer phenomena as it dictates the shape of the oil plugs in flow formation which directly affects the film thickness and hence the intensity of recirculations. Contact angle directly affect the heat transfer in sliding slug flow case as it can define the length of the oil touching the channel wall. Figure 4.18 shows the local Nusselt number variation in two slug flow cases with same flow rate but different contact angles. The slug flow with 140$^\circ$ contact angle gives higher heat transfer rates compared to the flow with 160$^\circ$ contact angle flow condition. This is due to the less curved front and back interfaces of oil droplets (as in Figure 4.10 (a)) which increase the recirculations within water slugs.
Figure 4.16: Bulk temperature variation along the channel with different flow conditions in the heated section.

Average slug flow $N_u$ and the theoretical single phase flow $N_u$ variation with $Re$ is shown in Figure 4.19. A significant increase of heat transfer rates are observed with oil-water slug flow as shown in Figure 4.19. An increase over 250% was observed with highest $Re$ flow. According to the average $N_u$ values obtained with different flow conditions, higher total flow rate flow (with higher oil to water flow rate ratio) has better heat transfer performance. This is due to the increased intensity of recirculations within the slugs when the slugs become shorter in length with high $Ca$ as shown in Figure 4.20. The higher magnitude of relative velocity implies the higher intensity of recirculations as explained earlier in the chapter.
Fluid in the longer slugs (at lower oil to water flow rates) tends to behave like single phase flow reducing the heat transfer enhancement.

4.3.3.3 Challenges in 3D simulations

The above mentioned average Nusselt numbers were calculated for the simulation cases run with the 4µm cubical structured base mesh elements with 1µm mesh elements at the fluid interfaces due to refinement with gradient adaptation. However, thin film formation for these cases is not clearly visible due to the insufficient refinement of mesh elements. Therefore, heat transfer simulations were also carried out for the smaller base mesh elements of 2 µm cubes. However, this refinement enormously increase the computational cost. The computational time for this case
Figure 4.18: Local Nusselt number variation along the channel wall for slug flow (water flow rate: 100 µl/min, oil flow rate: 30 µl/min) with different contact angles.

was proximately 2000 hours with 128 processors in a high performance cluster. Therefore, this was limited only for the 130 µl/min total flow rate case and two oil patches were used as explained in the boundary conditions section. The calculated local $Nu$ values for a unit cell with two different mesh sizes are shown in the Figure 4.21.

The local Nusselt number values are slightly different in two cases. This is due to the thin film formation with mesh refined as well as the errors encountered when defining the length of the oil plugs at the beginning of the simulations where the flow formation with cross junction is avoided for the simplicity. Approximate values for the oil plug and water slug lengths are defined at the beginning of the
simulations based on the 4 µm case which has flow formation. However, these lengths are slightly changed when the flow is formed and this will gives slightly changed slug lengths. The calculated average values for $Nu$ are, 7.62, and 6.30 for 4 and 2 µm mesh element cases respectively. A thin film is formed almost everywhere along the channel as shown in Figure 4.9 in the 2 micron mesh elements case. The calculated average $Nu$ for the refined mesh case is very close to the the lower margin of the experimental $Nu$ value (as in [9]) based on the associated experimental uncertainties. Therefore, it is challenging to simulate this type of flow as it is transient.

4.4 Summary

This chapter presented 3D numerical simulations of oil-water slug flow heat transfer in square cross sectional microchannels with VOF method. The numerical results showed large increase of heat transfer rates at presence of oil droplets in water.
Figure 4.20: Average Nusselt number variation in slug flow with magnitude of relative velocity.

The numerical results were verified with the experimental results obtained for the similar flow and boundary conditions. A test case was simulated in 2D space in order to check the possibility of predicting accurate physics in non-circular channel with 2D modelling. However, the results showed that 2D modelling gives lower $Nu$ values than the expected values proving the requirement of 3D modelling.

The study was limited to three different total oil-water flow rates 130, 120, and 110 $\mu$l/min and the contact angle effect on flow formation and heat transfer was studied based on the previous experiments. Flow focusing junction was included in the modelling in order to predict the slug lengths and plug lengths with the selected mesh elements throughout the simulation (4 micron). The numerical results showed a significant increase of heat transfer rates with slug flow mainly due to recirculations that are formed within the liquid slugs due to the presence of interfaces. The higher total flow rate flows which have shorter slugs give higher $Nu$ values compared to lower total flow rates flows, due to the higher intensity
An insight into the challenges regarding 3D modelling was presented as the important aspect of slug flow modelling is capturing the thin liquid film and considering the dynamic nature of the interfaces. However, finer mesh elements are required for this. Therefore, one case was simulated with 2 µm size of mesh elements to demonstrate the differences of heat transfer rates at presence of thin film. However, 3D modelling increases the computational time and new modelling techniques are required to predict the underlying physics accurately and efficiently.

![Figure 4.21: Local Nusselt number variation in a unit cell with different base mesh elements sizes (water flow rate: 100 µl/min, oil flow rate: 30 µl/min).](image)

Table 4.21: Values of $Nu$ and droplet radius for different mesh elements sizes.

<table>
<thead>
<tr>
<th>Axial Distance (m)</th>
<th>$Nu$ (2 micron base mesh)</th>
<th>$Nu$ (4 micron base mesh)</th>
<th>droplet radius (4 micron)</th>
<th>droplet radius (2 micron)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0029</td>
<td>10</td>
<td>2</td>
<td>0.003</td>
<td>0.0031</td>
</tr>
<tr>
<td>0.0031</td>
<td>15</td>
<td>4</td>
<td>0.0032</td>
<td>0.0033</td>
</tr>
<tr>
<td>0.0032</td>
<td>6</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0033</td>
<td>8</td>
<td>6</td>
<td></td>
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</tr>
</tbody>
</table>
Experimental study of slug flow heat transfer

This chapter will present the details of the experimental studies including setting up the experiments, methodology and the analysis of experimental results. The experimental studies were designed to achieve the main objective of the thesis, that is studying hydrodynamics and heat transfer characteristics of liquid-liquid two phase slug flow in microchannels. Therefore, experiments are divided into three main categories which are flow visualisation, pressure drop measurements and heat transfer studies.

5.1 Introduction

Experimental studies on slug flow heat transfer in microchannels have been mentioned extensively in literature review chapter. However, most of these studies are on gas-liquid two-phase flow while few studies have been carried out on liquid-liquid slug flow heat transfer. Heat transfer experiments are challenging at the micro scale due to the difficulties encountered in accurately measuring parameters such as bulk fluid temperature, wall temperature, and liquid flow rates. Different techniques have been used for measuring the wall temperature and bulk fluid temperature in microchannel flows. Some of them are thermocouples attached to wall, infra-red thermography for outer wall temperature measurement and laser induced fluorescence (LIF) microscopy for measuring the bulk fluid temperature.
By careful consideration of the existing methods and techniques the best possible methodologies were developed for conducting heat transfer and flow hydrodynamics experiments.

5.2 Experiment set up

Non-boiling liquid-liquid slug flow is made up of two non-mixing liquids. Therefore, in the present study, water and 1.5 cSt silicon oil were used as the working fluids. Their properties along with those of stainless steel are given in Table 5.1.

