Sensor Management for Multi-Target Tracking Using Random Finite Sets

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgment has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged.

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Acknowledgments

Until I know this sure uncertainty, I’ll entertain the offered fallacy.

William Shakespeare
The Comedy of Errors

After spending four years as a PhD student, I have learned one thing which is the fact that I could have never done any of this, particularly the research and writing that went into this thesis, without the support and encouragement of a lot of people.

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I would like to thank the coffee machine at the staff room for keeping the caffeine level high and enabling us to keep up our work throughout the days!
Finally, to the God, the author of the universe, and the establisher of the laws
of motion \(^1\) who initiated uncertainties in every bit of this universe. Through these
uncertainties this work exists.

\(^{1}\)Robert Boyle.
Credits

Portions of the material in this thesis have previously appeared in the following publications:


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<td>δ Generalized Labeled Multi-Bernoulli</td>
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<td>CB – MeMBer</td>
<td>Cardinality-Balanced Multitarget Multi-Bernoulli</td>
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<tr>
<td>CPHD</td>
<td>Cardinalized Probability Hypothesis Density</td>
</tr>
<tr>
<td>EAP</td>
<td>Expected a Posteriori</td>
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<tr>
<td>FOV</td>
<td>Field Of View</td>
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<td>FISST</td>
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<td>PEECS</td>
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\textit{PENTI} Posterior Expected Number of Targets of Interest

\textit{PENT} Posterior Expected Number of Targets

\textit{PIMS} Predicted Ideal Measurement Set

\textit{PHD} Probability Hypothesis Density

\textit{PGFl.} Probability-generating functionals

\textit{RFS} Random Finite Set

\textit{R – PEECS} Robust - Posterior Expected Error of Cardinality and States

\textit{SMC} Sequential Monte Carlo
Sensor management in multi-target tracking is commonly focused on actively scheduling and managing sensor resources to maximize the visibility of states of a set of maneuvering targets in a surveillance area. This project focuses on two types of sensor management techniques:

- controlling a set of mobile sensors (sensor control), and
- scheduling the resources of a sensor network (sensor selection).

In both cases, agile sensors are employed to track an unknown number of targets.

We advocate a Random Finite Set (RFS)-based approach for formulation of a sensor control/selection technique for multi-target tracking problem. Sensor control/scheduling offers a multi-target state estimate that is expected to be substantially more accurate than the classical tracking methods without sensor management. Searching for optimal sensor state or command in the relevant space is carried out by a decision-making mechanism based on maximizing the utility of receiving measurements.

In current solutions of sensor management problem, the information of the clutter rate and uncertainty in sensor Field of View (FoV) are assumed to be known in priori. However, accurate measures of these parameters are usually not available in practical situations. This project presents a new sensor management solution that is designed to work within a RFS-based multi-target tracking framework. Our
solution does not require any prior knowledge of the clutter distribution nor the probability of detection profile to achieve similar accuracy.

Also, we present a new sensor management method for multi-object filtering via maximizing the state estimation confidence. Confidence of an estimation is quantified by measuring the dispersion of the multi-object posterior about its statistical mean using Optimal Sub-Pattern Assignment (OSPA). The proposed method is generic and the presented algorithm can be used with any statistical filter.
1.1 Motivation


Relative to multi sensor systems, the information acquired by a single sensor system is very limited. Employing multiple sensors provides sufficient information of the environment in an integrated manner and could increase machine perception and improve awareness of the state of the world.

In recent years, due to the rapid progress in sensor technology development, sensors are becoming more common. In order to sense the complex nature of the environment in different scenarios, numerous sensors are needed. This increases the amounts of acquired data which is needed to be processed. This motivates
the emerging interest in research into intelligent management of sensor resources to improve performance of data fusion.

1.2 Multi-Sensor Management

Multi-sensor management is formally described as a system or process that seeks to manage the usage of a set of sensors or measurement devices in a dynamic, uncertain environment, in order to improve the performance of data fusion and ultimately to improve the perception of the environments [Ong and Ibanez-Guzman 2004].

Another advantage of multi-sensor management is to avoid increasing amount of storage and computational requirements in a sensor network system by controlling the data gathering process such that only the necessary data are collected and stored [Schaefer and Hintz 2000].

The why and what issues of both single-sensor and multi-sensor management were thoroughly discussed in the papers [Adrian 1993, Blackman and Popoli 1999, Lopez et al. 1998, Musick and Malhotra 1994] and [Ng and Ng 2000].

The basic objective of multi-sensor management is to select the right sensors to do the right task on the right object at the right time. The sensor manager is responsible for answering questions like [Xiong and Svensson 2002, Mahmoud and Xia 2014]:

- Which observation tasks are to be performed and what are their priorities?
- How many sensors are required to meet an information request?
- When are extra sensors to be deployed and in which locations?
- Which sensor sets are to be applied to which tasks?
- What is the action or mode sequence for a particular sensor?
- What parameter values should be selected for the operation of sensors?
The fundamental task of sensor management is to choose the optimal sensor parameter values given a set of sensors with respect to a given task, see for example, the paper [Tarabanis et al. 1995]. In general sensor management solutions are decision making procedures in which the algorithm should decide about what sensors to use and for which purposes, as well as when and where to use them.

Widely acknowledged is the fact that it is not realistic to continually observe everything in the environment, and therefore, selective perception becomes necessary, requiring the sensor management system to decide when to sense what and with which sensors. Typical temporal complexities, which must be accommodated in the sensor management process, were discussed in [Malhotra 1995].

1.3 Sensor Selection and Sensor Control in Target Tracking Scenarios

Sensor management solutions are broadly used in multi-target tracking scenarios usually in the form of sensor selection in sensor networks or sensor control for a set of mobile sensors. Tracking targets in a sensor network could result in a big-set of sensor measurement data. However, as it was mentioned earlier, due to physical and computational constraints, it is required that at each epoch, only a sub-set of sensors is selected to communicate with the central processor.

In such cases, sensor-selection problem is to select the right sensor nodes that maximize observability of the network for tracking multiple targets. Sensor-selection problem in general, comprises of two underlying frameworks,

- a multi-object filtering process,

- an optimal decision-making method.
CHAPTER 1: INTRODUCTION

Current nodes in sensor network
Candidate nodes for sensor selection
General nodes in sensor network

Figure 1.1: Sensor selection in a distributed sensor network for traffic monitoring: Due to energy and communication constraints, only the data provided by a limited number of nodes (one node in this example) can be processed to estimate the number and states of vehicles on the roads. At any time, out of a number of candidate nodes, one needs to be selected.

This is a sequential decision making process under stochastic uncertainties. These uncertainties stem either from the multi-target tracking process or from the effects of selecting different sensor nodes. An example of the sensor selection problem is demonstrated in Fig. 1.1 in which a sensor network is used for traffic monitoring.

The sensor-selection problem is fundamentally similar to the sensor-control problem. In sensor-control, a set of sensor commands is used to change mobile sensors’ state in which applying the right command would results in estimates of the number and states of targets with maximum expected accuracy. A practical example of sensor-control is shown in Fig. 1.2, in which a UAV needs to be navigated point by point in such a way that the number and states of multiple vehicle targets can be estimated with maximum expected accuracy.

In any sensor management framework, the selected actions/commands are the outputs of the decision-making component that selects the optimal sensor(s) state(s). This has steered the development of solutions towards devising and improving the
Objective function to address decision-making component, assuming that the multi-object filtering framework can return accurate estimates of the number and states of all targets. However, the multi-target tracking framework also plays a significant role in the overall performance of the scheme in terms of real-time accuracy and robustness.

A number of solutions proposed in the sensor management literature use a classical method for multi-target tracking [Hintz and McVey 1991, Kreucher et al. 2003; 2005, Li and Jilkov 2003a] in conjunction with an objective function to decide the next sensor(s) state(s).

The objective functions are commonly information-driven, i.e defined to quantify the information encapsulated in the posterior density. Examples of such objective functions include Rényi divergence [Kreucher et al. 2003], Kullback-Leibler divergence [Manyika and Durrant-Whyte 1995, Schmaedeke and Kastella 1994, Kastella 1997], and Shannon entropy [Hintz and McVey 1991, Hintz 1991].
CHAPTER 1: INTRODUCTION

Relatively new multi-sensor management solutions have been recently developed for multi-target tracking scenarios within *Finite Set Statistics* (FISST) framework [Mahler 2007b].

Unlike the classical multi-target tracking methods which struggle with two pertinent problems (*data association* and *combinatorial growth*) [Vermaak et al. 2005], FISST approach to multi-target filtering does not require explicit association of targets\-\tracks with measurements which results in a significant reduction in computational complexity. However, similar to classical approaches, the general form of Bayesian recursion involved in FISST-based methods is not computationally tractable [Mahler 2007b]. Consequently, several approximations are suggested, the Probability Hypothesis Density (PHD) filter and its extended version, Cardinalized PHD (CPHD) filter [Mahler 2003b; 2007a, Vo and Ma 2006, Vo et al. 2007; 2005], Multi-Bernoulli filters [Mahler 2007b, Vo et al. 2009] and $\delta$-GLMB filter [Vo and Vo 2013] are the most well-known instances of such approximations.


As it was mentioned before, at the core of these solutions is an objective function. To define an appropriate objective function, two common approaches have been introduced:

- *task-driven* approach, and
SECTION 1.4: SENSOR SELECTION AND SENSOR CONTROL IN TARGET TRACKING SCENARIOS

- *information-driven* approach.

In the task-driven approach, the objective function is formulated as a *cost function* which normally depends on the performance metrics such as error variance, cardinality variance and other distribution-dependent measures.

In information-driven approach, the aim of sensor management is to improve the information content of the multi-object distribution by optimizing some measure of information gain [Aoki et al. 2011b]. Examples of such measures are Rényi divergence [Kreucher et al. 2005, Ristic and Vo 2010, Ristic et al. 2011a], and Kullback-Leibler divergence [Manyika and Durrant-Whyte 1995, Schmaedeke and Kastella 1994, Kastella 1997].

This thesis studies sensor management problems in which quantities such as variance or estimated states are utilized to enhance estimation performance. The work has three folds:

- Utilizing a sensor management solution to be employed in Random Finite Set-based filter, in specific multi-Bernoulli and Labelled Multi-Bernoulli filters,

- Developing a sensor management solution within multi-Bernoulli filter with robust behaviour without having any prior knowledge of the clutter distribution or the probability of detection profile,

- Developing a novel sensor management solution in which a measure of multi-target tracking filter is employed to choose the best sensor command.

The primary problem of interest is multiple object tracking and identification and application areas including sensor network management and multifunction radar control.
1.4 Contributions

The contribution of this thesis is in the development, implementation and demonstration of Random Finite Set based sensor management.

- Devising a novel cost function called Posterior Expected Error of Cardinality and States (PEECS) that accounts for both localization and cardinality errors and its minimization can lead to a high quality updated density even with limited observability,

- Formulation and Monte Carlo (MC) implementation of a new sensor control/selection method that uses PEECS as its cost and works within a multi-Bernoulli multi-target tracking scheme,

- Formulation and MC implementation of a robust sensor control/selection methods based on an extension of PEECS, that does not need any prior knowledge of the clutter distribution or the probability of detection profile; and a robust information theoretic approach and its MC implementation for sensor control/selection in absence of clutter distribution and detection profile, and

- Proposing a method in which sensor actions are selected to maximize the confidence in multi-object state estimation using OSPA metric as a suitable distance for quantifying the dispersion of posterior about its mean.

1.5 Thesis Organization

This thesis is divided into two main parts. First part contains the background and second part a collection of our published papers.

- The background section provides the outline of the background theory which we utilize to develop our idea and results. The primary problem of interest
is that of detecting, tracking and identifying multiple objects, although many of the methods we discuss could be applied to any other dynamical process. Sensor management requires an understanding of several related topics: first of all, one must develop a statistical model for the phenomenon of interest; then one must construct an estimator for conducting inference on that phenomenon. One must select an objective that measures how successful the sensor manager decisions have been, and, finally, one must design a controller to make decisions using the available inputs.


This chapter presents a sensor-control method for choosing the best next state of the sensor(s), that provide(s) accurate estimation results in a multi-target tracking application. The proposed solution is formulated for a multi-Bernoulli filter and works via minimization of a new estimation error-based cost function. Simulation results demonstrate that the proposed method can outperform the state-of-the-art methods in terms of computation time and robustness to clutter while delivering similar accuracy.


This chapter addresses the sensor selection problem for tracking of multiple maneuvering targets within a sensor network. It is assumed that due to the bandwidth and energy constraints of the sensor network, it is not feasible to directly use the entire information provided by the large number of sensor nodes for detection and tracking of the targets, hence the need for sensor selection. We present a decision making solution for sensor selection in the
multi-Bernoulli random finite set framework. The proposed method selects a minimum subset of sensors which are most likely to provide reliable measurements. Our method is a robust method that works in the challenging uncertain scenarios where no prior information are available on clutter intensity or sensor detection profile. Simulation results demonstrate successful sensor selection in a challenging case where five targets move in a close vicinity to each other. Comparative results evidence the superior performance of our method in terms of accuracy of estimating the number of targets and their states.


A new sensor-selection solution within a Multi-Bernoulli-based multi-target tracking framework is presented. The proposed method is especially designed for the general multi-target tracking case with no prior knowledge of the clutter distribution or the probability of detection, and uses a new task-driven objective function for this purpose. Step-by-step sequential Monte Carlo implementation of the method is presented along with a similar sensor-selection solution formulated using an information-driven objective function (Rényi divergence). The two solutions are compared in a challenging scenario and the results show that while both methods perform similarly in terms of accuracy of cardinality and state estimates, the task-driven sensor-selection method is substantially faster.


The recently developed labeled multi-Bernoulli (LMB) filter uses better ap-
proximations in its update step, compared to the unlabeled multi-Bernoulli filters, and more importantly, it provides us with not only the estimates for the number of targets and their states, but also with labels for existing tracks. This paper presents a novel sensor-control method to be used for optimal multi-target tracking within the LMB filter. The proposed method uses a task-driven cost function in which both the state estimation errors and cardinality estimation errors are taken into consideration. Simulation results demonstrate that the proposed method can successfully guide a mobile sensor in a challenging multi-target tracking scenario.


In our previous work reviews we had received significant interest to investigate the relation of the PEECS objective function and OSPA metric. In other words how variance reduction in PEECS will result in better estimation and consequently error reduction in OSPA metric. This concern impelled us to rethink ways of creating pure multi-target-tracking-performance-measure-based objective function for sensor management system. In other words, how should we (and, indeed, should we) design a new class of objective function oriented to the OSPA metric?

This chapter presents a new sensor management method for multi-object filtering via maximizing the state estimation confidence. Confidence of an estimation is quantified by measuring the dispersion of the multi-object posterior about its statistical mean using Optimal Sub-Pattern Assignment (OSPA). The proposed method is generic and the presented algorithm can be used with any statistical filter. Implementation of the algorithm in conjunction
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with a labeled multi-Bernoulli filter is presented. Simulation studies demonstrate that the OSPA-based sensor control can successfully guide a sensor to achieve excellent results in tracking up to 16 targets, and outperforms the recent PEECS-based sensor control.

- The thesis closes with conclusion and outline the contributions made in this thesis.
Background

2.1 Introduction

Automated sensor management is the process of “directing the right sensors to the right targets at the right times” [Mahler 2014]. A sensor management solution must consider a number of factors that affect its performance:

- Constraints on target motion due to target dynamical limitations,
- The likelihood of the appearance and disappearance of targets,
- Network bandwidth,
- Energy consumption.

The multi-sensor management problem generally involves stochastic variations in the number of targets, measurement/process noise and detections, and can be formulated in the form of an optimal stochastic control problem. These stochastic variations are due to

- Randomly varying number of targets,
- Randomly varying number of measurements, and
CHAPTER 2: BACKGROUND

• Randomly varying number of sensors.

Consequently, the practical implementation of sensor management requires principled approach. In this thesis, we follow a control-theoretic approach for the proposed sensor management solution. In the control-theoretic approach the goal is to determine an optimal selection of a set of sensors by defining an objective function which should be maximized or minimized. The control-theoretic approach to sensor management problem is in stark contrast to rule based solutions. The main difference between rule based and control-theoretic approaches are [Mahler 2014]:

**Rule Based Approach**

- heuristic,

- deterministic,

- fast,

- stable.

**Control-Theoretic Approach**

- mathematically rigorous,

- stochastic,

- computationally intensive,

- potentially unstable.

However, in overall, the performance of the control-theoretic approach is more reliable due to [Mahler 2014]:

• accumulated decisions in a rule based approach does not necessarily leads to desired performance,

• control-theoretic approach inherently achieve near-optimal decisions,
• rule based approach is incapable of addressing the infinite number of possible contingencies that arise in practice,

• control-theoretic approach is capable of addressing this infinitude of contingencies.

From a theoretical point of view, the sensor management can be modeled as a Partially Observable Markov Decision Process (POMDP).

### 2.2 Sensor Management Solution Framework

This thesis focuses on solving the sensor management problem in the context of choosing the optimal or near-optimal sensor command among a finite number of admissible commands. In this context, the quality of a sensor command needs to be evaluated in terms of an objective function. The defined objective function can be in the form of reward function (to be maximized) or a cost function (to be minimized).

We formulate the sensor management solution in the Partially Observable Markov Decision Process (POMDP) framework. POMDP provides a rich framework for sequential decision-making under uncertainty in stochastic domains. POMDP is a generalized form of Markov Decision Process (MDP)[Kaelbling et al. 1998] in which there is no direct access to the states, and information about the states are only received from observations. The POMDP model comprises the following elements at any time $k$:

- a finite set $X_k$ of single-object states,
- a finite set of sensor commands $S = \{s_1, \cdots, s_n\}$
- a finite set of observations $Z$,
- a stochastic measurement model $g_k(z|x)$, and
Figure 2.1: The Partially Observable Markov Decision Processes (POMDPs) framework for sensor management and multi-object estimation problems.

- a function $\vartheta(s, X_k)$ that returns a reward or cost for transition from the multi-object state $X_k$ to the state $X_{k+1}$ via applying an action command $s \in \mathbb{S}$.

The overall procedure of sensor management solution in POMDP frameworks is shown in Fig. 2.1. Assume at time $k$ there are $m$ sensors with the following possible future states $\mathbb{S} = \{s_1, \ldots, s_n\}$. Assume that an unknown number of targets are maneuvering in the surveillance area. The targets’ motion model are based on Markov transition density of the form of $f_{k+1|k}(X_{k+1}|X_k)$. The general approach to sensor management is a closed-loop version of the multi-target Bayes filter. The procedure of sensor management are as follows:
• Prediction
Apply the multi-target filter prediction step to evolve the prior multi-target
distribution from $\pi_k$ to $\pi_{k+1|k}$ at time $k$.

• Pre-Estimation
Having the predicted multi-target distribution, estimate the number and states
of the targets. This information will be used to generate pseudo-measurements.

• Pseudo-Measurements
Generate a pseudo-measurement set for each sensor command. In sensor man-
agement solutions, the pseudo-measurement set is a function of the chosen sen-
sor command. In principle, the whole distribution of all possible measurement
sets is used to compute the update distribution. However, using whole mea-
surement distribution is computationally expensive if not intractable. Thus,
to reduce the computational complexity, for each sensor control command,
only the predicted ideal measurement set (PIMS) is used [Mahler 2004]. More
detail about PIMS approach is presented in section 2.3.2.

• Pseudo-Update
Using the generated pseudo-measurements as the actual measurements, apply
the multi-target filter update step to update the predicted multi-target distri-
butions to $\pi_{k+1}(X|Z,s)$ (where $Z$ is the pseudo-measurement set at the future
time $k+1$).

• Objective Function
From the Pseudo-updated distributions $\pi_{k+1}(X|Z,s)$ construct an objective
function $\vartheta(s,X_k)$ to quantify the information encapsulated in the posterior
density (for each possible command). To define the objective function, two
common approaches have been introduced in the sensor control literature:
task-driven approach and information-driven approach. More detail about objective function is presented in section 2.4

• Decision Making
The employed objective function is then directly computed from the updated distribution. The most rewarding sensor command is then selected by minimizing/maximizing the objective function over all measurements. The goal of sensor control is to find the control command \( \hat{s} \), which minimizes (or maximizes) the statistical expectation of the cost (or reward) function:

\[
\hat{s} = \arg\min_{s \in S} / \arg\max_{s \in S} \left\{ \mathbb{E}_Z [\vartheta(s, X_k)] \right\}.
\]

(2.1)

• Update
Collect the actual measurement sets for the chosen sensors and use the multi-target filter update step equation to update \( \pi_k \) to \( \pi_{k+1} \).

2.3 Assumptions In This Thesis

In this thesis, sensor management problem for multi target scenario has been studied under various assumptions.

2.3.1 Single-Step Look-Ahead

In this thesis we consider single-step look-ahead (myopic) approach to determine the optimal selection of sensors’ command. Myopic approach is in contrast to multi-step look-ahead approach where in multi-step look ahead approach the aim is to optimally select sensors’ command throughout a certain future time-step.

The multi-step look-ahead approach is possible with accurate target state estimation in the entire future time-steps. However, in practical situation this is not the case and due to complex nature of targets manoeuvring, the multi-target estimation
is not accurate enough. In such cases, myopic approach is more desirable. Also, multi-step approach is computationally expensive compared to myopic approach, especially in multi-target contexts [Mahler 2014].

2.3.2 Pseudo-Measurement Approximation

As it was mentioned, the objective function is a function of future measurements. The values of the future measurements are unknown thus, a set of pseudo-measurements for each hypothesized sensor command is generated. In principle, the whole distribution of all possible measurement sets is used to compute the objective function. However, using the whole measurement distribution is computationally expensive. In situations where the probability of detection is high and clutter rate is moderate, the multi target posterior density can be approximated under the assumptions of the PIMS (see [Mahler 2004]–section 4.1 and [Ristic 2013]). Thus, to reduce the computational complexity, for each sensor command, only the predicted ideal measurement set (PIMS) is generated.

Given the selected sensor command, the PIMS is comprised of the clutter-free and noise-free measurements that are most likely to be obtained from the selected sensor. Algorithm 1 shows a pseudocode to generate the PIMS returned by a selected sensor command.

2.4 Objective Function

To define the objective function, two common approaches have been introduced in the sensor control literature:

- Task-driven approach and
- Information-driven approach.
Algorithm 1 Generating Predicted Ideal Measurement Set [Mahler 2004].

**Inputs:** Estimates of the number of targets $\hat{M}_{k+1|k}$ and multi-target state $\hat{X}_{k+1|k} = \{x_{k+1|k}(\ell)\}_{\ell=1}^{\hat{M}_{k+1|k}}$ after a prediction step, a sensor node $s \in S$, and a single-target measurement likelihood function $g(z|x; s)$ which is dependent on the location of the sensor node $s$.

**Output:** A clutter-free set of noise-free measurements, $\hat{Z}_s$.

1: function $\text{PIMS}(\hat{M}_{k+1|k}, \hat{X}_{k+1|k}, s, g(z|x; s))$
2: $\hat{Z}_s \leftarrow \emptyset$
3: for $\ell = 1, \ldots, \hat{M}_{k+1|k}$ do
4: $\xi \leftarrow \arg\max_z g(z|x_{k+1|k}(\ell); s)$
5: $\hat{Z}_s \leftarrow \hat{Z}_s \cup \{\xi\}$
6: end for
7: return $\hat{Z}_s$
8: end function

**Task-driven approach**

In the *task-driven* approach, the objective function is formulated as a *cost function* which normally depends on the performance metrics such as error variance, cardinality variance and other distribution-dependent measures (e.g. [Kreucher et al. 2005]).

Task-driven objective functions attempt to devise mathematical formulas that, at least approximately, capture the intent underlying subjective tactical objectives. These objective functions are usually modelled using dynamic programming methods [Pierre 1986, Hernandez-Lerma 1989]. Task-driven objective functions tend to be heuristic and often do not end up adequately modelling actual mission objectives [Mahler 2014].

**Information-driven approach**

In *information-driven* approach, the aim of sensor management is to improve the information content of the multi-object distribution by optimizing some measure of information gain (uncertainty) [Aoki et al. 2011b]. Recently, due to the performance criteria independency of the information-driven methods [Kreucher et al. 2005], this

Objective of information-driven approach is to maximize the information gain produced by posterior multi-object densities [Aoki et al. 2011b]. The idea of measuring information gain as an objective function in sensor control related problem, first appeared in [Hintz and McVey 1991] and [Hintz 1991]. Later in [Manyika and Durrant-Whyte 1992] the expected value of information gain was utilized to tackle sensor management problem. Some examples of information measures are:

- Rényi divergence [Kreucher et al. 2003; 2005, Ristic and Vo 2010, Ristic et al. 2011a],
- Shannon entropy [Hintz and McVey 1991, Hintz 1991], and
- The Cauchy-Schwarz divergence [Mahler 2014, Hoang et al. 2013].

Information-driven objective functions are theoretically rigorous. However, they have no obvious relationship with operational requirements and do not even have a clear intuitive, physical interpretation. Another problem is the fact that there is a large indefinite quantity of information-driven objective functions. For example, each choice of $\alpha$ results in a different Rényi divergence objective function. Thus the question is which objective function should be choosed, when, and why [Mahler 2014]?
2.5 Objective Functions Studied In This Thesis

The aim of this thesis is to define an objective function in FISST framework in which a common ground between information-driven approach and physical interpretation of task-driven approach is established. The minimal objectives of any sensor management tasks are:

- Maximize the number of observed targets, and
- Maximize the accuracy of the estimated states of the target.

The defined objective function should carry the fundamental notions of the information theory as well as addressing the problem of physical interpretation. In chapters 3, 4, 5, 6 and 7, we defined objective functions using notion of information theory while they have physical interpretation of task-driven approach.

We then compared our method with the following methods in sensor management framework.

2.5.1 Rényi Divergence

The most common choice for an objective function in information-driven method is Rényi divergence function. Ristic et al [Ristic and Vo 2010, Ristic et al. 2011a] used Rényi divergence as the objective function in conjunction with random set filter and CPHD filter for the scenarios where clutter rate and uncertainty in sensor Field of View (FoV) are known.

In [Kreucher et al. 2003] and [Ristic et al. 2011a] Rényi divergence was employed as the generalization of the Kullback-Leibler divergence. Rényi divergence measures information gain between two densities:

\[ I_\alpha(f_1, f_2) = \frac{1}{\alpha - 1} \log \int [f_1(X)]^\alpha [f_2(X)]^{1-\alpha} \delta X. \]  

(2.2)
In target tracking applications, the first distribution is the updated distribution, \( \pi_{k+1}(X_{k+1}|Z_{1:k}, u_{0:k-1}, Z_{k+1}, s_k) \), and the second distribution is the prediction distribution, \( \pi_{k+1|k}(X_{k+1}|Z_{1:k}, u_{0:k-1}) \). Thus, Rényi divergence is given by:

\[
I_\alpha(Z_{k+1}, s_k) = \frac{1}{\alpha - 1} \log \int [\pi_{k+1}(X|Z_{1:k}, s_{0:k-1}, Z_{k+1}, s_k)]^\alpha \\
\cdot \left[ \pi_{k+1|k}(X|Z_{1:k}, s_{0:k-1}) \right]^{1-\alpha} \delta X. \tag{2.3}
\]

The \( \alpha \) parameter emphasizes on different parts of distributions.

The objective is to maximize information gain by computing divergence between the updated and predicted distributions via choosing the right \( s_k \) while any choice of \( s_k \) can lead to different measurements \( Z_{k+1} \). A common approach is to define, compute and maximize the reward function as the statistical mean of the Rényi divergence function,

\[
\mathcal{D}(s_k) = \mathbb{E}_{Z_{k+1}} [I_\alpha(Z_{k+1}, s_k)]. \tag{2.4}
\]

### 2.5.2 The Posterior Expected Number of Targets

In 2005, Mahler introduced a systematic and unified approach to sensor management using PHD filter\[Mahler 2004; 2005\]. In this approach, he employed an objective function called the posterior expected number of targets (PENT). In this approach, the aim is to maximize the value of the PENT, to direct the sensor FoV to those states where the sensors are most likely to collect the best PIMS. PENT can be generalized to include notions of target priority, in which case it is known as the predicted expected number of targets of interest (PENTI) \[Mahler 2007c; 2005\]. The direct effect of maximizing PENT is maximizing the expected number of targets in that region \( N_{k|k} \). The following points indicate the characteristic of the PENT objective function\[Mahler 2007c; 2014\].
1. PENT can be defined in a mathematically rigorous manner from the multi-target statistics of the multi-target system.

2. PENT is a computational approximation of abstract information-theoretic objective functions.

3. PENT is more computationally tractable than information-driven objective functions.

4. PENT can be modified to include the notion of the target of interest.

5. PENT can address the following properties:
   - Abstract information-theoretic.
   - Physical interpretation.

2.5.3 The Cardinality-Variance Based Objective Function

In previous section, it was mentioned that the PHD filters are used to devise PENT objective function. In other words, only the first-order information is used in PENT objective function. Also, the primary goals of the PENT objective functions are [Mahler 2014]:

- Computational tractability, and
- Physical intuitiveness.

Using the multi-Bernoulli filters, a cardinality distribution is also available, thus a similar but potentially more effective approach can be devised. This approach was proposed and applied in this thesis. Also, around the same time, Hoang et al [Gia Hoang and Tuong Vo 2013] defined a variance-based cost function. The main difference between our approach and Hoang et al is that in [Gostar et al. 2013a], statistical variance of cardinality around its mean is chosen as the cost function,
while Hoang et al [Gia Hoang and Tuong Vo 2013] used the variance of cardinality around its MAP estimate as the cost function.
Multi-Bernoulli Sensor-Control via Minimization of Expected Estimation Errors

This paper presents a sensor-control method for choosing the best next state of the sensor(s), that provide(s) accurate estimation results in a multi-target tracking application. The proposed solution is formulated for a multi-Bernoulli filter and works via minimization of a new estimation error-based cost function. Simulation results demonstrate that the proposed method can outperform the state-of-the-art methods in terms of computation time and robustness to clutter while delivering similar accuracy.
3.1 Introduction

Sensor-control techniques in multi-target tracking problems are designed to find the optimal control command from a set of admissible commands that results in the most rewarding observations. Sensor-control techniques usually involve addressing two main components:

- multi-object filtering, and
- sequential decision making.

To solve the above problems, we formulate the sensor-control problem in the partially observed Markov decision process (POMDP) framework by proposing to use a multi-Bernoulli filter for the multi-object filtering task. This filter enables us to devise a new cost function which has a more direct relation with the OSPA [Schuhmacher et al. 2008] multi-target miss-distance compared to information-theoretic-based objective functions commonly used in sensor-control solutions. Consequently, it results in better performance (in terms of the OSPA error).

