OPTIMISATION OF PARAMETRIC EQUATIONS FOR SHOCK TRANSMISSION THROUGH SURFACE SHIPS FROM UNDERWATER EXPLOSIONS

A thesis submitted in fulfilment of the requirements for the degree of Master of Engineering

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Declaration

Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of this thesis is the result of work which has been carried out since the official commencement date of the approved research program; and, any editorial work, paid or unpaid, carried out by a third party is acknowledged.

................................................

David James Elder

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<th>Definition</th>
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<tbody>
<tr>
<td>ALE</td>
<td>Arbitrary Lagrangian Eulerian</td>
</tr>
<tr>
<td>ALES</td>
<td>Arbitrary Lagrangian Eulerian with Smoothing</td>
</tr>
<tr>
<td>CAFE</td>
<td>Cavitating Acoustic Finite Element</td>
</tr>
<tr>
<td>CASE</td>
<td>Cavitating Acoustic Spectral Element</td>
</tr>
<tr>
<td>CRC-ACS</td>
<td>Cooperative Research Centre of Advanced Composite Structures</td>
</tr>
<tr>
<td>DAA</td>
<td>Doubly Asymptotic Approximation</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree-of-Freedom</td>
</tr>
<tr>
<td>DSTO</td>
<td>Defence Science and Technology Organisation</td>
</tr>
<tr>
<td>EFG</td>
<td>Element Free Galerkin</td>
</tr>
<tr>
<td>EOS</td>
<td>Equation of State</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>GRP</td>
<td>Glass Reinforced Polymer</td>
</tr>
<tr>
<td>HL</td>
<td>Hand Lay-up</td>
</tr>
<tr>
<td>JWL</td>
<td>Jones Wilkins Lee</td>
</tr>
<tr>
<td>KSF</td>
<td>Keel Shock Factor</td>
</tr>
<tr>
<td>MMALE</td>
<td>Multi-Material ALE</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NPL</td>
<td>National Physical Laboratory</td>
</tr>
<tr>
<td>ppt</td>
<td>Parts per thousand (by weight)</td>
</tr>
<tr>
<td>PLIC</td>
<td>Piecewise Linear Interface reConstruction</td>
</tr>
<tr>
<td>RMIT</td>
<td>Royal Melbourne Institute of Technology</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single Degree-of-Freedom</td>
</tr>
<tr>
<td>SMALE</td>
<td>Single Material ALE with void</td>
</tr>
<tr>
<td>SPH</td>
<td>Smooth Particle Hydrodynamics</td>
</tr>
<tr>
<td>SRI</td>
<td>Selectively Reduce Integration</td>
</tr>
<tr>
<td>TNT</td>
<td>Tri-Nitro-Toluene</td>
</tr>
<tr>
<td>UI</td>
<td>Under Integrated</td>
</tr>
<tr>
<td>USA</td>
<td>Underwater Shock Analysis</td>
</tr>
<tr>
<td>UNDEX</td>
<td>UNDERwater EXPlosion</td>
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### Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNESCO-IOC</td>
<td>United Nations Educational, Scientific and Cultural Organization-Intergovernmental Oceanographic Commission</td>
</tr>
<tr>
<td>UNO</td>
<td>Unless Noted Otherwise</td>
</tr>
<tr>
<td>USA</td>
<td>Underwater Shock Analysis</td>
</tr>
<tr>
<td>VBRI</td>
<td>Vacuum Bag Resin Infusion</td>
</tr>
<tr>
<td>WWFE</td>
<td>World Wide Failure Exercise</td>
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</table>
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Constant parameters in the JWL equation of state</td>
</tr>
<tr>
<td>A1</td>
<td>Pressure coefficient for similitude equations</td>
</tr>
<tr>
<td>A2</td>
<td>Constant coefficient for similitude equations</td>
</tr>
<tr>
<td>A3</td>
<td>Impulse coefficient for similitude equations</td>
</tr>
<tr>
<td>A_{EFF,R}</td>
<td>Effective area for velocity retardation pressure</td>
</tr>
<tr>
<td>A_H</td>
<td>Effective hull or plate area</td>
</tr>
<tr>
<td>B</td>
<td>Hull beam</td>
</tr>
<tr>
<td>B_{MOD}</td>
<td>Bulk Modulus</td>
</tr>
<tr>
<td>C</td>
<td>Acoustic speed of sound</td>
</tr>
<tr>
<td>C_0</td>
<td>Bulk viscosity constant</td>
</tr>
<tr>
<td>C_1</td>
<td>Bulk viscosity constant</td>
</tr>
<tr>
<td>D</td>
<td>Hull draft</td>
</tr>
<tr>
<td>D_c</td>
<td>Depth of charge</td>
</tr>
<tr>
<td>D_{V}</td>
<td>Detonation velocity</td>
</tr>
<tr>
<td>E</td>
<td>Isotropic modulus of elasticity or the energy term for the Gruneisen and JWL EOS</td>
</tr>
<tr>
<td>E_0</td>
<td>Initial internal energy</td>
</tr>
<tr>
<td>E_{11} and E_{22}</td>
<td>Orthotropic in-plane moduli of elasticity</td>
</tr>
<tr>
<td>E_{33}</td>
<td>Orthotropic out-of-plane modulus of elasticity</td>
</tr>
<tr>
<td>E_{CoG}</td>
<td>Eccentricity between load application and hull C of G</td>
</tr>
<tr>
<td>E_{FD}</td>
<td>Energy flux density</td>
</tr>
<tr>
<td>E_{Q}</td>
<td>LS-Dyna element formulation</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>F_{MAX}</td>
<td>Peak Force</td>
</tr>
<tr>
<td>F_{RET}</td>
<td>Retardation force on hull due to hull velocity</td>
</tr>
<tr>
<td>G</td>
<td>Isotropic shear modulus</td>
</tr>
<tr>
<td>G_{11}, G_{22} &amp; G_{33}</td>
<td>Orthotropic shear moduli</td>
</tr>
<tr>
<td>G_{MA}</td>
<td>Global Mean Acceleration</td>
</tr>
<tr>
<td>G_{MA,D}</td>
<td>Global Mean Acceleration for a hull with D &gt; H</td>
</tr>
<tr>
<td>H</td>
<td>Hull depth (V and circular hulls only)</td>
</tr>
<tr>
<td>H_{HP}</td>
<td>Hull plate thickness</td>
</tr>
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**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$h$</td>
<td>Enthalpy</td>
</tr>
<tr>
<td>$I$</td>
<td>Impulse</td>
</tr>
<tr>
<td>$I_{\text{LS-DYNA}}$</td>
<td>Impulse from LS-Dyna simulation</td>
</tr>
<tr>
<td>$I_{\text{SIMILITUDE}}$</td>
<td>Impulse from Similitude equation</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Shock wave coefficient for similitude equations</td>
</tr>
<tr>
<td>$K_2$</td>
<td>Decay coefficient for similitude equations</td>
</tr>
<tr>
<td>$K_3$</td>
<td>Impulse coefficient for similitude equations</td>
</tr>
<tr>
<td>$K_4$ to $K_{11}$</td>
<td>General equation constants</td>
</tr>
<tr>
<td>$L_F$</td>
<td>Effective horizontal section of a circular hull</td>
</tr>
<tr>
<td>$L_{\text{MF}}$</td>
<td>Effective vertical section of a circular hull</td>
</tr>
<tr>
<td>$M$</td>
<td>Hull mass</td>
</tr>
<tr>
<td>$M_D$</td>
<td>Hull mass for a hull with $D &gt; H$</td>
</tr>
<tr>
<td>$N'$</td>
<td>Local normal shock wave coordinate system</td>
</tr>
<tr>
<td>$P$</td>
<td>Shock wave pressure</td>
</tr>
<tr>
<td>$P_A$</td>
<td>Peak Acceleration</td>
</tr>
<tr>
<td>$P_{A_D}$</td>
<td>Peak Acceleration for a hull with $D &gt; H$</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Cavitation pressure</td>
</tr>
<tr>
<td>$P_{\text{Eff}}$</td>
<td>Effective shock wave reflected pressure</td>
</tr>
<tr>
<td>$P_{JC}$</td>
<td>Detonation pressure at the Chapman-Jouget in the JWL EOS</td>
</tr>
<tr>
<td>$P_{\text{MAX}}$</td>
<td>Peak shock wave pressure</td>
</tr>
<tr>
<td>$P_{\text{RET}}$</td>
<td>Effective retardation pressure due to hull velocity</td>
</tr>
<tr>
<td>$P_{\text{TV}}$</td>
<td>Peak Translation Velocity (or kick of velocity)</td>
</tr>
<tr>
<td>$P_{\text{TV_D}}$</td>
<td>Peak Translation Velocity for a hull with $D &gt; H$</td>
</tr>
<tr>
<td>$R$</td>
<td>Charge radius</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Reference pressure for tabulated EOS</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Constant parameters in the JWL equation of state</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Constant parameters in the JWL equation of state</td>
</tr>
<tr>
<td>$R_{\text{HULL}}$</td>
<td>Hull radius</td>
</tr>
<tr>
<td>$r$</td>
<td>Shock wave radius measured from the charge to the shock wave front</td>
</tr>
<tr>
<td>$r_1$ and $r_2$</td>
<td>Shock wave front radii at points 1 and 2</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Shock wave radius within the shock wave</td>
</tr>
<tr>
<td>$S$</td>
<td>Water salinity (ppt)</td>
</tr>
<tr>
<td>$S_1$</td>
<td>Coefficient for Gruneisen equation</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Coefficient for Gruneisen equation</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>S3</td>
<td>Coefficient for Gruneisen equation</td>
</tr>
<tr>
<td>T</td>
<td>Water temperature</td>
</tr>
<tr>
<td>T'</td>
<td>Local tangential shock wave coordinate system</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>t_A</td>
<td>Time to zero pressure from cavitation</td>
</tr>
<tr>
<td>t_FINAL</td>
<td>Time at which PTV is achieved</td>
</tr>
<tr>
<td>t_o</td>
<td>Time at first contact of shock wave</td>
</tr>
<tr>
<td>t_S</td>
<td>Total time for duration of shock load</td>
</tr>
<tr>
<td>t_Y</td>
<td>Time taken for the shock wave front to travel from the bottom of the hull to the water surface (i.e. $t_Y = D/C$ or $R_{hull}/C$) at zero slant</td>
</tr>
<tr>
<td>U</td>
<td>Flow velocity measured in the shock waves frame of reference</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
</tr>
<tr>
<td>ρ_o</td>
<td>Initial density (or density of un-reacted explosive)</td>
</tr>
<tr>
<td>V_o</td>
<td>Initial volume</td>
</tr>
<tr>
<td>V_R</td>
<td>Ratio of charge surface area to spherical charge ($V_R = 1$ Sphere, $V_R = 0.72$ Cubic) in JWL EOS</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
</tr>
<tr>
<td>v_{12}, v_{31}, v_{32}</td>
<td>Orthotropic Poisson’s ratios</td>
</tr>
<tr>
<td>v_{iso}</td>
<td>Isotropic Poisson’s ratio</td>
</tr>
<tr>
<td>υ'</td>
<td>Acceleration</td>
</tr>
<tr>
<td>v_p</td>
<td>Particle velocity</td>
</tr>
<tr>
<td>v_{PLATE}</td>
<td>Plate velocity</td>
</tr>
<tr>
<td>v_s</td>
<td>Shock wave velocity</td>
</tr>
<tr>
<td>v_{si}</td>
<td>Shock wave velocity of incident wave</td>
</tr>
<tr>
<td>v_{sr}</td>
<td>Shock wave velocity of reflected wave</td>
</tr>
<tr>
<td>w</td>
<td>Bulk density shape function</td>
</tr>
<tr>
<td>W</td>
<td>TNT charge weight</td>
</tr>
<tr>
<td>δρ</td>
<td>Change in density</td>
</tr>
<tr>
<td>δt</td>
<td>Change in time</td>
</tr>
<tr>
<td>δV</td>
<td>Change in volume</td>
</tr>
<tr>
<td>µ</td>
<td>Gruneisen EOS parameter</td>
</tr>
<tr>
<td>µ_D</td>
<td>Dynamic viscosity</td>
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>Constant parameter in the JWL equation of state</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Charge slant angle</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>Relative angle of incident shock wave for area 1</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>Relative angle of reflected shock wave for area 2</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Orthotropic material direction</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>Relative angle of incident shock wave for area 1</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>Relative angle of reflected shock wave for area 2</td>
</tr>
<tr>
<td>( 2\alpha )</td>
<td>V hull angle</td>
</tr>
<tr>
<td>( \psi_i )</td>
<td>Angle of incident shock wave</td>
</tr>
<tr>
<td>( \psi_r )</td>
<td>Angle of reflected shock wave</td>
</tr>
</tbody>
</table>

**Superscript** | **Definition**
--- | ---
X | X direction
Y | Y direction

**Subscript** | **Definition**
--- | ---
0 | Initial
1 | Intermediate
2 | Final
i | Incident
r | Reflected
t | Time
y | Vertical direction
x | Horizontal direction
Summary

This research has been undertaken to better understand the effect of hull shape on surface ships’ shock response to external UNDerwater EXplosions (UNDEX). A set of simple closed-form equations has been developed that accurately predicts the magnitude of dynamic excitation of different 2-D rigid-hull shapes subject to far-field UNDEX events. This research was primarily focused on the affects of 2-D rigid hull shapes and their contribution to global ship motions. A section of the thesis, “T-Joint”, considers the exacerbating affects that shock wave propagation has on a typical Glass Reinforced Polymer (GRP) laminated ship T-Joint with respect to its strength and the transmission of the shock to the adjacent bulkhead. This research considered the affects and sensitivities to the following variables:

- Hull motions
  - Charge mass
  - Charge stand-off and slant angle
  - Bulk cavitation
  - Local cavitation
  - Hull shape and mass
  - Ocean sea water properties
- GRP T-Joint
  - Joint material properties
  - Joint geometry
  - Strain rate effects

The hull motion parametric equations developed in this research are compared against computational fluid/structure interaction predictions obtained from non-linear, explicit Finite Element (FE) simulations using the LS-Dyna code. The equations are shown to predict the vertical acceleration and velocity of four basic hull shapes to within approximately ±15% of the FE model results. Addition error estimates obtained for sensitivity analyses, predicted that the LS-Dyna simulations were accurate to within ±11% when compared to real UNDEX events.
Summary

The resultant error of the closed-form solutions compared to real UNDEX events is the summation of the two error estimates at ±26%.

A number of different GRP T-Joint geometries were ranked with respect to their capability of withstanding UNDEX shock loading, the study included three basic geometries, allowing for the affects of non-monolithic construction due to jointing methods and material strain rate considerations. The study concluded that two of the joint geometries: the 45° chamfered and the 40 mm fillet preformed significantly better than the 22.5° chamfered geometry.
1. Introduction

1.1. General

The topic of this research was proposed by Defence Science and Technology Organisation (DSTO) in conjunction with the Cooperative Research Centre of Advanced Composite Structures (CRC-ACS), and sponsored by the Royal Melbourne Institute of Technology (RMIT) and the CRC-ACS. A set of closed-form solutions has been developed that predicts rigid hull motions of surface ships with various rigid hull geometries excited by an UNDERwater EXplosion (UNDEX). The thesis also addresses the affects of shock on typical bulkhead joints that are often used in Glass Reinforced Polymer (GRP) minehunter hulls.

The main reason for undertaking this research is to better understand and quantify the effect of basic hull shape on a ship’s rigid-body response to an UNDEX event. Such knowledge could allow a better prediction of a vessels response to an UNDEX using simple empirical based models and algorithms.

Hull motion estimates are based on computer simulations using the proprietary explicit finite element code LS-Dyna produced by the Livermore Software Technology Corporation.

1.2. Exclusions

The following considerations are not addressed in this thesis:

- Non-rigid hull dynamics
- Explosion bubble dynamics
- Charge casing effects
- Charge shape effects
- Near-field effects
- Seabed proximity effects
Introduction

1.3. Research Questions

The research questions proposed (and brief answers) are as per below:

**Question 1:** With respect to equipment and structural damage estimates within a ship subject to a shock loading, what motion characteristics (i.e. velocity and acceleration) are required and how are they to be expressed?

**Answer:** For the hull shapes considered, three types of characteristic motions were determined to be appropriate: linear, non-linear, and bi-linear.

**Question 2:** What form of parametric equation fits the computed data?

**Answer:** Two methods were developed: a simplistic approach (tier one) based on a non-dimensional approach and a complex approach (tier two) based on a classic differential equation solution.

**Question 3:** What are the methods for error estimation associated with hull motions?

**Answer:** The literature review identified statistical variations in charge and water property variables. These variations were stochastically analysed to determine their importance in the determination of hull excitation.

**Question 4:** With respect to a typical GRP hull-to-bulkhead structural joint, what are the mechanisms of shock transfer and what joint attributes minimise the detrimental effects of the shock loading?

**Answer:** The joint loading is considered to occur in three distinct phases: initial shock, reflected shock, and the hull-plate bending phase. Each loading phase develops characteristic stress regimes that are affected by the span of the hull-plate and the joint’s geometry. The 22.5° chamfered joint was found to be the least capable of resisting UNDEX loading when compared to the alternate geometries of the 45° chamfered and 40 mm filleted joints.
2. Aim

2.1. Rigid Hull Excitation

2.1.1. General

This research aimed to develop a set of closed-form equations that accurately predicts the magnitude of dynamic excitation of a rigid hull subject to an external UNDEX. A requirement for the closed-form equations is that they are easy to use. For this, the tier one equations were developed. However to investigate the possibility of developing equations of greater accuracy an alternate tier was considered; both tiers are discussed below:

- Tier one develops equations that predict salient attributes on the hull’s motion, such as Peak Translational Velocity (PTV), Peak Acceleration (PA) and Global Mean Acceleration (GMA). These solutions are semi-empirical and, due to their simplicity, are suitable for hand calculation.

- Tier two develops equations that predict the hull’s transient velocity. The equations are based on a semi-empirical solution to a modified Taylor’s differential equation solution.

Available from the current body of knowledge (using Finite Element (FE) analysis and shock trial results), it is known that the majority of a ship’s motion produced from an UNDEX is vertical, even if an oblique loading is encountered where the charge is not located directly under the ship. The reasons for this phenomenon will be discussed in a later section of the thesis, and accordingly, only vertical hull motions are considered in this research. However, relevant discussion on the effects of rotational and horizontal hull motions will be provided where required.
2.1.2. Hull Shapes Considered

The four hull shapes considered and the adopted coordinate system are as per Figure 2-1.

![Diagram of hull shapes: (a) rectangular, (b) circular, (c) & (d) V hulls]

The limits for hull dimensions are as per Table 2-1.

<table>
<thead>
<tr>
<th>Item</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hull Beam</td>
<td>B</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>Hull Draft</td>
<td>D</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>V Angle</td>
<td>$2\alpha$</td>
<td>45</td>
<td>90</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>1,023</td>
<td>1,023</td>
</tr>
</tbody>
</table>

*Figure 2-1: Hull shapes considered (a) rectangular, (b) circular, (c) & (d) V hulls*
2.1.3. Charge Limits

The geometric limits of the charge location for the research are as per Figure 2-2 and Table 2-2.

\[ \phi = 45^\circ \text{ for } 90^\circ \text{ V and } 22.5^\circ \text{ for } 45^\circ \text{ V hull} \]

Figure 2-2: Limits of $\phi$ for (a) rectangular and circular and (b) V hulls
Table 2-2: Limits of charge mass and location

<table>
<thead>
<tr>
<th>Item</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tri-Nitro-Toluene (TNT)</td>
<td>W</td>
<td>50</td>
<td>1,000</td>
</tr>
<tr>
<td>Charge stand off</td>
<td>R</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>Slant angle</td>
<td>$\phi$</td>
<td>0</td>
<td>45</td>
</tr>
</tbody>
</table>

2.1.4. General Methodology Used for Closed-Form Solutions

A deliverable of this research is a set of closed-form solutions that predicts the rigid body response of a surface ship to far-field UNDEX shock. Two tiers were developed: the first and simpler is a semi-empirical solution that predicts the critical transient attributes of the hull’s motions. The second tier is based on Taylor’s [1] first-order linear initial value differential equation that predicts the hull’s velocity at any time.

Both tiers simplify the hull dynamics to a Single Degree-of-Freedom (SDOF) system with the excitation shock pressure being a function from the similitude equations. The following effects are considered in this research:

- Charge mass
- Charge location
- Time
- Hull shape
- Hull size
- Bulk cavitation
- Instantaneous hull velocity
- Ocean sea water properties
- Local cavitation
Aim

2.1.5. Tier One Methodology

This approach characterises the transient velocity of each hull shape as being one of three curves; linear, bi-linear and non-linear. The three characteristic velocity shapes are described below, and are diagrammatically shown in Figure 2-3:

- Linear description of velocity has a relative constant acceleration over the duration of the shock excitation.
- Bi-linear description of velocity has a high acceleration over approximately the first 20% of the excitation duration followed by a much reduced acceleration.
- Non-linear description of velocity has the peak acceleration at the beginning of the excitation; this progressively reduces over the shock duration.

The approach also determines the salient motion variables of PTV, the time at which it occurs ($t_{\text{FINAL}}$), GMA and PA. PA and GMA are defined by Eqns (2-1) and (2-2).

$$PA = Peak\ slope\ of\ transient\ velocity\ curve \quad (2-1)$$

$$GMA = \frac{PTV}{t_{\text{FINAL}}} \quad (2-2)$$

The equation form used to determine PTV, PA and GMA is semi-empirical. The general form is shown in Eqn (2-3) for PTV where $K_4$ to $K_7$ are constants for the hull in question, $P_{\text{MAX}}$ and $\theta$ are shock wave parameters and $D$ is the hull draft.

$$PTV = (K_4)\cdot\left(\frac{P_{\text{MAX}}}{34.44}\right)^{K_5}\cdot\left(\frac{\theta}{0.9}\right)^{K_6}\cdot\left(\frac{12}{D}\right)^{K_7} \quad (2-3)$$
2.1.6. Tier Two Methodology

Tier two considers the use of a modified differential equation as solved by Taylor [1] for an air backed flat plate excited by a UNDEX. Taylor’s solution considers a plate excited by a shock wave as per Figure 2-4 (a). Taylor represents this as an exponential force with a viscous damper reducing the effective pressure due to the plate velocity as per Figure 2-4 (b). The initial excitation pressure (at t=0 and v=0) is twice that of the shock wave pressure due to the reflection of the wave. As the plate increases in velocity the excitation pressure wave is reduced by Eqn (2-4), where $P_{RET}$ is the reduction in pressure, $\rho$ is the water density, $C$ is the speed of sound, $A_H$ is the plate area and $v_{PLATE}$ is the instantaneous plate velocity.

$$P_{RET} = \rho \cdot C \cdot v_{PLATE} \quad (2-4)$$
Taylor’s simplification of an air backed plate excited by a shock wave

Using Newton’s third law of motion, the net force acting on the plate is equal to the shock force \( F \) subtracted from the retardation force \( F_{\text{RET}} \). It then follows that the acceleration at any time is as per Eqn (2-5) where \( v' \) is the plate acceleration, \( t \) is the time and \( M \) is the plate mass.

\[
v' = \left( \frac{2 \cdot P_{\text{MAX}} \cdot e^{-\frac{t}{H}} - \rho \cdot C \cdot v_{\text{PLATE}}}{M} \right) \cdot A_H
\]

\[(2-5)\]

Taylor solved Eqn (2-5) resulting in the closed form solution for the transient plate velocity \( \nu \) as per Eqn (2-6).
Aim

\[ v_{\text{PLATE}} = \left( \frac{P_{\text{MAX}} \cdot A_H}{M} \right) \left( \frac{e^{-\omega t} - e^{-\theta t}}{1 - \omega} \right) \]  

(2-6)

Where:

\[ \omega = \left( \frac{\rho \cdot C \cdot A_H}{M} \right) \]

In Tier two the possibility of using a modified Taylor type solution for determining the transient velocity of the four hull shapes was considered. This approach involved the review of alternate forcing functions for the hull excitation as shown in Figure 2-5; where K8 to K11 are hull dependent constants. The original Taylor solution used the excitation force as per Figure 2-5 (a), the alternate excitation forces (Figure 2-5 (b), (c) and (d)) and their application to the different hull geometries is detailed in Table 2-3.