<table>
<thead>
<tr>
<th>material</th>
<th>density [kg/m³]</th>
<th>specific heat [J/kg K]</th>
<th>thermal conductivity [W/m K]</th>
<th>viscosity [Pa.s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>998</td>
<td>4182</td>
<td>0.6</td>
<td>0.001002</td>
</tr>
<tr>
<td>silicon oil</td>
<td>853</td>
<td>1800</td>
<td>0.1047</td>
<td>0.0012795</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>8000</td>
<td>500</td>
<td>16</td>
<td>-</td>
</tr>
</tbody>
</table>

The interfacial tension between the silicon oil and water is 36.5 mN/m [130]. The viscosities of the two fluids changing significantly with temperature. Therefore, viscosity values were calculated as temperature dependent values during the heat transfer analysis section. Equations 5.1 and 5.2 were used for this calculation.

\[ \mu_w(T) = 0.0000002T^2 - 0.00003T + 0.0015 \] (5.1)

\[ \mu_o(T) = 0.00000001T^2 - 0.00002T + 0.0018 \] (5.2)

Here, \( \mu_w \), \( \mu_o \) and \( T \) are water viscosity, oil viscosity and temperature in °C respectively. As thermal conductivity is weakly dependent on temperature, a constant value was used.

5.2.1 Slug flow visualisation

Different techniques have been introduced in the literature for slug flow generation. Some of them are flow focusing devices, T-junction, Y-junction and needle injection method. When two immiscible liquids are brought into contact in any of the above
Figure 5.1: Experimental setup for droplet formations and visualisation

devices, the pattern of the out flow is determined by the ratio of two inlet flow rates, channel geometry and the interfacial tension. Based upon these conditions different types of two-phase flow patterns are generated at the outlet channel as discussed in Chapter 2. Therefore, suitable inlet flow conditions should be selected in order to obtain slug flow and Taylor flow regimes. A T-junction is a reliable and a inexpensive method to produce slug flow. Therefore, in the present study a T-junction was used for generating two-phase slug flow. The schematic of slug flow formation in a T-junction is shown in Figure 5.1.

There are commercially available ETFE T-junctions which can be used for generation of slug flow. However, extra effort is required in connecting tubes and pipes for visualising the flow as the flow is disrupted due to the changes in inside channel diameters, specially when the flow is introduced to the heating test section. Therefore, in this thesis a special T-junction made with polydimethylsiloxane(PDMS) is used for flow visualisation and introduction the flow into the heat transfer test
A schematic diagram of the PDMS T-junction is given in Figure 5.2 (b). In this T-junction, inlet and outlet channels were created with a diameter of 1 mm ($D_1$). However, slightly larger channel (diameter of $D_2$) which is equal to the outer diameter of the stainless steel tube (used for heat transfer study with 1 mm ID) is fabricated in order to connect the steel tube to the PDMS channel without a diameter change as shown in Figure 5.2 (a). This ensures no flow disruptions due to diameter mismatching between the channels. The selection of material is an important step in designing droplet based microfluidics devices as the wall wettability plays a key role in slug flow formation. PDMS channels are hydrophobic for organic solvents such as water as discussed in [131]. Therefore, silicon oil was selected as the continuous phase. The Phantom V1610 high speed camera was used for flow visualisation. The images obtained from the camera was analysed frame-by-frame in order to obtained the sizes of the oil and water slug lengths. The images were captured at 16000 frames per second (fps) with 256 x 576 pixels. The images were recorded with a 4x magnification lens. The plug velocity ($U_p$), slug length ($L_s$) and plug length ($L_p$) were calculated by image analysis.
Two different syringe pumps (Harvard apparatus PHD Ultra) are used for flow formation for having different flow rate ratios accurately. The syringe pumps have a manufacture’s accuracy of 0.3%.

5.2.1.1 Flow visualisation procedure

Following steps were carried out during flow visualisation experiments.

1. Experimental apparatus is set up as shown in Figure 5.1.
2. Camera controlling software is set.
3. The required flow rates are set in each syringe pump.
4. After starting, the flow is left to stabilise for 2-20 minutes depending on flow rate.
5. Once stable, the images are captured.
6. Process will be repeated for different flow rate ratios.

5.2.2 Pressure drop measurement

Heat transfer enhancement is generally associated the penalty of increased pressure drop. Thus it is necessary to measure the pressure drop accurately in order to have a optimum heat transfer enhancement. A separate test rig was designed for measuring the pressure drop in liquid-liquid slug flow. Introduction of secondary fluid into a continuous fluid, increases the pressure drop. Various types of studies have been carried out in the literature for pressure drop measurements and different type of correlations were developed to calculate the pressure drop. However, there is little agreement between these correlations as discussed in Chapter 2. A schematic diagram of the pressure drop measuring set up used here is shown in Figure 5.3. The same PDMS T-junction used for flow generation is used here for generation the slug flow and the outlet channel of PDMS will be directly connected to the steel tube of 1 mm ID which was used for the heat transfer studies.

A U-tube manometer was used for measuring the pressure drop over the channel length. The manometric fluid was water coloured to red with food die. The rest
of the tube was filled silicon oil. A T-type thermocouple connected to NI-9219 data logger together with Labview software was used to measure the temperature of fluid. The pressure drop for fully developed single phase flow in a capillary tube can be calculated from equation 1.4 as discussed in Chapter 1.

5.2.2.1 Pressure drop measuring procedure

Following steps were carried our in Pressure drop measurement experiments

1. Experimental apparatus is set up as show in Figure 5.3.

2. The required flow rates are set up in each syringe pumps.

3. After starting, the flow is left to stabilise for 2- 20 minutes depending on flow rate.

4. Once the flow is stable, the manometer is left to stabilise for another 20 - 30 minutes.

5. The height difference of water is recorded once the manometer is stable.
6. A sequence of images were captured once the flow is stable in order to measure the structure of the flow.

7. Process is repeated for different flow rate ratios after setting up the syringe pumps for required flow rates.

Range of non-dimensional parameters used in the pressure drop experiments are given in Table 5.2.

Table 5.2: Range of non-dimensional parameters used in pressure drop experiments

<table>
<thead>
<tr>
<th>non-dimensional parameter</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>3.3 - 101.7</td>
</tr>
<tr>
<td>L_s/D</td>
<td>2.5 - 6</td>
</tr>
<tr>
<td>L_p/D</td>
<td>0.9 - 1.5</td>
</tr>
</tbody>
</table>

5.2.3 Heat transfer study

Heat transfer measurements in small scale channels is a challenging task due to the difficulties encountered with the test procedure as discussed at the beginning of the chapter. The main challenges are heating up the channel in order to maintain constant wall heat flux or temperature boundary condition and accurately measuring temperature. A series of initial experiments were carried out with different test tubes in order to find a suitable method for setting up heat transfer experiments.