The purpose of multi-object filtering is to jointly estimate the randomly varying number and states of multiple objects. Several solutions for this problem have recently been developed in the finite set statistics (FISST) framework, in which the multi-object entity is treated as a random finite set (RFS) with its distribution being predicted and updated in every time step [Mahler 2007b]. Examples of such solutions include the PHD filter [Mahler 2003b; 2007b], CPHD filter [Vo et al. 2007, Mahler 2007a], MeMBeR filter [Mahler 2007b], cardinality balanced MeMBeR (CB-MeMBeR) filter [Vo et al. 2009] and Generalized labeled multi-Bernoulli filter [Vo and Vo 2013].

A number of sensor-control solutions have also been developed to work within the FISST multi-target filtering schemes[Mahler 1998; 2003a, Mahler and Zajic 2004, Zatezalo et al. 2008, Witkoskie et al. 2006, Ristic and Vo 2010, Ristic et al. 2011a,
Gostar et al. 2013a;c;b, Hoang 2012a]. In these solutions, a criterion is defined to evaluate the expected quality of the updated multi-target density after an admissible command is applied to the sensor. In this approach, the chosen control command is the one that provides the best updated density in terms of the defined criterion. Examples of this type of criteria include the Csiszár information-theoretic functional [Mahler 2004], posterior expected number of targets (PENT) [Mahler and Zajic 2004, Zatezalo et al. 2008] and the Cauchy-Schwarz divergence between the probability densities of two Poisson point processes [Hoang et al. 2013].

The most common criterion to evaluate the quality of the updated distribution is its divergence from the predicted distribution [Ristic and Vo 2010]. In two consecutive papers, Ristic et al [Ristic and Vo 2010, Ristic et al. 2011a] used Rényi divergence as a reward function to quantify the information gained via updating the predicted density in a random set filtering framework. In [Ristic and Vo 2010], the implementation of Rényi reward maximization was investigated for the “general form” of multi-target filters. Since this approach is computationally intractable even for a small number of targets, in the second paper [Ristic et al. 2011a], the Rényi divergence function was approximated based on i.i.d. cluster assumption for the multi-target distribution within a PHD filtering scheme. Later, in [Hoang 2012a] and [Gia Hoang and Tuong Vo 2013] the Rényi divergence objective function was implemented via the multi-Bernoulli filter.

In our recent study [Gostar et al. 2013a], reported independently from [Hoang 2012a] and [Gia Hoang and Tuong Vo 2013], we introduced the use of multi-Bernoulli filtering in a task-driven objective function. This criterion is introduced in the form of a cost that would be minimized over all admissible control commands. It is important to note that in multi-Bernoulli filter, estimation of the number and states of targets from the updated multi-Bernoulli distribution is straightforward and no clustering is needed. Furthermore, multi-Bernoulli filters have shown to be more accurate and involve less computation than other existing multi-target
tracking methods in various multi-target applications using radar, video and audiovisual measurements [Hoseinnezhad et al. 2010; 2011; 2013; 2012, Vo et al. 2009; 2010]. We also note that both in our earlier sensor-control method, and in the method introduced in this paper, the cardinality-balanced multi-Bernoulli filter (CB-MeMBer) [Vo et al. 2009] is used as an appropriate multi-object filtering scheme for the sensor-control method. CB-MeMBer is proven to result in unbiased cardinality estimates under high SNR [Vo et al. 2009].

In our previous work [Gostar et al. 2013a], the cost function was defined in terms of the estimation error of only the cardinality of the multi-target state with the assumption that in practice, an accurate estimate of the number of target would be followed by accurate estimates of the actual target states. Around the same time, a similar approach based on using a variance-based cost function was also pursued by Hoang et al [Gia Hoang and Tuong Vo 2013]. Unlike the cost function presented in [Gostar et al. 2013a], in which the statistical variance of cardinality around its mean is chosen as the cost function, Hoang et al [Gia Hoang and Tuong Vo 2013] used the variance of cardinality around its MAP estimate as the cost function.

In this paper, we present a new cost function called Posterior Expected Error of Cardinality and States (PEECS) that its minimization can lead to a high quality updated density even with limited observability. The proposed cost function is intended to account for both localization and cardinality errors. Inclusion of the extra terms for localization errors in PEECS (compared to cardinality variance-only costs used in [Gostar et al. 2013a, Hoang 2012a]) can enhance the performance of sensor-control, especially in challenging cases with numerous targets and high rates of clutter.

The rest of the paper is organized as follows. In Sec. 3.2, a formal statement of the sensor-control problem is presented. Since, our sensor-control solution is specifically formulated to work within multi-Bernoulli filtering schemes, the cardinality-balanced multi-Bernoulli filter is briefly reviewed in Sec. 3.3. Then, the details of
sensor-control method are presented in Sec. 3.4 followed by comparative numerical studies in Sec. 3.6. Sec. 3.7 concludes the paper.

3.2 Problem Statement: sensor-control

This paper focuses on developing an effective sensor-control solution for applications where mobile sensors are used for multi-object tracking while only instantaneous performance is considered (myopic approach).

The main aim of sensor-control is to choose the best control command(s) from a number of admissible commands. The application of the chosen command changes the sensor(s) state e.g. the location, heading or both to reach more accurate estimation of the number and states of targets via a set of noisy measurements which may include by clutter. The quality of a sensor-control command is usually evaluated in terms of a reward function (to be maximized) or a cost function (to be minimized).

Following [Castañón and Carin 2008, Ristic and Vo 2010, Ristic et al. 2011a], we formulated the sensor-control problem in the Partially Observable Markov Decision Process (POMDP) framework. POMDP is a generalized form of Markov Decision Process (MDP)[Kaelbling et al. 1998] in which there is no direct access to the states and the states information are only realized by noisy observations. The POMDP framework employed in this paper to formulate the sensor-control problem includes the following elements at any time $k$:

- a finite set $X_k$ comprising single-object states;
- a set of sensor-control commands (actions) $U$;
- a stochastic model for single-target state transition $f_{k|k-1}(x_k|x_{k-1})$;
- a finite set of observations $Z$, generally made by $N$ sensors$^1$;  

$^1$Without loss of generality, in our formulations and simulation studies, we consider multiple observations from one sensor and note that extension of the proposed solution to the general multi-sensor case is straightforward.
SECTION 3.2: PROBLEM STATEMENT: SENSOR-CONTROL

- a stochastic measurement model \( g_k(z|x) \); and

- a function \( \mathcal{V}(u; X_k) \) that returns a reward or cost associated with action command \( u \in U \).

In principle in a POMDP framework, the action space is infinite and continuous. To reduce computational complexity, we assume that the sensor(s) can choose from a finite set of actions or commands, namely \textit{admissible control commands} [Braziunas 2003].

The purpose of sensor-control is to find the control command \( \hat{u} \) which optimizes the cost (or reward) function. In stochastic filtering, where the multi-target states \( X_{k-1} \) and \( X_k \) are characterized by their distributions, the control command \( \hat{u} \) is commonly chosen to optimize the statistical mean of the objective function \( \mathcal{V}(u; X_k) \) over all observations. The function \( \mathcal{V}(u; X_k) \) is a real-valued objective function of the control command and estimated state of the targets. Depending on the nature of this function (being either a ”cost” or a ”reward”), the sensor-control solution would \textit{minimize} or \textit{maximize} it,

\[
\hat{u}_k = \operatorname*{argmin}_{u \in U} / \operatorname*{argmax}_{u \in U} \left\{ \mathbb{E}_{\mathcal{Z}_k(u)} [\mathcal{V}(u; X_k)] \right\}. \tag{3.1}
\]

The objective function usually depends on the statistical distribution of \( X_{k-1} \) and \( X_k \), which can be recursively computed in a Bayesian filtering scheme. The latest development in multi-target filtering is Mahler’s \textit{finite set statistics} (FISST) framework which provides a set of mathematical tools that allows direct application of Bayesian inferencing to multi-target problems. In a seminal paper [Vo et al. 2005], Vo et al established the relationship between FISST and conventional probability. The theory provides a rigorous paradigm for calculations involving RFSs based upon the notion of integration and density in \textit{point process} and \textit{stochastic geometry} [Moller and Waagepetersen 2003, Daley and Vere-Jones 2007, Stoyan and Kendall 2008]. This approach to multi-target filtering has influenced a variety of research areas.
such as autonomous vehicles and robotics [John Mullane 2011], radar tracking [Lee et al. 2010] and image processing [Maggio et al. 2008].

Since the general form of the multi-target Bayes filter is intractable [Mahler 2007b], the FISST framework also includes a number of computationally feasible approximations. Examples of such filters include the probability hypothesis density (PHD) [Mahler 2003b], the cardinalized PHD (CPHD) [Mahler 2007a], the multi-Bernoulli (MeMBer) [Mahler 2007b] and the cardinality balanced MeMBer (CB-MeMBer) [Vo et al. 2009] filters. PHD and CPHD filters are implemented via Sequential Monte Carlo (SMC) method [Vo et al. 2005, Whiteley et al. 2007, Zajic and Mahler 2003] or using Gaussian mixtures [Vo et al. 2007, Vo and Ma 2006]. The convergence of PHD and CPHD filters has been studied in [Vo et al. 2005, Clark and Bell 2006, Clark and Vo 2007, Caron et al. 2011]. SMC and Gaussian mixture implementations of CB-MeMBer filter have also appeared in [Vo et al. 2009].

As mentioned earlier, due to the better accuracy and lower computational requirements of multi-Bernoulli filtering schemes, in applications where SMC is necessary, this paper focuses on an effective sensor-control solution for CB-MeMBer filter. The common approach to sensor-control within multi-target filtering schemes is to choose the reward/cost function in terms of the predicted multi-target state and the expected update outcomes for every admissible control command. Before we present our choice of cost function, the multi-Bernoulli filtering scheme is briefly reviewed in the next section.

### 3.3 CB-MeMBer Filter

A multi-Bernoulli RFS is defined as the union of $M$ independent Bernoulli RFSs $X^{(i)}$, $i = 1, \ldots, M$. Each Bernoulli RFS is characterized by the parameter $r^{(i)}$ and function $p^{(i)}(\cdot)$, where $r^{(i)}$ is the probability of existence of an element in $X^{(i)}$ (being a singleton) and $p^{(i)}(\cdot)$ is the probability density function (pdf) of the state of the
single element of the set if it exists. It has been shown that all statistical properties of a multi-Bernoulli RFS can be completely characterized by the set of $M$ parameter pairs, existence probabilities $r^{(i)}$ and the distributions $p^{(i)}(\cdot)$, and a general multi-Bernoulli RFS is commonly denoted by $X \sim \{(r^{(i)}, p^{(i)})\}_{i=1}^{M}$ [Mahler 2007b].

The multi-Bernoulli Bayesian filter was first derived by Mahler [Mahler 2007b]. He showed that if the prior distribution of the multi-target random set state is multi-Bernoulli, the predicted and updated posteriors are also approximately multi-Bernoulli. Later, the original filter was shown to produce biased cardinality estimates [Vo et al. 2009] and a modified version, called Cardinality-Balanced MeMber (CB-MeMber) filter, was formulated in a numerically tractable form and implemented by the Sequential Monte Carlo (SMC) [Vo et al. 2009].

Suppose that at time $k-1$, the posterior multi-Bernoulli density of the multi-target state is given by $\{(r^{(i)}_{k-1}, p^{(i)}_{k-1})\}_{i=1}^{M_{k-1}}$ and each $p^{(i)}_{k-1}$ is approximated by a set of particles:

$$p^{(i)}_{k-1}(x) = \sum_{j=1}^{L^{(i)}_{k-1}} w_{(k-1)}^{(i,j)} \delta_{x_{k-1}^{(i,j)}}(x).$$  \hspace{1cm} (3.2)

For existing and new born targets, two proposal densities are given and denoted by $q^{(i)}_{k}(\cdot|x_{k-1}, Z_k)$ and $b^{(i)}_{k}(\cdot|Z_k)$. The predicted multi-target density (also a multi-Bernoulli density) is represented by the union of the survived targets and the newly born targets:

$$\pi_{k|k-1} = \left\{ \left( r^{(i)}_{P,k|k-1}, p^{(i)}_{P,k|k-1} \right) \right\}_{i=1}^{M_{k-1}} \cup \left\{ \left( r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,k} \right) \right\}_{i=1}^{M_{\Gamma,k}}$$ \hspace{1cm} (3.3)

where existence probabilities and distributions of the predicted Bernoulli components are given by:
CHAPTER 3: MULTI-BERNOULLI SENSOR-CONTROL VIA MINIMIZATION OF
EXPECTED ESTIMATION ERRORS

\[ r^{(i)}_{P,k|k-1} = r^{(i)}_{k-1} \sum_{j=1}^{L_{k-1}^{(i)}} w^{(i,j)}_{k-1} p_{S,k}(x^{(i,j)}_{k-1}) \]

\[ p^{(i)}_{P,k|k-1}(x) = \sum_{j=1}^{L_{k-1}^{(i)}} w^{(i,j)}_{P,k|k-1} \delta_{x^{(i,j)}_{P,k|k-1}}(x) \]  \hspace{1cm} (3.4)

\[ r^{(i)}_{\Gamma,k} = \text{birth model parameters} \]

\[ p^{(i)}_{\Gamma,k}(x) = \sum_{j=1}^{L_{\Gamma,k}^{(i)}} w^{(i,j)}_{\Gamma,k} \delta_{x^{(i,j)}_{\Gamma,k}}(x) \]

where

\[ x^{(i,j)}_{P,k|k-1} \sim q^{(i)}_{k} \left( x^{(i,j)}_{k-1}, Z_{k} \right), j = 1, \cdots, L_{k-1}^{(i)} \]

\[ w^{(i,j)}_{P,k|k-1} = \frac{w^{(i,j)}_{k-1} f_{k|k-1}^{(i)}(x^{(i,j)}_{P,k|k-1}, Z_{k})}{q^{(i)}_{k}(x^{(i,j)}_{P,k|k-1}, Z_{k})} \]

\[ w^{(i,j)}_{P,k|k-1} = w^{(i,j)}_{P,k|k-1} / \sum_{\ell=1}^{L_{k-1}^{(i)}} w^{(i,\ell)}_{P,k|k-1} \]  \hspace{1cm} (3.5)

\[ x^{(i,j)}_{\Gamma,k} \sim b^{(i)}_{k} \left( \cdot | Z_{k} \right), j = 1, \cdots, L_{\Gamma,k}^{(i)} \]

\[ w^{(i,j)}_{\Gamma,k} = p_{\Gamma,k}(x^{(i,j)}_{\Gamma,k}) / b^{(i)}_{k}(x^{(i,j)}_{\Gamma,k} | Z_{k}) \]

\[ w^{(i,j)}_{\Gamma,k} = w^{(i,j)}_{\Gamma,k} / \sum_{\ell=1}^{L_{\Gamma,k}^{(i)}} w^{(i,\ell)}_{\Gamma,k} \cdot \]

Let us denote the predicted multi-Bernoulli distribution by \( \{(r^{(i)}_{k|k-1}, p^{(i)}_{k|k-1})\}_{i=1}^{M_{k|k-1}} \).

The updated multi-Bernoulli is represented by the union of \textit{legacy tracks} and \textit{measurement-corrected tracks} [Mahler 2007b, Vo et al. 2009]:

\[ \pi_{k} = \left\{(r^{(i)}_{L,k}, p^{(i)}_{L,k})\right\}_{i=1}^{M_{k|k-1}} \cup \{(r^{(i)}_{U,k}(z), p^{(i)}_{U,k}(\cdot|z))\}_{z \in Z_{k}} \]  \hspace{1cm} (3.6)

with the following existence probabilities and singleton densities:
\[ r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{(1 - p_{L,k}^{(i)})}{(1 - r_{k|k-1}^{(i)} \delta_{L,k}^{(i)})} \]

\[ p_{L,k}^{(i)}(x) = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{L,k}^{(i,j)} \delta_{x_{k|k-1}^{(i,j)}}(x) \]

\[ \hat{r}_{U,k}(z) = \frac{\sum_{i=1}^{M_{k|k-1}^{(i)}} r_{k|k-1}^{(i)} (1 - r_{k|k-1}^{(i)} \delta_{U,k}^{(i)}(z))}{\sum_{i=1}^{M_{k|k-1}^{(i)}} \sum_{j=1}^{L_{k|k-1}^{(i)}} r_{k|k-1}^{(i)} \delta_{U,k}^{(i)}(z)} \]

\[ \hat{p}_{U,k}(x; z) = \sum_{i=1}^{M_{k|k-1}^{(i)}} \sum_{j=1}^{L_{k|k-1}^{(i)}} \hat{z}_{U,k}^{(i,j)}(z) \delta_{x_{k|k-1}^{(i,j)}}(x) \]

where

\[ \delta_{L,k}^{(i)} = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} P_{D,k}(x_{k|k-1}^{(i,j)}) \]

\[ w_{L,k}^{(i,j)} = \frac{w_{L,k}^{(i,j)}}{\sum_{j=1}^{L_{k|k-1}^{(i)}} w_{L,k}^{(i,j)}} \]

\[ w_{L,k}^{(i,j)} = w_{k|k-1}^{(i,j)} (1 - P_{D,k}(x_{k|k-1}^{(i,j)})) \]

\[ \delta_{U,k}^{(i)}(z) = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} P_{D,k}(x_{k|k-1}^{(i,j)}) q_k(z|x_{k|k-1}^{(i,j)}) \]

\[ \hat{z}_{U,k}^{(i,j)}(z) = \frac{\hat{w}_{U,k}^{(i,j)}(z)}{\sum_{i=1}^{M_{k|k-1}^{(i)}} \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{U,k}^{(i,j)}(z)} \]

\[ \hat{w}_{U,k}^{(i,j)}(z) = \frac{w_{k|k-1}^{(i,j)} r_{k|k-1}^{(i)} P_{D,k}(x_{k|k-1}^{(i,j)}) q_k(z|x_{k|k-1}^{(i,j)})}{1 - r_{k|k-1}^{(i)}} \]

To avoid degeneracy, the CB-MeMBer filter also includes a resampling step for each track. To prevent the numerical explosion of the generated particles, the existence probabilities are thresholded and the low probability tracks are removed while similar ones are merged – details of these steps are provided in [Vo et al. 2009].

### 3.4 Multi-Bernoulli Sensor-Control

In a multi-Bernoulli target tracking scheme, assume that at time \( k - 1 \), the multitarget distribution parameters are \( \{r_{k-1}^{(i)}, p_{k-1}^{(i)}\}_{i=1}^{M_{k-1}} \) where \( M_{k-1} \) indicates maximum
number of targets. This distribution is propagated through the multi-Bernoulli prediction and update steps, and turns into an updated multi-Bernoulli distribution with parameters \( \{ r_k^{(i)}(i) \}_{i=1}^M P_k^{(i)} \). As part of the multi-target tracking solution, at each time \( k \), the number and states of targets are extracted from the updated multi-Bernoulli parameters. We note that the sensor measurements are used within the update step of the recursion, and affect the quality of the updated distribution.

In practice, the quality of sensor measurements usually depends on a sensor state (e.g. the sensor location) which is assumed to be controllable, and the sensor-control problem is focused on choosing the command that would lead to the best sensor state. As it was mentioned earlier, most solutions are based on maximizing an information theoretic reward function such as Rényi divergence [Kreucher et al. 2003, Ristic and Vo 2010, Ristic et al. 2011a]. The main rationale behind choosing such reward functions is that the information encapsulated by the estimated multi-target distribution is expected to gradually increase as further measurements become available in time.

In this paper, we take a different approach in which the updated multi-Bernoulli distribution parameters are directly utilized to define a new cost function. Our approach is to define a cost function that quantifies the average uncertainty in all possible multi-target state estimates after each update step.\(^2\) The main difference here is that our focus is on measuring the quality of the updated density in terms of level of uncertainties, not the information gained from prediction to update (e.g. Rényi divergence function).

The updated distribution depends on the measurement set which is a function of the chosen sensor-control command. In principle, the whole distribution of all possible measurement sets is used to compute the update distribution. However, using whole measurement distribution is computationally expensive if not intractable.

\(^2\)It is important to note that this cost is not totally independent of the prediction outcomes, since state estimates extracted from predicted multi-Bernoulli density are used to calculate the proposed cost function.
Thus, to reduce the computational complexity, for each sensor control command, only the predicted ideal measurement set (PIMS) is used [Mahler 2004]. In PIMS approach only one future measurement sample set is generated for each control command under the ideal conditions of no measurement noise and no clutter where probability of detection is equal to one.\(^{3}\) In order to define the new cost function, we note that the PIMS depends on the chosen control command. For each command, we first compute the PIMS, then calculate an updated multi-object distribution \(\{r^{(i)}_k, P^{(i)}_k\}_{i=1}^{M_k}\) by considering the PIMS as the acquired measurement.

A linear combination of the normalized errors of the number of targets and their estimated states are considered as a measure of uncertainty associated with estimation of the multi-target state and as the cost function:

\[
V(u; X_k) = \eta \varepsilon^2_{|X|}(u) + (1 - \eta) \varepsilon^2_X(u),
\]

where \(\varepsilon^2_{|X|}(u)\) denotes the normalized error of estimated cardinality of the multi-target state and \(\varepsilon^2_X(u)\) denotes the normalized error of the multi-target state estimate. The details of defining and computing the normalized error terms, \(\varepsilon^2_{|X|}(u)\) and \(\varepsilon^2_X(u)\) for SMC implementation are presented in Sec. V through equations (3.10)–(3.16). Note that \(\eta \in [0, 1]\) is a user-defined constant parameter to tune the influence of the error terms on the total sensor control cost. It is also important to note that the expectation term in (3.1) does not appear as we use the PIMS approach [Mahler 2004] instead of sampling and averaging in measurement space.

### 3.5 Implementation

At time \(k - 1\), the multi-target distribution is modeled by a multi-Bernoulli RFS with parameters \(\{r^{(i)}_{k-1}, P^{(i)}_{k-1}\}_{i=1}^{M_{k-1}}\), where \(M_{k-1}\) indicates the maximum possible number

\(^{3}\)In situations where the probability of detection is high and clutter rate is moderate, the multi target posterior density can be approximated under the assumptions of the PIMS (see [Mahler 2004]–section 4.1 and [Ristic 2013]).
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of targets, and each \( p^{(i)}_{k-1} \) for \( i = 1, \ldots, M_{k-1} \) is approximated by \( L^{(i)}_{k-1} \) particles denoted by \( \{w^{(i,j)}_{k-1}, x^{(i,j)}_{k-1}\}_{j=1}^{L^{(i)}_{k-1}} \). In the prediction step, the multi-Bernoulli filter propagates the multi-Bernoulli components based on the temporal information from the transition density \( (f_{k|k-1}(\cdot)) \), the probability of survival \( (P_s) \), and the predefined multi-Bernoulli birth terms [Vo et al. 2009]. The predicted multi-Bernoulli density is denoted by \( \{r^{(i)}_{k|k-1}, p^{(i)}_{k|k-1}\}_{i=1}^{M_{k|k-1}} \) where \( p^{(i)}_{k|k-1} \) is approximated by \( L^{(i)}_{k|k-1} \) particles and represented by \( \{u^{(i,j)}_{k|k-1}, x^{(i,j)}_{k|k-1}\}_{j=1}^{L^{(i)}_{k|k-1}} \) for \( i = 1, \ldots, M_{k|k-1} \).

As it was mentioned before, the updated distribution would depend on the measurement set which in turn would be dependent on the chosen sensor control command. Similar to Ristic et al [Ristic 2013], for each control command, a single pseudo-measurement is generated for each single object under ideal conditions (perfect detection, zero measurement noise and no clutter). The ensemble of such pseudo-measurements forms the PIMS. To compute each of the ideal measurements, we need to know the single-target states which can be estimated from the predicted multi-target distribution via the following two steps. First, the estimated number of targets, denoted by \( \hat{M}_{k|k-1} \) is obtained by counting the predicted existence probabilities that exceed a given threshold (e.g. 0.5 as an unbiased value). Then, the EAP estimate of the \( \ell \)-th target state is given by \( \hat{x}^{(\ell)}_{k|k-1} = \sum_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{k|k-1} x^{(i,j)}_{k|k-1} \) where the indices \( i_{\ell} \) refer to the Bernoulli components that were counted in step I.

Having the ideal measurement set for each admissible sensor command, \( u_k \in U_k \), and considering it as the actual measurement, we can then run the update step and calculate the cost corresponding to that command. The cost defined in (3.9) combines two normalized error terms, \( \varepsilon^2_{|X|}(u) \) for the cardinality estimate and \( \varepsilon^2_{X}(u) \) for the multi-target state estimate. Both terms depend on the updated multi-object posterior \( \{r^{(i)}_{k,u_k}, p^{(i)}_{k,u_k}(\cdot)\}_{i=1}^{M_{k,u_k}} \) which in turn depends on the PIMS computed for the

\(^4\)It is important to note that the same procedure is used to finally extract the multi-target estimates after the control command is chosen and applied, the measurements are acquired from sensors and the multi-target state is updated. Furthermore, we note that alternative methods (such as the method suggested in [Vo et al. 2009]) could be also used for this purpose.
In [Delande et al. 2014a] and [Delande et al. 2014b], Delande et al showed that the regional variance of the number of targets quantifies the certainty of the filter estimates of the number of the targets that evolve in surveillance region. Following [Delande et al. 2014b], we chose the variance of the cardinality as a meaningful measure for its uncertainty or estimation error. In terms of the updated probabilities of existence this variance is given by:

$$\sigma^2_{|X|}(u_k) = \sum_{i=1}^{M_{k,u_k}} \left[ \eta_k^{(i)} \left( 1 - \eta_k^{(i)} \right) \right].$$  \hspace{1cm} (3.10)

It is important to note that in the linear combination presented in equation (3.9), in order for the weight $\eta$ to be dimensionless and bounded within the $[0, 1]$ interval (so that both $\eta$ and $1 - \eta$ weights are positive), the two error terms have to be dimensionless. We choose “normalized” error terms which are both dimensionless and bounded within $[0, 1]$ themselves. Thus, the linear combination forming the cost in (3.9) will be guaranteed to be bounded within $[0, 1]$. The normalized cardinality error term can be computed as follows:

$$\varepsilon^2_{|X|}(u_k) = \frac{\sigma^2_{|X|}(u_k)}{\max\{\sigma^2_{|X|}(u_k)\}}.$$  \hspace{1cm} (3.11)

The maximum variance occurs when $\forall i, \eta_k^{(i)} = 0.5$.

$$\max\{\sigma^2_{|X|}(u_k)\} = \frac{M_{k,u_k}}{4}.$$  

To arrive at a meaningful measure for the normalized state estimation error term $\varepsilon^2_X(u)$ in the cost defined in (3.9), we first compute the following total state estimation error:

$$\varepsilon^2_X(u_k) = \sum_{i=1}^{M_{k,u_k}} \left[ \eta_k^{(i)} \frac{2}{\eta_k^{(i)}} (u_k) \right] / \sum_{i=1}^{M_{k,u_k}} \eta_k^{(i)}.$$  \hspace{1cm} (3.12)

which is the weighted average of estimation errors of the states of single targets associated with each single Bernoulli component. The rationale behind this choice of
weights is that Bernoulli components with larger probabilities of existence contribute more strongly to the EAP estimate of the multi-object state –see section IV-A.4 in [Vo et al. 2009]. Although $\varepsilon_x^2$ is not directly related to estimation error, we named it as the “total state estimation error”. The philosophy behind this choice lies in the fact that reduction of $\varsigma_x^2(u_k)$ increases the confidence of estimation which implicitly increases the expected accuracy of estimation.

To compute the single Bernoulli component errors $\varsigma_x^2(u_k)$, we note that in practice, minimizing the estimation error of only some selected elements of target states maybe of the interest. For instance, in target tracking, usually the prime interest is in location, and target speed is included in the single-target state vector due to its appearance in motion and perhaps measurement models. In such target-tracking applications, an intuitive scalar measure for the single Bernoulli component error is given by the product of the variances of the target location coordinates. If the stochastic variations of target location coordinates are independent, this measure will translate into the absolute determinant of the covariance matrix of the target location. In case of tracking multiple-targets in two–dimensional space, the single Bernoulli component error term, $\varsigma_x^2(u_k)$, is given by:

$$\varsigma_x^2(u_k) \propto \sigma_x^2(u_k) \sigma_y^2(u_k)$$ (3.13)

where x and y denote the x and y-coordinates of the single-target location (part of its state vector $x$), and the proportionality (not equality) is chosen because the above product does not lead to a normalized measure. Having the updated particles and weights of each Bernoulli component, the single-coordinate errors can be calculated as follows:

$$\sigma_x^2(u_k) = \sum_{j=1}^{L_{k,uk}} w_{k,uk}^{(i,j)} x_{k,uk}^{(i,j)} - \left( \sum_{j=1}^{L_{k,uk}} w_{k,uk}^{(i,j)} x_{k,uk}^{(i,j)} \right)^2$$

$$\sigma_y^2(u_k) = \sum_{j=1}^{L_{k,uk}} w_{k,uk}^{(i,j)} y_{k,uk}^{(i,j)} - \left( \sum_{j=1}^{L_{k,uk}} w_{k,uk}^{(i,j)} y_{k,uk}^{(i,j)} \right)^2$$ (3.14)
where $x_{k,uk}^{(i,j)}$ and $y_{k,uk}^{(i,j)}$ denote the coordinates extracted from the particle $x_{k,uk}^{(i,j)}$ and power operation is element-wise. To normalize the total state estimation error term $\varsigma^2_{x^{(i)}}(u_k)$ in (3.13), we note that with equally weighted particles, i.e when $\forall j, w_{k,uk}^{(i,j)} = 1/L_{k,uk}^{(i)}$, the particles representing the $i$-th single Bernoulli component do not convey any information and the above estimation variances adopt their maximum values as follows:

$$\max\{\sigma^2_{x^{(i)}}(u_k)\} = \frac{1}{L_{k,uk}^{(i)}} (1 - \frac{1}{L_{k,uk}^{(i)}}) \sum_{j=1}^{L_{k,uk}^{(i)}} x_{k,uk}^{(i,j)}^2$$

$$\max\{\sigma^2_{y^{(i)}}(u_k)\} = \frac{1}{L_{k,uk}^{(i)}} (1 - \frac{1}{L_{k,uk}^{(i)}}) \sum_{j=1}^{L_{k,uk}^{(i)}} y_{k,uk}^{(i,j)}^2.$$  

The single Bernoulli error terms $\varsigma^2_{x^{(i)}}(u_k)$ in (3.13) can be normalized as follows:

$$\varsigma^2_{x^{(i)}}(u_k) = \frac{\sigma^2_{x^{(i)}}(u_k) \sigma^2_{y^{(i)}}(u_k)}{\max\{\sigma^2_{x^{(i)}}(u_k)\} \max\{\sigma^2_{y^{(i)}}(u_k)\}}$$

and the computed values are then used in (3.12) to calculate the normalized state estimation error term in the cost. Extension of the terms derived in (3.13)–(3.16) to the cases involving more than two dimensional location parameters is straightforward.

