Studies on three-dimensional metamaterials and tubular structures with negative Poisson’s ratio

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

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Abstract

Materials and structures with negative Poisson’s ratio exhibit counter-intuitive behaviour, i.e., under uniaxial compression (tension), these materials and structures contract (expand) transversely. The materials and structures that possess this feature are also called ‘auxetics’ by Evans. The terminology of ‘auxetic’ becomes a common adjective to describe materials and structures with negative Poisson’s ratio. Many desirable properties resulting from this unusual behaviour have been reported, e.g., higher shear resistance, indentation resistance, fracture resistance, improved acoustic and higher energy absorption, synclastic behaviour and variable permeability. These superior properties offer auxetics broad potential applications, e.g., smart filters, sensors, medical devices and protective equipment. However, there are still many challenging problems which impede wider applications of auxetics. First of all, most of the studied auxetic materials are two-dimensional (2D) and very few three-dimensional (3D) auxetic materials have been designed and investigated. Secondly, the base materials of the most existing auxetic metamaterials are rubber-like materials, hence these auxetic metamaterials are limited to the elastic deformation. In contrast to elastic auxetic metamaterials, metallic auxetic metamaterials exhibit some new features in mechanical properties, e.g., localisation of plastic strain, strain hardening and irreversible deformation. Furthermore, metallic metamaterials are usually stronger than those made of elastomers, which leads to superior loading resistance and energy absorption. Last but not the least, although most of the publications mention that auxetic materials possess many desirable properties, very few auxetic materials have been designed and fabricated to the practical stage. Therefore, it is worthy of carrying out more original research to explore new applications for auxetic materials.

In order to fill the research gap mentioned above and create better auxetic materials for applications, in this study, extensive numerical and experimental investigations have been conducted. Firstly, a novel methodology was proposed to generate a 3D metallic cubic auxetic material based on a cubic buckling-induced auxetic material. It was found that the base material affected the auxetic behaviour of the buckling-induced cubic auxetic materials. When the elastomer base material was replaced with a ductile metallic material, the previously observed auxetic behaviour of the buckling-induced auxetic
material would disappear. Inspired by this unexpected behaviour, a new methodology of generating 3D metallic auxetic materials was developed. The effectiveness of the methodology was then proved experimentally and numerically. The mechanical properties of the designed 3D metallic auxetic materials could be easily tuned by one single parameter of pattern scale factor (PSF). In the second part of this study, a simple auxetic tubular structure which exhibited auxetic behaviour both in compression and tension was generated by using the newly proposed PSF methodology. This simple auxetic tubular structure could also be tuned by one single parameter of the PSF. When the scale of PSF reached a certain value (around 60% in this study), the designed tubular structure exhibited an approximately identical auxetic effect both in tension and compression. In the third part of this study, a simple 3D auxetic metamaterial was designed which demonstrated the unexpected 2D auxetic behaviour. By utilising the proposed PSF methodology, the 2D auxetic behaviour could be successfully transformed to 3D auxetic behaviour. Hence, the functionality of the proposed PSF methodology was further extended. In the last part of this study, the first auxetic nails were designed, fabricated and experimentally investigated. According to the experimental results, the designed auxetic nails could not demonstrate superior mechanical performance to non-auxetic nails. Therefore, it would prudent to re-examine the potential applications for auxetic materials because many existing publications in the field might have overestimated the superiorities of auxetic materials and their limitations and disadvantages have been rarely discussed.
Publications and rewards list

Parts of this thesis have been presented in the following publications.

Journal papers


Conference papers


**Rewards**

1. **The Best Paper Award** for the paper entitled ‘Numerical simulations of 3D metallic auxetic metamaterials in both compression and tension’ during The 2nd Australasian Conference on Computational Mechanics, Brisbane, Nov 30 to Dec 1, 2015

2. The paper entitled ‘Experiments and parametric studies on 3D metallic auxetic metamaterials with tuneable mechanical properties’ was selected as the “Highlight of the Year 2015” by the journal *Smart Materials and Structures*, 2016

Chapter 1

Introduction

Background

The concept of metamaterials (meta means “beyond” in Greek) was originally defined as novel artificial materials with unusual electromagnetic properties that are not found in naturally occurring materials [1]. These superior properties created an avenue for the research field of transformation optics [2], which has many applications ranging from cloaking [3] to subdiffraction imaging [4] and super lens [5]. Recently, the concept of metamaterials has been extended to a class of materials whose effective properties are generated not only from the bulk behaviour of the materials which produce it but also from their internal structuring [6]. Metamaterials possess many superior and unusual properties in the aspects of static modulus, density, specific energy absorption, acoustic and phononic performance, heat transport performance, improved high strain rate mechanical behaviour, smart materials and negative Poisson’s ratio.

Poisson’s ratio ($\nu$), as a fundamental metric of material, is defined as the ratio between the longitudinal expansion and lateral contraction of a material under uniaxial loading. According to the classical elastic theory, for a homogeneous solid isotropic material, the range of Poisson’s ratio is between -1 and 0.5. However, the Poisson’s ratio value for most of the materials is in the range of 0 to
0.5. For gases, \( \nu = 0 \). Glasses and minerals are less compressible, and the \( \nu \) of these materials is slightly larger than 0. For most well-known solids like metals, polymers and ceramics, the \( \nu \) is in the range of 0.25 to 0.35. For weakly compressible materials like liquids and rubbers, where stress primarily results in shape change and \( \nu \) is close to 0.5 [7]. Materials which exhibit negative Poisson’s ratio are termed “auxetics or auxetic materials” [8].

The earliest auxetic materials dated back to 1987 when Lakes [9] reported the first foam structures which exhibit negative Poisson’s ratio. After four years, materials with negative Poisson’s ratio were termed as “auxetics” or “auxetic materials” by Evans et al. [8] for the sake of simplifying the long description of “negative Poisson’s ratio”.

As a branch of the most studied mechanical metamaterials, auxetic metamaterials exhibit counter-intuitive deformation behaviour. To be more specific, under uniaxial compression (tension), conventional materials expand (contract) in the directions orthogonal to the applied load. In contrast, auxetic materials contract (expand) in the transverse direction, as shown in Figure 1-1. Numerous desirable properties resulting from this unusual behaviour have attracted an increasing number of researchers to carry out the studies of auxetics. Accompanied by the unusual deformation pattern under compression and tension, auxetics are endowed with many desirable material properties, e.g., indentation resistance, shear resistance, fracture toughness, synclastic behaviour, variable permeability, acoustic and energy absorption. These aforementioned advantages of auxetic metamaterials make them potential candidates for applications that including but not limited to textiles, aerospace, military equipment, medical and protective devices, sensors and smart filters.
Figure 1-1 Deformation patterns in tensile and compressive load: (a) non-auxetic material; (b) auxetic material. [10]

1.1 Problem statement and objectives

Although the existence of auxetic materials was predicted for more than 160 years through classical elasticity theory, until 1944, Love [11] proved the existence of iron pyrite monocrystal and an estimated Poisson’s ratio of -1/7 was given. Afterwards, the progress in the field of auxetics was rare until a breakthrough was achieved by Lakes [9] who reported the first re-entrant foam structures with negative Poisson’s ratio in 1987. Since then, in the last three decades, a considerable amount of academic outcomes were achieved to explore the world of auxetics. Wojciechowski [12] reported that Poisson’s ratio value can be negative in the tilted phase of cyclic hexamers. Analytical methods were also employed to investigate different deformation mechanisms of auxetic materials, including 2D auxetics [13-21] and 3D auxetics [18, 22-25]. With the rapid development of computational capability of modern computers and the advancement of calculation software, the finite element simulation has become an increasingly significant method to investigate the mechanical performance of auxetic materials [26-30]. Limited by the fabrication techniques, few designs of 3D auxetic material were developed to the stage capable of using in practical applications [6]. One apparent disadvantage of most existing auxetic materials was the small effective strain for auxetic behaviour, e.g., less than 0.1 in the design of Bückmann et al. [22]. This effective strain was then extended to 0.3 by Babaee et al. [31] through proposing a novel design of ‘buckliball’ as the building cell. However, the geometries of their building cells were complicated and the fabrication processes included an extra bonding interface which resulted in a considerable scattering of the mechanical properties [26]. Nevertheless,
the base material of their models was elastomeric hence they can hardly sustain a considerable force and the deformation was limited in the elastic region. In contrast to elastic auxetic metamaterials, metallic auxetic metamaterials could offer some other desirable mechanical properties, e.g., localisation of plastic strain, strain hardening and irreversible deformation. More importantly, metallic metamaterials were usually much stronger than those made of elastomers, therefore metallic metamaterials could be superior to the non-metallic ones with respect to carrying capacity and energy absorption.

The study of metallic auxetic materials dated back to 1988 when Friis et al. [32] reported the first auxetic copper foam with non-periodic building cells. Employing resonant ultrasound spectroscopy, Li et al. [33] explored the auxetic property of copper foams. He et al. [34] reported a novel mechanism for auxetic behaviour in entangled materials with a spiral wire structure. Mitschke et al. [35] fabricated a linear-elastic cellular structure from Ti-6Al-4V alloy whose linear-elastic properties were measured in tensile experiments and a negative Poisson’s ratio of -0.75 was observed. In the work of Zhang et al. [36], an auxetic structure composed of tubes and corrugated sheets was proposed. The compressive test demonstrated in their study indicated that the proposed structure had a large auxetic effect. However, the work focusing on metallic auxetic metamaterials with periodic microstructures was still in infancy. Until recently, Taylor et al. [37] and Dirrenberger et al. [38] carried out the studies on metallic structures with auxetic behaviour. Dirrenberger et al. [38] concluded that the auxetic effect remained and became even more obvious with plastic yielding using the finite element method. However, their conclusion lacked convincing experimental support.

As one branch of auxetics, auxetic tubular structure or auxetic stent have attracted a considerable attention for scientists to explore their applications as foldable devices in the medical field, e.g., angioplasty stents [39-41], annuloplasty rings [42] and oesophageal stents [43, 44]. For the patterns of all the aforementioned medical devices, the cellular configurations were predesigned. The conventional and straightforward fabrication method was to roll the 2D auxetic sheets to tubular structures. Recently, Gatt et al. [45] proposed a 3D tubular system with a typical planar 2D sheet
constructed from rotating rigid units. They mentioned that the edge effect had a significant influence on the finite-sized 3D tubular structures. However, the cellular tubes designed using this method may not exhibit auxetic behaviour under large compressive strain. In addition, most of the existing auxetic stents only demonstrated auxetic behaviour in tension. To the author’s best knowledge, no systematic research was conducted on the auxetic tubular structures under large compressive strain which was of significance when stents scaffold were inserted into the blood vessels. Recently, Grima et al. [46] proposed a random cut technique which could convert a non-auxetic conventional sheet of rubber-like material to an auxetic metamaterial with a larger negative Poisson’s ratio. Their idea may deserve a trial to generate tubular structures with negative Poisson’s ratio.

Elastic stability, as one of the most basic mechanical behaviour, has attracted a considerable amount of attention with respect to the studies of elastic instabilities of many kinds [47], including snapping, buckling, wrinkling, crumpling, phase transitions and cavitation [48]. Among these, buckling is a phenomenon that has been known for centuries since a formula of the critical buckling of a column was derived by Leonhard Euler. It had long been a common sense that buckling was generally avoided through a special design and geometrical modifications, but recently, this phenomenon was regarded as favourable for many emerging applications, e.g., absorbers [49-51], dampers [52, 53] and isolators [54-57]. Many scientists have started to transform the buckling and post-buckling which have long been regarded as harmful towards beneficial. In this thesis, the mechanisms of buckling and post-buckling are involved in the methodology of designing 3D auxetic metamaterials and tubular structures.

Auxetic materials have drawn an increasing amount of attention because of their counter-intuitive behaviour in deformation. Under uniaxial compression (tension), auxetic materials would shrink (expand) laterally. This feature has also been mentioned to be useful for the applications of auxetic nails as an apparent conclusion [58]. However, no auxetic nail has been designed and no experimental work has been conducted. Therefore, it would be an interesting work to design the first auxetic nails
and conduct the first experimental investigation on auxetic nails to extend the potential applications of auxetic materials.

In summary, the primary objectives of this research are:

- Proposing a methodology of generating 3D metallic auxetic metamaterials with tuneable mechanical properties and with the capabilities of maintaining auxetic effect in a large effective strain range.
- Utilising the proposed pattern scale factor (PSF) methodology to design 3D auxetic tubular structures with tuneable mechanical properties.
- Exploring the effect of the base material on the 3D auxetic metamaterials and tubular structures.
- Carrying out a series of extensive parametric studies on the proposed 3D auxetic metamaterials and tubular structures.
- Proposing an alternative design of 3D auxetic metamaterials with a simple geometrical configuration of unit cells.
- Exploring the potential application of auxetic nails.

1.2 Significance

For the past three decades, a considerable progress has been achieved in the field of auxetics since Lakes [9] reported a breakthrough work of the first re-entrant foam structure with negative Poisson’s ratio in 1987. However, most of the existing publications are based on the studies of 2D auxetics. Attributed to the previously poor manufacturing techniques, very few 3D auxetics have been fabricated to the stage of practical application. 3D printing technique, also known as additive manufacturing techniques, rising up recently, is regarded as a signal of the third industrial revolution, has been employed to generate some 3D auxetics. Utilising this novel technique, Bückmann et al. [22] fabricated some 3D metamaterials with negative Poisson’s ratio. However, the effective strain range of auxetic behaviour for these metamaterials was less than 0.1. This effective strain range was
successfully extended to 0.3 by Babaee et al. [31] through proposing a novel design of ‘buckliball’ as the building cell. However, the base material of their models was silicone-based rubber, which made these models very weak with respect to mechanical performance and difficult to sustain a large loading impact and force. In contrast, metallic metamaterials are usually much stronger than those made of elastomers. This feature makes the metallic-based metamaterials have more potential for the applications of protective devices. Therefore, for the sake of combining the auxeticity and the advantages of metallic-based material, it is of significance to design and investigate 3D metallic auxetic metamaterials which could maintain auxetic effect in a large effective strain range.

Apart from the 3D auxetic metamaterials, 3D auxetic tubular structures also have drawn an increasing amount of attention due to their wide applications as foldable devices in the medical field, e.g., angioplasty stents, annuloplasty rings and oesophageal stents. For the patterns of all the aforementioned devices, the cellular configurations are predesigned. The conventional and straightforward method of generating auxetic tubular structure is to roll a 2D auxetic sheet to a tube. However, the cellular tubes designed in this method may not demonstrate auxetic behaviour in a large compressive strain. Nevertheless, most of the existing auxetic stents only exhibit auxetic behaviour in tensile deformation. Therefore, it is becoming vital to design auxetic tubular structures which not only exhibit auxetic behaviour in tension but also in compression. Meanwhile, the simplicity of tuning the mechanical properties of the auxetic tubular structures is crucial according to some specific functionalities and requirements of the mechanical performances. In addition, inspired by a recent work of Grima et al. [46], it is of significance to explore the mechanical performance of tubular structures with randomly cut perforated slits.

Most of the existing auxetic metamaterials and structures exhibit auxeticity based on the rotating mechanism. It has become popular to utilise buckling mechanism for exploring the mechanical performance of 3D auxetic metamaterials and structures. Therefore, it would be a promising work to design simple 3D auxetic metamaterials through combining the rotating mechanism and buckling mechanism. Although a successful 3D buckling-induced auxetic metamaterial has been designed and
reported by Shen et al. [26], the unit cell of this model is still too complicated and the success of this design is more like a coincidence. Therefore, based on the 3D rotating and buckling mechanism, investigating the 3D auxetic metamaterials with a very simple unit cell which is composed of a solid sphere and three cuboids would be a very interesting and important work.

Although a considerable progress has been made in the field of auxetics in the past three decades both theoretically and experimentally, the real application of auxetic materials is still in its infancy. More pioneering works of utilising auxetic materials to explore practical applications are in need. Choi et al. [59] reported that an auxetic fastener could be easier to insert and harder to pull out. Later on, Grima et al. [58] stated that auxetic nails could be potential applications for auxetic materials based on the concept that auxetic nails become thinner when knocked in and become fatter when pulled out. However, no auxetic nail has been designed and no related experimental work has been reported. Therefore, it would be a significant and pioneering work to carry out the first experimental study on auxetic nails.

1.3 Organisation of the thesis

This research aims at designing and exploring the mechanical performance of the 3D auxetic metamaterials and tubular structures. Also, exploring potential applications of auxetic materials is another purpose of this study. Therefore, the first auxetic nails are designed, fabricated and experimentally investigated.

In chapter two, the first and the most significant work is presented with an extensive literature review. It includes a thorough introduction with respect to six different types of cellular auxetic materials and structures, natural auxetic materials and structures, metallic auxetic materials and structures, multi-material auxetics and composite auxetics. In addition, six primary advantages of auxetic materials and structures are comprehensively presented, including shear resistance, fracture resistance, synclastic behaviour, variable permeability and energy absorption. After that, some typically existing or
potential applications of auxetic materials and structures are also reviewed. Last but not the least, the research gaps are discussed.

In chapter three, a kind of 3D metallic auxetic metamaterial with periodic configuration is proposed. This metamaterial also possesses easily controlled mechanical properties and could maintain auxetic behaviour in large effective strain. The effect of the base material on 3D buckling-induced auxetic metamaterials is investigated. Series of extensive parametric studies on the proposed modesl are also conducted using experimentally validated finite element analysis. More importantly, a methodology of generating 3D metallic auxetic metamaterials is established, and the effectiveness of the methodology is validated both numerically and experimentally. One primary advantage of the proposed methodology is that the mechanical properties of the 3D metallic auxetic metamaterials generated using this method could be easily tuned by one single parameter of pattern scale factor (PSF).

In chapter four, the methodology proposed in chapter three is further extended to generate 3D auxetic tubular structures which could demonstrate auxetic behaviour both in compression and tension in a large effective strain range. The effect of the base material on auxetic tubular structures is investigated. The mechanical properties of the proposed tubular structure could also be tuned by one single parameter of PSF. It is possible to achieve similar auxetic performance for the proposed auxetic tubular structure both in compression and tension. Inspired by an interesting work, recently proposed by Grima et al. [46], tubular structures with perforated cuts are generated and the mechanical properties of these tubular structures are preliminarily investigated.

In chapter five, a simple 3D auxetic metamaterial is designed. The unit cell of the proposed 3D auxetic metamaterial has a simple geometrical configuration composed of only one solid sphere and three cuboids. Although the auxetic behaviour of the proposed 3D metamaterial is triggered by the buckling-induced mechanism, the anticipated 3D auxetic behaviour is degraded to 2D auxetic behaviour. Employing the latest proposed PSF methodology as introduced in chapter three and
Chapter four, the degraded 2D auxetic behaviour could be successfully changed to 3D auxetic behaviour. Also, the auxetic behaviour of the proposed 3D auxetic metamaterial could be easily tuned by adjusting the dimension of the cuboids.

In chapter six, the first auxetic nails are designed, fabricated and experimentally investigated. The experiments are conducted using pine timber and medium-density fibreboard as the testing materials. Based on the experimental results, it is found that the designed auxetic nails do not always exhibit superior push-in and pull-out performance to non-auxetic nails. Also, the small auxetic behaviour of the designed auxetic nails is further identified through finite element method. The experimental and numerical results illustrate the limitations of auxetic materials for the nail applications and some suggestions are also provided for achieving the auxetic nail applications.

In chapter seven, a brief summary is made to cover all the works introduced in this thesis. In addition, the limitation of this work is introduced and the future work is also presented.
Poisson’s ratio, denoted $\nu$ and named after Siméon Denis Poisson (1787-1840), as a measure of the Poisson’s effect, is employed to characterise a material, which is the property of materials to expand (contract) in directions perpendicular to the direction of compression (tension). Poisson [60] defined the ratio $\nu$ between transverse strain ($e_t$) and longitudinal strain ($e_l$) in the elastic loading directions as $\nu = -e_t/e_l$. For isotropic materials, $\nu$ can also be presented using bulk modulus $B$ and the shear modulus $G$, which relate to the change in size and shape respectively [61]:

$$\nu = \frac{3(B/G - 2)}{6(B/G + 2)}.$$

This formula defines numerical limits of Poisson’s ratio for bulk materials as, $-1 \leq \nu \leq 0.5$ for $0 \leq B/G < \infty$. This corresponding numerical range is illustrated in Figure 2-1, where $\nu$ is plotted as a function of $B/G$ for many materials. Starting with compact, nearly incompressible materials, e.g., liquids and rubbers, where stress mainly results in shape change and $\nu$ is close to 0.5. For most well-known bulk materials, the Poisson’s ratio is in the range of 0 - 0.5 [62], e.g., metals and polymers, $-0.25 < \nu < 0.3$. Glasses and minerals are more compressible, and for these $\nu \to 0$. For gases and cork, $\nu = 0$. Re-entrant polymer foams and some metallic crystals can exhibit $\nu < 0$.

The materials with negative Poisson’s ratio were firstly named as ‘auxetics’ which was derived from the Greek word ‘auxetikos’ by Evans et al. [8] in 1991. Since Lakes [9] fabricated and reported the
first foam structures with negative Poisson’s ratio in 1987, an increasing amount of attention was
drawn to the field of auxetics. Unlike conventional materials, auxetic materials and structures could
exhibit counter-intuitive behaviour, i.e., under uniaxial compression (tension), auxetic materials and
structures could shrink (expand) in the direction perpendicular to the loading direction as shown in
Figure 2-2.

Accompanied by uncommon deformation pattern, auxetic materials and structures are endowed with
many desirable material properties, e.g., superior shear resistance, indentation resistance, fracture
resistance, synclastic behaviour, variable permeability and better energy absorption performance. All
the aforementioned properties draw many researchers to engage into studies of auxetic materials and
structures, but how to utilise these superior properties and transfer them into the real applications is
still a challenging work.

\[ \text{Figure 2-1 Numerical range of Poisson's ratio } \nu, \text{ from } -1 \text{ to } 0.5, \text{ plotted as a function of the ratio of the bulk and shear moduli } B/G \text{ for a wide range of isotropic classes of materials. [62]} \]

\[ \text{Figure 2-2 Deformation patterns under tension: (a) a non-auxetic material; (b) an auxetic material. [63]} \]
In this chapter, different cellular models of auxetic materials and structures are firstly reviewed in section 2.1. Then, the auxetic materials and structures which exist in nature are briefly discussed in section 2.2. As a primary objective of this chapter, in section 2.3, a detailed introduction of metallic auxetic materials and structures is presented. Afterwards, multi-material auxetic materials and structures are stated in section 2.4. The properties of auxetic materials and structures are extensively illustrated. After that, the potential applications of auxetic materials and structures are thoroughly discussed in section 2.6. Last but not the least, a comprehensive summary of advantages and disadvantages of previous studies is given as the last section of this chapter of 2.7.

### 2.1 Cellular auxetic materials and structures

Compared with solid materials, cellular materials have numerous superior mechanical and thermal properties, e.g., low density, enhanced energy absorption, high acoustic isolation and damping. Apart from the aforementioned advantages, cellular auxetic materials and structures also possess another special property when compared with most conventional materials and structures, they could shrink (expand) under perpendicular loading direction in compression (tension).

The Milton-Ashaby (bulk modulus (B), shear modulus (G) and mass density (ρ)) map of the auxetic materials could be utilised to demonstrate the relationship between ordinary solids and the auxetic materials, as shown in Figure 2-3. The black ellipses and the red space represent the property space of ordinary solids and auxetic cellular materials, respectively [64]. The properties of low-density materials mainly depend on their cellular configuration and the properties of the base material. An ultra-light weight metallic lattice with a density of 10mg cm\(^3\) was fabricated and reported by Schaedler et al. in 2011 [65].