![Figure 2-5: Alternate excitation functions for tier two](image)

*Figure 2-5: Alternate excitation functions for tier two*
Aim

Table 2-3: Hull shape and corresponding forcing function considered

<table>
<thead>
<tr>
<th>Hull shape</th>
<th>Slant angle</th>
<th>Forcing function</th>
<th>Forcing function description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>0°</td>
<td>(a)</td>
<td>Exponential 1</td>
</tr>
<tr>
<td>Rectangular</td>
<td>45°</td>
<td>(c)</td>
<td>Parabolic</td>
</tr>
<tr>
<td>Circular</td>
<td>0°</td>
<td>(b)</td>
<td>Exponential 2</td>
</tr>
<tr>
<td>Circular</td>
<td>45°</td>
<td>(b)</td>
<td>Exponential 2</td>
</tr>
<tr>
<td>V 90°</td>
<td>0°</td>
<td>(c)</td>
<td>Parabolic</td>
</tr>
<tr>
<td>V 90°</td>
<td>45°</td>
<td>(d)</td>
<td>Exponential + parabolic</td>
</tr>
<tr>
<td>V 45°</td>
<td>0°</td>
<td>(c)</td>
<td>Parabolic</td>
</tr>
<tr>
<td>V 45°</td>
<td>67.5°</td>
<td>(d)</td>
<td>Exponential + parabolic</td>
</tr>
</tbody>
</table>

2.2. T-Joint Sensitivity to Shock Loading

A review is presented on the propagation and mitigation of a shock wave through a typical GRP composite ship T-joint, as per Figure 2-6. The joint was reviewed to determine its strength sensitivity to the attributes of joint geometry and material strain rate effects.

Figure 2-6: Section through typical T-Joint
Investigations were undertaken in LS-Dyna to determine the joint’s sensitivity to the following attributes:

- Joint fillet geometry
- Strain rate effects
- Joint separation distance
- The inclusion of voids
- Shock intensity

### 2.3. Units and LS-Dyna Models

All units used in the thesis are as per Table 2-4. The LS-Dyna models ranged in size from unit cell to 15,000 elements with computation times up to 1 hr 30 minutes on a 1.8 GHz XP Windows computer. The relevant LS-Dyna models (203 models) are presented on the attached CD-ROM (or softcopy attachment) and the file name summary and description is as per Appendix D.

### Table 2-4: Adopted units

<table>
<thead>
<tr>
<th>Equation type</th>
<th>Length</th>
<th>Time</th>
<th>Mass</th>
<th>Force</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>All LS-Dyna models and general equations except as noted below</td>
<td>metre (m)</td>
<td>second (s)</td>
<td>kilogram (kg)</td>
<td>Newton (N)</td>
<td>Pascal (Pa)</td>
</tr>
<tr>
<td>Tier one equations and the Similitude equations</td>
<td>metre (m)</td>
<td>millisecond (ms)</td>
<td>kilogram (kg)</td>
<td></td>
<td>MegaPascal (MPa)</td>
</tr>
</tbody>
</table>
3. Literature Review

3.1. General

No single reference was found that addressed the specific aim of this research. The research into shock and its effect on structures allowed solutions for this thesis to be developed from first principles.

3.2. Principles of Shock Waves and their Interaction with Structures

3.2.1. Explosions

The chemical reaction within the TNT charge converts the solid charge material into a gas, with instantaneous temperatures in the order of 3000°C and pressures measured in hundred of thousands of atmospheres (Shin [2]). At this initial stage in the explosion, the conditions within the charge evolve in a very non-linear manner. Once the shock wave propagates into the water and shock wave speeds are reduced to about Mach 1.2 (Swisdak [3]), the shock wave properties become linear enough for dimensional scaling and acoustic theory to be used for shock wave modelling.

The sudden release of energy by the explosion gives a radial velocity to the adjacent water particles (molecules); the elastic collision between particles transfers the momentum from one particle to the next, propagating a compression shock wave in a radial direction. The actual particle motion is a direct representation of the specific kinetic energy per cubic metre at any point within the wave.

The energy contained in the charge is dissipated into three areas: shock wave energy, gas bubble energy, and the energy absorbed by the plastic deformation of the casing. Reid [4] indicates that the energy split for an uncased charge is approximately 53% of the charge energy being associated with the shock wave and 47% with the gas bubble. Jones and Northeast [5] show that steel casing the charge results in approximately 4% of the energy being consumed by the plastic deformation of the steel casing, while the bubble energy is reduced by 9%. This increases the shock wave energy by approximately 5%.

The affect of the proximity on the seabed will also significantly affect the shock wave intensity seen by a surface ship. The shock wave reflects off the seabed and back onto the
hull, the increase in energy is a function of the proximity and material of the seabed. A hard rock seabed will effectively reflect more energy, while a soft mud seabed will dissipate energy through plastic deformation of the mud. These effects are covered in Swisdak [3].

3.2.2. Shock Wave Front

The shock front is the distance that the pressure increases from ambient to the maximum shock pressure at the wave front. Cole [6] indicates that this distance is in the order of $10^{-7}$ to $10^{-8}$ m and is completely negligible. In essence, the shock front is a discontinuity and therefore may introduce singularities into a numerical analysis by means of a near infinite rate of change of pressure over the shock front distance. Often this results in spurious oscillations being developed within a numerical FE analysis. To remedy this problem, numerical codes such as LS-Dyna use an artificial bulk viscosity material attribute (Hallquist [7]); this is specifically designed to place an artificial quadratic slope on a large pressure gradient. The bulk viscosity achieves the reduction in slope by dissipating a small amount of energy over the shock front. The magnitude of energy may be determined by subtracting the total energy from the internal energy to confirm that the bulk viscosity energy loss is not a significant percentage of the total energy. The default value of the bulk viscosity used in LS-Dyna is as per Table 3-1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk viscosity</td>
<td>$C_0=1.5$ &amp; $C_1=0.06$</td>
<td>Dimensionless</td>
<td>Hallquist [8]</td>
</tr>
</tbody>
</table>

3.2.3. Shock Wave Reflection

The topic for shock wave reflection is definitively covered by Ben-Dor [9] and Ben-Dor et al. [10]. The mathematical treatment of shock wave and wave reflection can be broadly divided into two sub-divisions of regular and irregular. Weak shock waves with regular reflection occur at low pressures. Weak and strong shock waves with irregular reflection occur at high pressures or with compressible materials and are generally associated with aeronautical compressible shocks and fluid flows. The characterisation of shock waves are shown in Figure 3-1 as per Ben-Dor et al. [10]. During the compression
of the material at the shock front, the friction of the molecules increases the temperature. If significant energy (in the form of temperature) is able to migrate out of the wave front into the surrounding material, the wave is defined as a strong shock wave. In low pressure water shock, this is not the case and the waves are defined as an “isentropic” process that is thermodynamically reversible (or reversible adiabatic). The formulations for the two most common shock wave classifications, the two-shock, and the three-shock theory, are considered in Sections 3.2.3.1 and 3.2.3.2.
3.2.3.1. Two-Shock Theory

Ben-Dor [9] considers the case of an oblique incident wave reflecting off a rigid boundary A-B as per Figure 3-2. All velocity measurements are relative to the local coordinate system N'T', (Normal and tangential) that is attached to the incident shock wave. Using the conservation of mass, momentum, and energy, four equations can be written for both the incident and reflected waves as per Eqns (3-1) to (3-8). Where the shock velocity
incident velocity is $v_{si}$, reflected velocity is $v_{sr}$, angle of incidence is $\psi_i$, angle of reflection is $\psi_r$, local velocity attached to the T’N’ coordinate system is $U$, and wave angles in areas 1 and 2 are $\theta$ and $\phi$.

**Figure 3-2: Two-shock theory**

Conservation of mass across the incident wave, i:

$$\rho_0 \cdot U_0 \cdot Sin(\phi_i) = \rho_1 \cdot U_1 \cdot Sin(\phi_1 - \theta_i) \quad (3-1)$$

Conservation of normal momentum across the incident wave, i:

$$P_0 + \rho_0 \cdot U_0^2 \cdot Sin(\phi_i) = P_1 + \rho_1 \cdot U_1^2 \cdot Sin(\phi_1 - \theta_i) \quad (3-2)$$

Conservation of tangential momentum across the incident wave, i:

$$\rho_0 \cdot Tan(\phi_i) = \rho_1 \cdot Tan(\phi_1 - \theta_i) \quad (3-3)$$

Conservation of energy across the incident wave, i:

$$h_0 + \left(\frac{1}{2}\right) \cdot U_0^2 \cdot Sin^2(\phi_i) = h_1 + \left(\frac{1}{2}\right) \cdot U_1^2 \cdot Sin^2(\phi_1 - \theta_i) \quad (3-4)$$

Conservation of mass across the reflected wave, r:

$$\rho_1 \cdot U_1 \cdot Sin(\phi_2) = \rho_2 \cdot U_2 \cdot Sin(\phi_2 - \theta_2) \quad (3-5)$$

Conservation of normal momentum across the reflected wave, r:

$$P_1 + \rho_1 \cdot U_1^2 \cdot Sin(\phi_2) = P_2 + \rho_2 \cdot U_2^2 \cdot Sin(\phi_2 - \theta_2) \quad (3-6)$$

Conservation of tangential momentum across the reflected wave, r:
Literature Review

\[ \rho_1 \cdot \tan(\phi_2) = \rho_2 \cdot \tan(\phi_2 - \theta_\psi) \]  \hspace{1cm} (3-7)

Conservation of energy across the reflected wave, \( r \):

\[ h_1 + \left( \frac{1}{2} \right) U_1^2 \cdot \sin^2(\phi_2) = h_2 + \left( \frac{1}{2} \right) U_2^2 \cdot \sin^2(\phi_2 - \theta_\psi) \]  \hspace{1cm} (3-8)

In addition to the conservation laws, it is required that the thermodynamic stability of the fluid is maintained. This is achieved by an implicit relationship equating shock wave velocity to the materials compressibility and density.

Henderson [11] shows that for a perfect gas (where Pressure = gas density x gas constant x absolute temperature) the above equations can be reduced to a single six order polynomial. Four of the roots have no physical representation and can be disregarded, leaving two possible real solutions.

As the adiabatic compression of a fluid does not follow the perfect gas law, Ridah [12] solved for the two admissible solutions using the modified Tait Equation of State (EOS) for water up to Mach 3 by trial and error.

The degree of non-linearity of a shock wave can be determined by the Mach number. The Mach number is the ratio of the actual shock velocity to the acoustic speed of sound \( v_s/C \). At low Mach numbers, wave reflection is linear regular, that is to say the reflected shock wave attributes are identical to the incident wave with \( \psi_r = \psi_i \). The question of the effective pressure at the reflective boundary is covered by Ben-Dor [9] and Cole [6]. From acoustic theory the resulting pressure at the point of reflection is the sum of the pressures. Under a regular linear reflection where \( P_i = P_r \), acoustic theory predicts that the effective pressure at any incident angle (\( \psi \)) will be equal to 2\( P_i \). For waves of finite amplitude this is incorrect and is known as the acoustic paradox. The effective pressure imposed by a shock wave on a solid interface is determined by the momentum equations. Consider a particle within the shock wave with a shock pressure of \( P \), shock wave velocity \( C \), and a particle velocity of \( v_p \) as defined by Eqn (3-10) and as shown in Figure 3-3. As the particle reflects off the rigid wall it will induce a force on the wall due its change in momentum. Assuming a linear reflection (\( \psi_r = \psi_i \)), the induced pressure on the wall (\( P_{\text{EFF}} \)) is equal to the pressure in the incident wave, plus the change in momentum of the particle in the reflected wave, yielding the effective pressure in Eqns (3-9) and (3-10).
Literature Review

\[ P_{\text{Eff}} = P + \left[ \rho \cdot C \cdot \left( v_p - v_{\text{plate}} \right) \right] \cdot \cos(\psi) \]  
\[ P = \rho \cdot v_p \cdot C \]  

\[ (3-9) \]  
\[ (3-10) \]

Figure 3-3: Particle reflection

3.2.3.2. Three-Shock Theory

The three shock theory (or Von Newman theory, Nadamitsu et al. [13]) is similar to the two shock theory with the exception that a Mach stem is introduced that separates the incident and reflected waves from the surface as per Figure 3-4. This configuration is produced where large shock wave pressures are experienced.
3.3. Analytical Methods Used in the Literature for UNDEX Hull Motions

The methods used to solve fluid/structure interaction for shock loads can be divided into either, simple hand computations, classical theory, or numerical methods. A brief review of these methods follows.

3.3.1. Simple Hand Computations

A simple numerical integration technique is presented by Shin [2] where the problem is divided into horizontal strips within the depth of the ship. Geers et al. [14] presents two simplified methods that relate wave particle speed to the hull’s motion. These methods consider most variables with the exception of hull shape in determining the PTV (or kick-off velocity). A number of authors Reid [4], Reid and Burch [15] and Nilsson and Nuss [16] consider the use of empirical based shock factors. Using this approach Reid and Burch [15] define the Keel Shock Factor (KSF) as being a function of the UNDEX mean energy flux density ($E_{FD}$), charge mass (W), charge stand off, slant angle ($\phi$) and a surface cut-off factor. The PTV and KSF are related to a linear best fit relationship based on shock trial results as per Eqn (3-11), where the constants K4 and K5 are the derived for a particular vessel.

$$PTV = (K4) \cdot KSF + (K5)$$  \hspace{1cm} (3-11)
Literature Review

3.3.2. Classical Acoustic Theory

The classical methods mainly use linear acoustic theory and are restricted only to the simplest of geometries as per Lamb [17], Taylor [1] and Rayleigh [18]. Taylor [1] presents the closed solution of a differential equation for the excitation of an air backed plate that includes the effects of local cavitation.

3.3.3. Numerical Methods

3.3.3.1. General

Generally, numerical methods provide the most accurate solution for hull excitation from UNDEX due to their capability in accounting for:

- Complex hull geometries
- Non-linear effects of large structural displacements and/or strains.
- Fluid cavitation

Numerical approaches can be broadly categorised according to one of two types of time integration methods, namely implicit or explicit schemes. The implicit method assembles a set of linear equations relating all unknowns and solves them simultaneously. Although the process is linear, a non-linear problem can be solved by dividing the domain into a series of linear segments; if the segments are small enough the solution will converge to the non-linear solution. The explicit process uses a forward looking method iteratively solving for all unknowns at each time step. The strength of an implicit process is that it can solve a large range of problems, such as static, steady state, frequency domain, and time domain transient problems. In comparison the explicit process can only solve for transient problems, however it has the significant advantage over the implicit process in that it is extremely stable and capable of solving problems with large non-linear effects. Both the implicit and explicit methods can be applied to the numerical FE, Boundary Element (BE), Smooth Particle Hydrodynamics (SPH), and the Element Free Galerkin (EFG) methods as covered in Sections 3.3.3.2 to 3.3.3.5. While all the above methods can obtain a solution to any non-linear structural problem, differences in computational efficiency, solution stability, and the required accuracy for the particular problem at hand
will dictate the appropriate scheme used. Mair [19] and [20] reviews the computer codes used to evaluate UNDEX events.

3.3.3.2. Finite Element Method

The FE method is a numerical process extensively used in fluid/structure interaction problems, where both Lagrangian and Eulerian formulations are possible. In this process the problem is divided into a large number of regular volumes (elements of a finite size) with each volume defined by corner nodes. The regularity of the elements allows each region to be solved with the minimal computational effort; continuity between each region is provided by the fact that adjacent element regions share common boundary attributes, allowing compatibility between the elements. Many authors have used FE methods to predict UNDEX effects on structures such as, Shin [21], Shin and Santiago [22], Marconi and Baylor [23], Fiessler and Chwalinski [24], and Xing et al. [25].

The formulation of FE methods using acoustic equations has been adopted by a number of authors to reduce computational expense. Bathe et al. [26] mixed an FE method with acoustic continuum equations to reduce the computational expense of modelling the fluid half space. Felippa and Deruntz [27] developed the Cavitating Acoustic Finite Element (CAFE) method while Sprague and Geers [28] and [29] developed the Cavitating Acoustic Spectral Element (CASE) method. Both CAFE and CASE allow accurate modelling of fluid cavitation with reduced computation expense.

3.3.3.3. Boundary Element Method

The BE method is a particularly powerful method capable of representing complex 3-D shapes with minimum geometrical information. The mathematics for this method can be traced back to a number of authors in the nineteenth century. George Green developed a theorem that allowed a volumetric integral to be expressed as a surface integral, (now known as Green’s theorem), further work by Lord Kelvin and Somigliana allowed forces and displacements on a surface to be related to internal stresses. Additional work in the first half of the twentieth century and the advent of the high speed digital computer in the 1960s made the BE method a practical tool for engineers and scientists (Becker [30]). In the BE method the structure is represented by discrete integration points that lay its surfaces. The relationship between volume and stress is replaced by the “fundamental
solution”, a closed equation relating stress and strain. The significance of this is that all internal meshing and integration points are removed.

The use of the BE method is extensively used in linear acoustic problems to model shock waves (Haixiao [31]). The BE code, Underwater Shock Analysis (USA) computer code is often used to load FE models with UNDEX loads (Shin [21]).

3.3.3.4. Smooth Particle Hydrodynamics

SPH is a mesh-less Lagrangian technique that was developed to eliminate mesh tangle in fluid problems. The method discretises the fluid into evenly spaced particles. The effect that each particle has on any other particle in the fluid is approximated using a cubic-spline smoothing kernel (Monaghan [32]). Consider a one dimensional particle flow problem as per Figure 3-5: the prescribed velocity applied to particle ‘A’ will have an effect on neighbouring particles defined by an approximate relationship represented by a cubic-spline smoothing kernel. A smoothing radius, defines the limit of the kernel effects.

![Figure 3-5: One dimensional SPH particle velocity relationship](image)

This approach is similar to an implicit Lagrangian structural matrix method, where the effect of one node’s displacement can be related to any other node’s displacement through the flexibility matrix. Swegle and Attaway [33] consider the feasibility for using the SPH method coupled with FE methods for UNDEX shock and bubble dynamics,
concluding that additional advancement was required before bubble dynamics could be used. Liu et al. [34] used the method for high explosive detonation. This method has been used extensively in fluid/structure interaction problems such as bird strike impact (Johnson and Holzapfel [35]), where fragmentation is experienced. However, the literature strongly favours FE and BE methods for UNDEX applications.

3.3.3.5. Element Free Galerkin

The EFG is a mesh-free method that, from a user functionality perspective, is similar to the BE method; however, the mathematics of this method have more in common with the SPH method. In this method, the volumes surfaces are defined with nodes that may be coupled with an FE model; the internal volume of the structure is integrated with an approximate Galerkin function (Lu et al. [36] and Belytschko et al. [37]). The method is computationally expensive when compared to the Lagrangian method and is usually used in conjunction with Lagrangian models in areas high non-linearity, such as forming simulations. The author found no reference in the literature of EFG being used for UNDEX simulations.

3.3.3.6. Method Mixing and Compatibility

Generally the FE, BE, SPH and EFG methods can occur in different regions of the same model, providing that compatibility between the methods is provided by the software code.

3.4. Mathematical Methods for Eulerian and Lagrangian Computations

3.4.1. General

The literature generally uses two mathematical processes in conjunction with numerical methods to predict shock wave/hull interaction and cavitation. The water is represented by an Eulerian process and the hull by a Lagrangian process. The Eulerian and Lagrangian processes are named after two 18th century mathematicians Leonhard Euler (1707-1783) and Joseph Louis Lagrange (1736-1813) for their work in mathematics. The software programs available that provide fluid structure-interaction capabilities are loosely called hydro codes and are equipped with both Lagrangian and Eulerian solvers.
Eulerian mathematics describes the motion inside a fluid field. The variables of interest in this field are: velocity, density, pressure, and energy. In the FE representation of an Eulerian field, the mesh is fixed in space and does not move.

3.4.3. Lagrangian description of motion

Lagrangian mathematics describes the motion of a body using a spatial coordinate system that is fixed to the centre of mass of the body and, in result, moves with the body. For modelling explicit solids, six variables, three linear velocities, and three angular velocities are required to be solved.

3.4.4. Arbitrary Lagrangian Eulerian Description of Motion

The Arbitrary Lagrangian Eulerian (ALE) computational method has the capability of providing a Lagrangian (moving mesh), Eulerian (fixed mesh), or an arbitrary combination of Lagrangian and Eulerian solutions. The degree of mesh movement can vary depending on a user prescription or be attached to the centroid of a body. The ALE process can significantly reduce the computational expense for a group of problems where:

- A significant section of an Eulerian problem can be adequately solved with a Lagrangian process.
- An Eulerian body is required to expand (or contract) to an unknown size and location.

Both of these characteristics suitable for an ALE approach are diagrammatically considered in a soft body impact as shown in Figure 3-6. The computation economy of
the ALE method compared to the Eulerian is two fold. In this example, the initial velocity stage can be exactly modelled with a Lagrangian motion, hence, reducing the number of advection computations required. In addition, the number of elements required to model the impact as an ALE process is significantly reduced as shown in Figure 3-6 (c).
In LS-Dyna, the ALE process is achieved by two sequential steps: a Lagrangian step followed by a so-called advection step that relocates the nodes back to their starting position and remaps the nodal and mass, velocity, pressure, and energy. A pure Eulerian process can be produced by these two steps and is shown in Figure 3-7. With respect to the ALE methods ability for accurately modelling shock interaction, the method’s capability to solve the irregular Von Newman shock wave reflection is demonstrated by Nadamistsu et al. [13]. These predictions compared well with the experimental results, therefore ratifying the ALE method for shock and structure interaction.

The relationship between the LS-Dyna solver, numerical method, numerical process, and nodal movement is shown in Table 3-2.
Table 3-2: Relationship between solver, method, process and nodal movement

<table>
<thead>
<tr>
<th>LS-Dyna Solver</th>
<th>Numerical Method</th>
<th>Numerical Process</th>
<th>Nodal Movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagrangian</td>
<td>FE</td>
<td>Lagrangian</td>
<td>Full movement</td>
</tr>
<tr>
<td></td>
<td>SPH</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EFG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALE Solver</td>
<td>FE</td>
<td>Lagrangian with</td>
<td>Movement modified to reduce mesh tangle</td>
</tr>
<tr>
<td>using</td>
<td></td>
<td>smoothing (ALES)</td>
<td></td>
</tr>
<tr>
<td>Lagrangian</td>
<td></td>
<td>Lagrangian to</td>
<td>Movement defined by user, can vary from nil to full, such that computational expense is minimised</td>
</tr>
<tr>
<td>+ Advection</td>
<td></td>
<td>Eulerian</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eulerian</td>
<td></td>
<td></td>
<td>No movement</td>
</tr>
</tbody>
</table>

3.5. Explicit Fluid FE- Structure Interaction in LS-Dyna

3.5.1. General

An explicit FE fluid/structure interaction computation in LS-Dyna can be achieved by using a number of different strategies to produce a valid fluid/structure interaction approach (Benson [39], [40] and [41]). The different methods have both advantages and disadvantages in terms of accuracy and computational expense. To understand the methods available, an open container partly filled with water and having a movable piston on one side shall be considered as shown in Figure 3-8.
3.5.2. Fluid/Structural Interaction using ALES Process

The simplest method available in LS-Dyna to provide fluid/structure interaction is to model the interface with coincident nodes and to apply a smoothing process to the fluid nodes. The smoothing process reduces mesh tangling by the use of partial nodal advection, as shown in Figure 3-9; this is known as Arbitrary Lagrangian Eulerian with Smoothing (ALES). Of all the interaction methods it is the most computationally efficient. However, it is the least able to tolerate large mass transportation or non-linearity. The smoothing can be either a full automatic process, or a user defined master and slave nodal system, which further reduces mesh tangle.

