Cyclic Plasticity of Aerospace Metals: I. Modelling of Aluminium 7075-T6 for Structural Fatigue Analysis, II. Experimental Characterisation and Modelling of Additively Manufactured Ti-6Al-4V

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

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<tr>
<td>$a$</td>
<td>backstress ($X$) saturation level</td>
</tr>
<tr>
<td>$a^*$</td>
<td>multiplicative backstress ($X^*$) saturation level</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>threshold (multicomponent Armstrong-Frederick with threshold (MAFT))</td>
</tr>
<tr>
<td>$a^{cyc}$</td>
<td>cyclic backstress saturation level</td>
</tr>
<tr>
<td>$a^{cyc*}$</td>
<td>cyclic multiplicative backstress saturation level</td>
</tr>
<tr>
<td>$a^{mono}$</td>
<td>monotonic backstress saturation level</td>
</tr>
<tr>
<td>$AF$</td>
<td>asymmetric factor</td>
</tr>
<tr>
<td>$b$</td>
<td>rate of yield surface $R$ evolution</td>
</tr>
<tr>
<td>$c$</td>
<td>rate of backstress saturation</td>
</tr>
<tr>
<td>$c^*$</td>
<td>rate of multiplicative backstress saturation</td>
</tr>
<tr>
<td>$C$</td>
<td>linear backstress parameter</td>
</tr>
<tr>
<td>$C^{**}$</td>
<td>controls the size of the linear backstress coefficient ($C$)</td>
</tr>
<tr>
<td>$C^{rate}$</td>
<td>rate of decay in size of the linear backstress coefficient</td>
</tr>
<tr>
<td>$c_k$</td>
<td>rate of yield surface $k$ evolution</td>
</tr>
<tr>
<td>$c^{cyc}$</td>
<td>rate of cyclic backstress saturation</td>
</tr>
<tr>
<td>$c^{cyc*}$</td>
<td>rate of cyclic multiplicative backstress saturation</td>
</tr>
<tr>
<td>$c^{mono}$</td>
<td>rate of cyclic backstress saturation</td>
</tr>
<tr>
<td>$d\lambda$</td>
<td>plastic multiplier</td>
</tr>
<tr>
<td>$dp$</td>
<td>incremental equivalent plastic strain</td>
</tr>
</tbody>
</table>
\( dR \) \hspace{1cm} \text{incremental evolution of isotropic hardening}

\( dX \) \hspace{1cm} \text{incremental backstress evolution}

\( d\varepsilon^p \) \hspace{1cm} \text{incremental plastic strain}

\( E \) \hspace{1cm} \text{elastic modulus}

\( ERROR \) \hspace{1cm} \text{summation of differences equation (between \textit{EXP} and \textit{SIM} data points)}

\( EXP_i \) \hspace{1cm} \text{experimental data point}

\( f \) \hspace{1cm} \text{yield function}

\( f(x) \) \hspace{1cm} \text{experimental distribution}

\( f_n(x) \) \hspace{1cm} \text{simulated distribution}

\( FD \) \hspace{1cm} \text{Fréchet distance}

\( H \) \hspace{1cm} \text{cyclic hardening factor}

\( k \) \hspace{1cm} \text{yield stress}

\( k^s \) \hspace{1cm} \text{saturation value of yield stress} \( k \)

\( k_t \) \hspace{1cm} \text{stress concentration factor}

\( m \) \hspace{1cm} \text{ratcheting parameter (Ohno-Wang (OW) model)}

\( N_e \) \hspace{1cm} \text{cycle number corresponding to the end of cyclic softening}

\( N_{cyc} \) \hspace{1cm} \text{number of half cycles before encountering a maximum strain amplitude}

\( N_s \) \hspace{1cm} \text{cycle number corresponding to the start of cyclic softening}

\( p \) \hspace{1cm} \text{equivalent plastic strain}

\( R \) \hspace{1cm} \text{isotropic hardening}
\( R_s \) isotropic hardening saturation level

\( SIM_i \) simulated data point

\( T_L \) liquidus temperature

\( T_S \) solidus temperature

\( V_{cs} \) average cyclic softening rate

\( X \) total backstress

\( \bar{X} \) threshold (multicomponent Armstrong-Frederick with threshold (MAFT))

\( X^* \) multiplicative backstress (multiplicative Armstrong-Frederick model (MAFM))

\( \gamma \) rate of backstress (\( X \)) saturation

\( \gamma^* \) rate of multiplicative backstress (\( X^* \)) saturation

\( \delta_{max} \) previous maximum encountered strain amplitude

\( \delta_{min} \) previous minimum encountered strain amplitude

\( \frac{\Delta \varepsilon_p}{2} \) plastic strain amplitude

\( \varepsilon_a \) strain amplitude

\( \varepsilon_m \) mean strain

\( \varepsilon_{max} \) strain at the peak stress of the cycle

\( \varepsilon_{min} \) strain at the minimum stress of the cycle

\( \zeta \) reduces the monotonic micro-mechanism contribution

\( \eta \) alters the influence of the monotonic coefficients

\( \mu \) decaying rate of maximum strain amplitude
\( \nu \) alters the size of the decaying maximum strain amplitude

\( \sigma' \) deviatoric stress

\( \sigma_a \) stress amplitude

\( \sigma_c \) compression peak stress

\( \sigma_e \) elastic region of the hysteresis loop

\( \sigma_{eff} \) effective stress

\( \sigma_m \) mean stress

\( \sigma_{max} \) maximum stress

\( \sigma_{min} \) minimum stress

\( \sigma_{Ne} \) maximum tensile stress at the end of cyclic softening

\( \sigma_{Ns} \) maximum tensile stress at the start of cyclic softening

\( \sigma_{sat} \) stress amplitude of the first hardening cycle

\( \sigma_t \) tension peak stress

\( \sigma_{UTS} \) ultimate tensile strength

\( \sigma_y \) yield stress

\( \sigma_{y0} \) initial yield stress

\( \sigma_{yield} \) yield stress

\( \sigma_0 \) stress amplitude of any number of hardening cycles

\( \tau \) rate of decay in influence of the monotonic coefficients
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AA</td>
<td>Aluminium Alloy</td>
</tr>
<tr>
<td>AF</td>
<td>Armstrong-Frederick</td>
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<td>AM</td>
<td>Additive Manufacturing</td>
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<td>BCC</td>
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<td>CGAP</td>
<td>Crack Growth Analysis Program</td>
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<td>CT</td>
<td>Compact Tension</td>
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<td>DSTG/DST-Group</td>
<td>Defence Science and Technology Group</td>
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<td>DLD</td>
<td>Direct Laser Deposition</td>
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<td>EDS</td>
<td>Energy Dispersive Spectrometer</td>
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<td>ESED</td>
<td>Equivalent Strain Energy Density</td>
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<td>FAMS</td>
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<td>Hot Isostatic Heating</td>
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<td>Intrinsic Heat Treatment</td>
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<td>ISF</td>
<td>Incremental Space Filler</td>
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<td>Kernel Density Estimate</td>
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<td>LCF</td>
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<td>Multicomponent Armstrong-Frederick</td>
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<td>Multicomponent Armstrong-Frederick model with Multiplier</td>
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<td>Maximum Damage Method</td>
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<td>mMAFM</td>
<td>Multicomponent Armstrong-Frederick model with Multiplier and Linear Backstress</td>
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<td>Multi-Objective Genetic Algorithm</td>
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Abstract

Aircraft structures, as is the case for many engineering structures, contain discontinuities such as holes and notches, which harbour the potential for fatigue crack initiation. At the root of such discontinuities, localised plasticity can occur even though the rest of the structure is experiencing elastic deformation. In the cyclic regime, a number of different cyclic transient effects can occur depending on the control mechanism (stress, or strain), which include progressive relaxation of mean stress, accumulation of strain with cycles (strain ratcheting), plastic shakedown, and cyclic hardening/softening. Strain-life methodologies have routinely been used in the aeronautical industry for many years for fatigue analysis of aircraft. However, the current strain-life fatigue methods apply a simplified method of cyclic plasticity calculation, namely the Masing plasticity model. Consequently, cyclic transient effects on fatigue life are not considered in the strain-life fatigue predictions.

In addition, additive manufacturing (AM) presents a new challenge. AM is a promising new technology that can significantly alter how future aircraft structures are to be manufactured. One of the ground-breaking aspects of this evolving technology is the ability to manufacture material with microstructure composed of a wide range of crystallographic phases, which is only possible in traditional manufacture methods through post thermos-mechanical process. The technology begins to enhance the potential of tailoring the fabrication of components to contain specific microstructure to achieve the required mechanical properties for particular loading conditions, which has a significant effect on the future design process.

This thesis enhances the current understanding of the application of elastoplastic models to strain-life fatigue predictions, through an experimental and computational investigation of 7075-T6, and a parameter optimisation scheme. Furthermore, it begins to develop the cyclic elastoplastic and low cycle fatigue behaviour of the titanium alloy Ti-6Al-4V manufactured using the selective laser melting (SLM) additive manufacturing process. The main findings of this research are the following:
• **Cyclic elastoplastic behaviour of AA 7075-T6**

An in depth understanding of macroscopic behaviour of AA7075-T6 was successfully linked to the extensive micro-mechanism analysis of the material which occurred of the past 40 years. The first asymmetric stress-controlled analysis of the material was also conducted, where a plastic shakedown of the strain ratcheting occurred. An initial cyclic softening was also noticed for first time in AA7075-T6, which occurred during the low peak stress tests. A closer inspection of the evolution of the effective stress and backstress in asymmetric strain-controlled results showed that a difference in the micro-mechanism occurring in tension and compression contributed to the slowing down of the relaxation rate, which was dependent on the applied strain amplitude.

• **Elastoplastic constitutive model development and improvements to strain-life fatigue predictions**

Experimentally noticed plastic shakedown in AA7075-T6, can be successfully modelled through the modification of kinematic hardening rules to contain a linear backstress rule. A parameter optimisation workflow which could be applied to determine elastoplastic constitutive model parameters to be used to improve strain-life fatigue calculations was determined. The results of an extensive investigation into the potential improvements offered by the application of elastoplastic constitutive models to strain-life fatigue found that the Multicomponent Armstrong-Frederick (MAF) model improved the strain-life fatigue prediction accuracy. This was based on both statistical and deterministic methods of comparing the predicted and experimental fatigue lives calculated from the application of P-3C spectra. However, all the applied elastoplastic constitutive models improved the fatigue predictions compared to the traditional Masing model.
• **Cyclic elastoplastic investigation of SLM Ti-6Al-4V**
  Comparison with symmetric strain-controlled results obtained from mill-annealed Ti-6Al-4V coupons showed that the micro-mechanisms associated with an SLM $\alpha'$ martensite resulted in quite different mean stress relaxation behaviour, where the mean stresses in the SLM $\alpha'$ martensite microstructure were relaxed faster than the mill-annealed microstructure. Additionally, through an experimental program which used test coupons fabricated at different build orientations, the mechanical anisotropy of SLM Ti-6Al-4V was observed in both monotonic and cyclic results, where the diagonally manufactured coupon had the largest yield stress in both, while the diagonal and horizontal coupons were more ductile than the vertical coupons.

• **Elastoplastic constitutive modelling of SLM Ti-6Al-4V**
  Different uniaxial elastoplastic features of SLM vertically fabricated $\alpha'$ martensite Ti-6Al-4V were shown to be successfully captured through the application of phenomenological elastoplastic constitutive models. The simulation results gathered for each of the tested kinematic hardening models demonstrated very good agreement with symmetric strain-controlled hysteresis loop development corresponding to the experimentally gathered results at 1%, 1.5%, 2%, and 2.5%. Furthermore, good simulations results were achieved for mean stress relaxation and strain ratcheting.

**Keywords:** Aluminium Alloy, strain-life, fatigue, plasticity, constitutive model, kinematic hardening, isotropic hardening, mean stress relaxation, strain ratcheting, Ti-6Al-4V, additive manufacturing, cyclic hardening, cyclic softening, strain control, stress control
Introduction

Background

Aircraft structures, as is the case for many engineering structures, contain discontinuities such as holes and notches, which harbour the potential for fatigue crack initiation. Recent work in fatigue life calculations conducted by Hu and Wallbrink [1] from the Defence Science and Technology Group (DST Group) highlighted the potential challenges associated with a load spectrum containing occasional severe loading causing local plastic deformation at stress concentrations. Clipped and unclipped load spectra were used in the life assessment of notched coupons. Clipping a spectrum is a technique aimed at providing more conservative results (shorter fatigue lives), since a positive overload can cause compressive residual stresses ahead of the crack tip that retards crack growth. However, experimental results showed that the unclipped load spectrum had a shorter fatigue life than the clipped spectrum. Furthermore, computational simulations using a fatigue life and crack growth program developed by DST Group called CGAP (Crack Grown Analysis Program) showed no difference in fatigue life between the two load spectra, contradictory to the experimental results.

It was hypothesised that the difference between the experimental and simulated results was a consequence of the presence of local plastic deformation caused by the occasional severe overloads. During the work conducted by DST Group, a nonlinear kinematic hardening model developed by Chaboche [2], sometimes referred to as the multicomponent Armstrong-Frederick (MAF) model, was incorporated into the strain-life code FAMS [3]. The strain-life methodology is based on the concept that fatigue damage is dependent on the local plastic strains around discontinuities in the material. This methodology assumes that smooth laboratory specimens subjected to the same deformation as that at the notch root can be used to predict the life spent on crack nucleation and small crack growth of a notched specimen [4]. The behaviour at the root can be quite different to the rest of the material. This is due to stress concentrations at the notch which will cause local plastic deformation, which can
occur even though the rest of the structure remains elastic. The simulation results have revealed that accounting for mean stress relaxation and ratcheting using the cyclic plasticity models coupled with a notch correction method can produce promising results. Unlike the results obtained using the conventional method (Masing’s hypothesis) which predicted almost identical results for the two spectra, simulation results using the MAF model more accurately recognised the difference in fatigue lives between the two spectra. However, although results obtained using the MAF model were an improvement, there still was associated errors of up to 79% for one particular notch geometry. Also, the addition of the plasticity model was only tested using data obtained from one location on the aircraft, which translated to two spectra. Consequently, it is important to develop an understanding of how overloads and sequence affects in other spectra can be simulated using kinematic hardening models.

Kinematic hardening models are developed with the intention of improving the simulation of cyclic phenomena such as strain ratcheting and mean stress relaxation. In order to achieve this, the underlying mathematic formulation of the original Armstrong-Frederick model has been modified further using methods such as, increasing the number of backstress terms, adding threshold values and multiplicative terms. The difficulty which arises from the application of such models is in the calculation of the material parameters. The parameters can be approximated with reference to the loading branch of the stabilised symmetric strain-controlled curve as outlined in [5]. This method provides good simulation of symmetric strain-/stress-controlled results, however, the parameters require further adjustment to ensure an overall improvement in simulation, including cyclic phenomena such as strain ratcheting and mean stress relaxation. This is particularly important when applying such models to complex loading scenarios such as aircraft spectra. The complexity of the parameter selection procedure is enhanced further when attempting to adjust the parameters to improve mean stress relaxation and ratcheting, while trying to not reduce the accuracy of the symmetric stress/strain-controlled simulations. In some models, the adjustment of parameters to improve the simulation of ratcheting will have a detrimental effect on the
simulation of mean stress relaxation and vice versa, adding to the complexity of parameter selection.

In addition to the aforementioned difficulties in accounting for the mean stress relaxation and strain ratcheting in conventional aircraft materials, additive manufacturing (AM) presents a new challenge. AM is a promising new technology that can significantly alter how future aircraft structures are to be manufactured. Such benefits include the potential of manufacturing components quickly and cost effectively. It also provides a means of manufacturing geometrically complex structures which would otherwise require a considerable amount of time and money to manufacture conventionally. However, one of the ground breaking aspects of this evolving technology is the ability to manufacture material with microstructure with a wide range of crystallographic phases, which is only possible in traditionally manufacture methods through post thermos-mechanical process [6]. This stresses the need to develop AM processing parameters for the manufacture of material to be used in as-built conditions.

A considerable amount of research has been conducted on the qualification of parts manufactured using additive manufacturing (AM) technologies, with a particular focus on laser additive manufacturing (selective laser melting (SLM)) and the titanium alloy Ti-6Al-4V. The areas of primary research have been in the identification of the manufacturing aspects which have an influence on the quality and microstructure of the manufactured structure which include: process parameters, melt pool temperatures, gas flow in the build chamber, redistribution of powder, geometry of the manufactured parts on cooling rates, just to name a few. With this ever-growing database of vital information on the contribution of these different fabrication aspects on the quality and microstructure of the fabricated material, it is now vital to develop an understanding of the influence of the possible various crystallographic phases, residual stresses, and defects have on the mechanical properties of the material, such as monotonic properties, low and high cycle fatigue properties, and fracture mechanical behaviour.
Aims and Research Questions

The 7xxx-series aluminium alloys, which are precipitation-hardened Al-Zn-Mg-(Cu) alloys, have been very important in high stressed applications, particularly in the aerospace industry. AA7075-T6 has been experimentally analysed with respect to microstructure characteristics [7], contribution of micro-mechanisms to low cycle fatigue [8] and asymmetric loading [9], symmetric strain-control [10], and asymmetric strain-control [11]. However, no research into the asymmetric stress-controlled behaviour of the material has been considered, a phenomenon which is particularly important with respect to notch plasticity where a combination of both asymmetric-strain and stress control occurs [12]. Furthermore, although it has been previously recognised for other materials that mean stress relaxation and strain ratcheting are caused by the same micro-mechanisms [13-16], an understanding of the potential difference in the micro-mechanisms which occur during asymmetric strain-control compared to asymmetric stress-control is required in accurate elastoplastic constitutive model development for the cyclic transient effects which occur during both methods of control.

Initial findings made by Hu and Wallbrink [1] demonstrated potential improvements in strain-life fatigue calculations with the application of advanced plasticity models, highlighting the importance of recognising sequence affects in complex spectra. However, this was only a preliminary analysis, with the inclusion of a relatively simple plasticity model. Also, the experimental and simulated comparison was considered at only one location of the aircraft. A more in-depth understanding of sequence affects can be developed from the analysis of a number of different locations on the aircraft. Furthermore, elastoplastic constitutive model parameter developments have not been analysed for application to complex loading conditions, which is an important consideration when applying elastoplastic constitutive models to complex aircraft spectra.

A considerable amount of research has been undertaken in an attempt to understand the microstructure and defect influence on the monotonic tensile properties, high cycle fatigue (HCF), hardness, and electrochemical behaviour SLM Ti-6Al-4V [17-26]. The cyclic elastoplastic and LCF behaviour of SLM Ti-6Al-4V has not been researched, particularly in the
development of an understanding into the influence of build orientation on elastoplastic behaviour. This information is important in the formulation of elastoplastic constitutive models, which can accurately simulate cyclic plastic behaviour to ensure accurate fatigue life predictions and structural analysis capabilities.

The research questions which will be answered by the work presented in this thesis and the corresponding objective which outlines the process by which this was achieved are as follows:

- **What is the effect of strain or stress amplitude on the rate of mean stress relaxation and ratcheting?**

  Objective: investigate the effect of varying the applied strain and stress amplitude during asymmetric stress and strain-controlled tests and from a macroscopic point of view, determine the similarities/differences in micro-mechanism evolution during the two different control methods.

- **How does application of kinematic hardening models affect the accuracy of predicting the fatigue life under spectrum loading?**

  Objective: investigate whether the application of a nonlinear isotropic hardening model coupled with different kinematic hardening models applied to the strain-life fatigue process improves on fatigue predictions made at fatigue critical locations on the P-3C Orion aircraft.

- **What is the relationship between build direction and anisotropic elastoplastic behaviour of additively manufactured components?**

  Objective: investigate the elastoplastic behaviour of a commonly produced SLM Ti-6Al-4V microstructure at 0°, 45°, and 90° to the build plate using symmetric and asymmetric strain-control.
Assumptions and Limitations

The work presented in this thesis only considers uniaxial experimental results. For the application into the strain-life software, uniaxial stress-strains are required; consequently, the elastoplastic constitutive models were uniaxially formulated and the parameter optimisation procedures were developed for uniaxial versions of the models. However, it is intended to extend the parameter optimisation procedure to the development of parameters for multiaxial formulations of the tested models to further investigate the power of the parameter optimisation procedure and genetic algorithm settings developed through the research presented in this thesis.

Although it is intended to possibly extend the work presented in this thesis to experimental evaluation of the multiaxial elastoplastic behaviour of SLM Ti-6Al-4V, only a uniaxial experiment analysis was considered in this thesis. This is because no elastoplastic uniaxial investigation had been previously undertaken by researchers and an initial understanding of the elastoplastic uniaxial behaviour is important in developing an insight into the microstructure contribution to plastic deformation before advancing to more complex stress states.

Thesis Outline

The thesis contains two parts:

- **Part I Modelling of Aluminium 7075-T6 for Structural Fatigue Analysis**: Includes an elastoplastic experimental investigation of AA7075-T6, and follows the progression of the development of parameter optimisation for elastoplastic constitutive models for the final application of the elastoplastic models to strain-life fatigue predictions.

- **Part II Experimental Characterisation and Modelling of Additively Manufactured Ti-6Al-4V**: Elastoplastic experimental investigation of SLM Ti-6Al-4V and application of an elastoplastic constitutive model.
Chapter I.1

- Provides a review of the literature concerning the strain-life fatigue methodology, cyclic transient effects and the importance of their consideration in fatigue predictions, and elastoplastic constitutive models.
- This chapter provides context to research presented in this thesis.

Chapter I.2

- Experimental investigation of AA7075-T6 using symmetric strain-control, asymmetric strain-control, and asymmetric stress-control loading.

Chapter I.3

- Introduces a multi-objective parameter optimisation methodology by applying it to the multiplicative Armstrong Frederick model.
- Demonstrates the improvement to asymmetric stress-controlled plastic shakedown simulation with a modification to the multiplicative Armstrong-Frederick model.

Chapter I.4

- Initial investigation of the potential improvement to strain-life fatigue predictions through the application of elastoplastic constitutive models modified to improve plastic shakedown simulation.
- A sensitivity analysis of the type of experimental data required for elastoplastic model parameter develop to improve strain-life fatigue predictions.

Chapter I.5

- Application of the modified elastoplastic constitutive models with optimal parameters to strain-life fatigue predictions and evaluation of the improvement to life predictions.
- Further modification of the multiplicative Armstrong-Frederick model to improve hysteresis loop development and mean stress relaxation simulations in both constant and variable amplitude loading.
Chapter I.6

- Summarises the conclusion of the work presented in Part I of the thesis and provides some recommendations for future work.

Chapter II.1

- Provides a review of the literature concerning Ti-6Al-4V processed by selective laser melting, which includes the monotonic tensile, high cycle fatigue, low cycle fatigue, and fracture mechanical behaviour of various microstructures of Ti-6Al-4V produced by SLM.

Chapter II.2

- Investigates the elastoplastic behaviour of vertically manufacture SLM Ti-6Al-4V subjected to both symmetric and asymmetric strain-control loading, and the potential of elastoplastic constitutive models at simulating the cyclic transient effects occurring in additively manufactured material.
- Investigates the potential anisotropy in cyclic plasticity caused by the build orientation, using symmetric strain-controlled tests conducted on SLM Ti-6Al-4V at build orientations 0°, 45°, and 90° to the build plate.

Chapter II.3

- Investigates the ability of several nonlinear kinematic hardening rules of different formulations, to capture the cyclic transient effects of vertically manufactured SLM Ti-6Al-4V.

Chapter II.4

- Summarises the conclusion of the work presented in Part II of the thesis and provides some recommendations for future work.
References


Part I  Modelling of Aluminium 7075-T6 for Structural Fatigue Analysis
Chapter I.1  Part I Background and Literature Review

1.1 Classical Strain-Life Method

In many engineering structures, features that cause stress concentrations such as notches, holes and fillets are unavoidable. Consequently, it is important that the design accounts for effects of local plastic yielding at such discontinuities on the structural integrity. Finite element analysis (FEA) provides engineers with an important tool to characterise the non-linear deformation at stress concentrations. However, in some cases, the structures may be too geometrically complex to run FE analysis on a cycle by cycle basis, particularly for structures which are subjected to a large number of cycles. Consequently, alternative methods have been developed to enable rapid analysis. One such method is the local strain-life approach which is used in the FAMS software. This method falls under the banner of the safe-life techniques which assume fatigue life is governed by crack initiation. Underpinning this method is the theory that local deformation at discontinuities can be approximated by the cyclic stress-strain data and strain-life curves gathered from standard test coupons.

Material definition in FAMS uses the Ramberg-Osgood [1] equation coupled with Masing’s hypothesis [2]. Using remote stress or strains as input, the local stress and strains at the notch root are calculated using a notch correction method such as Neuber’s rule [3] and the Ramberg-Osgood equation (which is developed from the cyclic stress-strain curve of the material). Rainflow counting is used in conjunction with Masing’s hypothesis to create the local stress-strain hysteresis loops. The stress and strain amplitudes and mean stresses are determined from these hysteresis loops. The equivalent strain is calculated using these values which is then used to determine the fatigue life using Miner’s [4] rule. This calculation process is summarised in Figure 1.
Neuber’s rule is an approximation method used to calculate the local stress-strain values at geometric discontinuities. The rule was originally developed from experiments conducted on grooved shafts under monotonic torsional loading conditions. Experimental data demonstrated lower stress and a larger strain at the notch than that predicted by elastic solutions. A relationship based on the assumption of localised yielding at the notch root was developed to relate the stress and strain at the notch root to the nominal stress and strain of the component. The original derivation of the technique was developed from pure plane shearing; however, it was demonstrated by a number of researchers including Papirno [5], Papirno [5], Wu-Cheng [6], Eimermacher, et al. [7] and Gemma [8], that good results could be achieved with approximations made to plane stress and strain conditions and was extended to cyclic fatigue calculations by Topper, et al. [9]. Other methods, including the equivalent strain energy density (ESED) method [10] have been derived. The difference in the ability of the two methods at calculating the notch stress and strains was highlighted by Moftakhar, et al. [11]. Experimental and numerical data were used to test the performances of the two methods. It was found that stress and strains calculated by Neuber’s rule were larger than
the actual maximum principal stress and strains, while the ESED method under-predicted the response. Consequently, Neuber’s rule provides more conservative results.

Although the strain-life method has proven to be a valuable tool, there are some inaccuracies, particularly when considering variable amplitude loading. One of the inaccuracies was discovered by Hu and Wallbrink [12]. The fatigue-life results obtained from two different spectra (clipped and unclipped) showed almost identical fatigue life for varying notch geometries. This was contradictory to experimental results which reported a fatigue life for the clipped spectrum to be four times greater than the unclipped spectrum. Investigation demonstrated that the discrepancy could potentially be explained with consideration of the material modelling technique applied in FAMS. Due to the nature of the Masing’s hypothesis, closed stress-strain hysteresis loops are used in the calculation process. Consequently, transient effects such as cyclic hardening, cyclic softening, mean stress relaxation and ratcheting are not considered. The ratcheting phenomena occurs during asymmetric cyclic stress loading, while mean stress relaxation occurs during asymmetric cyclic strain loading. However, at the notch root, a combination of stress and strain control occurs, the extent of which is dependent on the notch geometry. Consequently, there is quite a significant deficiency by not considering such cyclic effects particularly when investigating variable amplitude loading (as is the case for this study) since the mean stresses at the notch root will in fact be different to those predicted by Masing-developed hysteresis loops, while strain ratcheting is not predicted by the Masing hypothesis at all.

1.2 Effect of Mean Stress in Fatigue

1.2.1 Stress-Control

It has been readily observed a tensile mean stress can cause a reduced fatigue life. Stress-controlled fatigue tests have demonstrated that tensile mean stresses reduce the fatigue life in several different materials [13-21]. This has been reported to be the result of crack opening effect which increases the accumulation of damage [14], as well as the accumulation of tensile strain due to the occurrence of ratcheting (also known as cyclic creep) which is an accumulation of strain with progressive cycles as shown in Figure 2. Ratcheting strain is
commonly measured as either the peak strain of each cycle or the mean strain \( \frac{1}{2} (\varepsilon_{\text{max}} + \varepsilon_{\text{min}}) \) per cycle. Consideration of the strain ratcheting influence has been necessary in the design for the aerospace [12, 22], rail [23], nuclear [17], and biomedical [16] industries to name a few. Uniaxial strain ratcheting has been observed to occur in a number of different materials such as stainless steel [24], magnesium alloy [25, 26], aluminium alloys [27], and steel alloys [28-30] to just name a few but further materials and their ratcheting strain evolution were critically reviewed by Kang [31].

![Figure 2 Example of the accumulation strain during asymmetric strain-controlled loading](image)

Ratcheting has been shown to be affected by the applied mean stress and/or stress amplitude, which can influence the cyclic hardening or softening occurring in the material, thus, the micro-mechanisms which contribute to the accumulation of ratcheting strain. The three main evolution mechanisms of ratcheting strain include:

1. Constant increase in ratcheting strain [28, 32, 33], which has been hypothesised to be the consequence of cyclic softening occurring during loading.
2. An initial increase in ratcheting strain before the onset of saturation in the accumulation of strain with cycles [21, 27, 34].
3. Tertiary evolution of ratcheting strain which can be separated into three main stages of evolution, characterised by the competing influence of strain hardening and creep softening [35].
These different strain ratcheting features are demonstrated in Figure 3.

![Diagram showing strain ratcheting](image)

**Figure 3** Example of three common strain ratcheting evolutions.

1.2.2 Strain-Control

The micro-mechanisms which cause strain ratcheting are also responsible for strain-controlled cyclic phenomenon known mean stress relaxation. During strain-controlled tests containing an initial mean strain, the mean stresses developed during cycling gradually decrease. This behaviour is demonstrated in Figure 4, where the mean stress relaxation is characterised by the gradual shifting of the hysteresis loops either upwards or downward shifting depending on the whether it was a compressive or tensile mean strain. The mean stress relaxation effects have been investigated in a number of different materials including aluminium alloys [36-38], stainless steels [39], superalloys [40], and carbon steel [41-43].
Figure 4 Mean stress relaxation initiating from a tensile mean strain.

The rate of mean stress relaxation is dependent on the applied strain amplitude, with the mean strain having less an influence on the amount of relaxation [24, 44]. It is also possible that for small strain amplitudes, the mean stress is not completely relaxed [36, 44]. This variation in relaxation is shown in Figure 5 which shows three possible mean stress relaxation curves obtained from strain amplitudes of varying size.

Figure 5 Varying amounts of relaxation dependent on the applied strain amplitude
Relaxation of mean stresses present in the material can be either beneficial or detrimental depending on the type of mean stress present. For example, the surface treatment processes such as shot peening can introduce beneficial fatigue crack initiation retarding compressive residual stress on the surface of the material, which could potentially be relaxed over time [45]. The effect of mean stresses on fatigue was considered by Bleuzen, et al. [46] who noticed an over-prediction in the fatigue life due the relaxation of mean stresses, while Dowling, et al. [47] and Arcari, et al. [36] reported a reduction in the benefit of induced residual stresses due to mean stress relaxation produced by cycles causing the appropriate level of plasticity to initiate relaxation. The rate at which plastic shakedown in strain-control occurs is another important consideration, since it too influences fatigue life. Plastic shakedown in strain-control describes the phenomena whereby the plastic stresses and strains reach a steady state (each following cycle results in the same level of tensile and compressive stress and there are no further accumulation of strain). The faster the shakedown process occurs, the quicker the mean stresses will relax. The importance of using plasticity models which can accurately predict this phenomenon was highlighted by Wang and Rose [48] whose work demonstrated how the rate of plastic shakedown varies between plasticity models subjected to the same loading case. Chiou and Yip [49] extended this and attempted to incorporate the relaxation of mean stress in fatigue life by developing a model to predict the stable hysteresis loop from an applied mean strain.

### 1.2.3 Fatigue-Life Mean Stress Incorporation

In order to incorporate the effects of mean stresses, models have been developed to quantify the effect of mean stresses on fatigue behaviour. During the strain-life fatigue calculations, calculated notch strains are used to find the equivalent fatigue life, which is determined with reference to the strain-life curve. These curves are formed from experimental data obtained from constant amplitude fully reversed fatigue tests at a number of different amplitudes on un-notched specimens. The total strain amplitude and corresponding fatigue cycles are plotted. The data are then curve fitted, providing ease of calculation of expected fatigue life for a corresponding strain. Traditionally, these equations were considered for zero mean stress case, thus, would not effectively calculate an accurate fatigue life for asymmetric
loading conditions. Consequently, in order to consider mean stress, equivalent strain equations have been developed which calculates an equivalent strain which takes into consideration the effect of the mean stress present. The exact form of these equations varies, with a number of versions being proposed each with their own associated strengths and weaknesses.

The success of three popular equivalent strain equations were considered by Dowling [50]. These three methods were the modified Morrow equation [51], the Smith-Watson-Topper (SWT) equation [52] and the Walker [53] equation. Eighteen data sets (formed from samples of titanium, aluminium alloys and steels) were examined using the four equivalent strain equations. The Walker equation was recommended by Dowling to be used for strain-based life estimates. The reason for this conclusion was based on the excellent results obtained from the eighteen data sets. The only disadvantage the Walker equation does pose is the requirement of a material parameter calculation ($\gamma$) from non-zero mean stress data.

1.3 Introduction to Constitutive Model Framework

In rate-independent plasticity theory, the fundamental components of the constitutive model include: the yield criterion, flow rule and hardening rule. To accurately simulate cyclic transient effects such as the Bauschinger effect [54], strain ratcheting, mean stress relaxation, cyclic hardening and softening, both nonlinear isotropic and kinematic hardening are required.

1.3.1 Yield Criterion

Classical plasticity theory assumes that yield is independent of the hydrostatic stress. Therefore, the yield function can be defined in terms of the deviatoric stress. The von Mises yield criterion is defined in the yield function given in Eq. 1.

$$f = \left( \frac{3}{2} \sigma' : \sigma' \right)^{1/2} - \sigma_{yield}$$

where $\sigma'$ is the deviatoric stress, and the bold representing its tensor properties.
1.3.2 Plastic Flow Rule

Associative flow assumes that the increment in plastic strain is normal to the yield surface, defined in Eq. 2.

\[ d\varepsilon^p = d\lambda \frac{\partial f}{\partial \sigma} \]  

where \( \frac{\partial f}{\partial \sigma} \) gives the direction of plastic flow and \( d\lambda \) defines the amount of plastic strain.

1.3.3 Hardening Rule

Hardening rules are used to describe the transformation of the yield surface with plastic flow. Isotropic hardening describes the expansion of the yield surface and kinematic hardening describes the translation of the yield surface. A translating yield surface recognises the Bauschinger effect, since the yield surface translates in the direction of loading reducing the yield the opposite direction, which is shown in Figure 6 where an initial tensile loading reduces the compressive yield stress. Application of a translating yield surface also ensure the simulation of ratcheting and mean stress relaxation due to the initial loading and reloading branches of the hysteresis loops are formed from different levels of deformation resulting in unclosed loops.

\[ \sigma = \sigma_1 \neq \sigma_2 \]

**Figure 6** A translating yield surface resulting in a reduction in the yield stress in the opposing direction to initial loading.
An expanding or contracting yield surface provides a mean of simulating the cyclic hardening or softening of the material. With an expanding yield surface, the cyclic yield increases with plastic strain which reduces the amount of predicted plastic strain with cycles, analogous cyclic hardening. A contracting yield surface results in a reduction of the cyclic yield stress with cycles which leads to an increase in plastic strain prediction, analogous to cyclic softening. The expanding yield surface and corresponding increase in the elastic region is demonstrated in Figure 7.

**Figure 7** Cyclic hardening modelled with an expanding yield surface which increases the size of the elastic region.

**Isotropic Hardening**

If low strain-rate and negligible temperature changes in the material occur, the assumption of an isothermal plastic deformation is permitted and a commonly applied nonlinear isotropic hardening model is that given in Eq. 3 [55].

\[
dR = b(R_s - R)dp
\]

where \(b\) and \(R_s\) are material constants. Integrating Eq. 3 gives Eq. 4 where the yield surface changes with accumulated plastic strain \(p\). The change in yield surface saturates at the value given by \(R_s\) and the rate at which saturation is reached is given by \(b\).

\[
R(p) = R_s(1 - e^{-bp})
\]
Kinematic Hardening

The multicomponent Armstrong-Frederick model developed by Chaboche, et al. [56] is a superposition of a number of Armstrong-Frederick (AF) [57] backstress terms. The AF model is a nonlinear kinematic hardening model which was an improvement on the Prager [58] model due to the introduction of a ‘dynamic recovery’ term used to reduce the gradient of plastic flow, thus developing a more nonlinear response. It is this combination of the hardening and dynamic recovery terms which has been related to the micro-mechanism of slip system process [59]. Chaboche noticed that an improvement to the AF model could be achieved with the addition of three back-stress terms which resulted in a more effective representation of the stabilised cyclic stress-strain curve. Each of the backstress terms were developed to model the different features of the stabilised curve. Initially, Chaboche suggested the use of a linear formulation of the third back-stress. However, this resulted in inaccurate ratcheting results due to a saturation in the amount of ratcheting. To overcome this, it was suggested the third backstress should have a slight nonlinearity [28]. The MAF model is comprised of 6 parameters given in Eq. 5 as $a_1, a_2, a_3, c_1, c_2, c_3$, where $a_i$ is the saturation of the backstress and $c_i$ is the rate at which saturation is reached.

$$\sum_{i=1}^{3} dX_i = c_i \left( \frac{2}{3} a_i d\varepsilon^p - X_i dp \right)$$  \hspace{1cm} (5)

The ratcheting simulation deficiencies of the MAF model at low stress amplitudes were improved by Chaboche with the introduction of a fourth backstress with a threshold [60] forming the multicomponent Armstrong-Frederick with threshold (MAF-T). The threshold stress level signifies a transition point whereby up to the threshold, the kinematic hardening rule develops linearly until beyond threshold, the stress-strain response develops according to the original Armstrong-Frederick rule (AF). The uniaxial backstress formulation of MAF-T is given in Eq. 6, where $\bar{X}$ is the threshold term and $\langle \hspace{.2cm} \rangle$ are the Macaulay bracket.

$$dX_4 = c_4 \left( \frac{2}{3} a_4 d\varepsilon^p - X_4 \left( 1 - \frac{\bar{X}}{X_4} \right) dp \right)$$  \hspace{1cm} (6)
The multiplicative Armstrong-Frederick model (MAFM) [61] was introduced as an improved alternative to the MAF-T model. As a consequence of the introduction of the threshold term in the original Chaboche model, an unphysical linear portion of the hysteresis loop was formed. However, the MAFM model was able to improve this issue without a reduction in ratcheting simulation accuracy. This improvement was achieved by introducing a multiplier into one of the AF back-stress terms. The multiplier is used to adjust the rate at which saturation (without affecting the saturation magnitude) of the backstress occurs, which is analogous to the MAF-T models threshold term. The uniaxial formulation of the back stress is given in Eq. 7, where $a_4^*$ and $c_4^*$ are the material parameters for the multiplicative term and $X_4^*$ is the multiplicative backstress given in Eq. 8.

$$dX_4 = [c_4^* + c_4^*(a_4^* - X_4^*)](a_4d\varepsilon^p - X_4dp)$$  \hspace{1cm} (7)

$$dX_4^* = c_4^*(a_4^*d\varepsilon - X_4^*dp)$$  \hspace{1cm} (8)

The Ohno-Wang (OW) model [62] originally acted as a multilinear model, which predicted no uniaxial ratcheting; however, to overcome these issues a multiplier was added to the dynamic recovery term of the AF model, which introduced a slight nonlinearity to each of the defined back-stress equations. However, unlike the other models presented so far, the OW model is essentially developed from the combination of several linear hardening rules, consequently, in order for a good representation of the hysteresis loop, a large number of backstress equations are required. The OW model has been shown to be very good at capturing the hysteresis loop as well as being very effective at simulating ratcheting rate [28]. The backstress formulation is given in Eq. 9, where $m$ is the ratcheting parameter.

$$dX_i = c_i \left( \frac{2}{3} a_i d\varepsilon^p - X_i dp \left( \frac{X_i}{a_i} \right)^m \right), \text{ for } i = 1,9$$  \hspace{1cm} (9)

1.3.4 Alternative Modelling Methods

Although the phenomenological modelling techniques presented in the previous sections are favoured in strain-life applications due to their computational efficiency when dealing with complex aircraft spectra containing millions of cycles, it is also worth mentioning the potential of other modelling techniques capable of successfully capturing cyclic transient
effects. Such methods include crystal plasticity models. These models simulate macroscopic behaviors through the incorporation of a physically-based modelling technique, which includes microstructure characteristics such as crystallographic texture, grain structures, solute content and dislocation structures. The application of crystal plasticity models was reviewed by Roters, et al. [63] where it was shown that a variety of constitutive formulations exists in the application to different mechanical problems. Physically-based models have continued to be developed to model mechanical behaviours in a variety of materials. Mandal, et al. [64] successfully applied the mechanical threshold stress (MTS) model proposed by Follansbee and Kocks [65], to incorporate dislocation mechanics, to simulate the stress-strain response of the titanium alloy Ti-5553 subjected to varying temperatures and strain rates. The dislocation mechanics of 9Cr steel were successfully incorporated by Barrett, et al. [66] through the development of a dislocation model based on important microstructural features. The applied physically-based model successfully simulated the cyclic elastoplastic behaviour, which included hysteresis loop development and stress relaxation. Furthermore, recent research by Bobbili, et al. [67] has demonstrated the significance of incorporating not only dislocation mechanics but also dynamic recrystallisation kinetics in physically-based constitutive models to accurately simulate the stress-strain development of titanium alloy Ti-10-2-3 during hot compression tests at varying strain rates. Additionally, as demonstrated by Murchú, et al. [68], physically-based models can be used to predict the elastoplastic behaviour of materials based on their precipitate and solute content. This provides an improved understanding of the influence of precipitates on different elastoplastic behaviours. Consequently, as the understanding of the underlying physical mechanisms for the deformation of different materials increases in conjunction with computational power, physically-based models offer the potential of improved elastoplastic predictive capabilities.

1.4 Strain Ratcheting Simulation

Over the past two decades there has been considerable research into the strain ratcheting behaviour of materials, and the need for improved ratcheting simulation accuracy has increased in order to simulate the nuances between materials and loading conditions. The development of the constitutive models for improved strain ratcheting simulations have been
widely based on the modification of the Prager’s kinematic hardening model. The direction taken in the development of the kinematic hardening models can be separated into two main groups: (1) modification and development of the A-F model, (2) multisurface plasticity theory based on the interaction of translating surfaces [69, 70] where elastoplastic behaviour occurs depending on the location of the stress state with respect to the surfaces. A significant amount of research has been conducted with respect to the first group of constitutive models through the development of the A-F model to improve ratcheting simulations. The emergence of the models have also progressively been reviewed by Ohno [71], Khan and Huang [72], and Chaboche [73]. In a review conducted by Bari and Hassan [28] into ratcheting simulation abilities of various classical plasticity models, it was found that a constant ratcheting rate was predicted by the AF model, contrary to experimental results. The ratcheting rate and strain is also over predicted by the model. However, for simulations using the MAF model, a significant improvement in ratcheting was achieved. The ratcheting rate was no longer constant but there were still issues with over-prediction of the accumulated strain and ratcheting rate. Bari and Hassan [28] reported improvements in lower ratcheting rate using the MAFT model. The MAFM model provided further improvements to those offered by the MAFT model with improved ratcheting rates without sacrificing the simulated shape of the hysteresis loop.

The superiority of the OW model at ratcheting prediction was shown by Jiang and Sehitoglu [74], Jiang and Sehitoglu [75] and Bari and Hassan [28], who noticed very good agreement with experimentally observed ratcheting rates. Bari and Hassan [28] also highlighted the OW models improvement to the simulation of the ratcheting stress-strain loop when compared to MAF and MAF-T models. Further improvements to ratcheting simulation was achieved with adjustments of the OW by Jiang and Sehitoglu [76], McDowell [59], and Abdel-Karim and Ohno [77].

Modification of the MAF-T model was performed by Voyiadjis and Basuroychowdhury [78], with the addition of a third term which results in the evolution of the yield surface to be influenced by the direction of the stress rate. Unlike the MAF-T and OW models, the modification to the A-F model does not take the form of a threshold modification. It was
reported that an increase in accuracy from results obtained from the MAF-T model were achieved.

Further modification to the MAF model was performed by Taheri and Lorentz [79]. A multisurface model was defined, whereby the first surface defines the loading surface incorporating plastic flow (kinematic and isotropic hardening); the second surface defines the evaluation of peak stress; the third defines the evolution of the definition of the plastic strain at unloading. Although results demonstrated an improvement in non-proportional simulations using only uniaxial data when compared to the MAF model, for simulations based on multiaxial data, the results still do not compare well with experimental data. The reason for this inaccuracy was attributed to the yield surface selection.

A modification to the MAF model was proposed by Kang, et al. [80]. It was proposed that the backstress terms should consider the effects of cyclic hardening/softening. This was achieved with the introduction of another term into the dynamic recovery component of the AF backstress equation. Uniaxial simulation results for SS304 stainless steel demonstrated good correlation for ratcheting rate, although the shapes of the hysteresis curves were not well simulated.

1.5 Mean Stress Relaxation Simulations

As demonstrated from the number of models presented thus far, a considerable amount of research has been undertaken into developing and modifying kinematic hardening models to more accurately define a relationship between ratcheting rate and strain amplitude. However, less research has been undertaken in assessing the accuracy of mean stress relaxation simulations. Some modifications to the A-F developed constitutive models have been made to improve the models’ mean stress relaxation capabilities. This included research conducted by Chaboche and Jung [81] into the relaxation of the residual stress field who modified the MAFT model, which showed good simulations of experimental results obtained from two strain amplitudes were considered. Further modification of the threshold definition was also considered by Chaboche, et al. [82], and Gustafsson, et al. [83] showed added mean stress relaxation prediction improvement. An alternate modification for improved mean stress
relaxation simulation by Lee, et al. [24] was through the application of the Burlet and Cailletaud [84] to modify the dynamic recovery term and by Wu, et al. [85] through the modification of the Abdel-Karim-Ohno model [77].

Further mean stress relaxation simulations were considered through the application of constitutive models developed for strain ratcheting simulations, which included the simulation of a mean stress relaxation case obtained from AA7050 by Kourousis and Dafalias [86] who applied the MAFM model, and Arcari and Dowling [87] who applied the Jiang model [76] and Wetzel model to AA7075-T6511 mean stress relaxation simulations.

1.5.1 Mean Stress Relaxation in Strain-Life Fatigue Predictions
The importance of considering mean stress relaxation in strain-life fatigue calculations was considered by Arcari and Dowling [87] and Arcari [88]. The Jiang model and the Wetzel models were used in AA 7075-T6511 simulations and compared to experimental strain-controlled results to determine the success of the models at predicting the extent of mean stress relaxation of particular strain amplitudes. Knowledge of the stabilised hysteresis loop was then reapplied into the strain-life fatigue calculation process. Doing so recognised the potential of some strain amplitudes to relax the mean stress. The study concluded that although there was an improvement compared to the results obtained using Masing’s hypothesis, there was still further improvement required.

1.5.2 Mean Stress Relaxation and Strain Ratcheting Simulations
An investigation into the ratcheting and mean stress relaxation simulation behaviour has been very limited. Lee, et al. [24] investigated the simulation accuracy of experimental ratcheting behaviour of a stainless steel using a newly proposed constitutive model. Uniaxial ratcheting simulations compared against experimental data obtained from several different stress amplitudes were very accurate. However, the mean stress relaxation simulation accuracy was only compared against one strain amplitude which induced complete relaxation of the mean stresses. The accuracy of the model at improving the mean stress relaxation was not analysed in depth, particularly with respect to strain amplitudes which do not fully relax to zero mean stress. Although the paper provided an in-depth experimental and computational investigation into the ratcheting phenomena, comparison with only one type of asymmetric
stain-controlled experimental data does not provide enough experimental evidence that the model can accurately simulate mean stress relaxation. Work conducted by Kourousis and Dafalias [86] showed successful simulation of both ratcheting and mean stress relaxation using the MAFM model; however, similar to Lee, et al. [24], the mean stress relaxation case was that which resulted in complete relaxation of the mean stress.

1.6 Parameter Optimisation

Simulation accuracy is very sensitive to the selection of the material parameters used in the calculation process of the plasticity models. Consequently, it is important to determine whether more accurate simulation results could be achieved with the use of parameters which are more effectively selected. One means of doing this is through the use of an optimisation procedure, which provides a means of selecting the most appropriate material values with respect to a number of different objectives.

The earliest work which utilising the optimisation process to improve plasticity simulation was considered by Mahnken and Stein [89] whereby improved simulations results using a combination $J_2$-flow theory and a nonlinear isotropic hardening model. This initial work was focused on the simulation of monotonic loading scenarios. In work by Sinaie, et al. [90] a multi-objective optimisation approach was utilised to determine the hardening parameter of the Chaboche model, including exponential isotropic hardening. The model outputs were matched against experimental data gathered from strain–controlled load histories, which were each followed by a tensile loading. Improvement in simulation accuracy with increasing number of back stresses was reported. However, it was also noticed that there was a diminishing extent of improvement with each additional backstress term. Plasticity parameter optimisation work conducted by Yun and Shang [91] who used a non-gradient based method coupled with a finite element model which used a user defined material subroutine to implement the plasticity model under investigation. It was found that the optimisation method was successful at capturing the experimental data. However, in both cases, the optimisation procedure did not consider the asymmetric loading cases; therefore, neither ratcheting nor mean stress simulation accuracy was considered in the optimisation of these parameters.
The genetic algorithm optimisation approach was used in research conducted by Badnava, et al. [92] to determine the most suitable Chaboche hardening parameters and isotropic hardening parameters for simulations to be compared with ratcheting, fully reversed strain-controlled and multiple step symmetric strain-controlled simulations. The parameters calculated were quite robust since simulations produced quite comparable results. However, this research was based on only one model and the variation of ratcheting and mean stress relaxation with different strain/stress amplitudes was not considered. The genetic algorithm optimisation process was also utilised in work conducted by Farrahi, et al. [93]. Fatigue simulations were run using the Chaboche, and Nagode and Fajdiga models using optimised parameters. This method of parameter calculation turned out to be quite good with simulated results matching experimental results quite well. Improved simulation results using the Chaboche model were also considered in [94]. An initial optimisation using the hysteresis loop as an objective was considered; however, it was noticed that in ratcheting simulation, the results were underperforming. This issue was improved by including ratcheting data as an objective. This improved the uniaxial and multiaxial ratcheting simulation.

Rate independent plasticity and creep effects in the same set of equations was considered by Grama, et al. [95] who recognised the need to incorporate material parameter optimisation techniques due to the complexity of parameter calculations. The features being modelled included isotropic hardening, kinematic hardening, hydrostatic pressure and temperature dependence and evolving micro-structural state of the material. The influence of the different parameters on simulation accuracy was also considered in this study by making use of the optimisation procedure.

Improved simulation results were also achieved by Simoni and Schrefler [96] by utilising a parameter optimisation procedure to calculate the parameters of a constitutive model used in the modelling of the mechanical behaviour of soil. Improvements to the simulation accuracy of a geologic material constitutive model was achieved by Desai and Chen [97] by employing a parameter optimisation procedure.
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Chapter I.2  Cyclic Elastoplastic Performance of Aluminium 7075-T6: Mechanical Testing
Cyclic Elastoplastic Performance of Aluminium 7075-T6 under Strain and Stress Controlled Loading

(Paper 1)

D. Agius, C. Wallbrink, K. I. Kourousis

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Cyclic Elastoplastic Performance of Aluminium 7075-T6 under Strain and Stress Controlled Loading

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Abstract

Elastoplastic investigations of aerospace aluminium are important in the development of an understanding of the possible cyclic transient effects and their contribution to the material performance under cyclic loading. Cyclic plasticity can occur in an aerospace aluminium component or structure depending on the loading conditions and the presence of external and internal discontinuities. Therefore, it is vital cyclic transient effects of aerospace aluminium are recognised and understood. This study investigates experimentally the cyclic elastoplastic performance of aluminium 7075-T6 loaded in symmetric strain-control, and asymmetric stress and strain-control. A combination of cyclic hardening and softening was noticed from high strain amplitude symmetric strain-controlled tests and at low stress amplitude asymmetric stress-controlled tests. From asymmetric strain-control results, the extent of mean stress relaxation depended on the size of the strain amplitude. Additionally, saturation of the ratcheting strain (plastic shakedown) was also found to occur during asymmetric stress-control tests. The experimental results were further analysed using published microstructure research from the past two decades to provide added explanation of the micro-mechanism contribution to the cyclic transient behaviour.
Keywords
Aluminium alloys, mean stress relaxation, ratcheting, cyclic hardening, cyclic softening, plasticity.

1. Introduction

Cyclic plasticity can occur in the bulk of the aerospace aluminium due to severe loading conditions, at the root of external discontinuities such as holes and notches and internal defects such as pores, as well as in the vicinity of a crack. Therefore, an in-depth understanding of the elastoplastic performance of aerospace aluminium under cycling is vital in the accurate design and sustainment of continuous structural integrity of aircraft. When the material is loaded in asymmetric stress-control an accumulation of strain can occur with progressive cycles which is known as strain ratcheting. Depending on the cyclic hardening/softening behaviour of the material, the micro-mechanisms which contribute to the accumulation of strain result in three main mechanisms of evolution:

- Constant increase in ratcheting strain [1-3];
- An initial increase in ratcheting strain before the onset of saturation in the accumulation of strain with cycles [4-6];
- Tertiary evolution of ratcheting strain [7].

Tensile mean stresses associated with asymmetric stress loading have been shown to reduce the fatigue life in various materials [6, 8-15], a consequence of a crack opening effect which increases the accumulation of damage [9]. Furthermore, strain ratcheting can have a detrimental effect on the life of a component due to additional fatigue damage associated with the accumulation of plastic strain [6].

When material is cyclically loaded in asymmetric strain-control a phenomenon known as mean stress relaxation occurs whereby the initial mean stress from the first load cycle progressively decreases in magnitude with cycles. The mean stress relaxation effects have been investigated in a number of different materials including aluminium alloys [16-18],
stainless steels [19], superalloys [20] and carbon steel [21-23]. Relaxation of mean stresses present in the material can be either beneficial or detrimental depending on whether a compressive or tensile mean stress is present. For example, the surface treatment processes such as shot peening can introduce beneficial fatigue crack initiation retarding compressive residual stress on the surface of the material, which could potentially be relaxed over time [24]. Alternatively, depending on the extent of relaxation, the influence of tensile mean stress on fatigue life can be reduced, slowing the rate of damage accumulation [19].

The 7xxx-series aluminium alloys, which are precipitation-hardened Al-Zn-Mg-(Cu) alloys, have been very important in high stressed applications, particularly in the aerospace industry. The mechanical performance of aluminium alloy 7075-T6 (AA 7075-T6) has been characterised and investigated experimentally with respect to its microstructure characteristics [25], the contribution of micro-mechanisms to low cycle fatigue [26] and asymmetric loading [27], mechanical behaviour under symmetric strain-control [28, 29], asymmetric strain-control [30], and asymmetric stress-control [31], as well as its fatigue behaviour [32]. However, a detailed understanding of the elastoplastic behaviour of aluminium 7075-T6 under both stress and strain control in conjunction with the micro-mechanisms which contribute to this behaviour has yet to be completed. This study presents an in-depth experimental analysis of the elastoplastic behaviour of aluminium 7075-T6 using symmetric strain and asymmetric stress/strain control coupon tests. The findings of the extensive microstructure analysis conducted over the last two decades is also used to provide an improved explanation of the macroscopic mechanical performance observations to better understand what contributes to the particular elastoplastic behaviour exhibited by aluminium 7075-T6. The experimental data, findings and analysis will be valuable for researchers and aerospace engineers developing and utilising elastoplastic constitutive model for the application of this aluminium alloy to fatigue life estimation methodologies, including safe-life (strain/stress-life) in the calculation of stress/strain amplitudes [33, 34] and damage tolerance in the modelling of the crack tip plastic zone [35].