In order to illustrate properties of cellular auxetic materials and structures with different cellular architecture, in the following sections, six primary kinds of models of cellular auxetic materials and structures are comprehensively reviewed.
2.1.1 Re-entrant models

Gibson et al. [66] proposed the first traditional cellular structure in the form of re-entrant honeycombs in 1982. The typical honeycomb with 2D re-entrant hexagons is shown in Figure 2-4. The re-entrant honeycomb can be deformed in the form of hinging the diagonal ribs under an applied tension load. Ideally, the externally vertical diagonal ribs are moved outwards when the re-entrant honeycomb is stretched along the horizontal direction. However, the flexure of the diagonal ribs also occurs and cannot be avoided for most of the honeycombs with re-entrant cellular configuration. Auxetic behaviour could also be attributed to the flexure of the ribs for the re-entrant hexagonal honeycomb system [67]. Masters et al. [68] developed a theoretical model for 2D re-entrant structures based on the deformation of the honeycomb cells by flexure, stretching and hinging.
Gibson et al. [70] provided a traditional two-dimensional model to illustrate the behaviour of conventional and auxetic honeycombs and foams. The Poisson’s ratio and Young’s modulus along the loading direction are presented as below:

\[ V_{12} = \frac{\sin \theta (h/l + \sin \theta)}{\cos^2 \theta} \quad (2-1) \]

\[ E_1 = k \frac{(h/l + \sin \theta)}{b \cos^3 \theta} \quad (2-2) \]

\[ k = E_s \frac{l^3}{b} \quad (2-3) \]

Where \( h, l, b, \theta \) are defined as shown in Figure 2-5 and \( E_s \) is the intrinsic Young’s modulus of the structure.

Larsen et al. [71] conducted a pioneering work to design compliant micromechanisms and structures with negative Poisson’s ratio using a numerical topology optimisation method. The method enables the user to specify the mechanical and geometrical advantages of the compliant mechanisms. Besides, engineers can conveniently interpret the resulting topologies. One typical optimised 2D re-entrant triangular model is shown in Figure 2-6, and the auxetic performance of the model can be tuned by the length of the ribs and angles between the ribs.
Unlike the 2D re-entrant triangular model as shown in Figure 2-6, the re-entrant honeycomb structures shown in Figure 2-5 could be easily patterned into 3D structures with sufficient unit cell connections and auxetic behaviours in multiple principal directions. By extending the concept of 2D re-entrant auxetic structure, Schwerdtfeger et al. [73] designed 3D structures with a hexagonal super-lattice pattern which exhibits negative Poisson’s ratio in multiple directions. The first orthotropic 3D re-entrant honeycomb auxetic structure as shown in Figure 2-7b was reported by Evans et al. [74]. Yang et al. [75] conducted a further analytical investigation, and the 3D re-entrant structure was presented by the unit cell as shown in Figure 2-7a. It was found that the mechanical properties of the re-entrant honeycomb auxetic structure could be controlled by the characteristic strut ratio and re-entrant angle.

A variety of 3D re-entrant cellular structure, shown in Figure 2-8, was reported by Hengsbach et al [76]. A promising approach for the development of auxetic metamaterials and devices using direct laser printing was presented in this study.
Wang et al. [77] developed a cylindrical auxetic structure using a 3D re-entrant triangle as shown in Figure 2-9. The effective Young’s modulus $E^*$ and the effective Poisson’s ratio $\nu^*$ along the vertical direction were calculated as below:

$$ E^* = \frac{\sigma}{\varepsilon_y} = \frac{HKE\alpha^3}{2K^2\beta\sin^2\phi + (K + K^2)\sin^3\phi} \quad (2-4) $$

$$ \nu^* = -\frac{\varepsilon_x}{\varepsilon_y} = \frac{\left(\frac{H}{KL}\right)\sin\phi\cos\phi + \left(\frac{\beta H}{L} - \frac{\beta H^2}{L}\right)\cos\phi - \frac{H^2}{L}\sin\phi - \frac{\beta L^2}{L}}{\left(\frac{L + \frac{L}{K}}{K}\right)\sin^3\phi + \left(3\beta L + \frac{\beta L}{K}\right)\sin^2\phi + 2\beta^2 L\sin\phi} \quad (2-5) $$

In these two formulas, most of the variables have been illustrated in Figure 2-9a. Besides, K and $\beta$ are length ratios, where $K = M/L$ and $\beta = N/L$; $\alpha$ is the wall thickness to length ratio and $\alpha = T_L/L = T_M/M$.

Another significant work regarding the cellular materials and structures with star-shape units was made by Grima et al. [78]. In this study, a technique based on force-field based methods (the EMUDA technique) was employed to explore the mechanical performance of the star-shape systems where the stars have rational symmetry of order 3, 4 or 6 as shown in Figure 2-10. This work is of importance mainly based on the following two points: Firstly, this study provided the convincing evidence that star-shape systems have a potential for auxetic behaviour and the magnitudes of the Poisson’s ratio
could be tuned by the stiffness of the hinges and the rod elements of the structure; Secondly, this work also demonstrated that the behaviour of periodic structures under applied loads could be easily investigated using the EMUDA technique, and in particular to distinguish auxetic and non-auxetic systems.

![Figure 2-9 3D re-entrant triangular models](image)

**Figure 2-9** 3D re-entrant triangular models: (a) the mechanical analytic model of a 2D re-entrant cell; (b) A layer of the cylindrical auxetic structure; (c) Cylindrical auxetic structure. [77]

![Figure 2-10 Various star-shape systems](image)

**Figure 2-10** Various star-shape systems with different rotational symmetry of order: (a) auxetic honeycomb; (b) Star-3 system; (c) Star-4 system; (d) Star-6 system. [78]

### 2.1.2 Rotating polygonal models

Grima et al. [79] proposed a novel mechanism to achieve a negative Poisson’s ratio based on an arrangement with rigid squares connected together at their vertices by hinges, and the unit cell of this mechanism is shown in Figure 2-11. This geometry could also be regarded as a projection of a specific plane of a three-dimensional structure or a two-dimensional arrangement of squares.
When two assumptions are made, i.e., the squares are assumed to be non-deformable along loading directions; the rotating square model is unable to shear. Hence, the Poisson’s functions, Young’s modulus and compliance matrix of the model could be presented using three formulas respectively as below:

\[
\nu_{12} = \nu_{21} = -1 \tag{2-6}
\]

\[
E_1 = E_2 = K_h \frac{8}{l^2} \cdot \frac{1}{[1 - \sin(\theta)]} \tag{2-7}
\]

\[
S = \begin{pmatrix}
S_{11} & S_{12} & 0 \\
S_{21} & S_{22} & 0 \\
0 & 0 & 0
\end{pmatrix} = \frac{1}{E} \begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix} \tag{2-8}
\]

Where \( K_h \) is the stiffness constant of the hinges, \( l \) is the length of the square and the \( \theta \) is the angle between the squares as shown in Figure 2-11, and \( S \) is the compliance matrix.
Apart from the rotating square model, Grima [80] also conducted a theoretical analysis, in which he concluded that the ‘rotating triangles’ mechanism can be a very effective way of introducing negative Poisson’s ratios in real materials. The formulas of Poisson’s ratio, Young’s modulus and the compliance matrix for the rotating triangle model shown in Figure 2-12 are presented as below:

\[ v_{12} = v_{12}^{-1} = -1 \]  \hspace{1cm} (2-9)

\[ E_1 = E_2 = K_h \frac{4\sqrt{3}}{l^2 \left[ 1 + \cos \left( \frac{\pi}{3} + \theta \right) \right]} \]  \hspace{1cm} (2-10)

\[ S = \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  \hspace{1cm} (2-11)

Where \( K_h \) is the stiffness constant of the hinges, \( l \) is the length of sides of the triangle.
The system composed of hinged 'rotating rectangles' of size \((a \times b)\) with a rectangular unit cell of dimensions \((X_1 \times X_2)\). \([81]\)

Considering the squares are special rectangles, Grima et al \([81]\) proposed a more general auxetic model of rotating rectangles as shown in Figure 2-13. Hence, the formulas of Poisson’s functions, Young’s modulus and compliance matrix of the rotating square model shown in Formulas 2-6 to 2-8 should be modified using three formulas respectively as below:

\[
V_{21} = (v_{12})^{-1} = \frac{a^2 \sin^2\left(\frac{\theta}{2}\right) - b^2 \cos^2\left(\frac{\theta}{2}\right)}{a^2 \cos^2\left(\frac{\theta}{2}\right) - b^2 \sin^2\left(\frac{\theta}{2}\right)}
\]

\[
E_1 = 8K_h \left(\frac{\left[a \cos\left(\frac{\theta}{2}\right) + b \sin\left(\frac{\theta}{2}\right)\right]}{a \sin\left(\frac{\theta}{2}\right) + b \cos\left(\frac{\theta}{2}\right)}\right)^2
\]

\[
E_2 = 8K_h \left(\frac{\left[a \cos\left(\frac{\theta}{2}\right) + b \sin\left(\frac{\theta}{2}\right)\right]}{a \sin\left(\frac{\theta}{2}\right) + b \cos\left(\frac{\theta}{2}\right)}\right)^2
\]

\[
\left[\begin{array}{cc}
\cos\left(\frac{\theta}{2}\right) & b \\
-a \sin\left(\frac{\theta}{2}\right) + b \cos\left(\frac{\theta}{2}\right) & a \sin\left(\frac{\theta}{2}\right) + b \cos\left(\frac{\theta}{2}\right)
\end{array}\right]
\]

\[
\left[\begin{array}{cc}
a \cos\left(\frac{\theta}{2}\right) + b \sin\left(\frac{\theta}{2}\right) & \left[a \cos\left(\frac{\theta}{2}\right) + b \sin\left(\frac{\theta}{2}\right)\right] \\
\left[a \cos\left(\frac{\theta}{2}\right) + b \sin\left(\frac{\theta}{2}\right)\right] & \left[a \cos\left(\frac{\theta}{2}\right) - b \sin\left(\frac{\theta}{2}\right)\right]
\end{array}\right]
\]
Where \( K_h \) is the stiffness constant of the hinges, \( a \) and \( b \) are the two sides of the rectangles.

Another subsequent work in terms of rotating polymeric models was reported by Grima et al. [82]. In this study, the rotating systems were constructed from either connected rhombi or connected parallelograms. Various rotating variants were generated and investigated as shown in Figure 2-14, the Poisson’s ratio of these systems can be positive or negative, is anisotropic and depend on the configuration of the parallelograms (rhombi) and the degree of openness of the system.

\[
S = \begin{pmatrix}
S_{11} & S_{12} & 0 \\
S_{21} & S_{22} & 0 \\
0 & 0 & 0
\end{pmatrix} = \frac{1}{E} \begin{pmatrix}
\frac{1}{E_1} & -\frac{\nu_{21}}{E_1} & 0 \\
\frac{\nu_{12}}{E_2} & \frac{1}{E_2} & 0 \\
0 & 0 & 0
\end{pmatrix}
\] (2-14)

\( E \) and \( \nu \) are the Young’s modulus and Poisson’s ratio of the materials used in the rhombi or parallelograms, respectively. For rhombi \( a = b \), whereas for parallelograms \( a \neq b \).

Figure 2-14 The rotating rhombi and rotating parallelograms systems: (a) rotating rhombi of Type \( \alpha \); (b–c) rotating parallelograms of Type I \( \alpha \) and Type II \( \alpha \) respectively; (d) rotating rhombi of Type \( \beta \); (e–f) rotating parallelograms of Type I \( \beta \) and Type II \( \beta \) respectively. [82]

Alderson et al. [83] explored the rotation and dilation deformation mechanisms for auxetic behaviour in the \( \alpha \)-cristobalite tetrahedral framework structure as shown in Figure 2-15. Three types of deformation mechanisms are assumed and analysed in this study. For the first one, the tetrahedrons are assumed to be rigid and free to rotate, and the auxetic response is caused by rotation (RTM). For
the second one, the tetrahedrons are assumed to maintain the shape and orientation but free to change size, and the auxetic response is caused by tetrahedral dilating (DTM). For the last one, tetrahedral rotation and dilation are assumed to act concurrently (CTM), the auxetic response is caused by both of RTM and DTM to act in a concurrent manner.

![The rotating tetrahedron model](image)

Figure 2-15 The rotating tetrahedral model. [83]

A new model system of ‘semi-rigid’ squares was also proposed by Grima et al. [84] to further extend the previous work of rigid rotating models. In this work, a simple modification of the idealised ‘rotating rigid squares’ model was made to allow the squares to deform by giving an additional degree of freedom. This small modification leads to significant changes in the mechanical properties of the proposed model, which makes the model more suitable for presenting the Poisson’s ratio of many real materials, e.g., zeolite and SiO₂.

Inspired by ancient geometric motifs, Rafsanjani et al. [85] proposed bistable auxetic mechanical metamaterials which exhibit auxeticity and structural bistability simultaneously. One typical model of their work is shown in Figure 2-16.
Chiral models are another kind of widely investigated cellular auxetic materials, and the word ‘chiral’ originally means a molecule that is non-superimposable on its mirror image. However, this term is often used to present a physical property of spinning. The basic chiral units are formed by connecting straight ligaments (ribs) to the central nodes as shown in Figure 2-17. Lakes [86] reported the first chiral hexagonal microstructure which exhibits auxetic behaviour. Prall et al. [87] conducted a theoretical and experimental investigation on a two-dimensional chiral honeycomb, and the result indicated that the in-plane Poisson’s ratio of the model was -1.

Grima et al. [88] analysed a novel class of structures (named as ‘meta-chiral’) which belongs to the class of auxetics constructed using chiral building blocks. The meta-chiral is also regarded as an intermediate structure between the ‘chiral’ and ‘anti-chiral’. Some examples of meta-chiral systems are shown in Figure 2-18. It should be noted that for all the systems, the ligaments are always attached.
tangentially to the nodes in a way that they protrude out from the circles in the same direction to form the ‘chiral’ sub-units but the ligaments are not attached to the rods in a rotationally symmetric manner where the order is equal to the number of rods.

![Figure 2-18 Meta-chiral systems with different number of ribs attached to each node: (a) six ribs; (b) four ribs; (c) three ribs. [88]](image)

Another important work regarding the auxetic chiral models was conducted by Gatt et al. [89], where the on-axis mechanical properties of the general forms of the flexing anti-tetrachiral system were investigated through both analytical and finite element methods. The results indicate that the geometry and material properties of the constituent materials have a significant impact on the mechanical properties of the flexing anti-tetrachiral system. To be more specific, the Poisson’s ratio of the general flexing anti-tetrachiral depends on the ratio of the ligament lengths and the thickness. Also, Gatt et al. [89] concluded that the rigidity of the anti-tetrachiral system can be adjusted without affecting the Poisson’s ratio in a form of changing the relative stiffness of the ligaments.

After that, Gatt et al. [90] explored the mode of the connection between nodes and ligaments in the anti-tetrachiral structure using finite element method. The amount of glueing material used to attach the ligaments to the node has little effect on the Poisson’s ratio but has a huge influence on the stiffness of the structure. Besides, the stiffness of the glue is proved to have a significant effect on the mode of deformation of the chiral system.

Rossiter et al. [91] presented a novel shape memory auxetic deployable chiral structure which could deform without external actuation mechanism. The contraction of chiral structures is shown in Figure
2-19. When the structure was heated the shape memory alloy beams converted from the programmed curled shape (as shown in Figure 2-19b) to straight beams (as shown in Figure 2-19a), resulting in a large overall expansion. Figures 2-19c and 2-19d present the maximum expansion and fully compressed state when the thickness of the connecting beam is regarded as negligible. The linear expansion ratio here is \( \frac{d}{2r} \) or \( \frac{\left(l^2 - 4r^2\right)^{1/2}}{2r} \), where \( l \) is the length of the connecting beam.

![Figure 2-19 Contraction of chiral structures: (a) expanded triangular element; (b) compressed triangular element; (c) a single structural element at maximum extension; (d) a single structural element at maximum contraction. [91]](image)

Mizzi et al. [92] carried on a pioneering study on the influence of translational disorder on the mechanical properties of hexachiral honeycomb systems through a finite element approach. The type of disorder was found to have minimal effect on the Poisson’s ratios of these systems when the ligament length to thickness ratio is large enough and the overall length to width ratio of the disordered system is same as that of its ordered counterpart.

Ha et al. [93] proposed 3D chiral lattices with tuneable Poisson’s ratio as shown in Figure 2-20. These chiral lattices were developed with a lot of cubical nodules and finite element analysis was employed in this study. The 3D chiral lattices exhibit stretch-twist coupling that increases with relative slenderness of ribs. The Poisson’s ratio of the chiral 3D lattices could be negative to zero and this value depends on the specific geometry.
Recently, Huang et al. [94] proposed a design of 3D chiral structure which exhibits auxetic behaviour. The experiment was conducted with good agreements with finite element analysis both in deformation patterns and Poisson’s ratio, as shown in Figure 2-21.

2.1.4 Crumpled sheets models

Alderson et al. [95] proposed a novel procedure for manufacturing thin auxetic flat sheet and curved foams by uniaxial compression. Auxetic behaviour is found as a consequence of a crumpled through the thickness microstructure using the detailed optical microscopy and Poisson’s ratio measurements.

Zhang et al. [96] fabricated and studied an auxetic structure made of tubes and corrugated sheets using a conventional method. The structural parameters and the deformation of the auxetic structure are shown in Figure 2-22. The equations for calculating Poisson’s ratio $\nu$ of the structure can be obtained as shown in formulas 2-15 based on the following assumptions: the auxetic structure is a perfectly
periodic structure; the shape of each corrugated sheet is formed by connecting straight ligaments with circular arcs; the straight ligaments of corrugated sheets are always kept straight under loading conditions; the effect of elastic deformation of the structure is not considered; the thickness of corrugated sheets h is ignorable; tubes are firmly connected with corrugated sheets so that no slippage takes place between pipes and corrugated sheets. In this study, the authors conclude that the auxetic behaviour of the proposed structure depends on its geometrical parameters. The auxetic behaviour could be increased when the crimped effect of the corrugated sheets is decreased, and the stability of the structure at the initial deformation is also reduced.

![Auxetic structure with corrugated sheets.](image)

\[
v = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\sin \theta_0 (b \cos \theta - 1) - \sin \theta_0 (b \cos \theta_0 - 1) - \sin \theta_0 (\theta - \theta_0)}{\sin \theta_0 (b \sin \theta - a) + \cos \theta (b \cos \theta_0 - 1) - \sin \theta_0 (\theta - \theta_0)}
\]

(2-15)

\[
\theta_0 = \arcsin \frac{ab - \sqrt{1 + a^2 - b^2}}{1 + a^2}
\]

(2-16)

where a and b are no-dimensional parameters defined as

\[
a = \frac{W_{2v}}{W_{ls}}, \quad b = \frac{D}{W_{ls}}
\]

(2-17)
By mixing single-walled and multi-walled nanotubes, Hall et al. [97] found that the in-plane Poisson’s ratio of carbon nanotube sheets (buckypaper) can be tuned from positive to negative with the consequence of a substantial increase in the density-normalised sheet toughness, strength and modulus.

Scarpa et al. [98] conducted a study on the effective elastic mechanical properties of single layer graphene sheets using analytical and numerical methods. They concluded that the shear loading seems to imply an equivalent auxetic behaviour for the bonds, with a significant negative Poisson’s ratio value when the bond material is regarded as an equivalent isotropic material. Grima et al. [99] conducted extensive molecular dynamics simulations which demonstrated that the conformation of graphene can be modified through the introduction of defects so as to make the modified graphene enable to exhibit a negative Poisson’s ratio. Typical images of a crumpled sheet of paper and a graphene sheet, shown in Figure 2-23, have illustrated the auxetic effect of a crumpled sheet conformation. Tan et al. [100] proposed a new kind of non-porous smooth curve sheet which demonstrates a negative Poisson’s ratio and a low-stress concentration factor.

The Japanese art of kirigami has inspired many researchers to carry out the studies of auxetic materials and structures. Eidini [101] created a kind of one-degree of freedom cellular mechanical metamaterials. The unit cell of the patterns made of two zigzag strips surrounding a hole with a parallelogram shape. The authors concluded that the dislocating zigzag strips of the Miura-ori along
the joining ridges preserve and/or tune the properties of the Miura-ori. Liu et al. [102] carried out extensive finite element analysis and experimental studies on the Miura-ori patterned sheet, particularly its deformation under three types of tests: out-of-plane compression, three-point-bending and in-plane compression. Although it is widely known that auxetic responses in paper structures are related to the cellulose fibre network structure in the sheet, the effects of materials and processing variables on auxetic behaviour still need to explore. Verma et al. [103] proposed a mathematical model aiming to explain auxetic behaviour in an idealised arrangement of fibres. Javid et al. [104] proposed a novel class of non-porous auxetic materials with periodically-arranged dimples. It was found that the auxetic behaviour resulted from a novel mechanism whereby the out-of-plane deformation of the spherical dimples was exploited. Interestingly, under tension, it was found that the sheet with dimples dented on one side could not demonstrate non-auxetic behaviour but the sheet with dimples dented on both sides could, as shown in the Figures 2-24a and 2-24b, respectively.

![Figure 2-24 The typical deformation pattern of two sheets: (a) sheet with dimples dented on one side; (b) sheet with dimples dented on both sides. [104]](image)

Bouaziz et al. [105] fabricated crumpled aluminium thin foils as shown in Figure 2-25, and an experimental investigation was conducted. Comparing with the compressive result of other cellular materials, the crumpled materials exhibit a hybrid mechanical behaviour, between foams and entangled fibrous materials. These materials exhibit a remarkable plasticity and a low hysteresis similar to conventional foams but have no plateau beyond the yield stress.
Duncan et al. [106] performed an extensive study on the fabrication, characterisation and modelling of uniform and gradient auxetic foam sheets. Several sheets were fabricated with uniform triaxial compression, with and without through-thickness pins, and also with different compression regimes in opposing quadrants. The quadrant of graded foam exhibited positive and negative Poisson’s ratios in tension and compression, respectively, accompanied by high and low in-plane tangent modulus.

### 2.1.5 Perforated sheets models

Perforated sheets models, as a novel kind of cellular auxetic materials and structures, have attracted considerable attention in the last several years. Grima et al. [107] reported the first conventional materials containing diamond or star shaped perforations which could exhibit auxetic behaviour both in tension and compression, and this mechanism could also be regarded as an extension of rotating rigid units. Some typical examples of perforated sheets are shown in Figure 2-26. The Poisson’s ratio function can be represented as below:

\[
\nu_{xy} = \left(\nu_{yx}\right)^{-1} = \frac{a^2 \cos^2 \left(\frac{\theta}{2}\right)-b^2 \sin^2 \left(\frac{\theta}{2}\right)}{a^2 \sin^2 \left(\frac{\theta}{2}\right)-b^2 \cos^2 \left(\frac{\theta}{2}\right)}
\]

(2-18)

Where a and b are the sides of the rectangles, and \(\theta\) represents the angle between the rectangles.
Examples of perforated sheets which may have negative Poisson’s ratio: (a) the angle between the direction of diamond-shaped inclusions and the sides of the square is 0°; (b) the angle between the direction of diamond-shaped inclusions and the corresponding sides of the square is 45°; (c) with two different sizes diamond-shaped inclusions; (d) with star-shaped inclusions. [107]

Through the finite element and analytical methods, Grima et al. [108] also reported the process of fabricating star or triangular shaped perforated sheets to exhibit Poisson’s ratio. Mizzi et al. [109] proposed a novel approach to design auxetic metamaterials using the patterned slit perforations. The maximum negative value of Poisson’s ratio of the proposed auxetic metamaterial can reach -13 as shown in Figure 2-27.

A typical auxetic perforated sheet: (a) Diagram of a typical auxetic perforated sheet at different degrees of strain in the y-direction; (b) Poisson’s ratio against engineering strain of the same sheet. [109]

Another significant study was reported by Grima et al. [46], where they proposed a novel class of perforated systems containing quasi-random cuts demonstrating the auxetic behaviour. Through finite element simulation and experiments, as shown in Figure 2-28, the authors concluded that despite the disorder and randomness in the orientation, the proposed perforated system still maintains auxetic properties. More importantly, this study indicated that a high degree of symmetry is not necessary for the system to exhibit auxetic behaviour, which tremendously released the design freedoms for
generating auxetic materials. Recently, Carta et al. [110] proposed a porous material with isotropic negative Poisson’s ratio, which was validated using experimental tests and numerical simulations. The planar auxetic and isotropic behaviour of the considered porous medium was observed on three specimens, characterised by a 45° rotation of the pores disposition relative to each other and loaded in the same direction. The parametric study indicated that the Poisson’s ratio value is highly influenced by the relative orientation of the pores.