A heat transfer apparatus with a 1 mm ID stainless steel tube was set up. The stainless steel tube was heated with Joule heating. A constant wall heat flux boundary condition commonly encountered in most of the heat exchange devices could then achieved. However, the drawback in using a steel tube is that the flow can not be visualised while the heat transfer experiments are running. Visualisation is required to determine flow parameters (slug lengths and plug lengths etc,) as the primary objective of the thesis to study heat transfer with different slug flow parameters. Therefore, a special arrangement was used in order to visualise the flow before entering to the stainless steel tube. The validation of the experimental set up is carried out using single phase flow heat transfer experiments. The following sections will explain in details the test facility, temperature measurement methods, and the testing procedure.
5.2.3.1 Heat transfer test facility

A schematic diagram of the experimental rig is given in Figure 5.4. The same PDMS T-junction which used for flow formation is used here for generating oil-water slug flow. The generated slug flow is visualised with a Phantom high speed camera before introducing the flow to the heat transfer test section. The length of the steel channel is 200 mm, a length of 190 mm is heated with Joule heating while 5 mm from each side is left for fitting the tubing. Another PDMS chip made with a bigger channel (outer diameter of the steel tube) at the beginning (5mm in length) and a 1 mm tube elsewhere is used at the end of the steel tube to visualise the flow. This is to ensure the flow is not disrupted passing into the steel tube. A series of flow visualisation experiments were carried out before each heat transfer experiments and images were captured before and after the steel tube and they were compared. They gave a maximum variation of 10% for the slug and plug lengths. These results confirmed that no changes to droplet shape due to the change of contact angle in steel tube. Furthermore, all the cases considered in the present study are in Taylor flow regime with a thin film surrounding the droplet. Thus the surface of the stainless steel tube will not interact with the droplet and
the contact angle will not affect the droplet shape. Scanning Electron Microscopic (SEM) images were captured in order to investigate the surface roughness of the stainless steel tubes. Figure 5.5 shows the inside roughness of the steel tube.

The next challenging task in setting up heat transfer experiments is to supply the power to the steel tube to obtain a constant wall heat flux boundary condition. The power is supplied to the steel tube with a ISO-Tech programmable DC power supply. Initial trials were carried out with connecting two wires to the steel tube by lead soldering. The two wires had a fixed and stable connection to the tube but due to the large contact resistance at the soldering joints excessive heat is generated. This makes it very difficult to generate a constant wall heat flux boundary condition. In order to avoid this problem, two wires were connected to two copper rings by silver soldering at two ends. These rings were also silver soldered to the tube. This minimised the contact resistance and a constant wall heat flux could be maintained with DC electric current. The temperature distribution in two different connection methods was observed with Flir IR camera. Those two different images are shown in Figure 5.6. According to Figure 5.6 (b) it is clear that there is a high temperature jump at two connections in the lead soldering setup. The images were captured while the fluid is passing through the channel and linear increase of wall temperature in silver soldered tube can be seen clearly in Figure 5.6 (a). The heat generated is measured under steady state conditions and the voltage drop across the heating section is measured in every experimental run.
in order to calculate the correct energy input to the channel. The steel tubes were selected with a minimum wall thickness (0.29 mm) in order to minimise the axial conduction along the channel. The axial conduction number which is the relative effect of axial conduction and axial convection presented by Maranzana et al. [132] was used to check the validity of avoiding axial conduction in the present study. The axial conduction number is given by equation 5.3.

\[
M = \frac{k_{st} A_{cond}}{L_h \rho Q c_p} \tag{5.3}
\]

where \(k_{st}\), \(A_{cond}\), \(L_h\), \(\rho\), \(Q\), and \(c_p\) are thermal conductivity of steel, area of heat conduction, heating length of the channel, density of fluid, fluid flow rate, and specific heat of fluid respectively. The axial conduction numbers calculated for the present study were in the range of \(4.5 \times 10^{-4}\) to \(4.1 \times 10^{-3}\) which are below \(10^{-2}\). Therefore, the axial conduction could be neglected in the present study. The whole heat transfer section was placed inside a well insulated box made of 7 cm thick Styrofoam.

The next challenging task of heat transfer experiments in small channels is measuring the wall temperature. In the present study, T-type 36 AWG (0.127 mm) thermocouple junctions are connected along the axial direction of the wall with double sided silicone adhesive tape. The other side of the thermocouple wires

Figure 5.6: Thermal images taken from Flir IR camera of the heated steel tube with power supply wires connected by (a) silver soldering and (b) lead soldering.
are connected to a TC-08 Pico data logger. Inlet and outlet temperatures are also measured with T-type thermocouples. A 97% porous copper block is inserted to the end of the steel tube in order to have a well mixed fluid flow just after the heating section where the outlet temperature is measured. All the thermocouples are calibrated with a standard RTD using a Julabo water bath for the temperatures ranging from 20° to 80°C. The flow rates are measured with the syringe pump.

5.2.3.2 Heat transfer experiment procedure

The following steps were followed during the heat transfer experiments.

1. The experimental apparatus is set up as in Figure 5.4.

2. The flow is started from two syringe pumps with required flow rates and the Pico data logger software is started.

3. After few minutes (once the flow has passed through the heating section) the power supply is turned on.

4. The temperature measurements are taken after the flow become thermally stable. This could be monitored through the pico logger software and the temperatures gave stabilised with ±0.2°C.

5. Then the voltage difference across the heating section is measured.

6. Finally the power supply is turned off and then the syringe pumps are turned off.

7. Steps from 2 to 6 are repeated for a new test case.

A range of non dimensional parameters were tested in heat transfer experiments as given in Table 5.3

Single phase flow experiments are carried with the same procedure. The two syringes are filled with the similar fluid and the flow rates are set in two pumps in order to get the required flow rate.
Table 5.3: Range of non-dimensional parameters used in heat transfer experiments

<table>
<thead>
<tr>
<th>non-dimensional parameter</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>10 - 70</td>
</tr>
<tr>
<td>Ca</td>
<td>0.0003 - 0.0029</td>
</tr>
<tr>
<td>Ls/D</td>
<td>0.6 - 6.0</td>
</tr>
<tr>
<td>Lp/D</td>
<td>0.9 - 3.5</td>
</tr>
</tbody>
</table>

5.2.3.3 Heat transfer calculation method

The steel tube is heated from Joule heating as described at the beginning of the section in order to maintain a constant heat flux boundary condition. The amount of heat supplied to the channel can be calculated from the voltage ($V$) and current ($I$) from equation 5.4.

$$q = VI$$ (5.4)

The total heat flux on the channel wall can be calculated assuming the losses are negligible from the heating channel in the Styrofoam insulation box. Thus, the total heat flux on the inner wall of the channel is calculated from,

$$q'' = \frac{q}{\pi D_i L}$$ (5.5)

where $D_i$ and $L$ are inner diameter of the heated channel and the length of the channel. Measuring inner wall and bulk fluid temperatures is challenging in microchannel heat transfer experiments while the outer wall temperature can be measured with fine thermocouple wires connected to outer wall. Therefore, the one dimensional heat conduction equation is used for calculating the inner wall temperature at each location of the outer wall temperature measurement. Thus, the inner wall temperature is given from equation 5.6

$$T_{wi_x} = T_{wo_x} - \left[ \frac{q \ln \left( \frac{D_o}{D_i} \right)}{2\pi L k_{st}} \right]$$ (5.6)

where, $T_{wi_x}, T_{wo_x}, D_o,$ and $k_{st}$ are inner wall temperature at distance $x$, outer wall temperature at distance $x$, outer diameter of the channel, and thermal conductivity
of the steel respectively. The bulk fluid temperature at each thermocouple position is calculated by considering the energy balance as in equation 5.7

\[ T_{mx} = T_{in} + \left[ \frac{q'' \pi D_i x}{Q \rho c_p} \right] \]  

(5.7)

where \( T_{mx} \), \( T_{in} \), \( Q \), \( \rho \) and \( c_p \) are bulk temperature at distance \( x \), inlet fluid temperature, total flow rate of fluid, density of fluid and the specific heat of fluid respectively. The properties are calculated as effective values in slug flow based on the volume fraction of each fluid. The convective heat transfer coefficient on the channel wall can be calculated from the calculated fluid and wall temperatures. Thus the local heat transfer coefficient (\( h_x \) at a distance \( x \) is given by equation 5.8.

\[ h_x = \frac{q''}{T_{wi} - T_{mx}} \]  

(5.8)

Finally, the local Nusselt number along the channel wall is calculated from equation 5.9

\[ Nu_x = \frac{h_x D_i}{k_f} \]  

(5.9)

where \( k_f \) is the thermal conductivity of working fluid. An effective value for \( k_f \) is calculated using the volume fraction of two liquids in slug flow experiments. An average \( Nu \) is calculated from the calculated local Nusselt numbers.

