Hybrid composite ply joints for integrating radiofrequency apertures in multifunctional aircraft structures

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Jasim Ahamed

1st Jul 2018
Abstract

Optimal utilisation of dissimilar materials is fundamental to the development of high-performance multifunctional structures, especially in lightweight aerospace applications. For multifunctional load-bearing antenna structures, utilisation of composite materials with dissimilar electrical properties enable the integration of radiofrequency transparent apertures in load-bearing aircraft structural skins. Aircraft skins are typically fabricated from advanced carbon fibre polymer composite, while the aperture requires an electrically non-conductive composite material such as quartz or glass fibre polymer composite. One method of achieving efficient load transfer between the structural skin and the aperture is to employ ply joints. Ply joints are co-cured composite joints produced by forming butt-splices and/or overlaps between individual plies while intelligently tailoring the relative positions and spatial distribution of the ply terminations. In this thesis, novel structural concepts for hybrid ply joints enabling multifunctional load-bearing antennas are developed.

The load-carrying capacity of hybrid ply joints depends strongly on several design parameters such as the distance between ply terminations (step length), overlap length, the spatial distribution of ply terminations, joint thickness and the mechanical stiffness and coefficients of thermal expansion of the dissimilar composite materials. An experimental program was created to investigate the influence of these key design parameters on the load-carrying capacity of hybrid ply joints. Several ply-interleaved and ply-overlap joint configurations were evaluated under quasi-static uniaxial tensile and compression loading conditions. Fractographic analysis is also performed to determine the failure mechanism. The findings of the experimental program are used to guide the development of predictive capability for the structural performance of hybrid ply joints. Both analytical and high-fidelity computational models were developed; the analytical models are based on strength of materials and linear elastic fracture mechanics methods while computational models employ continuum damage mechanics and cohesive element method.

The study revealed that the principal damage mode in hybrid ply joints is delamination emanating from the vicinity of ply terminations. The loss of load-carrying capacity is caused by transverse matrix as they promote the coalesce of delaminations. Through judicious selection of the spatial distribution of ply terminations and step/overlap length, hybrid ply joints can be designed to reach the design ultimate strength of the load-bearing skin structure. The analytical and computational predictive capability developed for hybrid ply joints enable the
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<tr>
<td>CFRP/GFRP/QFRP</td>
<td>Carbon/Glass/Quartz fibre-reinforced plastic</td>
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<tr>
<td>RF</td>
<td>Radiofrequency</td>
</tr>
<tr>
<td>DUL</td>
<td>Design Ultimate Load</td>
</tr>
<tr>
<td>OHT/OHC</td>
<td>Open-hole tension/compression</td>
</tr>
<tr>
<td>UD</td>
<td>Unidirectional</td>
</tr>
<tr>
<td>QI</td>
<td>Quasi-isotropic</td>
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<tr>
<td>FE</td>
<td>Finite element</td>
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<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>CDM</td>
<td>Continuum damage model</td>
</tr>
<tr>
<td>CZM</td>
<td>Cohesive zone model</td>
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<tr>
<td>SERR</td>
<td>Strain Energy Release Rate</td>
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Publications


Chapter 1 Introduction

1.1 Background

The proliferation of antenna systems required for communication, navigation and electronic warfare applications in modern aircrafts poses a significant technological challenge in delivering high functional capability without adversely affecting the structural performance. Traditionally, antennas are mounted externally onto the airframe or housed inside large fairings or radomes. However, this approach can impair both the aerodynamic performance and structural integrity of the aircraft. Protuberances caused by these antennas increases the parasitic drag thereby reducing the endurance of the aircraft. Antennas are mounted onto the airframe by machining cut-outs which are subsequently reinforced with mechanical fasteners. This mounting method not only reduces structural efficiency of the aircraft, but removal of load-bearing material also reduces the overall strength and stiffness of the airframe. Protruding antennas are also susceptible to foreign object damage during service and/or maintenance, thus affecting the damage tolerance of the airframe. To address the shortcomings of traditional aircraft antennas, considerable effort has been devoted to the development of structurally integrated load-bearing antennas.

Load-bearing antennas must support primary structural loads as well as perform the transmit and receive functions of an antenna. Therefore, load-bearing antenna design entails the consideration of structural requirements; such as strength and stiffness, as well as radiofrequency (RF) requirements; such as antenna gain and bandwidth. Optimising structural performance and RF performance can often be conflicting objectives. Hence, design trade-offs are often required for load-bearing antenna. In current literature, there are two commonly employed design methodologies for developing load-bearing antenna. One approach is to integrate conventional antenna elements such as metallic radiators and dielectric substrates into the structural skin. This is achieved by employing a multi-layered sandwich panel construction [1, 2] as shown in Figure 1. The sandwich structure consists of a honeycomb, or foam core embedded between an inner and outer structural skin. The inner skin is the primary load-bearing structure typically fabricated from carbon fibre reinforced plastic (CFRP) and the outer skin is fabricated from an electrically insulating (dielectric) composite material such as quartz/glass fibre reinforced plastic (QFRP/GFRP) to allow for RF transmission and reception. A cavity or recess feature is manufactured into the inner skin to accommodate antenna elements. The recess may be avoided if the inner skin also employs QFRP/GFRP [3,
However, this approach limits the placement of the load-bearing antennas on the aircraft to secondary structures which are designed to withstand much lower loads.

Another technique to impart antenna functionality to skin structures is by creating an array of radiating slots into the outer skin [5]. One embodiment of slotted array type load-bearing antenna is the Slotted Waveguide Antenna Stiffened Structure (SWASS) [6], shown in Figure 1. SWASS is a stiffened aircraft skin structure that contains an array of slots cut into the outer skin in which the structural stiffeners perform the dual function of mechanical reinforcement and a waveguide antenna. Figure 1 shows an example of a SWASS concept utilising a slotted CFRP structural skin with top-hat stiffeners. Slot shape and size is dictated by the desired antenna radiation pattern and antenna frequency. These slots are normally filled with...
dielectric material to provide environmental sealing and act as radar transparent apertures to allow for RF transmission and reception.

1.2 Research Rationale

The physical size of an antenna slot is inversely proportional to the frequency that the antenna operates, with conventional slot antennas being in the order of one half-wavelength long [7]. Slots for 8, 12 and 25 GHz applications are approximately 19, 13 and 6 mm long respectively [8]. From a structural standpoint, such slots would be treated as “open holes” because the dielectric fillers are typically low modulus epoxies that do not contribute to structural strength. Aircraft structures are commonly designed to withstand the design ultimate loads (DUL). The DUL is defined as the maximum load expected in service multiplied by the safety factor, typically 1.5. For primary aircraft structures, the DUL is often equal to open-hole tension/compression (OHT/OHC) strength of the material [9]. The OHT/OHC refers to the residual strength of the material containing a circular hole of 6.35 mm diameter. Thus, for antenna applications at 25 GHz or above, half-wavelength slots could be accommodated with no design changes. Many airborne RF systems however, operate at frequencies below this, the X-band (8-12 GHz) being very common [8]. Unfortunately, half-wavelength slots for these frequencies are larger than that possible using current design practice. Given the high notch sensitivity of CFRP, the presence of these slots can affect the structural strength significantly, especially in compression. Therefore, additional reinforcements are required to restore the load-bearing capacity of the structure, such as localised laminate thickening and/or employing mechanical reinforcements, which can incur a significant weight penalty.

A structurally efficient technique to increase the strength of a slotted load-bearing antenna would be to employ a structural dielectric material such as quartz/glass fibre composite in the slot. Straight-sided windows however will not significantly increase the load-carrying capacity of the antenna as the interface is dominated by weak resin. The formation of a joint between the dissimilar composite laminates could allow for efficient load transfer between the dielectric window and the structural skin. The need for additional reinforcement around the dielectric window may be eliminated entirely if the joint is designed to carry the DUL of the antenna structure.

One method of creating strong joints between the structural skin and the dielectric window is to employ ply joints. Ply joints are co-cured composite joints produced by forming butt-splices and/or overlaps between individual plies while intelligently tailoring the relative
positions and spatial distribution of the ply terminations [10]. The terminology used in this thesis for a butt-splice configuration is ply-interleaved and for an overlapped ply configuration is ply-overlap joints. A schematic illustration of a structurally integrated dielectric window created using hybrid composite ply-interleaved and ply-overlap joints are shown in Figure 2.

![Composite Window Diagram](image)

**Figure 2** Structurally integrated dielectric windows for load-bearing antenna using hybrid composite ply joints; Cross-sectional view of load-bearing antenna with ply-interleaved and ply-overlap joining technique

The concept of employing ply joints for creating dielectric windows for load-bearing antenna applications was originally proposed by Callus [6, 8]. The load-bearing capacity of ply joints depends on several joint design parameters; the geometry and size of ply terminations, distance between adjacent ply terminations, overlap lengths, the spatial distribution of the ply terminations and laminate stacking sequence. Joining dissimilar composite materials also requires careful consideration of the difference in thermal and mechanical properties. Manufacturing tolerances leading to geometric variability of the dielectric window affects the RF performance. Current literature does not address how the ply joining technique could be applied to joining dissimilar fibre-reinforced composite laminates for antenna applications.
1.3 Aim and scope of work

The aim of this research is to develop novel structural concepts for hybrid composite ply joints for multifunctional load-bearing antenna applications. The specific objectives of this study are to:

1. Develop a comprehensive understanding of the failure mechanism
2. Determine the effect of design parameters on ultimate strength
3. Develop an analysis methodology for predicting the failure mechanism and ultimate strength.

To achieve these objectives, the following fundamental scientific questions will be addressed:

1. What is the influence of spatial distribution of ply terminations on failure mechanism and ultimate strength, and what are some optimum configurations for high strength?
2. What is the relationship between step/overlap length and joint strength?
3. What are the major damage modes that cause structural failure?

Although RF requirements will be important for the optimisation of the multifunctional structure, the focus of this research is on the structural strength requirements.

1.4 Thesis Outline

Chapter 2 presents a critical review of current literature on ply joints with an emphasis on challenges associated with the manufacturing process, influence of joint topology on strength, failure mechanisms and analysis methodologies. Current research gaps in the study of ply joints are identified. Several studies on discontinuous laminates and tapered laminates are also included due to its physical similarity with ply joints.

Chapter 3 analyses the design requirements for this new load-bearing antenna. Firstly, the feasibility of this structure for the intended application is demonstrated using analytical methods. Secondly, the structural strength requirement for the joint to enable the integration of dielectric windows in primary aircraft structures is determined. Finally, several design configurations for hybrid ply joints are presented and their key design considerations are discussed.

A key topological feature critical to the strength of a ply joint are the resin rich pockets that form at the ply terminations. A parametric study of discontinuous laminates is presented in Chapter 4. The influence of resin pocket geometry, dissimilar materials and resin pocket
spacing on failure initiation stress and ultimate failure stress is investigated. The findings of the chapter guide development of design methodology for the joints.

The structural behaviour of hybrid ply joints is investigated in Chapters 5, 6 and 7. Details of the experimental program including manufacturing process, test methods and discussion of key results such as failure modes and ultimate strength are provided. The analytical models are derived from strength of materials and linear elastic fracture mechanics framework while the computational models employ continuum damage mechanics and cohesive element technique. The predictive capability of the two analysis methods are compared and discussed. Parametric analysis is also carried out to quantify the effect of joint design parameters and thermal residual stresses on the load-bearing capability. In Chapter 5, a preliminary study on hybrid ply joints is presented using unidirectional laminates. Load-bearing structures in aerospace applications commonly employ quasi-isotropic laminates. Chapter 6 and 7 present the investigation of hybrid ply-interleaved and hybrid ply-overlap joints fabricated from quasi-isotropic laminates.

Chapter 8 assesses the potential application of hybrid ply joints to structurally integrated dielectric windows in load-bearing antennas. Representative load-bearing antenna panels designed using optimal joint design parameters are fabricated and tested. The structural behaviour of the panels including failure modes and strength is compared with the two-dimensional hybrid ply joints.

The final chapter provides a comprehensive discussion of key results, summarises the major findings of the thesis and recommends areas of further research to the structural behaviour and design optimisation of hybrid ply joints.
Chapter 2 Literature review

2.1 Introduction

Joints are often potential sources of failure in structures due to the presence of stress concentrators arising from various material and geometric discontinuities. Joining composite materials is particularly challenging owing to the anisotropy, low strain-to-failure and the complexity in failure mechanism. Joint optimisation is therefore fundamental for achieving high structural efficiency and minimising the weight of composite structures for high performance aerospace applications. Traditional methods of joining composite materials include mechanical fastening and adhesive bonding. While mechanical fastening is a mature and reliable technique, persistent challenges remain for achieving satisfactory joint efficiencies. In addition to the weight penalty of fasteners, machining or drilling introduces fibre damage which creates high stress concentration around fastener holes. Complex damage mechanisms can arise from these discontinuities leading to premature structural failure. As composites exhibit a high degree of anisotropy, it is difficult to achieve high bearing and shear strengths without localised laminate thickening. In comparison, bonded joints result in lower parasitic weight and significantly lower stress concentration as minimal fibre damage occurs in the fabrication process. Especially for thin skin structures, bonding is a very attractive alternative to mechanical fastening. Bonded joints can be fabricated using structural adhesives in a single or two stage cure or co-cured in a single stage. Co-cured joints are a special type of adhesively bonded joints in which the excess polymer resin is utilised for bonding the adherends. For the fabrication of complex structural assemblies that utilise multiple materials, co-cured joining offers higher structural efficiency and lower manufacturing cost in comparison to traditional mechanical fastening and adhesive bonding techniques.

Integration of multiple materials enhances design flexibility and enables the optimal utilisation of materials with different properties to create affordable high-performance structures meeting diverse functional requirements. Hybrid structures combining dissimilar composite materials are also required for both structural and non-structural functions. For example, structural repair of a composite structure may be conducted with another composite material of dissimilar stiffness and strength [11]. Joining of dissimilar composite laminates may also be required for load-bearing antenna applications for integrating electrically insulating (dielectric) windows fabricated from dielectric composites such as glass fibre reinforced plastic (GFRP) or quartz fibre reinforced plastic (QFRP) in carbon fibre-reinforced
plastic (CFRP) aircraft skin structures [8]. The focus of this thesis is the design and development of novel co-cured joining techniques, referred to as ply joints, for efficiently integrating dissimilar composite materials.

Ply joints can be categorised as an integral fit joining technology in which the adherends are co-cured and therefore eliminates the use of adhesives or fasteners. Ply joints are formed by creating butt-splices or overlaps between uncured composite plies whilst tailoring the positions of the ply terminations for a given combination of materials and loading direction. The terminology adopted in this thesis containing only butt-splices is ply-interleaved whereas ply joints containing overlaps is referred to as ply-overlap. The key difference between the two types of ply joints is that ply-interleaved joints achieve a flush exterior whereas some eccentricity is expected with ply-overlap joints. It is proposed, in this thesis, that optimum design of ply joints could provide a structurally efficient method of transitioning between two dissimilar composite laminates. This thesis aims to investigate the fabrication, design, testing and analysis of these hybrid ply joints. It should be noted that the prefix “hybrid” refers to the use of dissimilar materials. The joint itself is a interlaminar hybrid laminate.

Although there is no current literature on hybrid ply joints, laminates conceptually similar to ply joints have been investigated. To increase the structural efficiency of highly loaded mechanically fastened composite joints, a local reinforcement technique employing the gradual substitution of specific composite plies with high strength thin metal foils was developed [12, 13]. Of relevance to this work, is the development of the transition region (Figure 3) which allows for efficient load transfer between the hybrid composite-metal laminate and pure composite laminate. The work undertaken Baucom et.al [14, 15] on discontinuous laminates referred to by the authors as “tiled” laminates provides fundamental understanding in structural behaviour of ply joints. As opposed to traditional continuously fibre-reinforced composite laminates, each lamina of a tiled laminate is subdivided into individual tiles. Although the authors proposed the joining of a cross-ply and quasi-isotropic CFRP laminate, most of the work was conducted on unidirectional CFRP laminates.

Discontinuous architectures containing ply terminations have also been purposefully introduced into continuously fibre-reinforced composite laminates to promote ductile failure mechanisms. While extensive research has been conducted on this topic to date, the work in some of the relevant publications are discussed. This will also include the work on new forms of aligned short fibre reinforced composites laminates referred to as unidirectional arrayed
chopped strand (UACS) laminates developed by introducing slits into conventional prepregs using compression molding techniques [16]. Ply terminations are sometimes necessary in the design of thickness tapered composite structures. Thickness tapering is commonly used in thin walled structures such as in the design of aircraft wing-skins, turbine blades and rotor flex beams to optimise the strength and stiffness depending on the magnitude of the loads and the primary loading condition [17, 18]. In tapered composite laminates, plies are “dropped” abruptly which leads to the formation of triangular resin rich zones analogous to that of ply-overlap joints.

The literature review presented in the following section is outlined as follows. Key findings from current research that reveal manufacturing process, key design parameters, failure mechanism and joint strength are presented. The next section critically evaluates the different analytical and numerical models employed for predicting the structural response of ply joints. Finally, the research gaps identified from literature, which outline the scope of this thesis, are summarised.

2.2 Review of ply joint literature

2.2.1 Hybrid composite-metal laminates
To increase the joint efficiency of highly loaded thin skin mechanically fastened composite joints, a local reinforcement technique employing the gradual substitution of specific composite plies with high strength thin metal foils was developed [12, 13, 19], as shown in Figure 3. Composite plies contributing to most of the load-carrying capacity remain uninterrupted and pass from the pure composite region through the transition region to the hybrid composite-metal reinforced fastened region. This ply substitution technique employed in the transition region, allows for efficient load transfer between the hybrid composite-metal laminate and pure composite laminate while eliminating any laminate thickening, thus avoiding eccentricities and secondary stresses analogous to ply-interleaved joints.

The authors were successful in developing several configurations of the transition region that had similar tensile strength of the CFRP laminate and a compressive strength that exceeded the CFRP laminate. This was achieved by tailoring the positions of the weaker carbon ply-titanium foil terminations while keeping 0° plies continuous. An example of an optimum configuration is shown in Figure 3 (bottom) in which +C45° and subsequently -C45° plies are replaced by titanium foils. These C-Ti terminations were separated by a lateral distance of 5 mm. Increasing the lateral distance beyond 5 mm had a negligible effect on strength. In the through-thickness direction, the C-Ti terminations were separated by a distance equal to two ply thickness. The authors also found that the manufactured methods had an impact of the strength of transition laminates. The strength of laminates manufactured by automatic tow placement were 10% lower in comparison to those manufactured using conventional hand-layup technique. This was attributed to the serrated shape of ±45° plies causing imperfect butt-splices with the corresponding titanium foils. Experiments showed that the major failure
modes for the transition laminates were delaminations emanating from C-Ti terminations and fibre fracture of C0⁰ ply.

To model the delaminations between CFRP and titanium, the fracture toughness for C-Ti interface was obtained using Transverse-Crack-Tension (TCT) specimens. It was assumed that the delamination was dominated by interlaminar shear stress. TCT test specimens are characterised by having some centrally cut plies placed within a continuous laminate. The TCT interlaminar fracture toughness, calculated using elementary linear fracture mechanics, was found to be more than twice the interlaminar fracture toughness obtained using traditional ENF specimens.

2.2.2 UACS laminates

Short fibre reinforced composites are attractive low-cost alternatives to continuous fibre reinforced composites. Their superior formability allows for easier manufacture of complex components. New innovative short aligned fibre composites referred to as unidirectional arrayed chopped strand (UACS) were developed by introducing slits into conventional prepregs using compression molding techniques [20, 21]. Both unidirectional and quasi-isotropic UACS laminates have been investigated under tensile loading by the authors. The dominant failure mode for UD UACS laminates were unstable delaminations emanating from resin rich zones. In QI UACS laminates, failure occurs through gradual initiation and growth of various damage modes. Micrographs of the damage development of a typical QI UACS test specimen at various levels of applied strain are shown in Figure 7. Transverse matrix cracks and resin pocket fracture occur at relatively low strain. This is followed by the initiation of delamination and increased transverse matrix cracking. The final failure of the laminate is caused by the rapid growth of delaminations which coalesce with the transverse matrix cracks. The strength of UACS laminates with optimal slit pattern and slit dimensions showed a 50% reduction in strength in comparison with continuous fibre composite laminates. Slight improvement in strength of UACS laminates was observed with the use of angled slits [16, 22]. Interlaminar toughening mechanisms [23] only led to a marginal improvement in the performance of UACS laminates.
Figure 4 Micrographs indicating the damage progression for QI UACS laminate under tensile loading. Each micrograph is captured at various strain levels.
2.2.3 Tiled laminates

Figure 5 Tiled laminate configurations [14, 15] developed by altering the spatial arrangement of ply terminations; Half-tile offset, Taper, Scarf (offset and non-offset) and Interleaved-scarf

Baucom et. al [14, 15] developed a new type of composite laminate in which a laminate could be subdivided into “tiles”. The purpose of creating such a discontinuous laminate was to...
expand the design space of composite laminates as each of these tiles can be tailored with differing fibre/matrix constituents, fibre orientation, length distribution etc… to meet specific design requirements. As with the ply-interleaved joints, resin pockets develop at the termination of each tile. The main aim of the work was to develop an optimum tiled configuration of high strength relative to traditional continuous laminates. Most the work was conducted on unidirectional CFRP laminates, primarily focusing on the tensile strength. Conventional hand-layup was used to fabricate the test specimens for tiled laminates. The authors reported that the resin pockets in fabricated specimens were not perfectly quadrangular due to the shear-strain induced by the motion of mould components during consolidation and distortion at ply edges caused by mechanical shearing of the fibres.

Several configurations were developed by employing various arrangements of resin pocket placement as shown in Figure 4. The authors successfully developed a tiled configuration, referred to as interleaved-scarf, which retained 92% of the strength of a continuous unidirectional composite laminate. The interleaved-scarf tiled laminate or joint consists of two oppositely directed scarf joints. The experimental results showed that in comparison with traditional scarf joints, the interleaved-scarf joint was 50% stronger. The high strength was attributed to the smaller bending deformations and lower maximal stress concentrations as shown by the linear FE analysis results in Figure 5. The step length recommended for achieving maximum strength was twice the length of the shear-lag zone or stress relaxation zone surrounding the ply termination. The size of the shear-lag zone, determined through linear FE analysis, represents the region where the local ply stress is 99% lower than the applied stress. Figure 5 shows that for laminates with sub-critical step lengths (less than the shear lag zone), stress fields of neighbouring resin pockets interact thereby reducing load-carrying capacity of the tiled laminates. This finding was experimentally validated for interleaved-scarf specimens, where sub-critical step lengths produced 50% lower strength.