![Images from finite element analysis and experiment: (a) numerical model with alternative perpendicular perforated slits; (b) numerical model with maximal rotating angle of perforated slits equal ±30°; (c) numerical model with the maximal rotating angle of perforated slits equals ±5°; (d) the experiment with maximal rotating angle of perforated slits equals ±30°. [46]](image)

2.1.6 Other models

Apart from the five types of cellular auxetic models which have been reviewed in the above, there are several auxetic models which are difficult to classify appropriately based on their geometrical varieties. Alderson et al. [111] proposed a nodule-fibril model to explain the auxetic microporous polymers employing concurrent fibril hinging and stretching deformation mechanisms. Smith et al. [112] proposed a novel mechanism of missing rib model, as shown in Figure 2-29a. Through comparing the conventional auxetic model and experimental data, the authors found that the missing rib model is superior in predicting the Poisson’s function and slightly better at predicting the stress-
strain behaviour of the experimental data. Grima et al. [113] proposed a simple analytical model based on a simplified and idealised system having the ‘egg rack’ geometry, as shown in Figure 2-29b, to explain the behaviour of this system when in plane force is loaded.

Gaspar et al. [114] proposed a new three-dimensional tethered-nodule model which could provide a better understanding towards the underlying principles of many three-dimensional auxetic models, as shown in Figure 2-30a. The auxetic behaviour of the model is mainly caused by the bending of beams in re-entrant angles which impose a limitation on the number of planes in which auxetic behaviour can be formed. Dirrenberger et al. [115] proposed a hexatruss model which has good results regarding indentation strength, particularly in the scenario when volume fraction or density is the key parameter for designing a structure.

Pasternak et al. [116] proposed two auxetic models of ball link model and thin shell model, as shown in Figure 2-31 which have a theoretical negative Poisson’s ratio of -1. However, the experimental
tests indicated that the ball link model was found to demonstrate a Poisson’s ratio close to -1, while the thin shell model only exhibited a Poisson’s ratio of -0.27. Recently, Rodney et al. [117] reported that a simple three-dimensional architected material made of a self-entangled single long coiled wire exhibited a considerable and reversible dilatancy in both tension and compression.

Figure 2-31 Auxetic models: (a) ball link model [116]; (b) thin shell model [116]; (c) entangled wire model [117].

2.2 Natural auxetic materials and structures

2.2.1 Molecular auxetics

Many scientists expressed a sceptical attitude towards the concept of auxetic behaviour at the beginning, although the existence of auxetic materials was proposed for more than 160 years through classical elasticity theory [118]. Natural materials are found to exhibit auxetic properties. The case of iron pyrite monocrystals was reported using experiments on the twisting and bending of mineral rods [119]. Until 1944, Love [11] proved the existence of iron pyrite monocrystal and an estimated Poisson’s ratio of -1/7 was given. Although the negative Poisson’s ratio is theoretically possible, this property is generally believed to be rare in crystalline solids [120]. In contrast to this belief, Baughman et al. [121] concluded that negative Poisson’s ratio is a common feature of many cubic metals, and 69% of the cubic elemental metals have a negative Poisson’s ratio when stretched along the [110] direction. Through investigating the elastic behaviour of α-cristobalite and other forms of silica with first-principles and classical interatomic potentials, Keskar et al. [120] conclude that α-quartz, the most common form of crystalline silica, also exhibit a negative Poisson’s ratio under large uniaxial tension.
By utilising the force-field-based molecular simulations, Grima et al. [122] reported that some idealised zeolitic cage structures possess negative Poisson’s ratios. The combination of framework geometry and simple deformation mechanisms can explain the auxetic behaviour for most idealised molecular structures. By “off-axis analysis” of experimental and simulated elastic constants for natrolite, Williams et al. [123] reported that the Poisson’s ratio $\nu_{xy}$ and $\nu_{yx}$ could be negative if the zeolite material were subjected to stress at $45^\circ$ along either the x or y crystallographic axes. The first direct experimental evidence for on-axis auxetic behaviour in a synthetic zeolite structure was reported by Sanchez-Valle et al. [124] through measuring the single-crystal elastic properties of MFI-silicalite using Brillouin scattering. Kimizuka et al. [125] carried on a molecular-dynamics study for investigating the mechanisms for the negative Poisson’s ratios over the $\alpha$-$\beta$ transition of cristobalite (SiO$_2$). It is found that the mechanisms differ between the $\alpha$ and $\beta$ phases. In the cubic $\beta$ phase, among the adiabatic elastic constants ($C_{ij}$) of SiO$_2$, $C_{44}$ has a value close to $C_{11}$ rather than $C_{12}$ which is in contrast to the Cauchy relation. Recently, through first-principle calculations, Kou et al. [126] reported that under a tensile strain in the armchair direction, a synthesised atomically thin boron sheet (i.e., borophene) exhibits an unexpected negative Poisson’s ratio results from its special triangle hinge structure and the related hinge dihedral angle variation.

### 2.2.2 Auxetic biomaterials

Although it is very difficult to obtain the accurate elastic properties of naturally occurring auxetic biomaterials, some classical examples have still been reported, e.g., Williams et al. [127] observed that a cancellous bone from the proximal tibial epiphysis exhibits a negative Poisson’s ratio. Veronda et al. [128] examined the mechanical characterization of cat skin and found that the cat skin is auxetic under a finite deformation, Lees et al. [129] conducted experiments on cow teat skin in uniaxial and biaxial strain and found that the cow teat skin can present a negative Poisson’s ratio at low strains. All the aforementioned auxetic effects are believed to result from the fibrillar structures at microstructure level.
Apart from the auxetic behaviour found in the macroscopic biomaterials mentioned above, auxetic behaviour also found at the microscopic scale of cells. Baughman [130] found that the membranes found in the cytoskeleton of red blood cells demonstrate a negative Poisson’s ratio. Wang [131] reported that the nuclei of embryonic stem cells (ESCs) extracted from mouse were found to be auxetic during the transitioning towards differentiation, as shown in Fig 2-32. However, the mechanism which drives this auxetic phenotype was not presented. Pagliara et al. [132] reported a similar finding that the nuclei of ESCs are auxetic. They also concluded that the auxetic phenotype of transition ESC nuclei is driven at least partly by global chromatin decondensation. Auxeticity could be a significant element in mechanotransduction through the regulation of molecular transformation in the differentiating nucleus by external forces. In a recent work of Yan et al. [133], through fabricating auxetic polyurethane scaffolds with various elastic modulus, Poisson’s ratio, microstructure and estimated neural differentiation of pluripotent stem cells, the authors concluded that the microstructure and Poisson’s ratio of auxetic scaffolds may enhance neural differentiation.

2.3 Metallic auxetic materials and structures

The base materials of the majority of the existing literature related to auxetic material and structures are polymeric materials, which tremendously restricts the application for auxetics where enhanced mechanical properties that can be provided by metallic materials, such as strength and stiffness. The pioneering work on metallic auxetics was a non-periodic copper foam with a negative Poisson’s ratio reported by Friis et al. [32] in 1988. Li et al. [33] extended this work using resonant ultrasound spectroscopy to explore the properties of copper foams with negative Poisson’s ratio, as shown in
Figure 2-33. The minimal Poisson’s ratio of around -0.7 was observed for the tested sample with a permanent compression strain in this study.

![Microstructures (optical, reflected light) of copper foam with different volumetric compression ratio: (a) 1; (b) 4.34; (c) 4.94. [33]](image)

The auxetic behaviour of crystalline solids was believed to be rare, however, Baughman et al. [121] stated that 69% of the cubic elemental metals exhibit a negative Poisson’s ratio when stretched along the [110] direction. Zhang et al. [134] explored the auxetic properties of iron-gallium and iron-aluminium alloys using both theoretical and experimental approaches. The good agreement between the experiment and theory indicated the validity and effectiveness of using the density functional calculations for determining auxetic properties. The negative Poisson’s ratios resulted from elastic anisotropy. In addition, Fe$_{75}$Ge$_{25}$ was predicted to have a significant negative Poisson’s ratio value as low as -0.9 through the proposed theoretical approach. Schwerdtfeger et al. [135] conducted a thorough investigation of the mechanical properties of a non-stochastic cellular auxetic structure. The base material of Ti-6Al-4V was employed to build experimental samples using the selective electron beam melting method. It was found that the Poisson’s ratio of the structure strongly depends on the relative density of the structure. Using selective electron beam melting, Mitschke et al. [35] fabricated a linear-elastic cellular structure from Ti-6Al-4V alloy whose linear-elastic properties were measured by tensile tests and a negative Poisson’s ratio of -0.75 was observed. Zhang et al. [36] proposed an auxetic structure made of tubes and corrugated sheets, and a real structure was fabricated using the aluminium material. The compression test exhibited that the proposed structure has an auxetic effect and a wide range of application due to its easy fabrication. Dirrenberger et al. [38] conducted full-
field simulations and concluded that the auxetic effect persists and becomes even stronger with plastic yielding. Besides, it was also found that the effect of plasticity on auxeticity reduces with the expansion of the plastic zone. Recently, Taylor et al. [37] demonstrated that low porosity metallic periodic structures exhibited a negative Poisson’s ratio. The experiments were performed on aluminium cellular plates, manufactured using the CNC machine as shown in Figure 2-34.

![Figure 2-34 Samples composed of different holes in the undeformed configuration: (a) with circular holes; (b) with elliptical holes. [37]](image)

### 2.4 Multi-material auxetics and auxetic composites

Although most of the existing literature in terms of auxetics is focused on one single base material, it is promising to investigate multi-material auxetics and auxetic composites which enable us to combine the desirable auxeticity with preferred properties which are not possessed by a single base material. Some of the works on multi-material auxetics and auxetic composites are discussed in this section.

#### 2.4.1 Multi-material auxetics

Hiller et al. [136] conducted a pioneering work to answer how the auxetic effect would be if multiple materials are used to generate auxetic materials and structures. In this work, using the digital material, the authors illustrated via simulation a dense auxetic material made of aluminium and acrylic voxels, as shown in Figure 2-35a, demonstrating a special microstructure of aluminium within acrylic. Theoretically, a negative Poisson’s ratio can be obtained by combining any stiff and flexible materials. A digital material made of $6 \times 8 \times 2$ voxel base unit consisting of 68% aluminium, 48% acrylic and 8% voids was observed to produce a minimal Poisson’s ratio of -0.63 as shown in Figure 2-35b. The
results also indicated that the Poisson’s ratio of the random pattern of voxels was positive. Using the struts of three different types of crystal structures (FCC, BCC and simple cubic configurations) with different elastic moduli, Hughes et al. [23] generated a periodic truss structure with tunable auxetic properties. They also concluded that it is possible to design an auxetic truss structure with a specific Poisson’s ratio, shear modulus and tensile modulus. Employing the computer-aided design and dual-material 3D printing techniques, Wang et al. [137] designed and fabricated some dual–material auxetic metamaterials which are remarkably different compared to the of traditional single-material based auxetic metamaterials. In this work, the effects of two novel design parameters introduced by the dual-material nature, e.g., the material selections and fraction of the stiff region were explored both computationally and experimentally. With the introduction of another material, researchers could adjust the mechanical properties of bulk metamaterials without changing the overall geometry of an auxetic unit cell. In the work of Vogiatzis et al. [138], topology optimization method was employed to generate auxetics with multi-material, and both numerical simulations and physical experiments of their study proved that the achieved design demonstrated a desirable auxetic behaviour.

![Figure 2-35 Images of a digital material with negative Poisson’s ratio: (a) a digital auxetic material; (b) the resulting Poisson’s ratio as a function of strain for material with random and auxetic voxel structure. [136]](image)

### 2.4.2 Composite auxetics

The main purpose of research on composite auxetics is to combine the advantages of composites and auxetics to expand the potential applications of auxetics. The existing researches on composite auxetics are based on the four aspects [72]: 1) generating negative Poisson’s ratio through sequential
piling of angle ply-reinforced laminates; 2) producing auxetic composites by introducing auxetic inclusions or using the auxetic matrix; 3) exploring and evaluating the properties of auxetic composites; 4) manufacturing of auxetic composites.

As a pioneer, Herakovich [139] reported an early work on composite laminates with negative through-the-thickness Poisson’s ratios in 1984. In this study, he also concluded that the laminate dilatation can be positive or negative, mainly depend on fibre orientation. In 1989, Miki et al. [140] investigated the unique behaviour of the Poisson’s ratio of laminated fibrous composites and observed a minimal value of Poisson’s ratio of -0.369 exists on unbalanced bi-directional laminates. Milton [141] reported that a kind of two-dimensional, two-phase, composite materials with hexagonal symmetry exhibits a Poisson’s ratio close to -1. He also concluded that by layering the component materials together in different directions on widely separated length scales, elastically isotropic two and three-dimensional composites with a Poisson’s ratio close to -1 could be easily generated. Chen et al. [142] conducted an experimental investigation of viscoelastic properties of composites made of traditional and re-entrant auxetic copper foam as a matrix. By employing finite element methods, Nkansah et al. [143] examined the elastic properties of continuous-fibre reinforced composites, and they found that an auxetic material can be used as the matrix in a continuous-fibre composite to increase the value of the transverse composite modulus without decreasing the longitudinal modulus. By conducting a series of experiments and comparing the laminate theory, Clarke et al. [144] concluded that laminate theory could make accurate predictions of the composite properties and validate the assumption of homogeneous strain. Using homogenization theory, Gibiansky et al. [145] proposed optimal piezocomposites for hydrophone applications, and they found that the optimal matrix is highly anisotropic and is characterised by negative Poisson’s ratios in certain directions. Theocaris et al. [146] investigated the variation of Poisson’s ratio in fibre composites using homogenization method. In this study, they concluded that the shape and the ratio of shear-to-bending of the beams have a significant influence on the value of Poisson’s ratio, and this conclusion is still valid for continua with voids, composites with irregular inclusions. Wei et al. [20] proposed that the relatively low Young’s modulus of existing auxetics could be improved by embedding an elastic material with sufficiently
high modulus with auxetics. This novel idea was theoretically validated, and they found that such composite materials exhibit auxetic behaviour when the inclusion volume fraction of embedded elastic material is below a certain value. Zhang et al. [147] reported that the ply orientations for achieving maximum transverse strain in a composite laminate are close to \([70^\circ/20^\circ]\). Using a specially designed software, Evans et al. [148] reported that researchers could match the mechanical properties of laminates with predicted negative Poisson’s ratio to those with similar mechanical properties but positive Poisson’s ratio. The fabricated samples and experiments demonstrated a good agreement with theoretical predictions. Alderson et al. [149] presented two different methods to fabricate auxetic composite. The first one was to use off-the-shelf prepregs and, by variations of the stacking sequence employed to design an auxetic composite with a through-the-thickness or in-plane negative Poisson’s ratio. The second one was to use auxetic constituents as part of the composite, and they concluded that the fibre pullout was resisted due to the auxetic deformation of the fibres. Tatlier et al. [150] proposed a modelling method to investigate the auxetic behaviour of compressed fused fibrous networks and they found that compression and anisotropy are the critical parameters that result in auxetic behaviour for these materials. Sigmund et al. [151] designed 1-3 piezo-composites with optimal performance for hydrophone applications by employing a topology optimisation method. Subramani et al. [152] explored the development of auxetic structures from composite materials, and the mechanical properties of these auxetic structures were also characterised. Based on the experimental and analytical result, they concluded that auxetic behaviour and tensile characteristics of the proposed structures significantly depend on their initial geometric configuration. A Poisson’s ratio range of -0.30 to -5.20 was observed for the proposed auxetic structures. Zhang et al. [153] explored the Poisson’s ratio behaviour of a further development of the helical auxetic yarn. The proposed three-component auxetic yarn was based on a stiff wrap fibre helically wound around an elastomeric core fibre coated by a sheath, as shown in Figure 2-36. They concluded that the coating thickness can be used as a new design parameter to tune both the Poisson’s ratio and modulus of this novel composite reinforcement. Recently, Jiang et al. [154] conducted a study on low-velocity impact response of multilayer orthogonal structural composite with auxetic effect, and they conclude that the auxetic composite had better energy absorption performance in medium strain range. Another interesting
work was conducted by Valentini et al. [155], where a biogenic successful method was reported to transform conventional silicone rubber composites to auxetic robust rubbers. In a recent work of Ghaznavi et al. [156], through a finite-element-based global-local layerwise theory and algorithm, the authors found that auxeticity of the core could significantly stiffen the core and plate of composites and reduce the lateral deflections of the plate.

![Figure 2.36 Three-component auxetic yarn. [153]](image)

### 2.5 Properties of auxetic materials and structures

Auxetic materials and structures possess counter-intuitive deformation behaviour which makes these auxetics endowed with many superiors comparing with conventional materials. The main properties of auxetic materials and structures are presented in the following sections.

#### 2.5.1 Shear resistance

Under shear forces, auxetic materials are known to be more resistant than regular materials. According to the classical theory of elasticity for three-dimensional isotropic solids, the elastic behaviour of a body can be presented by two of the four constants [69]: the Young’s modulus (E), the shear modulus (G), the bulk modulus (K) and the Poisson’s ratio (v) [157]. In three-dimensional cases, the relationship between these constants can be presented by two equations as below:

\[
G = \frac{3K(1-2v)}{2(1+v)} \tag{2-19}
\]

\[
G = \frac{E}{2(1+v)} \tag{2-20}
\]
It can be clearly seen that the value of the shear modulus increase when the Poisson’s ratio decreases, resulting in a consequent enhancement for shear resistance. The range of elastic modulus corresponding to instability and stability under different conditions is shown in the map in Figure 2-37. This map indicates that the Poisson’s ratio of the isotropic solid has to be in the range of -1 to 0.5. When the value of Poisson’s ratio approaches -1, the shear modulus would be infinity.

![Figure 2-37 Map of elastic material properties corresponding to different values of bulk modulus K and shear modulus G.](image)

### 2.5.2 Indentation resistance

Under an indentor local compression, the conventional material would spread in the direction perpendicular to the applied load [159] as shown in Figure 2-38a. In contrast, an indentation would occur if the same compression is applied on an isotropic auxetic material, and the material flows into the immediate region of an impact as shown in Figure 2-38b. Using holographic interferometry, Lakes et al. [160] reported that the indentation resistance of foams, both of conventional structure and the re-entrant structure resulting in auxetic behaviour. According to the classical theory of elasticity, the indentation resistance is closely related to the material hardness (H), which could be correlated to the Poisson’s ratio by the following equation:
Where $E$ is Young’s modulus, $\nu$ is the Poisson’s ratio of the base materials and $\gamma$ is assumed to be 1 or $2/3$ in the scenario of uniform pressure distribution or hertzian indentation, respectively. As can be clearly seen from equation 2-21, when the value of $\nu$ approaches -1, the indentation resistance tends to infinity [63]. When the value of $\nu$ reaches the maximal limit for 3D isotropic solids of 0.5, the indentation resistance would be much lower. However, because the maximal value of $\nu$ is 1 for the 2D isotropic system [161, 162], the materials with such positive Poisson’s ratio would possess an infinite hardness as well. Argatov et al. [163] conducted a theoretical work which was regarded as the first step towards an indentation and impact analysis of real auxetic materials. In the work of Coenen et al. [164], enhancements in indentation resistance were seen for the auxetic laminates with smaller, more localised damage areas for the two larger diameter indentors where delamination was concluded as the main failure mechanism. Dirrenberger et al. [115] conducted a series of numerical simulations of cylindrical and spherical elastic indentation tests to investigate effective elastic properties of auxetic microstructures. They concluded that auxetics can be superior to honeycomb cells in terms of indentation strength under certain conditions.

Figure 2-38 Indentation behaviours: (a) conventional material; (b) auxetic material. [165]
2.5.3 Fracture resistance

Materials which exhibit a negative Poisson’s ratio are reported to have a better fracture resistance than conventional materials [9, 166]. These auxetic materials were also reported that have low crack propagation [167]. Through an experimental investigation, Donoghue et al. [168] concluded that more energy was required to propagate a crack in the auxetic laminate. Maiti et al. [169] observed crack growth as shown in Figure 2-39.

![Figure 2-39 Schematic of crack extension manners in a cellular solid: (a) through the bending failure mode of the non-vertical cell elements; (b) through the tensile fracture of the vertical cell elements. [169]](image)

Liu [170] presented a detailed work on discussing the fracture mechanics side of auxetic materials.

The non-singular stress field at the distance $r$ for a crack of $2a$ with crack tip radius $r_{tip}$ and stress intensity factor $K_I$ is [169-172]:

\[
\sigma = \frac{K_I}{\sqrt{2\pi r}} + \frac{K_{II}}{\sqrt{2\pi r}} \left( \frac{r_{tip}}{2r} \right)
\]

(2-22)

Then, the force acting on the cell rib is:

\[
F = \int_{\frac{r_{tip}}{2}}^{\frac{r_{tip}}{2}} \left[ \frac{K_I}{\sqrt{2\pi r}} + \frac{K_{II}}{\sqrt{2\pi r}} \left( \frac{r_{tip}}{2r} \right) \right] r_{tip} \, dr
\]

(2-23)
Furthermore, with the thickness of the rib being \( t \), and the first order of the Taylor expansion, equation 2-23 can be simplified to:

\[
F = 2.38K_i^* \sqrt{\frac{l}{t}} \left( \frac{t}{l} \right)
\]  

(2-24)

Where \( K_i^* \) is the stress intensity of the conventional foam, and \( l \) is the rib length. The stress result from the bending moment is given by:

\[
F = 2.12 \frac{Fl}{t^3}
\]  

(2-25)

Substituting equation 2-24 the stress becomes:

\[
F = 5.05K_i^* \frac{1}{\sqrt{\pi}} \left( \frac{t}{l} \right)^2
\]  

(2-26)

The crack propagation will occur when \( \sigma \geq \sigma_f \), where \( \sigma_f \) is the fracture strength of the cell rib. The critical stress intensity factor or the fracture toughness can be calculated as:

\[
K_i^* = 0.20\sigma_f \sqrt{\pi d} \left( \frac{t}{l} \right)^2
\]  

(2-27)

Because \( \rho^* / \rho_i \propto (t/l)^n \) the stress intensity factor of conventional foams is proportional to the normalised density:

\[
\frac{K_i^*}{\sigma_f \sqrt{\pi d}} = 0.19 \left( \frac{\rho^*}{\rho_i} \right)
\]  

(2-28)

For the fracture toughness of re-entrant structure, the similar equations 2-27 and 2-28 become:
\[
\frac{K_{IC}^*}{\sigma_f \sqrt{\pi d}} = 0.10 \sqrt{\frac{1 + \sin \left( \frac{\pi}{2} - \varphi \right)}{1 + \cos 2\varphi} \left( \frac{\rho^*}{\rho_f} \right)}
\] (2-29)

Where \( K_{IC}^* \) is fracture toughness of re-entrant foams and \( \varphi \) is the rib angle as shown in Figure 2-39.

Choi et al. [172] conducted an experimental investigation. According to their observation, the following equation could be generated:

\[
\frac{K_{IC}^r}{K_{IC}} = 0.53 \sqrt{\frac{1 + \sin \left( \frac{\pi}{2} - \varphi \right)}{1 + \cos 2\varphi}}
\] (2-30)

In the work of Bhullar et al. [173], they concluded that when comparing with non-auxetic materials, auxetic materials have almost twice crack resistance to fracture.

### 2.5.4 Synclastic behaviour

When subjecting an out-of-plane bending moment, conventional materials exhibit a saddle shape and auxetic materials demonstrate a dome-shape, as shown in Figure 2-40. The dome-shape deformation pattern shown in Figure 2-40b can be also called synclasticity. This property is reported to be very useful [174] based on the fact that it provides a way to fabricate a dome-shaped structure with using damaging techniques nor additional machining [175]. This uncommon property is believed have the wide potential to be used in the medical areas.

![Figure 2-40 Deformation patterns for non-auxetic and auxetic materials under out-of-plane bending: (a) saddle shape (non-auxetic); (b) dome shape (auxetic). [176]](image)
2.5.5 Variable permeability

Because of the variations of the pores of auxetic materials during the compressive and tensile deformation, auxetic materials are believed to have a significant potential for the filter application. This behaviour can be illustrated by the Figure 2-41. In 2001, Alderson et al. [177] reported a pioneer work which illustrates that how auxetic materials offer improved filter performance from the macroscale to the nanoscale due to their unique pore-opening properties and characteristics. As an extension of their previous work, Alderson et al. [178] conducted glass bead transmission tests on auxetic polyurethane foams. They firstly confirmed the benefits in mass transportation applications because of auxeticity persists in 3D macroscale filters, and in 3D sieves at any scale which exhibit a significant tortuosity in the pore structure.