*Figure 3-8: Open top container (a) initial condition and (b) displaced piston*
Available elements in LS-Dyna for the Lagrangian and ALES processes typically used in UNDEX simulations are shown in Table 3-3, where the element equation (EQ) is defined in Hallquist [8].
### Table 3-3: FE element formulations for Lagrangian and ALES

<table>
<thead>
<tr>
<th>Element Description</th>
<th>LS-Dyna Element Identification</th>
<th>LS-Dyna Material Models</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D Shells</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-D 4 node shell</td>
<td>1, 10, 6 &amp; 16</td>
<td>Common metallic</td>
<td>Used for the solid modelling of the ship structures and mine casings. Element EQ 1 is the fastest that performs well under all actions except torsion (Belytschko et al. [42]). Element EQ 10 similar to EQ 1 with better torsional capabilities. Elements EQ 16 and 6 represent elements with greater accuracies and slower computational speeds.</td>
</tr>
<tr>
<td></td>
<td>Common elements shown only</td>
<td>Common composite models</td>
<td></td>
</tr>
<tr>
<td>2-D Volumes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plane strain volume</td>
<td>13</td>
<td>Common metallic, explosive and fluid models</td>
<td>Used for fluid and hull modelling in 2-D volumes.</td>
</tr>
<tr>
<td>Axi-symmetric volume</td>
<td>14 and 15</td>
<td>Common metallic, explosive and fluid models</td>
<td>Used for fluid and hull modelling in 2-D axi-symmetric volumes.</td>
</tr>
<tr>
<td>3-D Volumes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D volume</td>
<td>1, 2, 4, 15 and 16</td>
<td>Common metallic, explosive and fluid models</td>
<td>Selection of formulations in hexagon, petrahedron and tetrahedron shapes. Used for fluid and hull modelling in 3-D volumes.</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Acoustic material</td>
<td></td>
<td>This material (*MAT_90) and element EQ 8 is used in UNDEX simulations as is computationally efficient and can support a bi-linear cavitation model</td>
</tr>
</tbody>
</table>
3.5.3. Fluid/Structural Interaction using SMALE with Void

Although the smoothing process reduces mesh tangle, excessive mesh deflection will ultimately result in computational termination where large mass transportation is required. For this the Single Material ALE with void (SMALE) provides such a process with minimal computation expense as per Figure 3-10. An element can contain a one material or a void. Partially filled elements are approximated to an element fully filled with an equivalent material based on the rule of mixtures approach. As the model can only contain one material (air considered as a void in this case), the piston is modelled with a Lagrangian moving nodal constraint.

![Figure 3-10: (a) Initial SMALE model (single material and void) and (b) Deflected SMALE model (single material and void)](image)

Available elements in LS-Dyna for the SMALE process typically used in UNDEX simulations are shown in Table 3-4.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagrangian node</td>
<td></td>
</tr>
<tr>
<td>ALE Eulerian node</td>
<td></td>
</tr>
<tr>
<td>ALE porous Eulerian node</td>
<td></td>
</tr>
<tr>
<td>Void</td>
<td>Actual cell equilibrated to a full cell with reduced properties</td>
</tr>
<tr>
<td>Water</td>
<td></td>
</tr>
</tbody>
</table>
Table 3-4: FE element formulations used for the SMALE process

<table>
<thead>
<tr>
<th>Element Description</th>
<th>LS-Dyna Element Identification</th>
<th>LS-Dyna Material Models</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D Volumes</td>
<td></td>
<td>EQ 12</td>
<td>General element with one material and void in ALE and Eulerian processes.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EQ 5</td>
<td>ALE process where far-field conditions do not require a void (computationally more efficient than element EQ 12).</td>
</tr>
<tr>
<td>3-D volume</td>
<td></td>
<td>EQ 6</td>
<td>Eulerian process where far-field conditions do not require a void (computationally more efficient than element EQ 12).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EQ 7</td>
<td>Eulerian process where a pressure boundary condition is required with no void.</td>
</tr>
</tbody>
</table>

3.5.4. Fluid/Structural interaction using MMALE

Although the SMALE process is computationally efficient, it has the limitations of allowing only one material. It is also prone to material surface defusion where the material boundary can lose definition due to the rule of mixtures approximation. The Multi-Material Arbitrary Lagrangian/Eulerian (MMALE) method uses the ALE computational approach with the capability to accommodate a number of materials simultaneously within one grid. The material boundaries are defined using the Piecewise Linear Interface reConstruction (PLIC) method or, “onion skin” model (Benson [41]), shown in Figure 3-11. LS-Dyna can accommodate a total of 10 different materials simultaneously in one grid. Most of LS-Dyna’s materials (both solid and fluid) are allowed with the exception of the rigid material. This is shown in Figure 3-12.
Figure 3-11: PLIC method model of Eulerian material boundaries from Benson [41]

Figure 3-12: (a) Initial MMALE model and (b) deflected MMALE model

(a) Initial Multi-Material ALE model

(b) Deflected Multi-Material ALE model

Figure 3-12: (a) Initial MMALE model and (b) deflected MMALE model
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Available elements in LS-Dyna for the MMALE process typically used in UNDEX simulations are shown in Table 3-5.

<table>
<thead>
<tr>
<th>Element Description</th>
<th>LS-Dyna Element Identification</th>
<th>LS-Dyna Material Equation</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D Volumes</td>
<td>11 General element with 10 materials in ALE and Eulerian processes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D volume</td>
<td>5 Common metallic, explosive and fluid models</td>
<td>ALE process where far-field conditions require only one material (computationally more efficient than element EQ 11).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Eulerian process where far-field conditions require only one material (computationally more efficient than element EQ 11).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>Eulerian process where a pressure boundary condition is required with one material only.</td>
<td></td>
</tr>
</tbody>
</table>

From a post-processing point of view this method can be a little cumbersome due to the Lagrangian solid motion not being tied to specific node locations. This can be overcome by identifying tracer points that will follow the Lagrangian points of interest through the analysis.

3.5.5. Fluid/Structural Interaction using Lagrangian/Eulerian Coupling

The Lagrangian and Eulerian processes share the common variable of velocity, which allows the coupling of one domain to the other. Eulerian/Lagrangian coupling is shown in Figure 3-13. The Lagrangian and Eulerian nodes are non-coincident and the first computation cycle relocates the fluid to the solids interface. An iterative process is
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employed until the interfacing velocities between the fluid and structure have reached the required tolerance for compatibility. An example of this method for hull-fluid interaction is used by Shin [21] and Shin and Cisum [43].

The coupling of the Lagrangian and Eulerian velocities is analogous to the contact methods used in Lagrangian contact. Any Lagrangian structure can be coupled to either the SMALE or MMALE process.

Figure 3-13: Coupled Eulerian / Lagranian (a) initial and (b) deflected model

*Figure 3-13: Coupled Eulerian / Lagranian (a) initial and (b) deflected model*
3.6. Finite Element Formulations and Error Estimates

3.6.1. General

Two of the fundamental issues to be adequately addressed in UNDEX FE analysis is the adoption of an appropriate element formulation and an adequate integration point density to ensure a suitable accuracy. In the explicit FE field used for UNDEX, the desirable element capabilities are a high computational speed and the minimisation of errors. These two qualities tend to be mutually exclusive because high computational speed is usually achieved by minimising the number of computation steps, which in turn results in a less accurate element solution. Element performance and estimates in element errors can be measured through testing a single element or a patch of elements and is often referred to as “patch testing”. Patch testing allows element errors and relative computational speeds to be quantified (Zienkiewicz and Taylor [44] and Hugger [45]). An example of patch testing is given in Schwer et al. [46].

To guarantee the required solution accuracy, the current body of knowledge considers an adequate integration point (or mesh) density is required. This can be achieved by one of the following methods:

- Successive approximation is often used where the problem is modelled a number of times with different integration point densities used. This allows the solution results to be check for convergence with respect to the point density. The general methods for increasing integration point density are:
  - Defining a higher order element formulation
  - Remeshing the problem with a greater element density
- Comparing the FE solution with a comparable known classical solution
- Comparing the discontinuity magnitude of extrapolated element variables (of stress, strain or energy density) between adjacent elements gives a measure of the solution accuracy. G + D computing [47] considers that for Fully Integrated (FI) elements the discontinuity between adjacent elements should not exceed 10%.
Due to the large number of different modelling strategies available, few authors in the literature suggest appropriate element sizes to be used. Tran and Marco [48] have conducted studies on both element size and aspect ratio for the LS-Dyna fluid elements (Lagrangian with smoothing). These elements typically applied to far-field underwater explosions used in mine/hull interactions with similitude equation based explosion pressure boundary conditions. They considered that an element size of 20 mm to 50 mm and an aspect ratio of below 5:1 produced an accurate representation of the pressures within the water mesh with minimal spurious noise.

### 3.6.2. Lagrangian Elements

A common form of element formulation is known as an iso-parametric element, where the function that defines the shape of the element also defines the strain distribution within the element. These elements are known as FI and can suffer a problem known as shear lock. This is where, under a constant bending action, FI elements incorrectly calculate an associated shear effect, predicting an over stiff solution. To remedy this, many codes offer Under Integrated (UI) and/or Selectively Reduce Integration (SRI) elements. The UI elements are defined as an element where the function that defines the strain distribution within the element is one order less than that which defines the shape of the element (Key and Hoff [49]). For example, a linear function defines a four-node shell’s shape (between the nodes), hence, the strain distribution for a UI element is constant across the element. In large problems these elements are of considerable importance as they are designed to operate with the least possible computational steps while maintaining a robust configuration and allowing large geometric deformities to occur without suffering from numerical instabilities. These features allow large problems with a high degree of non-linearity to be solved with minimal computation time. Some typical examples of element formulation considerations for LS-Dyna are addressed below:

- The most computationally efficient of the 2-D shell elements is the Belytscho-Tsay, a UI formulation that preforms well under most stress regimes except torsional loading; to overcome this deficiency an alternative element the Belytschko-Wong-Chiang UI element was developed. It has better torsional capabilities, however is computationally slower than the Belytscho-Tsay element
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(Belytschko et al. [42]). The FI (very fast) element provides resistance to “hourglassing”, however is significantly slower than the UI elements. A number of SRI shells are available, they provide the accuracy advantages of the FI element without shear locking and are the slowest of all the elements.

- The hexagon 8-node UI element provides a robust element, with high computational speed however is subject to hourglassing. The SRI hexagon element is resistant to hourglassing and significantly slower that its UI counter part. This element is a constant stress element (same as the UI element) and suffers from shear lock at poor aspect ratios (Hallquist [8]). The hexagon 8-node UI acoustic element which can only be used in conjunction with *MAT_90 is a very cost effective formulation (Hallquist [8] and [7]). This element is used by Shin [21].

Under integrated elements can deform in a manner that does not consume energy, which allows the possibility of an incorrect deflected shape to be obtained while still maintaining the conservation of energy requirement. As mentioned above, this behaviour is known as hourglassing and is shown in Figure 3-14. It can be seen that although significant deflection has occurred, the single guass point, which measures strain energy mid-point along the element face, is unaware of this.

![Figure 3-14: Hourglassing of an UI element](image-url)
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The hourglassing energy can be calculated in LS-Dyna and, if it is considered to be excessive for a particular solution, the problem parameters need to be adjusted. This can be achieved by:

- Adopt a different element formulation
- Increase hourglass damping
- Remeshing the problem with a finer mesh

It should also be noted that UI elements are not guaranteed to converge to the correct solution (Key and Hoff [49]).

3.6.3. SMALE and MMALE elements

In LS-Dyna, SMALE and MMALE elements only have one formulation where a constant pressure and linear varying velocity is assumed within the element, making the element UI. In a pure Eulerian process these elements are impervious to hourglassing and do not suffer from shear lock.

3.7. Constitutive Fluid Models

Fluids subject to shock interaction with structures are considered compressible due to the large pressures generated in an UNDEX event. They can also experience a phase change (from fluid to a gas) when interacting with the free surface or a moving hull section. This section reviews the constitutive models available to model these properties.

3.7.1. Eulerian Fluid Materials

3.7.1.1. General

An Eulerian fluid can be uniquely defined by the following material attributes:

- Density ($\rho$)
- Compressibility (Bulk modulus [$B_{MOD}$])
- Cavitation pressure ($P_c$)
- Dynamic viscosity ($\mu_D$)

Note: The acoustic speed of sound ($C$) is a function of $\rho$ and $B_{MOD}$
### 3.7.1.2. Volumetric Stiffness and the Speed of Sound

The fluid can be defined by either a linear or non-linear constitutive material model. A linear elastic material is defined by Eqn (3-12) and relates the speed of sound \( C \), material compressibility \( B_{\text{MOD}} \), and density \( \rho \). In a real (or non-linear) fluid the shock wave pressure changes the density and the compressibility of the material. The speed of the wave is no longer constant and varies with the shock wave pressure. In this case the shock wave speed \( v_s \) is equal to the square root of the rate of change of pressure with respect to the volume as per Eqn (3-13). The linear and non-linear pressure/volume relationships are shown graphically in Figure 3-15.

\[
C = \sqrt{\frac{B_{\text{MOD}}}{\rho}} \quad \text{for linear elastic fluids} \tag{3-12}
\]

\[
v_s = \sqrt{\frac{\partial P}{\partial \rho}} \quad \text{non-linear fluids} \tag{3-13}
\]

![Figure 3-15: The relationship between pressure and volume](image)

From Figure 3-15 it can be seen that at low pressures the linear elastic assumption provides good agreement, while the more realistic non-linear material is generally used for compressible materials. To model the non-linearity of a compressible fluid the literature uses an EOS. The EOS approach approximates the material pressure/volumetric behaviour using line fitting methods. Different families of curves are available to suit
different material types. For water modelling the literature generally uses the Gruneisen EOS.

3.7.1.3. Bulk Cavitation

A shock wave passing through a material interface, where each material has a different speed of sound, will produce one reflected and one refracted shock wave this occurs at the water/air interface (free surface). The water borne shock wave will reflect and also reverse its polarity (changing into a tension wave), and, because water cannot take tension, it will vaporise which is known as cavitation. In addition, a small compression wave will propagate into the air, but this is usually ignored with minimal associated error. Bulk cavitation is where the region of cavitation is large (Shin [2]). As the reflected tension wave travels downward, it meets and nullifies the tail of the upcoming shock wave; this phenomenon is known as “surface cut-off”. The physics of bulk cavitation and surface cut-off are complex. On meeting the air/water interface, the shock wave detaches a thin layer of surface water, propelling it upwards with a velocity near to that of the peak particle velocity of the wave (Cole [6]). The resulting reflected tension wave travels downward, cavitating the water and producing large changes in its density and compressibility.

The literature considers two constitutive models for cavitation modelling. The constant density model uses linear acoustic assumptions while the variable density model allows the density and the shock wave velocity to vary with respect to pressure. Swisdak [3] references closed equations to predict the time at which any point will experience cavitation. Shin [2] references equations that predict the physical limits of the bulk cavitation, and both methods use constant density. In this treatment of bulk cavitation by Shin [2], the pressure at point A (in Figure 3-16) is the sum of the atmospheric and hydrostatic pressure, incident compressive wave, and the reflected tension wave pressures, where the radius of the shock is wave $r$, radius within the incident wave at the junction of the reflected wave is $r_i$, charge mass is $W$, speed of sound is $C$ and the shock pressure constants are $K_1$ and $\theta$. The effect of the refracted compressive air wave, which slightly reduces the intensity of the reflected tension wave, is neglected.
Expressing the total pressure at point A results in Eqn (3-14).

\[ P_{\text{TOTAL}} = P_{\text{ATMOSPHERIC}} + P_{\text{HYDOSTATIC}} + P_{\text{INCIDENT COMPRESSION}} + P_{\text{REFLECTED TENSION}} \]

where:

\[ P_{\text{INCIDENT COMPRESSION}} = K_1 \left( \frac{W^{2/3}}{r_1^2} \right)^{4l} e^{-\left( \frac{r-r_1}{c \theta} \right)} \]

\[ P_{\text{REFLECTED TENSION}} = K_1 \left( \frac{W^{2/3}}{r} \right)^{4l} \]

To aid comprehension the charge is mirror imaged, allowing easy visualisation of the tension wave at \( r_1 \) and \( r_2 \) as shown in Figure 3-17. At a constant angle of \( \phi \), the roots of Eqns (3-15) & (3-16) can be solved to determine the upper and lower limit of the bulk cavitation ray. Applying this to all values of \( \phi \) results in the determination of the bulk cavitation volume.

\[ P_{\text{TOTAL}} = 0 \]

\[ \frac{d}{dr} \left( r^{4l} P_{\text{TOTAL}} \right) = 0 \]
Shin [2] solves a number of bulk cavitation scenarios for different depths and charge sizes using a Pentolite explosive. The solution for 90.7 kg (200 lb) of Pentolite at a depth of 7.62 m (25 feet) is presented in Figure 3-18.

A number of authors use the variable density model: Bleich and Sandler [50], Shin and Santiago [22], Shin [21], Felippa and Deruntz [27], Sprague and Geers [29] and Galiev [51]. Bleich and Sandler [50] considers in detail the use of a bi-linear relationship between pressure and density as per Figure 3-19.
In addition to the bi-linear material model complying with the conservation laws of energy and momentum, it must also consider a uniqueness criterion. The interfaces between areas of different densities may exist as one or more discontinuities. A uniqueness criterion is required to ensure that one and only one solution is valid. Bleich and Sandler [50] solve a 1-D example of a steel plate floating on the surface that is excited by a shock wave, this is diagrammatically shown in Figure 3-20.
The model requires that the velocity and position of the three boundaries (plate/water interface, upper cavitation surface and lower cavitation surface) to be solved with respect to time. Shin and Santiago [22] verified the results of Bleich and Sandler for the plate velocity with and without cavitation using the USA code (CAFE model) coupled to MSC.Nastran code. Their results for the plate velocity are shown in Figure 3-21.
At some point in time the upper and lower cavitation boundaries will collapse in towards each other and collide ($Y(0)=0$). This phenomenon is known as cavitation closure and results in a compression wave being generated from the impact. Shin and Santiago calculate the pressure at the plate/water interface and capture the compression wave from the cavitation closure as per Figure 3-22.
3.7.1.4. Local Cavitation

It can be seen from Eqn (3-9) that the effective reflected shock pressure for the stationary plate is two times the shock pressure (at $\psi=0^\circ$), when $v_{\text{plate}}$ is equal to the $2v_p$, the effective pressure on the plate is equal to zero. For values of $v_{\text{plate}}$ greater than $2v_p$ the effective pressure is negative. Water cannot take tension and will aerate this is known as local cavitation.

3.7.1.5. Fluid Materials Available in LS-Dyna

For LS-Dyna UNDEX analysis the literature uses one of three materials to model water: *MAT_001_FLUID, *MAT_009 with an EOS or *MAT_90 as per below (Hallquist [8]):

- Material *MAT_001_FLUID is a linear material with the compressibility and density constant with pressure. It can support a constant density cavitation model but it cannot support viscosity.
- Material *MAT_009 coupled with an EOS is a non-linear material with the compressibility and density being a function of the pressure; a non-zero viscosity can also be used. The EOS can be used to support a CAFE like cavitation model.
- Material *MAT_90 is a designed to model weak acoustic waves in fluids and gasses and can accommodate a bi-linear cavitation model (Shin [21]).

3.8. Realistic Sea Water Properties

3.8.1. General

The properties of water referred to in this thesis are realistic estimates of the temperature, salinity, speed of sound, and the compressibility that may be encountered by a surface ship exposed to the threat of a mine induced shock wave loading. The following section is a review of the literature to determine the relevant Eulerian fluid properties to be used in the LS-Dyna simulations.
3.8.2. Water Temperature, Salinity, and Density

The main impurity in sea water is Sodium Chloride, but additional element ions of Magnesium, Sulphate, Calcium, Potassium, Bicarbonate and Bromide are also present. Dissolved gasses are also a potential impurity. The quantitative effect of the salts is generally lumped together to define the salinity \((S)\) as being the total mass of dissolved salts per unit mass of sea water. The surface temperature of the world's oceans vary from \(-1.9^\circ\) (blue) at the poles to \(37^\circ\) (pink) at the equator (National Aeronautics and Space Administration (NASA) [52]), as shown in Figure 3-23.

![Image: The world's ocean surface temperature from NASA [52]](image)

*Figure 3-23: The world’s ocean surface temperature from NASA [52]*

Beranek [53] gives a comprehensive envelope of extremes within the continental shelf region for temperature, density and acoustic speed of sound at one atmosphere of pressure, as per Table 3-6.
Table 3-6: Table of sea water variations in the continental shelf at 1 atmosphere

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Salinity (ppt)</th>
<th>Density (kg/m³)</th>
<th>Acoustic Sound Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea Water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>1023.8</td>
<td>1461</td>
</tr>
<tr>
<td>15(1)</td>
<td>31.6(1)</td>
<td>1023.4(1)</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>36</td>
<td>1026.8</td>
<td>1505</td>
</tr>
<tr>
<td>25</td>
<td>36</td>
<td>1024.1</td>
<td>1532</td>
</tr>
<tr>
<td>Pure Water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>999.1</td>
<td>1403</td>
</tr>
</tbody>
</table>

Notes:
(1) The standard mean within the earth’s continental shelf region.

The adopted temperature and salinity for this thesis shall be taken as the standard mean within the earth’s continental shelf region with:

- T=15° C
- S=31.6 ppt
- ρ=1023 kg/m³

3.8.3. Acoustic speed of sound in sea water

The relationship between the acoustic speed of sound, temperature, salinity, and depth are comprehensively covered by National Physical Laboratory (NPL) [54], Shin[2], and Freistel and Hagen [55]. Typical equation from NPL is the Mackenzie as per Eqn (3-17).

\[
C = \left(1448.96 + (4.591) \cdot T - \left(5.304 \times 10^{-3}\right) \cdot T^2 + \left(2.374 \times 10^{-4}\right) \cdot T^3 + (1.34) \cdot (S - 35) + \left(1.63 \times 10^{-2}\right) \cdot D + \left(1.675 \times 10^{-7}\right) \cdot D^2 - \left(1.025 \times 10^{-3}\right) \cdot T \cdot (S - 35) - \left(7.139 \times 10^{-13}\right) \cdot T \cdot D^3 \right)
\]

(3-17)

where:
- C= Speed of sound in m/s
- T = Temperature in °C
- S = Salinity in ppt
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\[ D = \text{Depth in m} \]

Shin [2] provides a simpler equation for the speed of sound as per Eqn (3-18).

\[ C = 4577 + (11.5) \cdot T + (4.6) \cdot S \quad (3-18) \]

where:

- \( C \) = Speed of sound in ft/s
- \( T \) = Temperature in °C
- \( S \) = Salinity in ppt

Comparison between the references at the adopted \( T, S, \rho \) and one atmosphere are as per Table 3-7.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Acoustic Speed of Sound ( C ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mackenzie</td>
<td>1503</td>
</tr>
<tr>
<td>Shin</td>
<td>1492</td>
</tr>
<tr>
<td>Freistel and Hagen</td>
<td>1501</td>
</tr>
<tr>
<td>Adopted ( C )</td>
<td>1500</td>
</tr>
</tbody>
</table>

Note: For each 100m of water depth \( C \) increases by 1 m/s.