2.2.4 Pseudo-ductile discontinuous laminates

High performance continuous fibre reinforced composites tend to fail in a brittle catastrophic manner. Thus, high operating safety margins are required for composite structures due to its poor residual strength. Extensive research work has been undertaken to develop high performance ductile composites. One of the techniques to induce ductile failure is the use of discontinuous ply architectures.
Malkin et al., 2013 [24] investigated a number of discontinuous configurations in unidirectional CFRP laminates with the purpose of inducing pseudo-ductile failure mechanism under flexural loading. Although the design configurations developed in Malkin et al. were optimised for failure and not strength, the study of laminates with a single cut ply are relevant to this thesis. Flexural tests were conducted on laminates with varying resin pocket sizes and the loads at which (a) resin pocket fracture and (b) delamination initiation occurred were recorded. The results revealed that increasing the resin pocket size delayed the fracture of the resin pocket; an increase of up to 30% between the smallest (0.2 mm) and largest (2 mm) resin pocket was observed. This was attributed to the interaction of stress concentrations emanating from ends of cut plies that form the resin pocket, as shown in Figure 6. The resin pocket size however had a negligible effect on the delamination onset load as it is controlled by critical strain energy release rate. The authors also implemented a vacuum assisted layup positioning guide which minimised the deviation in position of the resin pockets.

Czel et al. [25] developed a discontinuous laminate similar to the “half-tile offset” configuration (Figure 4). The authors were able to reduce the distortion of resin pockets in the manufacturing process by employing a circular ply cutter rather than standard staple knives. The mechanical behaviour of these joints differed fundamentally with the lateral distance between adjacent resin pockets also known as the joint length. For joints with joint lengths
less than the thickness of the ply, interlaminar shear stresses are nearly constant. Therefore, the failure of the joint occurs when the interlaminar shear stresses reach the interlaminar shear strength. For laminates with joint lengths in excess of the ply thickness, crack tips develop at ends of ply termination due to the interlaminar stress concentration and the interlaminar shear stresses away from the resin pocket are nearly zero. As a result, the joint strength is related to the interlaminar fracture toughness. The tensile strength of joints employing long joint lengths approached approximately 35% of UD laminate strength.

2.2.5 Tapered laminates

Figure 8 Hybrid carbon/glass tapered laminate for rotor hubs in helicopters [26]

Thickness tapering is commonly employed in the design of aircraft wing-skins, turbine blades and rotor flex beams (Figure 8) to optimise the strength and stiffness depending on the magnitude of the loads and the primary loading condition. In tapered composite laminates, plies are “dropped” abruptly which leads to the formation of triangular resin rich zones analogous to that found in non-flush ply-overlap joints investigated in this thesis. Geometric and material discontinuities caused by the presence of resin pockets at ply terminations promote the onset of delamination cracks. Under compression, tapered laminates can also fail by fibre kinking depending on the taper angle [27]. The failure strength of these laminates is influenced by several factors such as thickness of the dropped sub-laminate, dropped ply stiffness, taper angle, spacing between adjacent ply drops and the overall configuration of the tapered laminate [17, 28-33].
Design guidelines [15, 22-27] for tapered laminates recommend the use of symmetric layups with a combination of interleaved continuous and discontinuous plies as the optimum geometry for tapered laminates. Layup of an optimum QI laminate is shown in Figure 9. Symmetric layups can minimise undesirable secondary bending, therefore, reducing interlaminar peel stresses. Interleaving continuous plies with discontinuous plies ensures that ply terminations are vertically separated by a ply thickness. If the design requirements do not permit the presence of continuous plies, ply terminations must be separated by distance equal to 20 times the thickness of the dropped-plies in regions of high strain and 5 times the thickness of the dropped-plies in regions of low strain [28, 29]. The number of plies dropped simultaneously at any given intersection is also an important design consideration. Increasing the number of dropped-plies causes the taper angle to increase. Misalignments particularly in main load-bearing plies could cause fibre kinking in compression. Another reason to avoid thick ply drops is that thicker plies are more susceptible to delamination [35]. Therefore, it has been recommended that no more than two plies should be dropped-off at a single intersection.

### 2.3 Review of failure prediction techniques

Developing reliable and robust predictive modelling tools are fundamental to optimising the structural performance of hybrid ply joints. Analytical/semi-analytical models for strength prediction typically rely on simplified linear elastic theories. Moreover, empirical data that are dependent on loading conditions and material properties are required which limits the robustness of analytical models. However, these models are useful tools for preliminary structural design. To accurately predict the strength of composite structures, the initiation, progression and complex interaction of the various damage modes must be modelled. Progressive damage models incorporated into FE analysis software is currently the most
powerful failure analysis technique for composite structures due to its ability to model complex material and geometric behaviour. In the following sections, the most analytical methods and progressive damage models for ply joints and discontinuous laminates are discussed.

2.3.1 Analytical/semi-analytical methods

Analytical models used for strength prediction typically treat each failure mode independently. The dominant failure mode in discontinuous laminates are ply rupture due to fibre fracture/kinking and ply pull-out due to delamination [12, 16, 24, 25]. Although transverse matrix cracking causes the reduction in overall laminate stiffness, it does not lead to sudden loss of load-carrying capacity.

Ply rupture failure

Analytical methods for determining the failure of discontinuous laminate by ply rupture typically employ a stress-based failure criteria such as the point stress or average stress criteria [36]. In advanced polymer composites, the stiffness of resin matrix is much lower relative to the stiffness of the fibres and therefore highly localised stress concentrations develop in the plies above and below the resin pocket. The most common approach to determine the stress distributions around a resin pocket is using FE analysis [15, 24, 32, 37]. Figure 10 [15] depicts the stress concentrations in the vicinity of resin pocket in a discontinuous “tiled” laminate. Due to the stress singularity at the bi-material interface between ply and resin pocket, the peak stress is either determined by obtaining the stress at a point which is at a finite distance away from the interface or the average stress over a certain distance [36]. Ultimate failure of the laminate is assumed to occur when the local peak stress reaches the laminate or ply strength.
Figure 10 Stress concentrations/relaxations caused by presence of weak resin pockets in discontinuous "tiled" laminates [15]. The contour plots were computed by linear finite element analysis.

Although the point stress/average stress criteria [36] was initially developed for laminates containing a through thickness discontinuity [36, 38-41], Camanho et. al [12] also showed that the average stress method can be employed to accurately predict strength of hybrid CFRP/Titanium laminates (Figure 3). In this approach, tensile failure occurs when the 0° CFRP ply fails according to equation below:

$$\frac{1}{d_c} \int_0^{d_c} \sigma_{xx}(y) dy = X_T$$

where $\sigma_{xx}(y)$ is the longitudinal stress distribution, $d_c$ is the characteristic distance and $X_T$ is the longitudinal tensile strength of the composite. $d_c$ is the distance from the stress concentrator over which the stress distribution is averaged. $d_c$ is first calculated numerically by solving Eq.1 for a baseline configuration using the experimental failure stress ($X_T$) and stress distribution ($\sigma_{xx}(y)$), which is expressed as an appropriate polynomial function. Once $d_c$ is calculated, the failure stress is calculated as the far-field stress that leads to a stress distribution satisfying Eq.1.

**Pull-out failure**

High interlaminar stresses develop in the vicinity of resin pockets due to material and/or geometric discontinuities. As a result of weak interlaminar strength of advanced fibre-reinforced polymer composites, delamination is a prominent failure mode. The delamination problem of plies in a discontinuous composite laminate is analogous to the pull-out/debonding of fibres in discontinuous fibre polymer composites [42-45, 46]. Therefore, delamination models for ply joints are referred to in this thesis as pull-out models. Pull-out models typically employ strength-based or fracture mechanics-based theories [43, 47].

Strength theories assume that the pull-out occurs when the maximum interlaminar shear stress reaches the critical value i.e the interlaminar strength of the composite material. The delamination failure stress is the applied laminate stress at which interlaminar shear stress reaches the interlaminar strength. This approach assumes the matrix to be perfectly-plastic and stresses are transferred over resin pockets through yielding of the matrix under shear. Maximum shear stress can be obtained either analytically using the shear-lag method [48] or numerically using finite element analysis. Besides their application to discontinuous fibre
Composites [43, 47], strength-based models are commonly used to predict the strength of adhesively bonded joints [49, 50].

Meanwhile, the basic assumption of fracture mechanics-based models is to treat the resin pockets as voids or pre-existing cracks. In fracture mechanics theory, the growth of an existing crack is controlled by the rate of strain energy released in crack propagation in comparison to the maximum threshold strain energy release rate of the material i.e. the interlaminar fracture toughness of the material. The pull-out strength is then evaluated using the Griffith energy-criterion. Several authors have used LEFM approaches to successfully predict delamination strength in UACS laminates [20], discontinuous hybrid carbon/glass laminates [51, 52], hybrid titanium-CFRP ply joints [12, 13, 19] as well in tapered composite laminates [17, 28, 29, 53].

While there are several analytical equations used for both these pull-out models, the basic mathematical formulation can be found in Pimenta and Robinson [47] for a discontinuous composite laminate with a “brick and mortar” architecture as shown in Figure 11.

\[
\sigma_s = \frac{l^b S^m}{t^b} \tag{2}
\]

Strength-based equation:

\[
\sigma_s = \sqrt{2E^b G_{IIc}}/t^b \tag{3}
\]

Fracture mechanics-based equation:

where \(\sigma_s\) is the far field stress, \(\sigma^\infty\) at which pull-out out of the ply of thickness, \(t^b\) and ply length of \(2l^b\). \(S^m\) is the interlaminar shear strength of the matrix, \(E^b\) is the longitudinal stiffness and \(G_{IIc}\) is the interlaminar fracture toughness.
2.3.2 Progressive damage modelling

The most common method of idealisation of structural response of composite structures employed in progressive damage models is to employ the stacked-shell approach. This approach allows for the separation of in-plane or intralaminar ply stresses and interlaminar stresses. Each ply of the laminate is modelled as a homogenous anisotropic layer using shell elements while the ply interfaces are modelled using special purpose zero-thickness interface elements. The intralaminar damage modes are simulated primarily within the Continuum damage mechanics (CDM) framework while the interlaminar damage or delamination is modelled using Cohesive Zone Model (CZM). Both these models capture the initiation and propagation of damage modes that lead to structural collapse. The interaction or coupling of intralaminar and interlaminar damage models is achieved by integrating both damage models within a single finite element model.

2.3.2.1 Intralaminar damage model

CDM models for composite materials are based on the damage mechanics framework to describe the creep failure of metals by Kachanov [54] and Rabotnov [55]. Damage, here, is defined as an irreversible change to the material brought about by energy dissipating physical or chemical process, as a result of thermomechanical loading [56]. Several CDM models have been proposed in literature [57-60]. The main approach of CDM models is to quantify the loss in load-carrying capacity due to damage through gradual reduction in material stiffness. This is achieved by the introduction of damage variables into the material constitutive law that define the initiation and propagation of damage modes. These damage variables represent the state of damage in each of the damage modes and are implemented into the material constitutive laws to calculate the post-damage or effective stresses. Complete failure of material occurs when the area under the stress-strain curve reaches the fracture energy associated with each damage mode.

In this thesis, the CDM model in commercially available finite element package, Abaqus, is employed [61]. The Abaqus CDM model uses the 2-D Hashin failure criteria [62] developed for plane stress loading to define failure initiation in four intralaminar damage modes; fibre rupture in tension, fibre buckling and kinking in compression, matrix cracking under transverse tension and shearing and matrix crushing under transverse compression and shearing. Upon satisfying the initiation criterion, the effective stresses are calculated based on degraded stiffness of the material which are obtained by multiplying the undamaged material
stiffness with the corresponding damage variables. Four independent scalar damage variables corresponding to the different damage modes are calculated. Shear damage is expressed as a function of the remaining variables. The Abaqus stiffness degradation based damage evolution model is adapted from [63] and details of the Abaqus CDM model can be found in [59].

The material properties required as inputs for these models are the ply strengths (longitudinal and transverse tension, longitudinal and transverse compression, shear) and the fracture toughness (fibre fracture, fibre compression, matrix tension and matrix compression) associated with each damage mode. Ply strengths can be readily determined from standardised test methods. Although no standard method exists for measurement of fibre fracture toughness in tension/compression, a review of existing methods proposed by a number of authors can be found in [64]. Due to the complexities associated with these test methods, the fibre fracture toughness is usually calibrated with laminate configuration and loading condition being investigated [65, 66]. The matrix fracture toughness in tension is assumed to be equal to the mode-I interlaminar fracture toughness, this assumption has been confirmed by experimental tests [67]. The compressive matrix fracture toughness can be determined from the mode-II interlaminar fracture toughness using equation shown in [58]. A well-known issue with continuum damage models is that with stress-softening behaviour, leading to strain localisation, causes the energy dissipated to decrease with finite element size. To alleviate the mesh dependency of the numerical solution, a characteristic element length is recommended which ensures that the dissipated elastic energy before damage is less than the fracture toughness. The general formula proposed in literature is that the characteristic element length is directly proportional to the stiffness and fracture toughness and inversely proportional to the square of strength [58].

A number of limitations of the CDM models have been discussed literature [58, 68-71]. The fibre compression failure criterion does not consider the effect of shear stresses on fibre compression failure [72]. Also, the matrix compressive failure criterion does not account for the change in the shear strength of the ply and the change in fracture plane due to transverse compressive stress [73]. The Hashin failure criteria predicts matrix compressive failure to occur at a plane of 45° (maximum shear stress angle), however, experiments have shown the fracture plane to be close to 53° due to the presence of compressive stresses. Also, the transverse tensile and shear strengths used as inputs for the CDM model do not account for
“in-situ effects”. In-situ effects refer to constraints imposed by the neighbouring plies of different thickness and orientation on tensile and shear strengths [74, 75]. Also, failure of laminates under shear involves significant non-linearity which is not accounted for [76]. The fracture toughness associated with transverse matrix cracking does not account for the mod-en-mixity in the cracks since shear stresses also contribute to the evolution of matrix cracks, especially for off-axis plies. Despite these drawbacks, the Abaqus CDM model has been successfully used in the modelling of a variety of composite laminates under different loading conditions; notched laminates [66, 77], mechanically fastened joints [12, 78] and hybrid laminates [79, 80]. As an example, Figure 12 shows prediction of failure mode and failure stress for the compression failure of hybrid CFRP/Titanium laminate [12] by the interlaminar damage model in ABAQUS.
2.3.2.2 Interlaminar damage model

Two of the most common numerical methods for modelling interlaminar damage or delamination are the Virtual Crack Closure Technique (VCCT) and Cohesive Zone Model (CZM). VCCT is based on LEFM whereas CZM combines LEFM and CDM models discussed earlier. VCCT is computationally expensive compared to CZM as it constantly requires the calculation of nodal variables and topological information from nodes ahead and behind the crack front for the determination of strain energy release rates [81]. Ply joints contain several possible crack locations and the application of VCCT to predict delamination in ply joints could be very tedious. The CZM approach is therefore preferred to VCCT in this thesis.

The basic hypothesis of the CZM is that all irreversible damage events that occur at the vicinity of a crack tip can be lumped into a surface, known as the cohesive damage zone [82, 83]. CZM model uses a constitutive law to relate the interlaminar stresses or tractions to the displacement jumps at an interface where crack may occur. Damage initiates when the tractions reach interfacial strength. Further loading causes the reduction of interfacial tractions by gradual degradation of the interfacial stiffness. When the area under the traction-separation curve equal the fracture toughness of interface, the traction are reduced to zero and
a new crack surface is formed. The main advantage of CZM approach is the ability to model crack initiation and crack propagation.

The numerical implementation of CZM is achieved using special-purpose interface elements known as cohesive elements [82, 84, 85]. These elements discretise the mechanics of the cohesive zone to simulate the initiation and propagation of delamination. The Abaqus CZM model uses a bi-linear traction-separation law. The initial linear elastic (undamaged) portion of traction-separation law is defined by a penalty stiffness parameter that ensures a stiff connection between elements prior to initiation of damage. The cohesive stiffness which is generally a numerical artefact must be chosen carefully. The value must be high enough to ensure a stiff connection between undamaged elements but small enough to avoid spurious oscillations of tractions [85]. For mixed-mode delamination, damage is assumed to initiate when a quadratic interaction function involving the nominal stress ratios i.e. tractions to interlaminar strengths [86] reaches a value of 1. The interlaminar shear strength (ILSS) can be determined from standard test methods while the interlaminar peel strength is assumed to be equal to the transverse tensile strength of the ply. For the propagation of mixed-mode delamination the B-K law [87] is used, which has been shown to be more accurate for PEEK/epoxy matrix composites than other mixed-mode propagation criterion [85].

To accurately represent the stress gradients ahead of the crack tip, it is recommended that at least 3 elements be used within the cohesive zone length [85, 88]. The length of cohesive zone is defined as the distance from the crack tip to the point where maximum cohesive traction is attained. Numerous researchers have proposed equations for determining the cohesive zone length. The general formula of the length of the cohesive zone is that it is proportional to the fracture toughness and inversely proportional to the interfacial strength with an additional empirical parameter depending on the equations [88].

One of the technological barriers with current CZM models is the assumption that the mode-mixity is constant throughout propagation whereas experiments have shown that mode-changes during the process of damage [89]. Also, CZM models which primarily rely on the theory of LEFM do not account for the fracture resistance curve or R-curve. For example, in mode-I delamination crack growth is resisted by the increase in fracture toughness due to fibre bridging [90]. Despite these drawbacks, CZM still remains the most accurate method of analysing delamination in composite laminates in a variety of loading conditions; hybrid laminates [91, 92], laminates with defects [93, 94] and bonded/bolted joints [91, 95-97].
Figure 13 shows the prediction of delamination damage and failure stress by the interlaminar damage model in Abaqus for discontinuous laminates under a quasi-static compressive load from the study in [93].

Figure 13 Failure prediction of discontinuous laminates (a) using ABAQUS cohesive elements: Comparison of predicted (b) delamination failure path and (c) stress-strain response with experimental observations.

2.4 Summary

A comprehensive literature review that critically examines key aspects of ply joints has been presented. Literature on laminates with physical similarity to ply joints such as discontinuous fibre composite laminates, laminates with cut plies and tapered laminates have also been reviewed. Methods employed by different researchers to reduce manufacturing tolerances in the co-cure fabrication of ply joints have been discussed. The review has also highlighted the influence of joint topological parameters such as spatial distribution of ply terminations, joint length, taper angle and resin pocket geometry on the structural response of ply joints. It has been revealed that the failure of ply joints is driven by the complex interaction of damage modes that initiate from resin pockets situated at ply terminations. Finally, analytical and numerical methods for predicting strength of ply joints were reviewed and evaluated.
However, current literature does not address how ply joining techniques could be applied to joining dissimilar fibre reinforced composites. Incompatibility of mechanical and physical properties can affect both joint fabrication and the structural integrity of the joint. Residual thermal stresses that arise from differences in the coefficient of thermal expansion of two materials may cause structural damage and must be accounted for in design. To achieve high structural strength using ply joining, judicious selection of joint topological parameters such as spatial distribution of ply termination or joint configuration and step/overlap length is required. Joint configurations proposed by Camanho et.al [12] cannot be employed for load-bearing antenna applications as it will be necessary to terminate the conductive CFRP plies outside the dielectric window. Baucom et. al [14, 15] proposed a number of optimum joint configurations for joining unidirectional CFRP laminates. Designing ply joints with dissimilar composite materials is challenging due to the complexity in load path and failure mechanism of the joint. Therefore, new joint concepts for optimal structural performance are required for hybrid composite ply joints. In bonded joints, increasing the joint length typically increases the strength almost linearly up to certain length known as the transfer length and approaches an asymptotic value [98]. Thus, the critical transfer length must be determined for structurally efficient hybrid composite ply joints.

To optimise the design of load-bearing antennas, a robust, reliable and validated analysis methodology is required for the prediction of failure modes and ultimate strength of the joint. Analytical methods can be reliably used to predict strength based on linear elastic strength and fracture mechanics theories. High fidelity computational models is however required to capture the initiation and progression of the damage modes that eventually cause structural failure.
Chapter 3 Design requirements

3.1 Introduction

Load-bearing antennas are multifunctional structures that must (a) support primary/secondary structural loads and (b) perform transmit and receive (T/R) functions of a radar antenna [5]. These load-bearing antennas require electrically insulating (dielectric) windows to allow T/R of electromagnetic radiation. The dielectric windows, typically, are non-load bearing elements which require cut-outs in the structural skin, thereby, reducing the structural stiffness and strength. The hybrid composite joints proposed in this thesis could enable the dielectric windows to support the applied structural loads, thereby, alleviating the need for additional reinforcements to restore structural capability.

Most commonly available dielectric composite materials have a lower stiffness than CFRP. The difference in the stiffness of the two materials gives rise to a load-shedding effect [99] causing the stress inside the dielectric window and the joint region to be much lower than the far-field CFRP skin stress. However, the stress at edge of the window perpendicular to the loading direction is amplified. These stress concentration/reduction factors control the failure of load-bearing antenna. In this chapter, we employ finite element analysis to determine the stress factors using the bonded inclusion analogy [99] and then proceed to discuss its implications on the static strength requirements of this new load-bearing antenna.

Ply joints are advanced co-cured joints formed by either abutting or overlapping the plies while adjusting the position of the ply terminations. Novel structural concepts for hybrid composite ply joints which could enable the integration of large dielectric windows into CFRP laminates without the need for additional reinforcements.
3.2 Design feasibility study

3.2.1 Stress analysis

Consider a load-bearing antenna containing a structurally integrated dielectric window with an aperture radius of \( R \) and joint/transition region of length \( L \) as shown in Figure 14. Linear elastic FE analysis was performed to determine the stresses in (a) the aperture: region composed of entirely dielectric composite material, \( \sigma_w \) (b) the skin region at the transverse boundary of the window, \( \sigma_s^* \) and (c) the joint: transition/hybrid composite region between the skin and aperture, \( \sigma_j \) in relation to the applied stress. The dielectric window is assumed to be a rigidly bonded inclusion in the skin structure. An inclusion refers to a circular insert that is rigidly connected to a plate like structure i.e. no relative displacements between the skin and window occurs [99].