![Smart filters to demonstrate the variable permeability.][179]

2.5.6 Energy absorption

In terms of the performance of energy absorption, auxetic materials are reported to be superior to conventional non-auxetic materials. Chen et al. [180] conducted an investigation on the in-plane elastic buckling of hierarchical honeycomb materials. The study on the stress/strain law and deformation energy indicated that specific energy absorption would be enhanced when the hierarchical level $n$ is increased. Yang et al. [181] carried on a series of numerical simulations on ballistic resistance of sandwich panels with aluminium foam and auxetic honeycomb cores. They found that the auxetic-cored sandwich panel is far superior to the aluminium foam-cored panel in
ballistic resistance due to the material concentration at the impacted area resulting from the auxetic
behaviour. Mohsenizadeh et al. [182] conducted a comprehensive study both in simulations and
experiments to investigate the mechanical properties of the auxetic foam-filled tube under quasi-static
axial loading. They found that, in terms of all studied crashworthiness indicators, the auxetic foam-
filled square tube is superior to empty and conventional foam-filled square tubes. Recently, Imbalzano
et al. [183] reported that, under blast, auxetic composite panels could absorb double the amount of
impulsive energy via plastic deformation, and reduce up to 70% of the back facet’s maximum velocity
compared with monolithic ones. Scarpa et al. [184] investigated the acoustic properties of iron particle
seeded auxetic polyurethane foam and they found that the auxetic polyurethane foam possesses
intrinsic higher acoustic absorption properties comparing with conventional open-cell foams. In the
work of Ruzzene et al. [185], the attenuation capabilities of the auxetic lattice and their design
flexibility were demonstrated.

2.6 Application of auxetic materials and structures

Because of the counter-intuitive behaviour which auxetic materials and structures exhibit during
deformation, many desirable properties are offered to these smart materials which make them have a
huge potential in many applications.

In the aspect of medical and biomedical applications, auxetics can be used for stents, smart bandage
and blood vessels. In the work of Ali et al. [43], an auxetic structure film was designed and
manufactured and this film was configured as an auxetic stent for the palliative treatment of
oesophageal cancer, and for the prevention of dysphagia. Later on, Ali et al. [44] discussed the
manufacture of a small diameter auxetic oesophageal stent and stent-graft. The tensile test of the
auxetic polyurethane film exhibits a Poisson’s ratio of -0.87 to -0.963 at different uniaxial tensile load
values. And it was found that the diameter of the auxetic oesophageal stent expanded from 0.5 to 5.73
mm and the length of the stent extended from 0.15 to 1.83 mm at a certain pressure from the balloon
catheter. Based on the rotating rigid units, a new class of hierarchical auxetics was reported by Gatt et
al. [186]. The proposed stent with two levels hierarchical rotating square geometry, as shown in Figure 2-42, was believed can reduce inflammation occurring through reducing the actual surface area of the solid portion. Also, this hierarchical system was also reported to be very suitable for making skin grafts due to the relieved pressure on the swelling area.

Auxetic materials also possess a huge potential to be used for sports protective devices, e.g., pads, gloves, helmets and mats [72]. Using auxetic materials in impact protector devices could offer better conformability for support, and improved energy absorption for lighter and thinner components. Wang et al. [187] reported that auxetic cushions could reduce the pressure which could bring more comfort. The work of Michalska et al. [188] further proofed that auxetic materials could reduce contact stress concentrations by conducting numerical simulations on a seat with an auxetic polyamide spring skeleton. In the work of Sanami et al. [189], through numerical simulations, a new type of auxetic honeycomb was reported to have potential in helmet applications, along with indentation test of auxetic and non-auxetic foams for evaluating the applications of protective pads and running shoes.

![Hierarchical stent.](image)

One of the most promising applications of auxetic materials is manufacturing piezoceramic sensors, and it is reported that auxetic materials may improve the performance of piezoelectric actuators by more than an order of magnitude [190]. The piezoelectric composites including piezoelectric ceramic
rods within a passive polymer matrix are functioning by converting a mechanical stress into an electrical signal and vice versa, as shown in Figure 2-43.

The desirable property of various permeability which possessed by the auxetic material can be used to fabricate smart filters. In the work of Alderson et al. [191], simple experiments were performed which demonstrate that auxetic materials have superior properties than conventional materials in filter defouling and controlled pore size applications. Passage pressure could be controlled using smart filters [177].

Auxetic textile materials are increasingly popular because they could provide comfort, higher energy absorption, high volume change, wear resistance and drapeability [192]. The auxetic textile materials are generated through two methods. The first one is to use auxetic based fibres to knit and weave textiles directly [193]. The second method is to produce auxetic textiles is to use conventional fibres to weave or knit in a way which could make the textile production to be auxetic [192]. Recently, a novel kind of 3D auxetic fabric was reported to have auxetic behaviour in all the fabric plane directions. In the commercial market, auxetic textile has occurred, e.g., GoreTex and polytetrafluoroethylene [63], and some sports shoes recently released by Under Armour company as shown in Figure 2-44.
Besides, Prawoto [171] summarised the applications of auxetic materials from various work [32, 67, 165, 194-196], as shown in Table 2-1.

<table>
<thead>
<tr>
<th>Field</th>
<th>Application and rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerospace</td>
<td>Vanes for engine, thermal protection, aircraft nose-cones, wing panel, vibration absorber</td>
</tr>
<tr>
<td>Automotive</td>
<td>Bumper, cushion, thermal protection, sounds and vibration absorber parts, fastener</td>
</tr>
<tr>
<td>Biomedical</td>
<td>Bandage, wound pressure pad, dental floss, artificial blood vessel, artificial skin</td>
</tr>
<tr>
<td></td>
<td>drug-release unit, ligament anchors, Surgical implants</td>
</tr>
<tr>
<td>Composite</td>
<td>Fibre reinforcement (because it reduce the cracking between fibre and matrix)</td>
</tr>
<tr>
<td>Military (Defence)</td>
<td>Helmet, bullet proof vest, knee pad, glove, protective gear (better impact property)</td>
</tr>
<tr>
<td>Sensors / actuators</td>
<td>Hydrophone, piezoelectric devices, various sensors</td>
</tr>
<tr>
<td>Textile (Industry)</td>
<td>Fibres, functional fabric, colour-change straps or fabrics, threads</td>
</tr>
</tbody>
</table>

2.7 Concluding remarks

2.7.1 Research gaps

Because of the counter-intuitive behaviour which auxetic materials and structures could exhibit during compressive or/and tensile deformation, many superior properties, e.g., indentation resistance, shear resistance, synclastic behaviour, enhanced resilience, energy absorption, fracture toughness, vibration control and negative compliance could be offered by auxetics. Since Lakes [9] reported a first re-entrant foam structure with a negative Poisson’s ratio in 1987, an increasing number of researchers have engaged in the studies of these novel materials.

However, most of the earlier theoretical and numerical works on auxetics were based on periodic 2D geometries [197]. Afterwards, several analytical studies were conducted on 3D auxetic cellular
materials consisting of multi-pods, rigid units and networks of beams. Constrained by the fabrication techniques, very few designs of synthetic 3D auxetic materials have been developed into the stage capable of being used in practical applications [6]. One disadvantage of most existing auxetic materials was the small effective strain for auxetic behaviour, i.e., no more than 0.1 in the design of Bückmann et al. [22]. Babaee et al. [31] successfully extended the effective auxetic strain range to 0.3 by proposing a new design of ‘buckliball’ as the building cell. However, the geometry of their building cells was complicated and the fabrication processes included an extra bonding interface which resulted in the considerable scattering of its mechanical properties [26]. In addition, the base material of their model was silicone-based rubber, hence their results were limited to the elastic deformation. In contrast to elastic auxetic metamaterial, metallic auxetic metamaterials exhibit new features in their mechanical properties, e.g., localisation of plastic strain, strain hardening, and irreversible deformation. Furthermore, metallic metamaterials are usually much stronger than those made of elastomers, and therefore could carry more loads or absorb more impact energy. Therefore, it is of significance to design and fabricate 3D auxetic metallic metamaterials which could demonstrate a large effective strain range under deformation. Also, the geometry of the proposed design is expected to possess a good symmetry which is very useful under some conditions when the symmetrical auxetic effect is required, e.g., in the applications of sensors and smart filters.

In the aspect of the medical application, auxetics are reported to be of importance to act as foldable devices, e.g., angioplasty stents [39-41], annuloplasty rings [42] and oesophageal stents [43, 44]. For the patterns of all these medical devices, the cellular configuration is predesigned. The conventional straightforward method is to roll the 2D auxetic sheets to tubes. However, cellular tubes designed in this method may not have auxetic behaviour under large compressive strain. Besides, most of the existing auxetic stents only demonstrate auxetic behaviour in tension. To the best knowledge of authors, no systematic research has been conducted on auxetic tubular structures under large compressive strain which is required when stents scaffold are inserted into the blood vessels. Therefore, it is pivotal to design and explore the mechanical properties of novel tubular structures which could demonstrate negative Poisson’s ratio both in tension and compression.
2.7.2 Conclusions

This chapter has been aimed to provide a thorough literature review on auxetic materials and structures, including various types of cellular auxetics, natural and artificial auxetics, metallic auxetics, multi-material and composite auxetics. A series of superior properties which possessed by auxetics are comprehensively presented and some existing or potential applications are summarised in details.

Although remarkable progress has been made in the past three decades in the field of auxetics, including theoretical analysis, finite element simulations and experiments, there remain a heap of important and interesting problems which deserves further investigations and improvements. First of all, constrained by the poor manufacturing techniques, most of the previous studies of auxetics are based on simple 2D models. Besides, although some 3D auxetic materials have been reported, most base materials of these 3D auxetics are rubber-like materials which could only suffer a very limited loading force and impact. Apart from that, most of the existing 3D auxetic materials only exhibit negative Poisson’s ratio in a small effective strain range which tremendously constrains a wider application of these novel materials. More importantly, the geometries of the majority of the existing 3D auxetics are pre-designed which often creates difficulties for engineers towards tuning their mechanical properties. Lastly, all of the reported auxetic tubular structures may not demonstrate auxetic behaviour under compression which also constrains a wider application for auxetic tubular structures. Considering all the disadvantages of the state-of-the-art development of auxetics, a novel pattern scale factor (PSF) methodology will be introduced in the following sections, and successful cases will be presented using numerical simulations and experiments as solid evidence.
Chapter 3

Experiments and parametric studies on 3D metallic auxetic metamaterials

Designing the periodic microstructures of metamaterials allows us to achieve unusual and sometimes even unprecedented effective material properties. Those exotic properties are gained predominantly from the shape of microstructure rather than their chemical composition [22, 26, 31]. The cellular material with a negative Poisson’s ratio (NPR) is the most studied category of mechanical metamaterials. Under uniaxial compression, such materials contract (rather than expand) transversely. They are also known as “auxetic materials” by Evans [8].

An increasing interest has been found in the development of auxetic materials because of their many potential applications in various fields. Those applications include but are not limited to prostheses [198], piezoelectric sensors with desired performance [190], energy absorption, indentation and fatigue resistance [159, 199-201], smart filters [191], magnetic auxetic system [202], molecular sieves [122], seat cushions [187], superior vibration dampers [203] and acoustic isolators [204].

In the library of natural materials, 69% of the cubic elemental metals were reported to have negative Poisson’s ratio (NPR) when they were stretched in infinitesimal strain along a specific direction [196]. Auxetic behaviour was also observed in some materials at a temperature close to phase transition
[205-208]. Even for the well-known graphene, Grima et al. reported that NPR properties could be achieved by tailoring its spatial structures [99]. Recently, the nuclei of embryonic stem cells extracted from mouse were found to be auxetic during a metastable transition state [209], which further illustrated the functionality of auxetic behaviour in the natural world. However, more detailed analysis is required to implement those metamaterials in practical applications.

A variety of synthetic auxetic materials and structures were invented in the last three decades after Lakes [9] firstly presented a re-entrant foam with NPR in 1987. Wojciechowski [12] reported that Poisson’s ratio can be negative in the tilted phase of cyclic hexamers. Analytical solutions for both 2D [14-17, 19-21, 210, 211] and 3D [22-25, 211] auxetic materials were reported for different deformation mechanisms. Because computer simulations are more powerful than just analytical method, finite element (FE) methods have been widely used to investigate the performance of auxetic materials [26-30]. Limited by the fabrication techniques, few designs of 3D auxetic material were developed to the stage of practical applications [6]. One disadvantage of the most existing auxetic materials was the small effective strain for auxetic behaviour, i.e., no more than 0.1 in the design of Bückmann et al. [22]. Babaee et al. [31] extended the effective auxetic strain range to 0.3 by proposing a new design of “buckliball” as the building cell. However, the geometry of their building cells was complicated and the fabrication processes included extra bonding interface which resulted in the considerable scattering of its mechanical properties [26]. In addition, the base material of their model was silicone-based rubber, hence their results were limited to the elastic deformation. In contrast to elastic auxetic metamaterial, metallic auxetic metamaterial exhibits new features in their mechanical properties, e.g., localisation of plastic strain, strain hardening, and irreversible deformation. Furthermore, metallic metamaterials are usually much stronger than those made of elastomers, and therefore could carry more loads or absorb more impact energy.

The work on metallic auxetic materials dates back to Friis et al. [32] who reported the first auxetic copper foam with non-periodic building cells in 1988. Li et al. [33] investigated the auxetic property of copper foams via resonant ultrasound spectroscopy. He et al. [34] fabricated entangled materials
with a spiral wire structure which exhibited auxetic behaviour. Zhang et al. [96] reported a simple auxetic structure made of aluminium tubes and corrugated sheets. However, there was very limited research work focusing on metallic auxetic metamaterials with periodic microstructures. It is only very recently that Taylor et al. [37] and Dirrenberger et al. [38] investigated metallic structures with NPR. Dirrenberger et al. [38] concluded that auxetic effect remained and became even more obvious with plastic yielding using finite element (FE) method. However, their conclusion lacked experimental support.

To check whether the design of buckling-induced elastic metamaterials can be applied to metallic based materials, we started by simply replacing the rubber based material of the auxetic metamaterials in our previous work [26] with a metallic material. Unexpectedly, the original auxetic behaviour for elastomer samples disappeared in the metallic samples with identical periodic building cells in the uniaxial compression test. This finding demonstrated that the loss of auxetic behaviour in metamaterial from buckling-induced design was closely related to the base material. The design criteria developed by Babaee et al. [31] for buckling-induced auxetic metamaterials could not be used directly to metallic auxetic metamaterials.

In this chapter, we conduct an in-depth investigation, both experimentally and numerically, on a new type of 3D metallic auxetic metamaterials. A series of parametric studies using validated FE models were carried out to investigate the effects of microstructures and metal plasticity on 3D metallic auxetic metamaterials.

3.1 Designing the microstructures for 3D metallic auxetic metamaterials

The methodology of generating 3D metallic auxetic metamaterials can be divided into four steps: Firstly, designing buckling-induced auxetic metamaterial; Secondly, doing buckling analysis of the original FE model with linear elastic base material; Thirdly, identifying the desirable buckling mode; Lastly, altering the geometry of the representative volume element (RVE) using the desirable
buckling mode and repeating the altered RVE in three axial directions to form a 3D metallic auxetic metamaterials.

### 3.1.1 Designing buckling-induced auxetic metamaterial

The first step of the design approach starts with a buckling-induced auxetic metamaterial composed of a simple microstructure with regular shapes. Similar to the geometry configuration in our previous work [26], a simple cubic (SC) unit cell was generated with a diameter to edge length ratio of 1.248, which was within the limits of $1 < R < \sqrt{2}$ [212]. $R$ is the ratio of the diameter of the sphere to the edge length of the cube, as shown in Figure 3-1a. Figure 3-1b shows the representative volume element (RVE). The building cell was patterned in three normal directions to form the bulk metamaterial as shown in Figure 3-1c, where each edge of the material block was composed of eight unit cells to minimise the size effect in order to obtain reliable homogenised properties [213].

![Figure 3-1 Initial topology design of the 3D buckling-induced auxetic metamaterial with cubic symmetric microstructures ($L_{CE}$=6.25 mm, $R$=3.9 mm): (a) Building cell; (b) Representative volume element; (c) Bulk material for simulation and experiments.](image)

### 3.1.2 Buckling analysis of the original FE models with linear elastic base material

The second step is to conduct buckling analysis with proper loading and boundary conditions to trigger desired buckling patterns with auxetic behaviour under uniaxial compression. The material can be any arbitrary elastomer and a modulus of 87 GPa and Poisson’s ratio of 0.38 was used in our simulation. Subspace was chosen as the eigensolver during buckling analysis. The boundary condition
should be carefully selected to obtain valid buckling mode. In the present work, all degrees of freedom of nodes on the bottom and top surfaces of the FE models were constrained except for the nodal movements in the loading direction on the top surface.

A commercial finite element software package ABAQUS (Simulia, Providence, RI) was used for the buckling analysis. ABAQUS/Standard was employed for linear perturbation analysis using subspace eigensolver. The maximum number of eigenvalues of interest was set as 20. Models were built using solid elements (ABAQUS element type C3D8 with a mesh sweeping seed size of 0.31mm as shown in Figure 3-1a). Symmetrical mesh seed was distributed to the FE model and the analyses were performed under uniaxial compression condition.

3.1.3 Identifying the desirable buckling mode

Identifying the desired buckling pattern is based on experience from our previous and others’ works [37, 204, 214-218] where the alternating ellipsoidal pattern occurred for their auxetic materials and structures. The results of the two desirable buckling modes both for RVE and the entire model obtained from buckling analysis are presented in Figure 3-2. RVE was picked from the central part of the entire model.

Through buckling analysis, the first two overall buckling modes shown in Figure 3-2b were found to compare well with existing literature [37, 204, 214-218], where alternating ellipsoidal pattern occurred. Interestingly, by comparing the first and second modes of the central RVE, the better symmetry feature of the second mode was observed as presented in Figure 3-2a and Figure 3-2c. Hence the central RVE of the second pattern was selected as building cell.

3.1.4 Quantifying the shape of RVE of the desirable buckling mode and patterning the altered RVE to form the bulk metamaterial
Chapter 3

The buckling mode is a non-dimensional infinitesimal deformation vector of the original structure. A scale factor is required to quantify its shape so as to alter the original geometrical model. Conventionally, the post-buckling analysis is performed by converting the original bifurcation problem into a problem with the continuous response with a prescribed deformation pattern. This transition is accomplished by introducing a geometric imperfection into the “perfect” geometry. Besides, initial geometrical imperfection in a real structure is inevitable due to random errors and inaccuracy during manufacturing processes [219]. The conventional size for this imperfection is negligibly small, which will not change the overall shape of the original model. Therefore, the term “imperfection” used in our previous work [26] was unsuitable to present its true meaning as a way to change the shape of the original design. Alternatively, the term of pattern scale factor (PSF) was proposed, which could determine the magnitude of buckling mode to alter the initial geometry.

In order to define PSF in our design, the central RVE of the bulk material was selected as an indicator. After the buckling analysis, the normalised buckling mode could be demonstrated by simply adding the selected mode multiplied by a deformation scale factor (DSF) to the original coordinates of all nodes. When the void in the RVE at the centre of the bulk material was just closed as shown in Figure 3-2a, the corresponding deformation scale factor (0.0057 in this case) was defined as PSF=100%. Other percentages of PSF were defined accordingly, e.g., 50% with a deformation scale factor of 0.00285, and the 0% was the original geometry as shown in Figure 3-2a.

Rather than importing the overall pattern to the original bulk material as shown in Figure 3-2b, one representative volume element (RVE) at the centre of the bulk material was used as the building cell to form the bulk cellular material. This improvement resulted in a periodic metamaterial as shown in Figure 3-3.
Two most desirable buckling modes for both RVE and entire model from two perpendicular views (Eigenvalues of the 1st and 2nd modes are $2.08324 \times 10^{-3}$ and $2.08473 \times 10^{-3}$ respectively.): (a) Images of the Inner-most RVE of the second mode from buckling analysis at different PSF; (b) Views of full model of the first two patterns from buckling analysis at PSF of 100% and 20%; (c) Magnified views of Inter-most RVE of the first two patterns at PSF of 100% and 20%. (Green scale bars: 20 mm; Red scale bars: 5 mm)
3.2 Experiment

3.2.1 Fabrication of 3D cellular metallic materials for experiments

The specimens of the 3D cellular materials in Figure 3-4 were manufactured using 3D printing (Shapeways, New York) with raw brass as the base material. It should be noted that raw brass was chosen to print our 3D cellular metamaterials mainly because of its high ductility (with an elongation up to a strain of 0.3).

The detailed manufacturing process of the 3D bulk metamaterials was described by Shapeways as follows: First, the model was printed in wax using a specialised high-resolution 3D printer. It was then put in a container where liquid plaster was poured in around it. Once the plaster set, the wax was melted out in a furnace, and the remaining plaster became the mould. Molten brass was poured into this mould and set to harden. The plaster was then broken away, revealing the final sample.
The overall size of the test specimens is 50 mm x 50 mm x 50 mm, and the printed models are shown in Figure 3-4. The material properties of the printed brass base material were measured through standard tensile tests of six printed dog-bone specimens by using an MTS machine as shown in Figure 3-5. The results indicated that the constitutive behaviour could be accurately represented by a bilinear material model as shown in Figure 3-6. The average values from tensile tests of 3D printed brass dogbone specimens are summarised in Table 3-1.

![Figure 3-4 Two test samples for the metallic metamaterials from three different perspectives (scale bar: 10 mm): (a) front view; (b) side view; (c) isometric view. (PSF=0%: overall mass=192.52g, relative density=1540.16kg/m³, mass error=8.2%; PSF=20%: overall mass=190.00g, relative density=1520.00kg/m³, mass error=6.8%)](image)

<table>
<thead>
<tr>
<th>Table 3-1 Table summarising the main parameters measured from experiments of dog-bone specimens.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average value ± Error</strong></td>
</tr>
<tr>
<td><strong>Young’s modulus (GPa)</strong></td>
</tr>
<tr>
<td><strong>Yield strength (MPa)</strong></td>
</tr>
<tr>
<td><strong>Density (Kg/m³)</strong></td>
</tr>
<tr>
<td><strong>Strain hardening modulus (GPa)</strong></td>
</tr>
</tbody>
</table>
3.2.2 Uniaxial compression tests on 3D cellular metamaterials

The performance of our 3D auxetic metamaterials was tested using standard quasi-static uniaxial compression tests similar to those commonly used for other cellular materials [220], in which the strain rate of 10\(^{-3}\) s\(^{-1}\) was adopted using a Shimazu machine. A camera was used to capture the deformation in one lateral direction so as to evaluate the evolution of the Poisson’s ratio of the 3D metamaterial. As the performance of the metamaterials before densification was of major interest, compression process was stopped manually after specimens entered densification range.
Poisson’s ratio was calculated from 16 points of ligaments at the centre region of lateral surfaces, as shown in Figure 3-3 and Figure 3-9. Image processing method was used for calculating Poisson’s ratio of our 3D printed specimens after tests.

3.2.3 New findings from experiments

Experimental deformation processes of two metallic samples of PSF=0% and PSF=20% are demonstrated in Figure 3-7. The compression process of buckling-induced auxetic metamaterials indicated that the auxetic behaviour of the original geometry disappeared and was replaced by a localised deformation pattern with lateral deflection as shown in Figure 3-7a. However, the auxetic behaviour remained for the metallic cellular material with PSF=20% as shown in Figure 3-7b.

![Figure 3-7 Images of experimental deformation processes of two metallic samples (scale bar: 10 mm): (a) Deformation processes of the metallic sample with PSF=0%; (b) Deformation processes of the metallic sample with PSF=20%.](image)

The experimental values of Poisson’s ratio as a function of nominal strain are shown in Figure 3-8. The Poisson’s ratio of the originally designed model was positive (very close to zero), and localised deformation pattern occurred in the test which means the initial cellular metallic metamaterial is non-auxetic. After a PSF of 20% was introduced to the original geometry, the altered metamaterial exhibited auxetic behaviour from the very beginning as illustrated in Figure 3-8. The value of NPR was still decreasing when the nominal strain reached approximately 0.34.

