Numerical Simulation of Turbulence Modulation in Two-Phase Flows

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BE

A thesis submitted in fulfilment of the requirement for the degree of Doctor of Philosophy.

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Declaration

I, Krishna Mohanarangam, hereby submit the thesis titled “Numerical Simulation of Turbulence Modulation in Two-Phase Flows” for the degree of Doctor of Philosophy and certify that the work is my own work except where due acknowledge has been made; the work has not been submitted previously, in whole or in part, for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program.

I give consent for this copy of my thesis to be made available for loan and photocopying when handed to the University Library for archiving.

Krishna Mohanarangam
February 2008
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I would also like to thank my parents, my younger brother and my extended family, who helped me to keep my feet on ground when I was high up in the air searching for the unknown(s).

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Abstract

With the increase of computational power, computational modelling of two-phase flow problems using computational fluid dynamics (CFD) techniques is gradually becoming attractive in the engineering field. Two basic CFD approaches are used to simulate the two-phase flow, i.e. the Eulerian-Lagrangian model and the Eulerian-Eulerian model. The major aim of this thesis is to investigate the Turbulence Modulation (TM) of dilute two phase flows. In order to carry out this approach, an in house research code employing Two-Fluid model, with additional source terms to account for the presence of the dispersed phase in the turbulence equations has been employed.

Various density regimes of the two-phase flows have been investigated in this thesis, namely the dilute gas-particle flow, liquid-particle flow and also the liquid-air flows. While the density is quite high for the dispersed phase flow for the gas-particle flow, the density ratio is almost the same for the liquid particle flow, while for the air-liquid flow the density is quite high for the carrier phase flow. The study of all these density regimes gives a clear picture of how the carrier phase behaves in the presence of the dispersed phases, which ultimately leads to better design and safety of many two-phase flow equipments.

For the dilute gas-particle flows, particle-turbulence interaction over a backward-facing step geometry was numerically investigated. An Eulerian two-fluid model with additional turbulence transport equations for particles is employed in this investigation. RNG based k-ε model is used as the turbulent closure with additional transport equations solved, to better represent the combined gas-particle interactions. Two different particle classes with same Stokes number and varied particle Reynolds number are considered in this study. The turbulence modulation of the carrier phase in the presence of the dispersed particulate phase is simulated and compared against the experimental data. Despite the fact that the two particles used in this study share the same Stokes number their behaviour is found to be considerably different in the turbulent flow field, which basically underlines the fact that the Stokes number alone is not enough to fully describe the behaviour of particles, there by, herein particle Reynolds number is also investigated to fully understand
their behaviour. Two other turbulence modulation models along with the SATO model were tested against our own formulation and our model was found to compare better with the experimental findings.

A detailed study into the turbulent behaviour of dilute particulate flow under the influence of two carrier phases namely gas and liquid was also been carried out behind a sudden expansion geometry. The major endeavour of the study is to ascertain the response of the particles within the carrier (gas or liquid) phase. The main aim prompting the current study is the density difference between the carrier and the dispersed phase. While the ratio is quite high in terms of the dispersed phase for the gas-particle flows, the ratio is far more less in terms of the liquid-particle flows. Numerical simulations were carried out for both these classes of flows and their results were validated against their respective sets of experimental data. Qualitative results have been obtained for both these classes of flows with their respective experimental data, furthermore their response to their carrier phase has been investigated both at the mean and turbulence level for a range of Stokes number. While the particulate velocity seems to increase with the corresponding increase in Stokes number amidst both the carrier phases the particulate turbulence shows entirely a different pattern.

For the Liquid-Air flows the phenomenon of drag reduction by the injection of micro-bubbles into turbulent boundary layer has been investigated using an Eulerian-Eulerian two-fluid model. Two variants namely the Inhomogeneous and MUSIG (MUltiple-SIze-Group) based on Population balance models are investigated. The simulated results were benchmarked against the experimental findings and also against other numerical studies explaining the various aspects of drag reduction. For the two Reynolds number cases considered, the buoyancy with the plate on the bottom configuration is investigated, as from the experiments it is seen that buoyancy seem to play a role in the drag reduction. Well established theories of drag reduction from various experiments and high resolution numerical studies are scrutinised and explained in context to our numerical findings. The under predictions of the MUSIG model at low rates was investigated and reported, their predictions seem to fair better with the decrease of the break-up tendency among the micro-bubbles, this information was later used as a predictive tool for the two-fluid Inhomogeneous model.
Work Published During Candidature

