Time-dependent Reliability Analysis of Corrosion-induced Concrete
Cracking Based on Fracture Mechanics Criteria

A thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

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DECLARATION

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed. I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

Ian Lau

March 2019
ACKNOWLEDGEMENTS

To many whose guidance, assistance and support have played an integral role throughout the course of my PhD, I owe each and everyone of you a great debt of gratitude. I could not have accomplished this milestone without all your support.

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I am also grateful to Metro Trains Melbourne, Australia for their financial support and the opportunity to participate in this research. Also, to the technical staff in the Civil Engineering laboratory, Eric Gao, Shamir Bhuiyan and Pavel Ryjkov, I would like to thank everyone for their assistance at the Bundoora Labs.

While life as a PhD student can sometimes feel lonely, I am very grateful for my friends Mitchel, Ray, Geri, Vishak, Sam, James, Mat and Rintu. Thank you for the day-to-day cheers, conversations and revs. You have all made my experience in RMIT not just bearable but extremely enjoyable. I look forward to the times ahead when we cross paths again. To the bunch of incredibly remarkable people from Life* expedition whose hunger for truth has inspired me deeply, life is a journey and it is truly better doing it together. To the guys at Vert Engineering, Resh, Mark, Michael, Mia and Tony, thanks for all the good chats and the opportunity to work with you guys during this time.

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Last but not least, I would not be here without the providence of God.
LIST OF PUBLICATIONS

Journal Papers


Lau, I., Fu, G., & Li, C. Q. Prediction of time to corrosion-induced concrete cracking based on fracture mechanics criteria. *ASCE Journal of Structural Engineering* (Accepted 8th Dec 2018)

Lau, I., Li, C. Q. & Chen, F. J. Analytical and experimental investigation on corrosion-induced concrete cracking. *International Journal of Civil Engineering* (Submitted)


Conference Papers

ABSTRACT

Practical experience and observations suggest that corrosion affected reinforced concrete structures are more prone to cracking than other forms of structural deterioration. Around the world, maintenance and repairs resulting from premature concrete cracking and spalling are associated with very high running cost. Furthermore, the ever-increasing demand for greater load carrying capacity of existing reinforced concrete structure only exacerbate the issue. Consequently, this increases the probability of failure of corrosion-affected reinforced concrete structure. It is therefore vital to study corrosion-induced concrete cracking and to perform a service life prediction to avoid unwanted corrosion-induced failures and develop cost-effective methods for maintenance and rehabilitation of reinforced concrete structures.

This research attempts to examine the process of concrete cracking and determine the critical crack depth at which a corrosion-induced crack becomes unstable and suddenly propagate to the concrete surface. In the analytical model, a model for corrosion-induced critical crack depth has been derived based on the concept of stress intensity factor. In the numerical model, an extended finite element method has been used to predict the concrete cover capacity based on a maximum principal stress fracture criterion. To validate the developed models, an accelerated corrosion experiment was conducted and the time to corrosion-induced cracking and the growth of crack width was measured. With the developed model, a time-dependent remaining service life prediction for corrosion-induced cracking in RC was conducted.

It is concluded that the analytical method is one of the very few theoretical methods that can predict with reasonable accuracy corrosion-induced critical crack depth in reinforced concrete. It was also found that the extended finite element method can be used to model the concrete cover capacity which can then be used to predict the time to corrosion-induced concrete cracking. It was also found that the porous zone in concrete can significantly affect the time to corrosion-induced cracking. It was also found that corrosion rate and concrete cover are the most influencing factors that will affect the remaining service life of corrosion affected reinforced concrete structures.
With the developed models in this thesis, the information provided can help asset managers and engineers in making more informed decisions with regards to maintenance and rehabilitation strategies of corrosion affected reinforced concrete structures.
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<th>Description</th>
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<tbody>
<tr>
<td>CMOD</td>
<td>crack amount opening displacement</td>
</tr>
<tr>
<td>CN-E</td>
<td>cyanoacrylate</td>
</tr>
<tr>
<td>COV</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>CTL</td>
<td>chloride threshold level</td>
</tr>
<tr>
<td>CTOD</td>
<td>crack tip opening displacement</td>
</tr>
<tr>
<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>FEM</td>
<td>finite element method</td>
</tr>
<tr>
<td>FORM</td>
<td>first order reliability method</td>
</tr>
<tr>
<td>FPZ</td>
<td>fracture process zone</td>
</tr>
<tr>
<td>ITZ</td>
<td>interfacial transition zone</td>
</tr>
<tr>
<td>LHS</td>
<td>Latin hypercube sampling</td>
</tr>
<tr>
<td>LPR</td>
<td>linear polarisation resistance</td>
</tr>
<tr>
<td>LSM</td>
<td>level set method</td>
</tr>
<tr>
<td>MAXPS</td>
<td>maximum principal stress</td>
</tr>
<tr>
<td>MCS</td>
<td>Monte-Carlo simulation</td>
</tr>
<tr>
<td>PS</td>
<td>polyester</td>
</tr>
<tr>
<td>RC</td>
<td>reinforced concrete</td>
</tr>
<tr>
<td>SEM</td>
<td>scanning electron microscope</td>
</tr>
<tr>
<td>SI</td>
<td>sensitivity index</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
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<td>--------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>SIF</td>
<td>stress intensity factor</td>
</tr>
<tr>
<td>TPFM</td>
<td>two-parameter fracture model</td>
</tr>
<tr>
<td>XFEM</td>
<td>extended finite element method</td>
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# LIST OF SYMBOLS

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Diffusion coefficient; cross-sectional area</td>
</tr>
<tr>
<td>$a$</td>
<td>Inner radius; crack extension</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Stiffness reduction factor</td>
</tr>
<tr>
<td>$\alpha_{\text{rust}}$</td>
<td>Ratio of molecular weight of steel to corrosion products</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Stern and Geary constant; reliability index</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Cathodic tafel slope</td>
</tr>
<tr>
<td>$\beta_a$</td>
<td>Anodic tafel slope</td>
</tr>
<tr>
<td>$C(x,t)$</td>
<td>Chloride ion content at distance $x$ and time $t$</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Chloride content on concrete surface</td>
</tr>
<tr>
<td>$C_{SS}(t_i,t_j)$</td>
<td>Cross-covariance function</td>
</tr>
<tr>
<td>$D_{CTOD}$</td>
<td>Critical crack tip opening displacement</td>
</tr>
<tr>
<td>$D_e$</td>
<td>Evolution of damage</td>
</tr>
<tr>
<td>$D_0$</td>
<td>Diffusion constant</td>
</tr>
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<td>$d_0$</td>
<td>Thickness of porous band</td>
</tr>
<tr>
<td>$d_{cr}$</td>
<td>Critical amount of corrosion products</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Displacement</td>
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<tr>
<td>$e$</td>
<td>Crack depth</td>
</tr>
<tr>
<td>$\delta s$</td>
<td>Differential of arc-length along the contour $\Gamma$</td>
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<tr>
<td>$E_c$</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
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<td>Description</td>
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<td>-------------</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Strain softening modulus of elasticity</td>
</tr>
<tr>
<td>$E_{ef}$</td>
<td>Effective modulus of elasticity</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>Change in potential</td>
</tr>
<tr>
<td>$\bar{E}$</td>
<td>Plane stress</td>
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<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
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<td>$\text{erf}(x)$</td>
<td>Error function</td>
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<tr>
<td>$\varepsilon_0$</td>
<td>Strain at maximum stress</td>
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<tr>
<td>$\varepsilon_\theta(a)$</td>
<td>Tangential strain at $r = a$</td>
</tr>
<tr>
<td>$\varepsilon_\theta^e(a)$</td>
<td>Cracking strain at $r = a$</td>
</tr>
<tr>
<td>$F$</td>
<td>Faraday’s constant; External work done; Force applied</td>
</tr>
<tr>
<td>$\varepsilon_{corr}$</td>
<td>Strain of corrosion products</td>
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<tr>
<td>$F_n$</td>
<td>Influence coefficient</td>
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<td>$F_\alpha(x)$</td>
<td>elastic asymptotic crack tip functions</td>
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<tr>
<td>$f_t$</td>
<td>Tensile strength of concrete</td>
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<td>$f_c$</td>
<td>Compressive strength of concrete</td>
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<td>$f_{c,0}$</td>
<td>Compressive strength of concrete assuming zero porosity</td>
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<td>$f_{i,j}$</td>
<td>Dimensionless quantity</td>
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<td>$f_{R,S}$</td>
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<td>$f_R$</td>
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<td>$f_{x}(X)$</td>
<td>Joint probability density function</td>
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<tr>
<td>$f_\Gamma$</td>
<td>Cohesive breaking energy</td>
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xix
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$G$</td>
<td>Energy release rate</td>
<td>$G_{IC}$</td>
<td>Energy rate consumed by creating two cracked faces</td>
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<td>$G_f$</td>
<td>Fracture energy of concrete</td>
<td>$G_\sigma$</td>
<td>Energy rate to overcome cohesive forces</td>
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<td>$G_{ini}^i$</td>
<td>Initiation fracture energy</td>
<td>$G(R,S)$</td>
<td>Limit state function</td>
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<td>$G_{un}^i$</td>
<td>Unstable fracture energy</td>
<td>$h_c$</td>
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<td>$H(x)$</td>
<td>Discontinuous jump function</td>
<td>$\Delta I$</td>
<td>Change in applied current</td>
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<td>$I$</td>
<td>Current</td>
<td>$i_{corr}(t)$</td>
<td>Corrosion rate</td>
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<td>Maximum value for input parameter</td>
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<td>Minimum value for input parameter</td>
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<td>$J$</td>
<td>$J$-integral</td>
<td>$K_{ini}^i(e)$</td>
<td>Initial fracture toughness</td>
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<td>Stress intensity factor for mode I fracture</td>
<td>$K_{ini}^i$</td>
<td>Initial fracture toughness</td>
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<td>$K_{IC}^c$</td>
<td>Critical stress intensity factor</td>
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<td>$K_f^p(e)$</td>
<td>Stress intensity factor due to applied load</td>
<td>$K_n$, $K_r$</td>
<td>Stress intensity factor for given loading condition</td>
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<tr>
<td>$K_{cor}$</td>
<td>parameter</td>
<td>$L$</td>
<td>Original length of the structure</td>
</tr>
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<td>$K_{net}$</td>
<td>Net stress intensity factor</td>
<td>$M_1, M_2, M_3$</td>
<td>Weight function coefficients</td>
</tr>
<tr>
<td>$M$</td>
<td>Atomic weight of iron; weight loss</td>
<td>$N$</td>
<td>Number of trials</td>
</tr>
<tr>
<td>$m(x,e)$</td>
<td>Weight function</td>
<td>$N_l(x)$</td>
<td>Nodal shape functions</td>
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<tr>
<td>Symbol</td>
<td>Definition</td>
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<tr>
<td>--------</td>
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<tr>
<td>$n$</td>
<td>Coefficient; number of terms; number of load cases</td>
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<td>$O_{max}$</td>
<td>Maximum value for output parameter</td>
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<td>$O_{avg}$</td>
<td>Average value for output parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_i$</td>
<td>Internal pressure caused by corrosion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_0$</td>
<td>Magnitude of applied load</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Porosity; auto-correlation coefficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_f$</td>
<td>Probability of failure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{st}$</td>
<td>Density of steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Standard normal density function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Standard normal distribution function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>Coordinate within thick wall cylinder; Load system;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R(t)$</td>
<td>Resistance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td>Inner radius</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_p$</td>
<td>Polarisation resistance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\dot{S}(t)$</td>
<td>Time derivative process of $S(t)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_n$</td>
<td>Stress of corrosion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_A$, $\Omega_B$</td>
<td>Domains</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$O_{min}$</td>
<td>Minimum value for output parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Internal pressure; Maximum applied force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{lim}$</td>
<td>Concrete cover load bearing capacity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_S$</td>
<td>Probability of survival</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_a$</td>
<td>Acceptable risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{rust}$</td>
<td>Density of rust</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Potential energy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi_c$</td>
<td>Creep coefficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi(x,t)$</td>
<td>Level set function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(r, \theta)$</td>
<td>Polar coordinate of a point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\dot{R}(t)$</td>
<td>Time derivative process of $R(t)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td>Outer radius</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S(t)$</td>
<td>Load effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{g(x)}(X_i)$</td>
<td>Sensitivity index</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress; standard deviation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress tensor</td>
<td>$\sigma(x)$</td>
<td>Stress distribution on crack surface</td>
</tr>
<tr>
<td>$\sigma_c(x)$</td>
<td>Cohesive stress distribution along the crack surface</td>
<td>$\sigma_0^{\text{max}}$</td>
<td>Maximum principal stress of concrete</td>
</tr>
<tr>
<td>$\langle \sigma_{\text{max}} \rangle$</td>
<td>Maximum principal stress</td>
<td>$\sigma_p(x)$</td>
<td>Stress induced by corrosion</td>
</tr>
<tr>
<td>$\sigma_r(a)$</td>
<td>Radial stress distribution</td>
<td>$\sigma(w)$, $\sigma_w$</td>
<td>Tension softening curve</td>
</tr>
<tr>
<td>$T$</td>
<td>Tension vector; Indirect tensile strength</td>
<td>$T_c$</td>
<td>Time to end of service life</td>
</tr>
<tr>
<td>$T_{c1}$</td>
<td>Time to corrosion-induced cover cracking</td>
<td>$T_{c2}$</td>
<td>Time to permissible corrosion-induced crack width</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>$t_{cr}$</td>
<td>Time to cracking</td>
</tr>
<tr>
<td>$U$</td>
<td>Strain energy; displacement vector</td>
<td>$U_d$</td>
<td>Elastic energy density</td>
</tr>
<tr>
<td>$u_r(x,a)$</td>
<td>Crack opening displacement field</td>
<td>$\mu$</td>
<td>Mean</td>
</tr>
<tr>
<td>$\mathbf{u}$</td>
<td>Displacement vector</td>
<td>$\mathbf{u}_I$</td>
<td>Nodal displacement vector</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>Poisson’s ratio of concrete</td>
</tr>
<tr>
<td>$\nu_R^*$</td>
<td>Mean upcrossing rate</td>
<td>$W$</td>
<td>Energy available for crack formation; Elastic strain energy</td>
</tr>
<tr>
<td>$W_c$</td>
<td>Amount of rust in the porous band</td>
<td>$W_0$</td>
<td>Amount of rust filling the porous band</td>
</tr>
<tr>
<td>$W_{\text{rust}}(t)$</td>
<td>Total amount of corrosion</td>
<td>$W_s$</td>
<td>Amount of rust replacing</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Significance</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>--------------</td>
<td></td>
</tr>
<tr>
<td>( w )</td>
<td>Crack width</td>
<td>( w/c ) Water-cement ratio</td>
<td></td>
</tr>
<tr>
<td>( w_c )</td>
<td>Crack width limit</td>
<td>( X_n ) Basic variables</td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>Carbonation depth; coordinate along crack depth; chloride content distance; metal loss; query point</td>
<td>( x_i ) Coordinate of any point along the limit state function</td>
<td></td>
</tr>
<tr>
<td>( \hat{x}_i )</td>
<td>Sample value</td>
<td>( x^* ) Design point</td>
<td></td>
</tr>
<tr>
<td>( x_\Gamma )</td>
<td>Point on the discontinuity</td>
<td>( \Gamma ) Contour interface</td>
<td></td>
</tr>
<tr>
<td>( z )</td>
<td>Ionic charge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 1: Introduction

1.1 Background

Reinforced concrete (RC) is one of the most widely used materials in civil engineering construction. RC is used in buildings, bridges, tunnels and any physical infrastructure built above or below ground, all around the world. It was reported that twice as much concrete and mortar is used in construction, roughly 35 billion tonnes as the total of all other industrial building materials including wood, steel, plastic and aluminium (Van Damme, 2018). The extensive application of RC in the construction industry derives from its versatility, durability and low cost. As such, almost everything built today uses RC.

Until the 1950s, RC was considered a durable material, which means it does not degrade. Since then, there have been multiple cases in which a RC structure has prematurely failed. Figure 1.1 shows the partial collapse of a multistorey carpark in Wolverhampton in the UK. More recently, Figure 1.2 shows the collapse of a RC bridge in Genoa in Italy. Although corrosion may not be the sole factor contributing to the failure of the structures, combined with other factors, such as overloading and poor design, it can contribute to the failure of RC structures.
Since the 1970s, it has been accepted that concrete cover alone is inadequate for protecting and preventing the corrosion of reinforcing steel. As a result, a series of research was initiated to improve understanding of the corrosion of steel in concrete (Wilkins & Lawrence, 1980). From this research, it became evident that RC structures will eventually suffer from reinforcement corrosion, especially in chloride-
laden environments. Unfortunately, these environments have a large concentration of buildings and infrastructure, such as coastal cities and maritime structures.

Practical experience and experimental observations suggest that corrosion-affected RC structures deteriorate faster in terms of serviceability limit state (e.g., cracking) than ultimate limit state (e.g., strength) (Li, 2005). This situation is exacerbated as the loading demands for RC structures increases. Hence, many RC structures that may appear badly deteriorated or unserviceable, are in fact still structurally sound. Therefore, early detection of cracks will enable engineers to take appropriate measures to prevent or mitigate further deterioration of concrete structures. This will allow for cost-effective asset management plans to be developed to reduce public inconvenience resulting from possible interruptions required for maintenance of corrosion-affected RC structures.

Corrosion of reinforcing steel in concrete can be broadly divided into two phases: corrosion initiation and corrosion propagation. When steel depassivation occurs, corrosion is initiated and begins to propagate. During the propagation phase, corrosion products begin to form at the steel–concrete interface and exert an expansive pressure on the surrounding concrete. Due to the low tensile strength of concrete, the expansion of corrosion products cause concrete to crack. Once a crack is initiated, it steadily propagates to a critical depth at which the crack becomes unstable and suddenly propagates to the concrete surface. When the crack penetrates through the concrete cover, a path for rapid ingress of aggressive agents to the reinforcing steel is created. This leads to the progressive deterioration of the structure and to serviceability failure.

To date, considerable experimental investigations into corrosion-induced cracking have been conducted. Experimental investigation typically involves measuring corrosion-induced time to cracking and crack width on RC test specimens. The corrosion process in these tests is accelerated using either an impressed current technique or natural conditions (i.e., salt spraying) (Andrade, Alonso & Molina, 1993; El Maaddawy & Soudki, 2003; Liu & Weyers, 1998; Vu & Stewart, 2005). This is done so that the corrosion-induced effect on RC can be studied within a short time frame. Experimental investigation usually assumes that corrosion and porous
zones (i.e., voids) around the steel–concrete interface are uniformly distributed (Michel, Solgaard, Pease, Geiker, Stang & Olesen, 2013; Söylev & François, 2005). Parameters such as corrosion rate, concrete properties (i.e., strength), geometry (i.e., cover depth) and rebar diameter have been considered the main variables in experimental investigation of corrosion-induced cracking.

In numerical investigations, the finite element method (FEM) based on fracture mechanics is used to model the cracking behaviour of concrete. The fictitious crack model and the crack band model are among the most popular models used to study crack propagation. The fictitious crack model treats crack as a discrete crack and concrete fracture is analysed using the fracture energy concept. For example, Barpi and Valente (2000) analysed crack propagation in concrete dams based on the fictitious crack model and the results were verified with a scaled experimental test. In contrast, the crack band model treats the crack as a band of uniformly distributed microcracks. The stresses within this band is represented by stress–strain softening characteristic of concrete. For example, Molina, Alonso and Andrade (1993) used finite element techniques based on the crack band model to simulate the crack propagation rate by incorporating the linear softening characteristic of concrete and changing the elastic properties of elements to represent cracking. More recently, the extended finite element method (XFEM) has become more popular for the numerical investigation of crack propagation, as it allows crack propagation to be easily studied without remeshing. XFEM based on the fictitious crack model has been used to study crack propagation on wedge splitting, L-shaped panel specimens, three-point bending specimens and mixed mode fracture specimens (Sancho, Planas, Cendon, Reyes & Galvez, 2007; Unger, Eckardt & Konke, 2007). Further, Du and Jin (2014) and Thybo, Michel and Stang (2017) used XFEM based on the smeared crack model to simulate corrosion-induced damage in the form of corrosion-induced crack patterns.

In analytical investigations, corrosion-induced concrete cracking is usually modelled as a thick wall cylinder with the thickness as the cover thickness. The common derivation assumptions for the analytical solution to corrosion-induced concrete-cracking models are: (Andrade, Alonso & Molina, 1993; Bažant, 1979; Bhargava, Ghosh, Mori & Ramanujam, 2005; El Maaddawy & Soudki, 2003; Liu & Weyers,
1) concrete is an isotropic homogenous linear elastic material before cracking
2) corrosion products grow uniformly around the steel reinforcement
3) corrosion products must first fully fill the porous zone before expansive pressure on the concrete cover occurs
4) cracking in concrete is only caused by stresses resulting from the expansion of corrosion products

However, despite all these efforts, discrepancy between predictive models and observed data from the field and laboratory have been reported (Chen, Baji & Li, 2018). This may be attributed to factors such as modelling assumptions, lack of knowledge about the chemical composition and properties of corrosion products and simplification of the concrete-cracking process etc. It is therefore imperative to develop a more robust model based on mechanics to predict corrosion-induced concrete cracking.

With a realistic corrosion-induced cracking model, more accurate predictions about the serviceability conditions of corrosion-affected RC structures can be made. The literature review suggests that reliability assessments of corrosion-affected RC structures have focused more on strength deterioration than serviceability. Corrosion-induced strength failure of RC structures is typically considered by the loss of flexural, shear and bond strength. In a reliability assessment, the Monte-Carlo simulation (MCS) method is a popular method for determining the probability of failure of corrosion-affected RC structures due to its ease of implementation (Enright & Frangopol, 1998a; 1998b; Thoft-Christensen, 1998). For example, Val (2007) used the MCS method to assess the reliability of corrosion-affected RC beams based on flexural and shear strength failure. This is enhanced by research from Bhargava, Mori and Gosh (2011), in which Latin hypercube sampling (LHS) was used for efficient sampling of variability in the variables used in the MCS for reliability assessment of RC beams. It is accepted that, in RC structures, the corrosion rate is highly variable and random. To tackle this uncertainty, Marsh and Frangopol (2008) incorporated
spatial and temporal variations of probabilistic corrosion rate data to assessed the reliability of an RC bridge using MCS method based on a flexural failure mode.

In contrast, relatively fewer reliability studies have been conducted on serviceability failure modes (i.e., corrosion-induced cracking) and fewer employed a time-dependent reliability method (Li & Melchers, 2005, Wu & Ni, 2004, Val & Trapper, 2008). Because corrosion is time-dependent, it is only appropriate for the probability of serviceability failure to be determined using time-dependent reliability methods. Furthermore, time-dependent reliability study on corrosion-induced concrete cracking typically considers statistical variables to be normally distributed. This may give rise to unrealistic negative values and, in some cases, a lognormal distribution would be more appropriate (Li & Melchers, 2005).

1.2 Significance

Reinforcing steel encased in concrete is more durable than bare steel. Bare steel tends to corrode more easily, while steel encased in concrete is protected. The protection is attributed to the design of the concrete cover, as it provides a physical barrier against aggressive agents. This prevents/mitigates the migration of aggressive agents to the steel reinforcement. Further, the alkalinity of concrete helps to prevent corrosion initiation of reinforcing steel. However, due to factors such as poor quality management in construction methodology, corrosion of reinforcing steel remains a major problem affecting the durability of RC structures.

One of the first, and most significant, consequences of corrosion in RC is cracking. As such, corrosion-induced cracking is one of the most important parameters for the assessment of RC structures. Even with protective measures for steel reinforcement (e.g., paint systems, cathodic protection and sacrificial anode systems), maintenance remains unavoidable and the cost of these protections is considerable; maintenance and repairs of corrosion-affected RC structures costs approximately $100 billion per annum, worldwide. This cost is only expected to increase, as corrosion-induced failures of RC structures have not been effectively predicted and prevented. There is insufficient research into corrosion-induced failure mechanisms in concrete and a lack of advancement in assessment methods. This motivates research programs to advance
the theory of corrosion-induced failure in RC and develop accurate assessment methods.

There are many parameters, such as corrosion rate, material properties and loading conditions, that are uncertain. These can be time-variant during the service life of an RC structure. To consider the uncertainty and time-variance of these parameters, it is logical to represent either one or a combination of parameters as a stochastic process. To analyse the stochastic process against a threshold or limit, it is necessary to employ an upcrossing theory-based method into the failure assessment of corrosion-induced concrete cracking. By incorporating the time-variant characteristics of corrosion, the accuracy of failure prediction will significantly improve.

Although considerable research has been conducted on predicting the time to corrosion-induced cover cracking based on experimental results and numerical models, limited work has been undertaken to analytically investigate crack evolution in the concrete cover during the corrosion of reinforcing steel. So far, no models have been proposed to analytically determine the corrosion-induced critical crack depth at which a crack becomes unstable and suddenly propagates through the concrete cover. Corrosion cover cracking is one of the first indicators of premature degradation of RC. A direct method to calculate the propagation of a corrosion-induced crack can allow for more informed decisions about the design and maintenance of RC structures. Additionally, there is a lack of time-dependent reliability assessment methods of corrosion-induced cracking in RC. It is in this regard that this research is undertaken; to analytically, numerically and experimentally investigate corrosion-induced concrete cracking and the subsequent probability of failure using time-dependent methods.

1.3 Aims and Objectives

The aim of this research is to develop a new model to accurately predict the probability of failure of corrosion-induced concrete cracking using fracture mechanics criteria. Analytical, numerical and experimental studies have been conducted to investigate the cracking process of RC due to corrosion-induced pressure. With an accurate cracking model, the failure probability of the structure can
be determined and the remaining service life of corrosion-affected RC structures can be predicted.

The objectives of this research are to:

1) develop a thorough understanding of the concrete-cracking process subjected to corrosion-induced pressure
2) develop an analytical model for corrosion-induced crack propagation
3) develop an FEM based on an XFEM to simulate the cracking process in concrete
4) produce experimental data on time to corrosion-induced cover cracking and crack width with the concrete cover depth and corrosion rate as test variables
5) develop a time-dependent reliability method to predict the remaining service life of corrosion-affected RC structures
6) develop a program to execute the above computations.

1.4 Scope

The scope of this thesis is to study the corrosion effects on reinforced concrete using fracture mechanics and predict the remaining service life using time-dependent reliability analysis. This thesis consists of seven chapters.

Chapter 1 presents the background and significance of the research and outlines the aims and objective of the thesis.

Chapter 2 is the literature review. This chapter critically reviews the fundamental theories and methods that are necessary to carry out the research and identify the gaps in existing knowledge. This includes steel corrosion mechanisms in RC, corrosion experimentation, fracture mechanics theory, modelling of concrete cracking due to corrosion and reliability analysis.

Chapter 3 develops an analytical model to determine the corrosion-induced critical crack depth in RC. This model is derived from fracture mechanics, in which the stress intensity factor (SIF) for a single radial crack in a thick-walled cylinder is used to
model the crack depth. A worked example is presented to demonstrate the application of the developed method, followed by analysis and discussion.

Chapter 4 develops a numerical method to predict corrosion-induced cracking using the XFEM. ABAQUS is used to develop a cracking model that incorporates fracture mechanics and finite element techniques. A procedure to apply XFEM simulating corrosion-induced crack propagation is discussed. A worked example is presented to demonstrate the application of the developed method, followed by analysis and discussion.

Chapter 5 experimentally investigates the time to corrosion-induced concrete cover cracking and growth of corrosion-induced crack width. This is followed by a description of the accelerated corrosion experimental program. The measured time to corrosion-induced cracking, corrosion-induced crack width results and other results from the experiments are analysed and discussed.

Chapter 6 develops a new methodology to evaluate the probability of corrosion-induced concrete-cracking failure. A stochastic model with a non-stationary lognormal process is developed for corrosion-induced concrete cracking as a function of key contributing factors. The first passage probability method is employed to predict the time-dependent probability of corrosion-induced concrete cracking. A worked example is presented to demonstrate the application of the method, followed by analysis and discussion.

Chapter 7 concludes the thesis with recommendations for future work.
Chapter 2: Introduction to Fundamental Theories

2.1 Introduction

To model corrosion-induced cracking and subsequent remaining service life of RC structures, an in-depth knowledge of a broad range of disciplines is necessary. These include corrosion chemistry, corrosion experimentation, fracture mechanics, numerical modelling and structural reliability theory. This chapter critically reviews the elements required to model corrosion-induced cracking in RC. Because concrete cracking is caused by corrosion, the fundamentals of corrosion in concrete and experimentation on corrosion of reinforcing steel is discussed first. To analyse the cracking phenomenon in RC, fracture mechanics (the study of the mechanical behaviour of cracked structures) is reviewed. Numerical techniques on crack propagation are then evaluated. Finally, to evaluate the serviceability failure of RC cracking due to corrosion, structural reliability theory is discussed.

2.2 Corrosion of Reinforcing Steel in Concrete

RC is one of the most versatile, economical and widely used construction materials in the world. However, one of the main causes for premature deterioration of RC structures is the corrosion of reinforcing steel (Broomfield, 2002). Corrosion of reinforcing steel results in the expansion of corrosion products causing concrete to crack. In America, the cost of damage to highway bridges as a result of corrosion is approximately USD150 billion (Broomfield, 2002). In Australia, this cost is estimated to be between AUD36 billion and AUD60 billion (Moor & Emerton, 2010). The degradation of RC structures by corrosion is the result of rust growth and reinforcement reduction during the electrochemical process.

This section will provide an understanding of the basics of the corrosion of steel in concrete, the basics of concrete properties, the mechanisms of corrosion in concrete, corrosion condition evaluation techniques and a review of the corrosion life cycle in corrosion-affected RC. With a thorough understanding of the electrochemical process
of corrosion and the products formed, one can further understand how corrosion-induced cracking can lead to premature structural degradation of RC.

### 2.2.1 Basics of Corrosion of Steel

Concrete is an alkaline material. This alkalinity forms a passive layer around the steel reinforcement, preventing corrosion (Broomfield, 2002). Over time, the passive layer may break down through the process of either carbonation or chloride attack. Once the passive layer breaks down, corrosion initiates and rust begins to develop. The electrochemical reactions of steel reinforcement during corrosion can be expressed as:

- the anodic reaction: \( Fe \rightarrow Fe^{2+} + 2e^- \) (2.1)
- the cathodic reaction: \( 2e^- + H_2O + \frac{1}{2}O_2 \rightarrow 2OH^- \) (2.2)

The anodic reaction produces electrons that dissolve in pore water, which is then consumed by water and oxygen in the cathodic reaction to form hydroxyl ions. Depending on the distances between electrodes (i.e., anodes and cathodes), either macro-cell or micro-cell corrosion could occur. Macro-cell corrosion is when electrodes are separate. Micro-cell corrosion is when electrodes are immediately adjacent to one another (Berke, Bentur & Diamond, 2014). Macro-cell corrosion leads to a large section of steel acting as the electrical pathway to conduct electrons from anode to cathode. Corrosion in this case is usually not uniform, due to the inhomogeneity of cracks in concrete (Qian, Zhang & Qu, 2006). Regardless of macro or micro-cell corrosion, the electrochemical process of steel corrosion remains the same (see Figure 2.1). The free-flowing ferrous and hydroxyl ions in concrete can further react with water and oxygen to produce corrosion products. The electrochemical reaction is as follows:

\[
Fe^{2+} + 2OH^- \rightarrow Fe(OH)_2
\]  
(2.3)

\[
4Fe(OH)_2 + O_2 + 2H_2O \rightarrow 4Fe(OH)_3
\]  
(2.4)

\[
2Fe(OH)_3 \rightarrow Fe_2O_3 \cdot H_2O + 2H_2O
\]  
(2.5)
The chemical composition of corrosion products can be generally expressed as (Broomfield, 2002):

\[
m \cdot Fe(OH)_2 \cdot n \cdot Fe(OH)_3 \cdot p \cdot H_2O
\]  

(2.6)

where \( m, n, p \) are coefficients. During the oxidation process, either ferrous ion \( Fe^{2+} \) or ferric \( Fe^{3+} \) ions are formed and react with water and oxygen to form different types of corrosion products, such as ferrous oxide \( FeO \), ferric oxide \( Fe_2O_3 \) and magnetite \( Fe_3O_4 \). Different type of corrosion products results in different volume expansion. For example, fully dense ferric oxide has approximately twice the volume of the steel it replaces (Broomfield, 2002). Hydrated ferric oxide \( Fe_2O_3 \cdot H_2O \) has a volume approximately 10 times that of the steel it replaces. The expansion of corrosion products at the steel surface is the main cause of concrete cracking.

In some cases, when oxygen is not available, a different type of rust, known as black rust, can form. Unlike the typical red and brown rust, black rust does not expand in volume. This type of rust is particularly dangerous, as it can considerably weaken steel reinforcement without any surface indication of corrosion. Table 2.1 shows the volume expansion ratio of corrosion products relative to uncorroded steel (Liu & Weyers, 1998). Depending on the presence of oxygen and water, corrosion products with different densities, volume and colour will form.
Table 2.1: Volume Expansion of Corrosion Products (Liu & Weyers, 1998)

<table>
<thead>
<tr>
<th>Corrosion products</th>
<th>Colour</th>
<th>Volume increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferrous oxide, FeO</td>
<td>Black</td>
<td>1.7</td>
</tr>
<tr>
<td>Magnetite, Fe₃O₄</td>
<td>Black</td>
<td>2.2</td>
</tr>
<tr>
<td>Ferris oxide, Fe₂O₃</td>
<td>Red/brown</td>
<td>2.3</td>
</tr>
<tr>
<td>Fe(OH)₂</td>
<td>Blue/green</td>
<td>3.8</td>
</tr>
<tr>
<td>Fe(OH)₃</td>
<td>Red/brown</td>
<td>4.2</td>
</tr>
<tr>
<td>Fe(OH)₃·H₂O</td>
<td>Red/brown</td>
<td>6.4</td>
</tr>
</tbody>
</table>

The density of rust $\rho_{rust}$ is a significant parameter that will affect the volume expansion of rust and concrete cracking. Some researchers have attempted to quantify $\rho_{rust}$ using experimental tests such as the scratching test to determine the mechanical property of rust (Petre-Lazar & Gérard, 2000). A stress strain relation was developed (Lundgren, 2002) as:

$$s_n = K_{cor} \cdot \varepsilon_{cor}^p$$  (2.7)

where $\varepsilon_{corr}$ is the strain of corrosion products, $s_n$ is the stress and the parameters $K_{cor}$ and $p$ are chosen to provide reasonable agreement with the test results, which were 7.0 GPa and 7.0 respectively (Karin, 2002). In other studies, $\rho_{rust}$ was taken to be between $\frac{1}{4}$ and $\frac{1}{2}$ of steel density for macro-cell formation (Andrade, Alonso & Molina, 1993) and $\frac{1}{4}$ and $\frac{1}{8}$ for micro-cell formation (Gonzalez, Andrade, Alonso & Feliu, 1995). It was also found that the density of rust can be related to the composition or type of rust $\alpha_{rust}$, which is between 0.523 and 0.622 (Liu & Weyers, 1998).
2.2.2 Basics of Concrete Properties

The properties of concrete are important aspects that also need to be reviewed within the scope of this research. This is because concrete material can protect against the corrosion of reinforcing steel and the material properties of concrete (e.g., strength) will influence the cracking induced by corrosion. The main composition of concrete material is cement, aggregate, sand and water. With different mix designs, concrete with varying properties can be made.