Having the cost values computed for all admissible sensor control commands, the best command $\hat{u}_k$ is then chosen as the one incurring the smallest cost:

$$\hat{u}_k = \arg\min_{u_k \in U_k} \mathcal{V}(u_k; X_{k-1}).$$  

(3.17)

Algorithm 2 shows the pseudocode for the CB-BeMBe multi-target filtering scheme with the proposed sensor-control.
Algorithm 2 The CB-MeMBer multi-target filtering recursion with PEECS sensor-control.

**Inputs:** time $k$, dynamic model $f_{k|k-1}(\cdot|_{x_{k-1}})$, multi-Bernoulli birth model parameters, prior multi-Bernoulli parameters from time $k-1$, detection probability $p_D(\cdot)$, measurement likelihood function $g_k(\cdot|x)$, and clutter intensity $\nu(\cdot)$ and its integral $\lambda_c$, current sensor(s) location(s), finite set of admissible sensor-control commands $U$.

**Output:** The best control command $\hat{u}$ and updated multi-Bernoulli parameters.

**Prediction:**

1. Using equations (1)–(3), compute the predicted multi-Bernoulli component parameters and their particles $\{r_{k|k-1}^{(i)}, \{w_{k|k-1}^{(i,j)}, x_{k|k-1}^{(i,j)}\}_{j=1}^{L_{k|k-1}}\}_{i=1}^{M_{k|k-1}}$.

**Pre-estimation:**

2. $\ell \leftarrow 0$,
3. for $i = 0, M_{k|k-1}$ do
4. if $r_{k|k-1}^{(i)} > 0.5$ then
5. $\ell \leftarrow \ell + 1$
6. $z_{k|k-1}^{(\ell)} = \sum_{j=1}^{L_{k|k-1}} u_{k|k-1}^{(i,j)} x_{k|k-1}^{(i,j)}$
7. end if
8. end for
9. $\hat{M}_{k|k-1} \leftarrow \ell$
Sensor-control:

10: for all $u_k \in U$ do,
11: $\tilde{Z} \leftarrow \emptyset$.
12: for $\ell = 1, \ldots, N_{k|k-1}$ do
13: $\xi \leftarrow \text{argmax}_z g(z|\hat{x}_{k|k-1}^{(\ell)})$
14: $\tilde{Z} \leftarrow \tilde{Z} \cup \{\xi\}$
15: end for

Constructing the PIMS:

16: Use the PIMS $\tilde{Z}$ in (7)-(8) to update the multi-Bernoulli distribution parameters.

Updating for sensor control:

17: $\tilde{\varepsilon}_{|X|}^2(u_k) \leftarrow \left[ \sum_{i=1}^{M_{k;uk}} f_{k,uk}^{(i)} (1 - r_{k;uk}^{(i)}) \right] / M_{k;uk}.$
18: $\sigma_{x(i)}^2(u_k) \leftarrow \sum_{j=1}^{L_{k;uk}^{(i)}} w_{k,uk}^{(i)} x_{k,uk}^{(i,j)} - \left( \sum_{j=1}^{L_{k;uk}^{(i)}} w_{k,uk}^{(i)} x_{k,uk}^{(i,j)} \right)^2$.
19: $\sigma_{y(i)}^2(u_k) \leftarrow \sum_{j=1}^{L_{k;uk}^{(i)}} w_{k,uk}^{(i)} y_{k,uk}^{(i,j)} - \left( \sum_{j=1}^{L_{k;uk}^{(i)}} w_{k,uk}^{(i)} y_{k,uk}^{(i,j)} \right)^2$.
20: $\max\{\sigma_{x(i)}^2(u_k)\} \leftarrow \frac{1}{L_{k;uk}^{(i)}} (1 - \frac{1}{L_{k;uk}^{(i)}}) \sum_{j=1}^{L_{k;uk}^{(i)}} x_{k,uk}^{(i,j)}.$
21: $\max\{\sigma_{y(i)}^2(u_k)\} \leftarrow \frac{1}{L_{k;uk}^{(i)}} (1 - \frac{1}{L_{k;uk}^{(i)}}) \sum_{j=1}^{L_{k;uk}^{(i)}} y_{k,uk}^{(i,j)}.$
22: $\varsigma_{x(i)}^2(u_k) \leftarrow \frac{\sigma_{x(i)}^2(u_k)}{\max\{\sigma_{x(i)}^2(u_k)\}} \frac{\sigma_{y(i)}^2(u_k)}{\max\{\sigma_{y(i)}^2(u_k)\}}.$
23: $\hat{\varepsilon}_X^2(u_k) \leftarrow \left[ \sum_{i=1}^{M_{k;uk}} f_{k,uk}^{(i)} \varsigma_{x(i)}^2(u_k) \right] / \sum_{i=1}^{M_{k;uk}} f_{k,uk}^{(i)}$
24: $\mathcal{V}(u;X_k) \leftarrow \eta \hat{\varepsilon}_{|X|}^2(u) + (1 - \eta) \hat{\varepsilon}_X^2(u)$
25: end for
26: $\hat{u} \leftarrow \text{argmin}_u \mathcal{V}(u;X_k)$

Computing the cost:

27: Apply the control command $\hat{u}$ to change the sensor state.
28: Read the actual measurement set $Z_k$.

Measurement:

29: Use the set $Z_k$ as measurement set in equations (7)-(8) and compute the updated multi-Bernoulli parameters.

Update:

30: Prune and merge the updated Bernoulli components. More details in [Vo et al. 2009].
3.6 Simulation Results

The performance of the proposed PEECS sensor-control method was evaluated in two challenging case studies. Case study 1 is borrowed from [Ristic et al. 2011a] and [Gia Hoang and Tuong Vo 2013] and used to compare the performance of PEECS with the PHD-based sensor control method for different clutter rates. Case study 2 is a more challenging extension of case study 1 in which the targets are not pseudo stationary (they move and turn through the scenario), the motion model is more complex, and in addition to range, bearing measurements are also available.

3.6.1 Case study 1

A controllable range-only sensor is used to track multiple targets in a surveillance application. Each single target state is comprised of location and velocity components in x and y directions, denoted by \([x \ y \ \dot{x} \ \dot{y}]^\top\). In the particular scenario borrowed from [Ristic et al. 2011a, Gia Hoang and Tuong Vo 2013], there are five targets. The sensor location is denoted by \(s = [x_s \ y_s]^\top\). The sensor can detect an object in location \(o = [x_o \ y_o]^\top\) with the following location-dependent probability:

\[
p_D(s, o) = \begin{cases} 
1, & \text{if } ||o - s|| \leq R_0 \\
\max \{0, 1 - h(||o - s|| - R_0)\}, & \text{otherwise}
\end{cases}
\]  

(3.18)

To make the results comparable, we used the same parameters used in [Ristic et al. 2011a, Gia Hoang and Tuong Vo 2013]. Those are as follows:

- surveillance area: 1000 m × 1000 m,
- \(R_0 = 320\) m,
- \(h = 0.00025\) m\(^{-1}\),
- measurement model: \(z = ||o - s|| + e\) where \(e \sim \mathcal{N}(0, \sigma^2)\), \(\sigma = \sigma_0 + \beta||o - s||^2\), \(\sigma_0 = 1\) m and \(\beta = 5 \times 10^{-5}\) m\(^{-1}\).
The Poisson RFS for clutter has the intensity $\nu(z) = \lambda_c \ c(z)$ where $c(z) = U[0, 1000\sqrt{2}]$, and $\lambda_c = 0.5$.

Overall, there are five objects in the surveillance area, positioned relatively close to each other. Their initial state vectors are: $[800 \ 600 \ 1 \ 0]^\top$, $[650 \ 500 \ 0.3 \ 0.6]^\top$, $[620 \ 700 \ 0.25 \ 0.45]^\top$, $[750 \ 800 \ 0.6]^\top$, and $[700 \ 700 \ 0.2 \ 0.6]^\top$, where the units of $x$ and $y$ are meters and $\dot{x}$ and $\dot{y}$ are m/s. The objects move according to the constant velocity model [Li and Jilkov 2003a, Ristic et al. 2011a, Gia Hoang and Tuong Vo 2013, Gostar et al. 2013a].

The controllable mobile sensor initially enters the surveillance area at position $s = [10 \ m \ 10 \ m]^\top$. Other parameters such as the dynamic parameters of the motion model, probability of survival, target birth model, finite set of admissible control commands, and relative number of particles are also borrowed from [Ristic et al. 2011a, Gia Hoang and Tuong Vo 2013]. As it is also noted in [Ristic et al. 2011a], the range dependent sensor noise and relatively high rates of clutter and miss-detections make this a very challenging case of sensor-control for multi-target estimation in which many state-of-the-art techniques would fail.

Figure 3.1 shows the initial and final locations of the five targets in this scenario, and demonstrates how the sensor location is controlled towards locations with better accuracy and detection rate by PEECS method (Algorithm 2). We have compared PEECS sensor-control with recent PHD-based sensor control methods in terms of their OSPA errors. In addition, we compared our method with the multi-Bernoulli-based sensor-control method of Hoang et al [Gia Hoang and Tuong Vo 2013] which is more similar. Figure 3.1 shows the controlled sensor locations of this method compared to PEECS sensor-control method. We will further elaborate on this comparison later.

We computed the localization, cardinality and OSPA errors [Schuhmacher et al. 2008] at time steps $k = 1, \ldots, 35$. The estimation errors for three sensor-control methods using different reward/cost functions (PENT [Mahler 1998], Rényi diver-
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Figure 3.1: Sensor locations for PEECS cost function and MAP cardinality variance cost function [Gia Hoang and Tuong Vo 2013] during $k = 1, \ldots, 20$ for range-only measurement.

gence [Ristic et al. 2011a] and PEECS) are shown in Fig. 3.2 and Fig. 3.3 for the case of clutter rate $\lambda_c = 0.5$. Note that OSPA errors for Rényi divergence are computed and plotted for $\alpha = 0.5$ and $\alpha = 1$. We observe that during the initial time steps, the OSPA error returned by the PEECS sensor-control is significantly smaller than (about half of) the error of the two PHD-based sensor-control methods reported in [Ristic et al. 2011a] which have been demonstrated to outperform PENT and other existing methods. However, as it is reported in [Ristic et al. 2011a], the reason of poor performance of PENT lies in the fact that its objective function is designed for sensor-control problems in which the sensor has finite field of view,
which is not the case in this scenario.

Although PEECS and PHD-based methods converge to similar error values, the initially smaller error of the PEECS sensor-control demonstrates that the sensor is guided towards its optimum location faster and arrives there much earlier than the PHD-based methods that work based on maximizing Rényi divergence. A video of the live simulation run is attached as supplemental material. It is observed from
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Figure 3.4: Estimation errors with different clutter rate of our sensor-control method compared to the PHD-based sensor-control.

the run that after the first 13 time steps, our sensor-control drives the sensor to a close vicinity of the center point of the five targets, and during the rest of the simulation, the sensor stays in that region with minor movements. This observation is consistent with the OSPA errors of PEECS shown in figures 3.2 and 3.3 converging to its minimum at about time step 13.

To investigate the robustness of PEECS sensor-control to increased levels of clutter, we compared its OSPA errors with the PHD-based sensor-control method reported in [Ristic et al. 2011a] for different clutter rates of $\lambda_c = 0.5, 3, 5$. The errors were averaged over 200 Monte Carlo runs of each method and are shown in Fig. 3.4. It is observed that by increasing the clutter rate $\lambda_c$ from 0.5 to 3 then to 5, our proposed method returns OSPA errors that are generally smaller than the errors returned by the PHD-based sensor-control. We also note that the presented OSPA errors in the paper only provide a broad indication of sensor-control performance, and such results are scenario dependent.
The most significant advantage of PEECS sensor-control method is its low computational cost. Table 3.1 shows the computation times of PEECS sensor-control compared with the PHD-based sensor-control method of [Ristic et al. 2011a], averaged over 200 Monte Carlo runs, for different clutter rates.\(^5\) We observe that in average, PEECS sensor-control runs at least 4 times faster than competing methods. Note that the result of the computation times should only be considered as a broad indication of sensor-control performance, since these results are scenario and code dependent. We also note that PEECS does not consider possible constraints on the sensor field of view and needs to be revised in presence of such constraints. Comparison of the computational cost of the revised version of PEECS with methods like PENT [Mahler 2004] (that take such limits into consideration) is beyond the scope of this paper and left for future research.

The lower computational cost of PEECS sensor-control is mainly due to the fact that it is designed to work with the multi-Bernoulli filter. Extraction of cardinality and single-object state estimates in a multi-Bernoulli scheme is straightforward compared to PHD and CPHD methods. In the latter, the particles approximating the intensity function need to be grouped via a clustering algorithm. However, no such clustering step is required by multi-Bernoulli methods, hence substantial savings in computation is made particularly in applications that involve numerous objects. In such applications, the total number of particles, representing Monte Carlo approximations of the intensity function, is required to be very large otherwise substantial amount of information would be lost during the clustering step. The need for a large number of particles would impose heavy computation burden in those filtering schemes.

In Fig. 3.5, OSPA error of PEECS sensor-control method is compared with the method reported in [Hoang 2012a, Gia Hoang and Tuong Vo 2013]. Hoang et al [Gia

\(^5\)Both methods were coded in MATLAB and run on the same machine. For the PHD-based sensor-control, we used the code provided by the authors of [Ristic et al. 2011a].
Hoang and Tuong Vo [2013] used multi-Bernoulli filter in conjunction with Rényi divergence as a reward function. We observe that PEECS sensor-control performs better than the Rényi divergence-based sensor control method proposed in [Hoang 2012a, Gia Hoang and Tuong Vo 2013] in terms of OSPA error. This is mainly because PEECS cost function is structurally related to the error terms computed in OSPA. We note that the superior performance of PEECS in terms of OSPA error may not be necessarily valid in terms of information gain which is the major focus of Rényi divergence-based sensor control methods.

Hoang and Vo [Gia Hoang and Tuong Vo 2013] also proposed a multi-Bernoulli sensor-control method in which the variance of cardinality around its MAP estimate (termed “MAP cardinality variance”) is used as the cost function. This cost function is similar to PEECS. However, in PEECS, the variance term is the statistical variance (around the mean, not around the MAP estimate) of cardinality, and more importantly, the cost is penalized due to errors not only in cardinality estimates but also in state estimates. As it is shown in Fig. 3.5, our method outperforms the method reported in [Hoang 2012a, Gia Hoang and Tuong Vo 2013] in terms of OSPA errors.

The controlled sensor locations resulted by PEECS and the method reported in [Gia Hoang and Tuong Vo 2013] (with MAP cardinality variance as the cost function) are shown in Fig. 3.1. We note that in the case studies discussed in this paper, some measurement parameters depend on the sensor-target distance in such a way that a shorter sensor-target distance generally leads to more accurate measurements. To be more precise, based on the measurement models in case study 1, with a shorter sensor-target distance, higher probability of detection and smaller...
measurement noise are achieved. This seems to be a commonly accepted assumption as it is reported in many similar works on sensor control such as [Ristic and Vo 2010, Ristic et al. 2011a, Gia Hoang and Tuong Vo 2013]. With such a distance-dependent measurement model, an effective sensor control method is expected to guide the sensor towards the center of the targets, and maintain it in the vicinity of the center as the targets move. Figure 3.1 clearly shows that PEECS directly moves the sensor towards the center of the targets and maintains it in the center as the targets move. This is while the sensor trajectory created by the competing method of [Gia Hoang and Tuong Vo 2013] seems to be deviated from the path to the center of the targets and not to converge and remain in the center as the targets move. Hence, PEECS leads to the significantly improved OSPA errors shown in Fig. 3.5.

As stated by Hoang and Vo [Gia Hoang and Tuong Vo 2013], “the lack of observability of the full states, as we now have range-only measurements,” causes the poor performance of their proposed sensor control, especially when only the variance of cardinality around its MAP estimate is optimized. If the objective function “accounts for both localization and cardinality criteria”, which is indirectly the case when Rényi divergence is chosen as the objective function, sensor control is expected to result in “lower error” [Gia Hoang and Tuong Vo 2013]. We note that both cardinality and state estimation errors are explicitly considered in PEECS objective function.

### 3.6.2 Case study 2

To demonstrate the performance of our method with measurements that guarantee full observability of the targets, and to investigate the particular effect of the localization error term in the proposed cost function, we also designed a simulation involving a more complex scenario than case study 1. In this scenario, we chose the non-linear nearly-constant turn model employed in [Vo et al. 2009]. In this model,
CHAPTER 3: MULTI-BERNOULLI SENSOR-CONTROL VIA MINIMIZATION OF EXPECTED ESTIMATION ERRORS

Figure 3.5: The comparative results for PEECS method and the methods reported in [Gia Hoang and Tuong Vo 2013].

Each single target state $x = [\bar{x}^\top \omega]^\top$ is comprised of location and velocity in Cartesian coordinates, denoted by $\bar{x} = [x \ y \ \dot{x} \ \dot{y}]^\top$ and turning rate, denoted by $\omega$. The state dynamics are given by:

$$\bar{x}_k = F(\omega_{k-1})\bar{x}_{k-1} + G\epsilon_{k-1},$$

$$\omega_k = \omega_{k-1} + T\gamma_{k-1},$$

where

$$F(\omega) = \begin{bmatrix} 1 & 0 & \frac{\sin \omega T}{\omega} & -\frac{1-\cos \omega T}{\omega} \\ 0 & 1 & \frac{1-\cos \omega T}{\omega} & \frac{\sin \omega T}{\omega} \\ 0 & 0 & \cos \omega T & -\sin \omega T \\ 0 & 0 & \sin \omega T & \cos \omega T \end{bmatrix},
\quad G = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \end{bmatrix},$$

$$T = 1 \text{ s}, \ \epsilon_{k-1} \sim \mathcal{N}(\cdot; 0, \sigma^2_e I), \ \sigma_e = 15 \text{ m/s}^2, \ \text{and} \ \gamma_{k-1} \sim \mathcal{N}(\cdot; 0, \sigma^2_\gamma I), \ \sigma_\gamma = (\pi/180) \text{ rad/s}.$$  

The birth RFS is a multi-Bernoulli with density $\pi_\Gamma = \{(r^{(i)}_\Gamma, p^{(i)}_\Gamma)\}_{i=1}^4$ where $r^{(1)}_\Gamma =$...
\( r_1^{(2)} = 0.02, r_1^{(3)} = r_1^{(4)} = 0.03 \) and \( p_\Gamma^{(i)}(x) = N(x; m_\Gamma^{(i)}, P_\Gamma) \) where

\[
\begin{align*}
m_\Gamma^{(1)} &= [-1500 0 250 0 0]^T, \\
m_\Gamma^{(2)} &= [-250 0 1000 0 0]^T, \\
m_\Gamma^{(3)} &= [250 0 750 0 0]^T, \\
m_\Gamma^{(4)} &= [1000 0 1500 0 0]^T, \\
P_\Gamma &= \text{diag}(50^2, 50^2, 50^2, 50^2, (6 \times \frac{\pi}{180})^2).
\end{align*}
\]

Probability of survival, detection probability, initial sensor location and clutter rate are similar to the previous case study. In this case study, in addition to range, bearing information is also available and the observation model consists of noisy bearing and range measurements:

\[
z_k = \left[ \arctan\left(\frac{y_k}{x_k}\right), \sqrt{x_k^2 + y_k^2} \right]^T + \zeta_k,
\]

where \( \zeta_k \sim N(\cdot; 0, R_k) \) is the measurement noise with covariance \( R_k = \text{diag}(\sigma_\theta^2, \sigma_r^2) \) in which the scales of range and bearing noise are \( \sigma_\theta = (\pi/180) \text{ rad} \) and \( \sigma_r = 5 \text{ m} \).

The clutter RFS followed the uniform Poisson model over the surveillance region \([ -\pi/2, \pi/2 ] \text{ rad} \times [0, 2000] \text{ m} \), with \( \lambda_c = 1.6 \times 10^{-3} \text{ (rad m)}^{-1} \).

As it is shown in Fig. 3.6, the sensor starts moving toward the objects and remains between those. We ran PEECS sensor-control algorithm for both \( \eta = 1 \) and \( \eta = 0.5 \). When the parameter \( \eta \) equals 1, the cost function includes only the cardinality error term. With \( \eta = 0.5 \), both cardinality error and object state estimation error are equally weighted within the cost. We recorded the OSPA errors in both cases at time steps \( k = 1, \ldots, 50 \). The results are plotted in Fig. 3.7, this figure shows that although the targets maneuver in long paths (compared to the case study 1 borrowed from [Ristic et al. 2011a]), they are tracked with reasonably low error, indicating the reliability of PEECS. Comparison of OSPA errors in cases where \( \eta = 1 \) or \( \eta = 0.5 \) also demonstrates the benefit gained in terms of total error when both cardinality and state estimation errors were considered within the cost function, as it happened in PEECS with \( \eta = 0.5 \).
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Figure 3.6: Sensor and target locations in case study 2, during $k = 1, \ldots, 50$.

Figure 3.7: OSPA errors returned by PEECS in case study 2.
3.7 Conclusion

A sensor-control solution was proposed to be employed within a multi-Bernoulli multi-target filter. In this method, at each step, the next sensor control command is chosen by minimizing a new task-driven cost function called “Posterior Expected Error of Cardinality and States” (PEECS). The PEECS cost associated with each control command is defined as a linear combination of the normalized errors in cardinality estimate of the multi-target random finite set and the normalized error of localization of the elements in the multi-target RFS. Simulation results involving two challenging multi-target estimation and sensor-control scenarios, demonstrated that PEECS sensor-control can return multi-object state estimation accuracy and clutter tolerance that are similar to or better than competing methods, and generally performs faster than the state-of-the-art.
Robust Multi-Bernoulli Sensor Selection for Multi-Target Tracking in Sensor Networks

CHAPTER 4: ROBUST MULTI-BERNOULLI SENSOR SELECTION FOR
MULTI-TARGET TRACKING IN SENSOR NETWORKS

This paper addresses the sensor selection problem for tracking of multiple maneuvering targets within a sensor network. It is assumed that due to the bandwidth and energy constraints of the sensor network, it is not feasible to directly use the entire information provided by the large number of sensor nodes for detection and tracking of the targets, hence the need for sensor selection. We present a decision making solution for sensor selection in the multi-Bernoulli random finite set framework. The proposed method selects a minimum subset of sensors which are most likely to provide reliable measurements. Our method is a robust method that works in the challenging uncertain scenarios where no prior information are available on clutter intensity or sensor detection profile. Simulation results demonstrate successful sensor selection in a challenging case where five targets move in a close vicinity to each other. Comparative results evidence the superior performance of our method in terms of accuracy of estimating the number of targets and their states.
4.1 Introduction

Estimation and tracking of multi-target states, using a network of sensors with communication constraints is a challenging problem. The main challenge stems from the fact that while highly reliable measurements are required by multi-target tracking solutions, due to bandwidth and energy constraints of the sensor network, the system may need to select an optimal subset of sensors to communicate with, from which it is most likely to receive high quality target-related observations. This problem is termed sensor selection in the literature [Zemek et al. 2007].

Most sensor selection solutions predict the quality of measurements provided by sensor nodes before initiating the communication with those sensors. The results of those prediction are then used to compute an objective function for selecting the desirable sensor(s) that is expected to provide the most informative measurements. A common approach is to quantify the existing information embedded in the observations and use it as a reward [Mahler and Zajic 2004, Ristic et al. 2011a]. One possible choice for this reward function is the statistical mean of Rényi divergence between predicted and updated distributions [Ristic et al. 2011a, Hoang 2012b].

Information theoretic methods are mostly formulated in partially observed Markov decision process (POMDP) framework using a myopic approach. A set of candidate sensors generate synthetic measurements that are used to update the multi-target state distribution and compute the reward associated with each candidate. The most informative candidate is then selected based on its reward [Ristic et al. 2011a, Hoang 2012b].

In this paper, we propose a new sensor selection solution for multi-target tracking, which similar to [Ristic et al. 2011a, Gostar et al. 2013a] is formulated using the Finite Set Statistics (FISST) framework. Our contributions are two-fold. Firstly, our method does not need any prior information about the distribution of clutter measurements or the uncertainty in the sensor field of view. Secondly, with
multi-Bernoulli assumptions for the random set state distribution, and within the adaptive multi-Bernoulli filtering scheme of [Vo et al. 2013], we have been able to define a novel cost function that is directly relevant both to error of state estimation and to error of estimation of the number of existing targets (cardinality of the multi-target set state). Simulation results for a challenging multi-target tracking scenario demonstrate that our sensor selection method successfully selects the sensors proving most informative measurements and the estimation errors are less than competitive methods.

4.2 Sensor selection problem

Consider a POMDP defined as the tuple

\[ \Psi = \{ X_k, S, \pi_{k|k-1}(X_k|X_{k-1}), g_k(z|x), \vartheta(X_{k-1}, s, X_k) \} , \]

where \( k \) denotes the time step, \( X_k \) is a finite set of target states, \( S \) denotes a finite set of sensor nodes for selection, \( \pi_{k|k-1}(X_k|X_{k-1}) \) represents the multi-target dynamics modeled as multi-target state transition density, \( g_k(z|x) \) is a stochastic measurement model, and \( \vartheta(X_{k-1}, s, X_k) \) is an objective function that returns a reward or cost for transition from the multi-object state \( X_{k-1} \) to the state \( X_k \), given that the sensor located at \( s \in S \).

The aim of sensor selection is to find the candidate sensor node \( \hat{s} \) that optimizes the objective function (cost or reward). In stochastic filtering, the multi-target states are characterized by their statistical distributions, and for each given node \( s \), the objective function \( \vartheta(X_{k-1}, s, X_k) \) would not be entirely deterministic, instead varying according to a statistical distribution. Thus, \( \hat{s} \) is commonly chosen to optimize the statistical mean of the objective function \( \vartheta(X_{k-1}, s, X_k) \),

\[ \hat{s}_k = \arg\min_{s \in S} / \arg\max_{s \in S} \{ \mathbb{E}_{X_{k-1}, X_k} [\vartheta(X_{k-1}, s, X_k)] \} . \]  

(4.1)
In section 4.2.1 we describe an adaptive filtering method reported in [Vo et al. 2013], for formulating the multi-target state. Our novel objective function is also defined in this framework.

### 4.2.1 Adaptive multi-Bernoulli filtering

Unlike the PHD-based filters which are moment approximations [Vo and Ma 2006], the multi-Bernoulli filters are density approximations and are characterized by the probability of existence of a possible element and the probability density function of the state of that element \( \{ (r^{(i)}, p^{(\cdot)^{(i)}}) \}_{i=1}^{M} \) [Mahler 2007b, Vo et al. 2009].

Vo et al [Vo et al. 2013] have recently tackled the problem of multi-Bernoulli filtering in cases where clutter intensity and probability of detection profile are unknown and generally non-homogeneous. In their solution, the probability of detection is augmented to the multi-target state, and propagated as the time evolves. In addition, the adaptive multi-Bernoulli filter incorporates a set of clutter generators working based on Poisson assumptions with their transition and observation models formulated similar to actual objects in a hybrid space \( \tilde{\mathcal{X}} = \mathcal{X}^{(0)} \cup \mathcal{X}^{(1)} \) (0 for clutter generators state space and 1 for target space).

The Sequential Monte Carlo (SMC) implementation of the proposed solution is summarized in the following (see [Vo et al. 2013] for more details). The augmented multi-target state now includes a state \( a \) (denoting the probability of detection) and the multi-Bernoulli set state \( \mathcal{X} \). At any time, the multi-Bernoulli state is the union of an ensemble of \( M \) Bernoulli sets, each having a probability of existence, \( r^{(i)} \), and two single-object densities denoted by \( p^{(i)(u)}(a, x) \) where \( u = 1 \) corresponds to objects that are actual targets, and \( u = 0 \) corresponds to clutters.

At any time \( k \), the prior multi-target distribution at time \( k - 1 \) is input to a prediction step that incorporates utilization of dynamic and birth models for target states (with probability of detection now augmented) and clutters. The distribution
is then updated using current measurements received from sensors.