The reported research presents a summation of the elastoplastic behaviour of aluminium 7075-T6 under cyclic loading conditions. Although the published literature does contains a
number of papers reporting the elastoplastic response of the specific aluminium alloy, this study is an expansion on the stress-controlled induced elastoplastic response. In particular, the strain ratcheting graphs and their corresponding hysteresis loops stress-strain curves are reported and discussed in detail. Moreover, an in-depth analysis of the strain and stress controlled induced cyclic elastoplastic data is presented and discussed, an effort that has not been undertaken at this extent in the past. Overall, this study furthers the understanding of the observed elastoplastic behaviour by combining an extensive microstructure analysis on aluminium 7075-T6, to aid in the explanation of the cyclic transient behaviour, which is considered an addition to the current body of knowledge.

2. Experimental Procedure

Monotonic tensile and cyclic symmetric stress and asymmetric strain-/stress-controlled loading for various amplitudes were conducted on aluminium 7075-T6. The test coupons were machined from blanks cut from 12 mm thick plate, with the long axis of the specimen parallel to the rolling direction. The test coupon geometries and dimensions used for each load control are shown in Figure 1. All tests were conducted at room temperature using MTS servo-hydraulic closed-loop testing machine with a capacity of 100 kN. Monotonic tensile tests were performed using a strain rate of 0.5 mm/min, following ASTM E8/E8M [36] standard for coupon geometry. Strain-/stress-controlled tests were conducted using a 0.1 Hz sine wave, following ASTM E606/E606M [37] and ASTM E466-07 [38] standards respectively for coupon geometry. Strain measurements were collected using an Epsilon 10 mm extensometer at a 0.1 s time step. It is noted that the time step of the measurement was used deliberately instead of the strain increment, since the latter was not consistent throughout the test.
Figure 1 Test coupon geometries and dimensions used in monotonic tension and cyclic strain and stress-controlled tests.

The cyclic hardening/softening behaviour of material was investigated by employing symmetric strain-controlled tests until failure using two different strain amplitudes ($\varepsilon_a$) and zero mean strain ($\varepsilon_m$). In this study, the cyclic hardening and softening was identified by examining the change in the:

- Strain amplitude with cycles in stress-controlled loading;
- Stress amplitude with cycles in strain-controlled loading.

The load cases are summarised in Table 1.

Table 1 Symmetric strain-controlled load cases

<table>
<thead>
<tr>
<th>Test</th>
<th>$\varepsilon_a$ (%)</th>
<th>$\varepsilon_m$ (%)</th>
<th>Cycles to failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.65</td>
<td>0</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>0</td>
<td>104</td>
</tr>
</tbody>
</table>
The asymmetric strain-controlled load cases were selected to determine the influence of varying strain amplitude with constant mean strain and the influence of varying the mean strain with constant strain amplitude. Therefore, both low and high strain amplitudes were selected to investigate the extent of mean stress relaxation. It is noted that these tests were concluded upon the completion of 150 load cycles (the test coupons did not experience failure). The conditions used for the asymmetric strain-controlled tests are listed in Table 2.

Table 2 Asymmetric strain-controlled load cases

<table>
<thead>
<tr>
<th>Test</th>
<th>$\varepsilon_a$ (%)</th>
<th>$\varepsilon_m$ (%)</th>
<th>Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.65</td>
<td>0.50</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>0.70</td>
<td>0.90</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>0.80</td>
<td>0.75</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>0.80</td>
<td>0.85</td>
<td>150</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>0.65</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.90</td>
<td>150</td>
</tr>
</tbody>
</table>

Asymmetric stress-controlled tests were conducted to develop an understanding of the ratcheting behaviour of aluminium 7075-T6. The influence of stress amplitude ($\sigma_a$) and mean stress ($\sigma_m$) on the accumulation of ratcheting strain was investigated by applying load cases with constant mean stress and increasing stress amplitude, and load cases with constant stress amplitude and increasing mean stress. These tests were also concluded upon the completion of 150 cycles (before the test coupon failed). The load cases used in the experiments are summarised in Table 3.

Table 3 Asymmetric stress-controlled loading

<table>
<thead>
<tr>
<th>Test</th>
<th>$\sigma_a$(MPa)</th>
<th>$\sigma_m$(MPa)</th>
<th>Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>470</td>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>480</td>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>40</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>50</td>
<td>150</td>
</tr>
</tbody>
</table>
3. Results and Discussion

3.1 Monotonic Tensile Test

The monotonic tensile behaviour was experimentally investigated with the results (stress – strain curve) presented in Figure 2. Also, the corresponding tensile material properties obtained from this study and their comparison to other published results are summarised in Table 4.

Table 4 Aluminium 7075-T6 monotonic tensile mechanical properties obtained from present study compared to previously published data.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Elastic Modulus (MPa)</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Strength (MPa)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>67,776</td>
<td>523</td>
<td>570</td>
<td>13</td>
</tr>
<tr>
<td>Fatemi, et al. [39]</td>
<td>72,200</td>
<td>512</td>
<td>572</td>
<td>No data</td>
</tr>
<tr>
<td>Meggiolaro and Castro [40]</td>
<td>71,900</td>
<td>498</td>
<td>576</td>
<td>No data</td>
</tr>
<tr>
<td>Li, et al. [41]</td>
<td>No data</td>
<td>504</td>
<td>560</td>
<td>No data</td>
</tr>
<tr>
<td>Rao and Ritchie [42]</td>
<td>73,000</td>
<td>503</td>
<td>572</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure 2 Stress – strain curve obtained from the monotonic tensile test.
3.2 Cyclic Symmetric Strain Controlled Tests

Two cyclic symmetric strain-controlled tests were conducted until failure. The difference in the hysteresis loop development for the two different strain amplitudes between the first and last hysteresis loops are shown in Figure 3. Since only one coupon for each strain amplitude was tested, the reliability of the life results obtained may be impacted, as this is subject to scatter as a consequence of the potential occurrence of different failure mechanisms. Instead, what is of interest is the elastoplastic features of the obtained data, which is described and discussed in detail.

![Symmetric strain-controlled first and last hysteresis loop at (a) 1.5% and (b) 1.8% strain level.](image)

**Figure 3** Symmetric strain-controlled first and last hysteresis loop at (a) 1.5% and (b) 1.8% strain level.

3.2.1 Tension/Compression Asymmetry

The interesting element of the data is the asymmetry in the peak stresses in tension and compression (Figure 4), where the peak stresses in compression are larger than those in tension. In a past study, this material response was also noticed to occur in aluminium 7075-T6 at lower strain amplitudes (0.4% plastic strain amplitude) [28]. The phenomenon is the result of strain localisation, which has been known to influence the mechanical behaviour of the material, particularly with respect to inhomogeneous deformation [43]. The reason for the strain localisation is hypothesised to be the consequence of the aluminium aging process, which may have caused the formation of quench bands [28]. Quench bands are regions of...
localised plasticity caused by thermal residual stresses formed during the quenching process. Residual stresses, remaining in the material after such post processing treatments or due to rolling or forging, result during cyclic loading in a biased crystal stress response toward the residual stress induced state of stress [27]. Consequently, these areas of the material will have associated dislocations, prohibiting the homogenous nucleation of precipitates due to a reduction in the free-vacancy concentration. This results in the formation of coarse precipitates, leading in turn to the localisation of the strain.

Figure 4 Comparison of the peak stresses in tension and compression showing asymmetry at both 1.5% and 1.8% strain amplitudes

The asymmetry is further analysed by calculating the asymmetry factor (AF) [28, 44] defined by relation (1).

\[
AF = \frac{|\sigma_c| - |\sigma_t|}{(|\sigma_c| + |\sigma_t|)/2}
\]  

(1)

Where \(\sigma_c\) and \(\sigma_t\) are the peak stresses in compression and tension respectively.

The AF evolution per cycle is shown in Figure 5 for both strain amplitudes examined. It is noticed that the asymmetry increases quite drastically in the first 20 cycles before reaching a constant level (plateau). This is due to the compression peak stress increase (hardening) happening at a faster rate than the tension peak stress initially (before the rates become

54
equivalent). However, the hardening rates change at the end of both test coupon’s lives due to the formation of cracks, reducing in the tensile load-carrying capacity as indicated by the AF increase in the last cycles [45]. The asymmetry trend noticed in these results is consistent with that reported in the past for the lower plastic strain amplitudes (\(\Delta \varepsilon_p\)), namely \(\Delta \varepsilon_p = 0.4\%\) [28] and \(\Delta \varepsilon_p = 0.3\%\) [45]. The initial asymmetry is also higher for the total strain amplitude (\(\varepsilon_a\)) of \(\varepsilon_a = 1.8\%\), which corresponds to \(\Delta \varepsilon_p \approx 1.0\%\), consistent with the results obtained by Meininger, et al. [45], where increasing AF with increasing \(\Delta \varepsilon_p\) was noticed. Furthermore, the saturated AF value in the \(\varepsilon_a = 1.8\% (\Delta \varepsilon_p \approx 1.0\%)\) case is lower than the initial AF, while for the \(\varepsilon_a = 1.5\% (\Delta \varepsilon_p \approx 0.7\%)\) case, the saturated AF is higher. This trend was also reported by Meininger, et al. [45], where it was reported that the difference in the AF value between the initial and saturated values progressively decreased with increasing strain, until between \(\Delta \varepsilon_p = 0.8\%\) and \(\Delta \varepsilon_p = 1.0\%\), the saturated values became less than the initial value.

![Figure 5](image.png)

**Figure 5** Comparison of the asymmetry factor (AF) at both 1.5% and 1.8% strain amplitudes.
3.2.2 Cyclic Hardening/Softening

A closer inspection of the stress amplitude of both symmetric strain results in Figure 6 reveals a strain amplitude ($\varepsilon_a$) influence on the cyclic hardening and softening of the material. In particular, for the $\varepsilon_a=1.5\%$ case, the activation of the cyclic softening is only very slight, and occurs close to failure, as evidenced by the slight decrease in stress amplitude in the last 15 cycles before failure in Figure 6. Conversely, from Figure 6, for the larger strain amplitude of $\varepsilon_a=1.8\%$, an initially large amount of hardening takes place before the activation of cyclic softening at approximately 25 cycles. It has been previously reported that aluminium 7075-T6 at low plastic strain amplitudes (below 0.6%) experiences cyclic hardening and saturates before fracture. While, for plastic strain amplitudes greater than 0.6%, cyclic softening can occur [28, 45]. The $\varepsilon_a=1.5\%$ loading corresponds to $\sim0.7\%$ plastic strain amplitude and $\varepsilon_a=1.8\%$ corresponds to $\sim1\%$ plastic strain amplitude, therefore, the behaviour observed in this study is considered to support the aforementioned published results. While cyclic softening is apparent, there is an initial large amount of cyclic hardening before the initiation of softening. In the $\varepsilon_a=1.5\%$ case, cyclic softening is almost negligible. Therefore, the experimental results indicate that cyclic softening will occur, however, this will not happen until a large amount of initial hardening takes place.

![Figure 6](image_url)  
**Figure 6** Stress amplitude progression with cycles for symmetric strain amplitudes 1.5% and 1.8%, where cyclic hardening and cyclic softening is apparent.
At low strain amplitudes, the cyclic behaviour is dictated by the dislocation substructure, where it was noticed that in aluminium 7075-T6 cyclic hardening occurred due to developing backstress (center of yield surface) as a result of increasing dislocation-dislocation and dislocation-particle interactions leading to a rearranging of the dislocations to from dense dislocation bands [26, 28]. This means the stresses required to overcome the dislocation interaction increases [46]. This is the consequence of an ordered precipitant structure [47]. Most of the hardening occurred before approximately 100 cycles, which is consistent with the findings of [26] reporting the formation of the dislocation bands very early in the life of the tested coupons (approximately within 100 cycles). It is important to note that peak-aged matrix contains both \(\eta'\) particles and Guinier Preston (GP) zones [25, 26, 48, 49]. It is considered possible that the higher strain amplitudes, with progressive cycles, could cause the dissolution of the GP zones and \(\eta'\) particles and the nucleation and growth of \(\eta\) particles [26], removing the ordering component of hardening and, thus, resulting in cyclic softening [47].

### 3.3 Cyclic Asymmetric Strain-Controlled Tests

#### 3.3.1 Cyclic Response

The results of the six cyclic asymmetric strain-controlled test cases (Table 2) are presented in Figure 7, which shows the downward shift of the hysteresis loops - by comparing the 1\(^{\text{st}}\), 20\(^{\text{th}}\) and 100\(^{\text{th}}\) cycles for each test case. In Test 1 [Figure 7(a)] the \(\varepsilon_a=0.65\%\) which is just above the monotonic yielding strain, as evidenced from the narrow hysteresis loop associated with very low levels of plastic work. The variation in plastic work is also evident between the corresponding strain amplitudes in Figure 7(a), 7(b), 7(e), and 7(f). The similarity in plastic work between the tests with the same stress amplitude is evident in Figure 7(c), 7(e), and 7(f).
Figure 7 Cyclic stress-strain hysteresis loops of the 1<sup>st</sup>, 20<sup>th</sup> and 100<sup>th</sup> Cycle obtained for each of the cyclic asymmetric strain-controlled test cases examined: (a) Test 1, (b) Test 2, (c) Test 3, (d) Test 4, (e) Test 5 and (f) Test 6.
3.3.2 Plastic Strain

A closer examination of the changing plastic strain amplitude with cycles is shown in Figure 8. A decreasing amount of plastic strain with cycles for all tests is evidenced, which indicates the occurrence of cyclic hardening. As a consequence of cyclic hardening, the cyclic yield stress progressively increases, which increases the amount of elastic strain per cycle. Since the peak strains are fixed, the plastic strain decreases with cycles. The amount of plastic strain present in Test cases 3 to 5 is very similar, since these test cases have the same strain amplitude ($\varepsilon_a=0.8\%$).

![Figure 8](image.png)

**Figure 8** Plastic strain amplitude (%) progression with cycles for each cyclic asymmetric strain-controlled tests (Test 1 to 6).

3.3.3 Mean Stress Relaxation

The test load cases successfully demonstrated varying rates of mean stress relaxation, leading to different levels of relaxation (saturation) as illustrated in Figure 9(a). In Test cases 1 and 6, the different amounts of relaxation are attributed to the varying levels of plastic deformation, as indicated by the difference in the hysteresis loop sizes [Figure 7(a), 7(b), 7(c), and 7(f)] and supported by the amount of plastic strain induced, as compared in Figure 8. With reference to Figure 9(a) smaller strain amplitudes result in less relaxation, while the larger strain amplitudes relax towards zero. Noticeably, from these results, the amount of
relaxation for Tests 3 to 5 is very similar. In these test cases, the amplitudes are the same value \((\varepsilon_a=0.8\%)\) but have varying initial mean strain. The noticeable similar relaxation amounts for load cases with constant strain amplitude but varying mean strain has also been identified in stainless steel (S32750) [50]. This suggests that the strain amplitude has the most influence on the amount of relaxation that occurs in the examined aluminium 7075-T6 material. Figure 9(b) presents the variation in strain amplitude \((\varepsilon_a)\) for each load case (Test 1 to 6). Cyclic hardening is apparent in all tests cases, evidenced by the increasing strain amplitude, which supports the observed cyclic hardening noticed from the plastic strain amplitude plots in Figure 8.

![Figure 9](image)

**Figure 9** (a) Mean stress relaxation for each cyclic asymmetric strain-controlled test case (Test 1 to 6) and corresponding (b) strain amplitude variation.

Mean stress relaxation with progressive cycles in aluminium 7075-T6 has been identified to occur at very small strain amplitudes and even in macroscopic elastic conditions, as previously reported by Arcari, et al. [16] and hypothesised to be the consequence of two different physical phenomena causing mean stress relaxation. Plastic straining can occur in the crystals even if the macroscopic stress levels are lower than the yield strength. The crystal plastic strain results in redistribution of the crystal stresses in a manner which reduces the peak stress [27]. Therefore, the mechanisms which cause mean stress relaxation at low strain amplitudes are not related to a change in mechanical state associated with either texture or hardening evolution.
An interesting observation in the results presented in Figure 8(a) results is that mean stress relaxation occurs towards zero stress for all test cases except for Test 6 case. In particular, in Test 6 the relaxing tensile mean stress crosses into the compressive stresses regime. This is a phenomenon which has also been reported by Arcari, et al. [16], however the published literature cannot offer insight on any microstructural-related causes. Further to the present study, we have planned to undertake further investigation into the occurrence of this behaviour through additional coupon testing at slight variations of the Test 6 loading conditions. This may enable verification (or disproval) of this phenomenon, while, in conjunction to a microstructure analysis of the tests coupons, it may prove helpful in improving our understanding on the underlying microstructural mechanisms.

The backstress ($X$) and effective stress ($\sigma_{\text{eff}}$) can be obtained from the experimental data by applying Cottrell’s method [51], demonstrated in Figure 10. The $\sigma_{\text{eff}}$ represents the size (increasing or decreasing) of the yield surface, while (as mentioned) $X$ corresponds to the centre of the yield surface (translating in the stress space). At microstructure level, $\sigma_{\text{eff}}$ represents the local stress required for dislocation movement [52], while $X$ is related to the strain heterogeneities which influence mobile dislocations [52]. These values can be calculated for each loading branch using the peak stress ($\sigma_{\text{max}}$) and the stress at the end of the elastic region of the hysteresis loop ($\sigma_{e}$), in conjunction with $(\sigma_{\text{max}} - \sigma_{e})/2$ to calculate the effective stress and $(\sigma_{\text{max}} - \sigma_{e})$ to calculate the backstress. Therefore, the varying $X$ and $\sigma_{\text{eff}}$ (evolving in tension and compression) can be graphically presented (Figure 11).

![Figure 10 Backstress and effective stress method of calculation using hysteresis generation.](image)
The dislocation substructure can also be used to explain the occurrence of mean stress relaxation. The initial loading initiates dislocation nucleation and dislocation movement. This results in increasing dislocation-dislocation (pile-up, migration) and dislocation-particle interaction as a result of microstructural factors including volume fraction and size of grains and the presence of hard and soft phases [53] causing the development of a backstress. Slip activation is therefore favoured in the reverse direction due to the development of the backstress in the initial loading. This results in an earlier yield and increased plastic deformation. In Figure 11 (a), the tensile backstresses are decreasing, while in Figure 11 (b) the compressive backstress are increasing. In tension, the increased dislocation density known to occur in aluminium 7075-T6 [26, 28], leads to potential forest hardening and in an increase of the effective stress in tension, resulting in a hindrance to dislocation motion (adding to the reduction in backstress in tension). It is noticed that in compression, the backstress increases, with its magnitude being greater than that occurring in tension. This could be the consequence of different deformation modes being activated, such as multiplication of alternate dislocation-types [52]. Also, in Figure 11 it is noticed that the evolution in the tensile and compressive backstress for Tests 3 and 4 are very similar, while in Test 6 their difference is significant. Also, the effective stress evolution in tension is different for all tests, while very similar in compression. This observation illustrates the strain-amplitude dependency of the backstress micro-mechanisms that occur during deformation, as opposed to being mean strain dependent (since Test 3 and 4 have the strain amplitude and differing mean strain, while Test 6 has a large strain amplitude). However, the effective stress evolution micro-mechanisms, which occur in tension and compression, are neither strain amplitude nor mean stress dependent but are influenced potentially by the peak stress field in tension and compression.
Figure 11 Evolution of (a) backstress in tension, (b) backstress in compression, (c) effective stress in tension, and (d) effective stress in compression across the cycles in asymmetric strain-controlled tests 3, 4 and 6.

Although it is expected the mean stress relaxation rate will progressively decrease as the mean stress decreases (due to the mean stress being the source of the instability), micro-mechanism evolution can also be used to explain this phenomenon. With increasing strain hardening under constant amplitude loading the shear stress required for slip initiation (critical resolved shear stress) increases [27]. Consequently, the number of slip systems will decrease, which results in a reduction in the nucleation of further dislocations. This would ultimately lead to a slowing down in the rate of mean stress relaxation, which can be seen for all the Test cases presented in Figure 9(a).
3.4 Cyclic Asymmetric Stress-Controlled Tests

3.4.1 Cyclic Response

The varying levels of plastic work and ratcheting strain accumulation is presented in Figure 12 for each of the stress-controlled test cases examined (Table 3). This variation is as evidenced by the difference in the hysteresis loop area and the distance between progressive hysteresis loops.

![Figure 12](image)

Figure 12 Stress-strain hysteresis loops’ comparison between each of the cyclic asymmetric stress-controlled test cases, including 1st, 20th, and 100th cycle comparison for (a) Test 1, (b) Test 2, (c) Test 3 and (d) Test 4.

3.4.2 Plastic Strain

A closer examination of the plastic strain amplitude and total strain amplitude progression with cycles, shown in Figure 13 (a) and (b) respectively, reveals that at the lower stress
amplitudes there is an initial cyclic softening occurring before the transition to cyclic hardening (happening at approximately 20 cycles). The larger stress amplitudes in Tests 3 and 4 show cyclic hardening only. This is the only example of cyclic softening occurring in the material below a 0.6% plastic strain. Furthermore, for asymmetric strain-controlled tests inducing the same level of plastic strain, there is no evidence of cyclic softening.

Figure 13 (a) Plastic strain amplitude and (b) total strain amplitude progression with cycles for each asymmetric stress-controlled test cases (Test 1 to 4).

Cyclic softening may occur in a material as a consequence of a lower dislocation interaction energy and/or dislocation annihilation [54], which improves dislocation movement by reducing the local stress required to promote dislocation movement. As previously discussed, aluminium 7075-T6 typically hardens cyclically due to the rearrangement of dislocations forming dense dislocation bands. As shown in this study, at plastic strain amplitudes above 0.6% cyclic softening eventually initiates after cyclic hardening. However, the micro-mechanisms which contribute to the initiation of cyclic softening at plastic strain amplitudes above 0.6% are different to those which would be occurring in Test 1 and Test 2, where an initial cyclic softening is apparent. This is due to the fact that cyclic softening at large plastic strain amplitudes is a cyclically induced phenomenon caused by particle dissolution and nucleation. Nevertheless, cyclic softening in Test 1 and Test 2 occurs immediately and it, therefore, negates the previous explanation for cyclic softening. One possible explanation for the occurrence of this initial cyclic softening could be the consequence of a smaller amount of pre-straining, when compared to that occurring in Test 3 and 4. Due to the small amount
of initial strain in the first loading branch, there may be less dislocation nucleation happening in Test 1 and Test 2, which results in a lower dislocation interaction energy than the other test cases which undergo larger pre-straining and promote cyclic softening. However, with increasing cycles, dislocation nucleation and motion continues to occur until eventually the dislocation-dislocation and dislocation-particle interactions cause the material to cyclically harden. A comparison of the difference in total strain and initial peak stress for the first loading branch for all asymmetric stress and strain controlled tests is offered in Figure 14, where one may observe that the asymmetric stress-controlled Tests 1 and 2 have a noticeably different size of total strain induced in the initial monotonic loading.

![Figure 14](image)

**Figure 14** First loading branch peak stress and total strain comparison for all cyclic asymmetric (strain and stress) test cases, where the noticeable difference in total strain asymmetric stress Test 1 and Test 2 is observed.

### 3.4.3 Strain Ratcheting

The accumulation of strain (ratcheting) with cycles for different stress amplitude and mean stress can be calculated with the use of Eq. 2.
Ratcheting Strain $= \frac{\varepsilon_{MAX} + \varepsilon_{MIN}}{2}$ \hspace{1cm} (2)

Where $\varepsilon_{MAX}$ is the strain and the peak stress of the cycle and $\varepsilon_{MIN}$ is the strain at the minimum stress.

The material ratcheting strain curves for the four test cases examined is presented in Figure 15.

![Ratcheting strain curves for each cyclic asymmetric stress-controlled test case (Test 1 to 4).](image)

**Figure 15** Ratcheting strain curves for each cyclic asymmetric stress-controlled test case (Test 1 to 4).

As illustrated in Figure 15 (and previously in Figure 12), there is a significant difference in the magnitude of ratcheting strain accumulated during Test 1 and Test 4 due to the different level of stress amplitude. The stabilisation of the mean strain per cycle in the ratcheting results shown in Figure 15 (plateaued or near-plateaued behaviour) indicate the appearance of plastic shakedown. Plastic shakedown occurring under stress-control refers to the closing of the hysteresis loops (in the presence of plastic strain), resulting in no further accumulation of ratcheting strain as schematically presented in Figure 16. The ratcheting strain results in Figure 15 indicate that plastic shakedown initiates relatively quickly (within 30 cycles), which in turn prevents the accumulation of damaging ratcheting strain.
Figure 16 A schematic representation of the plastic shakedown phenomenon.

A comparison of the ratcheting rates for each test is presented in Figure 17, where the ratcheting rate for all test cases diminishes within the first 30 cycles. This very rapid decrease in ratcheting rate may be attributed to an increasing dislocation density [55].

Figure 17 Ratcheting rate comparison for each cyclic asymmetric stress test case (Test 1 to 4).

Ratcheting strain accumulation is affected by cyclic hardening and backstress evolution. The evolution of backstress and effective stress for Test 2, 3 and 4 are compared in Figure 18.
The backstress evolution is very different in all three cases, while the effective stress evolution is very similar in both compression and tension for the three tests. This suggests that the difference in stress field, in all three tests, influences the long-range mobile dislocations and microstructure interactions rather than short range interactions represented by the effective stress. The increasing effective stress in tension in all three tests cases (as visualised in Figure 18) suggests that there is a reduction in dislocation movement in tension, which results in a reducing amount of plastic strain in tension. However, in compression, the backstress and effective stress are relatively steady, which indicates the plastic strain in compression is relatively constant (unchanging). This eventually leads to a closing of the hysteresis loops and no further ratcheting strain accumulation (plastic shakedown), with the dislocations having effectively reached a stable configuration [5] -as indicated by the steady state reached in Figure 17.

**Figure 18** Evolution of (a) backstress in tension, (b) backstress in compression, (c) effective stress in tension, and (d) effective stress in compression across the cycles in the cyclic asymmetric stress-controlled tests cases 2, 3 and 4.
It is noted that this study focuses on and deals with the experimental investigation of the aluminium 7075-T6 mechanical behaviour under various cyclic loadings. Thus, it was deliberately decided not to include analytical investigation results, with the reader referred to the [31] and [34] recently published papers, which report the mathematical modelling and simulation results obtained for the cyclic elastoplastic response of aluminium 7075-T6.

4. Conclusion

An in-depth investigation of the cyclic behaviour of aluminium 7075-T6 has been undertaken using several different test controlling techniques including symmetric strain, asymmetric strain, and asymmetric stress. The findings of this investigation can be summarised as following:

- Symmetric stain controlled tests showed significant and rapid hardening occurring in the material followed by the onset of softening in both the 1.5% and 1.8% symmetric strain amplitude test, with a greater level of softening occurring in the 1.8% amplitude tests. This was consistent with previously conducted experiments reported in the literature which demonstrated that for plastic strain amplitudes above 0.6% cyclic softening of the material occurs.

- Asymmetric strain controlled tests showed varying rates of relaxation depending on the strain amplitude applied. Varying the mean stress while keeping the strain amplitude constant did not show any significant change in relaxation behaviour, which suggests that the strain amplitude has a profound effect on the mean stress relaxation behaviour. A closer examination of the evolution of effective stress and backstress suggests that a difference in the micro-mechanism occurring in tension and compression resulted in the deceleration of the mean stress relaxation. Finally, in all test cases, cyclic hardening occurs, as evidenced by the decreasing plastic strain amplitude and increasing stress amplitude with cycles.

- Plastic shakedown occurred within approximately 30 cycles in all asymmetric stress controlled tests which resulted in no further accumulation of ratcheting strain. This is a particularly important material phenomenon since ratcheting strain accumulation is detrimental to the fatigue life of aerospace structures and components. A small
amount of initial softening occurred at two low peak stress tests, which was hypothesised to be the consequence of the initial monotonic loading not nucleating many dislocations (resulting in a lower dislocation interaction energy). However, the cyclic hardening dominated the rest of the load case once the dislocations increased with cycles. The other two load cases showed only cyclic hardening.

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References


Chapter I.3  Elastoplastic Constitutive Model Development

This chapter contains two main components:

- I.3.1 Optimising the multiplicative AF model parameters for AA7075 cyclic plasticity and fatigue simulation
- I.3.2 Aluminum Alloy 7075 Ratcheting and Plastic Shakedown Evaluation with the Multiplicative Armstrong–Frederick Model

In Section I.3.1 Initial multi-objective parameter optimisations using the multiplicative AF model showed promising results with respect to the utilisation of parameter optimisation workflows for strain-life fatigue predictions. However, it also identified a competing accuracy of simulations for strain ratcheting and mean stress relaxation. This was identified in Section I.3.2 to be partly the consequence of the applied elastoplastic constitutive models being incapable of simulating asymmetric stress-controlled plastic shakedown. Therefore, this chapter introduces the potential application of parameter optimisation workflows for elastoplastic models to be applied to strain-life fatigue predictions and describes a modification required for the elastoplastic constitutive models to improve strain ratcheting simulations.
I.3.1 Optimising the multiplicative AF model parameters for AA7075 cyclic plasticity and fatigue simulation

(Paper 2)

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(Aircraft Engineering and Aerospace Technology)
Optimising the multiplicative AF model parameters for AA7075 cyclic plasticity and fatigue simulation

Abstract

Purpose - This study presents the improvements of the Multicomponent Armstrong-Frederick model with Multiplier performance through a numerical optimisation methodology available in a commercial software. Moreover, it explores the application of a multi-objective optimisation technique for the determination of the parameters of the constitutive models using uniaxial experimental data gathered from Aluminium Alloy 7075-T6 specimens. This approach aims to improve the overall accuracy of stress-strain response, not only for symmetric strain controlled loading but also for asymmetrically strain- and stress-controlled loading.

Design/Methodology/Approach - Experimental data from stress and strain controlled symmetric and asymmetric cyclic loadings have been utilised for this purpose. The analysis of the influence of the parameters on simulation accuracy has led to an adjustment scheme that can be used for focused optimisation of the MAFM model performance. The method was successfully employed to provide a better understanding of the influence of each model parameter on the overall simulation accuracy.

Findings - The optimisation identified an important issue associated with competing ratcheting and mean stress relaxation objectives; highlighting the issues with arriving at a parameter set that can simulate ratcheting and mean stress relaxation for load cases not reaching at complete relaxation.

Practical Implications - The study uses a strain-life fatigue application to demonstrate the importance of incorporating a technique such as the presented multi-objective optimisation method to arrive at robust parameters capable of accurately simulating a variety of transient cyclic phenomena.
Originality/Value - The proposed methodology improves the accuracy of cyclic plasticity phenomena and strain-life fatigue simulations for engineering applications. This study is considered a valuable contribution for the engineering community, as it can act as starting point for further exploration of the benefits that can be obtained through material parameter optimisation methodologies for models of the Multicomponent Armstrong-Frederick class.

Keywords: Cyclic plasticity; Fatigue; Kinematic hardening; Mean stress relaxation; Optimisation; aircraft structures.
1. Introduction

The application of advanced kinematic hardening models of cyclic plasticity can significantly improve the simulation accuracy, due to their ability to account for the effect of transient cyclic phenomena such as mean stress relaxation and ratcheting. Simulation accuracy is important for many engineering applications, particularly for fatigue life calculation of structures containing notches (Hu and Wallbrink, 2014). The motivation behind this is that a combination of strain and stress controlled loading occurs at the notch root, where localised plasticity is present (Hu et al., 1999). This highlights the importance of accuracy in simulating these two cyclic phenomena. Consequently, it is crucial that the parameters defining the plasticity model are determined accurately.

As the experimental understanding of materials increases, so has the level of sophistication and complexity of elastoplastic models, leading to increased parameter calculation requirements, as recognised by Grama et al. (2015). In order to improve the calculation process of elastoplastic constitutive model parameters, various optimisation techniques have been investigated, with two main optimisation strategies identified, the gradient-based (Mahnken and Stein, 1996, Saleeb et al., 2002, Desai and Chen, 2006) and the genetic algorithm (GA) methodologies (Rahman et al., 2005, Krishna et al., 2009, Badnava et al., 2012, Agius et al., 2017a, Mahmoudi et al., 2011, Farrahi et al., 2014, Rokonuzzaman and Sakai, 2010, Khademi et al., 2015, Cermak et al., 2015, Zhao and Lee, 2002, Khutia and Dey, 2014, Franulović et al., 2009). As highlighted by Furukawa et al. (2002), the disadvantage of the gradient-based approach in constitutive model parameter determination lies in the solution divergence, an issue not typically associated with GA methodologies, which are instead associated with poor solution efficiency. Therefore, a combination of the two strategies has also been applied in the past (e.g. Chaparro et al., 2008) to improve the parameter optimisation process, while further improved optimisation strategies have also been suggested and investigated by other researchers (Sinaie et al., 2014, Yun and Shang, 2011).
This study explores the application of a multi-objective optimisation technique for the determination of the parameters of the constitutive models using uniaxial experimental data gathered from Aluminium Alloy (AA) 7075-T6 specimens. Since this study aims to simulate cyclic transient effects of two different control methods of varying amplitudes (strain and stress), the difficulty at arriving at a parameter set capable of achieving accurate simulations of both phenomena justifies the application of a robust optimisation strategy, such as the GA, so as to limit the potential of solution divergence. This approach aims to improve the overall accuracy of stress-strain response, not only for symmetric strain controlled loading but also for asymmetrically strain- and stress- controlled loading. Moreover, this analysis aims to shed light on how the parameter values obtained from commonly used methodologies can be adjusted to improve the simulation results. Finally, the study uses a strain-life fatigue application to demonstrate the importance of incorporating a technique such as the presented multi-objective optimisation method to arrive at robust parameters capable of accurately simulating a variety of transient cyclic phenomena.

2. Cyclic Plasticity Model

In the rate independent plasticity theory, the fundamental components of a constitutive model include a yield criterion, a flow rule, and a kinematic, isotropic or combined hardening rule. In this work, the von Mises yield criterion and an associative flow rule were used, as appropriate for ductile metals. The Multicomponent Armstrong-Frederick model with Multiplier (MAFM) (Dafalias et al., 2008) was the kinematic hardening rule implemented in this study, due to its proven ability to simulate cyclic transient effects of aluminium alloys (Kourousis and Dafalias, 2013, Agius et al., 2017b). The MAFM is based on the widely used Multicomponent Armstrong-Frederick (Armstrong and Frederick, 1966) model (MAF) (Chaboche et al., 1979), with the difference between them being in the way hardening is modelled.

The uniaxial formulation of the MAF model is given in Eq. 1, where $X_i$ is the back-stress, $c_i$ and $\gamma_i$ are the material parameters and $de^P$ is the incremental plastic strain. The MAFM model has a fourth term as given Eq. 2, in addition to the three terms given by Eq. 1. The square brackets in Eq. 2 contain the so-called multiplier expressed in a non-dimensional back-stress $X^*$
which itself is given in Eq. 3, where $C^*$ and $\gamma^*$ are also material parameters.

For $i = 1,3$:

$$dX_i = C_i d\varepsilon^p - \gamma_i X_i |d\varepsilon^p|$$  \hspace{1cm} (1)

For $i = 4$:

$$dX_i = \left[ \frac{\gamma_i}{C_i} + \frac{C_i^*}{\gamma_i} \right] \left( \frac{C_i}{\gamma_i} d\varepsilon^p - X_i |d\varepsilon^p| \right)$$  \hspace{1cm} (2)

$$dX_i^* = C_i^* d\varepsilon^p - \gamma_i^* X_i^* |d\varepsilon^p|$$  \hspace{1cm} (3)

Furthermore, an isotropic hardening rule, as proposed by Chaboche (Chaboche, 1986), is given in Eq. 4, where $R$ is the magnitude of the yield stress and $R_s$, $b$ the material parameters controlling its evolution.

$$dR = b \left( R_s - R \right) |d\varepsilon^p|$$  \hspace{1cm} (4)

3. Parameter Determination Methodology

The parameter determination methodology presented in the next sub-sections is composed of two steps, determination of the baseline parameters using one of the established methods, and the optimisation of these parameters.

3.1 Baseline Parameters

The initial parameter calculation method is described in the work of Dafalias et al. (2008) and it is based on the methodology originally developed by Chaboche (1991) for the Multicomponent Armstrong-Frederick with Threshold (MAF-T) model (Chaboche, 1991). The formulation of the first three back-stresses is given by Eq. 1. Explicitly integrating Eq. 1 and assuming the initial conditions of the loading branch are given as $X_0 = -C_i/\gamma_i$ and $\varepsilon^{p0} = -\varepsilon^{p0}$, Eq. 5 can be used in conjunction with the loading branch of a saturated hysteresis loop (Bari and Hassan, 2000). The values of $C_i$ and $\gamma_i$ were determined by fitting Eq. 5 to the 1.8% strain-controlled stabilised cycle obtained from AA 7075-T6 experiments. The MAFM model parameters were adjustment with the technique outlined in (Dafalias et al., 2008), previously tested for AA 7050 (Kourousis and Dafalias, 2013). The obtained parameters are summarised
in Table 1.

\[
\sigma = \sigma_y + \sum_{i=1}^{4} \frac{C_i}{\gamma_i} \left(1 - 2\exp\left(\gamma_i \left(e^{\varepsilon_i^p} - (-e^{\varepsilon_i^p})\right)\right)\right) + R
\]

(5)

<table>
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<th>Table 1. Baseline MAFM parameters for AA7075-T6.</th>
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<td>Parameter</td>
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</tr>
<tr>
<td>Elasticity Modulus, E (MPa)</td>
</tr>
<tr>
<td>Yield Strength, ( \sigma_{yield} ) (MPa)</td>
</tr>
<tr>
<td>( C_1 ) (MPa)</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
</tr>
<tr>
<td>( C_2 ) (MPa)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
</tr>
<tr>
<td>( C_3 ) (MPa)</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
</tr>
<tr>
<td>( C_4 ) (MPa)</td>
</tr>
<tr>
<td>( \gamma_4 )</td>
</tr>
<tr>
<td>( C_4^<em>/\gamma_4^</em> )</td>
</tr>
<tr>
<td>( \gamma_4^* )</td>
</tr>
<tr>
<td>( R_s ) (MPa)</td>
</tr>
<tr>
<td>( b )</td>
</tr>
</tbody>
</table>

3.2 Optimising Parameters

The parameter optimisation methodology consists of coupling the MAFM material model with an optimisation engine provided by the commercial software package modeFrontier (ESTECO, 2015). In the applied optimisation workflow, there are three common sections for a typical problem: input, processing, and constraints and objectives. In total, 13 input parameters were used, with each attributed an allowable range as shown in Table 2. Although the yield stress is a material constant, it was allowed to vary in the optimisation process to improve the shape of the hysteresis loop by avoiding sharp and unrealistic elastic to plastic transition.
Table 2. Ranges and objectives for optimisation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
<th>Objective</th>
<th>Objective Loads (Max, Min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{yield}}$ (MPa)</td>
<td>(250,380)</td>
<td>Hysteresis loop shape</td>
<td>(1.8%, -1.8%)</td>
</tr>
<tr>
<td>$C_1, C_2, C_3, C_4$ (MPa)</td>
<td>$(1,10^5)$</td>
<td>Ratcheting</td>
<td>(540MPa, -460MPa)</td>
</tr>
<tr>
<td>$\gamma_1, \gamma_2, \gamma_3, \gamma_4$</td>
<td>$(1,10^5)$</td>
<td></td>
<td>(530MPa, -440MPa)</td>
</tr>
<tr>
<td>$\gamma_4^*$</td>
<td>$(1,10^6)$</td>
<td></td>
<td>(510MPa, -430MPa)</td>
</tr>
<tr>
<td>$C_4^<em>/\gamma_4^</em>$</td>
<td>$(1,10^6)$</td>
<td>Mean stress relaxation</td>
<td>(1.55%, -0.05%)</td>
</tr>
<tr>
<td>$R_s$ (MPa)</td>
<td>(0,65)</td>
<td></td>
<td>(1.60%, 0.20%)</td>
</tr>
<tr>
<td>$b$</td>
<td>$(0,10^5)$</td>
<td></td>
<td>(1.90%, -0.10%)</td>
</tr>
</tbody>
</table>

A schematic of the optimisation workflow employed in this study is given in Figure 1. Each stage of the workflow is outlined in the corresponding sections. Although the primary focus of this study is on the MAFM model, the optimisation workflow presented can also be applied to other elastoplastic constitutive models, through the following modifications:

- Modification of the initial population stage to include different parameter strings to recognise the alternate model formulation;
- Modification of the elastoplastic constitutive model stage to include the elastoplastic constitutive models required for the investigation.
3.2.1 Initial Population

The first stage of the optimisation workflow requires the formation of an initial population of 25 parameters. Strings of parameter values were determined by selecting from the ranges outlined in Table 2 using one or a combination of pseudo random sequence generator, which in this study was the Sobol sequence. An additional restriction was imposed on the parameter selection process to ensure that the parameters selected did not deviate significantly from the maximum stress attained under symmetric strain-controlled tests (Eq. 6). A relatively large
value of 600MPa was used as the maximum bound to exclude unrealistic solutions for the parameters and to avoid putting too much restriction on the search space for the parameters. A slight error in the approximation of the hysteresis loop shape is tolerated to ensure greater opportunity at reaching reasonable accuracy for other types of material behaviour (e.g. ratcheting).

\[ \sum_{i=1}^{4} C_i \gamma_i + R_S + \sigma_{\text{yield}} \leq 600\text{MPa} \]  \hspace{1cm} (6)

### 3.2.2 Objective Function

The input parameters which form the generation were then used to integrate the MAFM model using a backward Euler scheme, in conjunction with the nonlinear isotropic hardening model. The outputs of the simulation were compared to experimentally collected data to determine the accuracy of the selected parameters. The objectives of the optimisation workflow were used to minimise fitness values based on the difference between normalised simulated outputs and experimental data for ‘objective loads’ given in Table 2, which correspond to the stabilised hysteresis loop shape, the mean stress relaxation rate and the ratcheting rate. Moreover, a number of different loading cases were used to ensure that the optimised parameters were able to simulate effectively a diverse set of load cases.

The fitness value was defined according to the Fréchet distance (Alt and Godau, 1995), which is the maximum Euclidean distance of all possible ways to traverse the simulated and experimental curves. The Fréchet distance (FD) can be calculated using Eq. 7, where P and Q refer to the functions of the two curves being compared and i and j refer to points along those curves.

\[ FD[i, j] = \max \left[ \left\| P(i) - Q(j) \right\|, \min (FD[i-1, j-1], FD[i, j-1], FD[i-1, j]) \right] \]  \hspace{1cm} (7)

The obtained fitness scores were then submitted to the optimiser, which would generate a new set of values.
3.2.3 Optimiser

To concurrently achieve multiple objectives, a proprietary version of Multi-Objective Genetic Algorithm (MOGA-II) (ESTECO, 2015) was used as the optimisation engine in this study, which is a modified version of the MOGA (Poloni and Pediroda, 1997). As with the classical MOGA, the MOGA-II is based on a natural selection and genetics concept, whereby at the end of each optimisation iteration, the population of test parameter strings (or parameter sets) were evaluated to form a new population based on the best performing (with respect to the defined objectives) parameter strings. The new population consists of elite children, crossover children, and mutation children (‘children’ refers to a new parameter string, developed from previous parameter strings, or ‘parent’ strings). The children are formed from the combination of parent parameter strings, consequently, the next population will contain parameters selected from previous iterations. However, mutation children are formed from a random modification of a parent string, which demonstrates that the next population will also include new untested parameter strings. In MOGA-II, the amount by which the parent string is mutated can be set using DNA String Mutation Ratio. The MOGA-II settings used in this study are given in Table 3.

<table>
<thead>
<tr>
<th>Population</th>
<th>Selection Probability</th>
<th>Crossover Probability</th>
<th>Mutation Probability</th>
<th>DNA String Mutation Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Algorithm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Sobol</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3. MOGA-II settings used in the multi-objective study.

3.2.4 Optimised Parameters

The values of the MAFM parameters obtained from the optimisation method are provided in Table 4. Due to the nature of a multi-objective study, a number of possible solutions exist; however, in this study, selection of the most suitable parameter set is that based on attempting to achieve parameter robustness. The selected values in Table 4 corresponded to the solution which provided a relatively balanced accuracy across the different elastoplastic features.
considered in the optimisation (hysteresis loop shape, mean stress relaxation, strain ratcheting).

Table 4. Optimised MAFM parameters for AA7075-T6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity Modulus, $E$ (MPa)</td>
<td>69,000</td>
</tr>
<tr>
<td>Yield Strength, $\sigma_{yield}$ (MPa)</td>
<td>302</td>
</tr>
<tr>
<td>$C_1$ (MPa)</td>
<td>58,016</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>55,740</td>
</tr>
<tr>
<td>$C_2$ (MPa)</td>
<td>43,437</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>43,757</td>
</tr>
<tr>
<td>$C_3$ (MPa)</td>
<td>370</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>370</td>
</tr>
<tr>
<td>$C_4$ (MPa)</td>
<td>465</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>465</td>
</tr>
<tr>
<td>$C_4$/$\gamma_4^*$</td>
<td>513,447</td>
</tr>
<tr>
<td>$\gamma_4^*$</td>
<td>703,641</td>
</tr>
<tr>
<td>$R_s$ (MPa)</td>
<td>51</td>
</tr>
<tr>
<td>$b$</td>
<td>34</td>
</tr>
</tbody>
</table>

4. Results

4.1 Uniaxial Simulations

In order to determine the accuracy of the optimised parameters, symmetric/asymmetric strain controlled and asymmetric stress controlled simulations were conducted and compared to simulations using the baseline parameters. Error calculations using Eq. 8, where $M$ refers to the number of data points used in the comparison, were used as a means of comparing the accuracy of the base-line and optimised parameter outputs, with values given in Table 5.

$$\text{Error} = \frac{1}{M} \sum_{i=1}^{M} \left( \frac{Y_i^{EXP} - Y_i^{SIM}}{Y_i^{MAX}} \right)^2$$  (8)

Out of the seven load cases examined (four strain controlled and three stress controlled) an improvement was achieved in the five (from modest to drastic improvement) for the optimized simulations. A slight and very modest deterioration was observed for the (1.5%, -1.5%) and (1.65%, 0.05%) strain loading cases (-0.27% and -4.11% respectively). Overall,
the improvement achieved, as measured by the average error across all cases, was very significant (81.74%), which provides an indicator of the optimisation process effectiveness.

Table 5. Error calculation for each simulation for both baseline and optimised simulations.

<table>
<thead>
<tr>
<th>Loading condition</th>
<th>Baseline Simulations (%)</th>
<th>Optimised Simulations (%)</th>
<th>Improvement with Optimised simulations (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.5%, -1.5%)</td>
<td>0.01</td>
<td>0.28</td>
<td>-0.27</td>
</tr>
<tr>
<td>(1.35%, 0.05%)</td>
<td>65.17</td>
<td>7.42</td>
<td>+57.75</td>
</tr>
<tr>
<td>(1.60%, 0.20%)</td>
<td>38.10</td>
<td>8.99</td>
<td>+29.11</td>
</tr>
<tr>
<td>(1.65%, 0.05%)</td>
<td>9.92</td>
<td>14.03</td>
<td>-4.11</td>
</tr>
<tr>
<td>(510MPa, -430MPa)</td>
<td>281.09</td>
<td>0.20</td>
<td>+280.89</td>
</tr>
<tr>
<td>(520MPa, -440MPa)</td>
<td>162.60</td>
<td>0.29</td>
<td>+162.31</td>
</tr>
<tr>
<td>(540MPa, -460MPa)</td>
<td>46.51</td>
<td>0.01</td>
<td>+46.5</td>
</tr>
<tr>
<td>Average</td>
<td>86.2</td>
<td>4.46</td>
<td>+81.74</td>
</tr>
</tbody>
</table>

Figure 2 presents a comparison of the (AA7075-T6) stabilised 1.5% strain controlled experimental cycle (150th cycle) with the MAFM model simulated results for the parameters obtained by baseline and optimised methods.
A comparison between three different levels of mean stress relaxation test data with simulation results are given in Figure 3.

**Figure 2.** AA7075-T6 hysteresis loop (150th cycle): Experimental data (circle points) and MAFM model simulated results (line) with parameters obtained in (a) baseline and (b) optimised calculation methods.

**Figure 3.** AA7075-T6 mean stress relaxation: Experimental data (circle points) and MAFM model simulated results (lines) with parameters obtained in (a) baseline and (b) optimized calculation methods.

Significant improvement was achieved by the optimised set of parameters, as indicated by the ability of the simulation results to successfully improve the level of mean stress relaxation
saturation. However, the rate of evolution in relaxation of the mean stress was not improved as successfully, which is indicated by blue and black simulated relaxation curves reaching saturation considerably faster than the experimental results.

A comparison between the test data for ratcheting and the simulation results is given in Figure 4.

![Figure 4. AA7075-T6 ratcheting: Experimental data (circle points) and MAFM model simulated results (line) with parameters obtained in (a) baseline and (b) optimised calculation methods.](image)

Ratcheting strain was calculated as the strain at the peak stress of each cycle. Once again, significant improvement was achieved by the optimised set of parameters. The plastic shakedown, which is seen as the stabilisation of the hysteresis loop shape due to a diminishing rate of plastic accumulation with repeated cycles, was accurately simulated in all three load cases. The strain at which the plastic shakedown occurred was accurately simulated in all three load cases using the optimised set of parameters. This is an improvement on the baseline parameter simulation which were incapable of predicting plastic shakedown in any of the tested load cases.

Comparing the baseline and optimized simulation results, it has been shown that improved results are obtained for mean stress relaxation and separately for ratcheting but not both simultaneously. This can be partially attributed to the nature of the optimisation exercise.
itself, which is to achieve a simulation balance between very different load cases (stress / strain controlled, under different stress and strain levels). Moreover, given the emphasis of the MAFM model on ratcheting prediction (this feature was incorporated in this model by design), better performance is anticipated (and indeed achieved with parameter optimisation) for the corresponding cases, as opposed to mean stress relaxation.

4.2 Parameter Selection Influence

The optimisation process was further utilised to investigate the contribution of each of the MAFM model parameters in the simulation accuracy. In particular, starting from the baseline parameters, one parameter at a time was allowed to vary while the others remained constant. A least square fit of the simulated data to the experimental data was performed for each parameter variation. This resulted in a total fitness value across each objective for the stabilised hysteresis loop shape, mean stress relaxation rate and ratcheting rate. This procedure provided a means of assessing how the adjustment of one parameter influences the simulation accuracy of an alternate simulation output.

Using these results, it was found that the most influential parameter for the simulation of all cases examined (stabilised hysteresis loop, mean stress relaxation and ratcheting) was \( \gamma_i \ (i = 1, 2, 3, 4) \). Ranges over which these parameters improved simulation accuracy were compared with the other parameter ranges in order to identify any overlaps. An overlap suggests that adjustment of that parameter would lead to improved simulation accuracy of all objectives. The obtained results for this comparison are shown in Figure 5, where the different shades correspond to whether the parameter adjustment led to: improved simulation accuracy (positive), inaccurate simulations (negative), or slightly inaccurate simulations (slight negative). Values outside of the ranges displayed are not included since adjustment would result in violation of the Eq. 6 restriction.
Figure 5. Comparison of the MAFM model parameters’ ranges influence in simulation accuracy for hysteresis loop shape (shape), mean stress relaxation (relaxation) and ratcheting.

It is noticeable in Figure 5 that an overlap exists in the adjustment of $\gamma_4$ (range up to 400) between shape, relaxation, and ratcheting. Effectively, improvements to simulation accuracy can be achieved for all cases by altering this parameter. Alternatively, there is a particular range (value over 600) where the variation of $\gamma_3$ will improve ratcheting, but will result in inaccurate hysteresis loop shape and mean stress relaxation simulations. The term $\gamma_3$ corresponds to the back stress having a linear-type response (low slope and very quick saturation), utilised primarily for adjusting the ratcheting rate. Thus, this term ($\gamma_3$) has the potential to influence negatively the hysteresis loop shape and relaxation, which is indeed confirmed by the sensitivity analysis. Although this inverse proportionality overlap is also noticed in $\gamma_2$, the range over which this exists is significantly smaller (value up to 400).

Finally, there is an improvement overlap existing between shape and relaxation accuracy (around the 200 value). However, varying this parameter will have a detrimental effect on ratcheting accuracy. Therefore, based on this analysis, it is recommended to adjust $\gamma_1$, $\gamma_2$ and $\gamma_4$ when using the MAFM model, in order to achieve an improved simulation. It is also noted that, a more accurate solution could be obtained through the adjustment of $\gamma_3$, however the
inverse proportionality makes successful adjustment of this parameter very difficult.

4.3 Strain-Life Simulations

The optimisation method presented above provides a rational way of deriving a more robust set of parameters capable of characterising asymmetric strain-controlled and stress-controlled cyclic transient behaviour for varying loading conditions. In order to illustrate the importance of parameter selection on fatigue life prediction, the MAFM was implemented in the Defence Science and Technology (DST) Group developed fatigue analysis program called CGAP (Wallbrink and Hu, 2010) and used in strain-life fatigue calculations. Assessment of the performance of the model defined using the baseline and optimised parameters was conducted using past experimental data gathered as part of the P-3C Orion aircraft service life assessment program performed at DST Group (Mongru et al., 2010, Matricciani et al., 2016). Strain-life fatigue calculations were conducted for 21 different load spectra, using the baseline and optimised parameters, and compared to the prior experimental data gathered from notched coupons. The fatigue calculation process is summarised as following:

- In the strain-life fatigue calculation method the Neuber’s rule (Neuber, 1961) was used to relate the remote stress to local stress and strains.
- The equivalent strains, used in conjunction with the strain-life curve, were calculated using the modified Morrow equation (Dowling, 2009).
- Finally, fatigue damage accumulation was calculated using Miner rule (Miner, 1945).

Simulation results using the baseline and optimised parameter sets are compared in Figure 6, directly to the geometric mean calculated using the experimental fatigue lives gathered for each of the 21 load spectra. A majority of both simulation groups (baseline and optimised) are contained in the lower (green shaded) portion of the chart, which indicates that most of the simulations are conservative.
Figure 6. Simulated fatigue lives plotted against the corresponding geometric mean of the experimental results.

Given on Figure 6 is the accumulated difference for the MAFM optimised and baseline fatigue simulations. The results compare the geometric mean of the experimental data for each spectrum with the simulated fatigue life, as calculated using Eq. 8 with $M = 21$.