The FE model of the load-bearing antenna was created in Abaqus/CAE and meshed with plane stress quadratic elements (CPS8) (Figure 15). A uniform tensile load was applied at one end of the load-bearing antenna while the other end is fixed. An element size equal to 1/10th of the aperture radius is employed. The skin and the window are assumed to be isotropic materials with stiffness equal to the stiffness of QI GFRP and CFRP laminates (\( E_w = 17 \) GPa and \( E_s = 47 \) GPa) [100]. Two load-bearing antenna design configurations are considered; (a)
straight-sided window and a (b) jointed window in which the region stiffness increases step-wise from GFRP window to the CFRP load-bearing skin.

Figure 15 Finite element mesh of load-bearing antenna used for design feasibility study: (a) Straight sided window (b) Jointed window

Figure 16 Contour plots of y-stress normalised with applied stress, $\sigma_{yy}/\sigma^\infty$ for (a) Straight sided window (b) Jointed window
Figure 17 Graphical illustrations of $\sigma_{yy}/\sigma^\infty$ stress distribution along the $x$ and $y$ axis for (a) straight sided window and (b) jointed window.

The contour plots of the $y$-stresses, normalised by the far field applied stress, $\sigma_{yy}/\sigma^\infty$ are shown in Figure 16. Figure 17 shows the distribution of $\sigma_{yy}/\sigma^\infty$ from the centre of the window along the $x$ and $y$ axis; the distance is normalised with respect to the aperture radius $R$. The results show that the stress inside the aperture is much less than the far-field applied stress. The reduction in stress due to the load-shedding effect is proportional to the ratio of the stiffness of the two materials. The analytical expression for the stress reduction factor for a straight sided inclusion is as follows [99]:

$$k_t^w = \frac{3E_w/E_s}{1+2E_w/E_s}$$

(4)

Where $E_w$ and $E_s$ are the stiffnesses of the two materials. The stress inside the window, $\sigma_w$, is related to the far-field stress by $k_t^w$: 

$$\sigma_w = k_t^w \sigma^\infty$$
\[ \sigma_w = k_w^\infty \sigma^\infty \]  

As a result of the load-shedding effect, the maximum hoop stress at the boundary of the window, \( \sigma_s^* \), is higher than the applied stress. The stress concentration factor, \( k^s_t \), is given by the analytical equation for a straight sided inclusion:

\[ k^s_t = \frac{3}{1 + \frac{2E_w}{E_s}} \]  

(6)

The maximum hoop stress, \( \sigma_s^* \), is related to the far-field applied stress by \( k^s_t \):

\[ \sigma_s^* = k^s_t \sigma^\infty \]  

(7)

The analytical solutions for \( k^w_t \) and \( k^s_t \) compare reasonably well with FE results. For the jointed window, while there is no significant change in \( k^w_t \), \( k^s_t \) is significantly lower; \( k^s_t = 1.6 \) for straight sided window and \( k^s_t = 1.29 \) for a jointed window. The higher stress concentration for straight-sided window is caused the abrupt change in stiffness. Introducing a transition region in which the stiffness increases gradually from GFRP to CFRP reduces the hoop stress concentration significantly. The stress distribution graphs and stress contour plots indicate the load-shedding effect is also experienced in the joint region. Due to the varying stiffness of the joint region, the stress reduction is not identical to the aperture. In the example considered in this study, the joint stress varies between 67% to 72% of the applied stress along the length of the joint. Both the hoop stress concentration and joint stress reduction factor are dependent on the joint properties such as stiffness, configuration and joint length relative to the size of the aperture [101].

\[ k^j_t = \frac{3E_w/E_s}{1 + 2E_w/E_s} \]  

(8)

The stress in the joint is therefore given by:

\[ \sigma_j = k^j_t \sigma^\infty \]  

(9)

3.2.2  Static strength requirements

The static strength requirement for primary aircraft structures is that they must be able to withstand the DUL without failure. The DUL is very rarely the unnotched or pristine strength of the material. Manufacturing anomalies and process variations, especially in advanced composite materials, can be significant and is therefore accounted for in the material design allowable stress. Also, the structures must be able to sustain the DUL in extremes of
temperature and moisture, both of which have a significant effect on the composite material properties. Aircraft structures must also be damage tolerant, that is the structure must sustain the design ultimate loads in the presence of barely visible impact damage (BVID) which may be the result of accidents or foreign objects during service and maintenance. Mechanical fasteners are also extensively employed in assembly and repair and therefore structures must sustain the DUL in the presence of open or filled holes. For the reasons outlined, the DUL is typically the residual tensile or compressive strength of the material containing a 6.35mm hole in the Elevated Temperature Wet (ETW) condition [9].

Figure 18 Structural design requirement for new load-bearing antenna: The hybrid ply joints should enable a load-bearing antenna with large apertures to retain the DUL of the parent CFRP structure. Therefore, RF apertures equal to or smaller than allowable hole size (6.35 mm) can be accommodated into load-bearing CFRP skins as open-holes or slots without needing any additional reinforcement. At lower operational frequencies, such as in the X-band (8-12 GHZ), slots sizes can exceed 20 mm [8]. The structurally integrated dielectric window concept proposed in this thesis provides a structurally efficient method of restoring the DUL of the structure that requires a large aperture. Schematic illustration of design requirements for this new load-bearing antenna is shown in Figure 18.
Analysis presented in the previous section showed that failure stress of the load-bearing antenna will depend on the aperture stress, joint stress and hoop stress in the skin. The load-bearing antenna strength associated with each of these failure regions, identified in Figure 19, can be written as follows:

Failure of aperture (Region A):

\[ \sigma_a = \frac{\sigma_{w}^{all}}{k_t^w} \]  \hspace{1cm} (10)

Failure of the skin (Region B):

\[ \sigma_b = \frac{\sigma_{s}^s}{k_t^s} \]  \hspace{1cm} (11)

Failure of the skin (Region C):

\[ \sigma_c = \frac{\sigma_{j}^{all}}{k_t^j} \]  \hspace{1cm} (12)

where \( \sigma_{w}^{all} \) and \( \sigma_{j}^{all} \) is the design allowable stress of the window and joint respectively.

For the new load-bearing antenna to meet the static strength requirements, \( \sigma_a, \sigma_b, \sigma_c \) must all be less than the OHT/OHC strength of CFRP. If both failure mode A and mode B are avoided, the strength of the load-bearing antenna depends only the joint strength.

Considering that the allowable stress of the window is equal to OHT/OHC strength of dielectric composite material, the combination of materials that satisfy the following condition can ensure that the aperture does not become the weak link:

\[ \sigma_{w}^{all} > \sigma_{skin} \cdot k_t^w \]  \hspace{1cm} (13)
Setting the design allowable to the OHT strength of the CFRP QI laminate, $\sigma_{skin}^{all} = 450$ MPa and given most commonly available dielectric materials have half the stiffness of CFRP i.e $E_w/E_s = 0.5$, the required dielectric material strength must be approximately 70% as strong as the CFRP laminate. From literature on commonly used GFRP in aerospace applications, an ideal candidate for load-bearing antenna application would be S-2 glass GFRP which typically has a high strength [100]. Alternatively, some E-glass GFRP may also be suitable for this application.

Failure Mode B involves the rupture of the skin at the edge of the window due to the high stress concentration, $k_t^s$. The hoop stress must be lower the design allowable of the structure to meet the static strength requirement, $\sigma_b < \sigma_{skin}^{all}$. For an isotropic plate or laminate with a quasi-isotropic layup, $k_t^s = 3$. Analysis presented in the previous section showed that a load-bearing antenna in the weakest configuration, such as a straight sided window has a $k_t^s = 1.6$. Therefore, maximum load bearing capacity of the antenna is determined the hybrid ply joint strength. The strength requirement for the hybrid ply joint to satisfy the DUL of the load-bearing antenna is that the joint strength must exceed the greater of the skin or dielectric window design allowable.

$$\sigma_j^{all} > (k_t^s \cdot \sigma_{skin}^{all}) \text{ or } \sigma_w^{all} \quad (14)$$

### 3.3 Design of hybrid ply joints

#### 3.3.1 Design requirements

The primary function of the hybrid ply joint is to transfer loads efficiently between the structural skin and the dielectric window. To avoid unnecessary structural reinforcement surrounding the dielectric window, the joint strength must exceed the minimum of the design allowable strengths of the skin and the dielectric window. In other words, the joint strength must be greater than the strength of the weaker of the two laminates.

For the application of ply joining techniques to load-bearing antennas requiring a RF aperture, it is necessary to terminate all conductive CFRP plies outside the dielectric window, typically made from GFRP, for antenna requirements. Therefore, the hybrid ply joint configurations cannot have continuous plies. Another limitation on the scope of the study are the materials employed. Several types of dielectric composite materials are available; such as Quartz/E-glass/S-2 glass and advanced metamaterials. Material selection here is primarily driven by availability and cost. Most commonly used laminate configurations in highly
loaded aerospace structures are quasi-isotropic laminates with a $[45/0/-45/90]_{2s}$ stacking sequences. Unidirectional laminates are also investigated for preliminary design studies and evaluation of design parameters.

Fundamental to the efficiency of these joints is the joint thickness relative to the parent laminate thickness and overall joint length. Ideally, the joint must be designed to be flush with the window and skin. However, the ply-overlap technique introduces “swelling” of the joint region which introduces load path eccentricity resulting in undesirable secondary bending. Since the transition region in the load-bearing antenna is created by replacing CFRP plies with relatively lower stiffness GFRP plies, minimising the joint length increases the overall stiffness of the load-bearing antenna.

### 3.3.2 Design configurations

Several joint configurations can be developed for a given combination of materials and loading condition. Examples of ply-interleaved joint configurations are shown in Figure 20. The simplest configuration is a *finger* joint which is produced by offsetting every successive ply termination by a fixed distance. The main advantage of the finger joint is that it produces a very short transition region. One of the common techniques that are employed in adhesively bonded structures is the scarf joint. Since ply-interleaved joints are co-cured, complicated machining is not necessary. Therefore, it allows for a greater flexibility in design of scarf joint configurations. Figure 20 shows the traditional scarf or stepped-scarf joint configuration in which each ply termination is offset from each other by a fixed distance. The scarf joint can also be made symmetric about the mid-plane of the laminate. A novel scarf configuration is the interleaved-scarf, developed by Baucom et.al[15], involves offsetting successive ply terminations in opposing directions along the length of the joint. Both symmetrical and unsymmetrical interleaved-scarf configurations are shown in Figure 20.
Examples of ply-overlap joint configurations are shown in Figure 21. The simplest technique for creating a ply-overlap joint is to overlap every ply, with all plies being terminated at the same location. This design configuration is referred to in this study as aligned-overlap. The aligned-overlap is very the short joint length but also produces a very steep taper angle for the overlapped plies. Alternatively, the overlaps can be staggered in similar arrangement to the ply-interleaved joints. Two staggered overlap configurations are shown in Figure 21. An
offset-overlap and interleaved-scarf overlap has a staggered configuration equivalent to a finger and interleaved-scarf ply-interleaved joint configurations, respectively. The interleaved-scarf technique would minimise the taper angle of overlapped plies and distribute the joint thickness more uniformly over the hybrid region.

### 3.3.3 Design considerations

![Diagram of generic hybrid joint](image)

(a)

The generic laminate lay-up of a hybrid ply-interleaved and ply-overlap carbon/glass composite joint is shown in Figure 22. The carbon and glass plies are shaded in black and yellow respectively. The locations of resin pockets (shaded in blue) at ply terminations are denoted by $x_i$, with the subscript $i$ denoting the ply index. The structural performance of ply joints depends on design parameters such as the ply orientations of the carbon ($\theta_i^C$) plies and the glass plies ($\theta_i^G$), step/overlap length ($l_i$), and the spatial distribution of the ply terminations. Selecting an optimal spatial distribution of ply terminations, referred here as the joint configuration, is necessary for efficient load transfer and maximising the joint strength.

![Diagram of generic hybrid joint](image)

(b)

Figure 22 Structural model of a generic hybrid (a) ply-interleaved and (b) ply-overlap hybrid plies indicating key design parameters
The desired load transfer mechanism for ply joints is through interlaminar shear of the resin matrix. Peel stresses must be reduced as the interlaminar strength and toughness polymer resins are significantly weaker in Mode-I. The presence of the material discontinuities at ply terminations can give rise to secondary bending effects, even for joints with constant thickness. One method of alleviating peel stresses is to employ balanced and symmetric configurations where possible. The stiffness mismatch of the two composite materials requires the joint configuration to allow for gradual transition in stiffness. In addition to the mechanical considerations, the differences in coefficients of thermal expansion coefficients of CFRP and GFRP may produce thermal stresses/strains which must be accounted for in the design.

The critical design feature of ply joints is the carbon-glass ply terminations. Resin pockets formed at ply terminations act as damage initiation sites due to the presence of localised stress concentrations and the low strength of the matrix relative to the fibres. Using the finger and scarf ply-interleaved joints as an example, a simple linear FE analysis was carried out to understand the stress distributions around these resin pockets.

![Finite element mesh for linear static stress analysis of unidirectional finger and scarf joint: Mesh employs element size of 1/10th of ply thickness](image)

Finite element models of a unidirectional finger and scarf joint with 16 plies meshed with CPE8 plane strain quadratic elements were created in Abaqus/CAE. Orthotropic material properties of CFRP and GFRP (Appendix A) are assigned to each ply. An element size of 1/10th of ply thickness is employed to accurately capture the high stress gradients. Boundary conditions replicating a uni-axial static tensile test is applied to the model and analysed using Abaqus/implicit. Close-up view of the FE mesh indicating the size of the elements relative to the ply thickness is shown in Figure 23.
Figure 24 Longitudinal ply stress (normalised with applied stress) contour plots revealing the stress concentrations near the ply termination in a (a) Finger joint (b) Scarf joint and (c) Close-up view revealing the highly localised nature of the stress concentration.
Figure 24 shows the contour plots of longitudinal ply stresses near the ply termination. The presence of the weak resin pockets produces highly localised stress concentrations. The maximum stress concentrations are produced at the nodes that lie on the interface between the resin pocket and the composite ply. These nodes are present at geometric imperfection and material discontinuity and therefore produce mesh-dependent stress singularities. To alleviate these numerical singularities from FE analysis results, the elements directly above the resin pocket are suppressed. Figure 24 shows how the stresses get redistributed in the laminate because of the weak resin pockets and as a result the stresses are amplified in the neighbouring plies; particularly in the high stiffness CFRP plies. These high stresses could cause the failure of plies through fibre fracture.

The material and geometric discontinuities regions also give rise to interlaminar stresses. Figure 25 shows the contour plots of interlaminar shear stresses (normalised with applied stress). Although the magnitude of the interlaminar stresses relative to the applied tensile stress is low, the interlaminar strengths of CFRP/GFRP are also very low. Hence, these stress
concentrations can promote the initiation of delamination. Interlaminar peel stresses are not shown as they are compressive in tensile loading and thus do not contribute to delamination.

3.4 Concluding Remarks

The design of a structurally integrated dielectric windows using hybrid composite ply joints for load-bearing antennas has been investigated. It was found that large dielectric windows can be incorporated into CFRP skin structures, without needing any additional reinforcements to meet the static strength requirements. This is due to the load-shedding effect caused by the lower stiffness of dielectric composite materials in comparison to CFRP causing the stress inside the window to be lower than the applied stress. Depending on the mismatch in stiffness of the two materials, the dielectric material strength can be much lower than CFRP, roughly 60-70%. Provided the dielectric material strength requirement is met, the failure of load-bearing antenna is primarily controlled by the joint failure as hoop stresses are much lower.

For the load-bearing antenna to meet the static strength requirements, the joint must be at least as strong as the dielectric material. Additionally, the joint must employ a short transition (hybrid laminate) region and must be flush with the parent laminates. A number of viable joint concepts for ply-interleaved such as finger, scarf and interleave-scarf and ply-overlap joints such as aligned-overlap, offset-overlap and interleaved-overlap have been proposed. Fundamental to the optimum design of ply joints are the ply terminations at which highly localised concentrations lead to the initiation of intralaminar and interlaminar damage. A detailed investigation on damage and failure of discontinuous laminates will be presented in the following chapter.
Chapter 4 Discontinuous laminates

4.1 Introduction

Resin pockets develop during the manufacturing process of ply joints at end of a terminated ply [15, 24]. For most commonly available advanced fibre-reinforced polymer composites, the stiffness and strength of the polymer matrix is significantly lower than the reinforcing fibre. As a result, the resin pockets act as geometric and material discontinuities that distort the stress field in the vicinity giving rise to highly localised stress concentrations. These stress concentrations promote the initiation of various interacting damage modes. The growth and coalescence of damage can lead to overall loss of load-carrying capacity of ply joints. Therefore, the strength critical regions of ply joints are the resin rich pockets at ply terminations.

In this chapter, unidirectional laminates with discontinuities are investigated to aid the understanding of the fundamental failure mechanism of ply joints. The effect of several design parameters on structural behaviour are also studied. Analytical and numerical methods are also developed to predict stress concentrations and ultimate failure of discontinuous laminates. Understanding the fundamental structural behaviour of discontinuous laminates in informs the design optimisation process of hybrid ply joints.
4.2 Stress concentration analysis

4.2.1 Methodology

The FE model consists of a five-ply unidirectional laminate [0]; with a central discontinuous ply as shown in Figure 26. Each ply has a nominal thickness of 0.2 mm. The resin pocket is modelled as a void due to its relatively low stiffness and strength in comparison to the fibres. The length of the resin pocket was assumed to be equal to the ply thickness. Linear elastic orthotropic material properties of E-glass/MTM57 GFRP [102] are assigned to the laminate. Boundary conditions replicating a static tensile test were imposed on the laminate as shown in Figure 1. All nodes were also constrained in the width direction to simulate plane strain boundary conditions. A far-field stress of 100 MPa is imposed at one end. Linear static FE analysis is employed for this study.

The idealised geometry of the ply termination in the FE model contains sharp corners which will lead to numerical singularities. The peak stresses obtained at these corners will therefore be highly mesh dependent. To avoid the numerical singularities at the ply termination corners, peak stresses are queried at certain distance away from the corners. The peak interlaminar stresses are determined by querying the interlaminar node at 1/5th of the ply thickness ($t_p$) from the corner singularity while the peak longitudinal ply stress is determined by querying the maximum stress along a longitudinal path ($d$) offset by 1/5th of the ply thickness from the ply termination as depicted in Figure 27. This technique has previously used by other authors investigating discontinuous laminates [24, 103] to overcome the numerical singularities.
Figure 27 FE mesh near the ply termination (void). Peak stresses are queried at 1/5th of the ply thickness away from the corner of the ply termination.

\[
\sigma_{11}^* = \max \{ \sigma_{11}(d) \}; \quad d \in \left[ -t_p, \frac{1}{2}t_p; t_p, \frac{1}{2}t_p \right] \\
\sigma_{33}^* = \sigma_{33}(t_p) \\
\tau_{13}^* = \tau_{13}(t_p)
\]

(15)

(16)

(17)

A mesh sensitivity study was conducted to determine the appropriate element size for the analysis. Three different formulations of 3D solid elements; linear (C3D8), quadratic fully integrated (C3D20) and quadratic reduced integrated (C3D20R) for four different element sizes; 0.04, 0.02, 0.01 and 0.005 mm was investigated. Based on the results of the mesh sensitivity study (Figure 28), fully integrated quadratic solid elements (C3D20) of size 0.02 mm (1/10th of the ply thickness) were chosen for the stress concentration analysis.

Figure 28 Mesh sensitivity results for peak longitudinal ply stresses, \( \sigma_{11}^* \) (Top) and peak interlaminar shear stress, \( \tau_{13}^* \) (Bottom) using fully integrated linear, fully integrated quadratic and reduced integrated quadratic 3D solid elements
4.2.2 Results

4.2.2.1 Stress distribution

Figure 29 Stress contour plots of a discontinuous laminate: (a) Longitudinal ply stress, $\sigma_{11}$ (b) Interlaminar normal stress $\sigma_{33}$ (c) Interlaminar shear stress, $\tau_{13}$
Figure 30 Stress distributions in a discontinuous laminate: (a) Longitudinal ply stress, $\sigma_{11}$ along the $x$ axis at the interface and at half ply distance above the ply, (b) Interlaminar normal stress $\sigma_{33}$ and (c) Interlaminar shear stress, $\tau_{13}$ measured from the tip of the resin pocket.

Stress contour plots in Figure 29 (a) show that the ply termination produces a local disturbance of the longitudinal ply stress field. The discontinuous ply is unable to carry its full share of the applied tensile load over a large region near the resin pocket. This region is usually referred to as the “ineffective length” of the ply [45]. Over the ineffective length of the discontinuous ply, the stresses get redistributed to the neighbouring continuous plies. The highly localised nature of the stress concentration is also evident from the contour results. Away from the corner, the stresses are still significantly higher than the applied.

Over the ineffective length of the discontinuous ply, load transfer occurs primarily through interlaminar shear stresses, Figure 29 (c). For the loading case considered here, the interlaminar peel stress are mostly compressive except at the corner of the ply termination as shown in Figure 29 (b) and Figure 30 (b). The interlaminar shear stresses distribution in
Figure 30 (c) shows that the stresses are singular at the tip of ply termination and reduce to almost zero with increasing distance away from it.

4.2.2.2 Parametric studies

Finite element analysis was carried out to determine the influence of various parameters on the stress concentrations of discontinuous laminates. To appropriately characterise the effect of stress concentrations, the peak stress ratio is employed in the parametric analysis. The peak stress ratio is defined as the peak stress normalised with the applied stress.