3.8.4. Sea Water Compressibility

The most definitive work on this subject is covered by Freistel and Hagen [55] with comprehensive water properties for temperature, salinity, speed of sound, and the compressibility as shown in Table 3-8. Freistel and Hagen water properties are in good agreement with experimental results and the United Nations Educational, Scientific, and Cultural Organization-Intergovernmental Oceanographic Commission (UNESCO-IOC).
Table 3-8: Sea water density and acoustic speed of sound after Feistel and Hagen [55]

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Salinity (ppt)</th>
<th>Density (kg/m³)</th>
<th>Acoustic Speed of Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>30</td>
<td>1023.7</td>
<td>1464.3</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>1022.1</td>
<td>1500.9</td>
</tr>
<tr>
<td>15</td>
<td>40</td>
<td>1029.8</td>
<td>1512.5</td>
</tr>
<tr>
<td>25</td>
<td>40</td>
<td>1027.1</td>
<td>1539.8</td>
</tr>
<tr>
<td>Pure Water</td>
<td>15</td>
<td>0</td>
<td>999.1</td>
</tr>
</tbody>
</table>

The current body of knowledge considers the compressive bulk modulus of water for UNDEX events by either the Gruneisen EOS (Shyue [56], Brett [57] and Lu and Dorsett [58]) or the simple linear elastic model (Hammond and Flockhart [59] and Shin [21]) as per Eqn (3-12). In the accurate modelling of water, all the conservation laws and thermodynamic stability are required to be met. At any given pressure, the shock wave speed, internal particle speed, internal energy, and density are uniquely defined by the Gruneisen EOS Eqn (3-19). The Gruneisen EOS uses a cubic line fit to define the required relationship between particle and shock velocity as per Eqn (3-21) for a compressed material.

\[
P = \frac{\rho_0 \cdot C^2 \cdot \mu \cdot \left[ 1 + \left( 1 - \frac{\gamma_0}{2} \right) \cdot \mu - \left( \frac{a}{2} \right) \cdot \mu^2 \right]}{\left[ 1 - (S_1 - 1) \cdot \mu - S_2 \cdot \left( \frac{\mu^2}{\mu + 1} \right) - S_3 \cdot \left( \frac{\mu^3}{(\mu + 1)^2} \right) \right]^2} + \left( \gamma_0 + a \cdot \mu \right) \cdot E
\]  

(3-19)

\[
\mu = \frac{\rho}{\rho_0} - 1
\]  

(3-20)
For expanding material the Gruneisen equation is as per Eqn (3-22).

\[ P = \rho_0 \cdot C^2 \cdot \mu + \left( \gamma_0 + a \cdot \mu \right) \cdot E \]  (3-22)

In addition to the Gruneisen EOS relationship between pressure and volume is the additional requirement of cavitation. Under some negative pressure the water will cavitate and, in this state the pressure is constant and independent of volume. In LS-Dyna this condition is defined in the material definition and overrides the EOS under a tension pressure regime.

The Gruneisen coefficients are dependent on the water temperature, salinity, and the shock pressure range used. The coefficients used in the literature vary considerably as shown in Table 3-9.

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>( \gamma_0 )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75</td>
<td>0</td>
<td>0</td>
<td>0.28</td>
<td>[60]</td>
</tr>
<tr>
<td>2.56</td>
<td>-1.986</td>
<td>0.227</td>
<td>0.5</td>
<td>[57]</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>[56]</td>
</tr>
<tr>
<td>1.92</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>[58]</td>
</tr>
</tbody>
</table>

To determine the significance of the variations in the Gruneisen coefficients the adiabatic compressibility of sea water, at a temperature of 15\(^\circ\)C and salinity equal to 30 ppt, was calculated by a number of different approaches and compared to Gruneisen coefficients presented in the literature.

- Beyer [61] presents a truncated Taylor series to define the relationship between adiabatic pressure and volume change for sea water.
- Feistel and Hagen [55] provide a tabulated approach for the adiabatic pressure and volume change for sea water.
The Gruneisen coefficients of $S_1 = 1.75$ and $\gamma_0 = 0.28$ from Souli [60] were analysed for the relationship between volume change and adiabatic pressure using an LS-Dyna single cell model with a prescribed velocity compressing the cell.

The Gruneisen coefficients of $S_1 = 2.56$, $S_2 = -1.986$, $S_3 = 0.227$ and $\gamma_0 = 0.50$ from Brent [57] were analysed for the relationship between volume change and adiabatic pressure using an LS-Dyna single cell model with a prescribed velocity compressing the cell.

The acoustic method assumes that the density of water and the speed of sound remain constant for all pressures.


It should be noted that the Gruneisen parameters of $S_1$, $S_2$, and $S_3$ are cubic line fit coefficients. At the low pressures, the parabolic ($S_2$) and cubic ($S_3$) parts of the equation approach zero and have no significant effect for the pressure ranges used in this thesis.

The Gruneisen coefficients of $S_1 = 1.75$ and $\gamma_0 = 0.28$ give a pressure/volume relationship approximately equal to the average compressibility of Beyer, Feistel, and Kirkwood estimates for the compressibility of sea water at $T = 15^\circ$ C and $S = 30$ ppt as shown in Figure 3-24 and Figure 3-25, where the abscissa magnitude is defined as the the original volume divided by the actual volume.
Literature Review

Figure 3-24: Comparison of adiabatic compression of sea water at $T=15$ and $S=30$ ppt.

Figure 3-25: Enlarged view
Literature Review

Hence, it is considered that the appropriate compressibility of sea water within the continental shelf region, for pressures up to 180 MPa, is most accurately modelled using the Gruneisen coefficients as per Table 3-10.

Table 3-10: Adopted water compressibility for simulations using Gruneisen EOS

<table>
<thead>
<tr>
<th>Acoustic C (m/s)</th>
<th>Gruneisen Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.28</td>
</tr>
</tbody>
</table>

Alternatively, the compressibility of a linear elastic material determined from Eqn (3-12) is as per Table 3-11.

Table 3-11: Adopted water compressibility for simulations using a linear elastic material

<table>
<thead>
<tr>
<th>Acoustic C (m/s)</th>
<th>Bulk Modulus B_{Mod} (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>2.3 x 10^9</td>
</tr>
</tbody>
</table>

3.8.5. Cavitation Pressure

The ability of water to take small amounts of negative pressure depends on the water temperature and the amount of dissolved salts and gasses. Sprague and Geers [28] indicate that small amounts of dissolved gas markedly reduce the boiling point. Generally, the literature considered the magnitude of the actual water vapour pressure to be near or equal to zero. For theoretical and experimental comparison of a steel plate plastically deformed by a far-field blast, Hammond and Flockhart [59] used a cavitation pressure equal to zero with good agreement to the experimental results. Shin [2] considers a value of 2000 Pa (0.3 psi) being realistic. The possible range of vapour pressure from the literature is as per Table 3-12.

Table 3-12: Possible range of sea water vapour pressure

<table>
<thead>
<tr>
<th>Absolute Vapour Pressure (Pa)</th>
<th>Gauge Vapour Pressure (Pa)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0</td>
<td>-101,600</td>
</tr>
<tr>
<td>Maximum</td>
<td>2000</td>
<td>-99,600</td>
</tr>
</tbody>
</table>
3.8.6. Dynamic Viscosity

The possible range of dynamic viscosity is as per Table 3-13. Of the three LS-Dyna fluid models used in the literature only the non-linear *Mat_009 with an EOS can support a viscosity.

Table 3-13: Possible range of sea water viscosity

<table>
<thead>
<tr>
<th>Dynamic viscosity (kg/m.s)</th>
<th>Reference</th>
<th>LS-Dyna Material Compatibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0</td>
<td>*Mat_001_FLUID and *MAT_90</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.00113</td>
<td>*MAT_009 + EOS</td>
</tr>
</tbody>
</table>

3.9. Constitutive Composite Material Models

The LS-Dyna elastic orthotropic (*MAT_002) and isotropic (*MAT_001) material models were used for the T joint simulations.

3.9.1. Material Stiffness

3.9.1.1. Static Material Stiffness

The linear elastic stiffness of an orthotropic material is defined by Hooke’s law. Material constants in the constitutive matrix relate material stiffness to an orthogonal reference coordinate system with axis a, b, and c as per Figure 3-26 where E, G and ν are the orthogonal elastic moduli, shear modulus are Poisson’s ratio respectively.
Figure 3-26: Constitutive stiffness matrix from Hallquist [8]

The bulk modulus of the material is defined by Eqns (3-23) and (3-24)

\[
B_{\text{MOD}} = \frac{1}{w^T \cdot C^{-1} \cdot w}
\]

(3-23)

\[
w = [\lambda_a, \lambda_b, \lambda_c, 0, 0, 0]
\]

(3-24)

Where \( w \) is the shape function that defines a hydrostatic stress condition. For an isotropic material where \( E_a = E_b = E_c, G_{ab} = G_{bc}, v_{ab} = v_{bc} = v_{ca} \) and \( w = [1,1,1,0,0,0] \), Eqn (3-23) simplifies to Eqn (3-25).

\[
B_{\text{MOD}} = \frac{E}{3(1-2\nu_{iso})}
\]

(3-25)

3.9.2. Speed of Sound in a Solid

Due to the shear stiffness capacity of a solid, a number of wave formations can freely propagate through a structure with each wave type having a different speed. The energy associated for each wave formation depends on the structures shape and its initial
excitation. The types of wave formations and their descriptions are detailed by Clough and Penzien [63] as per Table 3-14.

<table>
<thead>
<tr>
<th>Wave type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Compression wave</td>
</tr>
<tr>
<td>S</td>
<td>Shear wave</td>
</tr>
<tr>
<td>Lamb</td>
<td>Compression/tension wave with particle motion perpendicular with direction</td>
</tr>
<tr>
<td>Love</td>
<td>Surface horizontal shear wave that varies with depth, usually associated with half spaces</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>Surface compression wave that varies with depth usually associated with half spaces</td>
</tr>
</tbody>
</table>

The orthogonal wave speeds in orthotropic materials can be determined using the well known Christoffel’s equations (Kaptsov [64]). For isotropic materials the P and S wave speeds are determined by Eqns (3-26) and (3-27) respectively.

\[
C = \sqrt{\frac{E(1-\nu_{iso})}{\rho(1+\nu_{iso})(1-2\cdot\nu_{iso})}} \quad (3-26)
\]

\[
C_s = \sqrt{\frac{G}{\rho}} \quad (3-27)
\]

3.9.3. Composite T-Joint Strength

3.9.3.1. Section Considered

The composite T-Joint material reviewed in this research is defined as a 2-D fabric construction, where multiple layers of a 2-D plain weave glass fibre fabric are embedded in a vinyl ester resin (Derakane 411). This type of construction typifies methods adopted in some non-metallic mine hunter vessels where the use of composite materials minimises the magnetic signature of the vessel while providing excellent resistance to the corrosive marine environment (Nilsson and Nuss [16], Trimming [65], and St John et al. [66]). For the joint considered, the hull plate and bulkhead are fabricated using a Vacuum Bag Resin Infusion (VBRI) process. After curing, these items are fitted up and bonded together with a chopped fibre filler and Hand Lay-up (HL) over-laminate material as shown in Figure 3-27. The bonded interface between the HL and the VBRI laminates has one layer of
chopped fibre mat that increases the fracture toughness of the interface, bolstering the joint’s resistance against delamination (Green et al. [67]).

3.9.3.2. Overview of General Material Strengths

In ductile metal structures, the deterministic estimates of its load carrying capacity are considered by the literature to be a mature science. A material’s ability to resist loading can be limited by either its capability to resist damage or fracture. Material damage is associated with a stress or strain regime and results in yielding while fracture is the fragmentation of the material by cracking. Fracture will occur if the growth of a crack results in a lower energy state of the system. That is to say the energy required to overcome the cohesive force of the atoms is equal to the dissipation of the strain energy that is released by the crack. In ductile metal structures, material damage and fracture are generally considered to be independent of each other. Damage will occur under excessive loads and will result in material yielding while fracture considers failure by the propagation of a crack from a cyclic loading where load magnitude is below that required to damage the material by yielding. Elder et al. [68] considers the discrete components of the 2-D composite may experience damage and fracture concurrently. A highly stressed composite typically experiences overstressing of the matrix, which results in sub-critical cracking (micro-cracking). The cracking redistributes the load and produces stress and energy concentrations at the inter-ply regions where large differences in material stiffness exist. These conditions are ideal for a fracture based inter-ply delamination to initiate and
grow. The onset and propagation of inter-ply delaminations result in sudden variations in section properties and load paths within the laminate. When comparing the two materials, the metallic deterministic methods are based on simple and well proven physics while the more complex laminate failure is still under development Elder et al. [68].

3.9.3.3. Overview of Current Predictive Composite Static Strength Methods

The predictive failure methods published in the literature apply various forms of physics to determine the magnitude of the peak external load that a laminate can sustain. A number of these are listed below:

- Stress or strain based damage failure methods
- Fracture base failure methods
- Probabilistic methods

In addition to the external loadings, Oosthuizen and Stone [69] quantify the internal stresses developed from differential thermal strains (between the glass and the epoxy vinyl ester resin) in the curing process. These strains are locked into the material by the manufacturing process and, due to the brittle nature of the laminate, will reduce its external load capacity. The factors that influence the development of internal residual stresses with the laminate are:

- Time interval between the lay-up of individual layers
- Temperature and humidity of the manufacturing process
- Glass reinforcement geometry, thickness, and direction
- Temperature gradients, resulting from the exothermic curing reaction

Due to the lack of faith in the failure criteria currently used, a co-ordinated study of 19 leading failure theories was undertaken. This is known as the World-Wide Failure Exercise (WWFE) (Hinton and Soden [70]). The WWFE progressed the application of current failure criteria for low strain rate loadings and ranked the accuracy of each theory against test results for particular stress regimes tested. It also provided an overall aggregate score for the best all round method (Hinton et al. [71] and [72]).
Due to the difficulty in quantifying the T-joints failure load and mode, the study in this research is limited to a comparative one. Local laminate stresses between models were compared to determine the leased stressed joint configuration. The failure criterion used to assess the comparative stress state was the maximum stress criterion. In this method the actual orthogonal stresses are compared to the allowables with no interaction being considered.

3.9.3.4. Strain Rate Effects of Laminate Strength and Stiffness

There is much evidence in the literature to indicate that the mechanical properties of composites are highly sensitive to the rate of loading. This is mainly driven by the matrix sensitivity. However, as of yet, no constitutive strain rate material models are available. For this thesis, the material testing by Akil et al. [73] was used to predict the approximate stiffness and compressive strength increase for this laminate at high strain rates. Akil et al. [73] measured the failure strength, failure mode, and stiffness of S-glass fabric reinforced vinyl ester 2-D laminates. This material was similar to that studied in this research, exception to this being that the S glass fibres have approximately a 20% greater stiffness and failure strain when compared to the E-glass fibres used in this thesis. The in-plane and through thickness moduli, failure stress, and failure strain were plotted on a linear-log strain X-Y plots as per Figure 3-28, Figure 3-29, and Figure 3-30. The relative changes in materials properties are as per Table 3-15.
Figure 3-28: Moduli verses log strain rate from Akil et al. [73]

Figure 3-29: Failure stress verses log strain rate from Akil et al. [73]
Figure 3-30: Failure strains verses log strain rate from Akil et al. [73]

Table 3-15: Laminate material properties after Akil et al. [73]

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Magnitude at Strain Rate 10&lt;sup&gt;-3&lt;/sup&gt; s&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>Magnitude at Strain Rate 500 s&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive moduli</td>
<td>E&lt;sub&gt;11&lt;/sub&gt; and E&lt;sub&gt;22&lt;/sub&gt;</td>
<td>24.5</td>
<td>27.5</td>
<td>+12%</td>
</tr>
<tr>
<td></td>
<td>E&lt;sub&gt;33&lt;/sub&gt;</td>
<td>8</td>
<td>14</td>
<td>+75%</td>
</tr>
<tr>
<td>Compressive failure stress</td>
<td>σ&lt;sub&gt;11&lt;/sub&gt; and σ&lt;sub&gt;22&lt;/sub&gt;</td>
<td>450</td>
<td>600</td>
<td>+33</td>
</tr>
<tr>
<td></td>
<td>σ&lt;sub&gt;33&lt;/sub&gt;</td>
<td>530</td>
<td>650</td>
<td>+23</td>
</tr>
<tr>
<td>Compressive failure strain</td>
<td>ε&lt;sub&gt;11&lt;/sub&gt; and ε&lt;sub&gt;22&lt;/sub&gt;</td>
<td>0.018</td>
<td>0.023</td>
<td>+28</td>
</tr>
<tr>
<td></td>
<td>ε&lt;sub&gt;33&lt;/sub&gt;</td>
<td>0.067</td>
<td>0.063</td>
<td>-6%</td>
</tr>
</tbody>
</table>

Mourtiz [74] conducted scaled down UNDEX shock tests on a GRP laminate with identical materials (E-glass and resin) and a similar lay-up as used in this thesis. It was found that if the shock pressure was under a threshold limit, the laminate experienced no damage. The actual damage in the test piece resulted from the shock induced bending and shear stress within the laminate however, these induced stresses were not measured in...
the experiment. Exceeding this limit resulted in marked reductions in the laminate’s residual strength, fatigue, and stiffness as per Figure 3-31 and Figure 3-32. Scanning electron micrograph of the laminates after the shock tests confirmed the damage extent. At shock pressures of up to 10 MPa, no noticeable damage was evident. At shock pressures between 15 and 35 MPa, gross structural damage in the form of delamination and fibre breakage was observed and complete failure at a shock pressure of approximately 50 MPa.

![Figure 3-31: Residual bending flexural strength verses shock pressure from Mouritz [74]](image-url)
Figure 3-32: Residual bending flexural stiffness verses shock pressure from Mouritz [74]

3.10. Realistic Joint Properties

The laminate material properties were taken from Gellert et al. [75] and are as per Table 3-16. Mouritz [74] indicates that for UNDEX events strain rates of between 10 and 100 s\(^{-1}\) are possible. On this basis the percentage increase (from static to 500 s\(^{-1}\)) of Akil et al. [73] materials was applied to Gellert’s static material properties, resulting in Table 3-18.

*Table 3-16: Adopted laminate stiffness properties at low strain rates after Gellert [75]*

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Shear (GPa)</th>
<th>Resin Content w/w</th>
<th>Laminate Density (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>VBRI</td>
<td>E(_{11}) 26.1</td>
<td>E(_{22}) 26.1</td>
<td>E(_{33}) 3.0</td>
<td>v(<em>{12}) &amp; v(</em>{12}) 0.165</td>
<td>v(<em>{11}) &amp; v(</em>{32}) 0.019</td>
</tr>
<tr>
<td>HL</td>
<td>23.5</td>
<td>23.5</td>
<td>3.0</td>
<td>0.165</td>
<td>0.021</td>
</tr>
<tr>
<td>Filler</td>
<td>4.0</td>
<td></td>
<td></td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>
The speed of sound in the orthotropic composite materials was determined from a 3-D shock tube analysis as per Table 3-17. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

### Table 3-17: Speed of sound for composite low strain rate T-Joint material

<table>
<thead>
<tr>
<th>Material</th>
<th>C from Eqn (3-26) (m/s)</th>
<th>C from Shock Tube Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C_{11} &amp; C_{22} (m/s)</td>
<td>C_{33} (m/s)</td>
</tr>
<tr>
<td>VBRI</td>
<td>4035</td>
<td>1360</td>
</tr>
<tr>
<td>HL</td>
<td>3970</td>
<td>1430</td>
</tr>
<tr>
<td>Filler</td>
<td>1940</td>
<td>1860</td>
</tr>
</tbody>
</table>

### Table 3-18: Adopted approximate laminate stiffness properties at high strain rates

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Shear (GPa)</th>
<th>Resin Content w/w</th>
<th>Laminate Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E_{11}</td>
<td>E_{22}</td>
<td>E_{33}</td>
<td>v_{12} &amp; v_{21}</td>
<td>G_{12}</td>
</tr>
<tr>
<td>VBRI</td>
<td>29.2</td>
<td>29.2</td>
<td>5.3</td>
<td>0.165</td>
<td>0.030</td>
</tr>
<tr>
<td>HL</td>
<td>26.3</td>
<td>26.3</td>
<td>5.3</td>
<td>0.165</td>
<td>0.033</td>
</tr>
<tr>
<td>Filler</td>
<td>7.0</td>
<td></td>
<td></td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3-19: Speed of sound for composite high strain rate T-Joint material

<table>
<thead>
<tr>
<th>Material</th>
<th>C from Eqn (3-26) (m/s)</th>
<th>C from Shock Tube Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C_{11} &amp; C_{22} (m/s)</td>
<td>C_{33} (m/s)</td>
</tr>
<tr>
<td>VBRI</td>
<td>4255</td>
<td>1810</td>
</tr>
<tr>
<td>HL</td>
<td>4165</td>
<td>1895</td>
</tr>
<tr>
<td>Filler</td>
<td>2575</td>
<td>2510</td>
</tr>
</tbody>
</table>

### 3.11. General Initial and Model Boundary Conditions

#### 3.11.1. Initial Hydrostatic Pressure Conditions

Current FE codes have the capability of preloading models with gravity and atmospheric loading. In an explicit analysis, LS-Dyna can initialise these loads using a relaxation
method built into the code. This method maximises the dissipation of kinetic energy using damping such that all the energy is quickly turned into internal strain. This method in LS-Dyna is known as dynamic relaxation and is controlled by *CONTROL_DYNAMIC_RELAXATION.

3.11.2. Restraint boundary conditions

As the ALE process is Lagrangian based, traditional Lagrangian restraints of displacement, velocity, and acceleration are used for all boundary conditions with the exception of fluid flow. If an ALE flow or pressure is required at the model boundary, a special donor cell (inexhaustible material reservoir) is required at the input point to feed the advection process. This element attribute is controlled by the ambient element type switch associated with the element definition.

3.11.3. Non-Reflective Boundary

LS-Dyna has the ability to define a non-reflective boundary condition. This maintains a constant pressure at the boundary preventing a reflected wave from being developed. This feature is found in most hydro-codes and is extensively used in the literature to allow effective termination of the fluid half space continuum.

3.12. Explosion Boundary Conditions

3.12.1. Similitude Equation

The similitude equation concept is based on dimensional similarity to test results (Cole [6]). The transient pressure history from a test explosion can be used to predict an explosion of different dimensions providing that the variables are scaled by the appropriate ratios. This leads to a set of equations known as the Hopkinson based Similitude equations. The equations are based on a spherical shock wave within an infinite medium (free field). Hammond and Saunders [76] review the Hopkinson scaling approach that is used by many authors to predict pressure/time characteristics of underwater and air borne blasts concluding that it is a well accepted approach. The similitude equations have a number of limitations that will be discussed below:

- The equations are based on spherical charges, as this is near to the truth for mine and torpedo devises, this assumption fits well with this thesis.
The charge is uncased.