The total accumulated error across all 21 spectra is lower for the optimised MAFM simulations, indicating an improvement to strain-life fatigue calculations. To provide a closer inspection of the improvement for each of the spectra tested, Figure 7 compares the calculated difference between the predicted value using the MAFM optimised and MAFM baseline to the geometric mean of the experimental data for each spectrum.
What is conveyed by examining Figure 7 is the improvement in fatigue accuracy. The blue line drawn at 50% error highlights how the optimised parameters have reduced the tested spectra with errors greater than 50% from 7 to 4. Overall, the fatigue accuracy has been improved in 14 of the tested spectra. The 7 tested spectra which do not provide an improvement in results are indicated by the red dots on top of the graph bars. However, out of the 7 cases, where the optimised parameters do not offer an improvement, the results are still very much comparable to the MAFM baseline simulations.

5. Discussion

It is observed that the optimised parameters have resulted in a slight decrease in the simulation accuracy of the stabilised strain-controlled hysteresis loop (Figure 2), when compared to the baseline parameters as demonstrated by the larger error calculation for the optimised parameter simulation. This reduction in accuracy can be explained with reference to the optimisation procedure. The parameter search in the optimisation spans multiple objectives and the narrowing of the search across the population of potential solutions is aimed at achieving comparable accuracy for all objectives. This could result in a reduction in accuracy in some of the objectives in order to provide a more robust parameter set for a
wider range of loading conditions, which explains the slight reduction in accuracy in the optimised MAFM symmetric strain-controlled simulations.

The optimisation process had difficulty in accurately simulating both the ratcheting and mean stress relaxation phenomena in strain-controlled load cases which do not induce complete relaxation of the mean stresses. The model was considerably more capable of accurately simulating the ratcheting behaviour than the mean stress relaxation behaviour. This is further supported by Figure 8 where the ratcheting and mean stress relaxation objectives are plotted for all valid iterations.

Figure 8. Evolution of multi-objective design iterations (mean stress relaxation versus strain ratcheting objectives).

Figure 8 was constructed from results gathered from a multi-objective analysis using three objectives: (1.5%, -1.5%) stabilised hysteresis loop, (540MPa, -460MPa) ratcheting strain, and (1.6%, 0.2%) mean stress relaxation. The smaller number of objectives was utilised in order to more effectively investigate the relationship between the accuracy in mean stress relaxation and ratcheting simulations. Indicated in Figure 8 is what is known as the Pareto front, which gives an understanding of the trade-off that exists between objectives.
Optimisation to improve accuracy for mean stress relaxation leads to a reduction in accuracy for ratcheting, and vice versa. At the midpoint of this front the two objective values are comparable; therefore, although the simulation accuracies for both objectives are comparable neither objective has been completely optimised. The shape of the Pareto front also provides an understanding of the relationship between objectives. The concave appearance of the Pareto front suggests a very strong trade-off between objectives; therefore, the existence of significant conflicting simulation accuracy between ratcheting and mean stress relaxation.

The strain-life fatigue application using the optimised parameters highlighted the importance of using an optimisation procedure to develop a more robust parameter set capable of simulating a variety of different cyclic transient phenomena. This was indicated by the improvement in the accuracy of fatigue life prediction for the majority of aircraft service loads tested. This improvement in accuracy also highlighted the potential of using uniaxial constant amplitude experimental data to develop kinematic and isotropic hardening parameters to be used in the strain-life fatigue analysis which implements complex variable amplitude loading.

6. Conclusions

Current engineering practice is generally limited to using plasticity models embedded within commercial finite element analysis packages (i.e. MAF model), which are known to have performance limitations. The simulation results presented in this study highlight the significant benefit of employing an optimisation procedure to determine the parameters for the advanced MAFM model. This is an important finding from the point of view of applied engineering, since sophisticated plasticity models (such as MAFM) can be employed more accurately by properly selecting (optimising) their parameters.

The simulation accuracy of cyclic phenomena occurring from a number of different loading cases was drastically improved with the application of a multi-objective optimisation method (included in the mode-Frontier software), in comparison to the standard protocols used for the determination of model parameters. The optimisation method was successfully employed to provide a better understanding of the influence that each of the different MAFM model
parameters has on the overall simulation accuracy. This implementation exercise has confirmed the suitableness of an optimisation process available in commercial software, which is accessible by engineers working on structural analysis and does not require specialist knowledge in the mathematical background, formulation and numerical algorithms involved.

Moreover, the optimisation identified an issue associated with competing ratcheting and mean stress relaxation objectives; highlighting the issues associated with arriving at a parameter set which can accurately simulate ratcheting and mean stress relaxation for strain-controlled load cases incapable of inducing complete relaxation. This finding can be useful when researchers and engineers employ advanced plasticity models (in the examined case the MAFM model) for problems requiring a more focused parameter selection strategy (i.e. if ratcheting is the main concern in a structural problem, the parameters’ variation window can be adjusted accordingly, etc).

Finally, the importance of parameter optimisation based on uniaxial stress/strain-controlled phenomena was demonstrated through the application of the MAFM optimised and baseline models to strain-life fatigue calculations. Again, from the standpoint of an overall improved performance (low and high cycle fatigue simulation), this analysis illustrated that advanced plasticity modelling can have a positive influence through a time efficient optimisation exercise.

Focused research on the use of advanced plasticity models can be very useful in engineering practice, where time and simplicity is of utmost importance. This study is considered a valuable contribution for the engineering community, as it can act as starting point for further exploration of the benefits that can be obtained through material parameter optimisation methodologies for models of the MAF class.
Acknowledgments

The financial support of the Defence Science and Technology Group (DSTG) of the Australian Department of Defence (DSTO-RMIT Research Agreement ref. 2014/1032188/1) is acknowledged.

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I.3.2  Aluminum Alloy 7075 Ratcheting and Plastic Shakedown Evaluation with the Multiplicative Armstrong–Frederick Model

(Paper 3)

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(The American Institute of Aeronautics and Astronautics)
Aluminum Alloy 7075 Ratcheting and Plastic Shakedown Evaluation with the Multicomponent Armstrong–Frederick Model

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DOI: 10.2514/1.J055833

This work investigates experimentally and computationally the uniaxial ratcheting strain and plastic shakedown of aluminum alloy 7075-T6. The experimental results illustrate the existence of both plastic shakedown and cyclic hardening, proving that both kinematic and isotropic hardening should be included in modeling. Although the multicomponent Armstrong–Frederick model with multiplier has demonstrated high accuracy in aluminum ratcheting simulation, as well implementation ease, the present results demonstrate poor performance when plastic shakedown is considered. This is attributed to the limited flexibility in varying the model parameters to balance the loading/unloading branches in the hysteresis loops and decelerate ratcheting pace. To improve the ability of the multicomponent Armstrong–Frederick model with multiplier in simulating plastic shakedown, a modification was made within the framework of the model that includes multiple backstress components, with each obeying its own kinematic hardening. A linear kinematic hardening backstress was added in the formulation, enabling the control of ratcheting pace and the occurrence of plastic shakedown. Simulations with the modified multicomponent Armstrong–Frederick model with multiplier demonstrate a significantly improved capability for ratcheting and plastic shakedown. Moreover, the modified multicomponent Armstrong–Frederick model with multiplier improved the life prediction for an actual aerospace structure. This provides a strong indication of the importance of achieving plastic shakedown accuracy when simulating cyclic elastoplastic behavior.

Nomenclature

\[\begin{align*}
a_i & = \text{saturation level of backstress } X_i \\
n_i & = \text{saturation level of multiplicative backstress } X_i^* \\
b & = \text{evolution pace saturation level of the yield surface size } R \\
c_i & = \text{saturation rate of backstress } X_i \\
c_i^* & = \text{saturation rate of multiplicative backstress } X_i^* \\
dp & = \text{incremental equivalent plastic strain} \\
dR & = b(R_e - R)dp \\
dX & = \text{incremental rate of backstress } X_i \\
dp^b & = \text{incremental plastic strain} \\
E & = \text{elasticity modulus} \\
R & = \text{yield surface size} \\
R(p) & = \text{yield surface size } R \text{ as a function of equivalent plastic strain } p \\
R_e & = \text{saturation level of the yield surface size } R \\
SIM_i & = \text{simulated data point corresponding to spectrum } i \\
X & = \text{total backstress} \\
X_i & = \text{Armstrong and Frederick backstress term } i \\
X_i^* & = \text{multiplicative backstress} \\
\sigma_a & = \text{stress amplitude} \\
\sigma_m & = \text{mean stress} \\
\sigma_{\text{max}} & = \text{maximum stress} \\
\sigma_{\text{min}} & = \text{minimum stress} \\
\sigma_{\text{yield}} & = \text{yield stress} \\
\end{align*}\]

ERROR = summation of differences equation (between EXP, and SIM, data points)

EXP, = experimental data point corresponding to spectrum i

I. Introduction

SIMULATING accurately the transient cyclic inelastic phenomena is an important consideration in the fatigue design of aerospace structures because it can significantly affect fatigue life [1]. Phenomena, such as ratcheting (plastic strain accumulation under repeated loading), can cause early exhaustion of the material ductility, leading to premature failure [2,3]. Moreover, understanding and predicting the effects of cyclic plasticity is critically important for an aerospace structure operating at high-stress and high-temperature regimes, such as hypersonic flight conditions [4]. A popular nonlinear kinematic hardening model, developed by Armstrong and Frederick [5], has been successful in modeling a number of...
II. Mechanical Testing Setup

A. Test Coupons

The material used in the test program was aluminum alloy 7075-T6. The coupons were manufactured from pieces cut in the rolling direction of a 25.4-mm-thick plate. The machined geometry outlined in Fig. 1 Aluminum alloy 7075-T6 test coupon geometry.

The American Society for Testing and Materials standard [39] guidelines was used in the design of the tensile strain-controlled [40] and stress-controlled [41] test coupons. The test coupons were manufactured with 25.4 mm tangentially blended fillets and a gauge length of 19 and a 6.35 mm diameter, shown in Fig. 1.

B. Loading Cases

An Instron servomechanical test machine was used for the performance of the uniaxial mechanical testing under tensile (monotonic) and cyclic loading cases (strain- and stress-controlled).

1. Strain-Controlled Tests

Symmetric strain-controlled tests at ±1.5 and ±1.8% strain were conducted with a 100 data points per load transition. These data were used to record the hysteresis stress–strain response and to assess the degree of cyclic hardening or softening in the tested aluminum alloy 7075-T6.

2. Stress-Controlled Tests

Four different asymmetric stress-controlled tests were conducted, inducing varying levels of plastic strain. The loads applied to the coupons in the stress-controlled tests are outlined in Table 1, with the following

1) $\sigma_{\text{min}}$ is the minimum stress.
2) $\sigma_{\text{max}}$ is the maximum stress.
3) $\sigma_{\text{a}}$ is the stress amplitude, given by
4) $\sigma_{\text{m}}$ is the mean stress, given by

\[ \sigma_a = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]
\[ \sigma_m = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]

III. Mechanical Testing Results

A. Tensile Tests

Tensile tests were performed to characterize the monotonic behavior of the aluminum alloy 7075-T6. The material properties determined from the tensile tests are summarized in Table 2 [42–45].

<table>
<thead>
<tr>
<th>Load cases used in the asymmetric stress-controlled tests</th>
<th>Stress, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>$\sigma_{\text{min}}$</td>
</tr>
<tr>
<td>1</td>
<td>−430</td>
</tr>
<tr>
<td>2</td>
<td>−440</td>
</tr>
<tr>
<td>3</td>
<td>−460</td>
</tr>
<tr>
<td>4</td>
<td>−450</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tensile properties of tested aluminum alloy 7075-T6</th>
<th>Elastic modulus, MPa</th>
<th>Yield strength, MPa</th>
<th>Ultimate strength, MPa</th>
<th>True fracture strength, MPa</th>
<th>True fracture strain, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>67,776</td>
<td>524</td>
<td>570</td>
<td>595</td>
<td>13.9</td>
<td></td>
</tr>
<tr>
<td>71,000 [42]</td>
<td>476–503</td>
<td>524–552</td>
<td>No data</td>
<td>No data</td>
<td></td>
</tr>
<tr>
<td>72,200 [43]</td>
<td>512</td>
<td>572</td>
<td>No data</td>
<td>No data</td>
<td></td>
</tr>
<tr>
<td>No data</td>
<td>489 [45]</td>
<td>567</td>
<td>No data</td>
<td>No data</td>
<td></td>
</tr>
<tr>
<td>73,000 [44]</td>
<td>503</td>
<td>572</td>
<td>No data</td>
<td>11.0</td>
<td></td>
</tr>
</tbody>
</table>
C. Asymmetric Stress-Controlled Tests

Ratcheting strains obtained from the stress controlled from the four test cases (outlined in Sec. II.B.2) are presented in Fig. 3. The ratcheting strain plotted corresponds to the maximum total (elastic and inelastic) strain at the peak of each loading cycle. Test 1 produced the least amount of ratcheting, whereas test 4 produced the largest amount of ratcheting strain. Ratcheting values for tests 2 and 3 were between those of test 1 (minimum) and test 4 (maximum).

The stabilization of the maximum strain per cycle in the ratcheting results presented in Fig. 3 (plateau behavior) provides an evidence of a plastic shakedown. However, it is noted that, in each of the asymmetric stress-controlled cases shown in Fig. 3 (tests 1 to 4), once the shakedown saturates, accumulation of plastic strain continues to take place, although at a heavily reduced rate. Therefore, the shakedown cannot be solely attributed to cyclic hardening because this would cause closed hysteresis loops due to the occurrence of purely elastic strains. The hysteresis loop evolution during ratcheting was also recorded during the stress-controlled tests. The changing shape of the hysteresis loops at various cycle counts for test 3 is presented in Fig. 4. The ratcheting (strain accumulation) and plastic shakedown (grouping of the hysteresis loops after 50 cycles) phenomena can be clearly distinguished in this graph.

The observed change (contraction) of the hysteresis loop shape is quite significant, indicating a need for further attention when it comes to fatigue calculation. Fatigue life predictions relying on stabilized data (shape) would be affected when material modeling does not account for this varying shape feature exhibited by aluminum alloy 7075-T6.

IV. Material Constitutive Modeling

To simulate the different phenomena observed in the experimental investigation, both kinematic and isotropic hardening models were required. Kinematic hardening is used to capture the transient cyclic phenomena present using a translating yield surface (yield surface center), whereas isotropic hardening attempts to model cyclic hardening or softening using expansion or contraction of the yield surface (yield surface magnitude).

A. Kinematic Hardening

1. Basic Multicomponent Armstrong–Frederick Model with Multiplier

The MAFM model was the selected nonlinear kinematic hardening model. The model controls the yield surface center shifting via a backstress. This model offers improved ratcheting simulations, through a concept based on the multiplication of one backstress component by a backstress-type entity, as opposed to the addition of backstresses employed in the classical MAF model. The multiplication feature provides a means of altering the rate at which (maximum/minimum) stress saturation occurs, without changing the saturation stress level.

The uniaxial formulation of the MAFM model is provided in this study because the simulations are limited for uniaxial loading cases. The total backstress $X$ (controlling the yield surface shifting) is given by Eq. (1):

$$X = X_1 + X_2 + X_3 + X_4$$

The definition of the first three AF backstress terms ($X_1, X_2, X_3)$ used in the MAFM model is based on the MAF model and is given in Eq. (2):

$$dX_i = c_i (a_{de}^p - X_i dp) \quad i = 1, 2, 3$$

where $de^p$, $dp$, $a_i$, and $c_i$ are the incremental plastic strain, equivalent plastic strain, saturation level, and rate of saturation, respectively.

The formulation of the fourth AF backstress $X_4$ with the multiplier term is given in Eq. (3):

$$dX_4 = [c_4 + a_4 (a_4^* - X_4)](a_{de}^p - X_4 dp)$$
where \(a_1^*\) and \(c_1^*\) are the multiplicative material parameters that control the saturation level and rate of saturation of the multiplicative backstress \(X_1^*\), respectively.

The \(X_4^*\) backstress is dimensionless and is controlled through a similar to all other backstresses nonlinear evolution rule, provided in Eq. (4):

\[
dX_4^* = c_2^*(a_2^*d\varepsilon^p - X_4^*dp)
\]

2. Modified Multicomponent Armstrong–Frederick Model with Multiplier

In the classic formulation of the MAF model, a linear backstress was used as the third backstress. A popular modification to this model is to replace the linear backstress with a nonlinear backstress of the same formulation as the other two backstresses [46]. This has the effect of allowing control of the shakedown process (where necessary, accelerate/decelerate). This modification was motivated by experimental observations on carbon steel, a material that does exhibit significant plastic shakedown.

As presented in Sec. IV.A.1, the MAFM model is built on the MAF model with the addition of a fourth backstress with a multiplier term, enabling enhanced ratcheting simulation. Because the present aluminum alloy 7075-T6 experimental results demonstrate a strong plastic shakedown, it is suggested that the third nonlinear AF backstress term is replaced by a linear hardening term as in the original MAF model. In particular, the first two AF backstresses \((X_1, X_2)\) and the fourth backstress with multiplier term \((X_4)\) remains unaffected, governed by Eqs. (2) and (3), respectively. The third backstress \(X_3\) is now evolving according to a linear hardening rule given by Eq. (5):

\[
dX_3 = c_3d\varepsilon^p
\]

where \(c_3\) is a material parameter controlling the evolution pace of the linear backstress.

This modification not only simplifies the model formulation but also allows for increased shakedown to occur (with its pace still controlled by the fourth backstress with the multiplier term). The modified model is denoted by mMAFM.

The evolution of each of the backstress terms of the basic MAFM model and the mMAFM model is presented in Figs. 5 and 6. The change in the third term from a nonlinear to a linear term is evident in Figs. 5c and 6c. The resulting effect on the other backstress terms is dramatic, with a far greater rate of shakedown observed.

B. Isotropic Hardening

In the present investigation, low strain rate and negligible temperature changes permit the assumption of an isothermal plastic deformation. Isotropic hardening, controlling the size \(R\) of the yield surface equation Eq. (6) as presented in [16], was used in this study:

\[
dR = b(R_t - R)|dp|
\]

where \(b\) and \(R_t\) are material constants controlling the evolution pace and saturation level of the yield surface size \(R\) with a varying equivalent plastic strain \(dp\).

Integrating Eq. (6) provides an analytical expression [Eq. (7)] of the yield surface size \(R\) evolution, where the yield surface changes through the accumulated equivalent plastic strain \((p)\):

\[
R(p) = R_t(1 - e^{-b(p)})
\]
V. Simulations

The methodology developed by Dafalias et al. [26] was used to generate the necessary MAFM and mMAFM model parameters to define the backstress evolution for ratcheting and hysteresis loop simulation. The initial monotonic loading curve was modeled using a different set of parameters from those used for the cyclic loading. The importance of employing such a strategy was highlighted in [47], where it was noticed that initial ratcheting strains are more accurately predicted that way. The isotropic hardening parameters were calculated using the procedure proposed by Chaboche [13]. The linear backstress parameter $c_3$ is the most critical parameter in simulating the plastic shakedown. Its value was selected to adjust the shakedown location but still ensure that the parameter evaluation procedures outlined in [26] were not violated. The monotonic and cyclic parameters used in the MAFM and mMAFM model are listed in Table 3.

A. Multicomponent Armstrong–Frederick Model with Multiplier Simulations

To demonstrate the influence of a linear backstress in the simulation of the plastic shakedown phenomenon, simulations using the basic MAFM model (which contains only nonlinear backstresses) were conducted. A backward implicit Euler integration scheme was used to generate the simulation results in Matlab. Initially, symmetric strain-controlled simulations were run to assess the calculated parameters’ performance in capturing the hysteresis loop shape. A comparison of the simulation and experimental results for the first cycle (Fig. 7a) and last (66th) cycle (Fig. 7b) in the 1.5% symmetric strain-controlled loading case is presented in Fig. 7. The results demonstrate an overall accurate simulation of the hysteresis loop shape. The apparent inaccuracy in the results is the overprediction in stress in the unloading branch, more evident in the first cycle.

Fig. 6 Backstress evolution for the mMAFM model: two nonlinear AF backstresses ($X_1, X_2$), one linear backstress ($X_3$), and one backstress with multiplier term ($X_4$).

Table 3 Monotonic and cyclic MAFM and mMAFM model parameters

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>Monotonic parameters</th>
<th>Cyclic parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress $\sigma_{yield}$, MPa</td>
<td>500 500</td>
<td>435 435</td>
</tr>
<tr>
<td>Elasticity modulus $E$, MPa</td>
<td>69,000 69,000</td>
<td>69,000 69,000</td>
</tr>
<tr>
<td>$a_1, a_2, a_3, a_4$, MPa</td>
<td>8, 39, 1, 2</td>
<td>8, 39, 2</td>
</tr>
<tr>
<td>$c_1, c_2, c_3, c_4$</td>
<td>20,304, 282, 34, 112</td>
<td>32,304, 282, 14, 6</td>
</tr>
<tr>
<td>$c_1, c_2, c_3, c_4$</td>
<td>20,304, 282, 112</td>
<td>48</td>
</tr>
<tr>
<td>$c_1, c_2, c_3, c_4$</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$R_s$, MPa</td>
<td>415 415</td>
<td>694 54</td>
</tr>
<tr>
<td>$b$</td>
<td>15 15</td>
<td>15 15</td>
</tr>
</tbody>
</table>
the result of attempting to adjust the parameters to limit the overprediction in ratcheting strain for the larger loading cycle cases.

Ratcheting simulations, for all loading cases examined, are presented in Fig. 8. The simulations for the lower stress ranges (tests 1 and 2, given in Table 1) demonstrate a slight overprediction of ratcheting. A similar issue was reported by Dafalias et al. [26] for the case of carbon steel 1026, where overprediction of ratcheting strain for the smaller-amplitude loading cases was also obtained. However, the two larger stress range simulations (tests 3 and 4, also given in Table 1) exhibit much higher overprediction of ratcheting. This is attributed to the difficulty in achieving high rates of plastic shakedown for larger stress ranges.

B. Modified Multicomponent Armstrong–Frederick Model with Multiplier Simulations

The mMAFM model simulations were conducted for the 1.5% symmetric strain-controlled tests. The obtained computational results have been compared against the experimental data, presenting a very good agreement (Fig. 9). The slight overprediction (Fig. 9a) at the cyclic yield location of the first cycle is due to a larger cyclic stress used to prevent ratcheting overshooting at smaller stress amplitudes. However, at higher cycle numbers (66th cycle, Fig. 9b), overshooting is almost diminished. Moreover, when comparing with the MAFM model simulations (Fig. 7), the overprediction of stress in the unloading branch is greatly reduced for the mMAFM model simulations.

The mMAFM model ratcheting simulations against the experimental results are presented in Fig. 10. The comparison shows that the mMAFM model appears to be capable of predicting the four load cases significantly better than the MAFM model. Ratcheting simulation for the larger stress amplitudes (loading cases 3 and 4 in Fig. 10) captures relatively successfully the plastic shakedown at the correct number of cycles. It is of note that the significant overprediction for the larger load cases noticed in the MAFM simulations (loading cases 3 and 4 in Fig. 8) has been drastically improved. It is noted that the MAFM material model, implemented in Matlab, requires negligible computational power because the analysis is limited to the simulation of uniaxial cases. In particular, both for the case of the basic model (MAFM) and its modified version (mMAFM), the total consumed CPU time required to obtain the strain ratcheting simulations (curves) shown in Figs. 8 and 10 is approximately 2.2 s.

C. Multicomponent and Modified Multicomponent Armstrong–Frederick Models with Multiplier Fatigue Life Simulations

To assess the importance of recognizing the plastic shakedown phenomenon, particularly when considering very complex variable load cases, the MAFM and mMAFM model was implemented in the Defense Science and Technology Group (DSTG) software Crack Growth Analysis Program (CGAP) [48], which is used for the assessment and analysis of fatigue damage on aircraft structures. The CGAP algorithm is based on the strain-life approach methodology to perform fatigue life calculations, which in turn relies on constitutive plasticity models simulating the cyclic elastoplastic response of the in-focus aerospace metal. The analysis performed with CGAP involved the utilization of the Australian Defence Force P-3C aircraft service life data, obtained from the relevant DSTG testing program [49,50]. No tests were conducted on the notched coupon as part of this study. The notched coupon test data presented were provided from DSTG (coming from a test campaign previously carried out for the Australian Defence Force P-3C Orion structural integrity program [49,50]). The load cases used in the notched tests were collected from fatigue critical locations on the P-3C aircraft. Fatigue life estimates were used to assess the influence of the plastic shakedown phenomena when accounting (complex) load cases arising from notched coupons tests. This program involves fatigue testing on notched aluminum alloy 7075-T6 test coupons subjected to 21 different load spectra, which were obtained from identified fatigue critical locations on the aircraft, determined from both the Royal Australian Air Force and the U.S. Navy full-scale fatigue tests. These spectra contain 15 completely different load sequences. The remaining six fatigue lives were determined by repeating spectra using notches with different stress concentration factors. A very small
extract of a spectrum is shown in Fig. 11 bottom-right corner to offer an insight into the variability of the spectra tested. Defense data restrictions are not allowing further details to be released around specificities of these service load spectra. These are very large and complex spectra with the least number of cycles being approximately 400,000, whereas the largest tested spectrum contained approximately 1,000,000 cycles. The geometric mean of the experimental fatigue life results (in flight hours), for each of these 21 spectra, were compared to the fatigue life results obtained from CGAP by implementing the MAFM and mMAFM model (Fig. 11). Figure 11 presents a comparison between predictions produced from the two models (MAFM and mMAFM). The horizontal axis is the experimental data (flight hours), and the vertical axis is the (MAFM/mMAFM) predicted data, with the diagonal line representing a 1:1 relation between the experimental and (MAFM/mMAFM) predicted data (100% accuracy).

For both the MAFM and mMAFM models, the majority of the fatigue life predictions obtained from CGAP are conservative, as indicated by the results shown in Fig. 11 (points residing below the diagonal). When comparing the performance of the MAFM and mMAFM model, it appears that the results of the latter lie closer to the geometric mean of the experimental data than those of the basic model’s (MAFM). Closer evaluation of the two MAFM models was performed by using a summation of differences (ERROR) equation, which compares the geometric mean of the experimental with the simulated data for each spectrum [Eq. (8)]:

\[
\text{ERROR} = \sum_{i=1}^{21} \frac{|\text{EXP}_i - \text{SIM}_i|}{\text{EXP}_i}
\]  

where EXP_i and SIM_i are correspondingly the experimental and simulated data point corresponding to spectrum i, with i = 1, 2, 3, ..., 21.

The calculated error results (shown in Fig. 11) suggest that mMAFM offers a 10% improvement in fatigue life prediction accuracy when compared to MAFM. This improvement, in fatigue life prediction, in conjunction with the enhanced simulation accuracy for the symmetrical hysteresis loops, ratcheting, and plastic shakedown, is highlighting the importance of the linear kinematic hardening term incorporation in the MAFM model.

A further example of the improved simulation capability offered by the mMAFM model is illustrated in Fig. 12. In particular, the variation of the stress concentration factor \( k_t \) in relation to fatigue life...
(in flight hours) was calculated for MAFM and mMAFM for two different spectra and compared against the experimental data points (where the notched test coupon \( k_t = 5 \) and \( k_t = 6.5 \) for spectra in Figs. 12a and 12b, respectively). The variations in fatigue life predictions particularly the improvement achieved for the notch geometry tested.

**VI. Discussion**

The comparisons performed between the ratcheting strain simulations using the MAFM model and its modified version mMAFM model demonstrate a significant improvement in the plastic shakedown prediction when using the latter (tests 3 and 4 in Figs. 8 and 10). The incorporation of the linear kinematic hardening term (backstress) provides a more robust method of parameter optimization for improved simulations. This was further supported by the negative impact that MAFM parameter adjustments had on the hysteresis loop shape when trying to reduce the ratcheting overshooting. However, the accurate prediction of plastic shakedown at lower load cases (tests 1 and 2 in Figs. 8 and 10) still remains an issue. The underlying cause for the overshooting in ratcheting strain for the lower load cases is the inability of the linear backstress to evolve quickly enough and reach equilibrium with the other constituent (nonlinear) backstresses. Plastic shakedown is predicted accurately for the larger load cases (tests 3 and 4) due to a greater amount of plastic strain being produced, resulting in larger nonlinear backstress progression, which ensures that equilibrium between the nonlinear and linear backstresses is reached at the required number of cycles. Increasing the evolution pace would ensure that shakedown occurs for the lower load cases (tests 1 and 2) but will result in significant ratcheting undershooting for the larger load cases (tests 3 and 4). Because the evolution of kinematic hardening (backstresses) is indirectly influenced by the evolution of isotropic hardening, the isotropic hardening rule could be adjusted to allow for greater cohesion between the load cases. This would provide greater control (flexibility) over the influence of the linear backstress and reach a stabilized state for all different load cases (lower, tests 1 and 2, and upper, tests 3 and 4). However, this would involve a more comprehensive optimization-material determination exercise, as opposed to the straightforward methodology currently in place. It is of note that, although the parameter selection for the mMAFM model was not capable of simulating plastic shakedown for all tested load cases, the improvement in shakedown accuracy contributed to an enhanced fatigue life prediction (Fig. 11).

What should also be noted is that, in some tested spectra, the lives predicted by the mMAFM model have the potential for further improvement. For example, for spectra corresponding to larger lives (greater than 30,000 h, far right side of Fig. 11), improvement could be achieved by adjusting the mMAFM parameters for more stress-controlled load cases. This is based on closer observation of the MAFM and mMAFM model hysteresis loops’ output for the spectrum corresponding to the predicted data enclosed by the dashed circle in Fig. 11 (where the MAFM point is considerably closer to the geometric mean than the MAFM point). When comparing the strain–plastic-strain outputs of the two models, shown in Fig. 13, what is noticeable is the negative offset in the mMAFM simulated plastic strain. The shape of the hysteresis loops are almost identical, but the progression of the hysteresis loops is different.

1) The MAFM model imposes shifting of the loops farther along the plastic strain axis, resulting in a larger accumulation of positive strain. Consequently, the model simulates larger tensile stresses.

2) The mMAFM model imposes less plastic strain accumulation and consequently less tensile and more compressive stresses.

Effectively, MAFM when compared to the mMAFM model causes an overestimation of the calculated damage, leading to lower predicted lives. Therefore, differences can be attributed either to underprediction of compressive stresses or to overprediction of tensile stresses. Taking into account that mMAFM has improved performance in ratcheting simulation over MAFM, if we focus the mMAFM model parameter determination toward capturing a larger number of stress-controlled load cases, then the overprediction of tensile stress and/or underprediction of compressive stresses is likely to be reduced. This is achieved by employing a set of parameters that is capable of simulating more accurately the various stress–strain (hysteresis loops) outputs produced by a larger number of load cases.

![Fig. 12 Fatigue life in (flight hours) simulations comparison between the MAFM and modified MAFM (mMAFM) model for varying stress concentration factors \( k_t \) for two spectra.](Image 62x563 to 523x737)

![Fig. 13 Comparison of plastic strain output for the MAFM and mMAFM models for the spectrum corresponding to the points enclosed by the dashed circle shown in Fig. 11.](Image 305x53 to 545x222)
VII. Conclusions

Experimental asymmetric stress-controlled load conditions identified a significant plastic shakedown phenomenon occurring in aluminum alloy 7075-T6. This occurred in all four load cases, after approximately 20 cycles. Simulation results using the MAFM model demonstrated a poor plastic shakedown prediction. Replacing one of the three nonlinear backstress terms with a linear term modified the model to a version called the mMAFM model. This modification improved the plastic shakedown simulation capability and the ratcheting prediction performance. Simulation comparisons using the modified model with symmetric strain-controlled experimental data were also analyzed, with the results proving that an improvement in simulation accuracy was achieved as well. Moreover, the enhancements in fatigue life calculations, using the mMAFM model, highlighted the importance of simulating accurately plastic shakedown, reiterating the relevance of the application of a linear backstress term.

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References


R. Ohayon
Associate Editor
Chapter I.4  Development of Parameter Optimisation Workflow

This chapter contains two main components:

- I.4.1  On the Utilisation of Nonlinear Plasticity Models in Military Aircraft Fatigue Estimation: A Preliminary Comparison
- I.4.2  Sensitivity and optimisation of the Chaboche plasticity model parameters in strain-life fatigue predictions

In Section I.4.1 an initial investigation into the potential improvements to strain-life fatigue predictions with the modified elastoplastic models was undertaken using a parameter optimisation procedure to develop the parameters with respect to some gathered AA7075-T6 experimental data. However, it was noticed that the elastoplastic parameters were very sensitive to the type of material data used to develop them. Consequently, in Section I.4.2 an in-depth investigation was undertaken in assessing the type of experimental data required for parameter development for elastoplastic constitutive models to be applied to strain-life fatigue calculations.
I.4.1 On the Utilisation of Nonlinear Plasticity Models in Military Aircraft Fatigue Estimation: A Preliminary Comparison

(Paper 4)


(Aerospace Science and Technology)
Short communication

On the utilisation of nonlinear plasticity models in military aircraft fatigue estimation: A preliminary comparison

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A B S T R A C T

Strain-life methodologies are commonly employed for fatigue estimation in military aircraft structures. These methodologies rely on models describing the elastoplastic response of the material under cycling. Despite the numerous advanced plasticity models proposed and utilised in various engineering problems over the past decades, the Masing model remains a popular choice in fatigue analysis software, mainly due to its simplicity. However, in the case of military aircraft load spectra including scattered overloads the Masing model fails to represent adequately transient cyclic phenomena, such as mean stress relaxation and strain ratcheting. In this study, four well-known constitutive plasticity models have been selected as potential substitutes for the Masing model within a defence organisation in-house developed fatigue analysis software. These models assessed were the well-known Multicomponent Armstrong–Frederick Model (MAF) and three of its derivatives: MAF with threshold (MAFT), Ohno–Wang (OW) and MAF with Multiplier (MAFM). The models were calibrated with the use of existing experimental data, obtained from aircraft aluminium alloy tests. Optimisation of the parameters was performed through a genetic algorithm-based commercial software. The models were incorporated in the fatigue analysis software and their performance was evaluated statistically and compared against each other and with the Masing model for a series of different flight load spectra for a military aircraft. The results show that all four models have achieved a drastic improvement in fatigue analysis, with the MAFT model giving a slightly better performance.

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1. Introduction

Fatigue damage can be influenced by transient phenomena exhibited in the cyclic plastic response of the material, especially under low cycle fatigue conditions. Aircraft structural fatigue assessment via strain-life methodologies has commonly been performed with the use of the Masing cyclic plasticity model [1]. The Masing model in conjunction with the Ramberg–Osgood curve fitting equation, define the hysteresis loops by using the stabilised stress–strain curve of the material. This modelling approach may be an acceptable solution for stabilised loading conditions, however it does not take into consideration transient cyclic plastic phenomena, such as strain ratcheting and mean stress relaxation.

Therefore, in the case of more complex loading conditions, where a benign load spectrum is interspersed with severe overloads leading to nonzero mean stress/strains, this could lead to a gross error in the prediction of fatigue life [2]. Improvements in fatigue life prediction can be achieved by adopting nonlinear kinematic hardening models, such as the Multicomponent Armstrong–Frederick (MAF) model [3], as opposed to approaches relying on the Masing model [4]. Since its inception, the MAF model has been a basis for numerous improvements modifications, including the following:

- MAF model with Threshold term (MAFT) [5] (addition of a fourth back stress containing a threshold term $\delta$);
- Ohno–Wang (OW) model [6] (superposition of a large number of back stress terms, each containing a slight nonlinearity introduced through a multiplier);
- Multicomponent Armstrong-Frederick model with Multiplier (MAFM) [7] (addition of a back stress multiplier $X^*$).

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Both the MAFT and MAFM model differ from the basic MAF model through an added fourth back stress term, aimed at improving hysteresis loop shape and strain ratcheting simulation accuracy. In the MAFT model the fourth term evolves in a dual way (nonlinearly below a threshold value and linearly above that value) [5], while in the MAFM model its evolution is controlled via a multiplier term, imposing a continuous nonlinear response [7,8]. The OW model differs from the MAF model in the modification of the dynamic recovery [6]. However, the OW model typically requires a larger number of back stresses terms (as opposed to MAFT and MAFM), all of which containing the modification of the dynamic recovery term.

In order to represent the transient cyclic plasticity behaviour exhibited in aircraft aluminium alloys during fatigue cycling, a plasticity model should be able to capture adequately the strain ratcheting and mean strain relaxation phenomena. The aforementioned nonlinear hardening models are considered to have the capacity to account for the strain hardening and strain ratcheting phenomena. This paper reports the results from the application of the MAFT, MAFT, O-W and MAFM model in strain-life fatigue predictions, extending ongoing research by the authors in this field [9–12].

2. Formulation of plasticity models

The formulation of the models is presented in its uniaxial form both for simplicity but also for the fact that their implementation in fatigue analysis (presented in the sequel) is typically performed in their uniaxial form.

The classical von Mises yield function \( f \) was selected in this study, described by Eq. (1),

\[
f = (\sigma - X(\varepsilon^p))^2 - [\sigma_{y0} - R(\varepsilon^p)]^2 = 0
\]

where \( \sigma \) is the stress, \( X \) the kinematic hardening back stress as a function of plastic strain \( \varepsilon^p \), \( \sigma_{y0} \) is the initial yield stress and \( R \) the isotropic hardening evolution rule, also as a function of plastic strain \( \varepsilon^p \).

The kinematic hardening rule (back stress \( X \)) is presented in its incremental form, as following:

\[
dX = \sum dX_i
\]

where \( dX_i \) the increment of the constituent back stresses \( X_i \) (with \( i = 1, 2, 3, \ldots \)) for the case of the MAFT model (summation of multiple back stresses).

In order to improve the model’s ability to simulate effectively plastic shakedown and cyclic hardening the following terms were included in the full model respectively:

- A nonlinear isotropic hardening rule given by Eq. (3) [13]:

\[
R = R_i(1 - e^{-b(\varepsilon^p)})
\]

where \( R_i \) and \( b \) are model parameters determined from experimental data.

- A Prager linear kinematic hardening term (back stress), given by Eq. (4) [14]:

\[
dX_i = c_1 \varepsilon^p
\]

where \( c_1 \) a model parameter.

The models’ kinematic hardening formulation (back stresses) are summarised in Table 1; readers can refer papers [3, 5–7] for further details. Moreover, the set of parameters \( \{ c_1, a_i, a, m, c_2, a_2 \} \) corresponding to each model is presented in Table 2 (labelled under kinematic and isotropic hardening parameters).

3. Optimisation of plasticity models’ parameters

The performance of the models is highly dependent on their calibration, which entails identifying their parameters through fit-
ting experimental (mechanical testing) data. For this reason, the parameters were determined through an optimisation process performed with the modeFrontier [15] commercial software, in conjunction with the basic methodologies applicable for each model described in [3,5–7] for the initial parameter value determination. This process, developed by the authors in previous works [10,11], is able to improve the simulation accuracy for symmetric strain-controlled and asymmetric stress/strain-controlled simulations.

In brief, a genetic algorithm is used in the parameter optimisation process which involves the following key stages:

- Parameter initialisation (initial population of 25);
- Model simulation;
- Objective evaluation.

In each iteration the chosen parameters are given a “fitness rating” based on the output of the objective functions (hysteresis loop shape and strain ratcheting). A new set of parameters is derived with the use of these ratings. This set is subsequently trialled in the next iteration, based on the best output obtained from the previous iteration. The full optimisation process is presented in detail in [11].

Each model’s parameters, obtained from this iterative optimisation process, are presented in Table 2.

4. Results

The results presented in this section were obtained from the numerical implementation of the models in Matlab and in the Defence Science and Technology (DST) Group CGAP fatigue analysis software [16].

4.1. Uniaxial cyclic elastoplastic simulations

Simulations were conducted for symmetric strain-controlled and asymmetric stress-controlled uniaxial test cases and compared to experimental data, as a verification of the selected optimal parameters. The comparison between the AA7075-T6 symmetric strain-controlled experimental data at the ±1.5% strain range with each of the model simulations for the first cycle and the stabilised cycles (150th cycle) demonstrate a relatively small amount of cyclic hardening (Fig. 1). Moreover, the results demonstrate that the parameters selected for each model capture accurately the shapes of both the first and the subsequently stabilised cycles.

The effectiveness of the models in simulating the ratcheting strain accumulation (maximum strain obtained at peak of each loading cycle) for varying stress amplitudes was also analysed using experimental data. In particular, the ratcheting strain outputs for the following combinations of imposed minimum and maximum (min, max) stresses were used:

1. (−450 MPa, 550 MPa);
2. (−460 MPa, 540 MPa);
3. (−440 MPa, 520 MPa);
4. (−430 MPa, 520 MPa).

The simulation results are presented in Fig. 2, where Fig. 2(a) compares the accuracy of each model against larger stress loading cases (pairs 1 and 2), while Fig. 2(b) shows the outputs for the lower stress cases (pairs 3 and 4). All models were able to predict the plastic shakedown phenomenon relatively well. However, under higher peak stresses the
models tend to underestimate the experimentally observed limits (curves 1 and 2 in Fig. 2), while for low peak stresses the models overestimate the experimentally observed limits (curves 3 and 4 the models Fig. 2). It is of note that only the MAFT model was successful in simulating the correct strain at which the plastic shakedown phenomena occur for both the examined lower load cases [curves 1 and 2 in Fig. 2(a)].

4.2. Strain-life fatigue simulations

Experimental fatigue data of AA7075-T6 notched coupons have been used in the CGAP software fatigue analysis, from 21 different load spectra obtained from the P-3C service life assessment program [17,18]. The CGAP simulations are compared to these experimental data. Although, in general the geometric mean of the experimental fatigue life under each spectrum is used to compare predictive accuracy, a statistical approach was adopted in addition. In particular, the comparison considered the probability distribution of experimentally determined fatigue lives for each spectrum, as a way to assess the experimental variability. The distributions of the experimental fatigue lives were generated with the bootstrap [19] non-parametric statistical method. Once distributions were generated, the bootstrap method was utilised to generate a characteristic distribution across selected fatigue critical locations. A similar distribution can be constructed using the predicted fatigue lives across the 21 locations. The Mean Integrated Squared Error (MISE) was used as the metric to compare the two distributions, where lower MISE corresponds to higher accuracy in the simulation results. MISE calculations were repeated in order to form a distribution of MISE values for each model. The MISE distributions allowed the assessment of the accuracy of the models relatively to each other, including the known variability in experimental test results.

In Fig. 3(a) the MISE distribution for each of the plasticity models is compared with the Masing model MISE distribution, while in Fig. 3(b) the geometric mean fatigue life data are presented for each model under comparison. From the results illustrated in Fig. 3(a), it is clear that the MAFT and MAFM models offer improved simulation accuracy when compared to the Masing model. It is particularly evident from the MAFT MISE distribution that the MAFT model achieves overall more accurate simulations than all the other models. It is also of note that the majority of the simulation results are located in the lower (green shaded) portion of Fig. 3(b), indicating that the predictions are conservative.

5. Discussion

From the preliminary comparison results, it was observed that the MAFT model is capable of achieving both the highest fatigue life estimation accuracy and the most accurate ratcheting simulation. However, one needs to consider that the optimisation process used in the parameters’ determination (for all models) was focused on identifying those set of parameters able to simulate the cyclic phenomena observed in the uniaxial experiments and not the fatigue life. Thus, the process may be inherently biased towards the uniaxial data and further research efforts are currently in progress, including a comprehensive sensitivity analysis for each of the models’ parameters and the fatigue data influence on the overall performance of the models.

The parameter selection for this study was based on achieving improved ratcheting simulation rather than simulating accurately the mean stress relaxation phenomenon. This choice was made on the basis of achieving an acceptable balance between these two cases but also due to a limitation on the number of experimental data that were used (mainly associated with the scarcity of representative available data for AA 7075-T6). Further work planned is aimed to determine the difference in fatigue life predictions when the models’ parameters are optimised for mean stress relaxation. The planned work will involve the collection of more extensive test data, which are essential for the validation of the models and a complete comparison study of the different cyclic plasticity models.

6. Conclusion

The fatigue estimation performance of four established cyclic plastic models (MAF, MAFT, OW, and MAFM) incorporated in the CGAP strain-life fatigue analysis software is currently under investigation within the DSTG. The assessment of the cyclic plasticity models was conducted using MISE as a metric, based on the statistical distribution of the experimental and the numerical results. The assessment showed that the MAFT and MAFM nonlinear kinematic hardening models, coupled with a Prager linear back stress and a nonlinear isotropic hardening rule, improved fatigue life predictions when compared to the Masing model.

Conflict of interest statement

None.

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References

I.4.2  Sensitivity and optimisation of the Chaboche plasticity model parameters in strain-life fatigue predictions  

(Paper 5)


(Materials and Design)
Sensitivity and optimisation of the Chaboche plasticity model parameters in strain-life fatigue predictions

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HIGHLIGHTS
• Sensitivity analysis showed that using appropriately selected uniaxial test data improves strain-life fatigue predictions;
• Sensitivity analysis useful for engineering practitioners in improving understanding in calibration of Chaboche model for fatigue calculations;
• Optimal uniaxial material data achieved with large maximum stress asymmetric stress-controlled hysteresis loops and strain ratcheting curve;
• Chaboche model parameters obtained from GA optimisation process improved significantly fatigue predictions, when compared to Masing predictions;
• Stress and strains calculated early in spectrum sequence influence significantly overall damage calculation with examined strain-life method;

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ABSTRACT
The aerospace industry is widely employing strain-life methodologies for structural fatigue predictions. Under spectrum loading, overloads significantly affect the fatigue, therefore it is very important to accurately account for the cyclic transient deformation phenomena. Describing these phenomena requires advanced plasticity models that involve a set of material parameters. Even for the well-known Chaboche model, there is lack of understanding of each parameter’s sensitivity in strain-life fatigue calculation among engineering practitioners. A parameter optimisation technique using a multi-objective genetic algorithm is applied for the Chaboche model parameters by employing varying strain and stress controlled uniaxial data from tests on Aluminium Alloy 7075-T6. The parameters obtained from each of the optimisations and for various workflows, are then used in strain-life fatigue calculations with a Defence Science and Technology Group in-house software. Fatigue life predictions for P-3C aircraft load spectra are compared against experimental lives obtained from the Masing model to ascertain the parameters offering the most accurate results. The optimum uniaxial material dataset for strain-life predictions employing...
1. Introduction

Cyclic transient effects such as strain ratcheting and mean stress relaxation can have an influence on the fatigue damage in a structure, particularly under low cycle fatigue. Consequently, it is important to account for these phenomena in fatigue life predictions. Strain-life approaches are an important fatigue life prediction methodology in the structural integrity management of aircraft. The most common material plasticity model applied in such methodologies is the Masing model [1]. In the Masing model, the elastoplastic hysteresis loops are constructed from a series of stabilised cyclic stress strain curves obtained from symmetrical strain controlled testing. The most significant drawback of the Masing model is its inability to recognise transient cyclic phenomena, such as strain ratcheting and mean stress relaxation, both having an influence on fatigue life [2–8]. Strain ratcheting, or else referred as cyclic creep (progressive accumulation of plastic strain under cycling), is a phenomenon which occurs during asymmetric stress controlled loading, while mean stress relaxation occurs during asymmetric strain-controlled loading (stabilisation of hysteresis loop evolution after a number of cycles). However, for components with stress raising features (such as notches or cut-outs), a combination of the two phenomena may occur at the notch root [9], which highlights the importance of taking into consideration both phenomena when it comes to fatigue life predictions. This is particularly important in service life loading since load sequence effects can have a significant influence on the fatigue life [10].

Constitutive plasticity models capable of accurately simulating cyclic transient behaviour have been shown to improve fatigue predictions [3, 11–14] and the importance of their application to specifically strain-life fatigue is becoming more apparent [14,15]. A difficulty associated with the implementation and wider usage of these cyclic plasticity models is in the identification of their parameters. As the models increase their level of sophistication and complexity, the number of material parameters has increased as well. This has led to the introduction of various material parameter optimisation techniques, in order to ease/streamline the identification process and improve the accuracy of the model predictions [16–27]. However, the sensitivity of the strain life fatigue predictions to the model parameters requires a thorough assessment, owing to the complexity of many real-life load spectra. This knowledge is important to the identification of the plasticity model parameters, obtained from uniaxial test data, which are able to provide the most accurate strain life fatigue predictions.

This paper presents a sensitivity analysis conducted to identify the optimal uniaxial dataset combinations that can be used to determine the parameters of a widely used cyclic plasticity model. Both symmetric and asymmetric strain and stress controlled test data from Aluminium alloy (AA) 7075-T6 have been used for the calibration and implementation of the model in strain life fatigue prediction. In this analysis, a number of genetic algorithm (GA) optimisation strategies have been employed to evaluate the effectiveness of material parameters for various operating load spectra. Moreover, out of this process, those optimisation strategies achieving higher accuracy in fatigue calculation are identified.

2. Research methodology

2.1. Mechanical testing

2.1.1. Test setup and coupons

An extensive knowledge of the AA 7075-T6 response under cycling loading was required for the calculation of the model parameters. This involved the execution of symmetric and asymmetric strain-controlled, as well as asymmetric stress-controlled tests. The tests were performed on a MTS servo-hydraulic closed-loop testing machine with a capacity of 100 kN, using 0.1 Hz sine waves. All strain measurements were made using a 10 mm extensometer. All tests were conducted at room temperature (consistent with past DST Group research work on notched coupon tests and tests conducted to develop the strain-life curves [28,29]).

The test coupons were machined from blanks cut from 12 mm thick plate, with the long axis of the coupon parallel to the rolling direction. Typically, rolled/extruded aerospace aluminium alloys (such as AA 7075) are expected to exhibit some anisotropy in the longitudinal/transverse direction, mainly due to the existence of residual stresses arising from heat treatments and plastic deformation imposed during machining [30]. This study was performed by using coupons from the rolling direction since the particular structural area under fatigue evaluation (aircraft skin) is loaded along the rolled direction of the material (also consistent with past research performed by DST Group on notched and un-notched AA7075-T6 coupon [28,29]). However, if other aircraft structural points are to be examined, where fatigue life may be influenced by plastic anisotropy, coupons from both the rolling and long/short transverse directions should be obtained and tested.

The geometry of the of the strain-controlled coupon was in accordance with the ASTM E606/E606M standard [31], while the stress-controlled coupons were in accordance with the ASTM E466-07 standard [32].

Fig. 1. AA 7075-T6 symmetric strain-controlled test results: hysteresis loop at (a) 1.5% (1st and 65th cycles) and (b) 1.8% (1st and 100th) strain level.
2.1.2. Symmetric strain-controlled tests

Two strain-controlled tests were performed, at 1.5% and 1.8% strain for 65 and 100 cycles respectively [Fig. 1(a) and (b)]. In both cases, a relatively small amount of cyclic hardening is observed.

2.1.3. Asymmetric strain-controlled tests

The load cases investigated include a combination of low and high strain amplitudes in order to assess a broad range of mean stress relaxation rates (Table 1). A total of 150 loading cycles have been performed in each test case.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Test & $\varepsilon_{\text{min}}$ (%) & $\varepsilon_{\text{max}}$ (%) & $\varepsilon_a$ (%) & $\varepsilon_m$ (%) \\
\hline
1 & -0.15 & 1.15 & 0.65 & 0.50 \\
2 & 0.20 & 1.60 & 0.70 & 0.90 \\
3 & -0.05 & 1.55 & 0.80 & 0.75 \\
4 & 0.05 & 1.65 & 0.80 & 0.85 \\
5 & 0.15 & 1.45 & 0.65 & 0.80 \\
6 & -0.10 & 1.90 & 1.00 & 0.90 \\
\hline
\end{tabular}
\caption{Asymmetric strain-controlled load cases.}
\end{table}

The corresponding mean stress relaxation curves are shown in Fig. 2, where increasing strain amplitude results in faster mean stress relaxation. Only one case (Test 6) relaxes completely, while other load cases relax at large mean stresses, a phenomenon noticed in other materials [9,12,33].

2.1.4. Asymmetric stress-controlled tests

The strain ratcheting behaviour of the material was investigated through a series of load cases (Table 2), which enabled the collection of a variety of ratcheting rates.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Test & $\sigma_{\text{min}}$ (MPa) & $\sigma_{\text{max}}$ (MPa) & $\sigma_a$ (MPa) & $\sigma_m$ (MPa) \\
\hline
1 & -430 & 510 & 470 & 40 \\
2 & -440 & 520 & 480 & 40 \\
3 & -460 & 540 & 500 & 40 \\
4 & -450 & 550 & 500 & 50 \\
\hline
\end{tabular}
\caption{Asymmetric stress-controlled load cases.}
\end{table}

In Table 2, $\sigma_{\text{min}}$ is the minimum strain, $\sigma_{\text{max}}$ the maximum strain imposed during cycling, $\sigma_a$ the strain amplitude, given by: $\sigma_a = (|\sigma_{\text{max}}| + |\sigma_{\text{min}}|)/2$, and $\sigma_m$ the mean strain, given by $\sigma_m = (|\sigma_{\text{max}}| - |\sigma_{\text{min}}|)/2$.

The strain ratcheting curves obtained from the Table 2 tests cases are presented in Fig. 4, where two different calculation methods are used:

- Maximum strain $\varepsilon_{\text{max}}$ at the peak of each cycle [Fig. 4(a)];
- Mean strain $\varepsilon_m$ for each cycle (where: $\varepsilon_m = (|\varepsilon_{\text{max}}| - |\varepsilon_{\text{min}}|)/2$) [Fig. 4(b)].

An early onset of plastic shakedown (stabilisation of the plastic strain accumulation per cycle) can be observed in the Fig. 4 results, as evidenced by the saturation of ratcheting strain after approximately 20 cycles, which is an important phenomenon to consider in fatigue design [34]. A closer inspection of the hysteresis loop development for Test 2, Test 3 and Test 4 (Table 2) is provided in Fig. 5. In particular, the plastic shakedown is evidenced by the proximity (pile-up) of the hysteresis loops between cycle 20 and 100. Moreover, strain hardening for all test cases is identified as the progressive decrease in the hysteresis loop size between cycle 1 and 100.

2.2. Cyclic plasticity modelling

2.2.1. Model formulation

The nonlinear kinematic hardening model used in the sensitivity analysis was the Multicomponent Armstrong-Frederick (MAF) model, commonly referred in the literature as the Chaboche model [35]. Kinematic hardening implies the translation of the yield surface in the stress

![Fig. 2. AA 7075-T6 mean stress relaxation curves for the asymmetric strain-controlled test cases (presented in Table 1).](image2)

![Fig. 3. AA7075-T6 hysterisis loops obtained from the asymmetric strain-controlled (a) Test 2 (b) Test 4 (presented in Table 1). Comparison of the first and last (150th) cycle.](image3)
space, as described by the movement of the yield surface centre. The Chaboche material model enjoys a very wide acceptance within the applied engineering and research community, mainly due to its simplicity, robustness and the fact that it is already embedded within commercial finite element (FE) analysis software. The Chaboche model has been the subject of many modifications aiming to improve the simulation ability of transient cyclic effects, such as mean stress relaxation and ratcheting, e.g.[36–41]. These modifications, offering enhanced simulation capabilities in the Chaboche model, are also able to improve fatigue results [12]. In the present study, the basic Chaboche model is selected as the test-bed for the strain-life fatigue sensitivity analysis.

Moreover, isotropic hardening was also included in the Chaboche model, so as to capture the cyclic hardening present in 7-series aerospace aluminium alloys [42]. Isotropic hardening implies the uniform expansion or contraction of the yield surface in the stress space, as described by the increase or the decrease of the yield surface radius. The isotropic hardening model chosen was the one proposed by Chaboche in [35], as it has been widely used in the past to model the hardening effect in various metallic materials.

