Variable Caster Steering in Automotive Dynamics

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

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my parents, my wife, Giang, and my son, Lam
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Love you all dearly.
Credits

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Abstract

The tyre lateral force is a function of side slip and camber. While the lateral force due to side slip is the main part of the total cornering force, it saturates at a certain level of side slip angle even when the available tyre-road friction has not been fully utilised. In such a situation, the use of an appropriate extra camber could increase the safety, stability, and manoeuvrability of the car. This research serves as a further step to put the theory of variable caster steering into effect. The key idea of the theory is actively varying the caster to produce the required extra camber in a good fashion. In this investigation, the theory was examined in a specific case: developing a variable caster scheme to counteract roll camber - a phenomenon that generally limits the tyre lateral force and hence the maximum lateral acceleration of the car. By applying the variable caster scheme, the lateral force is increased. Therefore, the lateral grip capacity of the car is expanded. The benefit of the variable caster was also further exploited to improve the steering returnability during low speed cornering.

The theory of variable caster steering began with the development of road steering wheel kinematics. In order to do that, a number of coordinate systems was introduced to sufficiently describe the steering motion of a road steering wheel. The homogeneous transformation was then utilised to map coordinates between the systems. By doing so, the kinematics was developed. It was then used in two ways. The first was to determine the camber, which is the orientation of the wheel, as a function of steering axis orientation, vehicle motion, suspension geometry, and steering angle, for a cornering car. The other was for developing a novel method to determine kingpin moment which affects the returnability of the steering wheels during low speed cornering.

Then a rollable vehicle model, which is capable of capturing important characteristics of a turning car such as load transfer and roll motion, was constructed. The Magic Formula was used for tyre force modelling to take the camber contribution and the non-linear characteristics of the tyre into account. MATLAB/Simulink was used to simulate the dynamic response of the vehicle to steering input. The steering wheel kinematics and the dynamics model of the car were later validated using both multi-body and experimental data. More specifically, the validation of the wheel kinematics was done by a road steering wheel model built in ADAMS software; the dynamic vehicle model was validated using data from field tests and from a full car model constructed in the CarSim environment.
A kinematics analysis of the camber function determined earlier was carried out. On the basis of the analysis, a scheme of varying the caster with the primary aim of countering the roll camber was proposed. The dynamic responses of the vehicle to different steering inputs were examined to evaluate its dynamic performance with and without the variable caster strategy. The simulation results show that the roll camber phenomenon, for the caster-controlled car, is reduced significantly. The associated lateral acceleration and yaw rate increase without compromising other handling characteristics. The variable caster strategy, therefore, provides a more manoeuvrable car with expanded turning capacity compared to the passive car. To take advantage of the variable caster, a caster configuration that can improve the returnability of the steering wheels in low speed cornering manoeuvres was also suggested. Using the novel method for determining the kingpin moment, we showed that the caster configuration provides a better steering returnability during low speed cornering.
1.1 Introduction to Variable Caster Steering

The lateral force of a tyre is a function of side slip and camber. The side slip, $\alpha$, is the angle between the tyre’s velocity vector and the tyre-plane when viewed from the top; and, the camber, $\gamma$, is the tilt angle of the tyre-plane from the vertical when viewed from the front or rear of the tyre, as illustrated in Figure 1.1.

![Figure 1.1: Side slip, $\alpha$, and camber, $\gamma$](image)

A typical sample of the lateral force curves is depicted in Figure 1.2. As is shown in the graph, while the lateral force due to side slip is the major part of the total lateral force, it reaches a saturation point at a certain level of side slip angle. This happens even when much of the available friction between the tyre and the road has not been utilised (Laws 2010); hence, the vehicle cannot make the most of the tyres’ potential; and its turning
capacity (lateral grip capacity) is limited. Although the lateral force due to camber is much less than the side slip force, the camber has the ability to utilise more available friction at the contact patch than the side slip. Unfortunately, when a conventional car negotiates a turn, its steering tyres usually lean away from the bend due to the roll motion of the car body. This general condition, as depicted in Figure 1.2, creates camber force in the opposite direction to the side slip force, as shown in the Figure, and makes the situation even worse. Consequently, the steering tyres may not be able to generate enough lateral force for the car to negotiate the bend; and it may deviate from the desired path of the motion. Therefore, controlling of the camber, especially when the tyre is working under a large side slip angle region, may expand the turning capacity; and hence enhance the safety, stability, and manoeuvrability of the vehicle.

![Figure 1.2: Lateral tyre force, $F_y$, is a function of side slip, $\alpha$, and camber, $\gamma$](image)

The camber of the steering tyres of a cornering car can be altered by changing the orientation of the kingpin axis, which is represented by caster and kingpin inclination (KPI) angles. The caster and the KPI, as shown in Figure 1.3, are the tilt angles of the steering axis from the vertical when viewed from the side and the front of the vehicle, respectively. Furthermore, as is demonstrated in this investigation, the caster is more effective than the kingpin inclination angle in generating the camber. Therefore, the idea of varying the caster in order to provide the steering wheel with an extra camber in some beneficial ways, which was suggested by Jazar et al. (2012), is examined in this research.
This current research acts as a further step to put into effect the idea called \textit{variable caster steering} theory. The investigation consists of two main parts: the kinematics of steering tyre with a variable caster, and the effects of a variable caster steering scheme on vehicle dynamics. In the former, the \textit{homogeneous transformation} is employed as a tool to develop the steering tyre kinematics in a general case which is applicable to the case of a variable caster. The latter deals with testing the theory with a specific variable caster scheme. This scheme of varying caster is proposed to counter the roll camber phenomenon which limits the lateral grip capacity of the car. The expected result is a more manoeuvrable car with better turning capacity. As an advantage of variable caster steering, we also show that the scheme can be further developed to improve the returnability of the steering system during low speed cornering.

1.2 Background and Literature Review

This section starts by providing the background to car lateral dynamics. This includes how a tyre generates lateral force by side slip and camber angles; the relationship between the tyre-road friction, the maximum lateral tyre force, and the turning ability of a car. It is then followed by a literature review on the topic of camber, camber control, variable caster and the related knowledge such as tyre and vehicle modelling.

1.2.1 Tyre Lateral Force and Vehicle’s Turning Ability

A car negotiates a turn by generating the lateral forces under its tyres, as shown in Figure 1.4. The turning ability of a car, which is characterized by lateral acceleration and yaw rate, is governed by the tyre lateral forces. In conventional cars, the lateral force arises principally from the side slip angle.
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Figure 1.4: A cornering car and its tyre lateral forces

To understand the limit on the turning ability, we review the fundamental of car lateral dynamics. This is done by utilising a simple vehicle model depicted in Figure 1.5. The model is called a two-wheel model or bicycle model. The model only deals with the vehicle motions in the plane parallel to the road plane with the roll motion being ignored. The vehicle has wheelbase $l$; its center of gravity (COG) is described by $a$ and $b$, the distances from the COG to the front and rear axles, respectively.

Figure 1.5: The bicycle car model
Accepting the small steering angle assumption, the approximated force equations are written as:

\[ F_x \approx F_{xf} + F_{xr} \]  \hspace{1cm} (1.1)
\[ F_y \approx F_{yf} + F_{yr} \]  \hspace{1cm} (1.2)
\[ M_z \approx aF_{yf} - bF_{yr} \]  \hspace{1cm} (1.3)

Therefore, the motion of a steady state cornering car is described by the following equations:

\[ ma_x = F_{xf} + F_{xr} \]  \hspace{1cm} (1.4)
\[ ma_y = F_{yf} + F_{yr} \]  \hspace{1cm} (1.5)
\[ I_z \dot{r} = aF_{yf} - bF_{yr} \]  \hspace{1cm} (1.6)

where \( m \) is the vehicle mass, \( I_z \) is the yaw moment of inertia, \( a_x \) is the longitudinal acceleration, and \( a_y \) presents the lateral acceleration.

In steady state cornering, the forward velocity and the yaw motion are constant, \( \dot{v}_x = 0, \dot{r} = 0 \), therefore, the two equations (1.5) and (1.6) become independent of the equation (1.4), and govern the motions of a steady state cornering car (Jazar 2014).

The maximum lateral force that can be generated at each tyre is determined by the road-tyre friction and the normal load, as shown in equations (1.7) and (1.8):

\[ F_{yf} \leq \mu F_{zf} \]  \hspace{1cm} (1.7)
\[ F_{yr} \leq \mu F_{zr} \]  \hspace{1cm} (1.8)

where \( \mu \) is the effective coefficient of friction between the tyre and the road.

Therefore, the maximum lateral acceleration of the car will only be reached if both lateral forces at the front and the rear axles are saturated simultaneously:

\[ ma_{y_{\text{max}}} = F_{y_{\text{max}}} = F_{y_{f_{\text{max}}}} + F_{y_{r_{\text{max}}}} = \mu F_{zf} + \mu F_{zr} = \mu F_z = \mu mg \]  \hspace{1cm} (1.9)

or

\[ a_{y_{\text{max}}} = \mu g \]  \hspace{1cm} (1.10)

Also, in the steady state, the yaw rate is proportional to the lateral acceleration. Thus, the maximum yaw rate is:

\[ r_{\text{max}} = \frac{a_{y_{\text{max}}}}{v_x} = \frac{\mu g}{v_x} \]  \hspace{1cm} (1.11)

where \( g \) is the acceleration due to gravity.

Although this simple two-wheel model is not capable of capturing other dynamic aspects and scenarios, the result is basically the same as that presented in the equations (1.10) and (1.11): the steady state turning capacity of the car is limited by the tyre-road friction.
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While the maximum lateral tyre force is limited by the friction between the road and the tyre, much of the available friction has not been fully exploited when the tyre lateral force is only generated by the side slip. In this situation, the use of the camber can increase the tyre lateral force and hence expand the turning capacity of the car. This is because the camber can utilise more of the available friction compared to the side slip. The generations of tyre lateral force by side slip and camber are explained by using a tyre brush model. This is presented in the next section.

1.2.2 The Lateral Force due to Side Slip

The formation of the side slip and the associated lateral force under the tyre contact patch can be explained by the tyre brush model presented in the literature on tyre modelling topic (Laws 2010, Pacejka 2012).

![Figure 1.6. The brush tyre model](image)

Figure 1.6. The brush tyre model

a) A side-slipping tyre with its ‘bristles’; b) The motion of a free rolling tyre; c) The motion of a side-slipping tyre with lateral force

The tyre brush model depicted in Figure 1.6 works on the assumption that there are small elastic ‘bristles’ on the tyre perimeter in the contact area. Their base is attached to the tyre while their tips touch the ground plane; the ‘bristles’ can horizontally deflect. When the tyre is in rolling motion, the first ‘bristle’ entering the contact patch is considered to be perpendicular to the ground plane. If the motion is free rolling (no driving action), the tyre will be travelling along a straight line in the direction of the
SECTION 1.2: BACKGROUND AND LITERATURE REVIEW

tyre plane (tyre velocity vector). In this case, all the ‘bristle’ elements stay vertical with respect to the ground. They move from the leading edge to the trailing edge with no horizontal deflection. Thus, there will be neither longitudinal nor lateral force developed. When the tyre velocity vector makes an angle with the tyre plane, the side slip exists. In that situation, the ‘bristles’ deform horizontally; and in particular, for pure side slip (no longitudinal slip), the deformations only occur in lateral direction. The restoring forces caused by the deformations give rise to a lateral force $F_y$. The deformation is zero when the element is entering the leading edge; it increases linearly as the element moves towards the trailing edge; finally, the deformation returns to zero as the element goes beyond the contact patch. The deformations result in a triangular distribution of the lateral force under the contact patch, as shown in Figure 1.6c.

Figure 1.6c: Lateral force distribution under the contact patch

Figure 1.7: Normal load distribution, sliding friction limit, and adhesion friction limit under the contact patch

The actual lateral force developed at the tyre contact patch, however, depends greatly on the normal load distribution. The distribution of the vertical load over the contact patch, which has been adopted largely in the literature, is illustrated in Figure 1.7. It is described by a parabola: the normal pressure is highest at the mid-point of the contact patch, and tends to zero at the leading and trailing edges. Consequently, the distributions of sliding friction and cohesion friction limits are the same pattern as that of the vertical load, as shown in the Figure. As long as the restoring force associated with a ‘bristle’ is less than the adhesion limit, the tip of that ‘bristle’ still adheres to the ground plane. If the restoring force exceeds the adhesion limit, that ‘bristle’ will slide; as a result, the actual restoring force will be restricted and equal to the sliding limit, which is less than the adhesion limit. The point at which the sliding starts to occur is called the sliding point; and the part of the contact patch from the sliding point to the rear is called the sliding part. In this sliding part, the actual lateral force distribution coincides with the sliding
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limit parabola. As the side slip angle increases, the sliding point moves from the rear to the front part of the contact patch. The distributions of the lateral force for different side slip angles illustrated in Figure 1.8 show that the available tyre-road adhesion friction is not fully utilised by the side slip. The side slip force is saturated at a certain level of the side slip angle. The distributions also show that the position of the resultant lateral force moves from the rear to the front as the side slip angle increases. These result in the tyre force and moment curves as illustrated Figure 1.9.

Figure 1.8: Side slip force distributions for small to large side slip angles

Figure 1.9: Typical lateral tyre force and aligning moment
The saturation is a downside of the side slip force, as the side slip cannot fully exploit the available friction between the road and the tyre. This, therefore, limits the turning ability of the vehicle. When the side slip force is saturated, the contribution of the camber force becomes notable. This is because the camber can utilise more available friction at the tyre contact. The formation of the camber force is presented in the next section.

### 1.2.3 The Lateral Force due to Camber

A free-rolling tyre with camber tends to rotate about point $O$, which is the intersection between the ground and its spin axis. Therefore, the base of the ‘bristles’ follows an arc-shape trajectory. Their tips are, however, constrained to move in a straight line on the road. This causes the lateral deformations of the ‘bristles’ an arc-shape profile. As a result, the associated camber force distribution is as shown in Figure 1.10.

![Figure 1.10: Camber force distribution](image)

Because of the distribution, the camber force profile matches with the available friction profile better than does the side slip force one. The matches of the camber force with the friction profile for different camber angles are depicted in Figure 1.11. Therefore, the camber has the ability to utilise more of the available road-tyre friction than the side slip.

![Figure 1.11: Camber force distributions for small to large camber angles](image)
1.2.4 Camber in Automotive Dynamics

When the tyre is cambered, a lateral force is produced at the tyre contact patch, as explained in the previous section. This force is called camber force or camber thrust. As a part of the total tyre lateral force, the camber force exerts either more or less effect on the dynamic behaviour of a car. If the camber thrust acts on the same direction as the side slip force, the total tyre lateral force will strengthen. If the camber force is in the opposite direction to that of the side slip force, the total lateral force will be reduced. Consequently, the lateral dynamics of the vehicle is affected.

The camber effects on vehicle dynamics have long been realised. One of the noticeable effects is that camber can change the handling characteristics of the car in terms of understeer, oversteer, and neutralsteer. More specifically, Gillespie (1992) suggested that, on passenger cars, camber thrust contributes to understeer behaviour, but normally as a secondary source. He also stated that, on vehicles with independent suspensions where significant camber may be achieved, the camber may contribute up to nearly 25 percent of the understeer gradient. Similar results were also found by Milliken (2006): in conventional car, although side slip provides most of the cornering force, the camber from independent suspension is useful in providing stability; for example, the positive outward camber on independent front suspension contributes to understeer.

The effect of camber on vehicle stability can be understood by considering Stability Factor, \( K \), which governs the car handling characteristics, as follows (Jazar 2014):

\[
K = \frac{m}{l^2} \left( \frac{a}{C_{\alpha f}} - \frac{b}{C_{\alpha r}} \right)
\]  

(1.12)

where \( C_{\alpha f} \) and \( C_{\alpha r} \) are the side slip coefficients for the front and rear tyres, respectively; the other parameters are given in the Figure 1.5.

The Stability Factor determines if the car is understeer (\( K > 0 \)), oversteer (\( K < 0 \)), or neutralsteer (\( K = 0 \)). The contribution of camber force to the total lateral force can be considered to be an additional equivalent side slip force. When the camber is taken into account, the side slip coefficients (\( C_{\alpha f} \) and \( C_{\alpha r} \)) in equation (1.12) are replaced by the equivalent side slip coefficients (\( C_{\alpha fe} \) and \( C_{\alpha re} \)). Therefore, the Stability Factor with camber being taken into consideration, and hence the handling characteristics of a car, can be changed. This effect was also claimed by Abe (2009). He argued that the camber thrust in a cornering car turns to be one of the forces that balance the centrifugal force acting at the centre of gravity of the vehicle. The lateral force caused by the camber may be in the same or opposite direction with the centrifugal force, resulting in the cornering force and the wheel side slip angles being larger or smaller, respectively. If the sign of the difference between side slip angles of front and rear tyres of the car changes, the vehicle steer characteristics will alter.

Another effect of camber is that it can change a car’s grip ability (Ammon 2002, Laws 2010, Milliken 2006). For example, Milliken (2006) found that negative camber on racing
cars can contribute to maximum lateral acceleration and steering effectiveness. It was also reported that, when the outer wheel in the concept car F 400 Carvings tilts inwards by 20 degrees, the maximum lateral acceleration is $1.28g$, which is higher than the conventional one (Ammon 2002). Moreover, camber not only has strong effects on the tyre lateral force as an additional equivalent lateral slip ratio but also leads to significant variation in the structural parameters of friction coefficient and cornering stiffness (Laws 2010). Cheng et al. (2011), Lu et al. (2006) also claimed similar results: the normal stress on the tyre contact zone increases with the increase of the camber angle; and the high normal stress increases with the increase of the camber and concentrates at the one-side shoulder. For off-road tyre, which lugs taken into account, it is shown that, together with side slip angle and soil hardness, camber has a strong effect on the tyre performance (Abd El-Gawwad et al. 1999).

Camber is required to have different values for different scenarios (Cuttino et al. 2008). In conventional cars, however, camber is set as a fixed value (known as static camber). Furthermore, camber is changed during the motion of the car. Therefore, not only does the fixed camber produce positive effects but also negative ones. The most common phenomenon that is considered to be a drawback of camber is roll camber, which is presented next.

![Figure 1.12: Cornering car and roll camber](image)

Roll Camber is the amount of camber change due to the roll of the chassis; this occurs in cornering cars. When a car is cornering, the lateral force together with suspension geometry causes the car body to roll. In most existing cars, due to the constraints in the suspension system, the body rolls towards the outer side of the bend; and the individually suspended wheels generally incline away from the centre of rotation, as depicted in Figure 1.12. Consequently, the camber force caused by the roll camber points towards the outer side, which is opposite to the side slip force. This general condition, as shown in Figure 1.2, might reduce the total lateral force required for cornering. This negative effect becomes critical in the limit region where the lateral acceleration is high and the side slip force is...
saturated. Therefore, the cornering capacity of the vehicle is limited even while much of the available friction has not been utilised. In such a situation, the car may move from its desired path and cause an accident.

Figure 1.13. Roll camber is reduced with the shorter upper arm  
a) Equilibrium condition; b) Lateral acceleration condition

Figure 1.14. Excessive camber during heave with the upper shorter arm  
a) Rebound condition; b) Jounce condition

The roll camber is a function of the car body roll and suspension geometry. In equal length double A arm suspension, for example, the roll camber (in relation to the road) is the same as the amount of the chassis roll. To lessen the undesirable effect of the roll camber, various designs of suspension geometry have been introduced. For example, in
double A arm suspension the upper arm is made shorter than the lower one. With this configuration, the outer wheel leans towards, while the inner wheel leans away from, the body of the cornering car. This counteracts the outward leaning of the wheels (-leaning away from the centre of rotation) occurring in a turn, as depicted in Figure 1.13. However, the drawback of this suspension is that it produces excessive camber during pure heave, as shown in Figure 1.14. This reduces longitudinal forces. Thus, there is still a trade-off between the desirable camber for cornering and that for pure heave; and this compromise is similar for most existing suspension systems (Cuttino et al. 2008). Therefore, the roll camber is still a disadvantage of the conventional suspension systems.

There has been research on developing suspension systems that can cancel out the outward inclination of the tyre in a turn, as well as provide zero camber during jounce and rebound (Boston 2011, Cuttino et al. 2008). However, the developed systems appear to be mechanically complex, as shown in Figure 1.15. They also exert some side effects, such as altering the roll centre, which in turn affects the vehicle dynamics. Therefore, those systems have not been put into practice; and the roll camber phenomenon remains unresolved.

Figure 1.15: The complexity of an optimized, passive suspension system suggested by Cuttino et al. (2008)

1.2.5 Camber Control

The topic of controlling camber has attracted many authors over the last decade (Horiguchi et al. 2013, Kuwayama et al. 2007, Laws 2010, Nemeth and Gaspar 2013, Park and Sohn 2012, Yoshino and Nozaki 2014). Its primary motivation is to alter camber in order to exploit the full potential of the tyre. The research shows that camber control generally improves vehicle stability, safety, and manoeuvrability.

Perhaps the most impressive example of camber control is that in the F400 Carving, a research car developed by Daimler Chrysler to showcase new technologies combined in the cars of tomorrow (Ammon 2002). The car is pictured in Figure 1.16. One of the most interesting developments in the car is the active camber control system, which can vary the
cambers of the outer wheels up to 20 degrees. This system, when used in conjunction with newly developed tyres, can provide 30 per cent more lateral stability than a conventional system with fixed camber and standard tyres. This enhances active safety, since better lateral stability equals improved grip and greater cornering stability.

Figure 1.16: F400 Carving concept car with active camber control

Kuwayama et al. (2007) developed a control strategy for a variable camber suspension in which the camber is optimised in three performance states: steady state behaviour, limit behaviour, and transient behaviour. Together with the camber control strategy, a 18 DoFs car model was developed to evaluate the efficiency of the active camber system compared to the passive one. They stated that the strategy demonstrates improved handling performance in the three states; and concluded that the variable camber system has a strong impact on vehicle handling performance.

In research conducted by Laws (2010), an active camber concept for extreme manoeuvrability was developed. The contribution of the research is the realization of active camber tyres and camber suspension systems. In the former, a new model for 2D shape and vertical pressure distribution of tyre contact patch was developed and validated; a brush model was extended to a 2D model to capture the effects of cambering tyres. In the latter, a set of design principles and design criteria for mechatronic suspension systems was presented; and a complete prototype suspension system for the active camber concept was presented.

An investigation into the effects of camber angle control of front suspension on vehicle dynamic behaviors was also carried out by Park and Sohn (2012). The aim of the research was to improve the cornering performance. In their study, the authors developed a variable camber mechanism in a double A arm type suspension. The camber control strategy was designed using the bicycle model. The yaw rate derived from the bicycle model was considered to be the reference for the control input. The simulation of the full car was conducted by using ADAMS/Car and MATLAB/Simulink. Through evaluating the dynamic responses of the camber-controlled vehicle to fishhook and single lane change
manoeuvres, the authors claimed that the controlled car provides better results than the passive car by improving the yaw rate, lateral acceleration, and dynamic behaviour; they also found that short cornering and fast straight forward ability are also improved.