Figure 31 Effect of resin pocket (RP) size (given by the aspect ratio, $l_r/l_p$) on stress concentrations in a discontinuous laminate (a) Graphs of peak stress ratio versus resin pocket size (b) Variation of longitudinal ply stress distributions with resin pocket size;
The effect of resin pocket size, defined by the aspect ratio \( (t_r/t_p) \), on the stress concentrations in a discontinuous laminate is shown in Figure 31 (a). The results show that as the resin pocket length reduces, both peak longitudinal stress, \( \sigma_{11}^* \) and peak interlaminar shear stress, \( \tau_{13}^* \) increase slightly. However, the increase in \( \sigma_{11}^* \) is much more significant than the increase in \( \tau_{13}^* \). The longitudinal ply stress distribution for various sizes of the resin pocket are shown in Figure 31 (b). It is evident show that the significant increase in \( \sigma_{11}^* \) is caused by the interaction of the stress concentrations produced at each corner of the resin pocket.

![Stress contour plots](image)

(a)

(b)

Figure 32 Stress contour plots of interlaminar shear stress in hybrid discontinuous laminates (a) C\(_2\)/C- G/ C\(_2\) (b) G\(_2\)/C-G/ G\(_2\)

<table>
<thead>
<tr>
<th>Laminate configuration</th>
<th>( \sigma_{11}^*/\sigma^\infty )</th>
<th>( \tau_{13}^*/\sigma^\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(_2)/C-C/ C(_2)</td>
<td>1.67</td>
<td>0.26</td>
</tr>
<tr>
<td>G(_2)/G-G/ G(_2)</td>
<td>1.71</td>
<td>0.35</td>
</tr>
<tr>
<td>C(_2)/C-G/ C(_2) (hybrid)</td>
<td>1.54</td>
<td>0.25</td>
</tr>
<tr>
<td>G(_2)/C-G/ G(_2) (hybrid)</td>
<td>1.92</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 1 Comparison of interlaminar shear stress concentrations in monolithic and hybrid discontinuous laminates

Stress concentration analysis was also performed on discontinuous laminates with dissimilar composite materials. Two hybrid configurations were analysed, one in which a carbon ply is
replaced by a glass ply in a discontinuous carbon laminate C₂/C-G/ C₂ and another in which a
glass ply is replaced by a carbon ply in a glass laminate in a discontinuous glass laminate
G₂/C-G/ G₂. The stress concentration results for different material configurations of
discontinuous laminates are shown in Table 1. The notations C-C, G-G and C-G indicate the
location of the resin pocket. In both hybrid configurations, maximum stresses occur near the
carbon ply, as shown by the stress contour plots in Figure 32. However, the peak interlaminar
shear stress $\tau_{13}$ is significantly higher in G₂/C-G/ G₂ in comparison to C₂/C-G/ C₂. Hart-
Smith [104] demonstrated that the peak shear stress in a dissimilar material joint occurs at the
end of the stiffer adhered tip and this stress concentration increases with the increase in
imbalance in stiffness of the two adherends. The peak longitudinal ply stress is only slightly
higher in G₂/C-G/ G₂ in comparison to C₂/C-G/ C₂.

(a)

Longitudinal stress distribution, $\sigma_{11}$

Interlaminar stress distribution, $\tau_{13}$

\[
\frac{l_s}{t_p} = 1
\]

\[
\frac{l_s}{t_p} = 3
\]
Discontinuous laminates containing two adjacent ply terminations were analysed to understand the effect of step length on peak stress concentrations. Figure 33 (a) indicates that $\tau_{13}$ increases sharply as the step length approaches the ply thickness of the material. The step length $l_s$ is normalised with the ply thickness $t_p$. The rapid increase in stress concentration is caused by the interaction of stress concentrations arising from each resin pocket. Stress contour plots in Figure 33 (b) depict the interaction of the stress concentrations at short step lengths i.e. $l_s \leq t_p$. Meanwhile, the longitudinal ply stresses concentrations do not increase significantly (less than 5%) with decreasing step length. However, at short step lengths the ineffective zones overlap causing a significant portion of the laminate to carry loads much less than the applied load. At large distances the resin pockets become “elastically isolated”. To maximise the strength of ply joints, it is necessary to determine the minimum step length at which the stresses from adjacent resin pockets are elastically isolated.

4.3 Strength analysis

4.3.1 Methodology

In section 4.2, it has been demonstrated that the two primary modes of damage that cause failure of discontinuous laminate are intralaminar damage; initiated by the peak longitudinal ply stress concentrations and interlaminar damage, initiated by the peak interlaminar shear stress concentration. Since the interlaminar strengths of fibre-reinforced polymers are an order of magnitude lower than the tensile/compressive ply strength, interlaminar damage is a more critical damage mode, particularly for unidirectional laminates. In the following section,
various analytical and numerical approaches to determine the delamination strength of discontinuous UD laminates are presented and compared with experimental results of Wisnom [105]. Incorporating ply rupture damage models will be considered in the following chapter.

4.3.1.1 Strength-of-materials

The strength-of-materials (stress-strength) approach for predicting delamination involves comparing the local stress state to the relevant material strength allowables or failure criteria. In our approach, the local stress state is obtained using linear elastic finite element analysis. Since the analysis showed that evaluating stresses at resin pocket interface is not meaningful due to the presence of numerical singularities, the point stress criterion [36] is used. In the point stress failure model uses the maximum stress criterion, delamination is said to have initiated when the interlaminar stress, $\tau_{13}$ at a finite distance, $d$, from the stress concentrator (resin pocket corner) reaches the interlaminar strength, $S_{13}$. The far-field stress at which delamination initiates is determined by multiplying the peak interlaminar shear stress ratio evaluated at $d = 0.5t_p$ from the FE model with interlaminar shear strength of the material.

$$\sigma_d^\infty = \frac{\tau_{13}}{\sigma^\infty} * S_{13}$$

The FE models used for the stress concentration analysis assumed that at the ply termination, a finite sized discontinuity or void exists. In reality, a resin rich pocket is formed at the ply termination due to the resin flow during curing process. Analysis was conducted using typical material properties of epoxy to model the resin pocket and compared with the baseline model.

4.3.1.2 Linear Elastic Fracture Mechanics

The resin pockets at the ply terminations have very low stiffness and strength and crack at very low loads. Therefore, the resin pockets can are treated as voids or pre-existing cracks and the delamination load can be calculated using the strain energy release rate (SERR) method under the linear elastic fracture mechanics framework [25, 105, 106]. This approach equates the critical SERR to the difference in strain energy of the laminate before and after pull-out based on energy conservation during crack growth. Assuming unit width and length of the laminate, the strain energy of the laminate before pull-out, $U_i$, under a constant applied load, $N^\infty$, can be written as follows:
\[ U_i = -\left( \frac{N^\infty}{h_i} \right)^2 \frac{h_i}{2E_i} \]  

\[ U_f = -\left( \frac{N^\infty}{h_f} \right)^2 \frac{h_f}{2E_f} \]

where \( E_i \) and \( h_i \) are the laminate stiffness and laminate thickness respectively. The strain energy after pull-out or delamination, \( U_f \), becomes:

\[ L = \frac{N^\infty}{h_i} \frac{h_i}{2E_i} + \frac{N^\infty}{h_f} \frac{h_f}{2E_f} \]

where \( E_f \) and \( h_f \) are the laminate stiffness and laminate thickness respectively after pull-out.

The critical value of SERR required for the unstable growth of a delamination crack is equal to the rate of change in strain energy of the laminate before and after delamination.

\[ nG_C = \frac{N^\infty}{h_i} \frac{h_i}{2E_i} + \frac{N^\infty}{h_f} \frac{h_f}{2E_f} \]

where \( n \) represents the total number of delaminating interfaces and \( G_C \) is the critical strain energy release rate. Rearranging the terms yields the formula for the applied load to cause delamination of the discontinuous ply:

\[ N^\infty = \sqrt{\frac{2nG_C E_i h_i h_f h_f}{E_i h_i + E_f h_f}} \]

For tensile loading, the delaminations in the discontinuous laminate are primarily in Mode II because the peel stresses are compressive, except at the tip of the ply termination as shown by the stress analysis in Figure 4(b). Therefore, the critical strain energy rate is equal to Mode II interlaminar fracture toughness (\( G_C = G_{IIc} \)). For the pull-out of a single ply from a discontinuous laminate \( n = 2 \) to account for fracture on both interfaces of the delaminating ply. The delamination stress, \( \sigma_{d}^\infty \), is then determined by dividing the Eq.(22) by the initial thickness of the laminate strength of a unidirectional discontinuous laminate.

### 4.3.1.3 Cohesive Zone Model

The creation and analysis of finite element models for delamination damage prediction of discontinuous laminates using the CZM were conducted using Abaqus version 6.12 [61]. The Abaqus CZM uses a linear traction-separation law to relate the interlaminar stresses to the displacements of two initially bonded nodes. The onset of cohesive damage is determined by the interlaminar strengths and the total energy under the traction separation curve is equal to
the interlaminar fracture toughness of the material. A mixed-mode quadratic failure criterion was used to define the initiation of damage while the B-K law [87] was employed for the damage evolution. The CZM is implemented in the FE model by using zero-thickness interface elements, known as cohesive elements (COH3D8) which are placed at the interfaces near the ply termination. As in 4.2.1, FE models containing physical resin pockets will be analysed and compared with the baseline FE model which assumes the resin pocket to be a void. The resin pockets are meshed with 3D reduced integrated solid element (C3D8R). A simple stress softening model was used to simulate the failure process of the resin pockets.

The geometry and boundary conditions of the FE model is identical to the FE model used for stress concentration analysis. The plies are meshed with layers of continuum shell elements (SC8R) which are 3-D volumetric elements with a plane stress formulation i.e. they can only capture in-plane ply stresses. Linear elastic behaviour is assigned to the continuum shell elements. Abaqus/Explicit dynamic solver was employed to overcome convergence difficulties associated with implicit solvers for quasi-static non-linear problems. No mass scaling was used. Upon conducting sensitivity analysis of different loading rates, a loading rate of 1000 m/s was found to produce a reasonable time increment size with minimum dynamic effects.

A mesh sensitivity study was conducted with three different element sizes; 0.1mm, 0.05mm and 0.025mm, to determine the element size that can accurately capture the stress gradients within reasonable computational time. The results in Table 2 show that the delamination failure load converges after 0.05 mm. Therefore, an element size of 0.05 mm was employed in the simulations.

<table>
<thead>
<tr>
<th>Element size (mm)</th>
<th>Failure Load (N)</th>
<th>Computational time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1168</td>
<td>35</td>
</tr>
<tr>
<td>0.1</td>
<td>1156</td>
<td>121</td>
</tr>
<tr>
<td>0.05</td>
<td>1156</td>
<td>712</td>
</tr>
</tbody>
</table>

Table 2 Mesh sensitivity study for CZM for discontinuous laminates
4.3.2 Results

4.3.2.1 Comparison with experimental results

Wisnom [105] conducted an experimental investigation into the failure of unidirectional E-glass/913 laminates with different ratios of discontinuous to continuous plies. All the laminates tested delaminated from the ply termination. No stable crack growth was observed i.e. once delamination crack had fully developed, it spread rapidly along the entire length of the test specimen. Comparison of the model predictions with experimentally determined delamination stress are shown in Figure 34. LEFM and CZM provide excellent correlation with the experimental data while the strength-of-materials model clearly under predicts the delamination stress.
4.3.2.2 Parametric studies

Figure 35 Effect of (a) resin pocket size and (b) step length on delamination strength as predicted by CZM

Figure 35 (a) shows that delamination stress is unaffected by the size of the resin pocket. This result is not entirely surprising given that the interlaminar stresses changed only marginally with changes in size of the resin pocket. However, the step length has a significant influence on the delamination stress as shown in Figure 35 (b). The delamination stress increases almost linearly with step length and asymptotes beyond a certain length. This phenomenon is due to the short and long joint length that is commonly observed in adhesively bonded joints.

<table>
<thead>
<tr>
<th>Laminate configuration</th>
<th>CZM</th>
<th>LEFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₄/C-C</td>
<td>1770</td>
<td>1754</td>
</tr>
<tr>
<td>G₄/G-G</td>
<td>1156</td>
<td>1145</td>
</tr>
<tr>
<td>C₄/C-G (hybrid)</td>
<td>1856</td>
<td>1754</td>
</tr>
<tr>
<td>G₄/C-G (hybrid)</td>
<td>626</td>
<td>611</td>
</tr>
</tbody>
</table>

Table 3 Comparison of delamination stress for monolithic and hybrid discontinuous laminates as predicted by CZM and LEFM

The analysis results of laminates with dissimilar materials produces some interesting findings. In both hybrid configurations, delamination failure is caused by the pull-out of the carbon ply. The higher stress concentration in the G₄/C-G laminate compared to the C₄/C-G laminate causes the delamination to initiate at much lower load in the G₄/C-G laminate. Although
stress concentrations are not captured in the LEFM model, the LEFM predictions compare extremely well with the CZM.

4.4 Concluding Remarks

Discontinuous laminates were investigated to understand the influence of various topological parameters on the initiation and propagation of damage in ply joints. Damage is initiated by the highly localised stress concentration around the resin pocket. Two damage modes, intralaminar and interlaminar, were analysed for unidirectional discontinuous laminates. Interlaminar damage or delamination is the critical damage mode due to low interlaminar strength of fibre-reinforced polymer composites. The results show that the size of the resin pocket did not influence peak interlaminar shear stress significantly, provided the resin pocket was greater than equal to the ply thickness. In dissimilar laminate configurations, it was shown that delamination would initiate in the stiffer ply. To elastically isolate the stress concentrations of adjacent resin pockets, the distance between the two adjacent resin pockets, known as the step length, must be at least greater than three times the ply thickness.

Analytical and numerical models were employed to determine the delamination propagation stress for discontinuous laminates. Model predictions were validated against the experimental results from Wisnom [105]. Analytical model based on LEFM and numerical model based on CZM showed excellent correlation with experimental results. Meanwhile, models based on stress concentrations and point stress criterion did not accurately predict the delamination stress. The validated models also showed that the resin pocket size had no effect on the delamination stress. The results also indicated that the delamination strength (stress at failure) continued to increase with increasing step length up to a certain threshold value.
Chapter 5 Comparative study of monolithic and hybrid ply joints

5.1 Introduction

Ply joints are advanced co-cured joints that are fabricated by butting and/or overlapping individual plies while intelligently tailoring the positions of the ply terminations. Baucom et.al [15] had previously investigated a number of joining techniques for unidirectional CFRP laminates. The aim of this chapter is to gain preliminary understanding of the structural behaviour of hybrid ply joints, in particular the effect of hybridization on mechanical properties. The finger joint configuration and unidirectional laminates are employed in this study. The effect of the step length on joint strength is investigated. The work presented in this chapter is outlined as follows. The following section provides details of the manufacturing technique and test method. Experimental results from the test program are outlined in Section 5.3. Analytical and computational models developed for strength prediction are described in section 5.4 and the model predictions are presented in Section 5.5. The major findings of this chapter are summarized in section 5.6.

5.2 Experimental Method

5.2.1 Specimen design

The finger joint configuration employed here constitutes a simple geometry, whereby alternate ply terminations are offset as shown in Figure 36. For comparison purposes, the traditional scarf joint configuration is also investigated. The scarf joint configuration employs a linear taper with each ply terminations being offset. The offset distance between the adjacent ply terminations, referred to as the step length is kept constant. Finger and scarf ply joints with similar composite materials were also investigated. All the joints investigated employed a unidirectional stacking sequence [0]_16.
5.2.2 Materials

The carbon and glass fibre composites chosen for this study were unidirectional prepreg of T700/VTM264 and E-Glass/MTM57 respectively supplied by Advanced Composites Group Australia [102, 107]. The elastic properties of the two materials obtained from manufacturer’s data sheet supplemented by additional tests are shown in Appendix 1. The cured ply thickness of carbon and glass plies were 0.21 and 0.26 mm respectively.

5.2.3 Manufacturing

Hybrid ply joints were fabricated using conventional hand prepreg lay-up technique. Prior to layup, the positions of the ply terminations were marked onto a template to assist in accurate positioning of the plies. Carbon and glass plies were cut from their respective prepregs using standard staple knives. Since 0° plies require cutting perpendicular to the fibres, care was taken to ensure the ply edges remained as straight as possible. Each layer of the joint panel was formed by abutting carbon and glass plies at their respective ply termination markings. In Chapter 4, it was demonstrated that leaving a finite gap between the cut plies had no beneficial effect on the strength. Therefore, no gaps are left between the cut plies. Debulking was performed every 3 layers to consolidate the layup and ensure the butted ply ends form together. This process involves applying full vacuum to the layup covered in perforated film.
and breather cloth for three minutes. The lay-up was then vacuum bagged and cured in the autoclave for 1 hr at 120 °C and 620 kPa.

The complete test matrix outlining the different joint configurations investigated in this study is shown in Table 2. For finger configuration, ply joints with step lengths 3, 6, 9 and 12 mm were fabricated. For scarf configuration, ply joints with step lengths of 3, 6 and 9 mm were fabricated.

A high-resolution optical image of a hybrid finger joint with a 6mm step length is shown in Figure 37. Comparing the actual joint dimensions in Figure 37 with idealised joint schematic in Figure 36, indicates a slight deviation in the position of the ply terminations. This deviation is a result of manufacturing tolerances and movement of plies during consolidation and curing. Also, there is a small taper in joint region thickness due to the difference in the two laminate thicknesses. This thickness taper of the joint region is more pronounced for finger joints due to the comparatively shorter joint length.

![Figure 37](image)

Figure 37 Micrograph of fabricated carbon-glass finger joint specimen: Deviation in positions of the ply terminations can be seen

### 5.2.4 Testing

Tensile test specimens measuring 300 mm in length and 25 mm in width were cut from the cured joint panels using a waterjet diamond wheel cutting machine in accordance with ASTM D3039 standard [108]. Joints were tested to failure under uni-axial quasi-static tension using a MTS hydraulic load frame equipped with a 100 kN load-cell. A constant cross-head speed of 1 mm/min was employed. Load-displacement data was recorded at a rate of 4 Hz. A minimum of four replicates for each configuration was tested. Stresses were calculated using the initial cross-sectional area of the GFRP laminate. The ultimate strength was calculated from the peak load and initial cross-sectional area of the GFRP laminate. Fractographic analysis was conducted to identify the failure modes through visual observation aided by high resolution optical images of tested specimens.
5.3 Experimental Results

Figure 38 Typical stress-displacement curves for monolithic and hybrid finger (left) and scarf (right) ply joints

Typical stress-displacement curves for ply joints are shown in Figure 38. Some non-linearity in the stress-strain response was observed. No discernible differences were observed in the stress-displacement response for different joint lengths for both similar and hybrid composite ply joints. Catastrophic brittle failure was observed for all configurations tested. Both scarf and finger joints exhibited a similar stress-displacement response.

The average tensile strengths with their coefficients of variation for all joint configurations are shown in Table 4. The highest coefficient of variation in the strength results was 11.1%. For the joint lengths considered in this study, the strength of hybrid finger joint is almost independent of the joint length as shown in Figure 39 (a). The scarf joints strength results are plotted against joint lengths. Joint lengths of 45, 90 and 135 mm correspond to 3, 6 and 9 mm step length respectively. For hybrid scarf joints, the strength increases by approximately 50% when the step length is increased from 3 mm to 6 mm as shown in Figure 39 (b). Further increase in joint length does not significantly increase the joint strength. It is interesting to note that both finger and scarf configurations produce similar maximum strengths, approximately 50% of un-notched GFRP laminate. However, the finger joint configuration achieves this joint strength with a significantly shorter joint length than the scarf joint configuration.

Testing of monolithic ply joints fabricated with laminates having identical stiffness and strength were aimed at providing upper and lower bound estimates of joint strength with finger and scarf configurations. For the step lengths tested, the monolithic carbon joints increase significantly with joint length whereas the monolithic glass joints exhibit similar strength behaviour to the hybrid ply joints.
Figure 39 Experimental results depicting the effect of joint length on failure stress of monolithic and hybrid ply joints; (Top) finger and (Bottom) scarf joint configuration

<table>
<thead>
<tr>
<th>Joint length [mm]</th>
<th>Joint strength [MPa] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Finger Joints</td>
</tr>
<tr>
<td></td>
<td>Carbon/Carbon Glass/Glass</td>
</tr>
<tr>
<td>3</td>
<td>605(6.0) 383(3.1) 433(3.6)</td>
</tr>
<tr>
<td>6</td>
<td>736(7.0) 387(0.8) 419(5.4)</td>
</tr>
<tr>
<td>9</td>
<td>774(7.0) 432(1.7) 409(3.9)</td>
</tr>
<tr>
<td>12</td>
<td>808(5.8) 402(4.3) 408(3.4)</td>
</tr>
</tbody>
</table>

Table 4 Summary of joint strength data for unidirectional monolithic and hybrid ply joints of various joint configurations

The common failure modes observed for all the joints were ply rupture, ply pull-out and resin pocket fracture. The finger joints exhibited a complex crack morphology owing to the positional variation of the resin pockets through the thickness and across the width of the specimen. Figure 40 (a) shows evidence of fibre fractures and fibre splitting in carbon and glass plies as well delaminations between carbon and glass plies. The sequence of these damage events was difficult to determine given the brittle nature of failure. The crack morphology of the failed specimens did not change considerably with joint length. Pull-out failure in glass/glass joint was more pronounced in comparison to carbon/carbon joint which
shows a more brittle failure process. For scarf joints, the principal failure mode was delamination between carbon and glass plies accompanied by some fibre splitting in glass plies (Figure 40 (b)).

Figure 40 Typical experimental failure modes of unidirectional hybrid (a) finger joints (b) scarf joints

5.4 Analysis Methods

5.4.1 Analytical Modelling

The major failure modes of an unidirectional finger ply joint is through rupture of the plies along the weakest intersection (intralaminar failure) or pull-out of the “fingers” from the hybrid/joint region (interlaminar failure). Schematic illustrations of the two failure modes are shown in Figure 41. Details of analytical models to predict the intralaminar and interlaminar failure of hybrid finger joints are presented in the following section.