### 5.2.4 Uncertainty analysis

This section presents the uncertainty values associated with the experimental measurements discussed in the previous sections. The uncertainty values were calculated using the method proposed by Coleman and Steele in 1999 [133]. All the experimental data should be reported with a systematic uncertainty (bias error), random uncertainty (precision error), and total uncertainty together with confidence level as suggested by Coleman and Steele [133]. The desired data is not measured directly in most of the cases and it will be calculated by combining a set of individual data in a form of data reduction equation given by

\[ r = r(X_1, X_2, \ldots, X_j) \]  

(5.10)
where \( r \) is the result and \( X_1, X_2, \ldots X_j \) are the \( j \) number of individual measured variables. If the bias error is \( B_r \) and the precision error is \( P_r \), the total error or uncertainty of the experimental measurement is given by the root-sum-square (RSS) of bias and precision errors.

\[
U_r = \sqrt{B_r^2 + P_r^2}
\]

(5.11)

The bias error can be calculated from the relationship given in the equation 5.12.

\[
B_r^2 = \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} \right)^2 B_{X_i}^2 + 2 \sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \left( \frac{\partial r}{\partial X_i} \right) \left( \frac{\partial r}{\partial X_k} \right) B_{X_i} B_{X_k}
\]

(5.12)

where \( B_{X_i} \) is the systematic uncertainty associated with the variable \( X_i \) and \( B_{X_i, X_k} \) is the covariance estimator for each pair of variables that share common bias errors.

The precision error for a single measured variable can be calculated from;

\[
S_{ri} = t \frac{S_X}{\sqrt{N}}
\]

(5.13)

where \( t \) statistical coverage factor (confidence level) which is determined from the degrees of freedom of the results, \( \nu = N - 1 \), and the \( t \)-distribution. \( N \) is the numbers of measured values for the variable \( X_i \) and \( S_X \) is the standard deviation of \( X_i \) variables which can be calculated by,

\[
S_X = \left[ \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})^2 \right]^{1/2}
\]

(5.14)

Here, \( \bar{X} \) is the mean of variable \( X \). The confidence level for the present study was selected as 95\% and therefore, \( t \approx 2 \). Thus, \( S_{ri} = \frac{2S_X}{\sqrt{N}} \). The total precision error \( P_r \) can be calculate from the equation,

\[
P_r = \sqrt{\sum_{i=1}^{J} S_{ri}^2}
\]

(5.15)

The uncertainties associated with the experiments are given in the Table 5.4.
Table 5.4: Uncertainties associated with the measured and derived variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate</td>
<td>0.35 %</td>
</tr>
<tr>
<td>Channel diameter</td>
<td>±0.05 mm (5 %)</td>
</tr>
<tr>
<td>Channel length</td>
<td>±1 mm (0.5 %)</td>
</tr>
<tr>
<td>Temperature</td>
<td>±0.1°C (2.3 %)</td>
</tr>
<tr>
<td>Primary variables</td>
<td>Material properties 3%</td>
</tr>
<tr>
<td>Pixels</td>
<td>±1</td>
</tr>
<tr>
<td>Current</td>
<td>±0.01 A (3.3 %)</td>
</tr>
<tr>
<td>Voltage</td>
<td>±0.01 V (3.3 %)</td>
</tr>
<tr>
<td>Pressure</td>
<td>±7 Pa</td>
</tr>
<tr>
<td>Derived variables</td>
<td>Re</td>
</tr>
<tr>
<td></td>
<td>Ca</td>
</tr>
<tr>
<td></td>
<td>slug length (L_s)</td>
</tr>
<tr>
<td></td>
<td>film thickness (δ)</td>
</tr>
<tr>
<td></td>
<td>Nu</td>
</tr>
</tbody>
</table>

5.3 Results and discussion

This section will present the results obtained from the experiments. The first section will present the flow visualisation results. The second part will present the pressure drop calculation for liquid-liquid two-phase flow, while the last section presents the heat transfer studies on liquid-liquid slug flow including the effect of different flow parameters on heat transfer.

5.3.1 Flow visualisation

Flow rate ratio (primary phase fluid flow rate to secondary phase fluid flow rate, Q_p/Q_s) of two non-mixing fluids was the only controllable parameter in the present study as the other parameters such as dimensions of T-junction and the fluid properties chosen to be fixed throughout the study. Therefore, a range of different flow rate ratios were chosen in order to get different slug lengths (L_s) and plug lengths (L_p). Table 5.5 shows the different oil and water flow rates and the respective Reynolds and capillary numbers. The Re and Ca were calculated based on the primary phase fluid properties.

A wide range of slug and plug lengths were obtained from the different flow rate combinations given in the Table 5.5. Different constant oil flow rates were
Table 5.5: Different oil and water flow rates

<table>
<thead>
<tr>
<th>oil flow rate (µl/min)</th>
<th>water flow rate (µl/min)</th>
<th>Re</th>
<th>Ca</th>
</tr>
</thead>
<tbody>
<tr>
<td>250.0</td>
<td>333.3</td>
<td>9.5</td>
<td>0.00029</td>
</tr>
<tr>
<td>250.0</td>
<td>416.7</td>
<td>10.6</td>
<td>0.00034</td>
</tr>
<tr>
<td>250.0</td>
<td>500.0</td>
<td>11.6</td>
<td>0.00038</td>
</tr>
<tr>
<td>500.0</td>
<td>500</td>
<td>15.2</td>
<td>0.00057</td>
</tr>
<tr>
<td>500.0</td>
<td>666.7</td>
<td>20.7</td>
<td>0.00076</td>
</tr>
<tr>
<td>500.0</td>
<td>833.3</td>
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<td>0.00086</td>
</tr>
<tr>
<td>500.0</td>
<td>1000.0</td>
<td>27.6</td>
<td>0.00095</td>
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<td>1000.0</td>
<td>166.7</td>
<td>17.4</td>
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</tr>
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<td>1000.0</td>
<td>250.0</td>
<td>19.1</td>
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<td>333.3</td>
<td>20.8</td>
<td>0.00094</td>
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<tr>
<td>1000.0</td>
<td>416.7</td>
<td>22.5</td>
<td>0.00099</td>
</tr>
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<td>1000.0</td>
<td>500.0</td>
<td>24.2</td>
<td>0.00104</td>
</tr>
<tr>
<td>1000.0</td>
<td>1333.3</td>
<td>41.3</td>
<td>0.00152</td>
</tr>
<tr>
<td>1000.0</td>
<td>1666.7</td>
<td>48.3</td>
<td>0.00172</td>
</tr>
<tr>
<td>1000.0</td>
<td>2000.0</td>
<td>55.2</td>
<td>0.00191</td>
</tr>
<tr>
<td>1500.0</td>
<td>500.0</td>
<td>31.2</td>
<td>0.00141</td>
</tr>
<tr>
<td>2000.0</td>
<td>500.0</td>
<td>38.2</td>
<td>0.00168</td>
</tr>
<tr>
<td>2000.0</td>
<td>666.7</td>
<td>41.6</td>
<td>0.00168</td>
</tr>
<tr>
<td>2000.0</td>
<td>833.3</td>
<td>45.0</td>
<td>0.00188</td>
</tr>
<tr>
<td>2000.0</td>
<td>1000.0</td>
<td>48.3</td>
<td>0.00207</td>
</tr>
<tr>
<td>2500.0</td>
<td>500.0</td>
<td>45.3</td>
<td>0.00215</td>
</tr>
<tr>
<td>3000.0</td>
<td>500.0</td>
<td>52.3</td>
<td>0.00252</td>
</tr>
<tr>
<td>4000.0</td>
<td>500.0</td>
<td>66.5</td>
<td>0.00327</td>
</tr>
</tbody>
</table>