Nemeth and Gaspar (2013) studied the control design of variable-geometry suspension with considering the construction system. In their investigation, the geometry of double-wishbone type suspension was varied to control the height of the roll center as well as the camber angles of front wheels. They suggested that, by changing the camber angles of the front wheels, the yaw rate of the vehicle is modified, and this can be used to reduce the tracking error from the reference yaw rate.

The topic of camber control was also considered in a research carried out by Yoshino and Nozaki (2014). Their research was intended to be applied to electric vehicles. The camber control strategy aimed at improving the cornering limit performance by implementing a negative camber angle proportional to the steering angle. They found that camber variation can be used to control the yaw moment and lateral acceleration at the turning limit. They also suggested that the critical lateral acceleration can increase by 20 percent when a negative camber (the wheel is tilted towards the bend, according to the sign convention used in their analysis) of 20 degrees is applied to the wheel.

Despite these promising results, the research suffers from some disadvantages. First, although motivated by using camber to exploit the tyre potential, most of the research ended up with improving the vehicle performance in the range where the side slip force is far less than its saturation point. In that range the side slip can still do the job well; and hence camber controlling may not be necessary. The second downside is that the variable camber mechanism compromises changing the suspension geometry, as illustrated in Figure 1.18. Because the suspension geometry determines the roll center of the car body, as shown in Figure 1.17, this suspension geometry variation may badly affect the vehicle dynamics. The mechanical complexity of the associated suspension systems, as shown in Figure 1.19, is another drawback of developing such systems. Furthermore, in those investigations the camber gained when the wheel is steered about the tilted kingpin axis was not taken into account; this may lead to the inaccuracy of the camber calculation.

There has also been a number of patents on variable camber suspension (Boston 2011, Choudhery 2005, Weiss 2001, Woo 2003). The primary motivation of those patented inventions is to provide a safer vehicle with better vehicle turning stability. This was done by providing more negative camber to the outer front wheel and zero/positive camber to the inner wheel (Boston 2011, Choudhery 2005). Although the suspension systems presented in the inventions showed considerable potential for controlling camber, the effects of such systems on vehicle dynamics have not been investigated.
CHAPTER 1: INTRODUCTION

Figure 1.17: Roll centre is determined by suspension geometry

Figure 1.18. Roll centre change due to suspension geometry variation
a) The variable camber mechanism proposed by Park and Sohn (2012); b) The variable camber mechanism proposed by Nemeth and Gaspar (2013)
1.2.6 The Steering Axis Orientation and Variable Caster Steering

The orientation of the steering axis is described by caster and kingpin inclination angles, as shown in Figure 1.3. In conventional cars, these angles are usually set as fixed. The idea of variable caster steering stems from the fact that when the wheel is steering around the tilted axis it gains camber. Therefore, camber of a steered wheel can be varied by changing the caster or/and KPI angles. As the caster is more effective than the KPI in generating camber of the steered wheel, it is chosen to be varied.

Kingpin Inclination is the angle between the kingpin axis and the vertical when viewed from the front or rear of the vehicle, as shown in Figure 1.3. The details of KPI with sign convention is provided in Chapter 2.
In most current cars, the KPI angle is configured such that the top of the steering pivot leans towards the car body. That configuration has two main effects. On one hand, it reduces the lateral distance between the kingpin axis and the tyre contact centre at road level, as shown in Figure 1.20. This distance is called the scrub radius. By reducing the scrub radius, the steering effort and tyre wear are decreased. On the other hand, that KPI angle provides the returnability of the steering wheel during low speed cornering. The self-centring moment caused by the kingpin inclination angle and the normal load of the tyre, as illustrated in Figure 1.21, tends to pull the steered wheel back to the straight position. The returnability is also explained by the lifting effect: the car body is raised as the wheel steers around the steering axis. This kingpin inclination angle also creates steer-camber, which is the camber generated by steering the wheel about the tilted steering pivot. However, with the symmetric geometry of the steering axes, the effects at the two steering wheels are opposite: the inner steered wheel leans towards, while the outer one leans away from, the bend. As the KPI angle creates much less steer-camber than the caster does, this effect is usually neglected.

Figure 1.21: The self-centring moment caused by normal force and KPI

Caster, or caster angle, is the angle between the steering axis and the vertical when
viewed from the vehicle’s side, as shown in Figure 1.3. The sign convention used in this research (presented in Chapter 2) determines that, if the top of steering axis tilts rearwards, the caster is negative. Note that this caster sign convention is opposite to the SAE systems (Jazar 2014).

The primary function of the caster in conventional cars is to provide a source of self-centring for steering. In order to do that, a certain amount of negative caster (following the sign convention in this thesis) is employed. The tyre lateral force and its lever arm provided by this negative caster generate a restoring moment, as illustrated in Figure 1.22, tending to pull the steered wheel back to the straight-running position. In this way, a negative caster angle can improve straight-line directional stability. This moment also makes steering out of the corner easier and produces good steering-in feel. The disadvantage of negative camber is that it requires a greater effort to turn a car into the corner. This drawback, however, can be neglected since power assist steering is now common.

Figure 1.22: The self-centring moment caused by lateral force and a negative caster

Another purpose of utilising a negative caster is to create the steer-camber that facilitates the cornering forces. With a negative caster, both left and right steering wheels gain camber that increases cornering force (both wheels lean towards the centre of rotation). Therefore, the negative caster can lessen the roll camber phenomenon so as to improve the vehicle grip capacity.
Variable caster: Although there have been very few research publications on the topic of the variable caster compared to camber control, the idea of varying the caster has been put forward for decades (Alberding et al. 2014, Chalin 2001, Harara et al. 1996, Jazar et al. 2012, Lee 2010, Lyu 1998). However, the motivation behind most of the research was not about controlling the wheel camber to increase the vehicle’s turning capacity. For example, Harara et al. (1996) were granted a patent for their invention of a caster control mechanism and method. They suggested a method to vary the caster in response to a set of parameters, namely, steering wheel angle, steering wheel angular velocity, road surface, and vehicle speed. The caster angle is increased in order to increase the righting moment of the steering wheels for steering manoeuvring. The caster angle is increased according to the vehicle speed in non-steering and initial steering conditions. This thereby improves vehicle stability in normal and transitional conditions. Lee (2010) held a patent on caster control apparatus, which can adjust the casters of the wheels to prevent the vehicle from tilting to one side due to the lateral slope of a road while traveling on that road, and hence can reliably maintain straight-line stability. Motivated by improving the steering behaviour of a trailer self-steering system, Alberding et al. (2014) introduced a variable caster into a trailer-self-steering system, where a considerable caster in the steering geometry is employed to produce a feedback for the lateral tyre force, searching for a zero side slip.

Despite the fact that the effect of the caster on the camber has long been acknowledged, the idea of varying the caster to control the camber was just recently prompted by Jazar et al. (2012), who investigated the kinematics of a steerable wheel with a changeable caster by employing the screw theory. The main focus of their investigation was the kinematics of a steerable wheel; the kinematic relationships between parameters such as caster-camber correlation was developed; also, some simple configurations of the steering axis were presented. However, they did not propose any scheme of varying the caster to control the camber. Therefore, the effects of variable caster steering on vehicle dynamics remain unknown. Also, the kinematics was only developed in some special cases with some assumptions that are only suitable for small angles; when large angles (caster, \(KPI\), and steering angles) are applied, the accuracy of the kinematics may become questionable. This research, therefore, aims at expanding the theory of variable caster steering originated by Jazar et al. (2012). Specifically, the kinematics of the steering wheel will be expanded such that it is applicable to the case of large caster, \(KPI\), and steering angles (by removing the small-angle assumptions); a scheme of varying the caster will also be proposed to enhance vehicle’s dynamic behaviour in some good manners.

### 1.2.7 Tyre Modelling

When conducting research on vehicle dynamics, one has to deal with tyre modelling. This is because tyres play the primary role in generating external forces to handle vehicle mo-
This is the reason why a brief review of tyre modelling is undertaken in this section. While the critical importance of tyre characteristics to road vehicle dynamics behaviour is widely recognised, tyre modelling has been a challenging task (Ammon 2005, Rauh and Mssner-Beigel 2008). Nonlinearity, dependency on many factors, such as vertical load, tyre structure, and road surface conditions, make it difficult to be modelled. Generally speaking, there have been three main types of tyre models developed for decades. In the first category, which is usually simple, the formulation of tyre characteristics is based on physical models. Depending on what physical model is employed, the level of accuracy is determined. A good example of this type of tyre models is the ‘brush model’, which consists of elastic bristles that contact the ground and can deflect in the horizontal plane. These simple physically-based models are mainly responsible for providing basic understanding of vehicle dynamics and tyre behaviour.

The second type of tyre models is complex physical models. These models represent tyres in great detail, usually by using a finite element method with the support of computer simulation. The RMOD–K model is an example in this category (Oertel and Fandre 2009). These tyre models are very effective for conducting research relating to transient and high frequency noise and vibration. However, because of the complexity of the models, they are not usually utilised to carry out vehicle handling research. (Ammon 2005, Laws 2010, Pacejka 2012)

The third type is empirically-based models. In this category, mathematical tyre models are used to present measured tyre characteristics through mathematical formulas or tables. The parameters in the formulas are assessed using regression techniques to obtain a best fit to the experimental data. These models are most suitable for vehicle handling simulation, real time application, and control system design (Bakker et al. 1987, Ammon 2005, Rauh and Mssner-Beigel 2008). The most widely used empirical tyre model is called the ‘Magic formula’ (Ammon 2005, Rauh and Mssner-Beigel 2008). The Magic formula relates lateral and longitudinal forces, as well as aligning moment, to a number of parameters, such as slip angles (lateral slip and longitudinal slip), camber angle, and vertical load; it also includes nonlinear characteristics of tyres. There have been different versions of the Magic formula for different purposes (Bakker et al. 1987, Pacejka and Bakker 1992, Pacejka and Besselink 1997, Besselink et al. 2010). One of the relatively simple versions was developed by Bakker et al. (1987). The formula utilises the data from experiments to calculate coefficients of stiffness, shape factor, peak factor, etc. Although this version of the Magic Formula was developed to predict the tyre force under steady state conditions, many researchers have used it in transient vehicle handling analysis because of its sufficient accuracy and relative simplicity (Farazandeh et al. 2012; 2013, Hegazy et al. 1999, Kuwayama et al. 2008, Mammar and Koenig 2002). Also, it is capable of describing camber, vertical load variation, and nonlinear characteristics which occur in this investigation. Therefore, we employ this version of the Magic formula to simulate the
1.2.8 Vehicle Modelling

The use of an appropriate vehicle model plays a central role in research on vehicle dynamics. Many different vehicle models, ranging from simple to complicated, have been developed and adopted to investigate the dynamic behavior of cars. The level of accuracy of a model depends on its complexity. Perhaps the simplest and most commonly used model is the linear ‘bicycle model’. This model works on the assumption that the difference between the left and right steering angles is small; and hence both tyres on each axle can be combined to form a single track model; the average steering angle for both steering tyres is used. The details of the model are presented in a large number of publications (Gillespie 1992, Jazar 2014, Milliken and Milliken 1995, Wong 2001). Although this model is simple, the linear bicycle model is sufficient to provide a basic understanding of road vehicle dynamics such as handling characteristics (understeer, oversteer, and neutralsteer). Steady state and transient responses of the vehicle to different types of steering inputs can also be obtained by using this type of vehicle model. Due to its simplicity and effectiveness the linear bicycle model has been extensively used by researchers. In more detailed research on vehicle dynamics where control strategies are developed, the parameters extracted from this simple model are used as referenced values to make the control inputs (Farazandeh et al. 2012, Nemeth and Gaspar 2013, Park and Sohn 2012). However, on the down side, this model is not effective in predicting vehicle dynamic behaviour for lateral acceleration greater than 0.3g. Moreover, it is not capable of modeling the vehicle dynamics when lateral load transfer, a phenomenon that affects the lateral forces at the inner and outer tyres, is taken into account.

On the other end of the spectrum are complicated multi-body models. In these models, the vehicle is described by a large number of bodies with many degrees of freedom. As the level of detail is high, the accuracy of the obtained results using those models is great. One example worth mentioning is the multi-body model developed by Hegazy et al. (1999). In their investigation, the vehicle model built in ADAMS software consists of 35 parts, including the ground; the parts are connected by a large number of joints, which equivalents to 110 constraints. Therefore, the number of degrees-of-freedom (DOF) the model possesses is 94. The suspension and steering systems as well as the tyres were modelled in great detail. The nonlinear characteristics of those systems were also taken into consideration. The authors suggested that this model is particularly useful for transient conditions arising from the application of a steering function. Another example of complex models is an 18-DOF model developed by Kuwayama et al. (2007; 2008). The motivation for constructing this model is to make the most of the advantages of the Matlab/Simulink platform, which is suitable for modelling electronic components integrated in advanced active vehicles. The drawbacks of the complicated multi-body models are
The motivation of this current study is to enhance turning ability by using variable caster. Thus, the investigation is expected to deal with the whole range of lateral acceleration, from very low to the limit. The dynamics of a car at high acceleration has to include many phenomena, such as the nonlinearity of tyre forces and the lateral load transfer from the inner to the outer wheels. Therefore, a vehicle model that is capable of capturing all those effects is required to conduct the current investigation. At the same time, to reduce simulation time and software-demand, the model should not be too complex. Considering all those requirements, a rollable vehicle model is chosen to carry out the research. This model can capture longitudinal, lateral, yaw, and roll motions of a cornering car. In fact, similar models have been available in the literature (Demerly and Youcef-Toumi 2000, Farazandeh et al. 2012, Salaani 1996); establishing the equations of motion in the different coordinate system used in this thesis (presented in Chapter 2) and integrating the variable caster/camber into the force system make it different from the available models.

1.3 Thesis Objectives

The literature reveals that actively controlled camber can generally enhance the safety, stability, and manoeuvrability of cars; it also shows that one of the functions of (negative) caster in conventional cars is to provide steer-camber of the two steering wheels that facilitates the turn. However, there has not been research on variable caster in regard to controlling camber. Therefore, the potential of variable caster in this regard has not been realised; and a scheme of varying caster in order to enhance vehicle dynamic performance in a beneficial way has not been available in the literature. This investigation is aimed at realising the new method of controlling camber: variable caster steering. Specifically, the objectives of the research are:

- **To develop the kinematics of a steering tyre that is appropriate to variable caster steering.** Variable caster steering involves the orientations of steering tyre and its steering axis in which a large value of caster is likely achieved. However, the kinematics of steering tyres available in the literature works on small-angle assumptions (caster, kingpin inclination, and steering angles are small). Therefore, when it comes to variable caster, those conventional kinematic models may not be capable of ensuring the required accuracy. To address the issue, we develop a kinematics model of steering tyres that is applicable to variable caster steering. This is done by removing the small-angle assumptions.

- **To propose a scheme of varying caster in some good manners to improve...**
the vehicle safety, stability, and manoeuvrability. Like most of the active systems applied in cars, the ultimate aim of variable caster is to enhance the vehicle safety, stability, and manoeuvrability. In order to demonstrate the beneficial effects of the variable caster steering on vehicle dynamics, we set out to develop a scheme of varying caster (to control camber) in order to address roll camber, a phenomenon that reduces the car’s turning capacity. The expected result is a more manoeuvrable car with increased turning capacity. It is also demonstrated that this variable caster scheme can also be expanded to improve steering returnability during low speed cornering.

1.4 Thesis Outline

The remainder of the thesis is divided into five parts, which also represent the methodology for conducting the research. Each part has a chapter, as follows:

- Kinematics of a steering tyre (Chapter 2)
- Vehicle modelling (Chapter 3)
- Variable caster steering in automotive dynamics (Chapter 4)
- Model validation (Chapter 5)
- Conclusion (Chapter 6)

Chapter 2 focuses on the kinematics of a tyre steering about a tilted kingpin axis. A number of coordinate frames sufficient to describe the motions of the tyre and the vehicle is introduced; homogeneous transformation is employed as a tool to express the steering motion and hence to develop the steering tyre kinematics. Unlike other models in the literature, which accepted small-angle assumptions, this tyre kinematics model is developed for the general case. Therefore, it is applicable to any value of the angles. This is suitable for the case of variable caster steering where a large caster angle may be achieved. From the steering tyre kinematics, we determine the camber of a steering tyre, with respect to the ground, as a function of steering axis orientation, steering angle, suspension geometry, and vehicle motion. A kinematic analysis is carried out which demonstrates that caster is more effective than its counterpart, kingpin inclination, in order to vary the steering tyre camber. The camber function, developed in this chapter, will be analyzed in Chapter 4 to calculate the caster required to cancel out the roll camber. The movement of the ground-to-tyre contact point, along the tyre perimeter, is also taken into consideration in this kinematics model. This allows us to accurately determine kingpin moment in low speed cornering with a large steering angle, presented in Chapter 3.
In Chapter 3, a rollable vehicle model, which is capable of representing important characteristics of a turning car, such as lateral load transfer and roll motion, was constructed. The model includes longitudinal, lateral, yaw, and roll motions of a cornering car. The Magic Formula is used for tyre force modelling in order to take the camber contribution and the nonlinear characteristics of the tyre into account. Based on the vehicle model and the tyre model, the equations of motion of the car, expressed in the body coordinate frame attached to the car body, are established. MATLAB/Simulink is used to simulate the dynamic responses of the vehicle to different typical steering inputs.

Chapter 4 presents variable caster steering and its effects on vehicle dynamics. This chapter serves as a demonstration of how variable caster steering can be used to enhance vehicle safety, stability, and manoeuvrability: a strategy of varying caster is proposed to counteract the roll camber in order to expand the grip capacity of the car. First, a kinematic analysis of the ultimate camber (with respect to the ground) is carried out. Based on the analysis, the mathematically required caster to neutralize the roll camber is calculated. A scheme of varying caster, with other constraints being considered, is then introduced. The program built in MATLAB/Simulink in Chapter 3 is used to examine the dynamic behaviour of the vehicle in two cases: the benchmark car and the caster-controlled car. A caster configuration is also suggested to improve returnability of the steering system for low speed cornering manoeuvres.

Chapter 5 is all about validation. The validation consists of two parts. In the first, the kinematics of the steering tyre developed in Chapter 2 is validated. To do so, a multi-body model of a steering tyre rotating about a kingpin with variable orientation is constructed in ADAMS software. The kinematic parameters derived from the model developed in Chapter 2 are compared with those extracted from the ADAMS model for validating purposes. The second part of the chapter is the validation of the dynamics vehicle model presented in Chapter 3. This is done by comparing the simulation data with publicly available test data for a full vehicle. We also construct an associate model in a multi-body environment - CarSim software, for reference.

Finally, Chapter 6 discusses the results, presents the highlights of the study and major conclusions, and develops recommendations for future work.
Kinematics of Steering Tyre

The tyre is the main component of a car that interacts with the road. When a tyre is steering, its orientation and location change. These changes affect the tyre-road interaction, and hence vehicle performance. This chapter shows how to develop the steering tyre kinematics for both stationary and cornering cars. The kinematics is applicable to the case of variable caster steering theory where large caster and steering angles might be employed. Homogeneous transformation is utilised as a tool, and the tyre is assumed to be a flat rigid disk.

2.1 Coordinate Systems and Sign Convention

The derivation of road steering wheel kinematics is carried out by using homogeneous transformation. This transformation allows us to map coordinates between different coordinate systems. Therefore, this section begins by introducing the coordinate frames required for developing the kinematics. An appropriate sign convention for steering wheel kinematics is also introduced here.

2.1.1 Coordinate Systems

To start with, a body coordinate frame $B(C_0xyz)$ is defined. This frame is attached to the vehicle at the mass center $C_0$, as depicted in Figure 2.1. The $x$-axis is longitudinal, goes through $C_0$, and is directed forwards. The $y$-axis goes laterally to the left of the driver’s viewpoint. The $z$-axis is perpendicular to the ground and upwards, so that it makes the coordinate system a right-hand triad. This coordinate system is also called $B$ frame.

To define coordinate systems at each road wheel with mathematical consistency, the wheels are numbered as follows: the front wheel at the left of the driver’s viewpoint is wheel number 1; the front right wheel is number 2; the rear right wheel is number 3; and
CHAPTER 2: KINEMATICS OF STEERING TYRE

wheel number 4 is the rear left wheel. In this investigation, we use ‘wheel’ and ‘tyre’ as equivalents.

At each wheel, we introduce four coordinate systems: tyre coordinate frame $T$, wheel coordinate frame $W$, upright-wheel coordinate frame $W_0$, and wheel-body coordinate frame $C$, as shown in Figure 2.2. The tyre coordinate frame $T(x_t, y_t, z_t)$ is the frame whose origin is at the tyreprint centre. The $x_t$-axis is the intersection of the ground and tyre planes. The tyre plane is made by centrally narrowing the wheel into a flat disk. The $z_t$-axis is always vertical to the ground and upwards. The $y_t$-axis goes laterally to the left and makes the system a right-hand triad. This frame only follows the steering motion of the wheel. We then attach another frame called wheel coordinate frame $W(x_w, y_w, z_w)$ to the wheel. The origin of this $W$ frame is the centre of the wheel. The $y_w$ is aligned with the spin axis and towards the left. The $(x_w, y_w, z_w)$ plane coincides with the tyre plane. At the straight-running position, the tyre plane makes a static camber angle $\gamma_0$ with the $z_t$-axis; and the $x_w$-axis is parallel to the $x_t$. However, when the wheel is steered with $\delta$ angle, the tyre plane makes a camber angle $\gamma$ with the $z_t$; and the $x_w$-axis is now not necessarily parallel to the $x_t$. The $z_w$ is determined such that it makes the $W$ frame a right-hand triad coordinate system. An upright-wheel coordinate frame $W_0(x_{w0}, y_{w0}, z_{w0})$ is also attached to the wheel. This $W_0$ frame is created by rotating the $W$ frame, at the straight-running position of the wheel, an angle of $-\gamma_0$ about the $x_t$ axis. Note that both $W$ and $W_0$ frames are attached to the wheel. Therefore, they follow every motion of the wheel except the spin. Finally, a wheel-body coordinate system $C(x_c, y_c, z_c)$ that coincides with the $W_0$ frame at zero steering angle position is attached to the car body. The wheel-body coordinate frame is motionless with the car body, and so it does not follow any motion of the wheel when steering.
As defined, the orientations and locations of the four coordinate frames are not affected by the spin motion of the wheel. Therefore, the spin is excluded from this kinematic analysis. Another aspect to note is that all the coordinate frames at each wheel are defined in the same way.

2.1.2 Sign Convention

The sign convention for wheel parameters that we adhere to in this thesis was introduced by Jazar et al. (2012). The convention is followed for all the wheels of the vehicle. Although it is different from the popular SAE convention, this current sign convention is employed to achieve the mathematical consistency of kinematic and dynamic analyses throughout the research.

Steering Angle

In this investigation, the steering angle is considered to be positive if the wheel is steered to the left of the driver’s viewpoint, and vise versa.

Side Slip

Side slip, $\alpha$, is the angle between the path of motion of the tyre and the tyre plane. The side slip angle is positive, $\alpha > 0$, if the tyre is turned about the $z_t$-axis to be aligned
CHAPTER 2: KINEMATICS OF STEERING TYRE

with the velocity vector $v$. Figure 2.3 depicts a practical situation where the side slip is negative associated with a positive steering angle (steering to the left).