The formulation of the complete cyclic plasticity model is presented in its uniaxial form, since the simulations are limited in the uniaxial stress space.

The yield surface applied was the Von Mises yield surface \( f \) given by Eq. (1):

\[
f = (\sigma - X)^2 - R^2
\]

where \( \sigma \) is the applied stress, \( X \) the back stress and \( R \) the yield stress evolving through the isotropic hardening rule [35] described by Eq. (2).

\[
dR = b[R_s - R]|ds|^p
\]

where \( R_s \) is the saturating value of yield surface expansion and \( b \) gives the rate at which saturation is reached.

Fig. 4. AA7075-T6 strain ratcheting curves for asymmetric-stress controlled tests (presented in Table 2): (a) maximum strain \( \varepsilon_{\text{max}} \) at peak of each cycle (b) mean strain \( \varepsilon_m \) at each cycle.

Fig. 5. AA7075-T6 Hysteresis loops obtained from the asymmetric stress-controlled (a) Test 2 (b) Test 3 and (c) Test 4 (presented in Table 2). Comparison of the first and last (150th) cycle.
The total back stress \( X \) (controlling the yield surface shifting) is given by Eq. (3).

\[
X = \sum_{i=1}^{3} X_i
\]  

(3)

The \( X_i \) back stress terms are obtained from Eq. (4) (\( d \) denotes the differential of each term).

\[
dX_i = \begin{cases} 
\gamma_i (a_i \Delta \varepsilon^p - X_i \Delta p) & i = 1, 2 \\
a_i \Delta \varepsilon^p & i = 3 
\end{cases}
\]  

(4)

where \( \Delta \varepsilon^p \), \( \Delta p \), \( a_i \) and \( \gamma_i \) are the incremental plastic strain, the equivalent plastic strain, the saturation level and rate of saturation respectively. Three back stress terms are defined as originally suggested by Chaboche [35], which is the minimum number needed to accurately simulate the cyclic behaviour. Moreover, selection of the minimum number of back stress terms was aimed at reducing computational times and ensuring ease of application of the model in the fatigue calculation software.

2.2.2. Determination of the model baseline parameters

The Chaboche model (kinematic and isotropic hardening) was implemented numerically using an implicit integration scheme. The kinematic hardening (back stresses \( X_i \)) parameters of the model were calculated by fitting Eq. (5) to the stabilised cyclic stress-strain curve (Fig. 6), according to the methodology proposed by Chaboche [35].

\[
\frac{\Delta \sigma}{2} = \sigma_{\text{yield}} + \sum_{i=1}^{3} a_i \tanh \left( \frac{\gamma_i \Delta \varepsilon^p}{2} \right) + a_3 \frac{\Delta \varepsilon^p}{2} + R
\]  

(5)

The isotropic hardening (\( R \)) parameters \( R_i \) and \( b \) were determined by nonlinear regression analysis to obtain the best fit to the cyclic stress range versus cycles data, with data collected from fully symmetric strain-controlled tests (Fig. 7) [9].

This technique was utilised to obtain the model baseline parameter values (shown in Table 3) that also acted as the range within which optimisation was performed (described in the sequel).

3. Single objective optimisation

A single objective optimisation was conducted using the commercial optimisation software package modeFRONTIER [44]. The objective of the optimisation was to minimise the difference between the simulated stress-strain curves and experimental data. The process involves an initial population development based on a select range for each of the plasticity model parameters. These values are then fed in the plasticity model, implemented numerically using an implicit integration scheme, to simulate the defined load case (hysteresis loop shape, strain ratcheting curves, etc.). The success of the trailed parameters in the generation are determined by comparing the simulation outputs with experimental data with the use of an objective function. Based on the objective function scores, the next generation evolves by employing an optimisation algorithm, which in this study is the genetic algorithm (GA). In this study the applied optimisation structure is referred to as a workflow. The sensitivity analysis required the development of a variety of workflows, where the main difference exists in the experimental curve definition and the imposed constraints. Varying the workflow definition allowed for the development of a number of parameter sets to be tested in strain-life fatigue predictions. In particular, the most suitable experimental curves are selected for the determination of those Chaboche model parameters, in terms of achieving better strain-life fatigue predictions. The overall optimisation process flow diagram is illustrated in Fig. 8, with its elements (objective function, constraints, etc.) described in detail in the following sections.

3.1. Population range

The range over which the population was developed is provided in Table 4, which includes the ranges for all workflows (1 to 6) used in the optimisation process.

The minimum and maximum values selected for the ranges were based on the parameter values given in Table 3, which were obtained by applying the manual calculation parameter technique as demonstrated in [35,45]. The large range selected for the genetic algorithm was to balance between search flexibility and convergence in solution.

3.2. Optimiser

A GA was utilised in the search for the most suitable combination of the parameters. In particular, the Multi-Objective Genetic Algorithm (MOGA-II) [44] was implemented. MOGA-II is an improved version of the MOGA, originally proposed by Poloni [46], utilising a smart multi search elitism and a set of operators (selection, classical one-point crossover, directional crossover and bit flip mutation). In order to customise and to improve the efficiency of MOGA-II for the elastoplastic constitutive model application, an extensive investigation into the probability of invocation of each operator was conducted. As with classical

---

**Table 3**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity modulus</td>
<td>( E = 69 ) GPa</td>
</tr>
<tr>
<td>Cyclic yield stress</td>
<td>( \sigma_{\text{yield}} = 465 ) MPa</td>
</tr>
<tr>
<td>Kinematic hardening</td>
<td></td>
</tr>
<tr>
<td>( X_1 )</td>
<td>( a_1 = 41 ) MPa, ( \gamma_1 = 35.304 )</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>( a_2 = 71 ) MPa, ( \gamma_2 = 242 )</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>( a_3 = 2100 ) MPa, N/A</td>
</tr>
<tr>
<td>Isotropic hardening</td>
<td></td>
</tr>
<tr>
<td>( R )</td>
<td>( R_i = 15 ) MPa, ( b = 6.8 )</td>
</tr>
</tbody>
</table>

---

**Fig. 6.** Curve fitting of the AA 7075-T6 experimental [43] and simulated stabilised cyclic stress-strain curve.

**Fig. 7.** Simulated isotropic hardening compared against 1.5% symmetric strain controlled stress amplitude.
MOGA, MOGA-II represents each parameter as a binary string onto which one of the operators (predefined by the operator probability) is applied during the reproduction process. The MOGA-II algorithm is summarised as following:

MOGA-II Pseudo Code
1. Generate initial random population \( P \) of size \( N \) and Elite set \( E \)
2. Evaluate objective values
3. Rank and sort based on the objective values (Pareto dominance)
4. Generate offspring population by the reproduction
   4.1. Combine both population and elite sets \( Q = P \cup E \)
   4.2. While \( Q \neq N \), resize \( Q \) by randomly removing individuals
   4.3. Based upon probability of invocation, randomly assign one operator (local tournament selection, directional crossover, one-point crossover or bit flip mutation) to compute the evolution from \( Q \) to \( R \)
   4.4. Evaluate objective values of population \( R \)
   4.5. Rank and sort based on the objective values (Pareto dominance)
   4.6. Extract the elite individuals
      4.6.1. Copy all non-dominated individuals of \( R \) to \( E \) and sort
      4.6.2. Update \( E \) by eliminating the duplicates and the dominated individuals
      4.6.3. While \( E \neq N \), resize \( E \) by randomly removing the individuals
5. Until termination, go to Step 4 with \( R \) as a new \( P \)

Table 4: Parameter value selection range for each optimisation scheme (Search Workflow).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Search workflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\text{init}} ) (MPa)</td>
<td>1 and 2</td>
</tr>
<tr>
<td>( a_1 ) and ( a_2 ) (MPa)</td>
<td>(1, 300)</td>
</tr>
<tr>
<td>( a_1 ) (MPa)</td>
<td>(1, 100,000)</td>
</tr>
<tr>
<td>( \gamma_1 ) and ( \gamma_2 )</td>
<td>(1, 500)</td>
</tr>
<tr>
<td>( b )</td>
<td>6.8</td>
</tr>
<tr>
<td>( E ) (GPa)</td>
<td>69</td>
</tr>
</tbody>
</table>
demonstrated in Fig. 12, where 10% mutation ratio of an 80 string parameter set results in the mutation of 8 strings.

The curves in Fig. 13(a) demonstrate a changing rate at which convergence is reached, as well as a changing convergence value between the tested mutation probabilities. The lower mutation probabilities converge prematurely but are significantly less noisy compared to the higher mutation probability curves. The reduced amount of mutation resulted in an inability of the optimiser to find a more suitable local optimal. The initial population has greater influence on the converging generation due to the lower influence of mutated genes. The optimisation outputs using mutation probabilities of 0.1 to 0.15 converge to the same objective value at approximately the same rate. The 0.05 mutation probability optimisation also converges to the same objective value but requires significantly more generations to achieve that. The convergence curve corresponding to the 0.1 mutation probability had the fastest convergence rate. Furthermore, the influence of DNA selection ratio on convergence is presented in Fig. 13 (b), which provides a comparison of the convergence curves. The results illustrate that the lower ratio of 0.01 mutation resulted in the less successful convergence value, which suggested that if too little of the DNA string is mutated, it is more difficult for the optimisation process to find the optimal location in the population of parameter combinations. The ratios of 0.03, 0.05 and 0.1 resulted in the same converged objective value, with the main difference being in the convergence rate. The 0.1 mutation had the less successful convergence rate, while 0.03 and 0.05 had very similar convergence rates. Consequently, values ranging between 0.03 and 0.05 would provide the optimal convergence.

3.2.4. Initial population
The great influence of the initial population on the convergence of heuristic types of searches is well recognised [47,48]. Therefore, to maximise the chances of finding optimal solutions, selecting the diversity and the optimal population size is very important. The curves in Fig. 14(a) provide an indication of the influence of generation size on convergence. An initial generation size of 25 is considerably more time efficient and converges to a lower objective than the other tested generation sizes. Different initial population sequence generator algorithms were investigated in order to maximise the diversity. In particular, a comparison between the random sequence generator, Sobol sequence, Uniform Latin Hypercube (ULH) and Incremental Space Filler (ISF) was performed [Fig. 14(b)]. These results suggest that the Sobol initial sequence was the most successful in achieving the lowest objective value.

![Fig. 10. Crossover of genes in two parameter sets.](image1)

![Fig. 11. Convergence curve (Objective value variation vs generations) comparison when altering crossover probability.](image2)

![Fig. 12. Mutation of a parameter set to produce a new individual to test in the next generation.](image3)
population exhibits significantly faster convergence rates compared to the other sequence generators.

3.2.5. Finalised optimiser

Based on this investigation, the GA settings for the elastoplastic constitutive model optimisation are listed in Table 5 and these were the values used in the optimiser for all workflows.

3.3. Objective function

The population of parameter combinations were fed into the model simulation stage, during which the parameters were used to generate the simulation outputs. The simulation outputs were then compared with experimental data and a fitness value was given by the objective function. The fitness value was defined according to the Frechet distance [49] which attempts to minimise the maximum Euclidean distance of possible ways to traverse the experimental curve and the simulated curve as summarised in Eq. (6), where the Frechet distance \( (FD) \) is calculated by comparing the functions of the curves \( P \) and \( Q \) (described in the MOGA-II Pseudo Code presented in Section 3.2), where \( i \) and \( j \) in Eq. (6) corresponding to points defining the curve.

\[
FD[i, j] = \max(\|P(i) - Q(j)\|, \min(FD[i-1, j-1], FD[i, j-1], FD[i-1, j]))
\]  

(6)

3.4. Constraints

It was important to ensure the gene development of the optimisation were not based on unrealistic simulation results. In some tested parameters, the results produced were deemed acceptable with regard to their fitness value but in fact produced unrealistic simulation results (e.g. prediction of decreasing ratcheting strain). Consequently, although

<table>
<thead>
<tr>
<th>Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genetic Algorithm (GA) optimal parameters.</td>
</tr>
<tr>
<td>Population Selection probability Crossover probability Mutation Probability DNA string mutation ratio</td>
</tr>
<tr>
<td>Size Algorithm</td>
</tr>
<tr>
<td>25 Sobol</td>
</tr>
</tbody>
</table>
the objective error was reduced, the actual appearance of the simulations curves were counterintuitive. To overcome this issue, constraints were added to the optimisation to prevent the emergence of unrealistic solutions. This ensured that genes associated with unrealistic results were not used in further chromosome development in future generations. For that purpose, constraints were imposed on the ratcheting outputs. In particular, negative gradients occurring between strain outputs were restricted, as indicated in the regions defined by dashed lines in Fig. 15. This avoided the appearance of potential unrealistic outputs, such as the curve anomalies shown in red lines in Fig. 15.

3.5. Search workflows

The workflow used in the optimisation strategy has a significant influence on the development of the parameters. It is possible that a number of alternate parameter combinations can achieve the same uniaxial simulation results. So, it is important to determine what influence these different optimised parameter sets have on the strain-life fatigue prediction accuracy. In order to determine this, a sensitivity analysis was conducted where a number of different workflows were used in the parameter optimisation. These varying workflows are summarised in Table 6, which provides an overview of how the experimental definition of the workflows varied across the six tested versions (detailed description is included in the Sections 3.5.1 to 3.5.5).

3.5.1. Workflow 1 and 2

The hysteresis loop shape and ratcheting strain simulation accuracy are important features to be used in the model parameter identification. Consequently, the experimental data used in the objective were:

Workflow 1:
- The stabilised 1.8% symmetric strain controlled hysteresis loop shown in Fig. 1(b).
- The two higher ratcheting strain curves labelled as Test 3 and Test 4 in Fig. 4(a).

Workflow 2:
- The stabilised 1.8% symmetric strain-controlled hysteresis loop shown in Fig. 1(b).
- The lower ratcheting strain curves labelled as Test 1 and Test 2 in Fig. 4(a).

Table 6
Summary of varying experimental curves used in each workflow.

<table>
<thead>
<tr>
<th>Workflow</th>
<th>Strain controlled loading cases</th>
<th>Stress controlled asymmetric loading cases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Various strain levels</td>
</tr>
<tr>
<td>1</td>
<td>Stabilised hysteresis loop Shape</td>
<td>(550, −450)</td>
</tr>
<tr>
<td>2</td>
<td>Stabilised hysteresis loop Shape</td>
<td>Cyclic stress-strain curve</td>
</tr>
<tr>
<td>3</td>
<td>Stabilised hysteresis loop Shape</td>
<td>Mean strain at Ratcheting Curve</td>
</tr>
<tr>
<td>4</td>
<td>Stabilised hysteresis loop Shape</td>
<td>Mean strain at Ratcheting Curve</td>
</tr>
<tr>
<td>5</td>
<td>Stabilised hysteresis loop Shape</td>
<td>Mean strain at Ratcheting Curve</td>
</tr>
<tr>
<td>6</td>
<td>Stabilised hysteresis loop Shape</td>
<td>Mean strain at Ratcheting Curve</td>
</tr>
</tbody>
</table>

Fig. 15. Ratcheting curve example (strain at peak of each loading cycle versus number of cycle): Dashed line segments define the constraint regions of the ratcheting output; dotted lines provide examples of what the constraints are attempting to prevent.
The isotropic hardening parameters were not included in the optimisation process. Instead, as proposed in [9], they were determined by nonlinear regression analysis to obtain the best fit to the cyclic stress range versus cycles data gathered from fully symmetric strain controlled tests.

The convergence curves obtained from the two workflow strategies are compared in Fig. 16, which is developed from the most elite (lowest objective value) of the generation.

3.5.2. Workflow 3

It was deemed necessary to also consider parameter development based on the accurate simulation of the evolution of the hysteresis loop shape. Due to the complexity of the loading in the tested spectra it was important to predict the maximum and minimum stress and strains present at each point in the spectrum. Therefore, a balance between the progression of the back-stress, as well as the yield and isotropic hardening had to be developed. In this workflow, the isotropic hardening parameters were allowed to fluctuate to allow better flexibility. This feature offered enhanced opportunity to recognise the progressive change in the hysteresis loop shape which was noticeable in the experimental data. The experimental data used in the objective were:

• The stabilised 1.8% symmetric strain controlled hysteresis loop [shown in Fig. 1(b)].
• The first and 100th cycle of the asymmetric strain-controlled Test 2 and Test 4 (Fig. 3).
• The mean stress relaxation curve obtained from Test 4, shown in Fig. 2.
• The cyclic stress strain curve, shown in Fig. 6.

Simulation results revealed that it was important to include the 100th cycle in the optimisation process to facilitate accurate development of the noticeable cyclic hardening phenomenon present in the material, which can be attributed to the narrowing of the hysteresis loops with cycles. The mean stress relaxation in the experimental curve definition was considered necessary in order to ensure that the parameters developed are able to simulate the progressive downward shift of the hysteresis loops between the 1st and 100th cycle. Finally, the cyclic stress-strain curve was also added to capture a variety of stabilised cycles.

The convergence curve corresponding to the optimisation is provided in Fig. 17. A solution was achieved (converged) within 240 generations.

3.5.3. Workflow 4

One key objective of this study was to determine whether the parameters developed from ratcheting hysteresis loops can provide improvement to strain-life fatigue calculations. The development of the hysteresis loops could be slightly different, leading to a different combination of kinematic and isotropic hardening parameters that may affect strain-life fatigue calculations. Due to the important influence of large overloads in the spectra, Workflow 4 was based on asymmetric stress-controlled data collected from large maximum stress load cases. Therefore, Workflow 4 included the following experimental curves:

• The stabilised 1.8% symmetric strain controlled hysteresis loop [shown in Fig. 1(b)].
• The asymmetric stress-controlled Test 3 and Test 4 strain ratcheting curves calculated by taking the mean strains by

\[
\varepsilon_m = \frac{1}{2} (|\varepsilon_{\text{max}}| + |\varepsilon_{\text{min}}|),
\]

which is shown in Fig. 4(b).
• The asymmetric stress-controlled Test 3 hysteresis loops at 1, 2, 20 and 100 cycles as per Fig. 5(b) and the first cycle from asymmetric stress-controlled Test 4, shown in Fig. 5(c).
• The cyclic stress strain curve, shown in Fig. 6.

The inclusion of one cycle from the asymmetric stress controlled Test 4 was used to ensure the parameter search was not restricted to the development of a good fit for solely one load case, broadening the parameter search and robustness of the converged parameters. The convergence curve from Workflow 4 is provided in Fig. 18. The rate of convergence was relatively fast (objective value reached within 35 generations).

3.5.4. Workflow 5

Due to the large loads contained in the spectrum coupled with the notch factor, the predicted stress and strain has the potential to violate the experimental bounds of the cyclic stress-strain curve. Predicting stresses or strains outside the bounds of this curve would lead to inaccurate results caused by the large negative stress and strains resulting in inaccurate damage calculations. The produced inconsistency is illustrated in the Fig. 19 example, showing how fatigue life (in flight hours) calculated at one particular critical location counterintuitively increases with increasing notch intensity factor \(K_t\). This is attributed to the prediction of an unrealistically large negative stress and strain, forcing...

---

**Fig. 16.** Convergence curves (objective value versus number of generations) obtained from workflows 1 and 2.

**Fig. 17.** Convergence curve (objective value versus generations) from workflow 3 incorporating asymmetric stress-controlled hysteresis loop experimental curves.

**Fig. 18.** Convergence curve (objective value versus generations) obtained from workflow 4 which incorporates asymmetric stress-controlled experimental curves.
subsequently developed hysteresis loops to predict no further damage accumulation.

To mitigate this issue, a constraint was added to the workflow whereby the end of the predicted cyclic-stress strain curve was forced to saturate according to Eq. (7), as indicated by the red line in Fig. 20. This constraint ensures that stresses outside the bounds of the cyclic stress-strain curve were not being over-predicted.

\[
\frac{\Delta \sigma_{\text{cyclic}}}{\Delta \varepsilon_{\text{plastic}}} < 343 \text{ MPa}\quad (7)
\]

In particular, the 343 MPa constraint corresponds to the (slope of the) limiting red-line curve shown in Fig. 20 and practically this curve is obtained through curve fitting. It is noted that the Fig. 20 (same as Fig. 6) curve is the cyclic stress - strain curve.

Workflow 5 had the same experimental curve definition as Workflow 4 (described in Section 3.5.3), with the only difference being the aforementioned additional constraint on the simulation outputs based on the saturation of the cyclic stress-strain curve.

The convergence curve obtained from Workflow 5 is shown in Fig. 21, which exhibits a very similar response as the other workflows employed in the optimisation (1 to 4).

3.5.5. Workflow 6

In order to determine the influence of lower maximum stress load cases, such as asymmetric stress controlled Test 1 and Test 2, on the parameter development for strain-life fatigue, Workflow 6 included additional experimental curves based on a low maximum stress load case. In particular, this workflow included the following experimental curves:

• The stabilised 1.8% symmetric strain controlled hysteresis loop [shown in Fig. 1(b)].
• The asymmetric stress-controlled Test 2, Test 3 and Test 4 strain ratcheting curves calculated by taking the mean strain, given by\( \varepsilon_m = (|\varepsilon_{\text{max}}| + |\varepsilon_{\text{min}}|)/2 \), which is shown in Fig. 4(b).
• The asymmetric stress-controlled Test 3 hysteresis loops at 1, 2, 20 and 100 cycles as per Fig. 5(b) and the first cycle from asymmetric stress-controlled Test 2, shown in Fig. 5(a) and Test 4, shown in Fig. 5(c).
• The cyclic stress strain curve, shown in Fig. 6.

The inclusion of experimental data collected from Test 2 enabled the parameter search to become broader in an attempt to find a set of parameters which can adequately recognise this variance in plastic behaviour between Test 2 and Test 4.

The objective value converged in approximately 50 generations, as demonstrated in Fig. 22.

4. Results

4.1. Output of optimisation process

The cyclic plasticity (Chaboche) model parameters obtained from the optimisation process, for each search workflows employed (1 to 6), are listed in Table 7. This set of parameters were then applied to perform the strain-life fatigue predictions, presented in Section 4.2 of this paper.
4.2. Strain-life fatigue predictions

Predicted fatigue lives were compared against experimental data gathered as part of the 2010 P-3C Orion aircraft service life assessment program conducted at the Australian Defence Science and Technology (DST) Group [29]. The programme collected notched coupon fatigue data using spectra obtained from fatigue critical locations identified in both the Royal Australian Air Force (RAAF) and the United States Navy (USN) full scale fatigue tests. In the analysis conducted in this work, a total of 21 spectra and corresponding experimental fatigue data were compared against strain-life predictions calculated using the DST Group developed fatigue analysis program CGAP [50], which employs a fatigue-life tool FAMS [51]. The main components of the CGAP strain-life calculation process are summarised below:

- Neuber’s rule is used to calculate the notch stress and strains using the remote loading condition.
- Equivalent strains are calculated using the strain-life curve and the modified Morrow Eq. (5).
- The total fatigue damage is calculated using the Miner’s rule [52].

The fatigue predictions using the six workflows were compared to the predictions obtained using the Masing model [1]. This was performed by integrating the Chaboche model into the strain-life calculation process by replacing the Masing model, resulting in the Chaboche model being used to calculate the local stress and strains at the notch root. This comparison was performed to determine whether the parameters obtained by applying the objective combinations considered in the present study were able to improve the strain-life prediction of the existing CGAP-incorporated model. This was performed by comparing the total accumulated error across the 21 different spectra, calculated using Eq. (8).

$$\text{ERROR} = \frac{\sum_{i=1}^{21} |\text{EXP}_i - \text{SIM}_i|}{\text{EXP}_i}$$  \hspace{1cm} (8)

where $\text{EXP}_i$ and $\text{SIM}_i$ are correspondingly the experimental and simulated data point corresponding to spectrum $i$, with $i = 1, 2, 3, \ldots, 21$.

An interesting finding is the anomalies produced by Workflows 2, 3, and 4, in terms of the obtained fatigue predictions. In particular, Fig. 23 demonstrates these issues, with respect to the results obtained from one large load spectrum. What is noticeable from at the areas enclosed by the dashed lines in Fig. 23, is the increase of the predicted fatigue lives (flight hours) with the increase in the notch intensity factors $K_t$. This is counterintuitive, indicating an issue with the calculation of the stress and strain as the loading increases. Therefore, on the basis of the resulting anomalies, the corresponding workflows (2, 3 and 4) were disqualified.

The predicted lives for the three workflows (1, 5, and 6) capable of successfully predicting the fatigue life for all spectra (i.e., without results containing anomalies) are presented in Fig. 24. The corresponding total accumulated error for each workflow is also shown in this figure. As indicated by the error value, Workflow 5 provides better overall improvement (16.41%) in life prediction among all workflows. This is also evidenced by the number of points concentrated closer to the diagonal (one-to-one line). Interestingly, Workflow 6, which is largely based on Workflow 5, resulted in significantly worse life predictions compared to the other two workflows (1 and 6) and the Masing model predictions.

In order to provide a better understanding on the accuracy achieved in each spectrum, the difference between experimental and predicted fatigue life was calculated with the use of Eq. (9).

$$\text{ERROR}_i = \frac{|\text{EXP}_i - \text{SIM}_i|}{\text{EXP}_i}$$  \hspace{1cm} (9)

where $\text{ERROR}_i$, $\text{EXP}_i$, and $\text{SIM}_i$ are correspondingly the error, experimental data and simulated data point corresponding to each spectrum $i$, with $i = 1, 2, 3, \ldots, 21$.

The results obtained for all 21 load spectra are plotted in Fig. 25. In all cases, Workflow 6 does not provide any improvement on the prediction. The two ratcheting based workflows (Workflow 1 and 5) demonstrate significant improvement compared to Masing. Considering the line drawn at 50% error, both Workflow 1 and 5 decreased the number of predicted lives exceeding 50% error from 8 to 3.
5. Discussion

The critical analysis of the Chaboche model parameter determination has demonstrated the level of influence that the type of uniaxial material data has on the accuracy of the strain-life fatigue predictions. This finding is not merely new, from a research point of view, however in light of the extensive use of the Chaboche model in actual fatigue life calculations, it is an issue that should be highlighted to the engineering practitioners employing this model on design and structural integrity management applications. Moreover, an interesting new finding is related to parameter determination with the use of asymmetric strain-controlled uniaxial data. This kind of data have caused anomalies in the prediction of the fatigue lives for the case of large notch intensity factors. This anomaly was the consequence of the imbalance between yield stress and kinematic hardening. In particular, if the yield stress is too low, then an increased number of loads in the spectrum will produce stress and strain. This offers an explanation as to why increasing the size of the loads in the spectrum would likely cause a decrease in fatigue life.

The accuracy improvement achieved with the use of Workflow 1 provides some evidence on the importance of developing parameters according to asymmetric stress-controlled data. However, as discussed, Workflow 4 fatigue predictions contradicts this finding, as life prediction increases with the increase of notch intensity factor. This anomaly can be attributed to the inclusion of the cyclic stress strain curve, since in some spectra the maximum stress was greater than the bounds of the cyclic stress-strain curve. Due to the small gradient at the end of the curve (having a slight positive value), the prediction of stresses outside of the bounds were constantly increasing in magnitude. Constrained these stresses to saturate outside the bounds, as was the case in Workflows 5 and 6, prevented the occurrence of the counterintuitive results. This resulted in Workflow 5 having the most accurate predictions, as evidenced by the total accumulated error and the 50% error improvement.

Although Workflow 2 highlighted the importance of considering only the larger maximum stress load cases in the model parameter determination, workflow 6 tested this further with the inclusion of low maximum stress experimental curves (in addition to those already defined in Workflow 5). The inclusion of the lower maximum stress load case resulted in a significant decrease in fatigue prediction accuracy, with reference to the accumulated error and Fig. 24 and Fig. 25 presented results. Comparing the developed parameters from Workflows 5 and 6 (shown in Table 7) it is apparent that the most significant difference is in the size of the yield stress. Workflow 6 employs a much larger yield stress to improve the simulation accuracy in the low maximum stress load case. Consequently, compressive loading in the spectrum may not produce any compressive plastic strain, which leads effectively to accumulation of (tensile) plastic strain. This effect is highlighted in Fig. 26 (extracted hysteresis loops from a tested spectrum), which demonstrates how plastic strain accumulation obtained through Workflow 6 leads to higher damage when compared with Workflow 5. An overcalculation in damage estimation leads to lower predicted fatigue lives, as it has been demonstrated for Workflow 6 (Fig. 24). The lower yield stress in Workflow 5 results in more compressive plastic strain, reducing the tensile plastic strain stacking noticed in Workflow 6 (clustering of hysteresis loops at lower plastic strain illustrated in Fig. 26).

The slight improvement in Workflow 5, in comparison to Workflow 1, can be explained by referring to the values of the added back stresses (\(a_u\)) and the saturation level obtained for the calculated stress. In particular, the saturation level for Workflow 5 is higher than that of workflow 1. This was due to the inclusion of the cyclic stress-strain curve in Workflow 5. Consequently, for the tested spectra, the predicted mean stresses in Workflow 1 can be lower than the one predicted by Workflow 5, which effectively reduces the influence on fatigue life. Moreover, Workflow 1 uses a higher yield stress than that of Workflow 5, which is a consequence of not including in the workflow the asymmetric stress-controlled hysteresis loop shape data. This causes the same issues as highlighted in Workflow 6 (illustrated in Fig. 26). The influence of the larger plastic strain accumulation is slightly negated by the lower predicted mean stress, since the larger plastic strain accumulation will decrease the predicted life, while the lower tensile mean stresses will raise the predicted life. Therefore, Workflow 5 provides...
the most accurate combination of both plastic strain and mean stress simulation.

Based on the results of this study, the optimal uniaxial material data to be used in the development of the elastoplastic constitutive model parameters for strain-life fatigue predictions is based on large maximum stress asymmetric stress-controlled hysteretic loops [load cases capable of inducing significant plastic strains as those given in Fig. 5(b) and (c)] and their corresponding strain ratcheting curve. For completeness, the cyclic stress-strain curve should also be included with a constraint to saturate outside the bounds of the curve, developed from known experimental data.

Conventional methods of parameter development as outlined by Chaboche [35] use stabilised hysteretic loops from symmetric strain-controlled results and/or the cyclic stress-strain curve to develop an initial set of parameters. These parameter values are then altered by fitting cyclic transient data from obtained from a large number of load cases. From the point of view of the effort required for this exercise, this is especially important when the Chaboche model is applied for strain-life fatigue analysis involving complex loading histories, such as those included in aircraft load spectra.

The conducted analysis showed that the Chaboche model parameter optimisation exercise can be more efficient when targeted to those experimental data (both symmetric and asymmetric stress/strain controlled) that can be the most influential in capturing successfully the transient effects. In particular, it was found that the material data can be narrowed down to the following:

- The cyclic stress-strain curve;
- The symmetric strain-controlled stabilised hysteretic loop, at one or two strain levels;
- The hysteresis loop development of at least two stress-controlled load cases inducing a large amount of plastic strain (e.g. 3–5% for aluminium alloys) and their corresponding ratcheting curves (plastic strain accumulation versus cycles).

In summary, this study showed that:

- The use of the Chaboche model parameters obtained from the optimisation process, has improved significantly the fatigue predictions, when compared to the CGAP-embedded Masing model predictions.
- The sensitivity analysis has showed that, using appropriately selected uniaxial material (test) data, improved strain-life fatigue predictions can be achieved, as opposed to randomly selected or too extensive test data.

6. Conclusion

The Chaboche nonlinear kinematic hardening model has been used successfully in a sensitivity analysis intended to determine the importance of the experimental curves’ type and constraints used in elastoplastic constitutive model parameter development for strain-life fatigue predictions. The application of an optimisation strategy using optimal GA settings, allowed for greater parameter search capabilities leading to a number of different parameter sets. The sensitivity of the strain-life calculation based on these different parameter sets was used to determine which material data provided the most accurate strain-life fatigue results, therefore, providing a better understanding into the importance of the type of material data used in plasticity model parameter determination. Based on these results, an optimisation strategy using optimal generic algorithm has been successfully developed to identify multiple model parameters that yield significantly improved prediction than conventional method. This study has also highlighted the importance of accurately simulating the hysteresis loop sequences. Stress and strains calculated early in the spectrum sequence are expected to have a significant influence on the overall damage calculation, due to their contribution to the size of future stress and strain later in the spectrum.

In future research work, the validity of this devised method will be tested on other aerospace alloys (aluminium and titanium), for fatigue predictions of spectra obtained from other aircraft fatigue critical locations (e.g. a number of control points of P-3C aircraft fatigue management programme). Moreover, it is noted that due to funding constraints the test campaign was focused on obtaining results from a broad spectrum of load cases, rather than limiting the cases and re-running experiments to check repeatability. Thus, further research efforts are planned to include repeat test runs of the symmetric strain, asymmetric strain and asymmetric stress-controlled loading cases conducted in this study.

Acknowledgements

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Naval Air Warfare Centre, Fatigue Analysis of Metallic Structures (FAMS) - a Computer Program to Calculate Fatigue Damage by Local Stress/Strain Approach, 1995.

Chapter I.5  Strain-Life Fatigue Prediction Improvements

This chapter contains two main components:

- I.5.1 Application of Elastoplastic Constitutive Models to Fatigue Calculations
- I.5.2 Further Improvement to and Mean Stress Relaxation and hysteresis loop development in Asymmetric Strain-Controlled Loading Simulations (constant and variable amplitude)

In Section I.5.1 With the most optimal GA settings for elastoplastic constitutive modelling determined, as well as the most accurate workflow to be used in parameter determination, the final step required the application of a number of different nonlinear kinematic hardening models to the strain-based fatigue calculation process in order to determine the most accurate model. In Section I.5.2 Further improvements in simulation accuracy of mean stress relaxation and hysteresis loop shape in both constant and variable amplitude loading was investigated by considering the influence of the linear backstress on the extent of the relaxation of mean stresses and the contribution of the monotonic backstress evolution.
I.5.1 Application of Elastoplastic Constitutive Models to Fatigue Calculations

(Supporting Analysis)
5.1.1 Statistical Method of Comparison

To assess the models, prior experimental data obtained as part of the P-3C service life assessment program performed at the DST Group [1, 2] was once again utilised. The comparison considered the probability distribution of experimentally determined fatigue lives for each spectrum. This provided a means to assess the experimental variability in the comparison. Although traditionally the geometric mean of the experimentally obtained fatigue lives under each spectrum is used to compare predictive accuracy, a statistical approach was also adopted in this investigation.

Non-Parametric Statistical Approach

The distributions of experimental fatigue lives were generated using a non-parametric statistical approach known as the kernel density estimate. Nonparametric density estimates make no assumptions of the underlying distribution, instead allow the data to ‘speak for itself’. Doing so allows for the potential of recognising multiple failure modes due to crack propagation responding uniquely to different overloads in the spectrum. Different failure modes will lead to different fatigue-life distributions. Consequently, the underlying distribution would not be a Gaussian distribution of log fatigue-lives, but instead a multi-peak distribution. Multiple fatigue modes were noticed by Underhill and DuQuesnay [3] in coupon tests using a spectrum from the CF-188 Hornet. The resulting distribution of fatigue lives is given in Figure 1. If the data is assumed to be a Gaussian distribution, then the red curve in Figure 1 shows the appearance of this distribution. What can be seen is a considerable difference in the fatigue-life distribution when assuming a Gaussian distribution. This hints at the suggestion that comparing simulated fatigue-lives with the geometric mean of the data could lead to a false understanding of the accuracy of the fatigue-life predictions.
**Kernel Density Estimate (KDE)**

The kernel density estimate uses a kernel data smoothing method. The method attempts to determine the governing distribution of experimental results without knowledge of the appearance of the underlying distribution but instead uses the gathered experimental sample to develop an estimate.

**Kernel and Bandwidth**

The accuracy of the estimated distribution is dependent on the selected kernel and bandwidth. Fatigue-lives are assumed to be log-normally distributed, consequently, taking the log of the experimental data and using a Gaussian kernel is based on the assumption that at each experimental point, there could potentially be further experimental data points existing in a log-normal distribution encompassing the known experimental point. The selection of the bandwidth is important in ensuring a good median is met between developing a distribution which is not too noisy with unrealistic peaks and a distribution which is too smooth, as demonstrated in Figure 2, which highlights how the selection of the bandwidth has a significant influence on the shape of the total density estimate.

**Figure 1** Comparison of the life distributions when assuming a normal distribution compared with a distribution formed using the kernel density estimate
A considerable amount of work has been conducted in the development of the most appropriate method of estimating the optimal bandwidth. Due to the selected application of the Gaussian kernel, Silverman’s rule [4] was used to calculate the optimal bandwidth. In Silverman’s work, the optimal bandwidth was found by finding the bandwidth which would reduce the Mean Integrated Squared Error (MISE) assuming a Gaussian population, which is demonstrated in Figure 3. Consequently, if the population is highly non-Gaussian, Silverman’s bandwidth may cause a slight over-smoothing of the estimated distribution. However, in this application, a Gaussian estimate is still in-line with the original assumption made by taking the geometric mean. The advantage the KDE offers is if there are peaks in the distribution due to the presence of multiple failure modes, the KDE is capable of recognising these, leading to a more accurate distribution.
Figure 3 Silverman's bandwidth derivation assuming a Gaussian population and attempting to minimise the MISE between the estimated and Gaussian distribution.

Once the distributions were generated, the kernel density estimate was utilised to generate a characteristic distribution across selected fatigue critical locations. A similar distribution can be constructed using the predicted fatigue lives across the 21 locations. This distribution formation across the 21 fatigue critical locations is explained with reference to Figure 4, which attempts to demonstrate how Gaussian kernels are developed from sampled values from each location distribution.

Figure 4 Kernel density estimate applied to the fatigue critical locations on the P-3C Orion.
Mean Integrated Squared Error (MISE)

The Mean Integrated Squared Error (MISE) as given in Eq.1, was used as the metric to compare the two distributions. The MISE was calculated by comparing the distribution formed across the 21 simulated lives with the distribution formed from the resampled array of 21 experimental values using the developed 21 distributions for each location. This process of MISE calculation is summarised in Figure 5 which involves sampling one fatigue life from each of the 21 formed experimental distributions and using these values to form another distribution to be compared with the distribution formed from the 21 simulated lives. In Eq.1, \( f_n(x) \) is the simulated distribution and \( f(x) \) is the experimental distribution. The lower the MISE, the more accurate the simulation results. Each resample of the 21 experimental distributions produced a MISE value and a final distribution of MISE values was formed. The MISE distributions allow assessment of the accuracy of the models relative to each other including the known variability in experimental test results.

\[
E\int (f_n(x) - f(x))^2 \, dx
\]  

(1)

**Figure 5** MISE calculation for each experimental distribution formation after resampling of the experimental location distributions
5.1.2 Strain-Life Predictions

Optimisation Setup

The initial population (and subsequent generations) were a size of 25 and were generated using the ranges given in Table 1 using the Sobol sequence.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{yield}}$ (MPa)</td>
<td>(250,500)</td>
</tr>
<tr>
<td>$a_i$ (MPa)</td>
<td>(1,100)</td>
</tr>
<tr>
<td>$C_n$ (MPa)</td>
<td>(1,100000)</td>
</tr>
<tr>
<td>$\gamma_{i-1}$</td>
<td>(1,100000)</td>
</tr>
<tr>
<td>$\bar{a}$ (MPa)</td>
<td>($10^{-3}$,1000)</td>
</tr>
<tr>
<td>$a^*_3$</td>
<td>(1, $10^6$)</td>
</tr>
<tr>
<td>$c^*_3$</td>
<td>(1, $10^6$)</td>
</tr>
<tr>
<td>$m$</td>
<td>($10^{-3}$,10)</td>
</tr>
</tbody>
</table>

As previously determined, the most appropriate experimental curves to be used in the development of parameters for elastoplastic constitutive models for strain-life fatigue predictions is large maximum stress asymmetric stress-controlled hysteresis loops (Figure 6) and their corresponding ratcheting curves (Figure 7) (ratcheting strain calculated as the mean strain ($\frac{\varepsilon_{\text{MAX}}+\varepsilon_{\text{MIN}}}{2}$) of each cycle). Therefore, this material behaviour was used in the optimisation workflow for each of the applied kinematic hardening models.
Figure 6 (a) (540MPa, -460MPa) load case and (b) (550MPa, -450MPa) load case hysteresis loop break down

Figure 7 Strain ratcheting curves.

The cyclic stress-strain curve was also used with a constraint applied to the shape of the gradient at the end of the curve in order to prevent the prediction of high stresses at strains larger than the bounds of experimental points obtained due to potential overloads present in the spectrum.
Optimisation Convergence

The optimisations were defined to run for 120 generations with all models converging within the specified number. The convergence curve for each model is given in Figure 8 and the corresponding objective value the optimisation converged for each model is listed in Table 3. The parameters obtained from the optimisation are given in Table 2.

Table 2 Converged parameters

<table>
<thead>
<tr>
<th></th>
<th>MAF (i=2, n=3)</th>
<th>MAFT (i=3, n=4)</th>
<th>MAFM (i=3, n=4)</th>
<th>OW (i=8, n=9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y$ (MPa)</td>
<td>377</td>
<td>247.7</td>
<td>332</td>
<td>398</td>
</tr>
<tr>
<td>$a_i$ (MPa)</td>
<td>7, 174</td>
<td>66, 185, 300</td>
<td>190, 49, 25</td>
<td>11, 121,0.072, 0.0097, 0.00032, 0.15, 0.0040, 0.0019</td>
</tr>
<tr>
<td>$\gamma_{i-1}$</td>
<td>99996, 171</td>
<td>97076, 253, 1</td>
<td>155, 99937, 73948</td>
<td>94194, 524, 82161, 90973, 99091, 45139, 10465, 74055</td>
</tr>
<tr>
<td>$C_n$ (MPa)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>203</td>
</tr>
<tr>
<td>$\bar{a}$ (MPa)</td>
<td>-</td>
<td>17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a^*_3$</td>
<td>-</td>
<td>-</td>
<td>470110</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma^*_3$</td>
<td>-</td>
<td>-</td>
<td>940650</td>
<td>-</td>
</tr>
<tr>
<td>$m$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.126</td>
</tr>
<tr>
<td>$R_s$ (MPa)</td>
<td>120</td>
<td>196</td>
<td>102</td>
<td>112</td>
</tr>
<tr>
<td>$b$</td>
<td>52</td>
<td>41</td>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>69910</td>
<td>73767</td>
<td>69135</td>
<td>69252</td>
</tr>
</tbody>
</table>
Figure 8 Optimisation convergence curves for each model

Table 3 List of converged objective values for each of the models

<table>
<thead>
<tr>
<th>Model</th>
<th>Objective value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAF</td>
<td>0.651</td>
</tr>
<tr>
<td>MAF-T</td>
<td>0.683</td>
</tr>
<tr>
<td>MAFM</td>
<td>0.636</td>
</tr>
<tr>
<td>OW</td>
<td>0.693</td>
</tr>
</tbody>
</table>

Uniaxial Simulations
The parameters developed using the optimisation procedure were used in simulations of a number of loading conditions. The tested loading conditions included asymmetric strain and stress -controlled.
MAF Simulations

The MAF simulation outputs for asymmetric stress-control loading for two loading conditions (550MPa, -450MPa) and (540MPa, -460MPa) are compared to the gathered experimental data in Figure 9. The hysteresis loop development is given in Figure 9 (a), where the first two cycles are well simulated, while the 20th and 100th cycles are not as well simulated due to development of the isotropic hardening parameters. This is a compromise in accuracy in an attempt to provide accurate simulations across a number of loading conditions. The ratcheting strain curves (where the ratcheting strain is measured as the mean strain per cycle) is given in Figure 9 (b) where a slight under-prediction in the amount of ratcheting strain for the (550MPa, -450MPa) loading condition.

![Hysteresis Loop](image1)

**Figure 9** (a) Cycle 1, 2, 20, and 100 simulation and experimental comparison between experimental and simulation results (b) ratcheting strain comparison between experimental and simulated results

The asymmetric strain-controlled MAF simulations outputs are given in Figure 10 for two different asymmetric strain-controlled loadings. The results suggest that the optimised parameters provide very good simulation accuracy for not only the first cycle but also the 100th cycle in both cases.
Figure 10 The 1st and 100th cycle comparison between simulated and experimental result for the (a) asymmetric strain-controlled loading (1.9%, -0.1%) (b) (1.55%, -0.5%).

MAFT Simulations
Similarly, the accuracy of MAFT simulations were checked using the same loading conditions. The asymmetric stress-controlled simulations are given in Figure 11 and asymmetric strain-controlled given in Figure 12. Unlike the MAF asymmetric strain-controlled simulations, the hysteresis loops developed in MAFT simulations do not compare as successfully to the experimental results.
Figure 11 (a) Cycle 1,2,20, and 100 simulation and experimental comparison between experimental and simulation results (b) ratcheting strain comparison between experimental and simulated results.

Figure 12 The 1st and 100th cycle comparison between simulated and experimental result for the (a) asymmetric strain-controlled loading (1.9%, -0.1%) (b) (1.55%, -0.5%).
MAFM Simulations
The MAFM simulations demonstrate improvement in overall simulation accuracy compared to the MAF and MAF-T simulations. This is particularly evident in Figure 13 (b) which shows very accurate simulation of the strain ratcheting.

**Figure 13** (a) Cycle 1, 2, 20, and 100 simulation and experimental comparison between experimental and simulation results (b) ratcheting strain comparison between experimental and simulated results.

**Figure 14** The 1st and 100th cycle comparison between simulated and experimental result for the (a) asymmetric strain-controlled loading (1.9%, -0.1%) (b) (1.55%, -0.5%).
**OW Simulations**

The OW simulations seem to be the least accurate, particularly in the asymmetric stain-loading case. The hysteresis loops for the first and 100th cycles in Figure 16 are not accurately simulated, with the mean stresses in both load cases relaxing far too quickly, portrayed as the lower hysteresis loops compared to the experimental loops.

![Figure 15](image)

**Figure 15** (a) Cycle 1, 2, 20, and 100 simulation and experimental comparison between experimental and simulation results (b) ratcheting strain comparison between experimental and simulated results.
Figure 16 The 1st and 100th cycle comparison between simulated and experimental result for the (a) asymmetric strain-controlled loading (1.9%, -0.1%) (b) (1.55%, -0.5%).

Accumulated Error
The accuracy of the simulations achieved by each of the models was determined by comparing the outputs with experimental data developed from a number of loading conditions, with a numerical value given for each using Eq.2. These values are listed in Table 4. Overall, the MAFM has the lowest total error, therefore, is the most accurate model in simulations.

\[
\text{Error} = \frac{1}{M} \sum_{i=1}^{M} \left( \frac{y_i^{\text{EXP}} - y_i^{\text{SIM}}}{y_i^{\text{MAX}}} \right)^2
\]  

(2)
Table 4: Accumulated error developed by comparing the simulation output with the experimental curves

<table>
<thead>
<tr>
<th>Model</th>
<th>(540, -460) Cycles</th>
<th>Ratcheting curves</th>
<th>Asymmetric strain cycles</th>
<th>Total values</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFM</td>
<td>0.0060</td>
<td>0.0069</td>
<td>0.034</td>
<td>0.047</td>
<td>1</td>
</tr>
<tr>
<td>MAFT</td>
<td>0.011</td>
<td>0.0078</td>
<td>0.032</td>
<td>0.051</td>
<td>2</td>
</tr>
<tr>
<td>MAF</td>
<td>0.0080</td>
<td>0.020</td>
<td>0.034</td>
<td>0.062</td>
<td>3</td>
</tr>
<tr>
<td>OW</td>
<td>0.012</td>
<td>0.035</td>
<td>0.047</td>
<td>0.094</td>
<td>4</td>
</tr>
</tbody>
</table>

Strain-Life Fatigue Predictions

Figure 17 gives an indication of the relative accuracy of each of the models. It also highlights the varying degrees of accuracy for each spectrum between the models, with the greatest difference being the predictions made by the MAF-T model. Listed in Table 5 is the accumulated difference error in order to provide a better understanding of the accuracy of life predictions.

![Graph showing simulated fatigue lives plotted against experimental values](image)

**Figure 17**: Simulated fatigue lives plotted against the corresponding experimental values.
Comparing the accumulated error, the MAF, MAFM, and OW are improvements on fatigue predictions compared to the Masing predictions as given by the lower accumulated error calculations obtained using Eq.3. The most improved predictions were obtained by applying the MAF and OW models.

\[
\text{Acummulated Error} = \sum_{i=1}^{21} \left| \frac{\text{EXP}_i - \text{SIM}_i}{\text{EXP}_i} \right|^{21}
\]

Where \( \text{EXP}_i \) and \( \text{SIM}_i \) are the correspondingly the experimental and simulated data point corresponding to spectrum \( i \).

Table 5 calculated accumulated error for each model

<table>
<thead>
<tr>
<th>Model</th>
<th>Accumulated Error</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAF</td>
<td>6.72</td>
<td>1</td>
</tr>
<tr>
<td>OW</td>
<td>6.74</td>
<td>2</td>
</tr>
<tr>
<td>MAFM</td>
<td>7.05</td>
<td>3</td>
</tr>
<tr>
<td>Masing</td>
<td>8.04</td>
<td>4</td>
</tr>
<tr>
<td>MAFT</td>
<td>8.27</td>
<td>5</td>
</tr>
</tbody>
</table>

Finally, the probabilistic comparison is given in Figure 18. The MAF model has once again shown to produce the most accurate fatigue life predictions, which is indicated by the lower MISE distribution. However, the order of accuracy between the other models is slightly different compared to the deterministic comparison. The probabilistic approach does not assume a distribution, instead, the experimental data is allowed to speak for itself. This is the reason why the geometric mean of the experimental data does not have a calculated MISE of zero. In the deterministic method of comparison, the model predicted lives are compared directly to the geometric mean of the experimental data. However, the probabilistic approach does not compare against the geometric mean but instead samples from distributions formed using the KDE which may not be log-normally distributed. Therefore, predictions which identify best with the actual experimental data rather than the geometric mean will be deemed more accurate according to the probabilistic method. This is the reason why the order of
model accuracy changes between comparison methods since some model predictions would compare well with the geometric mean while others will compare better with the KDE distributions of the experimental data.

The results also suggest that all nonlinear kinematic hardening predicted fatigue lives are considerably more accurate than those predicted using the Masing model as indicated by the bunching up of the nonlinear kinematic hardening model MISE distributions to the left side of the Figure 18.

![Figure 18 MISE distributions formed by each of the models](image)

**Comparing the Scatter in Error**

One final comparison to determine the success of the applied models to fatigue-life prediction improvement is to consider the difference in error (difference between experimental and predicted fatigue lives) scatter of the different models. This is a method of determining the consistency of prediction accuracy across the 21 different spectra. To do this, the error was considered to be normally distributed and the 95% confidence interval of each distribution was determined, as shown in Figure 19.
The confidence interval of each of the applied models was then compared to the confidence interval obtained using the Masing error distribution. The results of the difference in confidence interval size between the Masing model and the tested models are listed in Table 6. The results show that all models reduce the scatter in error between predicted and experimental lives across the 21 different spectra, with the MAF model having the greatest impact with 31% improvement compared to Masing predictions.

**Table 6** Comparison of 95% confidence interval for each model and the percentage difference in error distribution size.

<table>
<thead>
<tr>
<th>Model</th>
<th>95% Confidence Interval</th>
<th>% Difference to Masing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masing</td>
<td>[-31703, 63786]</td>
<td>-</td>
</tr>
<tr>
<td>MAF</td>
<td>[-25916, 50545]</td>
<td>31%</td>
</tr>
<tr>
<td>OW</td>
<td>[-26522, 49793]</td>
<td>20%</td>
</tr>
<tr>
<td>MAFM</td>
<td>[-25916, 50546]</td>
<td>20%</td>
</tr>
<tr>
<td>MAFT</td>
<td>[-46305, 34530]</td>
<td>15%</td>
</tr>
</tbody>
</table>
Identified Issue in Masing Predictions

Initial investigation into the potential increase in the accuracy of strain-life fatigue predictions was stimulated by the inaccuracy of predictions noticed in the investigation of a particular spectrum. Application of the MAF model by Hu and Wallbrink [5] demonstrated an improvement in fatigue accuracy. What needed to be tested was whether the new parameter calculation strategy did not affect the simulation accuracy of this particular spectrum. Figure 20 gives an indication of the improvement in accuracy of strain-life fatigue predictions using the MAF with optimised parameters. Although there is still an over-prediction of the life at a $k_t$ of 5 for the unclipped results, the clipped fatigue predictions are a significant improvement compared to the MAF traditional parameter calculation and Masing predictions. What is particularly important to take from the results is the MAF with optimised parameters does an exceptional job at recognising the anomaly noticed in notched coupon experimental results. As identified from experiments, the clipped spectrum produced less conservative results than the unclipped spectrum. The MAF with optimised parameters was able to very effectively recognise this anomaly in fatigue life between the clipped and unclipped spectra.

Figure 19 Example of the comparison of the confidence interval calculated for the Masing distribution compared to the applied model distributions.
Applying the simple accumulated difference error using Eq. 3 for fatigue life predictions given in Figure 20, the improvement in accuracy offered by the MAF optimised values is further reinforced as the lower accumulated difference. These values are listed in Table 7.

Table 7 Accumulated error for each model comparing predicted with experimental lives for the clipped and unclipped spectra

<table>
<thead>
<tr>
<th>Model</th>
<th>Accumulated Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAF Optimised</td>
<td>1.30</td>
</tr>
<tr>
<td>MAF Traditional</td>
<td>1.33</td>
</tr>
<tr>
<td>Masing</td>
<td>1.53</td>
</tr>
</tbody>
</table>
The reason for this difference in the experimental fatigue lives between the clipped and unclipped spectra can be explained with reference to the influence of overloads in the unclipped spectrum. These overloads force the cyclic transient effects to occur at larger plastic stress and strains, resulting in more damaging cycles than those in the clipped spectrum. This behaviour is demonstrated in Figure 21, where the overload pushes the damaging transient cyclic effects (given as dotted lines) to occur at larger plastic stress and strains.

**Figure 21** demonstrates the influence of the overload in the unclipped spectrum
5.1.3 Investigation Results

The results of this in-depth analysis of the potential improvements to strain based fatigue predictions has highlighted a number of important concepts when considering the application of elastoplastic constitutive models to strain-life fatigue predictions. Uniaxial experimental data obtained from asymmetric stress-controlled tests identified an early onset of plastic shakedown. In order to successfully simulate this, a linear back stress was introduced into the kinematic hardening formulations. This was shown to improve the strain ratcheting simulation for each of the models. Another modification came in the form of separating the monotonic and cyclic components of the experimental results, modelling each regime using different parameter sets. This once again improved the strain ratcheting simulations by ensuring the starting strain was more accurately predicted with the monotonic shaped curve.

The sensitivity analysis conducted based on the selection of the most appropriate uniaxial material data to be used in parameter calculation, gave rise to the importance of recognising sequence effects in fatigue life predictions. The fluctuation in fatigue life predictions between the parameter sets developed from the uniaxial material data gives an indication of the importance of accurately calculating plastic stress and strains early in the sequence, since the values of which will have an influence on the values calculated further in the sequence.