The deformation pattern of metallic material was compared with that of rubber material in our previous work [2]. The initial geometries of four samples before tests are shown in Figure 3-9 from (a)
to (d), and the deformed geometries of the testing specimens at the compression strain of 0.24 are illustrated in Figure 3-9 from (e) to (h) respectively. It should be noted that for the original design, two different deformation patterns were observed during the experiments, auxetic deformation pattern for linear elastic based material (rubber) and localised deformation pattern with a lateral deflection of metallic based material (brass).

![Figure 3-8 Numerical and experimental Poisson's ratio as a function of nominal strain for both models with PSF=0% and PSF=20%, where the corresponding shape of 3D printing models after test are shown.](image)

However, for the altered design, auxetic deformation pattern was observed on rubber and brass samples with a PSF of 20%. Therefore, the loss of the auxetic behaviour in buckling-induced auxetic metamaterials can be effectively recovered by altering the geometry of periodic RVE with the desired buckling mode as shown in Figure 3-7b and Figure 3-9h. It is because that the deformation pattern of the buckling-induced auxetic metamaterials is settled to the prescribe pattern when the alteration is large enough, i.e. PSF=20%. Those results also confirmed the robustness of the improved methodology of generating 3D metallic auxetic metamaterials. The subsequent PSF controlled microstructures led to a new type of 3D metallic auxetic metamaterials which is capable of undergoing large strain.

From the experimental observations, there are several new features for those new metallic auxetic metamaterials. In contrast to the elastic deformation, the plastic deformation is able to dissipate energy rather than storing it in the form of internal energy [221]. Therefore, this feature can play a
significant role in the fields of impact absorbing or acoustic damping related applications. In addition, the feature of irreversible deformation is necessary for some biomedical applications e.g. stents for blood vessels which require different stable patterns at different stages during the surgery operation [222]. According to previous research, metallic materials can be used to fabricate other negative-index metamaterials [223-225]. It can be expected that some other functional features will appear when a magnetic field is exerted on the metallic auxetic metamaterials [202]. Furthermore, the plastic hardening has been used to enhance the strength of metallic materials [226], and this feature could be exploited to adjust the stiffness and strength of the metallic auxetic metamaterials.

![Image of experimental deformation processes for both rubber and brass samples](image)

**Figure 3-9** Experimental deformation processes for both rubber and brass samples (scale bar: 10 mm): (a) Initial rubber model of original design; (b) Initial brass model of original design; (c) Initial rubber model of altered design; (d) Initial brass model of altered design; (e) Auxetic deformation pattern of rubber model after deformation for original design; (f) Localized deformation pattern of brass model after deformation for original design; (g) Auxetic deformation pattern of rubber model after deformation for altered design; (h) Auxetic deformation pattern of brass model after deformation for altered design.

To further explore these remarkable properties of the 3D metallic auxetic metamaterials, numerical investigations were carried out on our designed metamaterials. The effects of PSF dominated microstructures and metal plasticities on their mechanical properties were investigated.
3.3 Finite element analysis

3.3.1 FE model for metallic auxetic metamaterials

The geometries of the overall models for FE analysis are shown in Figure 3-3. ABAQUS/Explicit solver was used for the nonlinear post-buckling analysis for taking effects of large deformations and complex self-contacts into consideration [227]. The inertia effect had been minimised by applying a gradually increased velocity on the bottom surface of the model according to Hanssen et al. [227] to guarantee zero acceleration at the beginning. The bilinear elastic-plastic material model was used in FE models as shown in Figure 3-6. Young’s modulus of 87GPa and hardening modulus of 1.7 GPa were set for the material model. The verification of mesh density and loading rate were conducted during this work.

Through measuring the mass of the two printed brass models, slight difference of mass between 3D specimens and FE designed models was found. The printed models were heavier than the mass of initial design. The relative errors of the mass are 8.2% and 6.8% for the two specimens with a PSF of 0% and a PSF of 20% respectively. We adjusted our FE model by slightly decreasing the radius of cutting the sphere in order to match the mass of the 3D printed models.

3.3.2 FE model validation

The mesh dependence analysis has been conducted, which is similar to that by Pozniak et al. [228]. The resultant Poisson’s ratios from FE models with different mesh sizes are nearly identical. A mesh size with at least four layers of elements for the minimal link of RVE was used in all our FE models.

It is worth noting that the deformation process of our designed 3D metallic auxetic metamaterial was identical for views from two different perspectives as shown in Figure 3-10. The FE models were validated by comparing the deformation process and the stress-strain curves from simulation with that from experiments.
For designed metamaterials with a PSF of 20%, the numerical result shown in Figure 3-10 compares well with the experimental result shown in Figure 3-7b. The FE model with PSF of 20% was further validated by comparing the nominal stress-strain relationship. The comparison of these curves is shown in Figure 3-11. Despite the differences in the level of plateau stress, the overall trend and general characters of those curves are very similar. As a very important parameter, the value of densification strain (effective strain) from FE model marked with a vertical dashed line in Figure 3-11 are nearly the same as that from the experimental result. It should be noted that the densification strain is determined according to [220]. The difference in stress-strain curves was caused by the irregularity of the 3D printed models in all surfaces and geometrical imperfection during the manufacturing process. The curve obtained from numerical simulation is not very smooth, especially when it is close to the densification range as can be seen in Figure 3-11. By carefully checking the FE field output, we confirm that those kinks were caused by the complicated contacts of the building cells.

Figure 3-10 Deformation process of the FE model with PSF of 20%: (a) ZX-plane view; (b) ZY-plane view.

Figure 3-11 Comparison of nominal stress-strain curves of the improved auxetic metamaterial with PSF of 20% between experimental and FE models.
3.3.3 Comparison of rubber metamaterial and brass metamaterial

In order to emphasise the influence of the base material on the auxetic performance of buckling-induced design and our design, a direct comparison is made on auxetic performance between the rubber model and the brass model. Both numerical and experimental results are summarised in Figure 3-12.

For the buckling-induced design, rubber specimen exhibits auxetic behaviour while the brass specimen does not. According to our previous work [26], the auxetic behaviour of the rubber specimen starts from buckling of the microstructures of metamaterials. This is evidenced by the starting point of NPR behaviour. The negative value of Poisson’s ratio does not start from the very beginning as shown in Figure 3-12a. The corresponding magnitude of Poisson’s ratio still remains positive until the nominal strain reaches 0.075. At this strain, the brass material at the weak links undergoes plastic deformation in the brass model. This finding illustrates that the loss of auxetic behaviour for buckling-induced design with brass base material caused by the plastic deformation. The plastic deformation prevents the buckling of the microstructures of the brass model in a desirable pattern. In such a circumstance, the buckling-induced deformation pattern cannot be triggered for the buckling-induced brass model due to the sudden change of modulus of its base material. Alternatively, it crashes from the middle layers due to the localisation of plastic deformation initiated by random manufacture errors.

On the other hand, the auxetic behaviour occurs from the very beginning for both of the altered rubber and altered brass models with PSF=20%, as shown in Figure 3-12b. It should be noted that the magnitudes of Poisson’s ratio for altered rubber model and altered brass model are very close. This result demonstrates that the auxetic performance of the altered specimens is depended on their geometry rather than the chemical component of their base material. It also proves the effectiveness of the proposed methodology for designing the metallic auxetic metamaterials. This finding agrees well
with the conclusion drawn by Pozniak et al. [27], who stated that by applying harder joints one can reach lower Poisson’s ratios, i.e. foams with better auxetic properties.

Both numerical and experimental results demonstrate the excellent auxetic performance of the proposed design. The auxetic behaviour is active over a large compression strain up to 0.35. Further comparison with other existing auxetic materials indicates that our novel metallic metamaterial is superior to many existing auxetic foams as reported in [95, 229, 230] in terms of the effective strain range. Parametric studies have been performed to investigate effects of PSF and nonlinear stress-strain curve of the base material on the auxetic performance and other mechanical properties. The detailed results will be presented in the subsequent sections.

![Curves of Poisson's ratio as a function of nominal strain](image)

**Figure 3-12** Curves of Poisson’s ratio as a function of nominal strain: (a) curves of Poisson’s ratio as a function of nominal strain for rubber and brass models with PSF=0; (b) curves of Poisson’s ratio as a function of nominal strain for rubber and brass models with PSF=20%.

### 3.3.4 Parametric studies

#### 3.3.4.1 Effect of PSF on auxetic behaviour

The magnitude of PSF determines the shape of microstructure of our designed metallic metamaterial as shown in Figure 3-2. It is also determined the performance of auxetic behaviour of the metallic metamaterials. The validated FE models were used to analyse the auxetic performance for the different model with different values of PSF in a large strain range. Figure 3-12 illustrates the
variation of the auxetic performance with respect to the value of PSF. The pentagrams represent the corresponding end strain for auxetic behaviour. The original model with PSF=0% does not have auxetic behaviour. By increasing the value of PSF, the overall magnitude of NPR increases while the effective strain range decreases. The reason behind this phenomenon is the changes in the void of the design metamaterials. When PSF is imported into the initial geometry, the original spherical voids become ellipsoidal shapes. The solid material will contact earlier during uniaxial compression. Therefore, the result indicates that the value of NPR and effective strain range should be designed according to the requirements of its application. It is impossible to achieve a design with a large value of NPR with maximum effect auxetic strain.

![Figure 3-13](image)

**Figure 3-13** Effect of PSF on Poisson’s ratio: (a) curves of Poisson’s ratio as a function of nominal strain for models with different PSF; (b) curves of nominal stress as a function of nominal strain for models with different PSF; (c) curves of averaged Poisson’s and effective strain range as a function of PSF.

### 3.3.4.2 Effect of strain hardening on auxetic behaviour
Because of the ductility of the raw brass, the designed 3D metallic auxetic metamaterial is capable of undergoing large deformation without fracture. Under large deformation, the plastic properties of the base material of our metallic metamaterial will have a significant effect on their mechanical performance. Strain hardening, as a common feature for various metals, determines the instant loading modulus of the metallic base material. The effect of the strain hardening on the auxetic performance of our designed metallic auxetic metamaterial was investigated using the validated FE models. The bilinear elastic-plastic material model was used in FE models as shown in Figure 3-14c, where Es is elastic modulus and Ep is strain hardening modulus. Ep/Es=0 represents an extreme case where the true stress will remain constant after the yield stress. Ep/Es=1 represents the other extreme case where strain hardening modulus and elastic modulus are equal and it should behave like an elastomer. The practical values of strain hardening modulus are within the range of these two extreme cases.

Figure 3-14 Parametric study of hardening effect: (a) Curves of Poisson’s ratio as a function of nominal strain for models with PSF of 20% for the base materials with different values of Ep/Es; (b) Curves of nominal stress as a function of nominal strain for model with PSF of 20% for the base materials with different values of Ep/Es; (c) Image of base material model; (d) Curves of averaged Poisson’s ratio and effective strain range as a function of Ep/Es.
As can be seen in Figure 3-14a, the differences of Poisson’s ratio for FE models with the different ratios of $E_p/\varepsilon_s$ ranging from 0.2 to 1.0 are very small. It is safe to conclude that the influence of strain hardening effect on auxetic behaviour is negligible, especially when the ratio of $E_p/\varepsilon_s$ is larger than 0.2. The Poisson’s ratio is nearly unchanged when nominal strain is larger than 0.15. The nominal stress level at densification strain increases dramatically from 4.1 MPa to 81.7 MPa, when the ratio of $E_p/\varepsilon_s$ increases from 0 to 1.

### 3.3.4.3 Effect of volume fraction on auxetic behaviour

The volume fraction influences the auxetic behaviour of metamaterials according to our previous work [26]. A parametric study of volume fraction for the designed metallic metamaterials is carried out to study this effect. The volume fraction of the buckling-induced metamaterial is determined by the radius of the spherical void of unit cell as shown in Figure 3-1. The same value of PSF of 20% was used to all models. The radius of the cutting sphere varies from 3.4 mm to 4.2 mm with a fixed length of 6.25mm for the cubic unit cell. The RVEs of FE models with different volume fraction are presented in Figure 3-15. It should be noted that increasing the value of PSF will increase the volume fraction as well. In the case study, the maximal volume difference between FE models with or without PSF is about 8.5%. The volume fractions presented in the Figure 3-15 are calculated from original circular hollow models.

![Figure 3-15](image_url)

*Figure 3-15 Representative volume elements of FE models with and without PSF at different volume fraction.*
Figure 3-16a demonstrates clearly that the size of the voids has a significant effect on the value of NPR. When the volume fraction is larger than a certain value, the auxetic behaviour will disappear as well. For the model with the volume fraction of 34.5% and above, the models do not exhibit auxetic behaviour. When the volume fraction becomes smaller, the stiffness and strength will decrease rapidly. Thus a practical volume fraction range for the models with auxetic behaviour is selected as from 25.9% to 34.5%.

Figure 3-16 Effect of volume fraction on Poisson’s ratio: (a) curves of Poisson’s ratio as a function of nominal strain towards models in different volume fraction with PSF of 20%; (b) Curves of nominal stress as a function of nominal strain towards model in different volume fraction with PSF of 20%; (c) Curves of averaged Poisson’s ratio and effective strain range as a function of volume fraction.

And the auxetic performance can be improved by reducing volume fraction. The effective strain range of the models in the given volume fraction remains constantly around 0.3. From Figure 3-16b, the stress level of our models is very sensitive to volume fraction. When volume fraction of our models
reduces from 34.5% to 7.2%, the corresponding nominal stress in densification strain drops dramatically from 107 MPa to nearly 0.25 MPa.

3.4 The summary

The auxetic performance of a newly invented 3D metallic metamaterial was investigated experimentally and numerically in this chapter. Inspired by the loss of auxetic behaviour of buckling-induced brass auxetic metamaterials, a new methodology of generating 3D auxetic metamaterials was developed and its effectiveness was proved experimentally and numerically. The effects of several key parameters on the auxetic performance and mechanical properties were explored using validated FE models. Through these investigations, the following conclusions can be drawn:

(1) The buckling-induced auxetic metamaterial will lose its auxetic behaviour when the base material is changed from an elastomer to a ductile metal.

(2) The method of generating 3D metallic auxetic metamaterial by introducing PSF has been proposed and its effectiveness has been validated through both numerical and experimental methods.

(3) The loss of NPR behaviour in the previous auxetic metamaterial design is attributed to the localisation of plastic deformation of the metallic base material.

(4) Plastic strain hardening has a negligible effect on the auxetic behaviour of the designed 3D metallic auxetic metamaterial.

(5) The auxetic performance of our 3D metallic metamaterial can be controlled by PSF and the volume fraction.

The identified features for our metallic auxetic metamaterials are their large effective strain range, higher stiffness and strength than that of elastomer based metamaterials, periodic microstructure and simple tuning property by using one single control parameter of PSF. Based on these novel features, a wide range of potential applications of the designed 3D metallic auxetic metamaterial are expected. The auxetic performance of our metallic metamaterial is superior to the most conventional auxetic
foam and other non-periodic auxetic materials. Most of the buckling-induced auxetic metamaterials only exhibit auxetic behaviour under compression. It is worth noting that our 3D metallic metamaterials will exhibit auxetic behaviour under tension as well. Based on the novelty of the methodology and superior properties of our 3D auxetic metallic metamaterials in this work, it is expected that the developed methodology will inspire other researchers to generate more 3D metallic auxetic metamaterials. It should be noted that there are several limitations in the methodology presented in the chapter. The desired buckling mode with auxetic behaviour may not be suitable for all periodic cellular materials. An effective way to identify the desirable buckling pattern is required to facilitate the search for new types of auxetic structures. The metamaterial designed in the current chapter is anisotropic (cubic symmetric) and more symmetric constraints should be applied to obtain isotropic metamaterial. In the 3D auxetic metallic metamaterials, the value of NPR is not always constant over its effective strain range, which may limit their practical applications. Therefore, topology optimisation is necessary to improve those defeats in our design in our future research.
Chapter 4

Experiments and parametric studies on simple auxetic tubular structures

4.1 A simple auxetic tubular structure with tuneable mechanical properties

Materials and structures with negative Poisson’s ratio (NPR) exhibit counter-intuitive behaviour, i.e. under uniaxial compression (tension), these materials and structures contract (expand) transversely. They are also named as “auxetics” by Evans [8].

Because of the uncommon feature which is equipped by auxetics, these auxetic materials and structures are superior to conventional materials and structures in terms of indentation resistance [163, 231], shear resistance [167], synclastic behaviour [9], enhanced resilience [9], energy absorption [67, 232-234], fracture toughness [172], vibration control [235] and negative compliance [236-238]. Recently, Grima et al [46] found that a regular conventional sheet of rubber-like material can be converted to an auxetic metamaterial by using non-symmetric quasi-random cuts.

As one branch of auxetics, auxetic tubular structure or auxetic stent has attracted much research effort towards exploring its applications as foldable devices in the medical field, e.g., angioplasty stents [39-41], annuloplasty rings [42] and oesophageal stents [43, 44]. For the patterns of all these medical
devices, the cellular configuration is predesigned. The conventional straight forward method is to roll the 2D auxetic sheets to tubes. However, the cellular tubes designed in this method may not have auxetic behaviour under large compressive strain. Besides, most of the existing auxetic stents only demonstrate auxetic behaviour in tension. To the best knowledge of authors, no systematic research has been carried out on auxetic tubular structures under large compressive strain which is required when stents scaffold are inserted into the blood vessels. Gatt et al [45] proposed a three-dimensional tubular system with a typical planar two-dimensional system constructed from rotating rigid units. They mentioned that the edge effect had a significant influence on the finite-sized 3D tubular structures.

Recently, Mohsenizadeh et al [233] concluded that auxetic foam-filled square tube is superior to empty and conventional foam-filled square tubes in terms of crashworthiness indicators through experimental method, and Hou et al [234] obtained the optimal Poisson’s ratio of the filled material for three foam-filled tubes in terms of energy absorption through FE method. However, all the tubes they used were conventional ones, investigation on auxetic tubes are rare, particularly in compression. Grima et al [239] investigated the effect of Poisson’s ratio and Young’s modulus of 2D honeycomb structures on the formation of the tubular structures and indicated that the semi re-entrant honeycombs had a natural tendency to form cylindrical tubes.

Inspired by a planar auxetic metamaterial induced by elastic instability [204, 214, 240], we selected an available void fraction of 0.69 (larger than 0.34 in [204]), which demonstrated buckling-induced auxetic behaviour, to generate a tubular structure with normal circular holes. During FE simulations, we found that when the base material of the designed tubular structure is rubber, the tubular structure did show auxetic behaviour, as expected. However, this auxetic behaviour disappeared when the base material was replaced with brass.

In this section, we implement the latest methodology of generating buckling-induced auxetic metamaterials proposed in our previous work [241], to generate a simple auxetic tubular structure
which could be easily tuned by one parameter named as the pattern scale factor (PSF). An in-depth investigation on the designed auxetic tubular structure is carried out both experimentally and numerically. Although the focus point is on the compressive auxetic properties of the designed structure, tensile auxetic performance is also investigated. A series of parametric studies on the designed auxetic tubular structure are executed by using the experimentally validated FE models.

4.1.1 Designing auxetic tubular structure

Similar to our previous work [241], the methodology of generating auxetic tubular structures can be summarised as four steps. Firstly, designing buckling-induced auxetic tubular structure; secondly, carrying on buckling analysis of the initial tubular structure with the linear elastic base material; thirdly, identifying the desirable buckling mode; lastly, altering the initial tubular structure using the desirable buckling mode.

4.1.1.1 Designing buckling-induced auxetic tubular structure

The first step of the design framework is to generate a buckling-induced auxetic tubular structure. Similar to the geometry configuration of the Bertoldi’s work [204], a planar sheet is shown in Figure 4-1a with a void fraction of 0.69. Under planar constraint, it demonstrates a negative Poisson’s ratio behaviour induced by elastic instability. Using coordinate transformation method, the planar sheet can be transferred into a tubular structure as shown in Figure 4-1b. It should be noted that the shell model of the tubular structure is very similar to the finite-sized 3D tubular structure designed by Gatt et al [45].
4.1.1.2 Buckling analysis of the initial tubular structure with linear elastic base material

The second step is to perform buckling analysis to obtain desirable buckling modes with auxetic performance under uniaxial compression. The modulus of 110 GPa and the Poisson’s ratio of 0.38 were used in the simulation. Lanczos was chosen as the eigensolver in buckling analysis. In this work, the out-of-plane rotational degree of freedom on top and bottom nodes of the FE model was constrained. The degree of freedom of bottom nodes along compressive direction was also constrained. The nodal movement on the top nodes was allowed to move in the loading direction. It should be noted that one of the nodes in the bottom was fixed to avoid rigid rotation. Shell elements were used for buckling analysis.

The commercial finite element software package ABAQUS (Simulia, Providence, RI) was implemented for running buckling analysis. ABAQUS/Standard was adopted for linear buckling analysis using a Lanczos eigensolver. The number of eigenvalues requested was set as 10. The tubular structure was built using shell elements (ABAQUS element type SA) with a shell thickness of 6mm. Symmetrical mesh seed was distributed to the FE model which sustained the uniaxial compressive force.

4.1.1.3 Identifying the desirable buckling mode
The desirable buckling mode was selected based on similar patterns observed from previous research on planar auxetic structures. To be more specific, the configuration of the anticipated buckling modes should contain geometry similar to the alternating ellipsoidal pattern in our previous and other’s work [26, 37, 204, 214-218, 241]. The results of the first two buckling modes are presented in Figure 4-2 (Eigenvalues of the 1st and 2nd modes are $8.73143 \times 10^{-4}$ and $1.18656 \times 10^{-4}$ respectively). Apparently, the first buckling mode meets the requirement of selecting desirable modes, while the second buckling mode is unqualified because the holes are irregular.

![Figure 4-2](image)

Figure 4-2 The first two buckling modes of the initial tubular structure: (a) configuration of the first buckling mode; (b) configuration of the second buckling mode.

**4.1.1.4 Quantifying the desirable buckling mode and form the auxetic tubular structure**

The method of quantifying the desirable buckling mode is similar to our previous work [241]. The pattern scale factor (PSF) was employed to quantify the adjustment on the initial tubular structure using the desirable buckling mode. In the present work, when the edge of the elliptical void is just closed, as shown in Figure 4-3, the corresponding deformation scale factor (DSF) (0.01465 in this case) is defined as PSF=100%. Other values of PSF are defined accordingly, e.g., 50% with a deformation scale factor of 0.007325, and the 0% is the initial tubular structure without any adjustment.
Because the result of the first buckling mode obtained from buckling analysis was uniform. Unlike our previous work [241] where a representative volume element (RVE) was employed, the adjustment was directly applied to the whole configuration of the initial tubular structure in this study.

4.1.2 Experiment

4.1.2.1 Fabrication of metallic tubular structure for experiments

The specimens of the tubular structure in Figure 4-4 were fabricated using 3D printing (Shapeways, New York) technique with raw brass as their base material. The specific manufacturing procedure of the metallic tubular structure was same as our previous work [241]. The material properties of the printed raw brass material were measured through standard tensile tests which were completed in the previous work [241]. (PSF=0%: overall mass=164.10g, relative density=841.2kg/m³, mass error=9.0%, wall thickness=4.04 mm; PSF=20%: overall mass=165.56g, relative density=848.8kg/m³, mass error=12.3%, wall thickness=4.03 mm).
4.1.2.2 Uniaxial compression tests on tubular structures

The mechanical performance of the printed tubular structures was tested using standard quasi-static uniaxial compression tests, and the strain rate of $10^{-3}$s$^{-1}$ was employed using a Shimazu machine. A camera was used to record the deformation procedure to measure the evolution of the Poisson’s ratio of the tubular structures.