![Figure 2.3: Top view of a wheel with negative side slip](image)

**Caster**

Caster, as defined earlier, is the angle between the steering axis and a vertical line when viewed from the vehicle’s side. According to the sign convention introduced by Jazar et al. (2012), the caster is negative when the top of steering axis tilts rearward from the vertical, as shown in Figure 2.4. This sign definition is opposite to that of SAE.

![Figure 2.4: Negative caster](image)
Kingpin Inclination Angle

Kingpin inclination angle is the angle between the kingpin axis and the vertical when viewed from the front or rear of the vehicle. If a steering axis rotates around the positive direction of the $x_c$, it has a positive kingpin inclination angle. Figure 2.5 shows a left wheel with a positive $KPI$ angle.

![Figure 2.5: Front view of the left wheel with positive kingpin inclination angle](image)

Camber

Camber, $\gamma$, is the angle between the tyre plane and the $z_t$-axis. By definition, a wheel has a positive camber, $\gamma > 0$, if it rotates about the positive direction of the $x_t$-axis from the vertical. Also, a positive camber wheel is when the wheel leans to the right of the driver’s viewpoint, regardless of whether the wheel is on the left or right, at the front or rear. This is different from the $SAE$ definition where camber is positive if the wheel tilts outwards. A positive camber determined by this current sign convention is illustrated in Figure 2.6.

![Figure 2.6: Front view of a wheel with positive camber](image)
Lateral Force

A tyre lateral force is positive if it is in the same direction as the $x_t$-axis. Accordingly, a negative side slip produces a positive lateral side slip force; and a positive camber force is caused by a negative camber angle. The lateral force sign in relation to side slip and camber signs is illustrated in Figures 2.3 and 2.6. Therefore, the representation of the lateral force as a function of those angles in linear region is:

$$F_y = F_{y\alpha} + F_{y\gamma} = -C_\alpha\alpha - C_\gamma\gamma$$

(2.1)

where $C_\alpha$ and $C_\gamma$ are side slip and camber coefficients, respectively.

2.2 Homogeneous Transformation

In the first instance, we consider a simple situation in which a rigid body $B$ rotates about a point $O$ fixed to ground, as depicted in Figure 2.7. Two coordinate systems are defined here: the first, $B(Oxyz)$, is fixed to the rigid body and called the local coordinate frame; the other, the global coordinate frame, $G(OXYZ)$, is motionless with respect to the ground.

Figure 2.7: The local coordinate frame $B(Oxyz)$ rotates about a fixed point $O$ with respect to the global coordinate frame $G(OXYZ)$

The two frames, which share the same origin $O$, are employed to express a position vector $\mathbf{r}$ associated with point $P$. There is always a rotation matrix $^G R_B$ to transform the coordinates of the vector $\mathbf{r}$ from the reference frame $B(Oxyz)$ to the other frame $G(OXYZ)$ (Jazar 2010):
\[ G_r = G_{RB} B_r \]  

(2.2)

where \( B_r \) and \( G_r \) are the position vectors of point \( P \) expressed in the body coordinate frame and the global coordinate frame, respectively.

\[ B_r = \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix}; \quad B_r = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix}; \quad G_{RB} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \]  

(2.3)

Figure 2.8: Axis-angle rotation

More specifically, when the body rotates an angle of \( \delta \) about an axis denoted by its direction unit vector \( \hat{u} \) and passing through the origin \( O \), as illustrated in Figure 2.8, the rotation matrix becomes \textit{Rodriguez matrix} \( R_{\hat{u}, \delta} \); this Rodriguez matrix is represented by (2.4). In this case, the motion is also called axis-angle rotation (Jazar 2010).

\[ G_{RB} = \begin{bmatrix} u_1^2 \text{vers} \delta + \cos \delta & u_1 u_2 \text{vers} \delta - u_3 \sin \delta & u_1 u_3 \text{vers} \delta + u_2 \sin \delta \\ u_1 u_2 \text{vers} \delta + u_3 \sin \delta & u_2^2 \text{vers} \delta + \cos \delta & u_2 u_3 \text{vers} \delta - u_1 \sin \delta \\ u_1 u_3 \text{vers} \delta - u_2 \sin \delta & u_2 u_3 \text{vers} \delta + u_1 \sin \delta & u_3^2 \text{vers} \delta + \cos \delta \end{bmatrix} = R_{\hat{u}, \delta} \]  

(2.4)

where \( \text{vers} \delta = 1 - \cos \delta \), and \( \hat{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \)

Now the situation is made more complicated by giving the origin of the body coordinate frame \( B \) a freedom to move in the global coordinate frame \( G(OXYZ) \). Therefore,
the rigid body can now rotate in the global frame, while its origin $o$ can translate with respect to the origin $O$ of the frame $G$, as shown in Figure 2.9.

![Figure 2.9: The local coordinate frame $B(Oxyz)$ rotates and translates in the global coordinate frame $G(OXYZ)$](image)

If the position of the moving point $o$ with respect to the global frame is denoted by vector $^Gd$, then the coordinates of the point $P$ in the local coordinate frame will match those in the global coordinate frame according to the following equation:

\[
^G r = ^G R_B ^B r + ^G d \quad (2.5)
\]

or

\[
\begin{bmatrix}
X_P \\
Y_P \\
Z_P
\end{bmatrix} = \begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix} \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} + \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} = \begin{bmatrix} r_{11} x_P + r_{12} y_P + r_{13} z_P + X_o \\ r_{21} x_P + r_{22} y_P + r_{23} z_P + Y_o \\ r_{31} x_P + r_{32} y_P + r_{33} z_P + Z_o \end{bmatrix} \quad (2.6)
\]

In this equation, $^Gd$ is called the displacement or translation of $B$ with respect to $G$; $^G R_B$ is the rotation matrix to transform $^B r$ into $^G r$ when $^G d = 0$. Such a motion is called rigid motion (Jazar 2010), which is actually a combination of a rotation and a translation. Equation (2.5) shows that the location of a rigid body can be described by the position of the origin $o$ and the orientation of the body frame in the global frame.
If we add an element to the position vector and extend the rotation matrix as below:

\[ \begin{bmatrix} X_P \\ Y_P \\ Z_P \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]

\[ \begin{align*}
G_r P &= \begin{bmatrix} X_P \\ Y_P \\ Z_P \\ 1 \end{bmatrix} \\
B_r P &= \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \\
G_d &= \begin{bmatrix} X_o \\ Y_o \\ Z_o \\ 1 \end{bmatrix}
\end{align*} \quad (2.7)

\[ G_R B = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \rightarrow G_T B = \begin{bmatrix} r_{11} & r_{12} & r_{13} & X_o \\ r_{21} & r_{22} & r_{23} & Y_o \\ r_{31} & r_{32} & r_{33} & Z_o \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} G_R & G_d \\ 0 & 1 \end{bmatrix} \quad (2.8)
\]

we can establish the following equation:

\[ \begin{bmatrix} X_P \\ Y_P \\ Z_P \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & X_o \\ r_{21} & r_{22} & r_{23} & Y_o \\ r_{31} & r_{32} & r_{33} & Z_o \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11}x + r_{12}y + r_{13}z + X_o \\ r_{21}x + r_{22}y + r_{23}z + Y_o \\ r_{31}x + r_{32}y + r_{33}z + Z_o \\ 1 \end{bmatrix} \quad (2.9)\]

or

\[ G_r = G_T B G_r \quad (2.10) \]

Equation (2.10) demonstrates that the rigid motion can be expressed by only a single matrix transformation.

The new 4 x 4 matrix $G_T B$ as shown in (2.8) is called the homogeneous transformation matrix; and the new expression $G_r P$ as a 4 x 1 vector in (2.7) is called the homogeneous coordinates of the point $P$, expressed in the global coordinate frame $G$. The homogeneous expression is used only for simplifying numerical calculations. The physical coordinates of a vector remain the same as the first three homogeneous coordinates.

The homogeneous transformation matrix can be decomposed into two matrices, as follows:

\[ G_T B = \begin{bmatrix} r_{11} & r_{12} & r_{13} & X_o \\ r_{21} & r_{22} & r_{23} & Y_o \\ r_{31} & r_{32} & r_{33} & Z_o \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & X_o \\ 0 & 1 & 0 & Y_o \\ 0 & 0 & 1 & Z_o \\ 0 & 0 & 0 & 1 \end{bmatrix} = G_D B G_R \quad (2.11)\]

Equation (2.11) means that a transformation can be achieved by a pure rotation followed by a pure translation.

Two homogeneous transformations can also be compounded with each other as one homogeneous transformation. In order to prove that, we consider a situation in which there are three reference frames: $A$, $B$, and $C$, as shown in Figure 2.10.
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If the matrices to transform coordinates from the frame $B$ to the frame $A$, and from the frame $C$ to the frame $B$, respectively, are:

$$A_T^B = \begin{bmatrix} A_R & A_d \\ 0 & 1 \end{bmatrix}$$ (2.12)

$$B_T^C = \begin{bmatrix} B_R & B_d \\ 0 & 1 \end{bmatrix}$$ (2.13)

then the transformation matrix from the frame $C$ to the frame $A$ will be:

$$A_T^C = A_T^B B_T^C = \begin{bmatrix} A_R & A_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B_R & B_d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} A_R B_R & A_R B_d + A_d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} A_R C & A_d \\ 0 & 1 \end{bmatrix}$$ (2.14)

Figure 2.10: Compounding two homogeneous transformations

It is interesting that the homogeneous transformation can represent a rotation about an axis going through a point different from the origin. Figure 2.11 illustrates an axis with its direction unit vector, $\hat{u}$; the axis passes through point $P$ denoted by position vector, $d$. In order to express the transformation, a local frame $B$ is set at $P$ that is parallel to the $G$ frame. By doing so, the rotation can be expressed by a translation along $-d$, followed by a rotation about $\hat{u}$ and a translation $d$, as follows (Jazar 2010):
\begin{align}
G_{TB} &= D_{d,d}R_{\hat{u},\delta}D_{-d,-d} = \begin{bmatrix} I & d \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R_{\hat{u},\delta} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -d \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} R_{\hat{u},\delta} & d - R_{\hat{u}}d \\ 0 & 1 \end{bmatrix} \tag{2.15} \end{align}

This property makes the \textit{homogeneous transformation} a tool for conveniently developing the kinematics of a steering tyre. The next section deals with the derivation of the steering wheel kinematics using this \textit{homogeneous transformation}.

\section{2.3 Steering Motion of A Steering Tyre}

The assumption that the tyre is a flat and rigid disk is accepted throughout this research. Furthermore, we initially assume that the tyre has zero static camber; and the steering axis is a fixed line with respect to the car body. The last two assumptions will be removed afterwards to assure the generality of the steering tyre kinematics.

Because of the zero static camber, the $W$ frame coincides with the $W_0$ frame. At the straight-running position of the tyre (zero steering angle), the $W$ frame coincides with the $C$ frame. Therefore, the steering motion of the tyre is considered to be a $\delta$-angle-rotation of the $W$ frame (and also the $W_0$ frame), around the steering axis, with respect to the $C$ frame. The rotation is described by the steering axis and the steering angle, expressed in the $C$ frame. The steering axis is determined by its direction unit vector and a point on it. The intersection between the steering axis and the ground, $I(s_a, s_b, -R_w)$ is chosen to be that point. To find the direction unit vector, the steering axis is considered to be
the intersection line of two planes: caster plane, \( \pi_c \), and inclination plane, \( \pi_i \). The caster plane is a plane that has an angle \( \theta \) with \((z_c, x_c)\) plane; the inclination plane has an angle \( \phi \) with \((y_c, z_c)\) plane; and both planes contain the steering axis, as illustrated in Figure 2.12. The two planes can be denoted by their normal unit vectors in the wheel-body coordinate frame, as follows:

\[
\hat{n}_{\pi_c} = \begin{bmatrix} 0 \\ \cos \theta \\ \sin \theta \end{bmatrix} ; \quad \hat{n}_{\pi_i} = \begin{bmatrix} -\cos \phi \\ 0 \\ \sin \phi \end{bmatrix} \quad (2.16)
\]

The direction unit vector of the steering axis is:

\[
\hat{u} = \frac{\hat{n}_{\pi_c} \times \hat{n}_{\pi_i}}{|\hat{n}_{\pi_c} \times \hat{n}_{\pi_i}|} \quad (2.17)
\]

or

\[
C \hat{u} = \hat{u} = \frac{1}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \begin{bmatrix} \cos \theta \sin \phi \\ -\sin \theta \cos \phi \\ \cos \theta \cos \phi \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (2.18)
\]

![Figure 2.12: The orientation and location of kingpin axis for wheel number 1](image)

Mapping coordinates of a point in the wheel coordinate frame \( W \) onto the wheel-body coordinate frame \( C \) is governed by the following transformation:

\[
C_r = C T_W W_r \quad (2.19)
\]

where \( W_r \) and \( C_r \) are the homogeneous representations of a position vector associated with that point in the \( W \) frame and the \( C \) frame, respectively.
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\[
W_r = \begin{bmatrix} W_x \\ W_y \\ W_z \\ 1 \end{bmatrix}; \quad C_r = \begin{bmatrix} C_x \\ C_y \\ C_z \\ 1 \end{bmatrix}
\]  \tag{2.20}

and \( C_{TW} \) is a \( 4 \times 4 \) homogeneous transformation matrix:

\[
C_{TW} = \begin{bmatrix} R_{\hat{u},\delta}d_I - R_{\hat{u},\delta}d_I \\ 0 \\ 0 \\ 1 \end{bmatrix}
\]  \tag{2.21}

where \( d_I \) is the position vector of the point \( I \) expressed in the \( C \) frame.

\[
d_I = C_{dI} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} s_a \\ s_b \\ -R_{w} \end{bmatrix}
\]  \tag{2.22}

As discussed, the first three homogeneous coordinates of a position vector remain the same as physical coordinates of the vector. Therefore, we still use the regular vector and its homogeneous representation interchangeably in this investigation.

2.4 Kinematics of A Steering Tyre with Zero Static Camber

When the tyre is steered, its location and orientation in relation to the car body are changed. The *homogeneous transformation* enables us to determine the coordinates of any element on the wheel (fixed to the \( W \) frame) expressed in the wheel-body frame. By doing so, the steering tyre kinematics is derived. The parameters relevant to the topic of variable caster steering are determined as follows.

2.4.1 Wheel Centre

The homogeneous coordinates of wheel centre in the \( W \) coordinate frame are written as follows:

\[
W_{rw} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
\]  \tag{2.23}

They are transformed into the homogeneous coordinates expressed in the \( C \) frame by means of the following transformation:

\[
C_{rw} = C_{TW}W_{rw} = \begin{bmatrix} R_{\hat{u},\delta} \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_I - R_{\hat{u},\delta}d_I \\ 1 \end{bmatrix}
\]  \tag{2.24}
Substitution of (2.4) and (2.22) in (2.24) yields:

\[
C_{\text{rW}} = \begin{bmatrix}
    d_1(1 - u^2_1) \text{vers} \delta - u_1(d_2u_2 + d_3u_3) \text{vers} \delta + (d_2u_3 - d_3u_2) \sin \delta \\
    d_2(1 - u^2_2) \text{vers} \delta - u_2(d_3u_3 + d_1u_1) \text{vers} \delta + (d_3u_1 - d_1u_3) \sin \delta \\
    d_3(1 - u^2_3) \text{vers} \delta - u_3(d_1u_1 + d_2u_2) \text{vers} \delta + (d_1u_2 - d_2u_1) \sin \delta
\end{bmatrix}
\] (2.25)

If we substitute (2.18) and (2.22) in (2.25), then the coordinates of the steered wheel center in the wheel body frame will be:

\[
C_{xW} = [s_a + \frac{1}{4} \left(2R_w \sin 2\phi - 4s_a \sin^2 \phi \right) \cos^2 \phi + s_b \sin 2\theta \sin 2\phi] \text{vers} \delta
\]
\[
+ \frac{s_b \cos \theta - R_w \sin \theta}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \cos \phi \sin \delta
\] (2.26)

\[
C_{yW} = [s_b - \frac{1}{4} \left(2R_w \sin 2\theta + 4s_a \sin^2 \theta \right) \cos^2 \phi - s_a \sin 2\theta \sin 2\phi] \text{vers} \delta
\]
\[
- \frac{s_a \cos \phi + R_w \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \cos \theta \sin \delta
\] (2.27)

\[
C_{zW} = [-R_w + \frac{1}{2} \left(2R_w \cos^2 \theta + s_b \sin 2\theta \right) \cos^2 \phi - s_a \cos^2 \theta \sin 2\phi] \text{vers} \delta
\]
\[
- \frac{s_a \cos \phi \sin \theta + s_b \cos \theta \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta
\] (2.28)

\(C_{xW}\), \(C_{yW}\), and \(C_{zW}\) indicate how much the wheel centre longitudinally, laterally, and vertically displaces from the point \(C\) fixed to the car body, respectively. The \(C_{zW}\) can be used to determine the car body’s vertical displacement, as discussed in Section 2.4.3.

### 2.4.2 Wheel Camber

Wheel camber is the key parameter of the topic. Therefore, calculating the wheel camber as a function of steering angle and other parameters is required. The relationship between camber, steering angle, and other parameters has been established by researchers such as Dixon (2009), Jazar et al. (2012), and Alberding et al. (2014). However, it was either an approximated function or was expressed in special cases. The most common assumption was that the caster, kingpin inclination, and steering angles are small. Furthermore, authors assumed that the tyre-to-ground contact is a fixed point on the tyre perimeter. Therefore, the function may not be applicable to the case of variable caster steering where large values of those angles may be included. In this section, we express the relationship between those angles without small-angle approximations and in a general case.

A simple way to find the wheel camber without accepting the two mentioned assumptions is by using the normal vectors of the tyre plane and the ground plane, as depicted in
Figure 2.13. The camber angle, $\gamma$, is related to the angle between the two normal vectors, $\rho$, by the following equation:

$$\gamma = \frac{\pi}{2} - \rho \quad (2.29)$$

If the unit vectors in the directions of $x_c$, $y_c$, and $z_c$ of the $C$ frame are denoted by $\hat{I}$, $\hat{J}$, and $\hat{K}$; and the unit vectors in the directions of $x_w$, $y_w$, and $z_w$ of the $W$ frame are symbolized by $\hat{i}$, $\hat{j}$, and $\hat{k}$; then $\rho$ will be:

$$\rho = \cos C \hat{j} C \hat{K} \frac{C \hat{j}}{|C \hat{j}|} |C \hat{K}| \quad (2.30)$$

where

$$C \hat{K} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C \hat{j} = C_T W \hat{j} = \begin{bmatrix} R_{\hat{a},\delta} d_I - R_{\hat{a},\delta} d_I \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} u_1 u_2 (1 - \cos \delta) - u_3 \sin \delta \\ u_2 (1 - \cos \delta) + \cos \delta \\ u_2 u_3 (1 - \cos \delta) + u_1 \sin \delta \end{bmatrix} \quad (2.32)$$

By substituting (2.31) and (2.32) in (2.30) and then in (2.29), we have:

$$\gamma = \frac{\pi}{2} - \cos[u_2 u_3 (1 - \cos \delta) + u_1 \sin \delta] \quad (2.33)$$

Substituting (2.18) in (2.33) yields the wheel camber as a function of steering angle and steering axis orientation:

$$\gamma = \frac{\pi}{2} - \cos \left[ \frac{\cos \theta \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta - \frac{\cos^2 \phi \sin \theta \cos \theta}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \text{vers}\delta \right] \quad (2.34)$$
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As can be seen from the function, camber of a steered tyre in a stationary car solely depends on caster and kingpin inclination. As stated earlier, this analytical relationship is in a general case without any small-angle approximation. Having the function may enable us to choose the appropriate parameter among caster and KPI angles for varying the camber, and to propose a strategy for varying that parameter.

2.4.3 The Vertical Displacement of Car Body

When the tyre is steering, it makes a movement with respect to the car body. As the tyre is constrained to be on the ground plane, the vertical component of the movement is transferred to the car body. As a result, the point $C$ on the car body experiences a lift or drop. This affects the self-centring ability of the steering system at low speed. Therefore, this is an important parameter of steering tyre kinematics and needs to be derived. Here we calculate the vertical displacement of the point $C$ with respect to the ground (hereafter called car body displacement).

In order to determine the car body displacement, the tyre-to-ground contact point must be known. However, the tyre coordinate frame $T$, by definition, does not remain at a fixed point on the tyre; its origin is just the tyre-to-ground contact point, which can move along the tyre perimeter. Therefore, the homogeneous transformation cannot be used directly to determine the coordinates of the tyre-to-ground contact point $T$ in the $C$ frame. Fortunately, having camber angle, $\gamma$, and $C_zW$ is enough to define $C_zT$, as shown in the following equation:

$$\frac{C_zW - C_zT}{R_w} = \cos\gamma \iff C_zT = C_zW - R_w \cos\gamma$$  (2.35)

As we assumed that the ground is rigid and flat and the tyreprint centre (tyre-to-ground contact point) is at ground level, the point $C$ on the body of the car is displaced a distance of:

$$H = -C_zT - R_w = R_w (\cos\gamma - 1) - C_zW$$  (2.36)

Substituting (2.28) and (2.34) in (2.36) yields:

$$H = R_w \sin[acos(\frac{\cos\theta \sin\phi}{\sqrt{\cos^2\phi + \cos^2\theta \sin^2\phi}} \sin\delta - \frac{\cos^2\phi \sin\theta \cos\theta}{\cos^2\phi + \cos^2\theta \sin^2\phi} \vers\delta)]$$

$$- R_w \cos\delta - \frac{1}{2} (2R_w \cos^2\theta + s_b \sin^2\theta) \cos^2\phi - s_a \cos^2\theta \sin^2\phi \vers\delta$$

$$+ \frac{s_a \cos\phi \sin\theta + s_b \cos\theta \sin\phi}{\sqrt{\cos^2\phi + \cos^2\theta \sin^2\phi}} \sin\delta$$  (2.37)

$H$ shows how much the point $C$ fixed to the car body is raised by steering the wheel. As can be seen from the formula, $H$ depends on both steering axis’s orientation ($\theta$ and $\phi$) and its location ($s_a$ and $s_b$). This parameter will be used to evaluate the self-centring ability of the steering tyre at low speed.

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2.4.4 The Change of The Tyre-To-Ground Contact Point

As stated in the literature review, steering tyre kinematics has long been developed by many researchers. In their investigations authors either implicitly or explicitly accepted the assumption that the tyre-to-ground contact is a fixed point on the tyre perimeter. However, when a tyre is steering around a tilted kingpin axis and/or with a large steering angle, the two assumptions are no longer reasonable. Here we show that the tyre-to-ground contact point moves along the tyre perimeter when the tyre is steering, and that the displacement is significant for large steering and/or caster angles.