During the course of my PhD study, a number of papers have been produced based on the results described in this thesis. Three journal papers have been published, and another four journal papers have been submitted. Additionally, four conference/workshop papers have been presented in national and international conferences. A detailed publication list is presented below:

**Journal papers:**


Conference papers:


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Nomenclature

\( A_i \)  
convective flux

\( A_{ij}, A_s \)  
model constants for realizable \( k-\varepsilon \) turbulence model

\( B \)  
diffusion coefficient

\( B_{sp}, B_{e} \)  
model constants for the Eulerian two-fluid model

\( C_{\mu} \)  
coefficient in the \( k-\varepsilon \) turbulence model

\( C_D \)  
particle drag coefficient

\( C_{RNG} \)  
constant in RNG LES model

\( C_1, C_2 \)  
model constants for realizable \( k-\varepsilon \) turbulence model

\( C_{\varepsilon 1}, C_{\varepsilon 2} \)  
model constants for standard and RNG \( k-\varepsilon \) turbulence models

\( d_p \)  
particle diameter

\( e \)  
coefficient of restitution

\( e_n, e_t \)  
mean normal and tangential restitution coefficients

\( f \)  
correction factor

\( F_{Di} \)  
aerodynamic drag force

\( F_{Gi} \)  
gravity force

\( F_r \)  
Froude number

\( F_{WMI} \)  
wall-momentum transfer due to particle-wall collision force

\( g \)  
gravitational acceleration

\( h \)  
step height

\( H_r \)  
the mean roughness depth for wall surface

\( l_{sp} \)  
turbulence interaction between the gas and particle phases for the particle phase turbulent fluctuating energy

\( l \)  
length scale of energetic turbulent eddies

\( L_r \)  
the mean cycle of roughness

\( L_s \)  
characteristic length of the system

\( L_e \)  
eddy length scale

\( m \)  
ratio of particle to gas density
$\dot{m}$ mass of particles in per unit volume of the gas and particle mixture
$P_{A}$ the normal impulse due to adhesion during rebound
$P_{D}$ the normal impulse generated by deformation during approach
$P_{k}$ turbulence production by the mean velocity gradients of two phases
$P_{k}$ rate of production term of the turbulent kinetic energy
$P_{k}$ production term of the particle fluctuating energy
$q_{p}$ general source term
$r^{*}$ normalized radial co-ordinate
$r$ uniform random number
$Re$ Reynolds number
$R_{f}$ restitution coefficient in the absence of adhesion
$S$ source term
$St$ Stokes number
$S_{ij}, S_{jk}, S_{ki}$ strain rates
$t_{cross}$ eddy crossing time
$t_{int}$ eddy-particle interaction time
$t_{p}$ particle relaxation time
$t_{s}$ system response time
$T$ fluid temperature
$T_{L}$ fluid Lagrangian integral time
$u_{i}^{p}$ particle incident velocity in tangential direction
$u_{n}^{p}$ particle incident velocity in normal direction
$u_{i}, u_{j}, u_{k}$ velocity
$u_{o}$ free stream velocity
$V_{s}$ characteristic velocity of the system
$v_{i}^{p}$ particle rebound velocity in tangential direction
$v_{n}^{p}$ particle rebound velocity in normal direction
$v_{p}$ particle rebounding velocity
$x_{i}, x_{j}, x_{k}$ Cartesian coordinate system
\( a_{if} \)  
Interfacial area concentration