4.3 Adaptive Multi-Bernoulli Sensor Selection

Our proposed sensor selection solution is based on minimizing a novel cost function that can be defined and computed for every sensor node. Our cost directly quantifies the statistical mean of an error term over all possible updated multi-object states. The error term is a linear combination of average uncertainties in cardinality estimates, target state estimates, and estimated clutter intensity. Let us denote the posterior (updated) multi-object distribution at time $k$ by $\{(r^{(i)}_k, p^{(i)}(u)_k)\}_{i=1}^{M_k}$ and assume that each density $p^{(i)}(u)_k$ is approximated by a set of particles $\{(a^{(i,j)}_k, x^{(i,j)}_k)\}_{j=1}^{L^{(i)}_k(u)}$ with weights $\{w^{(i,j)}(u)_k\}_{j=1}^{L^{(i)}_k(u)}$. The expected a posteriori (EAP) estimate of the number of targets is given by $\mathbb{E}(|X|; s) = \sum_{i=1}^{M_k} r^{(i)(1)}_k$ and its uncertainty, quantified by its variance, is

$$\sigma^2_{|X|}(s) = \sum_{i=1}^{M_k} \left[ r^{(i)(1)}_k \left( 1 - r^{(i)(1)}_k \right) \right]. \quad (4.2)$$

The second term is indicative of average error in estimating the targets states. We note that we may be only interested in minimizing the average error of some state components. For instance, in the case study presented in the next section, only the variance of location of targets is considered as important. To quantify the uncertainty in single object state estimates, we first compute the average error of state estimates for each Bernoulli component, from the updated multi-Bernoulli distribution:

$$\hat{\sigma}^2_{x,i}(s) = \sum_{j=1}^{L^{(i)}_k} w^{(i,j)(1)}_k (x^{(i,j)(1)}_k)^2 - \left( \sum_{j=1}^{L^{(i)}_k} w^{(i,j)(1)}_k x^{(i,j)(1)}_k \right)^2 \quad (4.3)$$

where $(\cdot)^2$ is performed component-wise over the components of interest. The total error can be normalized by dividing with the weighted average of the errors of single Bernoulli components, where the weights are the updated probabilities of existence:

$$\hat{\sigma}^2_x(s) = \sum_{i=1}^{L_k^{(i)(1)}} \left[ r^{(i)(1)}_k \hat{\sigma}^2_{x,i} \right] / \sum_{i=1}^{L_k^{(i)(1)}} r^{(i)(1)}_k. \quad (4.4)$$
The variance associated with the EAP on the number of expected clutter samples can be approximated by updated particles for generated clutter samples:

\[ \hat{\sigma}^2_{\lambda}(s) = \sum_{i=1}^{M_k} \left[ r^{(i)(0)}_k \sum_{j=1}^{L_{\lambda,k}^{(0)}} \left\{ r^{(i,j)}_{\lambda,k} \left(1 - r^{(i,j)}_{\lambda,k}\right) \right\} \right] \]  

(4.5)

where \( r^{(i,j)}_{\lambda,k} = w^{(i,j)(0)}_k a^{(i,j)(0)}_k \). The proposed objective function is defined as the following cost comprising a weighted sum of the three error terms:

\[ \vartheta(s) = \eta_{|X|} \sigma_{|X|}^2(s) + \eta_x \sigma_x^2(s) + \eta_{\lambda} \sigma_{\lambda}^2(s) \]  

(4.6)

where \( \eta_{|X|}, \eta_x \) and \( \eta_{\lambda} \) are the user-defined importance weights for minimization of each of the variances and are positive values. To have a normalized weighted sum of the error terms in the cost, the weights are chosen to satisfy \( \eta_{|X|} + \eta_x + \eta_{\lambda} = 1 \). Since precision of the estimated number of objects and their locations have the highest priority, it is reasonable to put more emphasis on the weight of the variance of the number of targets and their state (\( \eta_{|X|} \) and \( \eta_x \)). The sensor node is then chosen by minimizing the above cost function:

\[ \hat{s} = \arg \min_s \vartheta(s). \]  

(4.7)

### 4.4 Simulation results

A challenging non-linear multi-target tracking scenario, similar to the one reported in [Ristic et al. 2011a], is employed to evaluate the performance of the proposed adaptive multi-Bernoulli sensor selection method. In this scenario, the sensor network includes sensor nodes laid out uniformly over a square of size 1000m × 1000m divided into 50m × 50m blocks. Each sensor regularly scans the surveillance area and returns a set of bearing and range measurements corresponding to detected targets, each in the form of \( z_k = [\theta_k, R_k]^T \). A total of five targets appear in the scene and maneuver in the surveillance area. One of the challenging aspects of this
problem, similar to the case presented in [Ristic et al. 2011a], is the fact that (due to energy and bandwidth constraints) at each time $k$, only one sensor measurement set is communicated with the central processor for multi-target detection and tracking purposes.\footnote{Extension of the algorithm for cases where more than one sensor nodes can be communicated and used in the update step of multi-target filtering is straightforward.}

At each time step, $k$, the set of sensor nodes, $S$, that are examined for sensor selection, is comprised of the previously chosen node $\hat{s}_{k-1} = [x_{s_{k-1}} \ y_{s_{k-1}}]^\top$, and its horizontal and vertical neighbouring nodes up to two blocks. The initially selected sensor node is assumed to start from the origin, i.e $\hat{s}_0 = [0 \ 0]^\top$.

### 4.4.1 Measurement model

The measurement data is synthetically generated according to the following distance-dependent detection probability:

\[
p_D(s, \mathbf{r}) = \begin{cases} 
1, & \text{if } ||\mathbf{r} - s|| \leq R_0 \\
\max\{0, \beta(||\mathbf{r} - s|| - R_0)\} & \text{otherwise}
\end{cases} \tag{4.8}
\]

where $s = [x_s \ y_s]^\top$ and $\mathbf{r} = [x \ y]^\top$ denote the locations of the sensor node and target, respectively, $|| \cdot ||$ denotes Euclidean distance in 2D space, and the range parameter $R_0$ and detection gain $\beta$ are selected as $R_0 = 320$ m and $\beta = 25 \times 10^{-5}$ m$^{-1}$.

Note that the $p_D(s, \mathbf{r})$ is only needed for generating the synthetic measurements.

The range and bearing measurements, included in each synthetic point measurement $z = [\theta \ \mathbf{r}]^\top$ in a scan (in case of detection) where $\theta = \arctan(\mathbf{r} - s) + e_\theta$ and $\mathbf{r} = ||\mathbf{r} - s|| + e_\mathbf{r}$ in which $e_\theta$ and $e_\mathbf{r}$ denote i.i.d. Gaussian measurement noise samples with zero mean, and the angle measurement follows the four-quadrant definition.

The angle measurement noise power is assumed to be constant at $e_\theta = \pi/180$, but the noise power for range measurements linearly increases with range according to:

\[
e_\mathbf{r} = \sigma_0 + \eta ||\mathbf{r} - s||^2 \text{ where } \sigma_0 = 1 \text{ m}, \eta = 5 \times 10^{-5} \text{ m}^{-1} \tag{4.9}
\]
4.4.2 Target states and their dynamic model

At any time \( k \), each single target state is comprised of the unknown detection probability, location and velocity components in \( x \) and \( y \) directions, and detection type label \( u_k \in \{0, 1\} \), together denoted by \([a_k, x_k, \dot{x}_k, y_k, \dot{y}_k, u_k]^\top\). The targets are positioned relatively close to each other in the surveillance area and their initial locations and velocities are at \([800, 600, 1, 0]^\top\), \([650, 500, 0.3, 0.6]^\top\), \([620, 700, 0.25, -0.45]^\top\), \([750, 800, 0, 0.6]^\top\), and \([700, 700, 0.2, 0.6]^\top\), where the units of \( x \) and \( y \) are meters and \( \dot{x} \) and \( \dot{y} \) are m/s.

Actual targets move according to the constant velocity model [Li and Jilkov 2003b], in which the transition density of target location and speed is given by a Gaussian density with the same parameters as used in [Ristic et al. 2011a]. The transition density of the augmented state, \( a_k \), is assumed to be a beta distribution [Vo et al. 2013]. The Beta density is used to model the unknown detection probability, since the density covers the range \([0, 1]\), and has sufficient flexibility to capture various detection probability profiles [Vo et al. 2013].

The survival probability for actual objects is fixed throughout the scenario \( P_s = 0.99 \) [Vo et al. 2013]. The birth process for the actual targets is a multi-Bernoulli process with five components \( \pi_{r, k}^{(1)} = \left\{ (r_{r, k}^{(1)(i)}, P_{r, k}^{(1)(i)}(\cdot, \cdot)) \right\}_{i=1}^5 \) where all probabilities of existence are equally chosen at 0.05, and the states of each of the five newly born targets are distributed uniformly in \([0.8, 1] \times X\). The parameters chosen for beta distributions were same as reported in [Vo et al. 2013]. In our simulation, clutter generators model and filter parameters are borrowed from [Vo et al. 2013].

4.4.3 Results

We ran the proposed adaptive multi-Bernoulli sensor selection method and computed the resulting estimates for number and locations of the targets for a sequence of 35 steps. In each step, one node of the sensor network was selected to communi-
Figure 4.1: Sensor selection during 35 scans.

cate with a central station where the scan provided by that node was used to update the multi-target state and to extract estimates for the number of targets and their states. As it was mentioned earlier, the initially selected node was at the origin. The sensor selection method is expected to select the sensor nodes that are closer to existing targets, ending up with selecting the nodes located in the vicinity of the targets. The rationale behind this expectation is that the noise power for range measurements increases with the distance between the sensor node and the actual target– see equation (4.9).

Figure 4.1 shows how our method selects sensor nodes that become closer to the five targets in the scene as the time evolves. It is important to note that sensor selection converges early at $k = 18$ and during the rest of the time, the selected node does not change within more than one block.

Fig. 4.2 shows average estimates of cardinality and clutter intensity calculated by 200 MC runs of the proposed adaptive sensor selection method. We observe that
the clutter intensity estimates shown in Fig. 4.2a gradually approach the ground-truth value of 10 and fluctuate around it with a relatively small standard deviation.

For the averaged cardinality estimates shown in Fig. 4.2b, we have also included the results returned by sensor selection using multi-Bernoulli filtering with known clutter intensity and probability of detection as reported in our previous work [Gostar et al. 2013a]. In that method, we used an overestimated value of 15 for the clutter intensity (compared to the ground-truth of 10) and a fixed value of 0.98 for probability of detection, which contrasts to the ground-truth probability of detection that decreases with sensor-target distance—see equation (4.8).

From Fig. 4.2b, we observe that in terms of estimating the cardinality, the proposed sensor selection method leads to better accuracy, as the averaged cardinality estimates over 200 MC runs of the scenario are closer to the ground-truth number of 5 targets. The superior performance of our method mainly lays in its adaptive nature. Indeed, in the absence of accurate knowledge of the measurement process, inaccurate assumptions for measurement process parameters would lead to inaccurate results and frequently missed targets. This is while without the need for such prior information, our method intrinsically adapts the multi-target filtering process to work best with the measurements received from the selected sensor node.

To evaluate the estimation accuracy, we computed the OSPA miss-distances [Schuhmacher et al. 2008] with parameters $p = 2$ and $c = 100$. Fig 4.3 includes a comparative analysis for the performance of our method against the state-of-the-art methods. The compared methods are the multi-Bernoulli [Gostar et al. 2013a] and CPHD-based [Ristic et al. 2011a] sensor selection methods both with known clutter intensity and detection probability profile.

For the CPHD filtering-based sensor selection method, the correct detection profile is given to the filter and clutter intensity is assumed to be equal to 2 (higher clutter intensities failed to converge). These results confirm that the proposed method significantly outperforms the state-of-the-art techniques in terms of esti-
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Figure 4.2: Averages of clutter intensity and cardinality estimates over 200 MC runs.

(a) Statistical mean and standard deviation of clutter intensity estimates.

(b) Statistical mean of cardinality estimates returned by our method, compared with the multi-Bernoulli method with known parameters [Gostar et al. 2013a].
4.5 Conclusion

The problem of multi-target tracking in a large sensor network with bandwidth and energy constraints was studied. A novel solution was presented in which a new criterion (in the form of a cost function) was introduced to select a minimum subset
of sensors which are most likely to provide the best informative measurements. The proposed cost function is defined and formulated within the robust multi-Bernoulli filter. The function is comprised of three terms, each quantifying uncertainties of an important estimate given by the multi-Bernoulli filter. Those terms are estimates of cardinality, object states and clutter intensity. The proposed sensor selection method is shown to work in challenging multi-target applications in which for the sensor nodes of the network, no prior information is available on their clutter intensity or their field-of-view parameters. Simulation results show that the proposed technique is able to select the sensors in locations where most accurate measurements can be acquired, leading to accurate estimates of the number of targets and their states and better detection of clutters compare to state-of-the-art techniques.

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Multi-Bernoulli Sensor-Selection for Multi-Target Tracking with Unknown Clutter and Detection Profiles

A new sensor-selection solution within a Multi-Bernoulli-based multi-target tracking framework is presented. The proposed method is especially designed for the general multi-target tracking case with no prior knowledge of the clutter distribution or the probability of detection, and uses a new task-driven objective function for this purpose. Step-by-step sequential Monte Carlo implementation of the method is presented along with a similar sensor-selection solution formulated using an information-driven objective function (Rényi divergence). The two solutions are compared in a challenging scenario and the results show that while both methods perform similarly in terms of accuracy of cardinality and state estimates, the task-driven sensor-selection method is substantially faster.
5.1 Introduction

Sensor network systems have been an appealing research topic in recent years due to their successful applications in a broad range of areas such as mobile sensor networks, vehicular networking, communication networks, and internet-based control [Yang et al. 2011; 2014, Mittal et al. 2015, Liu 2010, Ogren et al. 2004]. Multi-target tracking is one of the common applications of sensor networks including battlefield surveillance and enemy tracking in military applications, and habitat monitoring, environment observation, and traffic surveillance in civilian applications. In many practical situations, due to communication and computational constraints, it is required that at each time-step of the target tracking procedure, only a subset of sensors is selected to communicate with the central processor. In such cases, the problem of sensor-selection is to select the right sensor nodes that maximize the tracking observability with limited computing and communication resources.

In general, sensor-selection comprises two underlying components, a multi-object filtering process in conjunction with an optimal decision-making method. This is a sequential decision making process under stochastic uncertainties. These uncertainties stem either from the multi-target tracking process or from the effects of selecting different sensor nodes. An example of the sensor-selection problem is demonstrated in Fig. 5.1 in which a sensor network is used for traffic monitoring.

The sensor-selection problem is fundamentally similar to the sensor-control problem. Sensor-selection solutions are highly desirable to regulate and decide actions on real phenomena in many applications such as surveillance, factory instrumentation, defense and so on [Biagetti et al. 2009, Gastpar et al. 2006]. In sensor-control, a set of sensor commands is used to change mobile sensors’ states. The sensor-control problem is to find a member of the command set that can result in most accurate estimates of the number and states of targets, and best observability (if the sensors’ field-of-view is limited).
Figure 5.1: Sensor-selection in a distributed sensor network for traffic monitoring: Due to energy and communication constraints, only the data provided by a limited number of nodes (one node in this example) can be processed to estimate the number and states of vehicles on the roads. At any time, one out of a number of candidate nodes needs to be selected.

In sensor-selection problem, one or more selected sensors are the direct outputs of the decision-making component of the solution. As such, the focus has traditionally been placed on improving the decision-making component with underlying assumption that the multi-object filtering framework would return accurate estimates of the number and states of all targets. However, the multi-target tracking component also plays a significant role in the overall performance of the scheme in terms of accuracy and robustness.

A number of sensor-selection and sensor-control solutions have been recently developed for multi-target tracking scenarios within Finite Set Statistics (FISST) [Mahler 2007b, ?] framework [Ristic and Vo 2010, Ristic et al. 2011a, Gostar et al. 2013a;c;b, Gia Hoang and Tuong Vo 2013, Gostar et al. 2014]. Ristic et al [Ristic and Vo 2010] introduced a sensor-control solution to work with an FISST-based multi-object Bayesian filter. This solution is only computationally tractable in presence of few targets. Later, same authors presented a computationally tractable solution that would optimally select control commands within a PHD-based filter [Mahler 2003b;

As it was mentioned before, an important component of common sensor-selection solutions is a decision-making process, and most solutions decide on selecting a sensor node via optimization of an objective function. To define an appropriate objective function, two common approaches have been introduced: task-driven and information-driven approaches. In the former approach, the objective function is formulated as a cost function which usually depends on performance metrics such as variance of state and cardinality estimates and other distribution-dependent measures. In the latter, the objective function is a reward function that is directly related to the information content of the multi-object distribution. The most common choices of information-driven reward functions are based on some measure of information gain [Aoki et al. 2011a]. Examples of such measures are Kullback-Leibler divergence [Manyika and Durrant-Whyte 1995, Schmaedeke and Kastella 1994, Kastella 1997], Rényi divergence [Kreucher et al. 2005, Ristic and Vo 2010, Ristic et al. 2011a] and the Cauchy-Schwarz divergence between the probability densities of two Poisson point processes [Hoang et al. 2013]. It is important to note that in these solutions, clutter rate and uncertainty in sensor Field of View (FoV) are assumed to be known. However, accurate measures of these parameters are commonly unavailable in practical situations.

In our previous work [Gostar et al. 2013a], a sensor-selection solution was pro-
posed that is based on minimizing a cost function defined in terms of estimation uncertainty of cardinality of multi-target state (with estimation uncertainty quantified by statistical variance). Around the same time, a similar approach was proposed by Hoang et al [Gia Hoang and Tuong Vo 2013], based on using a variance-based cost function. The main difference is that in [Gostar et al. 2013a], statistical variance of cardinality around its mean is chosen as the cost function, while Hoang et al [Gia Hoang and Tuong Vo 2013] used the variance of cardinality around its MAP estimate as the cost function. Later, we revised and extended our previous work [Gostar et al. 2013a], defining a novel cost function for sensor control, called Posterior Expected Error of Cardinality and States (PEECS). As outlined by its name, the cost function is comprised of separate normalized error terms for cardinality and state estimates [Gostar et al. Accepted, to appear in 2015].

This paper presents a new sensor-selection solution to work within a robust multi-Bernoulli-based multi-target tracking framework where no prior knowledge of the clutter distribution or the probability of detection profile is available. The objective function of the presented method is a new task-driven cost function, called robust PEECS (or R-PEECS for short), which is reformulated from PEECS cost function to specifically suit the robust multi-Bernoulli object filtering framework (with extended states and clutter generators). Sequential Monte Carlo (SMC) implementation of the complete robust sensor selection is also presented. As another contribution, this paper introduces the SMC implementation of a different sensor-selection solution within the robust multi-Bernoulli filtering framework, that uses Rényi divergence as the objective function. These solutions are implemented and compared in challenging scenarios. The comparative simulation results show that while the solution using the task-driven R-PEECS cost function produces similar accuracies to the information-driven method, its computational cost is substantially lower.

The paper is written to be self-sufficient as much as possible. Therefore, since
the proposed sensor-selection method is formulated in the robust multi-Bernoulli framework, this framework is briefly reviewed in section 5.2. The PEECS cost and its robust extension, R-PEECS, as well as the overall sensor-selection framework are then presented in section 5.3. This section also includes SMC implementation of R-PEECS sensor selection and the information-driven method in detail. Simulation results are presented and discussed in section 5.5, followed by the conclusion presented in Section 5.6.

## 5.2 Robust Multi-Bernoulli Filtering

A Bernoulli random finite set (RFS) is a set that can be either empty or a singleton. Such an RFS is completely characterized by its probability of existence, \( r \), and probability density function of its possible element, \( p(\cdot) \). A multi-Bernoulli RFS is the union of \( M \) independent Bernoulli RFSs, each with given parameters \( r^{(i)} \) and \( p^{(i)}(\cdot) \) \( (i = 1, \ldots, M) \). Mahler showed that all statistical properties of a multi-Bernoulli RFS can be completely characterized by the set of \( M \) parameter pairs [Mahler 2007b; 2014]. He also introduced the idea of approximating the posterior density as a multi-Bernoulli distribution.

Vo et al [Vo et al. 2009] presented Cardinality-Balanced MeMBer (CB-MeMBer) filter to tackle the problem of bias in cardinality estimates produced by the original MeMBer filter [Mahler 2007b; 2014]. Other examples of filters approximated by multi-Bernoulli distribution are labeled multi-Bernoulli filter and its variations [Vo and Vo 2013, Vo et al. 2014, Reuter et al. 2014]. In these filters, as implemented in the above mentioned works, clutter intensity and probability of detection profile are assumed to be known a priori. Vo et al [Vo et al. 2013] have recently proposed a new implementation of the multi-Bernoulli filter that does not need any knowledge of clutter intensity and detection profile. The underlying idea is as follows.

The unknown probability of detection is separately estimated and propagated
by modifying the original filter via augmenting each state $x$ with a new element $a$ that represents the probability of detection in $[0, 1]$. The CB-MeMBer filter is also extended to incorporate a set of clutter generators which are implemented by multi-Bernoulli assumption for clutter generator random sets and modeling their behavior similar to actual objects in a hybrid space denoted by $\hat{X} = \mathcal{X}^{(0)} \cup \mathcal{X}^{(1)}$ (0 for clutter generators state space and 1 for actual objects state space). In this setup, the clutter generator has its own transition and observation model [Vo et al. 2013].

The sequential Monte Carlo (SMC) implementation of the proposed solution has been also presented in [Vo et al. 2013]. For the sake of completion, the SMC formulation of the robust multi-Bernoulli filter is presented as follows – see [Vo et al. 2013] for more details.

Assume that at time $k − 1$, the posterior multi-Bernoulli density of the multi-object state is given by $(r_{k-1}^{(i)}, p_{k-1}^{(i)(0)}, p_{k-1}^{(i)(1)})_{i=1}^{M_{k-1}}$ in which $p_{k-1}^{(i)(u)}$ is the density of the actual object state for $u = 1$, or the density of the clutter generator state for $u = 0$. These densities are approximated by sets of particles:

$$p_{k-1}^{(i)(u)}(a, x) = \sum_{j=1}^{P_{k-1}^{(i)(u)}} w_{(k-1)}^{(i,j)(u)} \delta_{a_{k-1}, x_{k-1}^{(i,j)(u)}}(a, x). \quad (5.1)$$

For existing and new born objects, two proposal densities are given and denoted by $q_{k}^{(i)(a)}(\cdot|a_{k-1}, x_{k-1}^{(u)}, Z_{k})$ and $b_{k}^{(i)(a)}(\cdot|Z_{k})$. The predicted multi-object density is also multi-Bernoulli and represented by the union of the survived objects and the newly born objects:

$$\pi_{k|k-1} = \prod_{u=0,1} \left\{ (r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)(u)}) \right\}_{i=1}^{M_{\Gamma,k}} \cup \left\{ (r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)(u)}) \right\}_{i=1}^{M_{k-1}} \quad (5.2)$$

where existence probabilities and distributions of the predicted Bernoulli components are given by:
\[ r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)} \sum_{u=0,1} L_{k-1}^{(i)(u)} \sum_{j=1}^{L_{k-1}^{(i)(u)}} w_{k-1}^{(i,j)(u)} p_{P,k}^{(u)}(x_{k-1}^{(i,j)(u)}) \]

\[ p_{P,k|k-1}^{(i)(u)}(a, x) = \sum_{j=1}^{L_{k-1}^{(i)(u)}} u_{P,k|k-1}^{(i,j)(u)} \delta_{a_{P,k|k-1}^{(i,j)(u)} x_{P,k|k-1}^{(i,j)(u)}}(a, x) \] (5.3)

\[ r_{\Gamma,k}^{(i)} = \text{birth model parameters} \]

\[ P_{\Gamma,k}^{(i)(u)}(a, x) = \sum_{j=1}^{L_{\Gamma,k}^{(i)(u)}} u_{\Gamma,k}^{(i,j)(u)} \delta_{a_{\Gamma,k}^{(i,j)(u)} x_{\Gamma,k}^{(i,j)(u)}}(a, x) \]

where for \( u = 0, 1 \)

\[
\begin{align*}
\delta_{a_{P,k|k-1}^{(i,j)(u)}, x_{P,k|k-1}^{(i,j)(u)}}, \; &j = 1, \ldots, L_{k-1}^{(i)(u)} \\
w_{P,k|k-1}^{(i,j)(u)} &\sim a_{P,k|k-1}^{(i,j)(u)} \quad (a_{P,k|k-1}^{(i,j)(u)}, x_{P,k|k-1}^{(i,j)(u)}, Z_k) \\
&\quad , 
\end{align*}
\]

\[
\begin{align*}
\delta_{a_{P,k|k-1}^{(i,j)(u)}, x_{P,k|k-1}^{(i,j)(u)}}, \; &j = 1, \ldots, L_{\Gamma,k}^{(i)(u)} \\
w_{\Gamma,k}^{(i,j)(u)} &\sim a_{\Gamma,k}^{(i,j)(u)} \quad (a_{\Gamma,k}^{(i,j)(u)}, x_{\Gamma,k}^{(i,j)(u)}, Z_k) \\
&\quad , 
\end{align*}
\]

\[
\begin{align*}
\delta_{a_{\Gamma,k}^{(i,j)(u)}, x_{\Gamma,k}^{(i,j)(u)}}, \; &j = 1, \ldots, L_{\Gamma,k}^{(i)(u)} \\
w_{\Gamma,k}^{(i,j)(u)} &\sim a_{\Gamma,k}^{(i,j)(u)} \quad (a_{\Gamma,k}^{(i,j)(u)}, x_{\Gamma,k}^{(i,j)(u)}, Z_k) \\
&\quad , 
\end{align*}
\]

If at time \( k \), the predicted multi-Bernoulli distribution is given by \( (r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)}(a, x)) \) and \( p_{k|k-1}^{(i)(u)}(a, x) = \sum_{j=1}^{L_{k|k-1}^{(i)(u)}} w_{k|k-1}^{(i,j)(u)} \delta_{a_{k|k-1}^{(i,j)(u)} x_{k|k-1}^{(i,j)(u)}}(x) \), then updated multi-Bernoulli is represented by the union of legacy tracks and measurement-corrected tracks as follows [Vo et al. 2013]:

\[
\pi_k = \left\{ (r_{L,k}^{(i)}, p_{L,k}^{(i)}) \right\}_{i=1}^{M_{k|k-1}} \cup \left\{ (r_{U,k}(z), p_{U,k}(\cdot; z)) \right\}_{z \in Z_k} \] (5.5)
with the following existence probabilities and singleton densities:

\[
\begin{align*}
    r_{L,k}^{(i)} &= \sum_{u=0,1} r_{L,k}^{(i)(u)} \\
    r_{L,k}^{(i)(u)} &= r_{k|k-1}^{(i)} \frac{(\tilde{q}_{L,k}^{(i)(u)}) - q_{L,k}^{(i)(u)}}{1 - r_{k|k-1}^{(i)}} \sum_{\ell=0,1} q_{L,k}^{(i)(\ell)} \\
    p_{L,k}^{(i)(u)}(a, x) &= \sum_{j=1}^{L^{(i)(u)}_{k|k-1}} w_{L,k}^{(i,j)(u)} \delta_{a^{(i,j)(u)}_{k|k-1} x^{(i,j)(u)}_{k|k-1}}(a, x) \\
    r_{U,k}(z) &= \sum_{u=0,1} r_{U,k}^{(u)}(z) \\
    r_{U,k}^{(u)}(z) &= \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} \left(1 - r_{k|k-1}^{(i)} \right) q_{U,k}^{(i)(u)}(z)}{\left(1 - r_{k|k-1}^{(i)} \sum_{\ell=0,1} q_{L,k}^{(i)(\ell)} \right)^2} \\
    p_{U,k}^{(u)}(a, x; z) &= \sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{L^{(i)(u)}_{k|k-1}} w_{U,k}^{(i,j)(u)}(z) \delta_{a^{(i,j)(u)}_{k|k-1} x^{(i,j)(u)}_{k|k-1}}(a, x)
\end{align*}
\]

where for \( u = 0, 1 \)
SECTION 5.3: SENSOR-SELECTION FRAMEWORK

\[ \hat{\rho}_{L,k}^{(i)}(u) = \sum_{j=1}^{L_{k|k-1}^{(i)(u)}} w_{k|k-1}^{(i,j)(u)} \]

\[ \hat{\theta}_{L,k}^{(i)}(u) = \sum_{j=1}^{L_{k|k-1}^{(i)(u)}} w_{k|k-1}^{(i,j)(u)} (a_{k|k-1}^{(i,j)(u)}) \]

\[ \tilde{w}_{L,k}^{(i,j)}(u) = w_{L,k}^{(i,j)(u)} \left( \sum_{u' \in \{0,1\}} \sum_{j'=1}^{L_{k|k-1}^{(i')(u')}} w_{L,k}^{(i,j')(u')} \right) \]

\[ w_{L,k}^{(i,j)}(u) = w_{L,k}^{(i,j)(u)} \left( 1 - \left( a_{k|k-1}^{(i,j)(u)} \right) \right) \]  

(5.7)

\[ \hat{\theta}_{U,k}^{(i)}(z) = \sum_{j=1}^{M_{k|k-1}^{(i)(u)}} w_{U,k}^{(i,j)(u)} g_k^{(u)} (z|x_{k|k-1}^{(i,j)(u)}) a_{k|k-1}^{(i,j)(u)} \]

\[ \hat{w}_{U,k}^{(i,j)}(z) = w_{U,k}^{(i,j)(u)} \left( \sum_{u' \in \{0,1\}} \sum_{i'=1}^{M_{k|k-1}^{(i')(u')}} \sum_{j'=1}^{L_{k|k-1}^{(i')(u')}} w_{U,k}^{(i',j')(u')} (z) \right) \]

\[ w_{U,k}^{(i,j)}(z) = \frac{r_{k|k-1}^{(i)(0)} \cdot w_{k|k-1}^{(i,j),(u)} g_k^{(u)} (z|x_{k|k-1}^{(i,j)(u)}) a_{k|k-1}^{(i,j)(u)}}{1 - r_{k|k-1}^{(i)(0)}} \]  

Having the updated multi-Bernoulli parameters, EAP estimates of the cardinality and state of the multi-Bernoulli set can be calculated in a similar fashion to CB-MeMBer but using the parameters with \( u = 1 \). EAP estimate of the clutter intensity can also be calculated by [Vo et al. 2013]

\[ \hat{\lambda}_k = \sum_{i=1}^{M_k} r_{k|k-1}^{(i)(0)} \sum_{j=1}^{L_{k|k-1}^{(i)(0)}} w_{k|k-1}^{(i,j)(0)} a_{k|k-1}^{(i,j)(0)}. \]  

(5.8)

5.3 Sensor-Selection Framework

In this paper, we formulate the sensor-selection problem in the Partially Observable Markov Decision Processes (POMDPs) framework in conjunction with a robust multi-Bernoulli filter [Vo et al. 2013] to concurrently solve the sensor-selection and multi-target tracking problems with unknown clutter intensity and sensor FoV.

POMDP is a generalized form of Markov Decision Process (MDP) [Kaelbling et al. 1998] which is a suitable framework for selecting an ideal sensor node by applying a series of decision criteria for a set of candidate nodes. Generally, this
is a problem with infinite horizon and its implementation for the large state space
problems is prohibitively expensive. Thus, in this paper POMDP framework is
implemented following a myopic policy—with one step-ahead planning. For the sake
of simplicity, we also assume that only one sensor node is chosen at each time
instance.\footnote{Generalization of this method for optimal selection of more than one sensor node is straightforward.}

In POMDP framework, there is no direct access to states, and decisions are only
made using uncertain observations. At any time step $k$, a POMDP can be defined
as a tuple

$$\Psi = \{X_k, S, f_{k|k-1}(x_k|x_{k-1}), Z, g_k(z|x; s), \vartheta(s, X_k)\},$$

where $X_k$ is a finite set of single-object states, $S$ defines an action space which here is
a finite set of candidate nodes, $f_{k|k-1}(x_k|x_{k-1})$ is a transition model for single-object
state, $Z$ comprises a finite set of observations, $g_k(z|x; s)$ is a stochastic measurement
model which is now explicitly dependent on the state of the sensor candidate $s \in S$
(note the difference from the simple notation $g_k(z|x)$ used so far), and $\vartheta(s, X_k)$ is
an objective function which returns either a reward or a cost for the multi-object
state transition from $X_{k-1}$ to $X_k$ by selecting a sensor node $s \in S$. In principle, in
POMDP framework, the action space is infinite and continuous. However, to reduce
computational complexity, we assume that the sensor node(s) can be chosen from a
finite set of admissible sensor nodes [Braziunas 2003].