The alkaline property of concrete material is due to the hydration process of cement (Neville, 1995). To make concrete, one of the main compositions is cement. Cement is the binding material that causes other entities, such as coarse and fine aggregates, to bond together. Portland cement is one of the most common classifications of cement and is frequently used around the world. Depending on the purpose of the concrete mix, different compositions of Portland cement can be used (e.g., fast-hardening Portland cement, sulphate resistance cement and low heat Portland cement). The raw material used to manufacture Portland cement consists of lime, silica and alumina (Neville, 1995).

In the presence of water, the cement compounds are hydrated and a cement paste is formed. Tricalcium silicate $C_3S$ and dicalcium silicate $C_2S$ are two of the main compounds in cement. The hydration process of these compounds is:

$$2C_3S + 6H' \rightarrow C_3S_2H'_3 + 3Ca(OH)_2$$  \hspace{1cm} (2.8)

$$2C_2S + 4H' \rightarrow C_3S_2H'_3 + Ca(OH)_2$$  \hspace{1cm} (2.9)

where $H'$ represents water.

The calcium hydroxide ($CaOH$) that is produced provides and maintains a protective layer on the steel, which is known as the passive layer. This passive layer helps to form a protective layer around the steel reinforcement, preventing corrosion. The amount of calcium hydroxide will vary depending on the type of cement used and the composition of the concrete mix. For example, Portland cement that is well-hydrated can contain up to 30% of calcium hydroxide by weight of the original cement.
(Pressler, Brunauer, Kantro & Weise, 1961). This approximates to concrete with a pH of 13.

The properties of concrete can also be understood by examining the mechanical stress–strain behaviour. To some degree, concrete can be considered an elastic material. Prior to peak stress, concrete in compression generally exhibits a stress–strain relationship similar to that shown in Figure 2.2. The tangent curve is known as the modulus of elasticity and the secant modulus corresponds to the trend that decreases as stress increases. The nonlinear behaviour of concrete after peak load is due to the development of microcracking between the interfaces of aggregate and cement paste (Shah, Swartz & Ouyang, 1995). It can be observed that when the cement paste and aggregates are loaded individually, the stress–strain curve is linear (see Figure 2.2).

![Figure 2.2: Stress–strain Relations for Aggregate, Cement Paste and Concrete (Neville, 1995)](image)

To mathematically represent the stress–strain curve, Desayi and Krishnan (1964) proposed a model to predict the entire stress–strain curve of concrete. This is expressed as:
\[ \sigma = \frac{E\varepsilon}{1 + \left(\varepsilon / \varepsilon_0\right)^2} \]  \hspace{1cm} (2.10)

where \( \sigma \) is stress, \( \varepsilon \) is strain, \( E \) is the modulus of elasticity and \( \varepsilon_0 \) is the strain at maximum stress. The pre-peak shape of the stress–strain relation depends on two factors: the compressive strength of the concrete and the loading rate during tests. A summary of the relationship between the secant modulus of elasticity and the compressive strength of normal and high strength concrete can be found in ACI 318-02 (2001) and ACI 363R-92 (1992).

**2.2.3 Mechanisms of Corrosion in Concrete**

Carbonation and chloride attack are the two most widely accepted mechanisms of corrosion in concrete. These mechanisms cause the reduction of alkalinity in concrete and the deterioration of the passive layer on the steel surface. Carbonation reduces the alkalinity of concrete through the reaction with carbon dioxide. Chloride attack breaks down the passive layer on the reinforcement surface through the reaction with chloride ions.

**2.2.3.1 Carbonation**

Carbonation is the reaction of alkaline hydroxide in concrete with carbon dioxide gas. The carbon dioxide in the atmosphere dissolves in water to form an acid known as carbonic acid, which reacts with calcium hydroxide in concrete. The carbonation reaction is:

\[ CO_2 + H_2O \rightarrow H_2CO_3 \]  \hspace{1cm} (2.11)

\[ H_2CO_3 + Ca(OH)_2 \rightarrow CaCO_3 + 2H_2O \]  \hspace{1cm} (2.12)

As calcium hydroxide reacts with carbonic acid, concrete pH levels reduce. This process causes the passive layer to break down and is termed depassivation. It is generally thought that having a low pH would immediately result in corrosion since it indicates depassivation. However, it was discovered that the electrical potential of
concrete also influences the occurrence of corrosion (Pourbaix, 1966). The relation between concrete pH, electrical potential and the onset of corrosion is summarised in Figure 2.3, for general corrosion caused by carbonation.

![Pourbaix Diagram for Steel in Concrete (Kay, 1992)](image)

**Figure 2.3: Pourbaix Diagram for Steel in Concrete (Kay, 1992)**

The Pourbaix diagram provides a guideline for the probable state of metal based on specific conditions. The passive zone refers to the stable passive layer formed on the surface of metal (Jones, 1996). Immune refers to the metal not being attacked and corroding refers to active corrosion of metal. In Figure 2.3, steel reinforcement will corrode when the pH drops below eight and the electrical potential is approximately higher than −750 mV. If the pH level is very low (i.e., < 8), but the electrical potential is equally low (i.e., lower than approximately −750mV), steel reinforcement remains immune to corrosion.

The diffusion of carbon dioxide ions into concrete follows the diffusion law and is expressed in terms of the rate that is inversely proportional to the concrete cover thickness (Broomfield, 2002):
\[
\frac{dx}{dt} = \frac{D_0}{x}
\]  
(2.13)

where \( x \) is the carbonation depth, \( t \) is time and \( D_0 \) is the diffusion constant, which depends on the quality of the concrete. Due to the presence of aggregates and general flaws in concrete, the carbonation penetration depth will vary depending on the position. The solution to Eq. (2.13) provides an estimated carbonation depth over time:

\[
x = A\sqrt{t}
\]  
(2.14)

where \( A \) is the diffusion coefficient and \( t \) is the time. A study on a range of structures was carried out. Some of the values of \( A \) were found to:

- range from 1.2 to 6.7, with an average of \( 3y^{1/2}/\text{mm} \) for 11 buildings aged between 8 and 24 years old
- range from 2.2 to 7.6 with an average of \( 4.27y^{1/2}/\text{mm} \) for 7 carparks aged between 14 and 41 years old (Broomfield, 2002).

Eq. (2.14) is only applicable to steady exposure conditions. If there are variations in exposure conditions, such as variable humidity, periodic wetting and drying, the carbonation rate will change.

2.2.3.2 Chloride Attack

Chloride attack occurs when chloride ions diffuse into concrete, causing corrosion. Chloride ions can come from external sources (e.g., externally diffuse into concrete) or internal sources (e.g., cast into concrete during mixing). In high chloride ion concentration, pitting or micro-cell corrosion often occurs. This type of corrosion is considered extremely dangerous, as it can cause deep pits that lead to rapid and significant loss of steel cross-sections (Batis & Rakanta, 2005). The relationship between concrete pH, electrical potential and the onset of corrosion is summarised in Figure 2.4 for general corrosion caused by chloride ions.
According to Figure 2.4, when chloride ions are involved, pitting corrosion occurs in high pH conditions. Pitting corrosion is when corrosion is localised, forming a pit and the adjacent steel surface is protected from corrosion due to cathodic protection. The ferrous ions remain in the pit solution and will not be oxidised into ferric ions, as oxygen is not easily soluble in concentrated solutions. Figure 2.5 is an illustration of pitting corrosion in which the anode is the pit and the cathode is the adjacent steel surface.

**Figure 2.4: Pourbaix Diagram for Chloride-induced Corrosion (Kay, 1992)**

**Figure 2.5: Pitting Corrosion Process (Broomfield, 1997)**
The diffusion of chloride ions follows Fick’s second law of diffusion. The estimation of chloride content from the concrete surface $x$ for a given time $t$ is determined as (Crank, 1979):

$$\frac{\partial C(x, t)}{\partial t} = D_c \frac{\partial^2 (x, t)}{\partial x^2}$$

where $C(x, t)$ is the chloride ion content at the distance $x$ from the surface of concrete at a time $t$ and apparent diffusion coefficient $D_c$, which is a property of concrete. The solution to Eq. (2.15) can be obtained for one-dimensional diffusion in a homogenous semi-infinite medium such as concrete (Bamforth, 1999):

$$C(x, t) = C_s \left[1 - erf \left( \frac{x}{2\sqrt{D_c t}} \right) \right]$$

where $C_s$ is the chloride content on concrete surface and $erf(x)$ is the error function. Therefore, the larger the concrete cover is, the longer it takes for chloride ions to diffuse to reinforcing steel; thus, mitigating corrosion. This research will focus on chloride-induced corrosion.

**2.2.3.3 Factors Affecting Corrosion in Concrete**

Water is one of the main components required to make concrete. The amount of water added in concrete will affect the properties of concrete and consequently the effect of corrosion on concrete. Water enables the hydration of cement and makes fresh concrete workable. When too little water is added, fresh concrete is less workable, but the strength of the concrete is high and vice versa. For fully compacted concrete, the strength of the concrete is inversely proportional to the water–cement ratio and can be calculated as (Abrams, 1919):

$$f_c = \frac{K_1}{K_2 w/c}$$

where $K_1$ and $K_2$ are constant and $w/c$ is the water–cement ratio.
The relationship between concrete compressive strength and the water–cement ratio, expressed in Eq. (2.17), is illustrated in Figure 2.6. For fully compact concrete, the compressive strength decreases when the water–cement ratio increases.

![Relationship Between Strength and the Water–Cement Ratio](image)

**Figure 2.6: Relationship Between Strength and the Water–Cement Ratio**

*(Neville, 1995)*

The amount of water added to concrete also affects the porosity of concrete. The porosity of concrete is the result of the presence of capillary pores, gel pores and air voids. Capillary pores are an interconnected system of randomly distributed pores throughout the cement paste. It is responsible for the permeability of concrete (Neville, 1995; Verbeck, 1955). Permeability of concrete is the ability for fluid to transport through the medium. Although porosity and permeability are related, they are not the same. Concrete with high porosity does not mean high permeability, as only interconnected pores can effectively transport fluids. This means that, even if concrete porosity is high, if the pores are not well-connected, the permeability of the concrete is low. The porosity of concrete will affect the rate at which carbon dioxide and chloride ions diffuse through the concrete material and cause corrosion. Hence, having concrete with lower porosity lowers the risk of corrosion of reinforcing steel, as aggressive ions cannot easily diffuse through concrete. This can be achieved by having a low w/c in concrete mix but this sacrifices workability. Similar to capillary
pores, gel pores are interconnected, but are significantly smaller, and have air voids, due to imperfect compactions. The volume of pores in concrete not only affect the diffusivity of aggressive ions but also the strength of the concrete. The relationship between porosity and concrete strength can be described as:

\[ f_c = f_{c,0} (1 - p)^n \]  \hspace{1cm} (2.18)

where \( f_c \) is the compressive strength of concrete, \( p \) is porosity, \( f_{c,0} \) is the compressive strength of concrete assuming zero porosity and \( n \) is a coefficient.

Concrete is considered a three-phase material, meaning that it consists of three phases: cement paste, aggregates and the interfacial transition zone (ITZ) (Li & Zheng, 2007). The ITZ is the interface between the aggregate and the cement paste. This is another factor that can influence corrosion in concrete. Microcracking always occurs along the ITZ, as the properties of the microstructure of the ITZ are different to the bulk cement paste. During compaction, the relatively large aggregates prevent cement paste and cement particles from moving as close to the aggregates. The ITZ is considered the weakest link in a concrete microstructure. Hence, microcracking always occurs along the ITZ. This leads to pore space, or voids, to be produced between them, which may accelerate corrosion.

In its natural state, concrete can prevent the corrosion of reinforcing steel, due to the hydration of calcium hydroxide. However, defects, microcracking and voids allow the migration of aggressive ions through concrete that breaks down the passive layer and initiates corrosion. Water is the most important single factor that not only affects the strength of concrete (see Eq. (2.17)) but also the concrete microstructure (i.e., porosity and permeability), which directly affects the process of corrosion. Therefore, having a high water–cement ratio leads to lower strength concrete, although the fresh concrete is highly workable. Higher water content also results in more porous concrete, which increases the diffusion process of aggressive ions through concrete. Further, high water content leads to a weaker ITZ form. As such, microcracking can occur within the concrete microstructure, increasing the diffusivity of aggressive ions, accelerating the corrosion process. Ideally, concrete would be impermeable and
aggressive ions would be unable to diffuse through concrete to initiate corrosion. However, this cannot be achieved and; as such, it is important to understand the properties of concrete and mechanisms in which corrosion occurs. This allows for improved decision-making during corrosion-induced deterioration of RC structures.

2.2.4 Corrosion Evaluation in Reinforced Concrete

The assessment of corrosion-affected RC structures can be tricky, as it may not be easily identified. Several techniques can be used to evaluate corrosion-affected RC structures. These can be broadly categorised into destructive or non-destructive methods. Depending on the condition and purpose of inspection, different techniques can be applied.

2.2.4.1 Visual Inspection

Visual inspection is an easy method that can provide an indication of the extent and severity of corrosion damage. It may help to inform the most appropriate techniques for more detailed inspection. Visual inspection should be conducted by a trained specialist, as the ability to interpret certain conditions relies on the knowledge and experience of engineers or technicians. Different specialists will have different approaches to visual inspection; however, in general, a systematic visual survey should always be planned prior to going on site along with any preliminary equipment, such as microscopes.

2.2.4.2 Gravimetric Weight-loss Method

The gravimetric weight-loss method is a destructive method that involves pre-weighing rebars before they are cast into concrete for testing. After the corrosion test, the corroded rebar is removed and weighed. The difference in weight indicates the quantitative average of corrosion penetration. This method is suitable for laboratory tests in which the corrosion and test conditions are controlled. A detailed test procedure is described in ASTM G1–03 (ASTM International, 2011). This method cannot be used to measure the instantaneous corrosion rate, as this parameter is determined over the course of the corrosion period. Due to the absence of relevant data, such as the weight of reinforcing steel, the corrosion rate for structures in use
cannot be determined using this method. Therefore, non-destructive methods are more appealing for monitoring the corrosion of reinforcing steel.

2.2.4.3 Chloride Content Measurement

Chloride content measurement involves two stages: sampling and analysis. Sampling involves sampling concrete core samples at various depths. After sampling, there are multiple ways to analyse the ingress of chloride ions in concrete. First, the ingress of chloride ions can be divided into two categories: free chloride ions (i.e., chloride ions existing in the pore solution) and bound chloride ions (i.e., chloride ions attached to various hydration products). The analysis of each chloride ion group requires a specific methodology. For free chloride ions, extracted dust samples can be boiled in accordance with ASTM C1218 (ASTM International, 1999). The accuracy of this method depends on variables such as the fineness of the dust samples, temperature and time allowed for extraction (Arya & Ofori-Darko, 1996). Another method is called the pore press method. This method uses high pressure to extract the pore solution from the cement paste to measure the amount of free chloride ions. In addition, the acid solution extraction method is used to measure the total chloride content in samples as a percentage by weight of cement. This method assumes that both free and bound chloride ions are soluble in acid, giving the total chloride content (British Standard, 2015). X-ray fluorescence is an expensive method that requires specialised skills and equipment. Aside from testing, theoretical models such as Fick’s second law of diffusion can be used to predict the rate of chloride ingress in concrete. However, this is not always applicable; for example, when $D_c$ is not constant or when the chloride binding of hydrated cement is nonlinear.

2.2.4.4 Half-cell Potential Measurement

During the corrosion process, an electrical potential is generated between the anode and cathode (i.e., half-cell). The half-cell potential measurement is a method that uses a voltmeter with the corroding reinforcement attached to the anode and a reference electrode attached to the cathode and placed on the top surface of the concrete. When steel reinforcement corrodes, steel in ferrous hydroxide is one half-cell and the other is the reference electrode, which is typically copper in a copper sulphate solution or
silver in silver chloride solution. Figure 2.7 is an illustration of the half-cell measurement.

![Diagram of half-cell measurement](image)

**Figure 2.7: Illustration of Half-cell Measurement**

When moving the reference electrode along the concrete surface, different readings from the voltmeter indicate different steel corrosion conditions. To interpret the measured potential, ASTM International (2009) presented one way of interpreting the half-cell potentials (see Table 2.2).

<table>
<thead>
<tr>
<th>Copper/copper sulphate</th>
<th>Silver/silver chloride/4M KCL</th>
<th>Calomel</th>
<th>Corrosion condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; –200 mV</td>
<td>&gt; –106 mV</td>
<td>&gt; –126 mV</td>
<td>Low (10% risk of corrosion)</td>
</tr>
<tr>
<td>–200 to –350 mV</td>
<td>–106 to –256 mV</td>
<td>–126 mV to 276 mV</td>
<td>Intermediate corrosion risk</td>
</tr>
<tr>
<td>&lt; –350 mV</td>
<td>&lt; –256 mV</td>
<td>&lt; –276 mV</td>
<td>High (90% risk of corrosion)</td>
</tr>
<tr>
<td>&lt; –500 mC</td>
<td>&lt; –406 mV</td>
<td>&lt; –426 mV</td>
<td>Severe corrosion</td>
</tr>
</tbody>
</table>

Half-cell potential measurement is a simple way to determine corrosion conditions. However, some of the limitations of this method are that:
1) it does not indicate the corrosion rate
2) it only detects active corrosion
3) concrete cover significantly affects the reading
4) rebar congestion can create inaccuracy

It may be necessary to use more than one evaluation technique, as each has different advantages and disadvantages. The engineers or technicians should use their knowledge and experience to decide what technique is most appropriate, given the circumstances of the structure.

2.2.4.5 Linear Polarisation Measurement

Linear polarisation resistance (LPR) measurement is an electrochemical technique that measures the rate of corrosion in real time. The device requires a probe that is attached to the corroding rebar and a reference electrode is placed on top (see Figure 2.8).

![Figure 2.8: Schematic of a Simple LPR System with an Unconfined Measurement Area](image)

The corroding reinforcement is characterised by the corrosion potential $E_{corr}$. The reinforcement potential may be shifted $\Delta E$ by an external perturbation (i.e., an externally applied current $\Delta I$). This process is called polarisation. Linear polarisation refers to the linear region of the polarisation curve, in which slight changes in the current applied to the corroding metal in an ionic solution causes corresponding
changes in the potential of the metal. The relationship between the steady state resistance that the reinforcement interface presents to a change in potential $\Delta E$ when the perturbation $\Delta I$ is small is called polarisation resistance $R_p$. $R_p$ is determined by:

$$R_p = \frac{\Delta E}{\Delta I} \quad \text{(2.19)}$$

For this relationship to be valid and remain linear, the change in potential must be kept to approximately less than 20 mV. Following the Stern-Geary equation, the instantaneous corrosion rate can be determined based on $R_p$ (Stern & Geary 1957):

$$i_{corr} = \frac{(\beta_a \beta_c)}{2.3 R_p (\beta_a + \beta_c)} = \frac{\beta}{R_p} \quad \text{(2.20)}$$

where $\beta_a$ is the anodic tafel slope, $\beta_c$ is the cathodic tafel slope and $\beta$ is Stern and Geary constant. The value of $\beta$ is a constant that ranges from 26 to 52 mV depending on the system. It is important to define the area of measurement to obtain an accurate corrosion rate measurement. Once corrosion rate $i_{corr}$ is obtained, the metal dissolution (or metal loss) can be determined based on Faraday’s law (Mangat & Molloy, 1992):

$$m = \frac{M \cdot I \cdot t}{z \cdot F} \quad \text{(2.21)}$$

where $m$ is the mass of steel consumed (g), $I$ is the current in (amps), $t$ is the time (s), $F$ is the Faraday constant, which is equal to 96500 C mol$^{-1}$, $z$ is the ionic charge and $M$ is the atomic weight of metal.

For iron with $M = 56$ g, the density of steel $\rho_{st} = 7.85$ g cm$^{-3}$ and $z = 2$. With the definitions that $A$ is the surface area of rebar (cm$^2$), $i_{corr}$ is the corrosion current density which is a measure of corrosion rate in $\mu$A/cm$^2$, $I = i_{corr} A$, $x$ is metal loss (mm):
\[ p_{st} A x = \frac{M_{corr} A t}{zF} \]  

\[ x = \frac{56 \times i_{corr} \times t}{2 \times 96500 \times 7.85} = 3.69 \times 10^{-7} i_{corr} t \]  

Therefore, metal loss \( x \) (\text{mm/year}) can be determined as:

\[ x = 3.69 \times 10^{-7} \times 365 \times 24 \times 60 \times 60 i_{corr} t = 0.0116 i_{corr} t \]  

Broomfield (2002) proposed guidelines to interpret the monitored corrosion rate. The interpretations of linear polarisation measurement with a sensor-controlled guard ring device are:

- **Passive condition:** \( i_{corr} < 0.1 \mu A/cm^2 \)
- **Low to moderate corrosion:** \( i_{corr} < 0.1 \text{ to } 0.5 \mu A/cm^2 \)
- **Moderate to high corrosion:** \( i_{corr} < 0.5 \text{ to } 1 \mu A/cm^2 \)
- **High corrosion rate:** \( i_{corr} < 1 \mu A/cm^2 \)

Without sensors in which 52 mV is used to describe ‘B’ (Clear, 1989):

- **No corrosion expected:** \( i_{corr} < 0.2 \mu A/cm^2 \)
- **Corrosion possible in 10 to 15 years:** \( i_{corr} < 0.2 \text{ to } 1.0 \mu A/cm^2 \)
- **Corrosion expected in 2 to 10 years:** \( i_{corr} < 1.0 \text{ to } 10 \mu A/cm^2 \)
- **Corrosion expected in two years or less:** \( i_{corr} < 10 \mu A/cm^2 \)

It is evident that the LPR method is effective for measuring the corrosion rate in a structure. However, there are some limitations to this method. These are:

1) It only detects the instantaneous corrosion rate. Hence, the readings can change with time, temperature and relative humidity.
2) The surface area is difficult to define, which may lead to significant errors.
Therefore, it is suggested that multiple readings should be taken over a period to obtain an average value of the corrosion rate. This can be used to predict the growth of corrosion products.

2.2.5 Corrosion Life Cycle

A life cycle is defined as the time at which actions of repairs or maintenance are required. In corrosion-affected RC structures, the life cycle can be broadly divided into two stages: corrosion initiation and corrosion propagation. To perform a whole life assessment of a corrosion-affected RC structure, each stage of the life cycle must be evaluated. Tuutti (1982) proposed one of the first models to predict the service life of corrosion-affected RC structures based on the degree of corrosion (see Figure 2.9).

![Figure 2.9: Tuuti’s Life Cycle Model](attachment:image.png)

2.2.5.1 Corrosion Initiation

Corrosion initiation is the first stage. This stage of service life ends when corrosion initiates. Recently, a phenomenological model for the corrosion of reinforcing steel in concrete was proposed (see Figure 2.10) (Melchers & Li, 2006). This model describes a more comprehensive process of reinforcement corrosion. It is suggested that, for corrosion to initiate, it will require an additional two stages: the time for chlorides to
reach a chloride threshold level (CTL) $t_i$ and the time for pH to reach a low level $t_{ac}$ by depassivation. This process explains the delay observed in fields in which corrosion does not immediately initiate once CTL is reached. This is logical, as there are other factors, such as electrical potential and pH, that affect the occurrence of corrosion.

The migration of aggressive ions through concrete cover is not as straightforward as Fick’s second law of diffusion. There are many other factors that affect the diffusion of aggressive ions and further delay the onset of corrosion. For example, the influence of chloride binding on the coefficient of chloride diffusion in plain concrete, the effect of time and space on the coefficient of chloride diffusion, temperature and humidity contribute to delay in corrosion initiation (Khan, Ahmad & Al-Ghatani, 2017). Assuming that all of the conditions are favourable for corrosion to occur, the time to corrosion initiation is determined as the time for aggressive ions to diffuse through concrete and reach a CTL. There have been many studies to determine the CTL. Hausmann (1967) proposed that the CTL was when chloride concentration exceeds 0.6 hydroxyl concentration. This approximates to a concentration of 0.4% chloride by weight of cement. The British Standard provides a chloride content limit of less than 0.4% by weight of concrete for RC and 0.1% for prestress or heat-cured RC (BS, 1997). The ACI document 222R-01 reports that the maximum chloride content for RC exposed to chloride in service is 0.1% by weight of cement compared to 0.2% for British Standard 8110 (ACI Committee 222, 2001; Ann & Song, 2007; BS, 1997). As for the ratio to concrete weight, the CTL is 0.06% (ASTM International, 2009; Thomas, 1996).
2.2.5.2 Corrosion Propagation

After corrosion initiation, the second stage is corrosion propagation. This stage is arguably more critical, as it results in structural deterioration. The corrosion propagation stage is when corrosion is active and rust begins to propagate. This stage is primarily controlled by the supply of oxygen, moisture content and resistivity of concrete (Tuutti, 1982). As corrosion propagates, rust begins to exert expansive pressure on the surrounding concrete. This results in various types of damage to RC such as loss of reinforcement bond strength, concrete cracking and reduction of the rebar cross-sectional area. The rate at which corrosion propagates is significantly influenced by the corrosion rate $i_{corr}$ of steel reinforcement. The type of rust produced is also an important factor. Hence, due to the direct structural effect in this stage, a considerable amount of research has been carried out to investigate the corrosion propagation stage (i.e., the effects of corrosion in RC), in particular, experimental investigations.

The corrosion of reinforcing steel is a slow and complex process. The study of the mechanical consequences of corrosion-affected RC structures can become long and expensive. Hence, corrosion of reinforcing steel in concrete experiments is often accelerated to reduce the time of testing and to gain control over the main variables of corrosion. One of the more comprehensive accelerated corrosion tests under natural
conditions was conducted by Liu and Weyers (1998), in which chloride ions were added to the concrete mix to accelerate the depassivation of steel. It was reported that the time to corrosion-induced cracking in RC was between 0.72 and 3.85 years. Accelerating corrosion under natural conditions comes with many uncertainties. For example, the corrosion rate which is a significant variable, is considerably affected by factors such as humidity, temperature, concrete quality and chloride content. To tackle the many uncertainties associated with natural corrosion, an alternative technique called the impressed current technique can be adopted to accelerate corrosion.

The impressed current technique accelerates corrosion by imposing an electrical current between the reinforcement and counter-electrode (i.e., a cathode) so that the oxidation of reinforcement is enhanced and the time to cracking is significantly reduced. This technique can help to reduce the uncertainties associated with corrosion by inducing a constant and specified corrosion rate. It is commonly used by researchers to study the effects of corrosion in RC (Alonso, Andrade & Gonzalez, 1988; Andrade, Alonso & Molina, 1993; Liu & Weyers, 1998; Lu, Jin & Liu, 2011; El Maaddawy & Soudki, 2003; Vu & Stewart, 2000; 2005; Wong, Zhao, Karimi, Buenfeld & Jin, 2010; Yalçyn & Ergun, 1996; Zhao, Yu, Wu & Jin, 2012). The test set-up of accelerating corrosion using this technique is straightforward; the anodes are connected to the reinforcement bar, which is connected to a cathode to complete the circuit. The metals that are most frequently used as a cathode are stainless steel, copper and titanium, due to their good electrical conductivity. Most researchers choose to use cathodes in the form of plates or mesh that cover the external faces of concrete specimens instead of cathode bars embedded in the concrete specimens (Azad, Ahmad & Azher, 2007; Fang, Lundgren, Chen & Zhu, 2004; Rio, Andrade, Izquierdo & Alonso, 2005). Though, it remains unclear which type and placement of cathode would better represent in-service corrosion conditions (Malumbela, Moyo & Alexander, 2012).

To use the impressed current technique to accelerate corrosion, one of the main parameters is the applied current. The applied current is directly calculated from the specified applied corrosion rate and the reinforcement area. It is argued that this
technique usually means that the corrosion rate applied is significantly higher than that observed in structures corroding in natural conditions, which may affect results (Andrade, Alonso & Molina, 1993). Despite this, the technique is still used by numerous researchers and has significantly contributed to improving understanding of the effects of corrosion on reinforcing steel in concrete. Research by Andrade, Alonso & Molina (1993) is one of the pioneering works that used this technique to accelerate corrosion. They proposed a time to first cracking model incorporating factors such as cover to diameter ratio, quality of concrete and corrosion rate. In addition, corrosion-induced crack width was also experimentally investigated using this method (Vu & Stewart, 2005; Zhao, Yu, Wu & Jin, 2012). It was discovered that using the impressed current technique, Faraday’s law for steel dissolution still holds true when the applied corrosion rate is between 100 and 500 $\mu A/cm^2$ (El Maaddawy & Soudki, 2003).

2.3 Fracture Mechanics of Concrete

Fracture mechanics is a branch of mechanics that addresses the separation of materials. To analyse the crack propagation and crack depth of concrete, methods based on this theory are required. In this section, the basics of fracture mechanics are introduced. This is followed by a discussion of linear and nonlinear fracture mechanics, which includes fracture parameters and widely accepted models. Fracture modelling methods, such as fracture resistance curves, effective elastic crack approach and the weight function method, are also reviewed. Finally, numerical techniques to model crack propagation are discussed.

2.3.1 Basics of Fracture Mechanics

The fracture process is when there is a local detachment of material cohesion in a solid body. In elastic mechanics theory, the presence of cracks in concrete alter the distribution of stresses in the material. Based on the principle of the conservation of energy, it was proposed that the energy required for a crack to propagate a unit surface is equal to the energy release (Griffith & Eng, 1921). However, it was later discovered that concrete material exhibits interlocking friction and plastic behaviour that influences the energy required to propagate a crack (Bažant & Planas, 1997).
Additionally, the size of the specimen affects the analysis of a crack. This is commonly referred to as the size effect phenomenon. For example, with increasing member size, the average stress at failure will decrease (see Figure 2.11) (ACI Committee 446.IR 1991; Bažant & Planas, 1997).

![Figure 2.11: Load-deflection Diagrams of Geometrically Similar Structures of Different Sizes (Bažant & Planas, 1997)](image)

The size effect phenomenon can be further explained using Figure 2.12. Assuming a uniaxially loaded rectangular body, a crack will initiate and propagate by \( \Delta a \). The strain energy release for a crack to initiate and propagate is determined by the cross-hatched area. Assuming an arbitrary crack band of thickness \( h \), the cross-hatched area can be calculated, and the corresponding strain energy release rate is determined. Therefore, in a larger medium, the cross-hatched area is larger for the same crack extension \( \Delta a \), as illustrated in Figure 2.12 (Bažant & Planas, 1997).
2.3.2 Linear Elastic Fracture Mechanics

Linear elastic fracture mechanics (LEFM) was first developed by Griffith & Eng (1921) and is the fundamental theory of fracture mechanics. It was discovered that the stress value may not be adequate for use as a failure criterion in the presence of a crack. This motivated the development of the strain energy release rate approach (Griffith & Eng, 1921). This concept was further developed by introducing the concept of the stress intensity factor, which can be related to Griffith’s energy approach for linear elastic fracture (Irwin, 1957).

For LEFM to be applied, the following assumptions must be met:

1) The whole material is elastic, except in the vanishing small region ahead of the crack tip.
2) The stress at the crack tip is high, due to the inelasticity that occurs.
3) The size of the inelastic zone must be small in relation to the linear performance of the entire structure.

2.3.2.1 Griffith’s Energy Approach
The fracture energy release rate developed by Griffith & Eng (1921) was used to examine crack propagation. The energy release rate is defined as the first order derivative of energy with respect to crack extension. Based on the principle of conservation of energy, the potential energy $\Pi$ must be kept constant and the first order derivative, with respect to crack extension, is zero. This is expressed as:

$$\Pi = F - U + W$$

(2.25)

$$\frac{\delta(U - F + W)}{\delta a} = 0$$

(2.26)

where $F$ is the external work done by applied load, $U$ is the strain energy of the structure, $W$ is the energy available for crack formation or fracture and $a$ is the crack extension. For crack formation, the energy available for crack formation $W$ is equal to the energy release rate $G$. As such, the energy release rate $G$ is determined by:

$$\frac{\delta F}{\delta a} - \frac{\delta U}{\delta a} = \frac{\delta W}{\delta a} = G$$

(2.27)

The energy release rate $G$ can then be used as a failure criterion to describe quasi-static crack propagation. This is achieved by introducing a crack growth resistance $R$ parameter as:

$$G = R$$

(2.28)

As such, the basic problem in fracture mechanics can be formulated by comparing the measured amount of energy required for a crack to propagate (i.e., $R$ and the calculated energy release rate $G$). $R$ is often expressed using alternatives such as $G_c$, $G_f$, $G_{ic}$, which are a material property and are sometimes referred to as the critical energy release rate.

### 2.3.2.2 Stress Intensity Factor Approach

The stress intensity factor $K$ is a fracture parameter that reformulates the LEFM problem in terms of the stress state around the crack tip (Irwin, 1957). It was
discovered that the stress distribution around the crack tip is the same, regardless of the crack model. However, the intensity of the stress concentration varies. The fracture of materials can be divided into three cracking modes: Mode I, II and III (see Figure 2.13). Mode I fracture refers to pure tension. Mode II fracture refers to in-place shear. Mode III fracture refers to out of plane shear. It should be emphasised that, in this research, only the Mode I fracture is discussed.