\( C \)  
Adjustable model constant

\( C_D \)  
Drag coefficient

\( C_{WE} \)  
Wake entrainment coefficient

\( d \)  
parent bubble diameter

\( d_i, d_j \)  
daughter bubble diameters

\( d_H \)  
Maximum bubble horizontal dimension

\( D \)  
Inner diameter of the pipe

\( D_B \)  
death rate due to break-up

\( D_C \)  
death rate due to coalescence

\( D_s \)  
Sauter mean bubble diameter

\( f_{BV} \)  
brkage volume fraction

\( f_i \)  
scalar variable of the dispersed phase

\( F_{C,F_B} \)  
Coalescence and Breakage calibration factors

\( F_{ig} \)  
Interfacial drag force

\( g \)  
Gravitational acceleration

\( \bar{g} \)  
Gravitational vector

\( h_0 \)  
initial film thickness

\( h_l \)  
critical film thickness

\( k \)  
Turbulent kinetic energy

\( n_i \)  
number density of the \( i \)th class

\( n_j \)  
number density of the \( j \)th class

\( P \)  
Pressure

\( P_B \)  
production rate due to break-up

\( P_C \)  
production rate due to coalescence

\( Re \)  
Flow Reynolds number

\( R_{ij} \)  
Net change rate of number density due to coalescence and break-up

\( t \)  
Physical time

\( u \)  
Velocity

\( u_t \)  
Turbulent Velocity
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<td>$\tilde{u}$</td>
<td>Velocity vector</td>
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<td>$U$</td>
<td>relative velocity between gas and liquid phase</td>
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<td>$U_r$</td>
<td>terminal velocity of bubbles</td>
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<td>$V_i, V_j$</td>
<td>volume corresponding to bubble group $i$ and $j$</td>
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**Greek letters**

- $\alpha$: volume fraction / void fraction
- $\beta$: model constant for RNG $\kappa$-$\varepsilon$ turbulence model
- $\Gamma$: diffusivity of the scalar
- $\varepsilon$: dissipation rate of turbulent kinetic energy
- $\varepsilon_0$: the direction of the relative velocity between particle surface and wall
- $\phi$: governing variable
- $\eta$: function defined in Equation (6)
- $\eta_o$: model constant for RNG $\kappa$-$\varepsilon$ turbulence model
- $k$: turbulent kinetic energy
- $\mu$: dynamic viscosity
- $\mu_0$: the static friction coefficient
- $\mu_d$: dynamic friction coefficient
- $\mu_{eff}$: effective turbulent viscosity
- $\mu_t$: turbulent viscosity
- $\nu$: kinematic viscosity
- $\theta$: angle between velocities of the particle and gas
- $\theta$: particle incident angle
- $\rho$: density
- $\rho_l$: adhesion coefficient
- $\sigma$: turbulence Prandtl number
- $\tau_e$: eddy life time
- $\tau_f$: fluid time scale
- $\tau_w$: wall shear stress
- $\tau_p$: particle relaxation time
- $\omega$: fluctuating vorticity
\( \omega_p \) particle initial angular velocity

\( \zeta \) normally distributed random number

\( \Pi_{gp} \) turbulence interaction between the gas and particle phases for the gas-particle

\( \Omega \) vorticity

\( \Omega_p \) particle rebounding angular velocity

**Subscripts**

*add* additional

*eff* effective

*\( g \)* gas phase

*\( gp \)* gas-particle

*\( n \)* normal direction

*\( p \)* particle phase

*\( s \)* solid phase

*\( t \)* tangential direction or turbulent phase

*\( \alpha \)* Void fraction

*\( \varepsilon \)* Turbulence kinetic energy dissipation

*\( \eta_{jki} \)* transfer coefficient between bubble groups arising from bubble breakup

*\( \lambda \)* Eddy size in the inertial subrange

*\( \mu \)* Effective viscosity

*\( \xi \)* size ratio between an eddy and a particle in the inertial subrange

*\( \rho \)* Density

*\( \Delta \rho \)* Density difference = \( \rho_i - \rho_g \)

*\( \sigma \)* Surface tension

*\( \tau_{ij} \)* Bubble contact time

*\( \chi_{ij} \)* Turbulent random coalescence rate
\[ \Omega(v) \quad \text{Bubble breakup rate} \]

*Superscript*

- \( g \): gas phase
- \( gp \): gas-particle
- \( p \): particle phase
- \( \varepsilon \): dissipation rate of turbulent kinetic energy
- \( \kappa \): turbulent kinetic energy
- \( \overline{\cdot} \): averaged or resolved parameters
- \( \cdot' \): fluctuation
- \( g \): Gas
- \( gl \): Transfer of quantities from liquid phase to vapour phase
- \( i \): Index of gas/liquid phase
- \( l \): Liquid
- \( lg \): Transfer of quantities from gas phase to liquid phase
- \( \min \): Minimum operator
- \( \max \): Maximum operator