Figure 22 attempts to demonstrate the importance of accurately predicting the shapes of the hysteresis loop when applied to strain-life fatigue. The blue and red curves in the figure have only one different predicted strain but subsequent predicted strains vary between the two hysteresis loop sequences and would be exaggerated further with continued loading. This anomaly between the two hysteresis loop developments would ultimately lead to different fatigue predictions. As the sensitivity analysis showed, these fluctuations in fatigue predictions happen even though the same elastoplastic constitutive model is applied. Therefore, the selection of the right material data to use in the development of the model parameters is very important. The results of the sensitivity analysis demonstrated how the traditional parameter calculation methods did not provide the most accurate fatigue predictions. Instead, parameter development according to stress-controlled hysteresis loop
shapes provided the most accurate results. The parameters developed in this way allowed for more accurate hysteresis loop tracking, limiting the influence of sequence effects.

![Hysteresis loop tracking](image)

**Figure 22** Influence of not accurately predicting plastic stress and strains early in the sequence

Isotropic hardening parameters were also developed using asymmetric stress-controlled results. The correct development of the isotropic hardening parameters is also important when applied to fatigue predictions. The changing yield surface will result in different predictions of plastic stress and strains compared to those predicted without the incorporation of isotropic hardening. Once again due to the complexity of the loading and number of cycles, the influence of isotropic hardening on the final fatigue prediction becomes even more significant.

Application of the different kinematic hardening models to strain-life predictions demonstrated that although the parameters for each of the models were developed using the same uniaxial material data and same parameter optimisation strategy, the application of these models and corresponding parameters had varying degrees of accuracy in strain-life predictions. The reason for this can be explained with reference to the complexity of the spectra used in the investigation. The spectra contain millions of cycles, which leads to significant interaction between the stress and strain-controlled transient cyclic behaviour. Although all models were capable of converging to a solution with very similar total
objective values in the parameter optimisation, the fluctuation in the yield stress values between the models is quite significant. This was due to the parameter development between the models to achieve the same simulation output being different. Consequently, the interaction and influence of the simulated strain ratcheting and mean stress relaxation during fatigue life calculations would vary leading to slightly different fatigue-life predictions between the models. Therefore, the results of this analysis highlight the importance of parameter development when it is intended that nonlinear kinematic hardening models are to be applied in situations which involve complex loading conditions.

It is very difficult to effectively capture correctly all material behaviour for all possible loading conditions the material may be subject to. The variation in fatigue results between the models supports this notion. Although the models had the same optimisation workflows, the models would vary in accuracy when attempting to simulate other loading conditions not including in the optimisation workflow. This is why complex spectrum simulation is so difficult, the sheer number of varying loading conditions would result in varying simulation outputs between the models. In the application of these models it is important to consider a sensitivity analysis using known results to determine which uniaxial material data to be used in the parameter development, since the results of this analysis demonstrate the potential variability in accuracy of predictions between the models with parameter development based on the same uniaxial material data.

The varying life accuracy can be explained using the hysteresis loop development for one of the 21 results. The closest to the geometric mean at this location is the MAF prediction with the next closest models ranked in Table 8. Figures 23, 24, and 25 compare the hysteresis development of the MAF model with the other three models in order to identify the features of the results which produce the difference in fatigue life calculations.
Table 8 Comparison of predicted fatigue lives at a particular location

<table>
<thead>
<tr>
<th>Model</th>
<th>Life (Flight hours)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric mean</td>
<td>10630</td>
<td></td>
</tr>
<tr>
<td>MAF</td>
<td>10169</td>
<td>1</td>
</tr>
<tr>
<td>MAFM</td>
<td>11904</td>
<td>2</td>
</tr>
<tr>
<td>OW</td>
<td>12064</td>
<td>3</td>
</tr>
<tr>
<td>MAFT</td>
<td>17881</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 23 compares a snapshot of a comparison of the MAF and MAFM hysteresis loops at a tested location. The predictions made by the two models are very comparable. This is supported by the hysteresis loop development. The main difference is in the lower simulated maximum stress made by the MAFM compared to the MAF. Therefore, the simulated tensile mean stresses are lower in the MAFM than the MAF, resulting in slightly larger fatigue lives predicted using the MAFM.

![Hysteresis loop comparison](image)

**Figure 23** Snapshot comparison of hysteresis loop development between MAF and MAFM simulations using a spectrum load case.
The hysteresis loop development of the OW model is quite different to the MAF as highlighted in Figure 24. Significantly less plastic cycles are simulated by the OW model, while the mean stresses are also higher than those predicted by the MAF. The lower predicted plastic cycles results in higher predicted fatigue life, while the larger mean stresses would result in a shifting towards lower predicted fatigue lives. The combination of this lead to the very similar predicted life compared to the MAF.

Figure 24: Snapshot comparison of hysteresis loop development between the MAF and OW simulations using a spectrum load case.

The MAFT hysteresis loop development had lower maximum stresses than the MAF hysteresis loop development. The compressive regime is also different between the models with the MAFT having larger compressive stresses. This resulted in lower calculated mean stresses than those predicted by the MAF model. This leads to the significantly larger fatigue lives predicted by the MAFT model.
These comparisons highlight the different influence of the sequence effects on the hysteresis loop development between the models. Although the model parameters were developed using the same material data, the influence of the complexity of the spectra results in significantly different hysteresis loop development.

5.1.4 Influence of Yield Stress

Further investigation into the influence of yield stress on FAMS simulation accuracy was undertaken. This was done by holding the yield stress constant and proceeding with the optimisation. The yield stress chosen was that corresponding to the yield stress developed from the optimisations of the MAF model. Consequently, the MAF model parameters remained consistent with the values developed from original simulations. However, the other model parameters were adjusted during the optimisation to recognise the new restriction on the parameter development based on the fixing of the yield stress. The converged parameters are listed in Table 9.


<table>
<thead>
<tr>
<th></th>
<th>MAF (i=2, n=3)</th>
<th>MAFT (i=3, n=4)</th>
<th>MAFM (i=3, n=4)</th>
<th>OW (i=8, n=9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{y0}$ (MPa)</td>
<td>377</td>
<td>377</td>
<td>377</td>
<td>377</td>
</tr>
<tr>
<td>$a_i$ (MPa)</td>
<td>7, 174</td>
<td>152, 8, 14</td>
<td>1, 203, 47</td>
<td>156, 1.56, 0.54, 0.061, 0.043, 0.023, 0.0028, 0.000055</td>
</tr>
<tr>
<td>$\gamma_{i-1}$</td>
<td>99996, 171</td>
<td>170, 97907, 228</td>
<td>84703, 162, 1</td>
<td>2592, 57623, 18296, 37759, 65594, 38633, 357, 18318</td>
</tr>
<tr>
<td>$C_n$ (MPa)</td>
<td>1</td>
<td>102</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{a}$ (MPa)</td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a'_3$</td>
<td>-</td>
<td>-</td>
<td>269671</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma'_3$</td>
<td>-</td>
<td>-</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>$m$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>$R_s$ (MPa)</td>
<td>120</td>
<td>121</td>
<td>101</td>
<td>91</td>
</tr>
<tr>
<td>$b$</td>
<td>52</td>
<td>47</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>69910</td>
<td>68892</td>
<td>69872</td>
<td>69190</td>
</tr>
</tbody>
</table>

These parameters were then applied in FAMS simulations to develop the predicted lives for each of the 21 spectra. Using the accumulated error method of calculation given in Eq.3, Table 10 lists the error between the experimental and predicted fatigue lives for each of the models. A slight overall improvement in accuracy is apparent for all models. The accuracy ranking was altered only slightly, with the OW model becoming slightly more accurate than the predictions made using the MAF model. The results of this optimisation highlight the influence of the yield surface on fatigue predictions. This is due to the amount of plastic strain produced during loading and supports the explanation in Section 1.5. However, although this method does improve simulations it does require prior knowledge of the yield strength before initiating the optimisations. Consequently, this method could only be applied...
once the initial optimisation strategy is employed, thus, could potentially be a method of finetuning the optimised parameters.

Table 10 accumulated error comparison and ranking of the models using parameters developed from the optimisations with constant yield.

<table>
<thead>
<tr>
<th>Model</th>
<th>Accumulated error (constant yield)</th>
<th>Rank (constant yield)</th>
<th>Accumulated Error</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAF</td>
<td>6.72</td>
<td>2</td>
<td>6.72</td>
<td>1</td>
</tr>
<tr>
<td>OW</td>
<td>6.67</td>
<td>1</td>
<td>6.74</td>
<td>2</td>
</tr>
<tr>
<td>MAFM</td>
<td>6.92</td>
<td>3</td>
<td>7.05</td>
<td>3</td>
</tr>
<tr>
<td>Masing</td>
<td>8.04</td>
<td>4</td>
<td>8.04</td>
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</tr>
<tr>
<td>MAFT</td>
<td>8.05</td>
<td>5</td>
<td>8.27</td>
<td>5</td>
</tr>
</tbody>
</table>

The influence of constant yield on hysteresis loop development was considered by comparing MAFT outputs. At a particular fatigue critical location, the original prediction by the MAFT was 2679 flight hours, however, with the constant yield developed parameters, this changed to 3179 flight hours. The reason for an increase in the predicted life can be explained by with reference to Figure 26 which compares hysteresis loops for each parameter set. Due to the smaller yield in the original parameters, a greater amount of plastic strain is predicted in Figure 26 (a) than Figure 26 (b) due to more load cases being above the cyclic yield, thus predicting greater damage. This again highlights the importance of cyclic yield strength definition when applied to complex load spectra.
5.1.5 Influence of the Search Algorithm

Although the GA has been used in similar optimisation strategies which makes it a very good choice in this study [6-15], the final point of investigation is to consider the influence of the search algorithm on convergence. Much of the previous investigations considered evolutionary algorithms but this section develops further the influence of the search algorithms by considering also a heuristic optimiser.

Figure 26 MAFT hysteresis loop development comparison between the (a) originally developed parameters and (b) parameters developed from a constant yield strength.
Single Objective Simplex Algorithm

The simplex algorithm available in ModeFrontier is that based on [16]. To test the ability of the simplex algorithm at reaching an acceptable convergence, the MAF model was applied using the optimal workflow. The convergence curve obtained using the simplex method is given in Figure 27. The convergence results demonstrate an inability of the algorithm to converge to a lower objective value than reached in the first 38 generations. After the first 38 generations, the next significant fluctuation in parameter values had no further improvement on the accuracy of parameters developed in the first 38 generations, the trend of which is seen with every further major fluctuation, which gives rise to the apparent upward stepping appearance of the converge curve. The shape of the convergence curve suggests that the optimisation requires considerably more generations to be developed before a solution is successfully converged. The optimisation in this case has been stopped prematurely, even before the objective error is in a state of reduction. However, for the purpose of comparison with the GA, the number of generations allowed in the optimisation using the simplex was fixed. Consequently, what this demonstrates is that linear based algorithms are not suitable for this kind of application if a solution is to be found in a reasonable time frame, which in this case was 120 generations.

![Figure 27 The convergence curve obtained from the optimisation using the simplex algorithm](image)
5.1.6 Conclusion

Application of the kinematic and isotropic hardening models to strain-life fatigue improved considerably the fatigue-life prediction accuracy for 21 different spectra. The use of an optimisation strategy ensured that the parameters used to define these models were accurate and unbiased, which allowed for effective comparison. The following major findings are summarised as follows:

- All elastoplastic constitutive models improved the fatigue predictions of 21 different P-3C spectra obtained from different fatigue critical locations on the aircraft. Not only this but the models also reduced the scatter in error between the predicted and experimental lives when compared with the Masing model.

- The MAF model is the most accurate of the kinematic hardening models which was proven using both statistical and deterministic methods of comparing the predicted and experimental fatigue lives. This is a particularly important finding since this model uses the least number of parameters making it considerably easy to implement.

- Closer inspection of the hysteresis loop development for each of the models outputted by simulating one of the tested spectrum, demonstrated quite different hysteresis loop progression. Although the same experimental data was used to develop the parameters, the different formulation of the kinematic hardening models resulted in different combinations of yield strength and backstress. This resulted in slight differences in the formation of the hysteresis loops due to the predictions of varying levels of plastic strains between the models. Consequently, the models predicted slightly different levels of damage for a variety of load sequences and due to the size of the load cases, the accumulated small differences in predicted damage ultimately leads to quite different fatigue life predictions. These findings highlighted the importance of accurately predicting the plastic strain, a finding which should highlight the importance of using asymmetric hysteresis loop shape experimental data in parameter development. Not only this but the parameters should be calculated using the most frequent large loading conditions in the spectrum. If the parameters are adjusted to these load cases, the plastic strain prediction and therefore the
hysteresis loop progression would be significantly improved, leading to more accurate damage calculations. Additionally, the material under investigation should be tested to determine the smallest stress amplitude to cause cyclic yielding. Equipped with this knowledge, the parameter optimisation can be adjusted to ensure that the parameters determined do not produce plastic strains at any amplitude less than the determine minimum. This prevents inaccurate plastic strain development causing shifting of the hysteresis loops.

References


I.5.2 A Modification of the Multiplicative AF Model for Aerospace Aluminium Enhanced Elastoplastic Simulation

(Paper 6)

D. Agius, K. I. Kourousis, C. Wallbrink

(In Review)
A Modification of the Multiplicative AF Model for Aerospace Aluminium Enhanced Elastoplastic Simulation

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Abstract

The multicomponent Armstrong-Frederick multiplicative (MAFM) model, which has demonstrated high simulation accuracy for uniaxial and multiaxial loading conditions for a number of different materials, has been modified to improve the phenomenological modelling of aluminium alloys 7075-T6 and 7050-T7451 under uniaxial constant and variable amplitude loading. To recognise the experimentally observed strain amplitude dependency of mean stress relaxation rate, the coefficient of the linear kinematic backstress was modified from a constant value to a strain amplitude dependent dynamic value. The modification improved the mean stress relaxation capability of the MAFM. Additionally, the hysteresis loop development has been enhanced with further modification to the MAFM to improve the monotonic stress-strain evolution of the initial loading branch of cyclic load cases by separating the kinematic backstress coefficients into two parts: contributions from cyclic and monotonic micro-mechanisms. The monotonic coefficients were allowed to decay with continued cycling, which captured the monotonic to cyclic transition of stress-strain development. Finally, the experimentally observed reversibility of the monotonic stress-
strain evolution is also incorporated through the introduction of decaying strain range memory parameter, which improved variable amplitude hysteresis loop development.

**Keywords**

Cyclic plasticity; kinematic hardening; hysteresis loops; mean stress relaxation; Multiplicative AF model; aerospace aluminium.

1. Introduction

Recent studies (Agius et al., 2017a; Agius et al., 2017b; Agius et al., 2016; Hu and Wallbrink, 2014; Novak et al., 2016) have demonstrated the potential improvements to strain-life fatigue predictions through the application of phenomenological modelling techniques to more accurately incorporate transient cyclic effects. These techniques are a branch of modelling which uses nonlinear kinematic hardening to predict the macroscopic stress and strain, with the most widely employed being those based on the original Armstrong-Frederick model (Armstrong and Frederick, 1966). These constitutive models are favoured for strain-life applications due to their computational efficiency, which is particularly important when considering fatigue life predictions of complex aircraft spectra. When applying constitutive models to strain-life predictions, it was shown by Agius et al. (2017a) that the hysteresis loop development is very important in accurate prediction of fatigue life using the strain-life approach; the combination of the applied isotropic hardening and kinematic hardening coefficients had a considerable influence on hysteresis loop predictions and simulation accuracy of both strain ratcheting and mean stress relaxation. Therefore, accurate simulation of not only the mean stress relaxation but also the hysteresis loop development is particularly important for strain-life applications.
One of the interesting features of the elastoplastic behaviour of many materials is the stress-strain progression in the initial loading branch compared to the stress-strain behaviour in the cyclic regime, which produces a noticeable difference in shape as indicated in Figure 1. In the constitutive modelling of materials, it is typically assumed the monotonic to cyclic transition is irreversible; therefore, two sets of coefficients are used to simulate the monotonic and cyclic behaviour. This was addressed by Wang et al. (2000) where in simulations of AA 7050-T7451 elastoplasticity, separate kinematic hardening coefficients were used for the monotonic and cyclic regimes, a concept which was further utilised in (Agius et al., 2017b; Hassan et al., 2008). Recently, Zhu et al. (2015), Zhu and Poh (2016), and Zhu et al. (2017) have managed to simulate both regimes without the need of a parameter switch, instead introduced a dynamic nonlinear kinematic hardening coefficient.

In an investigation conducted by Yu et al. (2014) using a variable amplitude load case applied to an aluminium alloy, the monotonic behaviour was reactivated by an overload (a larger strain amplitude), with stress-strain evolution almost identical to the initial monotonic loading branch. The peculiarity of the results is in the re-emergence of the monotonic deformation behaviour even after the formation of a very different dislocation microstructure. These results are further supported by the work conducted by Arcari and Dowling (2012); although it was not specifically reported, re-emergence of the monotonic behaviour is visible in variable amplitude hysteresis loop data in (Arcari and Dowling, 2012). These are examples of how the monotonic to cyclic stress-strain evolution is reversible and can appear later in variable amplitude loading.

The difference in backstress evolution between the initial monotonic loading and the cyclic loading could be explained with reference to the micro-mechanisms occurring during each stage of loading. Differences in dislocation density during the first few cycles compared with the dislocation density at a later stage of loading could result in a changing evolution of backstress. Huang et al. (2008) observed this difference in dislocation density at different stages of loading, and showed that the dislocation density eventually stabilised after a number of cycles. It has been previously identified that two types of effects give rise to backstresses in polycrystals, which are intergranular and intragranular effects(Sauzay, 2008).
Intragranular effects refer to backstress formed as a result of dislocation interactions which exist as a consequence of the formation of a dislocation microstructure of regions of high and low dislocation densities forming cell walls and cell interiors respectively (Muhammad et al., 2017). Intergranular effects are those which occur as a consequence of grain interaction (due to differing crystallographic orientations) or interactions between phases of alternate strength and/or stiffnesses (Wollmershauser et al., 2012). Since intragranular backstress is influenced by the dislocation density, before the onset of dislocation density stabilisation noticed by Huang et al. (2008), it is possible that there exists a difference in backstress evolution between the cyclic and monotonic regimes.

Furthermore, the micro-mechanism contribution to the re-emergence of the monotonic stress-strain evolution in a variable amplitude loading condition noticed by Yu et al. (2014) must also be considered. An explanation to this re-emergence could be in the influence of the dislocation structure formed during the benign cyclic loading. During the first few cycles of the benign load case, the material would have hardened as a consequence of dislocation interaction, particularly due to internal stresses formed from anchored dislocations within crystals (Hussain and De Los Rios, 1993). Therefore, when the overload occurs, an increased stress is required to overcome these dislocation interactions to initiate dislocation movement, as well as to nucleate dislocations, leading to the monotonic stress-strain evolution.

The re-emergence of the monotonic behaviour and the difference in stress-strain evolution between the monotonic and cyclic loading branches cannot be ignored in constitutive modelling of variable amplitude loading due to the influence the monotonic stress-strain evolution has on post load conditions. Additionally, it has previously been shown that mean stress relaxation is also dependent on the tensile plastic strain history (Wu et al., 2016); therefore, it is particularly important the re-emergence of the monotonic behaviour is considered to ensure accurate mean stress relaxation in variable amplitude loading.
Another feature of elastoplastic behaviour which affects stress-strain evolution and requires consideration in the development of constitutive models is the mean stress relaxation dependence on strain amplitude. For low strain amplitudes, the rate of mean stress relaxation has been identified to be considerably slower and at some applied amplitudes, saturates rather than completely relaxing. This phenomenon has been identified to occur in aluminium alloys, such as 7075-T6 (Agius et al., 2017a; Arcari et al., 2009), 7475-T651 (Arcari et al., 2009), 7050-T7451 (Hu et al., 1999), 2124-T851 (Hao et al., 2015), and 6060 (Hopperstad et al., 1995) but also in other materials such as steels (Landgraf and Chernenkoff, 1988a; Wu et al., 2016). The mean stress relaxation dependence was further recognised by Landgraf and Chernenkoff (1988b) in the development of their exponential model to predict mean stress values. The success of this model was further investigated by Arcari et al. (2009), where it was shown to provide good agreement with experimental mean stress relaxation data.

A number of different phenomenological constitutive models have been proposed over the last several decades as reviewed by Chaboche (2008). Although modifications to phenomenological models have primarily been focused on improving simulation accuracy of strain ratcheting, more recently, the need for improvements to the mean stress relaxation capabilities have drawn increased focus (Arcari and Dowling, 2012; Becker and Hackenberg, 2011; Brommesson et al., 2016; Wang et al., 2000; Wu et al., 2016). Limited modifications to the constitutive models have been made to capture the phenomenological behaviour of aluminium alloys (Arcari and Dowling, 2012; Wang et al., 2000) However, the modifications to these models do not take into consideration the beforementioned combination of elastoplastic behaviour experimentally investigated by several researchers.
In light of the elastoplastic behaviours presented and the need for a constitutive model to simulate these features accurately, this article evaluates the modification of a nonlinear kinematic hardening rule, specifically the multi-component Armstrong-Frederick multiplicative (MAFM) hardening rule (Dafalias et al., 2008), to accurately simulate the backstress evolution in the cyclic regime, while recognising the backstress evolution in the initial monotonic loading. Additionally, the modification incorporates the reversibility of the monotonic behaviour, allowing for its re-emergence in variable amplitude loading. Finally, this article also evaluates the modification of the hardening rule to incorporate the mean stress relaxation dependence on strain amplitude. The modification of the constitutive model improves the simulation of the phenomenological behaviour of two aluminium alloys to provide enhanced simulation accuracy of mean stress relaxation and hysteresis loop development in both constant and variable amplitude loading.

**Figure 1** Experimental asymmetric strain controlled first half cycle demonstrating the difference in shape between the monotonic and cyclic regions (experimental data from Agius et al. (2017a))
2. New Constitutive Model
The new constitutive model incorporates the following:

- A Kinematic hardening rule which contains dynamic coefficients to recognise the contribution of the monotonic micro-mechanisms, as well as the strain amplitude.
- A shrinking strain amplitude memory which allows for the reactivation of kinematic dynamic coefficients at strain amplitudes larger than previously encountered.

2.1 Development and Mathematical Formulation
2.1.1 Main Equations
Classical plasticity theory assumes that yield is independent of the hydrostatic stress. Therefore, the yield surface in terms of the deviatoric stress and using von Mises yield criterion can be described by,

\[ f = \frac{3}{2} (\sigma' - X): (\sigma' - X) \] 

\[ - R - \sigma_{yield} = 0 \]  

where \( \sigma' \) is the deviatoric stress, \( X \) is the centre of the yield surface, \( R \) representing the isotropic hardening, and \( \sigma_{yield} \) is the uniaxial yield stress.

The increment in plastic strain is defined according to the normality rule, where an increment of plastic strain is normal to the yield surface, as defined by,

\[ d\varepsilon^p = d\lambda \frac{\partial f}{\partial \sigma} \]  

(2)

Where \( d\lambda \) is the plastic multiplier, and \( d\varepsilon^p \) is the plastic strain rate.
Isotropic hardening, which controls the size of $R$ of the yield surface is mathematically expressed as,

$$dR = b(R_s - R)dp$$

Where $b$ and $R_s$ control the evaluation rate and saturation level of the yield surface $R$, and $dp$ is the equivalent plastic strain.

2.1.2 Monotonic Micro-Mechanism Equations

The first modification to the constitutive model is in the improved recognition of the monotonic micro-mechanisms in kinematic hardening parameter definition. The inequality in Eq 4 is used to determine the influence of the newly encountered strain amplitude compared to the maximum strain amplitude encountered. This is the memory modification to the constitutive model and allows for the re-emergence of the monotonic stress-strain evolution. Eq 4 ensures that if the newly encountered strain amplitude is large enough, the monotonic stress-strain evolution is reintroduced.

$$\delta_{max} - |\varepsilon_a| < 0$$

Where $\varepsilon_a$ is the current strain amplitude and $\delta_{max}$ is the previous maximum encountered strain amplitude, which slowly reduces with half cycles according to Eq 5.

$$\delta_{max} = \nu(|\varepsilon_a| \exp(-\mu N_{cyc})) + \delta_{min}$$

Where $\nu$ and $\mu$ are coefficients used to develop the size and rate of decay of the calculated maximum strain amplitude, $N_{cyc}$ is the number of half cycles which have not met the inequality in Eq 4, which is a value of 1 when the inequality is met, and $\delta_{min}$ is the value of the strain amplitude which does not meet the inequality in Eq 4. The inclusion of $\delta_{min}$ is used to ensure that the value of $\delta_{max}$ does not decrease lower than amplitudes which do not reintroduce the monotonic micro-mechanisms. The coefficients $\nu$ and $\mu$ dictate the number of cycles which will pass before the reactivation of the monotonic behaviour. Eq 5 prevents the premature activation of the monotonic stress-strain evolution by memorising the largest encountered strain amplitude but also allowing for decay to recognise the fading memory of the material with benign loading, as experimentally observed in (Yu et al., 2014). The strain range memory and modification presented is analogous to the plastic strain range parameter
introduced by Chaboche et al. (1979) and Nouailhas et al. (1983) to incorporate the dependency of the isotropic hardening on plastic strain range, while the fading memory of $\delta_{\text{max}}$ is analogous to the modification made to the strain range parameter by McDowell (1985).

If the inequality in Eq 4 is met, the monotonic micro-mechanism contribution is reduced according to the value of $\zeta$ which is a dimensionless term defined by,

$$\zeta = \eta \exp(-\tau N_{\text{cyc}})$$

Where $\eta$ and $\tau$ are coefficients used to develop the size and rate of influence of the contribution of the monotonic coefficients. Eq 6 recognises the difference in stress-strain evolution between the monotonic and cyclic regimes. As the value of $\zeta$ decreases, the stress-strain evolution becomes increasing more ‘cyclic’ in appearance.

The schematic representation of Eqs 4-6 is given in Figure 2, which provides a visual representation of the interaction of the developed equations (Figure 2 (a) ) and how the loading conditions interact with these equations (Figure 2(b)).

![Figure 2](image)

**Figure 2** A schematic representation of the Eqs 4-6 in terms of (a) the equation interactions and (b) the loading interaction, which provides a better understanding of the connection of influence on these equations on the development of the coefficients which control the monotonic micro-mechanisms.

The nonlinear kinematic hardening model implemented in this study is the multicomponent Armstrong-Frederick multiplicative (MAFM) model (Dafalias et al., 2008), which has been
previously shown to successfully simulate strain-controlled elastoplasticity of AA7050-T7451 (Kourousis and Dafalias, 2013), and has been proven to have simulation robustness through successful application to multiaxial elastoplastic simulations (Feigenbaum et al., 2012). The particular formulation of the MAFM model is the one which also contains a linear kinematic hardening rule modification introduced in (Agius et al., 2017b). The modification to include the monotonic micro-mechanism contribution is demonstrated in Eqs 8-11 where the $\zeta$ addition to the equations adjusts the contribution of the coefficients.

Eq 7 gives the total backstress $X$, which controls the translating yield surface, composed of four backstress equations,

$$X = X_1 + X_2 + X_3 + X_4$$ (7)

The first two backstress equations are formulated according to the Armstrong-Frederick model (Armstrong and Frederick, 1966),

$$dX_i = (c_i^{cy} + \zeta c_i^{mono}) \left( (a_i^{cy} + \zeta a_i^{mono}) d\varepsilon^p - X_i dp \right) \quad i = 1, 2$$ (8)

Where $c_i^{cy}$ and $c_i^{mono}$ are material parameters which dictate the rate at which the backstress saturates, which can be decomposed into cyclic and monotonic contributions respectively. $a_i^{cy}$ and $a_i^{mono}$ are also material parameters which dictate the point of backstress saturation, which can also be decomposed into cyclic and monotonic contributions respectively.

As proposed in (Agius et al., 2017b), the third backstress progresses linearly but now includes contributions from both cyclic and monotonic coefficients as defined by,

$$dX_3 = (C_3^{cy} + \zeta C_3^{mono}) d\varepsilon^p$$ (9)

Where $C_3^{cy}$ and $C_3^{mono}$ control the evolution pace of the linear backstress for both cyclic and monotonic contributions respectively.
The fourth backstress contains the multiplier,

\[ dX_4 = \left( [c_4^{cyc} + \zeta c_4^{mono}] + c_4^{cyc} (a_4^{cyc} - X_4^*) \right) \right) \left( \left( a_4^{cyc} + \zeta a_4^{mono} \right) d\varepsilon^p - X_4^* dp \right) \]  

(10)

Where \( c_4^{cyc} \) and \( a_4^{cyc} \) control the rate of saturation and level of saturation of the multiplicative backstress \( X_4^* \) respectively.

The multiplicative backstress \( X_4^* \) is a dimensionless backstress which progresses through,

\[ dX_4^* = c_4^{cyc} \left( a_4^{cyc} d\varepsilon^p - X_4^* dp \right) \]  

(11)

### 2.1.3 Strain Amplitude Dependent Equations

Another modification to the constitutive model is in the introduction of a strain amplitude dependent coefficient in the linear backstress rule, to recognise the noticed variation of relaxation rate for different strain amplitude. This methodology was applied due to the linear backstress coefficient as it was found by Agius et al. (2017b) that the linear backstress dictates the rate at which plastic shakedown occurs and since mean stress relaxation and strain ratcheting are caused by the same micro-mechanisms (Chaboche et al., 2012; Jhansale and Topper, 1971; Lee et al., 2014; Wu et al., 2016), it was hypothesised that the linear backstress could be used to manipulate the saturation of relaxation. This was done using Eq 12, which alters the linear coefficient depending on the strain amplitude, and is a decaying exponential to recognise the need of the linear coefficient to reduce in size for large strain amplitudes. As discussed by Zhang and Jiang (2005), dislocation substructures are dependent on the strain amplitude, therefore, backstress formed as a result of dislocation interactions will depend on the strain amplitude, which leads to the incorporation of a strain amplitude dependent backstress coefficient. This method of using the strain amplitude to adjust coefficients is analogous to the adjustment of the Ahmadzadeh-Varvani (A-V) hardening rule coefficients by (Ahmadzadeh and Varvani-Farahani, 2013), which was done according to stress amplitude to improve ratcheting simulations.
\[ C_3 = C_3^{**} \exp(-C_3^{*,rate}|\varepsilon_a|) \]  

(12)

Where \( C_3^{**} \) and \( C_3^{*,rate} \) are dimensionless coefficients which control the size of the linear backstress coefficient \( C_3 \).

The modifications made in the development of the new constitutive model is schematically summarised in Figure 3. The isotropic hardening component remains unchanged, while the kinematic hardening is altered in the parameter definition to introduce the monotonic contribution through the application of monotonic parameters \( (a_i^{mono}, c_i^{mono}) \). Further modification is introduced through the application of \( \zeta \) which dictates the contribution of the monotonic coefficients. Additionally, the strain range memory \( \delta_{max} \) allows for the re-emergence of the monotonic stress-strain evolution in variable amplitude loading.

**Figure 3** schematically demonstrates the modification to the constitutive model made in this study, where green represents the locations of modification, while blue represents unmodified components of the constitutive model.

2.2 Model Calibration and Material Parameter Determination

The Parameter determination process used to calculate the parameters used in the new formulation of the MAFM model is summarised in Figure 4 to provide further clarity to the
parameter identification process. Each of the headings in the parameter identification blocks in Figure 4 corresponds to a heading in this section where the process is expanding in more detail.

**MAFM Kinematic Hardening**

### Cyclic Coefficients
Calculated using the process identified in Dafalias, Kououis and Saridis (2008)

\[ a_1^{cyc}, c_1^{cyc}, c_2^{cyc}, a_4^{cyc}, c_3^{cyc}, C_4^{cyc}, c_4^{cyc}, a_4^{cyc}, c_4^{cyc} \]

### Monotonic Coefficients
Using the cyclic coefficients, the monotonic coefficients are added and adjusted to improve the monotonic loading branch

\[ a_1^{mono}, c_1^{mono}, a_2^{mono}, c_2^{mono}, C_3^{mono}, C_4^{mono}, a_4^{mono} \]

**Dynamic Linear Backstress (Monotonic) \([C_3^{mono}, C_3^{rate}]\)**
- Vary values of \(C_3^{mono, rate}\) to improve monotonic stress-strain evolution for three different loading branch strain amplitudes.
- Fit \(C_3^{mono} \exp(-C_3^{rate} |\varepsilon_a|)\) to the determined \(C_3^{mono}\) value with its corresponding \(\varepsilon_a\).

**Dynamic Linear Backstress (Cyclic) \([C_3^{cyc, rate}]\)**
- Vary values of \(C_3^{cyc}\) to improve mean stress relaxation predictions for three strain amplitudes.
- Fit \(C_3^{cyc} \exp(-C_3^{rate} |\varepsilon_a|)\) to the determined \(C_3^{cyc}\) value with its corresponding \(\varepsilon_a\).

**Monotonic Micro-Mechanism Contribution (\(\zeta\))**
- Adjustment of these coefficients should be performed to improve the transition from monotonic to cyclic stress-strain evolution

\[ \eta, \tau \]

**Strain Range Memory (\(\delta_{max}\))**
- Use variable amplitude data containing overloads.
- Adjust \(v, \mu\) to reactivate the monotonic stress-strain evolution at the correct overload.

**Figure 4** A summary of the material parameter calculation process.

### 2.2.1 Cyclic Coefficients

The MAFM parameters should be determined using the parameter identification process outlined in Dafalias et al. (2008), which will give the following coefficients:

\[ a_1^{cyc}, a_2^{cyc}, C_3^{cyc}, a_4^{cyc}, c_1^{cyc}, c_2^{cyc}, c_3^{cyc}, c_4^{cyc}, a_4^{cyc}, c_4^{cyc} \]. These coefficients should be calculated...
without the application of the monotonic coefficients or Eqs 4-6. Coefficients required for the simulation of AA7075-T6 are given in Table 1 (case 1) and the corresponding simulation results of the first loading and unloading condition compared with experimental data points in Figure 5. The results give an indication of level of undershoot which can occur due to the application of an inaccurate plastic modulus for the initial loading branch. This is the consequence of the defined parameter identification process in (Dafalias et al., 2008) being based on cyclic stress-strain data. Therefore, the backstress must be modified to also recognise the difference in plastic modulus between cyclic and monotonic loading.

**Table 1** Coefficients calculated during the parameter identification process which includes case 1 where only the cyclic coefficients are determined without the inclusion of the monotonic coefficients and Eqs 4-6, case 2 which includes the identification of both the monotonic and cyclic coefficients without the application of Eqs 4-6, and case 3 which includes the identification of monotonic and cyclic coefficients, as well as the those coefficients in Eqs 4-6.

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</table>
2.2.2 Monotonic Coefficients

![Simulated loading and loading branch using only cyclic coefficients](image)

**Figure 5** Simulated loading and loading branch using only cyclic coefficients

Modification to the simulation occurs by adjusting the values of $a_{1\text{mono}}, a_{2\text{mono}}, C_{3\text{mono}}, a_{4\text{mono}}, c_{1\text{mono}}, c_{2\text{mono}}, c_{4\text{mono}}$ to improve the initial loading branch. This is required to improve the backstress evolution to recognise the difference in evolution between cyclic and monotonic loading. The largest $a_{i\text{mono}}$ value should correspond to the cyclic developed backstress which contains the lowest $c_{i\text{cy}}$ value, since the backstress requires an improved saturation rate to meet the significantly faster rate of backstress saturation in the experimental results. The parameters developed accordingly are given in Table 1 (case 2), with the simulation results compared in Figure 6. The results show that there is a significant improvement in the loading branch which corresponds to the monotonic loading behaviour. However, the side effect is the cyclic backstress in the unloading branch is affected, resulting in an overshoot in stress compared to the cyclically developed coefficients and the experimental data.
2.2.3 Monotonic Micro-Mechanism Contribution ($\zeta$)

To improve the results and ensure the backstress can accurately simulate both behaviours the parameter $\zeta$ is added. The addition of $\zeta$ recognises that the micro-mechanisms which occur in the monotonic phase slowly reduce with cycles. Doing so will improve the simulations in the cyclic regime but importantly does not ignore the contribution of the monotonic micro-mechanisms. To successfully determine the coefficients of $\zeta$, asymmetric stain-controlled data of one complete cycle is required. The values of $\eta$ and $\tau$ are then adjusted to fit the varying plastic modulus through the first cycle. The parameters and corresponding simulation results are given in Table 1 (case 3) and Figure 7 respectively.
Figure 7 Loading and unloading branch comparison of experimental and simulated results using only cyclic coefficients (Simulated (cyclic)), cyclic and monotonic coefficients (Simulated (cyclic & monotonic)), cyclic, monotonic coefficients including $\zeta$ (Simulated (cyclic, monotonic & $\zeta$)).

With the inclusion of $\zeta$, slight adjustments to the monotonic and cyclic coefficients were required. The simulation results demonstrate a significant improvement to the simulation accuracy of both the monotonic and cyclic regimes.

2.2.4 Dynamic Linear Backstress

The next step is to formulate the coefficients in Eq 12, to incorporate the changing linear backstress coefficient to recognise variations in strain amplitude. The coefficients in Eq 12 can be calculated using asymmetric strain-controlled data, which includes hysteresis loop and mean stress relaxation data. At least three different strain amplitudes are required to successfully formulate these values, which must be for both the monotonic and cyclic loading regimes. Therefore, the selected asymmetric strain-controlled data used for parameter calibration must also have different maximum strains so the monotonic values can be successfully determined. A value of $C_3$ can be determined for each strain amplitude in both
the monotonic and cyclic regime to begin developing the values for which to fit Eq 12. This is achieved by keeping all other parameters constant and only varying values of $C_3$ since this value controls the rate of relaxation but will also influence the shape of the initial monotonic loading branch.

Firstly, the $C_3$ values were selected to achieve good monotonic fits of the asymmetric strain-controlled data. Once these values were selected, the values of $C_3$ were then selected to provide a good fit to experimental mean stress relaxation. The monotonic determined $C_3$ values and cyclic values of $C_3$ were separately fit using Eq 12 to give those in Table 2.

**Table 2** Values for the development of $c_3$ for different strain amplitude for both the monotonic and cyclic regimes.

<table>
<thead>
<tr>
<th>Values</th>
<th>Monotonic</th>
<th>Cyclic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_3^{**}$</td>
<td>6e+6</td>
<td>4e+9</td>
</tr>
<tr>
<td>$C_3^{rate}$</td>
<td>961</td>
<td>1471</td>
</tr>
</tbody>
</table>

### 2.2.5 Strain Range Memory ($\delta_{max}$)

The final coefficients to calculate are $\nu$ and $\mu$ which control the decay of the maximum recorded strain amplitude $\delta_{max}$ in Eq 5. To calculate these values, variable amplitude data is required. For this, the experimental results obtained by Yu et al. (2014) were used, more specifically, the results obtained for the loading condition given in Figure 8. This variable load case was chosen since it includes two positive overloads in close proximity which allows for improved determination of the rate of decay parameter $\mu$. The load case also includes a negative overload occurring directly after an initial positive overload which allows for a more accurate determination of $\nu$. The experimental results did not show a monotonic loading condition during the negative overload occurring directly after the positive overload which suggests that the value of $\nu$ should be large enough to ensure the monotonic coefficients are not reactivated during the negative overload but should also be altered in conjunction with $\mu$ to ensure that each positive overload (cycle B) reactivates the monotonic coefficients.
Figure 8 Variable amplitude load sequence used in the calibration of $\nu$ and $\mu$ in Eq 5.

The final determined coefficients for the two aluminium alloys 7075-T6 and 7050-T7451 used in simulations are given in Table 3.
Table 3 Calculated coefficients for two aluminium alloys 7075-T6 and 7050-T7451 used in simulations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>7075-T6</th>
<th>7050-T7451</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress $\sigma_{yield}$ (MPa)</td>
<td>350</td>
<td>200</td>
</tr>
<tr>
<td>Elasticity modulus E (MPa)</td>
<td>69000</td>
<td>69000</td>
</tr>
<tr>
<td>$a_1^{cyc}$, $a_2^{cyc}$, $a_4^{cyc}$, $a_1^{mono}$, $a_2^{mono}$, $a_4^{mono}$ (MPa)</td>
<td>116,54,50,148,3,1</td>
<td>100,49,85,161,1,1</td>
</tr>
<tr>
<td>$c_1^{cyc}$, $c_2^{cyc}$, $c_4^{cyc}$, $c_1^{mono}$, $c_2^{mono}$, $c_4^{mono}$</td>
<td>620,82,100,11004,21082,21</td>
<td>8,850,800,221004,71082,21</td>
</tr>
<tr>
<td>$a_4^{cyc\ast}$</td>
<td>100</td>
<td>0.6</td>
</tr>
<tr>
<td>$c_4^{cyc\ast}$</td>
<td>1</td>
<td>5000</td>
</tr>
<tr>
<td>$R_s$ (MPa)</td>
<td>35</td>
<td>71</td>
</tr>
<tr>
<td>$b$</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>$\eta$</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$\tau$</td>
<td>5</td>
<td>6.7</td>
</tr>
<tr>
<td>$C_3^{cyc\ast}$, $C_3^{mono\ast}$</td>
<td>4e+9, 6e+6</td>
<td>59898,2591</td>
</tr>
<tr>
<td>$C_3^{cyc\ast}$, $C_3^{rate}$, $C_3^{rate}$</td>
<td>1471, 961</td>
<td>138.1,0</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>

3 Results and Discussion

To validate the success of the proposed constitutive model, constant and variable amplitude symmetric strain and asymmetric strain controlled experimental data collected from Agius et al. (2017a), Yu et al. (2014) and Hu et al. (1999) were used as the basis for comparison. The simulations were conducted using the coefficients listed in Table 3. All simulations were performed using the algorithm outlined in (Hu et al., 1999), which employs the Newton method to solve the nonlinear equations.
3.1 Experimental and Simulated Comparison of AA7075-T6 (Constant Amplitude Loading)

The experimental results are compared to simulations using the proposed model and simulations using the original MAFM model with the optimised parameters developed in (Agius et al., 2015). These parameters were chosen to form a comparison with the proposed model since they were shown to be successful at achieving accurate mean stress relaxation results across a variety of load cases.

The symmetric strain-controlled simulation capabilities of the two models were compared with experimental data in Figure 9. The hysteresis loop development and transition from the cyclic elastic to the cyclic plastic regimes are considerably smoother in the simulation results obtained by the proposed model. The overpredictions in stress by the original MAFM model is indicative of coefficient formulation incapable of simulating both the monotonic and cyclic features.

![Figure 9](image)

**Figure 9** AA7075-T6 symmetric strain controlled experimental and simulated first cycle hysteresis loops for loading conditions (a) $\varepsilon_a = 1.5\%, \varepsilon_m = 0$ (b) $\varepsilon_a = 1.8\%, \varepsilon_m = 0\%$.

Mean stress relaxation simulation capabilities of the two models are compared in Figure 10. The results indicate a significant improvement in the simulation of mean stress relaxation with the proposed model demonstrating an enhancement in the prediction in amount of mean stress relaxed during each load case.
Figure 10 7075-T6 experimental and simulated mean stress relaxation comparison for four different loading conditions.

As stressed previously, not only is the relaxation of mean stresses important to accurately simulate but the hysteresis loop shapes are also very important to accurately simulate to improve strain-life fatigue calculations. For the load cases used in mean stress relaxation results in Figure 10, the corresponding hysteresis loop development (first and last cycle) of each model is compared to experimental data in Figure 11, Figure 12, Figure 13, Figure 14, and Figure 15. The results show a considerable improvement in hysteresis loop shape simulation for both the first and last cycles of loading.
Figure 11 AA7075-T6 experimental and simulated hysteresis loops for (a) first (b) last cycle of an asymmetric strain-controlled tests ($\varepsilon_a = 0.65\%$, $\varepsilon_m = 0.5\%$).

Figure 12 AA7075-T6 experimental and simulated hysteresis loops for (a) first (b) last cycle of an asymmetric strain-controlled tests ($\varepsilon_a = 0.7\%$, $\varepsilon_m = 0.9\%$).
Figure 13 AA7075-T6 experimental and simulated hysteresis loops for (a) first (b) last cycle of an asymmetric strain-controlled tests ($\varepsilon_a = 0.8\%, \varepsilon_m = 0.75\%$).

Figure 14 AA7075-T6 experimental and simulated hysteresis loops for (a) first (b) last cycle of an asymmetric strain-controlled tests ($\varepsilon_a = 0.8\%, \varepsilon_m = 0.65\%$).
Figure 15 AA7075-T6 experimental and simulated hysteresis loops for (a) first (b) last cycle of an asymmetric strain-controlled tests ($\varepsilon_a = 1\%, \varepsilon_m = 0.9\%$).

3.2 Experimental and Simulated Comparison of AA7075-T6 (Variable Amplitude Loading)

Experimental results obtained from variable amplitude load tests conducted by Yu et al. (2014) were used to compare the simulation accuracy of the two models. The two variable loading conditions L-03 and L-04 used are demonstrated in Figure 16 (a) and (b) respectively, with the overload cycles labelled ‘A’ and ‘B’.

Figure 16 Variable loading conditions used in simulations labelled (a) L-03 and (b) L-04.
A comparison of the experimental and simulated results obtained from the L-03 loading condition are given in Figure 17. From Figure 17 (b) and (d) it is immediately obvious that the proposed model can successfully simulate the reactivation of the monotonic behaviour induced by the overloads in the spectrum. As expected, the original MAFM model is incapable of recognising the re-emergence of the monotonic behaviour. Additionally, the simulation accuracy of cycles before and after the overload (Figure 17 (a) and (c) respectively) have been significantly improved with the proposed model.

**Figure 17** AA7075-T6 experimental and simulation comparison using variable experimental amplitude data from Yu, et al. [4] (a) one cycle before Cycle A (b) cycle A (c) one cycle after cycle A (d) cycle B.

The improvement is further highlighted in simulations of the L-04 loading condition in Figure 18. The slight monotonic behaviour introduced by the ‘A’ overload causes a change in the
appearance of the hysteresis loop in Figure 18 (a) which, unlike the proposed model, the unmodified MAFM model cannot simulate, resulting in an overprediction in the tensile stress. Similar to the L-03 results, simulation of the hysteresis loops of cycles before and after the overload (Figure 18 (b), (c), and (d)) are significantly improved by the proposed model.

![Hysteresis Loops Comparison](image)

**Figure 18** AA7075-T6 experimental and simulation comparison using experimental variable amplitude data from Yu et al. (2014) (a) Cycle A (b) One cycle after cycle A (c) three cycles after cycle A (d) one cycle before cycle B.

### 3.3 Experimental and Simulated Comparison of AA7050-T7451

To further determine the accuracy of the simulated results using the proposed new formulation of the MAFM model, an alternate aluminium alloy, 7050-T7451, was used. The experimental results used to compare with the simulated data was those originally gathered...
by Hu et al. (1999), which involved a load spectrum composed of segments of different strain ranges. This is summarised in Figure 19, where each strain range is labelled from Step 1-5. The accuracy of the proposed model was compared against the simulations produced using the original MAFM model with parameters determined in (Kourousis and Dafalias, 2013) which was used to simulate the same experimental AA7050-T7451 data. The proposed model parameters used in simulations are listed in Table 3.

**Figure 19** Coupon testing conducted by Hu et al. (1999) demonstrating the variation in strain range in one loading case.

The simulation accuracy of the hysteresis loop development in this variable amplitude load case has been significantly improved by the proposed model as evidenced in Figure 20. Both the monotonic and cyclic regimes in Figure 20 (a) have been improved by the proposed model compared to the simulations achieved by the original MAFM. Further improvements are noticed in Figure 20 (b), (c), and (d) in both the shape and position of the hysteresis loops.
Figure 20 Experimental from Hu et al. (1999) and simulation results using the proposed modified MAFM and the original MAFM for the first cycle of each strain range (a) Step 1 (b) Step 2 (c) Step 3 (d) Step 4.

The mean stress relaxation results of the models compared to the experimental results gathered by Hu et al. (1999) are shown in Figure 21. These results demonstrate that the proposed model is not only capable of improving mean stress relaxation simulations of constant amplitude load cases but also accurately simulating the variation in mean stresses in variable amplitude loading. There is a significant improvement in mean stress simulation accuracy using the proposed model compared to the original MAFM model, particularly in steps 2 and 3 where the original MAFM predicts complete relaxation of the mean stresses in these steps, which is contradictory to the proposed model and the experimental results.
Figure 21 Comparison of the experimental from Hu et al. (1999) and simulated results of mean stress relaxation for each step.

4. Conclusion
The extensive simulation and experimental comparisons made for two different aluminium alloys using both constant and variable amplitude load cases have highlighted the significant improvement to simulation accuracy of the MAFM model through the proposed modification of the model. The findings of the analysis include:

- The inclusion of the linear backstress and the modification to the linear backstress coefficient to recognise the variation in mean stress relaxation for different strain amplitudes has improved the mean stress relaxation simulation accuracy of the MAFM model in both constant and variable amplitude load cases in two aluminium alloys. Additionally, the improvement in mean stress relaxation has also enhanced the accuracy of the hysteresis loop development. This was particularly evident in the
AA7050-T7451 simulations of Step 4 where the location of the hysteresis loop is considerably more accurate when compared to the original MAFM results. This ensures that the stresses in variable amplitude loading are not underpredicted, which will significantly improve the calculated notch stress and strains.

- Modification of the MAFM coefficients to include a decaying contribution of coefficients used to simulate the initial monotonic loading branch has resulted in a significant improvement in hysteresis loop shape accuracy in both AA7075-T6 and AA7050-T7451. The results highlight the importance of a backstress evolution which can transition from monotonic to cyclic stress-strain development, rather than applying kinematic hardening coefficients determined using cyclic data, which assumes the cyclic backstress evolution is independent of the initial monotonic loading.

- The final modification to the MAFM model came in the form of a memory parameter. The maximum strain amplitude encountered was stored and allowed to decay with cycles to allow for the re-emergence of the monotonic loading behaviour from an overload in a variable amplitude load case, as identified by Yu et al. (2014). The incorporation of this modification improved the stress-strain development of the overload cycles, which prevents an overprediction of stress, and improves the calculation of notch stress-strain in strain-life fatigue predictions.

**Acknowledgement**

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Chapter I.6  Part I Summary and Future Work

6.1  Summary

The research conducted in Part I of this thesis has dealt with the three main areas associated with AA7075-T6: uniaxial experimental investigation, elastoplastic constitutive modelling with parameter optimisation, and improvements of strain-life fatigue predictions. The in-depth understanding of the elastoplastic behaviour of AA 7075-T6 in symmetric strain-control, as well as asymmetric strain/stress-controlled loading developed in Chapter I.2 was particularly important for several reasons. Firstly, the asymmetric stress-controlled behaviour of the material was yet to be investigated, the knowledge of which is important in understanding micro-mechanism influence on strain ratcheting accumulation to discern the severity of strain accumulation in the material. Furthermore, this material knowledge is particularly important for isotropic and kinematic hardening parameter development. In Chapter I.3, the need for a modification of the elastoplastic constitutive models was required in order to accurately simulate the strain ratcheting shakedown noticed in experimental results. In Chapter I.4 a parameter optimisation workflow for strain-life application was developed through a sensitivity analysis. Finally, in Chapter I.5, the developed elastoplastic constitutive models and corresponding parameters obtained from the sensitivity analysis was applied to strain-life predictions which showed an improvement in life prediction through application of elastoplastic constitutive models capable of incorporating transient cyclic effects. Based on the findings in Chapter I.5.1, a modification to the multiplicative Armstrong-Frederick model was suggested in Chapter I.5.2, which was shown to improve the symmetric and asymmetric hysteresis loop development. The major conclusions of Part I of the thesis are summarised as follows:

- An in depth understanding of macroscopic behaviour of AA7075-T6 was successfully linked to the extensive micro-mechanism analysis of the material which occurred of the past 40 years. The first asymmetric stress-controlled analysis of the material was also conducted, where a plastic shakedown of the strain ratcheting occurred. The
ability of the micro-mechanism evolution to halt the accumulation of ratcheting strain is a favourable material feature since ratcheting strain is detrimental to the fatigue life of a structure/component. An initial cyclic softening was also noticed for first time in AA7075-T6, which occurred during the low peak stress tests which was hypothesised to be the consequence of the initial monotonic loading not nucleating many dislocations resulting in a lower dislocation interaction energy. A closer inspection of the evolution of the effective stress and backstress in asymmetric strain-controlled results showed that a difference in the micro-mechanism occurring in tension and compression contributed to the slowing down of the relaxation rate, which was dependent on the applied strain amplitude.

- In order to accurately simulate the plastic shakedown of the material, the MAFM model was altered with the addition of a linear backstress term. This was further shown to be necessary to improve plastic shakedown simulations of the MAF, MAFT, and OW simulations. Initial multi-objective optimisations using the MAFM model showed the potential of using a parameter optimisation software to develop the constitutive model parameters. This was further extended by applying a sensitivity analysis to determine the optimal optimisation workflow for the development of the parameters of elastoplastic constitutive models to be applied to strain-life fatigue predictions. This included development of the genetic algorithm settings and the type of experimental data.

- The MAF model is the most accurate of the kinematic hardening models which was proven using both statistical and deterministic methods of comparing the predicted and experimental fatigue lives calculated from the application of P-3C spectra. This is a particularly important finding since this model uses the least number of parameters making it considerably easy to implement. However, all the applied new elastoplastic constitutive models improved the fatigue predictions compared to the traditional Masing model. Furthermore, hysteresis loop progression outputted from spectra simulations emphasised the importance of accurately predicting the plastic
strain, a finding which should highlight the importance of using asymmetric hysteresis loop shape experimental data in parameter development.

6.2 Recommendations for Future Work

The work conducted in Part 1 of the thesis has uncovered some important issues which need further investigation. These are listed as follows:

- **Notch stress-strain comparison**
  Development of an accurate means of measuring notch stress and strains of a spectrum loaded coupon to directly compare the simulated hysteresis loop development with experimental data.

- **Maximum load assessment**
  Determine whether there is an increase in the accuracy of fatigue predictions by using the most frequent load cases in the spectrum for parameter development. These load cases can replace the two stress-controlled load cases in the developed workflow. This is aimed at ensuring the parameters are developed to provide adequate tracking of the most damaging hysteresis loops in the spectrum.

- **Application to other materials**
  Application of this newly developed parameter optimisation workflow to other materials to determine the potential improvements to strain-life fatigue predictions for alternate materials and spectra.
Part II  Experimental Characterisation and Modelling of Additively Manufactured Ti-6Al-4V
Chapter II.1  Part II Background and Literature Review

(Paper 7)

D. Agius, K.I. Kourousis, C. Wallbrink

(In Review)
An Examination of the As-built SLM Ti-6Al-4V Mechanical Properties towards Achieving Fatigue Resistant Designs

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Abstract

Ti-6Al-4V has been widely used in the both the biomedical and aerospace industries, due to the mechanical properties of high strength, high corrosion resistance and fracture toughness, and light weight. Additive manufacturing (AM) of components made from Ti-6Al-4V is an attractive method of manufacturing, as it provides a low waste alternative for the manufacture of complex geometries. One AM fabrication method is selective laser melting (SLM) which is a layer based disposition method using a laser to selectively melt successive layers of powder in an inert gas filled chamber. As the technology progresses, the ability to manipulate the fabrication process in situ enhances the material engineering possibilities to design microstructure to contain combinations of particular crystallographic phases to achieve suitable mechanical properties for different loading conditions. With continued progress made in the SLM technology, the influence of build layers, grain boundaries and defects can be combined to enhance further the design process and to allow fabrication of components with improved strength in critical loading directions and fatigue performance. To initiate this possibility the mechanical properties including monotonic, low and high cycle fatigue and
fracture mechanical behaviour of machined as-built SLM Ti-6Al-4V have been reviewed to inform the research community. The corresponding crystallographic phases, defects and layer orientations have been analysed to determine the influence of these features on the mechanical behaviour, to provide further understanding of how these features can be manipulated and utilised to improve the fatigue resistance of components fabricated from Ti-6Al-4V using SLM.