![Figure 2.13: Different Types of Failure Modes](image)

The stress intensity factor for the Mode I fracture is represented by the coefficient $K_I$. $K_I$ is a function of several variables and, most notably, loading, crack size and structural geometry. This approach accounts for the stress singularity at the crack tip. The solution to $K_I$ is expressed as:

$$K_I = \sigma \sqrt{2\pi a} f_{ij}$$  \hspace{1cm} (2.29)

where $\sigma$ is the applied stress, $f_{ij}$ is the dimensionless quantity that varies with the load and geometry and $a$ is the crack length. A detailed expression of stress intensity factors for different geometries and loading conditions can be found in the stress intensity factor handbook (Murakami & Keer, 1993; Tada, Paris & Irwin, 1973). Using an example of an internal radial crack in a thick wall cylinder (see Figure 2.14), when the crack is under a polynomial crack face load, Eq. (2.29) is expressed as:

$$K_I = \sqrt{\pi a} \left( \frac{K_n}{\sqrt{a} \sqrt{\pi a}} \right) A_n$$ \hspace{1cm} (2.30)
where $K_n/(A_n\sqrt{\pi a})$ is a dimensionless value that can be obtained from tables in the stress intensity factor handbook (Murakami & Keer, 1993) and $A_n$ is a constant for a least square fit to the crack face loading as:

$$p(y) = \sum_{n=0}^{6} A_n\left[\frac{y}{R_2} - R_1\right]^n$$

(2.31)

![Figure 2.14: Thick-walled Cylinder Under Polynomial Crack Face Loading (Murakami & Keer, 1993)](image)

The stress intensity factor is equivalent to the Griffith energy approach in linear elastic conditions. The relationship between them for a Mode I fracture is:

$$G_I = \frac{K_I^2}{E}$$

(2.32)

This equilibrium is valid for all geometry conditions under plane stress. For plane strain problem, the $E$ is replaced with $(1 - \nu^2)/E$.

To determine stress intensity factors for a specific geometry and loading condition, there are three broadly categorised methods (Rooke, Baratta & Cartwright, 1981). The first method is the simplest, which is to reference books such as the stress intensity factor handbook by Tada, Paris & Irwin (1973) or Murakami & Keer (1993). If solutions cannot be obtained directly from a reference book, more analytical methods, such as superposition or the weight function method, can be used. In cases
in which the stress intensity factor is required repeatedly or if the loading and geometry conditions are complicated, numerical techniques, such as the FEM or boundary collocation method can be used. Depending on the application, different methods can be applied to obtain a solution for the stress intensity factor.

2.3.2.3 Weight Function Method

The weight function method is a powerful method to determine the stress intensity factor of cracked bodies in complex stress fields (Kumar & Barai, 2011). In concrete materials, the use of the weight function eliminates the need for numerical techniques to determine the SIF of the nonlinearity caused by the presence of the fracture process zone (FPZ). It provides a simple means to analytically calculate the stress intensity factor.

The weight function method was first introduced by Bueckner (1971) and was later modified by Rice (1972). This method expresses SIF as an integral of the product of applied stress and weight function. The unique feature of the weight function is that it is only dependent on the geometry. Therefore, if the weight function is determined for a cracked body, the SIF for any given loading condition applied to that body can be calculated using simple integration (Glinka & Shen, 1991). The SIF can be determined using the weight function as:

\[ K_I = \int_0^a \sigma(x)m(x,a)\,dx \]  \hspace{1cm} (2.33)

where \(\sigma(x)\) is the stress distribution on the crack surface and \(m(x,a)\) is the weight function of position \(x\) and \(a\). The determination of \(m(x,a)\) requires complex stress analysis of the crack body and can be simplified by the relationship derived by Bueckner (1971) and Rice (1972) in the form of:

\[ m(x,a) = \frac{E'}{2K_r} \frac{\partial u_r(x,a)}{\partial a} \]  \hspace{1cm} (2.34)
where \( u_r(x,a) \) is the crack opening displacement field, \( r \) represents the arbitrary given load system, \( K_r \) is the SIF corresponding to a certain \( r \) value, \( E' = E \) for the plane stress problem and \( E' = E/(1 - \nu^2) \) for the plain strain problem. \( E \) and \( \nu \) are the Young’s modulus and Poisson’s ratio, respectively. A generalised expression for weight function was proposed by Fett, Mattheck and Munz (1987), which can be expressed as:

\[
m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[ 1 + M_1 \left( 1 - \frac{x}{a} \right) + M_2 \left( 1 - \frac{x}{a} \right)^2 + M_3 \left( 1 - \frac{x}{a} \right)^3 + \cdots + M_n \left( 1 - \frac{x}{a} \right)^N \right]
\]  

(2.35)

where, \( M_N \) are weight function coefficients and \( n \) is the number of terms. In contrast, Sha and Yang (1986) introduced another form of the general expression of the weight function as:

\[
m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[ 1 + M_1 \left( 1 - \frac{x}{a} \right)^{1/2} + M_2 \left( 1 - \frac{x}{a} \right)^{3/2} + M_3 \left( 1 - \frac{x}{a} \right)^{3/2} + \cdots + M_n \left( 1 - \frac{x}{a} \right)^{N/2} \right]
\]  

(2.36)

By solving the weight function for a specific cracked body, the SIF can be determined by simple integration, as in Eq. (2.33).

### 2.3.3 Nonlinear Fracture Mechanics

Concrete is considered a quasi-brittle material, which means that it exhibits strain softening due to microcracking after peak load. This strain-softening characteristic is called the FPZ and occurs in the region ahead of the crack tip. For LEFM to be applicable, the FPZ must be zero or very small compared to the structure size. Figure 2.15 represents the stress-displacement behaviour under uniaxial tension for brittle, ductile and quasi-brittle materials. In quasi-brittle material (i.e., concrete), nonlinear
behaviour occurs just after peak load and the stress transfer capability of the material begins to reduce (Shah, Swartz & Ouyang, 1995).

![Stress-displacement Behaviour Under Uniaxial Tension](image)

**Figure 2.15: Stress-displacement Behaviour Under Uniaxial Tension**

*Note.* (a) brittle, (b) ductile and (c) quasi-brittle materials. Source: Kumar & Barai, 2011.

### 2.3.3.1 Fracture Process Zone and Toughening Mechanisms

For concrete materials, the small inelastic zone ahead of the crack tip is the FPZ. During crack propagation, many micro-failure mechanisms, such as microcrack shielding, grain bridging and crack branching (see Figure 2.16), can occur (Kumar & Barai, 2011). These micro-failure mechanisms—or toughening mechanisms—are responsible for stress transfer between crack faces, which results in additional energy consumed for crack propagation (Van Mier, 1991). Considerable research has been conducted to investigate toughening mechanisms. Kachanov (1985) discovered that microcracking shielding (i.e., the presence of microcracks ahead of the crack tip) consumed a small amount of external energy. Grain bridging is considered the most vital toughening mechanism, as stresses can transfer through aggregates across crack faces, even when the crack has progressed beyond the aggregates (Van Mier, 1991). Another toughening mechanism is crack branching (see Figure 2.16), which results in several crack branches forming due to the heterogeneity of concrete. Hence, more energy would be consumed to form new crack branches.
Figure 2.16: Illustration of Toughening Mechanisms in the Fracture Process Zone

(Shah, Swartz & Ouyang, 1995)

The influence of FPZ on the fracture behaviour of concrete cannot be ignored. Therefore, nonlinear fracture mechanics is more appropriate for analysing concrete cracking than LEFM. Some experimental studies discovered that LEFM may not be applicable to materials such as concrete, mortar or cement paste (Higgins & Bailey, 1976; Ohgishi, Ono, Takatsu & Tanahashi, 1986; Walsh, 1976). It was found that the stress intensity factor does not consider the stable crack growth associated with the FPZ. Hence, in quasi-brittle cracking, the energy release rate is the energy required to separate crack faces and the additional energy required to overcome the toughening mechanisms. Thus, the energy release rate for Mode I quasi-brittle cracking can be expressed as:

\[ G_q = G_{ic} + G_\sigma \]  \hspace{1cm} (2.37)
where $G_{ic}$ is the energy rate consumed by creating two cracked faces, which can be calculated using LEFM, and $G_\sigma$ is the energy rate to overcome cohesive forces (i.e., toughening mechanisms between the cracked faces).

2.3.3.2 J-integral

Since the development of Griffith’s energy approach, the $J$-integral was introduced as another fracture parameter (Rice 1968a; 1968b). The $J$-integral was treated as a form to express Griffith’s energy release rate, which is defined as (Shah, Swartz & Ouyang, 1995):

\[
J = \int_{\Gamma} \left[ U_d dy - T \frac{\delta U}{\delta x} \delta s \right]
\]

where $\Gamma$ is any closed contour following a counter-clockwise path surrounding the crack tip in a stressed body, $U_d$ is the elastic energy density, $T$ is the tension vector acting on the boundary, $U$ is the displacement vector and $\delta s$ is the differential of arc-length along the contour $\Gamma$. Figure 2.17 is an illustration of the $J$-integral concept.

![Figure 2.17: Illustration of the J-integral (Kumar & Barai, 2011)](image)

If the non-elastic zone reduces to a point in the interior of $\Gamma$, the crack faces are traction-free and the crack is plane and extends on its own, the $J$-integral is path-independent and equal to the energy release rate $G$ (Shah, Swartz & Ouyang, 1995). That is, the $J$-integral is path-independent when the material exhibits a linear stress–strain behaviour (Shah, Swartz & Ouyang, 1995). For quasi-brittle material such as concrete, the presence of the FPZ discounts the path independence of the $J$-integral.
Therefore, the $J$-integral can only account for the partial effect of the real energy release rate $G$.

### 2.3.3.3 Double $K$ and Double $G$ Criteria

The cracking process in concrete can be divided into three stages: crack initiation, stable crack propagation and unstable crack propagation (Xu & Reinhardt, 1999). Figure 2.18 shows a typical load $P$ and crack amount opening displacement (CMOD) curve obtained from a Mode I fracture test on a concrete specimen. From point O to point B, concrete material behaves linearly and the crack tip opening displacement (CTOD) is zero. At point B, a crack initiates when the stress intensity factor at the crack tip exceeds the initial crack toughness $K_{IC}^{ini}$ of concrete. From point B to C, the crack exhibits a stable propagation, as the crack only propagates with an increasing load. Point C is the critical point at which load $P$, crack length $a$ and CTOD become critical values and the stress intensity factor at the crack tip exceeds the initial crack toughness $K_{IC}^{ini}$ of concrete. After this point, the crack exhibits unstable propagation as the crack grows with the decreasing load. Based on this concept, the fracture criterion used to model the entire fracture process of concrete can be divided into two parts: $K_{IC}^{ini}$ and $K_{IC}^{un}$. This is called the double K fracture model (Kumar & Barai, 2011). The double K fracture parameters $K_{IC}^{ini}$ and $K_{IC}^{un}$ are size-independent (Xu & Reinhardt, 1999).

![Graphical Representation of Salient Points on P-CMOD Curve](image)

**Figure 2.18: Graphical Representation of Salient Points on P-CMOD Curve**

*(Kumar & Barai, 2011)*
Recently, a double $G$ fracture model was developed based on the concept of energy release rate. This model is similar to the double $K$ fracture model; the difference is the use of the energy release rate instead of the stress intensity factor (Xu & Zhang, 2008). Using the relationship between $G$ and $K$, the double $G$ fracture parameters can be converted to the equivalent double $K$ fracture parameters as:

$$K = \sqrt{EG}$$

(2.39)

The initiation fracture energy $G_{IC}^{ini}$ is defined as Griffith’s fracture energy when the material remains elastic. The unstable fracture energy $G_{IC}^{un}$ in this model consists of the initiation fracture energy and the cohesive breaking energy $G_{IC}^{c}$. This is expressed as:

$$G_{IC}^{un} = G_{IC}^{ini} + G_{IC}^{c}$$

(2.40)

The cohesive breaking energy is due to cohesive stress along the crack surface (i.e., FPZ after crack initiation). This requires additional energy for the crack to propagate. As such, during this period, the crack stably propagates. At the critical point, $G_{IC}^{un}$ is reached and unstable crack propagation occurs.

**2.3.3.4 Fictitious Crack Model**

The fictitious crack model—sometimes known as the cohesive crack model—by Hillerborg, Modeer & Petersson (1976) is represented by the cohesive stress-elongation relationship obtained from a uniaxial tensile test of a concrete plate (see Figure 2.19). It is interesting to note that once the tensile strength is reached, a crack initiates. After this point, the stress ahead of the crack tip reduces with increasing elongation, while unloading occurs outside the crack region. Consequently, the area under the curve is known as the specific fracture energy $G_f$ and can be obtained as:

$$G_f = \int_0^{w_c} \sigma(w)dw$$

(2.41)
where \( \sigma(w) \) is the tension softening curve. As such, there are three governing parameters in the cohesive crack model: material tensile strength, specific fracture energy and the shape of the \( \sigma(w) \) curve.

Considerable research has been conducted to determine the \( \sigma(w) \) curve, as it is an important parameter that influences the fracture energy of the material. Many different shapes of \( \sigma(w) \) curves have been proposed: linear, bilinear, trilinear, exponential and power functions (Shah, Swartz & Ouyang, 1995). Table 2.3 summarises the types of \( \sigma(w) \) curves proposed by different researchers (Du, Yon, Hawkins, Arakawa & Kobayashi, 1992; Gopalratnam & Shah, 1985; Hillerborg, Modeer & Peterson, 1976; Liaw, Jeang, Du, Hawkins & Kobayashi, 1990; Reinhardt, 1984; Roelfstra, 1986).

**Figure 2.19: Fictitious Crack Model by Hillerborg**

*Note. (a) a complete tensile stress-elongation curve (b) stress-crack width curve.*
### Table 2.3: Different Types of $\sigma(w)$ Curves

<table>
<thead>
<tr>
<th>Type</th>
<th>Expression</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear curve</td>
<td>$\sigma = f_t \left(1 - \frac{w}{w_c}\right)$</td>
<td><img src="image" alt="Linear Curve" /></td>
</tr>
</tbody>
</table>
| Bilinear curve | $\sigma = f_t - \left( f_t - \sigma_1 \right) \frac{w}{w_1}$ for $w \leq w_1$  
$\sigma = \sigma_1 - \frac{\sigma_1 (w - w_1)}{(w_c - w_1)}$ for $w_1 > w$ | ![Bilinear Curve](image) |
| Trilinear curve | $\sigma = \begin{cases} 
                        f_t & \text{for } w \leq w_1 \\
                        f_t - 0.7f_t(w - w_1)(w_2 - w_1) & \text{for } w < w_1 \leq w_2 \\
                        0.3f_t(w_c - w)/(w_c - w_2) & \text{for } w_2 < w \leq w_c 
                      \end{cases}$ | ![Trilinear Curve](image) |
| Exponential curve | $\sigma = f_t \exp(k \omega^\lambda)$  
where $n$ is a fitting parameter | ![Exponential Curve](image) |
| Power curve | $\sigma = f_t \left(1 - \frac{w}{w_c}\right)^n$  
where $n$ is a fitting parameter | ![Power Curve](image) |
| Power curve | $\sigma = 0.4f_t \left(1 - \frac{w}{w_c}\right)^{1.5}$ | ![Power Curve](image) |
The linear softening curve proposed by Hillerborg, Modeer & Petersson (1976) depends on concrete tensile strength and crack width. The disadvantage of this curve is that it may overestimate the energy release rate. The bilinear model is arguably the most widely accepted model, as it can reasonably approximate the fracture energy. However, there have been many debates on the location of the break point. The trilinear curve is similar to the bilinear curve, but remains at the tensile strength for some time before entering the bilinear phase. The exponential and power curves are straightforward; the values of the coefficients are easily determined from tests.

2.3.3.5 Crack Band Model

The crack band model developed by Bažant and Oh (1983) modelled the FPZ as a band of uniformly and continuously distributed microcracks $h_c$ (see Figure 2.20). The stable crack propagation is simulated by the progression of microcracks within this band and can be represented by a simple stress–strain curve relationship (Shah, Swartz & Ouyang, 1995). It is assumed that the stress–strain relationship is linear prior to peak load and after peak load; the strain softening is nonlinear. Therefore, for the crack band model, the main parameters are tensile strength, initial modulus of elasticity and strain-softening modulus. Under the same circumstances, the crack band model can adequately represent cracks compared to the cohesive crack model.

The crack band model was the main influence on the smeared crack approach, which is an effective numerical method representing cracking by changing the constitutive properties of the material (Rashid, 1968). This approach represents a crack by distributing small parallel cracks over a finite element, which reduces the material properties in the direction normal to the crack.
Figure 2.20. Illustration of the Stress–strain Relationship for the Crack Band Model

Similar to the cohesive crack model, the specific fracture energy is the energy under the curve. Assuming a linear softening relationship, the specific fracture energy is (Shah, Swartz & Ouyang, 1995):

\[
G_f = h_c(1 + \frac{E}{E_t})\frac{f_t^2}{2E}
\]  

(2.42)

where \(E_t\) is the strain-softening modulus of elasticity, \(E\) is the modulus of elasticity, \(f_t\) is concrete tensile strength and \(h_c\) is the crack band, which is estimated to be three times the maximum aggregate size. If \(h_cE = w\) the crack band model could be identical to the cohesive crack model.

2.3.3.6 Effective Elastic Crack Approach

The effective elastic crack approach represents the FPZ by using an equivalent, traction-free elastic crack. This means that the actual crack is replaced by an equivalent fictitious crack (Kumar & Barai, 2011). Based on this approach, Jenq and Shah (1985) proposed a two-parameter fracture model (TPFM) in which a notched three-point bending specimen was loaded to maximum value and then unloaded to obtain the elastic and plastic displacements (see Figure 2.21).
The two fracture parameters involved are the critical stress intensity factor $K_{IC}^s$ at the tip of the equivalent crack length at peak load and the corresponding critical crack-tip opening displacement ($CTOD_c$). A brief procedure to calculate the $K_{IC}^s$ and $CTOD_c$ can be found in the RILEM formula (RILEM, 1990). $K_{IC}^s$ and $CTOD_c$ are almost constant and size-independent and can be expressed as (Jenq & Shah, 1985):

$$K_I = K_{IC}^s$$

$$CTOD = CTOD_c$$

It should be noted that $K_I$ and $CTOD$ are functions of applied loads, structural geometry and crack length from the theory of LEFM. $K_{IC}^s$ and $CTOD_c$ are material constants.

### 2.3.3.7 Fracture Resistance Curve

The fracture resistance curve ($R$-curve) represents the complete crack growth process (see Figure 2.22). It was developed because the fracture models previously discussed cannot fully represent the entire crack growth process and are only applicable to a critical situation. In the concrete fracture process, the fracture criteria $G_q = R$ is used when $G_q$ is determined based on the applied load and crack length. $R$ is determined based on crack propagation.
For linear elastic material, fracture resistance is a material constant and independent of the crack extension. Hence, the $R$-curve is a straight line (see Figure 2.22). However, due to the FPZ, concrete experiences some form of inelasticity at the crack tip. Thus, the $R$-curve for inelastic material rises monotonically (Shah, Swartz & Ouyang, 1995). From Figure 2.22, it can be observed that when the applied load increases from $P_1$ to $P_2$, the $G_{q1}$ curve becomes the $G_{q2}$ curve and the crack extension $\Delta a_1$ occurs at point 2. As the $G_q$ curve changes from $G_{q1}$ to $G_{q2}$, the crack propagation is considered stable, as a crack only propagates when the applied load increases. This means that at point 0 and 2, the condition is that $G_q = R$ and $\frac{\partial G_q}{\partial a} < \frac{\partial R}{\partial a}$. Therefore, if the applied load does not increase, the crack will not propagate. However, at point 3, the condition becomes $G_q = R$ and $\frac{\partial G_q}{\partial a} = \frac{\partial R}{\partial a}$. This is known as the critical point, after which $\frac{\partial G_q}{\partial a} > \frac{\partial R}{\partial a}$ indicates an unstable crack propagation (i.e., the crack continues to propagate even with a decreasing load). However, with strain softening, the applied load can decrease, which leads to $G_q = R$ again.

### 2.3.4 Numerical Modelling of Concrete Cracking

Since the late 1960s, the FEM has been applied to predict the cracking of quasi-brittle materials (i.e., concrete). The two dominant methods implemented in the finite
element modelling of cracks are the smeared crack approach and the discrete crack approach. The discrete crack approach models the crack by introducing discontinuity in the geometry of the body. The crack path is usually assumed a priori and a mesh is arranged so that the crack path coincides with the element boundaries (Shah, Swartz & Ouyang, 1995). If the crack path is unknown, remeshing of elements is required, unless the XFEM is used. In that case, the crack path does not need to be clearly defined and remeshing is unnecessary. In the smeared crack approach, the crack is treated by considering the deterioration process through a constitutive relationship, thereby smearing the crack over a portion of the continuum (Borst, Remmers, Needleman & Abellan, 2004). There has been considerable discussion about the advantages and disadvantages of the models. In cases in which there is distributed damage or densely distributed parallel cracks, the smeared crack model can effectively represent the cracking phenomenon (Bažant & Planas, 1997; Shah, Swartz & Ouyang, 1995).

2.3.4.1 Discrete Crack Approach

The discrete crack approach was first introduced by Ngo and Scordelis (1967) and treats cracks as a geometric entity. In their study, cracks were introduced into a finite element mesh by separating elements along the crack path (Rots & Blaauwendraad, 1989). In the early version of this approach, the cracks were restricted to propagate along the boundary of elements. As such, this approach is strongly dependent on mesh type and size. In the 1980s, a new technique was developed that used automatic remeshing of elements when the crack extended by a small increment when the crack propagation criteria were breached (Ingraffea & Manu, 1980, Xu & Waas, 2016). Despite the new technique helping to reduce the mesh dependency of crack propagation, it requires complex code and cumbersome computation. The complexity lies in the continuous change of element topography as the crack grows. This has motivated the development of alternative approaches, such as the XFEM, which models crack growth without remeshing (Belytschko & Black, 1999; Moes, Dolbow & Belytschko, 1999). However, computational demand and complications that arise when describing phenomena such as crack branching and crack shielding limit the use of the discrete crack approach and favour the use of the smeared crack approach.
2.3.4.2 Smeared Crack Approach

The smeared crack approach was first introduced by Rashid in 1968 to model cracks in finite element analysis (Rashid, 1968). The smeared crack approach assumes that, within a finite element, the continuous distribution of small parallel cracks reduces the material properties in the direction normal to the cracks. This approach is advantageous, as it is more convenient to model cracking by changing the finite element properties than to change the topography of the finite element mesh. However, the disadvantages of this method are the instabilities during strain localisation and the sensitivity of mesh of finite element calculations (Bažant & Planas, 1997). This means that the approach depends on the fineness of the mesh and the orientation of the elements. The most widely used method based on this approach is the crack band model proposed by Bažant and Oh (1983). The main characteristic of this model is that the constitutive relationship with strain softening must be associated with a certain characteristic width of the crack band, as discussed section 2.3.3.5.

2.3.5 Application of Fracture Mechanics Theory

When a crack occurs in concrete material, the stress distribution changes and results in the stress at the crack tip to approach infinity. In this situation, strength-based elastic mechanics may not be appropriate. The Griffith’s energy approach, or the stress intensity factor approach, can be adopted. With the development of the $J$-integral, the energy release rate at the crack tip can be easily determined by integrating any closed contour anticlockwise from the crack tip in the stressed body. In contrast, the stress intensity factor approach considers the stress singularity at the crack tip and can be used as a fracture criterion to model crack propagation in corrosion-induced cracking. The possibility to derive analytical solutions for the stress intensity factor, using methods such as the weight function method, makes it convenient to analytically model crack propagation. The stress intensity factor is also additive. Therefore, the nonlinearity of concrete cracking due to the presence of the FPZ can be overcome using the stress intensity factor approach through the concept of superposition.
2.4 Structural Reliability Analysis

Corrosion-induced deterioration of RC is arguably the most dominant cause of premature failure. Corrosion-induced failure does not necessarily imply structural collapse, but the loss of structural serviceability characterised by concrete cracking. Corrosion-induced cracking in RC is a gradual process consisting of several phases such as corrosion initiation, corrosion propagation, crack initiation and crack propagation during the service life of the structure. The reliability of corrosion-affected RC structures can be defined as the probability that the structure will perform adequately during its service life. As discussed, damages caused by corrosion on RC structures is costly. Therefore, reliability analysis can be a useful tool to help structural engineers and asset managers make decisions regarding the serviceability condition and maintenance strategies of corrosion-affected RC structures. To evaluate the efficiency of repair and replacement of corrosion-affected RC structures, different reliability methods can be used. This section will discuss the basics of structural reliability, followed by the reliability problem, methods for reliability assessment and the application of reliability assessment methods.

2.4.1 Basics of Structural Reliability

A probabilistic approach is necessary for service life assessment, due to the uncertainty of loading and performance aspects of a structure. In a probabilistic approach, the safety and service or performance requirements are measured according to their reliabilities. As such, the reliability of a structure or component is defined as its probability of survival (Melcher, 1999):

\[ P_s = 1 - p_f \]  \hspace{1cm} (2.45)

where \( P_s \) is the probability of survival and \( p_f \) is the probability of failure.

In this case, failure is expressed as the limit state functions and reliability is expressed as the probability that the limit state functions do not exceed the probability of survival (i.e., \( 1 - p_f \geq P_s \)). The limit state function may consist of several variables, each corresponding to a probability density function with unique statistical properties.
Structural reliability analysis can be a useful tool for engineers and asset managers, as it can (Melchers, 1999):

1) predict the service life of existing structures for funding allocation to the most critical parts of the structure or infrastructure
2) evaluate the effect of repair, maintenance and rehabilitation actions on the service life of the structure (the ability to examine the consequences of potential action or inaction is relative to operational and maintenance procedures)
3) evaluate design choices and determine the effect of their implementation on service lives at the stage of conceptual design.

To accurately predict the service life of a structure, information, such as the condition of the structure, rates of degradation and past and future loading, is useful. With an accurate prediction of the remaining service life, a cost-benefit analysis can be made so that a more informed decision on the required action can be determined.

Traditionally, structural safety was defined through a ‘safety factor’ that is the ratio between strength and load. This factor is considered a deterministic approach and can be used to measure the reliability of a structure. Due to uncertainties, such as loading condition, strength and modelling systems, a probabilistic approach was deemed more appropriate.

Generally, the main steps in a reliability analysis for service life predictions of corrosion-affected RC structures are to:

1) identify the failure modes of the structure—in this case they are corrosion-induced cracking
2) decompose the failure modes in a series system or parallel systems of single components
3) formulate failure functions (i.e., limit state functions)
4) identify the stochastic variables and the deterministic parameters in the failure functions followed by the statistical parameters, distribution types and dependencies between them
5) estimate the reliability of each failure mode
6) evaluate the reliability result by performing a sensitivity analysis to identify major influencing variables.

The typical failure modes for corrosion-affected RC structures can be divided into three categories:

1) Ultimate limit states. This corresponds to the maximum load-carrying capacity, which can be related to excessive cracking due to corrosion, collapse due to corrosion or instability.
2) Conditional limit states. This correspond to the load-carrying capacity of a part of the structure that has failed (e.g., spalling of concrete caused by corrosion).
3) Serviceability limit states. This corresponds to the normal use of the structure (e.g., corrosion initiation and propagation, corrosion-induced cracking and corrosion-induced crack width).

The fundamental quantities that characterise the behaviour of a structure are called the basic variables and can be denoted as \( X_n \), where \( n \) represents the number of basic stochastic variables. Some examples of this are the corrosion rate, structural geometry size and material properties, which can be either dependent or independent. A stochastic process is defined as a random function of time in which, for any given point in time, the value of the stochastic process is a random variable. The uncertainty associated with a stochastic variable can be divided into physical uncertainty, measurement uncertainty, statistical uncertainty and model uncertainty. The definition of each can be found in Melchers (1999). These uncertainties can be treated by the reliability methods, which can be divided into four groups:

- Level I methods. The uncertain parameters are modelled by one characteristic value; for example, in codes of practice based on the partial safety factor concept.
- Level II methods. The uncertain parameters are modelled by the mean and standard deviation values and the correlation coefficients between the stochastic variables. The stochastic variables are implicitly assumed to be normally distributed. The reliability index method is an example of a level II method.
• Level III methods. The uncertain quantities are modelled by their joint
distribution functions. The probability of failure is estimated as a measure of their
reliability.
• Level IV methods. The consequences of failure are considered and the risk is
used as a measure of reliability. Different designs can be compared on an
economic basis considering uncertainty, costs and benefits (Madsen, Krenk &
Lind, 2006).

Level I methods can be calibrated using level II methods, level II methods can be
calibrated using level III methods and so on. Several techniques can be used to
estimate the reliability of level II and III methods. These include:

• Simulation techniques. For example, the MCS, in which samples of the
stochastic variables are generated and the relative number of samples
corresponding to failure are used to estimate the probability of failure.
• First order reliability methods (FORM). The limit state function is linearised
and the reliability is estimated using level II or III methods.
• Second order reliability methods. A quadratic approximation to the failure
function is determined and the probability of failure for the quadratic failure
surface is estimated.
• Time-dependent reliability techniques. When the limit state function is time-
dependent (i.e., one or more variables are a function of time).

Level IV methods are beyond the scope of this thesis and have not been included in
this chapter.
2.4.1.1 The Reliability Problem

The basic reliability problem considers only one load effect $S$, resisted by one resistance $R$. The load and resistance are expressed by a known probability density function $f_S$ and $f_R$, respectively. A structure will be deemed failed if the resistance $R$ is less than the load effect $S$ on it. Therefore, the probability of failure can be expressed as:

$$p_f = P[R - S \leq 0] = P[G(R, S) \leq 0]$$  \hspace{1cm} (2.46)

where $G(R, S)$ is termed the limit state function and the probability of failure is identical to the probability of limit state violation. Figure 2.23 illustrates Eq. (2.46) by the hatched failure domain $D$, so that the failure probability becomes:

$$p_f = P[R - S \leq 0] = \int \int f_{RS}(r, s)drds$$  \hspace{1cm} (2.47)

where $f_{RS}(r, s)$ is the joint density function.

Figure 2.23: Two Random Variable Joint Density Function $f_{RS}(r, s)$, Marginal Density Function $f_S$ and $f_R$ and Failure Domain $D$ (Melchers, 1999)
With the limit state function expressed as $G(X)$, the generalisation of Eq. (2.47) becomes:

$$p_f = P[G(X) \leq 0] = \int \ldots \int_{G(X) \leq 0} f_x(X) \, dx \tag{2.48}$$

Where $f_x(X)$ is the joint probability density function for n-dimensional vector $X$ of the basic variable. The generalisation of the reliability problem can be observed in Figure 2.24.

![Figure 2.24: Limit State Surface $G(X) = 0$ and its Linearised Version $G_L(X) = 0$ in the Space of the Basic Variables (Melchers, 1999)](image)

When both the load effect $S$ and resistance $R$ are independent and of normal distribution, the integral in Eq. (2.48) can be determined from (Melchers, 1999):

$$p_f(t) = \Phi \left[ -\frac{(\mu_R - \mu_S)}{(\sigma_S^2 + \sigma_R^2)^{1/2}} \right] = \Phi[-\beta] \tag{2.49}$$

where $\Phi$ is the standard normal distribution function, $\mu$ is the mean and $\sigma$ is the standard deviation of random variables. $\beta$ is known as safety index or reliability index.
2.4.2 Methods of Reliability Assessment

2.4.2.1 Reliability Index Method

The reliability index or safety index $\beta$ was initially proposed by Hasofer and Lind (1974) and was developed based on the FORM. For a linear limit state function, the solution to the reliability index was described as (Melchers, 1999):

$$\beta = \min \left( \sum_{i=1}^{n} x_i^2 \right)^{\frac{1}{2}} = \min(x^T . x)^{\frac{1}{2}}$$

(2.50)

where $x_i$ represents the coordinate of any point along the limit state surface. With the condition that the limit state function $g(x) = 0$ at design point $x^*$. The function can be expressed as:

$$g(x) = \beta + \sum_{i=1}^{n} \alpha_i x_i = 0$$

(2.51)

The design point is the point on the limit state function $g(x)$ closest to the origin of the standard normal coordinate system (i.e., the mean and standard deviation of all variables are zero and one). As such, the reliability index $\beta$ is the distance between the design point to the origin and can be determined as:

$$\beta = - \sum_{i=1}^{n} \alpha_i x_i = -x^* T \alpha$$

(2.52)

In cases in which the limit state function is nonlinear, the reliability index can be determined by linearising the limit state function using Taylor’s series approximation at the design point as (Hasofer & Lind, 1974):

$$G_L(x) \approx g(x^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} (x_i - x_i^*) = 0$$

(2.53)
Since \( x^* \) is on the limit state, the term \( g(x^*) = 0 \). Because \( \mu_{x_i} = 0 \) and \( \sigma_{x_i} = 1 \), the mean and standard deviation for the linearised limit state function is:

\[
\mu_{g_L}(x) = - \sum_{i=1}^{n} x_i^* \frac{\partial g}{\partial x_i} = - x^{*T} g(x)
\]

and

\[
\sigma_{g_L}^2(x) = \sum_{i=1}^{n} \left( \frac{\partial g}{\partial x_i} \right)^2 = g(x)^T g(x)
\]

where \( g(x) = \left( \frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \ldots \right) \). Because the reliability index \( \beta = \frac{\mu_{g_L}}{\sigma_{g_L}} \), it follows that:

\[
\beta = \frac{\mu_{g_L}}{\sigma_{g_L}} = \frac{- \sum_{i=1}^{n} x_i^* \frac{\partial g}{\partial x_i}}{\sqrt{\sum_{i=1}^{n} \left( \frac{\partial g}{\partial x_i} \right)^2}} = - x^{*T} g(x) = - x^{*T} \alpha
\]

### 2.4.2.2 Monte-Carlo Simulation

The MCS technique involves random sampling to artificially simulate a larger number of experiments and their results. For a structural reliability analysis, a sample value \( \hat{x}_i \) is generated based on the sampling of each random variable \( X_i \). The sample value is then applied to the limit state function \( G(\hat{x}_i) \) and \( G(\hat{x}_i) = 0 \) is achieved. If the limit state function is negative (i.e., \( G(\hat{x}_i) \leq 0 \)), the structural element or system fails. The experiment is repeated many times; each time there is a randomly selected vector \( \hat{X} \) of \( \hat{x} \) values. If \( N \) trails are implemented, the probability of failure of the structure is approximately given by:

\[
p_f(t) \approx \frac{n(G(\hat{x}_i) \leq 0)}{N}
\]
where \( n(G(\hat{x}_i) \leq 0) \) denotes the number of trails \( n \) for which \( G(\hat{x}_i) \leq 0 \). The number \( N \) trail required is linked to the ideal accuracy for \( p_f \). The smaller the expected probability of failure, the larger sample size is required to ensure the precision of the simulation. The accuracy of the estimation of the probability of failure using this method can be examined by their coefficient of variation (COV) (Melchers, 1999).