Figure 41 Analytical failure mode of a unidirectional hybrid ply joint in a finger joint configuration; (a) Ply rupture and (b) Pull-out
5.4.1.1 Ply rupture model

The weaker intersection for hybrid finger joint is $T_2$ which contains half the total number of the Glass plies, ignoring the contribution of resin pocket. To account for the stress concentration induced by the resin pocket, linear elastic finite element analysis was employed to determine the stress concentration factor. The details of the FE model have not been shown in this chapter for brevity but the methodology employed was similar to that in Chapter 4. The ply rupture strength, $\sigma_R$, can then simply be calculated by dividing the longitudinal tensile strength of the unidirectional glass ply, $X_{11,t}$, with the axial stress concentration factor, $K_{t11}$.

$$\sigma_R = \frac{X_{11,t}}{K_{t11}}$$

(23)

The maximum $K_{t11}$ for hybrid finger joint was 1.45 respectively. $X_{11,t}$ for carbon and glass plies are shown in Table 3. Similarly, the maximum $K_{t11}$ was determined for the scarf joint to occur at the topmost glass ply to be 1.73.

5.4.1.2 Pull-out model

Load transfer in the hybrid composite ply joints occurs through interlaminar shear. The interlaminar shear stress distribution can vary significantly with joint length as depicted in Figure 42. For short joint lengths (relative to a critical load transfer length), the interlaminar shear stress between the carbon and glass plies is approximately constant within the joint region. A short finger joint therefore fails when the interlaminar stress exceeds the interlaminar shear strength of the matrix, similar to the limit analysis method for determining structural collapse loads.

At “long” step lengths, the interlaminar shear stresses are zero almost everywhere in the interface, except at the ply termination where a stress singularity exists. Since the resin pockets have very low stiffness and strength (thus fracture well before the ultimate strength of the joint), they can be assumed to act as voided regions and the delamination strength can be
determined by using the SERR method under the linear elastic fracture mechanics framework outlined in Chapter 4.

![Figure 42 Schematic description of interlaminar shear stresses distribution between resin pockets of a hybrid ply joint; Left schematic shows a “short” joint and right schematic shows a “long” joint](image)

**Short joint:**

At very short step lengths, the difference in the carbon and glass laminate thicknesses results in geometric eccentricity which introduces interlaminar peel stresses. Figure 43 schematically illustrates the difference in interlaminar stresses in a flush and eccentric short finger joint. The peel stress, $\sigma_{33}$, can be expressed in terms of the shear stress $\tau_{13}$ by appealing to the equilibrium condition,

$$\sigma_{33} = \tau_{13} \tan \alpha$$  \hspace{1cm} (24)

where $\alpha$ is the average taper angle of the joint calculated using the two laminate thicknesses assuming linear change in thickness. At failure, the interlaminar shear and peel stresses obey the following quadratic failure criterion

$$\left( \frac{\tau_{13}}{S_{13}} \right)^2 + \left( \frac{\sigma_{33}}{S_{33}} \right)^2 = 1$$  \hspace{1cm} (25)

where $S_{13}$ and $S_{33}$ are the interlaminar shear and interlaminar peel strength of the composite ply. From equilibrium condition, the applied stress is

$$\sigma_s = n \left( \frac{l}{T} \right) \left( \tau_{13} \cos \alpha + \sigma_{33} \sin \alpha \right) = n \left( \frac{l}{T} \right) \frac{\tau_{13}}{\cos \alpha}$$  \hspace{1cm} (26)

$T$ denotes the GFRP laminate thickness, and $n$ denotes the number of debonding interfaces, assuming all the interfaces delaminate simultaneously and $l$ is the step length. Therefore, the interlaminar shear and peel stresses can be expressed in terms of the applied stress.

65
\[ \tau_{13} = \sigma_s \frac{T}{n_l} \cos \alpha \]  
(27)

\[ \sigma_{33} = \sigma_s \frac{T}{n_l} \sin \alpha \]  
(28)

The pull-out strength \( \sigma_s \) can be determined by inserting Eq.(27) and Eq.(28) into Eq.(26), yielding

\[ \sigma_s = \frac{m}{T} \frac{1}{\sqrt{\left(\frac{\cos \alpha}{S_{13}}\right)^2 + \left(\frac{\sin \alpha}{S_{33}}\right)^2}} \]  
(29)

where \( S_{13} \) and \( S_{33} \) are the interlaminar shear and interlaminar peel strength of the composite ply. The values of \( S_{13} \) and \( S_{33} \) for CFRP listed in Appendix A are used as a conservative measure. For the finger joint, \( n = 14 \), six plies with delamination occurring at both sides of the ply interface and one each at surface plies. For the scarf joint \( n = 15 \), one delamination for each carbon-glass interface.

![Figure 43 Effect of ply thickness mismatch on interlaminar stress state in hybrid ply joint; left image shows a flush joint and right image shows an eccentric joint](image)

**Long joint:**

The fractographic evidence in Figure 40 indicate that the delaminations are between the carbon and glass plies in the joint region. For example, a finger joint has two intersections from which delamination can initiate. At intersection T1 carbon plies pull-out from the hybrid laminate and at intersection T2 glass plies pull-out from the hybrid laminate as depicted in Figure 44. For a given intersection, it is assumed that all plies disbond at the same load. Recalling Eq.(22)
\[ N^\infty = \frac{2nGcE_iE_fh_ih_f}{E_ih_i - E_fh_f} \]

where \( h_i = T \) and \( h_f = T/2 \). \( n = 14 \), which is total number of delaminating interfaces. The laminate stiffnesses are determined using simple rule of mixtures (ROM); \( E_i = 0.5 \ast E_1(\text{carbon}) + 0.5 \ast E_1(\text{glass}) \). For T1: \( E_f = 0.5 \ast E_1(\text{carbon}) \) and for T2: \( E_f = 0.5 \ast E_1(\text{glass}) \).

Under tensile loading, the delaminations are driven by the Mode II fracture toughness (\( G_c = G_{IIc} \)) because the peel stresses are compressive, except at the tip of the ply termination. As a conservative measure \( G_{IIc} \) of carbon laminate is used for the carbon-glass delamination in finger and scarf joints.

![Analytical pull-out model (Long joint): Delamination of glass plies (left) at intersection T1 shown on the left image and delamination of carbon plies at intersection T2 shown the right image](image)

### 5.4.2 Computational modelling

#### 5.4.2.1 Finite element model description

Finite element models of the joints were created and analysed using Abaqus/Explicit. The geometry and mesh of the carbon/glass finger joint FE model for a joint length of 6mm is shown in Figure 45. Laminate thickness in the joint region tapers linearly from GFRP thickness to CFRP thickness. A thin slice model of the specimen of dimension 0.5 mm was used. The element dimensions, 0.2x0.5x0.1 mm, were chosen based on recommendations in [88] to alleviate mesh dependency of the numerical solution. Each ply was meshed using reduced integration continuum shell elements (SC8R). Transversely isotropic material properties, listed in Appendix A, were assigned to the elements of the resin pockets, Resin pockets were voids due to their low stiffness and hence modelled as discontinuities. The
average length of the resin pockets was set at 0.2 mm obtained from micrographs of joint specimens. All the nodes were constrained in the width direction to simulate plane strain boundary conditions. The model was fixed in all degrees of freedom at the carbon end and a monotonically increasing displacement was applied at the glass end.

5.4.2.2 Damage modelling approach

Ply rupture was modelled using the continuum damage mechanics (CDM) approach. The CDM model employed in this study uses the Hashin failure criteria for plane stress loading conditions [62] to define the onset of damage. A stiffness degradation approach based on the fracture energy associated with the damage mode is used to characterise the damage evolution process [63]. Details of the damage modelling approach can be found in [59]. Experimentally obtained ply strengths and fracture energies can be found in Appendix A. The cohesive zone model (CZM) was employed to simulate interlaminar damage. Cohesive elements [82-84] with a mixed-mode formulation [87] were inserted between plies. The traction laws of the cohesive elements were dependent on the respective bounding plies. For cohesive elements in carbon-glass interface, the carbon interlaminar properties were used as a conservative measure. Interlaminar strengths and fracture toughness are listed in Appendix A.

Figure 45 Geometry and mesh of finite element model for unidirectional hybrid finger ply joint
5.5 Analysis Results

5.5.1 FE damage progression

The deformed states of the FE model at the onset of damage and ultimate failure are shown in Figure 46. The maximum interlaminar stress concentrations occurred at carbon plies termination, thereby initiating damage between carbon and glass plies at approximately 95% of ultimate load. Delamination (highlighted in red) then propagated unstably across the entire length of the joint causing the complete separation of the carbon and glass plies. For finger joints with step lengths, \( l_s \geq 6 \, mm \), delamination is also accompanied by the fibre fracture of the glass plies as shown in Figure 46 (b).

Figure 46 FE damage model prediction of damage modes: (a) Deformed state at 95% of ultimate load showing initiation of delamination damage (left image) and deformed state ultimate load showing complete separation of carbon-glass interfaces (b) correlation of prediction of fibre fracture of glass plies at ultimate load with experiment (Contour plot legend - 1 is fully damaged and 0 is undamaged)
5.5.2 Comparison of strength predictions with experiments

Comparisons of the predictions of the analytical and computational models with experimental results are shown in Figure 47. Overall, the FE models employing CZM and CDM give good correlation with the experimental data. FE simulations conducted with linear-elastic ply behaviour and CZM yield slightly higher predictions to the FE (CZM+CDM) as they do not account for intralaminar damage resulting in the plies carrying stresses above their respective strengths. The pull-out model employing linear elastic analysis of interlaminar stresses clearly overpredicts the experimental results as the assumptions are only valid for joints of very short step length. The pull-out model based on LEFM and the FE (CZM) model give identical strength predictions. The ply rupture model based on strength-of-materials and a point stress criterion significantly overpredicts the glass fibre fracture stress in finger joints in comparison to the CDM model. This overprediction is due to the arbitrary choice of the characteristic distance and varying this distance will obviously produce a higher stress concentration and therefore lower the prediction.
5.6 Concluding Remarks

The structural behaviour of ply-interleaving joints using unidirectional CFRP and GFRP laminates were investigated in this study using experimental, analytical and numerical methods. Fractographic observations revealed that ply rupture of glass plies and interfacial pull-out between carbon and glass plies were the dominant failure modes. Both the finger and
scarf joints produced identical strength, approximately 50% of the un-notched unidirectional glass fibre composite strength. However, the hybridizing or joint length required for scarf joints to achieve optimum strength is much larger in comparison to finger joints. Analytical models provided accurate strength predictions for short joints (step lengths <1mm) and long joints (step lengths >3mm). Computational models employing both continuum damage model and cohesive zone model were found to correlate well with the experimental results.
Chapter 6 Hybrid ply-interleaved joints for load-bearing structures

6.1 Introduction

Load-bearing composite structures are primarily orthotropic laminates designed to withstand multiple loading conditions. Quasi-isotropic laminates are a special class of orthotropic laminates that have identical strength and stiffness in both in-plane directions. In this chapter, we present novel ply-interleaving techniques for joining quasi-isotropic dissimilar composite materials. To achieve high structural strength using the ply-interleaving technique it is necessary to optimise the design, such as the step lengths of interleaving plies and the spatial distribution of the ply termination locations. In addition to mechanical considerations, the differences in the coefficient of thermal expansion between carbon and glass fibre composites may produce residual thermal stresses/strains which must be accounted for in the design. The objectives of this chapter are to (1) investigate the failure mechanisms and the influence of key design parameters on joint strength and (2) develop an analysis methodology for joint strength prediction that is able to capture the effect of joint design parameters and residual thermal stresses. The work presented herein is outlined as follows. The methodology and results of the experimental testing program are outlined in section 6.2 and 6.3 respectively. The analytical and computational modelling methodology which builds on the methods previously presented in Chapter 4 and 5 are outlined in Section 6.4. The predictive capability of the two analysis methodologies are compared and discussed in Section 6.5. Finally, the conclusions of this chapter are summarised in the last section.

6.2 Experimental Method

6.2.1 Specimen Design

Five ply-interleaved joint configurations were investigated for joining QI CFRP and GFRP laminates with a matched stacking sequence of [45/0/-45/90]₂ₛ (Figure 48). Two finger joint configurations were investigated, aligned finger joint in which all four 0°-0° ply terminations lie at the same intersection and offset finger joint in which 0°-0° ply terminations are staggered symmetrically through the laminate thickness. A scarf joint with a linear taper and symmetric and asymmetric interleaved-scarf joint were also investigated. The distance between the terminations of adjacent plies or the step length is denoted as $l$, while the distance between the terminations of two adjacent 0° plies is denoted as $l_0$ and $L$ represents the total
joint length. For example, the interleaved-scarf joint $l_0 = 2l$ and $L = 7l$ whereas for offset finger joint $l_0 = l$ and $L = 2l$. The findings of chapter 4 revealed that stress concentrations were significantly higher for stiffer plies terminating in a softer laminate. Therefore, for interleaved-scarf and finger joints the critical $0^\circ-0^\circ$ ply terminations are located near the CFRP laminate.
6.2.2 Manufacturing

Joint panels were fabricated with unidirectional prepregs of carbon/epoxy (T700/VTM264) and glass/epoxy (E-Glass/MTM57) using conventional hand prepreg lay-up technique and autoclave cure as described in the previous chapter. Joints of varying step lengths were fabricated to investigate the effect of the joint length on the strength. The complete test matrix with the notations used for different configurations is summarised in Table 5. Tensile test specimens measuring 300 mm in length and 25 mm in width were cut from the cured joint panels using a waterjet diamond wheel cutting machine in accordance with ASTM D3039 standard [108].

Micrographs of a typical manufactured interleaved-scarf joint and a typical finger joint are shown in Figure 49. Comparison of the actual joint dimensions with the respective schematics in Figure 48, indicates a deviation in the position of the ply terminations. This deviation is a
result of the movement of plies during consolidation and curing. Similar observations have been reported by other authors [4], [5]. Hence the joints investigated in the present study are representative of practical manufacturing tolerances.

Figure 49 Optical images of fabricated joint specimens (a) Finger-aligned (FA-12) (b) Finger offset (FO-12) (c) Scarf (S-1) (d) Interleaved-scarf asymmetric (ISU-1) (e) Interleaved-scarf symmetric (IS-1)
6.2.3 Testing

Joints were tested to failure under uni-axial tension using an INSTRON 4465 load frame equipped with a 50 kN load cell. A constant cross-head speed of 1 mm/min was applied and the load-displacement response was recorded at a rate of 4 Hz. A minimum of four replicas for each joint configuration were tested. Fractographic analysis was conducted through visual observation aided by high resolution optical images of tested specimens.

6.3 Experimental Results

Typical stress-displacement response for each joint is shown in Figure 50. Stress was calculated using the initial cross-sectional area of the GFRP laminate. Some non-linearity in the stress-strain response was observed. Significant audible cracking was heard during the test for all joints at approximately 50% of the ultimate load although very little observable changes in the stress-displacement curves were detected. Catastrophic brittle failure was observed for all joints tested.

![Figure 50 Typical stress-displacement curves for QI hybrid ply-interleaved joints](image)

Optical images revealing the typical failure topography are shown in Figure 51. The common failure modes observed in all the joint configurations were interlaminar cracks or delamination and transverse matrix cracks. Very little fibre fracture was observed. The finger-aligned joints failed mostly by delamination in the carbon laminate and transverse matrix cracking of off-axis carbon plies ±C45°/C90°. The glass region is shown in to be largely
undamaged as all the 0°-0° ply terminations are situated near the carbon region. There were no discernible differences in the failure topography of interleaved-scarf joint (asymmetric and symmetric) and finger-offset joints. The fractographic evidence presented in Fig. 6 (a) and (b) reveal that the main failure mechanism is the delamination cracking between the glass fibre plies and carbon fibre plies. The crack paths are shown in Fig. 6 (c) and (d). The likely sequence of crack initiation and propagation in the hybrid joints is summarised as follows. Transverse matrix cracks occur at very low loads, particularly near the ply terminations. Delaminations then initiate from the 0°-0° ply terminations and grow between ±45°/0° ply interfaces. For joints with 0°-0° ply terminations in the carbon laminate, delamination growth is more pronounced between ±C45°/C0° ply interfaces compared to ±C45°/G0° ply interface. The transverse matrix cracks allow the coalescence of delamination cracks leading to ultimate failure of the joint. For scarf joints, the failure initiation can be traced to a cracked G45° ply below the 0°-0° ply termination. This crack then migrates into the glass plies and propagates rapidly along the length of the joint. Delaminations and fracture of off-axis plies can be seen, including fibre fracture of a 0° glass ply. The presence of glass plies on the carbon region of the joint provides further confirmation that the failure was mostly within the GFRP laminate.
Figure 51 Optical images of failure modes and schematic illustrations of crack paths for QI ply-interleaved joints: (a) Finger-Aligned (b) Finger-Offset (c) Scarf (d) Interleaved-scarf (IS-6); Red arrows indicate in (c) indicate the crack path.

The average tensile strengths with their coefficients of variation for all joint configuration are shown in Table 4. Apart from the FA joint which is sensitive to the misalignment of the 0°-0° ply terminations (13.1%), the highest coefficient of variation in strength results was found to be 5% which indicates low sensitivity to the manufacturing variations. Due to the close vertical alignment of the critical 0°-0° ply terminations, the FA joint strength is only about 50% of the unnotched GFRP laminate. The scarf joint has only a slightly higher strength than FA joint. Despite the large bond area, high secondary bending stresses cause the failure to occur in interlaminar peel rather than interlaminar shear. It can also be seen that employing a symmetric configuration as in the case of the interleaved-scarf increases the joint strength by 20%. Comparing the maximum strengths of the different joint configurations, the FO and IS produce the highest joint strength, approximately 75% of the unnotched GFRP laminate (Appendix A). With the exception of FA joint, the joint strength shows an asymptotic behaviour with step length. For example, IS joint strength increases by 58% when step length is increased from 1 mm to 3 mm. Further increase in step length does not significantly increase joint strength.
## Analysis Methods

### Analytical Modelling

Load transfer in the ply-interleaved joints occurs through interlaminar shear. The interlaminar shear stress distribution can vary significantly with joint length as shown in Chapter 5. For short joint lengths, interlaminar shear stresses are nearly constant along the joint length, away from corner singularities. Therefore, the maximum load-carrying capacity of the joint is proportional to the product of the interlaminar shear strength and the bond area. With increasing joint length, however, the interlaminar shear stress become much more highly localised at ply terminations. Therefore, for a long joint its strength is governed by the interlaminar fracture toughness. To predict the pull-out strength, a limit strength analysis for joints with “short” step lengths and a fracture mechanics approach for joints of “long” step lengths were developed. Details of the models are provided in the following sections. These analytical models build on the methods developed in Chapter 5. The analysis here will focus on the high performance interleaved-scarf (symmetric) and finger-offset joint. The finger-offset joint will simply be referred to as the finger joint in subsequent sections.

### Table 5 Summary of joint strength data for QI hybrid ply-interleaved joints of various joint configurations

<table>
<thead>
<tr>
<th>Joint configuration</th>
<th>Step length (mm)</th>
<th>Joint length (mm)</th>
<th>Average joint strength (MPa)</th>
<th>Standard deviation (MPa)</th>
<th>Coefficient of variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finger-Aligned (F-A)</td>
<td>3</td>
<td>3</td>
<td>171</td>
<td>17</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6</td>
<td>191</td>
<td>25</td>
<td>13.1</td>
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<td></td>
<td>9</td>
<td>9</td>
<td>170</td>
<td>19</td>
<td>11.2</td>
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<td></td>
<td>12</td>
<td>12</td>
<td>178</td>
<td>14</td>
<td>7.8</td>
</tr>
<tr>
<td>Finger-Offset (F-O)</td>
<td>1</td>
<td>4</td>
<td>190</td>
<td>8</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>12</td>
<td>210</td>
<td>6</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>24</td>
<td>241</td>
<td>14</td>
<td>5.8</td>
</tr>
<tr>
<td>Scarf (S)</td>
<td>1</td>
<td>15</td>
<td>158</td>
<td>5</td>
<td>3.2</td>
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<tr>
<td></td>
<td>3</td>
<td>45</td>
<td>199</td>
<td>4</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>90</td>
<td>201</td>
<td>4</td>
<td>2.0</td>
</tr>
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<td>7</td>
<td>155</td>
<td>8</td>
<td>5.4</td>
</tr>
<tr>
<td>symmetric (IS)</td>
<td>3</td>
<td>21</td>
<td>236</td>
<td>4</td>
<td>1.6</td>
</tr>
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<td>6</td>
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<td>247</td>
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<td>1</td>
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<td>170</td>
<td>7</td>
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<tr>
<td>unsymmetric (IS)</td>
<td>6</td>
<td>84</td>
<td>58</td>
<td>214</td>
<td>8</td>
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</tbody>
</table>

81
6.4.1.1 Pull-out model (short)

(a)

Figure 52 Pull-out model for short joints: (a) Schematic illustration of failure (b) Comparison of interlaminar stresses for a flush (interlaminar shear stress only) and eccentric joint (interlaminar shear and peel stresses) in interleaved-scarf joints

For short joint lengths, interlaminar shear stresses are nearly constant along the joint length, away from corner singularities. Therefore, the maximum load-carrying capacity of the joint is proportional to the product of the interlaminar shear strength and the bond area. The analytical pull-out failure mode for an IS joint of constant thickness and short joint length is shown in Figure 52 (a) where the two laminates debond or pull-out through interlaminar shear. At very short step lengths, the difference in the carbon and glass laminate thicknesses results in geometric eccentricity which introduces shear and normal stresses as shown in Figure 52 (b). Assuming nearly constant interlaminar shear and peel stresses, using the
derivations in Chapter 5, the tensile strength for a “short” joint of step length, \( l \), and thickness, \( T \):

\[
\sigma_s = \frac{L_s}{T} \left[ \frac{1}{\cos \alpha \cdot S_{13}} + \frac{\sin \alpha \cdot S_{33}}{2} \right]
\]

(30)

where \( S_{13} \) and \( S_{33} \) are the interlaminar shear and interlaminar peel strength of the composite ply. The values of \( S_{13} \) and \( S_{33} \) for CFRP listed in Appendix A are used as a conservative measure. \( L_s \) represents the total shear length of the joint. For instance, interleaved-scarf joint; \( L_s = 56l \) and for the finger joint, \( L_s = 8l \).