Here, $T_0$, and $T$ denote the tyre-to-ground contact points (on the tyre perimeter) associated with zero, and $\delta$ steering angles, respectively. The homogeneous representation of $T_0$ in the $W$ coordinate frame is:

$$W_rT_0 = \begin{bmatrix} 0 \\ 0 \\ -R_w \\ 1 \end{bmatrix} \quad (2.38)$$

They are transformed into those expressed in the $C$ frame as follows:

$$C_rT_0 = \begin{bmatrix} C_{xT0} \\ C_{yT0} \\ C_{zT0} \\ 1 \end{bmatrix} = C_TWrT_0 \quad (2.39)$$

Substituting (2.18) in (2.4), (2.22) and (2.4) in (2.21), and then (2.21) and (2.38) in (2.39) yields:

$$C_{zT0} = -R_w - \frac{1}{2} \frac{s_a \cos^2 \theta \sin 2\phi - s_b \sin 2\theta \cos^2 \phi}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \text{vers} \delta - \frac{s_a \cos \phi \sin \theta + s_b \cos \theta \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta \quad (2.40)$$

$C_{zT0}$ shows how much the point $T_0$ (on the tyre) displaces vertically from the point $C$ on the car body when the tyre is steering. It can be verified that when the caster and $KPI$ angles are zero, the point is always the wheel radius below the point $C$.

As shown in (2.35), the $C_z$-coordinate of $T$ can be determined by:

$$C_{zT} = C_{zW} - R_w \cos \gamma \quad (2.41)$$

where $C_{zW}$ and $\gamma$ are shown in (2.28) and (2.34), respectively.
Figure 2.14: The angular motion of the tyre-to-ground contact point on the steering tyre

The $C_z$-coordinates of $T_0$ and $T$ are sufficient to determine the angle $\beta$ that the tyre-to-ground contact point creates when it moves along the tyre perimeter.

$$\cos \beta = \frac{WT - HT}{WT_0} = \frac{R_w - HT}{R_w} = 1 - \frac{HT}{R_w} = 1 - \frac{C_zT_0 - C_zT}{R_w \cos \gamma} \quad (2.42)$$

Substituting (2.40) and (2.41) in (2.42), we have:

$$\cos \beta = \frac{\cos \delta}{\cos \gamma} + \frac{\cos^2 \theta \cos^2 \phi}{(\cos^2 \phi + \cos^2 \theta \sin^2 \phi) \cos \gamma} (1 - \cos \delta) \quad (2.43)$$

As indicated by (2.43), $\beta$ only depends on the steering angle and the orientation of the steering axis. The angular motion that the tyre-to-ground contact point makes on the tyre perimeter, for an exemplary configuration ($\phi = -9.6^0, \theta = 12.7^0$), is illustrated in Figure 2.15. The dependence of the motion on the caster and KPI angles, for a large steering angle ($-45^0$), is also visualised in Figure 2.16.

Figure 2.15: The angular motion of the tyre-to-ground contact point as a function of $\delta$
Figure 2.16: The angular motion of the tyre-to-ground contact point as a function of $\phi$ and $\theta$

It can be clearly seen in the graphs that the displacement of the tyre-to-ground contact point along the tyre perimeter is considerable for relative large values of the caster, KPI, and steering angles. Therefore, this effect should not be neglected in the analysis of variable caster steering.

Note that in Section 2.4.3 only the $z$-coordinate of $T$ in the $C$ coordinate system was determined. The angular motion, $\beta$, enables us to determine $x$-coordinate and $y$-coordinate of $T$. The homogeneous coordinates of $T$ in the $W$ frame can be calculated using Figure 2.14, as follows:

$$ W_rT = \begin{bmatrix} R_w \sin \beta \\ 0 \\ -R_w \cos \beta \\ 1 \end{bmatrix} \quad (2.44) $$

Its homogeneous coordinates in the $C$ frame will be:

$$ C_rT = C_TW_rT \quad (2.45) $$

The coordinates of the instantaneous tyre-to-ground contact point $T$ are used to more accurately determine the external forces and moments acting on the tyres. This is presented in Chapter 3.

### 2.4.5 Caster vs Kingpin Inclination in Generating Camber

Both caster and kingpin inclination angles contribute to camber of a the steered wheel. However, it is more practical to use only one of the two parameters to control the camber.
CHAPTER 2: KINEMATICS OF STEERING TYRE

This also reduces the complexity of the corresponding suspension mechanism. In order to select the most appropriate parameter for controlling camber, a kinematic analysis of the generated camber is performed; whether the caster or the KPI angle is the main source of the generated camber needs to be realised. To compare how much camber is created by the caster or by the KPI angle when steering, the two following configurations are considered:

The first case: a variable KPI with zero caster. Substituting $\phi = 0$ in (2.34), we have:

$$
\gamma = \frac{\pi}{2} - \cos[\sin \theta \cos \theta (\cos \delta - 1)]
$$

(2.46)

The second case: a variable caster with zero KPI. Substituting $\theta = 0$ in (2.34) yields:

$$
\gamma = \frac{\pi}{2} - \cos(\sin \phi \sin \delta)
$$

(2.47)

The graphs in Figures 2.17 and 2.18 illustrate how much camber of a steered wheel is generated for different caster and KPI angles, respectively.

Figure 2.17: The generated camber for a variable caster with zero KPI

(October 8, 2017)
Most noticeably of all, it can be seen that the caster creates much more camber than does the \textit{KPI} for the same practical steering angle. For example, with the steering angle of \(10^\circ\), the camber generated by the caster is approximately ten times greater than that of \textit{KPI}. It is also worth pointing out that the magnitude of the camber is proportional to that of the caster or \textit{KPI} angle. More specifically, when the caster rises from \(10^\circ\) to \(30^\circ\), the respective camber increases approximately threefold. Last but not least, for a specific steering angle we can achieve negative or positive camber by adjusting the caster to an appropriate value. These make the caster a better choice for controlling camber of a steered tyre.

2.5 Kinematics of A Steering Tyre with Static Camber

In the previous section, we assumed that the tyre has zero static camber. Therefore, the \(W\) coordinate frame coincides with the \(W_0\) coordinate frame; and we only need a transformation to relate coordinates from the wheel frame to the wheel-body frame. It is more realistic that the tyre has non-zero static camber. This static camber, \(\gamma_0\), is measured when the car is stationary and the tyre is at the straight-running position. This section presents the kinematics of such a steering tyre.

2.5.1 The Homogeneous Transformation Matrices

Because the \(W\) frame does not coincide with the \(W_0\) frame, mapping coordinates from the wheel frame \(W\) to the wheel-body frame \(C\) is achieved by two transformations: from \(W\) to \(W_0\), and from \(W_0\) to \(C\). As discussed earlier in Chapter 2, the \(W\) frame is actually
made by rotating the $W_0$ frame an angle of $\gamma_0$ about the $x_w$-axis when the tyre is in the straight-running position. The rotating axis is represented by vector $\hat{u}_0$ and the point $T_0$:

$$\hat{u}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(2.48)

$$W_0 d_{T_0} = \begin{bmatrix} 0 \\ 0 \\ -R_w \end{bmatrix}$$

(2.49)

where $\hat{u}_0$ and $T_0$ are, respectively, the direction unit vector of the rotating axis, $x_w$, and the point on it at the straight-running position of the tyre, both expressed in the $W_0$ coordinate frame.

The homogeneous transformation matrix from the $W$ frame to the $W_0$ frame is written as:

$$W_0 T_W = \begin{bmatrix} R_{\hat{u}_0,\gamma_0} & W_0 d_{T_0} - R_{\hat{u}_0,\gamma_0} W_0 d_{T_0} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma_0 & -\sin\gamma_0 & -R_w \sin\gamma_0 \\ 0 & \sin\gamma_0 & \cos\gamma_0 & R_w (\cos\gamma_0 - 1) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2.50)

The matrix to transfer coordinates from the $W_0$ system to the $C$ system, $C_{T_{W_0}}$, is the same as the matrix $C_{T_W}$ in (2.21):

$$C_{T_{W_0}} = \begin{bmatrix} R_{\hat{u},\delta} & d_I - R_{\hat{a},\delta} d_I \\ 0 & 1 \end{bmatrix}$$

(2.51)

As a result, the coordinates of a point in the $W$ frame can be transferred to those of the $C$ frame by the following transformation:

$$C_r = C_{T_{W_0}} W_0 T_W W r = \begin{bmatrix} R_{\hat{u},\delta} & C d_I - R_{\hat{a},\delta} C d_I \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{\hat{a}_0,\gamma_0} & W_0 d_{T_0} - R_{\hat{a}_0,\gamma_0} W_0 d_{T_0} \\ 0 & 1 \end{bmatrix} W r$$

(2.52)

By applying (2.52), one can derive tyre kinematic parameters similar to those presented in Section 2.4.

### 2.5.2 Wheel Camber

The camber of a steered wheel is also calculated using equation (2.30). Before applying the equation, the coordinates of the direction unit vector $\hat{j}$ in the $W$ frame have to be
transformed into those expressed in the $C$ frame by the following transformation:

$$
C^j = C^W T^W_0 T^W_0 T^W_0 W^j = \begin{bmatrix}
R_{u, \delta} & C d_I - R_{u, \delta} C d_I \\
0 & 1
\end{bmatrix} \begin{bmatrix}
R_{u_0, \gamma_0} & W^0 d T^0 - R_{u_0, \gamma_0} W^0 d T^0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
0 \\
1
\end{bmatrix} \tag{2.53}
$$

Substituting (2.53) and (2.31) in (2.30), and then (2.30) in (2.29), we have:

$$
\gamma = \frac{\pi}{2} - \arccos\left\{u_2 u_3 (1 - \cos \delta) + u_1 \sin \delta \cos \gamma_0 + u_2^2 (1 - \cos \delta) + \cos \delta \sin \gamma_0\right\} \tag{2.54}
$$

If we substitute (2.18) in (2.54), the camber of a steered wheel will be:

$$
\gamma = \frac{\pi}{2} - \arccos\left\{\frac{\cos \gamma \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta - \frac{\cos^2 \phi \sin \theta \cos \phi}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \cos \delta \cos \gamma_0 \right. + \left. \frac{\cos^2 \phi \cos^2 \theta}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \cos \delta \sin \gamma_0\right\} \tag{2.55}
$$

We can verify that, when $\gamma_0 = 0$, (2.55) reduces to (2.34).

### 2.6 The Orientation of A Steering Tyre in A Cornering Car

The tyre kinematics developed in the previous sections was for a stationary car, and the camber was in relation to the car body. The wheel camber with respect to the ground for a cornering car, however, is the ultimate parameter for determining the camber force. In this section, we present the development of the kinematics of such a tyre. The camber with respect to the ground is then extracted from the kinematics.

#### 2.6.1 The Homogeneous Transformation Matrices

When a car negotiates a turn, its body rolls. The roll motion causes a change in the steering axis orientation with respect to the ground. Because the steering axis and the wheel belong to the same rigid body, the roll motion gives rise to the same variation in the camber of the tyre associated with the straight-running position. This camber is similar to the static camber defined earlier. It is not called static camber because the car now is not stationary. Therefore, hereafter we call it the initial camber. The changes in steering axis orientation and the initial camber affect the value of the wheel camber with respect to the ground.

We first determine the dynamic orientation of the steering axis with respect to the ground. When the car is in roll motion, the dynamic kingpin inclination angle ($DKPI$) in relation to the ground is:
\[ \theta_D = \theta_S + \theta_\varphi = \theta + \theta_\varphi \]  

where \( \theta_S \) is the designed static kingpin inclination angle (SKPI), and \( \theta_\varphi \) is the inclination angle induced by the roll motion \( \varphi \) of the car body, as illustrated in Figure 2.19.

Figure 2.19: The change of steering axis and initial wheel camber in a cornering car

As the roll angle is usually small, the changes in the caster, longitudinal position, and lateral position due to the roll are negligible. Therefore, the dynamic values of the caster \( \phi_D \), the longitudinal position \( s_{aD} \), and the lateral position \( s_{bD} \), respectively, are:

\[ \phi_D = \phi_S = \phi; s_{aD} = s_{aS} = s_a; s_{bD} = s_{bS} = s_b \]  

where \( \phi_S \), \( s_{aS} \), and \( s_{bS} \) are the caster, the longitudinal position, and the lateral position of the static steering axis.

As discussed above, the roll motion of the car body causes a change in the initial camber of the wheel. Thus, the dynamic value of the initial camber is:

\[ \gamma_{0D} = \gamma_0 + \gamma_{0\varphi} \]  

where \( \gamma_0 \) is the static camber, and \( \gamma_{0\varphi} \) is the initial camber portion caused by the roll motion. Note that the inclination angle and the initial camber portion, both induced by
the roll motion, are the same:

\[ \theta_{\phi} = \gamma_{0\phi} \]  

(2.59)

The transformation of coordinates from the \( W \) frame into the \( C \) frame is governed by an equation similar to equation (2.52) in Section 2.5, but with the dynamic orientation of the steering axis and the dynamic initial camber:

\[
C_r = \begin{bmatrix} R_{\hat{u}_D, \delta}^C d_{I_D} - R_{\hat{u}_D, \delta}^C d_{I_D} \end{bmatrix} \begin{bmatrix} R_{\hat{u}_D, (\gamma_{0\phi} + \theta_{\phi})} W_0^d T_0 - R_{\hat{u}_D, (\gamma_{0\phi} + \theta_{\phi})} W_0^d T_0 \end{bmatrix} W_r \]  

(2.60)

where \( \hat{u}_D \) and \( d_{I_D} \) are the direction unit vector and the position vector, representing the dynamic steering axis in the \( C \) frame, which are similar to (2.18) and (2.22), respectively:

\[
\hat{u}_D = \begin{bmatrix} u_{1D} \\ u_{2D} \\ u_{3D} \end{bmatrix} = \frac{1}{\sqrt{\cos^2 \phi + \cos^2 (\theta + \theta_{\phi}) \sin^2 \phi}} \begin{bmatrix} \cos (\theta + \theta_{\phi}) \sin \phi \\ -\sin (\theta + \theta_{\phi}) \cos \phi \\ \cos (\theta + \theta_{\phi}) \cos \phi \end{bmatrix} \]  

(2.61)

\[
d_{I_D} = \begin{bmatrix} s_{aD} \\ s_{bD} \\ -R_w \end{bmatrix} \]  

(2.62)

Equation (2.60) can be used to derive the dynamic location and orientation of the steering tyre in a cornering car.

### 2.6.2 Wheel Camber with respect to The Ground

In order to determine the wheel camber with respect to the ground, the \( C \) frame is now redefined to make it always parallel to the ground during the turn. We introduce a new concept of no-roll-body. This no-roll-body is attached to the roll axis of the real car body but it does not follow the roll motion. At the stationary position of the vehicle, we attach the \( C \) frame to the no-roll-body at the same position as was previously defined. By definition, the \( C \) frame follows the horizontal motion of the car and is always parallel to the ground. Therefore, the orientation of the steering axis with respect to the ground is also in relation to the \( C \) frame. As the camber of a steered wheel only depends on the steering pivot’s orientation, the camber expressed in the new \( C \) frame is also that with respect to the ground.

In order to determine the wheel camber with respect to the ground using equation (2.30), the direction unit vector \( \hat{j} \) of the \( W \) frame is transformed into the \( C \) frame by the following transformation:

\[
C_j = \begin{bmatrix} R_{\hat{u}_D, \delta}^C d_{I_D} - R_{\hat{u}_D, \delta}^C d_{I_D} \end{bmatrix} \begin{bmatrix} R_{\hat{u}_D, (\gamma_{0\phi} + \theta_{\phi})} W_0^d T_0 - R_{\hat{u}_D, (\gamma_{0\phi} + \theta_{\phi})} W_0^d T_0 \end{bmatrix} \end{bmatrix} \]  

(2.63)
Substituting (2.63) and (2.31) in (2.30), and then (2.30) in (2.29), yields the camber function:

\[
\gamma = \frac{\pi}{2} - \cos\left[\frac{\cos(\theta + \theta_c)\sin\phi}{\sqrt{\cos^2\phi + \cos^2(\theta + \theta_c)\sin^2\phi}} \sin\delta - \frac{\cos^2\phi \sin(\theta + \theta_c)\cos(\theta + \theta_c)}{\cos^2\phi + \cos^2(\theta + \theta_c)\sin^2\phi} \text{vers}\delta\right]\cos(\gamma_0 + \gamma_0\phi)
\]

\[
+ \left[\frac{\cos^2\phi \cos^2(\theta + \theta_c)}{\cos^2\phi + \cos^2(\theta + \theta_c)\sin^2\phi} \text{vers}\delta + \cos\delta \sin(\gamma_0 + \gamma_0\phi)\right]
\]

(2.64)

It can be verified that (2.34) and (2.55) are special cases of (2.64). This camber function is analysed in Chapter 4 in order to propose a variable caster strategy.

2.7 A Potential Design of Variable Caster Mechanism

The theory of variable caster steering may be implemented by various designs of variable caster suspension; designing a mechanism for varying caster also lies beyond the scope of this research. A specific mechanism, however, is needed to examine how a variable caster scheme affects the vehicle dynamics beneficially. This is because the relationship between the wheel’s kinematic parameters depends on each specific mechanism. Therefore, in this section we propose a possible mechanism for varying the caster and extract the kinematics of the wheel coupling with it. The kinematics also provides a visual understanding of how wheel alignment parameters and steering angle are related by the variable caster mechanism.

The main idea of variable caster steering is to change the camber of a steered wheel, in a useful way, by varying the caster. The location and orientation of the wheel at the straight-running position (not steering), however, should not be affected by the variation of the caster. To ensure this, the steering axis must rotate about a line coincident with the \(y_c\) axis. Furthermore, a certain value of kingpin inclination angle may be needed to satisfy other requirements, such as self-centring at low speed (Milliken and Milliken 1995). Therefore, a mechanism that satisfies the two requirements is proposed. As illustrated in Figure 2.20, the caster in this suspension can be varied by rotating the steering pivot around the axis going through \(C(0, 0, 0)\) and \(M(0, L_b, 0)\) in the \(C\) frame. In that way, the steering axis always lies on the lateral surface of the cone made by the rotation, as depicted in Figure 2.21.
Here we show how caster and kingpin inclination angle are related by the variable caster mechanism. To do so, the two following positions of the steering axis are considered:

- When the steering axis is at the position of $PMPP'$ with zero caster, $\phi_0 = 0$, the associated KPI angle is $\theta_0$.
- When the axis moves to $QMQ'$ position with the caster $\phi$, the KPI angle is $\theta_\phi$. 
Based on the geometry in Figure 2.21 we have:

\[
\tan \theta_\phi = \frac{M_1 M}{M_1 Q_1}
\]

\[
\tan \theta_0 = \frac{M_1 M}{M_1 P}
\]

\[
\cos \phi = \frac{M_1 Q_1}{M_1 P}
\]  

(2.65)

because \(M_1 Q = M_1 P\), (2.65) leads to:

\[
\cos \phi = \frac{M_1 Q_1}{M_1 P} = \frac{\tan \theta_0}{\tan \theta_\phi}
\]

(2.66)

or

\[
\theta_\phi = \text{atan}\left(\frac{\tan \theta_0}{\cos \phi}\right)
\]

(2.67)

Figure 2.22 depicts how the kingpin inclination angle \(\theta\) and the caster \(\phi\) are related by this mechanism. As can be seen in the Figure, the variation of the KPI angle \(\theta\) versus the caster \(\phi\) is insignificant, around 2\(^0\) of KPI compared to 30\(^0\) of caster. This means that the variable caster only has a negligible effect on the KPI angle. Moreover, the main source of the generating camber using this mechanism remains the caster; the kingpin inclination angle only affects the camber to a small extent. This is also demonstrated in Figure 2.23, where the camber is plotted against the caster that is varied using the proposed mechanism. Comparing Figure 2.17 and Figure 2.23 shows that employing a certain kingpin inclination angle barely affects the value of the generated camber.

The above kinematic analysis indicates that the caster can be varied using this proposed mechanism; employing a certain amount of kingpin inclination angle, if need, barely affects the camber created by the variable caster.
2.8 Summary

The theory of variable caster steering starts with steering tyre kinematics. The existing kinematic tyre models in the literature worked on the basis of small-angle assumptions; in those models, the tyre-to-ground contact point was also assumed to be a fixed point on the tyre perimeter; furthermore, it was considered that the contact point is a wheel radius below the wheel centre. In the variable caster steering theory, large caster and camber may be achieved. Consequently, those assumptions make the kinematics of steering tyre less accurate, and hence the models become inapplicable. Therefore, a more accurate kinematics model of steering tyre which does not work on those assumptions is required in order to carry out the research on variable caster steering.

To develop the kinematics, first the sign convention for wheel alignment parameters used in this thesis was introduced. Although this sign convention is different from the commonly used SAE approach, it ensures mathematical consistency in developing the tyre kinematics. Then the fundamental of homogeneous transformation, which is used to transform coordinates between two coordinate frames, was presented. A number of coordinate systems required to develop the steering tyre kinematics was defined. By applying homogeneous transformation to the coordinate systems, the kinematics of the steering tyre was developed. The kinematic parameters of steering wheels relevant to the variable caster steering topic were derived. A kinematic analysis was carried out and a potential variable caster mechanism was proposed to provide a visual understanding. The results derived from this model are used in the remainder of the thesis.
Vehicle Modelling

The vehicle modelling needed for examining the dynamic behaviour of the car with and without a variable caster steering strategy, is presented in this chapter. An appropriate car model, and a tyre model integrated with the developed tyre kinematics, are employed in order to capture roll camber, camber contribution, and variable caster. At the end of the chapter, a novel technique for determining the kingpin moment for the steering wheel is also developed; this kingpin moment will be used to examine the steering returnability for low speed cornering manoeuvres discussed in Chapter 4.

3.1 Vehicle Dynamics Model

To simulate and evaluate the dynamic performance of a car, a vehicle dynamics model must be utilised. There have been various models in the literature, ranging from simple to very complex, for studying the motion of an automobile. Simple models are operating on many simplifying assumptions. Therefore, they do not require much information about the vehicle but still give a basic understanding of vehicle dynamics. Some are good enough to describe vehicle motion in specific circumstances. The most commonly used model in this group is the linear bicycle model, in which the left and right wheels on the same axle are transversely grouped into one wheel in the mid-line. The lateral motion and yaw motion are included. The relationship between the tyre lateral force and the side slip is linear. This model has been studied extensively in vehicle lateral dynamics and is mainly responsible for providing a good understanding of vehicle handling characteristics such as understeer, oversteer, and directional stability. The downside of simple models is that they are not capable of capturing necessary characteristics of vehicle motion, resulting in poor results for general use. For example, the linear bicycle model cannot predict vehicle behaviour at lateral acceleration higher than 0.3g; also, it fails to simulate the lateral load transfer phenomenon and roll motion of the car body, which clearly affect the dynamics
of a cornering car. Unlike simple models, very complex models cover many more degrees of freedom for a vehicle and use many fewer assumptions. Therefore, they can represent vehicle dynamic behaviours for a variety of scenarios, and provide very good results. Nevertheless, those models require much more vehicle information, calculation time; and are sometime not necessary. The literature on vehicle dynamics model was presented in Chapter 1. To balance the two trends, an appropriate vehicle dynamics model is employed in this section to assess the dynamic behaviour of the car with and without a variable caster strategy. This model can present longitudinal and lateral transfers of vertical loads under the tyres, which have a great influence on lateral tyre forces. It is also capable of capturing the roll motion of the car body. At the same time, the model should not be too complicated in order to avoid the cited disadvantages.