To improve the accuracy of approximating the probability of failure, importance sampling is a versatile tool that can be used. MCS requires a large amount of calculation or sample size to obtain accurate results. Importance sampling is a variance reduction technique that can effectively output results. Plain MCS generates random numbers that are close to the mean value of the distribution; however, the simulation cannot be achieved if the failure sets have a small volume or are in the tail of their distribution. With the use of important sampling, the interaction (or important region) of the failure sets can be highly utilised for analyses. Hence, the basic methodology in importance sampling is to choose a distribution that ‘encourages’ the important values. The use of ‘biased’ distributions will result in a biased estimator if it is applied directly to the simulation. However, the simulation outputs are weighted to the correct use of the biased distribution. This ensures that the new importance sampling estimator is unbiased (Mahmoodian & Alani, 2015; Melchers, 1999). As such, the fundamental issue in implementing importance sampling simulation is choosing a biased distribution, which can significantly reduce MCS run time (Kroese & Rubenstein, 2012).

2.4.2.3 Upcrossing Method

When one or more basic random variables are a function of time, the probability of failure should be calculated as time-dependent. Basic random variables can be a part of the resistance of the structure, which makes the resistance time-dependent; that is, \( R(t) \) or the applied load \( S(t) \). At any time \( t \), the limit state function \( G(R, S, t) \) is (Melchers, 1999):

\[
G(R, S, t) = R(t) - S(t)
\]  

(2.58)
Assuming $R(t)$ and $S(t)$ are statistically independent random variables, the probability of failure $p_f(t)$ is expressed as:

$$p_f(t) = P[G \leq 0] = \int_0^\infty f_R(x) f_S(x) dx$$

(2.59)

where $f_R(x)$ and $f_S(x)$ are the probability distribution functions of $R$ and $S$, respectively. Eq (2.59) is a quantitative measure of structural reliability and performance. The probability that failure occurs for any one load application is the probability of limit state violation. It may be estimated by the amount of overlap of the probability density function $f_R(x)$ and $f_S(x)$ (see Figure 2.25). Because this overlap may vary with time, $p_f$ is also a function of time.

![Figure 2.25: Schematic Time-dependent Reliability Problem](image)

Eq. (2.59) represents a typical upcrossing problem in which the probability of failure is determined as the time that is expected to elapse before the first occurrence of an excursion of the random vector $X(t)$ out of the threshold $R(t)$, which is defined as $G(X) > 0$. This is also known as ‘first passage probability’ and can be determined from Eq. (2.60):

$$p_f(t) = 1 - \left[1 - p_f(0)\right]e^{-f_0^t v dt}$$

(2.60)
where \( p_f(0) \) is the probability of concrete cracking at time \( t = 0 \) and \( v \) is the mean rate for \( S(t) \geq R(t) \). In many practical problems, the mean upcrossing rate \( v \) is small so that Eq. (2.60) can be approximated as:

\[
p_f(t) = p_f(0) + \int_0^t v d\tau \tag{2.61}
\]

The upcrossing rate in Eq. (2.61) can be determined from the Rice formula:

\[
v = v_R^+ = \int_R^\infty (S - R)f_{SS}(R, S)dS \tag{2.62}
\]

where \( v_R^+ \) is the mean upcrossing rate of the load effect \( S(t) \) relative to the resistance \( R(t) \), \( \dot{R} \) is the slope of \( R \) with respect to time, \( \dot{S} \) is the time-derivative process of \( S(t) \) and \( f_{SS}() \) is the joint probability density function of \( S(\quad) \) and \( \dot{R} \).

For a stationary normal process, \( f_{SS} \) can be determined as:

\[
f_{SS} = \frac{1}{2\pi \sigma_S \sigma_{\dot{S}}} \exp \left\{ -\frac{1}{2} \left[ \frac{(R - \mu_s)^2}{\sigma_s^2} + \frac{\dot{S}^2}{\sigma_{\dot{S}}^2} \right] \right\} \tag{2.63}
\]

where the variables of \( S(t) \) are normally distributed; that is, \( N(\mu_s, \sigma_s^2) \). However, the mean of \( \dot{S}(t) \) is zero; that is, \( N(0, \sigma_{\dot{S}}^2) \). The variance of \( \dot{S}(t) \) can be determined as:

\[
\sigma_{\dot{S}}^2 = \int_0^\infty \dot{S} \exp \left( -\frac{\dot{S}^2}{2\sigma_{\dot{S}}^2} \right) d\dot{S} \tag{2.64}
\]

Therefore, with Eq. (2.63) and Eq. (2.64), Eq. (2.62) is now expressed as:

\[
v_R^+ = \frac{\sigma_S}{2\pi \sigma_s \sigma_{\dot{S}}} \exp \left[ -\frac{(R - \mu_s)^2}{2\sigma_s^2} \right] = \frac{\sigma_S}{\sqrt{2\pi}} f_s() \tag{2.65}
\]

where \( f_s(\quad) = \left( \frac{1}{\sigma_s} \right) \phi \left[ \frac{(R - \mu_s)}{\sigma_s} \right] \), where \( \phi() \) is the standard normal density function.
For non-normal processes, the joint probability density function $f_{SS}(\cdot)$ will usually be significantly less amenable to definition and integration. For example, it is sometimes suggested that, for normal processes that are transformed nonlinearly, the upcrossing rate may be approximated by Eq. (2.65). However, this approximation can be seriously in error (Melchers, 1999). Further, all the above results—especially Eq. (2.62)—may be extended to smooth non-stationary processes by interpreting $v_R^+$ and $f_{SS}(\cdot)$ as time-dependent (Melchers, 1999). For the standardised Gaussian process, and assuming that $R$ is deterministic, the mean upcrossing rate can be calculated as:

$$v = v_R^+ = \frac{\sigma_{\dot{S}|S}}{\sigma_S} \varphi \left( \frac{R - \mu_S}{\sigma_S} \right) \left\{ \varphi \left( -\frac{\dot{R} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \right) - \frac{\dot{R} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \Phi \left( -\frac{\dot{R} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \right) \right\}$$

(2.66)

where $v_R^+$ is the mean upcrossing rate of the load effect $S(t)$ relative to the resistance $R$, $\dot{R}$ is the slope of $R$ with respect to time, $\dot{S}$ is the time-derivative process of $S(t)$. $\varphi(\cdot)$ and $\Phi(\cdot)$ are standard normal density and distribution functions, $\mu$ and $\sigma$ are the mean and standard deviation of random variables of $S$ and $\dot{S}$ and ‘|’ is the condition.

2.4.3 Sensitivity Analysis

In corrosion-induced cracking of RC, there are many variables that affect the cracking process and the subsequent limit state function. It is important to identify the most influential variables that will affect the probability of failure so that more research can focus on those variables. This can be achieved by conducting a sensitivity analysis. By performing a sensitivity analysis, quantitative information can be obtained and used to classify the random variables according to their influence on the model. These measures are essential for reliability-based service life prediction of deteriorating materials and structures. Various methods can be used for sensitivity analysis, including nominal range sensitivity analysis, differential analysis and reliability-based sensitivity index (SI).

The nominal range sensitivity analysis (also known as local sensitivity) evaluates the effects of each input variable $(X_1, X_2, \ldots, X_n)$ on the model results $Y$ by varying the parameters individually within a reasonable range while the other variables
remain their base value. The sensitivity is represented as a positive or negative change rate compared to their base value. This local sensitivity is expressed as:

\[ X_n = \frac{\partial Y}{\partial X_n} \bigg|_{X_n=x_n} \]  

(2.67)

Differential analysis (also known as the direct method) involves analysing the partial derivative of each input variable with respect to output. The partial derivative function is obtained from the best-fitting regression curve. By varying one input parameter \( X_n \), a new regression line \( Y \) is obtained. The average absolute value of the gradient of the partial derivative function can be calculated as:

\[ DA_{n,j} = \frac{1}{j} \sum_{1}^{j} \left| \frac{\partial Y}{\partial X_n} \right| = (X_{n,1}, X_{n,2}, X_{n,3}, \ldots, X_{n,j}) \]  

(2.68)

The reliability-based sensitivity analysis can provide a degree of variation of limit state functions at a specific point characterised by the realisation of all random variables. The SI \( S_{G(x)}(X_i) \) can be defined as (Kong & Frangopol, 2005):

\[ S_{G(x)}(X_i) = \frac{\partial G(X)}{\partial G(X_i)} = \lim_{\varepsilon \to 0} \frac{G(X + \varepsilon) - G(X)}{\varepsilon} \]  

(2.69)

where \( G \) is a performance function of \( X \), \( X \) and \( \varepsilon \) are vectors and \( \varepsilon \) is a small perturbation. An element of \( X_i \) of \( X \) can be any type of variable or parameter. For a complex system, the sensitivity measure can be computed by using the numerical differentiation method rather than an analytical approach (Kong & Frangopol, 2005).

**2.4.4 Application of Reliability Assessment Methods**

To have an optimum strategy for maintenance and rehabilitation plans in the management of corrosion-affected RC structures, accurate prediction of service life based on corrosion-induced deterioration is essential. The service life prediction is the time at which the structure becomes unserviceable due to corrosion. There are two classes of methods for service life prediction: deterministic and probabilistic.
Deterministic methods do not consider the effect of the variation of any variables. In contrast, probabilistic methods do consider the effect of variation of any variables. As discussed, the probabilistic approach accounts for the uncertainties in the parameters responsible for the deterioration of corrosion-affected RC structures. Hence, most researchers developed their performance models using this approach.

The MCS method is arguably the most popular, due to the ease of implementation. Software such as the Feasible Reliability Engineering Tool utilises this method to determine the service life assessment using both corrosion initiation and propagation period limit states (Vorechovska, Teply & Chroma, 2010). Similarly, the computer program Reliability of System Network utilises the MCS method to develop a probabilistic framework for forecasting the lifetime performance of RC slabs and girders of bridges that have been subjected to corrosion (Akgül & Frangopol, 2005). The MCS method can also be applied to nonlinear finite element analysis for reliability assessment when analytical deterioration models are combined with in situ monitoring to model degradation (Strauss, Bergmeister, Hoffman, Pukl & Novak, 2008). Melchers, Li and Lawanwisut (2008) used the MCS to model the nonlinear behaviour of corrosion variables and proposed a probabilistic approach for strength and performance deterioration modelling of RC structures. Val (2005) proposed a reliability-based method using MCS to evaluate the expected cost of failure of RC structures that were subjected to corrosion. Ying and Vrouwenvelder (2007) presented the service life prediction of RC structures subjected to corrosion by implementing a random spatial variation of property differences across the structures. This allowed the failure probabilities to be determined non-uniformly across the structure. Enright and Frangopol (2000) investigated corrosion-initiation time for steel reinforcement in bridge girders using the MCS method.

Because corrosion is time-dependent, corrosion-induced deterioration should be modelled as a time-varying function. The reliability analysis based on this scenario can be considered based on three aspects:

1) acquisition of statistical characteristics of input random variables from imperfect samples
2) model-based simulation
3) evaluation of variability and sensitivity measures of the non-deterministic performance model (Kong & Frangopol, 2005).

Mori and Ellingwood (1993) presented a probability-based method to evaluate time-dependent reliability of components and systems of reinforced or prestressed concrete structures. Stewart and Val (2003) investigated the structural reliability of a typical corrosion-affected RC slab bridge. In the analyses, it was found that serviceability failures, such as cracking and loss of reinforcement cross-sections, were significantly higher than ultimate strength limit states. Li, Lawanwisut and Zheng (2005) proposed a serviceability assessment methodology that directly relates to the structural response to design criteria of corrosion-affected RC structures. Under the same service conditions, corrosion-induced cracking was a more critical serviceability limit state than deflection. Vu and Stewart (2005) developed a two-dimensional spatial time-dependent reliability model to predict the likelihood and extent of corrosion-induced cracking and spalling, considering variables such as concrete cover, compressive strength and surface chloride concentration. Shao and Li (2007) proposed an asset management strategy by using a time-dependent reliability method to determine the probability of service life for corrosion-induced cracking and structural rupture assuming a Gaussian process.

The purpose of conducting a reliability assessment is to provide more information regarding maintenance strategies for deteriorating structures. Most of the reviewed probability assessments assume that statistical information is normally distributed. Sometimes, assuming a normal distribution may result in unrealistic negative values of variables. Therefore, a lognormal distribution may be more appropriate. Further, the corrosion process is time-dependent. Hence, the limit state function and probability of failure assessment should be determined using time-dependent reliability methods.

2.5 Summary

In this chapter, the literature concerning the corrosion of reinforcing steel, the effect of corrosion on RC, the properties of concrete, the fracture mechanics and reliability assessment methods were critically reviewed. It was found that, even with extensive
experimental investigations into the effects of corrosion on RC, there has been considerable debate over the accuracy and robustness of the proposed models. This is primarily due to the uncertainties of corrosion. For example, the corrosion rate is significantly affected by many factors, most notably humidity, temperature, concrete quality and chloride content. Additionally, the types of corrosion products form have different effects on concrete deterioration. Therefore, to make models more reliable and improve their practical significance. It is necessary to produce more experimental data so that models can become more accurate. When a crack occurs in concrete, the stress distribution in the bulk material changes. Therefore, a stress intensity factor approach is more adequate than a strength-based approach to analytically model crack propagation. The nonlinearity associated with concrete can also be considered when using this approach, as the stress intensity factor is additive through the concept of superposition. There is a lack of time-dependent serviceability assessment of corrosion-affected RC structures compared to ultimate limit states. Further, if statistical information is normally distributed, it may result in unrealistic negative values. As such, a lognormal distribution may be more appropriate. Based on this review, the limitations and gaps in the literature are:

- lack of research in analytically modelling corrosion-induced crack propagation considering the whole cracking process of concrete
- lack of knowledge in modelling the critical crack depth in corrosion-induced concrete cover cracking
- lack of knowledge in the investigation of stress intensity factors for corrosion-induced concrete cover cracking
- discrepancies between predictive model results and experimental or field data
- lack of reliability assessment that uses advanced upcrossing methods to model the time-dependent behaviour of corrosion-induced cracking.

This chapter highlighted these shortcomings in the existing literature and positions this thesis to address them.
Chapter 3: Development of the Analytical Model

3.1 Introduction

One of the main causes of premature deterioration in RC structures is the corrosion of reinforcing steel. Aggressive ions diffuse through the concrete cover and, over time, the passive layer protecting the steel rebar breaks down. This is known as depassivation. This is especially serious in chloride-laden environments where RC structures are constantly exposed to aggressive agents. When steel depassivation occurs, corrosion initiates and corrosion products begin to form at the steel concrete interface. Corrosion products have a higher volume than normal steel, which results in an expansive pressure exerted on the surrounding concrete, causing the concrete to crack (Li, Melchers & Zhang, 2006). A completely cracked cover provides a path for rapid ingress of aggressive agents to the reinforcing steel, accelerating corrosion. This leads to progressive deterioration and even spalling of concrete.

From the literature review, it was discovered that considerable research has been undertaken to investigate corrosion-induced cracking and the fracture parameters of concrete. Most of the focus has been on experimental and numerical investigations, as opposed to analytical studies. In experimental investigations, the corrosion process is usually accelerated by an impressed current technique or salt spray so that cracking is achieved within a reasonable time frame (Alonso, Andrade, Rodriguez & Diez, 1998; Andrade, Alonso & Molina, 1993; Bažant, 1979; El Maaddawy & Soudki, 2003; Liu & Weyers, 1998). These studies focused on determining the relationship between time to surface cracking and crack width to the amount of corrosion products for normal concrete. Experimental investigations were also conducted to study the critical crack depth, termed by some researchers as the critical crack length on three-point bending and wedge splitting concrete specimens (Jenq & Shah, 1985; Karihaloo & Nallathambi, 1989).

In numerical investigations, the FEM based on fracture mechanics theory is commonly used to model the fracture behaviour of concrete. The fictitious crack model developed by Hillerborg, Modeer & Petersson (1976) and the crack band
model developed by Bažant and Oh (1983) provide the basis for most research to study corrosion-induced concrete cracking. The convenience of using FEMs is that corrosion-induced crack patterns and crack width growth rates can be easily modelled by changing the properties of the elements to suit (Chen & Mahadevan, 2008; Molina, Alonso & Andrade, 1993; Qiao, Nakamura, Yamamoto & Miura, 2016).

In analytical studies, corrosion-induced cracking is typically conducted with the purpose of investigating the time to concrete cover cracking. These models are developed based on average stress across the concrete cover. The time to cracking occurs when the corrosion-induced pressure reaches the tensile strength of concrete (Liu & Weyers, 1998; El Maaddawy & Soudki, 2007; Chernin, Val & Volokh, 2010; Shodja, Kiani & Hashemian, 2010; Lu, Jin & Liu, 2011). However, analytical studies on corrosion-induced propagation is lacking. It is theorised that when a crack initiates at the interface between the rebar and concrete, it will steadily propagate and reach a critical crack at which the crack becomes unstable and suddenly propagates to the concrete surface. Currently, no work has been undertaken to analytically investigate this critical corrosion-induced crack depth. Hence, the purpose of this research arises.

This chapter develops an analytical model using the fracture mechanics concept to determine the critical depth at which a crack becomes unstable and suddenly propagates to the concrete surface during chloride-induced corrosion. The weight function method is employed with FEM to derive a solution of stress intensity factor at the crack tip. To consider the softening behaviour of concrete material, the net stress intensity factor between the stress intensity factor due to the applied load and concrete resistance is used as the fracture criterion to model crack propagation and determine the critical crack depth. An open crack in concrete will accelerate corrosion-induced deterioration and cause failures in the structures. Early detection of cracks will enable engineers to take appropriate measures to prevent further deterioration of concrete structures.

The main part of the work presented in this chapter has already been published in an international journal, *ACI Structural Journal* (Lau, Fu, Li, De Silva & Guo, 2018).
3.2 Problem Formulation

3.2.1 Corrosion-induced Cracking

It is well accepted that corrosion-induced cracking can be modelled as a thick wall cylinder (Tepfers, 1979; Pantazopoulou & Papoulias, 2001). During corrosion, the corrosion products exert expansive pressure on the surrounding concrete, causing the concrete cover to crack. The corrosion-induced cracking process can be described in three stages: no cracking, partial cracking and complete cover cracking. This is illustrated in Figure 3.1. Before complete cracking of the concrete cover occurs, a corrosion-induced crack propagates to a critical crack depth before the crack becomes unstable and suddenly causes cover cracking. Determining the critical crack depth is important, as it can provide engineers with information regarding time to cover cracking and information to better design a concrete cover.

![Illustration of the Corrosion-induced Cracking Process](image)

Figure 3.1: Illustration of the Corrosion-induced Cracking Process

The schematic of corrosion of reinforcing steel in concrete is shown in Figure 3.2. The inner radius of the thick wall cylinder is denoted as \(a\), which is the radius of reinforcing steel. The concrete cover is \(C\), the outer radius is \(b = C + a\), the internal pressure induced by corrosion is \(P\), the crack depth is denoted as \(e\) and the coordinate along the crack depth is \(x\).
During the corrosion process, the ingress of chloride ions through the concrete cover follows Fick’s second law diffusion and can be expressed as (Bamforth, 1999):

\[ C(x,t) = C_s \left[ 1 - \text{erf} \left( \frac{x}{2\sqrt{D_c t}} \right) \right] \]  

(3.1)

where \( C(x,t) \) is the chloride ion content at the distance \( x \) from the surface of concrete at a time \( t \). The apparent diffusion coefficient is \( D_c \), which is a property of concrete, \( C_s \) is the chloride content on concrete surface and \( \text{erf}(x) \) is the error function. Eq. (3.1) demonstrates that the larger the cover, the longer it takes for chloride ions to diffuse through initiating corrosion. As discussed in Chapter 2, once the chloride content reaches the threshold level, corrosion will initiate and propagate.

As corrosion propagates, the corrosion-induced pressure \( P \) increases. As a result, tangential tensile stresses develop within the concrete cylinder, which eventually leads to crack initiation and propagation. The corrosion-induced crack would propagate to a critical crack depth at which the crack becomes unstable and suddenly propagates, resulting in a visible crack on the concrete surface. To predict the critical crack depth in the concrete cylinder, it is essential to analyse the stress distribution in the concrete cylinder using a fracture mechanics approach (i.e., stress intensity factor).
As discussed in Chapter 2, corrosion products occupy a larger volume than steel. Prior to inducing pressure on the surrounding concrete, the corrosion products first fill the porous zone $d_0$ around the steel–concrete interface. As corrosion propagates, the thickness of rust $d_s(t)$ increases. According to Liu and Weyers (1998), the total amount of corrosion products $W_{\text{rust}}(t)$ can be assumed to distribute annularly around the bar, which can be determined based on three parts: the band of corroded steel, the porous zone $d_0$ and the thickness of rust $d_s(t)$ as:

$$W_{\text{rust}}(t) = W_s + W_0 + W_c$$  
(3.2)

where $W_s$ is the amount of rust replacing the corroded steel, $W_0$ is the amount of rust filling the porous band $d_0$ and $W_c$ is the amount of rust in the band $d_s(t)$. $W_s$, $W_0$ and $W_c$ can be derived from Eq. (3.3) to (3.5).

$$W_s = \alpha_{\text{rust}}W_{\text{rust}}\frac{\rho_{\text{rust}}}{\rho_{\text{st}}}$$  
(3.3)

$$W_0 = \pi\rho_{\text{rust}}d_0D$$  
(3.4)

$$W_c = \pi\rho_{\text{rust}}(D + 2d_0)d_s(t)$$  
(3.5)

$\alpha_{\text{rust}}$ is the ratio of molecular weight of steel to the molecular weight of corrosion products. It varies from 0.523 to 0.622 according to the different types of corrosion products (Liu & Weyers, 1998). $\rho_{\text{rust}}$ is the density of corrosion products and $\rho_{\text{st}}$ is the density of steel.

Because the influence of $2d_0d_s(t)$ is significantly small in comparison, it can be neglected when substituting Eq. (3.3) to Eq. (3.5) into Eq. (3.2). The thickness of corrosion products can be determined as:

$$d_s(t) = \frac{W_{\text{rust}}(t)}{\pi D}\left(\frac{1}{\rho_{\text{rust}}} - \frac{\alpha_{\text{rust}}}{\rho_{\text{st}}}\right) - d_0$$  
(3.6)
\( d_s(t) \) in Eq. (3.6) is the corrosion-induced expansion to the concrete cylinder, which can be used to determine corrosion-induced pressure in the cylinder.

In Eq. (3.6), \( W_{rust}(t) \) is related to the corrosion rate of the steel rebar and can be expressed as (Liu & Weyers, 1998):

\[
W_{rust}(t) = \sqrt{2 \int_0^t 0.105 \left( \frac{1}{\alpha_{rust}} \right) \pi D i_{corr}(t) \, dt}
\tag{3.7}
\]

where \( i_{corr} \) is the corrosion current density in \( \mu A/cm^2 \), which is widely used as a measure of corrosion rate.

The units of the parameters in Eq. (3.2) to Eq. (3.7) must remain consistent. These are specified in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>mm</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>mm</td>
</tr>
<tr>
<td>( \rho_{rust}/\rho_{st} )</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( W_{rust}/W_s/W_0/W_c )</td>
<td>mg/mm</td>
</tr>
<tr>
<td>( d_s )</td>
<td>m</td>
</tr>
<tr>
<td>( i_{corr} )</td>
<td>( \mu A/cm^2 )</td>
</tr>
<tr>
<td>( t )</td>
<td>year</td>
</tr>
</tbody>
</table>

### 3.3 Solution to Stress Intensity Factor

To model the propagation of cracks in concrete, fracture mechanics theory is essential or more specific the SIF approach. In typical engineering design, the rebar diameter ranges from 12 mm to 32 mm and the concrete cover \( C \) varies from 20 to 50 mm. This leads to a concrete cylinder wall ratio \( b/a \) ranging from 2 to 10. To date, there is no available solution to determine the stress intensity factor for a single radial crack.
that covers these practical ranges of wall ratios (Murakami & Keer, 1993; Tada, Paris & Irwin, 1973). To solve the stress intensity factor as the crack propagates across the concrete cylinder, the weight function method is employed.

Bueckner (1971) developed the weight function method, which was modified by Rice (1972). The weight function method expressed the SIF as an integral of the product of applied stress and the corresponding weight function as:

\[ K_I = \int_0^e \sigma(x)m(x,e)dx \] (3.8)

where \( K_I \) is the SIF for Mode I fracture, \( \sigma(x) \) is the stress distribution on the crack surface and \( m(x,e) \) is the weight function of position \( x \) and \( e \). The unique feature of the weight function method when solving SIF is that the weight function \( m(x,e) \) only depends on geometry. This means that once the weight function is solved for a cracked body, which in this case is the thick wall cylinder, the SIF for any loading distribution applied to that body can be calculated through simple integration in an analytical manner (Glinka & Shen, 1991).

The solution to the weight function \( m(x,e) \) in Eq. (3.8) can be generalised as:

\[ m(x,e) = \frac{2}{\sqrt{2\pi(e-x)}} \left[ 1 + M_1 \left(1 - \frac{x}{e}\right)^{1/2} + M_2 \left(1 - \frac{x}{e}\right) + M_3 \left(1 - \frac{x}{e}\right)^{3/2} + \cdots + M_n \left(1 - \frac{x}{e}\right)^{n/2} \right] \] (3.9)

where \( M_n \) are the weight function coefficients and \( n \) is a positive integer, which can be taken as three for sufficient accuracy of results (Shen & Glinka, 1991). To determine the weight function coefficients, a combination of two different load distributions and one boundary condition, or three different load distributions, can be used (Shen & Glinka, 1991). In this thesis, three different load distributions are selected.

The stress intensity factor \( K_I \) in Eq. (3.8) can now be expressed as:
\[ F_n P_0 \sqrt{\pi e} = \int_0^e \sigma_n(x) m(x, e) \, dx \quad (n = 0, 1, 2) \quad (3.10) \]

\[ m(x, e) = \frac{2}{\sqrt{2\pi(e-x)}} \left[ 1 + M_1 \left( 1 - \frac{x}{e} \right)^{1/2} + M_2 \left( 1 - \frac{x}{e} \right) + M_3 \left( 1 - \frac{x}{e} \right)^{3/2} \right] \quad (3.11) \]

Based on the three different loading cases; that is, uniform \((n = 0)\), linear uniform \((n = 1)\) and quadratic uniform \((n = 2)\), the stress distribution \(\sigma_n(x)\) is expressed as:

\[ \sigma_n(x) = P_0 \left( \frac{x}{C} \right)^n \quad (n = 0, 1, 2) \quad (3.12) \]

By solving Eq. (3.11), the weight function coefficients \(M_{1,2,3}\) can be determined and is expressed as a function of \(F_n\) as:

\[ M_1 = \frac{\pi}{\sqrt{2}} \left( 12 F_0 - \frac{78 \pi F_1 C}{e} + \frac{84 F_2 C^2}{e^2} \right) - \frac{48}{5} \quad (3.13) \]

\[ M_2 = \frac{\pi}{\sqrt{2}} \left( - \frac{105 F_1 C}{2F_0} + \frac{315 F_1 C}{e} - \frac{315 F_2 C^2}{e^2} \right) + 21 \]

\[ M_3 = \frac{\pi}{\sqrt{2}} \left( 48 F_0 - \frac{264 \pi F_1 C}{e} + \frac{252 F_2 C^2}{e^2} \right) - \frac{64}{5} \]

To solve for influence coefficients \(F_n\) and weight function coefficients \(M_n\), a widely used finite element software, ABAQUS is used (ABAQUS, 2011). ABAQUS can determine the SIF at the crack tip for a single radial crack propagating outward through the thick wall cylinder by using the \(J\)-integral method. The \(J\)-integral numerically determines the SIF using the domain integral method available in ABAQUS (ABAQUS, 2011; Irwin, 1997). The SIF is expressed as:

\[ K = \sqrt{J_0} \quad (3.14) \]
where $J$ is the strain energy release rate, $\bar{E} = E$ for plane stress and $\bar{E} = \frac{E}{1-\nu^2}$ for plane strain. $E$ and $v$ are the modulus of elasticity and Poisson’s ratio, respectively.

In ABAQUS, the $J$-integral in two dimensions is expressed as:

$$ J = \lim_{\Gamma \to 0} \int_{\Gamma} \mathbf{n} \cdot \mathbf{H} \cdot \mathbf{q} d\Gamma $$

(3.15)

where $\Gamma$ is a contour starting at the bottom crack surface and ending on the top surface (see Figure 3.3), the limit $\Gamma \to 0$ denotes that $\Gamma$ shrinks onto the crack tip, $\mathbf{q}$ is a unit vector in the virtual crack extension direction and $\mathbf{n}$ is the outward normal to $\Gamma$ and $\mathbf{H} = W\mathbf{I} - \boldsymbol{\sigma} \cdot \frac{\partial \mathbf{u}}{\partial \mathbf{x}}$. For elastic material, $W$ is the elastic strain energy, $\mathbf{I}$ is the identity tensor, $\boldsymbol{\sigma}$ is the stress tensor and $\mathbf{u}$ is the vector of displacements.

**Figure 3.3: Contour for Evaluation of the $J$-integral (ABAQUS, 2011)**

Based on the $J$-integral, and due to symmetry, half of the cylinder is modelled in ABAQUS to obtain the SIF in a thick wall cylinder, as shown in Figure 3.4.
At the crack front, fine triangular mesh with a three-node linear plane stress triangle element converging at the crack tip is adopted (see Figure 3.4). Outwards from the crack front, a four-node bilinear plane stress quadrilateral element with reduced integration was employed. To ensure that an efficient mesh size is used, a mesh convergence test has been carried out. Using an example of a cylinder wall ratio of 3 and $e/C = 0.5$, it has been found that a mesh size with seed number more than 15 would result in a difference of less than 1% as shown in Table 3.2. Therefore, a seed number of 25 has been used to ensure computational efficiency and accuracy.

Table 3.2: Mesh convergence test for thick wall cylinder

<table>
<thead>
<tr>
<th>$e/C$</th>
<th>seed no.</th>
<th>$F_0$</th>
<th>Andrasic and Parker (1984)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10</td>
<td>1.206932</td>
<td>1.179</td>
<td>2.34%</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>1.188954</td>
<td>1.179</td>
<td>0.84%</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1.183749</td>
<td>1.179</td>
<td>0.40%</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>1.180093</td>
<td>1.179</td>
<td>0.09%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1.178564</td>
<td>1.179</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

In principle, the larger the number of domains is, the more accurate the $J$-integral results are. However, in this study, five domains were deemed sufficient and were selected to determine the $J$-integral. Because the first domain shrinks to the crack line...
where the stress is infinite, it is neglected when calculating the $J$-integral. Consequently, the $J$-integral that is obtained is an average of the results from the remaining four domains.

![Figure 3.5: Application of Load in ABAQUS](image)

In ABAQUS, the thick wall cylinder is subjected to a load $P$ applied along the crack surface with the different load distributions defined in a local coordinate system along the crack direction (x-direction) (see Figure 3.5). The application of load distribution applied in ABAQUS is based on the following expression:

$$P = P_0 \left( \frac{x}{C} \right)^n$$  \hspace{1cm} (3.16)

where $n$ is 0, 1, 2 or 3 representing the different types of loading distributions, $P_0$ is the magnitude of the applied load. The stress intensity factor is generally expressed as a normalised function (Raju & Newman, 1982):

$$F_n = \frac{K_n}{P_0 \sqrt{\pi e}}$$  \hspace{1cm} (3.17)

where $F_n$ refers to the influence coefficient and $K_n$ is the SIF for each given loading condition.

Before solving the weight function coefficients in Eq. (3.13), the FEM must first be verified. The results of normalised SIF were compared with results from the literature. Andrasic and Parker (1984) obtained results of normalised SIF from the use
of the weight function generated with the accurate modified mapping shown in Eq. (3.13); that is, the collocation method. The normalised SIF from a single radial crack with $b/a = 3$ for four loading conditions (i.e., uniform, linear, quadratic and cubic) was used for comparison. The results in Table 3.3 show a maximum relative difference of 0.65%, which verifies the FEM that was developed.

Table 3.3: Verification of the Influence Coefficient $F_n$ with Andrasic and Parker (1984)

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>Loading condition</th>
<th>$e/C$</th>
<th>Andrasic and Parker (1984)</th>
<th>FEM</th>
<th>Diff %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Uniform $n = 0$</td>
<td>0.5</td>
<td>1.179</td>
<td>1.185</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7</td>
<td>1.385</td>
<td>1.393</td>
<td>0.57</td>
</tr>
<tr>
<td>3</td>
<td>Linear $n = 1$</td>
<td>0.5</td>
<td>0.356</td>
<td>0.358</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7</td>
<td>0.568</td>
<td>0.570</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>Quadratic $n = 2$</td>
<td>0.5</td>
<td>0.136</td>
<td>0.136</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7</td>
<td>0.297</td>
<td>0.296</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>Cubic $n = 3$</td>
<td>0.5</td>
<td>0.0569</td>
<td>0.0565</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7</td>
<td>0.171</td>
<td>0.170</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Based on the developed FEM, $F_0, F_1, F_2$ can be determined for cylinder wall ratios $b/a = 2$ to 10 and relative crack depth $e/C = 0.1$ to 0.9 associated with uniform, linear and quadratic load distributions. Results from the FEM analyses for $b/a = 4$ and 10 are shown in Table 3.4. For other $b/a$ values, similar tables can be produced, but are not included in this thesis.
Table 3.4: Finite Element Analysis Results of $F_n$ for Different $b/a$, $e/C$ and Load Distribution

<table>
<thead>
<tr>
<th>$e/c$</th>
<th>$F_n$ for $(b/a = 4)$</th>
<th>$F_n$ for $(b/a = 10)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uniform ($F_0$)</td>
<td>Linear ($F_1$)</td>
</tr>
<tr>
<td>0.1</td>
<td>1.01002</td>
<td>0.06393</td>
</tr>
<tr>
<td>0.2</td>
<td>0.99204</td>
<td>0.12664</td>
</tr>
<tr>
<td>0.3</td>
<td>1.00122</td>
<td>0.19126</td>
</tr>
<tr>
<td>0.4</td>
<td>1.03155</td>
<td>0.26066</td>
</tr>
<tr>
<td>0.5</td>
<td>1.08330</td>
<td>0.33854</td>
</tr>
<tr>
<td>0.6</td>
<td>1.15505</td>
<td>0.42679</td>
</tr>
<tr>
<td>0.7</td>
<td>1.26639</td>
<td>0.53744</td>
</tr>
<tr>
<td>0.8</td>
<td>1.46595</td>
<td>0.69924</td>
</tr>
<tr>
<td>0.9</td>
<td>1.91595</td>
<td>1.00970</td>
</tr>
</tbody>
</table>

From the FEM analysis results, an analytical solution for SIF for two dimensionless parameters $b/a$ and $e/C$ can be derived using the weight function method. By plotting the two dimensionless variables, $b/a$ and $e/C$, the solution to $F_n$ as a function of $e/C$ and $b/a$ can be obtained through mathematical regression using MATLAB (MathWorks Inc., 2013).