6.4.1.2 Pull-out model (Long):

![Figure 53 Pull-out model for long joints: Schematic illustration of failure in interleaved-scarf joints](image)

To determine the load required to cause unstable delamination or pull-out of plies for a “long” quasi-isotropic ply-interleaved joint, a linear elastic fracture mechanics approach is developed. This approach equates the critical strain energy release rate to the difference in strain energy of the laminate before and after pull-out (energy balance during crack growth). The resin pockets at ply terminations are assumed to have already been disbonded at low applied load well before the joint reaches its strength. The solutions presented here are derived using classical laminate theory and account for residual thermal stresses.

We will first derive the equations for strain energy of a laminate containing \( N \) number of plies with a nominal ply thickness, \( t_p \), and a resin pocket located in the \( k^{th} \) ply subjected to a remote tensile load per unit width of \( N^\infty \). Assuming plane stress conditions, the strain energy of the uncracked laminate is equal to the sum of strain energies of all the plies:
\[ U = \frac{1}{2} t_p \sum_{j=1}^{j=N} \sigma_{11}^j \varepsilon_{11}^j + \sigma_{22}^j \varepsilon_{22}^j \]  

(31)

where \( \sigma \) and \( \varepsilon \) are the ply stress and strain, with the subscripts 11 and 22 denoting the longitudinal and lateral components, and the superscript \( j \) denoting the ply index. After total of \( n \) plies disbond, the stresses and strains in the laminate get redistributed and strain energy of the resulting laminate containing \( N - n \) plies is written as:

\[ U^* = \frac{1}{2} t_p \sum_{j=1}^{j=N-n} \sigma_{11}^j \varepsilon_{11}^j + \sigma_{22}^j \varepsilon_{22}^j \]  

(32)

The energy balance equation can then be written as:

\[ 2nG_c = \sum_{j=1}^{j=N} \sigma_{11}^j \varepsilon_{11}^j + \sigma_{22}^j \varepsilon_{22}^j - \sum_{j=1}^{j=N-n} \sigma_{11}^j \varepsilon_{11}^j + \sigma_{22}^j \varepsilon_{22}^j \]  

(33)

where \( n \) represents the delaminating interfaces and \( G_c \) is the critical strain energy release rate. The factor of two is present on the left-hand side of the equation as delamination occurs on both interfaces at the ply termination. The delaminations are assumed to be Mode II dominant \((G_c = G_{IIc})\) as the Mode I normal stresses are compressive, except at tip of the ply termination. The delamination load is obtained from Eq. (33) using Newton-Raphson iteration method using a MATLAB script shown in Appendix C.

If thermal residual stresses are ignored, the strain energy of the laminate simplifies to:

\[ U = \frac{1}{2} \left( \sigma_1^\infty \varepsilon_1 + \sigma_2^\infty \varepsilon_2 \right) h \]  

(34)

where \( h \) is the laminate thickness. The lateral components of stress and strain are related to the longitudinal stress and strain and therefore, Eq. (8) can be rewritten as:

\[ U = \frac{1}{2} \left( \sigma_1^\infty \right)^2 k h \]  

(35)

where \( k = (E_1 + E_2 \nu_2^2) / E_1^2 \). Using the energy balance principle and rearranging equation in terms of the applied load gives

\[ N^* = \frac{4nG_c h h^*}{kh^* - k^* h} \]  

(36)

If the lateral stress \( \sigma_2 \) and strain \( \varepsilon_2 \) are negligible, the equation reduces to the form:
\[ N^* = \sqrt{\frac{4nG_j E_{\text{lam}}^* E_{\text{lam}}^* h h^*}{E_{\text{lam}}^* h^* - E_{\text{lam}}^* h}} \]  

(37)

The above equation reduces to the solution for unidirectional ply-interleaved joints, Eq.(22). Now as an example, let us consider the delamination cracking of the interleaved-scarf joint. From the fractographic evidence presented earlier, it is clear that the delaminations emanating from the 0°-0° ply terminations are most critical. There are four 0°-0° ply terminations in the joint, two each at intersection 1 and 2 as depicted in Figure 53. For a given intersection, it is assumed that both 0° plies disbond at the same load. The delaminations can occur between carbon plies or between carbon and glass plies, shaded in red and green respectively in Figure 53. The joint strength is the minimum of the two predicted values. Using Eq. (33) and Eq. (37), the delamination load for each of these different crack paths can be calculated.

### 6.4.2 Computational modelling

Analytical solutions presented in the previous section are essentially for very short and very long joints. Therefore, a more robust predictive model is required for joints of intermediate length. Furthermore, the analytical solutions do not consider effect of intralaminar matrix cracking on delamination growth which was consistently observed in the experimentally tested joint samples. To address the limitations of the analytical methodologies, the finite element (FE) analyses employing progressive damage models were used to simulate the onset and propagation of intralaminar and interlaminar damage and predict the ultimate strength. The creation of the finite element model and simulations were performed using commercially available finite element software, Abaqus version 6.12 [61, 109]. The cohesive zone model (CZM) [82-84] was used to simulate interlaminar damage or delamination. The Abaqus CZM uses a linear traction-separation law to relate the interlaminar stresses to the displacements of two initially bonded nodes. The onset of cohesive damage is determined by the interlaminar strengths and the total energy under the traction separation curve is equal to the interlaminar fracture toughness of the material. A mixed-mode quadratic failure criteria was used to define the initiation of damage while the B-K law [87] was employed for the damage evolution. To simulate intralaminar damage modes such as fibre failure and matrix damage the continuum damage model in Abaqus is used. The CDM model employed in this study uses the Hashin failure criteria for plane stress loading conditions [62] to define the onset of damage. A stiffness degradation approach based on the fracture energy associated with the
damage mode is used to characterise the damage evolution process [63]. Details of the
damage modelling approach can be found in [59].

A stacked-shell finite element model of the interleaved-scarf joint is shown in Figure 54.
Laminate thickness in the joint region tapers linearly from GFRP thickness to CFRP
thickness. The resin rich pockets were assumed to be voids due to their low stiffness and
strength. The average length of resin pockets was set at 0.2 mm obtained from microscopic
images of joint specimens. Local coordinates were established for each ply to facilitate the
assignment of the appropriate orthotropic material properties listed in Appendix A. The entire
specimen was modelled using three-dimensional continuum shell elements (SC8R) to capture
the in-plane ply stresses. In the joint region, three layers of continuum shell elements are
employed for each ply to improve the bending response of the model and capture the stress
gradient near the resin pockets. Away from the joint region, one layer of continuum shell
elements was employed for each ply. Cohesive elements (COH3D8) were placed at carbon-
carbon and carbon-glass interfaces in the joint region as shown in Figure 54 (b). For finger
joint models, cohesive elements were only placed at ply interfaces in the vicinity of 0°-0° ply
terminations.

Matrix damage and ply rupture failure were modelled using the continuum damage
mechanical (CDM) approach. In order to alleviate the mesh dependency of the FE model
prediction, the finite element size was chosen according to equations provided in [58, 88],
i.e., the necessary finite element size was found to be 0.2 mm. Mesh sensitivity study
(Appendix C) was then conducted with smaller element sizes of 0.1 mm and 0.05 mm and the
results differed by less than 4%. Therefore, the in-plane dimensions of the continuum shell
element and cohesive element size were chosen to be 0.2 mm by 0.2 mm.

Interlaminar and intralaminar properties for both materials can be found in Appendix A. For
delamination along carbon-glass interface, analysis was conducted using interlaminar
strengths and fracture toughness values of carbon ply as a conservative measure. The
transverse tensile, YT, and shear strengths, SL, were obtained from standard tests and the
literature [6]–[9]. Fibre tension strength XT, and fibre tension fracture energy, GfT, were
obtained using the calibration methods described in [66] by validating the material response
against notched and un-notched tests of quasi-isotropic CFRP and GFRP laminates,
respectively. The values of the matrix fracture energy, Gm, were assumed to be equal to the
Mode-I delamination fracture energy of the two materials.
One end of the FE model was fixed in all degrees of freedom and displacement control was applied at the other. Abaqus/Explicit dynamic solver was employed to overcome convergence difficulties associated with implicit solvers for quasi-static non-linear problems. No mass scaling was used. Upon conducting sensitivity analysis of different loading rates, a loading rate of 1000 m/s was found to produce a reasonable time increment size with minimum dynamic effects. Prior to the quasi-static analysis, a thermal analysis step with $\Delta T = -100^\circ C$ was performed to account for the residual thermal stresses and strains.
6.5 Analysis Results

6.5.1 FE damage progression

The FE model predicts that matrix damage initiates in the off-axis (±45°/90°) plies at relatively low stress, approximately 50% of the failure load. These cracks mostly occurred in the vicinity of the ply terminations. With increasing load, high interlaminar stresses develop at 0°-0° ply terminations leading to failure of the cohesive elements. Delamination propagation is initially stable, i.e. small increase in crack length with increasing load.

Delamination cracks grow to a size of 1 mm at approximately 75% of the failure load. Figure 55 (a) shows the delamination damage at various load levels in an interface between ±45° carbon ply and 0° carbon ply, where delaminations were observed predominately. As the cracks increase in length, redistribution of stresses in the laminate cause a significant increase in matrix damage in ±45°/90° plies. Figure 55 (b) shows the progression of matrix tensile damage in an interleaved-scarf joint model with increasing load. Failure of the joint occurs when the interlaminar cracks coalesce with intralaminar matrix cracking in
the off axis (±45°/90°) plies. The damage propagation behaviour in finger joint models was nearly identical to the interleaved-scarf joints and FE results are therefore omitted for brevity. The final deformed state of the FE model is shown in Figure 55 (c) which compares reasonably well with the optical images of failed specimens.
Figure 55 Finite element model prediction of damage modes in interleaved-scarf joint (a) Delamination progression at cohesive elements at the interface between 45° and 0° carbon ply and (b) Matrix tensile damage progression at different stages of loading c) Final deformed state with failed cohesive elements highlighted in red. Percentages in (a) and (b) indicate the different stages of loading.
6.5.2 Comparison of strength predictions with experiments

Table 6 Comparison of analytical predictions with experimental results for QI hybrid ply-interleaved joints; results expressed in MPa

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Pull-out (short)</th>
<th>Pull-out (Long) 1-D</th>
<th>Pull-out (Long) 2-D with thermal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interleaved-scarf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IS_2</td>
<td>155±8</td>
<td>579</td>
<td>309</td>
<td>254</td>
</tr>
<tr>
<td>IS_6</td>
<td>236±4</td>
<td>-</td>
<td>309</td>
<td>254</td>
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<tr>
<td>IS_12</td>
<td>245±11</td>
<td>-</td>
<td>309</td>
<td>254</td>
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<tr>
<td>Finger</td>
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<td></td>
</tr>
<tr>
<td>F_2</td>
<td>190±8</td>
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<tr>
<td>F_6</td>
<td>210±6</td>
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<td>360</td>
<td>298</td>
</tr>
<tr>
<td>F_12</td>
<td>241±14</td>
<td>-</td>
<td>360</td>
<td>298</td>
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Table 7 Comparison of FE model predictions with experimental results for QI hybrid ply-interleaved joints; results expressed in MPa

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>CDM+CZM (No thermal)</th>
<th>CZM</th>
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<tbody>
<tr>
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<td></td>
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</tr>
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<td>186</td>
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<td>F_12</td>
<td>241±14</td>
<td>235</td>
<td>242</td>
</tr>
</tbody>
</table>

![Pull-out model](image)
A comparison of the analytical and FE model predictions with experimentally obtained joint strength is presented in Table 6 and Table 7 respectively. The predictions of the FE model employing combined CDM for intralaminar damage and CZM for interlaminar damage (delamination) compares well with the experimentally measured strengths. The FE model does predict the gradual rise in joint strengths in the transition region between ‘short’ and ‘long’ joint lengths as shown in Figure 56. The FE model demonstrates that there is a gradual increase in strength followed by the intermediate region where the gradient change in strength is much lower. Above a certain length, the joint strength increases by less than 1%. This critical joint length, commonly referred to as the transfer length for adhesively bonded joints, is usually defined as the joint length at which 95% of the “maximum” load-carrying capacity of the joint is achieved [98]. From Figure 56, the transfer length for the interleaved-scarf and finger joint are 21 mm and 12 mm, approximately; which equates to a step length of 3 mm and 6 mm, respectively.

FE simulations conducted with linear-elastic ply behaviour and CZM consistently over-predict the experimentally measured strengths. The overprediction is due to the absence of interlaminar damage which results in the off-axis (±45°/90°) plies carrying non-physical stresses. At joint lengths of less than 2 mm and greater than 20 mm, analytical pull-out models give almost the same predictions as the FE model employing cohesive elements alone (i.e., no matrix damage is considered). The analytical models based on linear elastic theories are therefore useful in obtaining upper and lower strength limits of hybrid ply-interleaved
joints. Since the thermal residual stresses do not significantly affect the strength prediction as shown in Table 6, the simplified Eq.(10) can be employed. The predictions also reveal that the 1-D analytical equation Eq.(37) predicts 15% higher strength than the 2-D analysis Eq.(33). If the residual thermal stresses are not considered, the FE model gives 1% and 5% higher strength for finger and interleaved-scarf joints respectively. This difference is attributed to the matrix cracking that occurs in the off-axis (±45°/90°) plies due to the thermal stresses. While there exists a significant difference in the coefficient of thermal expansion of carbon and glass fibres, the effect of thermal stresses on the strength is low since the joints are fabricated from balanced quasi-isotropic laminates.

6.6 Concluding remarks

The structural behaviour of quasi-isotropic ply-interleaved hybrid composite joints has been investigated using experimental testing, analytical modelling and finite element analysis. Two optimum joint configurations, interleaved-scarf and finger-offset have been developed for application to high performance load-bearing antennas. The results reveal that joint failure is caused by delaminations emanating from ply terminations and transverse matrix cracking of off-axis plies. The strength of both interleaved-scarf and finger-offset joints are approximately to 75% of the un-notched GFRP laminate strength, provided the distance between 0°-0° ply terminations exceed a certain threshold value. Finite element analyses, employing a continuum damage mechanics model for intralaminar damage and cohesive zone model for capturing interlaminar failure, provide conservative predictions in comparison with experimental results.
Chapter 7 Hybrid ply-overlap joints for load-bearing structures

7.1 Introduction

The load-carrying capacity of ply-interleaved joints was found to be 31% lower than the un-notched strength of the GFRP laminate (Chapter 6). In this chapter, novel joints using ply-overlap techniques are developed to achieve higher joint strengths than ply-interleaved joints. The work presented herein is outlined as follows. Details of the experimental program including design, manufacture and testing of representative joint specimens. The key results of the experimental investigation is provided in Section 3. The analytical and computational modelling methodology and results are presented in Section 4 and 5 respectively. The main conclusions of this chapter are summarised in the final section.

7.2 Experimental Method

7.2.1 Specimen Design

Three ply-overlap design configurations were investigated for joining quasi-isotropic CFRP and GFRP laminates with a matched stacking sequence of \([45/0/-45/90]_2s\). Schematic illustrations of the ply-overlap joint configurations are shown in Figure 57. The aligned-overlap joint is the simplest design configuration in which every ply is overlapped with all plies being terminated at the same location. This joint requires the shortest joint length but produces an undesirable abrupt thickness change: the thickness of the overlapped region is equal to the sum of the thicknesses of the two laminates. A staggered overlap configuration analogous to a double scarf. This joint requires a comparatively long joint length but the taper angles of the overlap region are minimised. The third configuration which produces a flush joint is designed by selectively overlapping the main load-bearing plies aligned with the loading direction, while terminating the same number of weaker off-angle plies outside the joint region. For the quasi-isotropic layup considered in this study the 0° glass plies are overlapped with 0° carbon plies and ±45° glass plies are simply abutted with the carbon plies. The weakest plies which are the 90° plies are terminated away from the joint. This joint design has the additional benefit of being symmetric which will result in reduced interlaminar peel stresses compared with the aligned or staggered overlap joints. The flush ply-overlap joint will be the focus of further investigation and the non-flush designs used for comparison purposes.
Figure 57 Hybrid ply-overlap joint configurations investigated in this chapter: From top to bottom schematic- Aligned overlap, staggered overlap and flush overlap

7.2.2 Manufacturing

Hybrid ply-overlap joints were manufactured with unidirectional prepregs of carbon/epoxy (T700/VTM264) and glass/epoxy (E-Glass/MTM57) using conventional hand prepreg lay-up technique and autoclave cure as described in Chapter 4. Typical optical images of the fabricated joints are shown in Figure 58. The maximum joint thickness for both the aligned and staggered joint was the same. However, the aligned joint featured a steeper taper angle. There was an observable deviation in the position of ply terminations due to the manufacturing tolerances and movement of plies during consolidation and curing. A detail view of a typical resin pocket adjacent to a ply termination is shown in Figure 58 (c).
7.2.3 Testing

Tensile tests were conducted using an INSTRON 4465 load frame equipped with a 50 kN load cell. Schematics of the tension and compression test fixtures are shown in Figure 59. Tabs are added to the carbon fibre laminate end so that the two ends of the specimens are aligned. A constant cross-head speed of 1 mm/min was employed and the load-displacement response was recorded at a rate of 4 Hz. Compressive tests were performed using the NASA short block test fixture [110] in a MTS hydraulic load frame equipped with a 100 kN load.
cell. Compression test specimens, 50 mm long and 25 mm wide, were bonded ‘back-to-back’ to prevent bending due to eccentricity caused by different laminate thicknesses between the glass and carbon fibre laminates. A constant cross-head speed of 0.5 mm/min for compression was applied and the load-displacement response was recorded at a rate of 4 Hz. A minimum of four replicates for each configuration was tested. Stresses were calculated using the cross-sectional area of the GFRP laminate. Fractographic analysis was conducted to identify the failure modes through visual observation aided by high-resolution optical images of tested specimens. For comparison purposes, hybrid interleaved-scarf ply joints were also fabricated and tested in the identical compression test fixture.

Figure 59 Test fixture (a) Tension (b) Compression for hybrid ply-overlap joints
7.3 Experimental Results

Figure 60 Typical load-displacement curves (a) Tension (all configurations) (b) Compression (flush ply-overlap)
7.3.1 Tension

Typical stress-displacement responses for each joint configuration are shown in Figure 60 (a). The stress is calculated using the nominal thickness of the GFRP laminate. The test results indicate that the joints exhibited an approximately linear-elastic response apart from the initial slacking caused by the tightening of the grips. Significant audible cracking was heard during the test for all joints at approximately 60% of the ultimate strength although no discernible change in global stiffness was detected. Upon reaching ultimate strength, brittle failure was observed for all the joints tested. The average tensile strengths with their coefficients of variation for all joint configurations are given in Table 8. The strongest joint configuration, flush ply-overlap joint employing a 9 mm overlap approaches the un-notched tensile strength of GFRP (Appendix A).

Examples of fracture specimens of aligned and staggered ply-overlap joints are shown in Figure 61(a) and Figure 61(b), respectively. In addition to delaminations at the interface between carbon and glass plies, limited fracture of carbon fibre plies were also observed, likely due to the secondary bending caused by the steep tapering angles. The fracture modes for flush joints of 3 mm and 6 mm overlap length were very similar, with delaminations between the carbon 0° plies and glass 0° plies emanating from the root of the overlap region being the principal failure mode. Figure 61(c) shows the typical fracture topography of flush joints with 6 mm overlap length. The delaminated plies were then isolated and observed

<table>
<thead>
<tr>
<th>Test Method</th>
<th>Joint Configuration</th>
<th>Step/Overlap Length(mm)</th>
<th>Average strength(MPa)</th>
<th>Coefficient of variation(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>Aligned</td>
<td>9</td>
<td>278</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Staggered</td>
<td>9</td>
<td>316</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>Flush</td>
<td>3</td>
<td>302</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>Flush</td>
<td>6</td>
<td>330</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Flush</td>
<td>9</td>
<td>354</td>
<td>0.2</td>
</tr>
<tr>
<td>Compression</td>
<td>Flush</td>
<td>9</td>
<td>300</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Interleaved-scarf</td>
<td>6</td>
<td>216</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 8 Summary of joint strength data for QI hybrid ply-overlap joints of various joint configurations
under scanning electron microscopy (SEM) to determine the delamination crack path. For flush ply-overlap joint specimens with 9 mm step length, failure was observed in the GFRP laminate away from the joint region as shown in Figure 61(d). Failure in the glass laminate indicates that the joint strength is greater than the un-notched GFRP strength (Appendix A). Several carbon fibres remained bonded to the fracture surfaces of the glass plies as shown by the high resolution optical image and Scanning electron microscopy (SEM) image shown in Figure 62(a) and Figure 62(b). Carbon fibres on the glass plies is indicative of the delamination crack propagating primarily within the carbon fibre plies adjacent to the glass laminate.
Figure 61 Failure modes of joint specimens in tension. (a) Aligned, (b) staggered, (c) Flush 6 mm, and (d) Flush 9 mm

(a)

Figure 62 Delaminated glass ply in flush ply-overlap 6mm joint: (a) Optical image and (b) SEM image show a significant number of carbon fibres on glass ply.

7.3.2 Compression

Stress-displacement responses for all four specimens demonstrated approximately linear elastic behaviour followed by catastrophic brittle failure, referring to Figure 60. The failure modes and peak loads were consistent for all but one test specimen that buckled due to failure of the adhesive used to bond the specimens. The average compression strength of the flush joints was measured to be 300±3 MPa as shown in Table 8. The joint strength is approximately 90% of the un-notched GFRP laminate compression strength (Appendix A). Optical images shown in Figure 63 indicate the typical fracture surface topography of the
compression-tested joints. Delaminations can be seen in the joint region between the overlapped carbon and glass plies and in the GFRP laminate. The images also reveal some fractured 0° glass plies and transverse matrix cracks in the off-axis (±45°/90°) plies. Microbuckling of glass fibres may have occurred after delaminations. It has been reported that microbuckling occurs when steep taper angles are involved [27]. The failure mode of interleaved-scarf is driven primarily by matrix cracking and delaminations can be observed. The average compression strength of these joints was 216±8 MPa as shown in Table 8.