The purpose of sensor-selection is to find the ideal node $\hat{s}$ that optimizes the
objective function. In stochastic filtering where the multi-object states $X_{k-1}$ and
$X_k$ are characterized by their distributions, the ideal node $\hat{s}$ is commonly chosen to
optimize the statistical expectation of the cost or reward function $\vartheta(s, X_k)$ over all
measurements,

$$\hat{s} = \arg\min_{s \in S} \frac{\arg\max_{s \in S} \{E_z [\vartheta(s, X_k)]\}}{\{E_z [\vartheta(s, X_k)]\}}. \tag{5.9}$$
5.3.1 Robust multi-Bernoulli sensor-selection

Assume that the predicted multi-Bernoulli distribution, denoted by:

\[ \pi_{k|k-1} = (r_{k|k-1}^{(i)}, p_{k|k-1}^{(u)})^{M_{k|k-1}}_{i=1}, \quad u \in \{0, 1\}, \]

is approximated by \( L_{k|k-1}^{(u)} \) particles as described in (5.2) and (5.3). The aim is to find the most desirable (as defined by an objective function) sensor node from which measurements would be acquired and used to update the multi-object state, by optimizing the objective function. We will present our proposed objective function in Section 5.4.1 and the implementation of Rényi divergence function in Section 5.4.3.

The objective function is a function of future measurements (via the updated distribution), which has stochastic variations. Since, the values of the future measurements are unknown, a set of pseudo-measurements for each hypothesized sensor node is generated. In principle, the whole distribution of all possible measurement sets is used to compute the objective function. However, using the whole measurement distribution is computationally expensive. To reduce the computational cost, for each sensor node, only the predicted ideal measurement set (PIMS)\(^2\) is generated. Given the selected sensor node, the PIMS is comprised of the clutter-free and noise-free measurements that are most likely to be obtained from the selected sensor. Algorithm 3 shows a pseudocode to generate the PIMS returned by a selected sensor node. In section 5.5, we will present a measurement model and explain how the PIMS can be computed for a sensor at a given location.

Having a PIMS for each candidate node \( s \), the multi-Bernoulli multi-object density is updated using the PIMS corresponding to that sensor node. From each updated multi-object density, the objective function described in Section 5.4 is calculated and the most rewarding sensor node is then selected. A block diagram representing the steps involved in the proposed POMDP approach to sensor-selection

\(^2\)In situations where the probability of detection is high and clutter rate is moderate, the multi target posterior density can be approximated under the assumptions of the PIMS (see [Mahler 2004]–section 4.1 and [Ristic 2013]).

Inputs: Estimates of the number of targets $\hat{M}_{k|k-1}$ and multi-target state $\hat{X}_{k|k-1} = \{x_{k|k-1}^{(\ell)}\}_{\ell=1}^{\hat{M}_{k|k-1}}$ after a prediction step, a sensor node $s \in S$, and a single-target measurement likelihood function $g(z|x;s)$ which is dependent on the location of the sensor node $s$.

Output: A clutter-free set of noise-free measurements, $\hat{Z}_s$.

1: function PIMS($\hat{M}_{k|k-1}, \hat{X}_{k|k-1}, s, g(z|x;s)$)
2:  $\hat{Z}_s \leftarrow \emptyset$.
3:  for $\ell = 1, \ldots, \hat{M}_{k|k-1}$ do
4:     $\xi \leftarrow \arg \max_z g(z|x^{(\ell)}_{k|k-1};s)$
5:     $\hat{Z}_s \leftarrow \hat{Z}_s \cup \{\xi\}$
6:  end for
7:  return $\hat{Z}_s$
8: end function

using PIMS method is shown in Fig. 5.2. Note that in order to discriminate the two update blocks in each iteration, when the multi-object distribution is updated using the PIMS, we use the term “pseudo-update”, and when the measurement set acquired from the selected sensor node is used to update the multi-object distribution, we use the term “update”.

5.4 Objective Function

In order to select the best sensor node, an objective function is defined and computed in each iteration, as depicted in Fig. 5.2. As it was mentioned earlier, there are two approaches to define an objective function, namely the task-driven and information-driven approaches. In the task-driven approach, the objective function is usually a cost function that is directly related to the performance of the multi-object filter. A common performance measure is the accuracy of estimation. However, since the cost function for estimation is generally intractable, we introduce a simpler cost function named Robust-Posterior Expected Error of Cardinality and States (R-PEECS), which is the statistical expectation of an error calculated over all possible updated multi-
object states. The definition of this cost function is described in Section 5.4.1, and its intuition and formulation are presented in Section 5.4.2.

A common choice for the information-driven approach is Rényi divergence, which has been employed as a reward function in [Kreucher et al. 2005, Ristic and Vo 2010, Ristic et al. 2011a]. To compare the performance of Rényi divergence objective function with R-PEECS, we have re-formulated Rényi divergence in conjunction with the robust multi-Bernoulli filter within an SMC implementation to form an alternative sensor-selection solution. The details of the SMC formulation and implementation of the Rényi divergence are presented in Sections 5.4.3 and 5.4.4, respectively.
CHAPTER 5: MULTI-BERNOLULLI SENSOR-SELECTION FOR MULTI-TARGET TRACKING WITH UNKNOWN CLUTTER AND DETECTION PROFILES

5.4.1 R-PEECS Cost function

Let us assume that PIMS for each candidate sensor node \( s \in S \) is generated, and using each PIMS as the actual measurement, the multi-object distribution is updated (pseudo-update). The proposed cost function, called R-PEECS is then directly computed from the updated distribution parameters. The cost is defined as a linear combination of the statistical mean of three normalized errors, in terms of statistical variances: number of targets, their states, and clutter intensity. Denoting the normalized error terms by \( \varepsilon_{|S|}^2(s), \varepsilon_{X}^2(s), \varepsilon_{\lambda}^2(s) \), the R-PEECS cost function is defined as:

\[
\vartheta V(s,X_k) = \eta_{|S|} \varepsilon_{|S|}^2(s) + \eta_{X} \varepsilon_{X}^2(s) + \eta_{\lambda} \varepsilon_{\lambda}^2(s),
\]

(5.10)

where the positive coefficients \( \eta_{|S|}, \eta_{X} \) and \( \eta_{\lambda} \) are user-defined importance weights for minimization of each error terms. In order to have a normalized weighted sum of the error terms, the weights are chosen to satisfy \( \eta_{|S|} + \eta_{X} + \eta_{\lambda} = 1 \). The most rewarding sensor node is then selected by minimizing the above cost function over all \( s \in S \):

\[
\hat{s} = \arg \min_{s \in S} \vartheta(s,X_k).
\]

(5.11)

5.4.2 Implementation of the error terms

The proposed cost function defined in (5.10) is a sum of three normalized error measures that intuitively capture the overall accuracy of the multi-object estimation. The precise definition of each term and its normalization procedure are as follows.

**Normalized error of cardinality estimate** \( (\varepsilon_{|S|}^2) \): In [Delande et al. 2014a] and [Delande et al. 2014b], Delande et al showed that the regional variance of the number of targets quantifies the certainty of the filter estimation of the number of targets that evolve in surveillance region. Following [Delande et al. 2014a;b], we quantify the error associated with the cardinality estimation by the statistical expectation of the
SECTION 5.4: OBJECTIVE FUNCTION

cardinality variance. Having an updated multi-Bernoulli distribution parameters at each step, this variance can be computed as:

\[ \sigma^2_{|X|}(s) = \sum_{i=1}^{M_{k,s}} \left[ r_{k,s}^{(i)(1)}(1 - r_{k,s}^{(i)(1)}) \right]. \tag{5.12} \]

We note that only the updated existence probabilities of actual targets (i.e, the set \( \{r_{k,s}^{(i)(u)}\}_{i=1}^{M_{k,s}} \) with \( u = 1 \)) are used to compute the variance. This is to ensure that only the estimation accuracy of the number of actual targets (and not clutter generator objects) is captured by this measure. The above error variance is maximum when \( \forall i, r_{k,s}^{(i)(1)} = 0.5 \), which leads to \( \max\{\sigma^2_{|X|}(s)\} = \frac{M_{k,s}}{4} \). Thus, a normalized form of the cardinality error term can be computed as follows:

\[ \varepsilon^2_{|X|}(s) = 4 \times \frac{\sigma^2_{|X|}(s)}{M_{k,s}}. \tag{5.13} \]

Normalized error of state estimates: The normalized multi-target state error term \( \varepsilon^2_X(s) \) of the cost function (5.10) is defined as a weighted average of estimation errors of the states of all targets:

\[ \varepsilon^2_X(s) = \sum_{i=1}^{M_{k,s}} \left[ r_{k,s}^{(i)(1)} \zeta^2_x(s) \right] / \sum_{i=1}^{M_{k,s}} r_{k,s}^{(i)(1)}. \tag{5.14} \]

Since Bernoulli components with larger probabilities of existence have larger contributions to the EAP estimate of the multi-object state, weights in (5.14) are chosen to be the (pseudo) updated probabilities of existence for the actual targets \( (u = 1) \).

One way to compute the single Bernoulli component errors \( \zeta^2_{x(i)}(s) \) is to minimize the estimation error of the main states of interest for the given application. In target tracking applications, the main state of interest is usually the location, and target speed is included in the single-target state vector because it appears in the motion and measurement models. In those applications, an intuitive scalar measure for the single Bernoulli component error would be the product of the variances of the target location coordinates. Our intuition stems from the fact that if the stochastic
variations of target location coordinates were independent, this measure would have been translated into the absolute determinant of the covariance matrix of the target location.

Using the above intuition, for tracking multiple-targets in two–dimensional space, the single Bernoulli component error term, $\varphi_{x(t)}(s)$, would be given by:

$$\varphi_{x(t)}(s) \propto \sigma_{x(t)}^2(s) \sigma_{y(t)}^2(s)$$  \hspace{1cm} (5.15)

where $x$ and $y$ denote the horizontal and vertical coordinates of the single-target location (part of its state vector $x$), and the proportionality (instead of equality) is chosen because the above product does not lead to a normalized measure. Having the updated particles and weights of each Bernoulli component, the single-coordinate errors can be calculated as follows:

$$\sigma_{x(t)}^2(s) = \sum_{j=1}^{L_{k,s}^{(1)}} w_{k,s}^{(i,j)(1)} x_{k,s}^{(i,j)(1)} - \left( \sum_{j=1}^{L_{k,s}^{(1)}} w_{k,s}^{(i,j)(1)} x_{k,s}^{(i,j)(1)} \right)^2$$

$$\sigma_{y(t)}^2(s) = \sum_{j=1}^{L_{k,s}^{(1)}} w_{k,s}^{(i,j)(1)} y_{k,s}^{(i,j)(1)} - \left( \sum_{j=1}^{L_{k,s}^{(1)}} w_{k,s}^{(i,j)(1)} y_{k,s}^{(i,j)(1)} \right)^2$$  \hspace{1cm} (5.16)

where $x_{k,s}^{(i,j)(1)}$ and $y_{k,s}^{(i,j)(1)}$ denote the coordinates extracted from the particle $x_{k,s}^{(i,j)(1)}$. To normalize the total state estimation error term $\varphi_{x(t)}^2(s)$ in (5.15), we note that with equally weighted particles, i.e when $\forall j, w_{k,s}^{(i,j)} = 1/L_{k,s}^{(1)}$, the particles representing the $i$-th single Bernoulli component do not convey any information and the above estimation variances adopt their maximum values, given by:

$$\max\{\sigma_{x(t)}^2(s)\} = \frac{1}{L_{k,s}^{(1)}} \left( 1 - \frac{1}{L_{k,s}^{(1)}} \sum_{j=1}^{L_{k,s}^{(1)}} \left[ x_{k,s}^{(i,j)(1)} \right]^2 \right)$$

$$\max\{\sigma_{y(t)}^2(s)\} = \frac{1}{L_{k,s}^{(1)}} \left( 1 - \frac{1}{L_{k,s}^{(1)}} \sum_{j=1}^{L_{k,s}^{(1)}} \left[ y_{k,s}^{(i,j)(1)} \right]^2 \right).$$  \hspace{1cm} (5.17)

Thus, the single Bernoulli error terms $\varphi_{x(t)}^2(s)$ in (5.15) can be normalized as follows:

$$\varphi_{x(t)}^2(s) = \frac{\left( \sum_{j=1}^{L_{k,s}^{(1)}} \left[ x_{k,s}^{(i,j)(1)} \right]^2 \right)^2 \sigma_{x(t)}^2(s) \sigma_{y(t)}^2(s)}{\left( \sum_{j=1}^{L_{k,s}^{(1)}} \left[ x_{k,s}^{(i,j)(1)} \right]^2 \right)^2 \left( \sum_{j=1}^{L_{k,s}^{(1)}} \left[ y_{k,s}^{(i,j)(1)} \right]^2 \right)^2}$$  \hspace{1cm} (5.18)
Using the above values, the normalized state estimation error $\varepsilon_{X}^{2}$ can be calculated by (5.14). Extension of the terms derived in (5.15)–(5.18) to the cases involving more than two dimensional locations or non-location states (such as speed, acceleration, turn rate, etc.) is straightforward.

**Normalized error of the estimated clutter:** Using the updated particles for clutter generator objects ($u = 0$), the variance of clutter intensity estimate is approximated by [Vo et al. 2013]:

$$
\sigma_{\lambda}^{2}(s) = \sum_{i=1}^{M_{k,s}} \left( r_{k}^{(i)(0)} \sum_{j=1}^{L_{k}^{(i)(0)}} \left( r_{\lambda,k}^{(i,j)} (1 - r_{\lambda,k}^{(i,j)}) \right) \right)
$$

(5.19)

where $r_{\lambda,k}^{(i,j)} = w_{k}^{(i,j)(0)} a_{k}^{(i,j)(0)}$. Similar to the case of cardinality variance, here, the term $r_{\lambda,k}^{(i,j)} (1 - r_{\lambda,k}^{(i,j)})$ adopts its maximum value when $\forall i, j, r_{\lambda,k}^{(i,j)} = 0.5$, and all the clutter generators are existent, i.e $\forall i, r_{k}^{(i)(0)} = 1$. In such an extreme scenario, the above variance would be at its maximum given by $\frac{1}{4} \sum_{i=1}^{M_{k,s}} L_{k}^{(i)(0)}$. Thus, the normalized clutter error term can be computed as follows:

$$
\varepsilon_{\lambda}^{2}(s) = \frac{4 \sum_{i=1}^{M_{k,s}} \left( r_{k}^{(i)(0)} \sum_{j=1}^{L_{k}^{(i)(0)}} \left( r_{\lambda,k}^{(i,j)} (1 - r_{\lambda,k}^{(i,j)}) \right) \right)}{\sum_{i=1}^{M_{k,s}} L_{k}^{(i)(0)}}.
$$

(5.20)

Having all the three error terms computed for every candidate sensor node, the best node can then be chosen as the one returning the lowest cost. Actual measurements can then be acquired from the selected sensor node, and used to update the multi-object distribution. This process of filtering and sensor-selection can be iteratively repeated for every sampling time. We call the sensor-selection method that uses the R-PEECS cost function within a robust multi-Bernoulli filtering scheme, as R-PEECS sensor-selection. Algorithm 4 shows the step-by-step pseudocode for the robust multi-Bernoulli filter combined with R-PEECS sensor-selection solution.
Algorithm 4 The robust multi-Bernoulli multi-target filtering recursion with sensor-selection via R-PEECS

**Inputs:** time $k$, dynamic model $f_{k|k-1}(\cdot|x_{k-1})$, multi-Bernoulli birth model parameters, prior multi-Bernoulli parameters from time $k-1$, measurement likelihood function $g_k(\cdot|x; s)$, currently selected sensor node $\hat{s}_{k-1}$, finite set of sensor candidates $S$, number of Monte Carlo samples of measurements $L_z$.

**Output:** The best sensor node(s) $\hat{s}_k$ and updated multi-Bernoulli parameters.

**Prediction:**
1. Compute the predicted multi-Bernoulli component parameters and their particles $\{r_{k|k-1}^{(i)}, \{u_{k|k-1}^{(i,j)(u)}, x_{k|k-1}^{(i,j)(u)}\}_{j=1}^{M_{k|k-1}}\}$ for $u = 0, 1$ (eqs. (5.2), (5.3)).

- **Pre-estimation:**
  
  2. $\ell \leftarrow 0$
  3. $X_{k|k-1} \leftarrow \emptyset$
  4. **for** $i = 0, \ldots, M_{k|k-1}$ **do**
  5. **if** $r_{k|k-1}^{(i)} > 0.5$ **then**
  6. $r \leftarrow \sum_{j=1}^{L_z} u_{k|k-1}^{(i,j)(1)} x_{k|k-1}^{(i,j)(1)}$
  7. $\hat{X}_{k|k-1} \leftarrow \hat{X}_{k|k-1} \cup \{r\}$
  8. $\ell \leftarrow \ell + 1$
  9. **end if**
  10. **end for**
  11. $\hat{M}_{k|k-1} \leftarrow \ell$

- **Sensor-selection:**
  
  12. **for** all $s \in S$ **do**
  13. $\hat{Z} \leftarrow \text{PIMS}(M_{k|k-1}, \hat{X}_{k|k-1}, s, g(z|x; s))$
  14. Run the update step —eqs. (5.5)- (5.6)— with $\hat{Z}$ as measurement set.
  
- **Updated distribution parameters are dependent on $s$,**
  
- **Existence probabilities:** $\{r_{k|s}^{(i)(u)}\}_{i=1}^{M_{k|s}}$, Particles:

\[
\left\{ \left\{ w_{k|s}^{(i,j)(u)}, x_{k|s}^{(i,j)(u)} \right\} \right\}_{j=1}^{L_z}.
\]

15. $\varepsilon^2_{X(i)}(s) \leftarrow 4 \sum_{i=1}^{M_{k|s}} r_{k|s}^{(i)(1)} (1 - r_{k|s}^{(i)(1)}) / M_{k|s}$
16. **for** $i = 1, \ldots, M_{k|s}$ **do**
17. $\sigma^2_{x(i)}(s) \leftarrow \sum_{j=1}^{L_z} w_{k|s}^{(i)(j)} x_{k|s}^{(i)(j)}^2 - \left( \sum_{j=1}^{L_z} w_{k|s}^{(i)(j)} x_{k|s}^{(i)(j)} \right)^2$
18. $\sigma^2_{y(i)}(s) \leftarrow \sum_{j=1}^{L_z} w_{k|s}^{(i)(j)} y_{k|s}^{(i)(j)}^2 - \left( \sum_{j=1}^{L_z} w_{k|s}^{(i)(j)} y_{k|s}^{(i)(j)} \right)^2$
19. $\varepsilon^2_{z(i)}(s) \leftarrow \frac{\left( L_z^{(i)(1)} \right)^4 \sigma^2_{z(i)}(s) \sigma^2_{y(i)}(s)}{\left( 1 - L_z^{(i)(1)} \right)^2 \left( \sum_{j=1}^{L_z} w_{k|s}^{(i)(j)} x_{k|s}^{(i)(j)} \right)^2 \left( \sum_{j=1}^{L_z} \left[ x_{k|s}^{(i)(j)} \right]^2 \right)}$
20. **end for**
5.4.3 Reward function

The information-driven approach to sensor-selection problem is based on choosing the sensor node that likely to result in a posterior multi-object density that has maximum information encapsulated in it [Aoki et al. 2011b]. Rényi divergence has recently appeared as the measure of choice since it effectively quantifies the reduction in entropy of the posterior distribution induced by measurements [Kreucher et al. 2005]. The Rényi divergence function directly measures the information difference between two density functions $f_1(\cdot)$ and $f_2(\cdot)$:

$$I_\alpha(f_1, f_2) \triangleq \frac{1}{\alpha - 1} \log \int f_1(X)^{1-\alpha} f_2(X)^{\alpha} \delta X.$$  \hfill (5.21)

In target tracking applications, the first distribution is the predicted multi-target distribution, $\pi_{k|k-1}(X|Z_{1:k-1})$ and the second is the updated distribution, $\pi_k(X|Z_{k,s})$. Thus, at any time $k$, the Rényi divergence from prediction to update is given by:

$$\mathcal{V}(s, X) = I_\alpha(f_1, f_2)$$

$$= \frac{1}{\alpha - 1} \log \int \left[ \pi_{k|k-1}(X|Z_{k,s}) \right]^{1-\alpha} \left[ \pi_k(X|Z_{k,s}) \right]^\alpha \delta X.$$  \hfill (5.22)
in which \( \alpha \) is a parameter to control the emphasis on tails of two distributions in the metric.

The objective is to maximize information gain between prediction and update by choosing the right future sensor node \( s \). It is noted that any choice of \( s \) can lead to a different measurement set \( Z_k \). A common approach is to define, compute and maximize the reward function as the statistical mean of the Rényi divergence function [Ristic and Vo 2010]. Branko et al [Ristic and Vo 2010] used Rényi divergence as a reward in conjunction with the general form of the FISST-based multi-target filter. This reward function was then approximated by:

\[
\mathcal{V}(s, X_k) \approx \frac{1}{\alpha - 1} \log \frac{\sum_{i=1}^{n} w_i^k [g_k(Z_k|X^i_{k|k-1}, s)]^\alpha}{\sum_{i=1}^{n} w_i^k g_k(Z_k|X^i_{k|k-1}, s)}.
\]  

(5.23)

We note that in order to compute the likelihood function \( g_k(Z_k|X^i_{k|k-1}, s) \), one requires the knowledge of clutter intensity and probability of detection profiles, and the derived formula is not directly applicable in our intended applications where the detection profile and clutter rate are not known. To compute Rényi divergence as part of a sensor-selection solution for the applications intended in this paper, we propose to use Monte Carlo sampling of both predicted and updated multi-object multi-Bernoulli densities as explained in Section 5.4.4.

5.4.4 Implementation of the reward function

Consider a given multi-Bernoulli distribution with its parameters denoted by \( \{r^{(i)}, p^{(i)}(\cdot)\}_{i=1}^{M} \) where the distribution of each Bernoulli component, \( p^{(i)}(\cdot) \) is approximated by \( L^{(i)} \) particles, i.e \( p^{(i)}(x) \approx \sum_{j=1}^{L^{(i)}} w^{(i,j)} \delta(x - x^{(i,j)}) \). After resampling each Bernoulli component distribution, \( L_{\text{max}} \) particles are created with all their weights equal to \( \frac{1}{L_{\text{max}}} \), and with some repeated particles depending on their original weights.\(^3\)

\(^3\)For simplicity of notation, we assume that the number of resampled particles is constant and independent of \( i \). The proposed MC sampling method can be extended to the general case where the number of resampled particles varies from one Bernoulli component to another.
We need to generate $L$ Monte Carlo samples of the above multi-Bernoulli distribution. Algorithm 5 shows our proposed approach to generate such a set samples.

Monte Carlo sampling of the updated multi-Bernoulli distribution results in $L$ sets, each in the form of
\[
X^\ell = \{x^\ell,1, \ldots, x^\ell,n^\ell\},
\]
with the following update density:
\[
\pi_k(X|\hat{Z}_s) \approx \frac{1}{L} \sum_{\ell=1}^{L} \delta(X - X^\ell),
\]
where $n^\ell \leq M$ is the cardinality of the $\ell$-th set. Assuming that the error term in the particle approximation given by (5.24) has a linear order of $O(L^{-1})$, we have:
\[
\forall h(\cdot), \int h(X)\pi_k(X|\hat{Z}_s) \, \delta X = \sum_{\ell=1}^{L} \frac{1}{L} h(X^\ell) + O(L^{-1}).
\]
Substituting $h(X)$ with $\left[\frac{\pi_{k|k-1}(X)}{\pi_k(X|\hat{Z}_s)}\right]^{1-\alpha}$ leads to:
\[
\mathcal{V}(s, X) = \frac{1}{\alpha - 1} \log \int \left[\frac{\pi_k(X|\hat{Z}_s)}{\pi_{k|k-1}(X)}\right]^\alpha \left[\frac{\pi_{k|k-1}(X)}{\pi_k(X|\hat{Z}_s)}\right]^{1-\alpha} \, \delta X
\]
\[
= \frac{1}{\alpha - 1} \log \left[\frac{1}{L} \sum_{\ell=1}^{L} \left[\frac{\pi_{k|k-1}(X^\ell)}{\pi_k(X^\ell|\hat{Z}_s)}\right]^{1-\alpha}\right] + O(L^{-1}).
\]
In Eq. (5.26), the multi-Bernoulli distribution terms can be directly calculated, and the reward function is then computed by discarding the $O(L^{-1})$ terms. For general multi-Bernoulli parameters $\{r^{(i)}, p^{(i)}(\cdot)\}_{i=1}^{M}$, the density at $X = X^\ell$ is given by [Mahler 2007b] p. 369:
\[
\pi_{\tau}(\emptyset) = \prod_{i=1}^{M} (1 - r^{(i)}),
\]
\[
\pi_{\tau}(X^\ell) = \pi_{\tau}(\emptyset) \sum_{1 \leq i_1 < \ldots < i_j \leq M} \prod_{j=1}^{[X^\ell]} \frac{r^{(i_j)} p^{(i_j)}(x^\ell_{i_j})}{1 - r^{(i_j)}}.
\]
It is important to note that in the above calculations for computing the R\`enyi divergence, we only consider the target-generated Bernoulli components, i.e the components with $u = 1$. For the sake of simplicity, the $u$ indices are omitted from formulations given in this section. In the following, we describe how the density terms $p^{(ij)}(\cdot)$, are calculated for the cases: (1) $\tau = k$ and (2) $\tau = k|k - 1$. 

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CHAPTER 5: MULTI-BERNOULLI SENSOR-SELECTION FOR MULTI-TARGET TRACKING WITH UNKNOWN CLUTTER AND DETECTION PROFILES

Algorithm 5 Monte Carlo sampling of a multi-Bernoulli distribution with given parameters and particles.

**Inputs:** probabilities of existence \( r = [r_1 \cdots r_M]^T \), particles matrix \( P = [x_{ij}]_{M \times L_{\text{max}}} \), and number of output samples (sets) denoted by \( L \).

**Outputs:** A set \( X \) comprised of \( L \) sets, each being a Monte Carlo sample of the multi-Bernoulli distribution, in the form of \( X_\ell = \{x_{\ell,1}, \ldots, x_{\ell,n_\ell}\} \), where \( n_\ell \leq M \) is the cardinality of the \( \ell \)-th set.

1: function MB-SMC\((r,P,L)\)
2: \( X \leftarrow \emptyset \)
3: From the size of the particles matrix \( P \), find \( M \) and \( L_{\text{max}} \).
4: for \( \ell = 1, \ldots, L \) do
5: \( X_\ell \leftarrow \emptyset \)
6: for \( i = 1, \ldots, M \) do
7: \( u \sim \mathcal{U}(0,1). \)
8: if \( u < r_i \) then
9: \( v \sim \mathcal{U}(0,1). \)
10: \( j \leftarrow \lceil L_{\text{max}}v \rceil. \)
11: \( X_\ell \leftarrow X_\ell \cup \{x_{ij}\}. \)
12: end if
13: end for
14: \( X \leftarrow X \cup \{X_\ell\}. \)
15: end for
16: return \( X \)
17: end function

**Case 1:** \( \tau = k \)

In order to calculate the \( p_k^{(i,j)}(x_{\ell,j}) \) terms, we note that each \( X_\ell \) is a Monte Carlo sample of the updated distribution, and from Algorithm 5, we note that the elements \( x_{\ell,j} \) of the sample set \( X_\ell \) each must coincide with one of the particles approximating one of the Bernoulli components of the updated multi-Bernoulli multi-object distribution. That particle can be found and its weight would quantify the \( p_k^{(i,j)}(x_{\ell,j}) \) density term. In other words, if \( \left\{ w_k^{(i,j)}(x_{k,j}^{(i,j)}) \right\}_{j=1}^{L_k^{(i,j)}} \) represents a particle approximation of the \( i_j \)-th Bernoulli component of the updated multi-Bernoulli distribution (before resampling), we have:
Case 2: $\tau = k|k - 1$

In order to calculate the $p^{(ij)}_\tau(x_{\ell,j})$ terms for $\tau = k|k - 1$, we note that there is no guarantee that the $x_{\ell,j}$ component of the $\ell$-th Monte Carlo sample set coincides with any of the particles that approximate the $i_j$-th Bernoulli component in the predicted multi-Bernoulli distribution. Thus, we suggest that among all the particles representing the $i_j$-th Bernoulli component of the predicted distribution, the nearest to $x_{\ell,j}$ is found and its weight is used to quantify the density term $p^{(ij)}_{k|k-1}(x_{\ell,j})$. Indeed, if \[ \{w^{(ij,j)}_{k|k-1}, x^{(ij,j)}_{k|k-1}\}_{j=1}^{L_{k|k-1}} \] represents a particle approximation of the $i_j$-th Bernoulli component of the predicted multi-Bernoulli distribution, we have:

\[
p^{(ij)}_{k|k-1}(x_{\ell,j}) = \begin{cases} 
0 & \text{if } D_M \left( x_{\ell,j}, \{x^{(ij,j)}_{k|k-1}\}_{j=1}^{L_{k|k-1}} \right) > D_{\text{max}} \\
w^{(ij,j)}_{k|k-1} & \text{otherwise}
\end{cases}
\]  

(5.29)

where $D_M(x_{\ell,j}, \{x^{(ij,j)}_{k|k-1}\}_{j=1}^{L_{k|k-1}})$ represents the Mahalanobis distance between the point $x_{\ell,j}$ and the set of particles $\{x^{(ij,j)}_{k|k-1}\}_{j=1}^{L_{k|k-1}}$, and

\[
j = \arg\min_{j=1:L_{k|k-1}} d_M(x_{\ell,j}, x^{(ij,j)}_{k|k-1})
\]  

(5.30)

where $d(x_{\ell,j}, x^{(ij,j)}_{k|k-1})$ represents the normalized distance between the two points $x_{\ell,j}$ and $x^{(ij,j)}_{k|k-1}$ in which the normalization is based on the statistics (mean and covariance) of the set of particles $\{x^{(ij,j)}_{k|k-1}\}_{j=1}^{L_{k|k-1}}$ as follows:

\[
d(x_{\ell,j}, x^{(ij,j)}_{k|k-1}) = \left( \frac{x_{\ell,j} - \mu^{(ij)}_{k|k-1}}{\sqrt{\Sigma^{(ij)}_{k|k-1}}} \right)^\top \left( \Sigma^{(ij)}_{k|k-1} \right)^{-1} \left( \frac{x_{\ell,j} - \mu^{(ij)}_{k|k-1}}{\sqrt{\Sigma^{(ij)}_{k|k-1}}} \right)
\]  

(5.31)
in which $\mu^{(ij)}_{k|k-1}$ and $S^{(ij)}_{k|k-1}$ denote the statistical mean and covariance matrix of the particles approximating the density of the $i_j$-th predicted Bernoulli component of the filter, respectively.