Thus, the solution to $F_n$ is:

$$F_0 = A_1 s^4 + A_2 s^3 + A_3 s^2 + A_4 s + A_5$$  \hspace{1cm} (3.18)

where

$$A_1 = 0.00877y^4 - 0.2363y^3 + 2.301y^2 - 9.854y + 26$$

$$A_2 = -0.02015y^4 + 0.5327y^3 - 5.046y^2 + 20.68y - 47.84$$

$$A_3 = 0.01454y^4 - 0.3771y^3 + 3.473y^2 - 13.64y + 29.71$$

$$A_4 = -0.002274y^4 + 0.05395y^3 - 0.4316y^2 + 1.353y - 3.794$$
\[ A_5 = 0.0001557y^4 - 0.003851y^3 + 0.03437y^2 - 0.1539y + 1.442 \]

\[ F_1 = B_1s^4 + B_2s^3 + B_3s^2 + B_4s + B_5 \quad (3.19) \]

where

\[ B_1 = 0.0008028y^4 - 0.02834y^3 + 0.3624y^2 - 2.081y + 9.874 \]
\[ B_2 = -0.002427y^4 + 0.07614y^3 - 0.8776y^2 + 4.541y - 17.12 \]
\[ B_3 = 0.002488y^4 - 0.07245y^3 + 0.7736y^2 - 3.675y + 11.12 \]
\[ B_4 = -0.0005379y^4 + 0.01517y^3 - 0.1556y^2 + 0.7011y - 1.535 \]
\[ B_5 = 3.376 \times 10^{-5}y^4 - 0.0009468y^3 + 0.009686y^2 - 0.04419y + 0.1394 \]

\[ F_2 = C_1s^4 + C_2s^3 + C_3s^2 + C_4s + C_5 \quad (3.20) \]

where

\[ C_1 = -0.0005611y^4 + 0.009381y^3 - 0.01981y^2 - 0.3475y + 5.402 \]
\[ C_2 = 0.001021y^4 - 0.01699y^3 + 0.03623y^2 + 0.5966y - 8.507 \]
\[ C_3 = -0.0003707y^4 + 0.00333y^3 + 0.04885y^2 - 0.6641y + 5.673 \]
\[ C_4 = 9.639 \times 10^{-5}y^4 - 0.001009y^3 - 0.009296y^2 + 0.1431y - 1.093 \]
\[ C_5 = -1.2 \times 10^{-5}y^4 + 0.0001933y^3 - 0.000333y^2 - 0.006718y + 0.06737 \]

where \( s = \frac{e}{c} \) and \( y = \frac{b}{a} \)
With the weight function coefficients determined, the stress intensity factors for single radial cracks for cylinder wall ratios $b/a$, ranging from 2 to 10, and the relative crack depth $e/C$, ranging from 0.1 to 0.9, can be calculated.

### 3.4 Solution to Critical Crack Depth

![Figure 3.6: Illustration of the Crack Growth Process](image)

During corrosion-induced cracking, a crack will initiate at the steel–concrete interface and propagate to a critical depth, which is defined as the critical point at which the maximum load is reached. At this point, the crack becomes unstable and suddenly propagates to the concrete surface. For a crack to propagate in concrete material, the following fracture criteria are adopted:

$$K_{net} = K_I^P(e) - K_I^C(e) = K_I^{ini}$$

(3.21)

where $K_{net}$ is the net stress intensity factor at the crack tip (see Figure 3.6), $K_I^P(e)$ is the SIF due to the applied load $P$. In this case, this is the expansion of corrosion products. $K_I^C(e)$ is the SIF due to the closure force or cohesive stress $\sigma(x)$ acting on the crack faces in the FPZ due to the softening behaviour of concrete and $K_I^{ini}$ is the initial fracture toughness of concrete (Foote, Mai & Cotterell, 1986).

With the derived weight function in Eq. (3.13), $K_I^P(e)$ and $K_I^C(e)$ can be determined once the stress distribution is known. Because $K_I^P(e)$ corresponds to the stress
distribution $\sigma_p(x)$, which is due to the expansion of corrosion products, it can be expressed as (Timoshenko & Goodier, 1970; Yang, Ni & Li, 2013):

$$K_i^p(e) = \int_0^e \sigma_p(x) m(x, e) dx$$  \hspace{1cm} (3. 22)

$$\sigma_p(x) = P_i + \frac{P_i a^2}{b^2 - a^2 (a + x)^2}$$  \hspace{1cm} (3. 23)

where $P_i$ is the internal pressure caused by corrosion.

$K_i^C(e)$ is the SIF due to cohesive stress distributed along the crack surface and can be expressed as:

$$K_i^C(e) = \int_0^e \sigma_c(x) m(x, e) dx$$  \hspace{1cm} (3. 24)

where $\sigma_c(x)$ is the cohesive stress distributed along the crack surface. The cohesive stress distribution changes with the increase in crack depth and width, as shown in Figure 3.7 (Xu & Reinhardt, 1998).

![Figure 3.7: Illustration of the Variation of Cohesive Stress Along the Crack Depth](image)

At any point along the crack depth, when the crack width exceeds the limit value, the stress transfer between the crack surface changes from a linear relationship to a
bilinear relationship. However, as the crack depth growth under investigation is in a relatively small domain (i.e. concrete cover depth), it would be logical to assume that the crack width at the rebar will not exceed a critical value that will result in a bilinear cohesive stress distribution on the crack surface. Therefore, in this study, a linear relationship for cohesive stress between the steel–concrete interface and the crack tip is adopted. The cohesive stress distribution $\sigma_c(x)$ is expressed as:

$$
\sigma_c(x) = \sigma_w + (f_t - \sigma_w) \frac{x}{e} 
$$

(3.25)

where $\sigma_w$ follows the traction–separation relation for exponential softening curve of concrete and can be expressed as (Gopalaratnam & Shah, 1985):

$$
\sigma_w = f_t \exp \left( \frac{f_t G_f w}{w} \right) 
$$

(3.26)

where $f_t$ is the tensile strength of concrete, $G_f$ is the fracture energy of concrete and $w$ is the crack width at the steel–concrete interface, which is determined by:

$$
w = 2\pi a [\varepsilon_0(a) - \varepsilon_6(a)] 
$$

(3.27)

Figure 3.8: Illustration of the Variation of Crack Width Along the Crack Depth

With the continuing growth of corrosion products, the crack will initiate at the steel–concrete interface and propagate through the cover of the concrete. Because the crack
The fracture criterion in Eq. (3.21) can now be expressed as:

\[
\int_0^w P_t + \int_0^w \sigma_w \left[ \frac{2}{2\pi(e-x)} \right] \left[ 1 + M_1 \left(1 - \frac{x}{e} \right)^2 + M_2 \left(1 - \frac{x}{e} \right) + M_3 \left(1 - \frac{x}{e} \right)^2 \right] dx \\
- \int_0^w \sigma_{\text{cr}} \left( f_t - \sigma_w \right) \left[ \frac{2}{2\pi(e-x)} \right] \left[ 1 + M_1 \left(1 - \frac{x}{e} \right)^2 + M_2 \left(1 - \frac{x}{e} \right) + M_3 \left(1 - \frac{x}{e} \right)^2 \right] dx \\
= K_I^{\text{ini}}
\]  

(3.30)

Using Eq. (3.30), the critical crack depth and corresponding critical pressure at which a corrosion-induced crack becomes unstable and suddenly propagates to the concrete surface can be determined. This will be illustrated in the worked example to follow.
3.5 Worked Example

Based on Eq. (3.30), an algorithm was developed in MATLAB (MathWorks Inc., 2013) (see Figure 3.9) to carry out all computations. To demonstrate the application of the critical crack depth model, an example was carried out using the basic variables in Table 3.5. Based on the fracture criterion in Eq. (3.21), for a given relative crack depth $e/c$, the corresponding load $P$ can be determined using Eq. (3.30). At the maximum load, the corresponding crack depth would be the critical crack depth discussed herein.

Table 3.5: Basic Variables Used in Critical Crack Depth Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete cover</td>
<td>$C$</td>
<td>30 mm</td>
<td>Assumed</td>
</tr>
<tr>
<td>Rebar diameter</td>
<td>$D$</td>
<td>12 mm</td>
<td>Assumed</td>
</tr>
<tr>
<td>Inner radius</td>
<td>$a$</td>
<td>$D/2$</td>
<td></td>
</tr>
<tr>
<td>Outer Radius</td>
<td>$b$</td>
<td>$a + C$</td>
<td></td>
</tr>
<tr>
<td>Fracture energy</td>
<td>$G_f$</td>
<td>0.088 N/mm</td>
<td>Pantazopoulou and Papoulia (2011)</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>$f_t$</td>
<td>2 MPa</td>
<td>Assumed</td>
</tr>
<tr>
<td>Effective modulus of elasticity</td>
<td>$E_{ef}$</td>
<td>18820 MPa</td>
<td>Li (2003)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$v_c$</td>
<td>0.18</td>
<td>Li (2003)</td>
</tr>
<tr>
<td>Initial fracture toughness</td>
<td>$K_{i,ini}$</td>
<td>0.778 MPa√m</td>
<td>Xu and Reinhardt (1999)</td>
</tr>
</tbody>
</table>
The computation begins by assuming that there are no defects between the steel and concrete interface. Thus, the initial crack depth $e = 0$ is used. The weight function coefficients $M_1, M_2, M_3$ and corresponding stress distributions $\sigma_p(x)$ and $\sigma_c(x)$ are determined from Eq. (3.13), Eq. (3.23) and Eq. (3.25). Due to the difficulty of analytically solving the integration of $K_I^p(e)$ and $K_I^c(e)$, numerical integration is adopted. A specified range of $P$ from 0 to 30 MPa was sufficient for the
computations. From Eq. (3.30), $K_{net}$ is solved for the range of $P$ values. When $K_{net} = K_f^{ini}$, the corresponding $P$ value associated with the crack depth is determined through the algorithm in Figure 3.9. From Figure 3.10, it is found that, given the variables listed in Table 3., the critical crack depth occurs at 6 mm from the rebar at a maximum pressure of 6.4 MPa. After this critical point, the internal pressure decreases, as, once the critical crack depth is reached, the crack will unstably propagate to the surface, which leads to an unloading situation.

![Figure 3.10: Crack Depth v. Internal Pressure](image)

**3.5.1 Verification of the Developed Model**

To verify the developed model, the derived solution to the stress intensity factor through the weight function method must first be verified. To do this, results from the finite element analyses are compared with the results obtained analytically using the weight function method. Because the weight function is load-independent, the SIF
can be determined for any load distribution applied to the crack surface. Therefore, a different load distribution (i.e., cubic load distribution) was applied to the crack surface in the FEM in ABAQUS. The SIF results were compared with the results using the weight function method (see Figure 3.11). A good agreement has been achieved with a maximum difference of 6.5%. Therefore, the SIF obtained through the derived weight function method is accurate for determining the stress intensity factor for a single radial crack propagating in a thick wall cylinder.

![Figure 3.11: Comparison of SIF from the Derived Weight Functions and FE Analysis at Crack Tip for Cubic Stress Distribution](image)

When verifying the developed model for determining critical crack depth $e$, it would be ideal to have experimental corrosion-induced cracking data on the critical crack depth. However, this has been difficult to obtain, as monitoring data for the cracking process are scarce, due to the difficulty of monitoring internal crack propagation and measuring the crack depth that changes with applied load. The lack of data on crack depth $e$ vindicates the significance of this research. Therefore, even with these difficulties, an experimental program was developed to indirectly verify the model. This is discussed in Chapter 5. For now, indirect verification of the model is achieved by comparing the maximum pressure obtained with results published in literature.
Zhang and Su (2017) proposed an empirical function for determining maximum pressure, which is represented as a function of concrete cover, tensile strength of concrete and rebar diameter $D$. The empirical function is:

$$P_r = -0.00338f_tDc + 0.11308f_tc + 0.00118D^2c - 0.03689Dc$$

$$+ 0.02319D^2f_t - 0.68993f_tD + 3.9058f_t - 0.10141D^2$$

$$+ 0.22599 + 3.0511D - 15.418$$

where $D$ ranges from 12 mm to 20 mm, $c$ ranges from 25 mm to 80 mm and $f_t$ ranges from 2.8 MPa to 4.3 MPa.

Munoz, Andrade and Torres (2007) proposed an experimental model through regression based on results from experiments. The model predicts the pressure required to cause cover cracking and is expressed in a dimensionless form as:

$$\frac{P_r}{f_t} = \left(2.3384 \frac{C}{D}\right)^{0.7017}$$

where $C/D$ ranges from 1 to 10.

<table>
<thead>
<tr>
<th>Table 3.6 Comparison of the Pressure Required to Cause Concrete Cover Cracking with $f_t = 3$MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>Developed model</td>
</tr>
<tr>
<td>$C = 30$</td>
</tr>
<tr>
<td>$D = 12$</td>
</tr>
<tr>
<td>$C = 40$</td>
</tr>
<tr>
<td>$D = 12$</td>
</tr>
<tr>
<td>$C = 40$</td>
</tr>
<tr>
<td>$D = 16$</td>
</tr>
</tbody>
</table>

From Table 3.6, it can be observed that increasing the rebar diameter from 12 mm to 16 mm leads to a decrease in the maximum pressure required to cause concrete cover
cracking. This could be due to the larger surface area exerted on the surrounding concrete as corrosion products expand. In addition, the maximum pressure obtained from the developed model is compared with Eq. (3.31) and Eq. (3.32). It can be observed that there is a significant difference between the developed model and Eq. (3.31). However, the same difference can be found between Eq. (3.31) and Eq. (3.32). This could be due to the lack of data, particularly experimental data, to calibrate the model developed by Munoz, Andrade & Torres (2007). Again, this indicates the novelty and necessity of this research into concrete internal cracking. To apply the developed model to predict the critical crack depth in corrosion-induced cracking, further verification is required through more specific experimental investigations, which will be discussed in Chapter 5. The applicability of the analytical model assumes that the cylinder wall ratio \( b/a \) falls within the range of 2 to 10 and the crack depth ratio \( e/c \) within 0.1 to 0.9. The model also assumes that corrosion-induced cracking results in a single radial crack propagating outward and the corrosion-induced pressure acts uniformly on the surrounding concrete.

### 3.5.2 Sensitivity Analysis

With the developed model, it is of interest to identify the influential factors affecting the corrosion-induced cracking process. To do so, a sensitivity analysis can be performed. Various methods have been reported and applied for sensitivity analysis, including the nominal range sensitivity analysis, differential analysis, SI and response surface methods. These methods are of different complexity, data requirements and representation of sensitivity (Chen, Baji & Li, 2018). In this section, the SI method is used. The SI is determined by calculating the output change in percentage when varying one input parameter from its minimum value to its maximum value. As such, the SI can be calculated as:

\[
SI = \frac{O_{\text{max}} - O_{\text{min}}}{I_{\text{max}} - I_{\text{min}}} \frac{I_{\text{avg}}}{O_{\text{avg}}} \tag{3.33}
\]
where $I_{\text{max}}$ and $I_{\text{min}}$ are the maximum and minimum values of the input parameters. $O_{\text{max}}$ and $O_{\text{min}}$ are the corresponding maximum and minimum output or results. $I_{\text{avg}}$ and $O_{\text{avg}}$ are the average value of the input parameters and output results.

To implement the SI method, a practical range for each of the variables involved in determining the critical crack depth is required. Only concrete geometry and concrete tensile strength are used for the sensitivity analysis. Other parameters, such as $G_f, E_{\text{ef}}$ and $K_{I\text{ini}}$, are not considered, as it is difficult to find a practical range for these variables. Further, these variables are random and depend on many other factors, such as concrete mix design, aggregate distribution, type of composition and test conditions. For this reason, only concrete geometry and concrete tensile strength is considered, as they can be considered more stable variables. The ranges of the input parameters used in this study are listed in Table 3.7.

**Table 3.7: Input Parameters for Sensitivity Analysis**

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Unit</th>
<th>Base-case value</th>
<th>Input range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>mm</td>
<td>30</td>
<td>25–50</td>
</tr>
<tr>
<td>$D$</td>
<td>mm</td>
<td>12</td>
<td>12–36</td>
</tr>
<tr>
<td>$f_t$</td>
<td>MPa</td>
<td>3</td>
<td>2–5</td>
</tr>
</tbody>
</table>

The results of the sensitivity analysis are summarised in Figure 3.12. It can be observed that the most influential variable affecting critical crack depth is concrete cover depth. From Figure 3.12, it is interesting to observe that increasing the rebar diameter has a negative effect on the results. This has been discussed previously, when increasing the rebar would result in a larger surface area of corrosion-induced pressure exerted on the surrounding concrete. Based on the analysis, it can be deduced that corrosion-induced cover cracking can be mitigated by increasing the cover depth and tensile strength and decreasing the rebar diameter. However, as engineering designs are typically dictated by immediate cost and performance, long-term corrosion effects on RC structures are not usually considered. This further vindicates this research, as an accurate corrosion-induced cracking model can help
engineers and asset managers conduct a reliability assessment of corrosion-affected RC structures and help provide more sustainable designs.

Figure 3.12: Sensitivity Analysis Results

3.5.3 Parametric Analysis

Based on the sensitivity analysis, a parametric study can be carried out to investigate the effect of tensile strength and geometry represented by the concrete cover \( c \) and rebar diameter \( D \). The results are shown in Figure 3.13 to Figure 3.15. From Figure 3.13, it can be observed that increasing the tensile strength does not affect the critical crack depth, but increases the maximum pressure required to cause concrete cover cracking. From Figure 3.13 to Figure 3.15, the critical crack depth increases from 6 mm to 7 mm to 9 mm, respectively. This is due to the increase in concrete cover, which results in the crack having to propagate further to reach the critical depth. The critical crack depth implies that, for a particular geometry (e.g., concrete cover depth is 30 mm), when the crack reaches the critical point, it becomes unstable and suddenly propagates to the concrete surface. Although the tensile strength changes, the location of the critical crack depth remains the same. Instead, the maximum pressure required to propagate the crack to reach the critical crack depth changes. Hence, the critical crack depth can be considered a factor of geometrical properties, such as rebar diameter and concrete cover depth than material properties. Figure 3.16 indicates that the geometry is represented by the cover to rebar diameter ratio \( C/D \), which affects the maximum pressure more than the tensile strength. An increase in
$C/D$ will result in a proportional increase in the maximum pressure required to cause cover cracking, as the crack must propagate further before unstable propagation; thus, requiring more pressure to cause cover cracking.

![Graph showing Crack Depth vs. Internal Pressure with $D = 12$ mm and $C = 30$ mm.]

**Figure 3.13: Crack Depth v. Internal Pressure with $D = 12$ mm and $C = 30$ mm**
Figure 3.14: Crack Depth v. Internal Pressure with $D = 12$ mm and $C = 40$ mm
Figure 3.15: Crack Depth v. Internal Pressure with $D = 16$ mm and $C = 50$ mm

Figure 3.16: Effect of Concrete Cover to Rebar Diameter Ratio $C/D$ and Tensile Strength on Internal Pressure
3.6 Summary

An analytical model has been developed to determine the critical crack depth in corrosion-induced concrete cracking. The developed model employs fracture mechanics concepts to model crack propagation. The fracture criterion used is the net stress intensity factor at the crack tip, which is the difference between the stress intensity factor from corrosion-induced pressure and concrete material’s resistance to cracking. The solution to the stress intensity factor has been derived based on the weight function method as a function of two dimensionless parameters, cylinder wall ratio \((b/a)\) and relative crack depth \((e/C)\). After verifying the developed model with existing literature, a sensitivity analysis and parametric study has been conducted. The effects of tensile strength, concrete cover depth and rebar diameter on the critical crack depth has been investigated. Increasing the tensile strength does not change the critical crack depth but increases the pressure required to cause concrete cover cracking. Increasing the cover not only increases the critical crack depth but also the pressure required to cause cover cracking. For the same cover depth, increasing the rebar diameter leads to a decrease in the pressure required to cause concrete cover cracking. It can be concluded that the model derived can determine the critical crack depth in corrosion-induced cracking of RC with reasonable accuracy.
Chapter 4: Development of the Numerical Method

4.1 Introduction

The FEM is a powerful numerical tool for analysing the mechanical behaviours of structures. For concrete structures, fracture mechanics theory has been implemented into the FEMs to study concrete cracking since the late 1970s (Hillerborg, Modeer & Petersson, 1976; Leibengood, Darwin & Dodds, 1984). DIANA, ANSYS and ABAQUS are popular FEM software that have been used to model crack propagation using fracture mechanics. In this research, ABAQUS is used to analyse corrosion-induced crack propagation in concrete due to its ease of implementation and sound ability to solve nonlinear finite element analysis.

Most numerical studies on concrete cracking has investigated external bending problems or simulated concrete fracture experiments (i.e., the three-point bending test). For example, Roesler, Paulino, Park and Gaedicke (2007) developed a finite element-based cohesive zone model to predict the load and crack mouth opening displacement of a concrete three-point bending beam. Similarly, Barpi and Valente (2000) used FE-based cohesive zone model to analyse crack propagation in a concrete dam. More recently, a new method called the XFEM has become popular in crack propagation studies. This method is attractive, as it does not require remeshing as crack propagates, which greatly reduces computation time. For example, Unger, Eckardt & Konke (2007) used XFEM to model a discrete crack on a variety of specimens using a customised crack growth algorithm. Sancho, Planas, Cendon, Reyes & Galvez (2007) also used XFEM to analyse the load and crack mouth opening displacement of a concrete three-point bending specimen. In contrast to previous studies, corrosion-induced crack propagation has also received considerable attention over the past decade. For example, many of the studies have focused on simulating crack propagation during corrosion, such as Thybo, Michel & Stang (2017), used a FE-based smeared crack model to simulate the damage caused by corrosion. The results were compared with experimental results to validate the model and sound agreement was achieved. In addition, Du and Jin (2014) developed a numerical solution incorporating the heterogeneities of concrete to more realistically
model the crack patterns of corrosion-induced concrete cracking. Given the advancement in FE techniques, the use of XFEM to model corrosion-induced cracking is well justified.

This chapter attempts to predict corrosion-induced cracking using XFEM. ABAQUS is used to develop a cracking model that incorporates fracture mechanics and FEMs. First, the fracture process zone of concrete is introduced followed by the concept and fundamentals of XFEM. After validation of the XFEM model, a worked example is used to model corrosion-induced crack propagation. The results from the XFEM model are compared to the analytical model developed in Chapter 3.

4.2 Fracture Process Zone

The failure mechanisms of materials can be classified into three categories: brittle, ductile and quasi-brittle. An illustration of the stress-displacement behaviour of the materials are shown in Figure 4.1. These failure mechanisms depend on the stress–strain relationship of the material. Hence, different models should be applied, depending on the material. For example, with brittle material, the Griffith model based on LEFM can be used. For ductile material, the Drucker-Prager, or von Mises model, can be used. For quasi-brittle material, such as concrete, the cohesive zone model based on nonlinear fracture mechanics can be used.

![Figure 4.1: Stress-displacement Behaviour Under Uniaxial Tension](image)

Note. (a) brittle, (b) ductile and (c) quasi-brittle materials.
4.2.1 Softening Behaviour of Concrete

Concrete is considered a quasi-brittle material, meaning the tensile stress slowly decreases with increasing strain after the critical stress. This behaviour is termed strain softening. This concept was developed from plasticity, in which the post-critical stress decline is considered a gradual decrease with continued increase in strain. For concrete material, the strain softening is due to the presence of the FPZ ahead of the crack tip, as shown in Figure 4.2.

![Diagram of the Mechanism of the Fracture Process Zone](image)

**Figure 4.2: Schematic of the Mechanism of the Fracture Process Zone**

During crack propagation in concrete, stresses can be transferred across the cracked surface due to various toughening mechanisms such as aggregate bridging, void formation or microcrack shielding (Shah, Swartz & Ouyang, 1995). Therefore, cracked surfaces may be able to sustain some stresses. In concrete, this is characterised by the softening degradation curve. The FPZ in concrete is typically surrounded by a nonlinear hardening zone, which can be considered negligible (Bažant & Planas, 1997). As shown in Figure 4.2, the FPZ is illustrated by the hatched region. The area beyond point B is the true crack, where the crack surfaces are completely separated. No stress transfer can occur between them. Point A is the crack initiation point where, when the crack initiation criterion is reached, the crack will propagate. Point B is the true crack tip where the tensile stress at that point is zero.
When concrete is subjected to a uniaxial tensile force (see Figure 4.3), the full stress-elongation curve will be similar to that shown in Figure 4.4. At the beginning, the stress will linearly increase with elongation until critical stress. In this stage, the strain is uniformly distributed throughout the structure. After critical stress, strain localisation occurs. This is where the strain is localised into a narrow region, which is the cohesive crack shown in Figure 4.3.

Figure 4.3: Schematic of Strain Localisation for Unloading a Structural Element

Beyond the strain localisation area, the structure unloads. A graphical representation of this phenomenon is illustrated in Figure 4.4 where, when a concrete cylinder with a notch in the middle is subjected to a pull-out force at two ends, the stress concentrates the notch where the crack initiates. Once crack initiation occurs, the strain localises to a notch band while the other two parts unload. By neglecting the inelastic strain in the

Figure 4.4: Stress Elongation and Crack Width Relationships
loading and unloading cycle, the total elongation under uniaxial tensile $\delta$ can be expressed as (Hillerborg, Modeer & Petersson, 1976):

$$\delta = L\varepsilon + w = L\frac{\sigma}{E} + w$$  \hspace{1cm} (4.1)

where $\varepsilon$ is the strain on the structure beyond the cohesive crack, $L$ is the original length of the structure in the direction of tension, $E$ is the modulus of elasticity, $\sigma$ is the residual stress and $w$ is the cohesive crack width.

The FPZ can be represented by the cohesive interface. The thickness of the interface should be very small or close to zero. A traction separation law can be used to describe the stress-displacement relationship of the interface as:

$$\sigma = f_{T-S}(\delta)$$  \hspace{1cm} (4.2)

where $f_{T-S}$ is a softening function. A number of studies have been conducted on this to define a relationship (Du, Hawkins, Arakawa & Kobayashi, 1992; Gopalratnam & Shah, 1985; Hillerborg, Modeer & Petersson, 1976; Liaw, Jeang, Du, Hawkins & Kobayashi, 1990; Reinhardt, 1984; Roelfstra, 1986). As discussed in Chapter 2, the earliest representation of the softening function is a linear curve, which was expanded to a bilinear, then trilinear and then exponential curve. Because $\delta$ is related to $w$, $f_{T-S}$ can also be expressed in terms of $w$. In the FEM, due to ease and simplicity, a linear softening curve is used and expressed as:

$$\sigma = \frac{2G_f}{f_t}$$  \hspace{1cm} (4.3)

where $G_f$ is concrete fracture energy and $f_t$ is concrete tensile strength.

4.2.2 Material Properties

The material parameters required to represent the full tensile-displacement relationship of plain concrete consist of the fracture energy $G_f$ and the tensile strength $f_t$. The term full implies that tensile stress will first increase to the tensile strength
and then steadily decreases until the tensile stress reaches zero. In addition to the fracture energy, a failure displacement \( \delta \), which is the displacement limit when the stress drops to zero, can also be used. The details of the parameters are described herein.

### 4.2.2.1 Tensile Strength \( f_t \)

The tensile strength of concrete material is commonly used as a criterion to determine if a cohesive crack is initiated. For a Mode I fracture, once the tensile stress at any point of a structure reaches its tensile strength, a crack is initiated and the material of that point begins to degrade. The tensile strength of concrete can be obtained using three types of tests: splitting test, flexural test and direct tensile test. The strengths measured from these tests vary considerably. \( f_t \) should ideally be determined using direct tensile test, because in the splitting and flexural tests, the distributed stresses are not pure tension. Therefore, the strength determined from these tests is not the true tensile property of concrete.

### 4.2.2.2 Fracture Energy \( G_f \)

The fracture energy \( G_f \) is the energy absorbed per unit area of the crack with the unit of N/mm or N/m. It can be regarded as the external energy supply required to create and fully break a unit surface area of a cohesive crack (Elices, Guinea, Gomez & Planas, 2002). \( G_f \) can be calculated as the area under the softening curve shown in Figure 4.4b and expressed as:

\[
G_f = \int_0^{w_c} f_{T-S}(\delta) \, d\delta \tag{4.4}
\]

Because the entire stress-displacement curve \( f_{T-S}(\delta) \) is considered a material property, \( G_f \) is also a material parameter, which is independent of structural geometry and size. \( G_f \) is used as an energy balance that controls stable crack propagation; that is, a crack will propagate when the strain energy release rate is equal to \( G_f \).

### 4.2.2.3 Shape of the Softening Curve
Cohesive crack initiation is followed by strain softening, which can be represented by a range of forms (e.g., linear, bilinear and nonlinear softening). Without knowing the shape of the softening curve, it is difficult to determine the entire stress-displacement curve. The shape of the curve is important for predicting the structural response and local fracture behaviour (i.e., the crack width is particularly sensitive to the shape of the softening curve) (Shah, Swartz & Ouyang, 1995).

4.3 Extended Finite Element Method

There have been persistent difficulties with analytically solving sets of high order differential equations that govern the mechanical and geometric behaviours of concrete that are necessary for studying concrete cracking (ACI Committee 466.3R, 1997). Thus, numerical approximations must be used. In the numerical approach, there are two main options: the boundary element method and the FEM. However, a relatively new method that extends the classic FEM has been developed to study concrete cracking. The XFEM was first introduced by Belytschko and Black (1999) and is a numerical method that extends the classic FEM. The XFEM uses enrichment functions to model singularities and discontinuities around the crack, such as the asymptotic near-tip functions, which are sensitive to singularities, and the jump function, which simulates the discontinuity when the crack opens (de Oliveira, 2013). In addition to the enrichment functions, the level set method (LSM) is implemented to track moving interfaces by representing cracks as zero-level set functions of one dimension higher. In this section, the basics of XFEM is discussed, followed by the LSM. After that, a detailed procedure and validation for XFEM modelling of cracking in ABAQUS is presented.

4.3.1 Basics of the Extended Finite Element Method

The XFEM is based on the concept of partition of unity, which allows local enrichment functions to be easily incorporated into a finite element approximation. Special functions, in conjunction with additional degrees of freedom, are used to ensure discontinuities. Crack modelling using XFEM allows for the simulation of both stationary and propagating cracks in which the initial crack and crack path definition is not required to conform to the structural mesh. The cracks can propagate
through elements allowing the modelling of the fracture of the bulk material. The main idea of XFEM is to enrich standard finite element spaces with additional degrees of freedom such that displacement across the crack surface can be discontinuous (Zhai, Wang, Kong, Li & Xie, 2017).

For fracture analysis, the enrichment functions typically consist of the near tip asymptotic functions that capture the singularity around the crack tip and a discontinuous function that represents the jump in displacement across the crack surfaces. The approximation for a displacement field can be described as (Peng, 2009):

$$\mathbf{u} = \sum_{i=1}^{N} N_i(x) \left[ \mathbf{u}_i + H(x) \mathbf{a}_i + \sum_{a=1}^{4} F_a(x) \mathbf{b}_a^\alpha \right]$$  \hspace{1cm} (4.5)

where $N_i(x)$ are the nodal shape functions, $\mathbf{u}_i$ is the nodal displacement vector associated with the continuous part of the finite element solution, $\mathbf{a}_i$ is the product of the nodal-enriched degree of freedom vector, $H(x)$ is the discontinuous jump function across the crack surface, $\mathbf{b}_a^\alpha$ is the product of the nodal-enriched degree of freedom vector and $F_a(x)$ is the elastic asymptotic crack-tip function, which is given as:

$$F_a(x) = \left[ \sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \theta \sin \frac{\theta}{2}, \sqrt{r} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right]$$  \hspace{1cm} (4.6)

where $(r, \theta)$ is the polar coordinate of a point.

In a domain, ABAQUS differentiates stationary and propagating cracks based on the number of enriched nodes and enrichment functions adopted. For stationary cracks, both Heaviside and crack tip singularity functions are included in the XFEM discretisation (see Figure 4.5). For propagating cracks, only the Heaviside function is included in the enrichment scheme (see Figure 4.5). It is also required for cracks to propagate along the boundary of the element and completely cut an element. Therefore, once a crack begins to propagate, the crack tip motion cannot arrest within an element.
The XFEM is implemented in ABAQUS based on the phantom nodes method, meaning that phantom nodes are superposed to the standard nodes to reproduce discontinuity (Gigliotti, 2012). Therefore, phantom nodes are tied to their corresponding real nodes when the enriched element is intact. During cracking, the phantom and real nodes separate. Cracking only occurs when the stresses or strains satisfy a specified crack initiation criterion. In ABAQUS, the crack initiation criteria can be the maximum principal stress (MAXPS) or strain criterion, the maximum nominal stress or strain criterion, the quadratic traction–interaction criterion and the quadratic separation–interaction criterion.

In the numerical XFEM model, the MAXPS of material is used as a crack initiation criterion and set at the value of the commencement of damage. Therefore, damage is assumed to initiate when the MAXPS ratio reaches a value of one, which is expressed as:

\[
D_e = \left\{ \frac{\langle \sigma_{max} \rangle}{\sigma_{max}^0} \right\}
\]

where \(\sigma_{max}^0\) is the MAXPS of concrete material. The evolution of damage \(D_e\) monotonically evolves from zero to one upon further loading after the initiation of damage.

Although XFEM is a convenient method to model crack propagation. It has its limitations. These are (Gigliotti, 2012):

1) only general static and implicit dynamic analyses can be performed
2) only linear continuum elements can be used with or without reduced integration
3) parallel processing of elements is not allowed
4) fatigue crack growth phenomenon cannot be modelled
5) only single or non-interacting cracks can be contained in the domain
6) no crack branching is allowed
7) a crack cannot turn more than 90 degrees within an element.