In order to construct the model, the following assumptions are accepted:

- The examined vehicle is a four-wheel car with two front steering wheels. It consists of three rigid bodies: one for sprung mass, two for unsprung masses. The suspension system is characterized by its effective roll stiffness and damping properties. The effects of aerodynamics on the vehicle motion are also ignored.

- Because this investigation only considers the cornering motion of the car with constant longitudinal velocity, the longitudinal acceleration/deceleration is small, and the pitch motion is also ignored.

- The vehicle is running on a flat and rigid road. The wheels are rigid flat disks; and they always connect to the ground. The vertical motions of the car body and the wheels with respect to the road are ignored. This is conventionally considered to be acceptable for examining lateral dynamics of a cornering car.

- The vehicle is laterally symmetrical. Therefore, the centre of gravity is in the vertical middle plane.

- The compliance steer effect is ignored. This means that the road steering wheel angle is only kinematically related to the hand steering wheel angle. The reason for this assumption is that the steering angle and the orientation of the road steering wheel are most important in the context of the current research.

- The spin motion of the tyre and the tyre’s inertia are not considered.

- While the the relationship between the left and the right steering wheels can be established for any type of steering mechanism, for simplification reason the steering angles of the left and the right front wheels are assumed to be the same.

Figure 3.1 illustrates the nonlinear dynamics model of the vehicle (NLDM). It has four degrees of freedom: translation in the longitudinal direction, translation in the lateral
Section 3.1: Vehicle Dynamics Model

Direction, yaw motion - rotation about the vertical axis, and roll motion of the car body around the roll axis.

Figure 3.1: Nonlinear dynamics model of the vehicle

3.1.1 Equations of Motion

The Newton-Euler equations of motion for the vehicle expressed in the body coordinate frame $B$ are:

\[
F_x = m(\dot{v}_x - rv_y) + m_s h \dot{r} \dot{\phi} 
\]

\[
F_y = m(\dot{v}_y + rv_x) - m_s h \dot{\phi} 
\]

\[
M_z = I_{zz} \dot{r} + I_{xz} \ddot{\phi} 
\]

\[
M_x = I_{xx} \ddot{\phi} + I_{xz} \dot{r} - m_s h (\dot{v}_y + rv_x) 
\]

where the left-hand side of the equations represents the force system acting on the vehicle:

\[
F_x = -F_{y1} \sin \delta_1 - F_{y2} \sin \delta_2 + F_{x1} \cos \delta_1 + F_{x2} \cos \delta_2 
\]

\[
F_y = F_{y1} \cos \delta_1 + F_{y2} \cos \delta_2 + F_{x1} \sin \delta_1 + F_{x2} \sin \delta_2 + F_{y3} + F_{y4} 
\]
\[ M_z = (M_{z1} + M_{z2} + M_{z3} + M_{z4}) + a(F_y1\cos\delta_1 + F_y2\cos\delta_2) + a(F_{z1}\sin\delta_1 + F_{z2}\sin\delta_2) \\
+ \frac{wf}{2}(F_y1\sin\delta_1 - F_y2\sin\delta_2) - \frac{wf}{2}(F_{z1}\cos\delta_1 - F_{z2}\cos\delta_2) - b(F_{y3} + F_{y4}) \]  
(3.7)

\[ M_x = m_xgh\sin\varphi - (K_{x1} + K_{x2})\varphi - (C_{x1} + C_{x2})\dot{\varphi} \]  
(3.8)

Respectively substituting the equations (3.5), (3.6), (3.7), and (3.8) in the equations (3.1), (3.2), (3.3), and (3.4), the equations of motion can be rewritten as:

\[ \dot{v}_x = \frac{1}{m}(-F_y1\sin\delta_1 - F_y2\sin\delta_2 + F_{x1}\cos\delta_1 + F_{x2}\cos\delta_2 - m_xh\dot{\varphi}) + rv_y \]  
(3.9)

\[ \dot{v}_y = \frac{1}{m}(F_y1\cos\delta_1 + F_y2\cos\delta_2 + F_{x1}\sin\delta_1 + F_{x2}\sin\delta_2 + F_{y3} + F_{y4} + m_xh\dot{\varphi}) - rv_x \]  
(3.10)

\[ \dot{r} = \frac{1}{I_{xz}}[(M_{z1} + M_{z2} + M_{z3} + M_{z4}) + a(F_y1\cos\delta_1 + F_y2\cos\delta_2) + a(F_{x1}\sin\delta_1 + F_{x2}\sin\delta_2) \\
+ \frac{wf}{2}(F_y1\sin\delta_1 - F_y2\sin\delta_2) - \frac{wf}{2}(F_{x1}\cos\delta_1 - F_{x2}\cos\delta_2) - b(F_{y3} + F_{y4}) + I_{xx}\ddot{\varphi}] \]  
(3.11)

\[ \ddot{\varphi} = \frac{1}{I_{xx}}[I_{xz}\dot{r} + m_xgh\sin\varphi + m_xh(\dot{v}_y + rv_x) - (K_{x1} + K_{x2})\varphi - (C_{x1} + C_{x2})\dot{\varphi}] \]  
(3.12)

The external forces and moments on the right-hand side of the equations (3.9) to (3.12) will be determined to establish the equations of motion in their final form. The determination of those forces and moments is presented as follows.

### 3.1.2 Tyre Normal Load

The lateral tyre force is a function of the vertical load on the tyre. The vertical load consists of static and dynamic components. The static vertical load is given by the weight of the vehicle distributed between the tyres. The dynamic component results from the load transfer due to longitudinal acceleration, lateral acceleration, and the roll motion of the car body. Therefore, the total normal load under each tyre is written as:

\[ F_{z1} = F_{z10} + F_{z1i}(a_x) + F_{z1i}(a_y) + F_{z1i}(\varphi) \]  
(3.13)

**Static normal load:** the static normal loads of tyres are determined when the car is stationary or moving at a constant speed on a level and straight road:

\[ F_{z10} = F_{z20} = mg \frac{b}{2L} \]  
(3.14)

\[ F_{z30} = F_{z40} = mg \frac{a}{2L} \]  
(3.15)

**The longitudinal transfer of normal load:** the longitudinal transfer of normal load is a result of accelerating or braking. When the car accelerates, the normal load is transferred from front wheels to the rear wheels. When it is under braking condition, the normal load
is transferred from the rear end to the front end. As we assumed that the longitudinal acceleration/deceleration is small, and the pitch motion is ignored, the car is considered to be a mass \( m \) for the longitudinal direction. The load transfer due to the longitudinal acceleration/deceleration is determined by taking the moment, in the longitudinal vertical plane, about any of the road-tyre contact points. When this longitudinal load transfer is considered, the normal loads under the tyres become:

\[
F_{z10} + F_{z1}(a_x) = mg \frac{b}{2L} - ma_x \frac{h_y}{2L}
\]  
\[
F_{z20} + F_{z2}(a_x) = mg \frac{b}{2L} - ma_x \frac{h_y}{2L}
\]  
\[
F_{z30} + F_{z3}(a_x) = mg \frac{a}{2L} + ma_x \frac{h_y}{2L}
\]  
\[
F_{z40} + F_{z4}(a_x) = mg \frac{a}{2L} + ma_x \frac{h_y}{2L}
\]

The lateral transfer of normal load: when the car makes a turn, the centrifugal force acting at the centre of gravity tends to pull it away from the centre of rotation. Furthermore, the car body usually leans away from the turn as a result of centrifugal force and suspension geometry. The two phenomena give rise to a normal load transfer between the two sides of the vehicle: the vertical loads under the inner wheels reduce, those of the outer wheels increase. Figure 3.2 illustrates the model for calculating the lateral load transfer. We assume that the roll centres of the front and the rear suspensions lie on the vertical middle plane of the vehicle; the sprung mass and the unsprung masses have the same lateral acceleration.

First, we determine the lateral load transfer due to the lateral acceleration. This consists of two parts: one is associated with the unsprung masses, the other is connected with the sprung mass. The centrifugal force acting at each unsprung mass gives rise to a lateral load transfer across the corresponding axle. Using the same method as in calculating the longitudinal load transfer, the lateral load transfers due to the unsprung masses are written as:

\[
F_{z1}(a_y, u) = -m_{uf} a_y \frac{h_{uf}}{w_f}
\]  
\[
F_{z2}(a_y, u) = m_{uf} a_y \frac{h_{uf}}{w_f}
\]  
\[
F_{z3}(a_y, u) = m_{ur} a_y \frac{h_{ur}}{w_r}
\]  
\[
F_{z4}(a_y, u) = -m_{ur} a_y \frac{h_{ur}}{w_r}
\]
CHAPTER 3: VEHICLE MODELLING

The centrifugal force, $m_s a_y$, and the vertical force, $m_s g$, acting at the centre of gravity of the sprung mass, $C_s$, can be moved to $C_{sr}$ with the addition of a moment. Here, $C_{sr}$ is the projection of $C_s$ on the roll axis; this moment causes the roll motion of the sprung mass. The lateral force, $m_s a_y$, is distributed between the front and rear wheels. Therefore, the load transfers due to the sprung mass on the axles are:

$$F_{z1}(a_y, s) = -\frac{m_s a_y b_s}{L} \frac{h_f}{w_f}$$  \hspace{1cm} (3.24)

$$F_{z2}(a_y, s) = \frac{m_s a_y b_s}{L} \frac{h_f}{w_f}$$  \hspace{1cm} (3.25)

$$F_{z3}(a_y, s) = \frac{m_s a_y a_s}{L} \frac{h_r}{w_r}$$  \hspace{1cm} (3.26)

$$F_{z4}(a_y, s) = -\frac{m_s a_y a_s}{L} \frac{h_r}{w_r}$$  \hspace{1cm} (3.27)

The moment acting on the sprung mass due to $m_s a_y$ and $m_s g$ is:

$$M_s = m_s a_y h cos \varphi + m_s g h sin \varphi$$  \hspace{1cm} (3.28)
This roll moment is transmitted to unsprung masses through suspension systems (springs, shock absorbers, and anti-roll bars) causing lateral load transfer due to the roll motion. The amount of that moment is determined by the reacted moment from the suspension system to the sprung mass:

\[ M_{s2s} = -(K_{p} \phi + C_{p} \dot{\phi}) - (K_{r} \phi + C_{r} \dot{\phi}) = -M_s \] (3.29)

We are working on the assumption that the sprung mass is torsionally rigid. Therefore, the distribution of the roll moment is considered to be the same as the ratio of roll stiffness between the axles. The corresponding lateral load transfers on front and rear axles due to the roll motion are:

\[ F_{z1}(\phi) = -\frac{K_{f} \phi + C_{f} \dot{\phi}}{w_f} \] (3.30)

\[ F_{z2}(\phi) = \frac{K_{f} \phi + C_{f} \dot{\phi}}{w_f} \] (3.31)

\[ F_{z3}(\phi) = \frac{K_{r} \phi + C_{r} \dot{\phi}}{w_r} \] (3.32)

\[ F_{z4}(\phi) = -\frac{K_{r} \phi + C_{r} \dot{\phi}}{w_r} \] (3.33)

Combining the equations from (3.13) to (3.33) yields the dynamic normal load under each tyre of the vehicle, as follows:

\[ F_{z1} = \frac{mgb}{2L} - \frac{mh_y}{2L} a_x - \frac{a_y}{w_f} \left( \frac{m_s b_h f}{L} + m_u h a_f \right) - \frac{1}{w_f} (K_{f} \phi + C_{f} \dot{\phi}) \] (3.34)

\[ F_{z2} = \frac{mgb}{2L} + \frac{mh_y}{2L} a_x + \frac{a_y}{w_f} \left( \frac{m_s b_h h}{L} + m_u h a_f \right) + \frac{1}{w_f} (K_{f} \phi + C_{f} \dot{\phi}) \] (3.35)

\[ F_{z3} = \frac{mga}{2L} + \frac{mh_y}{2L} a_x + \frac{a_y}{w_r} \left( \frac{m_s a h r}{L} + m_u h u r \right) + \frac{1}{w_r} (K_{r} \phi + C_{r} \dot{\phi}) \] (3.36)

\[ F_{z4} = \frac{mga}{2L} + \frac{mh_y}{2L} a_x - \frac{a_y}{w_r} \left( \frac{m_s a h h}{L} + m_u h u r \right) - \frac{1}{w_r} (K_{r} \phi + C_{r} \dot{\phi}) \] (3.37)

These normal loads will be used to determine lateral forces and moments generated at the tyres of the cornering car.

### 3.1.3 Side Slip Angles

The lateral force and self-aligning moment developed under each tyre are functions of the side slip angle. The side slip angle greatly affects the lateral dynamics of the vehicle, as it is the main source of the lateral force. Using the model in Figure 3.1, we can calculate...
the side slip angles for all the tyres based on vehicle longitudinal ($v_x$), lateral ($v_y$), yaw ($r$) velocities, and steering angles ($\delta_1, \delta_2$):

$$\alpha_1 = \delta_1 - \tan^{-1} \frac{v_y + ar}{v_x - \frac{w_f}{2}r}$$ (3.38)

$$\alpha_2 = \delta_2 - \tan^{-1} \frac{v_y + ar}{v_x + \frac{w_f}{2}r}$$ (3.39)

$$\alpha_3 = -\tan^{-1} \frac{v_y - br}{v_x + \frac{w_r}{2}r}$$ (3.40)

$$\alpha_4 = -\tan^{-1} \frac{v_y - br}{v_x - \frac{w_r}{2}r}$$ (3.41)

The side slip angles determined here will be the inputs to the side slip forces and self-aligning moments presented in the following section.

### 3.2 Tyre Model

The tyre is the most important part for generating force to handle a road vehicle. Therefore, tyre modelling plays a significant role in simulating vehicle dynamics. There have been three main approaches to modelling a tyre, as discussed in Chapter 1. The simplest tyre model for studying vehicle’s lateral dynamics, which is often employed in basic vehicle handling analyses, is the linear model. The linear tyre model can well represent tyre forces and moments for low lateral acceleration. However, it fails to capture the nonlinear characteristics of the tyre when the vehicle is moving with moderate and high lateral accelerations. This study deals with camber contribution, especially in the region where the side slip force is near the saturation level. Therefore, a tyre model that can predict the tyre forces and moments over the whole range of vehicle handling, and can take the camber into account, is required. For those reasons, we employ the Magic Formula tyre model (Bakker et al. 1987), which appears to be popular for tyre modelling. This model can include the contribution of the tyre longitudinal slip. However, as this analysis only focuses on the vehicle’s lateral dynamics, we utilise the formula for the case of pure lateral slip. This model determines lateral force and aligning moment as functions of normal load, side slip angle, camber angle, and other parameters, such as tyre-road friction. The lateral force and moment are presented by the following formulae:

- **Lateral force:**

  $$F_y(\alpha, \gamma) = D\sin[C\tan^{-1}(B\Phi)] + \Delta S_v$$ (3.42)

  where,

  $$\Phi = (1 - E)(\alpha + \Delta S_h) + \frac{E}{B}\tan^{-1}[B(\alpha + \Delta S_h)]$$ (3.43)
\[
D = a_1 F_z^2 + a_2 F_z
\]  
(3.44)

\[
C = 1.30
\]  
(3.45)

\[
B = \frac{a_3 \sin[a_4 \tan^{-1}(a_5 F_z)]}{CD} (1 - a_{12} |\gamma|)
\]  
(3.46)

\[
E = a_6 F_z^2 + a_7 F_z + a_8
\]  
(3.47)

\[
\Delta S_h = a_9 \gamma
\]  
(3.48)

\[
\Delta S_v = (a_{10} F_z^2 + a_{11} F_z) \gamma
\]  
(3.49)

- **Self-aligning moment:**

\[
M_z(\alpha, \gamma) = D \sin[C \tan^{-1}(B \Phi)] + \Delta S_v
\]  
(3.50)

where,

\[
\Phi = (1 - E)(\alpha + \Delta S_h) + \frac{E}{B} \tan^{-1}[B(\alpha + \Delta S_h)]
\]  
(3.51)

\[
D = a_1 F_z^2 + a_2 F_z
\]  
(3.52)

\[
C = 2.40
\]  
(3.53)

\[
B = \frac{a_3 F_z^2 + a_4 F_z}{\left| a_5 F_z \right|} (1 - a_{12} |\gamma|)
\]  
(3.54)

\[
E = \frac{a_6 F_z^2 + a_7 F_z + a_8}{1 - a_{13} |\gamma|}
\]  
(3.55)

\[
\Delta S_h = a_9 \gamma
\]  
(3.56)

\[
\Delta S_v = (a_{10} F_z^2 + a_{11} F_z) \gamma
\]  
(3.57)
where B, C, D, and E are called stiff factor, shape factor, peak factor, and curvature factor, respectively; $\Delta S_h$ and $\Delta S_v$ are horizontal and vertical shifts; $a_i$ (i = 1..13) are the coefficients obtained through regression techniques to best match the mathematical formula with the experiment data; they can be found in Bakker et al. (1987), Demerly and Youcef-Toumi (2000), and Salaani (1996).

Figures 3.3, 3.4, 3.5, and 3.6 depict the lateral tyre force and moment as functions of side slip angle, camber angle, and vertical load for an exemplary stand-alone tyre.

**Figure 3.3:** Lateral tyre force as a function of side slip and camber ($F_z = 4000$ N)

**Figure 3.4:** Lateral tyre force as a function of side slip and vertical load ($\gamma = 0$ deg)
In this investigation, the side slip angle, which is the main source of the lateral force and self-aligning moment, is computed using the equations (3.38) to (3.41). The camber with respect to the ground, which is a function of steering pivot orientation, vehicle motion, suspension geometry, and steering angle, is presented by equation (2.64). The vertical force for each tyre is determined by equations (3.34) to (3.37). Other parameters

Figure 3.5: Aligning moment as a function of side slip and camber ($F_z = 4000$ N)

Figure 3.6: Aligning moment as a function of side slip and vertical load ($\gamma = 0$ deg)
can be found in Bakker et al. (1987), Demerly and Youcef-Toumi (2000), and Salaani (1996). The tyre model, and the steering tyre kinematics are incorporated into the vehicle model. A program is built in Matlab/Simulink environment to solve the equations of motion. In this way, the dynamic behaviour of the car with and without a variable caster strategy can be evaluated.

3.3 The Determination of Kingpin Moment for Low Speed Cornering

In this section, a method of determining the kingpin moment acting on steering wheels is introduced. This method is based on the novel kinematics model of the steering wheel developed in Chapter 2. Although the dynamics of the steering system itself is not included in this research, the determination of the kingpin moment plays an important role in conducting this investigation. The kingpin moment, as a function of steering angle and wheel alignment parameters, will be assessed to propose a caster configuration such that the self-centring ability of steering system during low speed cornering can be improved. The kingpin-moment-related topic has long attracted researchers’ attention (Avachat et al. 2008, Cho 2009, Gough 1953, Kurishige et al. 2000, Pfeffer and Harrer 2008, Sharp and Granger 2003). In those investigations, it was assumed that the tyre-to-ground contact is a fixed point on the tyre perimeter (note that the spin motion of the wheel is excluded, as it does not affect the wheel’s orientation and location). However, in subsection 2.4.4 we showed that the tyre-to-ground contact point moves along tyre perimeter when steering, and that the movement is significant for a large steering angle. Therefore, the assumption leads to an inaccuracy in calculating the kingpin moment when the steering angle is large.

Here we present the determination of kingpin moment based on the novel tyre kinematics model that takes the movement of the tyre-to-ground contact into consideration. All the parameters in this calculation will be expressed in the wheel-body coordinate frame. Therefore, if a vector is expressed in the \( C \) frame and there is no confusion, we will drop the left superscript.

Kingpin moment of a road steering wheel results from the tyre forces and moments developed at the contact patch. The moment of a force \( \mathbf{F} \) that acts at point \( T \), about the kingpin axis, is calculated as:

\[
\mathbf{M}_k = \mathbf{\hat{u}} \cdot [(r_T - r_I) \times \mathbf{F}] 
\]  

where \( r_T \) and \( r_I \) are position vectors of the points \( T \) and \( I \), respectively; the kingpin axis is represented by the direction unit vector \( \mathbf{\hat{u}} \) and the point \( I \) on it.

The moment about the kingpin axis caused by an arbitrary moment \( \mathbf{M} \) is:

\[
\mathbf{M}_k = \mathbf{\hat{u}} \cdot \mathbf{M}
\]
By applying (3.58) and (3.59), the contents of the total kingpin moment are calculated as
follows.

### 3.3.1 Moment Caused by Longitudinal Force

Longitudinal force may be traction force, braking force, and rolling resistance. It is usually
assumed that the longitudinal force vector acts at the tyre print centre \( T \), and is along the
intersection line between the tyre plane and the ground plane. If the length of the vector
is denoted by \( F_x \), the longitudinal force vector expressed in the wheel frame will be:

\[
W F_x = \begin{bmatrix}
F_x \cos \beta \\
0 \\
F_x \sin \beta \\
0
\end{bmatrix}
\]

Hence, the longitudinal force vector expressed in the wheel-body frame will be:

\[
F_x = C F_x = C_{TW} W F_x
\]

The moment about the kingpin axis caused by the longitudinal force is written as:

\[
M_{\hat{u}(F_x)} = \hat{u}. [(r_T - r_I) \times F_x]
\]

### 3.3.2 Moment Caused by Lateral Force

Lateral force may result from side slip, camber angle, lateral wind, and the lateral slope of
the road. When a car turns in a bend, the side slip is the main source of the lateral force.
The lateral force is usually at a longitudinal distance from the tyre print centre. However,
it can be treated as a force acting at the tyre print centre plus a so-called aligning torque.
Therefore, the lateral force vector acts at the point \( T \), and is perpendicular to both the
vertical and the longitudinal forces. If the length of the vector is denoted by \( F_y \), the lateral
force vector expressed in the wheel frame will be:

\[
W F_y = \begin{bmatrix}
F_y \sin \gamma \sin \beta \\
F_y \cos \gamma \\
-F_y \sin \gamma \cos \beta \\
0
\end{bmatrix}
\]

Hence, the lateral force vector, expressed in the wheel-body frame, is:

\[
F_y = C F_y = C_{TW} W F_y
\]

The moment about the kingpin axis caused by the lateral force is:

\[
M_{\hat{u}(F_y)} = \hat{u}. [(r_T - r_I) \times F_y]
\]
CHAPTER 3: VEHICLE MODELLING

The lateral force is a function of the vertical load, side slip angle, camber angle, and other parameters. The effects of these factors are considered by utilising the Magic Formula for tyre modelling, and the rollable model for car modelling presented earlier.

3.3.3 Moment Caused by Vertical Force

The vertical force vector is always perpendicular to the ground. Therefore, its expression in the wheel-body frame is independent of the steering motion of the wheel:

\[
\mathbf{F}_z = C \mathbf{F}_z = \begin{bmatrix}
0 \\
0 \\
F_z \\
0
\end{bmatrix}
\]  

(3.66)

where \( F_z \) is the length of the vertical force vector.