The assumption that the pressure has an exponential decay is not entirely valid, in real charges significant variations in the decay rate can occur after 1 decay constant due to the explosive burn rate (Cole [6] and Swisdak [3]). To allow for this factor a double decay curve can be used as per Swisdak [3] or an additional constant pressure term can be added to the equation as per Lawrence [77].

Swisdak [3] indicates that the accurate similitude pressure range for TNT blast predictions is between the magnitudes of 3.4 to 138 MPa.

The rate of decay is generally considered to be an exponential function as shown in Figure 3-33 where the decay constant (θ) is defined as the time taken for the pressure to reach 0.3679 or (e^-1) of the peak pressure.

\[ \text{Time} \]
\[ \text{Shock Pressure} \]
\[ \text{Peak pressure} \]
\[ \text{Peak pressure} \times e^{-1} \]

In the design of a charge, the burn rate is often modified by the inclusion agents allowing the energy split between the shock and bubble components to be optimised. Lawrence [77] quantifies the effects of particle size and additives that change the burn rate of the explosive. Using the pressure/time history from Lawrence [77] a typical double decay constant is 2.9. The resulting pressure/time history is shown in Figure 3-34.
The equations that determine the pressure at any time \( P \), the speed of decay \( \theta \), Impulse \( I \), and the energy flux density \( E_{FD} \) are detailed in Eqns (3-28) to (3-32), where the distance from the charge to the point under consideration is \( R \) and the actual time is \( t \).

\[
P = K1 \cdot \left( \frac{W^3}{R} \right)^{d1} \cdot e^{-\frac{R}{\theta}}
\]

(3-28)

\[
\theta = K2 \cdot W^3 \cdot \left( \frac{W^3}{R} \right)^{d2}
\]

(3-29)

The Impulse \( I \) is the time integral of the pressure curve for pure exponential decay results in the following equations:

\[
I = \int_0^\infty P(t) \cdot dt = K3 \cdot W^3 \cdot \left( \frac{W^3}{R} \right)^{d3}
\]

(3-30)

Where \( K3 = K1 \cdot K2 \) and \( A3 = A1 + A2 \)
Integrating the pressure for zero to infinity Eqn (3-30) can be simplified to Eqn (3-31).

\[ I = \theta \cdot P_{\text{MAX}} \]  \hspace{1cm} (3-31)

The Energy Flux Density (\(E_{FD}\)) is a measure of the work done on the surface or the energy behind the shock front per unit area.

\[ E_{FD} = \left( \frac{1}{\rho \cdot C} \right) \int_0^\infty P^2 \cdot t \cdot dt = K\cdot A \cdot W^{\frac{1}{3}} \cdot \left( \frac{W^{\frac{1}{3}}}{R} \right)^{44} \]  \hspace{1cm} (3-32)

The variables for the similitude equations are presented in Table 3-20.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius to the point in space under consideration</td>
<td>(R)</td>
<td>m</td>
</tr>
<tr>
<td>Weight of TNT use in charge</td>
<td>(W)</td>
<td>kg</td>
</tr>
<tr>
<td>Time at which the shock wave reaches the point</td>
<td>(t_0)</td>
<td>ms</td>
</tr>
<tr>
<td>Time at which the shock wave pressure acts</td>
<td>(t)</td>
<td>ms</td>
</tr>
<tr>
<td>Shock pressure at time = (t)</td>
<td>(P_t)</td>
<td>MPa</td>
</tr>
<tr>
<td>Decay constant</td>
<td>(\theta)</td>
<td>ms</td>
</tr>
<tr>
<td>Impulse</td>
<td>(I)</td>
<td>kPa-sec</td>
</tr>
<tr>
<td>Energy flux density</td>
<td>(E_{FD})</td>
<td>m-kPa</td>
</tr>
</tbody>
</table>

For TNT and Pentolite the coefficients for the similitude equations are presented in Table 3-21 from Reid [4]. LS-Dyna has the capability of applying the the Similitude pressure boundary conditions through a user defined curve (*DEFINE_CURVE) or the Sub-sea loading method using the key word *LOAD_SSA (Hallquist [8]).
Table 3-21: TNT and Pentolite coefficients for the Similitude Equations (after Reid [4])

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TNT</th>
<th>Pentolite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock-wave Pressure</td>
<td>K1: 52.12</td>
<td>K1: 56.21</td>
</tr>
<tr>
<td></td>
<td>A1: 1.18</td>
<td>A1: 1.194</td>
</tr>
<tr>
<td>Decay Constant</td>
<td>K2: 0.092</td>
<td>K2: 0.086</td>
</tr>
<tr>
<td></td>
<td>A2: -0.185</td>
<td>A2: -0.257</td>
</tr>
<tr>
<td>Impulse</td>
<td>K3: 4.795*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A3: 0.995*</td>
<td></td>
</tr>
<tr>
<td>Energy flux density</td>
<td>K4: 94.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A4: 2.155</td>
<td></td>
</tr>
</tbody>
</table>

Note: * Impulse parameters determined from first principles as per Eqn (3-30) assuming pure exponential decay. Published values of K3 and A3 may differ from the above if non-exponential decay is obtained by experimental or simulation methods.

3.12.2. Application of Similitude Equation Boundary Condition

Tran and Marco [48] use the combination of a Similitude pressure boundary together with a non-reflective boundary to reduce the model size. This allows the pressure boundary to be located close to the hull. In this configuration, the reflected hull wave can pass back through the application surface (A-B) to be absorbed by the non-reflective boundary, as shown in Figure 3-35. This reduces the model size significantly by eliminating any reflected wave from the pressure application surface that would otherwise interact with the hull.
3.12.3. Boundary Element Loadings using Underwater Shock Analysis (USA)

Underwater Shock Analysis (USA) is a BE code that can be coupled to many popular FE codes, allowing shock boundary loads to interact with a FE analysis, Shin [21]. This allows the wetted boundary of the hull (or water model) to be loaded with the UNDEX loading. USA also allows the use of Doubly Asymptotic Approximation (DAA) boundary elements. The shock wave boundary conditions can be applied to an arbitrarily shaped boundary in a FE model. The principle of coupling DAA boundary elements to a FE model is diagrammatically shown Figure 3-36.

Figure 3-35: Diagrammatic representation of Tran & Marco [48] loading
3.12.4. Modelling the Explosive Charge with an EOS

The FE method can be used to model both the explosive charge and its interaction with the water and the hull. This approach uses an EOS to model the detonation propagation and energy release within the explosive. LS-Dyna has a number of material models and EOSs that allow this to be considered. In this research, the Jones-Wilkins-Lee (JWL) EOS is used in conjunction with the *HIGH EXPLOSIVE BURN MATERIAL. The JWL equation is a curve fitted approximation relating volume to pressure. The JWL equation is as per Eqn (3-33) and the JWL factors are listed in Table 3-22 after Kloster [78].

\[
P = A_{JWL} \left(1 - \frac{\omega}{R_1 \cdot V}\right) e^{-R_1 \cdot V} + B_{JWL} \left(1 - \frac{\omega}{R_2 \cdot V}\right) e^{-R_2 \cdot V} + \frac{\omega \cdot E}{V}
\]  

(3-33)

Where:

- \(A_{JWL}, B_{JWL}, R_1, R_2\) and \(\omega\) are constant parameters in the JWL EOS
- \(E\) is the internal energy
- \(V\) is the volume
Table 3-22: Adopted JWL Constants after Kloster [78]

<table>
<thead>
<tr>
<th>(\rho_0) (kg/m(^3))</th>
<th>(Dv) (m/s)</th>
<th>(P_{CJ}) (Pa)</th>
<th>(A) (Pa)</th>
<th>(B)</th>
<th>(R_1)</th>
<th>(R_2)</th>
<th>(\omega)</th>
<th>(E) (J/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1630</td>
<td>6930</td>
<td>21.00</td>
<td>373.77</td>
<td>3.747</td>
<td>4.15</td>
<td>0.9</td>
<td>3.5</td>
<td>6.00</td>
</tr>
<tr>
<td>E+9</td>
<td>E+9</td>
<td>E+9</td>
<td>E+9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>E+9</td>
</tr>
</tbody>
</table>

Where:

\(D_v\) is the detonation velocity

\(P_{CJ}\) is the detonation pressure at the Chapman-Jouget state

\(\rho_0\) is the unreacted density

A typical FE model of charge, water and hull is shown in Figure 3-37.

\[Figure~3-37:~FE~model~of~charge,~water~and~hull\]
3.13. Conclusion

The available numerical methods of FE, BE, and SPH, combined with the Lagrangian and Eulerian mathematical processes, provide the ability to realistically model all of physics involved in an UNDEX. The adopted modelling strategies employed must address the following issues:

- Transient shock wave pressures
- Shock wave front discontinuity
- Shock wave reflection and refraction
- Water infinite half space
- Discrete hull
- Local water cavitation
- Bulk water cavitation and surface cut-off

The literature favours the FE and BE methods due to their maturity and innate abilities in modelling the water continuum and the discrete structure of the ship. Although SPH has been used by a number of authors for fluid/structure interaction, its computational strength lies in the area of fragmentation, where one body may be broken into many. This occurs in the areas of projectile impacts and charge casing fragmentation. Due to the high non-linearity and short time intervals of an UNDEX, most of the literature uses the explicit time integration method with the water modelled as an Eulerian process and the ship as a Lagrangian process. The two processes are coupled to produce compatibility at their interface.

The type of computations used, largely depend on the size of the problem, solution accuracy required, computer speed, and the available time to complete the computation. For near field problems where the charge is close to the hull all of the components, charge, water and hull, can be represented in the model. In this case, non-linear water properties should be considered as explosive charge acts in a non-linear manner. For large far-field problems where a complete ship is required to be modelled, the use of a model containing the charge, water and hull would produce a prohibitively large model. In this case, the ship and a section of the water around the ship would be modelled and the shock wave pressures mapped onto the external water surfaces. This can be done by
the use of the Smilitude equations, boundary conditions or the propriety BE code USA which can interface with many structural codes. The interface between the water and the hull can be achieved in either a continuous or discontinuous way. The continuous method requires the water and hull mesh to have coincident nodes, which often represents a significant and time consuming meshing challenge. Many authors use a coupling of the water and hull where a discontinuous mesh is allowed. In this case, the FE code uses nodal velocity interpolation to provide nodal compatibility between the water and the hull elements; this is known as Eulerian/Lagranian coupling. The advantage of this method is that it significantly reduces the model meshing time, complexity and allows elements of different sizes to be coupled.
4. Error Estimates

4.1. General

The aim of this chapter is to critically review the current body of knowledge covered in Section 3, ratifying the relevant methodologies, and answering the research question in Chapter 1 with respect to error estimation. The section includes various analytical results using LS-Dyna simulations. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

4.2. Sensitivity of Hull motions to Sea Water Variations

4.2.1. Basic Numerical Models used for Water Property Sensitivity Analysis

As per Section 3.8 the properties of sea water vary depending on the temperature, salt concentration, temperature, and density. To investigate the affects of these variations on the hull motions, a number of sensitivity analyses were preformed using LS-Dyna. The sensitivity simulations performed in this section are summarised in Table 4-1.

<table>
<thead>
<tr>
<th>Eulerian water parameter</th>
<th>Symbol</th>
<th>Method of Determining a Parameters</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>ρ</td>
<td>Shock tube with TNT, water, and hull modelled</td>
<td>4.2.2</td>
</tr>
<tr>
<td>Linear compressibility</td>
<td>$B_{MOD}$</td>
<td>Shock tube with TNT, water, and hull modelled</td>
<td>4.2.2</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>$\mu_D$</td>
<td>Shock tube with water and hull modelled</td>
<td>4.2.3</td>
</tr>
</tbody>
</table>

The sea waters attributes of density and stiffness are embedded in the similitude equations pressure predictions. To investigate the sensitivity of these parameters required the modelling of the explosive charge and its interaction with the water. For this the JWL EOS was used. To verify its performance, a 77.8 kg spherical charge of TNT (R=0.225 m) was considered using an axi-symmetric model for a 4.825 m water sphere, comparing its results to the single decay and double decay (2.9 factor) Similitude solutions. The axi-symmetric element formulation in LS-Dyna allows either a Lagrangian smoothing or EFG process, (MMALE formulations are not supported by the...
2-D volume elements). The ALES process consistently resulted in the water mesh size approaching zero width resulting in premature computation termination. The use of the ALES process for the charge and pure Lagrangian for the water produced a suitable model configuration. The TNT charge was modelled using the JWL EOS as per Table 3-22. The initial model is shown in Figure 4-1 with a mesh size of 15 mm normal to the shock wave. The water pressure at time 2.2 ms and charge boundary location is shown in Figure 4-2.

*Figure 4-1: Initial model geometry of spherical blast*
The comparison between the FE simulation and the similitude equations at $R = 2.19$ m for the single decay are as per Figure 4-3. The double decay comparison at $R = 2.19$ m is as per Figure 4-4. The double decay comparison at $R = 4.218$ m is as per Figure 4-5. It can be seen from these comparisons that the double decay similitude equation fits the FE simulations well.
Error Estimates

Figure 4-3: Transient pressure comparison between the JWL and Smilitude at 2.193 m
Figure 4-4: Transient pressure comparison between the JWL and Similitude at 2.193 m
4.2.2. Sensitivity of Sea Water Density and Compressibility

To determine the sensitivity of the water density and compressibility to hull excitation, a LS-Dyna shock tube model was developed with the TNT, water, and also included a rigid hull included. The model arrangement, meshing, ignition point and change pressures shortly after ignition are shown in Figure 4-6. The ignition of the TNT produced a shock wave that propagated through the water column and reflected off the rigid body. By changing the water attributes, the affect on the hull motions was determined with respect to these properties. The two extremes of water properties used were pure water at 15°C (density=999.1 kg/m$^3$ and a bulk modulus of 1.97 x 10$^9$ Pa) and sea water at 25°C with a salinity of 40 ppt (with a density=1027.1 kg/m$^3$ and a bulk modulus of 2.43 x 10$^9$ Pa). It was found that increasing the water density and stiffness impeded the expansion of the TNT charge, resulting in an average pressure increase of about 7% in the TNT. This interaction with the water resulted in the TNT burning at a high rate for a shorter time, typically resulting in a PA increase of 9% and a PTV reduction of 3%.
4.2.3. Sensitivity to Dynamic Viscosity

The fluid viscosity will affect the transmission of shock pressures into the hull by prolonging the confinement of the shock wave pressures at an exposed hull corner. At the hull corner the shock wave becomes discontinuous and viscosity related fluid shears are produced. To test the effect of fluid viscosity at an exposed corner (i.e. the corner of a rectangular hull), a shock tube was developed with a compressive shock wave of 60 MPa in LS-Dyna as shown in Figure 4-7. The peak shear within the fluid was negligible at $451.5 \times 10^{-6}$ MPa which makes no measurable difference to the hull motion.
4.3. Boundary Conditions

4.3.1. Application of Shock Wave to Models

The application of the shock wave pressures to the LS-Dyna models used in the thesis was the similitude equation method. The similitude equations allow models loaded with the shock pressure to be applied at the extremity of the model using either a segment pressure or nodal velocity based boundary condition. Tran and Marco [48] elaboration of this method allows the pressure boundary condition to be placed very close to the hull, resulting in significant reduction in model size (Section 3.12.2). To verify its suitability, a shock tube simulation was developed with a shock wave pressure applied at line A-B (applied shock pressure = 68.4 MPa) as per Figure 4-8. The pressure is split, propagating a compression wave towards point C (material in this area capable of cavitating) and a tension wave towards the non-reflective boundary (material in this area not capable of cavitating). Defining the materials in the shock tube with an EOS, results in different material densities being produced within the tension and compression regions. This produces a refractive boundary at the line A-B, and results in spurious pressure
Error Estimates

oscillations occurring within the shock tube. The use of an elastic fluid (density constant with pressure) greatly reduces the spurious pressure oscillations as showing in Figure 4-9. The EOS with default bulk viscosity values (EOS_STD_BV) produces large spurious pressure oscillations. Increasing the bulk viscosity values 2.5 times the default (EOS_Non_STD_BV) reduces the spurious pressure oscillations.

Figure 4-8: Shock Tube with Tran and Marco’s [48] non-reflective boundary conditions
The pressure split between the compressive and tension waves was found to be relatively constant over a wide range of models with 64% of the wave magnitude at A-B being converted into a compression shock wave. The LS-Dyna simulation indicates that the shock front is approximation 0.2 ms in duration. In the model, ten element lengths (i.e. 0.03 m in length) were required to capture the shock front shape. Cole [6] indicates that in reality, the shock wave front distance is in the order of $10^{-7}$ to $10^{-8}$ m and is completely negligible. To standardise the simulation with the similitude results and allow for LS-Dyna’s over estimation of the shock front, the $P_{\text{MAX}}$ developed by LS-Dyna will be calibrated such that $I_{\text{LS-Dyna}} = I_{\text{SIMILITUDE}}$.

### 4.3.2. Half Space Boundary Condition

LS-Dyna non-reflective boundaries are available in both 2-D and 3-D geometries. The 3-D boundary has the option to discriminate between normal and shear waves. The functionality of the LS-Dyna non-reflective boundary was investigated by modelling a 10 m cubic section of water with a shock pressure applied on line A-B, a non-reflective boundary on surfaces A-D-C, and a rigid boundary on surface C-B as shown in Figure 4-10.
The adjacent non-reflective boundary (D-A) significantly affects the shock wave peak pressure ($P_{\text{MAX}}$) and Impulse ($I_{\text{MAX}}$) magnitude within the model. The progressive erosion of the shock waves edge results in 50% of the original volume no longer containing a realistic representation of the shock wave this is shown in Figure 4-11.
The area occupied by $P_{\text{MAX}}$ is reduced to approx 65% (A-1-C-D) of the total area due to the affects of the non-reflective boundary D-A

(a)

The area occupied by $I_{\text{MAX}}$ is reduced to approx 50% (A-C-D) of the total area due to the affects of the non-reflective boundary D-A

(b)

Figure 4-11: (a) Affect of non-reflective boundary A-D on $P_{\text{MAX}}$ and (b) $I$

The adopted half space boundaries for this research were the combination of the rigid boundaries (located at a sufficient distance from the hull section) and non-reflective boundaries. All model boundary parameters were adjusted if necessary after a sensitivity analysis was performed to ensure that they do not cause erroneous reflected waves, or significant reductions in shock pressures.

4.4. Cavitation

4.4.1. General

A review is presented here of the quantitative cavitation work by:

- Shin [2] for 3-D bulk cavitation with a constant density model cavitation. The example considers a 90.7 kg (200 lb) Pentolite charge at a depth of 7.62 m (25 feet) and uses the material *MAT_001 with a cavitation pressure of -1 Pa.
- Bleich and Sandler [50] and Shin and Santiago [22] for 1-D cavitation bi-linear material cavitation example using a tabulated EOS.
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In this thesis, their results are duplicated for verification and error estimates due to cavitation. The 3-D simulations in LS-Dyna are an axi-symmetric FE formulation that represents a conical 3-D section of water. Volumes with and without hull shapes are considered. In the axi-symmetric formulation, the coordinate system is radial and the variables that require solution are independent of the angle \( \theta \). This allows a 3-D shape as per Figure 4-12 to be modelled as a 2-D volume. In LS-Dyna the shell element formulation EQ 14 and 15 are axi-symmetric 2-D volumes. The 1-D cavitation models use the 2-D plane strain volume (shell element formulation EQ 12). For all models in this section the application of gravity was applied using *LOAD_BODY_Y and *CONTROL_DYNAMIC_RELAXATION. The pressure application surface was restrained from vertical downward motion by a contact surface using *CONTACT_2D_AUTOMATIC_SURFACE_TO_SURFACE and was free to move upward under the application of the pressure loading.

![Figure 4-12: 3-D model](image)

4.4.2. 3-D Cavitation Depth on the Charge Centre Line

The LS-Dyna constant density cavitation peak depth estimate was verified against Shin [2] for the 90.7 kg (200 Ib) Pentolite charge at a depth of 7.62 m (25 feet). The initial
conditions, charge size, location and gravity loads are shown in the axi-symmetric LS-Dyna as per Figure 4-13.

Figure 4-13: Initial gravity and pressure loads in LS-Dyna axi-symmetric model

The propagation of the shock wave just prior to the free surface reflection is shown in Figure 4-14.
The progressive development of the bulk cavitation is shown in Figure 4-15 and Figure 4-16. The peak depth of cavitation of 2.7m is as per Figure 4-17. This agrees with the approximate magnitude as scaled from Shin [2].

Figure 4-15: (a) Shock wave and (b) cavitation at time =0.00289 s
Figure 4-16: (a) Shock wave and (b) cavitation at time = 0.003995 s

Figure 4-17: Peak centre line depth of cavitation at time = 0.004889 s
4.4.3. 3-D Cavitation Development for 90° V Hull

4.4.3.1. General

The model in Figure 4-13 was modified to include a 90° V hull with a 2 m draft. The hull is restrained against motion in the model such that local cavitation was not activated in the analysis. The progressive development of the bulk cavitation is shown in Figure 4-18 and Figure 4-20. The peak depth of cavitation of 3.4m is as per Figure 4-20. This analysis shows that the inclusion of a hull in the half space significantly affects the boundaries of bulk cavitation.

Figure 4-18: Shock wave loading the hull

Figure 4-19: (a) Shock wave surface reflection (b) cavitation extent at time =0.003096 s
4.4.3.2. Surface Cut-off

As the bulk cavitation develops, it passes through the water, reducing the pressure in its envelope to the cavitation pressure (near zero). This truncation of the shock wave tail is known as “Surface cut-off”. The transient pressure profile of two elements are comparable: element 7210 near the surface shows the classic tail truncation of surface cut-off while the deeper element 7643 is not affected as shown in Figure 4-21.
4.4.3.3. Local Cavitation Visualisation

The local cavitation is demonstrated by releasing the hull in the vertical direction and reviewing the cavitation extent before the shock reflects off the free surface as per Figure 4-22. The velocity of the hull reduces the peak pressures in the shock wave/hull intersection and generates an area of local cavitation where the total pressure falls below the cavitation pressure.

Figure 4-22: (a) Shock wave loading hull (b) local cavitation from hull velocity
4.4.4. Cavitation at Slant Angles of Non-Zero

With the boundary conditions as per Figure 4-13, a LS-Dyna model was produced to consider the shape of the cavitation region experienced by a 3-D rigid 45° hull face. The reflected shock pressure is shown in Figure 4-23 and the bulk cavitation area and its bottom boundary (at depth \( \approx 2.7 \) m) are shown in Figure 4-24.

![Figure 4-23: Pressure of reflect 3-D shock wave at time=0.00569 s](image)

---

**Figure 4-23: Pressure of reflect 3-D shock wave at time=0.00569 s**
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Figure 4-24: Cavitation region at time=0.00569 s
4.4.5. Review of 1-D cavitation case considered by Bleich and Sandler [50]

1-D cavitation example by Bleich and Sandler [50] and Shin and Santiago [22] as described in Section 3.7.1.3 and Table 4-2 is duplicated using a LS-Dyna model. The model was using an ALES process. The depth of the water column was 15.24 m which prevented unwanted base shock reflections.