selected (250, 500, 1000, and 2000 µl/min) while changing the water flow rate in order to study the effect of total flow rate on slug and plug lengths at the same flow rate ratios. Furthermore, secondary phase was also kept constant while changing oil flow rates. Different flow rate combinations generated different flow patterns as illustrated in the following figures.

Figure 5.7 shows the variation of slug length and plug length with different water flow rates for 250 µl/min constant oil flow rate. The slug length decreases with increasing water flow rate where the size of water plug increases. Figure 5.8 (b), (c), and (d) show the variation of slug and plug lengths for the same flow rate ratios as in 5.7 but with constant oil flow rate of 500 µl/min. This combinations follow the same trend as in 250 µl/min constant oil flow rates. However, the slug and plug lengths for each respective flow rates ratios are slightly different and this
(a) Oil flow rate 250 µl/min, $Q_p/Q_s = 0.75$

(b) Oil flow rate 250 µl/min, $Q_p/Q_s = 0.6$

(c) Oil flow rate 250 µl/min, $Q_p/Q_s = 0.5$

Figure 5.7: Slug flow with 250 µl/min constant oil flow rate.
Figure 5.8: Slug flow with 500 µl/min constant oil flow rate.
(a) Oil flow rate 1000 µl/min, \( Q_p/Q_s = 6 \)

(b) Oil flow rate 1000 µl/min, \( Q_p/Q_s = 4 \)

(c) Oil flow rate 1000 µl/min, \( Q_p/Q_s = 3 \)

(d) Oil flow rate 1000 µl/min, \( Q_p/Q_s = 2.4 \)

(e) Oil flow rate 1000 µl/min, \( Q_p/Q_s = 2 \)

Figure 5.9: Slug flow with 1000 µl/min constant oil flow rate.
Figure 5.10: Slug flow with 2000 µl/min constant oil flow rate.
(a) Water flow rate 500 µl/min, $Q_p/Q_s = 3$

(b) Water flow rate 500 µl/min, $Q_p/Q_s = 5$

(c) Water flow rate 500 µl/min, $Q_p/Q_s = 6$

(d) Water flow rate 500 µl/min, $Q_p/Q_s = 8$

Figure 5.11: Slug flow with 500 µl/min constant water flow rate.
will be discussed further later. Figure 5.8 (a) shows the slug and plug lengths for the flow rate ratio of one.

Figure 5.9 and 5.10 illustrate the variation of slug and plug lengths with varying water flow rates with constant oil flow rates 1000 and 2000 µl/min respectively. The slug length increases with increasing $Q_p/Q_s$ in these cases similar to lower total flow rate cases (Figures 5.7 and 5.8). However, the shape of the water plugs changes with the increasing total flow rate. At low total flow rate cases, the plug has a similar shape in the front and tail of the plug. However, at higher total flow rate cases the front of the plug maintains a sharp curvature while the tail becomes less curved. Increasing total flow rate creates thicker films which become visible even with the low magnification lenses (in this study 4X is used for flow visualisation).

Figure 5.11 shows the variation of slug and plug lengths with increasing oil flow rates for a constant water flow rate of 500 µl/min. The flows with varying primary fluid flow rates also follow the same trend in changing the slug and plug lengths similar to changing water flow rate cases. However, the changes of shape of the plug which discussed before become significant with increasing oil flow rates as shown from Figure 5.11 (a) to (d). The thickness of the liquid film varies along the plug and it becomes thicker at the front of the water plugs compared to the rear area.

The lengths of liquid slugs and plugs were measured using the images captured during the flow visualisation experiments. The pixel values of the images were used to measure the slug and plug length using the pixel value for the known diameter of the channel. The quantitative illustration of the lengths are given Figure 5.12 and Figure 5.13.

Variation of slug and plug lengths with the flow rate ratio, $Q_p/Q_s$ for a constant oil flow rate of 1000 µl/min is given in Figure 5.12. Increasing $Q_p/Q_s$ leads to increased slug lengths while the plug lengths become shorter. The unit cell length decreases up to a certain $Q_p/Q_s$ ratio and then starts increasing with flow rate ratio beyond this point. Similar slug length variations have been observed in the experimental studies by Kashid et al [134].

Similar to the slug and plug length variation in constant oil flow rate flows, the slug length is increased with increasing $Q_p/Q_s$ in flow combinations with constant
water flow rate as shown in Figure 5.13. The plug length and the unit cell length also follow the same trend as in constant oil flow rate cases. The both types of flows have the equal slug and plug lengths approximately at the same $Q_p/Q_s$ which occurs when $Q_p \approx 1.5 Q_s$. This is similar to the results of Xu et al.[135] liquid-liquid slug flow experiments.

As discussed at the beginning of the section, there is a slight variation of slug and plug lengths for the flow cases with equal primary to secondary flow rate ratios as shown in Figure 5.14. The plug lengths with 0.5 and 0.6 volume fractions are higher for the 500 µl/min constant oil flow rate cases as shown in Figure 5.14 (a). This is due to the thicker film with higher flow rate case.
5.3.1.1 Film thickness measurements

The film thickness measurement was carried out by analysis of the images captured from the Phantom camera with the 16X optical magnification. The pixel value variation across the diameter of the channel is used for calculation of the film thickness. The thickness of the film varies along the water plug as explained in the flow visualisation section. However, this variation is smaller at low $Q_p/Q_s$ flows. Figure 5.15 shows the film thickness around a water plug in a slug flow with 1000 $\mu$l/min oil and 500 $\mu$l/min water flow rate. The film thickness is almost uniform along the plug in this particular case where it becomes slightly thinner at the rare of the film region.

However, this film has a large variation along the water plug in higher capillary number flows as shown in Figure 5.16. The front of the film region become wider
Figure 5.14: Variation of slug/plug lengths with $Q_{p}/Q_{s}$ for different primary phase flow rates, (a) slug length ($L_{s}$) and (b) plug length ($L_{p}$).

Figure 5.15: Liquid film around a water plug/droplet for the case of oil flow rate 1000 $\mu l/min$ and water flow rate 500 $\mu l/min$ and the length of the droplet/plug become smaller as discussed previously in Figure 5.11 (c) and (d) for the high $Q_{p}/Q_{s}$ ratio flows. Further increase of this ratio leads to the plugs becoming spherical bubbles.