Hence, the moment about the kingpin axis caused by the vertical force will be:

\[
\mathbf{M}_u(F_z) = \mathbf{u} \cdot [(r_T - r_I) \times \mathbf{F}_z]
\]

(3.67)

When the car is cornering, there is the load transfer from the inner to the outer tyre. As a result, the vertical forces change, leading to the variation in the respective moment. This effect is taken into account in this analysis by using the appropriate car model presented in Section 3.1.

3.3.4 Moment Caused by Aligning Torque

‘Aligning’ torque vector, as a result of moving the lateral force to the tyreprint centre \( T \), is perpendicular to the ground. If the length of its vector is \( M_z \), the vector expressed in the wheel-body frame will be:

\[
\mathbf{M}_z = C \mathbf{M}_z = \begin{bmatrix}
0 \\
0 \\
M_z \\
0
\end{bmatrix}
\]

(3.68)

The moment about the kingpin axis caused by the ‘aligning’ torque will be:

\[
\mathbf{M}_u(M_z) = \mathbf{u} \cdot \mathbf{M}_z
\]

(3.69)

Similarly to the lateral force, the aligning torque depends on the vertical force, slip angle, and camber angle, and it is also simulated by using the Magic Formula.
3.3.5 Resultant Kingpin Moment

The kingpin moment for a steering wheel is determined as the summation of all the moments around its steering axis. The contribution of the overturning moment is only minor so it is ignored (Cho 2009). Therefore, the moment around the kingpin axis is:

\[ M_k = M_k(F_x) + M_k(F_y) + M_k(M_z) \]  

(3.70)

As the steering angles are assumed to be the same for the left and right road steering wheels, the resultant kingpin moment for the steering system can be approximated by the summation of the moments for the two wheels.

3.4 Summary

A vehicle model needed for examining the dynamic response of a car with and without a variable caster strategy was constructed in this chapter. In order to capture the important phenomena occurring when the car is cornering, such as the lateral load transfer due to the lateral acceleration, and roll motion, we employed a rollable vehicle model. This model possesses four degrees of freedom: translations in the longitudinal and lateral directions, yaw and roll motions.

The basic idea of variable caster steering is controlling camber (the orientation of the tyre) by mean of variable caster (the orientation of steering axis). Therefore, the relationship between the steering axis orientation and the wheel orientation, and the correlation of the vehicle motion with the wheel orientation, are taken into account. To do so, we incorporated the tyre kinematics developed in Chapter 2 into the vehicle modelling.

As this thesis deals with increasing tyre lateral force limit using the camber, a tyre model that can include camber contribution is required. Furthermore, in the limit region the tyre force is highly nonlinear; therefore, this characteristics needs to be captured. For those reasons, the Magic Formula was chosen for tyre force modelling.

Finally, a novel method for calculating the kingpin moment was developed. This was introduced with the intention that the method is adopted in the next chapter, where a caster configuration can be proposed to improve steering returnability for low speed cornering. The novelty of the method lies in the fact that the movement of the tyre-to-ground contact point when steering is taken into account; and hence it is more accurate than those in the literature.
Variable Caster Steering in Automotive Dynamics

The key idea of variable caster steering theory is controlling the camber by means of a variable caster. Depending on the aims of controlling the camber, the caster is varied in different ways, following various strategies. This chapter demonstrates some benefits of the variable caster by proposing a scheme of varying the caster to counter the roll camber - a phenomenon that limits the grip capacity of a car; the expected result is a more manoeuvrable car with an increased lateral grip capacity. We also show other benefits that a variable caster can provide: improving the returnability of the steering system during low speed cornering.

4.1 Variable Caster Steering for Countering Roll Camber

4.1.1 Development of The Variable Caster Strategy

In Chapter 1 it was showed that the roll camber occurs mostly in a way that both front steering tyres lean away from the bend. In this way, the turning ability of the car is limited, because in this case the roll camber reduces the total lateral force for a car to corner. This is illustrated in Figure 1.2, where the steering wheel is cambered in a way described by the ‘general condition’. Chapter 2 also demonstrated that the caster can be used, and much more effectively than the kingpin inclination angle, to control the camber of the steering tyre. Here we propose a scheme of varying the caster to cancel out, or at least to counter the roll camber phenomenon in order to expand the lateral grip capacity of the car.

Developing the variable caster strategy to counter the roll camber begins with the camber, with respect to the ground, as a function of other parameters. This camber
CHAPTER 4: VARIABLE CASTER STEERING IN AUTOMOTIVE DYNAMICS

function is described by equation (2.64), which can be rewritten as follows:

$$
\sin \gamma_G = \left[ \frac{\cos(\theta + \theta_\varphi) \sin \phi}{\sqrt{\cos^2 \phi + \cos^2(\theta + \theta_\varphi) \sin^2 \phi}} \sin \delta - \frac{\cos^2 \phi \sin(\theta + \theta_\varphi) \cos(\theta + \theta_\varphi)}{\cos^2 \phi + \cos^2(\theta + \theta_\varphi) \sin^2 \phi} \cos \delta \right] \cos(\gamma_0 + \theta_\varphi)
\right. 
+ \left[ \frac{\cos^2 \phi \cos^2(\theta + \theta_\varphi)}{\cos^2 \phi + \cos^2(\theta + \theta_\varphi) \sin^2 \phi} \cos \delta \right] \cos \delta \sin(\gamma_0 + \theta_\varphi)
\right]
\right] 
\approx \frac{\cos(\theta + \theta_\varphi)}{\sqrt{\cos^2 \phi + \cos^2(\theta + \theta_\varphi) \sin^2 \phi}} \approx 1
\sin \gamma_G \approx \gamma_G
\sin \delta \approx \delta
\cos \delta \approx 1
\right]
vers \delta = 1 - \cos \delta \approx 0
\cos(\gamma_0 + \theta_\varphi) \approx 1
\sin(\gamma_0 + \theta_\varphi) \approx \gamma_0 + \theta_\varphi
\right]
\right]
\right]
(4.1)

The reduction of the lateral force and hence the grip ability of the car caused by the roll camber phenomenon only becomes critical in the limit region, which is associated with high speed cornering. The steering angles applied in high speed cornering scenarios are usually small. Therefore, we may make some small-angle approximations. Note that these approximations are only applied to this specific variable caster strategy for simplicity reason; in general, the approximations may not be necessary for any variable caster steering strategies. Therefore, the generality of the tyre kinematics developed in Chapter 2 is still valued. The approximations made are as follows:

$$
\left. \frac{\cos(\theta + \theta_\varphi)}{\sqrt{\cos^2 \phi + \cos^2(\theta + \theta_\varphi) \sin^2 \phi}} \approx 1
\sin \gamma_G \approx \gamma_G
\sin \delta \approx \delta
\cos \delta \approx 1
\right]
vers \delta = 1 - \cos \delta \approx 0
\cos(\gamma_0 + \theta_\varphi) \approx 1
\sin(\gamma_0 + \theta_\varphi) \approx \gamma_0 + \theta_\varphi
\right]
(4.2)

Applying the approximations, the camber function is then reduced to:

$$
\gamma_G = \delta \sin \phi + \gamma_0 + \theta_\varphi
\right]
(4.3)

Figure 4.1: A comparison between exact camber and approximated camber

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.1}
\caption{A comparison between exact camber and approximated camber}
\end{figure}
For practical values of the parameters in the function, the error arising from using the approximations is just a few per cent. Moreover, the camber only contributes a minor part to the total lateral force. Therefore, the error and the approximated function are considered to be acceptable for this variable caster scheme. Figure 4.1 depicts an exemplary comparison between the exact function and the approximation.

To cancel out the outward inclination of the front steering wheels (roll camber) during the turn, the caster must be varied such that the camber with respect to the ground is zero. Substituting $\gamma_G = 0$ in (4.3) yields:

$$\phi_{var} = -\arcsin\left(\frac{\gamma_0 + \theta_\varphi}{\delta}\right)$$ (4.4)

Equation (4.4) suggests that the roll camber phenomenon can be addressed by varying the caster, based on the static camber $\gamma_0$, the KPI angle induced by roll motion $\theta_\varphi = \theta(\varphi)$, and the steering angle $\delta$.

The effects of such a variable caster strategy on vehicle dynamics are examined for a specific configuration which serves as an example: the vehicle is a front-wheel-steering car, has zero static camber ($\gamma_0 = 0$), and the suspensions are equal length double A arm ($\theta_\varphi = \varphi$). For this exemplary configuration, the equation (4.4) becomes:

$$\phi_{var} = -\arcsin\left(\frac{\varphi}{\delta}\right)$$ (4.5)

By the sign convention presented in Chapter 2, the required caster determined by (4.5) generally has a negative sign: for example, when the car is steering to the left ($\delta > 0$), the body rolls to the right ($\varphi > 0$), and hence the required caster, $\phi_{var}$, is negative. Theoretically, if the caster is varied to satisfy equation (4.5), the roll camber of the wheel will be canceled out. However, the caster should be limited to be within a specific range, say $[\phi_L, \phi_U]$. This is due to mechanical constraints such as the required room for the steering tyre. Furthermore, the aim of the variable caster scheme is to address the roll camber occurring during cornering; therefore, during straight-running conditions, the fixed caster as used in the original suspension should be retained. Considering these aspects, a strategy for varying the caster to counter the roll camber is proposed as follows:

$$\phi_{var} = \begin{cases} 
-arcsin\left(\frac{\varphi}{\delta}\right) & \text{if } \{\delta \neq 0, \phi_L \leq-arcsin\left(\frac{\varphi}{\delta}\right) \leq \phi_U\} \\
\phi_U & \text{if } \{\delta \neq 0, -arcsin\left(\frac{\varphi}{\delta}\right) > \phi_U\} \\
\phi_L & \text{if } \{\delta \neq 0, -arcsin\left(\frac{\varphi}{\delta}\right) < \phi_L\} \\
\phi_U & \text{if } \delta = 0 
\end{cases}$$ (4.6a) (4.6b) (4.6c) (4.6d)

where $\phi_L$ and $\phi_U$ are the lower, and the upper limits of the caster, respectively. Note that the lower limit $\phi_L$ is more strongly negative than the upper one $\phi_U$. The $\phi_L$ can be chosen as the maximum achievable value of the caster required by the available room
for the steering tyre. The \( \phi_U \) can be chosen as the fixed caster value of the original car. Therefore, when the car is running straight, its performance stays the same as that of the original car. The car with the variable caster only works differently from the passive one when it negotiates a turn.

The preliminary simulation results, run with the proposed variable caster scheme, showed that, when the steering angle changes its sign (as in lane change manoeuvres), the variable caster jumps from \( \phi_L \) to \( \phi_U \) in no time (from point A to point B in Figure 4.2). The reason for this jump lies in the fact that, between \( t_2 \) and \( t_3 \), the variable caster, \( \phi_{var} \), is set to \( \phi_L \) (because \(-\arcsin(\frac{\delta}{\phi}) < \phi_L\), as stated in 4.6c); at \( t_3 \), the steering angle is going though its zero-crossing point. Between \( t_3 \) and \( t_4 \), the steering angle has changed its sign to negative (steering right), while the roll angle remains positive (rolling right) due to a delay in the response of the car. Therefore, the computed caster, for the period between \( t_3 \) and \( t_4 \), satisfies 4.6b, and is set to \( \phi_U \).

![Figure 4.2: Steering-angle-zero-crossing point and the variable caster](image)

Moving between those two points in no time is impractical for any potential variable caster mechanism. Therefore, this must be addressed. One of the simple ways to do this is plotting a curve, based on the roll motion \( \phi \), to connect point A (associated with \( \delta = 0 \)) and point C (associated with \( \phi = 0 \)). The curve passing through the two points is represented by the dotted line and is written as:

\[
\phi = (\phi_L - \phi_U) \frac{1}{\phi_0} \phi + \phi_U
\]

(4.7)

where \( \phi_0 \) is the roll angle associated with zero steering angle, \( \delta = 0 \).

By incorporating equation (4.7) into the previous strategy (4.6a,4.6b,4.6c,4.6d), we have the tuned variable caster scheme described in the Table 4.3. This variable caster
strategy will be tested in the next section.

Table 4.1. Variable caster scheme

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Variable Caster, $\varphi_{var}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_{L} \leq \arcsin(\frac{\varphi_{\delta}}{\varphi_{\delta}}) \leq \varphi_{U}$</td>
<td>$-\arcsin(\frac{\varphi_{\delta}}{\varphi_{\delta}})$</td>
</tr>
<tr>
<td>$-\arcsin(\frac{\varphi_{\delta}}{\varphi_{\delta}}) &lt; \varphi_{L}$</td>
<td>$\varphi_{L}$</td>
</tr>
<tr>
<td>$-\arcsin(\frac{\varphi_{\delta}}{\varphi_{\delta}}) &gt; \varphi_{U}$</td>
<td>$\varphi_{U}$</td>
</tr>
<tr>
<td>$\varphi = 0$</td>
<td>$\varphi_{U}$</td>
</tr>
<tr>
<td>${\varphi \neq 0, \varphi_{\delta} \leq 0}$</td>
<td>$(\varphi_{L} - \varphi_{U}) \frac{1}{\varphi_{\delta}} \varphi + \varphi_{U}$</td>
</tr>
</tbody>
</table>

### 4.1.2 Dynamics of The Vehicle with The Variable Caster Strategy

In this section, the dynamic behaviour of the vehicle with the variable caster strategy is examined and compared with that of the original vehicle. Because designing control algorithm is beyond the scope of this thesis, a simple proportional control algorithm is used for simplicity. Since this research does not focus on any particular car, we selected the 1994 Ford Taurus GL, a sedan car that had been parameterized, for analysis. Another reason for selecting this car is that the field test data for it were also publicly available (Demerly and Youcef-Toumi 2000, Salaani 1996), which are used later for validation. The vehicle parameters used for the simulation are presented in Table 4.2.

Table 4.2. Vehicle parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(kg)$</td>
<td>1704.7</td>
<td>$I_{zz}(kgm^2)$</td>
<td>21.09</td>
<td>$h_{g}(m)$</td>
<td>0.542</td>
<td>$C_{\varphi 1}(\frac{Nm}{rad})$</td>
<td>2823</td>
</tr>
<tr>
<td>$m_{s}(kg)$</td>
<td>1526</td>
<td>$L(m)$</td>
<td>2.69</td>
<td>$h_{f}(m)$</td>
<td>0.13</td>
<td>$K_{\varphi 2}(\frac{Nm}{rad})$</td>
<td>37311</td>
</tr>
<tr>
<td>$m_{af}(kg)$</td>
<td>98.1</td>
<td>$a(m)$</td>
<td>1.035</td>
<td>$h_{r}(m)$</td>
<td>0.11</td>
<td>$C_{\varphi 2}(\frac{Nm}{rad})$</td>
<td>2653</td>
</tr>
<tr>
<td>$m_{ur}(kg)$</td>
<td>79.7</td>
<td>$a_{s}(m)$</td>
<td>1.015</td>
<td>$h_{af}(m)$</td>
<td>0.313</td>
<td>$R_{w}(m)$</td>
<td>0.313</td>
</tr>
<tr>
<td>$I_{xx}(kgm^2)$</td>
<td>440.911</td>
<td>$w_{f}(m)$</td>
<td>1.54</td>
<td>$h_{ur}(m)$</td>
<td>0.313</td>
<td>$\varphi_{U}(deg)$</td>
<td>-5</td>
</tr>
<tr>
<td>$I_{zz}(kgm^2)$</td>
<td>3048.1</td>
<td>$h(m)$</td>
<td>0.445</td>
<td>$K_{\varphi 1}(\frac{Nm}{rad})$</td>
<td>47298</td>
<td>$\varphi_{L}(deg)$</td>
<td>-35</td>
</tr>
</tbody>
</table>

The dynamic responses of the vehicle to three types of steering inputs, step steer, slowly increasing steer, and sinusoidal steer, are examined to evaluate the handling performance in terms of steady state, limit, and transient behaviour.

**Step Steer Input**

Step steer input is a manoeuvre used to assess both steady state and transient behaviours of a car. The manoeuvre consists of straight running followed by a sudden turn, which results in steady state cornering at a particular lateral acceleration. In this investigation, the car speed is kept constant, and the amplitude of the steering angle is that which
produces a relatively high lateral acceleration, around 0.6g. To make the step steering more realistic, it is rounded by using the following function:

\[
\delta = \begin{cases} 
0 & \text{if } t < t_0 \\
\frac{\delta_0}{2} + \frac{\delta_0}{2} \sin\left(\frac{t - t_0}{t_1 - t_0} \pi - \frac{\pi}{2}\right) & \text{if } t_0 \leq t \leq t_1 \\
\delta_0 & \text{if } t > t_1
\end{cases} \tag{4.8a}
\]

\[
\delta_0 = \begin{cases} 
0 & \text{if } t < t_0 \\
\frac{\delta_0}{2} + \frac{\delta_0}{2} \sin\left(\frac{t - t_0}{t_1 - t_0} \pi - \frac{\pi}{2}\right) & \text{if } t_0 \leq t \leq t_1 \\
\delta_0 & \text{if } t > t_1
\end{cases} \tag{4.8b}
\]

\[
\delta_0 = \begin{cases} 
0 & \text{if } t < t_0 \\
\frac{\delta_0}{2} + \frac{\delta_0}{2} \sin\left(\frac{t - t_0}{t_1 - t_0} \pi - \frac{\pi}{2}\right) & \text{if } t_0 \leq t \leq t_1 \\
\delta_0 & \text{if } t > t_1
\end{cases} \tag{4.8c}
\]

Figure 4.3 shows the time history of the steering input described by the function for the following parameters:

\[
\delta_0 = 0.1 \text{ rad} \approx 5.7 \text{ deg}; t_0 = 0 \text{ sec}; t_1 = 1.8 \text{ sec}; v_x = 60 \text{ km/h} \approx 16.67 \text{ m/s} \tag{4.9}
\]

The car’s dynamic reaction to the step steer input, with and without the variable caster strategy, is illustrated in Figures 4.4 to 4.8. More specifically, the dashed curve in Figure 4.4 represents the fixed caster employed in the benchmark car. When this passive car negotiates the turn, its body rolls towards the outer side of the bend. This roll results from the centrifugal force acting at its centre of gravity. The roll motion of the car body for the passive car is described by the dashed curve in Figure 4.8. This roll motion gives rise to the roll camber phenomenon, where the front steering wheels lean away from the centre of rotation. Figure 4.5 illustrates the camber of the two front steering wheels: the dashed curve is the camber of the inner wheel, and the dotted curve is that of the outer wheel. As is shown, the camber of the inner wheels is slightly higher than that of the outer one; this is because the \textit{KPI} angles of the two steering axes are opposite (symmetric suspension geometry). The roll camber will limit the lateral acceleration and the yaw rate, because it reduces the total lateral force for a car to turn into a corner.
SECTION 4.1: VARIABLE CASTER STEERING FOR COUNTERING ROLL CAMBER

The solid curve in Figure 4.4 shows the caster when it is made variable in accordance with the proposed strategy. As can be seen in Figure 4.5, the cambers of the front steering wheels associated with the controlled car (the dotted dashed curve and the solid curve), experience great reduction of around 90 per cent compared to those of the passive car. The roll camber is not cancelled out completely because we use the approximations in (4.1). Furthermore, there is a slight difference between the controlled camber of the left and the right wheels due to the opposite $KPI$ angles for the two, as mentioned earlier.

Figure 4.4: Caster for step steer input at 60 km/h

Figure 4.5: Camber for step steer input at 60 km/h
The reduction of the roll camber leads to the increases in the steady state lateral acceleration and yaw rate, as shown in Figures 4.6 and 4.7, respectively. Here, the solid curves are the ones associated with the caster-controlled car. To be more specific, the steady state lateral acceleration of the car with the variable caster increases to nearly 6.86 m/s$^2$ from the reference value of about 6.28 m/s$^2$. Similarly, the yaw rate for the controlled car experiences an improvement of around 8 per cent. However, as the lateral acceleration increases, the roll motion of the car body also rises, from 3.14 degrees to 3.43 degrees, as shown in Figure 4.8. Although the increase in the body roll is not desired,
it is just a minor disadvantage and is greatly outweighed by the advantage gained from the improvement in lateral acceleration and yaw rate. This is because, the maximum lateral acceleration for passenger cars is generally much less than the level at which the roll-over can occur, which means the car will tend to go out of the desired path of the motion long before the roll-over occurs (Gillespie 1992, Wiegand 2011). Figures 4.6 to 4.8 also indicate that, while the steady state lateral acceleration and yaw rate are increased, the transient characteristics, such as overshoot value, settling time, and response time, are almost unaffected by the variable caster. This is considered to be an upside of the variable caster strategy.

The controlled caster car, however, is expected to experience a larger tendency to oversteer, which may badly affect the stability of the car. This is because the front wheels with less outward inclinations will create more lateral forces, which is equivalent to larger front cornering stiffnesses compared to those of the reference vehicle. As shown in equation (1.12), the greater the front wheel cornering stiffness, the higher the tendency towards oversteer. Therefore, to assess the change in the understeer gradient of the caster-controlled car, a simulation constant speed test is carried out in the next section.

**Slowly Increasing Steer Input**

The slowly increasing steer manoeuvre can be used to assess the steady state behaviour of the car from a low acceleration level to the handling limit. In this investigation, we also use this manoeuvre to conduct a constant speed test for examining the handling characteristics of the car. As indicated by the name, the steering angle is slowly and
linearly increased while the vehicle speed is kept constant. Figure 4.9 illustrates this steer input with the following parameters:

Table 4.3. Slowly increasing steer parameters

<table>
<thead>
<tr>
<th>Start time [s]</th>
<th>End time [s]</th>
<th>Steering Ramp [rad/s]</th>
<th>Velocity [km/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>0.01</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 4.9: Slowly increasing steer input at 50 km/h

Figures 4.10 to 4.14 show the dynamic responses of the car to the slowly increasing steer input for the benchmark car, and for the car equipped with the controlled caster.

Figure 4.10: Caster for slowly increasing steer input at 50 km/h
SECTION 4.1: VARIABLE CASTER STEERING FOR COUNTERING ROLL CAMBER

Figure 4.11: Camber for slowly increasing steer input at 50 km/h

Figure 4.12: Lateral acceleration for slowly increasing steer input at 50 km/h

It can be seen that the lateral acceleration (Figure 4.12), the yaw rate (Figure 4.13), and the body roll (Figure 4.14) of the passive car (the dashed curves) increase with the steering angle. The roll motion causes the front wheels to lean away from the turn, as shown by the dashed line and the dotted line in Figure 4.11. The outward inclinations of the steering wheels, however, are counteracted by the camber gained when the caster of the steered wheel is appropriately varied. Therefore, the cambers of the front steering wheels (the solid line and the dashed dotted line) reduce to approximately 10 per cent of those of the passive car. This leads to the increases in lateral acceleration and yaw
rate compared to the non-controlled car, as illustrated in Figures 4.12 and 4.13 (the solid lines). These increases appear to be proportional to the steering angle. The roll motion of the car body also experiences a similar increase (the solid line in Figure 4.14), but again this is just a minor downside, as explained in the previous section.