As can be seen in Figure 4-4, the centres of two rotation part of six layers were marked with small points. The experimental value of Poisson’s ratio was calculated using image processing method from 6 layers of the tubular structures, as shown in Figure 4-5. The equation of calculating Poisson’s ratio of one layer of the tubular structure is shown in formula 4-1 and the equation of calculating overall Poisson’s ratio of the tubular structure is shown in formula 4-2.

\[
\nu_i = -\frac{\varepsilon_y}{\varepsilon_z} = -\frac{\Delta l / D}{\Delta z_i / Z_i} \quad (2 \leq i \leq 5) \tag{4-1}
\]

\[
\overline{\nu} = \frac{1}{4} \sum_{i=1}^{5} \nu_i \quad (2 \leq i \leq 5) \tag{4-2}
\]
Where \( \Delta d = d - D \), \( \Delta z = z_i - z_{i+1} \), \( Z_i = z_i - Z_{i+1} \). \( D \) is the diameter of the tube before deformation, \( d \) is the real-time diameter of the tube during deformation, \( Z_i \) is the height of the \( i \)-th layer before deformation, \( z_i \) is the height of the \( i \)-th layer during deformation.

![Diagram](image1)

**Figure 4-5** Calculating Poisson’s ratio using an imaging processing method from two perspective views: (a) top view; (b) front view.

### 4.1.2.3 Comparison of auxetic behaviour of the two tubular structures from experiments

According to the Bertoldi’s finding regarding negative Poisson’s ratio behaviour induced by elastic instability [204], here we utilised a similar geometry mentioned in their work to generate a tubular structure as shown in Figure 4-4a. In our previous study [241], we found that the loss of auxetic behaviour in metallic auxetic metamaterials. To verify the loss of auxetic behaviour will also occur towards tubular structures, here we made a comparison experimentally.

The experimental deformation processes for two tubular metallic samples of PSF=0% and PSF=20% are illustrated in Figure 4-6. The initial metallic tubular structure with PSF=0% is non-auxetic, and the altered metallic tubular structure with PSF=20% illustrates auxetic behaviour. This result further verified that the phenomenon of the loss of auxetic behaviour not only occurs for 3D metamaterials but also exists for tubular structures. In addition, using the latest methodology of generating 3D metallic metamaterials, the loss of auxetic behaviour towards tubular structure could be easily obtained again.
According to the method of calculating Poisson’s ratio described in Figure 4-5 and formulas (1)-(2), the value of Poisson’s ratio for the tubular structure with PSF=0% could not be calculated properly because the marked points in the same layer were not in the same horizontal level. The ideal auxetic deformation pattern is that the diameter of the tubular structures in different height could change evenly under uniaxial deformation. However, from the deformation pattern shown in 6(a), we can define the metallic tubular structure with PSF=0% is non-auxetic. The experimental values of Poisson’s ratio as a function of displacement for the specimen with PSF=20% are shown in Figure 4-7.

Figure 4-6: Experiments on two metallic tubular structures (scale bar: 10 mm): (a) global buckling of the metallic sample with PSF=0%; (b) auxetic behaviour of the metallic sample with PSF=20%.

Figure 4-7: Experimental results of Poisson’s ratio as a function of displacement for the model with PSF=20%. 
So far, the experimental result has confirmed that the methodology of generating 3D metallic metamaterials can be extended to design auxetic tubular structure as well. To the best knowledge of authors, no one has conducted studies on auxetic tubular structures about his or her compressive mechanical performance. To illustrate the nonlinear effect on metallic auxetic tubular structures, numerical investigation both on compressive and tensile auxetic performance was executed on our designed tubular structures. The influence of PSF and metal plasticity on their mechanical properties was explored.

4.1.3 Finite element analysis

4.1.3.1 FE model for metallic tubular structure

The geometric configuration of the FE model is shown in Figure 4-5. ABAQUS/Explicit solver was employed in the post-buckling analysis for considering large deformation and self-contacts [227]. Although the shell elements were used in the stage of generating auxetic tubular structures, for obtaining the accurate result in FE simulations, solid elements (ABAQUS element type C3D8) were adopted for large deformation analysis. Because the base material of the experimental specimens was same as that of the printed models in our previous work [241], the same bilinear elastic-plastic material model with Young’s modulus of 87 GPa and a hardening modulus of 1.7 GPa were used in FE simulations.

Through comparing the FE models and the printed tubular models, we found that the printed tubular models were lighter than our initial design. The relative mass errors were 9.0% and 12.3% for the two printed samples with PSF of 0% and 20%, respectively. We found that the wall thickness of the printed models was smaller than our initial design. Adjustment of the FE models was made by decreasing the thickness of the tubular structures when it is compared with experimental results.
4.1.3.2 FE model validation

The mesh dependence analysis was performed, which is similar to the work conducted by Pozniak et al [228]. A mesh size with four layers of elements for the minimal link of the FE models was adopted. The FE model was validated by comparing the deformation process and force-displacement curves from the simulation with that from experiments. The deformation process of the tubular structure with PSF=20% was shown in Figure 4-8, which is nearly identical to the deformation process from experiments shown in Figure 4-6b.

![Deformation process of the FE model with PSF=20%](image)

The FE model was further validated by comparing the force-displacement curves as shown in Figure 4-9. Although the peak force of the experimental result is higher than that of FE result, the overall trends of these two curves are similar. Through checking the geometries of the printed sample after the test, we found some broken parts on it, as shown in the green dashed circles. However, the failure criterion was not defined in the FE simulations. Therefore, we attributed the difference between these two curves to the imperfection of the printed specimen and the fracture of the minimal links in the experiment, which are difficult to simulate using FE models.
4.1.3.3 Comparison of rubber and brass tubular structures

According to the previous findings of our work [241], the initial auxetic behaviour of the buckling-induced metamaterial disappeared when the base material of rubber was replaced by brass. Based on the work of Bertoldi et al [204], we selected the similar geometry with a void fraction of 0.69, to generate a tubular structure which should possess the auxetic behaviour induced by elastic instability. To verify the effect of the base material on the tubular structure with PSF=0%, we conducted a comparison using FE method by changing material model in ABAQUS setting. The linear elastic material model for rubber with Young’s modulus of 1 MPa was chosen in FE simulations. The deformation patterns for the same geometry with two different material models were shown in Figure 4-10.

The FE results illustrate that the base material has a significant effect on the auxetic performance of the buckling-induced tubular structure. When the base material is replaced from rubber to brass, the initial auxetic behaviour of the buckling-induced tubular structure will disappear. The finding is similar to the finding we have observed for the test of 3D auxetic metamaterials.
4.1.3.4 Effect of PSF on auxetic behaviour

The magnitude of the PSF not only determines the geometric configuration of the designed tubular structure but also affects the auxetic performance of the metallic tubular structure. The auxetic performance for different models with various values of PSF was investigated using the validated FE models.

The variation of the auxetic performance on the values of PSF is shown in Figure 4-11. We can see clearly that the auxetic performance of the tubular structure can be adjusted by changing the values of PSF. By increasing the value of PSF, the effective strain range for the tubular structure under tension could be enlarged, while the effective strain range under compression will become smaller. Therefore, to obtain an auxetic tubular structure which has a similar effective auxetic strain both in compression and tension, a proper value of PSF should be chosen. As can be seen in Figures 4-11a and 4-11b, when the tubular structure has a PSF of 60%, the trends of the curves of Poisson’s ratio-displacement and force-displacement are nearly the same.
4.1.3.5  Effect of plastic strain hardening on auxetic performance

Because of the high ductility of raw brass (with an elongation up to 0.3), it was chosen as the base material of the printed tubular structures. Strain hardening, as a fundamental feature of metal plasticity, has a significant effect on the mechanical performance of cellular structures and materials.

The effect of plastic strain hardening on auxetic performance and load-bearing capability of the designed tubular structures were investigated by using the validated FE models. A bilinear elastic-plastic material model was employed in FE models. The FE results of the parametric study for strain hardening are shown in Figure 4-12, where $E_s$ is the elastic modulus and $E_p$ is the strain hardening modulus.
We can see that the differences in Poisson’s ratio for FE models with different ratio of $E_p/E_s$ from 0.2 to 1.0 are very small, especially when the ratio of $E_p/E_s$ is over 0.2. The magnitude of the force at the point of densification strain increases significantly from around 1 KN to 17 KN when the ratio of $E_p/E_s$ enhances from 0 to 1.

4.1.4 The conclusion of this section

In this section, the latest methodology for generating 3D auxetic metamaterials was successfully extended to the development of metallic auxetic tubular structures. The performance of a simple metallic auxetic tubular structure under both compressive and tensile loading was investigated experimentally and numerically. The effects of the pattern scale factor (PSF) and the plastic strain hardening on the auxetic performance and other mechanical properties were examined using the validated FE models. From the obtained results, the following conclusions could be drawn:

(1) A simple auxetic tubular structure has been designed, fabricated and tested, which exhibits auxetic behaviour in both compression and tension.

(2) The buckling-induced auxetic tubular structure would lose its auxetic behaviour when the base material is changed from an elastomer to a ductile metal.

(3) The latest methodology for generating 3D metallic auxetic metamaterials has been further developed to create a simple auxetic tubular structure, and the effectiveness of the design approach has been validated experimentally and numerically.

(4) The mechanical properties of the proposed tubular structure can be tuned by adjusting the magnitude of the PSF. When the PSF is set at a certain value (~60% in this study), it is possible to achieve a similar auxetic performance under compression and tension.
4.2 Simple auxetic tubular structures generated using cutting technique

The term of “auxetic” is firstly proposed by Evans [232] to represent a cumbersome phrase of “negative Poisson’s ratio”, which is used to describe materials and structures with a counter-intuitive behaviour, i.e. under uniaxially vertical compression (tension), these materials and structures contract (expand) transversely.

Because of the uncommon behaviour, auxetics have been widely explored ever since Lakes [9] reported the first re-entrant foam with negative Poisson’s ratio (NPR) in 1987. Compared with traditional materials and structures, the superior properties of auxetics include shear resistance [242], indentation and fatigue resistance [159, 163, 199-201, 231], energy absorption [67, 183, 232], synclastic behaviour [9], improved resilience [9], fracture toughness [172], acoustic absorption [204], vibration control [235] and smart filters [191].

As foldable and expandable devices, auxetic tubular structures or auxetic stents have attracted significant interest because of their important role in the medical field, e.g. annuloplasty rings [42], angioplasty stents [39-41] and oesophageal stents [43, 243]. However, for the patterns of the most existing auxetic tubular structures, the cellular configuration is predesigned. Recently, Grima et al. [46] reported an interesting work. Using a random cut method, they successfully converted a non-auxetic conventional sheet of rubber-like material to an auxetic metamaterial with a large negative Poisson’s ratio. In our previous work [244], we also suggested that random cut method could be used to generate auxetic tubular structures.

In this section, two kinds of tubular structural models utilising random cut method [46] and vertical-horizontal (VH) cut method were firstly generated, respectively. Then validated FEA was carried out to explore the mechanical properties of the designed tubular structures under quasi-static tension. Lastly, the auxetic performance of the VH cut tubular structure was compared with an auxetic tubular structure [244] generated using PSF adjustment method [241, 244-246].
4.2.1 Designing auxetic tubular structures

4.2.1.1 Designing auxetic planar sheet using cutting method

Similar to the work of Grima et al. [46], two slit perforated planar sheets were generated, one with a highly ordered pattern of alternating vertical and horizontal slits, the other one with a disordered pattern of randomly oriented slits.

The geometries of the slit perforated sheets are shown in Figure 4-13, where $L$ is the length of the unit cell equals 20 mm; $l$ is the length of the cutting slit equals 18 mm; $t$ is the thickness of the cutting slit equals 0.2 mm; $s_0$ is separation between the slits in the ordered system equals 0.9 mm. For the disordered planar sheet, same as [46], the disorder is introduced to the sheet by a random change in the angular orientation of the cutting slit. The rotational angle is defined by $d$ where $-30^\circ \leq d \leq 30^\circ$, with $5^\circ$ as an increment. That means $d$ is set to $0^\circ$, $\pm 5^\circ$, $\pm 10^\circ$, $\pm 15^\circ$, $\pm 20^\circ$, $\pm 25^\circ$, $\pm 30^\circ$. It should be noted that $s_0$ is always larger than 0, i.e., none of the slits is overlapped with each other.

4.2.1.2 Converting sheets to auxetic tubular structures

According to the work from Grima et al. [46], the planar sheets shown in Figure 4-13 have auxetic behaviour under quasi-static tension. But how these planar sheets would behave under tension when they were transformed to tubular structures was still unknown. Same as our previous work [244], a coordinate transformation method was used so that the planar sheets could be transferred into tubular structures as shown in Figure 4-14.
Figure 4.13 Planar sheets with perforated slits: (a) the representative unit cell of an auxetic perforated sheet consisting of perpendicularly arranged alternating slits; (b) the ordered perforated sheet made up of 8×6 representative unit cell using VH cut method; (c) the disordered perforated sheet with $d_{\text{max}} = 30^\circ$ using random cut method.

Figure 4.14 Cutting patterns of the tubular structures from different views: (a) the front view of the tubular structure generated using VH cut method; (b) the front view of the tubular structure generated using the random cut method; (c) the perspective view of the tubular structure generated using the random cut method.

4.2.2 Finite element analysis

4.2.2.1 FE model for tubular structures

ABAQUS/Explicit solver was employed for the large deformation. Shell elements (ABAQUS element type S4) with the thickness of 2 mm were adopted for large deformation analysis. In order to minimise the inertia effect of the models, a gradually increased velocity on the top layer of the models was applied to guarantee its zero acceleration at the beginning of the loading process. Similar to the rubber material setup in our previous work [26], the linear elastic material was used with an isotropic Young’s modulus of 1 MPa and Poisson’s ratio of 0.47. All freedoms of nodes on the bottom and top
layer were constrained except for the loading direction on the top layer which was set as \( z = 1 \). The loading rate and mesh density were determined by a convergence check.

4.2.2.2 Comparisons of auxetic performance of tubular structures in tension

According to the work of Grima et al. [46], planar sheets with ordered or disordered random cuts have an auxetic behaviour under axially quasi-static tension. To extend their work from planar sheets to tubular structures, the FEA method was implemented and the FE results of deformation processes both for VH cut and randomly cut tubular structures were shown in Figure 4-15. The normalised strain was defined by the vertical displacement of the top layer of a tubular structure divided by the initial height of the tubular structure.

As can be seen from Figures 4-15a and 4-15b, both of the two tubular structures seemed to exhibit auxetic behaviour because the overall diameters of these two tubular structures increased when compared with the initial diameters. It should be noted that the diameters at the different height of the tubular structure with ordered slits increased identically, as shown in Figure 4-15a. However, the diameters at different height of the tubular structure with random slits increased non-uniformly, as shown in Figure 4-15b.
When observed from the top view, the differences of the deformation processes for the two tubular structures became more obvious, which can be seen in Figures 4-15c and 4-15d. The tubular structure with ordered slits was auxetic, which was confirmed by its deformation process from two different views, as shown in Figures 4-15a and 4-15c. Interestingly, we found that the diameter of the tubular structure with disordered slits was not increased with the tensile deformation which could be clearly seen from the top view, as shown in Figure 4-15d. This FE result indicated that the tubular structure generated from the randomly cut method was non-auxetic, or at least, not the strictly auxetic.

For the planar sheet with perpendicular ordered slits, as shown in Figure 4-13b, the planar system itself made up a rotating square system. For the planar sheet with disordered slits, as shown in Figure 4-13c, the system was no longer a uniformly shaped rotating square system under tensile deformation,
and it could be regarded as a system consisting of irregular quadrilaterals rotating units. The irregularity of the rotating unit was not an important factor for the 2D planar sheet with disordered slits, because the auxetic performance for the two planar sheets was very close, which was concluded in Ref [46].

For a 3D tubular structure, although it could be generated by rolling a planar sheet, one more constraint was implemented, e.g., the previous two unconnected vertical edges of a planar sheet were merged when the planar sheet was transferred to form a tubular structure.Attributed to this extra constraint, the mechanical effect of irregularity of the rotating units was enlarged for a 3D tubular structure resulted in an out-of-plane deformation. It is beyond the scope of this work to fully understand the enlarging effect of irregularity of rotating units on 3D tubular structure, but we could still conclude that the random cut method is unsuitable for generating auxetic tubular structures.

### 4.2.2.3 Comparisons of auxetic performance for the tubular structures generated by VH cut method and PSF adjustment method

To evaluate the performance of VH cut auxetic tubes, deformation of the tubular structures generated by VH cut method was compared with that by PSF adjustment method. Followed by our previous work [244], an auxetic tubular structure with PSF of 100% was generated as shown in Figure 4-16. And the same Abaqus configurations as the previous two tubular structures were used for exploring its auxetic performance under tension. The mesh density was determined by an independent convergence analysis for this FE model. The deformation process for the tubular structure generated by PSF adjustment method under quasi-static tension, with a PSF=100% is shown in Figure 4-17.

The auxetic performance of two kinds of tubular structures, which were generated by VH cut method and PSF adjustment method, with a PSF=100%, is demonstrated in Figure 4-18.

As can be seen in Figure 4-18a, the Poisson’s ratio for the tubular structure with a PSF=100% was nearly constant over the normalised strain range of -0.9 to -0.6. The Poisson’s ratio value for the VH
cut tubular structure decreased dramatically at the beginning of the tensile deformation, with a maximal negative Poisson’s ratio of -3.8. Then this value increased rapidly to -1.2, then the Poisson’s ratio value increased moderately at the following normalised strain. At the normalised strain of 0.339, this value exceeded that of the tubular structure with a PSF=100%.

For the auxetic tubular structure, the uniformed expansion ratio of diameter was a significant factor to estimate its auxetic performance. As can be seen in Figure 4-18b, the expansion ratio of diameter for a tubular structure with VH cut was larger than that of a tubular structure with a PSF=100% before the normalised strain reached 0.355. After this normalised strain point, the expansion ratio of diameter for the tubular structure with a PSF=100% exceeded the value of tubular structure with VH cut until the end of the given normalised strain. The maximal expansion ratio of diameter for the tubular structure with a PSF=100% was around 1.35 which was higher than the value of VH cut tubular structure around 1.30. It should be noted that the right part of the curve for VH cut tubular structure was unsmooth, but the overall curve for a tubular structure with a PSH=100% was very smooth.

![Geometries of the tubular structure generated by PSF adjustment method, with a PSF=100%: (a) the front view; (b) the perspective view.](image)
Figure 4-17 Deformation process for the tubular structure generated by PSF adjustment method under quasi-static tension, with a PSF=100%.

Figure 4-18 Curves of auxetic performance as a function of normalised strain for two kinds of tubular structures: (a) curves of Poisson’s ratio as a function of normalised strain; (b) curves of expansion ratio as a function of normalised strain.

Figure 4-19 Configurations of rotating unit cells generated using two methods: (a) PSF adjustment method; (b) VH cut method.

We attributed the considerable turbulence of the curve for the VH cut tubular structure in Figure 4-18a and the unsmooth curve for the VH cut tubular structure in Figure 4-18b to the unsmooth
transition of the geometry of the unit cell, as can be seen in Figure 4-19a. Compared with the geometry of unit cell for VH cut tubular structure, the unit cell for the tubular structure with a PSF=100% has a smooth transition at the joint rotating parts, as can be seen in Figure 4-19b. Therefore, the overall tensile deformation of the tubular structure with PSF=100% is more moderate and result in the smooth curves as shown in Figure 4-18.

4.2.3 The conclusion of this section

Inspired by the latest work of Grima et al. [46], where they reported that a conventional non-auxetic sheet could be transformed into an auxetic metamaterial through a random cut method. As an extension work, we investigated the auxetic performance of the tubular structures generated by cutting technique through FEA method. Also, we compared the auxetic performance of the vertical and horizontal (VH) cut tubular structure with our designed auxetic tubular structure [244] using the pattern scale factor (PSF) adjustment method [241], with a PSF=100%. From the obtained FEA results, the following conclusions could be drawn:

1. The tubular structure generated using the random cut method has been investigated for the first time. It is found that the random cut method is unsuitable for generating auxetic tubular structures.

2. The tubular structure with PSF=100% has a larger maximal expansion ratio of diameter than the tubular structure generated using the VH cut method.

3. The unit cell of the tubular structure generated using PSF adjustment method has smoother transition curves than that of the tubular structure generated using the VH cut and the random cut methods.

4. The auxetic performance of tubular structures generated using the PSF adjustment method is more stable than that of tubular structures generated using the VH cut method.
4.3 The summary

The most significant feature of the designed auxetic tubular structure in this chapter is its tunability by simple control parameters, i.e. the PSF and the plastic strain hardening ratio of the base material. The mechanical properties of the tubular structure could be easily adjusted by these two parameters individually. Most of the existing auxetic stents only exhibit auxetic behaviour under tension. The designed simple tubular structure has auxetic behaviour in both compression and tension. Therefore, our designed metallic tubular structure not only has potential to be used in the medical field but could also be employed in other structures (e.g. in armoured vehicles to absorb impact loading). The same design approach could be extended to the development of new composite materials and structures with auxetic behaviour.

Also, it is a fascinating work to investigate the mechanical performance of auxetic tubular structures composed of 2D auxetic metamaterial proposed by Grima et al [46] where a conventional sheet with randomly oriented cuts are found to have auxetic behaviour during tensile deformation. Therefore, in the section of 4.2, simple tubular structures generated using cutting techniques including vertical-horizontal and randomly cut methods are designed. By employing the finite element method, the mechanical properties of three types of tubular structures, e.g., tubular structures with vertical-horizontal and randomly cut perforated slits and with a PSF of 100% are investigated. The tubular structure consisting randomly oriented cuts does not exhibit auxetic behaviour which indicates that the randomly cutting method is unsuitable for generating auxetic tubular structures. The tubular structures with a PSF of 100% and vertical-horizontal perforated cuts demonstrate the auxetic effect in tensile deformation. The study also demonstrates that the tubular structure with a PSF=100% has a larger maximal expansion ratio of diameter than the tubular structure generated using the VH cut method which further validates the merits of the proposed methodology in generating auxetic tubular structures.
Chapter 5

Design and characterisation of a tuneable 3D buckling-induced auxetic metamaterial

Lakes [9] first reported a re-entrant foam with negative Poisson’s ratio in 1987. Such metamaterials, which contract laterally under uniaxial compression, were called “auxetics” by Evans [8] in 1991.

For the last three decades, 2D auxetic metamaterials [13, 16, 19, 21, 37, 46, 82, 84, 107, 152, 202, 236, 246-259] were mainly studied. The boom of 3D printing technique facilitated the fabrication of 3D auxetic metamaterials [31, 93, 241, 244, 245, 260-265], which tremendously released the design freedom of 3D auxetic metamaterials. Babaee et al. [31] proposed a 3D soft auxetic metamaterial using “buckliball” as the unit cell. Huang et al. [94] designed chiral auxetic metamaterials by reconfigurable connections. Shen et al. [26] reported a 3D soft auxetic metamaterial using solid sphere cutting method, but the success of generating this auxetic unit cell was more like a coincidence.

Auxetic materials and structures have been investigated for protective purposes since their discovery [266-268]. Many researchers looked into the behaviour of auxetic and conventional foams, demonstrating that auxetic foams present higher yield strength, lower stiffness and better energy absorption [183, 266, 267, 269, 270]. Other studies of auxetic structures in composite panels demonstrated improvements in static indentation, specifically in terms of stiffness, low impact
velocities and resistance to fibre pull-out with localisation of damage, therefore overall requiring less
maintenance. Sandwich panels with certain auxetic cores have been analysed under static and
dynamic loadings, including blast-induced shockwaves. Reduction of deformation, localisation of
damage, better flexure response with lower effective shear modulus and higher maximum effective
shear strain and better energy absorption have been obtained in different studies. Auxetic
metamaterials also have great potential for biomedical applications [271].

Ideally, under one principal axis direction of the uniaxial compression or tension, 3D auxetic
metamaterial exhibits negative Poisson’s ratio in the other two principal axis directions. Here, we
name this deformation effect as the 3D auxetic behaviour of the 3D auxetic metamaterials. In contrast,
in some scenarios, under one principal axis direction of the uniaxial compression or tension, 3D
auxetic metamaterial only exhibits negative Poisson’s ratio in one of the other two principal axis
directions. This deformation effect could be regarded as the 2D auxetic behaviour of the 3D auxetic
metamaterials.