The variation of liquid film thickness with $Ca$ is illustrated in Figure 5.18 which shows how the thickness of film increases with increasing $Ca$. The solid line represents the calculated film thickness for these particular flow conditions using Bretherton’s correlation given by equation 2.2 [20]. There is a close agreement with the measured and calculated film thickness values. However, few experimental values are significantly deviated from the theoretical values. These films have a varying thickness along the droplet (thinner at the front and back of the droplet
Figure 5.16: Liquid film around a water plug/droplet for the case of oil flow rate 3000 µl/min and water flow rate 500 µl/min

Figure 5.17: Liquid film around a water plug/droplet for the case of oil flow rate 2000 µl/min and water flow rate 1000 µl/min

and thicker in between) as shown in Figure 5.17. As Bretherton’s correlation is 1D, this variation causes a higher value than the theoretical prediction when calculating the thickness as a mean value.

5.3.2 Pressure drop measurements

The pressure drop for single phase and slug flow was measured using the U-tube manometer. The experimental apparatus was set as explained in section 5.2.2 and the experimental method was followed as outlined there. Initially the experiments were carried our for single phase flow. The single phase pressure drop was measured for both silicon oil and water. Height change of the manometric fluid was noted down when it became stable and it was converted to pressure measure-
Figure 5.18: Thickness of liquid film with \( Ca \)

ment units. Figure 5.19 shows the variation of pressure drop with \( Re \) for single phase flow. The measured pressure drop values had a good agreement with theoretically predicted pressure drop values from equation 1.4. The pressure drop values in silicon oil flow is higher than the water flow in all the flow rates due to the comparatively high viscosity of silicon oil.

The two phase flow pressure drop was measured for a combination of flow rates. The flow was generated using the same T-junction used in the flow formation experiments. The variation of measured slug flow pressure drops are shown in Figure 5.20. According to the figure, the slug flow pressure drop is approximately 2.5 times higher than the single phase flow. Slug flow pressure drop consists of three components namely, frictional pressure drop due to primary phase, frictional pressure drop due to secondary phase and the interfacial pressure drop as discussed in
detail in literature review chapter. The interfacial pressure gives the higher contribution compared to other two components in slug flow pressure drop specially at low \( Re \) flows. The measured pressure drop values were compared with the calculated pressure drop values from the Jovanovic correlation [17] given in equation 2.10. There are some discrepancies between the measured and calculated values. This can be probably due to the wide variety of variables associated with slug flow pressure drop which is not accounted for in the correlations. Pressure drop is a penalty in heat transfer enhancement with two phase slug flow, so it is important that to be minimum.
5.3.3 Heat transfer analysis

5.3.3.1 Single phase flow heat transfer

Initially single phase flow heat transfer experiments are carried out in order to validate the experimental set up. Experiments were carried out for Reynolds numbers from 10 to 295 for both water and oil as the working fluid. Thermo-physical properties of water were determined from IAPWS-IF97 database, while the properties of silicone oil were determined by measurements and some from manufacturers data. At steady state the overall heat balance for the heating system can be expressed
as

\[ q_f = \dot{m}C_p(T_{out} - T_{in}) = q_{in} - q_{loss} \]  \hspace{1cm} (5.16)

where \( \dot{m} = \dot{Q}\rho \) is the mass flow rate. \( q_f, \dot{Q}, \rho, C_p, T_{in}, T_{out}, q_{in}, \) and \( q_{in} \) are heat transferred to the fluid, volume flow rate, density of fluid, specific heat of fluid, inlet fluid temperature, outlet fluid temperature, power input from Joule heating and the heat loss to the ambient respectively. In general, the heat loss was very small due to the well insulated experimental set up and this value was below 10% of the input power indicating the power was predominantly transferred to the fluid stream.

The bulk temperature calculated from equation 5.7 should increase linearly along the channel as shown in the Section 1.2.1.2 for constant heat flux wall boundary condition. Variation of measured outside wall temperature and calculated inside channel wall and bulk fluid temperatures along the channel for a flow with total flow rate of 5000 \( \mu l/\text{min} \) is given in Figure 5.21. According to Figure 5.21 the temperatures are varying linearly as explained previously and the temperature difference between outer and inner walls is very small due to the small thermal resistance of the stainless steel. However, at lower \( Re \) flows, this temperature variation is not completely linear along the channel due to the scaling effects such as axial conduction and viscous heating as discussed in the literature review chapter. The local Nusselt number along the channel wall is calculated from the measured and calculated temperatures together with power input as discussed previously. The calculated local \( Nu \) values at different thermocouple locations are given in Figure 5.22. The \( Nu \) closer to the channel inlet is little higher than the values far from the inlet which is due to the thermally developing flow at the beginning of the channel. The thermally entry length was approximately 3 - 10.5 mm for the water flow and 3 to 8 mm for silicon oil flow. The \( Nu \) values are almost constant along the channel even though they are comparatively smaller at lower flow rate cases.

The average Nusselt number for different flow rates/Reynolds numbers can be calculated from those local \( Nu \) values. The variation of average \( Nu \) with \( Re \) is illustrated in Figure 5.23. The data series with open symbols represents the present study while the others are experimental studies in the literature on single phase flow heat transfer. The black colour line represents the theoretical single
phase flow $Nu$ for this particular boundary condition. The calculated $Nu$ values at lower Reynolds numbers are little lower than the theoretically expected value of 4.36 independent on $Re$. This could be due to the axial conduction within the fluid at very low $Re$ flows. The present result showed good agreement with the theoretically expected value except at few of those low $Re$ values as explained above. Most of the previous studies showed very lower $Nu$ values at low $Re$ flows showing that scaling effect plays a key role in lower flow rate flows. However, some of these reported low values could be due to the difficulties encountered during measurements at this small scale other than the scaling effects.
5.3.3.2 Slug flow heat transfer

The homogeneous flow model which considers the two-phase as a single-phase having average fluid properties, which depend upon the mixture quality is used in slug flow heat transfer analysis. The bulk fluid temperature was calculated similar to single phase flow calculation using the energy balance. However, the fluid properties were calculated as effective values based on the volume fraction of each liquid in the mixture as given in the following equations:

$$\rho_{eff} = \phi \rho_s + (1 - \phi) \rho_p$$

(5.17)
Figure 5.23: Comparison of single phase flow Nusselt numbers obtained from present study with experimental data from the literature.

\[ \mu_{eff} = \phi \mu_s + (1 - \phi) \mu_p \] (5.18)

\[ C_{peff} = \phi C_{ps} + (1 - \phi) C_{pp} \] (5.19)

\[ k_{eff} = \phi k_s + (1 - \phi) k_p \] (5.20)

where, \( \rho_{eff} \), \( \mu_{eff} \), \( C_{peff} \), and \( k_{eff} \) are effective density, effective viscosity, effective specific heat, and effective thermal conductivity of mixture respectively. The subscripts \( p \) and \( s \) denote the primary and secondary fluid while \( \phi \) is the volume fraction of secondary phase in the mixture. The local \( Nu \) values at different
Figure 5.24: Local Nusselt number variation along the channel in slug flow for some selected flow rate combinations.

temperature measurement locations along the channel wall can be calculated using the above effective properties. The calculated $Nu$ values are shown in Figure 5.24. Local $Nu$ values have slight variations along the channel in each different flow conditions. This can be due to the uncertainties associated with attaching thermocouples at each position.