![Figure 4.13: Yaw rate for slowly increasing steer input at 50 km/h](image1)

![Figure 4.14: Body roll for slowly increasing steer input at 50 km/h](image2)

When the caster is varied, the caster trail and hence the kingpin moment (the moment around the kingpin axis) alters. The formation of this moment was presented in Section 3.3. The change of the kingpin moment will affect the steering effort and the returnability.
of the steering system. The slowly increasing steer can be used to assess the kingpin moment. Figure 4.15 illustrates the kingpin moment as a function of the road steering wheel for the two cases, fixed caster (the dashed line) and the variable caster (the solid line).

![Figure 4.15: The kingpin moment increases when the caster is varied](image)

Figure 4.15: The kingpin moment increases when the caster is varied

It is observed that, when the caster is varied in accordance with the proposed strategy the kingpin moment increases compared to that of the fixed caster, but the pattern remains the same: the kingpin moment increases almost linearly with the steering angle. The increased kingpin moment exerts two effects. On the upside, it improves the steering returnability, which in turn enhances the directional stability of the car. On the downside, the increased kingpin moment requires a greater steering effort to turn the steering wheel. However, since the assist steering system is now used extensively, the increased steering effort can be addressed appropriately. Therefore, this downside is considered to be outweighed by the advantages.

The slowly increasing steer manoeuvre is also used in this research to conduct simulation constant speed tests for examining the handling characteristics of the car. The test was carried out with the vehicle speed being kept constant, which is similar to real driving conditions. The steering angle of the hand steering wheel is linearly increased to the level at which the lateral acceleration is saturated in order to assess the handling characteristics up to the limit and also to determine the lateral acceleration capacity. In the test, the steering angle and the lateral acceleration (or equivalently the radius of turn or the yaw rate) are logged.
Figures 4.16 and 4.17, respectively, depict the lateral acceleration gains at 40 km/h and 80 km/h for the car with the constant caster (the dotted line) and with the variable caster (the dashed line). To determine the handling characteristic of the car, a control line associated with neutral steer is also drawn in the Figures (the solid line). It can be clearly seen from the graphs that at any steering angle the controlled car produces a higher level of
lateral acceleration compared to the passive car. Most noticeably, the car’s grip capacity (maximum lateral acceleration) is expanded by approximately 5 per cent when the variable caster scheme is employed. Although the understeer gradient of the caster-controlled car deceases as predicted, the car with the variable caster still possesses an understeer characteristic (the curve is above the neutral steer line). Furthermore, the patterns of the handling curves in the two cases are similar: the understeer gradient increases with the increase of the car’s lateral acceleration. This is considered to be desirable for a passenger car: the small level of understeer at low lateral acceleration provides a sensitive steering response in most cornering manoeuvres; and the high understeer level at high lateral acceleration enhances stability during tight turns. The increase of maximum lateral acceleration, along with the understeer characteristics of the controlled caster car, shows that the variable caster scheme provide a expanded grip capacity without impairing the directional stability of the car.

**Sinusoidal Steer Input**

Sinusoidal steer, a lane-change approximation, has been used by a number of organizations to evaluate vehicle transient handling behaviour; it is considered to be challenging to the vehicle’s response and representative of actual driving situations. In this analysis, it is used to assess not only the behaviour of the caster-controlled car but also how the caster is varied. The sinusoidal steering test can be expressed by the following function:

\[
\delta = \begin{cases} 
0 & \text{if } t < t_0 \\
\delta_0 \sin\left(\frac{t - t_0}{t_1 - t_0}\pi\right) & \text{if } t_0 \leq t \leq t_1 \\
0 & \text{if } t > t_1 
\end{cases} 
\]  

To generate a relatively high lateral acceleration, the sinusoidal steer input is selected with the following parameters:

\[
\delta_0 = 0.1 \text{ rad} \approx 5.7 \text{ deg}; t_0 = 0 \text{ sec}; t_1 = \pi \text{ sec}; v_x = 70 \text{ km/h} \approx 19.44 \text{ m/s} 
\]  

Figure 4.18 shows the time history of the steering angle for this manoeuvre.
The responses of the vehicle to the input are shown in Figures 4.19 to 4.23. More specifically, Figure 4.19 illustrates the caster of the passive car (the dashed line which presents a constant value) and that of the controlled car (the solid curve). The graph indicates that the tuned variable caster strategy described by Table 4.3 enables the controlled caster to avoid the jump from $\phi_L$ to $\phi_U$ in the area associated with the zero-crossing point of steering angle. This is desirable, as jumping from the two values in no time is unfeasible for any potential suspensions with the variable caster, as discussed earlier. Because the caster is appropriately varied, the roll camber of the controlled car generally reduces com-
pared to that of the non-controlled car: this is illustrated in Figure 4.20. In particular, the reduction of roll camber is significant in the regions near the curves’ vertices, which correspond to large steering angles. In the vicinity of the steering-angle-zero-crossing point, however, the roll camber change between the two cases is inconsiderable. The reason is that the steering input in that region is approaching zero (and changing sign), while the roll motion remains considerable (caused by a delay in the response). Therefore, the required caster to cancel the roll camber, \(-\frac{\phi}{\delta}\), is too large; because the variable caster is set to be within the range \([\phi_L, \phi_U]\), the caster in this region is not sufficient to compensate for the roll camber.

![Figure 4.20: Camber for sinusoidal steering input at 70 km/h](image)

Figure 4.20: Camber for sinusoidal steering input at 70 km/h

![Figure 4.21: Lateral acceleration for sinusoidal steering input at 70 km/h](image)

Figure 4.21: Lateral acceleration for sinusoidal steering input at 70 km/h
As the roll camber is reduced, the lateral acceleration and the yaw rate, respectively shown in Figures 4.21 and 4.22, are enhanced. The improvements of lateral acceleration and yaw rate are considerable, at approximately 9 per cent, occurring in the high lateral acceleration region (greater than 0.6g). When the lateral acceleration is low (less than 0.4g), the improvements are subtle. However, the marginal improvements of those responses are not considered to be a drawback of the variable caster, because this solely occurs where the lateral acceleration is low, and hence do not affect the limit performance. The considerable increases of lateral acceleration and yaw rate associated with high acceleration level do give the variable caster an advantage, because they improve the grip capacity of the car. The car body also experiences a higher roll motion similar to the
lateral acceleration, as illustrated in Figure 4.23, but, as explained earlier, this downside is minor and is overcome by the advantage of increasing the grip capacity.

4.2 Caster for Low Speed Cornering

The primary motivation of the variable caster scheme developed in this thesis is controlling the camber of steering wheels. However, because the caster is made variable, the scheme can be further expanded in a way that improves the steering returnability during low speed cornering. This section discusses the possibility of such an expanded variable caster scheme; we propose a caster configuration utilised during low speed cornering that provides better steering returnability.

4.2.1 Steering Returnability

Steering returnability or the self-centering of the steering system is the ability of road steering wheels to automatically return to the straight-running position after the bend has been negotiated. Steering returnability is a necessary characteristic of a steering system; without it the stability of the vehicle will be impaired. Steering returnability is assured by a moment about the kingpin axis (kingpin moment) that has a restoring effect on the steered tyre; it is, therefore, called the self-centering moment. The self-centering moment is produced by tyre forces and wheel alignment. This moment helps the driver to be able to return the steering to the straight-running position fast enough when coming out of the turn, avoiding a delay in handling the car. Also, the self-centering moment provides the driver with steering feel during cornering - the most important feedback on the road conditions. Furthermore, when the wheel is unexpectedly steered by disturbances such as uneven road surface or lateral wind, the self-centering moment can also help stabilising the straight-running motion. The downside of the self-centering moment is that, it requires greater effort to turn the car into the corner. This drawback, however, is easily overcome since power-assisted steering is common in existing cars; the drawback, therefore, far outweighed by the advantages.

The steering self-centering ability involves two manoeuvres: low speed cornering with large steering angles, and high speed cornering with small steering angles (on-centre area). This section suggests a caster configuration to improve the steering returnability for low speed cornering manoeuvre.

4.2.2 Steering Returnability during Low Speed Cornering

Although steering returnability is shaped by different aspects such as steering gear box design and friction in the steering system, the self-centering moment generated by tyre forces and wheel alignment plays the most important role. This is because the self-centering moment is the source of steering returnability. This investigation, therefore,
only focuses on improving the self-centering moment, with the assumption that other aspects can be addressed separately and are beyond the scope of this investigation.

The self-centring moment in conventional cars is usually created by using ‘proper’ wheel alignment parameters. In this regard, the wheel alignment configuration is set in a way that it, together with the tyre forces, generates a moment acting in the direction opposite to the steering motion of the wheel. Normally, a certain degree of negative caster (again, the top end of the steering axis leans rearwards, in accordance with the sign convention used in this thesis) is usually employed in passenger cars. Generally speaking, the tyre lateral force and its lever arm provided by that caster will generate a restoring moment, tending to pull the steered wheel back to the straight-running position, as illustrated in Figure 1.22.

For high speed cornering motion, this negative caster produces restoring moments for both the inner and the outer steering wheels. The reason is that during high speed cornering the lateral forces at both steering tyres point towards the inner side of the bend, as depicted in Figure 4.24. Therefore, a negative caster is beneficial to returnability during high speed cornering.

![Lateral forces under two front steering tyres during high speed cornering](Figure is generated by CarSim)

Unlike high speed cornering, the lateral forces at the two wheels during low speed cornering are directed towards the mid-plane of the vehicle (Cho 2009), as illustrated in Figure 4.25. Consequently, the moment generated by the lateral force and its lever arm at the inner tyre is in the same direction with the steering motion; and hence it facilitates the steering motion. Although the moment generated by the lateral force at the outer wheel is in opposite direction to the steering, its restoring effect is normally outweighed by the facilitating effect at the inner wheel. Thus, this caster impairs returnability during low speed cornering.
The steering returnability for low speed cornering manoeuvres normally relies upon the kingpin inclination angle; it has been stated in the literature that a \( KPI \) angle that the steering axis’s top end leans towards the car body gives a righting moment for both inner and outer wheels when steering (Milliken and Milliken 1995, Milliken et al. 2002, Reimpell et al. 2000). This righting moment generated during the steering is caused by the tyre normal load and its lever arm. As indicated by the name, this moment tends to pull the steered wheel back to the straight-running position. The restoring effect can also be explained by the lifting effect in which the car body is raised when the wheel is steering away from the straight-ahead position. Because the car body always seeks the lowest position with the lowest potential energy, the lifting effect is beneficial to returnability (Milliken and Milliken 1995, Stone and Ball 2004).

### 4.2.3 A Caster Configuration for Improving Steering Returnability during Low Speed Cornering

While the kingpin inclination angle is used in the existing passenger cars to provide the returnability for steering during low speed cornering manoeuvre, it is stated that the lifting effect (created by that \( KPI \) angle) that restores the steering motion is perfectly true with the zero caster (Reimpell et al. 2000); if a negative caster exists, the car body at the outer wheel with much more weight than the inner wheel will sink when steering; and hence, instead of returning, the outer steered wheel will be turned further (Milliken and Milliken 1995, Reimpell et al. 2000). This is also reinforced by the kinematics developed in this thesis. In order to show that, we examine the vertical displacement of the car body when steering for different caster values, with the \( KPI \) being kept constant. The result is shown in Figure 4.26. As can be clearly seen, when a non-zero caster is employed, the car body is lifted when steering the wheel towards one side and lowered for the other side; the
only configuration that provides the lifting effect for both steering left and right is the one with zero caster (the solid curve). This also means that only with zero caster, the lifting effect caused by steering occurs for both steering wheels. Therefore, it is highly likely that the zero caster is the best configuration for returnability during low speed cornering.

In conventional cars, however, the caster is generally non-zero. The reason is that the wheel alignment parameters in those cars are set as fixed values. Consequently, there is always a trade-off between the returnability for high speed cornering and that for low speed cornering. If a highly negative caster is used in order to improve the returnability and hence the directional stability at high speed, the returnability for low speed cornering will be weakened; and if a zero caster (in combination with a KPI) was employed to maximise the returnability during low speed cornering, the self-centering and hence the directional stability at high speed would be impaired. That is the reason why a negative caster in combination with a KPI is usually used in conventional cars. As a result, the returnability for low speed cornering is compromised. Specifically, the total restoring moment for a steering system only exists up to a relatively small steering angle, beyond which the moment changes its sign and hence impairs the returnability. This is undesirable, as a righting moment is necessary, at any event, to provide the driver with correct information about the actual state of the vehicle and the road (Mastinu and Ploechl 2013). Therefore, it is desirable that the returnability is assured with larger steering angles.

As the caster is made variable to address the roll camber for high speed cornering, it can also be set to a desired value during low speed cornering to improve steering returnability. From the above analysis it is highly likely that the restoring moment to assure the
steering returnability during low speed cornering is highest with a zero caster configuration. However, the moment about the kingpin axis (hereafter called the kingpin moment) is generated by not only the vertical force but also the lateral force, longitudinal force, and ‘aligning’ torque. Therefore, we have to look into how these components form the kingpin moment and hence affect the low-speed-steering-returnability.

The model built in Section 3.3 is utilised to examine how the tyre forces and wheel alignment parameters generate the kingpin moment. A slowly increasing steering input with the vehicle speed of 5 km/h is selected to mimic the low speed cornering manoeuvre. The kinematic relationships between the wheel alignment parameters are derived from the mechanism presented in subsection 2.7. The vehicle parameters are presented in the Table 4.2. Figures 4.27 to 4.30 show how the longitudinal, lateral, vertical forces, and ‘aligning’ torque form the kingpin moment during low speed cornering for the benchmark configuration with its negative caster. In accordance with the sign convention used in this thesis, a positive moment (about the kingpin axis) is the one that acts in the same direction of steering motion, and hence facilitates the turn; a negative moment tends to pull the steered wheel towards the straight-running position, and hence has a self-centering effect. The negative kingpin moment is, therefore, called the restoring moment, righting moment, or self-centring moment.

![Image](image URL)

**Figure 4.27:** Moment caused by $F_x$ for the negative caster

The moment about the kingpin axis caused by the longitudinal force is illustrated in Figure 4.27. The tyre longitudinal force is to overcome the braking portion of the cornering force to maintain a constant longitudinal velocity. As the lateral force increases with the steering angle, its braking portion and hence the longitudinal force rise. As a result,
the moment caused by this longitudinal force increases. However, while the moment at
the inner wheel (the dotted curve) has a self-centring effect, that of the outer wheel (the
dashed curve) tends to push the wheel in the steering direction. It is clearly seen that the
opposite effects nearly balance out each other. Also the magnitudes of those moments are
insignificant. Therefore, the total effect of the longitudinal forces (the solid curve) on the
returnability is negligible.

Figure 4.28 shows the moment generated by the tyre lateral force. It is apparent that
the total effect of the lateral forces for the steering system, during low speed cornering,
is facilitating the steering motion rather than righting it which occurs in high speed
cornering. This is attributed to the fact that the lateral forces under the two tyres, during
low speed cornering, are directed towards the vehicle’s vertical mid-plane. Consequently,
the lateral force at the inner wheel, together with its lever arm (caused by the caster trail),
gives rise to a moment acting in the same direction with the steering motion, and hence
facilitates the turn. This moment is represented by the dotted curve. Although the lateral
force under the outer tyre tends to pull the steered wheel towards the straight-running
position (the dashed line), this effect is exceeded by the opposite one of the inner wheel.
Furthermore, because the road-tyre contact point displaces along the wheel perimeter
when steering (as discussed in Section 2.4.4), the moment caused by the lateral force at
the outer wheel changes its sign at about 22 degrees of steering angle. Therefore, the
total moment caused by the lateral forces impairs returnability for this manoeuvre. This
undesirable effect appears to be proportional to the steering input. This is in agreement
with the finding reported by Cho (2009).

Figure 4.28: Moment caused by $F_y$ for the negative caster
The moments generated by normal forces at the inner and the outer tyres are shown in Figure 4.29. It can be seen in the graph that the returnability benefits from the vertical force under the inner steering tyre (the dotted line). The vertical force under the outer tyre only aids in returnability for steering angles beyond about 22 degrees; for steering angles smaller than 22 degrees it strengthens the steering motion. At the straight-running position, the two opposite effects associated with the two tyres cancel out each other due to the symmetry of the steering geometry. As the steering angle is increased, the restoring effect produced by the inner vertical force overcomes the steering-facilitating effect at the outer wheel. Therefore, the normal loads provide a source of self-centring ability. This result is in agreement with what is shown in the literature (Cho 2009, Reimpell et al. 2000).

The so-called ‘aligning’ torque resulting from moving the lateral force to the tyreprint centre, acts about a vertical axis (rather than about the kingpin pivot). Although named ‘aligning’ torque, it does not always have an aligning effect. This is because the lateral force is not only at a distance behind but also in front of the tyreprint centre, as shown in Figure 1.8. Furthermore, the lateral force is not always directed towards the inner side of the bend (as shown earlier). Figure 4.30 shows the moment component that is achieved by projecting the ‘aligning’ torque on the steering axis for each wheel. As is shown by the graph, the kingpin moment caused by the ‘aligning’ torque for the outer tyre has a righting effect. By contrast, that moment for the inner tyre aids in steering the tyre further. The reason for the opposite effects on the two tyres lies in the fact that, while the outer tyre lateral force points towards the center of rotation, the lateral force at the inner tyre points
away from the corner. The two effects nearly cancel out each other for a steering angle up to about 27 degrees, beyond which the total moment facilitates the turn.

The kingpin moment for each wheel is determined by (3.70), as the summation of the above four moments. As we assumed the same steering angle for the left and the right wheels, the resultant kingpin moment for the steering system can be approximated as the summation of the kingpin moments for the left wheel and for the right wheel. The kingpin moment for each wheel and the resultant kingpin moment for the steering system are illustrated in Figure 4.31. The graph shows that the kingpin moment at the inner wheel only works as a self-centring moment up to about 14 degrees of steering angle; when the wheel is steered further, this kingpin moment strengthens the steering motion. The reason is that, when the steering angle is less than 14 degrees, the lateral force and the ‘aligning’ torque at the inner wheel are still small. Therefore, their facilitating-steering effect, as shown in Figures 4.28 and 4.30, are marginal. However, the self-centring effect produced by the vertical force is considerable (the dotted curve in Figure 4.29) and outweighs the opposite effect produced by the lateral force and ‘aligning’ torque. When the steering angles are larger, the moments caused by the lateral force and the ‘aligning’ moment at the inner wheel become significant and exceed the self-centering moment caused by the vertical force. In contrast, the kingpin moment at the outer wheel facilitates the turn up to approximately 10 degrees of steering angle, and has a righting effect for larger steering angles. This is because, at the outer wheel, the moment caused by the vertical force dominates the kingpin moment for a small steering angle, and the moment caused by the lateral force dominates the kingpin moment for larger steering angle.

Figure 4.30: Moment caused by $M_z$ for the negative caster

![Diagram showing moment caused by $M_z$ for the negative caster](image-url)
The resultant kingpin moment for the steering system is denoted by the solid curve. It is observed that this moment only provides the source of self-centring up to nearly 20 degrees of steering wheel; beyond that value, its sign changes, and hence the moment impairs the steering returnability. The is why the returnability during low speed cornering is very limited for the passive vehicle.

Figure 4.31: Resultant kingpin moment for the negative caster

A low speed cornering manoeuvre, such as getting the car out of a parking lot, involves large steering angles. Therefore, it is desirable to have a restoring moment within a larger steering angle range to help the driver to return to the straight-running state fast enough. It has also been stated that a righting moment is necessary, at any event (Mastinu and Ploechl 2013). That is the motivation for us to propose a steering axis configuration that can assure a restoring moment within a wider steering range. As is shown in Figures 4.27 to 4.30, the longitudinal forces and the ‘aligning’ torques have negligible effects on the resultant kingpin moment during low speed cornering; the resultant kingpin moment for the steering system is mainly dominated by the moments caused by lateral and vertical tyre forces. Therefore, we expect that, by increasing the restoring effect due to the vertical force, the returnability is improved. The restoring effect caused by the vertical force is maximum with a zero caster, as discussed earlier. Thus, whether a zero caster can also improve the resultant kingpin moment for the steering system is examined. Figures 4.32 to 4.35 show the moment components caused by the tyre forces with a zero caster while other parameters are kept constant. The results from the benchmark that have been shown above are also put together for comparison: the curves with asterisks are associated with the zero caster configuration; those without the asterisks are for the benchmark car with
the negative caster.

As can be seen in Figure 4.32, the moment caused by the longitudinal force for the zero caster (the asterisked curves) is slightly higher than that of the negative caster. The trend, however, stays the same as that of the benchmark: the moment at the inner wheel has a restoring effect, and is neutralised by the opposite effect caused by the moment at the outer wheel. Therefore, the change in the caster almost does not affect the moments caused by the longitudinal forces; and therefore the longitudinal forces for the two cases have negligible effect on the kingpin moment, and hence the returnablity.

![Figure 4.32: Moment caused by $F_x$ for different casters](image)

The moments caused by the lateral forces with the proposed zero caster are represented by the asterisked curves in Figure 4.33. In general, the total effect of the lateral forces is to strengthen the steering motion with a somewhat larger moment than that of the passive car. To be more specific, the moments caused by the lateral forces at both wheels are positive, which facilitate the turn. With the zero caster configuration, the moment at the inner wheel (the dotted-asterisked curve) experiences a reduction compared to that of the benchmark (the dotted curve), which is considered to be better for returnability; however, there is an increase in the moment at the outer wheel (the dashed-dotted-asterisked curve) in comparison with the benchmark (the dashed-dotted-curve). Because the increase in the outer moment is more than the decrease in the inner moment, the total moment generated by the lateral forces for this zero caster in fact makes returnability worse than
the benchmark configuration.

Figure 4.33: Moment caused by $F_y$ for different casters

Figure 4.34: Moment caused by $F_z$ for different casters
In contrast to the lateral forces, the moments caused by normal loads under the two tyres for the zero caster, as illustrated in Figure 4.34, both exert a restoring effect. The moment caused by the normal load of the inner wheel (the dotted-asterisked curve) is negative for the whole steering angle range, and is the same as that of the outer one (the dotted-dashed-asterisked curve). As a result, the total restoring effect caused by the normal loads for the proposed caster represented by the solid-asterisked curve is much stronger than that of the benchmark configuration (the dashed curve).

The projection of the ‘aligning’ torque on the steering axis for the two cases (negative and zero casters) is shown in Figure 4.35: there is a slight difference between them for the whole range of steering angles, with a better restoring effect associated with the zero caster.

The resultant kingpin moments for the passive and caster-controlled car are shown in Figure 4.36. The first thing to note is that, at any steering angle, the total kingpin moment associated with the zero caster is more strongly negative than that for the negative caster. This means that, with the zero caster configuration, the restoring effect is improved. The reason is that, when the caster is set to zero, the improvement of the restoring effect caused by the vertical forces (Figure 4.34) and ‘aligning’ torques (Figure 4.35) clearly exceeds the decline of the restoring effect caused by the lateral forces (Figure 4.33). More importantly, with the zero caster configuration, a negative kingpin moment (which also
SECTION 4.3: SUMMARY

means a restoring moment) is assured up to approximately 26 degrees of steering angle compared to about 20 degrees in the case of the negative caster. This means that the restoring effect for the zero caster is better than that for the baseline configuration.