Keywords
Selective laser melting; Ti-6Al-4V; additive manufacturing; mechanical properties; fatigue; fracture.

1. Introduction
Additive manufacturing (AM) is a promising new technology that can significantly alter how components for many different industries are manufactured, with the potential to drastically improve manufacturing efficiency. It also provides a means of manufacturing geometrically complex structures which would otherwise require a considerable amount of time and money to manufacture with other methods. However, one the exciting aspects of this evolving technology is the ability to manufacture material with microstructure having a wide range of crystallographic phases, previously only possible in traditionally manufacture methods through post thermos-mechanical process (Murr et al., 2009). Furthermore, taking advantage of the intrinsic heat treatment (IHT) of the AM fabrication process, which is associated with cyclic reheating of layers through continued layer disposition, there is the potential to induce precipitation growth in situ (Kürnsteiner et al., 2017), which offers this technology the potential for material inclusion engineering for material properties’ enhancement. This application of the AM technology therefore stresses the need to develop processing parameters for the manufacture of material to be used in as-built conditions.
A considerable amount of research has been conducted on the qualification of parts manufactured using AM technologies, with a focus on laser additive manufacturing [e.g. selective laser melting (SLM)] for titanium alloy Ti-6Al-4V. The areas of primary research have been in the identification of the manufacturing aspects which have an influence on the quality and the microstructure of the manufactured part/structure. With this ever-growing database of vital information on the contribution of these different fabrication aspects on the quality and microstructure of the fabricated material, further research has continued in the vital area of improving our knowledge on the influence of the possible various crystallographic phases, residual stresses, layer orientation, and defects on the mechanical properties of the material (monotonic properties, low and high cycle fatigue properties, and fracture mechanical behaviour). This information is important to gain understanding on which microstructural characteristics are required to improve the fatigue resistance of the material, such as the combination of phases, orientation of grains and build layers, and orientation and the types of defects. Moreover, this knowledge can be used to manufacture components with tailored microstructures which are capable to improve the fatigue resistance in critical areas (Morton et al., 2015) by applying these properties to enhance crack nucleation time or crack retardation capabilities of the component in the most critical loading directions.

This paper reviews the various Ti-6Al-4V microstructures obtained using SLM and the resulting monotonic, low and high cycle fatigue and fracture mechanical behaviour of machined as-built SLM Ti-6Al-4V. The corresponding microstructure, processing parameters, porosity level, build strategy and knowledge of whether it was a single or multiple build are also reported since this information is vital for the repeatability of the fabrication process and the obtaining of consistent mechanical properties. The properties presented are discussed with respect to the micro-mechanism which may contribute to the results obtained, providing a more in-depth understanding of how the build orientation, defects, and microstructure can be utilised by selecting the most appropriate crystallographic phases, layer and build orientation in the manufacture of fatigue resistant Ti-6Al-4V.
2. Microstructure of Ti-6Al-4V Manufactured by Selective Laser Melting (SLM)

2.1 $\alpha$ Microstructure

As indicated by Vrancken et al. (2012b), during the SLM process, the microstructure development is dependent on the processing parameters, including the layer thickness, scan strategy, scan spacing and speed and laser power. A widely reported microstructure of SLM Ti-6Al-4V is an acicular $\alpha'$ martensite in columnar prior-$\beta$ grains process [e.g. (Chen et al., 2017; Qiu et al., 2013; Shi et al., 2017; Simonelli et al., 2014a; Xu et al., 2017)]. The development of this microstructure is commonly attributed to the processing parameters during manufacture, resulting in a cooling rate greater than 410 K s$^{-1}$ (Ahmed and Rack, 1998) above the martensite start temperature, which promotes $\alpha'$ martensite growth (Al-Bermani et al., 2010). The prior $\beta$ grain boundaries are elongated in the build direction due to the heat conduction.

The type of $\alpha'$ martensite present can be combinations of primary $\alpha'$ martensite, secondary $\alpha'$ martensite, tertiary $\alpha'$ martensite, and quartic $\alpha'$ martensite, which vary in size from 1-3$\mu$m of primary $\alpha'$ martensite to less than 20nm for quartic $\alpha'$ martensite. It was shown by Yang et al. (2016b) that the presence and amount of the different sized $\alpha'$ martensite depends on the influence of thermal cycles caused by the reheating of deposited layers. The reheating aiding in the $\alpha'$ martensite transformation, is represented in the graph of Figure 1, where $T_L$ is the liquidus temperature, $T_L-T_S$ is the solidus temperature, $T_S-T_\beta$ is the $\beta$ transus temperature and $T_\beta-M_S$ is martensite transformation temperature. As discussed by Yang et al. (2016b), in the initial cycle, upon cooling from the liquidus state, the fast cooling rates result in the microstructure being primarily composed of acicular primary $\alpha'$ martensite and prior $\beta$ grains. In the second cycle, the formation of secondary $\alpha'$ martensite occurs from residual $\beta$ phase at peak temperatures. In the fourth cycle, some previously developed $\alpha'$ martensites decompose into metastable $\beta$ phase which upon further cooling can form ternary $\alpha'$ martensites. Finally, in the fifth cycle, the majority of the formed $\alpha'$ martensite remains, however, a very small amount may transform into $\beta$ which upon cooling can form the fine $\alpha'$ martensite characteristic of quartic $\alpha'$ martensite.
There is also the possibility of the microstructure containing a combination of lamellar and acicular $\alpha/\alpha'$, which was shown by Yang et al. (2016a) to be caused by the melt pool geometry increasing the deposited layer thermal cycles promoting diffusive transformation of $\beta \rightarrow \alpha$.

2.2  $(\alpha + \beta)$ Microstructure

Improved ductility in Ti-6Al-4V, without sacrificing too severely the yield strength of the material, can be achieved having a microstructure which contains lamellar $(\alpha + \beta)$ (Lütjering, 1998). In-situ transformation of the $\alpha'$ martensite to lamellar $(\alpha + \beta)$ has been previously achieved by Xu et al. (2015a) and Xu et al. (2015b) by altering the focal offset distance (FOD) and energy density (E) in a way as to promote intrinsic heat treatment (IHT). Simonelli et al. (2014b) demonstrated the potential to achieve varying $\alpha+\beta$ microstructures by utilising the cyclic reheating associated with layer disposition, in conjunction with the build platform temperature, and thermal stresses. A combination of lamellar $(\alpha+\beta)$, equiaxed $(\alpha+\beta)$, bimodal lamellar $(\alpha+\beta)$ and equiaxed $\alpha$ was noticed through the height of fabricated specimens, which recognises the varying influence of build platform temperature and thermal stresses on microstructure development. This was further supported by the work of Ali et al. (2017), in an investigation of the effects of powder bed preheating, which highlighted the
significant influence that the powder bed temperature has on SLM Ti-6Al-4V microstructure development. In particular, preheating the powder bed to different temperatures resulted in slowing of the cooling rate, promoting $\alpha'$ decomposition into $\alpha$ and the growth of $\beta$ between $\alpha$ laths. Although it is possible to achieve such a microstructure from an initial $\alpha'$ martensite microstructure using post heat treatment or hot isostatic heating (HIP), as previously reported by (Qiu et al., 2013; ter Haar et al., 2016; Vilaro et al., 2011; Vrancken et al., 2012a), the advantage of in-situ microstructure tailoring is important for a number of reasons, including manufacturing cost and time reduction, as well as tailoring of the microstructure (e.g. in order to improve performance characteristics of the component).

Further improvements to the in-situ transformation of $\alpha'$ martensite was developed by Xu et al. (2017) due to the difficulty associated with the FOD adjustment and the high microstructure sensitivity to FOD. Transformation was achieved by employing a scanning strategy which uses the residual heating of new layers on previous layers to maintain the temperature profile to ensure $\alpha'$ martensite transformation into $\alpha + \beta$. This provided the correct environment for the development of different microstructure, which included a combination of $\alpha' + (\alpha + \beta)$, ultrafine lamellar ($\alpha + \beta$) and coarse lamellar ($\alpha + \beta$). Depending on the support structure, layer thickness and part dimension, varying $\alpha$ lath width can be achieved. In conjunction with this work, Barriobero-Vila et al. (2017) proposed a laser scanning strategy which exploited the benefits of longer IHT exposure times to ensure $\alpha'$ martensite decomposition into $\alpha + \beta$ by using porosity-optimised processing parameters achieved by Kasperovich et al. (2016) and a tight hatch distance. Both studies further demonstrate the growing microstructure design control methods and importance of localised cyclic reheating to build the foundation for tailor making particular microstructure. With the progression of the technology and continued understanding of the SLM processing ability, the significant advantages of the technology are now becoming more apparent. Rather than attempting to manufacture Ti-6Al-4V with comparable mechanical properties to that of the wrought Ti-6Al-4V, the technology is on the cusp of being able to offer tailored microstructures providing optimised mechanical properties.
2.3 Microstructures Examined

The main Ti-6Al-4V microstructures discussed in this paper are:

- Bimodal (duplex) microstructure, composed of lamellar (α+β) colonies and interconnected equiaxed primary α.
- Lamellar (α+β) microstructure, composed of α lamellae within β grains.
- Equiaxed (globular) microstructure, consisting of equiaxed primary α with β along the grain boundaries.
- Acicular α′ martensite.

3. Build Defects and Residual Stresses

3.1 Build Defects and the Influence of Gas Flow

The occurrence of different defects in additively manufactured metals has been studied by a number of researchers and has been recently critically reviewed by Grasso and Colosimo (2017). The formation of spherical pores is a commonly occurring defect, which was indicated by Vilaro et al. (2011) to be caused by gas between powder particles dissolving in the melt pool, which remains in the material due to rapid solidification. Spherical defects have also been shown to be the consequence of processing parameters, for example, Gong et al. (2013) showed that the processing parameters have a significant influence on the size and density of defects due to the changes in the melt pool characteristics. In particular, Gong et al. (2013) found that at higher energy density, the defects were spherically shaped and were the consequence of mechanically scraping ejected solidified particles. Irregular shaped defects are usually the consequence of lack of fusion/melting. As discussed by Vilaro et al. (2011), this is caused by the incorrect processing parameters which leads to insufficient reheating of the previous layer preventing optimal fusion between the layers. Processing parameter influence has also been reported by Panwisawas et al. (2015) who explored the different ways that spherical pores can become elongated through increased laser scan speed. Furthermore, Gong et al. (2013) showed that at low energy densities, the defects are more irregularly shaped due to insufficient layer melting. The occurrence of such defects was
Further supported by Kasperovich and Hausmann (2015) who noticed elongated voids perpendicular to the build direction, occurring from insufficient energy density.

The SLM manufacturing process requires an inert atmosphere with a gas designed to flow across the build platform. This gas flow in the chamber can also have an influence on the porosity level of the manufactured components. It has previously been reported by Kong et al. (2011) and Ferrar et al. (2012) that the gas flow is not uniform across the build chamber, which effectively causes a build-plate location dependence. Inadequate gas flow can cause a reduction in the removal of vaporised powder (condensate) produced by the melting process. As reported by Ferrar et al. (2012), the low gas flow regions correspond to a variation in the intensity, spot diameter, and energy of the laser beam as a consequence of the presence of the condensate. This leads to a reduction in the effectiveness of the originally defined processing parameters on successive layer melting and fusion, potentially introducing lack of fusion defects. In addition, it was reported by Ladewig et al. (2016) that the layer thickness can be affected by the inefficient removal of by-products (condensate, ejected powder and spatter) occurring during the melting process, which increases the potential of lack of fusion defects. Ladewig et al. (2016) also linked areas of lack of fusion defects with regions where the effects of low gas flow rate induced laser beam and by-product interaction had occurred.

3.2 Residual Stresses

Residual stresses of the most significant importance are those associated with a variation across the whole part (referred as type I residual stresses), rather than on the atomic level (Mercelis and Kruth, 2006). As indicated by Mercelis and Kruth (2006) and Knowles et al. (2012), these residual stresses develop during the build process due to the expansion and contraction interaction between layers, and have been measured in a number of SLM Ti-6Al-4V builds, e.g. as reported by Leuders et al. (2013) and Simonelli et al. (2014a). Nickel et al. (2001) and Shiomi et al. (2004) demonstrated that the development of the residual stress is dependent on laser scan, while Klingbeil et al. (1998) showed that is also geometry dependent. Furthermore, it is also important to note that residual stress can vary across the thickness of the built part, as measured by Casavola et al. (2009). However, it is becoming more evident that it is increasingly possible to reduce the residual stresses in situ (during the
fabrication process). For example, a residual stress reduction scheme has been recently presented by Ali et al. (2017), whereby it was demonstrated that the residual stresses can be reduced by increasing the preheat temperature of the powder bed (reducing effectively the thermal gradient). Furthermore, it was shown by Masoomi et al. (2017), through numerical models simulating heat transfer, that employing optimised scanning strategies can reduce the build-up of residual stress.

4. Influence of the Microstructure, Residual Stress and Porosity on Mechanical Properties

4.1 Laser Scanning Strategy and Build Orientation of Typical Test Coupons

The laser scanning strategy used in the fabrication of SLM-produced parts and mechanical test coupons varies between researchers. Commonly reported strategies in the literature are illustrated in Figure 2.

![Image of laser scanning strategies]

**Figure 2** Laser scanning strategies commonly reported in the literature.
Moreover, the tensile and fatigue test coupons used in axial tests vary in build orientation and type (flat or cylindrical). Each of the build orientations used in coupon fabrication with a corresponding reference label to indicate the direction are summarised graphically in Figure 3.

**Figure 3** Flat and cylindrical tensile and fatigue test coupons used in axial tests with corresponding reference labels indicating the build orientation.

Compact-tensile (CT) test coupon build orientations commonly used by researchers are also summarised in Figure 4. The corresponding build direction is also indicated in the figure which corresponds to the Z direction.

**Figure 4** Compact-tensile (CT) coupon build orientations and corresponding build orientation reference label.
4.2  Monotonic Properties

4.2.1 Behaviour Under Tension

The monotonic properties of various as-built SLM Ti-6Al-4V microstructures have been extensively analysed at different test coupon build orientations, as reported recently by Lewandowski and Seifi (2016) and Tong et al. (2017). The microstructures investigated include (summarised literature presented):

- $\alpha'$ martensite (Cain et al., 2015; Facchini et al., 2010; Gong et al., 2015; Kasperovich and Hausmann, 2015; Leuders et al., 2014; Leuders et al., 2013; Mertens et al., 2014; Murr et al., 2009; Qiu et al., 2013; Rafi et al., 2013a; Simonelli et al., 2014a; Vilaro et al., 2011; Vrancken et al., 2012a);
- Coarse lamellar ($\alpha+\beta$) (Simonelli et al., 2014b; Xu et al., 2017)
- Ultrafine lamellar ($\alpha+\beta$) (Xu et al., 2015a; Xu et al., 2015b)

The yield stress and elongation is plotted in Figure 5 to provide a better visualisation of the difference in the results obtained from varying microstructure, build orientation and porosity, which are summarised in Table 1 (for each of the test results shown Figure 5). Moreover, the processing parameters that have been used for the fabrication of the test coupons are listed in Table 2. The improvement in elongation achieved with a lamellar ($\alpha+\beta$) is evidenced in the results obtained by Xu et al. (2017) (Xu 1) and Xu et al. (2015a) (Xu 2), presented in Figure 5. What is also observed in Figure 5 is the significant scatter in the results obtained from $\alpha'$ martensite coupons, which suggests that defects may have a profound influence on the mechanical properties.
Figure 5 Comparison of the yield strength and elongation of SLM Ti-6Al-4V coupons of different microstructure, build orientation, and porosity levels obtained from the literature listed in Table 1 (circle points represent an α’ martensite coupons and square points represent a lamellar (α+β) microstructure).
**Table 1** Material features of the coupons used in monotonic tensile tests (summarised literature).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compared to standard density of Ti-6Al-4V 4.43g/cm³)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cain Kasperovich</td>
<td>Cain et al. (2015)</td>
<td>Flat XY, ZX, Cylindrical Z</td>
<td>α' martensite</td>
<td>&lt;1 0.077</td>
<td>None None</td>
</tr>
<tr>
<td>Leuders</td>
<td>Leuders et al. (2014)</td>
<td>Cylindrical Z</td>
<td>α' martensite</td>
<td>NR</td>
<td>None None</td>
</tr>
<tr>
<td>Simonelli</td>
<td>Simonelli et al. (2014a)</td>
<td>Flat XY, ZX, XZ, Z</td>
<td>α' martensite</td>
<td>NR</td>
<td>None None</td>
</tr>
<tr>
<td>Facchini</td>
<td>Facchini et al. (2010)</td>
<td>Flat Z</td>
<td>α' martensite</td>
<td>0.3</td>
<td>None None</td>
</tr>
<tr>
<td>Gong 1</td>
<td>Gong et al. (2015)</td>
<td>Cylindrical Z</td>
<td>α' martensite</td>
<td>0.45 1.37 5.23 1.37 5.48 &lt;1</td>
<td>None None</td>
</tr>
<tr>
<td>Gong 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gong 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gong 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gong 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vilaro</td>
<td>Vilaro et al. (2011)</td>
<td>Flat (X or Y), Z</td>
<td>α' martensite</td>
<td>&lt;0.5</td>
<td>None None</td>
</tr>
<tr>
<td>Mertens</td>
<td>Mertens et al. (2014)</td>
<td>Flat XZY</td>
<td>α' martensite</td>
<td>&lt;0.5</td>
<td>None None</td>
</tr>
<tr>
<td>Hollander</td>
<td>Hollander et al. (2006)</td>
<td>Cylindrical (X or Y)</td>
<td>NR</td>
<td>&lt;0.5</td>
<td>None None</td>
</tr>
<tr>
<td>Qui</td>
<td>Qiu et al. (2013)</td>
<td>Cylindrical Z, (X or Y)</td>
<td>α' martensite</td>
<td>&lt;0.1</td>
<td>None None</td>
</tr>
<tr>
<td>Xu 1</td>
<td>Xu et al. (2017)</td>
<td>Cylindrical Z</td>
<td>Coarse lamellar (α+β)</td>
<td>NR</td>
<td>None None</td>
</tr>
<tr>
<td>Xu 2</td>
<td>Xu et al. (2015a)</td>
<td>Cylindrical Z</td>
<td>Ultrafine lamellar (α+β)</td>
<td>&lt;0.5</td>
<td>None None</td>
</tr>
</tbody>
</table>

NR=Not Recorded
Table 2 SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in monotonic tensile results (summarised literature).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (µm)</th>
<th>Layer thickness (µm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cain 1</td>
<td>Cain et al. (2015)</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>1</td>
<td>Single</td>
</tr>
<tr>
<td>Kasperovich</td>
<td>Kasperovich and Hausmann (2015)</td>
<td>200</td>
<td>1250</td>
<td>NR</td>
<td>40</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Leuders</td>
<td>Leuders et al. (2014)</td>
<td>400</td>
<td>NR</td>
<td>NR</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Simonelli</td>
<td>Simonelli et al. (2014a)</td>
<td>157</td>
<td>225</td>
<td>100</td>
<td>50</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Facchini</td>
<td>Facchini et al. (2010)</td>
<td>195</td>
<td>225</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 1</td>
<td>Gong et al. (2015)</td>
<td>120</td>
<td>960</td>
<td>540</td>
<td>40</td>
<td>1260</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 2</td>
<td></td>
<td>160</td>
<td>100</td>
<td>600</td>
<td>200</td>
<td>120</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 3</td>
<td></td>
<td>175</td>
<td>710</td>
<td>120</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 4</td>
<td></td>
<td>175</td>
<td>710</td>
<td>120</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 5</td>
<td></td>
<td>175</td>
<td>710</td>
<td>120</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Vilaro</td>
<td>Vilaro et al. (2011)</td>
<td>160</td>
<td>100</td>
<td>600</td>
<td>200</td>
<td>120</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Mertens</td>
<td>Mertens et al. (2014)</td>
<td>175</td>
<td>710</td>
<td>120</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Hollander</td>
<td>Hollander et al. (2006)</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Qui</td>
<td>Qiu et al. (2013)</td>
<td>150-200</td>
<td>800-1500</td>
<td>75</td>
<td>20</td>
<td>NR</td>
<td>Chessboard</td>
<td>Multiple</td>
</tr>
<tr>
<td>Xu 1</td>
<td>Xu et al. (2017)</td>
<td>175-375</td>
<td>800-1500</td>
<td>75</td>
<td>20</td>
<td>NR</td>
<td>Chessboard</td>
<td>Multiple</td>
</tr>
<tr>
<td>Xu 2</td>
<td>Xu et al. (2015a)</td>
<td>175-375</td>
<td>800-1500</td>
<td>75</td>
<td>20</td>
<td>NR</td>
<td>Chessboard</td>
<td>Multiple</td>
</tr>
</tbody>
</table>

NR=Not Recorded
4.2.2 Behaviour Under Torsion

The monotonic torsion behaviour of vertically (Z axis) fabricated SLM Ti-6Al-4V has been recently reported by Fatemi et al. (2017b), with as-built, as-built annealed and machined annealed material being examined. The results of the machined coupons are listed in Table 3, with the microstructure characteristics and processing parameters summarised in Table 3 and Table 4 respectively. The SLM material results are compared against the wrought bimodal Ti-6Al-4V results. In the Fatemi et al. (2017b) it was reported that the yield strength of the SLM coupons was higher than that of the wrought material, while the ductility of the SLM coupons was less, analogous to results reported for monotonic tension tests obtained from $\alpha'$ martensite SLM coupons.

Table 3 Monotonic torsion results for SLM Ti-6Al-4V obtained from (Fatemi et al., 2017b).

<table>
<thead>
<tr>
<th>Coupon</th>
<th>Shear Modulus (GPa)</th>
<th>Shear Yield Strength (MPa)</th>
<th>Shear Ultimate Strength (MPa)</th>
<th>Shear Fracture Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrought</td>
<td>42</td>
<td>523</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SLM</td>
<td>42.5</td>
<td>634</td>
<td>699</td>
<td>4.56</td>
</tr>
</tbody>
</table>

Table 4 Material features of the coupons used in monotonic torsion tests conducted by (Fatemi et al., 2017b).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compare to standard density of Ti-6Al-4V 4.43 g/cm³)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi et al. (2017b)</td>
<td>Tubular coupon, z</td>
<td>NR</td>
<td>NR</td>
<td>Annealed at 700°C for 1 hour</td>
</tr>
</tbody>
</table>

NR=Not Recorded

Table 5 SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in monotonic torsion test conducted by (Fatemi et al., 2017b).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (µm)</th>
<th>Layer thickness (µm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi et al. (2017b)</td>
<td>400</td>
<td>1000</td>
<td>160</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>NR</td>
</tr>
</tbody>
</table>

NR=Not Recorded
4.2.3 Micro-Mechanism Contribution

As discussed by Simonelli et al. (2014a), the higher yield stress in monotonic tensile tests in the $\alpha'$ martensite microstructures and ultrafine lamellar ($\alpha+\beta$) and the monotonic torsion tests $\alpha'$ martensite microstructure may be attributed to the smaller $\alpha$ colony size which slows down the onset of plastic deformation, a consequence of a greater hindrance to dislocation movement enforced by dislocation tangling at grain boundaries. Furthermore, it was explained by Simonelli et al. (2014a) that the increase in ductility in the lamellar ($\alpha+\beta$) can be attributed to the plasticity of the $\alpha+\beta$ phases; the effective slip length in the $\alpha'$ martensite microstructure is within single grains, while the $\beta$ phase present in the lamellar ($\alpha+\beta$) allows for slip transfer between the two phases. This increases the dislocation movement and therefore the ductility of the material.

The influence of the microstructure on mechanical properties is further complicated with the potential of inclusions, formed during the fabrication process. These inclusions may cause the deformation behaviour of an $\alpha'$ martensite Ti-6Al-4V microstructure developed using SLM to be different from a similar acicular $\alpha'$ martensite Ti-6Al-4V microstructure formed from classical heat treatment and rapid cooling. The inclusions form at the melt pool surface and include hard-alpha ($\alpha$), amorphous CaO, and microcrystalline Al$_2$O$_3$ as discussed by Huang et al. (2016). The influence of these inclusions on the miro-mechanical behaviour of the material requires further investigation. Rather than a detriment to the fabrication process, as indicated by Hennig et al. (2005), the phase transformation and stability controlling potential of impurities could be harnessed to favourably modify the mechanical properties.

The influence of build orientation is also an issue requiring closer attention. The anisotropy in monotonic yield strength is the consequence of a number of different microstructural characteristics. The $\alpha$ phase present in the test coupons contributes to the yield strength anisotropy, since $\alpha$ titanium is plastically anisotropic (Banerjee and Williams, 2013), therefore, orientation of the grains to a preferred slip system will promote dislocation movement. This is further supported by Yang et al. (2017) who indicated that vertically built coupons contain a larger number of grains in a stress state which are easier to slip than horizontal coupons.
It is also important to recognise the influence of defects on anisotropy, since it was suggested by Gong et al. (2015) that defects can influence the yield strength of the material depending on orientation of the defects to the loading direction, which may explain the anisotropy in yield strength between the orientations.

The presence of defects can also explain the difference in ductility between the build orientations. As suggest by Vilaro et al. (2011), defects associated with lack-of-fusion are aligned with the layers; therefore, the defects will have a greater influence on the vertically built material due to the loading pulling apart the defects, compared to the horizontally built material where the loading is pulling it closed. Due to the variability in the number of defects in builds between studies, the influence of defects on ductility may offer an explanation as to why some studies report lower ductility in vertically built coupons such as that reported by Vilaro et al. (2011), while others, such as Qiu et al. (2013), report higher ductility. Moreover, the possible reason for some studies reporting similar ductility in both vertically and horizontally manufactured coupons, such as Rafi et al. (2013a), could be the consequence of a balance existing between the influence of slip surfaces and defects.

4.3 Fracture Behaviour

Comparison of the fatigue crack growth rate (FCGR) \( (\frac{da}{dN}, \text{where } a \text{ is the crack length and } N \text{ the number of cycles}) \) in the Paris-region between results obtained from published literature from machined as-built coupons, tested using \( R=0.1 \), show very similar trends, noticeable from the results summarised in Figure 6. These results were obtained from test coupons having the microstructure characteristics presented in Table 6, fabricated using the processing parameters of Table 7. It was reported by Cain et al. (2015) that the XZ and ZX coupons had very similar FCGR. The XZ orientation corresponds to the crack growth occurring perpendicular to the build layers but parallel to the prior columnar \( \beta \) grains. The ZX orientation corresponds to crack growth parallel to the build layers but perpendicular to the prior columnar \( \beta \) grains. This was contradictory to what was reported by Leuders et al. (2013) who noticed an increased crack growth resistance in the ZX orientation when compared to that of XZ. The highest crack growth resistant orientation reported by both Cain
et al. (2015) and Van Hooreweder et al. (2012) was the XY orientation, which corresponds to crack growth occurring perpendicular to both the build layers and prior columnar β grains.

Figure 6 Comparison of the fatigue crack growth rate (FCGR) data in the Paris-region for various results obtained from the literature.

Table 6 material features of the coupons used in crack propagation experiments (summarised literature).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compared to standard density of Ti-6Al-4V 4.43g/cm³)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Hooreweder</td>
<td>Van Hooreweder et al. (2012)</td>
<td>XY</td>
<td>α′ martensite</td>
<td>0.27</td>
<td>None</td>
</tr>
<tr>
<td>Leuders 1</td>
<td>Leuders et al. (2013)</td>
<td>ZY</td>
<td>α′ martensite</td>
<td>0.23</td>
<td>None</td>
</tr>
<tr>
<td>Leuders 2</td>
<td>Cain et al. (2015)</td>
<td>ZY</td>
<td>α′ martensite</td>
<td>&lt;1</td>
<td>None</td>
</tr>
<tr>
<td>Cain 1</td>
<td>Van Hooreweder et al. (2012)</td>
<td>ZY</td>
<td>α′ martensite</td>
<td>0.27</td>
<td>None</td>
</tr>
<tr>
<td>Cain 2</td>
<td>Leuders et al. (2013)</td>
<td>ZY</td>
<td>α′ martensite</td>
<td>0.23</td>
<td>None</td>
</tr>
<tr>
<td>Cain 3</td>
<td>Edwards and Ramulu (2015)</td>
<td>XY</td>
<td>α′ martensite</td>
<td>NR</td>
<td>None</td>
</tr>
</tbody>
</table>

NR= Not Recorded
Table 7 SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in crack propagation experiments (summarised literature).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (µm)</th>
<th>Layer thickness (µm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Hooreweder</td>
<td>Van Hooreweder et al. (2012)</td>
<td>250</td>
<td>1060</td>
<td>60</td>
<td>30</td>
<td>NR</td>
<td>1</td>
<td>NR</td>
</tr>
<tr>
<td>Leuders 1/</td>
<td>Leuders et al. (2013)</td>
<td>400</td>
<td>NR</td>
<td>NR</td>
<td>30</td>
<td>100</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Leuders 2</td>
<td>Cain et al. (2015)</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>1</td>
<td>Single, Multiple</td>
</tr>
<tr>
<td>Cain 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cain 3</td>
<td>Edwards and Ramulu (2015)</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>Multiple</td>
</tr>
<tr>
<td>Edwards 1/</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edwards 2/</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edwards 3</td>
<td></td>
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<tr>
<td>NR=Not Recorded</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3.1 Micro-Mechanism Contribution

As reported by Yadollahi and Shamsaei (2017), microstructure features, such as the grain size and morphology, can influence the crack growth of the material. Increased grain boundaries due to larger grain size can create resistance to crack propagation, therefore, varying the size of the grains will influence crack growth. However, crack propagation can also be dependent of the build orientation. In the recent work of Galarraga et al. (2017), on Ti-6Al-4V manufactured by electron beam melting (EBM), crack propagation rate parallel to the build direction was lower than that corresponding to the perpendicular build direction. This was attributed to the prior β grain boundaries and the deflection of the crack caused by the scanning layers, which increased the tortuosity of the crack path (reducing the rate of propagation). This phenomenon is likely to explain the difference in FCGR between the Leuders 1 and Leuders 2 (Leuders et al., 2013) results shown in Figure 6, where Leuders 2 (Leuders et al., 2013) has a slower crack growth rate. Moreover, this explanation can be further utilised to understand the low FCGR values observed in Cain 3 (Cain et al., 2015) and Van Hooreweder (Van Hooreweder et al., 2012). However, Cain 1 (Cain et al., 2015) has the same orientation to Leuders 2 (Leuders et al., 2013) but the FCGR in Cain 1 is comparable to Cain 2, which has a grain and layer orientation, which according to Galarraga et al. (2017), should produce higher FCGR values. This may be attributed to the influence
of residual stresses. As reported by Leuders et al. (2013), the two main factors influencing crack growth in SLM Ti-6Al-4V are residual stresses and microstructure. Further analysis of the results obtained by Cain et al. (2015) highlighted the potential occurrence of tensile residual stress near the lateral edges of the fracture plane, which would ultimately add to the tensile loading and, therefore, increasing the rate of crack propagation. However, the Edwards 1,2,3 results (Edwards and Ramulu, 2015) in Figure 6 show very little difference in FCGR between the orientations, which was hypothesised by Edwards and Ramulu (2015) to be the consequence of the influence of residual stresses. Consequently, further progress into the understanding of the influence of grain boundaries and residual stresses has to be made in order to investigate the contribution of these micro-mechanisms on crack growth.

The currently limited crack growth data on AM Ti-6Al-4V has only been concerned with long cracks, however, fatigue cracks can exist as also physically-small cracks, and microstructurally-small cracks (Zhai et al., 2016). The difference lies on their crack length and width (Nalla et al., 2002). A long crack has a crack length and width much larger than the characteristic microstructural size scale. A physically-small crack once again has crack length and width larger than the characteristic microstructural size scale but has a crack length less than the equilibrium shield zone (region which is associated with crack closure effects (Nalla et al., 2002; Zhai et al., 2016)). Microstructurally-small cracks are those which have a crack length and width smaller than the shield zone and characteristic microstructural size scale. The fatigue crack growth behaviour of all these cracks can be different, which is why it is important to develop an understanding of the fatigue crack growth behaviour of all types of cracks in the material to develop a broader understanding of fatigue resistance capabilities of the material, as recognised by Zhai et al. (2016).

Although no investigation has been undertaken into analysing the fracture mechanical behaviour of other possible as-built SLM microstructures, it would be useful to progress our understanding on the potential influence of possible phases present in a SLM Ti-6Al-4V microstructure using knowledge from the wrought counterpart. As suggested by Nalla et al. (2002), crack propagation of microstructurally-small cracks is slower in the fine grained wrought bimodal and equiaxed (α+β) microstructures due to higher grain boundary density.
However, as reported by Tao et al. (2016), acicular α’ microstructures have longer slip paths, which, in turn, promote microstructurally-small crack growth; while coarse lamellar (α+β) microstructures contain aligned α, which aids in microstructurally-small crack propagation due to long planar slip bands. Long cracks in a coarse lamellar (α+β) microstructure was shown by Nalla et al. (2002) to have slower growth rates than a bimodal microstructure, which is the consequence of a more tortuous crack path. One may also expect that long crack growth in acicular α’ martensite microstructure will be faster than a bimodal or lamellar microstructure, since it was reported by Tao et al. (2016) that the crack path of an acicular α’ martensite microstructure is less tortuous than a bimodal microstructure.

4.4 High Cycle Fatigue Behaviour

4.4.1 Axial Fatigue Loading

In high cycle fatigue (HCF) assessment it is considered important to consider the influence of the different features of the manufactured material. In a recent critical review by Li et al. (2016), the HCF performance of various SLM coupons was compared and assessed. Due to the varying stress ratios (R) between tests, the effective maximum applied stress (σ_eff) at a stress ratio R=-1.0 is calculated according to Li et al. (2016), which is given by Eq. 1, where σ_max is the maximum applied stress for an applied stress ratio (R).

\[
σ_{eff} = σ_{max} \left( \frac{1 - R}{2} \right)^{0.28}
\] (1)

In Figure 7, various HCF test results from the literature are compared. It is noted that the comparison is limited to results obtained from machined surface coupons, to negate the influence of surface defects on the fatigue life results.
Figure 7 Axial high cycle fatigue (HCF) performance of SLM Ti-6Al-4V for machined coupons, where cycle points are $\alpha'$ martensite microstructures and square points are lamellar ($\alpha+\beta$) (lines of best fit also included to help with visualisation of results).

In Table 8 the build orientation, microstructure and porosity are summarised for each of the test coupons corresponding to the results shown in Figure 7. Also, the processing parameters used in the manufacturing of the test coupons is summarised in Table 9.
Table 8 Material features of the coupons used in high cycle fatigue (HCF) tests (summarised literature).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compared to standard density of Ti-6Al-4V 4.43g/cm³)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eric 1</td>
<td>Eric et al. (2013)</td>
<td>Cylindrical coupon at 45° to build plate</td>
<td>α’ martensite</td>
<td>0.40</td>
<td>Heat treatment - 650°C for 3 hours</td>
</tr>
<tr>
<td>Gong 1</td>
<td>Gong et al. (2015)</td>
<td>Cylindrical coupon Z</td>
<td>α’ martensite</td>
<td>0.45</td>
<td>None</td>
</tr>
<tr>
<td>Gong 2</td>
<td>Gong et al. (2015)</td>
<td>Cylindrical coupon Z</td>
<td>α’ martensite</td>
<td>1.37</td>
<td>None</td>
</tr>
<tr>
<td>Gong 3</td>
<td>Gong et al. (2015)</td>
<td>Cylindrical coupon Z</td>
<td>α’ martensite</td>
<td>5.23</td>
<td>None</td>
</tr>
<tr>
<td>Gong 4</td>
<td>Gong et al. (2015)</td>
<td>Cylindrical coupon Z</td>
<td>Lamellar (α + β)</td>
<td>1.37</td>
<td>None</td>
</tr>
<tr>
<td>Gong 5</td>
<td>Gong et al. (2015)</td>
<td>Cylindrical coupon, horizontal</td>
<td>Lamellar (α + β)</td>
<td>5.48</td>
<td>None</td>
</tr>
<tr>
<td>Xu 1</td>
<td>Xu et al. (2015b)</td>
<td>Cylindrical coupon, Z</td>
<td>Lamellar (α + β)</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Xu 2</td>
<td>Xu et al. (2015b)</td>
<td>Cylindrical coupon, X</td>
<td>Lamellar (α + β)</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Xu 3</td>
<td>Xu et al. (2015b)</td>
<td>Cylindrical coupon, Z</td>
<td>Lamellar (α + β)</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Rafi 1</td>
<td>Rafi et al. (2013b)</td>
<td>Vertical</td>
<td>α’ martensite</td>
<td>NR</td>
<td>Heat treatment - 650°C for 4 hours</td>
</tr>
<tr>
<td>Edwards 1</td>
<td>Edwards and Ramulu (2014)</td>
<td>Flat coupon, X</td>
<td>α’ martensite</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Edwards 2</td>
<td>Edwards and Ramulu (2014)</td>
<td>Flat coupon, Y</td>
<td>α’ martensite</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Edwards 3</td>
<td>Edwards and Ramulu (2014)</td>
<td>Flat coupon, Z</td>
<td>α’ martensite</td>
<td>NR</td>
<td>None</td>
</tr>
</tbody>
</table>

NR = Not Recorded
Table 9 SLM parameters used in test coupon manufacturing for each version of Ti-6Al-4V used in high cycle fatigue (HCF) tests (summarised literature).

<table>
<thead>
<tr>
<th>Label</th>
<th>Parameter</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (μm)</th>
<th>Layer thickness (μm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eric 1</td>
<td>Eric et al. (2013)</td>
<td>179</td>
<td>1250</td>
<td>100</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
<tr>
<td>Gong 1</td>
<td>Gong et al. (2015)</td>
<td>120</td>
<td>960</td>
<td>400</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 2</td>
<td>Gong et al. (2015)</td>
<td>120</td>
<td>1260</td>
<td>100</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Gong 3</td>
<td>Gong et al. (2015)</td>
<td>120</td>
<td>1500</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
<td></td>
</tr>
<tr>
<td>Gong 4</td>
<td>Gong et al. (2015)</td>
<td>120</td>
<td>30</td>
<td>NR</td>
<td>NR</td>
<td>Multiple</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gong 5</td>
<td>Gong et al. (2015)</td>
<td>120</td>
<td>60</td>
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<td>Multiple</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xu 1</td>
<td>Xu et al. (2015b)</td>
<td>375</td>
<td>180</td>
<td>120</td>
<td>60</td>
<td>NR</td>
<td>Multiple</td>
<td></td>
</tr>
<tr>
<td>Xu 2</td>
<td>Xu et al. (2015b)</td>
<td>375</td>
<td>120</td>
<td>60</td>
<td>NR</td>
<td>Multiple</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xu 3</td>
<td>Xu et al. (2015b)</td>
<td>375</td>
<td>60</td>
<td>NR</td>
<td>Multiple</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rafi 1</td>
<td>Rafi et al. (2013b)</td>
<td>170</td>
<td>1250</td>
<td>100</td>
<td>30</td>
<td>NR</td>
<td>3</td>
<td>NR</td>
</tr>
<tr>
<td>Edwards 1/</td>
<td>Edwards and Ramulu (2014)</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>Multiple</td>
</tr>
<tr>
<td>Edward 2/</td>
<td>Edwards and Ramulu (2014)</td>
<td>200</td>
<td>200</td>
<td>180</td>
<td>50</td>
<td>NR</td>
<td>2</td>
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<td>200</td>
<td>200</td>
<td>180</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>Multiple</td>
</tr>
</tbody>
</table>

4.4.2 Torsional Fatigue Loading
Torsional HCF investigation was undertaken by Fatemi et al. (2017b) of annealed SLM Ti-6Al-4V hypothesised to be composed of an α’ martensite microstructure (material features and processing parameters given in Table 10 and Table 11 respectively). These results were compared to bimodal Ti-6Al-4V in both shear strain and stress control. The results of this analysis are presented in Figure 8. Under both control methods, the wrought material resulted in having higher HCF resistance, as opposed to the SLM Ti-6Al-4V, as indicated by the higher cycles to failure for higher strain and stress amplitudes.
Figure 8 Fully reversed torsional high cycle fatigue (HCF) test results in (a) shear strain-controlled tests (b) shear stress-controlled tests (adopted from Fatemi et al. (2017b)).

Table 10 Material features of the coupons used in torsion high cycle fatigue (HCF) tests (after Fatemi et al. (2017b)).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.% (compared to standard density of Ti-6Al-4V 4.43g/cm³))</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi et al. (2017b)</td>
<td>Tubular coupon, Z</td>
<td>NR NR</td>
<td>NR</td>
<td>Annealed at 700°C for 1 hour</td>
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</tbody>
</table>

NR=Not Recorded

Table 11 SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in the torsional HCF tests performed by Fatemi et al. (2017b).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (µm)</th>
<th>Layer thickness (µm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi et al. (2017b)</td>
<td>400</td>
<td>1000</td>
<td>160</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>NR</td>
</tr>
</tbody>
</table>

NR=Not Recorded
4.4.3 Micro-Mechanism Contribution

Comparing the Gong 1 and Gong 5 results (Gong et al., 2015) in Figure 7, it is noticed that the influence of porosity on HCF is profound, where the higher porosity content of the Gong 5 test coupons show significantly less cycles to failure than the denser Gong 1 coupons. This can be attributed to the pores affecting the rate at which the crack initiation stage activates, with the definition of crack initiation adopted from (Chan, 2010) referring to the formation of a crack of length grain size or less. Prolonging the occurrence of this stage, by reducing the size and/or number of pores, is likely to lead to longer fatigue life. Furthermore, in a two-dimensional analysis conducted on cast Al-Si alloys by Fan et al. (2003), it was demonstrated that the crack incubation was influenced by both the pore size and the space between the pores.

Anisotropic HCF behaviour has been observed in the Edwards 1,2,3 results (Edwards and Ramulu (2014). Referring to the work conducted by Yadollahi et al. (2017), the defects formed between layers due to lack of fusion in the vertical coupon used in Edwards 3 (Edwards and Ramulu (2014) will be more detrimental to crack initiation due to exposure of a larger area of the defect, than Edwards 1 and 2, which explains the difference in HCF life shown in Figure 7. This was an anticipated phenomenon, attributed to the orientation of the defects. Comparison of the Xu 1 and Xu 2 results (Xu et al., 2015b) provide an indication of the impact of microstructure on fatigue performance. In particular, the lamellar (α+β) (Xu 2) test coupons demonstrated greater fatigue resistance than the α’ martensite (Xu 1) coupons. This suggests that although porosity has a considerable effect on the fatigue resistance of the material, the improvement in ductility offered by a lamellar (α+β) microstructure can reduce the influence of the pores on the crack initiation, as proposed by Edwards and Ramulu (2014).

If the presence of voids in the material is small, the microstructure features will influence the crack initiation. The HCF behaviour of the material is dependent on the time spent in crack nucleation and crack growth. As demonstrated by Mall et al. (2004), crack initiation resistance in Ti-6Al-4V is increased with finer and more homogenous microstructures. Fatigue cracks typically nucleate due to irreversible slip in the longest crystallographic slip bands in the microstructure (Nalla et al., 2002). Therefore, a finer microstructure is likely to
contain a lower quantity of long slip bands. Consequently, coarse lamellar (\(\alpha+\beta\)) have less resistance to crack nucleation than fine lamellar (\(\alpha+\beta\)) microstructures due the presence of large slip lengths. The orientation of the grains will also affect crack initiation, especially if crack initiation occurs in slip bands within the grains (Bantounas et al., 2007). This is due to the fact that different build directions have the maximum resolved shear direction at different orientations to planes which contain the easiest and most common slip systems (Yadollahi and Shamsaei, 2017). In a comprehensive analysis of the HCF results of wrought Ti-6Al-4V obtained from 21 different published research results, it was shown by Wu et al. (2013) that a bimodal microstructure will have superior HCF resistance over a lamellar microstructure, followed by a equiaxed microstructure. Analysis of the primary \(\alpha\) content in the bimodal microstructure demonstrated the importance of having a primary \(\alpha\) volume fraction of between 30-50\% and a primary \(\alpha\) size of between 0 and 5\( \mu m\) to achieve the greatest fatigue resistance. Additionally, in order of fatigue resistance, the following microstructures followed:

- The Lamellar microstructure, where it was suggested that an \(\alpha\) lamellae width of less than 1\( \mu m\) could improve HCF resistance;
- The Equiaxed microstructure, with the greatest resistance occurring when the primary \(\alpha\) size was less than 6\( \mu m\).

This comprehensive study Wu et al. (2013) ultimately suggested that a finer grain size will improve the HCF behaviour of the material, which is consistent with the previously presented understanding with respect to improving crack initiation resistance. However, the long crack growth analysis showed a preference to the lamellar microstructure, particularly with respect to coarser lamellae width. Therefore, this suggests that the HCF behaviour of Ti-6Al-4V is weighted towards the time spent in crack nucleation and short crack growth. However, it is particularly important to recognise the influence of defects on crack initiation times. As previously discussed, the volume of defects in SLM fabricated material is significantly higher than wrought material. Consequently, the time spent in crack nucleation would be significantly less in SLM than wrought material due localised slip induced crack initiation due to the high stress concentration at the pores (Holmes and Queeney, 1985). Therefore,
greater weighting is given to crack growth properties of the material when it comes to the HCF resistance, since this is a property which can be used to improve the fatigue resistance of the material. Microstructures which improve crack growth rates will be considerably more valuable than those which improve crack initiation.

4.5 Low Cycle Fatigue Behaviour

4.5.1 Axial Fatigue Loading

The cyclic elastoplastic behaviour of SLM Ti-6Al-4V has recently been investigated by Kourousis et al. (2015), Phaiboonworachat and Kourousis (2016) and Agius et al. (2017). The microstructure features and corresponding fabrication parameters used in these studies are listed in Table 12 and Table 13 respectively. In these studies, the elastoplastic behaviour of α’ martensite SLM Ti-6Al-4V under symmetric strain-control is compared to results obtained from wrought bimodal Ti-6Al-4V. Hysteresis loop development across a number of different strain amplitudes used in the investigation show a much narrower hysteresis loop formed from the micro-mechanisms evolving in the SLM Ti-6Al-4V compared to the wrought Ti-6Al-4V. This suggests that there is significantly more plastic work occurring in the wrought Ti-6Al-4V than the SLM Ti-6Al-4V, which is associated with the higher cyclic yield in the SLM Ti-6Al-4V. Moreover, it was noticed that the stress amplitudes during symmetric strain cycling decreased with cycles in the SLM Ti-6Al-4V, a phenomenon known as cyclic softening.

In an investigation into the mechanical anisotropy associated with build orientations, Agius et al. (2017) conducted symmetric strain-controlled tests on cylindrical coupons manufactured at horizontal, diagonal, and vertical build orientations. Results of the investigation revealed a difference in not only the monotonic tensile behaviour but also the elastoplastic behaviour, with the diagonally manufactured coupon having the largest monotonic tensile and cyclic yield stresses.
In conjunction with this work, elastoplastic constitutive modelling capabilities were investigated by Kourousis et al. (2016), that included the implementation of an advanced model (Dafalias et al., 2008) aiming to simulate accurately the material cyclic phenomena under strain and stress controlled loading histories.

Axial low cycle fatigue (LCF) behaviour of SLM Ti-6Al-4V requires further investigation, however currently very limited understanding exists in this area, despite the engineering significance of these phenomena in design and operation as indicated by Al-Bermani et al. (2010). Despite the lack of understanding of the LCF effects in SLM Ti-6Al-4V, some work has been conducted on the LCF behaviour of other materials additively manufactured, such as that conducted by Gribbin et al. (2016) using Inconel alloy 718, where a build orientation dependence on strain amplitude in symmetric strain-controlled results was observed. Further LCF analysis was conducted by Yadollahi et al. (2017) on 17-4 PH stainless steel using symmetric strain-controlled tests, which showed that crack initiation sites on the LCF fracture surface were smaller than those on the HCF fracture surface. It was suggested by Yadollahi et al. (2017) that the defect distance from the surface was more important than the actual defect size in LCF.

**Table 12** SLM Ti-6Al-4V features of the coupons used in axial elastoplastic investigative tests performed by Kourousis et al. (2016), Phaiboonworachat and Kourousis (2016), and Agius et al. (2017).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compared to standard density of Ti-6Al-4V 4.43g/cm³)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kourousis et al. (2016)</td>
<td>Cylindrical Z</td>
<td>α’ martensite</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Phaiboonworachat and Kourousis (2016)</td>
<td>Cylindrical X,Z, 45°</td>
<td>Coarse and Ultrafine α’ martensite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agius et al. (2017)</td>
<td></td>
<td></td>
<td>NR</td>
<td></td>
</tr>
<tr>
<td>NR=Not Recorded</td>
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<td></td>
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</tr>
</tbody>
</table>
Table 13 SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in axial elastoplastic investigative tests of Kourousis et al. (2016), Phaiboonworachat and Kourousis (2016), and Agius et al. (2017).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (μm)</th>
<th>Layer thickness (μm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kourousis et al. (2016)</td>
<td>175</td>
<td>710</td>
<td>120</td>
<td>30</td>
<td>200</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Phaiboonworachat and Kourousis (2016)</td>
<td>175</td>
<td>710</td>
<td>120</td>
<td>30</td>
<td>200</td>
<td>NR</td>
<td>Multiple</td>
</tr>
<tr>
<td>Agius et al. (2017)</td>
<td>100</td>
<td>375</td>
<td>130</td>
<td>30</td>
<td>200</td>
<td>Stripe pattern</td>
<td>Multiple</td>
</tr>
</tbody>
</table>

NR=Not Recorded

4.5.2 Torsion Fatigue Loading

Similar to the lack of LCF understanding in axial loaded SLM Ti-6Al-4V, LCF of torsion loaded material has been given even less attention. Strain-life results of SLM Ti-6Al-4V reported by Fatemi et al. (2017b) showed that the SLM Ti-6Al-4V had considerably shorter LCF lives than its wrought bimodal counterpart (Figure 9 (a)). However, stress-life results showed the SLM Ti-6Al-4V had a higher shear fatigue strength than the wrought bimodal coupons in LCF (Figure 9 (b)). The reason for the difference in results could be the consequence of the higher associated yield stress in α’ martensite dominated coupon, resulting in the tested stress amplitudes being below the macroscopic yield of the SLM coupons compared to the wrought coupons, which may be in the plastic region. In Table 14, the build orientation, microstructure and porosity are summarised for each of the coupons corresponding to the test results in Figure 7. The processing parameters used in the manufacturing of the test coupons are also summarised in Table 15.
**Figure 9** Fully reversed torsional low cycle fatigue (LCF) test results in (a) shear strain-controlled tests (b) shear stress-controlled tests [data obtained and adapted from Fatemi et al. (2017b) and Fatemi et al. (2017a)].

**Table 14** Material features of the coupons used in torsion low cycle fatigue (LCF) tests [conducted by Fatemi et al. (2017b) and Fatemi et al. (2017a)].

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compared to standard density of Ti-6Al-4V 4.43g/cm³)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi 1</td>
<td>Fatemi et al. (2017b)</td>
<td>Tubular coupon, Z</td>
<td>NR</td>
<td>NR</td>
<td>None</td>
</tr>
<tr>
<td>Fatemi 2</td>
<td>Fatemi et al. (2017a)</td>
<td>α' martensite</td>
<td>0.1-0.6</td>
<td>Heat treatment - 700°C for 1 hour</td>
<td></td>
</tr>
</tbody>
</table>

**Table 15** SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in torsion low cycle fatigue (LCF) tests [conducted by Fatemi et al. (2017b) and Fatemi et al. (2017a)].

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (μm)</th>
<th>Layer thickness (μm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi 1</td>
<td>Fatemi et al. (2017b)</td>
<td>400</td>
<td>1000</td>
<td>160</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>NR</td>
</tr>
<tr>
<td>Fatemi 2</td>
<td>Fatemi et al. (2017a)</td>
<td>400</td>
<td>1000</td>
<td>160</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>NR</td>
</tr>
<tr>
<td>Fatemi 3</td>
<td>Fatemi et al. (2017a)</td>
<td>400</td>
<td>1000</td>
<td>160</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>NR</td>
</tr>
<tr>
<td>NR=Not Recorded</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Elastoplastic investigation of the SLM coupons compared to those obtained from a wrought bimodal microstructure was also undertaken by Fatemi et al. (2017b) at a shear strain amplitude of 2%. The shear stress amplitude progression with cycles compared to the wrought material identified similar softening rates between the two. However, a greater magnitude of softening was observed in the wrought material, due to the premature fracture in the SLM coupon. Comparison of the mid-life hysteresis loops identified a much narrower hysteresis loop developed from the SLM material than the wrought material, which is consistent with axial elastoplastic results obtained in (Agius et al., 2017; Kourousis et al., 2016; Phaiboonworachat and Kourousis, 2016), which is associated with the higher cyclic yield in the tested SLM material.

4.5.3 Micro-Mechanism Contribution

The primary and secondary slip planes for titanium alloys are the prism planes \{10\bar{1}0\} and basal planes \{0001\} respectively (Evans et al., 2005). The majority of deformation at low temperatures occurs by \{10\bar{1}0\}(1\bar{2}10) slip (Kailas et al., 1994). The difference in the elastoplastic behaviour between the tested \(\alpha'\) martensite microstructure compared to other possible Ti-6Al-4V microstructures is anticipated due to differences in the micro-mechanism which govern the plastic behaviour. Gil et al. (2003) compared the elastoplastic and LCF behaviour of wrought bimodal, lamellar, and martensitic microstructures of Ti-6Al-4V. Cyclic softening rates varied between the microstructures due to differences in the micro-mechanisms which cause cyclic softening. In the bimodal and lamellar microstructure, cyclic softening occurred predominantly due to the existence of mobile dislocations, while in the martensite microstructure, mobile dislocations and induced twinning resulted in cyclic softening. Furthermore, Gil et al. (2003) reported that the LCF resistance was higher in the bimodal microstructure, where crack nucleation occurred at \(\alpha\) grains for low strain amplitude and \(\alpha-\beta\) interfaces at high strain amplitudes. The lamellar microstructure demonstrated the second most highest resistance to LCF. The resistance increased with increasing number of colony boundaries and decreasing grain size, which increased the tortuosity of the crack path. The martensitic microstructure demonstrated the least LCF resistance due to crack propagation occurs along martensite plate boundaries, reducing the tortuosity of the crack.
path. A comparison of the LCF and elastoplastic behaviour of a equiaxed and lamellar structure was conducted by Tan et al. (2015) using the titanium alloy TC21. This investigation revealed that crack nucleation at LCF occurred easier in the lamellar microstructure than the equiaxed microstructure.