Table 4-2: 1-D cavitation example considered for Bleich and Sandler [50]

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Bleich and Sandler in Imperial Units</th>
<th>Adopted Metric Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of sound in uncavitated water</td>
<td>C1</td>
<td>4670 ft/sec</td>
<td>1,423.4 m/s</td>
</tr>
<tr>
<td>Speed of sound in cavitated water (Note: Non-zero value adopted for numerical stability)</td>
<td>C2</td>
<td>≈0 ft/sec</td>
<td>≈0 m/s</td>
</tr>
<tr>
<td>Cavitation pressure</td>
<td>P₀</td>
<td>0 psi</td>
<td>0 Pa</td>
</tr>
<tr>
<td>Initial water density</td>
<td>ρ₀</td>
<td>1.94 slug/ft³</td>
<td>999.8 kg/m³</td>
</tr>
<tr>
<td>Gravitational constant</td>
<td>g</td>
<td>32.2 ft/s²</td>
<td>9.815 m/s²</td>
</tr>
<tr>
<td>Atmospheric pressure</td>
<td>P_ATMOS</td>
<td>14.7 psi</td>
<td>101,353 Pa</td>
</tr>
<tr>
<td>Peak shock pressure</td>
<td>P_MAX</td>
<td>103 psi</td>
<td>710,160 Pa</td>
</tr>
<tr>
<td>Decay constant (length)</td>
<td>L</td>
<td>4.74 ft</td>
<td>1.4447 m</td>
</tr>
<tr>
<td>Decay constant (time)</td>
<td>θ</td>
<td>4.74/4670= 1.015 x10⁻³ s</td>
<td>1.015 x10⁻³ s</td>
</tr>
<tr>
<td>Steel plate UDL</td>
<td>W</td>
<td>0.921 slug/ft²</td>
<td>144.68 kg/m²</td>
</tr>
<tr>
<td>Steel thickness at ρ_S=7870 kg/m³</td>
<td>t</td>
<td>0.724 in</td>
<td>0.01838 m</td>
</tr>
</tbody>
</table>

The above LS-Dyna model was initialised with atmospheric pressure and gravitational acceleration using dynamic relaxation and was then loaded with a pressure pulse to simulate the shock loading and a cavitation pressure of zero. Two water models were considered and compared to the Bleich and Sandler [50] and Shin and Santiago [22] results. The water models are as per Table 4-3.
Table 4-3: LS-Dyna models for 1-D Bleich and Sandler [50] example

<table>
<thead>
<tr>
<th>File name</th>
<th>LS-Dyna Material</th>
<th>EOS</th>
<th>Element Length (mm)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bleich_mat_001.k</td>
<td>*MAT_001</td>
<td></td>
<td>8.5</td>
<td>Constant density cavitation model</td>
</tr>
<tr>
<td>Bleich_Bi_eos_09.k</td>
<td>*MAT_009</td>
<td>Tabulated</td>
<td>25.4</td>
<td>Bi-linear density definition as per Appendix C</td>
</tr>
</tbody>
</table>

The variable density model provided good agreement with the Bleich and Sandler [50] and Shin and Santiago [22] velocity results by predicting a more accurate estimate of cavitation closure when compared to constant density cavitation model. The transient velocity comparison between the three is shown in Figure 4-25.

![Figure 4-25: Velocity comparison between Shin [22] and LS-Dyna models](image)

The LS-Dyna constant density cavitation model performed with no significant convergence problems during the gravity and atmospheric initialisation stage of the analysis. The variable density model posed significant convergence problems which is why the larger element (25.4 mm) size is adopted in these models. Additional to the difficulty in initialisation convergence, the variable density models also suffered from
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spurious transient pressure oscillations. The transient pressure at the steel plate-water interface between Shin and Santiago [22] and the constant density cavitation model shows an \( \approx 0.002 \) s error in the cavitation closure time estimate as per Figure 4-26. This time error is consistent with the speed of sound difference between the cavitated regions of the two models.

\[ 
\begin{align*}
\text{Pressure (Pa)} \\
& 1,200,000 \\
& 1,000,000 \\
& 800,000 \\
& 600,000 \\
& 400,000 \\
& 200,000 \\
& 0 \\
\end{align*}
\]

\[ 
\begin{align*}
\text{Time (s)} \\
& 0 \\
& 0.002 \\
& 0.004 \\
& 0.006 \\
& 0.008 \\
& 0.01 \\
& 0.012 \\
& 0.014 \\
\end{align*}
\]

\textit{Compression wave, a result of cavitation closure}

\textit{Mat_001.k estimate}

\textit{After Shin\_CAFE cavitation} \quad \textit{MAT_001.k}

\textit{Figure 4-26: Transient pressure comparison after Shin and Santiago [22] and Mat_001.k}

The similar comparison between Shin and Santiago [22] and the bi-linear model shows a greater accuracy in the prediction on the cavitation closure as per Figure 4-27.
4.4.6. Simulation Error Associated with Cavitation

The assessment of the error associated with cavitation was addressed by applying the variations in the cavitation pressure and different models (bi-linear and constant density). The worst scenario for cavitation effects was considered to be the smallest V hull with a slant angle of 45° as shown in Figure 4-28 and Figure 4-29. The model was loaded with atmospheric pressure and gravity and three load cases were considered as per Table 4-4. The variation in PTV between the load cases was only 5%.
Error Estimates

Figure 4-28: Shock pressure for 2 m V hull at time = 0.0015789 s

Figure 4-29: Shock pressure for 2 m V hull at time = 0.0021436 s
Table 4-4: Sensitivity of PTV to cavitation models

<table>
<thead>
<tr>
<th>Load Case</th>
<th>$P_{\text{MAX}}$ (MPa)</th>
<th>$\theta$ (s)</th>
<th>Cavitation Pressure (Pa)</th>
<th>Cavitation Model</th>
<th>PTV (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Constant density</td>
<td>16.204</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>34.44</td>
<td>0.9</td>
<td>2000</td>
<td>Constant density</td>
<td>16.202</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td></td>
<td>Bi-linear EOS</td>
<td>16.960</td>
<td></td>
</tr>
</tbody>
</table>

4.5. Variations due to Atmospheric and Gravity Loading

The variations in gravity and atmospheric pressure experienced at different locations on the earth or in variable water conditions have no significant affect on hull motions. However, to reduce the computational expense of each LS-Dyna model which increases the number of models generated, it was concluded that the error associated with not considering gravity and atmospheric pressure was minimal. To quantify the error, the model in Section 4.4.6 was run without gravity and pressure, resulting in a 0.5% change in the PTV.

4.6. Numerical Method Review

To compare the different ALE methods available in LS-Dyna, the acceleration response for the three different modelling methods were compared. The hull considered was a 45° V of draft 7.0 m with a 1,000 kg explosion at 39.231 m below the hull base. The typical model components are shown in Figure 4-30 for the coupled Lagrangian/Eulerian model. The shock boundary conditions were applied using the similitude equation method.
The transient hull accelerations of the three methods can be seen in Figure 4-31. The MMALE approach cannot accommodate a rigid material. As a substitute to the rigid material, the hull was modelled in steel for the MMALE analysis; hence, periodic oscillations can be seen from hull flexing. All methods gave similar results. The advantages and disadvantages attributed each method are discussed in Table 4-5.

Figure 4-30: Isometric view of 3-D Eulerian and multi-material models
Figure 4-31: Transient hull acceleration comparison of ALE methods
### Table 4-5: Advantages and disadvantages of ALE methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALES</td>
<td>Most computationally efficient</td>
<td>Least able to accommodate large mass transfers. The coincident fluid/structure nodes increase meshing complexity between hull and fluid.</td>
</tr>
<tr>
<td>MMALE</td>
<td>Can accommodate large mass transfers. Has density as an output variable. Can generate complex solid shapes by an enclosed shell geometry</td>
<td>The modelling of ships shell type structures not appropriate, all elements require 3-D solid models. Can not model 2-D solids.</td>
</tr>
<tr>
<td>Lagrangian</td>
<td>Can accommodate large mass transfers. Fluid/structure nodes not required to be coincident and hence can decrease meshing complexity between hull and fluid.</td>
<td>Coupling between hull and fluid requires an iterative numerical procedure that may be an error source if penalty variables not optimized.</td>
</tr>
</tbody>
</table>


4.7. Finite Element Size

4.7.1. Finite element size adopted

To review the affects of element size, a shock tube analysis was conducted with the boundary conditions equivalent to a 590 kg TNT charge at 34 m. This results in the boundary conditions are defined by Eqns (4-1) to (4-3):

\[ P_{\text{MAX}} = K_1 \left( \frac{W^\frac{1}{3}}{R} \right)^{41} = 52.12 \left( \frac{590^\frac{1}{3}}{34} \right)^{1.18} = 10 \text{ MPa} \quad (4-1) \]

\[ \theta = K_2 \cdot W^\frac{1}{3} \cdot \left( \frac{W^\frac{1}{3}}{R} \right)^{42} = (0.092) \cdot (590)^\frac{1}{3} \cdot \left( \frac{590^\frac{1}{3}}{34} \right)^{-0.185} = 1 \text{ ms} \quad (4-2) \]

\[ I = K_3 \cdot W^\frac{1}{3} \cdot \left( \frac{W^\frac{1}{3}}{R} \right)^{43} = (4.795)(590)^\frac{1}{3} \cdot \left( \frac{590^\frac{1}{3}}{34} \right)^{0.995} = 10 \text{ kPa} \cdot \text{s} \quad (4-3) \]

The above boundary conditions were placed at one end of two 10 m long shock tubes. The mesh in each shock tube varied: one with 25 mm mesh and the other with a 250 mm mesh as per Figure 4-32. This allowed the direct comparison of the effects of mesh density with respect to shock wave definition to be determined.
The shock tube results produced significant pressure oscillations in the coarse mesh (250 mm) while the finer 25 mm mesh produced pressure responses, smoother and similar in shape to the applied boundary loads, as shown in Figure 4-33. The results show that the magnitude of the pressure reduces and broadens along the shock tube length. This effect is synonymous with the transient evolution of pressure waves with distance from the explosion source.
Error Estimates

However, the Impulse remains constant and is independent of mesh size and the distance along the tube ($I=10 \text{ kPa.s}$). The theoretical value of $I$ is the same as calculated by the LS-Dyna integration of the results from Figure 4-33; this can be seen in Figure 4-34.

![Figure 4-33: Results from shock tube analysis](image)

![Figure 4-34: Impulse ($I$) =10 kPa.s at time = infinity](image)
Error Estimates

The method of successive approximation determines the affect of element size on hull motions for two hull shapes: the flat bottom hull and the 90° V hull section. These simulations were conducted using a shock tube model for the flat bottom and a half model for the 90° V hull (4 m deep) with zero slant as shown in Figure 4-35. From observation, the element size is critical under maximum acceleration; This occurs under peak pressure and minimum hull mass (at $P_{\text{MAX}} = 68.43$ MPa and $D = 4$ m).

Figure 4-35: 90° V hull, 4 m deep with varying mesh densities
Error Estimates

The flat bottom hull motion with respect element size is shown in Table 4-6 and Figure 4-36.

Table 4-6: Element size and resulting PTV and PA % error for the flat bottom

<table>
<thead>
<tr>
<th>Element Size (mm)</th>
<th>PTV % Error Compared to an Element Size of 12.5 mm</th>
<th>PA % Error Compared to an Element Size of 12.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>-0.2</td>
<td>-4.4</td>
</tr>
<tr>
<td>50</td>
<td>-0.4</td>
<td>-15.1</td>
</tr>
<tr>
<td>100</td>
<td>-1.0</td>
<td>-30.0</td>
</tr>
<tr>
<td>125</td>
<td>-1.2</td>
<td>-35.5</td>
</tr>
<tr>
<td>150</td>
<td>-2.1</td>
<td>-36.6</td>
</tr>
<tr>
<td>250</td>
<td>6.0</td>
<td>-54.8</td>
</tr>
<tr>
<td>500</td>
<td>-2.0</td>
<td>-62.7</td>
</tr>
<tr>
<td>1000</td>
<td>0.4</td>
<td>-76.1</td>
</tr>
</tbody>
</table>

Figure 4-36: Element size versus hull motion error for the flat bottom hull
Error Estimates

The 90° V hull motion with respect element size is shown in in Table 4-7 and Figure 4-37.

**Table 4-7: Element size and resulting PTV and PA % error for the 90° V hull**

<table>
<thead>
<tr>
<th>Element Size (mm)</th>
<th>PTV % Error Compared to an Element Size of 58 mm</th>
<th>PA % Error Compared to an Element Size of 58 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>117</td>
<td>-0.5</td>
<td>-0.6</td>
</tr>
<tr>
<td>236</td>
<td>-2.0</td>
<td>-2.4</td>
</tr>
<tr>
<td>742</td>
<td>-24.2</td>
<td>-19.7</td>
</tr>
<tr>
<td>1114</td>
<td>-33.5</td>
<td>-30.6</td>
</tr>
</tbody>
</table>

**Figure 4-37: Element size versus hull motion error for the 45° hull**

The successive approximation simulations indicate that the mesh density significantly affects the model’s ability to predict the peak shock pressures, eliminate spurious pressure oscillations, and produce realistic hull motions. For this research, the element dimensions were adopted as per Table 4-8. Also, in areas of element size transition normal to the shock wave, the element size increase between elements shall not exceed 100%.
4.8. Hull Rotation due to off Centre Loading

A hull loaded with an off centre shock was of particular interest in this research with respect to its effect on the average vertical motions of a hull. Hence, the effect of the cross coupling of the vertical and rotational hull motions is required to be understood for off centre shock loadings. Consider a 45° V hull loaded with an off centre shock loading as per Figure 4-38.

Figure 4-38: Section considered for off centre shock loading WRT vertical motion
The vertical motion at points A and B with respect to time shows that significant hull rotation occurs. This is characterised by the different magnitudes of vertical velocities experienced by points A and B, as per Figure 4-39.

Figure 4-39: Vertical motions of points A & B and the average

Comparing different degrees of fixity of the hull (restraint against rotation and restraint against rotation and lateral) with the average vertical velocities shows that the rotational DOF can be uncoupled from the problem as per Figure 4-40.
The finding that hull rotation has little effect on the average vertical hull motions indicates that the 2-D work carried out by this research can be related to real 3-D problems without being encumbered with hull rotation considerations.

4.9. **Verification of Similitude Equations Parameters in the Literature**

The published similitude equations parameters contain small variations from author to author. This section will compare the final $P_{oj}$, $\theta$, and $I$ adopted in this research compared to Swisdak [3]. The comparison for results for a 1000 kg TNT charge at a distance of 10 m are shown in Table 4-9.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Section 3.12.1</th>
<th>Swisdak [3]</th>
<th>Units</th>
<th>% Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Pressure</td>
<td>$P_{\text{MAX}}$</td>
<td>52.12</td>
<td>52.4</td>
<td>MPa</td>
<td>+1</td>
</tr>
<tr>
<td>Decay Constant</td>
<td>$\theta$</td>
<td>0.92</td>
<td>0.84</td>
<td>ms</td>
<td>-10</td>
</tr>
<tr>
<td>Impulse</td>
<td>$I$</td>
<td>47.9</td>
<td>57.5</td>
<td>kPa-s</td>
<td>+16</td>
</tr>
<tr>
<td>Energy flux</td>
<td>$E_{\text{FD}}$</td>
<td>942.7</td>
<td>884</td>
<td>m-KPa</td>
<td>-7</td>
</tr>
</tbody>
</table>

*Table 4-9: Comparison of similitude explosion characteristics*
The variations between the above two Similitude parameters represent typical fitting of scaling results to the similitude equations by different authors. It should also be noted that the definition of explosive charges by name is not definitive. There can be significant variations in the performance of charges associated with manufacturing processes and the inclusion of additives.

4.10. Error Estimate

The estimation of the errors associated with the LS-Dyna models used in this thesis compared to the actual test depends on ten modelling assumptions. The different assumptions may not produce uniform variations across the cases considered. For example, bulk cavitation affects on V hulls with no-zero slant is considerably more than for rectangular hull with zero slant. In this treatment of error estimation, the peak error determined for each variable will be summated to produce an upper bound estimate shown in Table 4-10. The error will be expressed as a plus or minus magnitude and is the result of the research completed in this section.
### Table 4-10: Error estimate of LS-Dyna models compared to real hulls

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Reference</th>
<th>PTV Error Estimate (%)</th>
<th>Plus or Minus PTV Error Estimate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Water properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>• Water density and compressibility</td>
<td>3</td>
<td>± 1.5</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>• Water viscosity</td>
<td>0</td>
<td>± 0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Shock wave properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>• Published peak pressure</td>
<td>1</td>
<td>± 0.5</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>• Decay variation</td>
<td>3</td>
<td>± 1.5</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>• Mine casing</td>
<td>5</td>
<td>± 2.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Cavitation</td>
<td>5</td>
<td>± 2.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Element density</td>
<td>2.4</td>
<td>± 1.2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Omission of atmospheric and gravity in models</td>
<td>1</td>
<td>± 0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Summation (rounded to the nearest %)</td>
<td></td>
<td></td>
<td>± 11</td>
</tr>
</tbody>
</table>

### 4.11. Conclusion

The LS-Dyna methodology and modelling strategy used in this thesis was verified against a known solution obtained from the literature. This example was dominated by local cavitation effects. The ALES method used for this research provides the most suitable process for the modelling of 2-D hulls because it can support a rigid material and it is not sensitive to the coupling parameters. For accurate results, the simulations require an appropriate mesh size to predict peak acceleration. This is particularly important for the rectangular hull with zero slant angle or V hulls with a slant angle. These hulls are loading with the peak shock pressure applied instantaneously over the effective hull section, requiring a high node density to capture the peak shock pressure. For hulls in which the shock is applied progressively, a larger mesh size is adequate. V hulls at zero slant and 3-D hulls are examples of hulls that experience a progressive shock loading.
Error Estimates

because the shock wave takes some time to envelope the hull surface. For this research, the adopted mesh depends on the incident angle $\psi$ between the wave and the hull surface. Where $\psi = 0^\circ$, a mesh size of 25 mm to 50 mm was required. For values of $\psi \geq 45^\circ$, the mesh can be increased up to 200 mm. These values result in errors of -15% for PA and 2% for PTV predictions. For PTV predictions the two largest errors were considered to be the shock wave definition accounting for an error of $\pm 4.5\%$ and the cavitation error at $\pm 2.5\%$.

The sensitivity of UNDEX to the sea water properties can be considered to be three fold:

- Developing an UNDEX shock boundary condition by modelling the charge detonation and the surrounding water is best achieved using a non-linear material for modelling the water. In this scenario the expanding charge compresses and densifies the surrounding water, retarding the charge expansion. This increases the burn rate within the charge, resulting in a high peak pressure with a reduced duration. The most recognised constitutive material model for non-linear density and stiffness in the literature is the Gruneisen EOS.

- Developing an UNDEX shock boundary condition for a far-field scenario by mapping the shock pressure onto the external boundary of the model requires rudimentary water parameters, such as a linear acoustic model. This approach is not overly sensitive to the water properties of stiffness or density.

- Hulls of limited draft with non vertical hull segments near the surface (such as V hulls with large slant angles) are sensitive to bulk cavitation and surface cut-off effects. When considering these types of hulls, a variable density cavitation model provides a significant advantage due to its accuracy. The smallest rectangular hull (of draft 4m) covered in this thesis is not significantly affected by bulk cavitation and a constant density cavitation model is considered adequate.

In was found that the effect of hull rotation from off center shock loading has little effect on the average vertical hull motion. This allows the application of this research to be applied to 3-D hulls without rotational inertia of the entire ship being considered.
5. Modelling Strategy and Equation Forms

5.1. Introduction

This chapter details the software versions used, the construction and keyword commands used in the LS-Dyna files, upper and lower limits of the hulls and charge sizes and the forms of the two tiered hull motion equations.

5.2. Software Used

For FE predictions of hull motions, the explicit FE software LS-Dyna was used. It was complemented by the pre- and post-processing software as per Table 5-1.

<table>
<thead>
<tr>
<th>Software</th>
<th>Version</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS-Dyna</td>
<td>970</td>
<td>Explicit solver</td>
</tr>
<tr>
<td>LS-Post</td>
<td>1.4</td>
<td>Post-Processing</td>
</tr>
<tr>
<td>Ansys</td>
<td>6.0</td>
<td>Pre-Processing</td>
</tr>
</tbody>
</table>

5.3. LS-Dyna Model Construction

5.3.1. General

All models, unless noted otherwise, are constructed using a 2-D UI element formulation solved using the ALES explicit process. The water model used was the acoustic *MAT_001_FLIUD in conjunction with a constant density cavitation model. The shock loading pressure boundary conditions were applied using the Similitude equation predictions.

5.3.2. Model Pressure Initialisation

Only selected models shall be initialized with gravity and atmospheric pressure. For these cases the following key word commands were used:

- *CONTROL_DYNAMIC_RELAXATION allows the model to be initialized with gravity and atmospheric pressure such that dynamic energies are quickly damped
out and converted into internal stain energy. This procedure produces the static initial condition for the shock pressure application.

- \*LOAD_SHELL_SET loads 2-D shell elements with out of plane pressure loadings.
- \*LOAD_SEGMENT_SET loads 2-D and 3-D volume elements with the pressure loadings.
- \*LOAD_BODY_Y loads the model with the a gravitational acceleration.
- \*CONTACT_2D_AUTOMATIC_SURFACE_TO_SURFACE_TITLE allows the fluid section of the model to be constrained by a rigid part such that, under the initialization stage, the fluid elements are restrained from downward motions and are free to move upward under the shock pressure loading.

5.3.3. Shock Load Application in Non-Gravitational Models

The shock loading was applied using a pressure developed from the \*LOAD_SEGMENT_SET command in conjunction with the \*DEFINE_CURVE. A standard unity shock pressure load curve was used for all models with $\theta=0.001$ s and $P_{\text{MAX}}=10$ units. This curve was adapted to the correct $\theta$ and $P_{\text{MAX}}$ for the specific model by the scaling factors of SFA and SFO. Where the non-reflective pressure application boundary condition was used, the split between the compressive and tension waves was proportioned by the factor SF. Typically a pressure split of $\left(\frac{1}{1.5443}\right)$ was used as shown in Figure 5-1.
5.3.4. Shock Load Application in Gravitational Models

For these models, a method similar to that used in Section 5.3.3 was adopted except that SFO=1 and OFFO equals the total average initialization pressure at the contact boundary.

5.4. Differential Equation Solution Method

The differential equation form used in this research to provide a closed solution for the tier two hull motion is known as a linear first-order initial value problem, it was solved using the integration factor method, Bronson [79] as per Appendices A and B. Numerical integration shall also be used to check the correctness of the closed form solution by using the truncated Taylor series method (Bronson [79]).
5.5. **UNDEX Limits and Variables**

5.5.1. **General**

This section defines the hull and explosion geometries required to uniquely define the range of limits required for this thesis. The variables that contribute to the hull motion depend on the hull shape, hull size, charge size, and location. This equates to seven variables. However, considering only far-field UNDEX events, the number of variables can be reduced to five as per Table 5-2. In addition, the use of five far-field variables provides a greater flexibility for the parametric equation development. In using this approach, the decay constant ($\theta$) can be uncoupled from the TNT coefficients allowing charges with significantly different burn rates to be easily considered. Considering the UNDEX event to be far-field will over estimate the hull motions, producing conservative motion estimates for near field events.