It appears to be very straightforward to design buckling-induced auxetic metamaterial using the
rotating mechanism [13, 15, 16, 79, 256]. Following our previous design approach [26, 241], we
designed a simple 3D unit cell which was composed of a solid sphere and three cuboids. Then the 3D
metamaterial composed of the designed unit cells was numerically simulated. When the radius of the
sphere and length of the unit cell were fixed, it was found that bulk metamaterial composed of the
simple unit cells exhibited auxetic performance when the thickness of the connecting bars was below
a certain value. Surprisingly the auxetic behaviour only exhibited in one of the lateral directions.
Compression tests were performed on the 3D printed models to explore this finding.

5.1 Design the unit cell of 3D auxetic metamaterial

The simple 3D unit cell was designed based on the concept of rotating mechanism. The unit cell was
composed of three perpendicularly intersectional cuboids with a solid sphere as the joint as shown in
Figure 5-1, where R is the radius of the solid sphere, T is the thickness of the connecting bars and L is
the overall length of the unit cell. According to our previous work [26] for a 3D buckling-induced metamaterial, a 3D metamaterial should exhibit auxetic behaviour when volume fraction is below 31%. Hence two sets of parameters for the proposed unit cell were selected, a slender unit cell (SUC) with thin connecting bars with the dimensions of T=2 mm, L=12.5 mm, R=4 mm and a thick unit cell (TUC) with thick connecting bars with the dimensions of T=4 mm, L=12.5 mm and R=4 mm. The volume fraction of the SUC was 16.5%, and the volume fraction of the TUC was 26.3%.

![Figure 5-1 The geometries of the unit cell from two perspectives: (a) side view; (b) isometric view.](image)

5.2 Finite element analysis

ABAQUS/Explicit solver was employed for considering the large deformation effect and complex self-contacts. For the contact property, the normal behaviour of hard contact was chosen and the contact domain was set as all with self. A linear elastic material model with Young’s modulus of 1 MPa was used in finite element models. The models were built using solid elements (ABAQUS element type C3D8 with a mesh sweeping seed size of 0.7 mm). The symmetrical mesh was distributed to the finite element model and the analyses were performed under uniaxial compression. The verification of loading rate and mesh density were conducted similarly to our previous work [241].

Two typical deformation patterns of the designed models are shown in Figure 5-2. The model with SUC clearly demonstrates an auxetic behaviour in the ZX-plane as shown in Figure 5-2a, and a zero Poisson's ratio behaviour in the ZY-plane as shown in Figure 5-2b. On the other hand, the model with TUC demonstrates a non-auxetic behaviour in both of the transverse directions as can be seen in
Figures 5-2c and 5-2d. This 2D auxetic behaviour of the designed metamaterial with SUC was unexpected and interesting which triggered us to conduct an experimental study to validate this finding.

![Figure 5-2](image)

**Figure 5-2** Two deformation patterns: (a) model with SUC, ZX-plane; (b) model with SUC, ZY-plane; (c) model with TUC, ZX-plane; (d) model with TUC, ZY-plane.

### 5.3 Experimental study and discussion

Object Connex 350 with a silicone-based rubber material (TangoPlus) and an insoluble supporting material were used to fabricate two bulk metamaterials with different unit cells, one model with the SUC and the other model with the TUC, as shown in Figures 5-3a and 5-3b. Then, the insoluble supporting material of these two models was removed manually. However, during the process of the manually removing supporting material, some unit cells at the edges of the model with the SUC were damaged, as shown in the red circles in Figure 5-3a, due to the slender connection parts with a thickness of 2 mm were brittle and difficult to handle manually. In order to avoid the possible negative effect from the manual work of removing supporting material, another model with the SUC was fabricated using a soluble supporting material (SUP706) with the support from Objective 3D (a Melbourne based company). This soluble supporting material could be easily removed by using a caustic soda and sodium metasilicate solution. The newly printed model using soluble supporting
material demonstrated a cleaner appearance compared with the model printed using the insoluble supporting material which was difficult to clean manually. The newly printed model presented an intact geometry nearly identical to the original configuration designed using a computer-aided design software, as shown in Figure 5-3c.

![Figure 5-3 Photographs of the three printed models: (a) model with the SUC printed using insoluble supporting material; (b) model with the TUC printed using insoluble supporting material; (c) model with the SUC printed using soluble supporting material. (scale bar: 10 mm)](image)

The auxetic performance of the three printed models was tested using standard quasi-static uniaxial compression test and the strain rate of 10^-3 s^-1 was adopted using a Shimazu machine. A camera was employed to record the deformation process to measure the evolution of the Poisson’s ratio of the bulk metamaterials. The typical deformation patterns of the three printed models in the two lateral directions are shown in Figure 5-4.
As can be seen from Figure 5-4a, the model with the SUC printed using insoluble supporting material demonstrated auxetic behaviour in one lateral direction. However, the overall deformation was not symmetric which was indicated by non-horizontal alignment of the marked green points in the same height. It could be resulted from the broken parts of the model, as shown in Figure 5-3a. In contrast, the model with the SUC printed using soluble supporting material exhibited a symmetric deformation as can be seen in the first row of the Figure 5-4b. It was also compared well with the finite element results in Figure 5-2a. From ZY-plane, although the model with the SUC printed using insoluble supporting material also exhibited a non-auxetic behaviour which was same as the numerical result, the specific deformation pattern was different. In the ZY-plane of finite element simulation, as can be seen in Figure 5-2b, the two middle rows of the unit cells deformed towards each other which were compared well with the experimental result of the model using soluble supporting material shown in Figure 5-4b. However, the two middle rows of the unit cells of the model printed using insoluble supporting material deformed in the opposite directions as can be seen in Fig 4(a). Therefore, for the models with the SUC, the model printed using soluble supporting material exhibited a better agreement with finite element results than that of the model using insoluble supporting material. For the model with the TUC, because the connecting bars with a thickness of 4 mm were relatively strong,
and no broken parts were found after the process of the manually removing supporting material, this model was not printed using soluble supporting material. The experimental result, shown in Figure 5-4c demonstrated a global buckling deformation pattern which was same as the numerical result as shown in Figures 5-2c and 5-2d.

In order to quantify the variation of the Poisson’s ratio of the two models, similar to our previous work [26, 241], image processing method was employed by using the marked green points. The experimental and numerical results were summarised in Figure 5-5. It can be clearly seen that the Poisson’s ratio of the model with the TUC was positive from the beginning to be the end of the compressive deformation. Before the buckling of the model with the SUC, the Poisson’s ratio values in two different lateral directions were close to each. However, after the buckling behaviour of this model, Poisson’s ratio value in ZX-plane became negative and the value in ZY-plane was maintained close to zero. Therefore, the 2D auxetic behaviour of the model with the SUC was further quantified through the calculation of Poisson’s ratio.

5.4 Altered unit cell with pattern scale factor to recover 3D auxetic behaviour

In our previous work [241, 244], a methodology was proposed which could recover the loss of the auxetic behaviour when the elastomer base material of buckling-induced metamaterials and tubular
structures was replaced by metal. Here, this methodology was employed to change the 2D auxetic behaviour to 3D auxetic behaviour. A similar procedure was employed starting with buckling analysis on the proposed model with the SUC.

A linear elastic material model with Young’s modulus of 1 MPa and Poisson’s ratio of 0.47 was used in the simulation. Lanczos was chosen as the eigen solver in the buckling analysis. The boundary condition was carefully selected similar to that in the experiments. In this work, all degrees of freedom of nodes on the bottom and top surfaces of the finite element models were constrained except that the nodal movements along the loading direction on the top surface. A commercial finite element software package ABAQUS (Simulia, Providence, RI) was employed for performing buckling analysis. The number of eigenvalues requested was set as 10. Models were built using solid elements (ABAQUS element type C3D8 with a mesh sweeping seed size of 0.7 mm as shown in Figure 5-6). A symmetrical mesh seed was distributed to the finite element model and analyze was conducted under uniaxial compression condition.

The obtained first 10 buckling modes of the central representative volume element (RVE) of the bulk material are shown in Figure 5-7a, where the second and the sixth modes were found to be cubic symmetric. Also, it was found that the eigenvalues occurred as pairs and the second mode was chosen as the desirable mode to adjust the initial geometry. According to the definition of our previous work [241, 244], the desirable mode with various PSF values is shown in the Figure 5-7b. Similar to our
previous work [241], the RVE with PSF of 20% was selected and patterned in three principal directions to form a bulk model with $8 \times 8 \times 8$ unit cells. Then a post-buckling analysis was conducted. The finite element results are shown in Figure 5-7c. It can be clearly seen that through employing our proposed methodology [241, 244], the previous 2D auxetic behaviour of the initial model, as shown in Figures 5-2a and 5-2b, was successfully changed to 3D auxetic behaviour. The Poisson’s ratio of the altered model is presented in Figure 5-8. It is worth to note that the model with a PSF of 20% demonstrates a nearly identical auxetic behaviour in the two lateral directions and the effective auxetic strain range is extended around 2.5 times from only 0.1 to 0.25. It should be noted that the reason for selecting the PSF value of 20% was to make this work to be consistent with our previous work [241, 244] where the PSF value of 20% was also chosen. Other PSF values could also be selected for the sake of changing the 2D auxetic behaviour of the initial model to 3D auxetic behaviour.
Figure 5.7 Finite element analysis results of the altered design: (a) the first ten buckling modes of the central RVE of the bulk material; (b) the second mode of the central RVE of the bulk material with different PSF values; (c) the compressive deformation process of the model with a PSF value of 20%.
5.5 The summary

In this study, a simple unit cell composed of a solid sphere and three cuboids which cross the sphere was proposed as the building cell for 3D auxetic metamaterials. Two metamaterials were generated by linearly patterning the unit cell along three principal directions. By conducting finite element simulations, we found that the metamaterial with SUC exhibited 2D auxetic behaviour under compression and the one with TUC was non-auxetic. To validate the finite element analysis results, two prototypes were fabricated using 3D printing technology where a manual procedure of removing the supporting material was involved. To eliminate the effect of the damaged edge cells caused by manually removing supporting material, another model with the SUC was manufactured using 3D printing with soluble supporting material. Furthermore, our previously developed methodology to alter the auxetic unit cell using the PSF was used to change the degraded 2D auxetic behaviour of a given design to 3D auxetic behaviour. Combining the numerical and the experimental results, the following conclusions could be drawn:

(1) Design buckling-induced auxetic metamaterials with 3D auxetic behaviour may not always work by applying the commonly used rotating mechanism.

(2) The designed simple 3D metamaterial with 2D auxetic behaviour can be changed to exhibit 3D auxetic behaviour by employing our previously proposed PSF methodology.
(3) The auxetic behaviour of the designed simple buckling-induced metamaterial can be easily controlled and tuned by adjusting the thickness of the cuboids.

From this study, 3D printing with a soluble supporting material is recommended to manufacture the 3D rubber-like material. This feature enables engineers to design a complex model without the time-consuming and burdensome work of removing insoluble supporting material. Utilising the soluble supporting material could also avoid possible damage to the printed model during the process of removing supporting material manually.
Chapter 6

Auxetic nail: design and experimental study

Auxetic materials exhibit uncommon deformation behaviour, e.g., under uniaxial compression (tension), rather than expanding (shrinking) in the lateral direction as conventional materials, auxetic materials would shrink (expand) [62, 241, 244-246, 259, 272-274]. Along with this counter-intuitive behaviour, auxetic materials are regarded to possess many desirable properties, e.g., shear resistance [69, 157], indentation resistance [115, 160, 161, 231], fracture resistance [9, 166, 167, 172], synclastic behaviour [175, 176], variable permeability [177, 178] and energy absorption [183, 233].

As a pioneer in the field of auxetic materials, Lakes [9] successfully fabricated and reported the first re-entrant foam structure with negative Poisson’s ratio in 1987. Since this breakthrough contribution, significant efforts have been made towards investigating the novel materials with negative Poisson’s ratio. Friis et al. [32] reported the first metallic re-entrant foam with negative Poisson’s ratio in the year after. Theoretically, for further investigating the underlying mechanism of auxetic behaviour, various auxetic cellular models were proposed and analysed. Re-entrant model, as a traditional cellular structure was firstly proposed by Gibson et al. [66] in 1982. Grima et al. [79] put forward a novel mechanism to achieve negative Poisson’s ratio, which was based on an arrangement with rigid squares connected together at their vertices by hinges. Lakes [86] reported a first chiral hexagonal microstructure which exhibited auxetic behaviour. The theoretical and experimental investigations on
a two-dimensional chiral honeycomb were conducted by Prall et al. [87]. A crumpled sheet model was also reported which could be adjusted to demonstrate auxetic behaviour [99]. As a typical type of perforated sheets model, conventional materials containing diamond or star shaped perforations could demonstrate auxetic behaviour both in compression and tension [107]. In addition, other models were also reported to have auxetic behaviour, e.g., nodule-fibril model [111], missing rib model [112, 152], egg rack model [113], tethered-nodule model [114], hexatruss model [115] and entangled wire model [117].

The contraction behaviour of auxetic materials and structures against compressive loadings makes them potential candidates for the protective structure and defence applications [183, 266-270]. Researchers have compared the dynamic behaviours between auxetic and conventional foams and demonstrated that auxetic foams present higher yield strength, lower stiffness and better energy absorption. The auxetic structures and materials have also shown enhanced static indentation resistance and stiffness improvement. In the studies [183, 269, 270], sandwich panels with auxetic cores have also been investigated against the impulsive loading, e.g., blast-induced shockwave and low-velocity impact. It has been evidently shown that the deformation, localised damage and structural flexure of the auxetic panels could be reduced noticeably.

Although a considerable progress has been made in the field of negative Poisson ratio metamaterials in the last three decades both theoretically and experimentally, most of the previous work focused on two-dimensional auxetic materials and structures. Attributed to the rapid development of 3D printing technique in the past years, manufacturing the 3D auxetic models with complicated geometrical configuration was no longer a technical constraint and resulted in several successful fabrications of 3D auxetic materials and structures [26, 137, 241, 244, 262, 275]. Unfortunately, the real application of auxetic materials is still in its infancy and more pioneering works of utilising auxetic materials towards practical application are in need.
Choi et al. [59] reported that an auxetic fastener could be easier to insert and harder to pull out. Later on, Grima et al. [58] stated that auxetic nails were a potential application for auxetic materials based on the concept that auxetic nails become thinner when knocked in and become fatter when pulled out, as shown in Figure 6-1. Inspired by their work and based on our previous work of auxetic tubular structures [244], the first auxetic nails were designed, fabricated and experimentally investigated.

![Figure 6-1 Illustration of auxeticity for auxetic nails: (a) during push-in; (b) during pull-out. (The nails in grey and red colour represent the configurations of the nails before and after deformation, respectively)](image)

**6.1 Design of auxetic and non-auxetic nails**

Based on our previous work of auxetic tubular structures [244], four groups (A, B, C, D) of nails with three categories of nails in each group were designed. The three categories of nails included an auxetic nail (AN) with alternatingly patterned elliptical holes, a non-auxetic nail with circular holes (CN) and another non-auxetic nail of the solid nail (SN) without holes. The geometrical configurations of the three categories of nails in different perspectives are shown in Figure 6-2. The nail bodies of ANs and CNs were hollow. Also, the number of holes and superficial area of the holes between ANs and CNs were kept the same.
In total, twelve different nails were designed. The geometric parameters of these nails are shown in Figure 6-3, where H is the overall length, h is the length of the nail head, L is the length of the nail body, D1 is the diameter of the nail head, D2 is the outer diameter of the nail body, T is the wall thickness of the nail body, θ is the angle of the nail bottom, N is the number of the rows of holes in the longitudinal direction. It should be noted that the three categories of nails which belong to the same nail group (A, B, C, D) have all the identical parameters shown in Figure 6-3 except for the parameter of T based on the fact that the nail type of SN was solid, but the parameter of T for the AN and CN was still identical.

In order to make our experiments more convincing, the effects of six parameters of H, h, L, T, θ and N were investigated. Two parameters of D1=8 mm and D2=6 mm were fixed to satisfy the acceptable
range of the clip of the Shimadzu machine, i.e., from 4 to 9 mm. The specific parameters for the four groups of nails (A, B, C, D) are listed in Table 6-1. For the simplicity of labelling the nails, three capital letters were used to represent one of the twelve nails with different nail group and nail type. The first two represent the nail type and the third one represents the nail group, i.e., ANA is the auxetic nail in group A, CNA is the nail with circular holes in group A, and ANC is the auxetic nail in group C. It should be noted that most of those parameters were selected due to the manufacturing capacity of the available 3D printing machine.

<table>
<thead>
<tr>
<th>Nail group</th>
<th>D₁</th>
<th>D₂</th>
<th>H</th>
<th>h</th>
<th>L</th>
<th>T</th>
<th>θ (°)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8</td>
<td>6</td>
<td>26.7</td>
<td>3</td>
<td>18.5</td>
<td>1.5</td>
<td>30</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>6</td>
<td>25.7</td>
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<td>18.5</td>
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<td>15</td>
<td>0.0</td>
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</table>

6.2 Fabrication of the designed nails

The 3D printing technique (Shapeways, New York) was employed to fabricate the designed twelve types of nails. The designed twelve types of nails were not only printed using stainless steel but also printed using brass due to the high ductility of brass was beneficial for demonstrating larger auxetic deformation, as shown in Figure 6-4. The nails with gold colour were printed using brass material and the nails with silver colour were printed using stainless steel. It should be noted that the minimal link length of the designed auxetic nail was only 1.1 mm and this value approached the limit (1 mm) of the 3D printing company (Shapeways, New York) for manufacturing brass and stainless steel production.
Figure 6-4 3D printed twelve different types of nails in four nail groups using brass and stainless steel materials: (a) auxetic nails (ANs); (b) nails with circular holes (CNs); (c) solid nails (SNs). (The nails with gold colour and silver colour are printed using brass and stainless steel, respectively) (scale bar: 10mm)

6.3 Experiments

6.3.1 Pine timber experiments

Considering the overall stiffness of the printed nails and the available materials, timber was selected as a candidate for carrying out push-in and pull-out tests. One piece of cuboid plantation pine timber with a length of 2.7 metres and cross section of 88 × 88 millimetres was selected, as shown in Figure 6-5a. Then the four rectangular timber surfaces were marked with different numbers of straight lines from one end to the other end. After that, the long timber was cut into many pieces of small cubic timbers with the same length of 50 millimetres, as shown in Figure 6-5b. It should be noted that there is glue bond appears between the second and fourth pine timber surfaces as indicated by the dashed line in Figure 6-5b. In order to minimise the influence caused by the inhomogeneous properties of timbers, the push-in and pull-out performance of different nails was firstly tested on the small timbers surface with one straight line. The procedure of marking the timber with straight lines was to identify the surfaces of the small pieces of the timbers after the cut because each nail was started to test on an intact piece of small timber.
The testing material of pine timber: (a) the whole piece of timber before cut; (b) one small piece of timber after the cut. (According to the Australian timber standard AS 1720.1 [272], the pine timber belongs to the MGP12 stress grade with a characteristic compressive strength value of 22-24 MPa) (scale bar: 10 mm)

The experimental setup is shown in Figure 6-6a, where a Shimadzu machine was used to drive the nail to carry out compressive and tensile tests. A webcam and a square ruler were employed to observe the front and back angle of the nail to guarantee the nail was perpendicular to the timber top surface. Two bolts and one steel plate were used to fix the timber to avoid the movement of timbers during the push-in and pull-out nail tests. It should be noted that the steel plate on the timber had a circular hole with the diameter of 15 millimetres in the central area. Also, the specific definitions of the red code marked on the timber are illustrated in Figure 6-6b.

The loading procedure for testing one nail can be divided into four steps. Firstly, adjusted the machine head to the position when the bottom of the nail just attached the timber. Secondly, started to
compress the nail into a timber at a constant speed of 7.5 mm/min and stopped at a certain displacement, i.e., 15 mm for nails in A, B and D group, 13 mm for nails in C group. This setting was to avoid the clip to touch the steel plate because the overall length of the nails in C group was shorter than the value of nails in other groups, as shown in Figure 6-7a. The raw displacement and force were saved and the maximum compressive force was recorded. Thirdly, started the pull-out test by lifting the machine head at the constant speed of 7.5 mm/min until nearly all parts of the nail were pulled out from the timber, as shown in Figure 6-7b, then the raw displacement and force were saved and the maximum tensile force was recorded. Fourthly, in order to minimise the influence caused by the inhomogeneous properties of timber, the nail was tested on different surfaces of one small cubic timber unless the nail was deformed too much after a test and not suitable for conducting another test.

Figure 6-7 Demonstration of experimental procedures: (a) push-in test; (b) pull-out test; (c) repetitive tests on different surfaces of one small cubic timber.

In total, 27 nails were tested using pine timber, including 15 brass nails (5 auxetic nails, i.e., 2 ANA, 1 ANB, 1 ANC, 1 AND; 10 non-auxetic nails, i.e., 2 CNA, 1 CNB, 1 CNC, 1 CND and 2 SNA, 1 SNB, 1 SNC, 1 SND) and 12 steel nails, i.e., one for each type of the 12 designed nails. Typical force and displacement curves are shown in Figure 6-8, where the maximum compressive forces and maximum tensile forces are represented using pentagram symbols. As can be seen in the pine timber
experiments, the maximum compressive force appeared in the middle of the push-in test for the ANA_B1 nail and in the end of the push-in test for the ANA_B2 nail. It should be noted that the gaps between compressive and tensile force curves were caused by the slow procedure of the wood creep during the short period of the inversion of controlled forces in Shimadzu machine. Also, the undiscussed green curves were obtained from the medium-density fibreboard (MDF) experiments which would be presented in the next section.

![Image of force and displacement curves](image)

*Figure 6-8 Typical force and displacement curves for the nails tested using pine timber and medium-density fibreboard (MDF).*

The maximum compressive force and maximum tensile force for each tested nail in push-in and pull-out experiments using pine timber are summarised in Table 6-2, where FMC is the maximum compressive force and FMT is the maximum tensile force. It should be noted that the auxetic nails were compared with the non-auxetic nails in the same group. To minimise the effect caused by the inhomogeneous property of pine timber, the maximum compressive forces and the maximum tensile forces of ANs were compared with the values of the corresponding CNs and SNs recorded in the same timber surface, as the values shown in the same row of Table 6-2. The lowest maximum compressive force and the highest maximum tensile force in the same row are highlighted in red colour (If the designed auxetic nails (ANs) could exhibit superior push-in and pull-out performance to the non-auxetic nails (CNs and SNs), the red numbers should always appear in the columns with orange shading colour).
It can be seen that all the steel nails were tested on four different timber surfaces. All of the steel nails remained intact after tests due to their high stiffness and they were tested on four surfaces of pine timbers. Some of the brass nails with the nail body wall thickness of 1 mm were only tested on the first and second timber surfaces due to their low stiffness and excessive deformation.

Table 6-2 The maximum compressive and maximum tensile forces of the 27 tested nails in different timber surfaces (The data of AN, CN and SN are in the orange, blue and green shading colours, respectively) (unit: N).

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<th>Timber surface</th>
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<th>$F_{MC}$</th>
<th>$F_{MT}$</th>
<th>Nail name</th>
<th>$F_{MC}$</th>
<th>$F_{MT}$</th>
<th>Nail name</th>
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Ideally, when performing push-in and pull-out tests for the nails on timber, it was anticipated that auxetic nails would experience an apparently lower maximum compressive force and a larger maximum tensile force compared to the non-auxetic ones. However, according to the experimental results summarised in Table 6-2, this anticipated result was not confirmed. We assumed that the surface roughness of all steel and brass SNs were very similar. The data of SNs were used to estimate the variation of the pine timber, as shown in Figure 6-9. Those data demonstrate a significant
variation, e.g., for the SNs, the maximum compressive force is $1551.6 \pm 701.3$ N and the maximum tensile force is $197.1 \pm 199.1$ N.

![Figure 6-9](image)

Figure 6-9 Experimental data of SNs on four different pine timber surfaces: (a) maximum compressive force; (b) maximum tensile force.

For the data analysis, the effects of $H$, $h$, $L$, $T$, $\Theta$ and $N$ were indistinguishable due to the scattering of mechanical properties of the pine timber. Only four repeating tests were conducted for most type of nails, which was a bit weak to support this conclusion. However, a large amount of data could be used to analyse the three primary factors that may contribute to the variations of maximum push-in and pull-out forces, i.e., mechanical properties of timber, the surface roughness of nails and the auxetic deformation of nails. It can be clearly seen, as shown in Figure 6-9, the maximum compressive forces are much larger than the maximum tensile forces, which indicated the nails would have a larger compressive deformation and a smaller tensile deformation if the designed nails possess the same compressive stiffness and tensile stiffness.