The constant parameter $D_{\text{max}}$ in equation (5.29) is the thresholding maximum distance used to rule out any particles within $X_\ell$ that are far from the ensemble of particles $\{x^{(ij)}_{k|k-1}\}_{j=1}^{L_{k|k-1}}$. Similar to Mahalanobis distance, the threshold $D_{\text{max}}$ is normalized. With Gaussian distribution assumption for the cloud of points $\{x^{(ij)}_{k|k-1}\}_{j=1}^{L_{k|k-1}}$ in the single-target state space, 98% of the points will be in Mahalanobis distances of 2.5 or less, i.e. $D_{\text{max}} = 2.5$ will include at least 98% of the particles. In practice, the Gaussian assumption may not hold and the actual distribution of the particles may have longer tails. Thus, in our simulations, we chose $D_{\text{max}} = 5$.

Algorithm 6 shows the pseudocode to implement our suggested sensor-selection solution via maximizing Rényi divergence reward function. In the algorithm, after the prediction step, a pre-estimate for the number of targets and their estimates is obtained from the particles of the actual targets (with $u = 1$). This pre-estimated multi-object state $\hat{X}_{k|k-1}$ is then applied to generate an ideal measurement set $\hat{Z}$ corresponding to each sensor node $s \in S$, by calling the PIMS function implemented in Algorithm 3. For each of such measurement sets, the update step is performed and the updated multi-Bernoulli particles are resampled then used for Monte Carlo sampling of the updated multi-object distribution via calling the MB-SMC function implemented in Algorithm 5. The Monte Carlo sample sets are then used to approximate the Rényi divergence as the reward value for choosing the sensor node. The most rewarding sensor node is chosen, from which actual measurements are acquired and utilized for actual update step followed by the final step of pruning and merging of the updated Bernoulli components.
Algorithm 6 The robust multi-Bernoulli multi-target filtering recursion with sensor-selection via Rényi divergence.

**INPUTS:** time $k$, dynamic model $f_{k|k-1}(\cdot|x_{k-1})$, multi-Bernoulli birth model parameters, prior multi-Bernoulli parameters from time $k-1$, measurement likelihood function $g_k(\cdot|x; s)$, currently selected sensor node $s_{k-1}$, finite set of sensor candidates $\mathcal{S}$, number of Monte Carlo samples of multi-object finite sets $L$, number of resampled particles for each updated Bernoulli component $L_{\text{max}}$.

**OUTPUT:** The best sensor node(s) $s_k$ and updated multi-Bernoulli parameters.

**Prediction:**
1: Compute the predicted multi-Bernoulli component parameters and their particles $\{r_{(i)}_{k|k-1}, \{w_{(i,j)(u)}_{k|k-1}, x_{(i,j)(u)}_{k|k-1}\}_{j=1}^{L_{(i)(u)}}\}_{i=1}^{M_{k|k-1}}$ for $u = 0, 1$ (eqs. (5.2),(5.3)).

**Pre-estimation:**
2: $\ell \leftarrow 0$
3: $\hat{X}_{k|k-1} \leftarrow \emptyset$
4: for $i = 0, \ldots, M_{k|k-1}$ do
5: if $r_{(i)}_{k|k-1} > 0.5$ then
6: $r \leftarrow \sum_{j=1}^{L_{(i)(1)}} w_{(i,j)(1)} x_{(i,j)(1)}$
7: $\hat{X}_{k|k-1} \leftarrow \hat{X}_{k|k-1} \cup \{r\}$
8: $\ell \leftarrow \ell + 1$
9: end if
10: end for
11: $\hat{M}_{k|k-1} \leftarrow \ell$

**Sensor-selection:**
12: for all $s \in \mathcal{S}$ do,
13: $\hat{Z} \leftarrow \text{PIMS}(\hat{M}_{k|k-1}, \hat{X}_{k|k-1}, s, g(z|x; s))$
14: Run the update step – eqs. (5)-(6) – with $\hat{Z}$ as measurement set.
15: Resample the particles of each updated Bernoulli component.

\[ \Rightarrow \] Updated multi-Bernoulli parameters are dependent on $s$,

Existence probabilities: $\{r_{(i)(u)}_{k,s}\}_{i=1}^{M_{k,s}}$,

Resampled Particles: $\left\{ \left\{ x_{(i,j)(u)}_{k,s} \right\}_{j=1}^{L_{\text{max}}} \right\}_{i=1}^{M_{k,s}}$.

16: $r_{k,s} \leftarrow \left[ r_{(1)(1)}_{k,s} \cdots r_{(M_{k,s})(1)}_{k,s} \right]^T$
17: $P_{k,s} \leftarrow \left[ x_{(i,j)(1)}_{k,s} \right]_{M_{k,s} \times L_{\text{max}}}$
18: $X \leftarrow \text{MB-SMC}(r_{k,s}, P_{k,s}, L)$
19: $\mathcal{V}(s, X) \leftarrow 0$
for all $X_\ell \in X$ do
  Compute $\pi_k(X_\ell|\tilde{Z}_s)$ \hfill $\triangleright$ Using eqs. (5.27) & (5.28).
  Compute $\pi_{k|k-1}(X_\ell)$ \hfill $\triangleright$ Using eqs. (5.27) & (5.29).
  $\mathcal{V}(s, X) \leftarrow \mathcal{V}(s, X) + \frac{1}{L} \left[ \frac{\pi_{k|k-1}(X_\ell)}{\pi_k(X_\ell|\tilde{Z}_s)} \right]^{1-\alpha}$
end for

$\mathcal{V}(s, X) \leftarrow \frac{1}{\alpha-1} \log \mathcal{V}(s, X)$

Select the sensor node according to the $\hat{s}_k$.

Read the actual measurement set $Z_k$.

**Update:**

Use the set $Z_k$ as measurement set in equations (5)-(6) and compute the updated multi-Bernoulli parameters.

Prune and merge the updated Bernoulli components. \hfill $\triangleright$ More details in [Vo et al. 2013].

*Note that before Monte Carlo sampling of the updated multi-Bernoulli distribution, the particles approximating each of its Bernoulli components need to be resampled.*

### 5.5 Numerical Studies

In this section, two different scenarios are employed to evaluate the performance of the proposed R-PEECS and Rényi divergence approaches for sensor-selection. The first scenario involves pseudo-stationary targets, and serves to visualize the performance of sensor-selection in terms of gradual transition of the selected sensor towards the center of the targets. The second scenario involves targets moving according to a more complex motion model in which the effects of each individual term in the R-PEECS method are investigated.

Both cases involve five targets maneuvering in a surveillance area within a network of sensors. At each step, a single sensor node denoted by $s = [x_s, y_s]^T$ is selected to communicate with a central processor.

The sensor regularly scans the surveillance area and returns a set of bearing and range measurements in the form of $z = [\theta \ \Re]^T$ where:
\[ \theta = \theta_r(t) + e_{\theta} \]
\[ R = \|r - s\| + e_R, \]  
(5.32)

in which \( r = [x \ y]^\top \) denotes the location of a target, \( \| \cdot \| \) denotes Euclidean distance, \( e_{\theta} \) and \( e_R \) are samples of i.i.d. Gaussian measurement noise with zero mean. The angle measurement noise power is assumed to be constant at \( e_{\theta} = \pi/180 \), but the noise power for range measurements is distance-dependent according to:

\[ e_R = \sigma_0 + \eta \|r - s\|^2 \]  
(5.33)

where \( \sigma_0 = 1 \text{ m} \) and \( \eta = 5 \times 10^{-5} \text{ m}^{-1} \). In the sensor-selection step, measurements are synthetically generated as sets, each containing target-generated point measurements and possibly clutter and misses. In each set, every target can be detected with the distance-dependent detection probability \(^4\)

\[ p_D(s, x) = \begin{cases} 1, & \text{if } \|r - s\| \leq R_0 \\ \max\{0, 1 - b(\|r - s\| - R_0)\} & \text{otherwise} \end{cases} \]  
(5.34)

where \( R_0 = 320 \text{ m} \) and \( b = 25 \times 10^{-5} \text{ m}^{-1} \).

### 5.5.1 R-PEECS and Rényi divergence performance comparison

A challenging non-linear multi-target tracking scenario, similar to the one reported in [Ristic et al. 2011a] and [Gostar et al. 2013b], is used to evaluate the performance of the proposed robust multi-Bernoulli sensor-selection method. The initial sensor node in the surveillance area is at position \( (10 \text{ m}, 10 \text{ m}) \). The locations of the sensor node and targets are shown in Fig. 5.3 for iterations \( k = 1, \ldots, 35 \).

**Target states and their dynamic model:** At any time \( k \), each single target state is comprised of the unknown detection probability, location and velocity components in \( x \) and \( y \) directions denoted by \( x = [a_k \ x_k \ \dot{x}_k \ y_k \ \dot{y}_k]^\top \). Actual targets

\(^4\)Note that the \( p_D(s, x) \) is not assumed to be available in practice and in our simulations, it is only used to generate the synthetic measurements.
move according to the constant velocity model [Li and Jilkov 2003b], in which the transition density of target location and speed is given by a Gaussian density with the same parameters as used in [Ristic et al. 2011a] and [Gostar et al. 2013b].

The transition density of the augmented state, $a_k$, is assumed to be a beta distribution [Vo et al. 2013]. The survival probability for actual objects is fixed thorough the scenario $P_s = 0.99$ [Vo et al. 2013]. The birth process for the actual targets is a multi-Bernoulli process with five components $\pi_{\Gamma,k}^{(1)} = \{(r_{\Gamma,k}^{(1)(i)}, p_{\Gamma,k}^{(1)(i)}(\cdot, \cdot))\}_{i=1}^5$ where all probabilities of existence are equally chosen at 0.05, and the states of each of the five newly born targets are distributed uniformly in $[0.8, 1] \times X$. The parameters
chosen for filter, beta distributions and clutter generators model are same as ones used in [Vo et al. 2013].

**Results:** We implemented the proposed robust multi-Bernoulli sensor-selection methods (R-PEECS and Rényi) to estimate the number and states of the targets for a sequence of 35 steps. The x and y components of the actual tracks and the estimated position of those for both methods (R-PEECS and Rényi) are shown in figures 5.4 and 5.5, respectively. As the plots demonstrate, both methods are able to identify and track the objects with similar errors.

Intuitively, the sensor-selection method is expected to select those sensor nodes that are closer to existing targets, and to end up in the vicinity of the targets. The
Figure 5.5: True tracks and multi-Bernoulli filter estimates for information-driven method based on Rényi divergence reward maximization.

The rationale behind this expectation is that more accurate measurements are expected from sensor nodes that are closer to the targets, as the noise power for range measurements increases with the distance and the detection probability decreases for large distances.

Figure 5.6 shows cardinality and clutter intensity estimates averaged over 200 Monte Carlo runs of the proposed sensor-selection methods (R-PEECS and Rényi). It is observed that the clutter intensity estimates shown in Fig. 5.6a gradually approach the ground-truth value of 10 and fluctuate around the true value with a relatively small standard deviation for both methods.

In order to show the advantage gained from robustness of our sensor-selection
solution to unknown clutter intensity and probability of detection, we also tried multi-Bernoulli sensor-selection with given clutter intensity and probability of detection as reported in [Gostar et al. 2013b]. We used an overestimated value of 15 for the clutter intensity (compared to the ground-truth of 10) and a fixed value of 0.98 for probability of detection, which contrasts to the ground-truth probability of detection that decreases with sensor-target distance—see (5.34). The average cardinality estimates returned by the method is also included in the plots shown in Fig. 5.6b. We observe that in terms of cardinality estimation, the robust sensor-selection methods presented in this paper had better accuracy, as the average cardinality estimates over 200 MC runs were closer to the ground-truth number of 5 targets.

The superior performance of the robust methods (both Rényi and R-PEECS) mainly lays in their adaptive nature. Indeed, in the absence of accurate knowledge of the measurement process, inaccurate assumptions for measurement process parameters would lead to inaccurate results and frequently missed targets. This is while without the need for such prior information, the proposed robust methods intrinsically adapt the multi-target filtering process to work best with the measurements received from the selected sensor node.

In order to evaluate the estimation accuracy, we computed the OSPA miss-distances [Schuhmacher et al. 2008] with parameters $p = 2$ and $c = 100$. Figure 5.7 presents comparative results for the performance of the R-PEECS and Rényi methods. The plot shows how the error metric decreases as the sensor moves toward the targets. It also demonstrates similar performance achieved from both methods. Although the R-PEECS and Rényi divergence-based approaches in robust multi-Bernoulli sensor-selection perform similarly in terms of accuracy of estimation, the R-PEECS method substantially outperforms the other in terms of computational cost. We recorded the runtime at the end of each iteration. The results are presented in Fig. 5.8 (note the logarithmic scale of the vertical axis), which shows that on average, the R-PEECS method is about 500 times faster than the Rényi
CHAPTER 5: MULTI-BERNOULLI SENSOR-SELECTION FOR MULTI-TARGET TRACKING WITH UNKNOWN CLUTTER AND DETECTION PROFILES

Figure 5.6: Clutter intensity and cardinality estimates averaged over 200 MC runs.

(a) Comparison of the statistical mean of clutter intensity estimates for R-PEECS and Rényi divergence-based methods.

(b) Comparison of the statistical mean of cardinality estimates of R-PEECS and Rényi divergence-based methods as well as multi-Bernoulli method with known parameters [Gostar et al. 2013b].

5.5.2 Effect of error terms in R-PEECS

In order to investigate the particular effect of the localization clutter estimation error terms in the proposed cost function, a more complex scenario was designed. In this scenario, we chose a non-linear nearly-constant turn model [Li and Jilkov...
Figure 5.7: OSPA errors results for the R-PEECS and Rényi divergence-based methods.

Figure 5.8: Runtime of R-PEECS and Rényi divergence-based sensor-selection solutions plotted versus iteration number. Note that the vertical axis has logarithmic scale.
same as the one reported in [Vo et al. 2009]. In this model, each single target state \(x = [\bar{x}^\top \omega]^\top\) is comprised of location and velocity in Cartesian coordinates, denoted by \(\bar{x} = [x \ y \ \dot{x} \ \dot{y}]^\top\) and turning rate, denoted by \(\omega\). The state dynamics were chosen to be:

\[
\bar{x}_k = F(\omega_{k-1})\bar{x}_{k-1} + G\epsilon_{k-1},
\]

\[
\omega_k = \omega_{k-1} + T\gamma_{k-1},
\]

where

\[
F(\omega) = \begin{bmatrix}
1 & 0 & \frac{\sin \omega T}{\omega} & -\frac{1-\cos \omega T}{\omega} \\
0 & 1 & \frac{1-\cos \omega T}{\omega} & \frac{\sin \omega T}{\omega} \\
0 & 0 & \cos \omega T & -\sin \omega T \\
0 & 0 & \sin \omega T & \cos \omega T
\end{bmatrix},
\]

\[
G = \begin{bmatrix}
\frac{T^2}{2} & 0 \\
0 & \frac{T^2}{2} \\
T & 0 \\
0 & T
\end{bmatrix},
\]

\(T = 1\, \text{s}, \epsilon_{k-1} \sim \mathcal{N}(\cdot; 0, \sigma^2 I), \sigma_\epsilon = 15\, \text{m/s}^2,\) and \(\gamma_{k-1} \sim \mathcal{N}(\cdot; 0, \sigma^2 I), \sigma_\gamma = (\pi/180)\, \text{rad/s}.\)

The birth RFS is a multi-Bernoulli with density \(\pi = \{(r^{(i)}_\Gamma, p^{(i)}_\Gamma)\}_{i=1}^4\) where \(r^{(1)}_\Gamma = r^{(2)}_\Gamma = 0.02, r^{(3)}_\Gamma = r^{(4)}_\Gamma = 0.03\) and \(p^{(i)}_\Gamma(x) = \mathcal{N}(x; m^{(i)}_\gamma, P_\gamma)\) where

\[
m^{(1)}_\gamma = [-500\ 0\ 250\ 0\ 0]^\top,
\]

\[
m^{(2)}_\gamma = [-250\ 0\ 1000\ 0\ 0]^\top,
\]

\[
m^{(3)}_\gamma = [\ 250\ 0\ 750\ 0\ 0]^\top,
\]

\[
m^{(4)}_\gamma = [1000\ 0\ 1500\ 0\ 0]^\top,
\]

\[
P_\gamma = \text{diag}(50^2, 50^2, 50^2, 50^2, (6 \times \frac{\pi}{180})^2).
\]

The probability of survival, detection probability, initial sensor location, clutter rate and measurement model are similar to the previous scenario. As it is shown in Fig. 5.9, the initial sensor node in the surveillance area is at the position (0m, 1500m). Those sensor nodes which are closer to the targets are gradually selected as time evolves. We ran the R-PEECS sensor-selection algorithm with three sets of values for the cost function parameters \(\eta_{|X|}, \eta_X, \eta_\lambda\) as listed in Table 5.1.

We recorded the OSPA errors for these cases at time steps \(k = 1, \ldots, 50\). The results are plotted in Fig. 5.10. Comparison of OSPA errors in the three cases
demonstrates that in Case I, where all error terms in the R-PEECS cost function have positive weights, the multi-target estimation error is generally better than Case II (where the clutter error term is omitted from the cost) and significantly improved with respect to Case III where only the cardinality estimation error term is considered as the cost.

Table 5.1: List of cost function parameters in scenario 2.

<table>
<thead>
<tr>
<th></th>
<th>( \eta_{x_1} )</th>
<th>( \eta_x )</th>
<th>( \eta_\lambda )</th>
<th>Short Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
<td>Recommended case: all terms included.</td>
</tr>
<tr>
<td>Case II</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>Clutter rate error term disabled.</td>
</tr>
<tr>
<td>Case III</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Only cardinality estimation error included.</td>
</tr>
</tbody>
</table>
5.6 Conclusion

This paper introduced a sensor-selection solution especially designed to work with a robust multi-Bernoulli multi-object filter that needs no prior knowledge of clutter distribution or detection profile. The proposed sensor-selection technique employs a novel cost function that takes the accuracy of cardinality, state and clutter estimation into account. SMC implementation of this sensor-selection routine, along with SMC implementation of an information-theoretic sensor-selection solution (based on Rényi divergence) were both presented in detail. The two solutions were examined in simulation studies which revealed that they produce similar estimation accuracies, but the task-driven sensor-selection method is substantially faster.
Sensor Control for Multi-Object Tracking Using Labeled Multi-Bernoulli Filter

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The recently developed labeled multi-Bernoulli (LMB) filter uses better approximations in its update step, compared to the unlabeled multi-Bernoulli filters, and more importantly, it provides us with not only the estimates for the number of targets and their states, but also with labels for existing tracks. This paper presents a novel sensor-control method to be used for optimal multi-target tracking within the LMB filter. The proposed method uses a task-driven cost function in which both the state estimation errors and cardinality estimation errors are taken into consideration. Simulation results demonstrate that the proposed method can successfully guide a mobile sensor in a challenging multi-target tracking scenario.
6.1 Introduction

In the context of multi-target tracking, the aim of sensor-control is to direct sensor(s) (by applying a set of admissible control commands) toward an unknown number of targets to maximize observability. In general, each control command positions sensor(s) to a new state(s) which results in generating different sets of measurements. Each set of the generated measurements contains information which differ from other sets. The generated information can be analyzed via a decision making process (e.g. optimizing an objective function) and as a result, the right control command could be determined in order to maximize the utility of the measurements.

The complexity of this procedure is caused by uncertainty embedded in both state and measurement spaces. In control theory, such problems are addressed by the stochastic control theory in which the number of targets may vary randomly when the time evolves. Also the observation is affected by noise, false alarm or miss detection. A natural choice to model sensor control problem is the *Partially Observed Markov Decision Processes* (POMDPs) framework in which an observer (e.g., mobile sensor) cannot reliably identify the underlying actual state (e.g. target states).

Recently, the finite set statistics (*FISST*)[Mahler 2007b] has received substantial attention to address the underlying state estimation process in POMDP framework [Mahler 1998; 2003a, Mahler and Zajic 2004, Zatezalo et al. 2008, Witkoskie et al. 2006, Ristic and Vo 2010, Ristic et al. 2011a, Gostar et al. 2013a;c;b, Gia Hoang and Tuong Vo 2013]. FISST is based on considering the multi-target entity in both state and measurement spaces as a random finite set (RFS). Several solutions for multi-target tracking problems have been proposed and implemented in FISST framework, such as the *PHD* [Mahler 2007b], *CPHD* [Vo et al. 2007], *MeMBER* [Mahler 2007b], *CB-MeMBER* [Vo et al. 2009], Labeled Multi-Bernoulli (*LMB*) [Reuter et al. 2014] and its general version δ-Generalized Labeled Multi-
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Bernoulli ($\delta$-GLMB) [Vo and Vo 2013] filters.

In FISST-based sensor control framework, a criterion is defined to evaluate the quality of the updated multi-target density after a control command is applied to the sensor. In this approach, the control command is chosen to provide the best updated density based on the defined criterion. Example of this approach is employing the Csiszár in Mahler’s solution for sensor-control problem [Mahler 2004]. He utilized the Csiszár as the reward function within a FISST filtering scheme. However, he later introduced a new reward function and forged it as the “Posterior Expected Number of Targets” (PENT) [Mahler and Zajic 2004, Zatezalo et al. 2008].

The commonly used criterion to evaluate the quality of the updated distribution is its divergence from the predicted distribution [Ristic and Vo 2010]. In two consecutive papers, Ristic et al. [Ristic and Vo 2010, Ristic et al. 2011a] used Rényi divergence as a reward function to quantify the information gained via updating the predicted density using sensor-control technique. In those works, Mahler’s FISST [Mahler 2007b] was used as the framework for multi-target Bayesian filtering. In [Ristic and Vo 2010], the implementation of Rényi reward maximization was investigated for the general form of multi-target filters with random finite set (RFS) assumptions for the multi-target state. Since this approach is computationally intractable even for a small number of targets [Ristic et al. 2011a], in the second paper [Ristic et al. 2011a] the PHD-based filter was used to propagate the multi-object posterior, which facilitates approximation of the Rényi divergence function via i.i.d. assumption.

Recently, a number of solutions have been developed for sensor control within a multi-Bernoulli filter [Gostar et al. 2013a;c;b, Hoang 2012a]. In these works, new task-driven objective functions are defined and optimized, as a result of which, sensor control is aimed to directly minimize cardinality and state estimate errors. This approach is in stark contrast to sensor control with information driven objective functions (such as Rényi divergence) where the enhancement in quality of measure-
ments is expected to be resulted from gaining the most informative posterior density. Gostar et al. [Gostar et al. 2013b] defined a new objective function for the sensor-control problem in the multi-Bernoulli filter framework. This objective function is based on the statistical mean of cardinality variance in conjunction with state estimate errors. In a similar work, Hoang [Hoang 2012a] used the “MAP” cardinality variance of the multi-Bernoulli filter.

In this paper we propose an alternative approach for the solution of multi-target sensor control problem by exploiting a new family of the RFS and its related filter. The Labeled Multi-Bernoulli RFS (which is the special case of δ-GLMB RFS) is a new family of RFS which conjugate with respect to the multi-object observation likelihood and is closed under Chapman-Kolmogorov equation [Vo and Vo 2013, Reuter et al. 2014]. In [Reuter et al. 2014], the Labeled Multi-Bernoulli (LMB) RFS is employed to construct a multi-object filter which is able to produce track-valued estimates. We use LMB filter to estimate the states of the unknown number of targets. Also, we employed the parameters of LMB filter as the variables in the cost function introduced in [Gostar et al. 2013b] for the purpose of sensor resource allocation in sensor-control problem. Our simulation results confirm that our proposed method is more accurate than the state-of-art RFS-based sensor-control methods even for scenarios with high clutter rate.

The rest of the paper is organized as follows. In Sec. 6.2 an overview of the sensor-control framework is given. Then in Sec. 6.3 we briefly review the Labeled Multi-Bernoulli filter which is used to address the underlying multi-target state estimation problem. Section 6.4 is dedicated to describe the defined cost function and implementation of Labeled Multi-Bernoulli sensor-control. Numerical results are presented in section 6.5. Section 6.6 conclude the paper.
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6.2 Problem Statement: Sensor-Control

Following [Gostar et al. 2013a], we formulate the sensor-control problem in the POMDP framework. The POMDP is a generalized form of Markov decision process (MDP) [Kaelbling et al. 1998] in which there is no direct access to the states and the states information are only realized by noisy observations. The elements of the POMDP formulation in this paper are: a finite set of single-object state denoted by \( X_k \), a set of sensor-control commands denoted by \( S \), a stochastic model for single-target state transition, a finite set of observations denoted by \( Z \), a stochastic measurement model, and a cost function \( V(s; X) \) that returns a reward or cost for transition of the multi-object state to \( X \) via applying an action command \( s \in S \).

The purpose of sensor-control is to find the control command \( \hat{s} \in S \) which minimizes the defined cost function. In stochastic filtering, where the multi-target states \( X_{k-1} \) and \( X_k \) are characterized by their distributions, the control command \( \hat{s} \) is commonly chosen to minimize the statistical mean of the cost function \( V(s; X) \) over all observations,

\[
\hat{s} = \arg\min_{s \in S} \{ \mathbb{E}_{Z(s)} [V(s; X)] \}. \tag{6.1}
\]

In POMDP, a Bayesian filtering scheme is commonly utilized as the framework to formulate target evolution. The latest development in multi-target Bayesian filtering is the Generalized Labeled Multi-Bernoulli (GLMB) filter [Vo and Vo 2013] and its special case the Labeled Multi-Bernoulli (LMB) filter [Reuter et al. 2014]. The LMB filter is a solution to the multi-object Bayes filter [Vo 2012] and it produces track-valued estimates in a mathematically principled manner [Reuter et al. 2014]. In terms of accuracy of estimation, the LMB filter outperforms the multi-Bernoulli filter. The main reason is that the LMB filter uses less approximations than the multi-Bernoulli filter [Reuter et al. 2014]. Indeed, the LMB only involves one approximation of the posterior density, while the multi-Bernoulli filter requires two approximations on the multi-target posterior probability generating functional [Reuter et al. 2014]. Also,
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the performance of LMB filter in terms of OSPA error values is similar to the δ-GLMB filter [Reuter et al. 2014] which has already proved to outperform the PHD and CPHD filters [Vo and Vo 2013, Vo et al. 2014, Reuter et al. 2014]. The reason lies in the fact that the δ-GLMB filter propagates a parameter approximation of the multi-object posterior, whereas the PHD and CPHD filters are the first moment approximation to the multi-target Bayes filter.

Due to the above mentioned advantages of LMB filter, this paper focuses on an effective sensor-control solution for LMB filter using measurements of controlled sensors. The task-driven approach to sensor-control within multi-target filtering schemes is to choose the cost function in terms of the predicted multi-target state and the expected update outcomes for every admissible control command. Before we present our choice of cost function, the LMB filter is briefly reviewed in the next section – see [Vo and Vo 2013] for details.

6.3 Labeled Multi-Bernoulli Filter

In this section the summary of the notion and formulation of the Labeled Multi-Bernoulli filter, which was introduced in [Reuter et al. 2014], is presented. The notion of Labeled Multi-Bernoulli (LMB) RFS was introduced for the first time in [Vo and Vo 2013]. LMB is a special case of generalized labeled multi-Bernoulli RFS. It is shown that LMB RFS is a conjugate prior with respect to the multi-object observation likelihood, and it is closed under the multi-target Chapman-Kolmogorov equation [Vo and Vo 2013, Reuter et al. 2014].

In the following, we adopt the same notation used in [Reuter et al. 2014] where the single-object states are denoted by lower-case letters, e.g. \( x, \ x \) and multi-object states by upper-case letters, e.g. \( X, \ X \). In order to distinguish between labeled and unlabeled states and their distributions, the labeled one is shown by bolded letters e.g. \( x, \ X, \) etc, spaces by blackboard bold e.g. \( X, \ L, \ C, \) etc, and the class of finite
subsets of a space \( \mathbb{X} \) by \( \mathcal{F}(\mathbb{X}) \). Following [Reuter et al. 2014], throughout the paper, the standard inner product notation is used and denoted by

\[
\langle f, g \rangle \triangleq \int f(x)g(x)dx,
\]

the generalized Kronecker delta is denoted by

\[
\delta_Y(X) \triangleq \begin{cases} 
1, & \text{if } X = Y \\
0, & \text{otherwise}
\end{cases},
\]

and the inclusion function, a generalization of the indicator function, by

\[
1_Y(X) \triangleq \begin{cases} 
1, & \text{if } X \subseteq Y \\
0, & \text{otherwise}
\end{cases}.
\]

The multi-object distribution of a GLMB RFS with state \( \mathbb{X} \) and discrete label space \( \mathbb{L} \) is given by

\[
\pi(\mathbb{X}) = \Delta(\mathbb{X}) \sum_{c \in C} w^c(\mathcal{L}(\mathbb{X})) [p^c]^\mathbb{X}, \tag{6.2}
\]

where

\[
\Delta(\mathbb{X}) = \delta_{|\mathbb{X}|}(|\mathcal{L}(\mathbb{X})|)
\]

and \( C \) is a discrete index set and \( w^c(L) \) is the non-negative weights that only depends on the labels of multi-object state and satisfies \( \sum_{L \subseteq \mathbb{L}} \sum_{c \in C} w^c(L) = 1 \). Each \( p^c(x, \ell) \) is a probability density and satisfies \( \int p^c(x, \ell)dx = 1 \). In (6.2), \( h^X \triangleq \prod_{x \in \mathbb{X}} h(x) \), denotes the multi-object exponential, where \( h \) is a real-valued function, with \( h^\emptyset = 1 \) by convention. Thus, the multi-object distribution of a GLMB RFS presented in (6.2), can be interpreted as a mixture of multi-object exponentials. Each term in this mixture consists of a weight \( w^c \) that only depends on the labels of the multi-object state, and a multi-object exponential \( [p^c]^\mathbb{X} \) that depends on the entire multi-object state. The projection \( \mathcal{L} : \mathbb{X} \times \mathbb{L} \to \mathbb{L} \) is given by \( \mathcal{L}(x, \ell) = \ell \) and \( \mathcal{L}(\mathbb{X}) = \{ \mathcal{L}(x) : x \in \mathbb{X} \} \) is the set of object labels of \( \mathbb{X} \). A labeled RFS with state
space $X$ and discrete label space $L$ is an RFS on $X \times L$ such that each realization has distinct labels [Vo and Vo 2013, Vo et al. 2014].