![Fibre rupture](image)

![Delaminations](image)

![Transverse matrix crack](image)

(a)

![Delaminations](image)

(b)

Figure 63 Typical experimental failure modes of (a) flush ply-overlap joint and (b) interleaved-scarf ply-interleaved joint in compression
7.4 Analysis Methods

7.4.1 Analytical Modelling

The principal failure mode of flush ply-overlap joints was pull-out of the carbon plies from the joint as schematically depicted in Figure 64(a). To predict the delamination strength, a limit strength analysis for joints with “short” overlaps as shown in Figure 64(b) and a fracture mechanics approach for joints of “long” overlaps as shown in Figure 64(c) were developed. Details of the models are provided in the following sections. These analytical models build on the methods developed in Chapter 6.

Figure 64 Analytical pull-out failure model: (a) Schematic illustration of delamination path, (b) short overlap, and (c) long overlap
7.4.1.1 Pull-out model (short overlap)

For short overlap lengths (relative to a critical load transfer length), the interlaminar shear stress between the overlapped plies is approximately constant within the overlap region. Due to the difference in laminate thickness of the two materials, peel stresses must be considered. Assuming nearly constant interlaminar shear and peel stresses, using the derivations in Chapter 5, the tensile strength for a “short” joint of overlap length $L$, and thickness, $T$:

$$
\sigma_s = \frac{nL}{T} \frac{1}{\sqrt{\left(\frac{\cos \alpha}{S_{13}}\right)^2 + \left(\frac{\sin \alpha}{S_{33}}\right)^2}}
$$

where $S_{13}$ and $S_{33}$ are the interlaminar shear and interlaminar peel strength of the composite pl. and $n$ denotes the number of debonding interface, assuming all the interfaces delaminate simultaneously. The values of $S_{13}$ and $S_{33}$ for CFRP (Appendix A) are used as a conservative measure. For the flush joint in this study, $n = 8$ since there are four carbon 0° plies and debonding occurs on both sides of each ply.

7.4.1.2 Pull-out model (Long):

At long overlap lengths, the interlaminar shear stresses are zero almost everywhere in the interface, except at the ply termination where a stress singularity exists. Since the resin pockets have very low stiffness and strength (thus fracture well before the ultimate strength of the joint), they can be assumed to act as voided regions and the delamination strength can be determined by using the strain energy release rate (SERR) method described in Chapter 6. Under tensile loading, the delaminations are driven by the Mode-II fracture toughness ($G_c = G_{IIc}$) because the peel stresses are compressive, except at the tip of the ply termination. Under compression loading, the peel stresses are tensile and therefore delamination is affected by the peeling mode. To account for this mixed-mode delamination in compression, we employ the B-K law [87] to calculate the mixed-mode fracture toughness

$$
G_C = \begin{cases} 
G_{IIc} & \sigma_{33} \leq 0 \\
G_{Ic} + (G_{IIc} - G_{Ic}) \left( \frac{\sigma_{II}}{G_{II} + G_{I}} \right)^\eta & \sigma_{33} > 0 
\end{cases}
$$

where $\sigma_{33}$ is the interlaminar peel stress at a ply thickness distance away from the ply termination and the mode-mixity ratio is estimated from the ratio of interlaminar shear and peel stresses at the same distance using linear finite element analysis. Using a value of $\eta=2$ [84] and mode-mix ratio of 0.58, obtained from linear FE analysis, the critical SERR for
compression is 0.625 N/mm². The delamination load is obtained from Eq. (33) using Newton-Raphson iteration method using a MATLAB script shown in Appendix II.

7.4.2 Computational modelling

The creation and analysis of finite element models was conducted using commercially available finite element software, ABAQUS Explicit version 6.12 [61]. The geometry and mesh of the FE model for a flush ply-overlap joint of 9 mm overlap length is shown in Figure 65. The FE model consists of the joint region and a 25 mm length of the CFRP and GFRP laminate regions of the test specimen. The entire width (25 mm) of the test specimen was modelled. Plies in the overlap region were assumed to taper linearly from the nominal GFRP laminate thickness to the nominal CFRP laminate thickness. The sizes of the triangular and rectangular resin pockets were estimated from the micrographs of the specimens to be approximately 1 mm and 0.2 mm in length respectively. Using nominal ply thickness of GFRP (0.26 mm) the taper angle for the overlapped 0° glass plies at the triangular resin pocket is determined to be 14.5°. The joint geometry employed here assumes perfect alignment of the resin pockets. The effect of this geometric idealisation will be investigated later.

Each ply of the hybrid composite joint was discretised with eight node reduced integration continuum shell elements (SC8R) [61] while the resin pockets were discretised with eight node reduced integration continuum solid elements (C3D8R) [61]. The resin pockets in ply-overlap joints are much larger than the ply-interleaved joints and therefore were not modelled as voids. Linear elastic behaviour was assigned to the plies while a stress-softening model was used to simulate the failure process of resin pockets. Cohesive elements [82-84] were inserted at every dissimilar interface in the joint region and extended to 3 mm in the laminates to capture the delamination damage. The dimensions of the cohesive element, 0.2 mm by 0.5 mm, were chosen based on the recommendations in [85, 88] to alleviate mesh dependence of the numerical results. The in-plane dimensions of the shell elements were identical to the cohesive elements. Each ply was discretised with three elements through the thickness to improve the bending response and capture the stress gradient near the resin pockets more accurately. The properties of the cohesive elements representing delamination within CFRP and GFRP laminates are listed in Appendix A. The interlaminar strength and fracture toughness of carbon laminate were used for the cohesive elements between carbon-glass ply interfaces, based on the experimental observations that delamination occurred primarily within the carbon laminate. The B-K law [87] was employed to simulate effect of mixed
fracture mode. The CFRP laminate end of the model was fixed in all degrees of freedom and the GFRP laminate end was prescribed a displacement in the loading direction with other degrees of freedom fixed. Prior to the static analysis, a thermal analysis step was performed with $\Delta T = -100^\circ\text{C}$ to account for the effect of residual thermal stresses and strains resulting from the cure.

Since fractographic evidence presented in previous section for compression experiments showed both delaminations and fibre fracture of glass plies, the ABAQUS progressive damage model [59] was used to simulate compressive fibre failure. The model requires the fibre fracture strength $X_{11,c}$ for damage initiation and fibre compressive fracture energy $G_{fc}$ for damage propagation. The values of $X_{11,c} = 1000$ MPa and $G_{fc} = 40$ N/mm$^2$ were obtained from previous publications on the same material system [111, 112].

Figure 65 Finite element model mesh for progressive damage analysis of flush ply-overlap joint
7.5 Analysis Results

7.5.1 FE damage progression

7.5.1.1 Tension

The deformed states of the FE model at three stages damage onset, propagation and final failure for the ply-overlap joint under tensile loading is shown in Figure 66. The maximum interlaminar stress concentrations were adjacent to the terminations of off-axis glass plies, thereby, initiating damage between the G0°-C0° interfaces (here G and C denote glass and carbon fibre plies) at approximately 80% of the ultimate load. The delamination propagation was initially stable, i.e. a small increase in crack length with increasing load. Fatal delaminations that cause the separation of the glass and carbon laminates were at the interface between glass and carbon laminates.
7.5.1.2 Compression

Figure 67 FE model prediction of damage evolution of flush ply-overlap joint under compression; 80% failure load – damage onset, 90% final load – damage propagation and final failure

Under compressive loading, delamination failure was observed to initiate in the G0°-C0° interfaces at a lower load due to the presence of tensile peel stresses, approximately 60% of the ultimate compression strength. The deformed states of the FE model showing the damage onset, propagation and final failure under compressive loading are presented in Figure 67. Delaminations were predicted to occur within the GFRP laminate. The delamination grew in a stable manner initially until upon reaching a critical size and then propagated unstably at the ultimate load.

7.5.2 Comparison of strength predictions with experiments

<table>
<thead>
<tr>
<th>Test Method</th>
<th>Overlap Length (mm)</th>
<th>Experimental strength (MPa)</th>
<th>Pull-out (short)</th>
<th>Pull-out (Long)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>302±12</td>
<td>427</td>
<td>343</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>330±4</td>
<td>-</td>
<td>343</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>354±1</td>
<td>-</td>
<td>343</td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td>9</td>
<td>300±2.8</td>
<td>-</td>
<td>236</td>
</tr>
</tbody>
</table>

Table 9 Comparison of analytical model predictions with experimental results
Comparison of strength predictions obtained from the analytical models and the FE model with the experiments are shown in Table 9 and Table 10. The pull-out model employing

<table>
<thead>
<tr>
<th>Test Method</th>
<th>Overlap Length (mm)</th>
<th>Experimental strength (MPa)</th>
<th>CZM Strength (MPa)</th>
<th>No thermal (CZM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>3</td>
<td>302±12</td>
<td>232</td>
<td>239</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>330±4</td>
<td>344</td>
<td>372</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>354±1</td>
<td>385</td>
<td>415</td>
</tr>
<tr>
<td>Compression</td>
<td>9</td>
<td>300±2.8</td>
<td>284</td>
<td>278</td>
</tr>
</tbody>
</table>

Table 10 Comparison of FE model predictions with experimental results

Figure 68 Comparison of FE and analytical model predictions flush ply-overlap joint for different joint length in (a)tension and (b) compression
linear elastic analysis of interlaminar stresses clearly overpredicts the experimental results as the assumptions are only valid for joints of very short step length. The long overlap analytical model based on fracture mechanics approach gives almost identical predictions as the FE model employing CZM for 9 mm flush joints. In compression, the analytical pull-out models were conservative as it underpredicted the joint strengths. This may be due the assumption of constant mode-mixity ratio which may vary with delamination size. Overall, the FE model can predict the experimentally measured joint strengths. The effect of various parameters that influence the FE strength prediction are discussed below.

7.5.2.1 Effect of thermal residual stresses

Comparison of strength predictions for analysis with and without the effect of thermal residual stresses is shown in Table 10. Inclusion of the residual thermal stresses resulted in an increase in tensile strength of 7% and a reduction in compressive strength of 2.5%. This is expected as the thermal residual stress is small for balanced joints.

7.5.2.2 Effect of spatial distribution of ply termination

The computational modelling considers perfectly aligned geometry for the flush overlap joint. Manufacturing variations can however affect the positions of the overlaps which may in turn influence the joint strength. Two possible misaligned spatial distribution of critical ply terminations have been investigated: a symmetric offset configuration in which overlaps were offset by a fixed distance while retaining joint laminate symmetry as shown in Figure 69 (a) and an asymmetric offset configuration in which overlaps are arbitrarily dispersed as shown in Figure 69 (b).

Comparison of the strength predictions of the baseline geometry (Figure 65) with the non-idealised configurations (Figure 69) are shown in Figure 70. The results indicate that employing a symmetric offset configuration increased the tensile and compressive strengths by 2%. The increase in strength is small because the strength critical ply terminations are several ply thicknesses away from each other and therefore stress concentrations do not interact or are “elastically isolated” as discussed in [29]. For the asymmetric configuration, the predicted tensile strength was unaffected whilst the compressive strength reduced by 9%. This reduction in compressive strength can be attributed to the increased bending loads due to joint asymmetry that give rise to increasing detrimental peel stresses.
Figure 69 Finite element models used for studying the effect of spatial distribution of ply terminations on flush ply-overlap joint strength prediction (a) Symmetric offset geometry (b) Asymmetric offset geometry.

Figure 70 Comparison of FE results for the effect of spatial distribution of ply terminations; No offset, symmetric offset and asymmetric offset. Note: Thermal stress not included in these predictions.
7.5.2.3 Effect of overlap length

FE models of different overlap lengths were used to predict joint strength versus overlap length with the results shown in Figure 68. The simplified analytical pull-out models are able to provide upper and lower strength limits, however, the FE model allows the intermediate region between short and long overlaps to be accurately predicted. The transfer length required for the flush joint is estimated as 9 mm, referring to Figure 68(a), in tension and 6 mm in compression as shown in Figure 68(b).

7.6 Concluding remarks

The structural behaviour of hybrid CFRP/GFRP ply-overlap joints were investigated using experimental, analytical and numerical methods. The principal failure mode observed in the experiments was delamination emanating from ply terminations in the vicinity of the overlapped plies. Ply-overlap joints can be designed to reach a strength approaching the un-notched GFRP laminate strength through judicious selection of the spatial distribution of ply terminations and overlap length. A stacked shell modelling FE methodology has been found to accurately predict the delamination path and ultimate loads recorded during experimental testing. The validated FE modelling approach provides a valuable design tool that can be used to efficiently design an integrated radar transparent window for load-bearing antenna applications using the ply overlapping technique, discussed in the following chapter.
Chapter 8 Application to load-bearing antennas

8.1 Introduction

Load-bearing antennas are multifunctional structures that must meet both structural and radar performance requirements. One method of imparting antenna functionality to load-bearing structural skins fabricated from advanced CFRP is to create slots in those skins. The slots are filled with dielectric material to provide environmental sealing and allow for T/R of radar signals. As the operating frequency of the antenna decreases, the required size of the slot increases. The hybrid ply joints allow for the integration of large radar transparent apertures without significantly degrading the structural strength of the load-bearing structure. The design methodology for the load-bearing antenna, thus far, has been developed based on analysis of the joint. The joint is a two-dimensional representation of the structure oriented in the direction of the maximum load, i.e. along the longitudinal fibre direction or 0° ply. This representation however does not consider the load by-pass effect that occurs due to difference in stiffness of dielectric window and surrounding host structure and ignores possible 3-D effects on failure modes. In this chapter, representative load-bearing antenna panels employing a hybrid ply-overlap joint design are studied experimentally. The differences and correlation between the failure loads and failure modes hybrid joints and 3-D structure are presented and discussed.

8.2 Experimental Details

The design of the load-bearing antenna is based on conventional slot antennas [113] consisting of a single circular dielectric window reinforced with a hybrid ply-overlap joint in which only the main load-bearing 0° plies are overlapped. The diameter of the dielectric window was chosen to be 25 mm which would be in the required range for an antenna operating in low frequencies such as in the X-band (1-10 GHZ). An overlap length of 10 mm was employed. Parametric analysis on the effect on overlap length in Chapter 7 revealed that the joint strength does not increase significantly beyond 9 mm. The dielectric window and host structure were eight ply quasi-isotropic laminates of CFRP and GFRP with a [45/0/-45/90]s stacking sequence. Schematic illustration of the layup of antenna structure is shown in Figure 71.

The fabrication procedure involved the following steps:
1. CFRP plies measuring 300 mm x 75 mm were cut from the prepreg. A rotary cutter was used instead of the standard staple knives to minimise shearing of the fibres.

2. Circular cut-outs measuring 25 mm were made at the centre of each carbon ply using a wad punch backed by a commercial medium density fibreboard.

3. Similarly, circular glass plies were cut from the prepreg; six plies measuring 25 mm and two plies measuring 35 mm.

4. An aluminium base plate with aligning pins at each corner was prepared to allow for accurate positioning of the plies. The ply orientations were marked onto the base plate to ensure ply angles cut CFRP plies and circular GFRP inserts matched up accurately.

5. The first layer is formed by placing the +45° glass ply at the centre of the cut +45° carbon ply.

6. A cut carbon 0° ply was then laid onto the base plate carefully using the aligning pins. A glass ply of 35 mm is then placed at the centre of the +45° glass ply, thereby creating an overlap of 10 mm.

7. Steps 5 and 6 are repeated depending on the ply orientation to build the laminate thickness. The layup was consolidated after every two layers.

Four panels measuring 270 mm by 50 mm were manufactured and tested to failure under uniaxial tension using an MTS hydraulic load frame equipped with a 250 kN load cell. Example of fabricated test panel is shown in Figure 72. A constant cross-head speed of 1 mm/min was applied and the load-displacement response was recorded at a rate of 4 Hz.

![Diagram of laminate layup](image)

Figure 71 Layup schematic of representative load-bearing antenna test panel
Figure 72 Typical load-bearing antenna test panel. Note: Panel shown in figure employed a similar joint configuration but was fabricated using different materials for DSTG tests.

8.3 Experimental Results and Discussion

Figure 73 Load-displacement curves for load-bearing antenna test panels

Figure 74 Typical failure modes for load-bearing antenna test panel. Image on the left shows the development of delamination damage at 95% of ultimate and image on right shows the final failure
modes. Note: Panel shown in figure was employed a similar joint configuration but was fabricated using different materials for DSTG tests.

<table>
<thead>
<tr>
<th>Panel thickness (mm)</th>
<th>Panel width (mm)</th>
<th>Peak Load (kN)</th>
<th>Failure stress, (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.62</td>
<td>49.84</td>
<td>41.7</td>
<td>516</td>
</tr>
<tr>
<td>1.67</td>
<td>51.77</td>
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<td>393</td>
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<tr>
<td>1.75</td>
<td>51.60</td>
<td>35</td>
<td>388</td>
</tr>
<tr>
<td>1.74</td>
<td>49.81</td>
<td>36.3</td>
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<tr>
<td>Average failure stress (MPa)</td>
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<td>429</td>
</tr>
<tr>
<td>Coefficient of variation (%)</td>
<td></td>
<td></td>
<td>(14)</td>
</tr>
</tbody>
</table>

Table 11 Summary of experimental strength results of representative load-bearing antenna panels

Load-displacement curves for all test panels are shown in Figure 73. The test results indicate that the panels exhibited an approximately linear-elastic response apart from the initial slacking caused by the tightening of the grips for one of the panels. Significant audible cracking could be heard at approximately 70% of ultimate strength. All test panels failed catastrophically. The failure modes were identical to the joints; delamination cracking and transverse matrix cracks as shown in Figure 74. The failure stress was calculated using the peak load and initial cross-sectional area of CFRP laminate. The average failure stress of the load-bearing antenna was 429 ±14 MPa.

8.4 Analysis of results

To investigate the differences in the structural behaviour of the 3-D antenna and the 2-D hybrid ply joints, the stress at failure in the CFRP and GFRP laminate obtained from experimental tests are compared. Using the bonded inclusion analogy [99], the stress inside a soft inclusion, \( \sigma_w \), in a rigid host-like structure is related to the far-field stress by Eq.(3) from Chapter 3:

\[
\sigma_w = \frac{3E_w/E_S}{1+2E_w/E_S} \sigma_\infty
\]

To account for the difference in the thickness of the two laminates, the stress inside the GFRP window becomes:

\[
\sigma_w = \frac{3E_w t_w/E_S t_S}{1+2E_w t_w/E_S t_S} t_s \sigma_c
\]

(40)
where $t_s$ and $t_w$ are the laminate thickness of the CFRP and GFRP. Using classical laminate theory, the QI stiffness of CFRP and GFRP are calculated to be 45.7 GPa and 16.5 GPa. Eq.(40) gives us the stress in the dielectric window, $\sigma_w = 292$ MPa. The failure stresses for the joint stresses based on both the GFRP and CFRP laminate thickness was calculated and compared with the antenna test results in Table 12. The results show that while the structural skin stress is very similar, the load-shedding effect reduces the stress inside the window significantly.

<table>
<thead>
<tr>
<th>Joint failure stress (MPa)</th>
<th>Antenna failure stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin structure (CFRP), $\sigma^\infty$</td>
<td>433</td>
</tr>
<tr>
<td>Dielectric window (GFRP), $\sigma_w$</td>
<td>350</td>
</tr>
</tbody>
</table>

Table 12 Comparison of the experimental failure stresses in 2-D hybrid joint and 3-D antenna structure

The hybrid composite joint has been developed to allow load-bearing antennas requiring large apertures to withstand the design ultimate load (DUL) without the need for additional reinforcements. A conservative estimate of the DUL of a composite laminate can be taken as the open-hole tension OHT (or open hole compression for structures under compression-dominated loading). The CFRP considered in this study has an OHT strength of 462 MPa. The experimental results show that the strength of this new load-bearing antenna approaches the DUL of host skin structure. However, being an integral part of the skin structure, the dielectric window must also meet the same design requirements of the host structure. The OHT strength of the GFRP is 205 MPa. The antenna produces a stress of 292 MPa in the dielectric window which clearly exceeds its design allowable. This implies that for the host structure to retain the DUL, a dielectric composite of similar OHT/OHC strength such as the S-2 glass/epoxy composite in [100] is required. However, the lower stiffness of the dielectric composite is advantageous as it deflects the load away from the window and joint.

8.5 Concluding remarks

Representative load-bearing antennas employing dielectric windows with hybrid composite ply-overlap joints were fabricated and tested to failure under uniaxial quasi-static tension. The results show that strength of the antenna approaches the DUL of CFRP structural skin. Furthermore, the antenna exhibited similar failure modes and ultimate strength as the hybrid ply joint. This shows that the analysis methodologies developed for hybrid ply-overlap joint,
which is a two-dimensional representation of the load-bearing antenna along the most highly loaded direction can be used with reasonable accuracy for optimising the design of the three-dimensional structures.
Chapter 9  Discussion, Conclusions and Future Work

9.1  Discussion

9.1.1  Manufacturing tolerances

One of the challenges in joint fabrication is achieving tight tolerances with respect to the position of resin pockets that form at ply terminations. High resolution optical micrographs of fabricated specimens show that the resin pockets deviate from their idealised positions. The deviation is a result of the movement of plies during consolidation and curing. Parametric studies on the effect of misalignment of resin pockets in Chapter 7, Section 7.5.2.2, has shown that the deviation does not significantly affect the tensile strength of joint. However, the compressive joint strength is affected as composite materials are much more sensitive to geometric irregularities under compressive loads. For instance, it was shown that for ply-overlap joints asymmetric configurations had a much lower strength than symmetric configurations due to increased secondary bending. Besides mechanical considerations, there are strict tolerances that must be adhered to for radar performance. For X-band applications, the resin pockets should have a dimensional tolerance of $\pm t_p$ (ply thickness) [8]. Techniques such as using circular cutting blades to reduce shearing of plies and vacuum assisted layup positioning guide that reduces movement of layup during consolidation process, showed that good alignment can be achieved [24, 25].