4.3.2 Level Set Method

To numerically track cracks, the LSM is a useful technique that simplifies the implementation of crack propagation and determination of enriched elements (Osher & Sethian, 1988; Zhai, Wang, Kong & Xie, 2017). In this method, the interested interface is represented as a zero-level set of a function \( \phi(x, t) \). As shown in Figure 4.6, assuming an interface \( \Gamma \) to be an open or closed interface that divides the domain into two distinct domains \( \Omega_A \) and \( \Omega_B \) and is moving outward with a velocity \( F \) normal to the interface (Ahmed, 2009), the evolution equation for the interface using the time-derivative can be expressed as:

\[
\frac{\partial \phi}{\partial t} + F \left| \nabla \phi \right| = 0
\]

\[
\phi(x, t = 0) = \text{given}
\]

The initial condition is usually taken as the signed distance function to the initial curve:

\[
\phi(x, t) = \pm \min_{x_{\Gamma} \in \Gamma(t)} \| x - x_{\Gamma} \|
\]

where \( x \) is any query point and \( x_{\Gamma} \) is a point on the discontinuity \( \Gamma \). The sign of the minimum distance depends on which side of the interface a point \( x \) is located.
There are three advantages to using the LSM for tracking an interface: The motion of the interface is computed on a fixed mesh. The method handles changes in the topology of the interface naturally. The evolution equation is of the form shown in Eq. (4.8), regardless of the dimension of the interface. Therefore, extending the method to higher dimensions is easily accomplished. Finally, the geometric properties of the interface can be obtained from the level set functions $\phi$ (Stolarska, Chopp, Moes & Belytschko, 2001).

4.3.3 Extended Finite Element Modelling

As discussed in Chapter 3, the corrosion of reinforcing steel in concrete can be modelled as a thick wall cylinder. Therefore, due to symmetry, half of the cylinder is modelled in ABAQUS as a two-dimensional deformable shell (see Figure 4.7).
The geometry of the cylinder is defined in the way shown in Chapter 3 (i.e., inner radius $a$, outer radius $b$). It is assumed that only one crack will initiate and propagate from the inner boundary of the cylinder to the outer boundary. The procedure for creating a XFEM model for ABAQUS is quite unique. As such, the procedure to set up an XFEM crack propagation model is:

1) Define and build the geometry. As specified in Table 4.1, the two-dimensional thick wall cylinder shell can be implemented. The bulk material would be a two-dimensional shell element and the initial crack is a wire element with a length of 2 mm.

2) Establish the material model. The elastic properties of concrete and Poisson’s ratio are implemented based on the basic variables. To model the crack initiation, a damage criterion based on the MAXPS is defined as the tensile strength of concrete material (see Figure 4.8). The damage evolution criteria are implemented assuming a linear softening behaviour with a fracture energy of 88 N/m. When the material properties are defined, they can be assigned to the entity (i.e., thick wall cylinder).

![Figure 4.8: Material Definition in ABAQUS](image)

3) Create an instance. An instance is an assembly of the parts. The purpose of the assembly is to define a global coordinate system, as every part is created in its own coordinate system. There are two options for the instance type: dependent on
part or independent from part. For this study, the instance chosen is dependent on part.

4) Define the time step for analysis (see Figure 4.9). In this case, the definition of step is straightforward. A general linear static step is defined with NGLEOM turned on for more stable convergence during crack propagation. The viscous regularisation is set to 0.0002. Aside from the default output parameters, PHILSR, PHILSM and STATUS XFEM must be requested in the field output to visualise the crack propagation. A detailed description of these parameters can be found in the ABAQUS manual (ABAQUS, 2011).

![Figure 4.9: Defining the Step in ABAQUS](image)

5) Define an interaction assignment. This is when ABAQUS implements XFEM under the special tab for crack propagation analysis. Under XFEM, the crack domain is selected as the entire geometry. The crack location is where the crack is defined to initiate. To define this location, a wire part approximately 2 mm in length is defined in the part section. The wire has no material property and the instance created is dependent on the part. This line is translated to the edge of the inner cylinder (see Figure 4.10) and is defined as the crack location.
6) Define the load and boundary condition. Because half the cylinder is modelled, a boundary condition was defined to restrain the y-direction (see Figure 4.11) to prevent vertical movement. A displacement load is applied to the inner surface of the cylinder in a local coordinate system.

7) Establish mesh on instance. Prior to generating the mesh, it is important to establish an appropriate mesh element, element type and mesh size. For this geometry, a structured quadrilateral mesh element is assigned and a four-node bilinear plane stress quadrilateral element with reduced integration is adopted. The mesh size is an important parameter, as it affects the distribution of stress on the body. Mesh that is too coarse underestimates the results. Mesh that is too fine results in unrealistic stress distribution. To determine optimum mesh size, a mesh
convergence test was carried out. By applying a 3 MPa pressure in the cylinder, it was expected that the stress output from ABAQUS at the inner cylinder would be close to 3 MPa. As shown in Figure 4.12, finer mesh produced more accurate results but required longer time for computation. From Figure 4.12, it can be observed that applying a global seed size of 0.3 resulted in the stress at the inner cylinder to be 2.99 MPa, compared to 2.85 MPa when a global seed size of 0.7 was applied. Applying a global seed size of 0.3 increased accuracy, but significantly increased the time for computation. Adopting a global seed size of 0.3 resulted in convergence issues during crack propagation. Therefore, in this work, a global seed size of 0.7 was adopted (see Figure 4.13) and the mesh is assigned to the instance (see Figure 4.14).

![Figure 4.12: Mesh Convergence Test](image)

![Figure 4.13: Assignment of Mesh and Seed in Crack Propagation Analysis](image)
8) Create and submit the job to the solver of ABAQUS/Standard. The ‘monitor’ tab can be used to check on the progress of the increment generation (see Figure 4.15).
9) Review results. When the job is successfully completed, a result file is generated. The results can be visualised by opening the file in the module of visualisation. All the default and defined output parameters of every increment can be obtained in this module.

4.3.4 Validation of the Extended Finite Element Method

Prior to modelling corrosion-induced cracking in ABAQUS, a benchmark model is first developed to validate the proposed modelling technique. The most common numerical model for concrete crack propagation analysis is modelling a three-point bending specimen. Sancho, Planas, Cendon, Reyes & Galvez (2007) modelled this using a cohesive crack approach and a subroutine was developed to model the propagation of cracks using displacement-controlled loading. The details of the model are described in Sancho, Planas, Cendon, Reyes & Galvez (2007). The results obtained using XFEM are compared with the literature to assess the validity of XFEM to model corrosion-induced crack propagation. Following Figure 4.16, the dimensions of the beam are: length = 2000 mm, thickness = 100 mm, depth = 500 mm and notch depth = 200 mm. The material properties are tensile strength \( f_t = 2.5 \text{MPa} \), Young’s modulus \( E = 20\text{GPa} \), Poisson’s ratio \( \nu = 0.15 \) and fracture energy \( G_f = 0.1\text{N/mm} \).

![Figure 4.16: Schematic of the Three-point Bend Test](image)

In the XFEM model, a linear softening material property was assigned, due to its convenience and ease of implementation. The mesh assigned to the model was a structured quadrilateral mesh type. A triangular mesh type was not chosen, as it is not
suitable for crack propagation using XFEM. The applied load was displacement controlled with a displacement limit of 2 mm. The results of the analysis are presented in Figure 4.17.

**Figure 4.17:** (a) Crack Propagation at 50%, (b) Crack Propagation at 100% and (c) Load v. Displacement Results

Based on the XFEM analysis, the critical load for the three-point bending test is 9.1kN. In comparison, the critical load in the literature is approximately 8.1kN. The difference in results could be due to the assumption in modelling techniques and
material property used. For example, the implementation of an exponential softening curve in Sancho, Planas, Cendon, Reyes & Galvez (2007), versus a linear softening in ABAQUS, or the use of cohesive element approach, versus the XFEM method. Nonetheless, the difference is not considered large and the main purpose is to assess the adequacy of XFEM to model crack propagation. Based on these results, it can be concluded that XFEM can accurately model crack propagation.

### 4.4 Worked Example

Table 4.1: Values of the Basic Variables for Numerical Analysis

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete cover</td>
<td>$c$</td>
<td>30 mm</td>
</tr>
<tr>
<td>Diameter of rebar</td>
<td>$D$</td>
<td>12 mm</td>
</tr>
<tr>
<td>Inner radius</td>
<td>$a$</td>
<td>$D/2$</td>
</tr>
<tr>
<td>Outer radius</td>
<td>$b$</td>
<td>$a + c$</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>$G_f$</td>
<td>88 N/m</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>$f_t$</td>
<td>2.0 MPa</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>$E_c$</td>
<td>30000 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$v_c$</td>
<td>0.18</td>
</tr>
<tr>
<td>Initial stress intensity factor</td>
<td>$K_{I}^{ini}$</td>
<td>0.778 MPa√m</td>
</tr>
</tbody>
</table>

Using the values of the variables in Table 4.1, and following the proposed procedure, the change in pressure acting on the inner radius causing crack propagation can be determined and plotted. From Figure 4.18 it can be observed that the stress distribution is symmetrical and concentrated at the crack tip. The wire element was introduced to encourage stress concentration at a point so that the location of crack initiation is set. This will help the crack to propagate stably across the cylinder. It is also logical to assume that there are small defects in the concrete, which allow stresses to concentrate at a location. When the maximum principal tensile stress is achieved, the crack will initiate and begin to propagate (see Figure 4.18). To make the
crack propagation visible, the following field output parameters must be defined: PHILSR, PHILSM and STATUS XFEM in the step module.

(a) 25% crack propagation

(b) 75% crack propagation
When analysis is complete, the change in pressure acting on the inner radius over displacement can be plotted by creating an XY data under the tools tab in the visualisation module. The inner elements are selected and averaged to obtain a plot of maximum pressure over applied displacement. In this case, the actual applied displacement is an assumed value and is not significant. It is of greater importance to obtain the critical pressure; that is, the pressure at which the crack propagates through the entire cylinder. In this example (see Figure 4.19), it can be observed that the pressure will linearly increase to a point at which nonlinearity occurs due to the presence of the FPZ. After critical stress (approximately at 8.58 MPa), the model experiences a linear softening behaviour until the stress reaches zero. From the results, the critical pressure at which a corrosion-induced crack penetrates through the cover is approximately 8.58 MPa given that $C = 30\text{mm}, D = 12\text{mm}$ and $f_t = 2.5\text{MPa}$.

Figure 4.18: Visualisation of Crack Propagation

(c) 100% crack propagation
Based on this analysis, it is important to investigate the effects of some parameters on the XFEM model. Geometry and material properties have been established as the main parameters that affect critical pressure to cause cover cracking. Therefore, a parametric study was carried out to investigate the effects of concrete cover depth, tensile strength and rebar diameter on the critical pressure to cause concrete cracking. The results are shown from Figure 4.20 to Figure 4.22. From the results, increasing the tensile strength leads to an increase in critical pressure, as the strength of the concrete is greater. If Figure 4.20 and Figure 4.21 are compared, it can be observed that increasing the thickness of the concrete cover depth leads to an increase in critical pressure. For example, given $f_t = 2$ MPa, the critical pressure for $C = 30$ mm is $6.98$ MPa, while the critical pressure for $C = 40$ mm is $9.27$ MPa. If Figure 4.21 and Figure 4.22 are compared, it can be observed that increasing the rebar diameter $D$ leads to a decrease in critical pressure. This is also observed in the results in Chapter 3.

**Figure 4.19: XFEM Results for Pressure-induced Cracking**
Figure 4.20: XFEM Results for Crack Propagation for $C = 30\text{mm}, D = 12\text{mm}$

Figure 4.21: XFEM Results for Crack Propagation for $C = 40\text{mm}, D = 12\text{mm}$
The model developed in Chapter 3 determines the critical crack depth to cause concrete cover cracking. This critical crack depth corresponds to a critical pressure. Therefore, the critical pressure obtained from XFEM analysis is compared with the results obtained from the analytical model developed in Chapter 3. From Table 4.2, it can be observed that the results from the numerical model are typically higher than those from the analytical model. This discrepancy could be due to the modelling assumptions and modelling techniques. For example, the analytical model is developed based on the assumption of a smeared crack approach, while the numerical model is based on the cohesive zone approach. This means that the analytical model represents the crack as a band of distributed microcracks whilst the cohesive zone approach represents the crack as a discrete crack. Even though there is a difference in modelling technique, numerous researches have proven that both approaches are appropriate in modelling crack propagation as they form the fundamental basis in fracture mechanics investigations (Shah, Swartz & Ouyang 1995). Another difference between the numerical and analytical model is the use of fracture criterion. The numerical model uses the maximum principal tensile stress whilst the analytical model uses the stress intensity factor criterion. These criterions may not necessarily lead to significant differences because both criterions are related and acceptable for
the use to model fracture. The implementation of the softening distribution of concrete material is arguably the most influential. This is because the one of the input parameters in the softening distribution is fracture energy. Fracture energy is determined as the area under the stress-strain curve in concrete. Therefore, assuming a linear distribution in the FE model would result in an overestimation as compared to an exponential curve in the analytical model. This is exhibited in Table 4.2 where the numerical results are consistently higher than the analytical results. However, to input the same softening distribution of concrete into a finite element model required specialized subroutine codes which is cumbersome and beyond the scope of this research. The primary purpose of this model is to verify the adequacy of XFEM as a tool to model crack propagation in a thick wall cylinder and to verify the analytical model developed in Chapter 3. As such, although there are discrepancies between results, the difference is sufficiently large to cause concern and the numerical model developed based on XFEM can still be used to predict concrete crack propagation and the critical pressure at which concrete cover cracks.

**Table 4.2: Results for Critical Pressure Required to Cause Concrete Cover Cracking**

<table>
<thead>
<tr>
<th>Variables</th>
<th>$f_c$</th>
<th>Numerical model results</th>
<th>Analytical model results</th>
</tr>
</thead>
<tbody>
<tr>
<td>C = 30 mm D = 12 mm</td>
<td>2.0 MPa</td>
<td>6.6 MPa</td>
<td>6.4 MPa</td>
</tr>
<tr>
<td></td>
<td>2.5 MPa</td>
<td>8.6 MPa</td>
<td>6.8 MPa</td>
</tr>
<tr>
<td></td>
<td>3.0 MPa</td>
<td>10.2 MPa</td>
<td>7.1 MPa</td>
</tr>
<tr>
<td>C = 40 mm D = 12 mm</td>
<td>2.0 MPa</td>
<td>9.3 MPa</td>
<td>8.3 MPa</td>
</tr>
<tr>
<td></td>
<td>2.5 MPa</td>
<td>10.8 MPa</td>
<td>8.8 MPa</td>
</tr>
<tr>
<td></td>
<td>3.0 MPa</td>
<td>12.3 MPa</td>
<td>9.4 MPa</td>
</tr>
<tr>
<td>C = 40 mm D = 16 mm</td>
<td>2.0 MPa</td>
<td>7.2 MPa</td>
<td>7.2 MPa</td>
</tr>
<tr>
<td></td>
<td>2.5 MPa</td>
<td>8.7 MPa</td>
<td>7.7 MPa</td>
</tr>
<tr>
<td></td>
<td>3.0 MPa</td>
<td>10.0 MPa</td>
<td>8.1 MPa</td>
</tr>
</tbody>
</table>
4.5 Summary

A numerical model has been developed to determine the critical pressure required to cause concrete cover cracking. The model employs fracture mechanics theory and crack propagation modelling using XFEM. The FPZ is simulated using a linear softening curve with fracture energy and tensile strength specified as fracture parameters. The pressure applied to the inner cylinder radius is displacement controlled in a local coordinate system. From the analysis, it was discovered that increasing the tensile strength and concrete cover leads to an increase in critical pressure required for the crack to penetrate through the cylinder model, as more force is required to propagate the crack. It has also been found that an increase in the rebar diameter results in a decrease in critical pressure, as a larger surface area causes more stress to be generated to propagate a crack. The slight discrepancy between the results from the numerical and analytical models are due to the difference in modelling assumptions (i.e., smeared crack approach v. cohesive zone approach). Nonetheless, the numerical model can be used to model both crack propagation and the critical pressure required to cause concrete cover cracking due to corrosion.
Chapter 5: Experiment on Corrosion

5.1 Introduction

The application of direct current (DC) or impressed current is one of the most common techniques used to accelerate corrosion in experimental investigations. This technique accelerates corrosion by increasing the electron flow in the circuit by varying the duration and amount of current applied to reinforcement. Typical experimental investigations using the impressed current technique involve establishing a relationship between structural responses (i.e., concrete cracking, crack width, stiffness degradation and bond loss) to the amount or degree of corrosion (Alonso, Andrade, Rodriguez & Diez, 1998; Andrade, Alonso & Molina, 1993; El Maaddawy & Soudki, 2003; Lu, Jin & Liu, 2011; Rasheeduzzafar, Al-Saadoun & Al-Gahtani, 1992; Vu & Stewart, 2005; Wong, Zhao, Karimi, Buenfeld & Jin, 2010; Zhao, Yu, Wu & Jin, 2012). Based on the experimental results, some empirical models for predicting the time to corrosion-induced cover cracking have been proposed (Alonso, Andrade, Rodriguez & Diez 1998; Morinaga, 1988; Vu & Stewart, 2005). Corrosion-induced crack width models have also been proposed using this technique (Andrade, Alonso & Molina, 1993; Vu & Stewart, 2005; Cao, Cheung & Chan 2013; Pedrosa & Andrade, 2017). Aside from accelerating corrosion using the DC approach, some researchers have adopted a more natural method for accelerating corrosion; salt solution spraying on specimens with prescribed wetting and drying cycles (Li, 2001; Liu & Weyers, 1998; Zhang, Castel & François, 2010). These studies mainly involve investigating the effect of corrosion on RC cracking.

In general, the time taken for corrosion to affect RC using natural acceleration is in years, as opposed to days when using the DC method. The main advantages of accelerating corrosion using the DC method is that the effects of corrosion can be achieved in a short time frame. Further, the rate of corrosion—which usually varies depending on resistivity, oxygen concentration, humidity and temperature—can be easily controlled. For example, change in concrete resistivity due to change in temperature or humidity can be overcome by supplying a greater voltage through the
impressed current circuit. The justification for using the impressed current to accelerate corrosion is strong, as it can greatly reduce initiation time and control the corrosion rate without compromising the reliability of the corrosion products (Austin, Lyons & Ing, 2004). Although the impressed current technique is advantageous for accelerating corrosion, this technique does not fully simulate naturally induced corrosion. Austin, Lyons & Ing (2004) reported that using the impressed current technique would artificially polarise the rebar, increasing the potential for a value greater than the transpassive potential in naturally occurring corrosion. Using this method, the corrosion products are said to be uniformly distributed around the rebar compared to where the corrosion products mainly occur on the concrete surface facing a natural corrosive environment (Yuan, Ji & Shah, 2007).

Based on the literature review, it was discovered that there are considerable discrepancies between predictive models and those obtained from laboratory or field data. This is due to the complicated nature of corrosion and concrete-cracking processes. However, it can be agreed that concrete cover and corrosion rates are the most critical factors that affect the time to concrete cracking. This has motivated an experimental investigation into corrosion-induced cracking using the impressed current technique. Although there are drawbacks to this technique, the advantage of time, a controlled rate of corrosion and the ability to effectively simulate damage to RC justifies the suitability to accelerate corrosion.

This chapter experimentally investigates the time to corrosion-induced concrete cover cracking and the growth of corrosion-induced crack width. First, the design and preparation of test specimens is presented. This is followed by a description of the accelerated corrosion experimental program and the methods to collect data. The measured time to corrosion-induced cracking and corrosion-induced crack width from experiments are used to validate the models discussed in this chapter. The data produced from the experiments can be used to validate future corrosion-induced cracking models.

The main work presented in this chapter has already been submitted to an international journal, *International Journal of Civil Engineering*. 
5.2 Experimental Program

An experimental program was designed and conducted to produce data for corrosion-induced cracking. The corrosion process in RC can be divided into two parts. The first is corrosion initiation, which can be estimated using Fick’s second law of diffusion. However, it is assumed that when the rebar is impressed by a DC, corrosion immediately initiates; hence, corrosion-initiation time is not relevant here. The second is corrosion propagation; whereby, with the supply of oxygen and water, corrosion propagates, resulting in concrete cracking. In this test, three corrosion rates and two cover depths are the main variables used to study its effect on time to cracking and growth of crack width.

5.2.1 Design and Preparation Test Specimens

Concrete specimens (200 mm x 200 mm x 300 mm in size with a single reinforcing bar in the middle) were cast (see Figure 5.1). During casting, the specimens were cast upside down to ensure a smooth finish on the top surface for corrosion-induced crack observation. The reinforcement that was used was a deformed bar with a diameter of 12 mm.

![Figure 5.1: Schematic of Concrete Specimens](image)

General purpose Portland cement was used with a content of 350 kg/m³. Sand was used as the fine aggregates and crushed rock with a size of 7 mm and 14 mm was used as coarse aggregate. The concrete mix proportions by weight of cement, sand and gravel were 1:1.77:3.25 and a water–cement ratio (w/c) of 0.6 was used. A w/c of 0.6 was used, as it represents one of the worst-case scenarios for concrete mixes,
which may be used in practice. A high \( w/c \) would also result in a more permeable concrete, which would increase the chloride ions diffusion time through the concrete cover.

As discussed, the main variables that affect corrosion-induced cracking are corrosion rate and cover depths. Therefore, three corrosion rates and two cover depths were used. The corrosion rates applied to accelerate corrosion in RC specimens were 100, 200 and 300 \( \mu A/cm^2 \). These values were used so that the effects of corrosion on test specimens could be achieved in a short amount of time. These values have also been used in previous studies (Andrade, Alonso & Molina, 1993; El Maaddawy & Soudki, 2003; Val, Chernin & Stewart 2009; Vu & Stewart, 2005). The amount of DC required to achieve the expected corrosion rate is calculated by multiplying the surface area of steel with the desired corrosion rate. For example, a 12 mm diameter x 350 mm long steel rebar has a surface area of 13.42 cm\(^2\). Therefore, for a desired corrosion rate of 100 \( \mu A/cm^2 \), the applied current will be 13.42 mA (See Table 5.1). The cover depths used to cast the specimens were 30 mm and 50 mm. These values were selected based on practical experiences with typical concrete cover depths. In general, 30 mm is a typical cover depth used in RC design to ensure sufficient performance from rebar. However, for high corrosion risk environments, such as coastal structures, a 50 mm cover depth is usually used. Based on this, the test was divided into two groups. Group 1 represented specimens with a cover depth of 30 mm and group 2 represented specimens with a cover depth of 50 mm (see Table 5.1). The groups were further subdivided into three subgroups representing the different
applied corrosion rates. For each subgroup, three specimens were cast so that average values could be obtained. A total of 18 specimens were cast.

<table>
<thead>
<tr>
<th>Group</th>
<th>Subgroup</th>
<th>Applied current (mA)</th>
<th>Corrosion rate (µA/cm²)</th>
<th>Cover depth (mm)</th>
<th>w/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1</td>
<td>13.42</td>
<td>100</td>
<td>30</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>26.84</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>40.26</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>13.44</td>
<td>100</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>26.88</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>40.32</td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The concrete specimens were cast in two batches, with batch 1 representing group 1 specimens and batch 2 representing group 2 specimens. The specimens were cast in two batches due to the size of the concrete mixer (see Figure 5.2). The procedure to cast concrete first involved weighing the materials. As mentioned previously, the concrete mix proportions by weight of cement, sand and gravel is 1:1.77:3.25 with a w/c of 0.6. This means that, nine RC prism specimens with a cement content of 350kg/m³ and a w/c of 0.6 result in an approximate water content of 210 kg/m³, sand content of 620 kg/m³ and total gravel concrete of 1140 kg/m³ to make normally weighted concrete. The raw materials were then added to the concrete mixer and water was added slowly as the concrete mixed. Once the fresh concrete was thoroughly mixed, it was poured into custom square prism moulds for curing test
specimens. The custom moulds were made with the hole for the rebar placed from the bottom so that when the specimens were removed from the moulds, a smooth surface finish was achieved (see Figure 5.3).

Figure 5.2: Concrete Mixer
The moulds were made with 25 mm thick plywood and were reused to cast the batch 2 specimens. In addition to the nine corrosion specimens, three small cylinders and three large cylinder specimens were made using standard moulds with a nominal diameter of 100 mm x 200 mm high and a nominal diameter of 150 mm x 300 mm high, respectively. The small cylinder was used for compression tests and the large cylinder was used for indirect tensile test (see Figure 5.4). All the specimens follow the Australian Standard 1012.8.1:2014 (Standards Australia, 2014). All the moulds were applied with mould oil at the inner surface before pouring the fresh concrete mix. This is to prevent concrete from sticking to the mould, allowing for easier removal later. The specimens were compacted on a vibrating table in the laboratory before being set aside to cure. The specimens were left for at least 24 hours at a room temperature of approximately 20°C to 22°C before being demoulded. During this time, the specimens were covered with moist hessian clothes and polyethylene sheets.
to prevent excessive moisture loss (see Figure 5.5a). After demoulding, the specimens were water cured in a water tank for 28 days as shown in Figure 5.5b.

![Figure 5.5: Curing of Specimens](image)

### 5.2.2 Test Procedure

To accelerate the corrosion of reinforcing steel, a direct electric current was applied to the rebar using a laboratory DC power supply IPS2303. The DC power supply converts the alternating current to DC and has a maximum current and voltage output of 3A and 30V, respectively. The power supply was then connected to a custom-built split channel box that splits the DC power source into nine channels. The split channel box has a maximum current capacity of 700 mA in increments of 0.01 mA, which allows for a constant current to be applied to the specimens (see Figure 5.6). Having a split channel box allowed each concrete specimen to be connected in a single impressed current circuit (see Figure 5.7). This eliminated the risk of any loss of current; in some tests the specimens were connected in series or in parallel.
Before beginning the corrosion test, the concrete specimens were fully submerged in 3% NaCl solution by weight in a tank for three days. This was to ensure that the chloride ions had sufficiently saturated the concrete specimens. During the test, the specimens were immersed in the same concentration of NaCl. The top surfaces of the specimens were regularly wetted using wet hessians sheets. To complete the DC circuit for accelerated corrosion, the anode was connected to the rebar and the cathode was connected to the copper sheets of 200 mm x 300 mm, which were placed on both sides of the specimens (see Figure 5.7). To ensure sufficient electrical contact between the copper sheet and concrete, a sponge was placed in-between. Throughout the test, the applied current was regularly checked for each channel using a voltmeter to ensure that the desired current was accurately applied. The set-up of the corrosion test is shown in Figure 5.8.
To measure the mechanical property of concrete (i.e., compressive and tensile strength), the compressive strength of concrete was measured using the compressive strength test of the small concrete cylinder. This test was carried out in accordance with Australian standards (Standards Australia, 2000) using an MTS machine with a loading capacity of 1000 kN and a loading rate of 20 + 2 MPa/minute. The small cylindrical specimens (100 mm diameter x 200 mm length) were tested for compressive strength 28 days after casting. The average results of the three cylindrical specimens were recorded. The compressive strength of the specimen was calculated using Eq. (5.1):
\[
\sigma = \frac{F}{A}
\]  
\hspace{1cm} (5.1)

where \( F \) is the force applied and \( A \) is the cross-sectional area of the cylinder.

The tensile strength of concrete is calculated using the measured indirect tensile strength (see Figure 5.9). The indirect tensile strength test was carried out in accordance with Australian standards (Standards Australia, 2000) using an MTS machine with a loading capacity of 1000 kN and a loading rate of \( 1 + 0.1 \) MPa/minute, provided with indirect tensile strength test equipment. Three cylindrical specimens (150 mm diameter x 300 mm length) were tested 28 days after casting. The average results of the three cylindrical specimens was recorded. The indirect tensile strength \( T \) of the specimen was calculated using Eq. (5.2):

\[
T = \frac{2000P}{\pi LD}
\]  
\hspace{1cm} (5.2)

where \( P \) is the maximum applied force indicated by the testing machine, \( L \) is the length and \( D \) is the diameter. Therefore, the tensile strength of concrete \( f_t \) is \( 0.9 T \).

![Figure 5.9: Indirect Tensile Strength Apparatus (Standards Australia, 2000)](image-url)


5.2.3 Mass Loss Test

To further verify the impressed current technique for accelerating corrosion, a mass loss analysis was performed on a separate accelerated corrosion test. This separated corrosion test was carried out on reinforcement bars that were not cast in concrete (i.e., expose rebars). The purpose of corroding reinforcement bars that are not cast in concrete is to eliminate any uncertainty of corrosion, as, with reinforcement embedded in concrete, the presence of concrete material may affect the corrosion process (i.e., oxygen and water diffusion through the concrete material and alkalinity of concrete). Therefore, by having an exposed reinforcement bar, it can be assumed that, once the current is applied, the reinforcement will begin to corrode due to the abundance of oxygen and water.

Before corroding the specimens, nine reinforcement bars 300 mm in length and 12 mm in diameter were weighed and recorded (see Figure 5.10). The purpose of nine rebars was to monitor the change in mass loss over 28 days. The reinforcement bars were subjected to an applied corrosion rate of 150 $\mu A/cm^2$, which resulted in an applied current of 17.3 mA. The choice of applied corrosion rate was not critical in this test scenario, as the purpose was to verify the theoretical mass loss using Faraday’s law (see Eq. (5.3)) and actual measured mass loss.

$$i_{corr} = 87.6 \left( \frac{M}{\rho_{st}At} \right)$$

(5.3)

where $M$ is the weight loss in $mg$, $\rho_{st}$ is the density of metal in $g/cm^2$, $A$ is the area of steel sample in $cm^2$ and $t$ is the exposure time in hours.

The set-up of the test is shown in Figure 5.11 and Figure 5.12. Similar to the concrete corrosion test, each rebar was connected to its own individual circuit with the anode connected to the rebar and cathode connected to the copper sheet. Throughout the test, the rebar was kept moist by regular spraying of 3% salt solution by weight.
Figure 5.10: Weighing the Rebar

Figure 5.11: Reinforcement Bar Corrosion Set-up for Mass Loss Analysis

Figure 5.12: Connection of Reinforcement Bar for Corrosion
5.3 Measurement of the Data

To measure the time to concrete cover cracking, strain gauges 90 mm in length were attached to the top of the concrete surface (see Figure 5.13). In addition, 10 mm long strain gauges were attached around the rebar on the front and back faces of the specimens to measure the strain responses. The 90 mm strain gauges were P series and the 10 mm strain gauges were PFL series polyester (PS) single element wire strain gauges with a gauge resistance of 120 Ω. The strain gauges were installed according to the manufacturer recommendations. The concrete surface was prepared by sanding the area so that a layer of PS adhesive could effectively bond to the concrete surface. The PS adhesives were applied as a pre-coat for bonding the strain gauge. The pre-coat helped to prevent moisture from seeping from the concrete surface, which can damage the electrical components of the strain gauge. The pre-coat cured for 24 hours before being attached the strain gauges. To attach the strain gauge, cyanoacrylate (CN-E) adhesive was used and allowed to cure. The concrete specimens were constantly wetted throughout the corrosion test. Therefore, to protect the strain gauge from external moisture, a layer of wax was applied to the top surface (see Figure 5.13). To check if the strain gauge is installed properly, a voltmeter is used to check the resistance to ensure a reading of 120 Ω is obtained. A total of 42 strain gauges were attached for each group and measured using a datalogger. Due to the number of strain gauges used for each group, two datalogger CEM 20 expansions connected to one datalogger DT80 were required (see Figure 5.14).
Figure 5.13: Strain Gauge Set-up for (a) Top Surface and (b) Front and Back Surface
In addition to measuring time to cover cracking, the change in surface crack width over time was measured. To measure the change in crack width is relatively straightforward. A DEMEC mechanical gauge with a gauge length of 100 mm and accuracy of up to 0.001 mm was used (see Figure 5). To calibrate the device, a reference gauge was used to install the DEMEC location disks exactly 100 mm apart. The DEMEC locating disc was attached to concrete using CN-E adhesive. The locating discs were placed on the top surface of the concrete surface at the front, middle and back portions of the specimen (see Figure 5). Measurements were recorded every three to four days after surface cracking to measure the change in crack width over time. It is common practice to allow 0.3 mm as a permissible crack width for concrete structures (American Concrete Institute, 1999; BS, 1997). Therefore, the experiment was stopped after the crack width reached approximately 0.3 mm.

Figure 5.14: Datalogger Set-up
Corrosion-induced internal crack mapping was also recorded. This was conducted by cutting the cross-section of the corroded concrete specimens using a circular saw at regular intervals (see Figure 5.16). During cutting, the movement speed of the saw was kept slow, while the revolution speed of the saw was high. This was to minimise damage to the specimen and ensure a smooth cut through the cross-section. Despite these precautions, the saw cutting cause some minor damage to the cut specimens, which can make quantitative measurements of internal cracking unreliable. As such, at this point, only a qualitative crack map could be observed.

Figure 5.16: Cutting the Cross-section of the Specimens
5.4 Results for Time to Cracking

When the concrete cracked, the strain measurements on the top surface were analysed and averaged (see Figure 5.17 and Figure 5.18). The specimens exhibited different concrete strain behaviour due to different concrete cover thicknesses and applications of impressed current densities. It was evident that, for the same time, higher strains were measured with higher applied corrosion rates; that is, the strain responses from 300 µA/cm² specimens were significantly higher than 100 µA/cm². It was also evident that the larger the concrete cover was, the longer the strain response became (see Figure 5.18 compared to Figure 5.17). It can also be observed that, in Figure 5.17, a corrosion current density of 300 µA/cm² did not exhibit a time to concrete cracking that was three times faster than 100 µA/cm².

![Figure 5.17: Average Top Surface Strain of Group 1](image)
The side strain measurements on the left, right, top and bottom side of the rebar were recorded for some specimens. It can be observed that the strain response around the rebar follows a similar trend. On the left and right side of the rebar there is a higher strain response compared to the top and bottom surface at a given time (see Figure 5.19 and Figure 5.20). This means that, at the rebar, multiple microcracks are occurring around the rebar, as opposed to just one single crack propagating to the surface. This is more prominent when the cover thickness is 50 mm and the side strain around the rebar is significantly higher than the top surface strain (See Figure 5.20).
Figure 5.19: Side Strain Response for Subgroup 1.1 With Cover Thickness of 30 mm

Figure 5.20: Side Strain Response for Subgroup 2.3 With Cover Thickness of 50 mm

By taking the concrete tensile strain limit of $180 \times 10^{-6}$ (Carreira & Chu, 1986), the time of cracking for the specimens can be obtained from Figure 5.17 and Figure 5.18.
From Figure 5.18, it was observed that the time to cracking of 300 µA/cm² and 200 µA/cm² was approximately three and two times faster than that of 100 µA/cm². It is reasoned that the time to concrete cracking, as suggested by most models, are directly proportional to corrosion current density. However, this was not the case for group 1, in which the corrosion current density was 300 µA/cm² (see Figure 5.17). One of the reasons for the observed disproportionality of time to cracking could be due to the difference in porous zone size. The porous zone is the microscopic void between the steel and concrete interface. The size of the porous zone has been reported to range from 10 µm to 100 µm and can significantly affect the time to cracking (Chen, Baji & Li, 2018; Chernin, Val & Volokh, 2010). However, in most literature, the porous zone is assumed to be 12.5 µm, which could underestimate the time to cracking. The size and distribution of the porous zone depends on many factors, such as the water–cement ratio, aggregate size and distribution, cement content and concrete casting direction. Because the concrete mix design has a water–cement ratio of 0.6, a larger porous zone can be expected.