In order to obtain a better understanding of the influence of defects in LCF of AM material, the Åkerfeldt et al. (2016) study is considered helpful. In this study the influence of defects on the LCF of Ti-6Al-4V coupons manufactured using laser metal wire disposition is analysed. In the LCF tests conducted using strain-control, it was noticed that coupon fracture at lower strain amplitudes was dominated by crack initiation due to pores and lack-of-fusion defects in the material, while at high strain amplitudes, the cracks initiated at the surface. This was the consequence of the stress concentration being higher at the surface of the coupon when subjected to higher strain amplitudes than the stress concentration at defects and vice-versa, when the material was subjected to lower strain amplitudes. Additionally, the horizontally manufactured coupons had better LCF properties. No failure occurred from cracks initiating at lack-of-fusion defects in horizontal coupons, which is a consequence of the defect being located on a plane parallel to the loading direction. This results in the aspect ratio being lower than that of a round pore, causing a lower stress concentration in the vicinity of the defect (Kumar et al., 2016). However, lack-of-fusion defects are considerably more dangerous in vertically manufactured coupons due to the aspect ratio being greater than a round pore, which leads to greater stress concentrations in the vicinity of the lack-of-fusion defect. Consequently, favourably orientated lack-of-fusion defects can be less detrimental to not only HCF life but also LCF life.
4.6 Multiaxial Fatigue Behaviour

A multiaxial fatigue investigation using both proportional and non-proportional loading on vertically fabricated tubular coupons has been recently published by Fatemi et al. (2017a). The coupons have the microstructure characteristics shown in Table 16, manufactured using the process parameters provided in Table 17. The results of the tested SLM Ti-6Al-4V are summarised in Figure 10, where a comparison to a wrought bimodal microstructure is also presented (using von Mises equivalent stress). It is noticed from these results that the multiaxial fatigue resistance of the wrought material is superior to the acicular $\alpha'$ martensite SLM Ti-6Al-4V for both in-phase and out-of-phase loading conditions.

![Figure 10](image)

**Figure 10** Multiaxial axial torsion loading results comparison between SLM Ti-6Al-4V and wrought Ti-6Al-4V for (a) in-phase loading and (b) out-of-phase loading, compared using von Mises equivalent stress, for tests conducted by Fatemi et al. (2017a).

**Table 16** SLM Ti-6Al-4V material features of the coupons used in multiaxial tests conducted by Fatemi et al. (2017a).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Build orientation</th>
<th>Microstructure</th>
<th>Porosity (vol.%) (compared to standard density of Ti-6Al-4V 4.43g/cm$^3$)</th>
<th>Post treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi</td>
<td>Fatemi et al. (2017a)</td>
<td>Tubular coupon, Z</td>
<td>$\alpha'$ martensite</td>
<td>0.1-0.6</td>
<td>Heat treatment - 700°C for 1 hour</td>
</tr>
<tr>
<td>NR= Not Recorded</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

252
Table 17 SLM parameters used in coupon manufacture for each version of Ti-6Al-4V used in multiaxial tests conducted by Fatemi et al. (2017a).

<table>
<thead>
<tr>
<th>Label</th>
<th>Reference</th>
<th>Laser Power (W)</th>
<th>Scan speed (mm/s)</th>
<th>Hatch spacing (μm)</th>
<th>Layer thickness (μm)</th>
<th>Baseplate temperature (°C)</th>
<th>Scanning strategy</th>
<th>Multiple or single build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatemi</td>
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<td>400</td>
<td>1000</td>
<td>160</td>
<td>50</td>
<td>NR</td>
<td>2</td>
<td>NR</td>
</tr>
</tbody>
</table>

NR=Not Recorded

4.6.1 Micro-Mechanism Contribution

It was observed by Fatemi et al. (2017a) that all failures of the SLM coupons occurred on the maximum principal stress plane. This is an expected path of a brittle material, which is based on both the understanding that acicular α’ martensite microstructures typically have lower ductility than wrought bimodal Ti-6Al-4V, and the monotonic results gathered during the investigation. The complexity of the SLM microstructure presents some concerns with respect to how the crack propagates during both proportional and non-proportional loading. This is further complicated by non-proportional loading since the principal stress and strain axes rotate during cycling, which not only actives more slip planes as indicated by Nakamura et al. (2011) but can also result in defects having an even greater influence than they would have in proportional loading. Additionally, the defects in the material will result in localised stress concentrations, causing shear stresses to promote slip and therefore to lead to crack nucleation. Thus, it is anticipated that the defects within the material will have an effect on the plane on which the cracks initiates. The interaction of the defects and the layers with the progressive crack propagation will also influence the plane on which the crack propagates until the occurrence of failure. Further test data analysis is considered highly important in order to understand the influence of porosity, the orientation of lack-of-fusion defects and build layer direction on the orientation of the crack propagation plane (critical plane). Furthermore, this information will be of value in the potential use of techniques that can determine the orientation of the critical plane, e.g. Maximum Damage Method (MDM) (Bannantine and Socie, 1991) and Maximum Variance Method (Macha, 1989).
Closer examination of possible Ti-6Al-4V microstructure in multiaxial fatigue conditions is also an unexplored area that requires attention, especially when this material is used in complex geometry structures and parts prone to HCF. Other than the SLM acicular $\alpha'$ martensite microstructure investigation by Fatemi et al. (2017a), Ti-6Al-4V multiaxial fatigue investigations have considered mainly the wrought bimodal Ti-6Al-4V microstructure, such as in the study of Nakamura et al. (2011), Wu et al. (2012), and Morishita et al. (2016). An understanding of the microstructural influence on the critical plane orientation is particularly important in the development of the microstructure for fatigue resistance, since the underlying microstructure can have a significant influence over the fatigue fracture plane characteristic of the material (Marciniak et al., 2014); therefore, is vital in the tailoring of microstructure during SLM to improve component resistance to multiaxial fatigue.

5. Conclusion

Examination of the published SLM Ti-6Al-4V literature has provided an insight into the current knowledge surrounding the influence of microstructure, defects and residual stresses on the monotonic tensile, HCF, LCF, fracture mechanisms, and multiaxial fatigue. An overview of the influence of the different combination of possible phases of Ti-6Al-4V achieved through SLM on monotonic properties was provided. In conjunction to this, the microstructure and defect influence on monotonic yield stress and ductility was also summarised and critically assessed. From this examination, a significant understanding has been developed around these matters.

An overview of the current HCF behaviour of SLM Ti-6Al-4V has demonstrated that a significant amount of research has been undertaken with respect to the $\alpha'$ martensite dominated microstructure. Less attention has been paid to the possible lamellar ($\alpha+\beta$) microstructure. During the crack nucleation stage, finer microstructures are favoured due to a lower quantity of long slip bands, therefore, less irreversible slip leading to fewer nucleation sites. However, the extent of defects in the material will affect the crack nucleation stage due to stress raisers in the vicinity of defects, which influence the rate at which slip is activated. Consequently, the crack growth behaviour of the material becomes increasingly important,
due to the increased rate and presence of crack nucleation and microstructurally-small crack growth. SLM crack growth studies have been focused on $\alpha'$ martensite microstructure; however, more favourable microstructures to crack growth have been shown to be bimodal and lamellar microstructures. In order to develop an understanding of the influence of grain boundaries and layer orientation in conjunction with a lamellar microstructure on crack growth, crack growth studies by Galarraga et al. (2017) obtained from material manufactured using EBM were analysed. Based on this study and the effect of build orientation developed from SLM results, it is important crack growth occurs perpendicular to the build layers and if possible orientated in the same direction as the elongated grains. Obtaining both of these microstructure features is expected to aid in the crack deflection and reduce the growth rate. Orientating the layers parallel to the load direction will also significantly reduce the influence of lack-of-fusion defects on crack nucleation, a consequence of lower the aspect ratio and the subsequent localised stress concentration.

Acicular $\alpha'$ martensite microstructures have also been the primary focus of elastoplastic and LCF behaviour of SLM Ti-6Al-4V. Further investigations have to be undertaken to develop an understanding of the influence of SLM defects on LCF resistance of not only acicular $\alpha'$ martensite but also of the possible lamellar microstructures. Extensive analysis of the literature using wrought Ti-6Al-4V, as well as alternate AM fabrication methods, highlighted the potential improvement to LCF of SLM Ti-6A-4V with a lamellar structure and also reiterated the importance of the axial LCF loading direction being parallel to build layers to improve fatigue resistance.

Very limited research has been conducted so far into the understanding of the multiaxial fatigue behaviour of additively manufactured materials. It is considered particularly important to investigate the influence of build layers and defects on the plane of crack initiation and propagation. Furthermore, the microstructure contribution to multiaxial fatigue failure has to further investigated, particularly with respect to lamellar microstructures. Knowledge of how the crack propagates through different Ti-6Al-4V microstructures will assist in achieving more suitable design for SLM fabricated parts, through an informed selection of microstructures which can mitigate crack propagation, ultimately improving the
fatigue life of the structure. Additionally, knowledge of the plane orientations that are more susceptible to damage and developing and testing methods of predicting these orientations is very important in the design process. As proposed by Yadollahi and Shamsaei (2017) weaker planes can be aligned with areas expecting lower loads, which will improve the multiaxial fatigue resistance of the material.

Furthermore, reporting of the results obtained from SLM fabricated Ti-6Al-4V should at least also include:

- SLM processing parameters;
- Microstructural information, including phases present and size;
- Porosity;
- Location on the build platform;
- Whether it was a single or multiple build;
- Scanning strategy.

This knowledge is vital in providing sufficient information on the type of material fabricated and will give considerably more value to the results being reported. This will be very important in the development of a database for SLM Ti-6Al-4V which can be referred to in the design of components, particularly in the optimisation and tailor design of microstructure to achieve the required design criteria.

The extensive research conducted into SLM fabricated Ti-6Al-4V and supporting analysis from published Ti-6Al-4V research for both traditional manufacture methods and alternate AM methods has provided an in-depth understanding of how the microstructure, build orientation, defect percentage can be manipulated and applied in the development of fatigue resistance materials and what further research and investigation have to be undertaken to improve on this ever-growing database of knowledge.
References


Chapter II.2  Cyclic Elastoplastic Performance of SLM Ti-6Al-4V: Experimental Characterisation

This chapter contains two main components:

- II.2.1 Elastoplastic Response of As-Built SLM and Wrought Ti-6Al-4V Under Symmetric and Asymmetric Strain-Controlled Cyclic Loading.
- II.2.2 Cyclic Plasticity and Microstructure of As-Built SLM Ti-6Al-4V: The Effect of Build Orientation.

Section II.2.1 investigates the elastoplastic behaviour of vertically fabricated SLM Ti-6Al-4V. The micro-mechanisms associated with the SLM $\alpha'$ martensite microstructure observed from scanning electron microscopy contributed to the differences in elastoplastic behaviour both in symmetric and asymmetric strain-control loading compared to the results obtained from mill-annealed Ti-6Al-4V. Section II.2.2 investigates the anisotropy in cyclic plasticity caused by the build orientation. This involved stepped symmetric strain-controlled tests on coupons manufactured at 0°, 45°, and 90° to the build plate. Scanning electron microscopy and x-ray diffraction was also used to investigate the microstructure fabricated by the SLM processing parameters.
Chapter II.2.1 Elastoplastic Response of As-Built SLM and Wrought Ti-6Al-4V
Under Symmetric and Asymmetric Strain-controlled Cyclic Loading

(Paper 8)

D. Agius, K.I. Kourousis, C. Wallbrink

(Rapid Prototyping Journal)
Elastoplastic Response of As-built SLM and Wrought Ti-6Al-4V under Symmetric and Asymmetric Strain-controlled Cyclic Loading

Abstract

Purpose - Using as-built SLM Ti-6Al-4V in engineering applications requires a detailed understanding of its elastoplastic behaviour. This preliminary study intends to create a better understanding on the cyclic plasticity phenomena exhibited by this material under symmetric and asymmetric strain-controlled cyclic loading.

Design/methodology/approach – This paper investigates experimentally the cyclic elastoplastic behaviour of as-built SLM Ti-6Al-4V under symmetric and asymmetric strain-controlled loading histories and compare it to that of wrought Ti-6Al-4V. Moreover, a plasticity model has been customised to simulate effectively the mechanical behaviour of the as-built SLM Ti-6Al-4V. This model is formulated to account for the SLM Ti-6Al-4V specific characteristics, under the strain-controlled experiments.

Findings - The elastoplastic behaviour of the as-built SLM Ti-6Al-4V has been compared to that of the wrought material, enabling characterisation of the cyclic transient phenomena under symmetric and asymmetric strain-controlled loadings. The test results have identified a difference in the strain-controlled cyclic phenomena in the as-build SLM Ti-6Al-4V when compared to its wrought counterpart, due to a difference in their microstructure. The plasticity model offers accurate simulation of the observed experimental behaviour in the SLM material.

Research limitations/implications – Further investigation through a more extensive test campaign involving a wider set of strain-controlled loading cases, including multiaxial (biaxial) histories, is required for a more complete characterisation of the material performance.

Originality/value – The present investigation offers an advancement in the knowledge of cyclic transient effects exhibited by a typical α' martensite SLM Ti-6Al-4V under symmetric
and asymmetric strain-controlled tests. The research data and findings reported are among the very few reported so far in the literature.

**Key words:** Plasticity; Cyclic loading; Mean stress relaxation; Titanium alloys; Selective laser melting.

1. **Introduction**

Metal additive layer manufacturing offers several benefits to engineering applications with the potential to manufacture geometrically complex components with limited material wastage. Focusing on Ti-6A-4V manufacturing, selective laser melting (SLM) has been a technique heavily utilised for high value and performance aerospace and biomedical components. SLM offers significant benefits to components made from Ti-6Al-4V alloys due to the cost of the material difficulties in traditional manufacturing processes. However, if SLM is to expand more within the manufacturing industry, the structural integrity of the components fabricated by this method must be considered. Monotonic tensile properties and high cycle fatigue have been the focus of research efforts to characterise the performance of SLM Ti-6Al-4V (Rafi et al., 2013; Eric et al., 2013; Edwards and Ramulu, 2014; Xu et al., 2015; Gong et al., 2015; Zhao et al., 2016; Gunther et al., 2017). Low cycle fatigue (LCF) and transient elastoplastic cyclic effects have not been thoroughly considered, despite the engineering significance of these phenomena in design and operation (Li et al., 2016; Lewandowski and Seifi, 2016).

Cyclic plasticity can occur in a component (or a structure) in the bulk material due to severe loading conditions, in the vicinity of a crack or at the root of external discontinuities, such notches and holes, and internal defects, such as pores. Furthermore, in regions of localised plasticity the imposed loading is a combination of both stress and strain-controlled loadings (Hu et al., 1999a), which offers further justification on the importance of developing a broad understanding of SLM material LCF and elastoplastic behaviour (under both stress and strain loading conditions). These experimental (mechanical testing) data are necessary to employ effectively material constitutive models for the purposes of finite element and fatigue life
analyses, which include modelling of the crack tip plastic zone in the application of damage tolerance methods (Antunes and Rodrigues, 2008). Moreover, the simulation of stress/strain hysteresis loops and the calculation of the stress/strain amplitudes is necessary when performing fatigue life estimation with the use of safe-life (strain/stress-life) methodologies (Agius et al., 2017a). This is becoming increasingly important through the shown success of microstructure-sensitive fatigue (MSF) models in strain-life fatigue predictions employed for additively manufactured Ti-6Al-4V (Torries et al., 2016).

The strain-controlled response of additively manufactured materials has been given less attention, an issue highlighted by Shamsaei et al. (2015) in their comprehensive review of the mechanical behaviour of materials manufactured using direct laser deposition (DLD) and further recognised by Sterling et al. (2016), through their extensive symmetric strain-control program, investigating the cyclic and fatigue behaviour of DLD Ti-6Al-4V. Limited published research exists on the elastoplastic behaviour of SLM Ti-6Al-4V under symmetric/asymmetric strain and stress controlled loading (Kourousis et al., 2016; Phaiboonworachat and Kourousis, 2016, Agius et al., 2017b). The vast majority of material constitutive models, embedded in or custom-built for finite element and fatigue analysis software, require input (calibration data) obtained from cyclic uniaxial tests. This (cyclic uniaxial properties) dataset is currently unavailable to researchers and engineering practitioners.

A main drawback of the additive layer manufacturing techniques, including SLM, is the need for significant post-processing of the produced parts, which is a major cost raiser for the manufacturing industry. Post-processing includes costly heat treatment, machining and surface finishing, for the improvement of the quality and performance characteristics of the fabricated parts. However, there are engineering applications where as-built material, that has undergone no or very limited post-processing, can be used effectively. One of the exciting elements of this advancing technology is the potential to manufacture material with microstructure having a wide range of crystallographic phases, which can only be achieved in conventional manufacturing methods by applying post thermo-mechanical processes (Murr et al., 2009). Consequently, as our knowledge on the influence of processing
parameters, geometry and scan strategy on the intrinsic heat treatment capabilities (associated with cyclic reheating of layers advances), the in-situ tailoring of the as-built material microstructure is becoming a real option in our effort to improve fatigue resistance in critical areas (Morton et al., 2015; Yadollahi and Shamsaei, 2017). Thus, identifying cyclic elastoplastic features of the currently manufactured as-built material can be helpful in eliminating/tailoring those microstructure characteristics that limit the as-built material usability. Therefore, it is important, for material qualification purposes, to examine in more detail and assess the monotonic and cyclic mechanical properties of the as-built material.

This paper investigates experimentally the cyclic elastoplastic behaviour of as-built SLM Ti-6Al-4V under symmetric and asymmetric strain-controlled loading histories, extending the work of Agius et al. (2017b) which investigated the influence of build orientation on plastic anisotropy when applying symmetric strain-controlled cyclic loading. Our ongoing research on as-built SLM Ti-6Al-4V aims to characterise and model the elastoplastic behaviour of this material under various symmetric and asymmetric strain/stress-controlled cyclic loading conditions. The present preliminary study intends to create a better understanding on the cyclic plasticity phenomena exhibited by this class of material under strain-controlled loading. Moreover, a plasticity model has been customised in order to simulate effectively the mechanical behaviour of the as-built SLM Ti-6Al-4V. This model is formulated to account for the specific characteristics exhibited by this material, under the various strain-controlled experiments performed.

2. Experimental methodology

2.1 Test Coupons

Cylindrical cross-section test coupons were used for the monotonic and cyclic tests, fabricated with 1.0 mm excess on the geometry (dimensions) shown in Figure 1 to allow for machining and surface finishing. The test coupons were compliant with the ASTM E606 standard.
Two sets of Ti-6Al-4V test coupons were obtained, one manufactured via SLM and one from mill-annealed wrought material. The SLM coupons were fabricated with the SLM Solutions 250HL machine at the Centre for Additive Manufacturing of RMIT University using vertical layering, with the SLM processing parameters used presented in Table 1. The SLM processing variables used in (Kourousis et al., 2016) and (Phaiboonworachat and Kourousis, 2016) are also provided for cross-reference.

### Table 1 SLM processing parameters

<table>
<thead>
<tr>
<th>Processing Parameter</th>
<th>Present Study</th>
<th>Phaiboonworachat and Kourousis, 2016; Kourousis et al., 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser power (W)</td>
<td>100</td>
<td>175</td>
</tr>
<tr>
<td>Laser scanning velocity (mm/s)</td>
<td>375</td>
<td>710</td>
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<tr>
<td>Hatch Spacing (μm)</td>
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<td>120</td>
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<tr>
<td>Layer thickness (μm)</td>
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<tr>
<td>Focal offset distance (mm)</td>
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<td>0</td>
</tr>
<tr>
<td>Preheat Temperature</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

### 2.2 Mechanical Testing Setup

The tests on both the SLM and wrought Ti-6Al-4V coupons were conducted with a 100kN MTS servo-hydraulic closed-loop testing machine. The monotonic (tensile) test were conducted at strain rate of 0.5mm/min, with strain readings obtained with a 10mm extensometer. The cyclic (tension-compression symmetric and asymmetric) tests were
conducted using a 0.1 Hz sine. One test coupon for each loading condition was used to obtain the material behaviour results.

2.3 Cyclic Tests Load Histories

2.3.1 Symmetric Strain-Controlled Tests

Symmetric strain-controlled tests were performed on both wrought and SLM coupons in order to identify and compare the cyclic hardening/softening phenomena. The experiments followed a stepped loading pattern at ±1.0%, ±1.5%, ±1.8% and ±2.0% strain, with 80 cycles performed at each of the strain levels.

2.3.2 Asymmetric Strain-Controlled Tests

Asymmetric strain-controlled tests, on both the SLM and wrought tests coupons, were conducted using the four load cases given in Table 2, with 150 cycles performed at each test case. These tests were performed in order to determine the mean stress relaxation behaviour of the SLM and wrought material under fixed strain amplitude (Eq. 1) and varying mean strain (Eq. 2).

\[
\text{Strain Amplitude} = \frac{|\text{Maximum Strain}| - |\text{Minimum Strain}|}{2} \quad (1)
\]

\[
\text{Mean Strain} = \frac{|\text{Maximum Strain}| + |\text{Minimum Strain}|}{2} \quad (2)
\]

<table>
<thead>
<tr>
<th>Test</th>
<th>Maximum Strain (%)</th>
<th>Minimum Strain (%)</th>
<th>Mean Strain (%)</th>
<th>Strain Amplitude (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
<td>-0.8</td>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>1.8</td>
<td>-0.6</td>
<td>0.6</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>2.2</td>
<td>-0.2</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>2.4</td>
<td>0.0</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 2 Asymmetric strain-controlled load cases
3. Elastoplastic Modelling of the SLM Ti-6Al-4V Behaviour

Further to the experimental investigation of the elastoplastic behaviour of the as-built SLM Ti-6Al-4V under strain-controlled loading histories, it was important to select appropriate constitutive models capable of simulating this behaviour. Moreover, simulation is required for calculating fatigue on structural components with the use of strain life methodologies. To capture the various cyclic phenomena observed in the experiments both kinematic and isotropic hardening rules have to be adopted. Kinematic hardening employs a translating yield surface, in stress space, which in turn can capture the transient cyclic effects such as mean stress relaxation under cycling. Isotropic hardening, on the other hand, imposes a uniform expansion/contraction to the yield surface, a feature necessary to simulate strain hardening/softening correspondingly.

The selected kinematic hardening model was the Multicomponent Armstrong & Frederick (AF) model with Multiplier (MAFM) (Dafalias et al., 2008), which has not only shown to be successful in SLM Ti-6Al-4V simulations (Kourousis et al., 2016), but also in conventionally manufactured aluminium (Kourousis and Dafalias, 2013) and steel alloys (Dafalias et al, 2008). The MAFM model controls the yield surface centre shifting via a back stress. Since the experimental work of this study was limited to uniaxial loading, the uniaxial formulation of the MAFM model is provided.

The yield surface applied was the Von Mises yield surface \( f \) given by Eq. 5:

\[
f = (\sigma - a)^2 - R^2
\]  

(5)

Where \( \sigma \) is the applied stress, \( a \) back stress and \( R \) the yield stress evolving through the isotropic hardening rule (Chaboche, 1986) described by Eq. 6.

\[
\dot{R} = b (R_s - R) |\dot{\varepsilon}^p|
\]  

(6)

Where \( b \) and \( R_s \) are material constants and \( \varepsilon^p \) the plastic strain. Dot notation over a term corresponds to the time derivative of the term.

The MAFM back stress \( a \) is given by:
\[
\dot{a} = \sum_{i=1}^{6} \dot{a}_i = \left\{ \sum_{i=1}^{6} c_i \left( a_i^* \mp a_i \right) + \left[ c_7 + c_7^* \left( a_7^* \mp a_7 \right) \right] \left( a_i^* \mp a_i \right) \right\} + c_8
\]  
(7)

There are three distinct components of the back stress formulation (Eq. 7):

The first component is the traditionally Armstrong and Frederick (AF) nonlinear hardening back stress (Armstrong and Frederick, 1966), which in this case corresponds to six different AF back stresses:

\[
\dot{a}_i = c_i \left( a_i^* \mp a_i \right) \dot{\varepsilon}^p \quad i = 1, 2, ..., 6
\]  
(8)

Where \( c_i \) and \( a_i^* \) are material parameters and \( \dot{\varepsilon}^p \) is the plastic strain rate.

The second component is the back stress containing the multiplier:

\[
\dot{a}_7 = \left[ c_7 + c_7^* \left( a_7^* \mp a_7 \right) \right] \left( a_i^* \mp a_i \right) \dot{\varepsilon}^p
\]  
(9)

Where \( c_7, c_7^* \) and \( a_7^* \) are the material parameters defining the evolution of the multiplier back stress. The multiplier in Eq. 9 is term \( a_7^* \) with its evolution given by the dimensionless Eq. 10.

\[
\dot{a}_7^* = c_7^* \left( a_7^* \mp a_7 \right) \dot{\varepsilon}^p
\]  
(10)

Where \( a_7^{*,*} \) also a material parameter.

The third component is a linear hardening back stress (Prager, 1956), given by Eq. 11, which was added to reduce the saturation rate of the mean stresses, as proposed in (Yeom et al., 2001).

\[
\dot{a}_8 = c_8 \dot{\varepsilon}^p
\]  
(11)

Where \( c_8 \) a material parameter.
4. Results and Discussion

4.1 Experimental Results

4.1.1 Monotonic Test

The tensile properties of the SLM and wrought Ti-6Al-4V materials tests are provided in Table 3. In this test, the as-built SLM material exhibited a higher yield and ultimate tensile strength.

<table>
<thead>
<tr>
<th></th>
<th>Elasticity Modulus (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Tensile Strength (MPa)</th>
<th>Ultimate Tensile Strength / Yield Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrought</td>
<td>110</td>
<td>955</td>
<td>1,036</td>
<td>1.08</td>
</tr>
<tr>
<td>SLM</td>
<td>110</td>
<td>1,120</td>
<td>1,220</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 3 Tensile mechanical properties of SLM and wrought Ti-6Al-4V

4.1.2 Cyclic Symmetric Strain-Controlled Tests

A comparison between the SLM and wrought Ti-6Al-4V hysteresis loops obtained for each of the symmetric tests (stepped tests at 1.0%, 1.5%, 1.8% and 2.0% strain level) is given in Figure 2, where the first cycle for each step is compared in Figure 2(a) and the 80th cycle for each step is compared in Figure 2(b). Moreover, a decomposed view of the last (80th) hysteresis loop for each strain level is provided in Figure 3. The noticeable difference in the hysteresis loop shape between the SLM and wrought material results offers an indication on the significantly higher plastic work occurring in the wrought material. This is an expected outcome coming out from this investigation, as very similar observations have been reported in the past for the monotonic tensile behaviour of SLM Ti-6Al-4V (higher yield stress and ultimate stress at the price of lower ductility and elongation at fracture, as opposed to that exhibited by the annealed wrought material). It is also noted that at the highest strain level tested (2.0%) the wrought coupon failed at 73 cycles (out of the 80 programmed), while the SLM coupon completed the test without failing.
Figure 2 SLM and wrought Ti-6Al-4V hysteresis loops obtained from the stepped symmetric strain controlled tests at 1.0%, 1.5%, 1.8% and 2.0% strain: (a) first cycle and (b) last cycle (80th)
Figure 3 SLM and wrought Ti-6Al-4V hysteresis loops obtained from the last (80th) cycle of the stepped symmetric strain controlled tests at (a) 1.0%, (b) 1.5%, (c) 1.8% and (d) 2.0% strain.

In order to determine whether cyclic hardening or softening occurred during the tests, the stress amplitude was calculated and plotted against the loading cycles (Figure 4).

To visualise the cyclic softening behaviour of the SLM and wrought material, the average cyclic softening rate $V_{cs}$ of the two materials was calculated for each strain amplitude using Eq. 12 (Tan et al., 2015) and plotted in Figure 5.

$$V_{cs} = \frac{\sigma_{Ne} - \sigma_{Ne}}{N_e - N_s}$$

(12)
Where $\sigma_{Ns}$ and $\sigma_{Ne}$ is the maximum tensile stress at the start and end of cyclic softening respectively; $N_e$ and $N_s$ is the cycle number corresponding to the start and end of cyclic softening respectively.

**Figure 4** Variation of the stress amplitude for SLM and wrought Ti-6Al-4V during the stepped symmetric strain controlled tests at (a) 1.0%, (b) 1.5%, (c) 1.8% and (d) 2.0% strain.
Figure 5 Comparison of the average softening rate using symmetric strain controlled results for each tested amplitude for both the SLM and Wrought material.

For both materials, cyclic softening is evidenced by a decrease in the stress amplitude for the 1.5%, 1.8% and 2.0% strain level loading cases. Cyclic softening is an expected phenomenon, since the calculated ratio of monotonic ultimate tensile strength and yield strength ($\frac{\sigma_{UTS}}{\sigma_y}$) is for both material less than 1.2 (shown in Table 2). In particular, as reported in (Smith et al., 1963), the occurrence of either cyclic softening or cyclic hardening is dependent on the ($\frac{\sigma_{UTS}}{\sigma_y}$) ratio, where hardening is expected with ratios greater than 1.4 and softening less than 1.2. It is of note that at the 1.0% strain level, the SLM material exhibits cyclic hardening, with a slight softening effect starting to appear after approximately 50 cycles (leading to softening response overall). On the contrary, the wrought material results demonstrate a clear softening behaviour in all tested strain amplitudes, due to the strain amplitude being above the critical value of 0.6%, below which cyclic stabilisation occurs rather than the reported softening (Nag et al., 2006; Wang et al., 2013; Wang et al., 2013). The initial hardening followed by softening effect observed in symmetric strain-controlled tests has been reported for mill-annealed Ti-6Al-4V (Gil et al., 2003; Wang et al., 2013), as well as other titanium alloys (TC21, LT26A, etc) (Nag et al., 2006; Wang et al., 2013; Tan et al., 2015), which
supports the results reported in this study. The mill-annealed microstructure is composed of globular $\alpha$ grains with $\beta$ phase present at the grain boundaries (Mulay et al., 2016). Therefore, cyclic softening occurs due to an increasing number of mobile dislocations (Steele and McEvily, 1976) which are activated in both the $\alpha$ and $\beta$ phases (Tan et al., 2015).

Scanning electron microscopy (SEM) micrographs, obtained from the SLM specimens, demonstrate the existence of acicular martensite ($\alpha'$ phase), which is typical of an SLM material (Mur et al., 2009; Thijs et al., 2010; Facchini et al., 2010; Vrancken et al., 2012; Simonelli et al., 2014; Xu et al., 2017; Chen et al., 2017; Shi et al., 2017) (Figure 6). A complementary X-ray diffraction (XRD) profile obtained (Figure 7), shows significant $\alpha$ phase, with limited amounts of $\beta$ phase. Thus, the tested SLM specimen microstructure is different to that of a typical wrought (mill annealed) material, which in turn differentiates the mechanisms inducing the cyclic softening to these materials. In particular, in a martensitic Ti-6Al-4V, the cyclic softening occurs due the presence of mobile dislocations in the undeformed microstructure and cyclic induced twinning (Gil et al., 2003). In the SLM material, these martensitic cyclic softening mechanisms required approximately 50 cycles to take effect in the 1.0% strain amplitude test. While, for the wrought material tested at the same strain amplitude (1.0%), the softening mechanisms occurred instantly, resulting in the noticeable difference in the stress amplitude [shown in Figure 4(a)]. The average softening rates become progressively more comparable in the subsequent stepped strain amplitudes (1.5%, 1.8% and 2.0%), as shown in Figure 5, due to the fact that the softening mechanisms in both materials have already been activated, which results in a continued relaxation.
Figure 6 Scanning electron microscopy (SEM) micrographs of the microstructure of the tested SLM Ti-6Al-4V coupons at two different magnifications (a) x10,000 and (b) x20,000, showing a $\alpha'$ martensite structure.

Figure 7 X-ray diffraction (XRD) profile for the tested SLM Ti-6Al-4V
Further investigation, through additional tests (both at the same strain level but at other strain levels, progressively starting from 0.5% for example) is expected to determine whether initial cyclic hardening is still apparent at both lower and larger strain amplitudes.

4.1.3 Cyclic Asymmetric Strain-Controlled Tests

The hysteresis loops obtained from each of the asymmetric tests (Tests 1 to 4 shown in Table 2) are presented in Figure 8 (Test 1), Figure 9 (Test 2), Figure 10 (Test 3) and Figure 11 (Test 4). The size of the hysteresis loops in all tests suggests that the SLM material is less ductile than the wrought material as it undergoes less plastic work. The size of hysteresis is also a measure of the material capacity for plastic energy dissipation, which can have an influence on the fatigue life of components, especially at points of high stress concentration. Thus, these findings are very important from a fatigue evaluation point of view and can be utilised in conjunction with past research findings reported on the high cycle fatigue (HCF) performance of SLM Ti-6Al-4V. Examining more closely the Test 4 results (Figure 11) one may observe an increase in size of the hysteresis loop (Figure 12), which is another confirmation on the existence of cyclic softening in both materials.

![Hysteresis Loops](image)

**Figure 8** SLM and wrought Ti-6Al-4V hysteresis loops obtained from the asymmetric strain controlled Test 1 [Table 2: (strain max, min) = (1.6%, 0.8%)]: (a) first cycle, (b) last (150th) cycle.
**Figure 9** SLM and wrought Ti-6Al-4V hysteresis loops obtained from the asymmetric strain controlled Test 2 [Table 2: (strain max, min) = (1.8%, -0.6%)]: (a) first cycle, (b) last (150th) cycle.

**Figure 10** SLM and wrought Ti-6Al-4V hysteresis loops obtained from the asymmetric strain controlled Test 3 [Table 2: (strain max, min) = (2.2%, -0.2%)]: (a) first cycle, (b) last (150th) cycle.
Figure 11 SLM and wrought Ti-6Al-4V hysteresis loops obtained from the asymmetric strain controlled Test 4 [Table 2: (strain max, min) = (2.4%, 0%)]: (a) first cycle, (b) last (150th) cycle.

Figure 12 Asymmetric strain controlled Test 4 hysteresis SLM and wrought Ti-6Al-4V loops comparison for 1st cycle and last (150th) cycle.
A comparison of mean stress (Eq. 13) and stress amplitude (Eq. 14) variation for the four tested load cases (Test 1 to Test 4) was also conducted.

\[
\text{Mean Stress} = \frac{\text{Maximum Stress} + \text{Minimum Stress}}{2}
\]  
(13)

\[
\text{Stress Amplitude} = \frac{\text{Maximum Stress} - \text{Minimum Stress}}{2}
\]  
(14)

Mean stress relaxation was observed in both material, with the relevant results presented in Figure 13. SLM Ti-6Al-4V exhibits a significantly higher relaxation rate than the wrought material. This is an important fatigue property, since tensile residual stresses can contribute to faster crack initiation; however, if the material can relax these tensile residual stresses faster, crack initiation can be delayed.

**Figure 13** Mean stress relaxation for asymmetric strain controlled Tests 1, 2, 3 and 4 (Table 2) for (a) wrought Ti-6Al-4V and (b) SLM Ti-6Al-4V

The stress amplitude variation for the four asymmetric strain load cases (Table 2) was also examined for both materials, presented in Figure 14. When comparing the results, an interesting difference is noticeable. In particular, for the SLM material, the stress amplitude exhibits a steady increase over the first 40 cycles (out of 150 in total), with a subsequent slow decrease (almost saturating to a level). This feature specific to the SLM material is especially important when modelling its elastoplastic behaviour, such as the computational
investigation conducted in this study. On the other hand, the wrought material experiences a steady decrease in the stress amplitude, except for the very first cycles (where a slight increase is observed before decrease starts to happen).

![Figure 14](image)

**Figure 14** Variation of the stress amplitude for wrought and SLM Ti-6Al-4V during the asymmetric strain controlled (a) Test 1, (b) Test 2, (c) Test 3 and (d) Test 4.

In order to examine more closely the wrought – SLM material differences in the variation of the stress amplitude, the maximum and minimum stress is plotted for all test cases (Figure 15 to 18). From these results, it can be observed that the interaction between the maximum and minimum stresses during each cycle, leads to a different evolution of the stress amplitude [Figure 15(b), 16(b), 17(b) and 18(b)]. In particular, it is the SLM material minimum stress values (obtained from compressive loading) that drive the widening of the stress amplitude gap between SLM and wrought material. It is of note that the SLM material exhibits a
saturated behaviour for all asymmetric load cases examined (as indicated by the stabilised mean stress and stress amplitude values – Figure 13 and 14 correspondingly.). However, this is not apparent if the hysteresis loops (Figure 8, 9, 10 and 11) are examined alone, since the curvature of the final cycles does not indicate a saturated response (SLM: high slope; wrought: near-plateaued slope).

**Figure 15** Variation of (a) maximum stress and (b) minimum stress for SLM and wrought Ti-6Al-4V subjected to asymmetric strain controlled Test 1 (strain 1.6%, -0.8%).

**Figure 16** Variation of (a) maximum stress and (b) minimum stress for SLM and wrought Ti-6Al-4V subjected to asymmetric strain controlled Test 2 (strain 1.8%, -0.6%).
Figure 17 Variation of (a) maximum stress and (b) minimum stress for SLM and wrought Ti-6Al-4V subjected to asymmetric strain controlled Test 3 (strain 2.2%, -0.2%).

Figure 18 Variation of (a) maximum stress and (b) minimum stress for SLM and wrought Ti-6Al-4V subjected to asymmetric strain controlled Test 4 (strain 2.4%, 0%).
4.1.4 Simulations

The MAFM model was implemented numerically in Matlab, through an implicit integration scheme. The material parameters for the as-built SLM Ti-6Al-4V have been obtained with the use of the methodology developed by Dafalias, et al. [19]. This model calibration methodology utilises strain-controlled cyclic test results, which were obtained from the experiments conducted. The parameters of the complete (kinematic and isotropic hardening) plasticity model, referred as MAFM model for simplicity, are listed in Table 4.

<table>
<thead>
<tr>
<th>Elastic modulus</th>
<th>$E = 100,048$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclic yield stress</td>
<td>$\sigma_{\text{yield}} = 60$ MPa</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$a_1^* = 65$ MPa</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$a_2^* = 156$ MPa</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$a_3^* = 208$ MPa</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$a_4^* = 127$ MPa</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$a_5^* = 201$ MPa</td>
</tr>
<tr>
<td>$a_6$</td>
<td>$a_6^* = 200$ MPa</td>
</tr>
<tr>
<td>$a_7$</td>
<td>$a_7^* = 5,000$ MPa</td>
</tr>
<tr>
<td>Multiplier</td>
<td>$a_7^* = 0.009$</td>
</tr>
<tr>
<td>$a_8$</td>
<td>$c_8 = 8,505$ MPa</td>
</tr>
<tr>
<td>Isotropic softening</td>
<td>$R_s = -40$ MPa</td>
</tr>
<tr>
<td>$b$</td>
<td>$b = 1.2$</td>
</tr>
</tbody>
</table>
The four symmetric strain-controlled experimental results for the SLM material (presented in Figure 3) were compared with the computational data obtained from the MAFM plasticity model. In particular, the last cycle of all four symmetric strain-controlled SLM experimental results is compared to the simulated cycles, shown in Figure 19. In all cases, the simulations can capture the various features of the hysteresis loops accurately, despite the increasing variation in the linearity of the curves for the different strain levels examined (e.g. higher linearity at 1.0% strain, lower at 1.8% and 2.0% strain).

**Figure 19** Experimental and computed (MAFM model) SLM Ti-6Al-4V hysteresis loop at last cycle (80th) of the stepped symmetric strain controlled tests at (a) 1.0%, (b) 1.5%, (c) 1.8% and (d) 2.0% strain.
The capability of the MAFM model in capturing the strain-controlled experimental data is further illustrated by the simulation of the asymmetric test data (Test 1 to Test 4, shown in Table 2). The hysteresis loops of the SLM material are compared against the computed, by MAFM model, data (Figure 20). Moreover, a simulation of the cyclic softening behaviour of the SLM material is conducted (Figure 21). The obtained computed results for all four test cases are in very good agreement with the experimental data, with the exception of Test 1 mean stress relaxation data undershooting [Figure 21(a)].

Figure 20 Experimental and computed (MAFM model) SLM Ti-6Al-4V first and 150th cycle hysteresis loops for asymmetric strain controlled loading (a) Test 1, (b) Test 2, (c) Test 3 and (d) Test 4.
5. Conclusions

The performed test campaign has revealed that the mean stress relaxation in as-built SLM Ti-6Al-4V occurs faster than in its wrought counterpart. Moreover, the symmetric strain-controlled tests have demonstrated that the SLM material experiences significantly less plastic work than the wrought material, as indicated by the consistently smaller hysteresis loops produced at the same strain levels, indicative of the SLM materials $\alpha'$ martensite content. The present investigation offers an advancement in the knowledge of cyclic transient effects exhibited by a typical $\alpha'$ martensite SLM Ti-6Al-4V under symmetric and asymmetric strain-controlled tests. However, this preliminary study has to be extended further through a more extensive test campaign involving a wider set of strain-controlled loading cases, including multiaxial (biaxial) histories.
The observed cyclic transient behaviour under uniaxial loading cases (both symmetric and asymmetric strain-controlled cases) has also been simulated accurately with the use of a combined kinematic (MAFM) and isotropic hardening model. The computational results have captured the particularities of the SLM material, verifying the general capability of existing advanced models to simulate cyclic plasticity phenomena exhibited by additively manufactured metals. This study has verified the general capability of a modelling method to predict the behaviour of as-built SLM Ti-6Al-4V, which is of interest to both the academic and industry community. This finding can be very useful for HCF calculation with software utilising strain-life methodologies.

Further investigation, jointly undertaken by the University of Limerick, RMIT University and the Defence Science and Technology (DST) Group, is conducted with regard to the as-built SLM and wrought material pre and post testing microstructure characteristics. Moreover, this continuation project is going to include further mechanical tests (with more specimens, to account for possible variability and stress-controlled cases examined in addition) and supporting microstructural and residual stresses’ analyses of the currently and new tested coupons. The results of this investigation, when completed, will be reported in a separate paper.

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References


II.2.2 Cyclic Plasticity and Microstructure of As-Built SLM Ti-6Al-4V: The Effect of Build Orientation

(Paper 9)

D. Agius, K.I. Kourousis, C. Wallbrink, T. Song

(Materials Science & Engineering A)
Cyclic plasticity and microstructure of as-built SLM Ti-6Al-4V: The effect of build orientation

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A R T I C L E   I N F O

Keywords:
Additive manufacturing
Titanium
Cyclic plasticity
Hardening
Softening

A B S T R A C T

This paper investigates the cyclic elastoplastic anisotropy of Ti-6Al-4V manufactured via selective laser melting, caused by different build orientations. Tensile monotonic and cyclic stepped symmetric strain-controlled tests on coupons manufactured at 0°, 45°, and 90° build orientations were performed for both the SLM material and for a mill annealed material. The microstructure characteristics of the SLM material were examined through optical and electron microscopy revealing a unique α′ martensite microstructure. The examination of the evolving tensile and compressive maximum stresses identified an interesting phenomenon, that of asymmetric cyclic softening. This phenomenon was observed only in the SLM Ti-6Al-4V, while its wrought counterpart confirmed the findings of past research reported in the literature. The residual stresses present in the SLM coupons had a significant influence on the cyclic behaviour of the material. Mechanical anisotropy in both monotonic and cyclic tests was noticed with the diagonal (45°) coupon having the largest yield stress in both loading conditions. The findings of this research study can be very useful in engineering applications utilising as-built SLM materials.

1. Introduction

As metal additive layer manufacturing continues to grow, it is important in engineering material design to develop an understanding of how layer based material will behave in low cycle fatigue (LCF). LCF can occur in a structure for a number of reasons, including the presence of sharp corners or notches, where stress raisers can cause local plasticity [1–3]. Also, complex spectrum loading [4] and surface treatments, such as grit blasting [5] or shot peening [6], can lead to LCF behaviour due to the high associated loading conditions causing elastoplastic behaviour. Titanium alloys such as Ti-6Al-4V have been widely used in both the biomedical and aerospace industries, due to their weight-to-strength ratio and superior mechanical properties, i.e. high strength, high corrosion resistance and fracture toughness. Additive manufacture (AM) of components made from Ti-6Al-4V is an attractive method of manufacture as it provides a low waste alternative for the manufacture of complex geometries. One commonly used AM fabrication method is selective laser melting (SLM), which is a layer-based deposition method using a laser to selectively melt successive layers of metal powder in an inert gas filled chamber. However, due to the complexity of the manufacturing process, the mechanical properties of the Ti-6Al-4V can vary depending on the microstructure produced, the different phases present and the nature/extent of the defects (i.e. porosity, inclusions). Consequently, a considerable amount of research has been conducted in understanding the microstructure and defect influence on the monotonic tensile properties, high cycle fatigue (HCF), hardness, and electrochemical behaviour SLM Ti-6Al-4V [7–16]. However, the cyclic elastoplastic and LCF behaviour of SLM Ti-6Al-4V has not been as thoroughly researched [17,18], particularly in understanding the influence of build orientation on the elastoplastic behaviour. This information is important in the formulation of elastoplastic constitutive models for accurate simulation of the associated phenomena and, in turn, fatigue life prediction for structural analysis problems. Equipped with a more in-depth understanding of the microstructure influence on cyclic plasticity and LCF, in conjunction with constitutive modelling capabilities, the advantages of AM can be further utilised through customised design (i.e. adjustment of geometry, adaptation of the safety factor, etc.). With advancing knowledge of SLM Ti-6Al-4V, particularly in the possible microstructures which can be produced through SLM [15] and the influence on mechanical properties such as monotonic loading, HCF and LCF, the potential to tailor the microstructure of components to improve their fatigue resistance for the required service is becoming a real possibility [19].

This research aims to enrich the body of knowledge in the area of
cyclic plasticity and LCF characteristics of as-built SLM Ti-6Al-4V. For that purpose, an optical and electron microscopy microstructure analysis was performed on polished and etched samples obtained at transverse and longitudinal directions at 0°, 45°, and 90° relatively to the build platform. This analysis revealed the existence of a unique α′ martensite microstructure, not previously reported in the literature. The influence of the particular microstructure characteristics and residual stresses on the cyclic elastoplastic behaviour was investigated by conducting a strain-controlled tests campaign. These tests results were compared to strain-controlled tests results from mill-annealed Ti-6Al-4V. This comparison offers a reference point for the elastoplastic behaviour of SLM and mill-annealed Ti-6Al-4V having different microstructures. The findings of this research study can be very useful in engineering applications utilising the as-built SLM material, as opposed to applications with SLM material that has undergone heat treatment or other substantial (in terms of cost and time) post processing.

2. Materials and methods

2.1. Mechanical testing coupons

Nine strain-controlled (fatigue) test cylindrical rods and three monotonic tensile cylindrical rods were manufactured using spherical gas atomized Ti-6Al-4V powder (25–45 µm, ELI, 0.1% O, 0.009% N, 0.008% C, 0.17% Fe, 0.002% H, TLS Technik GmbH & Co.), at a height of 113.6 and 114.28 mm and diameter of 14.7 and 14 mm respectively. The strain-controlled coupons were machined from the cylindrical rods according to ASTM E606/E606M [20] and the monotonic tensile coupons manufactured according to ASTM E8/E8M [21], using the dimensions in Fig. 1(b). The cylindrical rods were manufactured at three orientations with respect to the build direction: horizontal (0°), diagonal (45°), and vertical (90°) as shown in Fig. 1(a). For manufacture, the powder bed was preheated to 200 °C and the chamber was filled with argon gas until the oxygen level was reduced to 0.1%. The processing variables used in the manufacture of the cylindrical rods were laser power = 100 W; laser scanning velocity = 375 mm/s; hatch spacing = 130 µm; layer thickness = 30 µm; focal offset distance = 0 mm, similarly to a previous study work by Kourousis et al. [17]. All coupons were fabricated by employing a stripe pattern scanning strategy, which practically divides the layers into parallel stripes. Stripes are then rotated by 79° for each consecutive layer, where, as indicated in Fig. 2, the dashed red lines represent the stripes and the blue and green arrows show the change in scanning direction for each layer.

The location of the cylindrical rods was recorded and is labelled in Fig. 3, where letter ‘F’ denotes the strain-controlled (fatigue) coupons and letter ‘T’ the tensile coupons, with 0°, 45° and 90° providing the build orientation (0°, 45° and 90°) and subscript index coupon number (1, 2, 3). The first batch contained the vertical and diagonal coupons (Fig. 3(a)), while the second contained the horizontal rods (Fig. 3(b)).

2.2. Load cases

All tests were conducted at room temperature using MTS servo-hydraulic closed-loop testing machine with a load capacity of 100 kN. Monotonic tensile tests were performed using a strain rate of 0.5 mm/min, while the strain-controlled tests used a 0.1 Hz sine wave. Strain measurements were collected with the use of an Epsilon 10 mm extensometer at an interval of 0.1 s.

2.2.1. Monotonic tensile tests

Monotonic tensile tests were conducted for each build orientation to allow for a comparison of the mechanical properties resulting thereof.

2.2.2. Symmetric strain-controlled tests

The cyclic elastoplastic behaviour of the three build orientations was investigated by employing a strain-controlled test program considering tension-compression cyclic load cases at no mean strain but varying strain amplitudes. The strain-controlled test program is provided in Table 1.

3. Results

3.1. Microstructural observations

All microstructural observations were conducted using a FEI Scios scanning electron microscope (SEM) with energy dispersive spectrometer (EDS) and the UHL VMM 200 optical microscope (OM). Polished and etched (in 10 mL hydrofluoric acid, 5 mL nitric acid, 85 mL water)
The grain morphology and phases present in the fabricated coupons was analysed for each build orientation, which identified the presence of \( \alpha' \) martensite typical of that formed during the SLM process \([14-16,22]\). This is due to the processing parameters and coupon geometry achieving a cooling rate greater than 410 K s\(^{-1}\) (which has been suggested to be the correct rate for martensite \( \alpha' \) formation \([23]\)) beginning from above the martensite start temperature \([24]\). OM micrographs of the samples taken in the longitudinal and transverse directions to the build orientation from the \( F_0 \), \( F_{45} \), and \( F_{90} \) coupons are presented in Figs. 4–6, where prior columnar \( \beta \) grains, containing acicular \( \alpha' \) martensite, are orientated in the build direction.

The XRD profiles obtained for the transverse samples for each build orientation indicate significant \( \alpha/\alpha' \) phase with very limited \( \beta \) phase (Fig. 7).

Using SEM on the etched samples, the different features of the microstructure for the horizontal, diagonal, and vertical coupons are visible in Figs. 8–10, respectively. These features include primary \( \alpha' \), secondary \( \alpha' \), tertiary \( \alpha' \), quartic \( \alpha' \), nano-dispersolds, lamellar growth, and \( \alpha \) rods and dots. The nano-dispersolds are the initial phase of lamellar growth \([25]\), while the \( \alpha \) rods and dots are suggested to be different morphologies of secondary \( \alpha \) \([25]\). Primary, secondary, tertiary and quartic \( \alpha' \) martensite are visible, all of which have been previously shown to exist in SLM Ti-6Al-4V \([26]\). The etched sample, shown in Fig. 11, showed potential \( \beta \) phase precipitating from martensite surrounding \( \alpha' \) martensite, which has been previously observed in \([27,28]\). The \( \beta \) phase can be identified from SEM micrographs in backscattered electron mode as the bright spots in the micrographs, since the higher vanadium content in \( \beta \) phase results in a brighter aspect \([28]\).

In order to collect further information on the microstructure, SEM of the polished samples was performed in the transverse direction. In Fig. 12(a) micrograph the typical \( \alpha' \) martensite is present, composed predominately of coarse \( \alpha' \). However, in Fig. 12(b) and (c) micrographs, there is significantly less coarse \( \alpha' \) and a greater volume of ultrafine \( \alpha' \). This could be explained by considering the transformation process of the hierarchical \( \alpha' \) martensite, which is affected by the laser deposition process, and the reheating and cooling process \([26]\). The evolution of the \( \alpha' \) type martensite depends on the reheating temperature reached during the layer deposition induced thermal cycles. This interaction leads to varying volume fractions in primary, secondary, tertiary and quartic \( \alpha' \) martensite. Since the processing parameters were kept constant during each build orientation, the different \( \alpha' \) transformation is not dependent on the processing parameters. The transformation process of the hierarchical \( \alpha' \) martensite is likely to be influenced by the

---

**Table 1**

Strain-controlled test program.

<table>
<thead>
<tr>
<th>Test number</th>
<th>Number of cycles</th>
<th>Load level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Test 1</td>
<td>200</td>
<td>± 1.0%</td>
</tr>
<tr>
<td>Test 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

*Fig. 3. Location of the manufactured cylindrical rods on the build platform and corresponding label to help identify the coupon (e.g. \( F_{45} \), \( F_{90} \), \( F_0 \)) (a) First batch containing the diagonal and vertical rods; (b) Second batch containing horizontal cylindrical rods.*

*Fig. 4. Optical microscopy (OM) images of the etched horizontal coupons (\( F_0 \)) showing prior columnar \( \beta \) grains (indicated by red continuous lines) containing acicular \( \alpha' \) taken in the: (a) transverse and (b) longitudinal directions (relative to the build direction). (Coordinates included on the figure based on those in Fig. 3). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)*
number of layers deposited during the fabrication. The horizontal coupon has less associated layers than the diagonal and vertical builds due to the majority of its area being in close proximity to the build platform. Effectively, fewer deposited layers reduce the number of induced thermal cycles, which could lead to a reduction in the transformation of tertiary and quartic α′ martensite. However, the lower number of deposited layers may be counterbalanced by the influence of the preheated build platform. As was demonstrated by Ali et al. [29], the build platform temperature has a significant influence on the cooling rate of the fabricated material and the corresponding microstructure development. The influence of the build platform temperature on deposited layers will last longer in the horizontal build than the diagonal and vertical builds, due to the close proximity of the horizontal layers to the build platform. Therefore, although there are less layers in the horizontal build (reducing the transformation of primary and secondary α′ martensite to tertiary and quartic α′ martensite), the greater influence of the build platform temperature in the horizontal build increases the potential of deposited layer reheating to promote tertiary and quartic α′ martensite transformation.

To investigate further the reason for the ultrafine martensite microstructure formation, vertical coupons previously manufactured using the same processing parameters were examined through SEM. The obtained SEM results are provided in Fig. 13, where two different microstructures were observed in the same fabricated batch of vertical coupons. These microstructures included an ultrafine α′ microstructure (Fig. 13(a)) and a predominantly coarse α′ microstructure (Fig. 13(b)), which are very similar to the microstructures observed in this most recent study (Fig. 12). Therefore, it is possible to obtain the ultrafine microstructure in both horizontal and vertically fabricated coupons, which suggests that the occurrence of the ultrafine α′ microstructure is not solely build orientation dependent.

Another possible influencing factor on the formation of the ultrafine martensite was hypothesised to be attributed to the coupon’s location on the build platform. Past research has confirmed that the inert gas flow in the build chamber is not uniform throughout the chamber [30,31]. It was found that the extent of the removal of vapourised powder (condensate) from the laser path varied across the chamber due to a variation from high to low gas flow, where high gas flow regions were effective in condensate removal, while low gas flow regions were not. Consequently, the defined laser beam parameters are affected, leading to a variation in the intensity, spot diameter and energy of the laser beam across the build platform. The influence of gas flow velocity has been also investigated in the past by considering the impact of by-products on beam scatter, where it was found that the low gas velocity areas resulted in significantly more scatter [32]. Scattering of the laser beam lowers the available energy, reducing the depth of heating of the substrate. The flow velocity not only varies from inlet to outlet (right to
left) on the build plate but also from front to back [30]. Since the samples used in metallography were taken from diagonal and vertical coupons manufactured toward the front left side of the build plate and the horizontal coupon toward the back left, it is reasonable to assume that a difference in gas flow velocity existed between the coupons. The horizontal coupon would have suffered more from the low gas flow velocity effects described here, leading to scattering of the laser beam and reduced heating of previous layers. Therefore, for the same layer thickness and focal offset distance, there will be different energy levels, where subtle changes in energy can have an influence on phase transformation [13]. The reduction in substrate heating could lessen the thermal cycles required for hierarchal \( \alpha' \) martensite formation.