Figure 4.36: Resultant kingpin moment for different casters

Increasing steering returnability leads to a greater effort being required to turn the steering wheel. Since the power-steering systems are now common in vehicles, this negative effect can be addressed by an appropriate assist steering system; and we expect that this matter will be resolved separately. It was also stated that, while power-steering systems have been extensively developed to reduce steering effort, there has been hardly any attention to augmenting returnability (Kurishige et al. 2002). Therefore, the improvement in restoring moment achieved by varying the caster is appreciated.

4.3 Summary

This chapter has demonstrated that the caster can be varied in some good manners. Specifically, the roll camber, a phenomenon that limits the lateral grip capacity of the car, was chosen to be addressed. In order to do so, a kinematic analysis of the wheel camber with respect to the ground was conducted. On the basis of the analysis, a strategy of varying the caster to counter the roll camber was proposed. Three types of steering input were employed to assess the dynamic behaviour of the vehicle. The responses of the car
with the variable caster strategy to the steering inputs were compared with those of the baseline configuration. The results show that, by countering the roll camber of the front steering tyres, the lateral acceleration and the yaw rate are improved. The improvements lead to an increase of 5 per cent in the lateral grip capacity while the car remains stable (understeer). This means, the caster-controlled car is more manoeuvrable. The increased manoeuvrability gives the car numerous advantages such as a higher collision avoidance capability, better performance, and higher ability to manoeuvre under poor conditions. It is acknowledged that the side slip is the main source of cornering force, and the camber contribution is generally insignificant. However, in the limit region, where the lateral force due to side slip is nearly saturated, a few per cent improvement in grip ability provided by the extra camber is of critical importance and can save lives.

While the limited lateral capacity of the conventional car is a disadvantage associated with high speed cornering, the drawback of the conventional fixed caster configuration in the passive car is poor steering returnability during low speed cornering. Motivated by this fact, a caster configuration to improve the returnability of the steering system during low speed cornering was proposed. To do that, an analysis of the source of steering returnability was carried out; the novel model for calculating kingpin moments (caused by the tyre forces) presented in Chapter 2 was applied. On the basis of the analysis, a zero caster was proposed for low speed cornering. With this caster configuration, the kingpin moment, and hence returnability for low speed cornering, are improved. The result is a higher returnability assured in a wider steering angle range.

The downside of the variable caster strategy is creating a greater steering effort. However, this issue can be addressed appropriately since the assist steering is now extensively used in vehicles. Therefore, this downside is greatly outweighed by the advantages provided by the variable caster strategy.
Comparing simulation results with field test data on the dynamics of the vehicle equipped with the variable caster appears to be the most convincing option for validation purposes. However, since this research is still at a theoretical stage, the experimental data on the dynamic behaviour of the controlled caster vehicle have not been available. Therefore, the tyre kinematic model developed in Chapter 2 and the vehicle dynamics model constructed in Chapter 3 are validated.

5.1 Validation of Steering Tyre Kinematics Model

The novelty of the steering tyre kinematics developed in this investigation lies in the greater generality and accuracy of the model. Unlike other models in the literature, this current model does not work on small-angle assumptions; hence, it is more accurate and applicable to a large caster/camber used in the variable caster steering theory. Furthermore, the displacement of the tyre-to-ground contact point along the tyre perimeter is taken into account. In order to confirm the mathematical correctness of the kinematic model, a multi-body model of a steering tyre was built using ADAMS software. The camber, which is the most important kinematic parameter, extracted from the two models were compared for validation.

ADAMS is developed by the MSC Software Corporation. It is a multi-body program representing the model at component levels. This software enables us, for example, to describe kinematics of a steering tyre by linkages, joints, and motions. ADAMS has been used extensively in automotive dynamics analyses, such as Ikhsan et al. (2015), Kuwayama et al. (2008), Park and Sohn (2012), Xiu-Qin et al. (2012), and Liu et al. (2015). For fairness in comparison, the methods for determining parameters used in the ADAMS model must be the same as those for the analytical model; for instance, to include the camber sign consistent with the one in Chapter 2, the camber angle is defined through
the angle between the normal vectors of the ground plane and the tyre plane, as shown in Figure 5.1. The camber was compared for different steering axis configurations. The steering axis orientation was changed to suit the intended configurations. To show the exactness of the kinematics extracted from the novel model, the comparison was made for the steering angles from $-180^\circ$ to $180^\circ$, even though they are much more than practical steering angles. In order to do so, some physical constraints in the ADAMS model were removed, such that the wheel can rotate about the steering axis for $360^\circ$. To illustrate the generality of the model, we also compared the results with those in the literature.

\[ \gamma = \frac{\pi}{2} - \rho \]

Figure 5.1: Determining camber in the ADAMS-tyre model

Figures 5.2 to 5.6 show the camber generated when the wheel is steering about a tilted steering axis from four sources: using the homogeneous transformation method (the plus sign); extracting from the ADAMS model (the solid curve); and employing the formulas developed by Alberding et al. (2014) (the dashed curve), and Dixon (2009) (the dotted curve). The comparison was made for different orientations of the steering axis. The graph is arranged in descending order of caster and ascending order of $KPI$ angle. Note that the signs of some parameters in the two references were equivalently changed to suit the sign convention used in this thesis.

At the first glance it is clear that the camber extracted from the current tyre model is wholly consistent with that of the ADAMS model. More specifically, the analytical data obtained by using the homogeneous transformation method fits the multi-body data for all the configurations over the whole range of steering angle.
SECTION 5.1: VALIDATION OF STEERING TYRE KINEMATICS MODEL

Figure 5.2: Camber of a steered wheel for $\phi = -25^0$, $\theta = 0^0$

Figure 5.3: Camber of a steered wheel for $\phi = -20^0$, $\theta = 5^0$
CHAPTER 5: MODEL VALIDATION

Figure 5.4: Camber of a steered wheel for $\phi = -15^0$, $\theta = 10^0$

Figure 5.5: Camber of a steered wheel for $\phi = -10^0$, $\theta = 15^0$
SECTION 5.1: VALIDATION OF STEERING TYRE KINEMATICS MODEL

The four sets of data are only show good agreement for a zero \textit{KPI} configuration, as seen in Figure 5.2. When \textit{KPI} is non-zero, as shown in Figures 5.3 to 5.6, there are discrepancies between the ADAMS data and those derived from the literature. To be more specific, the difference between the ADAMS results (the solid curve) and the camber values using the formula in Alberding et al. (2014) (the dashed curve) becomes substantial when large \textit{KPI} or/and steering angles are applied. Although the data corresponding to Dixon (2009) (the dotted curve) predicts the ADAMS model much better than that from Alberding et al. (2014), it is still not completely consistent with the multi-body data, especially at large steering angles.

It is also observed that the two analytical formulas (from the literature) are still in good agreement with the multi-body model for small steering angles. This is because the two analytical models work on the small-angle assumptions. Another feature to note is that, as we reduce the longitudinal tilt angle (caster) of the steering pivot and increase the lateral tilt angle (\textit{KPI}), the steering angle range within which the agreement assured narrows, as can be seen in Figures 5.3 to 5.6 (the magnified graphs).

The comparison demonstrates that the steering tyre kinematics developed in Chapter 2 is mathematically correct. Importantly, without accepting the small-angle assumptions, this current tyre kinematics holds great generality; unlike those in the literature, this kinematics is applicable any steering angle and any steering axis orientation which may be achieved in the variable caster steering theory. Because of the absolute consistency
with the ADAMS model, the tyre kinematics developed in this thesis is validated.

5.2 Validation of Vehicle Dynamics Model

Since the study presented here is still at a theoretical stage, it does not focus on any particular car. It is also not easy to obtain both parameters and full test data from a specific car. Therefore, one good option is to utilise available data on a specific car’s dynamics to conduct the validation of the non-linear dynamics model (NLDM) developed in Chapter 3. For that reason, we chose to use the publicly available data on a sedan car named, the 1994 Ford Taurus GL (Demerly and Youcef-Toumi 2000, Salaani 1996). Moreover, a vehicle model constructed in CarSim software was used to demonstrate the similarity between the two models. CarSim is a multi-body symbolic dynamics analysis software which describes a car and its systems with hierarchical structure. The hierarchical characteristics enables every ‘child’ inherits its ‘parents’s’ degrees of freedom. This software has been used by many researchers to conduct vehicle dynamics research (Dahmani et al. 2014, Emrler et al. 2015, Hasagasioglu et al. 2012). The parameters used in this current CarSim model were chosen as close as possible to those in the NLDM. Because the simulation data is compared with the available real car data, the actual inputs including the steering wheel angle and car speed from the experiment were used. The responses of the car to three types of steering input, slowly increasing steer, J-turn, and lane change manoeuvre, were compared for the NLDM, CarSim model, and experimental data. The responses are shown in Figures 5.7 to 5.12.

**Responses to Slowly Increasing Steer**: As indicated by its name, this manoeuvre was carried out by slowly increasing the input steering angle while the vehicle speed was kept constant, until vehicle lateral acceleration limit was achieved. The test is used to assess the ability of a model to predict the steady state gain of the car for the whole range of lateral acceleration (from very low to the limit). Therefore, the hand wheel steering angle was increased at a rate of 10 deg/sec in order to eliminate dynamic effect and maintain steady state characteristics. The test was conducted at two speeds: 40 km/h and 80 km/h (Demerly and Youcef-Toumi 2000, Salaani 1996). The steady state lateral acceleration gain in response to the steering input at the two speeds, for the three sets of data, is depicted in Figures 5.7 and 5.8.
SECTION 5.2: VALIDATION OF VEHICLE DYNAMICS MODEL

Figure 5.7. Lateral acceleration gain at 40 km/h

Figure 5.8. Lateral acceleration gain at 80 km/h
CHAPTER 5: MODEL VALIDATION

Generally speaking, there is a good correlation between the simulation results and those of the experiment. More specifically, the two models predict the real vehicle response with great accuracy up to nearly $0.75g$ of lateral acceleration. Furthermore, the lateral acceleration gain, for the three cases, increases with the vehicle speed. However, there are slight differences between the three sets of data in the limit zone (around $0.8g$) with a higher lateral acceleration associated with the experiment. The difference between the $NLDM$ results and the field data may be attributed to the greater actual friction in the experiment. The discrepancy between the two simulation results in the limit zone may caused by the fact that some differences remain (such as the steering and suspension kinematics) between the two models; while the CarSim model is more complex than the $NLDM$, some sub-system parameters may not be accurately determined due to the lack of information. Also, not all the parameters in the two models are exactly the same, even through the parameters in the CarSim model were chosen to be as close as possible to those of the present model.

Responses to J-turn Steering Input: This test is employed to examine the steady state and transient responses of a vehicle. The test is conducted at a constant speed, with an approximate step steering angle being applied at the hand steering wheel. The J-turn steering input, presented in the references (Demerly and Youcef-Toumi 2000, Salaani 1996), was chosen to produce a desired level of lateral acceleration. The responses of the vehicle in terms of lateral acceleration, yaw rate, body roll angle, and body roll rate to the J-turn steering inputs at 40 and 80 km/h are depicted in Figures 5.9 and 5.10, respectively. As can be seen from the graphs, the vehicle model developed in this thesis shows a high similarity to the CarSim model and the actual vehicle data. In addition, the $NLDM$ model predicts the real vehicle behaviour better than the CarSim model. However, there is a discrepancy in the steady state values of roll response across the three data sets. This may be attributed to the fact that not all the suspension kinematic parameters in the two model are accurately simulated: for example, the roll axis in the $NLDM$ is simulated as a fixed line while it is movable for the real car. Despite the difference in the steady state value of roll response, the curves for the three cases have the same trend. The $NLDM$, therefore, predicts the vehicle response well, especially for the case of 80 km/h.
Figure 5.9. Lateral responses for J-turn manoeuvre at 40 km/h
Figure 5.10. Lateral responses for J-turn manoeuvre at 80 km/h

Responses to Lane Change Steering Input: This test is considered to be a complex manoeuvre which mimics real world driving situations. As stated earlier, the steering inputs for the *NLDM* and the CarSim model were taken from the publicly available experimental data (*Demerly and Youcef-Toumi 2000, Salaani 1996*); the lane change manoeuvre had been tested at two different constant speeds (15.5 m/s and 20 m/s); the steering angle amplitude had been chosen such that the car performs with a medium severity at 15.5 m/s and high severity (limit zone) at 20 m/s. The responses of the car to the lane change steering inputs at the two vehicle speeds are illustrated in Figures 5.11 and 5.12.
It can be observed that, at the vehicle speed of 15.5 m/s, the real car dynamic behaviour is well-predicted using the *NLDM*. The responses at the 20 m/s speed show a lower accuracy but are still reasonable. The *NLDM* is better than the one constructed in CarSim software in predicting real car responses; the lower accuracy of the CarSim model was explained earlier. It appears that the *NLDM* can predict the vehicle response up to the limit zone reasonably.
The comparison of the vehicle responses for the three steering input types, namely, slowly increasing steering, J-turn, and lane change manoeuvre, has shown that the dynamic model of the vehicle developed in Chapter 3 can reasonably predict the real vehicle responses from low levels of lateral acceleration up to the grip capacity. The model is, therefore, considered to be validated.

5.3 Summary

The validation of the tyre kinematic model and the vehicle dynamic model was presented in this chapter. There are two reasons for choosing the two models for validation. The first is that the variable caster steering theory developed in this investigation is built on these two models. The other lies in the fact that a car equipped with the variable caster system, and hence the field data on its dynamics, have not been available at this stage of research.
In the first part of the chapter, the kinematics model of the steering tyre developed in Chapter 2 was validated. In order to do that, a model of steering tyre with its tilt steering axis was built, using ADAMS software. The kinematic camber calculated using the homogeneous transformation method was compared with that extracted from the ADAMS model for validating purpose. To show the greater accuracy of the current kinematic model compared with those in the literature, the results from other investigations were also used in the comparison. The comparison has demonstrated that the current analytical data exactly fits the multi-body approach while those in the literature do not. This means the tyre kinematic model developed in this thesis is validated.

The remaining part of this chapter is to validate the vehicle dynamic model shown in Chapter 3. To do so, field test data on the dynamics of a specific car that had been made publicly available was utilised; this car had also been characterized such that its parameters had become available for use in the simulation. The responses of the car to different steering inputs for the two cases, namely, the simulation in this thesis and the real car, were compared for validation purposes. The steering inputs taken from the experimental data were also used as the inputs of the simulation. To show the correlation between different models, a vehicle model built in CarSim software was also used. The comparison has demonstrated that the vehicle dynamic model can reasonably predict the responses of the real car, which also means that it was validated.
CHAPTER 6

Conclusion

The variable caster steering theory was prompted by Reza Jazar et al. in 2012. The main idea of the theory is actively varying caster, a steering axis orientation parameter, to control the camber of the steering tyre in a beneficial way. Although the idea is interesting, the effects of variable caster steering on vehicle dynamics remains unknown. Also, a steering tyre kinematics model that is appropriate to the theory has not been available. Therefore, from 2013 this current PhD research was initiated with the aim of taking several steps towards bringing the theory into effect. Specifically, in this investigation we developed the kinematics of the steering tyre with variable caster. A variable caster strategy was then proposed to examine and hence demonstrate some positive effects of variable caster steering on vehicle dynamics.

The starting point of the theory is the kinematics of the steering tyre. This is because, as indicated by its name, variable caster steering deals with the orientations of steering tyre and steering axis. There have been various kinematics models of steering tyres available in the literature. Those models were developed based on small-angle assumptions. Furthermore, the tyre-to-ground contact point was assumed to be a fixed point on the tyre perimeter. However, these two assumptions become erroneous in the context of variable caster steering, where large caster and camber might be achieved. Therefore, in this thesis, we developed the kinematics of the steering tyre without accepting the two assumptions. In order to do that, first a number of coordinate frames sufficient to describe the orientation and location of the steering tyre was defined. Then homogeneous transformation was employed to transform coordinates between the frames. Thereby, the orientation and location of the tyre when steering were determined. Because it does not use small-angle assumptions, this model is applicable to any value of caster, kingpin inclination, and camber angles. This generality makes the developed kinematics novel feature; therefore, the new tyre kinematics is considered to be a notable contribution to the literature. Not only is the developed kinematics applicable to the variable caster in this investigation but also
it can be used to conduct any investigation that requires a more accurate tyre kinematics than those in the literature.

The tyre camber, which is the key parameter for the current theory, was then derived from the kinematics. At first, it was determined for a stationary car; then it was extended to include the case of a cornering car. This dynamic camber, which is a function of the vehicle’s roll motion, suspension geometry, caster, kingpin inclination angle, and steering angle, was then used to develop a variable caster strategy in Chapter 4. The coordinates of the tyre-to-ground contact point were also determined, which shows that the contact point moves along the tyre perimeter during steering (the spin motion of the tyre is ignored). This point was later used in Chapter 3 to determine kingpin moment for examining steering returnability during low speed cornering. Although designing the suspension with variable caster is outside the scope of this thesis, an exemplary variable caster mechanism with its kinematics was also proposed to introduce the idea of the task. The steering tyre kinematics with variable caster was presented in Chapter 2, and validated by an ADAMS model in Chapter 5. Parts of Chapter 2 have also been published as an article in *International Journal of Vehicle Design*, a chapter in the book *Nonlinear Approaches in Engineering Applications*, and a SAE technical paper, as stated at the beginning of the thesis.

A vehicle dynamics model is required to examine the dynamic behaviour of the caster-controlled car. This is the reason that vehicle modelling was covered in Chapter 3. A rollable model of vehicle was constructed. With four degrees of freedom describing longitudinal and lateral translations, yaw motion, and roll motion, this model allows the capture of the important physical phenomena that occur during a turn. One of those phenomena is the lateral load transfer, in which the normal load at the outer tyre increases, while that at the inner one decreases when the car is turning; this altered normal load affects the lateral force, which in turn determines the vehicle lateral dynamics. Another aspect that needs to be considered in this current investigation is the roll motion of the car body. This is because the roll affects the wheel camber - the key parameter in the variable caster steering theory. The correlation between the roll motion of the car body and the orientation of the wheel is described by incorporating the developed tyre kinematics into the vehicle modelling. A tyre model, known as the ‘Magic Formula’, was adopted to simulate the tyre forces and moments generated at the vehicle tyres. One of the reasons for selecting this model is that it can include the camber contribution as well as the nonlinearity of the tyre force/moment - two dominant characteristics with regard to the current analysis. The car model was then validated by field test data, as presented in Chapter 5.

The remainder of Chapter 3 presented a novel method for determining kingpin moment for low speed cornering manoeuvring. Based on the determination of the tyre-to-ground contact point carried out in Chapter 2, the moments about the steering axis caused
by tyre forces were computed. This part has also been published as an article in *SAE International Journal of Passenger Cars - Mechanical Systems*. The kingpin moment was then examined for suggesting a caster configuration that can enhance the steering returnability during low speed cornering (presented in Chapter 4).

In Chapter 4, the *variable caster steering* theory was examined with a specific variable caster strategy: varying caster to address roll camber - a phenomenon that generally reduces the tyre lateral force and hence the vehicle’s lateral grip capacity. The expected result was a more manoeuvrable car with expanded grip capacity. In order to propose the strategy for varying caster, an analysis of the camber function was carried out. The analysis shows that the roll camber can be addressed by varying the corresponding caster based on the static camber, the roll motion, the suspension geometry and the steering angle. Then the strategy for varying caster in order to counter the roll camber was proposed. The responses of the car to three steering inputs, namely, step steer, slowly increasing steer, and sinusoidal steer, were examined for the two cases: the benchmark configuration, and the variable caster. The results demonstrate that the roll camber is countered by the camber generated using the variable caster strategy. In this way the lateral acceleration and the yaw rate increase. Most importantly, the variable caster strategy provides about 5 per cent more lateral grip capacity with the caster-controlled car still being stable, which means the controlled car is more manoeuvrable. While it is acknowledged that the camber contribution is generally insignificant compared to that of the side slip, a few per cent improvement of grip capacity provided by the extra camber is of critical importance and can save lives. This is because, at the limit, the lateral force due to side slip is saturated.

The lateral dynamics of a car can be enhanced in different ways, using various active systems such as active steering, yaw moment control, or lateral tracking control, etc. While those controlled systems have shown their notable contributions to the safety, stability, and manoeuvrability of the car, the variable caster strategy for addressing the roll camber is still appreciated. This is because realising the full potential of a car, in general, as well as expanding its lateral grip capacity, in particular, are the goals that researchers/engineers have been pursuing. The benefits of the improved grip capacity (with the car being stable) are numerous. For example, they could be an enhanced collision avoidance capability, a higher performance, and a higher ability to manoeuvre under bad weather conditions. With the expanded grip capacity provided by the variable caster, the advantage of the other controlled systems can also be strengthened. Therefore, we expect that this variable caster scheme can be utilised in combination with other control strategies/systems to further enhance the safety, stability, and manoeuvrability of cars.

The advantage of variable caster was also exploited such that the steering returnability during low speed cornering is improved. To do that, an analysis of the source of steering returnability was carried out; the novel model for calculating kingpin moments
presented in Chapter 2 was applied. On the basis of the analysis, a zero caster configuration was proposed for low speed cornering. The kingpin moment, and hence returnability for low speed cornering is improved when the zero caster is utilised. The result is a higher returnability assured up to a larger steering angle. Although, as stated in Chapter 4, other aspects will need to be considered in regard to steering returnability (such as steering effort), being able to provide a caster configuration that benefits steering returnability is an advantage of the variable caster.

Chapter 4 has shown that the caster can be actively varied so as to improve the safety, stability, and manoeuvrability of the car. Parts of this chapter have also been submitted to Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, as a manuscript; and it has been accepted.

To further develop the variable caster steering theory, the work done in this thesis could be extended by:

- **Variable caster mechanism/suspension development**: This investigation did not address the design of variable caster mechanisms/suspensions. Therefore, on the basis of the steering tyre kinematics developed in this thesis, the variable caster mechanism/suspension should be developed in the future.

- **Variable caster steering and tyre wear**: As proven in Chapter 2, the tyre-to-ground contact displaces horizontally and rotationally when the tyre is steering about the tilted steering axis. The displacements and the camber change will affect tyre wear and hence service life. Therefore, we hope that the tyre wear in variable caster steering will be investigated in the future.

- **Full-car control development**: The scheme of varying caster developed in this thesis only serves as an illustration of the caster steering theory. For simplicity reason, we only used a proportional control algorithm for examining the effects of the variable caster scheme on the vehicle dynamics. This is a limitation of the analysis. We expect that better control algorithms will be developed in the future. Also, variable caster control strategies should be incorporated into other control systems to improve the safety, stability, and manoeuvrability of the car.

- **Full-car development**: Once the variable mechanisms/suspensions and appropriate/combined control strategies have been developed, they should be combined together in a car to provide a test-bed for variable caster steering development.

With several steps towards bringing variable caster steering into effect being made in this thesis, it is hoped that this research can be helpful for those who further develop the theory. We also anticipate that a real variable caster car will be developed in the near future.
Bibliography