The average Nusselt number for each flow combinations is calculated using the local $Nu$ values along the channel. Figure 5.25 shows the calculated average $Nu$ variation with $Re$ for all the considered flow combinations within the study. The $Nu$ is higher than the value of its single phase counterpart at each slug flow cases, showing the heat transfer enhancement due to the introduction of a secondary phase plugs. Nusselt number increases with $Re$ up to a certain point and then it starts decreasing with increasing $Re$ as shown in Figure 5.25. However, it is
necessary to analyse the heat transfer rates against the flow parameters such as slug length, $Ca$ and void fraction as only the $Re$ will not represent the state of the slug flow.

- **Effect of slug and plug lengths on heat transfer**
  The normalised Nusselt number ($Nu^* = Nu_{TP}/Nu_{sp}$) variation with slug length is shown in Figure 5.26. The Nusselt number increases up to a certain slug length and then starts decreasing with further increase of slug length. The highest $Nu$ was observed for the slug flow with $Q_p/Q_s = 3$ with total flow rate of 2000 $\mu l/min$. The numerical studies on effect of slug length on heat transfer showed that lower slug lengths give highest heat transfer rates (in Section 3.3.3.5). Experimental results follow the same trend starting from 2.13 mm slug length which corresponds to the highest $Nu$ which is 3.8 times higher than for single phase flow. Similar findings were reported on variation of $Nu$ in gas-liquid two phase flow experiments in the literature [5, 81, 8, 21] with up to a 2.9 times (in [21]) higher than single phase flow.
Figure 5.26: Variation of $Nu^*$ (Nusselt number normalised with single phase flow $Nu$) with slug length, where the water flow rate is constant value of 500 $\mu$l/min.

However, $Nu$ values are comparatively lower at first three slug lengths. This could be due to the lower total flow rate of these cases which gives lower heat transfer rates. The plug lengths are decreased with increasing slug lengths in the above cases as explained in flow visualisation section. In these cases, the water flow rate was chosen to be a constant value of 500 $\mu$l/min while the oil flow rate was changing from 250 to 4000 $\mu$l/min. The slug lengths could not be changed while the other parameters such as void fraction set to be constant as one T-junction was used for flow formation throughout the study. Therefore, the parameters such as slug length, void fraction and $Ca$ numbers will have a combined effect on heat transfer rates rather than an individual parameter in this type of flow system. The above results show that the slug length plays a key role in heat transfer rates in non-boiling slug flow. Thus, understanding the slug flow generation with controlled slug and plug lengths will be important in practical applications. Future experiments
Effect of void fraction on heat transfer

The variation of normalised Nusselt number \((Nu^*)\) and slug length with void fraction (the volume fraction of water, \(\phi\)) is shown in Figure 5.27. \(Nu^*\) increases with increasing \(\phi\) until a maximum reached, beyond which \(Nu^*\) declines steadily towards the value of 1 when it becomes water only flow (when \(\phi = 1\)). In the flows with high volume fraction of water, longer water droplets/plugs are generated as shown in Figure 5.7 and these longer water plugs together with low \(Ca\) lead for poorer heat transfer rates. However, the slug length decreases monotonically with increasing volume fraction of water as shown in the figure. Similar variation of \(Nu^*\) and slug length with void fraction was reported in the experiments studies carried out by Leung et al. [21] for gas-liquid slug flow.

Heat transfer enhancement in slug flow is mainly caused by the internal recirculation within the slugs/plugs and the shorter slugs have higher intensity.
recirculation as discussed in numerical chapters. This promotes the radial mixing of the fluid by continuously moving the cold fluid in the middle of the channel to wall and the heated fluid near the wall towards the centre. Therefore, higher heat transfer rates are expected at higher $\phi$ values where it has shorter slug lengths. However, there are optimum slug and plug lengths that give highest heat transfer rates due to the overall heat absorption in the slug region is higher than that in the thin film region as described in 3.3.3.2. Therefore, these higher $\phi$ flows have poor heat transfer performance due to the longer water plug lengths within a unit cell compared to slug length and low flow velocities of the mixture. This can be well explained with the $Nu^*$ variation with void fraction for different mixture velocities as in Figure 5.28. According the figure, it is clear that the heat transfer rates are higher at higher flow rates cases. The maximum heat transfer enhancement is observed when $0.2 < \phi < 0.3$ (as in Figure 5.27) which gives optimum slug and
plug lengths. A similar range of $\phi$ for maximum performance was reported in [21]. At lower $\phi$ values, $Nu^*$ is comparatively low due to their longer slugs which reduce the intensity of recirculation. Thus, the void fraction is an important parameter in heat transfer enhancement and understanding of the ranges which gives higher heat transfer rates is important in applications.

- **Effect of $Ca$ and film thickness on heat transfer**

![Graph](image)

**Figure 5.29: Variation of $Nu^*$ and film thickness with $Ca$**

Figure 5.29 shows the variation of normalised Nusselt number and film thickness with $Ca$. Heat transfer rates increases with increasing $Ca$ as shown in the figure. However, this enhancement is only taking place up to a certain $Ca$ and start decreasing the heat transfer rates capillary numbers beyond this point as the thickness of the film increases with $Ca$ as shown in the figure. Secondary phase also makes a significant contribution in heat removal process unlike in gas-liquid flow due to the high thermo-physical properties of water in the present study. The thicker films around the water plugs create a thermal barrier between the water plugs and heated wall. The fluid
within the film region has slower velocities than the fluid within the slug region. Therefore, the heat transfer rates are lower in film region while the fluid temperatures are comparatively high. However, the secondary phase also has recirculation within the plugs and transfer heat continuously to the water plugs. Thus, the $Nu$ values are reasonably high with thicker film cases than the lower $Ca$ flows. However, controlling $Ca$ itself does not increase heat transfer rates as these slug flow parameters have a combine effect on heat transfer enhancement.

According to the above analysis the flow parameters are inter-related to each other and they have a combined effect on heat transfer. Therefore, understanding of optimum flow conditions is important in order to achieve higher heat transfer enhancement with slug flow.

5.3.3.3 Comparison of experimental and numerical results

A few selected experimental cases were numerically simulated with ANSYS Fluent in 2D axisymmetric space. The simulation boundary conditions were used similar to the cases in Chapter 3. However, channel wall thickness was considered with volumetric heating. Two oil droplets were patched based on the experimental slug and plug lengths. The numerically calculated $Nu$ values are compared with experimental $Nu$ values as shown in Figure 5.30. Numerical and experimental results showed a good agreement with less than 10% difference in all simulated cases. These higher $Nu$ values in numerical simulations are due to the slightly shorter slug lengths than initially specified, created with flow development.