A scanning electron microscope (SEM) analysis has been carried out to measure the size of the porous zone of one of the specimens (see Figure 5.21). It has been observed that the porous zone ranges from 50 µm to 100 µm. It has also been observed that the distribution of the porous zone is not uniform around the rebar.
Figure 5.21: SEM Analysis of the Porous Zone (Not to Scale)

SEM results show that corrosion is more aggressive where there is a large porous zone. At locations with a very small porous zone, little to no corrosion products were observed. This could be because, without a porous zone, there was no space for oxygen and water to come into contact with the rebar for corrosion to occur. The non-uniform distribution of $d_0$ could also have contributed to the observation of non-uniform corrosion in the test specimens. Therefore, it can be argued that corrosion by impressed current may not necessarily produce uniform corrosion, as it depends on the distribution of the porous zone.

The results measured in the experiments were compared to the analytical time to cover cracking model. The analytical time to cover cracking model—an extension of the developed analytical model in Chapter 3—can be expressed as:

$$t_{cr} = \frac{365 \left( \frac{\rho_{rust}\rho_{st}[\pi d(d_{crit} + d_0)]^2}{\rho_{st} - \rho_{rust}\alpha_{rust}} \right)^{1/2}}{2 \times 0.105(1/\alpha_{rust})\pi d_{corr}} \tag{5.4}$$
where \( d_{\text{crit}} \) is the critical amount of corrosion products required to cause cover cracking. \( d_{\text{crit}} \) is calculated as:

\[
d_{\text{crit}} = \frac{P_{\text{lim}} a(1 + \varphi_c) \left[ \frac{b^2 + a^2}{b^2 - a^2 + v_c} \right]}{E_c}
\]  

(5.5)

where \( P_{\text{lim}} \) is the critical pressure required to cause concrete cover cracking or in other words, the concrete cover load bearing capacity can be determined from Chapter 3. One of the main input parameters in the model is the strength of concrete. The compressive and tensile strength of concrete can be obtained using standard tests on cylinder specimens following Australian Standard 1012.8.1:2014 (Standards Australia, 2014). From the test, the 28-day average compressive and tensile strength of concrete were measured as 38 MPa and 3.12 Mpa, respectively. Therefore, with the average measure size of porous zone, the strength of concrete and using the basic variables presented in Table 5.2, the time to concrete cover cracking can be calculated. Using Eq. (5.4), the model results and experimental results are shown in Table 5.3. From the analysis, it can be found that the influence of the porous zone on the time to cracking is significant and consistent with findings from Chen, Baji & Li (2018). Nevertheless, the model can predict the time to cracking. However, the accuracy of the model requires more thorough investigation into the influence and distribution of the porous zone.
### Table 5.2: Values of Basic Variables

<table>
<thead>
<tr>
<th>Basic variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>12 mm</td>
</tr>
<tr>
<td>$C$</td>
<td>30–50 mm</td>
</tr>
<tr>
<td>$d_0$</td>
<td>0.05–0.1 mm</td>
</tr>
<tr>
<td>$i_{corr}$</td>
<td>100, 200, 300 µA/cm$^2$</td>
</tr>
<tr>
<td>$E_c$</td>
<td>30100 MPa</td>
</tr>
<tr>
<td>$f_t$</td>
<td>3.12 MPa</td>
</tr>
<tr>
<td>$\varphi_{cr}$</td>
<td>2</td>
</tr>
<tr>
<td>$v_c$</td>
<td>0.18</td>
</tr>
<tr>
<td>$G_f$</td>
<td>0.088 N/mm</td>
</tr>
<tr>
<td>$K_{ini}$</td>
<td>0.778 MPa.m$^{0.5}$</td>
</tr>
<tr>
<td>$\alpha_{rust}$</td>
<td>0.57</td>
</tr>
<tr>
<td>$\rho_{rust}$</td>
<td>3600 kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{st}$</td>
<td>7850 kg/m$^3$</td>
</tr>
</tbody>
</table>

### Table 5.3: Time to Cracking Results

<table>
<thead>
<tr>
<th>Corrosion current density (µA/cm$^2$)</th>
<th>Eq. (5.4) model results (days)</th>
<th>Experimental result (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>28–99</td>
<td>39</td>
</tr>
<tr>
<td>200</td>
<td>14–50</td>
<td>23</td>
</tr>
<tr>
<td>300</td>
<td>10–33</td>
<td>22</td>
</tr>
</tbody>
</table>

Time to concrete cracking, $T_{cr}$ for $c = 50$mm

<table>
<thead>
<tr>
<th>Corrosion current density (µA/cm$^2$)</th>
<th>Eq. (5.4) model results (days)</th>
<th>Experimental result (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>33–110</td>
<td>96</td>
</tr>
<tr>
<td>200</td>
<td>17–55</td>
<td>50</td>
</tr>
</tbody>
</table>
5.5 Results for Crack Width

The experimental data on corrosion-induced crack width were also recorded. The results are presented in Table 5.4 and Table 5.5 and Figure 5.22 to Figure 5.24. Using the basic variables presented in Table 5.2, corrosion-induced crack width over time was plotted and compared with an analytical time-dependent corrosion-induced crack width model (Li, Melchers & Zhang 2006). The analytical model is a corrosion-induced crack width model developed based on elastic and fracture mechanics and is also time-dependent. The advantage of an analytical model is that it can be conveniently used for comparison, as the model is not developed based on a specific experimental condition, like many empirical models based on mechanical theories. The analytical time-dependent corrosion-induced crack width model is:

\[ w = \frac{4\pi d_s(t)}{(1 - v_c)(a/b)^{\sqrt{\alpha}} + (1 + v_c)(b/a)^{\sqrt{\alpha}}} - \frac{2\pi b f_t}{E_{ef}} \]  \hspace{1cm} (5.6)

where \( v_c \) is Poisson’s ratio, \( \alpha \) is the stiffness reduction factor, \( f_t \) is concrete tensile strength and \( E_{ef} \) is the effective modulus of elasticity of concrete. The stiffness reduction factor \( \alpha \) is introduced to consider the inelastic zone ahead of the crack tip. The residual tangential stiffness \( \alpha \) along the crack surface follows the traction–separation relationship of concrete and can be expressed as (Li, Melchers & Zhang, 2006):

\[ \alpha = \frac{f_t \exp \left(-2\pi r \frac{f_t}{\bar{u}_f} \left( \varepsilon_\theta(r) - \varepsilon_\theta^{em}(r) \right) \right)}{E_{ef} \varepsilon_\theta(r)} \]  \hspace{1cm} (5.7)

where \( \varepsilon_\theta(r) \) is the residual tangential strain of cracked concrete and \( \varepsilon_\theta^{em}(r) \) is the maximum elastic tangential strain at any radius \( r \).
Table 5.4: Measured Crack Width Results for Concrete Cover $C = 30$ mm

<table>
<thead>
<tr>
<th>$i_{corr}$ = 100 $\mu A/cm^2$</th>
<th>$i_{corr}$ = 200 $\mu A/cm^2$</th>
<th>$i_{corr}$ = 300 $\mu A/cm^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (days)</td>
<td>Crack width (mm)</td>
<td>Time (days)</td>
</tr>
<tr>
<td>39.5</td>
<td>0.041</td>
<td>23.5</td>
</tr>
<tr>
<td>44</td>
<td>0.09</td>
<td>26</td>
</tr>
<tr>
<td>47</td>
<td>0.117</td>
<td>27</td>
</tr>
<tr>
<td>50</td>
<td>0.15</td>
<td>30</td>
</tr>
<tr>
<td>53</td>
<td>0.175</td>
<td>32</td>
</tr>
<tr>
<td>58</td>
<td>0.225</td>
<td>34</td>
</tr>
<tr>
<td>60</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>0.285</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>0.373</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: Measured Crack Width Results for Concrete Cover $C = 50$ mm

<table>
<thead>
<tr>
<th>$i_{corr}$ = 100 $\mu A/cm^2$</th>
<th>$i_{corr}$ = 200 $\mu A/cm^2$</th>
<th>$i_{corr}$ = 300 $\mu A/cm^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (days)</td>
<td>Crack width (mm)</td>
<td>Time (days)</td>
</tr>
<tr>
<td>97</td>
<td>0.05</td>
<td>51</td>
</tr>
<tr>
<td>100</td>
<td>0.067</td>
<td>55</td>
</tr>
<tr>
<td>104</td>
<td>0.133</td>
<td>57</td>
</tr>
<tr>
<td>107</td>
<td>0.166</td>
<td>62</td>
</tr>
<tr>
<td>111</td>
<td>0.179</td>
<td>65</td>
</tr>
<tr>
<td>121</td>
<td>0.322</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>71</td>
</tr>
</tbody>
</table>
Figure 5.22: Experimental Verification of Crack Width Over Time for $i_{corr} = 100 \mu A/cm^2$
Figure 5.23: Experimental Verification of Crack Width Over Time for $i_{corr} = 200 \mu A/cm^2$
Figure 5.24: Experimental Verification of Crack Width Over Time for $i_{\text{corr}} = 300 \, \mu A/cm^2$

As can be observed from Figure 5.22 to Figure 5.24, the analytical model is in agreement with the experimental results. In general, the experimental results are
lower than the model. This could be due to the analytical model assuming a uniform corrosion state in which it may not necessarily be the case. The model also assumes that there is only a single crack and that corrosion-induced pressure fully contributes to the increase in crack width, which may overestimate the expected results. However, it has been discovered that there are multiple internal cracks at larger crack widths (see Section 5.6). Therefore, the corrosion-induced pressure would not fully contribute to the increase in surface crack width, but also propagates internal cracks. It has also been found that corrosion rate is the most important factor affecting the growth of crack width. It can be seen from Figure 5.22 to Figure 5.24, that there is a rapid increase in crack width at the end part of the curve. This is because, with an open crack, the corrosion process is further accelerated due to the increased supply of water and oxygen. Hence, the growth of crack width is more prominent especially in specimens with 30mm cover than specimens with 50mm cover because specimens with 30mm cover crack much earlier. It is interesting to note that this rapid increase occurs after the crack width is larger than 0.3mm which is also considered the serviceability crack width limit in most design codes. It would be ideal to further compare crack width results with other authors such as Andrade, Alonso & Molina (1993), Vu and Stewart (2005), Zhao, Yu, Wu & Jin (2012) and Pedrosa and Andrade (2017). However, it would not be a fair comparison, as the results from other authors are either empirical or the experimental data are based on different parameters such as cover thickness, rebar diameter and corrosion rate. Therefore, it would only be appropriate to compare these results with analytical models based on mechanics, such as Li, Melchers & Zhang (2006), as opposed to empirical models based on specific conditions or parameters. Based on the analysis shown in Figure 5.22 to Figure 5.24, it is clear that the analytical crack width model is accurate and can provide guidance to practitioners and researchers in their investigation and assessment of corrosion-affected RC structures.
5.6 Other Results

5.6.1 Results for Mass Loss

After corroding the reinforcement bar for approximately 7, 14 and 28 days, at each time point, the corroded rebars were cleaned using Clarke’s solution, as per ASTM G1–03 standards (ASTM International, 2011), as shown in Figure 5.25. After cleaning, the rebars were weighed and recorded in Table 5.6 and shown in Figure 5.26. Using Eq. (5.3) the new rebar weight was used to calculate the theoretical corrosion rate $i_{corr}$ in units (mm/year) during the exposure period. Therefore, it was expected that the theoretical corrosion rate would be close to the applied corrosion rate of 150 $\mu A/cm^2$.

![Figure 5.25: Corroded Rebar and Clarke Solution Water Bath](image1)

<table>
<thead>
<tr>
<th>Rebar no.</th>
<th>Weight before corrosion (g)</th>
<th>Weight after corrosion (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>164 hours</td>
</tr>
<tr>
<td>1</td>
<td>254.57</td>
<td>251.63</td>
</tr>
<tr>
<td>2</td>
<td>254.91</td>
<td>251.58</td>
</tr>
<tr>
<td>3</td>
<td>254.6</td>
<td>252.09</td>
</tr>
<tr>
<td>4</td>
<td>255.06</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.6: Mass Loss Results
Based on the mass loss, and using Eq. (5.1), the actual corrosion rate was averaged to be 2 mm/year equivalent to 172 $\mu A/cm^2$ compared to the actual expected impressed corrosion rate of 1.74 mm/year equivalent to 150 $\mu A/cm^2$. These results show that the impressed current technique is adequate for accelerating corrosion, although it may result in an overestimation of the corrosion process.

5.6.2 Corrosion and Crack Observation

At regular intervals, a cross-section of the concrete specimens was cut using a circular saw to observe internal cracking. It was observed that the first visible crack on the surface of the test specimen was approximately 0.05 mm. These cracks randomly appeared on the surface of the concrete but were always parallel to the reinforcing bars (see Figure 5.27a).
At the end of the test, when the crack width reached 0.3 mm, some of the concrete samples were cut open and the internal cross-section of the samples were analysed. In some cases, non-uniform corrosion was observed. As discussed previously, this non-uniformity could be due to the non-uniform distribution of the porous zone at the interface of steel and concrete. Therefore, accelerated corrosion by an impressed current may not always lead to uniform corrosion, as previously thought. Further, severe internal cracking was observed when the crack width reached 0.3 mm. In some cases, the internal crack almost spread to the edge of the specimens (see Figure 5.27b and Figure 5.27c). Therefore, although a crack width of 0.3 mm may be considered small, if it is caused by corrosion, the internal cracking could be in a critical state.

In general, there were four types of crack patterns that appeared when the concrete samples were cut open (see Figure 5.28). A type 1 crack pattern is when there is one main crack that propagates to the surface and two minor side cracks. This is the type of crack pattern observed in Figure 5.27. A type 2 crack pattern is when there is one main crack followed by another minor crack propagating in the opposite direction. A type 3 crack pattern is when there is one main crack and one minor side crack. A type 4 crack pattern is uncommon, but is when there is one main crack and three other minor internal cracks.

Figure 5.27: (a) Surface Crack Width; (b) Illustration of Internal Crack; (c) Internal Cracking
The cracks observed from the specimens were consistent with the strain gauge results (see Figure 5.19 and Figure 5.13) when there is a large side strain response from the rebar. It was observed that the higher the applied corrosion rate, the more obvious the internal cracks were. Similarly, the larger the cover thickness, the larger the internal cracks. This could be because there was more time for internal cracks to form with larger concrete covers. In all the test specimens, it was observed that corrosion cracks occurred parallel to the steel reinforcing bar, regardless of the density. There was no apparent association between the types of crack patterns and the level of current density.

5.7 Summary

In this chapter, an accelerated corrosion test was carried out to measure time to corrosion-induced concrete cracking and crack width changing with time. Internal corrosion-induced crack pattern and a reinforcement mass loss analysis were also analysed. Corrosion in concrete was accelerated using an impressed current technique in which the intended corrosion rate was induced by varying the applied current. Time to corrosion-induced cracking was measured using strain gauges through a datalogger. It was discovered that the time to corrosion-induced cracking may not be directly proportional to the induced corrosion rate. For example, inducing a corrosion rate of 300 µA/cm² and 100 µA/cm² may not necessarily imply a time to cracking that is three times faster. This is because the porous zone can significantly affect the time to cracking. An SEM was carried out on one of the samples and it was observed that the porous zone was non-uniformly distributed around the rebar and with thickness ranging from 50 to 100 µm. A comparison between results obtained from

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**Figure 5.28: Illustration of Corrosion-induced Crack Patterns**

The cracks observed from the specimens were consistent with the strain gauge results (see Figure 5.19 and Figure 5.13) when there is a large side strain response from the rebar. It was observed that the higher the applied corrosion rate, the more obvious the internal cracks were. Similarly, the larger the cover thickness, the larger the internal cracks. This could be because there was more time for internal cracks to form with larger concrete covers. In all the test specimens, it was observed that corrosion cracks occurred parallel to the steel reinforcing bar, regardless of the density. There was no apparent association between the types of crack patterns and the level of current density.
the analytical model and experimental results on time to cracking showed some discrepancies. These discrepancies were associated with the non-uniform distribution and thickness of the porous zone. It was also discovered that accelerating corrosion using an impressed current technique may not lead to uniform corrosion. This has been associated with the distribution of the porous zone. The change in corrosion-induced crack width over time was measured and compared with an analytical model. In general, the experimental results were lower than the model results for corrosion-induced crack width. This was attributed to the modelling assumption of a uniform corrosion state and the existence of only a single crack, which is not the case in reality. An analysis of the concrete specimen cross-section demonstrated severe internal cracking. There are four types of internal crack patterns that could occur during corrosion-induced cracking. A mass loss analysis was also carried out and demonstrated that the impressed current technique was adequate for accelerating corrosion, although it can result in an overestimation of the corrosion process. It can be concluded that both the time to cracking and crack width models can be used to predict the time to corrosion-induced cracking and crack width. However, the porous zone distribution and thickness must be carefully considered in the time to corrosion-induced cracking model.
Chapter 6: Reliability Analysis of Corrosion-induced Concrete Cracking

6.1 Introduction

The corrosion process is not only random but also changing with time. Therefore, a stochastic approach using time-dependent reliability methods is essential to assess the service life of corrosion-affected RC structures. The service life of corrosion affected RC structures is the time at which the structure becomes unserviceable such as excessive corrosion-induced cracking. Time-dependent reliability methods are advantageous, as they have the capability to determine the time for intervention; that is, repairs and strengthening for the deteriorated structures. The information provided from the assessment can help engineers and asset managers in maintenance strategies of corrosion affected RC structures.

The literature review (see Chapter 2) suggests that reliability assessments of corrosion-affected RC focused more on strength deterioration than serviceability. Enright and Frangopol (1998a; 1998b) studied the loss of flexural strength in concrete bridges due to the corrosion of reinforcing steel using the MCS to determine the probability of failure. The loss of flexural strength due to corrosion was considered by the reduction of the steel cross-sectional area. Similarly, Thoft-Christensen (1998) conducted a reliability assessment (also using the MCS) on corrosion-affected concrete bridges considering the yield line, shear, crack width and deflection as limit states. A sensitivity analysis was carried out and it was discovered that the most important variables affecting the reliability of the structure were the thickness of the slab, reinforcement yield strength and model uncertainty. Val & Trapper (2008) used the MCS to study the effect of corrosion of reinforcing steel (both general and pitting) on flexural and shear strength and on the reliability of RC beams. They found that high corrosion rates (i.e., greater than 1 $\mu A/cm^2$) caused significant pitting corrosion on stirrup reinforcement, which greatly affected the reliability of the RC beam. Marsh and Frangopol (2008) used the reliability index method, incorporating spatial and temporal variations of probabilistic corrosion rate data, to better predict the
probability of failure of an RC bridge deck by applying a flexural limit state. In this model, the special and temporal variation of corrosion was assumed through the interpolation of corrosion sensors throughout the structure. Bhargava, Mori & Ghosh (2011) carried out reliability analysis on shear and flexural strength of corrosion-affected RC beams using the MCS, in which the LHS was used for efficient sampling. It was discovered that the probability of failure becomes sensitive to the COV associated with degradation functions if it is more than 4%.

In contrast to most research on ultimate failures, few studies on corrosion-affected RC structures have used corrosion-induced cracking as a failure criterion. Fewer employed time-dependent reliability methods (e.g., the first passage probability method). Li and Melchers (2005) are among the few researchers who employed the time-dependent reliability method to predict the probability of corrosion-induced concrete cracking. In this study, the basic variables (e.g., strength, geometry, loading and other physical properties) are assumed to be normally distributed. This may give rise to unrealistic negative values. It would be more appropriate to assume a lognormal distribution for variables that are inherently positive, such as physical properties.

In this chapter, a new methodology is developed to evaluate the probability of corrosion-induced concrete-cracking failure. Cracking described herein can be divided into two parts: concrete cover cracking and crack width. Concrete material is considered quasi-brittle and corrosion-induced cracking is determined based on fracture mechanics criteria (i.e., cracks are assumed to be smeared). A stochastic model with a non-stationary lognormal process is developed for corrosion-induced concrete cracking as a function of key contributing factors. The first passage probability method is employed to predict the time-dependent probability of corrosion-induced concrete cracking from which the time for the structure to become unserviceable can be determined with confidence. Factors that affect the failure due to corrosion-induced cracking are also studied. The significance of this work is that fracture mechanics criteria are used to determine concrete cracking and the first passage probability method is employed to predict serviceability failure, which is modelled as a non-stationary lognormal stochastic process.
The main work presented in this chapter has already been accepted in an international journal, *ASCE Journal of Structural Engineering*.

### 6.2 Formulation of Service Life Prediction

In this section, the definition of service life, the time-dependent reliability methods and the stochastic model are presented. The model provides the basis for predicting the service life of corrosion-affected RC structures.

#### 6.2.1 Definition of Service Life

The service life of a structure is defined as the time period at the end of which the structure stops performing its intended function (Li & Mahmoodian, 2013). When assessing the risk of failure for a structure, a performance criterion should be established. In the theory of structural reliability, this criterion is expressed in the form of a limit state function as:

\[
G(R, S, t) = R(t) - S(t)
\]  
(6.1)

where \(S(t)\) is the load effect and \(R(t)\) is the resistance at time \(t\).

With the limit state function in Eq. (6.1), the probability of corrosion-induced failure in RC structures \(p_f\) can be determined as:

\[
p_f = P[G(R, S, t) \leq 0] = P[S(t) \geq R(t)]
\]  
(6.2)

where \(P\) denotes the probability of an event.

At a time when \(p_f(t)\) is greater than the maximum acceptable risk in terms of the probability of failure \(p_a\), it is the time the structure fails. This can be determined as:

\[
p_f(T_c) \geq p_a
\]  
(6.3)

where \(T_c\) denotes the time at which a phase of service life ends. In principle, the acceptable risk \(p_a\) can be determined from a risk–cost optimisation of the structure.
during its entire service life. This is beyond the scope of this work and will not be discussed. Previous research can be referred to by Li & Zheng (2007) and Thoft-Cristensen and Sorensen (1987)

For corrosion-affected RC structures, there are two phases in corrosion progress in terms of its effect on structures (see Figure 6.1). The first phase is the corrosion initiation stage, which is defined as the time period from the completion of a new structure to the beginning of the corrosion of reinforcing steel in concrete. This is known as stage I and can be denoted as \([0, T_i]\).

![Diagram showing the stages of corrosion and the service life of reinforced concrete](image)

**Figure 6.1: Service Life of Reinforced Concrete Subjected to Corrosion (Tutti, 1982)**

Corrosion initiation has been the focus of research in RC corrosion for the past decade. The initiation time has been extensively studied (Li, 2000; 2001; 2002; Mangat & Molloy, 1992; Val & Trapper, 2008). Therefore, the following corrosion initiation criterion based on Faraday’s law can be used (Bamforth, 1999):

\[
C(x, t) = C_s \left[ 1 - erf \left( \frac{x}{2\sqrt{D_c t}} \right) \right]
\]  

(6.4)
where \( C_s \) is the chloride content on the concrete surface, \( \text{erf}(x) \) is error, \( t \) is time and apparent diffusion coefficient \( D_c \), which is a property of concrete. Corrosion initiates once the chloride concentration on the steel surface reaches a CTL. Many different types of threshold values have been proposed based on different structural types and exposure conditions. In this example, a CTL of 0.06% by weight of concrete can be used as the limiting value (ASTM International, 2009; Thomas, 1996).

The second phase (i.e., stage II) is defined as the time period from corrosion initiation to corrosion-induced concrete cracking, denoted as \([T_i, T_c]\). There are two parts to corrosion-induced concrete cracking: time to corrosion-induced cover cracking \( T_{c1} \) and time to permissible corrosion-induced crack width \( T_{c2} \).

### 6.2.2 First Passage Probability

Eq. (6.2) represents a typical upcrossing problem, which can be determined using time-dependent reliability theory (Li & Melchers, 2005). In time-dependent reliability methods, the corrosion-induced failure depends on the time that is expected to elapse before the first occurrence of the stochastic \( S(t) \) upcrossing an acceptable limit \( R(t) \) during the service life of the structure \([0, T_c]\). This is measured by \( \frac{S(t)}{R(t)} \geq 1 \). Equivalently, the probability of the first occurrence of such an event is the probability of failure \( p_f(t) \) during that period. This is known as the ‘first passage probability’ and can be determined from Eq. (6.5) (Melchers, 1999):

\[
p_f(t) = 1 - [1 - p_f(0)] e^{-\int_0^t v dt}
\]

where \( p_f(0) \) is the probability of concrete cracking at time \( t = 0 \) and \( v \) is the mean rate for \( S(t) \geq R(t) \). In most practical problems, the mean upcrossing rate \( v \) is small so that Eq. (6.5) can be approximated as:

\[
p_f(t) = p_f(0) + \int_0^t v dt
\]

(6.6)
The upcrossing rate in Eq. (6.6) can be determined from the Rice formula (Rice, 1944):

\[ v = v_R^+ = \int_{-\infty}^{\infty} (\dot{S} - \dot{R}) f_{SS}(R, \dot{S}) d\dot{S} \]  \hspace{1cm} (6.7)

where \( v_R^+ \) is the mean upcrossing rate of the load effect \( S(t) \) relative to the resistance \( R(t) \), \( \dot{R} \) is the slope of \( R \) with respect to time, \( \dot{S} \) is the time-derivative process of \( S(t) \) and \( f_{SS}() \) is the joint probability density function of \( S(t) \) and \( \dot{S} \). An analytical solution to Eq. (6.7) was derived in Li and Melchers (1993) when \( R \) is deterministic. The solution is:

\[ v = v_R^+ = \frac{\sigma_{\dot{S}|S}}{\sigma_S} \varphi \left( \frac{R - \mu_S}{\sigma_S} \right) \left\{ \varphi \left( -\frac{\dot{R} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \right) - \frac{\dot{R} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \Phi \left( -\frac{\dot{R} - \mu_{\dot{S}|S}}{\sigma_{\dot{S}|S}} \right) \right\} \]  \hspace{1cm} (6.8)

According to the theory of stochastic processes, all variables in Eq. (6.8) can be determined for a given stochastic process with the mean function \( \mu_S(t) \) and autocovariance function \( C_{SS}(t_i, t_j) \) as (Papoulis & Pillai, 2002):

\[ \mu_{\dot{S}|S} = E[\dot{S}|S = R] = \mu_\dot{S} + \rho \frac{\sigma_\dot{S}}{\sigma_S} [R - \mu_S] \]  \hspace{1cm} (6.9)

\[ \sigma_{\dot{S}|S} = \sqrt{\sigma_\dot{S}^2 (1 - \rho^2)} \]  \hspace{1cm} (6.10)

\[ \mu_\dot{S} = \frac{d\mu_S(t)}{dt} \]  \hspace{1cm} (6.11)

\[ \sigma_S = \sqrt{\frac{\partial^2 C_{SS}(t_i, t_j)}{\partial t_i \partial t_j} \bigg|_{i=j}} \]  \hspace{1cm} (6.12)

\[ C_{\dot{S}\dot{S}}(t_i, t_j) = \frac{\partial^2 C_{SS}(t_i, t_j)}{\partial t_i \partial t_j} \]  \hspace{1cm} (6.13)
\[
\rho = \frac{C_{SS}(t_i, t_j)}{\sqrt{C_{SS}(t_i, t_i) \times C_{SS}(t_j, t_j)}}
\]

(6.14)

and the cross-covariance function is:

\[
C_{SS}(t_i, t_j) = \frac{\partial C_{SS}(t_i, t_j)}{\partial t_j}
\]

(6.15)

It is unlikely that corrosion-induced cracking occurs at the beginning of structural service life. Therefore, the probability of serviceability failure at \( t = 0 \) is zero; that is, \( p_f(0) = 0 \). Because the resistance \( R \) is assumed as a constant, the solution to Eq. (6.6) can be expressed, after substituting Eq. (6.8) into Eq. (6.6):

\[
p_f(t) = \int_0^t \left\{ \frac{\sigma_{\tilde{S}|S}(t)}{\sigma_S(t)} \varphi \left( \frac{R - \mu_S(t)}{\sigma_S(t)} \right) \varphi \left( -\frac{\mu_{\tilde{S}|S}(t)}{\sigma_{\tilde{S}|S}(t)} \right) \right. \\
+ \left. \frac{\mu_{\tilde{S}|S}(t)}{\sigma_{\tilde{S}|S}(t)} \Phi \left( -\frac{\mu_{\tilde{S}|S}(t)}{\sigma_{\tilde{S}|S}(t)} \right) \right\} \, dt
\]

(6.16)

The solution in Eq. (6.16) is derived by assuming that the statistical variables follow a normal distribution. However, this may not be appropriate for some cases; assuming a normal distribution may result in unrealistic negative values for some basic variables. Therefore, a lognormal distribution would be more appropriate, as it would eliminate unrealistic negative values for values that are inherently positive, such as physical parameters. Hence, for a lognormal process, \( S \), the joint probability density function of \( S \) and \( \tilde{S} \) (i.e., \( f_{SS}() \)) is expressed as \( f_{SS}(R, \tilde{S}) = f_S(R) \cdot f_{\tilde{S}|S}(\tilde{S}|R) \), where \( f_{\tilde{S}|S} \) is the conditional probability density function of \( \tilde{S} \) given \( S = R \). Thus, Eq. (6.7) can be written—assuming a lognormal distribution—as (Melchers, 1999):

\[
v_R^+(t) = f_{\tilde{S}}[R] \int_R^\infty (\tilde{S}(t) - \check{R}) f_{\tilde{S}|S}(\tilde{S}|R) \, d\tilde{S}
\]

(6.17)
To derive a solution to Eq. (6.17), the stochastic process $S(\cdot)$, can be viewed as a function of two variables $\omega$ and $t$, $S(\omega, t)$. For each fixed $\omega$, a random trajectory of the stochastic process $S$ can be realised over time. The state of stochastic process $S(\omega,t)$ at any given point of time $t$. That is, $S(t)$ is a random variable that follows a lognormal distribution with parameters $\varepsilon(t)$ and $\lambda(t)$; $S(t) \sim \ln[\varepsilon(t), \lambda(t)]$. Thus, the probability density function of $S$ can be expressed as (Papoulis, 2002):

$$f_S[R] = \frac{1}{R\varepsilon(t)} \varphi\left(\frac{\ln[R] - \lambda(t)}{\varepsilon(t)}\right)$$  

(6.18)

where the parameters $\varepsilon(t)$ and $\lambda(t)$ can be written as a function of the mean; that is, $\mu_S(t)$. The standard deviation; that is, $\sigma_S(t)$, of the lognormal random variable $S(t)$ can be expressed as (Papoulis & Pillai, 2002):

$$\varepsilon = \sqrt{\ln\left(\frac{\sigma_S^2}{\mu_S^2} + 1\right)}$$  

(6.19)

$$\lambda = \ln\left(\frac{\mu_S^2}{\sqrt{\mu_S^2 + \sigma_S^2}}\right)$$  

(6.20)

By substituting Eq. (6.18) into Eq. (6.17), the following extended form is yielded:

$$v_R^+(t) = \frac{1}{R\varepsilon(t)} \varphi\left(\frac{\ln[R] - \lambda(t)}{\varepsilon(t)}\right) \int_{\hat{R}}^{\infty} (\hat{S}(t) - \hat{R}) f_{\hat{S}|S}(\hat{S}|R) d\hat{S}$$  

(6.21)

To derive a close form solution for Eq. (6.17), an analytical solution to the integral in Eq. (6.21) must be determined. Based on the theory of stochastic process, the statistic of the derivative process $\hat{S}$ is conditional on any general stochastic process $S$ can be calculated. For a lognormal random variable $\hat{S}$, as per the definition, the variable $u = \frac{\ln(\hat{S}|S) - \lambda_{\hat{S}|S}}{\varepsilon_{\hat{S}|S}}$ follows the standard normal distribution $\varphi(u)$. By substituting...
\( du = \frac{1}{\varepsilon_{\hat{S}|S}} (\hat{S}|S) d\hat{S} \) and \( \hat{S}|S = e^{\left(\varepsilon_{\hat{S}|S} u + \lambda_{\hat{S}|S}\right)} \), the integral in Eq. (6.21) can be expressed as:

\[
\int_{\hat{R}}^{\infty} (\hat{S}(t) - \hat{R} f_{\hat{S}|S}(\hat{S}|R)) d\hat{S} \tag{6.22}
\]

\[
= \int_{\ln(\hat{R}) - \lambda_{\hat{S}|S}}^{\infty} e^{\left(\varepsilon_{\hat{S}|S} u + \lambda_{\hat{S}|S}\right)} \left[ \frac{1}{\hat{S}(t)\varepsilon_{\hat{S}|S}} \varphi(u) \right] \left[ \varepsilon_{\hat{S}|S}(\hat{S}|S) du \right] \\
- \hat{R} \int_{\ln(\hat{R}) - \lambda_{\hat{S}|S}}^{\infty} \varphi(u) du \\
= e^{\left(\varepsilon_{\hat{S}|S}\right)} \left( \int_{\ln(\hat{R}) - \lambda_{\hat{S}|S}}^{\infty} e^{(u)} \varphi(u) du \right) \\
- \left[ \frac{\hat{R} - e^{\left(\lambda_{\hat{S}|S}\right)}}{e^{\left(\varepsilon_{\hat{S}|S}\right)}} \right] \Phi \left\{ - \left[ \ln(\hat{R}) - \lambda_{\hat{S}|S} \right] \right\}
\]

Therefore, to completely derive a close form solution for Eq. (6.17), the main effort lies in determining a solution to another integral in Eq. (6.22). Because \( u = \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \), it follows that:
\[
\int_{\ln(\hat{R})-\lambda_{S|S}}^{\infty} e^{(u)} \varphi(u) du = \int_{\ln(\hat{R})-\lambda_{S|S}}^{\infty} \sqrt{2\pi} e^{-u^2/2} du = \int_{\ln(\hat{R})-\lambda_{S|S}}^{\infty} \sqrt{2\pi} e^{(2u-u^2)/2} du = \sqrt{e} \int_{\ln(\hat{R})-\lambda_{S|S}}^{\infty} \sqrt{2\pi} e^{-v^2/2} dv = \sqrt{e} \left( 1 - \Phi \left( 1 - \frac{\ln(\hat{R}) - \lambda_{S|S}}{\varepsilon_{S|S}} \right) \right)
\]

Therefore, with Eq. (6.22) and (6.23), the solution to Eq. (6.21) is:

\[
v_R^* (t) = \frac{e^{\varepsilon_{S|S}(t)}}{R \varepsilon(t)} \varphi \left( \frac{\ln[R] - \lambda(t)}{\varepsilon(t)} \right) \left[ \sqrt{e} \left( 1 - \Phi \left( 1 - \frac{\ln[\hat{R}] - \lambda_{S|S}(t)}{\varepsilon_{S|S}(t)} \right) \right) \right. \\
\left. - \left( \frac{\hat{R} - e^{\lambda_{S|S}(t)}}{e^{\varepsilon_{S|S}(t)}} \right) \Phi \left( \frac{\ln[\hat{R}] - \lambda_{S|S}(t)}{\varepsilon_{S|S}(t)} \right) \right]
\]

This is the closed form solution to the upcrossing rate in Eq. (6.7), which calculates the upcrossing rate of a lognormal process S over a positively sloped barrier level R. For structural deterioration with a fixed barrier level (i.e., R is a constant that yields \( \hat{R} = 0 \)), this function loses its utility. In practice, it can be assumed that \( \hat{R} = \epsilon \), in which \( \epsilon \) is a small positive real number. The parameters \( \varepsilon_{S|S} \) and \( \lambda_{S|S} \) can be calculated using Eq. (6.19) and Eq. (6.20) as a function of the corresponding mean \( \mu_{S|S} \) and standard deviation \( \sigma_{S|S} \) of conditional stochastic process \( \hat{S}|S \) while these parameters can be determined using Eq. (6.9) to Eq. (6.14). Therefore, all variables in Eq. (6.24) can be determined.
At the beginning of structural service, it can be assumed that the resistance \( R(t) \) is greater than the load effect \( S(t) \) (i.e., probability of failure at \( t = 0 \) is zero). Therefore, by substituting Eq. (6.24) into Eq. (6.6), the probability of failure can be rewritten as:

\[
p_f(t) = \int_0^t \frac{e^{\varepsilon_S(t)}}{R \varepsilon(t)} \varphi \left( \frac{\ln[R] - \lambda(t)}{\varepsilon(t)} \right) \times \left[ \sqrt{\varepsilon} \left( 1 - \Phi \left( 1 - \frac{\ln[\hat{R}] - \lambda S|S(t)}{\varepsilon S|S(t)} \right) \right) \right. \\
\left. - \left\{ \hat{R} - e^{\lambda S|S(t)} \right\} \Phi \left( - \frac{\ln[\hat{R}] - \lambda S|S(t)}{\varepsilon S|S(t)} \right) \right] dt \tag{6.25}
\]

6.2.3 Stochastic Processes

For Eq. (6.25) to be applied to practical corrosion-affected RC structures, a stochastic model for corrosion-induced cracking (i.e., the load effect) \( S \) must be developed. Because corrosion-induced cracking is not only random but also time-variant, it is justifiable to model corrosion-induced cracking as a stochastic process expressed in terms of primary contribution factors, which are treated as basic random variables. It follows that the load effect \( S(t) \) can be modelled as a function of the basic random variables and time and can be expressed as (Li & Melchers, 2005):

\[
S(t) = f(X_1, X_2 \ldots X_n, t) \tag{6.26}
\]

where \( X_1, X_2 \ldots X_n \) are basic random variables, the statistical information of which are (presumed) available. With this treatment, the statistics of \( S(t) \) can be obtained using the MCS. An appropriate sample size in the MCS can be determined such that an acceptable COV for the simulation is achieved. This is done by trial and error, in which, for a given stochastic process, an MCS is run for a random number of sample size \( N \). The sample size increases until the results converge or when an acceptable COV is achieved.
To account for the randomness of load effect, $S(t)$, a random variable $\xi_S$ with an assumed lognormal distribution is introduced. According to Li and Melchers (2005), the random variable $\xi_S$ is defined such that its mean is unity i.e., $E(\xi_S) = 1$ and COV, $\lambda_S$ is constant. Therefore, the load effect $S(t)$ can be expressed as a lognormal stochastic process as:

$$S(t) = S_c(t) \cdot \xi_S$$ (6.27)

where $S_c(t)$ is treated as a pure time function. The mean and auto-covariance functions of $S(t)$ can be determined as (Li & Melchers, 2005):

$$\mu_s(t) = E(S(t)) = S_c(t)E[\xi_S] = S_c(t)$$ (6.28)

$$C_{SS}(t_i, t_j) = \rho \lambda_S^2 S_c(t_i)S_c(t_j)$$ (6.29)

where $\rho$ is the (auto-) correlation coefficient for the load effect $S(t)$ between two points in time $t_i$ and $t_j$. From Eq. (6.28) and (6.29), other statistics of Eq. (6.27) that are used in Eq. (6.25) can be determined based on the theory of stochastic process, as presented in Eq.(6.9) to (6.14).