Fig. 8. Typical SEM images of the etched samples taken in the transverse direction from the horizontal coupon (P05) with labelled examples of the different sized \( \alpha' \) martensite.

Fig. 9. SEM images of the etched samples taken in the transverse direction from the diagonal coupon (P45t) with labelled examples of the different sized \( \alpha' \) martensite.
associated with layer reheating, reducing the amount of tertiary \( \alpha' \) and quartic \( \alpha' \) present. For the formation of a greater level of ultrafine martensite, the increased depth of reheating of the solidified layers could lead to increased tertiary \( \alpha' \) and quartic \( \alpha' \) transformation due to a greater number of thermal cycles promoting \( \alpha' \) martensite transformation into \( \beta \) and then fine \( \alpha' \) martensite, occurring below the \( \beta \) transus temperature. Therefore, the combination of cyclic reheating, associated with layer disposition, in conjunction with build platform preheating and the location of the coupon on the build platform may influence the extent of the intrinsic heat treatment behaviour of the fabrication process that leads to varying levels of ultrafine martensite.

3.2. Tensile tests

Fig. 14 presents the comparison of the SLM materials’ monotonic tensile tests for the three build orientations. The horizontal and diagonal coupons showed significantly more ductility than the vertical coupon. The diagonal coupon displayed the greatest yield strength while the horizontal and vertical coupons had very comparable yield strength. The mechanical properties pertaining to the three built orientations are summarised in Table 2.

3.3. Strain-controlled tests

Table 3 presents the number of cycles to failure (endurance life) for each of the three loading conditions for each coupon, including the wrought material coupon. The endurance life of the coupons when entering into the final step of the load case varied not only between the coupons manufactured at the same build orientation but also between orientations. In all cases, the SLM coupons had a lower number of cycles to failure than their wrought counterpart. It is noted that the premature failure of the diagonal (45°) coupon in Test 3 (F452) occurred due to the influence of residual stresses in the coupon, since during this test there was significant bending of the gauge section. An uneven distribution of the residual stresses through the gauge cross-section would lead to uneven stresses on the sides of the coupon, resulting in bending of the gauge section. Once a bend exists in the gauge section, a bending moment forms during each succeeding compression cycle, ultimately leading to premature failure.

The progression of the stress amplitude with cycles is demonstrated in Fig. 15. In particular, in Fig. 15(b) and (c) cyclic softening is evidenced by the decreasing stress amplitude. However, in Fig. 15(a) the strain amplitude initially increases and then reduces in the SLM material, which suggests that an initial hardening occurs in the first few cycles before the activation of cyclic softening. However, the wrought material consistently showed cyclic softening. As expected based on the monotonic yield results, the diagonal coupon produces the greatest cyclic stress, followed by the horizontal and vertical coupons.

The SLM and wrought material hysteresis loop development is presented in Figs. 16–18, showing the first and last cycle (as indicated in Table 3) of each loading step (for the three build orientations studied). Cyclic softening is apparent in all load cases, in both the SLM and
wrought material, as evidenced by the increase in area of the hysteresis loop. It is also apparent from Fig. 16 that the wrought material has a larger hysteresis loop area compared to that of the SLM material. This is due to the higher cyclic yield strength in SLM material, owing to the microstructure being composed of α’ martensite, compared to the mill-annealed Ti-6Al-4V material used in this research being composed of globular α grains with β phase present at the grain boundaries [33]. The α’ phase has been shown to have high dislocation density [34,35], which causes dislocation tangling at the α’ grain boundaries. It also contains staking faults and twin structures [34], all of which contribute to the high strength associated with this kind of microstructure. On the other hand, since the wrought material is composed of both α and β there is less volume fraction of α present, therefore the wrought material does not have the same dislocation tangling characteristics as the SLM material. Additionally, the globular α grains in the wrought material is coarser than the α’ martensite which reduces dislocation hindrance.

![Fig. 12. SEM micrographs of polished samples showing the microstructure of (a) horizontal (F01), (b) diagonal (F45°) and (c) vertical (F90°) coupon.](image)

![Fig. 13. SEM micrograph of previously manufactured coupons (past research of the authors) in the vertical orientation showing (a) a mixture of fine and ultrafine α’ martensite and (b) coarse α’ martensite both of which are very similar to the SEM micrographs obtained in this study using the same machine processing parameters.](image)

![Fig. 14. Comparison of the monotonic tensile behaviour of the three SLM Ti-6Al-4V coupons manufactured at the three orientations [horizontal (0°), diagonal (45°), and vertical (90°)].](image)
4. Discussion

4.1. Asymmetric cyclic softening

A closer examination on the evolving tensile and compressive maximum stresses identifies an interesting phenomenon, that of asymmetric cyclic softening. Normally, during cyclic softening the maximum absolute tensile and compressive stresses will progressively decrease and similarly during cyclic hardening, the maximum absolute tensile and compressive stresses progressively increase. However, in the obtained experimental results, the anticipated behaviour was not observed in the SLM Ti-6Al-4V but only in the wrought material. Fig. 19 presents the progression of the peak stresses in tension and compression for the 1.0% strain load case in both Test 1 and Test 2, demonstrating the difference in peak stress progression, and identifying the hardening/softening behaviour. The severity of the asymmetry is apparent from a closer inspection of the hysteresis loop progression, where in Test 1 there is progressive shifting of the hysteresis loop upwards in all build orientations, as shown in Fig. 20(a)–(c). However, this is not evident in the wrought coupon, as shown in Fig. 20(d), where the 200th hysteresis loop has reduced absolute maximum and minimum stresses, as expected of a material exhibiting softening.

Comparison of the peak stresses of the 1.5% loading level in Test 1 (Fig. 21) shows a reduction in the number of coupons showing asymmetric softening from six to two. Consequently, with progressive load steps, the asymmetric softening behaviour is reduced.

In order to determine the influence of strain amplitude on asymmetric softening, a comparison of the peak stress evolution of Test 3 (1.5%) is presented in Fig. 22. The asymmetric softening phenomena are still evident but less aggressive, as opposed to the results of Test 1 at 1.0% strain. However, the asymmetry level is still high enough to cause the hysteresis loop shifting, as illustrated in Fig. 23.

The extent of the asymmetric softening is calculated using the asymmetry factor [36,37] given by Eq. (1):

\[
\text{Asymmetry Factor} = \frac{|\sigma_+ - \sigma_-|}{\sigma_+ + \sigma_-}
\]

Table 2

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Elastic modulus (MPa)</th>
<th>0.2% Yield strength (MPa)</th>
<th>Ultimate strength (MPa)</th>
<th>Strength at failure (MPa)</th>
<th>Strain at failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal (0°)</td>
<td>95,238</td>
<td>1000</td>
<td>1182</td>
<td>942</td>
<td>24%</td>
</tr>
<tr>
<td>Diagonal (45°)</td>
<td>100,000</td>
<td>1100</td>
<td>1254</td>
<td>988</td>
<td>24%</td>
</tr>
<tr>
<td>Vertical (90°)</td>
<td>103,700</td>
<td>1037</td>
<td>1181</td>
<td>1102</td>
<td>7%</td>
</tr>
</tbody>
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Table 3

<table>
<thead>
<tr>
<th>Test coupon</th>
<th>Load level</th>
<th>1 ± 1.0%</th>
<th>2 ± 1.5%</th>
<th>3 ± 2.0%</th>
<th>Number of cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>F01</td>
<td>200</td>
<td>200</td>
<td>39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F02</td>
<td>200</td>
<td>200</td>
<td>84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F03</td>
<td>200</td>
<td>200</td>
<td>121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F451</td>
<td>200</td>
<td>200</td>
<td>63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F452</td>
<td>200</td>
<td>31</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>F453</td>
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<td>200</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F901</td>
<td>200</td>
<td>200</td>
<td>26</td>
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<td></td>
</tr>
<tr>
<td>F902</td>
<td>200</td>
<td>200</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F903</td>
<td>200</td>
<td>200</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrought</td>
<td>200</td>
<td>200</td>
<td>159</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 16. Test 1 first and last cycle comparison for each load level for the SLM material built at (a) horizontal (0°) (b), diagonal (45°) and (c) vertical (90°) orientation and (d) the wrought material.

Fig. 17. Test 2 first and last cycle comparison for each load level for the SLM material built at (a) horizontal (0°) (b), diagonal (45°) and (c) vertical (90°) orientation.
\[
A = \frac{|\sigma_c| - |\sigma_t|}{(|\sigma_c| + |\sigma_t|)/2}
\]  

where \(\sigma_c\) and \(\sigma_t\) are the maximum compressive and tensile stresses respectively.

Considering the influence of the strain amplitude on the extent of asymmetric softening, the relative asymmetry factors have been plotted for both the horizontal and vertical coupons, shown in Fig. 24. The asymmetry factor is significantly less in the 1.5% Test 3 load case than the 1.0% Test 1 load case, which suggests the asymmetric softening effect diminishes with increasing strain amplitude.

This asymmetry may be attributed to a thermal residual stress, which in turn induces an asymmetric yield stress. Residual stresses are developed during the build process due to the expansion and contraction interaction between layers [38,39] and have been measured in a number of SLM Ti-6Al-4V builds [14,40,41]. To better understand the influence of the residual stresses on the plastic behaviour of the material, the macroscopic characteristics of isotropic hardening and kinematic hardening evolution were examined. Isotropic hardening describes the expansion or contraction of the yield surface (yield stress in the uniaxial stress space), while kinematic hardening accounts for the Bauschinger effect (represented by a back stress \(X\)). The size of the yield surface is defined as the effective stress \(\sigma_{\text{eff}} = R + \sigma_0\), where \(R\) is the isotropic hardening stress and \(\sigma_0\) is the initial yield stress. The contraction and translation of the yield surface and the corresponding hysteresis loop development is presented schematically in Fig. 25.

At microstructure level, effective stress represents the local stress required for dislocation movement [42]. Similarly, the back stress is...
related to the strain heterogeneities which influence mobile dislocations [42]. The effect of the residual stress on the effective stress and back stress evolution can be indirectly witnessed in Fig. 26. In particular, Fig. 26 compares the evolution of effective stress/back stress in compression and tension for 1.0% strain (Test 1), where the influence of the residual stresses are apparent from the significant asymmetry in evolution in compression and tension. However, this asymmetry is reduced when comparing the effective stress and back stress obtained from the 1.5% strain step in the Test 1 loading sequence. This suggests that the influence of the residual stresses are diminishing with progressive cycles, which is the consequence of a relaxation of the residual stresses [43].

Another interesting research finding has to do with the influence of the initial strain amplitude on the residual stresses. In particular, it was observed that a larger initial strain amplitude causes a reduction in the residual stress. This is identified in Fig. 27 as the significant reduction in asymmetry during the evolution of the effective stress and back stress in compression and tension. This is the consequence of increased tensile

Fig. 20. First and 200th cycle comparison for Test 1 step loading for the SLM Ti-6Al-4V (a) horizontal (0°), (b) diagonal (45°) and (c) vertical (90°) build orientation and (d) wrought material.

Fig. 21. Peak stress evaluation with cycles for Test 1 at 1.5% strain-controlled load case for (a) tension and (b) compression.
cold work which can cause faster relaxation of the residual stress [44].

From this analysis, one can assume that although the build platform was heated to 200°C, remaining residual stresses may still be present. These can be large enough not only to cause premature failure for the diagonal (45°) test coupon in Test 3 but also to induce an asymmetric yield stress. However, this assumption would have to be investigated further.

4.2. Variation of the Elasticity modulus

In order to investigate the formation of material internal defects occurring during cyclic loading, the change in Elasticity Modulus across loading cycles was examined. This choice (Elasticity Modulus measurement) is based on the understanding that a reduction in the effective cross section due to the formation of defects, will lead to a decreasing material stiffness [45,46]. In Fig. 28 graph, the reduction in the Elasticity Modulus is apparent during all loading steps in the SLM material. In the wrought material, one can observe the negligible change in the Elasticity Modulus at the 1.0% strain amplitude test. This
suggests that no defects had appeared in the material during this loading, or the defects which did start to nucleate were too small to have any macroscopic effect.

4.3. Build orientation anisotropy

The experimental results of this study have also identified that the build orientation has an influence on both the monotonic and cyclic yield stress of the material. This is illustrated by plotting the loading branch of the final cycle of each load case (for each build orientation), shown in Fig. 29.

As with the case of the monotonic results, the diagonal (45°) coupon had the highest cyclic yield stress, followed by the horizontal (0°) and the vertical (90°) coupons.

It has been previously reported that the horizontally (0°) SLM manufactured coupons have a slightly higher yield strength than the vertically (90°) manufactured coupons in monotonic yielding [14,47,48]. This is consistent with the present study's monotonic and cyclic test results. However, what has also been identified from these research results is that the Ti-6Al-4V diagonal (45°) coupon has the largest yield strength in both monotonic and cyclic loading, which has also been observed in the case of SLM fabricated material Inconel alloy 718 diagonal (45°) coupons (both monotonic and cyclic results) [49], which suggests that the improved yield strength in the diagonal (45°) coupon could be related to build orientation dependent factors.

The monotonic and cyclic yield strength anisotropy is the consequence of a number of different microstructure characteristics. The α phase present in the coupons contributes to the yield strength anisotropy since α titanium is plastically anisotropic [50]. The orientation of the grains to a preferred slip system will promote dislocation movement; therefore, the mechanical properties are dependent on the crystallographic direction. Vertically (90°) built coupons have been shown to contain a larger number of grains in a stress state which are easier to slip than horizontal (0°) coupons [51]. Using the same method as that in [51], the plastic deformation ability of the diagonal coupon was determined by calculating the Schmid factor for the diagonal (45°) build coupon, since it gives an indication of the ease of slip system activation [52]. The Schmid factor for some typical slip systems [51] was calculated using Eq. (2) [51,53]:

$$m = \cos \theta \cos \lambda$$

Where θ is the angle between the slip plane normal and loading direction, and λ is the angle between the slip and loading directions.

The Schmid factors calculated for a 45° force (loading 2) are presented in Table 4. These values are compared with those obtained for a vertical (loading 3) and horizontal (loading 1) loading force.

As previously reported there is a large volume fraction of the crystallographic orientation shown in Fig. 30 in the SLM Ti-6Al-4V [51]. In horizontal (0°) coupons, this orientation is subjected to Loading 1, where the Schmid factors are significantly smaller than in Loading 2 and 3. Since this common crystallographic orientation is subjected to Loading 1 and 3 in diagonal (45°) and vertical (90°) coupons respectively, slip occurs more readily in diagonal (45°) and vertical (90°) coupons than horizontal coupons. This can explain the larger yield strength in horizontal (0°) coupons than vertical (90°) coupons. However, this does not explain why the diagonal coupon showed the largest yield strength of the three orientations, therefore, the slip systems cannot effectively explain this phenomenon.

During the SLM process, voids or pores can form due to insufficient...
laser energy [54,55], which can also influence the removal of gaseous bubbles from the melt pool, resulting in various levels of pore formation due to entrapped gas [48,56]. Further defects exist associated with lack of fusion between layers [48]. These defects can influence the yield strength of the coupons [57], depending on orientation of the defects to

![Fig. 29. Comparison of the cyclic stress-strain loading branches formed for each of the three tests for each build orientation (refer to Table 3 summarized test data) (a) Test 1 first cycle Loading branch for 1%, 1.5%, and 2% (b) Test 1 last cycle loading branch for 1% (200th cycle), 1.5% (200th cycle), 2% (26th cycle) (c) Test 2 first cycle loading branch for 1%, 1.5%, and 2% (d) Test 2 last cycle loading branch for 1% (200th cycle), 1.5% (200th cycle), and 2% (10th cycle) (e) Test 3 first cycle loading branch for 1.5%, and 2% (f) Test 3 last cycle loading branch for 1.5% (200th cycle), and 2% (63rd cycle).]

![Fig. 30. Common crystallographic orientation in SLM Ti-6Al-4V with loading directions.]

<table>
<thead>
<tr>
<th>Slip system</th>
<th>Loading 1</th>
<th>Loading 2</th>
<th>Loading 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basal (&lt; a ) slip [0002] (&lt; , , , , , 11\overline{2}1 ) &gt;</td>
<td>0</td>
<td>0.245</td>
<td>0</td>
</tr>
<tr>
<td>Prismatic (&lt; a ) slip [10\overline{1}2 , 0] (&lt; , , , , , 11\overline{2}1 ) &gt;</td>
<td>0</td>
<td>0.173</td>
<td>0.433</td>
</tr>
<tr>
<td>Pyramidal (&lt; a ) slip [10\overline{1}2 , 1] (&lt; , , , , , 11\overline{2}0 ) &gt;</td>
<td>0</td>
<td>0.2828</td>
<td>0.224</td>
</tr>
<tr>
<td>Prismatic (&lt; a + c ) slip [11\overline{2}2] (&lt; , , , , , 11\overline{2}3 ) &gt;</td>
<td>0.327</td>
<td>0.588</td>
<td>0.286</td>
</tr>
</tbody>
</table>

![Table 4. Schmid factors for slip systems identified by Yang et al. [51] for three different loading directions.]

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the loading direction, which could explain the anisotropy in yield strength between the orientations. The presence of defects can also explain the difference in ductility between the build orientations. The vertical (90°) coupon is expected to have lower ductility due to loading opening up the defects, while the diagonal (45°) and horizontal (0°) coupons should have greater ductility due to the orientation of the defects resulting in the loading pulling on the defects rather than opening them [48].

Finally, it is also possible that the difference in yield strength and ductility can be explained with respect to the combination of varying volumes of coarse and ultrafine α′ martensite. The grain size can have a significant influence on the yield strength of Ti-6Al-4V, where ultrafine grained Ti-6Al-4V can have much higher yield strength than a coarse grained version of the same phase distribution [58]. Therefore, slight variations in yield strength can occur with alterations in the volume fraction of ultrafine α′. The increased yield strength with grain refinement is typically associated with the loss of ductility [59]. However, with reference to the monotonic tension results listed in Table 2, the diagonal coupon had not only the highest yield strength but also the largest elongation. This may be the consequence of a combination of both ultrafine grains and coarse grains in the microstructure. A low volume content of coarse grains has the potential to reduce the loss of elongation expected from an ultrafine grained microstructure by increasing the strain hardening capacity through improved dislocation storage [60]. This is expected to delay the onset of necking instability in tension as governed by the Considere criterion described by Eq. (3), where σ and ε are the true stress and true strain respectively, while the dot over strain denotes rate (time derivative of strain).

\[
\frac{\partial \sigma}{\partial \varepsilon}_t \leq \sigma
\]  

(3)

Consequently, the anisotropy and improved yield strength and ductility in the diagonal coupon could be possibly attributed to an improved distribution of ultrafine and coarse α′ martensite, which was preferentially developed through the intrinsic heat treatment associated with build orientation and build platform location.

5. Conclusion

The microstructure and LCF behaviour of SLM Ti-6Al-4V manufactured at horizontal (0°), diagonal (45°), and vertical (90°) orientations were investigated experimentally. From this research the following conclusions can be drawn:

1. Etched samples obtained from the three build orientations showed a number of different microstructure features, including: α rods and dots, primary α′, secondary α′, tertiary α′, quartic α′, nano-dispersoids, lamellar growth, and β particles.

2. Diagonal (45°) and vertical (90°) orientated coupons contained ultrafine α′ martensite, while SEM images of the horizontal (0°) coupon manufactured using the same processing parameters as the other two orientations showed a coarser α′ martensite structure. The difference in microstructure was hypothesised to be affected by the coupons position on the build plate, with gas flow effects influencing laser beam energy, and, therefore, the formation of martensite.

3. The residual stresses present in the SLM coupons induced during the fabrication process had a significant influence on the cyclic behaviour of the material. Closer investigation of the absolute minimum and maximum stresses showed an asymmetric softening behaviour, which lead to a vertically shifting hysteresis loop under symmetric strain loading. This was the consequence of the residual stresses influencing the yield stress development, which was further validated by comparing the effective stress and back stress evolution in compression and tension. The residual stress influence progressively diminished with increasing cycles and was less severe with increasing initial strain amplitude.

4. Mechanical anisotropy in both monotonic and cyclic tests was noticed with the diagonal (45°) coupon having the largest yield stress in both loading conditions. The diagonal (45°) and horizontal (0°) coupons also showed significantly more ductility than the vertical (90°) coupon.

Acknowledgments

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References


Chapter II.3  Cyclic Plasticity Modelling of SLM Ti-6Al-4V

This chapter investigates the ability of several nonlinear kinematic hardening rules of different formulations, to capture the cyclic transient effects of vertically manufactured SLM Ti-6Al-4V. The applied hardening rules were included in an elastoplastic constitutive model, which also contained an isotropic softening rule to capture the noticed cyclic softening in the experimentally gathered results. The success of the kinematic hardening rules at capturing different elastoplastic features were compared to determine the most appropriately formulated constitutive model to simulate the elastoplasticity.
Constitutive Modeling of Additive Manufactured Ti-6Al-4V Cyclic Elastoplastic Behaviour

(Paper 10)

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(Technische Mechanik)
Constitutive Modeling of Additive Manufactured Ti-6Al-4V Cyclic Elastoplastic Behaviour

K. I. Kourousis, D. Agius, C. Wang, A. Subic

Metal additive manufacturing techniques have been increasingly attracting the interest of the aerospace and biomedical industry. A particular focus has been on high value and complexity parts and components, as there the advantages offered by additive manufacturing are very significant for the design and production organisations. Various additive manufacturing techniques have been tested and utilized over the past years, with laser-based technology being among the preferred solutions – e.g. selective laser melting / sintering (SLM / SLS). Fatigue qualification, as one of the primary design challenges to meet, imposes the need for extensive material testing. Moreover, this need is amplified by the fact that currently there is very limited in-service experience and understanding of the distinct mechanical behaviour of additively manufactured metallic materials. To this end, material modelling can serve as a mediator, nevertheless research particular to additively manufactured metals is also quite limited. This work attempts to identify the cyclic elastoplastic behaviour characteristics of SLM manufactured Ti-6Al-4V. A set of uniaxial stress and strain controlled mechanical tests have been conducted on as-built SLM coupons. Phenomena critical for engineering applications and interrelated to fatigue performance (mean stress relaxation, ratcheting) have been examined under the prism of constitutive modeling. Cyclic plasticity models have been successfully employed to simulate the test results. Moreover, a preliminary analysis has been conducted on the differences observed in the elastoplastic behaviour of SLM and conventionally manufactured Ti-6Al-4V and their possible connection to material performance in the high cycle fatigue regime.

1 Introduction

Additive manufacturing of three-dimensional metallic components involves laser assisted depositing and melting of powder metallic materials layer-by-layer, starting from computer aided designs. As additive manufacturing (unlike traditional subtractive manufacturing) is not constrained by geometry or shape, the design paradigm behind additively manufactured components is fundamentally different as it is driven by function (with form following function). This approach offers significant opportunities for light-weighting by identifying key functional features and then organically developing the minimal geometry that will support these functions. Furthermore, this allows for multiple parts to be replaced by a single more complex component by integrating or merging a number of different functions into a single component. To achieve this, we use different computational methods, such as structural and shape optimisation methods based on evolutionary algorithms or topographic optimisation algorithms (Leary et al., 2009; Huang et al., 2012; Leary et al., 2013a; Leary et al., 2013b).

In the past 20 years, additive manufacturing technology has developed from simple three dimensional (3D) printers used to generate non-structural resins into a sophisticated manufacturing process capable to produce objects without the use of tools. Most recently, the focus has been shifted from polymers to metals, primarily aiming to cover the needs of the aerospace, biomedical and automobile industry. Weight reduction of metallic parts using additive manufacturing processes and topographic structural optimisation could go beyond 50% compared to traditional computational and manufacturing methods applied to the same part and material.

High precision manufacturing of Titanium alloys, such as the Ti-6Al-4V, is of primary importance for the biomedical and aerospace industry, due to the high cost involved and the engineering significance of the applications. Various additively manufacturing techniques have been developed, with selective laser melting (SLM) being one of the most commonly used. The SLM process emerged between the late 1980s and early 1990s (Vrancken et al., 2012). The SLM technique uses an infrared fiber laser which assembles solid layers out of loose powder material. In principle, a thin layer of loose powder is initially levelled across a processed platform and selected areas of the powder are melted and consolidated, via a scanning laser beam, in a serial pattern (Simonelli et al., 2012). In comparison with the conventional manufacturing techniques, SLM offers many advantages, such as the reduction of production steps, high level of flexibility, high efficiency in the use of material use and a near
net shape production. However, there are several important effects that need to be avoided: internal stresses development, occurring from steep temperature gradients and high cooling rates during the manufacturing process, and increased porosity. All the effects have an impact on the mechanical behaviour of the material under cyclic loading and low or high cycle fatigue.

Ti-6Al-4V is one of metal alloys employed for SLM produced parts, mainly for applications of high value, performance and complexity, due to its high strength and strength-to-weight ratio. A wide spectrum of mechanical properties over a range of temperatures can be achieved by varying the microstructure of the dual-phase Ti-6Al-4V through appropriate heat treatment and thermomechanical processing (Zhang et al., 2007). However, heat treatment adds time and cost to the production process, reducing partly the advantages offered by additive manufacturing. Moreover, customisation of the microstructure may be achieved through altering the SLM manufacturing parameters. In this case the as-built material can be utilised without further processing or treatment, which is highly preferable from a manufacturing cost point of view. Various researchers have examined the monotonic and high cycle fatigue behaviour of SLM Ti-6Al-4V, nevertheless its elastoplastic response under cyclic loading has not been investigated in much detail, e.g. Leuders et al. (2013). In this context, modelling and simulation of the cyclic elastoplastic behaviour of as-built SLM Ti-6Al-4V is expected to complement the knowledge of researchers and engineers aiming to identify the characteristics of such alloys. This study aims to provide a set of preliminary results from a computational work conducted on the simulation of the cyclic elastoplastic response of as-built SLM Ti-6Al-4V through the implementation of rate independent plasticity models. For this purpose, experimental results are utilised, obtained from an ongoing test campaign conducted at the Centre for Additive Manufacturing of RMIT University.

2 Plasticity Models

Many rate-independent plasticity models for metal material simulation have been developed over the past thirty years. However, only a relatively small fraction of these models enjoy today a wide acceptance by the research community and the industry. Three well established plasticity models (Multicomponent Armstrong-Frederick model, Multicomponent Armstrong-Frederick model with threshold term and Ohno-Wang model) are utilised to model and simulate the behaviour of as-built SLM Ti-6Al-4V. Moreover, a more recently proposed plasticity model (Multicomponent AF with Multiplier) is included in this investigation, in an effort to compare and contrast its performance with this set of widely accepted models.

2.1 Multicomponent Armstrong-Frederick (MAF)

The Multicomponent Armstrong-Frederick (MAF) model is in practice a superposition of a number of Armstrong-Frederick (AF) (Armstrong and Frederick, 1966) back-stress terms (Chaboche et al., 1979). This addition of AF terms, commonly three for most applications, produces a more effective representation of the transient and stabilised stress-strain curves when compared to a model with a single AF term. In practice, each of the different back-stress terms captures the different features of the hysteresis loops. The uniaxial formulation of the MAF model is provided by the following set of equations:

\[ a = \sum a_i, \text{ for } i = 1, 2, 3 \]

(1)

Where, \( a \) is the total backstress and \( a_i \) the added backstresses, each governed by equation:

\[ \dot{a}_i = c_i \left( a_i^k - a_i \right) \dot{\varepsilon}^p \]

(2)

Dot over quantities shown in equation (2) and elsewhere denotes the time derivative (rate) of backstress \( a_i \) and plastic strain \( \dot{\varepsilon}^p \). The model material parameters are represented by \( c_i \) (backstress evolution pace) and \( a_i^k \) (backstress saturation value).

The MAF model has been employed extensively in many applications by both scientists and engineers. It is incorporated as a standard feature in most commercial finite element analysis software (e.g. Abaqus, ANSYS), however it has very limited capabilities in simulating ratcheting effectively. A series of modifications have been proposed to account for this drawback, especially in relation to the improvement of multiaxial ratcheting simulation.
2.2 Multicomponent Armstrong-Frederick with Threshold term (MAF-T)

The ratcheting deficiencies associated with the MAF model were improved with the introduction of a fourth backstress which included a threshold term (Chaboche, 1991). This model is abbreviated as MAF-T (MAF with Threshold term). The threshold stress level, controlled by the corresponding term, signifies a transition point whereby up to it the kinematic hardening rule develops linearly and over that point it evolves according to the original AF rule (nonlinearly). The motivating factor behind the introduction of the threshold term was to reduce the rate of ratcheting predicted by the MAF model by stiffening the loading curve and relaxing the unloading curve. Similarly to the MAF model the total back stress in the MAF-T model is given by the addition of different backstress, as per equation (1). For the uniaxial case, for four backstresses, the following set of equations apply:

\[ \dot{a}_i = c_i \left( a_i^s - a_i \right) \dot{\epsilon}^p, \text{ for } i = 1, 2, 3 \]  
\[ \dot{a}_4 = c_i \left( a_i^s - a_i \left( 1 - \frac{\eta_i}{a_i^s} \right) \right) \dot{\epsilon}^p, \text{ for } i = 4 \]

Where, the model material parameters are represented by \( c_i \) (backstress evolution pace), \( a_i^s \) (backstress saturation value) and \( \eta_i \) (threshold term). The symbol \( \{ \} \) represents the Macaulay brackets.

2.3 Ohno-Wang (O-W)

The Ohno-Wang (O-W) model (Ohno and Wang, 1993a; Ohno and Wang, 1993b) originally acted as a multilinear model, unlike to the MAF and MAF-T models, which predicted no uniaxial ratcheting. In order to overcome this issue a multiplier was added to the dynamic recovery term of the AF backstress, which introduced a slight nonlinearity to each of the defined backstress equations. The uniaxial formulation of the O-W model is as follows [again, for four added backstress terms, as per equation (1)]:

\[ \dot{a}_i = c_i \left( a_i^s - a_i \right) \dot{\epsilon}^p, \text{ for } i = 1, 2, 3 \]  
\[ \dot{a}_4 = c_i \left( a_i^s - a_i \left( \frac{a_i}{a_i^s} \right)^m \right) \dot{\epsilon}^p, \text{ for } i = 4 \]

Where, the model material parameters are represented by \( c_i \) (backstress evolution pace), \( a_i^s \) (backstress saturation value) and \( m \) (multiplier). The multiplier is calculated with reference to ratcheting or mean stress relaxation data. As shown by Bari and Hassan (2000) for small values of \( m \), the higher the predicted ratcheting rate is.

2.4 Multicomponent AF with Multiplier (MAFM)

The Multicomponent AF with Multiplier (MAFM) model was introduced as an improved alternative to the MAF-T model (Dafalias et al., 2008). As a consequence of the incorporation of the threshold term in the original MAF model, an un-physical linear portion of the hysteresis loop was formed. However, the MAFM model is able to improve this issue without affecting the ratcheting simulation accuracy (Dafalias et al., 2008). This was achieved with the introduction of a multiplier term into one of the AF backstresses. The multiplier is used to adjust the rate at which saturation of the backstress occurs, without affecting the saturation level. This operation is analogous to the MAF-T model’s threshold term, without the need to check the exceedance of a threshold (which is a computationally expensive task). The uniaxial formulation of MAFM model, for four backstresses [obeying to an additive decomposition, as per equation (1)], is given by the following set of equations:

\[ \dot{a}_i = c_i \left( a_i^s - a_i \right) \dot{\epsilon}^p, \text{ for } i = 1, 2, 3 \]  
\[ \dot{a}_4 = \left[ c_i + c_i^* \left( a_i^{s*} - a_i^* \right) \right] \left( a_i^s - a_i \right) \dot{\epsilon}^p, \text{ for } i = 4 \]

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With $a^*_4$ (dimensionless multiplier) given by:

$$a^*_4 = c^*_4 \left( a^*_4 - a^* \right) \dot{\epsilon}^p$$  \hspace{1cm} (9)

Where, the model material parameters are represented by $c_i$ (backstress evolution pace), $a^*_{i}$ (backstress saturation value), $c^*_4$ ($a^*_4$ multiplier backstresess evolution pace) and $a^*_{s}$ ($a^*_s$ multiplier backstress saturation value).

### 2.5 Isotropic Hardening

A nonlinear isotropic hardening rule was also incorporated along each of the kinematic hardening models examined. This addition was deemed necessary for the effective simulation of the strong cyclic softening characteristics of the material under study (Ti-6Al-4V). The governing equation (in uniaxial stress space) for the yield stress ($k$) is provided below:

$$k = c_k \left( k^* - k \right) \dot{\epsilon}^p$$  \hspace{1cm} (10)

Where, $c_k$ the parameter controlling the evolution pace and $k^*$ the saturation value of the yield stress ($k$).

### 3 Experimental Data

The experimental data utilised for the purposes of this computational study were obtained from mechanical tests previously conducted at the RMIT University Centre for Advanced Manufacturing. These test data come from tests performed on cylindrical cross section specimens of as-built SLM Ti-6Al-4V material manufactured with an SLM Solutions SLM® 250 HL machine. This manufacturing equipment has a rated power of 400 W under continuous laser mode and a minimal laser spot size of 80 microns. The specimens have been fabricated with a laser power of 175 W, at a laser scanning speed of 710 mm/s, 120μm hatch spacing and a 30μm powder layer thickness. The platform was preheated to 200°C prior to the building process and each specimen had a total of 2,746 built layers.

The microstructure of the obtained specimens was examined with the use of optical microscopy (Phaiboonworachat, 2014). Ti-6Al-4V alloys are composed of two phases, the α-phase (Hexagonal Close Packed, HCP structure) and the β-phase (Body Centered Cubic, BCC), while a $α^*β$ phase can be present simultaneously. The examined SLM specimens’ samples revealed the presence of a martensitic microstructure ($α^*$-phase). In particular, the characteristic needle shaped thin lamellas (lath martensite) of the $α^*$-phase have been observed (Figure 1a). Moreover, prior $β$-phase acicular shaped columns, with random orientation, have been identified on the SLM material surface (Figure 1b). The length of the prior $β$-phase column has been measured as 183.8μm.

![Figure 1. SLM Ti-6Al-4V microstructure micrographs presenting (a) needle shaped thin lamellas $α^*$-phase and (b) characteristic $β$-phase acicular shaped columns.](image-url)
A comparison of the microstructure of the SLM material to a wrought material was also conducted. Unlike to the needle shaped lamellas observed in the SLM material (Figure 2a), the wrought material is dominated by a spheroidite microstructure containing α-phase and β-phase (bright areas and dark spots respectively, shown in Figure 2b).

![Figure 2. Comparison of the microstructure of (a) SLM Ti-6Al-4V and (b) wrought Ti-6Al-4V.](image)

The specimens were tested under strain and stress controlled loading histories, summarised in the Table 1 and Table 2 correspondingly. The test control (input) parameters for both loading histories are the following:

- **Strain Controlled Tests**: Symmetric (zero mean strain) cycling at different strain amplitudes ($\varepsilon_a$) for a given number of cycles (150 cycles or lower, where the specimen experienced a failure) (Table 1);
- **Stress Controlled Tests**: Non-symmetric cycling under different combinations of mean stress ($\sigma_m$) and stress amplitude ($\sigma_a$) pairs, until specimen failure (Table 2).

<table>
<thead>
<tr>
<th>Loading History</th>
<th>Strain Amplitude $\varepsilon_a$</th>
<th>Number of Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>$\pm 1.0%$</td>
<td>150</td>
</tr>
<tr>
<td>Test 2</td>
<td>$\pm 1.5%$</td>
<td>150</td>
</tr>
<tr>
<td>Test 3</td>
<td>$\pm 2.0%$</td>
<td>150</td>
</tr>
<tr>
<td>Test 4</td>
<td>$\pm 2.5%$</td>
<td>88*</td>
</tr>
</tbody>
</table>

*specimen failure

Table 1. Strain controlled loading histories.
Table 2. Stress controlled loading histories.

<table>
<thead>
<tr>
<th>Loading History</th>
<th>Stress Amplitude $\sigma_a$</th>
<th>Mean Stress $\sigma_m$</th>
<th>Minimum, Maximum Stress $(\sigma_{\text{min}}, \sigma_{\text{max}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>865 MPa</td>
<td>15 MPa</td>
<td>(-850, +880) MPa</td>
</tr>
<tr>
<td>Test 2</td>
<td></td>
<td>35 MPa</td>
<td>(-830, +900) MPa</td>
</tr>
<tr>
<td>Test 3</td>
<td>875 MPa</td>
<td>30 MPa</td>
<td>(-845, +905) MPa</td>
</tr>
<tr>
<td>Test 4</td>
<td></td>
<td>40 MPa</td>
<td>(-835, +915) MPa</td>
</tr>
</tbody>
</table>

The test data (results) are not presented independently in this section but in conjunction with the obtained simulation results in the next sections of this paper. However, some important observations are highlighted here, as these are relevant to the objective and focus of this study. In particular, the cyclic tests performed on the as-built SLM material, resulting in elastoplastic behaviour, signify important features related to the fatigue performance of such materials. Figure 3 presents the hysteresis loops obtained from the as-built SLM Ti-6Al-4V strain controlled tests in comparison with previously published experimental data (Mayeur et al., 2008).

By examining Figure 3 it is noticed that the SLM material exhibits, for each of the four strain levels, a strong cyclic softening behaviour. Moreover, the yield stress, ranging from approximately 917 MPa to 1,096 MPa, is achieved at relatively low strains. This causes a compression in the shape of the hysteresis loops, which is consistent with the brittle nature of the as-built SLM material and its low capacity to undertake plastic deformation during cyclic loading (low plastic deformation dissipation). This particular feature of the studied SLM material is evident in the hysteresis loops corresponding to ±1% strain level, as compared to experimental results for wrought Ti-6Al-4V, obtained from the literature (Mayeur et al., 2008), both shown in Figure 3. It is also noticed that the as-built SLM material exhibits a higher load capacity at the expense of reduced ductility. In summary, one may conclude that:

- High strength and low ductility under cyclic loading are the primary characteristics observed, which are opposed to lower values applicable for wrought Ti-6Al-4V;
- Cyclic softening behaviour observed at all strain levels is also of notice;
- Plastic deformation dissipation, approximated by the area enclosed by the hysteresis loops, is lower for the SLM Ti-6Al-4V as compared to wrought alloy. This, in turn, is believed to have a negative effect on the low and high cycle fatigue life of the materials, as previously reported by Leuders et al. (2013) and Xu et al. (2015).
4 Numerical Implementation

Each of the plasticity models, namely MAF, MAF-T, O-W and MAFM, was implemented numerically in Matlab. The model parameters were obtained with the use of the applicable (for each model) methodology and consequently optimised through a manual iterative fine-tuning process. In particular, the process outlined in Bari and Hassan (2000) and Dafalias et al. (2008) was used to determine the material parameters. The values for the $c_i$ and $\alpha_i$ parameters were calculated with reference to the stabilised cyclic stress-strain curve, while the parameters which control the ratcheting rate ($\alpha_4^*, \beta_4^*, \alpha_5^*$ and $m$) were determined with reference to ratcheting data. Adjustment of these parameters was made until the models could capture sufficiently the amount of ratcheting for a particular load case. It was important that such parameter adjustment was double-checked against the stabilised cyclic stress-strain response to ensure the simulation of the stabilised stress-strain curve was not compromised. The isotropic hardening values ($c_k$, $k^3$) were estimated by fitting the uniaxial integrated isotropic hardening equation to stress range versus accumulated plastic strain data in order to define the relationship between these two measures. The final set of material parameters obtained through the aforementioned process, summarised in Table 3, was utilised for the simulation of the experimental results.

<table>
<thead>
<tr>
<th>Backstress</th>
<th>MAF</th>
<th>MAF-T</th>
<th>O-W</th>
<th>MAFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial yield stress ($\sigma_y$)</td>
<td>490 MPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity Modulus ($E$)</td>
<td>101 GPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>$\alpha_1$</td>
<td>105.00 MPa</td>
<td>122.50 MPa</td>
<td>38.96 MPa</td>
</tr>
<tr>
<td></td>
<td>$c_1$</td>
<td>400</td>
<td>400</td>
<td>1,540</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$\alpha_2$</td>
<td>69.52 MPa</td>
<td>79.52 MPa</td>
<td>74.74 MPa</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>300</td>
<td>300</td>
<td>553</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$\alpha_3$</td>
<td>242.10 MPa</td>
<td>140.65 MPa</td>
<td>21.40 MPa</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>69</td>
<td>69</td>
<td>100</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$\alpha_4$</td>
<td>295.7 MPa</td>
<td>153.85 MPa</td>
<td>9.57 MPa</td>
</tr>
<tr>
<td></td>
<td>$c_4$</td>
<td>10</td>
<td>26</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$\beta_4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_4^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta_4^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_4^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isotropic hardening parameters</td>
<td>$c_k$</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k^3$</td>
<td>180 Mpa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Material parameters for plasticity models (MAF, MAF-T, O-W and MAFM)
5 Results

The simulation results obtained for the different models are compared against test data for each of the different loading histories (strain / stress controlled) shown in Table 1 and Table 2. These are presented and discussed in detail in the following sections.

5.1 Strain Controlled Results

The stabilised hysteresis loops at each strain level (1.0%, 1.5%, 2.0% and 2.5%) are presented in Figure 4. It is observed that all models are capable to capture effectively the shape and characteristics of the loops.

A closer look at the cyclic hardening behaviour of the material was considered of interest. This was performed through the calculation of the cyclic hardening factor \( H \) at different strain levels. The cyclic hardening factor \( H \) is given by the following equation:

\[
H = \frac{\sigma_{\text{sat}} - \sigma_0}{\sigma_0}
\]  

(11)

Where, where \( \sigma_{\text{sat}} \) the stress amplitude is in the first cycle and \( \sigma_0 \) is the stress amplitude of any number of cycles.

The results (experimental and computational) are presented in Figure 5. All models perform fairly acceptable. However, one may note that the last point of the experimental curve (corresponding to 2.5% strain) was not captured by any model. This was attributed to the fact that this point corresponds to the prematurely failed specimen (at 88 cycles), which did not allow the cyclic phenomenon evolve fully (no stabilisation occurred).
The material mean stress relaxation was also investigated. For this purpose, the stress amplitude variation was plotted against the number of cycles until saturation, both for experimental and computational results. Figure 6 presents the obtained results for each of the different strain levels and for all models compared. One may note that the models perform better at higher strain levels (2.0% and 2.5%), capturing quite well the experimental curves. All models have similar behaviour in terms of curve fitting, however a closer look at each model’s performance is discussed in greater detail in the sequel.
An interesting analysis, from the point of view of material fatigue performance, has been attempted via the calculation of the plastic deformation energy dissipation. The capacity of the SLM material to absorb plastic energy was considered important for the completeness of this study, since no such results have been reported in the past, to the best knowledge of the authors. To this end, the plastic energy density (corresponding to the hysteresis loop area) was plotted against the number of cycles, both for test and simulated data. Figure 7 presents these results, while Figure 8 presents the percentile error (%error) between test and simulated data for each model compared. A variation between models, in terms of their capability to capture the test data, is quite evident from these results. This observation provides a useful way to assess more accurately and perhaps in a more representative way, the plasticity models’ performance in reference to fatigue behaviour. However, an overall comparison of the models is necessary. This needed to be put under the prism of capturing the various test cases (strain and stress controlled) and characteristic response resulting from each loading cases. For this purpose, a ranking scheme is proposed as a simple way to obtain quantitative results on the overall performance of the models. This scheme is presented in the sequel.
Figure 7. Plastic Strain Energy Density, per cycle, for experimental and simulated results.

Figure 8. Plastic Strain Energy Density percentile error (%Error), per cycle, between experimental and computed results for each of the models compared.
5.2 Stress Controlled Results

The investigation of ratcheting behaviour (plastic strain accumulation under cyclic loading) of the SLM material (as-built Ti-6Al-4V) was a focal point for this research. Such results, experimental and computational, have not been reported so far, to the best knowledge of the authors, and were considered important for the completeness of this work. As mentioned, a set of different loading histories was utilised to obtain the ratcheting curves (as detailed in Table 2). Figure 9 presents the experimental and computational ratcheting results for each stress pair, while Figure 10 presents the percentile error (%Error) between computational and experimental results (for each of the models compared). One may notice that, again, the performance of the different models is varied, especially when observing the %Error curves. These findings are instrumental for the overall comparison and assessment of the plasticity models.

![Ratcheting Results](image)

Figure 9. Ratcheting strain per cycle for each of the stress controlled loading cases for experiments and computed results, for each of the models compared.
5.3 Overall Performance of the Models

The need for a simple way to conduct an overall comparison is apparent by examining the various results obtained, for both strain and stress controlled loading histories. The four plasticity models have proven to be, in general, capable to simulate effectively complex elastoplastic phenomena. However, when it comes to electing a model to be used in actual engineering applications it is not apparent or at least easily identifiable which would be the most suitable one. For the engineering practitioner such a decision would be done on the basis of a good trade-off between the different characteristics, advantages and disadvantages of each model. Effectively, a ranking scheme could be considered as a good tool to assist the decision making process. Such a ranking scheme is exemplified in this study.

The proposed scoring scheme relies on a weighting matrix which assigns different points at the various levels of percentile error (%error) achieved by each of the models. Lower performance (greater %Error) is assigned with lower points and the opposite. For simplicity, four different error levels and discrete point values were used for this example ranking scheme (Table 4).

<table>
<thead>
<tr>
<th>Error Level</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error &lt; 5%</td>
<td>3</td>
</tr>
<tr>
<td>5% &lt; Error &lt; 10%</td>
<td>2</td>
</tr>
<tr>
<td>10% &lt; Error &lt; 20%</td>
<td>1</td>
</tr>
<tr>
<td>Error &gt; 20%</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Weighting Matrix for scoring scheme

Employing the weighting matrix in all different cases leads to a scoring table for each of the models compared (MAF, MAF-T, MAFM and O-W). These results are presented in Table 5, where each model’s performance is noted (in terms of the %error achieved). The different colours represent the number of points assigned for each case and when summed a total (overall) score results for each model (Table 6).
Table 5. Score Matrix results for each model compared - Average Error% for each test case examined.

<table>
<thead>
<tr>
<th>Model</th>
<th>Stress Controlled</th>
<th>Strain Controlled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratcheting Rate</td>
<td>Mean Stress Relaxation</td>
</tr>
<tr>
<td></td>
<td>Test 1 Test 2 Test 3 Test 4</td>
<td>Test 1 Test 2 Test 3 Test 4</td>
</tr>
<tr>
<td>MAF</td>
<td>40.6% 70.5% 55.7% 10.4%</td>
<td>6.2% 6.8% 0.8% 4.4%</td>
</tr>
<tr>
<td>MAFT</td>
<td>25.4% 34.1% 23.6% 6.1%</td>
<td>5.6% 6.3% 0.8% 7.7%</td>
</tr>
<tr>
<td>MAFM</td>
<td>7.3% 5.2% 5.0% 16.7%</td>
<td>5.1% 6.7% 1.0% 0.8%</td>
</tr>
<tr>
<td>O-W</td>
<td>11.0% 12.2% 8.0% 17.7%</td>
<td>4.4% 7.7% 0.8% 1.4%</td>
</tr>
</tbody>
</table>

Table 6. Overall score for each model

<table>
<thead>
<tr>
<th>Model</th>
<th>MAF</th>
<th>MAFT</th>
<th>MAFM</th>
<th>O-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>30</td>
<td>29</td>
<td>37</td>
<td>34</td>
</tr>
</tbody>
</table>

The total (overall) score obtained for each model may be used as a descriptor of the overall performance of the model, in comparison to other models. From this example one may notice that the difference between models may be rather small (in terms of points collected), however some models do tend to perform better when examining their overall characteristics (e.g. MAFM and O-W, as compared to the simplest MAF and the widely used MAF-T). Of course, this scoring scheme is not the only applicable or the best available (in terms of accuracy) to conduct a representative comparison. This example intends primarily to highlight the importance of such (simple) scoring schemes for decision making processes. These can be particularly useful for engineering practitioners considering to employ advanced plasticity models, as opposed to built-in models in commercial finite element software (such as the rather simple MAF model).

6 Conclusions

This study aimed to identify the basic cyclic elastoplastic characteristics of as-built SLM Ti-6Al-4V and moreover assess the capability of constitute models to simulate this behaviour. The four models employed in this regards were successful in simulating complex phenomena arising from uniaxial strain and stress controlled loading histories. This research is one of the few investigations reported to date on SLM material. Phenomena associated with the fatigue performance of this class of metals can be captured with reasonable accuracy with this set of models, with some of the models being more promising (as indicated by this rough comparison). Engineering design considerations, especially for applications where high and low cycle fatigue is a critical operating factor, can benefit from these mechanical testing and simulation results. Moreover, these findings can be helpful as a basis and initiator for further studies on as-built SLM Ti-6Al-4V, for which very few published data exist.

Another interesting finding was that the cyclic plasticity models compared, quite widely used by industry and academia, have a varied performance under different loading cases and parameters. A scoring scheme, for the assessment of their overall performance, can be utilised for a quick and easy comparison. This can be especially useful for practicing engineers wishing to go beyond the classic MAF model or, its derivative, MAF-T model. A more sophisticated scoring scheme (e.g. utilising discrete and non-discrete values weighing matrices), as opposed to the example used, could lead to the development of a more comprehensive tool for researchers and engineers. This study has also provided the grounds for the MAFM model to confirm its value not only as an alternative to the MAF-T model but moreover as a versatile model capable to simulate the behaviour of various classes of metal alloys (Kourousis and Dafalias, 2013; Dafalias et al., 2008). However, a more detailed analysis and implementation of the MAFM model for other metal alloys, and in the multiaxial regime, is considered necessary for its further validation.
7 Acknowledgments

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References


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Chapter II.4  Part II Summary and Future Work

4.1 Summary

Part II of this thesis has dealt with two main areas associated with SLM Ti-6Al-4V in extending the body of knowledge to begin to understand how the various microstructure features can contribute to different LCF behaviour. In Chapter II.2.1 an in-depth analysis of vertical SLM fabricated $\alpha'$ martensite subjected to varying asymmetric strain-controlled loading conditions was conducted to develop an understanding of the micro-mechanisms which contribute to the cyclic transient phenomena. In Chapter II.2.2, this initial understanding was further developed by considering the influence of build orientation on the LCF behaviour of the material, which was analysed with respect to two different $\alpha'$ martensite microstructures. This was achieved by investigating the symmetric strain-controlled loading response to coupons manufactured at 0°, 45°, and 90° to the build plate. Finally, in Chapter II.3, the potential of the application of phenomenological elastoplastic constitutive models to simulate cyclic transient effects in an $\alpha'$ martensite Ti-6Al-4V fabricated using SLM was investigated. The major conclusions of Part 2 of the thesis are as follows:

- Comparison with symmetric strain-controlled results obtained from mill-annealed Ti-6Al-4V coupons showed that the micro-mechanisms associated with an $\alpha'$ martensite microstructure produced quite different cyclic softening behaviour than the wrought mill-annealed microstructure. However, it was found that after continued cycling at increasing strain amplitude, the softening rates become progressively similar. A difference in the micro-mechanisms was also shown to result in quite different mean stress relaxation behaviour, where the mean stresses in the $\alpha'$ martensite microstructure were relaxed faster than the mill-annealed microstructure. Finally, the elastoplastic behaviour of the SLM Ti-6Al-4V material was successfully simulated using the MAFM plasticity model, extending the potential of application of elastoplastic constitutive models to additively manufactured materials. Overall, the
investigation provided an advancement in the knowledge of the cyclic transient effects and potential elastoplastic modelling capability of typical $\alpha'$ martensite SLM Ti-6Al-4V.

- A novel SLM fabricated ultrafine $\alpha'$ martensite microstructure was developed which was hypothesised to be the result of layer reheating promoting continued $\beta$ transformation into quartic $\alpha'$ martensite. A more typical coarser $\alpha'$ microstructure was also observed in some of the coupons manufactured using the same processing parameters. The formation of the two different microstructures was hypothesised to be the result of build plate dependency and the influence of varying gas flow velocity in the chamber. Residual stresses present because of the build process were shown to have a significant influence on the symmetric strain-controlled behaviour of the material at all three build orientations. The residual stresses caused an asymmetric cyclic softening behaviour, which resulted in a vertical shifting of the hysteresis loops under symmetric strain-controlled loading. With progressive cycling and/or increasing the strain amplitude, reduced the influence of the residual stresses. Finally, mechanical anisotropy was observed in both monotonic and cyclic results, where the diagonally manufactured coupon had the largest yield stress in both, while the diagonal and horizontal coupons were more ductile than the vertical coupons.

- Different uniaxial elastoplastic features of SLM vertically fabricated $\alpha'$ martensite Ti-6Al-4V were shown to be successfully captured through the application of phenomenological elastoplastic constitutive models. The simulation results gathered for each of the tested kinematic hardening models demonstrated very good agreement with symmetric strain-controlled hysteresis loop development corresponding to the experimentally gathered results at 1%, 1.5%, 2%, and 2.5%. Furthermore, good simulations results were achieved for mean stress relaxation and strain ratcheting. Therefore, the study highlights the ability of phenomenological constitutive models at simulating the macroscopic stress-strain evolution. This suggests that although the
evolution of micro-mechanisms in SLM fabricated material is different, the robustness of the phenomenological constitutive modelling approach allows for its application to achieve accurate simulations of transient cyclic effects.

4.2 Recommendations for Future Work

The investigation into the elastoplastic behaviour of $\alpha'$ martensite material fabricated by SLM has uncovered some further important issues. These include:

- **Asymmetric stress/strain controlled testing**
  Further analysis of the elastoplastic behaviour of the $\alpha'$ martensite SLM Ti-6Al-4V through both asymmetric stress and strain controlled loading at varying build orientations is required. This is important in building on the knowledge gained in this research to develop elastoplastic constitutive models capable of simulating the potential anisotropic elastoplastic behaviour. It is also important in understanding the influence of micro-mechanisms on elastoplastic behaviour at different orientations to develop a database, which can be used in the tailoring of microstructure design to take advantage of the different elastoplastic behaviours.

- **Repeating tests for alternate SLM Ti-6Al-4V microstructures**
  The elastoplastic and LCF analysis conducted in this study should be extended to other possible SLM fabricated Ti-6Al-4V microstructures such as ultrafine and coarse lamellar ($\alpha+\beta$). This is important in growth of the database which can be used for the microstructure tailoring to achieve different mechanical properties at different locations on the component. As was demonstrated in the literature review, the database has been growing with respect to monotonic tensile, HCF, and fracture mechanics for various SLM Ti-6Al-4V microstructures. Although the work presented in this thesis is valuable for the growth of this database, this work only analysed $\alpha'$ martensite, therefore, needs to be extended to other SLM Ti-6Al-4V microstructures.
**Damage mechanisms**

To successfully achieve this smarter design process, a more in depth understanding of the microscopic damage mechanisms is required for the possible Ti-6Al-4V microstructures formed from varying the processing parameters during SLM. Developing an understanding of the damage mechanisms associated with each of these microstructures, including slip systems, twinning, as well as dislocation interactions is vital in the development of elastoplastic constitutive models capable of accurately simulating a variety of cyclic transient phenomena.