<table>
<thead>
<tr>
<th>Table 5-2: Variables affecting hull motions for each hull shape</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Item</strong></td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td><strong>Near and far-field variables</strong></td>
</tr>
<tr>
<td>Hull draft</td>
</tr>
<tr>
<td>Hull beam</td>
</tr>
<tr>
<td>Hull density</td>
</tr>
<tr>
<td>Charge mass</td>
</tr>
<tr>
<td>Charge radius</td>
</tr>
<tr>
<td>Hull Mass</td>
</tr>
<tr>
<td>Slant angle</td>
</tr>
<tr>
<td><strong>Far-field variables</strong></td>
</tr>
<tr>
<td>Peak pressure</td>
</tr>
<tr>
<td>Decay constant</td>
</tr>
<tr>
<td>Slant angle</td>
</tr>
<tr>
<td>Hull draft</td>
</tr>
<tr>
<td>Hull beam</td>
</tr>
</tbody>
</table>

Notes:
(1) $P_{\text{MAX}}$ from Eqn (3-28) at $t=0$ s, $W=50$ kg and $R=100$ m
(2) $P_{\text{MAX}}$ from Eqn (3-28) at $t=0$ s, $W=1000$ kg and $R=8$ m
Modelling Strategy and Equation Forms

(3) $\theta$ from Eqn (3-29) at $W=50$ kg and $R=8$ m

(4) $\theta$ from Eqn (3-29) at $W=1000$ kg and $R=100$ m

The reference points for the measurement of $R$ and $\phi$ are diagrammatically shown in Figure 5-2. The choice of these points is as per Table 5-3.

![Figure 5-2: Reference points for radius and slant angle](image)

**Table 5-3: Reason for the chose of hull reference hull location**

<table>
<thead>
<tr>
<th>Hull shape</th>
<th>Slant Angle</th>
<th>Reference Point Location</th>
<th>Reason for Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>$0^\circ$</td>
<td>On hull surface</td>
<td>All shock-hull interaction acts on this surface</td>
</tr>
<tr>
<td></td>
<td>$45^\circ$</td>
<td>On hull surface centre line</td>
<td>The average shock-hull interaction acts at this point</td>
</tr>
<tr>
<td>Circular</td>
<td>$0^\circ$ &amp; $45^\circ$</td>
<td>D/6 from the hull base</td>
<td>The shock wave front location at this point represents the peak force developed on the hull</td>
</tr>
<tr>
<td>V</td>
<td>$0^\circ$</td>
<td>D/2 from hull base</td>
<td>The average shock-hull interaction acts at this point</td>
</tr>
<tr>
<td></td>
<td>$45^\circ$</td>
<td>On Hull front surface centre line</td>
<td>Most of the shock-hull interaction acts on this front surface, the far-surface has limited effect on peak accelerations</td>
</tr>
</tbody>
</table>
5.6. **Equation Forms for the Two Tiered Approach**

5.6.1. **Tier One**

This approach characterises the transient velocity of each hull shape as being one of three curves; linear, bi-linear and non-linear. The three characteristic velocity shapes are described below, and are diagrammatically shown in Figure 5-3:

- Linear description of velocity has a relative constant acceleration over the duration of the shock excitation.
- Bi-linear description of velocity has a high acceleration over approximately the first 20% of the excitation duration followed by a much reduced acceleration.
- Non-linear description of velocity has the peak acceleration at the beginning of the excitation; this progressively reduces over the shock duration.

The approach also determines the salient motion variables of PTV, the time at which it occurs ($t_{FINAL}$), GMA and PA. PA and GMA are defined by Eqns (5-1). And (5-2)

\[
PA = \text{Peak slope of transient velocity curve} \quad (5-1)
\]

\[
GMA = \frac{PTV}{t_{FINAL}} \quad (5-2)
\]
The form of the tier one equations is based on a rectangular hull with a draft D, beam B, density \( \rho \), peak shock pressure \( P_{MAX} \), decay constant \( \theta \) with no cavitation and a unit width. It then follows:

\[
\text{Acceleration} = \frac{\text{Force}}{\text{Mass}} \quad (5-3)
\]

\[
\Rightarrow PTV = \int_0^{\infty} \left( \text{Acceleration} \right) \cdot dt \quad (5-4)
\]

\[
\Rightarrow PTV = \frac{2 \cdot P_{MAX} \cdot B}{D \cdot B \cdot \rho} \int_0^{\infty} \left( e^{-\frac{t}{\rho}} \right) \cdot dt \quad (5-5)
\]

\[
\Rightarrow PTV = \frac{2 \cdot P_{MAX} \cdot \theta}{D \cdot \rho} \quad (5-6)
\]

Applying a constant \( K4 \) for a specific hull shape and water density yields Eqn (5-7):

\[
\Rightarrow PTV = (K4) \cdot \left( P_{MAX} \right) \cdot \left( \frac{\theta}{D} \right) = (K4) \cdot \frac{I}{D} \quad (5-7)
\]
Modelling Strategy and Equation Forms

Centring the equation on the mean $P_{MAX} = 34.44$ MPa, mean decay constant $\theta = 0.9$ ms, and mean depth of $D=12$ m, and permitting the curve fitting constants $K4$ to $K7$ to allow for cavitation in accordance with the LS-Dyna simulations, yields the general form of the equation as per Eqn (5-8). The constant $K4$ represents the magnitude of PTV for the mean hull.

$$PTV = \left( K4 \right) \cdot \left( \frac{P_{MAX}}{34.44} \right)^{K5} \cdot \left( \frac{\theta}{0.9} \right)^{K6} \cdot \left( \frac{12}{D} \right)^{K7}$$

(5-8)

5.7. Tier Two

5.7.1. General

Tier two considers the use of a modified differential equation as solved by Taylor [1] for an air backed flat plate excited by a UNDEX. Taylor’s solution considers a plate excited by a shock wave as per Figure 5-5 (a). Taylor represents this as an exponential force with a viscous damper reducing the effective pressure due to the plate velocity as per Figure 5-5 (b). The initial excitation pressure (at $t=0$ and $v=0$) is twice that of the shock wave pressure due to the reflection of the wave. As the plate increases in velocity the excitation pressure wave is reduced by Eqn (5-9), where $P_{RET}$ is the reduction in pressure, $\rho$ is the water density, $C$ is the speed of sound, $A_H$ is the plate area and $v_{PLATE}$ is the instantaneous plate velocity.

$$P_{RET} = \rho \cdot C \cdot v_{PLATE}$$

(5-9)
Using Newton’s third law of motion, the net force acting on the plate is equal to the shock force \( F \) subtracted from the retardation force \( F_{RET} \).

Where the excitation force \( F \) is a function of:
- Charge mass
- Charge location
- Time
- Hull shape
- Water properties
- Bulk cavitation

And the retardation force \( F_{RET} \) is a function of:
- Hull shape
- Hull velocity
- Local cavitation

**Figure 5-4:** *Taylor’s simplification of an air backed plate excited by a shock wave*
Modelling Strategy and Equation Forms

It then follows that the acceleration at any time is as per Eqn (5-10) where $v'$ is the plate acceleration, $t$ is the time and $M$ is the plate mass.

$$v' = \left( \frac{2 \cdot P_{\text{MAX}} \cdot e^{\frac{t}{C}} - \rho \cdot C \cdot v_{\text{plate}}}{M} \right) \cdot A_t$$  \hspace{1cm} (5-10)

In Tier two the possibility of using a modified Taylor type solution for determining the transient velocity of the four hull shapes was considered. This approach involved the review of alternate forcing functions for the hull excitation as shown in Figure 5-5; where $K_8$ to $K_{11}$ are hull dependent constants. The original Taylor solution used the excitation force as per Figure 5-5 (a), the alternate excitation forces (Figure 5-5 (b), (c) and (d)) and their application to the different hull geometries is detailed in Table 5-4.

![Figure 5-5: Alternate excitation functions for tier two](image)

5.7.2. Forcing Function ($F(t)$) Estimate

A generalised empirical estimate of $F$ was determined for each hull shape and slant angle considered. This was produced by developing an FE model of the water geometry only and fully restraining the nodes at the hull/water interface. This is shown in for a circular model with a zero slant angle (half model shown only). By requesting the *DATABASE_NODAL_FORCE_GROUP output in LS-Dyna the total vertical transient
Modelling Strategy and Equation Forms

force was determined for a stationary hull, allowing the appropriate equation form to be determined for \( F \).

An appropriate equation form for the circular hull at zero slant was found to be Eqn (5-11).

\[
F_{(t)} = F_{\text{MAX}} \left( e^{\frac{-t}{\theta}} - e^{\frac{-t}{k(0.6)\theta}} \right)
\]  

\[ (5-11) \]

\( F_{\text{MAX}} \) and \( k \) are a function of the explosion decay time, hull radius and peak shock pressure. A typical comparison of the actual FE total vertical force and the empirical force estimate is as per Figure 5-7.
Figure 5-7: Comparison of the actual FE vertical force and empirical estimate

The relationship between hull shapes, forcing functions and resulting transient PTV shape are summarised in Table 5-4

Table 5-4: Summary of hull shape and adopted corresponding forcing function

<table>
<thead>
<tr>
<th>Hull shape</th>
<th>Slant angle</th>
<th>Forcing function Figure 5-5</th>
<th>Forcing function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>0°</td>
<td>(a)</td>
<td>Exponential 1</td>
</tr>
<tr>
<td>Rectangular</td>
<td>45°</td>
<td>(c)</td>
<td>Parabolic</td>
</tr>
<tr>
<td>Circular</td>
<td>0°</td>
<td>(b)</td>
<td>Exponential 2</td>
</tr>
<tr>
<td>Circular</td>
<td>45°</td>
<td>(b)</td>
<td>Exponential 2</td>
</tr>
<tr>
<td>V 90°</td>
<td>0°</td>
<td>(c)</td>
<td>Parabolic</td>
</tr>
<tr>
<td>V 90°</td>
<td>45°</td>
<td>(d)</td>
<td>Exponential + parabolic</td>
</tr>
<tr>
<td>V 45°</td>
<td>0°</td>
<td>(c)</td>
<td>Parabolic</td>
</tr>
<tr>
<td>V 45°</td>
<td>67.5°</td>
<td>(d)</td>
<td>Exponential + parabolic</td>
</tr>
</tbody>
</table>
6. Hull Motion Equations

6.1. Introduction

This chapter develops the closed-form equations for hull motion predictions. A requirement for the closed-form equations is that they are easy to use. For this, the tier one equations were developed. However to investigate the possibility of developing equations of greater accuracy an alternate tier (tier two) was considered; both tiers are discussed below:

- Tier one develops equations that predict salient attributes on the hull’s motion, such as Peak Translational Velocity (PTV), Peak Acceleration (PA) and Global Mean Acceleration (GMA). These solutions are semi-empirical and, due to their simplicity, are suitable for hand calculation.
- Tier two develops equations that predict the hull’s transient velocity. The equations are based on a semi-empirical solution to a modified Taylor’s differential equation solution.

6.2. Rectangular Hull Motions at Zero Slant

Four LS-Dyna models were developed with a 25 mm element size and an axis of symmetry: three of the models with exposed corners, and one without an exposed corner as per Figure 6-1.
6.2.1. Tier One Equations

6.2.1.1. Hull Motions with No Corner Effects

The following parametric Eqs (6-1) to (6-3) predict the salient hull to with $\pm 8.4\%$ of the LS-Dyna solutions.

The characteristic hull motion for this scenario is non-linear as per Figure 5-3.

\[
    t_{\text{FINAL}} = (2.2) \cdot \left( \frac{\theta}{0.9} \right)^{0.68} \cdot \left( \frac{D}{12} \right)^{0.35} \text{ (ms)} \quad (6-1)
\]

\[
    PTV = (3.79) \cdot \left( \frac{P_{\text{MAX}}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.83} \cdot \left( \frac{12}{D} \right)^{0.76} \text{ (m/s)} \quad (6-2)
\]
Hull Motion Equations

\[ PA = \left( \frac{P_{\text{MAX}}}{[1.023 \cdot D]} \right) \cdot 1000 \quad (m/s^2) \]  

(6-3)

6.2.2. Hull Motions with Corner Effects

The LS-Dyna simulations predict that hull corners significantly affect the magnitude of PTV due to the loss of confinement for the reflected pressure wave near a corner. In this area of the hull, the advancing shock and the reflected shock wave become separated, allowing the particles on the edge of the reflected wave to flow from high to low pressure regions. This reduces the reflected shock wave pressure in the vicinity of the hull corner as shown in the area A-B of Figure 6-2. In the LS-Dyna simulations, the PA was marginally affected; however, in theory, the hull acceleration will not be reduced by corner affects owing to the shock wave front having an effective shock width of zero.
Figure 6-2: 4 x 4 m hull pressure contours, and horizontal particle velocity.

The reduction in PTV for hulls with beams of 4 m, 12 m, and 20 m are presented in Table 6-1.
### Hull Motion Equations

**Table 6-1: PTV reduction due to corner affects**

<table>
<thead>
<tr>
<th>Hull Width (m)</th>
<th>% Reduction of PTV Compared to an Infinite Length Hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9.5</td>
</tr>
<tr>
<td>12</td>
<td>3.2</td>
</tr>
<tr>
<td>20</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Allowing for corner effects modifies Eqn (6-2) to Eqn (6-4)

\[
PTV = (3.79) \cdot \left( \frac{P_{\text{MAX}}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.83} \cdot \left( \frac{12}{D} \right)^{0.76} \cdot \left( 1 - \frac{0.38}{B} \right) \ (m/s) \tag{6-4}
\]

#### 6.2.3. Tier Two Equations

A flat bottom hull excited by a distant charge (at zero slant) results in the shock wave reflecting off all points simultaneously, which develops a purely exponential loading. This is shown in Figure 6-3 and can be modelled using Taylor's [1] original solution Eqn (6-9). The general solution for Taylor’s equation is as per Appendix A.

*Figure 6-3: Rectangular hull excited by a distant explosion*
Hull Motion Equations

- Peak force estimate
  \[ F_{\text{MAX}} = P_{\text{MAX}} \cdot B \]  
  (6-5)
- Peak acceleration estimate
  \[ \Omega = \left( \frac{F_{\text{MAX}}}{M} \right) \]  
  (6-6)
- Retardation area estimate
  \[ A_{\text{RET}} = B \]  
  (6-7)
  \[ \varpi = \left( \frac{\rho \cdot C \cdot A_{\text{RET}}}{M} \right) \]  
  (6-8)
- Transient vertical velocity equation
  \[ v(t) = \left\{ \frac{\Omega \cdot \left( e^{-\omega t} - e^{-\varpi t} \right)}{\left( \frac{1}{\varpi} - \varpi \right)} \right\} \]  
  (6-9)

Consider the full width rectangular hull excited by a shock wave as shown in Figure 6-4.

Figure 6-4: LS-Dyna model
The comparison between Taylor’s solution and the LS-Dyna simulation for velocity is shown in Figure 6-5 and for acceleration in Figure 6-6.

**Figure 6-5**: Comparison between Taylor velocity and LS-Dyna simulation

**Figure 6-6**: Comparison between Taylor acceleration and LS-Dyna simulation
Hull Motion Equations

The results compare well with an 8% error in the acceleration prediction and 0.5% in the velocity. To allow for corner affects the Taylor solution Eqn (6-9) has been modified with the following empirical variable to produce Eqn (6-10).

\[
\nu(t) = \left( \frac{\Omega \cdot \left( e^{-\omega t} - e^{-\sigma t} \right)}{1 - \left( \frac{1}{\theta - \sigma} \right)} \right) \cdot \left( 1 - \frac{(0.38) \cdot t}{B \cdot t_{\text{FINAL}}} \right)
\]  

(6-10)

Where \( t_{\text{FINAL}} \) is as per Eqn (6-1).

6.2.4. Conclusion to the Rectangular Hull Zero Slant

Both tier one and two approaches produce accuracy estimate for this scenario providing exposed corners are made allowance for. The affect of bulk cavitation has no significant affect on the solution with a hull depth of 4 m or more. The characteristic hull motion for this scenario is non-linear as per Figure 5-3
6.3. **Circular Hull Motion with Zero Slant (H=D)**

LS-Dyna models were developed with element lengths (normal to shock wave) ranging from 16 mm to 80 mm and an axis of symmetry as per Figure 6-7 is used to reduce the model size. The model boundary conditions D-A were extended to check the proximity effects on the translational boundary condition as per Figure 6-8. It was found that extending the boundary had no significant effect on the hull motion. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

![Figure 6-7: Half symmetry circular model A](image)

*Figure 6-7: Half symmetry circular model A*
6.3.1. Pressure Application

All models split the pressure with 64% of the pressure being converted into a shock compression wave. The theoretical shock wave of $P_{\text{MAX}} = 34.44$ MPa and $\theta = 0.9$ ms is compared to the LS_Dyna wave as per Figure 6-9 where $I_{\text{LS-DYNA}} = I_{\text{THEORETICAL}}$. Typical shock hull interaction is shown in Figure 6-10.
Hull Motion Equations

Figure 6-9: Comparison between the LS-Dyna and theoretical transient pressure

Figure 6-10: Shock wave evolution for 6 m hull (Model A) and $P_{\text{MAX}} = 34.44$ MPa
6.3.2. Tier One Equations

Equations (6-11) to (6-13) develop hull motion estimates for PTV and PA to within \( \pm 9\% \) of the LS-Dyna simulations while \( t_{FINAL} \) and GMA are within \( \pm 15\% \).

The characteristic hull motion for this scenario is non-linear as per Figure 5-3

\[
\begin{align*}
  t_{FINAL} & = (5.6) \cdot \left( \frac{\theta}{0.9} \right)^{0.26} \cdot \left( \frac{D}{6} \right)^{0.82} \quad (ms) \\
  PTV & = (6.9) \cdot \left( \frac{P_{MAX}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.67} \cdot \left( \frac{6}{D} \right)^{0.67} \quad (m/s) \\
  PA & = (4000) \cdot \left( \frac{P_{MAX}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.29} \cdot \left( \frac{6}{D} \right)^{1.34} \quad (m/s^2)
\end{align*}
\]

6.3.3. Tier Two Equations

6.3.4. General

The premise of this section is that the transient force on a stationary hull from a shock loading will have the form as per Eqn (6-14) where K1, K2, and K3 are constants.

\[
F_{(t)} = F_{MAX} \cdot K3 \cdot \left[ e^{\frac{-t}{K1\cdot\theta}} - e^{\frac{-t}{K1(K2)\cdot\theta}} \right] \quad (6-14)
\]

Solving Eqn (6-14) for transient velocity (Refer Appendix B) results in Eqn (6-15).

\[
v_{(t)} = \frac{F_{MAX} \cdot K3}{M} \cdot \left[ \left( e^{\frac{-t}{K1\cdot\theta}} - e^{-\sigma} \right) + \left( e^{\frac{-t}{K1(K2)\cdot\theta}} - e^{-\sigma} \right) \right] \quad (6-15)
\]
6.3.5. Stationary Hull Load

Consider the transient force on a hull as per Figure 6-11.

\[ F(t) = F_{MAX} \cdot K3 \cdot \left( e^{-t/K1} - e^{-t/(K1K2)} \right) \]

Differentiating Eqn (6-15) to determine the relationship between the time of maximum hull force \( T_{MF} \), \( K1 \), and \( K2 \) yields Eqn (6-16).
Hull Motion Equations

\[
T_{MF} = \frac{1 - \frac{1}{K2}}{K1} \cdot \theta \cdot LN(K2)
\]

(6-16)

From the LS-Dyna simulation of a restrained hull the empirical estimates of \(T_{MF}, F_{MAX}, L_F\) and \(K2\) were estimated by Eqns (6-17) to (6-20).

\[
T_{MF} = \left(0.642 - (0.432) \cdot \left(\frac{\theta}{T_f}\right)\right) \cdot \theta \ (ms)
\]

(6-17)

\[
F_{MAX} = \left(250 \times 10^6\right) \cdot \left(\frac{P_{MAX}}{34.44}\right) \cdot \left(\frac{\theta}{0.9}\right)^{0.3} \cdot \left(\frac{D}{6}\right)^{0.5} \cdot \left(\frac{L_F}{2.888}\right)^{0.244} \ (N)
\]

(6-18)

\[
K2 = (0.18) \cdot \left(\frac{6}{D}\right) \cdot \left(\frac{\theta}{0.9}\right)
\]

(6-19)

\[
L_F = \sqrt{D^2 - (D - L_{MF})^2}
\]

(6-20)

The solution for \(K3\) is obtained by substituting \(T_{MF}, K1, K2, F_{MAX}\), and \(\theta\) back into Eqn (6-14). The transient velocity can be solved using Eqn (6-15) where:

\[
\omega = \frac{\rho \cdot C \cdot A_{EFF, R}}{M}
\]

(6-21)

\[
A_{EFF, R} = \frac{L_F}{3.3} \ (m)
\]

(6-22)

6.3.6. Errors

The tier two equations produce transient estimations within ±7% for PTV, PA and GMA. The least accurate result is for a 6m hull with \(P_{MAX}=67.82\) MPa and \(\theta=1.4086\) ms. The transient plots of velocity and acceleration are shown in Figure 6-13 and Figure 6-14.
Figure 6-13: Transient velocity results, LS-Dyna verses equation

Figure 6-14: Transient acceleration results, LS-Dyna verses equation
6.4. **V Hulls Motion with Zero Slant (H=D)**

A typical LS-Dyna meshed model is shown in Figure 6-15. The model uses half symmetry with no non-reflective boundaries. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

![Figure 6-15: Typical V model using half symmetry](image)

Consider a V hull being progressively enveloped by a shock pressure wave as per Figure 6-16. At (a) the shock wave begins to load on the hull, (b) the net excitation force increases to a maximum, (c) the increasing hull velocity results in a reduction in the differential particle velocity between the hull and the water particles, resulting in a decreasing net excitation force, and at (d) the force quickly reduces to zero as the bulk cavitation reflects off the free surface, nullifying the shock wave. The net force acting on the hull is approximately parabolic for small hulls. However, for large hulls with drafts greater than 8 m, the net force resembles a stepped function as shown in Figure 6-17.
Hull Motion Equations

Figure 6-16: Progressive enveloping of V hull by a shock wave

Figure 6-17: Net hull force for a large hull at $D > 8$ m
Hull Motion Equations

6.4.1. Tier One Equations 90° V

The characteristic hull motion for this scenario is linear as per Figure 5-3

\[ t_{\text{FINAL}} = (5.6) \left( \frac{\theta}{0.9} \right)^{0.07} \left( \frac{H}{6} \right)^{0.88} \text{ (ms)} \]  
\[ (6-23) \]

\[ PTV = (9.7) \left( \frac{P_{\text{MAX}}}{34.44} \right) \left( \frac{\theta}{0.9} \right)^{0.67} \left( \frac{6}{H} \right)^{0.58} \text{ (m} / \text{s)} \]  
\[ (6-24) \]

\[ PA = (2569) \left( \frac{P_{\text{MAX}}}{34.44} \right) \left( \frac{\theta}{0.9} \right)^{0.482} \left( \frac{6}{H} \right)^{1.686} \text{ (m} / \text{s}^2) \]  
\[ (6-25) \]

6.4.2. Tier One Equations 45° V

The characteristic hull motion for this scenario is linear as per Figure 5-3

\[ t_{\text{FINAL}} = (9.2) \left( \frac{\theta}{0.9} \right)^{0.07} \left( \frac{H}{12} \right)^{0.88} \text{ (ms)} \]  
\[ (6-26) \]

\[ PTV = (5.28) \left( \frac{P_{\text{MAX}}}{34.44} \right) \left( \frac{\theta}{0.9} \right)^{0.67} \left( \frac{12}{H} \right)^{0.72} \text{ (m} / \text{s)} \]  
\[ (6-27) \]

\[ PA = (692) \left( \frac{P_{\text{MAX}}}{34.44} \right) \left( \frac{\theta}{0.9} \right)^{0.71} \left( \frac{12}{H} \right)^{1.71} \text{ (m} / \text{s}^2) \]  
\[ (6-28) \]