### 6.3 Limit State Functions

To assess the risk of corrosion-induced failure in RC structures, a performance criterion must be established. The performance criterion for corrosion-induced deterioration of reinforced structures is expressed in the form of a limit state function and is presented in this section. The limit state function may consist of several variables, each corresponding to a probability density function with unique statistical properties.

As discussed, the corrosion process has two phases. The first phase is corrosion initiation, in which the load effect is the ingress of chloride ions $Cl(t)$. This can be calculated using Eq. (6.4). The resistance is the CTL $Cl_{lim}$, in which several different threshold values have been proposed. Therefore, for corrosion-initiation failure, the limit state function can be written as:
For corrosion-induced cracking failure, failure is divided into two parts. The first part is corrosion-induced cover cracking, in which the load effect is corrosion-induced pressure $P_{lim}$ and the resistance is the critical pressure required to cause cover cracking $P_{lim}$. The second part is corrosion-induced crack width, in which the load effect is the corrosion-induced crack width $w(t)$ and the resistance is the acceptable crack width limit $w_c$. The concrete cover must first crack before a crack width can be visible on the surface. Thus, for corrosion-induced cracking failure, the limit state function can be written as:

$$G(P_{lim}, P, t) = P(t) - P_{lim} \quad (6.31)$$

$$G(w_c, w, t) = w(t) - w_c \quad (6.32)$$

The values of $Cl(t)$, $P(t)$, $w(t)$ and $P_{lim}$ depend on specific basic variables, which are presented in Section 6.4. Therefore, the time-dependent probability of failure for each performance criterion of corrosion-affected RC structure can be calculated based on Eq. (6.25).

### Table 6.1: Reformat of Limit State Functions

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>S(t)</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrosion initiation</td>
<td>$Cl(t)$</td>
<td>$Cl_{lim}$</td>
</tr>
<tr>
<td>Corrosion-induced cover cracking</td>
<td>$P(t)$</td>
<td>$P_{lim}$</td>
</tr>
<tr>
<td>Corrosion-induced crack width</td>
<td>$w(t)$</td>
<td>$w_c$</td>
</tr>
</tbody>
</table>

### 6.4 Worked Example

To illustrate the method for predicting the probability of failure for corrosion-affected RC structures, a worked example is conducted using the basic variables listed in Table 6.2.
The service life of a corrosion-affected structure \([0, T_c]\) includes the time to corrosion initiation \(T_i\) and the time to concrete cover cracking \(T_{c1}\) and ends when an acceptable crack width limit is achieved \(T_{c2}\). The time to corrosion initiation \(T_i\) is defined as when the chloride ions at the steel surface reach a threshold amount of initiation corrosion. The two factors affecting initiation time are concrete quality and cover thickness. Higher quality concrete with high cover thickness significantly increases the time taken for chloride ions to diffuse by initiating corrosion and vice versa. Therefore, one of the first and easiest ways to mitigate corrosion in RC structures is to have an optimum design. However, even with proper design, corrosion issues with RC structures still arise as a result of poor construction and management. Because the time to corrosion initiation \(T_i\) is not the focus of this research, it is considered to be zero (i.e., \(T_i = 0\)).

**Table 6.2: Values of Basic Variables**

<table>
<thead>
<tr>
<th>Basic variables</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Distribution (assumed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(D)</strong></td>
<td>32 mm</td>
<td>0.15</td>
<td>Lognormal</td>
</tr>
<tr>
<td><strong>(C)</strong></td>
<td>50 mm</td>
<td>0.2</td>
<td>Lognormal</td>
</tr>
<tr>
<td><strong>(d_0)</strong></td>
<td>0.0125 mm</td>
<td>0.15</td>
<td>Lognormal</td>
</tr>
<tr>
<td><strong>(i_{corr})</strong></td>
<td>0.2 to 1.0 (\mu)A/cm²</td>
<td>0.2</td>
<td>Lognormal</td>
</tr>
<tr>
<td><strong>(E_c)</strong></td>
<td>30100 MPa</td>
<td>0.12</td>
<td>Lognormal</td>
</tr>
<tr>
<td><strong>(f_t)</strong></td>
<td>3.0 MPa</td>
<td>0.3</td>
<td>Lognormal</td>
</tr>
<tr>
<td><strong>(\varphi_{cr})</strong></td>
<td>2.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(v_c)</strong></td>
<td>0.18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(G_f)</strong></td>
<td>0.088 N/mm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(K_{ini})</strong></td>
<td>0.778 MPa.m(^{0.5})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(\alpha_{rust})</strong></td>
<td>0.57</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(\rho_{rust})</strong></td>
<td>3600 kg/m³</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(\rho_{st})</strong></td>
<td>7850 kg/m³</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The second stage is the time to corrosion-induced cover cracking $T_{c1}$, which is defined as when a crack initiates at the steel–concrete interface and propagates to a critical crack depth, resulting in the sudden cracking of the concrete cover. The limit state function for this failure mode is expressed in Eq. (6.31), in which the load effect is $P(t)$ and the resistance is $P_{\text{lim}}$. Using the same concept of the thick wall cylinder (see Figure 6.2), $P(t)$ and $P_{\text{lim}}$ can be derived.

\[ P(t) = \frac{E_{ef}d_s(t)}{a \left[ \frac{b^2 + a^2}{b^2 - a^2 + v_c} \right]} \]  

Figure 6.2: Schematic of a Thick Wall Cylinder

When the reinforcing bar corrodes in concrete, the corrosion products completely fill the pore zone and exert radial pressure on the surrounding concrete. The thickness of corrosion products can be determined as (Liu & Weyers, 1998):

\[
d_s(t) = \left( \frac{2}{\pi} \int_0^{105} \frac{1}{\alpha_{\text{rust}}} \pi D_i(t) dt \right)^{1/2} \left( \frac{1}{\rho_{\text{rust}}} - \frac{\alpha_{\text{rust}}}{\rho_{\text{st}}} \right) \]  

(6.33)

where $\alpha_{\text{rust}}$ is a coefficient related to the type of corrosion products, $\rho_{\text{rust}}$ is the density of corrosion products, $\rho_{\text{st}}$ is the density of steel and $I_{\text{corr}}(t)$ is the corrosion current density ($\mu A/cm^2$), which is a measure of the corrosion rate.

Considering that concrete is a homogenous material and a thick wall cylinder, the pressure $P(t)$ at the corrosion product and concrete interface can be expressed as (Timoshenko & Goodier, 1970):

\[ P(t) = \frac{E_{ef}d_s(t)}{a \left[ \frac{b^2 + a^2}{b^2 - a^2 + v_c} \right]} \]  

(6.34)
where $v_c$ is Poisson’s ratio of the concrete, $E_{ef} = E_c/(1 + \varphi_{cr})$ is the effective elastic modulus of concrete, $E_c$ is the elastic modulus of concrete and $\varphi_{cr}$ is the creep coefficient of concrete.

Over time, as $P(t)$ increases, a corrosion-induced crack will propagate to a critical crack depth. The critical crack depth is defined as the critical point at maximum load upon which the concrete cover suddenly cracks. It is at this point that the crack becomes unstable and instantly propagates to the concrete surface. This is termed the concrete cover load bearing capacity $P_{lim}$, which can be determined based on the following fracture criterion, as discussed in Chapter 3 (Wu, Wu, Zheng, Wu & Dong, 2014):

$$P_{lim} = K_I^P(e) - K_I^C(e) = K_I^{ini}$$  \hspace{1cm} (6.35)

where $K_I^P(e)$ is the stress intensity factor due to applied load, $K_I^C(e)$ is the stress intensity factor due to concrete material’s resistance to cracking and $K_I^{ini}$ is the initial fracture toughness of concrete.

$K_I^P(e)$ can be determined based on the corrosion-induced stress distribution on the crack surface. As the thickness of the corrosion products increase, pressure is exerted on the surrounding concrete, which can be translated to a tangential stress distribution $\sigma_p(x)$ on the crack surface. $K_I^P(e)$ can be determined as (Lau, Fu, Li, De Silva & Guo, 2018):

$$K_I^P(e) = \int_0^e \sigma_p(x)m(x,e)dx$$  \hspace{1cm} (6.36)

$$\sigma_p(x) = P + \frac{Pa^2}{b^2-a^2(a+x)^2}$$  \hspace{1cm} (6.37)

where $m(x,e)$ is the weight function to which the solution is derived in Chapter 3 and $x$ is the coordinate along the crack depth.

In Eq. (6.35), $K_I^C(e)$ can be determined based on the softening behaviour of concrete, as represented by cohesive stress distribution $\sigma_c(x)$ along the crack surface. By
assuming an exponential distribution for $\sigma_c(x)$, $K_f^C(e)$ can be determined as (Gopalaratnam & Shah, 1985; Xu & Reinhardt, 1998):

$$K_f^C(e) = \int_0^e \sigma_c(x)m(x,e)dx \quad (6.38)$$

$$\sigma_c(x) = f_t \exp \left( \frac{f_t}{G_f} w(x) \right) + \left[ f_t - f_t \exp \left( \frac{f_t}{G_f} w(x) \right) \right] \frac{x}{e} \quad (6.39)$$

$$w(x) = -2\pi a [\epsilon^t_\theta(a) - \epsilon^c_\theta(a)] \frac{x}{e} + 2\pi a [\epsilon^t_\theta(a) - \epsilon^c_\theta(a)] \quad (6.40)$$

where $\epsilon^t_\theta(a)$ is the tangential strain at the steel–concrete interface and $\epsilon^c_\theta(a)$ is the cracking strain of concrete at $r = a$, which is determined as (Timoshenko & Goodier, 1970):

$$\epsilon^t_\theta(a) = \frac{1 + v_c}{E_{ef}} \frac{P_{cr}a^2}{b^2 - a^2} \left( 1 - 2v_c \frac{b^2}{a^2} \right) \quad (6.41)$$

$$\epsilon^c_\theta(a) = \frac{f_t - v_c \sigma_r(a)}{E_{ef}}$$

$$\sigma_r(a) = \frac{P_{cr}a^2}{b^2 - a^2} \left( 1 - \frac{b^2}{a^2} \right)$$

From Eq. (6.36) and Eq. (6.38), Eq. (6.35) can now be expressed as:

$$P_{lim} = \int_0^e P + \int_0^e \frac{P_{cr}a^2}{b^2 - a^2} \left( 1 + \frac{b^2}{a^2} \right) \frac{1}{(a+x)^2 \sqrt{2\pi(e-x)}} \left[ 1 + M_1 \left( 1 - \frac{x}{e} \right)^2 + M_2 \left( 1 - \frac{x}{e} \right) + M_3 \left( 1 - \frac{x}{e} \right)^3 \right] dx$$

$$= K_{f \text{ini}}$$
Based on Eq. (6.42), an algorithm has been developed in MATLAB (MathWorks Inc., 2013) to determine the concrete cover load bearing capacity $P_{\text{lim}}$.

When the concrete cover completely cracks, a visible crack width is formed on the surface. The continual growth of corrosion products increases the crack width until it reaches a permissible crack width limit. When this occurs, the structure is considered to have failed. The limit state function for this failure mode is expressed in Eq. (6.32), in which the load effect is the corrosion-induced crack width $w(t)$ and the resistance is the crack width limit $w_c$. Using the same thick wall cylinder concept, the corrosion-induced crack width on the surface of the concrete can be determined as:

$$w(t) = 2\pi b \left[ \varepsilon_{\theta}(b) - \varepsilon_{\theta}^c(b) \right]$$  \hspace{1cm} (6.43)

where $\varepsilon_{\theta}(b)$ is the tangential strain at the concrete surface and $\varepsilon_{\theta}^c(b)$ is the cracking strain, which can be calculated as (Li, Melchers & Zhang, 2006; Timoshenko & Goodier, 1970):

$$\varepsilon_{\theta}(b) = \frac{2d_s(t)/b}{(1 - v_c)(a/b)^{1/\alpha} + (1 + v_c)(b/a)^{1/\alpha}}$$ \hspace{1cm} (6.44)

$$\varepsilon_{\theta}^c(b) = \frac{f_t}{E_{ef}}$$ \hspace{1cm} (6.45)

where $\alpha$ is the tangential stiffness reduction factor, which is related to the average tangential strain at the cracked surface and the concrete properties. Detailed derivation of $\alpha$ can be found in Li, Melchers & Zhang (2006). Using Eq. (6.44) and Eq. (6.45), the corrosion-induced crack width $w(t)$ can be expressed as:

$$w(t) = \frac{4\pi d_s(t)}{(1 - v_c)(a/b)^{1/\alpha} + (1 + v_c)(b/a)^{1/\alpha}} - \frac{2\pi bf_t}{E_{ef}}$$ \hspace{1cm} (6.46)

The acceptable limit for crack width $w_c$ is in the range of 0.1 to 0.5 mm (Andrade, Alonso & Molina, 1993; Thoft-Christensen, 2001; Vu & Stewart, 2002). However, most design codes and standards prescribe a maximum permissible crack width of 0.3 mm for flexural members (American Concrete Institute, 1999; BS, 1997). Therefore, with
$w_c = 0.3\text{mm}$, the probability of serviceability failure due to corrosion-induced crack width assuming a lognormal stochastic process can be determined.

To carry out a time-dependent reliability assessment, an appropriate sample size must be determined for statistical accuracy. To achieve this, the MCS can be used to determine an acceptable COV for the simulation. In this example, the sample size $N = 5000$ was acceptable (see Figure 6.3).

**Figure 6.3: Monte-Carlo Simulation for Different Sample Size**

With the values of basic variables given in Table 6.2 and using the MCS, the mean function $\mu$ and standard deviation $\sigma$ of load effect $P(t)$ and $w(t)$ can be calculated as a function of time $t$. The upcrossing rate can be obtained using Eq. (6.24) for a given auto-correlation coefficient $\rho$, followed by the calculation of the probability of failure (i.e., the concrete cracking) $p_f$ using Eq. (6.25). The results are shown in Figure 6.4 to Figure 6.6. Figure 6.4 shows that the effect of the auto-correlation coefficient $\rho$ of the cracking process for an assumed corrosion rate of 0.2 $\mu\text{A/cm}^2$ between two points in time on cracking failure can be negligible. This may be of practical significance, as $\rho$ is not readily available. Therefore, the assumption of no correlation may not lead to a significant difference. Research (Li & Melchers, 1993) suggests that the assumption of no auto-correlation between different time points leads to larger estimates of the probability of the occurrence of events, which is conservative for the assessment of structural deterioration. Figure 6.5 demonstrates that the higher the corrosion rate, the larger the probability for concrete to crack for a given cover thickness and correlation.
coefficient. Figure 6.6 demonstrates that increasing the cover depth will reduce the probability of concrete cracking, as more time is required for the concrete cover to crack for a given corrosion rate and correlation coefficient.

Figure 6.4: Probability of Failure With Various Auto-correlation ($i_{corr} = 0.2 \mu A/cm^2$)

Figure 6.5: Probability of Corrosion-induced Cover Failure ($\rho = 0.5$)
Figure 6.6: Probability of Corrosion-induced Cover Failure with (\(i_{\text{corr}} = 0.5 \, \mu\text{A/cm}^2\) and \(\rho = 0.5\))

With the probability of failure known over time, the time to corrosion-induced concrete cracking (i.e., \(T_{c1}\)) can be obtained for a given acceptable risk \(p_a\). Figure 6.5 demonstrates that, with a given acceptable probability of failure of \(p_a = 0.1\), it can be determined that \(T_{c1} = 45\) days and \(T_{c1} = 206\) days for a given corrosion rate of \(1.0 \, \mu\text{A/cm}^2\) and \(0.2 \, \mu\text{A/cm}^2\) with \(\rho = 0.5\) and \(C = 50\)mm.

When the concrete cover cracks, a corrosion-induced crack width is formed. As such, a complete picture of the serviceability assessment of the corrosion-affect concrete structure can be determined for a given acceptable risk \(p_a\). Following Figure 6.7, it can be determined that, for a corrosion rate of \(1 \, \mu\text{A/cm}^2\), and given an acceptable probability of failure of \(0.1\), \(T_{c2} \approx 15\) years. This means that, if there are no interventions, such as maintenance or repair, during the service period of up to 15 years, the risk of corrosion-induced failure due to crack width is 0.1. Similar to Figure 6.5, it can be observe that \(i_{\text{corr}}\) is a significant parameter that affects corrosion-induced cracking. Increasing \(i_{\text{corr}}\) from 0.1 to \(1 \, \mu\text{A/cm}^2\) significantly increases the probability of failure from approximately 50 years to 15 years, given that \(p_a = 0.1\). \(i_{\text{corr}}\) can only be obtained from site-specific measurements of the structure.
Therefore, an accurate measure of $i_{corr}$ is essential to accurately predict the serviceability of corrosion-affected concrete structures based on the criteria of cover cracking and crack width.

Figure 6.7: Probability of Corrosion-induced Crack Width ($\rho = 0.5$)

Figure 6.8: Probability of Corrosion-induced Crack Width Failure ($i_{corr} = 1.0 \mu A/cm^2$ and $\rho = 0.5$)
Figure 6.8 shows the effect of concrete cover $C$ on corrosion-induced crack width. An increase in $C$ does not significantly affect the probability of corrosion-induced crack width failure compared to the probability of corrosion-induced cover failure (see Figure 6.6). This is because concrete cover has a more important role for affecting the time to concrete cover cracking, as a crack would need to propagate a further distance for complete cover failure. Hence, once concrete cover fails, the main driving force for increasing corrosion-induced crack width is the corrosion rate $i_{corr}$. As such, after concrete cracks, the effect of concrete cover on the growth of the crack width is negligible. From the analysis, it can be observed that $T_{c1}$ and $T_{c2}$ is of practical importance to structural engineers and asset managers. It will help to provide more information regarding maintenance and rehabilitation strategies, which are usually carried out based on the circumstances of the budget.

6.5 Sensitivity Analysis

Considering the large number of variables that affect corrosion-induced cracking, it is important to identify the most influential variables so that more research can focus on them. This can be achieved by sensitivity analysis, using a probability SI to demonstrate the contribution of each random variable to the probability of concrete cracking. The probability SI $\alpha_i$ can be represented as (Nowak & Collins, 2012):

$$\alpha_i = \frac{-\frac{\partial G}{\partial Z_i}}{\sqrt{\sum_{k=1}^{n} \left(\frac{\partial G}{\partial Z_k}\right)^2}}$$  \hspace{1cm} (6.47)

In Eq. (6.47) the variables can be expressed as:

$$\frac{\partial G}{\partial Z_i} = \frac{\partial G}{\partial X_i} \frac{\partial X_i}{\partial Z_i} = \frac{\partial G}{\partial X_i} \sigma_{x_i}$$  \hspace{1cm} (6.48a)

$$\sum_{i=1}^{n} (\alpha_i)^2 = 1$$  \hspace{1cm} (6.48b)

$$Z_i^* = \beta \alpha_i$$  \hspace{1cm} (6.48c)
In Eq. (6.48) $X_i$ is the random variable ($i = 1, 2, \ldots, n$), $n$ is the number of design variables, $\mu_{X_i}$ and $\sigma(X_i)$ are the mean and standard deviation of the random variable, $X_i$ and $\beta$ is the shortest distance between the origin of the ‘standard form’ variables $Z_i$ and the limit state function. As such, $(Z_1^*, Z_2^*, \ldots, Z_n^*)$ is the design point (also known as the checking point) on the surface of the limit state function. To determine the $2n + 1$ unknowns (i.e., $\alpha_i$, $\beta$ and $Z_i^*$), an iterative method is used. The method requires the following steps:

1) Initialise the design point $\{x_i^*\}$ for $n-1$ of the mean values of the random variables $X_i$.

2) Solve the limit state function $G = 0$ for the remaining random variables.

3) Determine the reduced variates $\{z_i^*\}$ corresponding to the design point $\{x_i^*\}$ using the following equation:

$$z_i^* = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}}$$  \hspace{1cm} (6.49)

4) Calculate the partial derivatives of the $G_i$ with respect to the reduced variates using Eq. (6.48a). For convenience, define a column vector $\{G\}$, as this vector, whose elements are these partial derivatives, is multiplied by $-1$:

$$\{G\} = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{pmatrix} \ \text{where} \ \left. G_i = -\frac{\partial G}{\partial Z_i} \right|_{\text{evaluated at design point}}$$  \hspace{1cm} (6.50)

5) Calculate an estimate of $\beta$ using the following formula:
\[
\beta = \frac{G^T z^*}{\sqrt{G^T G}} \text{ where } \{z^*\} = \begin{bmatrix} z_1^* \\ z_2^* \\ \vdots \\ z_n^* \end{bmatrix}
\]

(6.51)

6) Determine the column vector of the SI using the following equation:

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\vdots \\
\alpha_n
\end{bmatrix} = \frac{G}{\sqrt{G^T G}} (6.52)
\]

7) Determine a new design point for n–1 of the variates \(\{z_i^*\}\) and calculate the original variates according to Eq. (6.49) in step 2, as:

\[
z_i^* = \alpha_i \beta \\
x_i^* = \mu x_i + z_i^* \sigma x_i
\]

(6.53)

8) Repeat the previous steps until convergence is achieved (Nowak & Collins, 2012).

Based on the statistical information provided in Table 6.2, the probability sensitivity indices of different random variables are calculated followings steps 1 to 7. The results are plotted in Figure 6.9 and Figure 6.10 for corrosion-induced cover cracking and the crack width limit state function. Figure 6.9 demonstrates that, out of the five random variables, three demonstrated a negative index. This implies that the probability of concrete cracking decreases. The two variables with a positive index imply that the probability of concrete cracking increases when the values increase. This is logical, in terms of the limit state function in Eq. (6.31) and practical experience, as increasing the random variables D, C and \(d_0\) lead to an increase in the probability of concrete cracking, while increasing \(E_{ef}\) and \(i_{corr}\) decreases the probability of concrete cracking. Figure 6.9 demonstrates that, before 50 days, \(E_{ef}\), \(i_{corr}\) and D have the most influence on the probability of concrete cracking.
However, the importance of \( i_{corr} \) and \( D \) reduces with time, while \( E_{ef} \) increases. It can also be observed that \( C \) and \( d_0 \) have a relatively minor effect on the probability of failure among these variables. From Figure 6.10, out of the six random variables, only two exhibited a positive index, which means that increasing \( i_{corr} \) and \( E_{ef} \) increases the probability of corrosion-induced crack width failure, although the effects of \( E_{ef} \) are small. While it is observed that increasing the diameter of rebar \( D \) and cover \( C \) decreases the probability of corrosion-induced crack width failure. Based on the sensitivity analysis of both limit state functions, it can be deduced that \( i_{corr} \) is the single most significant parameter affecting the probability of failure for corrosion-induced failures. The information in Figure 6.9 and Figure 6.10 has considerable practical significance, as it will guide engineers and asset managers to focus on studying the most influential variables on the probability of corrosion-induced concrete cracking (i.e., cover cracking and crack width).

![Figure 6.9: Change of Probability Sensitivity Index With Time](image)
This chapter proposes a new methodology for predicting the service life of corrosion-affected RC structures. The service life was divided into two parts: the time to corrosion-induced cover cracking and the time to acceptable corrosion-induced crack width. A stochastic model with a non-stationary lognormal process was developed for corrosion-induced cover cracking and corrosion-induced crack width functions. The first passage probability method was employed to predict the time-dependent probability of corrosion-induced cracking from which the time to unserviceability could be determined with confidence. Based on the time-dependent probability of failure for corrosion-induced cover cracking, it was discovered that increasing the corrosion rate led to a greater probability of corrosion-induced cover failure. Increasing the cover depth reduced the probability of corrosion-induced failure. Based on the time-dependent probability of failure for corrosion-induced crack width, a similar trend was discovered, in which increasing the corrosion rate and effective modulus of elasticity increased the probability of failure. Increasing the rebar diameter, tensile strength and cover depth had the opposite effect. Through a sensitivity analysis for both failure modes, it became evident that the corrosion rate is
the single most significant factor affecting the probability of corrosion-induced failure. The proposed methodology can serve as a rational tool for serviceability assessment of corrosion-affected RC structures, with a view to determine the appropriate times for repairs and maintenance.
Chapter 7: Conclusion and Recommendations for Future Work

7.1 Conclusion

The objective of this research is to develop a new model to accurately predict the probability of failure of corrosion-induced concrete cracking using fracture mechanics criteria. The work includes analytical model development, numerical simulations, laboratory corrosion tests and a corrosion-induced failure assessment of RC. Although considerable research has been conducted on corrosion-affected RC, the occurrence of corrosion-induced deterioration has not been effectively prevented. As such, the current understanding of corrosion-induced cracking in concrete remains limited. Most of the reported research is based on empirical and strength-based modelling, rather than fracture mechanics. Further, the analytical investigation into crack propagation using fracture mechanics, and the subsequent failure assessment using time-dependent reliability method, is limited. With these considerations in mind, an analytical model capable of modelling the critical crack depth during corrosion-induced crack propagation in the concrete cover has been developed. Numerical simulations using XFEM has been performed to investigate crack propagation in a thick wall cylinder. An experimental program using the impressed current technique to accelerate corrosion was conducted to verify the developed models and produce corrosion-induced cracking data that can be used by others. The upcrossing method was employed in failure probability prediction by modelling corrosion-induced cracking and crack width as a non-stationary lognormal process. This research provides crucial insight into corrosion behaviour in RC and the affecting factors. It contributes to the understanding of material deterioration and corrosion-induced cracking failure mechanisms, providing guidelines for the repair and maintenance of corrosion-affected RC structures.

Based on the research presented in this thesis, the following conclusions can be drawn:

- An analytical model based on fracture mechanics was developed to predict the critical crack depth and corresponding critical pressure required to cause
corrosion-induced concrete cover cracking. From the analysis, it has been found that increasing concrete cover results in an increase in critical crack depth and critical pressure. It has also been found that for the same geometry, increasing tensile strength only increases the critical pressure. This is because the critical crack depth is a factor of geometrical properties, therefore, increasing the strength would not affect the critical crack depth but only the critical pressure. It has also been found that increasing the rebar diameter results in a decrease in critical pressure required to cause concrete cover cracking.

- A numerical model was developed using the XFEM to model crack propagation in concrete cover. From the analysis, increasing the tensile strength and concrete cover results in an increase in critical pressure required for the crack to completely penetrate the thick wall cylinder model. It has been found that increasing the rebar diameter resulted in a decrease in critical pressure required for the crack to completely penetrate the model. From comparison of the XFEM model and the developed analytical model results, it has been found that the type of softening model such as the use of linear or exponential concrete softening can influence the propagation of cracks and resulting critical pressure.

- An experiment was conducted to measure corrosion-induced time to cracking and crack width. The corrosion process was accelerated by an impressed current technique so that the effects of corrosion could be achieved within a reasonable time frame. It was observed that, using this technique, corrosion is not uniformly distributed. This has been associated with the non-uniform distribution of the porous zone. The thickness of the porous zone ranges from 50 to 100 µm. This means that most models that assume a 12.5 µm uniform thickness distribution overestimate the corrosion-induced time to cover cracking.

- The non-uniform distribution of the porous zone is deduced to also contribute to the variation in time to cover cracking. It has been observed that inducing a
corrosion rate of 300µA/cm² and 100µA/cm² does not imply a time to cracking that is three times faster, as it depends on the distribution and size of porous zone.

- A mass loss test was conducted on exposed reinforcement to analyse the adequacy and accuracy of the impressed current technique to accelerate corrosion. Based on Faraday’s law, the actual corrosion rate was averaged as 172 µA/cm² or 2 mm/year. The actual impressed corrosion rate was 150 µA/cm² or 1.74 mm/year. These results show that the impressed current technique is adequate for accelerating corrosion, although it may result in an overestimation of the corrosion process.

- Corrosion-induced crack width was measured in the experiment and the results were compared with results from an analytical model. The experimental results were lower than the model results. This is attributed to the assumption in the analytical model of a uniform corrosion state and the existence of only a single crack. This has not been the case, in which non-uniform corrosion and severe internal crack patterns were observed.

- In the assessment of the probability of corrosion-induced cracking failure, a time-dependent reliability method (i.e., an upcrossing method) was employed. The corrosion-induced time to cover cracking and crack width models were modelled as a non-stationary lognormal process. An example was undertaken to illustrate the application of the proposed method. The corrosion rate is the single most significant parameter affecting the probability of corrosion-induced serviceability failure. Concrete cover has the most influence on the time to cover cracking failure, compared to corrosion-induced crack width failure. A reliability-based sensitivity analysis was carried out and the time-dependent influence of the basic variables was identified.
7.2 Recommendation for Future Work

- Corrosion-induced pressure has been assumed as uniformly applied to the concrete cylinder. Corrosion may be non-uniformly distributed; that is, corrosion on some sides may be more than other sides. This leads to non-uniform pressure on the concrete. Further research is required to consider the non-uniform corrosion-induced pressure in analytical and numerical solutions.

- The numerical XFEM model only considers the FPZ as linear softening. This may cause a inaccuracy of numerical results. It may be more appropriate to input an exponential softening curve, which requires specific subroutine codes. Alternatively, concrete fracture tests can be carried out to obtain the full stress–strain curve, which can be input into the numerical model. Further research is required to implement a more accurate softening curve to better represent concrete material in the numerical model.

- Both the numerical and analytical models are developed assuming only Mode I fractures. In real service conditions, other deformation modes of fracture can be found and the SIF of corrosion-induced cracking under mixed modes must be investigated. Further, the models should be extended from two to three dimensions to consider longitudinal cracking in RC.

- A full size corrosion test should be conducted, considering more variables such as different water–cement ratios, reinforcement bar diameter and concrete aggregate size, so that a more robust model can be derived.

- In the corrosion-induced failure assessment, the load effect was modelled as a lognormal process. However, in the reliability analysis, the stochastic process can have different distributions. Further research is required to derived analytical solutions based on different statistical distributions to first passage probability; that is, the mean upcrossing rate.

- The mean upcrossing rate has been derived based on the assumption that resistance is constant. In reality, this may not be the case. RC material can
deteriorate due to other mechanisms, such as overloading. Therefore, further research is required to discover a solution to first passage probability, assuming resistance is time-dependent.
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