Because there is glue bond existed in the between of the second and forth pine timber surfaces, as shown in the dashed line of Figure 6-5b, which may significantly affect the data analysis, only the data collected from the first and the third pine timber surfaces were analysed, as shown in Figure 6-10. It should be noted that according to the curves shown in Figure 6-8, the compressive forces are positive and the tensile forces are negative. For the convenience of analysing, only the absolute values of these forces were compared, and the tensile forces shown in Figure 6-10 became positive as well. It
can be clearly seen that the average maximum compressive forces for the three categories of brass nails were close to each other, i.e., less than a variation of 9 N as shown in the upper section of Figure 6-10a. But the values of the average maximum compressive forces for the three categories of steel nails exhibited a significant variation, i.e., larger than 100 N, which may mainly attribute to the inhomogeneous of pine timber. The average maximum tensile forces for the three categories of brass nails had a small variation, i.e., around 22 N as shown in the lower section of Figure 6-10a. The average maximum tensile forces for the three categories of steel nails had a larger variation, i.e., larger than 53 N as shown in the lower section of Figure 6-10b. Also, it can be clearly seen that the average maximum tensile forces for CNs were larger than the values for the other two categories of nails (ANs and SNs), which may indicate that the surface roughness of CNs was larger than the surface roughness of the other two categories of nails.

Figure 6-10 Maximum compressive force and maximum tensile force of brass and steel nails tested using pine timber: (a) brass nails; (b) steel nails.

6.3.2 Medium-density fibreboard experiments

To reduce the negative effect caused by testing material, the testing material of medium-density fibreboard (MDF) was selected which was considered to have better homogeneous properties. Before nail tests were performed, the homogeneous property of the MDF was checked by using a steel SN (solid nail). The curves of compressive force and displacement, shown in Figure 6-11a, demonstrating a similar tendency and the variations were kept within 12%. Therefore, the homogeneity of the MDF
was confirmed superior to that of pine timber and this feature was significant for the nail tests to obtain more reliable conclusions. The small MDF panel after the trial tests is shown in Figure 6-11b, where the red numbers indicate the order of the trial tests. The location of the holes was far enough from each other to eliminate any boundary effects.

Figure 6-11 Initial homogeneity check on the first piece of MDF: (a) compressive force and displacement curves using a steel solid nail; (b) the MDF after 10 trial compressive tests. (scale bar: 10 mm)

In order to eliminate the possible inhomogeneity caused by different pieces of MDF, before the push-in and pull-out nail tests, the trial tests were also conducted five times for the second piece of MDF using the same steel solid nail, as shown in Figure 6-12, where the position of the trial tests was marked with black numbers. The variations of the compressive force of these five tests were also maintained within 12% which compared well with the result shown in Figure 6-11a. Then all the remaining nails were tested on the second piece of MDF, and the position of the corresponding tests was recorded in red marks as shown in Figure 6-12.

Figure 6-12 The second piece of MDF after tests (scale bar: 10 mm)
In total, 21 printed nails were tested using MDF, including 10 brass nails (4 auxetic nails, i.e., 1 ANA, 1 ANB, 1 ANC, 1 AND; 6 non-auxetic nails, i.e., 1 CNB, 1 CNC, 1 CND and 1 SNB, 1 SNC, 1 SND) and 11 steel nails, i.e., one for each type of the 12 designed nails except for the SNA. Because of the good homogeneity of MDF, the force and displacement curves of all tested nails had a similar tendency. Only one typical force and displacement curves are shown in the green dashed line in Figure 6-8, where the maximum compressive force appears at the end of the compressive test. Also, it can be clearly seen that the MDF was stiffer than pine timber as the maximum compressive force recorded for MDF was higher than that of pine timber.

The maximum compressive force and maximum tensile force for each tested nail in push-in and pull-out tests using MDF are summarised in Table 6-3, where FMC is the maximum compressive force and FMT is the maximum tensile force. Also, same as Table 6-2, the lowest maximum compressive force and the highest maximum tensile force in the same row are highlighted in red colour.

### Table 6-3 The maximum compressive and maximum tensile forces of the 21 tested nails using MDF (The data of AN, CN and SN are in the orange, blue and green shading colours, respectively) (unit: N).

<table>
<thead>
<tr>
<th>Nail name</th>
<th>FMC</th>
<th>FMT</th>
<th>Nail name</th>
<th>FMC</th>
<th>FMT</th>
<th>Nail name</th>
<th>FMC</th>
<th>FMT</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANA_B1</td>
<td>2163.1</td>
<td>-195.5</td>
<td>CNA_B1</td>
<td>2055.3</td>
<td>-207.4</td>
<td>SNA_B1</td>
<td>1853.3</td>
<td>-205.9</td>
</tr>
<tr>
<td>ANA_S1</td>
<td>1869.5</td>
<td>-178.0</td>
<td>CNA_S1</td>
<td>2097.5</td>
<td>-265.0</td>
<td>SNA_S1</td>
<td>1853.3</td>
<td>-205.9</td>
</tr>
<tr>
<td>ANB_B1</td>
<td>2037.7</td>
<td>-177.1</td>
<td>CNB_B1</td>
<td>2023.6</td>
<td>-202.9</td>
<td>SNA_B1</td>
<td>1853.3</td>
<td>-205.9</td>
</tr>
<tr>
<td>ANB_S1</td>
<td>1961.9</td>
<td>-182.8</td>
<td>CNB_S1</td>
<td>1856.5</td>
<td>-192.1</td>
<td>SNA_S1</td>
<td>1853.3</td>
<td>-205.9</td>
</tr>
<tr>
<td>ANC_B1</td>
<td>1918.0</td>
<td>-152.5</td>
<td>CNC_B1</td>
<td>1709.9</td>
<td>-124.4</td>
<td>SNC_B1</td>
<td>1796.9</td>
<td>-122.7</td>
</tr>
<tr>
<td>ANC_S1</td>
<td>1638.1</td>
<td>-92.6</td>
<td>CNC_S1</td>
<td>1709.9</td>
<td>-124.4</td>
<td>SNC_S1</td>
<td>1796.9</td>
<td>-122.7</td>
</tr>
<tr>
<td>AND_B1</td>
<td>1581.6</td>
<td>-127.2</td>
<td>CND_B1</td>
<td>1973.9</td>
<td>-192.6</td>
<td>SND_B1</td>
<td>1856.8</td>
<td>-121.9</td>
</tr>
<tr>
<td>AND_S1</td>
<td>1597.8</td>
<td>-108.8</td>
<td>CND_S1</td>
<td>1611.9</td>
<td>-97.5</td>
<td>SND_S1</td>
<td>1846.7</td>
<td>-81.9</td>
</tr>
</tbody>
</table>

Similar to the pine timber experiments, the data of the maximum compressive forces and the maximum tensile forces of brass nails and steel nails tested using MDF were analysed as shown in Figure 6-13. It can be clearly seen that the average maximum compressive forces and average maximum tensile forces of CNs were larger than the corresponding values of ANs and SNs. Also, the average maximum compressive forces and average maximum tensile forces of ANs were larger than the corresponding values of SNs. These results confirmed with the conclusion that the surface roughness of CNs was larger than the other two categories of nails. Because of the good homogeneous of the MDF, the phenomenon that the surface roughness of ANs was larger than SNs was also
demonstrated. Therefore, it can be concluded that the nails with holes had larger surface roughness than solid nails. Also, when the superficial area of holes was maintained the same, the surface roughness of nails with circular holes was larger than the nails with alternately patterned elliptical holes. Similar to the result of pine timer experiments, the maximum compressive forces are much larger than the maximum tensile forces, as shown in Figure 6-13. Therefore, an ideal auxetic nail should have high stiffness in compression and low stiffness in tension to avoid too large deformation when pushed in and to obtain large enough auxetic deformation when pulled out for the nail applications.

![Figure 6-13 Maximum compressive force and the maximum tensile force of the brass and steel nails tested using MDF: (a) brass nails; (b) steel nails.](image)

### 6.4 Finite element analysis

The auxetic deformation of the designed auxetic nails could not be detected when the nails were pushed into and pulled out from the testing material. To check the scale of auxetic deformation and clarify the reason behind the unanticipated results observed in the previous tests, the auxetic deformation of the designed ANs was further investigated using the finite element models. One typical nail of ANC (auxetic nail in group C) was selected to check the auxetic deformation of the designed nails. Because the auxetic deformation occurred in the nail body rather than the nail head or
bottom, only the nail body of the ANC was simulated as shown in Figure 6-14a, which was similar to our previous work of auxetic tubular structures [244].

The ABAQUS/Explicit solver was employed for considering large deformation. The mesh dependence analysis was conducted and the solid element (ABAQUS element type C3D8) was adopted with a mesh size of 0.2 mm for the finite element model. As for the base material of brass, the same bilinear elastic-plastic material model as our previous work [244], with Young’s modulus of 87 GPa and a hardening modulus of 1.7 GPa was used in finite element model. All the degrees of freedom of the top and bottom nodes were constrained except for the nodal movement on the top nodes, which was allowed to move in the longitudinal direction. The control of displacement was defined during the simulation which could be divided into two steps: firstly, the top nodes of the nail body were defined to move downwards along the longitudinal direction (compressive simulation), then when the maximum compressive force of around 1918 N was reached, the corresponding time was recorded. Secondly, the displacement before the time was maintained to simulate the overall compressive deformation, then the opposite displacement was added to the overall displacement control, i.e., the top nodes of the nail body were defined to move upwards along longitudinal direction (tensile simulation) until the maximum tensile force of around 152.5 N was reached. The force and displacement curves both in compression and tension are shown in Figure 6-15. It can be seen that when the nail body was under a vertical compressive force of around 1918 N, the height of the nail body decreased 0.87 mm. Then when a maximum tensile force of around 152.5 N was reached, the displacement of nail body decreased a little bit from 0.87 mm to 0.83 mm.
In order to represent the auxetic deformation of the model, the three principal displacements of the twelve red points were recorded in deformation to calculate the change in outer diameter of the nail body during the compressive and tensile deformation, as shown in Figs. 14b and 14c. It was unsuitable to define the Poisson’s ratio for a nail body to show the auxetic deformation. Alternatively, the average outer diameter was used to illustrate the Poisson’s ratio behaviour, which was equal to the average value of the six instantaneous outer diameters in each marked point layer. The original outer diameter of the nail body was 6 mm. The average outer diameter of the nail body during the simulation is shown in blue dashed line in Figure 6-15, where can be seen that the average outer diameter was less than 6 mm and maintained to decrease in the compression from 6 mm to 5.920 mm, which indicated the auxetic behaviour exhibited in compression. Also, it can be seen that the average outer diameter of the nail body became to increase from 5.920 mm to 5.924 mm during the tensile deformation. However, the auxetic behaviour was too small to be able to identify during the experiments.

Figure 6-14 One typical auxetic nail body: (a) the geometrical configuration of nail body; (b) nail body with marked red points in front view; (c) nail body with marked red points in top view.
Through a further check on the designed ANs using the experimentally validated finite element model, the reasons why the anticipated superior [58, 59] push-in and pull-out performance of the designed ANs could not exhibit through the push-in and pull-out experiments can be explained. Firstly, the base materials of the printed nails are metals that have very limited deformation under a certain loading force and this assumption has been confirmed by the finite element simulations. Secondly, although best efforts have been made to avoid the inhomogeneous properties of testing materials, this negative effect could not be totally avoided. Thirdly, although the holes of the designed nails of ANs and CNs have the identical superficial area, the surface roughness for these two categories of nails could not be maintained the same.

Because the nail would experience a large compressive force and a relatively lower tensile force during the push-in and pull-out tests, the ideal auxetic nails should not only possess large auxetic behaviour but also have high compressive stiffness and low tensile stiffness to satisfy their practical application. In order to meet these two criteria at the same time, two possible methods are proposed. Firstly, based on the pattern scale factor (PSF) methodology [241, 244], the auxetic nail body with a full densification property under compression (approximately equivalent to the nail body with 100% PSF value [244]) may work for the applications of auxetic nails. Secondly, the minimal link of the ideal auxetic nail body should be small enough to achieve a large tensile auxetic deformation but also the ideal auxetic nail should be strong enough to sustain a certain compressive force. However, it is a
technical problem which may be solved by using a more advanced manufacturing technology and materials in the future.

6.5 The summary

In this study, the first auxetic nails were designed, fabricated and experimentally investigated. According to the previous work [58, 59, 244], compared with conventional nails, when knocked in and pulled out from a timber, auxetic nails could be easier to be pushed in and more difficult to be pulled out based on the auxetic behaviour. The push-in and pull-out performance of the printed auxetic and non-auxetic nails was investigated through loading on pine timber and medium-density fibreboard, respectively. The maximum compressive force and maximum tensile force were regarded as the two key parameters to estimate the push-in and pull-out performance of the designed nails. Based on the experimental results and further investigation using finite element method, the following conclusions could be drawn:

(1) Auxetic nails do not always exhibit superior push-in and pull-out performance to conventional nails. It is still a challenge to design and fabricate metallic auxetic nails with better push-in and pull-out performance.

(2) The surface roughness of the nails plays an important role in the push-in and pull-out performance of auxetic nails which has been underestimated before.

(3) The shape of the holes on the surface of nails affects the surface roughness of nails. With the identical surface area, the nails with circular holes are rougher than those with alternatingly patterned elliptical holes.

(4) The ideal auxetic nails should have high compressive stiffness and low tensile stiffness.

Most of the literature mainly focused on the desired performance of auxetic materials and stated or speculated that they could be superior for numerous applications including smart filters, sensors, protective and medical devices. Few of them realised or admitted limitations and disadvantages of auxetic materials, such as lower stiffness due to the high porosity and high manufacturing cost
because of the complicated geometrical configuration which requires, e.g., 3D printing. We believe this work could offer readers a new perspective to re-examine real advantages and limitations of auxetic materials and structures, especially when many publications in the field might have overestimated their superiorities.
Chapter 7

Conclusion, limitation and future work

In this thesis, a novel pattern scale factor (PSF) methodology of generating 3D auxetics has been developed. The main concept of the methodology is to import desirable buckling mode to an initially unaltered buckling-induced auxetics. Employing this methodology, a 3D metallic auxetic metamaterial and a 3D auxetic tubular structure have been designed, fabricated and investigated. The effectiveness of the proposed methodology has been validated through numerical simulations and laboratory experiments. In addition, the methodology has been confirmed to have the capability to recover 2D auxetic behaviour to 3D auxetic behaviour for the 3D buckling-induced metamaterial composed of simple unit cells. As a pioneering work to extend the potential applications of auxetic materials, the first auxetic nails have been designed, fabricated and experimentally investigated.

In this chapter, the aforementioned new research findings are firstly summarised, followed by a brief description of the limitation of this research and an introduction to the future work of auxetics.

7.1 Conclusion

The auxetic properties of the auxetic metamaterials and structures were previously regarded as mainly depend on the geometrical configuration rather than chemical composition or base material. By
contrast, the work in this thesis has offered solid evidence that base material has a significant effect on the auxetic properties of 3D auxetic metamaterial and tubular structure. It is also found that when the elastomeric base material is replaced with the metallic material, the auxetic effect of the buckling-induced 3D auxetics would disappear. This finding has also been validated numerically and experimentally for the proposed 3D auxetic metamaterial and 3D auxetic tubular structure.

In order to regain auxetic properties when the elastomeric base material of the buckling-induced 3D auxetics is replaced by metal, a novel methodology of generating 3D auxetics has been developed and its procedures can be summarised as four steps as shown in Figure 7-1. Firstly, design buckling-induced 3D auxetics; secondly, conduct buckling analysis on the original finite element model of 3D buckling-induced auxetics with linear elastic base material; thirdly, identify the desirable buckling mode for the 3D auxetics; lastly, alter the geometry of the representative volume element (RVE) using the desirable buckling mode and pattern the altered RVE in three principal directions to form a 3D metallic auxetic metamaterial, or alter the original tubular structure using the desirable buckling mode to form an auxetic tubular structure.

The proposed 3D metallic auxetics have superior properties compared to conventional auxetics. One of the most significant advantages of the proposed auxetics is that the mechanical properties of these models could be easily tuned by one single parameter of the pattern scale factor (PSF). In addition, these models could exhibit auxetic behaviour both in compression and tension. When a certain value of PSF is imported to the original geometrical configurations, the proposed models could demonstrate an approximately identical auxetic performance in compression and tension. This feature is crucial for some applications, e.g., sensors and smart filters. Another advantage of the proposed 3D auxetics is that the base material of these models is metal which makes these 3D metallic auxetics stronger than elastomeric auxetics to satisfy some potential applications, e.g., protective devices to reduce impact loading and absorb more energy. In addition, the proposed 3D metallic auxetics could maintain auxetic effect in a large effective strain range, which is useful for biomedical applications, e.g., oesophageal stent and blood vessel. Last but not the least, good geometric symmetry is an apparent
advantage of the proposed auxetics, especially for some applications which require symmetrical and precise auxetic effect, e.g., 3D sensors.

Figure 7-1 The methodology of generating 3D metallic auxetic metamaterial and 3D metallic tubular structure: (a) 3D auxetic metamaterial; (b) 3D auxetic tubular structure.

Apart from the function of generating 3D auxetic metamaterials and 3D auxetic tubular structures with tuneable mechanical properties, the PSF methodology has also been utilised to recover 2D auxetic behaviour to 3D auxetic behaviour for a simple 3D auxetic buckling-induced metamaterial, as shown in Figure 7-2. It is found that design buckling-induced auxetic metamaterials with 3D auxetic behaviour may not always work by applying the commonly used rotating mechanism. The auxetic behaviour of the designed simple buckling-induced metamaterial can be easily controlled by adjusting the thickness of the cuboids.
This auxetic property has been speculated to be useful for the applications of auxetic nails as an apparent conclusion. However, no related work has been reported. In this study, the first auxetic nails are designed, fabricated and experimentally investigated. It is found that the auxetic nails do not always exhibit superior mechanical performance to non-auxetic ones. It is still a challenge to design and fabricate metallic auxetic nails with better push-in and pull-out performance. The surface roughness of the nails plays an important role in the push-in and pull-out performance of auxetic nails which has been underestimated before. The shape of the holes on the surface of nails affects the surface roughness of nails. With the identical surface area, the nails with circular holes are rougher than those with alternatingly patterned elliptical holes. The ideal auxetic nails should have high compressive stiffness and low tensile stiffness.

### 7.2 Limitation

In this thesis, a novel pattern scale factor (PSF) methodology has been proposed, which has been utilised to generate 3D metallic auxetic metamaterials and 3D auxetic tubular structures with tuneable mechanical properties. Also, this methodology has been confirmed to have the capacity to change 2D auxetic behaviour of a simple 3D metamaterial to 3D auxetic behaviour, as shown in Figure 7-2.

**Figure 7-2** Illustration of utilising pattern scale factor (PSF) methodology to change 2D auxetic behaviour to 3D auxetic behaviour for a 3D metamaterial.
However, this methodology still has some disadvantages. One of the most crucial disadvantages of the PSF methodology is that it has to start with 3D buckling-induced auxetics. In other words, the varieties of the possibly designed 3D metallic auxetics highly depend on the varieties of the 3D buckling-induced auxetics. However, it is difficult to find many buckling-induced 3D auxetics. The second disadvantage of the PSF methodology is that although the mechanical properties of the designed 3D auxetic metamaterials and auxetic tubular structures could be easily tuned by one single parameter of PSF, the volume fraction of these auxetics would change when the PSF value varies. Therefore, using PSF methodology to adjust mechanical properties of 3D auxetics may not work in scenarios where a specific volume fraction is fixed or preferred.

More importantly, all of the proposed auxetics have substantial porosities in their geometrical configurations which inevitably reduce their mechanical capability, e.g., stiffness and strength. In other words, obtaining some desirable properties of auxetic materials, such as enhanced shear resistance, improved indentation resistance and superior energy absorption, is actually at the cost of sacrificing the mechanical performance in the beginning. However, in most scenarios, the obtained auxetic behaviour could not compensate for the loss of the mechanical performance, which tremendously impedes wider applications for auxetics, e.g., energy absorption. However, the disadvantage of the porous microstructure of auxetic materials is rarely discussed.

Last but not least, the cost of manufacturing auxetic materials is still very high. For most 3D auxetic materials with reliable auxetic behaviour, due to their complicated geometrical configurations, fabrication often requires, e.g., 3D printing. However, manufacturing auxetics using 3D printing often costs a lot, especially for 3D metallic auxetics with large volumes. Therefore, it is still a challenge to realise the mass production of auxetics at a low cost which significantly impedes wide applications of auxetics.

7.3 Future work
For the last three decades, since Lakes reported the first re-entrant foam structures with negative Poisson’s ratio in 1987, a significant progress has been made. Although many potential applications of auxetics have been proposed and some preliminary application has been reported, most of the reported auxetics still maintain in their infancy, and very few auxetics have been developed to the stage of practical application. Therefore, it is worthy for researchers to make efforts towards exploring the potential applications of auxetics and enable more people could witness and experience the advantages of auxetics.

Recently, hierarchical auxetic materials and structures have attracted considerable attentions. Sun et al. [276] reported that hierarchical tubes exhibited auxetic behaviour under longitudinal axial tension through theoretical method. Based on rotating units mechanism, Gatt et al. [186] reported a hierarchical 2D system with auxetic behaviour by employing the numerical method. Tang et al. [253] proposed a design of hierarchically cut hinges and the auxetic behaviour of this design was validated both numerically and experimentally. A hierarchical configuration composed of rectangular perforation that exhibited auxetic behaviour was reported by Billon et al. [277]. However, all of their works are mainly limited to the 2D cases, and it would be an interesting and vital work to design a 3D hierarchical system which could exhibit auxetic behaviour under deformation.

Most of the researchers only focus on auxetic behaviour either in compression or tension. Also, most of the reported auxetic materials and structures could not demonstrate auxetic behaviour in both compression and tension, let alone an auxetic material and structure which could exhibit an identical auxetic behaviour both in compression and tension. In our previous work [244, 245], using the proposed PSF methodology[241, 244], when PSF value of 60% was chosen, the 3D auxetic metamaterial and 3D auxetic tubular structure could demonstrate a nearly identical auxetic behaviour both in compression and tension. When this auxetic material is compressed and stretched at the same strain, the deformed vertical strain is the same, as shown in Figure 7-3. Therefore, the PSF methodology could inspire researchers to design and investigate more auxetic materials and structures
which have identical auxetic compressive behaviour and tensile behaviour. This feature could be very useful for the application of smart devices, e.g., 3D smart sensors and biomedical devices.

Another promising work is to combine the auxetic property with other negative indexes, e.g., negative compressibility (NC) or negative thermal expansion (NTE) to generate some novel and functional advanced materials, e.g., satisfy the requirements for multifunctional and multipurpose devices. As a pioneering work, Ai et al. [278] reported that a metallic metamaterial with biomaterial star-shaped re-entrant planar lattice structure demonstrated an auxetic NTE behaviour through numerical simulations. The relationships of the three negative indexes are presented in Figure 7-4. As stated in the work of Huang and Chen [272], further investigations of the inter-relations among NPR, NC, and NTE are necessary.
In summary, most of the existing designs of auxetic materials are mainly based on the experience of engineers. The PSF methodology of generating 3D metallic auxetic materials and structures has been proposed, and the mechanical performance of the designed auxetics could be easily tuned by only one single parameter of PSF. However, this methodology has many limitations. The most limitation is that the PSF methodology has to start with a geometrical configuration with the buckling-induced mechanism. Therefore, the design freedom is tremendously constrained. However, using topology optimisation method to design auxetic materials with preferred mechanical performance is a promising direction for auxetics. Several successful optimised auxetic materials have achieved in recent years [279-281], but all of these designs still remain in 2D. Therefore, more 3D auxetic materials with preferable and superior performance could be designed using advanced topology optimization algorithms, such as evolutionary structural optimization (ESO) [282, 283] and bi-directional evolutionary structural optimization (BESO) [284, 285].
References


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