5.3.3.4 Predicting Nusselt number with flow parameters

There is a number of correlations available in the literature as discussed in Chapter 2 for predicting Nusselt number in slug flow based on flow parameters. However, most of those correlations are based on gas-liquid slug flow models and there are discrepancies in values when applied to liquid-liquid slug flow systems. Thus a
A correlation based on flow parameters was developed as in equation 5.21.

\[ Nu = (1 - \phi) \left[ 4.36 + 25 \left( \frac{L_s}{D} \right) ^{-0.139} \right] ^{0.523} \]  

(5.21)

This correlation was based on the one by Walsh et al. [8] to predict \( Nu \) in their gas-liquid slug flow experiments. The predicted and experimentally obtained Nusselt numbers are shown in Figure 5.31. There is a good agreement between the predicted and experimentally obtained \( Nu \) values showing that over 85% of experimental \( Nu \) values are within \( \pm 25\% \) range of predicted values. This correlation can be used to predict the \( Nu \) values in liquid-liquid slug flows.
Figure 5.31: Comparison of experimental $Nu$ and predicted $Nu$ values from equation 5.21

## 5.4 Summary

Experiments on slug flow were performed under three main categories; flow visualisation, pressure drop measurement and heat transfer study. A steel channel with 1 mm internal diameter was used for pressure drop and heat transfer studies while the flow formation and visualisation were carried out on a PDMS T-junction. Water and 1.5 cSt silicon oil were used as the working fluids and silicon oil was selected as the primary fluid due to the wettability properties of PDMS. A wide range of liquid-liquid slug flow combinations were generated during the experiments and slug and plug lengths variation followed the similar trends reported in the literature. The film thickness was also measured for all the flow cases and they showed a good agreement with the theoretically predicted film thickness values.
The pressure drop measurements were carried out for both single phase and slug flow using a U-tube manometer. The measured pressure drop values varied from the predicted values with the Jovanovic correlation [17]. The slug flow pressure drop values are significantly higher than the single phase pressure drop values due to the frictional pressure drop from each phases and the interfacial pressure drop from the non-mixing interfaces.

Heat transfer under constant wall heat flux was studied experimentally for both single phase and slug flow. The channel was heated with Joule heating and the experimental set up was well insulated in a Styrofoam box to minimise the heat losses to the environment. Heat transfer experiments were carried out for a wide range of capillary (0.0003 < $Ca < 0.0029$) and Reynolds numbers (10 < $Re < 70$ for slug flow and 10 < $Re < 295$ for single phase flow). Single phase flow Nusselt numbers showed a good agreement with the theoretical value for this particular boundary conditions except little lower values at low Reynolds numbers. The slug flow heat transfer was carried out for different flow combinations in order to study the effect of slug length, void fraction, and $Ca$ on heat transfer and a significant increase of $Nu$ (up to 400%) was observed during the study. The heat transfer rates were analysed with different flow parameters and they showed a huge impact on heat transfer. However, these parameters are inter-related and they have a combined effect on heat transfer. Therefore, a new correlation was developed to calculate the Nusselt number for liquid-liquid slug flow based on the flow parameters. It is necessary to find out the optimum values for each parameter to have the maximum heat transfer rates in engineering applications where the non-boiling slug flow plays a key role.
Chapter 6

Conclusions and recommendations

This Chapter will present the conclusions of the study and future research directions and recommendations. The objective of the research was to study the effect of liquid-liquid slug flow parameters on heat transfer in micro/minichannels. Thus both numerical and experimental studies were carried out to understand the hydrodynamics of liquid-liquid slug flow and heat transfer with slug flow. The following conclusions were made from the research project.

6.1 Conclusions

A significant increase of heat transfer rates in circular microchannels was observed numerically with slug flow with up to 400 % maximum increase in $Nu$ compared to single phase flow. Slug length has a significant effect on heat transfer rate with shorter slug increasing the heat transfer rates. The contact angle also plays a key role in slug flow heat transfer as it defines the shape of the droplet in sliding slug flow cases. Higher heat transfer rates were observed with $140^\circ$ static contact angle case when the other flow parameters are set to be constant. This occurs during sliding slug flow where the intensity of recirculation is higher compared to that of Taylor flow with a film around the droplet. The film thickness and $Ca$ has a combined effect on heat transfer in slug flow as the film thickness increased with increasing $Ca$. Increasing $Ca$ increases heat transfer rates up to a certain $Ca$ flow, after which the heat transfer rates decreases as the thickness of film increases with $Ca$. 
Heat transfer in a square cross sectional microchannels was also studied numerically with ANSYS Fluent. The simulations were carried out in 3D space as they required 3D modelling to predict physics accurately. The study was limited to three different flow rate combinations and two different contact angle cases due to the high computational cost. The simulation results were validated with previous experiments and showed a good agreement between experimental and numerical $Nu$ values. The slug flow gives significantly higher heat transfer rates compared to single phase flow similar to circular microchannels. The flows with shorter slugs increase heat transfer rate due to higher intensity of recirculation in shorter slugs. Again the walls with 130° static contact angle showed higher heat transfer rates due to the less curved interface of droplets compared to 160° contact angle case. New modelling techniques are required to model slug flow heat transfer in non-circular microchannels due to high computational cost.

The experimental studies were designed to study the hydrodynamics of slug flow and heat transfer. The experiments were carried out in a 1 mm ID stainless steel tube. The slug and plug lengths were measured with flow visualisation results and showed a good agreement with the available droplet formation data in the literature. The liquid film thickness was measured from the images captured with 16X magnification. The measured film thickness values showed a good agreement with the Bretherton’s film thickness correlation [20]. Measured single phase flow pressure drop showed a good agreement with the theoretical values. Slug flow increases pressure drop by up to two times compared to single phase flow due to the frictional pressure drop from two liquid phases and the interfacial pressure drop. The measured values were compared with the correlations available in the literature but a significant difference between the values was observed. This is probably due to the wide variety of variables associated with two-phase flow which is not accounted for in the correlations.

Heat transfer experiments with slug flow were carried out for a wide range of flow combinations in order to study the effect of slug length, volume fraction, film thickness and $Ca$ on heat transfer. The experimental set up was validated with single phase flow experiments and single phase flow $Nu$ showed a good agreement with the theoretical values. A significant increase of $Nu$, up to 400% was observed with slug flow experiments. The effect of slug length showed a similar trend to
numerical results up to a certain slug length and then heat transfer rates were
decreased if the slug length become smaller than that length. The volume fraction
of fluids, $Ca$ and film thickness also have strong effects on heat transfer. However,
these parameters have combined effects on heat transfer. A correlation based on
the slug length and the volume fraction was developed for predicting the Nusselt
number. The predicted $Nu$ values generally agree within a 25% with experimental
values. The findings of this research will be important in designing the liquid-liquid
slug flow based systems for the cooling applications in small scale.

6.2 Recommendation

Beyond the fundamental aspects of traditional liquid-liquid Taylor flow systems
being understudied, there are many opportunities to incorporate a range of con-
vergent technologies and systems to take heat transfer enhancement to the next
level. These include:

- Detailed study of slug flow velocity using micro-PIV which has not been
done this research project will enable to understand the heat transfer process
in slug flow, particularly during sliding slug flow where there is no data
available.

- Detailed studies on the impact of thermal conductivities of the two phases
including systems with liquid metal, for example.

- Heat transfer with Taylor flow in non-straight or uniform microchannels. The
slugs can be manipulated using flow through various geometries such as dif-
fuser/nozzle shapes, and curved and meandering channels. The asymmetry
and chaotic advection within the slug can improve mass and heat transfer
significantly.

- Solid/liquid/liquid system. Solid particles dispersed in at least one of the
liquid phases has the potential to improve the heat transfer rate. Nanofluids
with higher bulk thermal conductivity would increase the heat transfer rate.
However dispersed nano/microparticles will affect the interfacial phenomena
(including wetting, film thickness, liquid/liquid interfacial tension), so there is much work to be done in understanding their overall effect on heat transfer.

• Complex fluids and surfaces. A ferrofluid, for example, could be used for the slug and possibly the interface shape and the internal recirculating flow could be altered autonomously by the use of an external magnetic field. Similarly for the sliding slug condition, external methods such as electro-wetting could be used to control the contact line dynamics and slug interface shape and thus the heat transfer.
Appendix A

Published work

Journal papers


Peer reviewed conference papers


Conference poster presentations

Bibliography