The LMB RFS is a special case of GLMB RFS and similar to the multi-Bernoulli RFS it is completely described by its components $\pi = \{(r^{(i)}, p^{(i)}) : \zeta \in \Psi\}$. The LMB RFS density is given by

$$\pi(X) = \Delta(X) w(L(X)) [p]^X, \quad (6.3)$$

where

$$p(x, \ell) = p^{(\ell)}(x) \quad (6.4)$$

$$w(L) = \prod_{i \in L} (1 - r^{(i)}) \prod_{\ell \in L} \frac{1_{L}(\ell) r^{(\ell)}}{1 - r^{(\ell)}} \quad (6.5)$$

comprising a single component [Reuter et al. 2014].

Similar to the general multi-target Bayes filter, the LMB multi-target Bayes recursion propagates multi-target posterior density at each time according to the Chapman-Kolmogorov (prediction step) and the Bayes rule (update step).

### 6.3.1 Prediction

Reuter et al [Reuter et al. 2014] proved that a LMB RFS is closed under the Chapman-Kolmogorov equation which means if the current multi-object posterior is of the form of LMB, then the predicted multi-object distribution is still LMB. Assume that the prior and birth labeled multi-Bernoulli sets are modelled as follows:

$$\pi(X) = \Delta(X) w(L(X)) [p]^X \quad (6.6)$$

$$\pi_B(X) = \Delta(X) w_B(L(X)) [p_B]^X \quad (6.7)$$
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where

\[ w(L) = \prod_{i \in L} \left( 1 - r^{(i)} \right) \prod_{\ell \in L} \frac{1}{1 - r^{(\ell)}} \left( \frac{r^{(\ell)}}{1 - r^{(\ell)}} \right), \]  

(6.8)

\[ w_B(L) = \prod_{i \in B} \left( 1 - r^{(i)} \right) \prod_{\ell \in L} \frac{1}{1 - r^{(\ell)}} \left( \frac{r^{(\ell)}}{1 - r^{(\ell)}} \right), \]  

(6.9)

\[ p(x, \ell) = p^{(\ell)}(x) \]  

(6.10)

\[ p_B(x, \ell) = p^{(\ell)}_B(x). \]  

(6.11)

with state space \( X \) and label space \( L_+ = \mathbb{B} \cup \mathbb{L} \) and with the condition \( \mathbb{B} \cap \mathbb{L} = \emptyset \).

The predicted multi-object distribution is then a LMB RFS and given by

\[ \pi_+(X) = \Delta(X)w_+(\mathcal{L}(X)) [p_+]^X \]  

(6.12)

where

\[ w_+(I_+) = w_s(I_+ \cap L)w_B(I_+ \cap B) \]  

(6.13)

\[ w_s(L) = (1 - r^{(\cdot)})\eta_s(\cdot) \left( \frac{r^{(\cdot)}\eta_s(\cdot)}{1 - r^{(\cdot)}\eta_s(\cdot)} \right)^L, \]  

(6.14)

\[ \eta_s(\ell) = \langle p_s(\cdot, \ell), p(\cdot, \ell) \rangle \]  

(6.15)

\[ p_+(x, \ell) = 1_L(\ell)p_{+S}(x, \ell) + 1_B(\ell)p_B(x, \ell) \]  

(6.16)

\[ p_{+S}(x, \ell) = \frac{\langle p_s(\cdot, \ell)f(x|\cdot, \ell), p(\cdot, \ell) \rangle}{\eta_s(\ell)} \]  

(6.17)

where \( p_s(\cdot|\ell) \) is the survival probability of an object and \( f(x|\cdot, \ell) \) is the single-object transition model. Thus, if the multi-target posterior density is an LMB RFS with parameter set \( \pi = \{(r^{(\ell)}, p^{(\ell)}): \ell \in L\} \) with state space \( X \) and label space \( L \) and the birth model is also an LMB RFS with parameter set \( \pi_B = \{(r^{(B)}_B, p^{(B)}_B): \ell \in B\} \) with state space \( X \) and label space \( B \) then the predicted multi-target density is also an LMB RFS with state space \( X \) and label space \( L_+ = \mathbb{B} \cup \mathbb{L}(\mathbb{B} \cap \mathbb{L} = \emptyset) \) and it is given by

\[ \pi_+ = \{(r^{(\ell)}_{+S}, p^{(\ell)}_{+S}): \ell \in L\} \cup \{(r^{(\ell)}_B, p^{(\ell)}_B): \ell \in B\} \]  

(6.18)
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where

\[ r_+^{(\ell)} = \eta_\ell \rho_+^{(\ell)}, \quad (6.19) \]

\[ p_+^{(\ell)} = \frac{\langle p_+^{(\cdot, \ell)} f(x, \cdot, \ell), p(\cdot, \ell) \rangle}{\eta_\ell}, \quad (6.20) \]


6.3.2 Update

In update step, if the multi-object density is an LMB RFS, then the multi-object posterior is not necessarily still an LMB RFS. Similar to multi-Bernoulli RFS [Vo et al. 2009], Reuter et al [Reuter et al. 2014] approximate the updated LMB RFS by its first moment. Thus, if the predicted multi-target density is an LMB RFS with parameter set \( \pi_+ = \{(r_+^{(\ell)}, p_+^{(\ell)}) : \ell \in L_+\} \), the multi-target posterior is then given by

\[ \pi(\cdot|Z) = \{(r^{(\ell)}, p^{(\ell)}(\cdot) : \ell \in L_+\} \quad (6.21) \]

where

\[ r^{(\ell)} = \sum_{(I_+, \theta) \in F(L_+) \times \Theta_{I_+}} w^{(I_+, \theta)}(Z) 1_{I_+}(\ell), \quad (6.22) \]

\[ p^{(\ell)}(x) = \frac{1}{r^{(\ell)}} \sum_{(I_+, \theta) \in F(L_+) \times \Theta_{I_+}} w^{(I_+, \theta)}(Z) 1_{I_+}(\ell) p^{(\theta)}(x, \ell), \quad (6.23) \]

where \( \Theta_{I_+} \) denotes the space of mapping \( \theta : I_+ \to \{0, 1, \ldots, |Z|\} \) and,

\[ w^{(I_+, \theta)}(Z) \propto w_+(I_+) \left[ \eta^{(\theta)}_Z \right]^{I_+} \quad (6.24) \]

\[ p^{(\theta)}(x, \ell|Z) = \frac{p_+(x, \ell) \psi_Z(x, \ell; \theta)}{\eta^{(\theta)}_Z(\ell)}, \quad (6.25) \]

\[ \eta^{(\theta)}_Z(\ell) = \langle p_+(\cdot, \ell), \psi_Z(\cdot, \ell; \theta) \rangle, \quad (6.26) \]

\[ \psi_Z(x, \ell; \theta) = \delta_0(\theta(\ell)) q_0(x, \ell) \]

\[ + (1 - \delta_0(\theta(\ell))) \frac{p_D(x, \ell) g(z_\theta(\ell)|x, \ell)}{\kappa(z_\theta(\ell))} \quad (6.27) \]
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where, \( g(z|x) \) is the single-sensor measurement likelihood, \( p_D(\cdot, \ell) \) denotes probability of detection, \( q_D(\cdot, \ell) = 1 - p_D(\cdot, \ell) \) is the probability of a missed detection, and \( \kappa(\cdot) \) intensity function of the Poisson distributed clutter process.

6.3.3 Implementation

Details of sequential Monte Carlo implementation of the LMB filter are presented in [Reuter et al. 2014]. In the implementation, the number of hypotheses grows exponentially. For computational reduction, targets and measurements are subjected to spatial grouping and gating and the update step is run in parallel for those groups. In order to keep only the most significant hypotheses, several methods of truncation are proposed in the literature [Vo and Vo 2013, Reuter et al. 2014, Vo et al. 2014].

In the prediction step, the \( K \)-shortest path algorithm is used to truncate the predicted LMB without computing all the prediction hypotheses and their weights [Eppstein 1994]. To avoid computing all the hypotheses and their weights in the update step, the updated LMB multi-target posterior is truncated, via the ranked assignment algorithm. Murty’s method is employed for the ranked assignment process in which only the \( M \) most significant association hypotheses are evaluated [Murty 1968]. For more details see [Reuter et al. 2014].

6.4 Labeled Multi-Bernoulli Sensor-Control

As it was mentioned earlier, the POMDP approach to the sensor-control problem comprises of a multi-target tracking framework and a stochastic decision making solution to choose the optimal command via an objective function. Due to estimation accuracy of the LMB filter, in this study we choose LMB filter to carry out the multi-target tracking problem. The only drawback of this filter compare to the other RFS-based filters is the computational complexity of its update step.
To reduce the complexity of our proposed method, in the sensor-control step, instead of using the update formulation of the LMB filter, the multi-Bernoulli update [Vo et al. 2009] is employed. In order to use the update step of the multi-Bernoulli filter, first the predicted parameters of the LMB are computed and the augmented label state is discarded. Note that the unlabeled version of the LMB parameters is equal to the multi-Bernoulli parameters. Having the predicted parameters of the multi-Bernoulli distribution, the number and states of the targets are pre-estimated. For each sensor-control command, a set of pseudo-measurements are generated (according to the pre-estimated targets) using the Predicted Ideal Measurement Set (PIMS) approach [Mahler 2004], then the multi-Bernoulli update is performed. By acquiring posterior multi-Bernoulli densities for each admissible command, the command that maximizes the utility of the measurement is chosen and applied. After changing the state of the sensor(s) and receiving the actual set of measurements, the LMB update is performed. The main steps of the sensor-control with LMB filter are given in Algorithm 7.

6.4.1 Cost function

The most common approach to choose the optimal control command in the sensor-control solutions are based on maximizing an information theoretic reward function such as Rényi divergence [Kreucher et al. 2003, Ristic and Vo 2010, Ristic et al. 2011a]. The main rationale behind choosing such reward functions is that the information encapsulated by the estimated multi-target distribution is expected to gradually increase as further measurements become available by time.

Following our preliminary study [Gostar et al. 2013a], we take a different approach in which the updated parameters of the multi-Bernoulli filter are used to define a new cost function. Note that the multi-Bernoulli parameters are updated by extracting the unlabeled version of the predicted LMB parameters. In the sensor-
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Algorithm 7 The LMB multi-target filtering recursion with sensor-control.

**INPUTS:** dynamic model $f(x|\cdot, \ell)$, LMB birth model parameters, prior LMB parameters from time $k-1$, detection probability $p_D(\cdot)$, measurement likelihood function $g_k(\cdot|x)$, and clutter intensity $\nu(\cdot)$ and its integral $\lambda_c$, current sensor(s) location(s), finite set of admissible sensor-control commands $S$.

**OUTPUT:** The best control command $\hat{s}$ and updated LMB parameters.

**Prediction:**
1: Compute the predicted LMB component parameters.
2: Extract unlabeled version of LMB parameters.

**Pre-estimation:**
3: Compute the prediction estimates of the number and states of objects.

**Sensor-control:**
4: for $s \in S$ do
5: Construct the PIMS, $\hat{Z}(s)$.
6: Update the multi-Bernoulli distribution parameters.
7: Compute the cost $\mathcal{V}(s; X)$
8: end for
9: $\hat{s} \leftarrow \text{argmin}_s \mathcal{V}(s; X)$

**Measurement:**
10: Apply the control command $\hat{s}$ to change the sensor state
11: Collect the actual measurements from controlled sensor(s).

**Update:**
12: Use the measurement set to update the LMB parameters.

control step, these parameters are then updated by using the update formulation of the multi-Bernoulli filter. Our approach is to consider a cost function that quantifies the average uncertainty in all possible multi-target state estimates after each update step. This cost is not totally independent of the prediction outcomes, and state estimates extracted from predicted multi-Bernoulli density are used to calculate the proposed cost function. The main difference here is that our focus is on the quality of the updated density in terms of level of uncertainties, not the information gained from prediction to update (e.g. Rényi divergence function).

The updated distribution depends on the receiving measurement. The gener-
ated measurements set is also a function of the chosen sensor control command. In principle the whole distribution of all possible measurement sets is used to compute the update distribution. However, to reduce the computational complexity, we adopt the predicted ideal measurement set (PIMS) [Mahler 2004] for the purpose of updating the multi-target distribution and computing the cost from it. To define the new cost function, we note that the predicted ideal measurement set depends on the chosen control command. For each command, we first compute the PIMS, then calculate an updated multi-object distribution by considering the PIMS as the acquired measurement. A linear combination of the normalized errors of the number of targets and their estimated states is considered as a measure of uncertainty associated with estimation of the multi-target state and as the cost function:

\[ V(s; X) = \eta \varepsilon_{|X|}^2(s) + (1 - \eta) \varepsilon_X^2(s), \]

where \( \varepsilon_{|X|}^2(s) \) denotes the normalized error of estimated cardinality of the multi-target state, \( \varepsilon_X^2(s) \) denotes the normalized error of the multi-target state estimate, and \( \eta \in [0, 1] \) is a user-defined constant parameter to tune the influence of the error terms on the total sensor control cost. Appearance of the \( X_{k-1} \) as an argument of the cost function is to emphasize that the cost not only depends on the selected control command, but also on the prior distribution. It is important to note that the expectation term in (6.1) does not appear as we use the predicted ideal measurement set (PIMS) approach [Mahler 2004] instead of sampling and averaging in measurement space. The details of computing the PIMS, and defining and computing the normalized error terms, \( \varepsilon_{|X|}^2(s) \) and \( \varepsilon_X^2(s) \) for SMC implementation are presented in Sec. 6.4.2.

The quality of sensor measurements usually depends on a sensor state (e.g. the sensor location) which is assumed to be controllable, and the sensor-control problem is focused on choosing the command that would lead to the best sensor state.
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6.4.2 Implementation

Suppose that at the time \( k - 1 \), the multi-target distribution is modelled by a LMB RFS with parameters \( \pi = \{(r^{(\ell)}, p^{(\ell)}) : \ell \in L\} \) and \( \pi_B = \{(r_B^{(\ell)}, p_B^{(\ell)}) : \ell \in B\} \), in which each single target density \( p^{(\ell)}(\cdot) \) is represented by a set of weighted samples \( \{(\omega_\ell^{(i)}, x_\ell^{(i)})\}_{i=1}^{J_\ell} \) and the birth density \( p_B^{(\ell)}(\cdot) \) is represented by \( \{(\omega_B^{(i)}, x_B^{(i)})\}_{i=1}^{J_B^{(\ell)}} \). In the prediction step, the LMB filter propagates the LMB components based on the temporal information from the transition density, the probability of survival, and the predefined LMB birth terms. The predicted LMB density is denoted by \( \pi_+ = \{(r_+^{(\ell)}, p_+^{(\ell)}) : \ell \in L_+\} \) where each single target density \( p_+^{(\ell)}(\cdot) \) is represented by a set of weighted samples \( \{(\omega_+^{(i)}, x_+^{(i)})\}_{i=1}^{J_+^{(\ell)}} \).

For each label \( \ell \in L_+ \), if the probability of existence \( r_+^{(\ell)} \) is greater than a user-defined threshold (chosen at 0.5 in our simulation studies), the EAP estimate of a single-object state is computed as follows:

\[
\hat{x}_+^{(\ell)} = \sum_{i=1}^{J_+^{(\ell)}} \omega_+^{(i)} x_+^{(i)}.
\]  

(6.29)

Each of the above estimates represent a predicted target. Following the PIMS approach [Mahler 2004], an ideal set of measurements are then generated from the predicted target state estimates. This hypothetical set of measurement would depend on not only the predicted number and states of targets, but also the new state of the sensor(s) after a sensor command is applied. Indeed, for each possible sensor control command \( s \in S \), a different set of ideal measurements, \( \hat{Z}(s) \), is computed. Considering this set as the actual measurement set, we can now run the update step and calculate the cost corresponding to that command.

We note that the LMB prediction step “actually coincides with performing the prediction on the unlabeled process and interpreting the component indices as track labels.”–Remark 3 from [Reuter et al. 2014]. Therefore, we can simply remove the labels from the predicted LMB multi-object state, and update the existence proba-
abilities and density particles and weights through the CB-MeMBer update step [Vo et al. 2009] in which \( Z(s) \) is taken as the actual measurement set. It is important to note that this update step needs to be repeated for each hypothetical measurement set \( Z(s) \). By using the CB-MeMBer update on the unlabeled components we avoid to repeatedly run the computationally expensive update of LMB filter. Hence, substantial savings are achieved in terms of computational cost of our sensor-control method.

**Computing the cost:** The cost defined in (6.28) comprises two normalized error terms, \( \varepsilon^2_{|X|}(s) \) as the error for the cardinality estimate, and \( \varepsilon^2_X(s) \) as the error for the multi-target state estimate. Both terms depend on the updated multi-object posterior which in turn depends on the PIMS computed for the command \( s \).

Assume that for each control-command \( s \in \mathcal{S} \), the CB-MeMBer updated unlabeled multi-Bernoulli is given by \( \{r^{(i)}(s), p^{(i)}(s, \cdot)\}_{i=1}^M \) where each single Bernoulli density \( p^{(i)}(s, \cdot) \) is approximated by particles \( \{\omega_j^{(i)}(s), x_j^{(i)}(s)\}_{j=1}^J \).

We choose and calculate the statistical expectation of the cardinality variance as a meaningful measure for its estimation error. In terms of the updated probabilities of existence, it is given by:

\[
\sigma^2_{|X|}(s) = \sum_{i=1}^M \left[ r^{(i)}(s)(1 - r^{(i)}(s)) \right].
\]

(6.30)

The above given value is maximum when \( \forall i, r^{(i)}(s) = 0.5 \) which leads to \( \max \{\sigma^2_{|X|}(s)\} = \frac{M(s)}{4} \). Thus, the normalized cardinality error term can be computed as follows:

\[
\varepsilon^2_{|X|}(s) = \frac{4\sigma^2_{|X|}(s)}{M(s)}.
\]

(6.31)

To arrive at a meaningful measure for the normalized state estimation error term \( \varepsilon^2_X(s) \) in the cost defined in (6.28), we consider the following total state estimation error:

\[
\varepsilon^2_X(s) = \sum_{i=1}^M \left[ r^{(i)}(s)\varepsilon^2_{x^{(i)}}(s) \right] / \sum_{i=1}^M r^{(i)}(s)
\]

(6.32)
which is the weighted average of normalized estimation errors of the states of single targets associated with each single Bernoulli component. Before we present how the normalized error terms $\epsilon^2_{x(i)}(s)$ are computed, we note that the averaging weights are the updated probabilities of existence. The rationale behind this choice of weights is that Bernoulli components with larger probabilities of existence contribute more strongly to the EAP estimate of the multi-object state – see section IV-A.4 in [Vo et al. 2009].

To compute the normalized single Bernoulli component errors $\epsilon^2_{x(i)}(s)$, we first formulate the actual error denoted by $\varsigma^2_{x(i)}(s)$, then its maximum, upon which a normalized measure will be given by $\varsigma^2_{x(i)}(s)/\max \varsigma^2_{x(i)}(s)$. In practice, we are commonly interested in minimizing the estimation error of selected elements of target states. For instance, in some applications, the prime interest is in location, and target speed is included in the single-target state vector due to its appearance in motion and perhaps measurement models. In such target-tracking applications, an intuitive scalar measure for the single Bernoulli component error is given by the product of the variances of the target location coordinates. If the stochastic variations of target location coordinates are independent, this measure will translate into the absolute determinant of the covariance matrix of the target location.

In case of tracking multiple-targets in 2D space, the single Bernoulli component error term, $\varsigma^2_{x(i)}(s)$, is given by:

$$\varsigma^2_{x(i)}(s) = \sigma^2_{x(i)}(s) \sigma^2_{y(i)}(s)$$  \hspace{1cm} (6.33)

where x and y denote the x and y-coordinates of the single-target location (part of its state vector $x$). Having the updated particles and weights of each Bernoulli
component, the single-coordinate errors can be calculated as follows:

\[
\sigma^2_{x(i)}(s) = \sum_{j=1}^{J(i)(s)} \omega_{j}^{(i)}(s) \left( x_{j}^{(i)}(s) \right)^2 - \left( \sum_{j=1}^{J(i)(s)} \omega_{j}^{(i)}(s) x_{j}^{(i)}(s) \right)^2
\]

\[
\sigma^2_{y(i)}(s) = \sum_{j=1}^{J(i)(s)} \omega_{j}^{(i)}(s) \left( y_{j}^{(i)}(s) \right)^2 - \left( \sum_{j=1}^{J(i)(s)} \omega_{j}^{(i)}(s) y_{j}^{(i)}(s) \right)^2
\]

(6.34)

where \( x_{j}^{(i)}(s) \) and \( y_{j}^{(i)}(s) \) denote the coordinates extracted from the particle \( x_{j}^{(i)}(s) \) and power operation is element-wise operation. To normalize the total state estimation error term \( \varsigma^2_{x(i)}(s) \) in (6.33), we note that with equally weighted particles, i.e. when \( \forall j, \omega_{j}^{(i)}(s) = 1/J(i)(s) \), the particles representing the \( i \)-th single Bernoulli component do not convey any information and the above estimation variances adopt their maximum values as follows:

\[
\max\{\sigma^2_{x(i)}(s)\} = \frac{1}{J(i)(s)} \left( 1 - \frac{1}{J(i)(s)} \right) \sum_{j=1}^{J(i)(s)} \left( x_{j}^{(i)}(s) \right)^2
\]

\[
\max\{\sigma^2_{y(i)}(s)\} = \frac{1}{J(i)(s)} \left( 1 - \frac{1}{J(i)(s)} \right) \sum_{j=1}^{J(i)(s)} \left( y_{j}^{(i)}(s) \right)^2.
\]

(6.35)

Thus, the single Bernoulli error terms \( \varsigma^2_{x(i)}(s) \) in (6.33) can be normalized as follows:

\[
\epsilon^2_{x(i)}(s) = \frac{\sigma^2_{x(i)}(s) \sigma^2_{y(i)}(s)}{\max\{\sigma^2_{x(i)}(s)\} \max\{\sigma^2_{y(i)}(s)\}}
\]

(6.36)

and the computed values can be used in (6.32) to calculate the normalized state estimation error term in the cost.

Having the cost values computed for all admissible sensor control commands, the best command \( \hat{s} \) is then chosen as the one incurring the smallest cost:

\[
\hat{s} = \arg\min_{s \in S} \mathcal{V}(s; X).
\]

(6.37)

As the cost function of the proposed method is a combination of posterior expected errors of cardinality and states, henceforward, we call it the Posterior Expected Error of Cardinality and States (PEECS).
6.5 Numerical Studies

To demonstrate the performance of our method with measurements that guarantee full observability of the targets, we have run a case study involving a complex scenario. In this scenario, we choose a non-linear nearly-constant turn model reported in [Vo et al. 2009]. In this case, each single target state \( x = [\bar{x}^\top \omega]^\top \) is comprised of location and velocity in Cartesian coordinates, denoted by \( \bar{x} = [x \ y \ \dot{x} \ \dot{y}]^\top \) and turning rate, denoted by \( \omega \). The state dynamics are given by:

\[
\begin{align*}
\bar{x}_k &= F(\omega_{k-1})\bar{x}_{k-1} + G\epsilon_{k-1}, \\
\omega_k &= \omega_{k-1} + T\gamma_{k-1},
\end{align*}
\]

where

\[
F(\omega) = \begin{bmatrix} 1 & 0 & \sin \omega T \omega & -1 - \cos \omega T \omega \\ 0 & 1 & -1 - \cos \omega T \omega & \sin \omega T \omega \\ 0 & 0 & \cos \omega T & -\sin \omega T \\ 0 & 0 & \sin \omega T & \cos \omega T \end{bmatrix},
\]

\[
G = \begin{bmatrix} T^2 \omega & 0 \\ 0 & T^2 \omega \\ T & 0 \\ 0 & T \end{bmatrix},
\]

\( T = 1 \) s, \( \epsilon_{k-1} \sim \mathcal{N}(:, 0, \sigma_\epsilon^2 I) \), \( \sigma_\epsilon = 15 \) m/s², and \( \gamma_{k-1} \sim \mathcal{N}(:, 0, \sigma_\gamma^2 I) \), \( \sigma_\gamma = (\pi/180) \) rad/s.

The birth RFS is a multi-Bernoulli with density \( \pi_\Gamma = \{(r^{(i)}_\Gamma, p^{(i)}_\Gamma)\}_{i=1}^4 \) where \( r^{(1)}_\Gamma = r^{(2)}_\Gamma = 0.02, r^{(3)}_\Gamma = r^{(4)}_\Gamma = 0.03 \) and \( p^{(i)}_\Gamma(x) = \mathcal{N}(x; m^{(i)}_\gamma, P_\gamma) \) where

\[
\begin{align*}
m^{(1)}_\gamma &= [-1500 0 250 0 0]^\top, \\
m^{(2)}_\gamma &= [-250 0 1000 0 0]^\top, \\
m^{(3)}_\gamma &= [250 0 750 0 0]^\top, \\
m^{(4)}_\gamma &= [1000 0 1500 0 0]^\top, \\
P_\gamma &= \text{diag}(50^2, 50^2, 50^2, 50^2, (6 \times \pi/180)^2).
\end{align*}
\]

The sensor can detect an object in location \( o = [x_o \ y_o]^\top \) with the following probability that depends on the location of both the sensor and object locations:

\[
p_D(s, o) = \begin{cases} 1, & \text{if } ||o - s|| \leq R_0 \\ \max\{0, 1 - b(||o - s|| - R_0)\}, & \text{otherwise} \end{cases} \quad (6.38)
\]
Overall, there are five objects in the surveillance area, positioned relatively close to each other. Their initial state vectors are: $[800 \ 600 \ 1 \ 0]^{\top}$, $[650 \ 500 \ 0.3 \ 0.6]^{\top}$, $[620 \ 700 \ 0.25 \ 0.45]^{\top}$, $[750 \ 800 \ 0 \ 0.6]^{\top}$, and $[700 \ 700 \ 0.2 \ 0.6]^{\top}$, where the units of $x$ and $y$ are meters and $\dot{x}$ and $\dot{y}$ are m/s. The objects move according to the constant velocity model.

Each measurement includes a set of ranges and bearings, and the observation model is given by:

$$z_k = \left[ \arctan\left(\frac{y_k}{x_k}\right) \sqrt{x_k^2 + y_k^2} \right]^{\top} + \zeta_k,$$

where $\zeta_k \sim \mathcal{N}(\cdot; 0, R_k)$ is the measurement noise with covariance $R_k = \text{diag}(\sigma_\theta^2, \sigma_r^2)$ in which the scales of range and bearing noise are $\sigma_\theta = (\pi/180) \text{ rad}$ and $\sigma_r = 5 \text{ m}$.

The clutter RFS follows the uniform Poisson model over the surveillance region $[-\pi/2 \text{ rad}, \pi/2 \text{ rad}] \times [0, 2000 \text{ m}]$, with $\lambda_c = 1.6 \times 10^{-3} \text{ (rad m)}^{-1}$.

The sensor initial position is at $x = 0$ and $y = 1500$. Targets enter the scene with the following position, velocity, turning velocity, birth and death time:

- $x_{T1} = [1000 \ -20 \ 1500 \ -20 \frac{\pi}{180}]^{\top}$, $k_{b_{T1}} = 1$, $k_{d_{T1}} = 35$,
- $x_{T2} = [-250 \ 20 \ 1000 \ 3 \frac{\pi}{180}]^{\top}$, $k_{b_{T2}} = 5$, $k_{d_{T2}} = 50$,
- $x_{T3} = [-500 \ 11 \ 250 \ 10 \frac{\pi}{180}]^{\top}$, $k_{b_{T3}} = 20$, $k_{d_{T3}} = 50$,
- $x_{T4} = [-500 \ 14 \ 250 \ 0 \ 0]^{\top}$, $k_{b_{T4}} = 15$, $k_{d_{T4}} = 50$,
- $x_{T5} = [250 \ 11 \ 750 \ 5 \frac{\pi}{180}]^{\top}$, $k_{b_{T5}} = 10$, $k_{d_{T5}} = 50$.

Intuitively, we expect the sensor start moving toward the targets and for each time step remains in vicinity of them. As it is shown in Fig. 6.1, sensor start moving from the initial position and as it was expected, after a few steps it remains among the manoeuvring targets.

The estimation error are computed based on the Optimal SubPattern Assignment (OSPA) metrics introduced in [Schuhmacher et al. 2008] (cutoff parameter $c = 100$ and order parameter $p = 2$). The comparative averaged error performance of LMB PEECS sensor control and CB-MeMBer PEECS sensor control are shown.
Figure 6.1: Sensor and target locations during $k = 1, \ldots, 50$.

in Fig. 6.4 over 200 Monte Carlo runs. Figure 6.2 and 6.3 show the cardinality and localization errors for both methods respectively. At first the cardinality and localization errors are high due to uncertainty in the number and states of targets. After the time step $k = 20$ both the cardinality and localization errors are fixed through the rest of the simulation. The LMB PEECS errors are comparatively lower than the CB-MeMBer PEECS error. The total OSPA error are shown in Fig. 6.4. The superiority of the LMB PEECS method is due to accuracy of the LMB filter which is the result of proper approximation in update step of the LMB filter. As it was mentioned in Sec. 6.2, unlike multi-Bernoulli filter the LMB filter uses a more accurate update approximation. More precisely, the LMB filter uses a more accurate update approximation than the CB-MeMBer filter by exploiting the conjugate prior labeled RFSs [Reuter et al. 2014].
6.6 Conclusions and future studies

A novel sensor-control method was proposed in this paper, for controlling mobile sensor(s) in such a way that minimum expected errors are achieved in a multi-target tracking application. The proposed method works based on choosing the sensor-control command that is expected to lead to the lowest cost, and the cost is defined and formulated in terms of cardinality and single-target state estimation er-
Implementation of the cost computation and its minimization within a labeled multi-Bernoulli filter was elaborated and a step-by-step algorithm was presented. In a challenging simulation scenario, our proposed method demonstrated success in optimal guidance of a mobile sensor in tracking of up to 5 targets which can appear and disappear in/from the scene. Compared to the scenario where a similar cost function was used within a CB-MeMBer filter, we showed that our new method developed for the labeled multi-Bernoulli filter performs better in terms of OSPA errors in the same challenging scenario. This can also be due to the advantageous nature of the labeled multi-Bernoulli filter in terms of its better accuracy in approximating the update step.