9.1.2  Joint design considerations

Joint configuration

Ply joints are formed by creating butt-splices (ply-interleaved) or overlaps (ply-overlap) between uncured composite plies whilst tailoring the positions of the ply terminations. One of the major design considerations for ply joints is the spatial distribution of the ply terminations or the joint configuration. An optimal joint configuration must (a) alleviate undesirable secondary bending (b) produce a short hybridizing region (joint length) and (c) be flush with parent composite laminates. Since the joint is formed between dissimilar materials, the joint configuration must ensure a gradual transition in stiffness. A comparative assessment of several joint configurations has been conducted in this thesis. For ply-interleaved joints, the finger configuration in which only the $0^\circ-0^\circ$ ply terminations are offset is ideal for loading scenarios where the direction of maximum load is known. The finger joint produces a much shorter transition region than the interleaved-scarf. However, for quasi-isotropic laminates
that must achieve the identical strength in both in-plane directions, the interleaved-scarf configuration is preferred. For ply-overlap joints, the flush joint design which overlaps only the main load-bearing plies outperforms the aligned and staggered joint design as eccentric loading is minimised. However, the staggered joint may be preferred for implementation to 3D load-bearing antenna structure as the flush joint design will require non-circular/irregular shaped GFRP inserts. The staggered joint could be optimised by introducing plane symmetry to the design thereby reducing the overall joint length.

Comparing ply-interleaved and ply-overlap joints, it is evident that ply-overlap joints produce strengths approaching the un-notched laminate strength. The study of Darby et.al [114] reveal that the introduction of a single ply cut in a unidirectional laminate reduces the tensile strength by 15% . The interleaved-scarf configuration can achieve 75% of unnotched laminate strength which is close to theoretical maximum strength indicated by Darby et.al. However, one of the drawbacks of ply-overlap joints is significant localised laminate thickening which could severely affect its compressive properties. Therefore, the judicious selection of the joint configuration must be in appropriation with both geometric and strength requirements.

**Joint length**

Analogous to the structural behaviour of adhesively bonded composite joints, the strength of ply joints exhibits an asymptotic relationship with joint length. Joint strength initially increases almost linearly with joint length up to a certain limit. This is followed by an intermediate region where joint strength increases gradually but the gradient change is much lower. Above a certain length, the joint strength increases by less than 1%. This threshold value which varied in different joint configurations is referred to as the transfer length of the joint, which is typically defined as the joint length at which 95% of the “maximum” load-carrying capacity of the joint is achieved. The relationship between strength and joint length of ply joints is fundamental to increasing the structural efficiency of load-bearing antennas as it allows designers to select the joints based on hybdrizing length which could be limited by the radar requirements. Moreover, minimising the joint length enables the load-bearing antenna to retain higher mechanical stiffness.

**Thermal Residual stresses**

High performance FRPs require high processing temperatures and the subsequent cooling of to room temperature can cause residual thermal stresses and strains due to the mismatch in
coefficients of thermal expansion of fibres and matrix. For hybrid carbon/glass ply joints, CTE differences also exist between the carbon and glass fibres (Appendix A). Carbon fibres have almost zero CTE in comparison to glass fibres have a CTE of \(5-6 \times 10^{-6}/\degree\text{C}\). The epoxy matrices of the CFRP and GFRP prepregs also have different CTEs; MTM57 has a CTE of roughly 1.5 times that of VTM264. As a result of differences in the CTE, residual compressive strains are induced in the laminate. These residual thermal strains can cause matrix cracks and also influence delamination initiation load. In tension, delamination initiation load increases whereas in compression, delamination initiation load decreases. The overall effect of residual thermal strains on the joint strength was found to be minimal because of the balanced symmetric laminate stacking sequence employed.

9.1.3 Computational damage modelling

Several factors affect the predictive capability of the computational damage models. One of them is the idealisation of joint geometry used in the FE analysis. In the joint region, geometric features such as ply thickness and resin pocket size vary considerably. Optical microscopy, for instance, it was revealed that for ply-interleaved joints, resin rich pockets varied between 0.1-0.2 mm. For ply-overlap joints, the length of resin pockets was 0.6 mm however they were not right-angled triangles. The model also does not capture subtle changes in ply thickness and variation in fibre waviness around resin pockets. The modelling approach also treats ply terminations and resin pockets as having sharp corners whereas these locations may be blunt in reality. Therefore, the stacked-shell approach which employs smeared homogenous material properties may not be truly representative of the actual variability in stiffness and strength properties that are present in the hybrid laminate region. While highly accurate representative geometric models can be achieved by employing image processing tools to characterise the microstructural features in the as manufactured joint, the computational effort would also increase substantially.

Fractographic evidence has demonstrated that the transverse matrix cracking is a critical damage mode in ply joints. However, one of the current limitations of the Abaqus continuum damage model is that it does not simulate non-linear in-plane shear behaviour, mode-mixity and size effects (in-situ) that influence transverse matrix cracking [68, 71]. The existing shell-element based modelling technique does not account for any through-thickness damage as it captures only in-plane failure mechanisms.
9.2 Conclusions

A novel integrally fit (co-cured) joining technology has been proposed in thesis for hybridizing dissimilar composite laminates to enable the structural integration of dielectric windows in a conductive composite skin structure for load-bearing antenna applications. Through fractographic analyses, a comprehensive understanding of failure mechanism of the hybrid joints was gained. The load-carrying capacity of ply joints and the failures depends strongly on the distance between ply terminations, the spatial distribution of ply terminations, overlap length and the mismatch in the stiffness and coefficients of thermal expansion of the dissimilar composite material. Through judicious selection of the spatial distribution of ply terminations and joint length, the load-bearing antennas can be designed to reach the DUL strength of the structural skin. An analysis methodology has been developed using analytical models and high-fidelity computational models. The analytical models based on linear elastic fracture mechanics and computational models employing cohesive damage model and continuum damage model were able to accurately predict the failure modes and strength of the hybrid ply joints. These modelling tools pave the way for optimising hybrid joint concepts for multifunctional load-bearing antennas. The major conclusions addressing the key research questions set out in Chapter 1 are summarised below.

9.2.1 Joint design

Ply joints are formed by creating butt-splices (ply-interleaved) or overlaps (ply-overlap) between uncured composite plies whilst tailoring the positions of the ply terminations for a given combination of materials and loading direction. Ply-interleaved joints produce flush joints whereas ply-overlapping produces localised thickening of the hybrid region. For the composite materials (T700/VTM264 and E-glass/MTM57), laminate stacking sequence (Quasi-isotropic laminates) and loading condition (uni-axial static tension/compression), ply-interleaved joints were able to achieve 75% of the DUL strength of load-bearing CFRP skin structure. Ply-interleaved joints employing interleaved-scarf and finger-offset configurations were able to reach the maximum strength provided the distance between provided the distance between 0°-0° ply terminations exceed the transfer length, calculated to be approximately 30 times the ply thickness. In comparison, ply-overlap joints can be designed to exceed the DUL strength. Flush ply-overlap joints which can designed by selectively overlapping the main load-bearing plies aligned with the loading direction, while terminating the same number of weaker off-angle plies outside the joint region. bearing plies were found
to achieve the maximum strength, provided the overlap length was approximately greater than 20-30 times the ply thickness.

9.2.2 Failure mechanism

The major failure modes of hybrid ply joints are delaminations emanating from ply terminations and transverse matrix cracking of off-axis plies. The likely sequence of damage initiation and propagation in the hybrid ply joints is summarised as follows. Transverse matrix cracks occur within off-axis plies at very low loads, particularly in the region of ply terminations. Delaminations then initiate from the ply terminations, primarily from ply terminations of principal load-bearing plies, due to high stress concentrations. The transverse matrix cracks allow for the coalescence of delamination cracks leading to ultimate failure of the joint. For delamination between dissimilar composite material interface, the delamination crack was primarily within composite with lower fracture toughness.

9.3 Recommendations for future work

The joint design configurations developed in this thesis were optimised for joining QI laminates with identical stacking sequences for primarily uni-axial tension/compression loading. Varying the ply orientations of the laminates will give rise to new joint concepts for a wider range of orthotropic laminates. It would also be interesting to apply ply joining techniques to non-UD prepgs; short fibre composites and woven composites. Consideration of multiaxial loading, which are more realistic loading scenarios for load-bearing antennas must be considered.

The analysis methodology employs continuum damage mechanics (CDM) and cohesive zone model (CZM) for predicting the intralaminar and interlaminar damage modes, respectively. Studies have shown that this combined approach [115-118] may not accurately represent sequence of damage events in a composite structure as there is a strong interaction and coupling of these damage modes. Methods such as the extended Finite element Method (X-FEM) [119] allow for arbitrary crack paths could improve the predictive capability of the failure mechanism.

Load-bearing antenna structures must meet fatigue life and damage tolerance requirements in addition to static strength requirements. GFRP typically has poorer fatigue life than CFRP due the lower stiffness of glass fibres but a higher impact resistance in comparison to CFRP due the higher strain-to-failure. Previous studies hybrid carbon/glass laminates have shown that
the number of cycles to failure at a particular stress level is higher than monolithic GFRP laminate[120-122] and impact resistance; peak impact loads and absorbed energies where higher than monolithic CFRP laminate [122-124]. This synergistic or “hybrid” effect requires more investigation in the context of ply joints. Extensive work is also required to characterise the RF performance, in terms of impedance, bandwidth and antenna gain. The joint or the hybrid region reduces the effective antenna aperture since it contains CFRP plies which are not RF transparent. The effect of joint design on antenna performance needs to be characterised. RF requirements must therefore also drive material selection. Advanced design trade-offs need to be performed to optimise both structural and RF requirements.

Ply joining is a structurally efficient method that can potentially replace mechanically fastened joints as well as adhesively bonded joints. It allows for greater flexibility in joint design without significant weight penalty. Further work will need to be conducted to improve the delamination strength through various interlaminar toughening techniques to increase the load carrying capacity of ply joints. Load-bearing structures for aerospace, naval, automotive and civil applications commonly employ dissimilar composite materials. Ply joining techniques provide a novel method of transitioning efficiently between dissimilar materials without the need for additional reinforcement. Ply terminations which are problematic in co-cured structures could also be interleaved and/or overlapped leading to lower stress concentrations.
References


102. Advanced Composites Group, ACG MTM®57 series prepreg system. 2009.


Appendix A Material Properties

Materials employed in this thesis are unidirectional prepregs supplied by Advanced Composites Group Australia. The conductive structural composite material is a T700 carbon fibre impregnated in a VTM264 epoxy matrix and the dielectric composite material is an E-glass fibre impregnated in a MTM57 epoxy matrix. The elastic, thermal, strength and toughness properties are listed in the tables below.

<table>
<thead>
<tr>
<th></th>
<th>T700/VTM264 (CFRP)</th>
<th>E-glass/MTM57 (GFRP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (MPa)</td>
<td>120200</td>
<td>34160</td>
</tr>
<tr>
<td>$E_2, E_3$ (MPa)</td>
<td>7467</td>
<td>7870</td>
</tr>
<tr>
<td>$G_{12}, G_{13}$ (MPa)</td>
<td>3904</td>
<td>3700</td>
</tr>
<tr>
<td>$G_{23}$ (MPa)</td>
<td>2304</td>
<td>3098</td>
</tr>
<tr>
<td>$\nu_{12}, \nu_{13}$</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.33</td>
<td>0.27</td>
</tr>
<tr>
<td>$\alpha_{11}$ ($10^{-6}/^\circ C$)</td>
<td>-0.3</td>
<td>6</td>
</tr>
<tr>
<td>$\alpha_{22}$ ($10^{-6}/^\circ C$)</td>
<td>25</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 13 Ply elastic and thermal properties

<table>
<thead>
<tr>
<th></th>
<th>T700/VTM264</th>
<th>E-glass/MTM57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal tensile strength,$X_{11,t}$ (MPa)</td>
<td>2500 [125]</td>
<td>1000*</td>
</tr>
<tr>
<td>Longitudinal compressive strength,$X_{11,c}$ (MPa)</td>
<td>1235 [125]</td>
<td>900*</td>
</tr>
<tr>
<td>Transverse tensile strength,$Y_{22,t}$ (MPa)</td>
<td>40 [125]</td>
<td>47 [126]</td>
</tr>
<tr>
<td>Transverse compressive strength,$Y_{22,c}$ (MPa)</td>
<td>182 [125]</td>
<td>150 [59]</td>
</tr>
<tr>
<td>Longitudinal Shear strength,$S_{12}$ (MPa)</td>
<td>85.7 [125]</td>
<td>120 [8]</td>
</tr>
<tr>
<td>Longitudinal tensile fracture toughness,$G_{ft}$ (KJ/m$^2$)</td>
<td>70**</td>
<td>40**</td>
</tr>
<tr>
<td>Longitudinal compressive fracture toughness,$G_{fc}$ (KJ/m$^2$)</td>
<td>78 [58]</td>
<td>12.5 [7]</td>
</tr>
<tr>
<td>Transverse tensile fracture toughness,$G_{mt}$ (KJ/m$^2$)</td>
<td>0.3 (Assumed equal to $G_{lc}$)</td>
<td>0.4 (Assumed equal to $G_{lc}$)</td>
</tr>
</tbody>
</table>
Transverse compressive fracture toughness, $G_{mc}$ (KJ/m$^2$)  

<table>
<thead>
<tr>
<th></th>
<th>T700/VTM264</th>
<th>E-glass/MTM57</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.76 [58] {Maimi, 2007 #172}</td>
<td></td>
</tr>
</tbody>
</table>

*Tested at RMIT University

**Calibrated with OHT laminate tests

Table 14 Intralaminar strengths and fracture toughness

<table>
<thead>
<tr>
<th>Property</th>
<th>T700/VTM264</th>
<th>E-glass/MTM57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interlaminar peel strength, $S_{33}$</td>
<td>40*</td>
<td>47*</td>
</tr>
<tr>
<td>Interlaminar shear strength, $S_{13}$</td>
<td>77*</td>
<td>78*</td>
</tr>
<tr>
<td>Interlaminar shear strength, $S_{23}$</td>
<td>77 (Assumed equal to $S_{13}$)</td>
<td>78 (Assumed equal to $S_{13}$)</td>
</tr>
<tr>
<td>Mode I fracture toughness, $G_{IC}$</td>
<td>0.3 *</td>
<td>0.4 *</td>
</tr>
<tr>
<td>Mode II fracture toughness, $G_{IIC}$</td>
<td>1.6*</td>
<td>2.4*</td>
</tr>
<tr>
<td>Mode III fracture toughness, $G_{IIIc}$</td>
<td>1.6 (Assumed equal to $G_{IIC}$)</td>
<td>2.4 (Assumed equal to $G_{IIC}$)</td>
</tr>
<tr>
<td>Cohesive stiffness, $K_I, K_{II}, K_{III}$ (N/mm$^3$)</td>
<td>$1\times10^6$ [84]</td>
<td>$1\times10^6$ [84]</td>
</tr>
</tbody>
</table>

*Tested at RMIT

Table 15 Laminate strength data obtained by testing to relevant ASTM standards

<table>
<thead>
<tr>
<th>Test type</th>
<th>Configuration</th>
<th>Average strength(MPa)</th>
<th>Coefficient of variation(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>CFRP QI</td>
<td>833</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>GFRP QI</td>
<td>354</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>CFRP QI OHT</td>
<td>463</td>
<td>4.71</td>
</tr>
<tr>
<td></td>
<td>GFRP QI OHT</td>
<td>205</td>
<td>1.5</td>
</tr>
<tr>
<td>Compression</td>
<td>CFRP QI</td>
<td>520</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>GFRP QI</td>
<td>331</td>
<td>5.1</td>
</tr>
</tbody>
</table>
Table 15: Open-hole tension test data for QI CFRP and GFRP laminates

Table 15: Unnotched QI CFRP and GFRP laminates
Appendix B  MATLAB script: Analytical Pull-out model

%----------------------
%Analysis ignoring thermal effects
%----------------------

hc=3.36;hg=4.16;
GI=0.3;GII=1.6;eta=2;Modemix=0.58;

%G for compression
Gcomp=GI+((GII-GI)*(Modemix)^eta);

%laminate thickness at intersection of delamination
% ht=hc+(hg-hc)/2;
% hi=ht;
% hf=ht-(2*(ht/16));
% For overlap joint thickness =GFRP ply thickness
hi=hc;
hf=(14/16)*hc;

%Number of plies being pulled out
%For overlap joints d=8, for flush joints d=4
dop=4;dfl=2;

%Ply thickness based on carbon ply thickness
[Exi,Eyi,vxyi]=Laminatestiffness('Input.dat');
[Exf,Eyf,vxyf]=Laminatestiffness('Input_afterpullout.dat');

%----------------------
%1D
%----------------------

%Failure load per unit width
Faiload=sqrt((4*dfl*GII*Exi*Exf*hi*hf)/(abs(hf*Exf-hi*Exi)));
Strength1D=Faiload/hc;

%----------------------
%2D
\[ ki = \frac{(Exi + Eyi \cdot (vxyi^2))}{(Exi^2)}; \]
\[ kf = \frac{(Exf + Eyf \cdot (vxyf^2))}{(Exf^2)}; \]

% Failure load per unit width

\[ F\text{aiload} = \sqrt{\frac{(4 \cdot df1 \cdot GII \cdot hi \cdot hf)}{(\abs{ki \cdot hf - kf \cdot hi})}}; \]

Strength2D = Failload/hc;

%----------------------

%----------------------

% Thermal analysis using CLT

%----------------------

%----------------------

% Number of iteration points

it = 50;

% Load

Nx = linspace(0, 5000, it);

% Energy array definition

% \text{U1} = 2D analysis with axial stresses and strains only;

% \text{U2} = 2D analysis, \text{U3} = \text{U3 }\text{including shear components}

% Initial energy

U1i = zeros(it, 1); U2i = zeros(it, 1); U3i = zeros(it, 1);

\text{for} \ j = 1:it

[U1i(j), U2i(j), U3i(j)] = Calcenergy(Nx(j), 'Input_QICarbon.dat');

\text{end}

% Final energy

U1f = zeros(it, 1); U2f = zeros(it, 1); U3f = zeros(it, 1);

for \ j = 1:it

[U1f(j), U2f(j), U3f(j)] = Calcenergy(Nx(j), 'Input_QICarbon_final2zeropullout_outer.dat');

\text{end}
dU2D1 = abs(U1f-U1i);
dU2D2 = abs(U2f-U2i);
dU2D3 = abs(U3f-U3i);
Strength2DT1 = plotfn(dU2D1,Nx'/hc,2*dfl*GII);
Strength2DT2 = plotfn(dU2D2,Nx'/hc,2*dfl*GII);
% Strength2DT3 = plotfn(dU2D3,Nx'/hg,2*dop*GIIc);

Strain energy calculation

function [U1,U2,U3] = Calcenergy(Nx,Inputfile)
Ny = 0; Nx = 0; Mx = 0; My = 0; Mxy = 0; dT = 0;
M = [Mx; My; Mxy];
[th, t, E1, E2, G12, v12, Alpha1, Alpha2] = textread(Inputfile,'%f %f %f %f %f %f %f %f','headerlines',5);
N = [Nx; Ny; Nxy];
n = size(t,1); Q = cell(n,1); S = cell(n,1); Alpha = cell(n,1);
% Calculation of reduced stiffness matrix for each ply
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R = [1 0 0; 0 1 0; 0 0 2];
for i = 1:n
   c = cosd(th(i)); s = sind(th(i));
   T = [c^2 s^2 2*c*s; s^2 c^2 -2*c*s; -c*s c*s c^2-s^2];
   v21(i) = v12(i)*E2(i)/E1(i);
   S(i) = [1/E1(i), -v21(i)/E2(i), 0; -v12(i)/E1(i), 1/E2(i), 0; 0, 0, 1/G12(i)];
   St(i) = R*inv(T)*inv(R)*S(i)*T;
   Alpha(i) = [Alpha1(i); Alpha2(i); 0];
   Alpha(i) = R*inv(T)*inv(R)*Alpha(i);
   Q(i) = inv(St(i));
end
% Setting h matrix

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
h = zeros(n+1,1); h(1,1) = -sum(t)/2;

for k = 2:n+1
    h(k,1) = h(k-1,1) + t(1);
end

Calculation of Thermal Loads and Moments

Nthx=0;Nthy=0;Nthxy=0; Mthx=0;Mthy=0;Mthxy=0;
Nth=[Nthx;Nthy;Nthxy]; Mth=[Mthx;Mthy;Mthxy];

for i=1:n
    Nth = Nth + dT*Q{i}*Alpha{i}*(h(i+1,1)-h(i,1));
    Mth = Mth + 1/2 * dT*Q{i}*Alpha{i}*(h(i+1,1)^2-h(i,1)^2);
end

Total Loads and Moments

NM=[N+Nth;M+Mth];

Calculating A matrix

A=0;B=0;D=0;

for i=1:n
    A = A + Q{i}*(h(i+1,1)-h(i,1));
    B = B + 1/2 * Q{i} * (h(i+1,1)^2 - h(i,1)^2);
    D = D + 1/3 * Q{i} * (h(i+1,1)^3 - h(i,1)^3);
end

Laminate stiffness

Ex=(A(1,1)*A(2,2)-A(1,2)^2)/(sum(t)*A(2,2));
Ey=(A(1,1)*A(2,2)-A(1,2)^2)/(sum(t)*A(1,1));
Gxy=A(3,3)/sum(t);
vxy=A(1,2)/A(1,1);

Calculation of Mid-plane strains and curvatures

ABD = [A B;B D];
MID = inv(ABD)*NM;
MIDs = MID(1:3);
MIDk = MID(4:6);
%Calculation of global strains and stresses in the middle of each ply

Z = zeros(n,1); Strain = cell(n,1); Stress = cell(n,1);

for k = 1:n
    Z(k,1) = h(k,1)+t(i)/2;
    Strain{k} = MIDs+(Z(k,1)*MIDk)-dT*Alpha{k};
    Stress{k} = Q(k)*Strain{k};
end

%Energy Calculation before delamination

U1=0; U2=0; U3=0;

for i=1:n
    U1 = U1 + 0.5*t(i)*Stress{i,1}(1,1)*Strain{i,1}(1,1);
    U2 = U2 + 0.5*t(i)*(Stress{i,1}(1,1)*Strain{i,1}(1,1) + abs(Stress{i,1}(2,1)*Strain{i,1}(2,1)));
    U3 = U3 + 0.5*t(i)*(Stress{i,1}(1,1)*Strain{i,1}(1,1) + abs(Stress{i,1}(2,1)*Strain{i,1}(2,1)) + abs(Stress{i,1}(3,1)*Strain{i,1}(3,1)));
end