This work can be extended to applications where detection and tracking of targets of interest are required. In this case, a new task-driven cost function would be needed in which not only the estimation errors are considered but also the track labels produced by the labeled multi-Bernoulli filter are utilized. Such a sensor-control routine would guide the sensor(s) towards sensor states which are likely to lead to better estimates of the states of the targets of interest (with particular labels) only.
Sensor Management via
Minimization of OSPA-Based
Dispersion of Posterior

CHAPTER 7: SENSOR MANAGEMENT VIA MINIMIZATION OF OSPA-BASED DISPERSION OF POSTERIOR

This paper presents a new sensor management method for multi-object filtering via maximizing the state estimation confidence. Confidence of an estimation is quantified by measuring the dispersion of the multi-object posterior about its statistical mean using Optimal Sub-Pattern Assignment (OSPA). The proposed method is generic and the presented algorithm can be used with any statistical filter. Implementation of the algorithm in conjunction with a labeled multi-Bernoulli filter is presented. Simulation studies demonstrate that the OSPA-based sensor control can successfully guide a sensor to achieve excellent results in tracking up to 16 targets, and outperforms the recent PEECS-based sensor control.
7.1 Introduction

Sensor management in multi-object filters, is usually employed to control or select one or more sensors with the aim of acquiring the best measurements for filtering purposes [Mallick et al. 2012, Krishnamurthy 2002, Krishnamurthy and Evans 2001, Berenguer et al. 2005, Koch 1999]. There are two approaches in defining what the best measurement is[Koch 2007, White et al. 2008, Boers et al. 2008, Kreucher et al. 2005, Aughenbaugh and LaCour 2011, Bar-Shalom and Li 1995]. One approach, called information-driven, uses the information content to build the criterion for goodness, which is usually quantified via a divergence function such as Rényi divergence [Kreucher et al. 2003, Ristic and Vo 2010, Ristic et al. 2011a, Ristic 2013, Gia Hoang and Tuong Vo 2013]. In information-driven methods, statistical expectation of the chosen divergence function (over the measurement space) is selected as a reward function, which is maximized to solve the sensor management problem.\(^1\)

The second approach, called task-driven, is to consider the expected performance of the multi-object filter as the measure of goodness. A good measurement is assumed to be the one that reduces the dispersion of the multi-object filter posterior. The smaller dispersion (around the statistical mean) is taken to convey more confidence in the estimation process. In task-driven methods, a cost function to measure the multi-object posterior dispersion, is formulated and minimized to solve the sensor management problem. Examples of such cost functions include estimated target cardinality variance [Gostar et al. 2013a, Gia Hoang and Tuong Vo 2013] and posterior expected error of cardinality and states (PEECS) [Gostar et al. 2013b; 2015b].

Divergence-based solutions are reported to produce good results [Kreucher et al. 2003; 2005, Ristic and Vo 2010, Ristic et al. 2011a]. In situations where improve-

\(^1\)We note that in PENT [Mahler 2004], instead of a divergence-based reward function, the expected cardinality of the multi-object estimate is chosen as the reward function. In multi-object applications involving sensors with limited field of view, extra detected targets indicate extra information content in the measurements [Mahler 2014].
ment in confidence of estimation is the main focus, task-driven methods seem to be the preferred option, as the divergence-based sensor management solutions do not necessarily increase the confidence of estimation. As an example, consider a visual target tracking scenario, involving a single target with one-dimensional state space. The state, denoted by $x$, is the horizontal location of the detected center of the target in a global coordinate system.\(^2\) Let us assume that at time $k$, the prior is given by $\pi(x|y_{1:k-1}) = \mathcal{N}(x; 5.0 \text{ m}, 0.5 \text{ m})$ where $y_{1:k-1}$ denotes all the recorded images acquired before time $k$, and $\mathcal{N}(\cdot; \mu, \sigma^2)$ is the Gaussian distribution with mean $\mu$ and variance $\sigma^2$. There are a number of camera control (panning) commands. The no panning command $u_1$ put uses a measurement with likelihood function $g(y_k|x; u_1) = \mathcal{N}(x; 5.0 \text{ m}, 0.3 \text{ m})$ which means the target is detected around $x = 5.0 \text{ m}$ with variance of $0.3 \text{ m}$. Another panning commands (panning to the left) put uses $g(y_k|x; u_2) = \mathcal{N}(x; 7.0 \text{ m}, 0.8 \text{ m})$ which demonstrates an unrealistic shift in the detected location of the target from $x = 5.0 \text{ m}$ to $x = 7.0 \text{ m}$ in a single frame, possibly due to the object being partially occluded after the move. Figure 7.1a shows the prior and the two possible likelihood functions. The posteriors associated with each of the two control commands (after applying Bayes’ rule using the likelihood functions) are presented in Fig. 7.1b. This figure shows that control command $u_1$ leads to a narrower posterior (i.e a more confident single object estimate\(^3\)). Despite the fact that the Rényi divergence from prior to posterior is larger for the command $u_2 (0.38072)$ than $u_1 (0.20626)$.

In our previous work reviews we had received significant interest to investigate the relation of the PEECS objective function and OSPA metric. In other words how variance reduction in PEECS will result in better estimation and consequently error reduction in OSPA metric. This concern impelled us to rethink ways of creating

\(^2\)We assume that pixel coordinates are transformed to global coordinates via camera calibration.

\(^3\)We note that in a single-object filtering application, posterior variance is the obvious choice for quantifying the confidence in estimation.
Figure 7.1: (a) The prior distribution and likelihood functions of the measurements associated with two sensor control commands, $u_1$ and $u_2$. (b) The posteriors resulting via Bayesian update using the two likelihood functions shown in (a).
pure multi-target-tracking-performance measure-based objective function for sensor management system. In other words, how should we (and, indeed, should we) design a new class of objective function oriented to the OSPA metric?

In [Aoki et al. 2011], Aoki et al showed that performance measurement, estimation and task-driven sensor management are intrinsically related problems, i.e. that the choice of a performance metric leads to some corresponding optimal estimate, and those lead to some corresponding optimal (task-driven) sensor management criterion.

In this paper, we introduce a task-driven sensor management method that minimizes the dispersion of the multi-object posterior, quantified based on the well-known OSPA distance [Schuhmacher et al. 2008]. This approach can be applied in conjunction with various multi-object filters such as Probability Hypothesis Density (PHD) [Mahler 2007b], Cardinalized PHD (CPHD) [Vo et al. 2007], MeMber [Mahler 2007b], CB-MeMber [Vo et al. 2009], Labeled Multi-Bernoulli (LMB) [Reuter et al. 2014], GLMB [Vo and Vo 2013] and Marginalized δ-GLMB [Fantacci et al. 2015] filters. The Sequential Monte Carlo (SMC) implementation of the devised cost function in conjunction with the LMB filter is also presented. The utility and advantages of the proposed approach are demonstrated through simulation studies. The approach can also be extended to sensor management problems involving non-standard non-separable measurement models by using the GLMB-based recursion [Papi et al. 2014].

7.2 Problem Statement

In multi-object tracking applications, the aim of sensor management is to direct the right sensors toward the right targets at the right times [Mahler 2004]. The multi-object filtering problem generally involves stochastic variations in the number of targets, measurement/process noise and detections, and can be formulated in the
form of an optimal stochastic control problem [Mahler 2014]. In this context, the sensor management problem is usually tackled via formulating the stochastic process as a Partially Observable Markov Decision Process (POMDP). Recent task-driven solutions are designed based on the POMDP framework depicted in Fig. 7.2. In each iteration of the multi-object Bayesian filter, the predicted prior, denoted by $\pi_{k|k-1}$ is used to (pseudo-)estimate the multi-object state $X_{k|k-1}$.

Given the multi-object estimate, for each sensor action command candidate (control command or selection command), $u \in U = \{u_1, \ldots, u_n\}$, the predicted ideal measurement set (PIMS) [Mahler 2004] is calculated which comprises the clutter- and noise-free measurements for the given multi-object state. For each PIMS, denoted by $\hat{Z}_u$, the multi-object posterior is (pseudo-)updated, then a cost function $V(u; X_{k|k-1})$ is computed. The best action command is selected as the one returning the minimum cost. Once the action command is chosen, the action takes place and actual measurements are acquired. Hence, the posterior is updated and the algorithm moves to the next iteration.

Devising a proper cost function is at the core of formulating task-driven sensor management techniques, and as it was mentioned earlier, recent solutions include cost functions that return a quantity that somehow indicates a measure of dispersion in the pseudo-updated posterior. For instance, in [Gostar et al. 2013a;c] the variance of cardinality, and in [Gia Hoang and Tuong Vo 2013], the Maximum A Posteriori
(MAP)-variance of cardinality are proposed as cost functions. In [Gostar et al. 2013b; 2014; 2015b] a normalized linear combination of cardinality variance and state estimation confidence (quantified as the product of single state variances) is proposed as the cost function. The next section presents a novel cost function that is directly related to the widely adopted multi-object estimation error, OSPA metric.

7.3 OSPA-Based Cost Function

Consider a single-object filtering scenario in which the aim of sensor management is to acquire measurements that maximizes the confidence of the object state estimation. The confidence of estimation can be quantified by measuring the dispersion of the updated posterior around its statistical mean. In a single-object filtering scenario, a straightforward measure of dispersion is the statistical expectation of Euclidean distance of the state from its mean. In case of multi-object filtering, where both the object states and their number need to be estimated, a set distance is need for the quantification.

In multi-target tracking literature, OSPA [Schuhmacher et al. 2008] has been promoted as a suitable metric for set distance measurement [Schuhmacher et al. 2008, Ristic et al. 2011b, Beard and Arulampalam 2012, Reuter et al. 2013, Crouse et al. 2011]. The calculation of OSPA requires ground truth and as such it is only used to quantify the multi-object estimation error. In this paper, we however use OSPA metric to quantify the dispersion of a multi-object distribution.

Let us denote the OSPA distance between two sets, $X$ and $Y$ by $d_p^c(X,Y)$ where $p$ and $c$ are the user-defined parameters of the metric [Schuhmacher et al. 2008]. We also denote the pseudo-updated distribution (using the PIMS computed from the action command $u$) by $\pi_{k,u}(\cdot)$ and its corresponding multi-object random set by $X_{k,u}$. Our proposed cost function, the OSPA-based dispersion of $\pi_{k,u}(\cdot)$ around its mean, is defined as:
\[ V(u; X_{k|k-1}) = \mathbb{E} \left[ \tilde{d}^c_p(X_{k,u}, X_{k,u}) \right] \] (7.1)

where the expectations are over the posterior \( \pi_{k,u} \), i.e.

\[ V(u; X_{k|k-1}) = \mathbb{E} \left[ \tilde{d}^c_p(X_{k,u}, \bar{X}_{k,u}) \right] = \int \tilde{d}^c_p(X, \bar{X}_{k,u}) \pi_{k,u}(X) \delta X \] (7.2)

in which \( \bar{X}_{k,u} = \mathbb{E} [X_{k,u}] = \int X \pi_{k,u}(X) \delta X \).

To calculate the above cost function, the posterior distribution is constructed using \( L \) Monte Carlo samples, \( \pi_{k,u}(X) \approx \frac{1}{L} \sum_{i=1}^{L} \delta_{X_{k,u}}(X) \), and we have:

\[ V(u; X_{k|k-1}) = \frac{1}{L} \sum_{i=1}^{L} \tilde{d}^c_p(X_{k,u}^{(i)}, \bar{X}_{k,u}). \] (7.3)

The pseudocode of our OSPA-based sensor management method is presented in Algorithm 8. The derivation of functions \( \text{EAP}(\pi_{k,u}) \) and \( \text{MC}(\pi_{k,u}, L) \), which return the \( \text{EAP} \) estimate and \( L \) samples for a multi-Bernoulli distribution is outlined in the following section.

### 7.4 LMB Implementation

The proposed sensor management solution presented in Algorithm 8 can be used with any Bayesian multi-object filter. To demonstrate its performance, the algorithm is implemented with an LMB filter [Reuter et al. 2014] (a special case of the GLMB filter [Vo and Vo 2013, Vo et al. 2014]) which produces track-valued estimates. An LMB Random Finite Set (RFS) is completely described by its components \( \pi = \{(r^{(\ell)}, p^{(\ell)}): \ell \in \Psi\} \) where each Bernoulli component with label \( \ell \) has a probability of existence \( r^{(\ell)} \) and a density \( p^{(\ell)}(\cdot) \).

Similar to the general multi-target Bayes filter, the LMB multi-target Bayes recursion propagates the multi-target posterior density at each time according to the Chapman-Kolmogorov and the Bayes rule. Interested readers are referred to
CHAPTER 7: SENSOR MANAGEMENT VIA MINIMIZATION OF OSPA-BASED DISPERSION OF POSTERIOR

Algorithm 8 The OSPA-based sensor management with SMC implementation.

**INPUTS:** predicted multi-object distribution \( \pi_{k|k-1}(\cdot) \), measurement likelihood function \( g_k(\cdot|x,u) \), current sensor(s) location(s), finite set of admissible commands \( U \), OSPA parameters \( p \) and \( c \).

**OUTPUT:** The best action command \( \hat{u} \).

**Pre-estimation:**
Estimate the number and state of targets based on predicted distribution

1: \( \hat{X}_{k|k-1} \leftarrow \operatorname{EAP}(\pi_{k|k-1}) \)  
   \( \triangleright \) Compute the EAP estimate from \( \pi_{k|k-1} \).
2: \( \hat{M}_{k|k-1} \leftarrow |\hat{X}_{k|k-1}| \)  
   \( \triangleright \) Record estimated number of targets.

**Cost Calculation:**

3: for all \( u \in U \) do,
4: \( \hat{Z} \leftarrow \emptyset \)  
   \( \triangleright \) Constructing the PIMS
5: for \( \ell = 1, \hat{M}_{k|k-1} \) do
6: \( \xi \leftarrow \operatorname{argmax}_z g(z|\hat{x}_{k|k-1}^{(\ell)},u) \)  
   \( \triangleright \) Note: \( \hat{x}_{k|k-1}^{(\ell)} \in \hat{X}_{k|k-1} \).
7: \( \hat{Z} \leftarrow \hat{Z} \cup \{\xi\} \)
8: end for
9: \( \pi_{k,u} \leftarrow \text{Updated posterior with } \hat{Z} \text{ as measurement set.} \)
10: \( \hat{X}_{k,u} \leftarrow \operatorname{EAP}(\pi_{k,u}) \)  
    \( \triangleright \) Compute the EAP estimate from \( \pi_{k,u} \).
11: \( \{X_{k,l}\}_{l=1}^L \leftarrow \operatorname{MC}(\pi_{k,u},L) \)  
    \( \triangleright \) Generating \( L \) samples from \( \pi_{k,u} \).
12: \( \mathcal{V}(u; X_{k|k-1}) \leftarrow 0 \)
13: for all \( l \in \{1,\ldots,L\} \) do
14: \( \mathcal{V}(u; X_{k|k-1}) \leftarrow \mathcal{V}(u; X_{k|k-1}) + \frac{1}{L}d_p(\bar{X}_{k,l}, \hat{X}_{k,u}) \)
15: end for
16: end for

**Decision Making on Sensor Action:**
17: \( \hat{u} \leftarrow \operatorname{argmin}_u \mathcal{V}(u; X_{k|k-1}) \)

the original paper by Reuter et al [Reuter et al. 2014] for the formulation of the propagation of LMB parameters. The prediction in LMB filter is identical to the prediction in the multi-Bernoulli filter [Reuter et al. 2014], and after discarding the label information, the predicted LMB density can be translated to the original predicted multi-Bernoulli density [Vo et al. 2009]. To save computation, for the purpose of sensor management, Algorithm 8 was implemented with the unlabeled version of the predicted LMB distribution as the input distribution \( \pi_{k|k-1}(\cdot) \).

Let us denote the predicted (unlabeled) multi-Bernoulli density by \( \{r_k^{(i)}|x_k^{(i)}\}_{i=1}^{M_{k|k-1}} \) which each density is approximated by weighted particles. The EAP estimate of the multi-object state (see line 1 in Algorithm 8) can be computed as follows. The
EAP estimate of cardinality is given by \( \hat{M}_{k|k-1} = \sum_{i=1}^{M_k} r^{(i)} \). The probabilities of existence can be sorted in a descending order, and EAP estimate of the state of the \( \ell \)-th object (corresponding to the Bernoulli component with the \( \ell \)-th largest probability of existence) can be calculated by the weighted sum of the particles associated with the Bernoulli component’s density. The same approach is used to calculate the EAP estimate of the multi-Bernoulli posterior (pseudo-updated using the PIMS measurement set – see line 10 in Algorithm 8).

When the unlabeled predicted multi-Bernoulli distribution is pseudo-updated (using the PIMS), the resulting posterior is still an unlabeled multi-Bernoulli. We denoted the pseudo-updated distribution parameters by \( \{r^{(i)}_k, p^{(i)}_k(\cdot)\}_{i=1}^{M_k} \) where the distribution of each Bernoulli component \( p^{(i)}(\cdot) \) is approximated by \( L^{(i)} \) particles, i.e \( p^{(i)}(x) \approx \sum_{j=1}^{L^{(i)}} w^{(i,j)} \delta_{x^{(i,j)}}(x) \). After resampling each Bernoulli component distribution, \( L_{\text{max}} \) particles are created with identical weights \( w^{(i,j)} = \frac{1}{L_{\text{max}}} \). Since, the pseudo-updated multi-Bernoulli distribution is fully parametrized by its probabilities of existence \( r = [r_1 \cdots r_M]^T \) and its particle matrix \( P = [x_{ij}]_{M \times L_{\text{max}}} \) we can devise the function \( MC(\pi_{k|k-1}) \) that returns \( L \) Monte Carlo samples of the above multi-Bernoulli distribution (see line 11 of Algorithm 8). The detail implementation of the function \( \text{MB-MC}(r,P,L) \) is presented in Algorithm 9.

Unlabeled multi-Bernoulli distributions are only calculated within the sensor management routine (the steps of Algorithm 8). In each iteration of multi-object filtering, once the best sensor action command \( \hat{a} \) is found and sensors are managed accordingly, an actual measurement set from sensors is acquired and used to update the predicted LMB distribution, resulting in an updated LMB posterior. In our simulations, for the pseudo-update operations needed by Algorithm 8, we have used the update formula of cardinality-balanced multi-Bernoulli filter [Vo et al. 2009], as well as the update formula of LMB filter [Reuter et al. 2014] for the final LMB posterior.
Algorithm 9 Monte Carlo sampling of a multi-Bernoulli distribution with given parameters and particles.

**INPUTS:** probabilities of existence \( r = [r_1 \cdots r_M]^\top \), particles matrix \( P = [x_{ij}]_{M \times L_{\text{max}}} \), and number of output samples (sets) denoted by \( L \).

**OUTPUTS:** A set \( X \) comprised of \( L \) sets, each being a Monte Carlo sample of the multi-Bernoulli distribution, in the form of \( X_\ell = \{x_{\ell,1}, \ldots, x_{\ell,n_\ell}\} \), where \( n_\ell \leq M \) is the cardinality of the \( \ell \)-th set.

1: \( \text{function } \text{MB-MC}(r, P, L) \)
2: \( X \leftarrow \emptyset \)
3: From the size of the particles matrix \( P \), find \( M \) and \( L_{\text{max}}. \)
4: for \( \ell = 1, \ldots, L \) do
5: \( X_\ell \leftarrow \emptyset \)
6: for \( i = 1, \ldots, M \) do
7: \( u \sim U(0,1). \)  
\( \triangleright U(0,1) \) denotes uniform random distribution between 0 and 1.
8: if \( u > r_i \) then
9: \( v \sim U(0,1). \)
10: \( j \leftarrow [L_{\text{max}}v]. \)  
\( \triangleright \) Index \( j \) randomly generated in \([1, L_{\text{max}}]. \)
11: \( X_\ell \leftarrow X_\ell \cup \{x_{ij}\}. \)
12: end if
13: end for
14: \( X \leftarrow X \cup \{X_\ell\}. \)  
\( \triangleright \) The set of MC sets is gradually completed.
15: end for
16: return \( X \)
17: end function

7.5 Simulation Results

To demonstrate the performance of our method we devised and run a case study, where we chose a pseudo stationary model for the targets. In this case, each single target state \( x = [\bar{x}^\top \omega]^\top \) was comprised of location and velocity in Cartesian coordinates, denoted by \( \bar{x} = [x \ y \dot{x} \dot{y}]^\top \) and turning rate, denoted by \( \omega. \) Values of the velocities were assumed to be very small. The state dynamics were given by:

\[
\bar{x}_k = F(\omega_{k-1})\bar{x}_{k-1} + \mathcal{G}_{\epsilon_{k-1}},
\]

\[
\omega_k = \omega_{k-1} + \mathcal{T}_{\gamma_{k-1}},
\]

where
\[ F(\omega) = \begin{bmatrix}
1 & 0 & \frac{\sin \omega T}{\omega} & -\frac{1-\cos \omega T}{\omega} \\
0 & 1 & \frac{1-\cos \omega T}{\omega} & \frac{\sin \omega T}{\omega} \\
0 & 0 & \cos \omega T & -\sin \omega T \\
0 & 0 & \sin \omega T & \cos \omega T 
\end{bmatrix},
\]
\[ G = \begin{bmatrix}
\frac{T^2}{2} & 0 \\
0 & \frac{T^2}{2} \\
T & 0 \\
0 & T 
\end{bmatrix},
\]
and \( T = 1 \text{s} \), \( \epsilon_{k-1} \sim N(\cdot; 0, \sigma^2 \epsilon I) \), \( \sigma_\epsilon = 15 \text{m/s}^2 \), and \( \gamma_{k-1} \sim N(\cdot; 0, \sigma^2 \gamma I) \), \( \sigma_\gamma = (\pi/180) \text{rad/s} \). The birth RFS is a multi-Bernoulli with density \( \pi^\Gamma = \{(r_\Gamma^{(i)}, p_\Gamma^{(i)})\}_{i=1}^{16} \) where \( r_\Gamma^{(i)} = 0.03 \) and \( p_\Gamma^{(i)}(x) = N(x; m_\Gamma^{(i)}, P_\gamma) \) where \( m_\Gamma^{(i)} \) are given in Table 7.1.

Probability of survival, detection probability, initial sensor location and clutter rate were chosen to be similar to [Gostar et al. 2014]. The observation model consisted of noisy bearing and range measurements, \( z_k = \left[ \arctan\left(\frac{y_k}{x_k}\right) \sqrt{x_k^2 + y_k^2} \right]^T + \zeta_k \), where \( \zeta_k \sim N(\cdot; 0, R_k) \) was the measurement noise with covariance \( R_k = \text{diag}(\sigma^2_r, \sigma^2_\theta) \) in which the scales of range and bearing noise were \( \sigma_r = \sigma_0 + \eta_r ||X-u||^2 \) and \( \sigma_\theta = \theta_0 + \eta_\theta ||X-u|| \) where \( \sigma_0 = 1 \text{m}, \eta_r = 5 \times 10^{-5} \text{m}^{-1}, \theta_0 = \pi/180 \text{rad}, \eta_\theta = 1 \times 10^{-5} \text{m}^{-1} \).

Measurements were synthetically generated as sets, each containing target-generated point measurements and possibly clutter and misses. In each set, every target could be detected with the distance-dependent detection probability,
\[
p_D(u, X) = \begin{cases} 
0.99, & \text{if } ||X-u|| \leq R_0 \\
\max\{0, 0.99 - b(||X-u|| - R_0)\} & \text{otherwise}
\end{cases}
\]
where \( R_0 = 400 \text{ m} \) and \( b = 25 \times 10^{-5} \text{ m}^{-1} \). The clutter followed the uniform Poisson model in the surveillance area \([-\pi/2, \pi/2] \text{rad} \times [0,2000] \text{m} \), with \( \lambda_c = 1.6 \times 10^{-3} \text{ (rad m)}^{-1} \). The sensor initial position was at \([0,0]\). Targets entered the scene with different initial positions, velocities and turning rates.

Intuitively, we expected the sensor to started moving toward the targets and remained in vicinity of those. As it is shown in Fig. 7.3, the sensor moved from the initial position and as expected, after a few steps, it remained among the targets. Figure 7.4 shows the OSPA errors for the proposed OSPA-based sensor control. We use the Euclidean metric with parameters \( c = 100 \) and \( p = 2 \). In general,
Figure 7.3: Sensor locations for OSPA-based (red) and PEECS-based (blue) sensor control, for $k = 1, \ldots, 40$.

Table 7.1: Birth parameters and labels $\left( m_{B}^{(t)}, \ell \right)$ at time $k$.

<table>
<thead>
<tr>
<th>$m_{B}^{(t)}$</th>
<th>$\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[1500 \ 0 \ 1900 \ 0 \ 0]^T, (k, 1)$</td>
<td></td>
</tr>
<tr>
<td>$[1217 \ 0 \ 1782 \ 0 \ 0]^T, (k, 3)$</td>
<td></td>
</tr>
<tr>
<td>$[1400 \ 0 \ 1800 \ 0 \ 0]^T, (k, 5)$</td>
<td></td>
</tr>
<tr>
<td>$[1117 \ 0 \ 1682 \ 0 \ 0]^T, (k, 7)$</td>
<td></td>
</tr>
<tr>
<td>$[1100 \ 0 \ 1500 \ 0 \ 0]^T, (k, 9)$</td>
<td></td>
</tr>
<tr>
<td>$[1782 \ 0 \ 1217 \ 0 \ 0]^T, (k, 11)$</td>
<td></td>
</tr>
<tr>
<td>$[1000 \ 0 \ 1400 \ 0 \ 0]^T, (k, 13)$</td>
<td></td>
</tr>
<tr>
<td>$[1682 \ 0 \ 1117 \ 0 \ 0]^T, (k, 15)$</td>
<td></td>
</tr>
</tbody>
</table>

OSPA errors for the proposed method is substantially lower than the errors returned by the recently developed PEECS-based sensor control [Gostar et al. 2015b] in the same multi-target tracking scenario.
Figure 7.4: OSPA errors returned by OSPA-based sensor control versus the PEECS-based method.
7.6 Conclusions

A new sensor management method for multi-object filtering was presented. In the proposed method, sensor actions are selected to maximize the confidence in multi-object state estimation. The dispersion of the multi-object posterior about its statistical mean was employed as an intuitive choice for measuring confidence in estimation. We proposed to use the well-known OSPA metric as a suitable distance for quantifying the dispersion of posterior about its mean.

A step-by-step algorithm was presented for the proposed sensor management scheme, with details of its implementation within a labeled multi-Bernoulli filter. In a simulation study, involving sensor control in a challenging multi-target tracking scenario, we examined the performance of the proposed OSPA-based method in comparison with the recently developed PEECS-based sensor control. The results demonstrate that the proposed method guides the sensor successfully producing excellent tracking results (in terms of OSPA errors) and outperforms the recently published PEECS-based sensor control method.
In this thesis, we defined the objective functions in FISST framework in which establishes a common ground between information-driven approach and physical interpretation of task-driven approach. The objectives of these sensor management tasks are:

- Maximize the number of observed targets, and
- Maximize the accuracy of the estimated states of the target.

The defined objective function carry the fundamental notions of the information theory as well as addressing the problem of physical interpretation. In chapters 3, 4, 5, 6 and 7, we defined objective functions using notion of information theory while they have physical interpretation of task-driven approach. We then compared our method with the following methods in sensor management framework.

8.0.1 Summery of the Contributions

The original summary of contributions in this thesis are:

Chapter 3:
CHAPTER 8: CONCLUSION

In Chapter 3 a sensor management solution was proposed to be employed within a multi-Bernoulli multi-target filter. In this method, at each step, the next sensor control command is chosen by minimizing a new task-driven cost function called “Posterior Expected Error of Cardinality and States” (PEECS). The PEECS cost associated with each control command is defined as a linear combination of the normalized errors in cardinality estimate of the multi-target random finite set and the normalized error of localization of the elements in the multi-target RFS. Simulation results involving two challenging multi-target estimation and sensor-control scenarios, demonstrated that PEECS sensor-control can return multi-object state estimation accuracy and clutter tolerance that are similar to or better than competing methods, and generally performs faster than the state-of-the-art.

Chapter 4 and 5:

Later in Chapters 4 and 5, we introduced a sensor management solution especially designed to work with a robust multi-Bernoulli multi-object filter that needs no prior knowledge of clutter distribution or detection profile. The proposed sensor management technique employs a novel cost function that takes the accuracy of cardinality, state and clutter estimation into account. SMC implementation of this sensor management routine, along with SMC implementation of an information-theoretic sensor management solution (based on Rényi divergence) were both presented in detail. The two solutions were examined in simulation studies which revealed that they produce similar estimation accuracies, but the task-driven sensor-selection method is substantially faster.

Chapter 6:

In Chapter 6, we introduced a novel sensor management, for controlling mobile sensor(s) in such a way that minimum expected errors are achieved in a multi-target tracking application. The proposed method works based on choosing the sensor command that is expected to lead to the lowest cost, and the cost is defined and formulated in terms of cardinality and single-target state estimation errors.
Implementation of the cost computation and its minimization within a labeled multi-
Bernoulli filter was elaborated and a step-by-step algorithm was presented.

In a challenging simulation scenario, our proposed method demonstrated suc-
cess in optimal guidance of a mobile sensor in tracking of up to 5 targets which
can appear and disappear in/from the scene. Compared to the scenario where a
similar cost function was used within a CB-MeMBeR filter, we showed that our new
method developed for the labeled multi-Bernoulli filter performs better in terms of
OSPA errors in the same challenging scenario. This can also be due to the advan-
tageous nature of the labeled multi-Bernoulli filter in terms of its better accuracy
in approximating the update step.

This work can be extended to applications where detection and tracking of
targets of interest are required. In this case, a new task-driven cost function would
be needed in which not only the estimation errors are considered, but also the track
labels produced by the labeled multi-Bernoulli filter are utilized. Such a sensor-
control routine would guide the sensor(s) towards sensor states which are likely to
lead to better estimates of the states of the targets of interest (with particular labels)
only.

Chapter 7:

In our previous works’ reviews, we had received significant interest to investigate
the relation of the PEECS objective function and OSPA metric. In other words how
variance reduction in PEECS will result in better estimation and consequently error
reduction in OSPA metric. This concern impelled us to rethink ways of creating
pure multi-target-tracking-performance-measure-based objective function for sensor
management system. In other words, how should we (and, indeed, should we) design
a new class of objective function oriented to the OSPA metric?

In Chapter 7 we introduced a new type of objective function with explicit
relation to multi-target performance metric. A new sensor management method
for multi-object filtering was presented. In the proposed method, sensor actions are
selected to maximize the confidence in multi-object state estimation. The dispersion of the multi-object posterior about its statistical mean was employed as an intuitive choice for measuring confidence in estimation. We proposed to use the well-known OSPA metric as a suitable distance for quantifying the dispersion of posterior about its mean.

A step-by-step algorithm was presented for the proposed sensor management scheme, with details of its implementation within a labeled multi-Bernoulli filter. In a simulation study, involving sensor control in a challenging multi-target tracking scenario, we examined the performance of the proposed OSPA-based method in comparison with the recently developed PEECS-based sensor control. The results demonstrate that the proposed method guides the sensor successfully producing excellent tracking results (in terms of OSPA errors) and outperforms the recently published PEECS-based sensor control method.


CHAPTER 8: BIBLIOGRAPHY