6.4.3. Tier Two Equations

Due to the variability of the forcing function shape between small and large hulls, the tier two approach is not suitably for this hull shape.
### 6.5. Rectangular Hull Motions with 45° Zero Slant

The flat bottom hull excited by a distant charge with a slant angle is diagrammatically shown in Figure 6-18. A typical LS-Dyna meshed model is shown in Figure 6-19. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

![Figure 6-18: Section considered](image)

![Figure 6-19: Typical LS-Dyna meshed model](image)
Hull Motion Equations

6.5.1. Tier One Equations

6.5.1.1. Average Motions

The characteristic hull motion for this scenario is linear as per Figure 5-3

\[ t_{\text{FINAL}} = (7.53) \cdot \left( \frac{\theta}{0.9} \right)^{0.071} \cdot \left( \frac{H}{12} \right)^{0.88} \cdot \left( \frac{B}{H} \right)^{0.83} \text{ (ms)} \] \hspace{1cm} (6-29)

\[ PTV = (3.15) \cdot \left( \frac{P_{\text{MAX}}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.68} \cdot \left( \frac{12}{H} \right)^{0.73} \text{ (m/s)} \] \hspace{1cm} (6-30)

\[ PA = (525) \cdot \left( \frac{P_{\text{MAX}}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.69} \cdot \left( \frac{12}{H} \right)^{1.76} \cdot \left( \frac{H}{B} \right)^{0.81} \text{ (m/s}^2) \] \hspace{1cm} (6-31)

6.5.1.2. Peak motions at point B

\[ PTV_b = (1.15) \cdot PTV \cdot \left( \frac{B}{H} \right)^{0.13} \text{ (m/s)} \] \hspace{1cm} (6-32)

\[ PA_b = (1.17) \cdot PA \cdot \left( \frac{B^{1.4}}{H} \right)^{0.3} \text{ (m/s}^2) \] \hspace{1cm} (6-33)

6.5.2. Tier Two Equations

Similar to Section 6.4.3, the variability of the forcing function shape between small and large hulls results in this approach not being suitable for this hull shape.
6.6. Circular Hull Motion 45° Slant (H=D)

The load imposed on the circular hull with a charge slant of 45° is similar to that of a zero slant with the exemption that bulk cavitation plays a larger role in the reduction of the excitation force as shown in Figure 6-20. A typical LS-Dyna model is shown in Figure 6-21. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

Figure 6-20: A circular hull loaded with a charge at slant angle 45°
Hull Motion Equations

6.6.1. Tier One Equations

6.6.1.1. Average Hull Motions

The characteristic hull motion for this scenario is non-linear as per Figure 5-3.

\[
T_{FINAL} = \left(6.29\right) \left(\frac{b}{0.9}\right)^{0.068} \left(\frac{H}{6}\right)^{0.88} (ms) \tag{6-34}
\]

\[
PTV = \left(6.29\right) \left(\frac{P_{MAX}}{34.44}\right) \left(\frac{\theta}{0.9}\right)^{0.68} \left(\frac{6}{H}\right)^{0.65} (m/s) \tag{6-35}
\]

\[
PA = \left(2960\right) \left(\frac{P_{MAX}}{34.44}\right) \left(\frac{\theta}{0.9}\right)^{0.35} \left(\frac{6}{H}\right)^{1.287} (m/s^2) \tag{6-36}
\]

6.6.1.2. Peak Motions at Point B

\[
PTV_B = (1.3) \cdot PTV \tag{6-37}
\]

\[
PA_B = (2.22) \cdot PA \tag{6-38}
\]

6.6.2. Tier Two Equations

Tier two equations were not developed for this hull scenario.
6.7.  **90°V hull 45 slant (H=D)**

A typical LS-Dyna meshed model is shown in Figure 6-22. The model uses a non-reflective pressure application boundary. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

![Figure 6-22: Typical LS-Dyna meshed model](image)

### 6.7.1. Tier One Equations

#### 6.7.1.1. Average Hull Motions

The characteristic hull motion for this scenario is bi-linear as per Figure 5-3.

\[
F_{\text{FINAL}} = (5.9) \cdot \left( \frac{\theta}{0.9} \right)^{0.09} \cdot \left( \frac{H}{6} \right)^{0.94} \text{ (m/s)} \tag{6-39}
\]

\[
PTV = (8.25) \cdot \left( \frac{P_{\text{MAX}}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.6} \cdot \left( \frac{6}{H} \right)^{0.6} \text{ (m/s)} \tag{6-40}
\]

\[
PA = (9000) \cdot \left( \frac{P_{\text{MAX}}}{34.44} \right) \cdot \left( \frac{\theta}{0.9} \right)^{0.15} \cdot \left( \frac{6}{H} \right)^{1.06} \text{ (m/s²)} \tag{6-41}
\]

#### 6.7.1.2. Peak Motions at Point B

\[
PTV_B = (1.45) \cdot PTV \tag{6-42}
\]
Hull Motion Equations

\[ PA_B = (2.22) \cdot PA \]  
(6-43)

The hull is excited by a normal shock wave on the near hull face. This produces a short period of high acceleration followed by a longer, less intense period where the far face is loaded as per Figure 6-23.

![Figure 6-23: Typical velocity of the hull](image)

6.7.2. Tier Two Equations

Tier two equations were not developed for this hull scenario.
6.8.  45°V slant (H=D)

A typical LS-Dyna meshed model is shown in Figure 6-24. The model uses a non-reflective pressure application boundary. A large section of the far side water has been modelled to accurately capture the low intensity, loner duration far side shock wave refraction. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

6.8.1. Tier One Equations

6.8.1.1. Average Hull Motions

The characteristic hull motion for this scenario is bi-linear as per Figure 5-3

\[
H(t) = \theta \cdot \left( \frac{H}{12} \right)^{0.72} \cdot \left( \frac{H}{12} \right)^{0.72} (ms)
\]  

(6-44)
The hull is excited by the shock wave being normal to the near hull face. This produces a short period of high acceleration followed by a longer, less intense period where the trailing face is loaded as per Figure 6-25.

Figure 6-25: Typical velocity of the hull

6.8.2. Tier Two Equations

Tier two equations were not developed for this hull scenario.
6.9. **Modified Hull Forms**

6.9.1. **General**

Three of the four basic hull shapes considered can exist in a modified form. This occurs when the draft of the hull is larger than the hull depth (i.e $D > H$). In this scenario the effective vertical area loaded by the shock wave remains unchanged, however the hull mass is increased with the inclusion of a rectangular section as per Figure 6-26. Three additional tier one equations were developed allowing the characteristic motions of PTV, PA, and GMA for the circular and V hulls to be adjusted for this additional mass.

![Figure 6-26: Alternate forms for circular and V hulls](image-url)
Hull Motion Equations

6.9.2. Parameter Definitions

The following definitions were adopted for this section as per Table 6-2.

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Mass of hull as per Sections 9, 10, 12, 13 and 14 where H=D</td>
</tr>
<tr>
<td>M_D</td>
<td>Mass of hull with additional rectangular section</td>
</tr>
<tr>
<td>PTV</td>
<td>PTV as per Sections 9, 10, 12, 13 and 14 where H=D</td>
</tr>
<tr>
<td>PTV_D</td>
<td>PTV with additional rectangular section</td>
</tr>
<tr>
<td>PA</td>
<td>PA as per Sections 9, 10, 12, 13 and 14 where H=D</td>
</tr>
<tr>
<td>PA_D</td>
<td>PA with additional rectangular section</td>
</tr>
<tr>
<td>GMA</td>
<td>GMA as per Sections 9, 10, 12, 13 and 14 where H=D</td>
</tr>
<tr>
<td>GMA_D</td>
<td>GMA with additional rectangular section</td>
</tr>
</tbody>
</table>

6.9.3. Equations

The equations for the variable mass are governed by local cavitation. As the hull increases in mass the hull velocity decreases, reducing the retardation force from local cavitation. This results in the power factor \( k_P \) being less that unity for PTV and GMA estimates. The effects on PA are less pronounced as peak acceleration occurs well before PTV is achieved. In the case of V hulls with slant, the peak acceleration occurs at time equals zero. This results in the power factor being equal to unity.

The motion for the modified hull form is as per Eqns (6-49) to (6-51) where the power constants for each hull shape are as per Table 6-3. The equations predict the modified hull motions to within an additional ±4% error within the mass range of \( 0.5 \leq \frac{M}{M_D} \leq 1.0 \).

LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

\[
P_{TV_D} = PTV \cdot \left( \frac{M}{M_D} \right)^{K_P} \quad (6-49)
\]

\[
P_{GMA_D} = GMA \cdot \left( \frac{M}{M_D} \right)^{K_P} \quad (6-50)
\]
Hull Motion Equations

\[ PA_D = PA \cdot \left( \frac{M}{M_D} \right)^{Ka} \]  

(6-51)

Table 6-3: Power constants for hull with \( H < D \)

<table>
<thead>
<tr>
<th>Hull shape</th>
<th>Slant angle</th>
<th>Kp</th>
<th>Ka</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>0°</td>
<td>0.78</td>
<td>0.9</td>
</tr>
<tr>
<td>Circular</td>
<td>45°</td>
<td>0.78</td>
<td>0.9</td>
</tr>
<tr>
<td>V 90°</td>
<td>0°</td>
<td>0.78</td>
<td>0.9</td>
</tr>
<tr>
<td>V 90°</td>
<td>45°</td>
<td>0.78</td>
<td>1.0</td>
</tr>
<tr>
<td>V 45°</td>
<td>0°</td>
<td>0.78</td>
<td>0.9</td>
</tr>
<tr>
<td>V 45°</td>
<td>67.5°</td>
<td>0.78</td>
<td>1.0</td>
</tr>
</tbody>
</table>

6.10. Discussion on Hull Motions

The tier one equations provided the salient motions of PTV, PA, and GMA to within ±15% of the LS-Dyna simulations for all of the eight hull scenarios considered. An estimate of the error associated with the LS-Dyna simulations and compared to real hull motions increased the uncertainty by an additional ±11%, resulting in a total error between the tier one equations and real hull motions of ±26%. The two largest errors were considered to be the shock wave definition accounting for a ±4.5% and the cavitation error at ±2.5%.

The hull excitations are significantly varied between the eight hulls considered. The rectangular hull at zero slant is loaded instantaneously with the full peak shock force, resulting in extremely high initial accelerations which produce a characteristic, non-linear PTV curve. The V hulls with a slant angle produce an initial high acceleration with similarly shaped PTV curve to that of the rectangular hull. However, the shock loading continues for a significantly longer time because it loads the far side of the hull. This produces a bi-linear PTV curve. The circular hulls experience the peak acceleration at the time that the shock wave has engaged the hull by one sixth of the beam dimension which results in a non-linear PTV curve. The V hull at a zero slant is loaded by a relatively constant shock force of low intensity resulting in linear PTV excitation.
Hull Motion Equations

The tier two approach was only 100% successful for one out of the eight hulls considered. The rectangular hull with zero slant angle compared favourably with the classic air backed plate differential equation solution. For the hull depths considered, the shock excitation force was terminated by local cavitation before surface cut-off occurred. Hence, the classic solution provided the required physics with the exception of corner affects that slightly reduced hull motions. For this, an additional term was added to the equation. The circular hull equations provided good agreement for all hull sizes and charge geometries with the exception of the 4 m beam hull. In this case, owing to the hull small radius of 2 m, the shock wave envelope a large position of the hull, significantly changing the required empirical constants. For the remaining hull shapes, tier two equations were developed as was considered that they are overly complicated with no gain in accuracy when compared to the tier one equations.
7. T-Joint

7.1. Introduction

This section considers the effects of an UNDEX shock loading on a GRP ship of beam 9 m and a hull plate span of 4 m. The methods of load transfer from the external side of the hull plate through the bonded joint and into the bulkhead are identified. A number of different joint geometries were considered to determine the optimal shape of the over-laminate with respect to material stresses. The dimensions of the bulkhead, hull plate, and over-laminate are detailed in Section 7.3.1 with the material properties defined in Section 3.10. LS-Dyna *.k and Excel summary *.xls file names are detailed in Appendix D.

7.2. Consideration of Water-Solid and Solid-Air Shock Interfaces

In the previous chapters, it was assumed that the hull acted in a rigid manner providing complete reflection of the shock wave at the water-hull interface. In considering the composite to water interface, additional research was required to develop an appropriate LS-Dyna model. To progress this line of investigation, consider three different modelling strategies for the application of the shock wave to the water/hull interface as per Figure 7-1. All models were loaded with a $P_{\text{MAX}} = 15$ MPa and $\theta = 0.1$ ms at the boundary A-B and the models were free bodies (unrestrained). The pressure abscises for each model has been adjusted so that all interface elements at point 1 are loaded at the same time.
Consider the pressure interface at point 1 for each model. The shell model faithfully duplicates the transient shock pressure applied while the solid model’s transient shock pressure is the sum of the applied component and a reflected component. In the solid model the air-solid interfaces at points 1 and 2 results in a reflected wave oscillating between the two surfaces, this can be seen as the tension wave spikes shown in Figure 7-2. The realistic water/solid model allows the shock wave to travel through the interface at point 1 with negligible reflection owing to the speed of sound between the materials being similar (C=1500 and C_{33}=1480 m/s). The shock wave is, reflected at point 2, reversing its sign and direction and passes back through point 1, cavitating the water as observed in Figure 7-2.
The velocities of the structures interface at point 1 shed additional insight into the shock wave hull interaction. With no through thickness dimension, the shell model acts as a free body loaded with a force. The solid model’s velocity is the summation of free body motion and the oscillating wave that is continuously reflected between points 1 and 2. In a real structure, damping would consume the shock wave energy; however, in the LS-Dyna model no damping was included. The water/solid model includes the allowance of cavitation and the ability of the shock wave to migrate out of the solid back into the water as per Figure 7-3 which allows a more realistic model than either the shell or the solid model.
When considering local cavitation at the water/solid interface, the physics of local cavitation will depend on the difference in wave speeds between the water and the solid. Consider two simplistic models: model A where the speed of sound within the solid is equal to the water and model B where the speed of sound in the solid is much greater than the water’s speed of sound as per Figure 7-4.
When considering a rigid material ($C_{33} >> C$), local cavitation will occur when the velocity at the solid-water interface is two times that of the water particle velocity (as per Section 3.7.1.4). When the speed of sound in the water is approximately equal to the through thickness speed of sound in the hull ($C_{33} \approx C$), local cavitation occurs at the time taken for the shock wave to travel from point A1 to the free face and back to point A1 as per Eqn (7-1). This behaviour can be seen by comparing the water pressure at points 1A and 1B in the above models as shown in Figure 7-5, where the time at which the model experiences cavitation at point A1 is $t_A$ and the solid thickness is $H_{HP}$.

$$t_A = \frac{2 \cdot H_{HP}}{C} = \frac{2 \times 0.0585}{1500} = 7.85 \times 10^{-5} \text{ (s)}$$  \hspace{1cm} (7-1)
7.3. Hull Loading and Modelling Strategy

7.3.1. Section Considered

For the T-joint loading, it was assumed that the hull of beam 9 m was loaded with a charge of a zero slant. This UNDEX loads each side of the ship simultaneously, generating equal shock waves within the bulkhead that travel towards each other, passing through each other and migrating to the opposite side T-joint. This scenario was modelled in LS-Dyna using half symmetry. The model was fully restrained at its centre line (4.5 m from the T-joint), resulting in the shock wave reflection and mimicking the shock wave from the opposite hull side. The global model dimensions are as per Figure 7-6 and the joint dimensions are as per Figure 7-7.
Figure 7-6: General arrangement of model
7.3.2. Model Elements

A number of plane strain T-Joint models were constructed using a combination of UI and SRI 8 node hexahedral solid and 4 node FI shell elements. The bulkhead and a local section of the T-joint being modelled with the solid elements that are translational restrained in the global Z-direction to produce a plane strain scenario. The models have a thickness of 5mm in the Z-direction. To reduce the computational expense, most of the far-field hull section used shell elements embedded into the solids to produce the required moment compatibility as shown in Figure 7-6 and Figure 7-8. The bulkhead span of 4.5 m represents the half span of the 9 m beam hull; the hull span varied from 1m to 4m. The magnitude of these spans and their importance will be discussed in detail later in this section. The model included a section of the water and a non-reflective boundary in the solid section of the joint to allow the shock wave within this section to freely pass in and out of the 3-D section as discussed in Section 7.2.
T-Joint

7.3.3. Model Restraints

The hull plate section is restrained at its centre line by a global Z rotational restraint, resulting in a pure bending load path from the far-field hull plate to the bulk head with no membrane effects.

7.3.4. Material Orientation and Loading

The solid sections of the model used the orthotropic material *MAT_ORTHOTROPIC_ELASTIC with the exception of the filler material (Part102) which used *MAT_ELASTIC. The material orientation of the orthotropic materials was defined using a global vector (AOPT=2) and the stress output in the material directions (CMPFLG=1). The correctness of the vectors and stress output was checked with unit cell models. The fillet areas of the over-laminate change their origination as the fillet angle changes. To allow for this, the fillet sections for all of the models are divided into six parts (Parts 104, 105, 106, 114, 115, and 116) with each part’s material orientation being the average angle of that part. This discretization of material angles does produce some degree of inaccuracy; however, the method has been proven by the unit cell simulations. An alternate method of assigning material direction to the local element axis system is available in LS-Dyna, but the post processor used (LS-Post) does not have the capability of visualizing brick element orientation. On this basis, the discretization of fillets into a number of constant sections was adopted. The external load applied to the hull plate was a $P_{\text{MAX}}$ of 15 MPa with a $\theta$ of 0.1 ms. This produced a maximum average axial stress within the over-laminate of 162.2 MPa and a peak of 332 MPa for the 22.5° chamfer model.
The orthotropic material orientation axis for the models is defined in Figure 7-9 and the different joint geometries with material angles are shown in Figure 7-10 to Figure 7-13. A typical meshed model is as per Figure 7-14.
T-Joint

Figure 7-10: 45 deg chamfer with 25 mm fillet joint geometry

Figure 7-11: 45 deg chamfer with 25 mm fillet joint geometry plus isolation void
Figure 7-12: 40 mm fillet joint

Figure 7-13: 22.5 deg chamfer with 25 mm fillet joint geometry
7.4. Criticality of Joint Locations with Respect to Stress Levels

To determine the most damage resistant joint geometry, particular components of stress at locations A, B, C, D, and E were evaluated as shown in Figure 7-15.

![Figure 7-15: Critical joint stress locations](image)
7.5. General Joint Loadings

From the analysis of the $45^\circ$ chamfer joint, it is considered that the joint loading occurs in three distinct phases. The initial phase is associated with the migration of the shock wave into the joint from the near side of the hull. The second phase occurs on the arrival of the shock wave from the far side of the hull. Both these phases are near identical with the second phase displaced in time. Both phases are independent of the hull span and are the result of locally induced shock loads. For the 9 m beam hull, the second phase occurs at a displaced time of 0.00223 s as per Eqn (7-2).

$$Time \ of \ arrival \ of \ far \ side \ shockwave = \frac{B}{C_{11}} \approx \frac{9\, (m)}{4035\, (m/s)} \approx 0.00223\, (s) \quad (7-2)$$

The axial stress at point A is shown for the two initial phases for the $45^\circ$ chamfer joint with a comparison of spans 1 m and 4 m as per Figure 7-16.

The phase one loading can be visualised by reviewing the deflected shape of the joint. The undeformed shape and the amplified deformed joint are compared at three times showing the progression of the shock wave into the joint. At time equal to 0.024 ms the shock wave has progressed through to the hull plate to over-laminate interface. At this stage, deflection only occurs in the hull plate as shown in Figure 7-17.
The over-laminate shock wave speed is over twice that of the filler, resulting in the over-laminate providing the main load path for the shock wave. The arrival of the shock wave at the bulkhead via the over-laminate is shown in Figure 7-18.
T-Joint

At time equals 0.078 ms the shock wave was fully developed in the bulkhead due to its progression through the over-laminate and filler material. This is shown in Figure 7-19.

The third phase is the span dependent bending phase shown in Figure 7-20 with all phases shown for the 1 m and 4 m spans up the 0.03 s.

Figure 7-19: Deflected shape phase 1 at time equals 0.078ms

Figure 7-20: Stress 11 at point A showing phase 1, 2, and 3 for the 1 m and 4 m spans
T-Joint

7.6. T-Joint 45° Chamfer 4m Span

7.6.1. Stress at Point B

Figure 7-21: Stress 11 at point B

7.6.2. Stress at Point C

Figure 7-22: Peak shear stress at point C
T-Joint

![Figure 7-23: Through thickness stress 33 stress at point C](image)

7.6.3. Stress at Point D

![Figure 7-24: Peak stress 11 at D](image)
T-Joint

7.6.4. Stress at Point E

Figure 7-25: Peak shear stress at point E

Figure 7-26: Peak shear 33 at point E
7.6.5. Joint Separation Equal to Zero

Continuing the bulkhead material to the hull plate as per Figure 7-27 (a separation distance of zero) produces no significant stress difference in the joint.

![Figure 7-27: Section considered with a separation distance of zero](image)

7.6.6. Joint with Void

The isolation void reduces the effectiveness of the shock wave load path through the filler. In doing so, it concentrates the stress away from the filler into the over-laminate for phases 1 and 2. This increases the stresses in the over-laminate by approximately 10 MPa as shown in Figure 7-28.
7.6.7. High Strain Rate Effects

The use of the high strain rate elastic stiffness properties has a limited effect on the joint, generally increasing all the stress in the components by approximately 8%. This is due to the increased magnitude of the structures natural frequency producing a larger dynamic amplification factor associated with bending actions. This is demonstrated by the comparison of the over-laminate axial stresses for the low and high strain rate material models shown in Figure 7-29. The shear stress at point C reduced by approximately 20% to 35% with the high strain rate material model. It is considered that this was due to the filler increasing its stiffness to a higher proportional magnitude than the over-laminate stiffness increase. (At high strain rates the filler increased its stiffness to 175% of the lower strain rate. This is compared to the over-laminate which increased its 11 stiffness to 112% of the lower strain rate). This results in the filler becoming a stiffer load path, taking its greater load share.
Figure 7-29: Comparison of the over-laminate stress for normal and high strain rate

7.7. Joint with 40 mm Fillet (4 m Span)

7.7.1. Stresses at Points A and B

The stresses at points A and B for this model were not significantly different to that of the 45° chamfer model. The greatest difference between the two models was a 6 % increase in the over-laminate stress in the 40 mm fillet model when compared to the 45° chamfer model.

7.7.2. Stress at Point C

The stresses in the 40 mm fillet joint were reduced by 28 % for phase 1, 2, and 35% of phase 3 when compared to the 45° chamfer joint.

7.7.3. Stress at Point D

The stresses in the 40 mm fillet joint were reduced by approximately 5% for all phases when compared to the 45° chamfer joint.

7.7.4. Stresses at Point E

Significant stress differences between the 40 mm fillet and the 45° chamfer joint were found at this point. The shear stress was markedly reduced in the 40 mm fillet joint by
T-Joint

22% for phase 1, 2, and 50% for phase 3. The through thickness compressive stress increased by approximately 50% in the 40 mm fillet joint.

7.8. Joint with 22.5° Chamfer

7.8.1. Stress at Point A

No significant difference stresses occur in this joint at this location

7.8.2. Stress at Point B

The over-laminate stress is increased from 109.2 MPa in the 45° model to 162.2 MPa in the 22.5° model as per Figure 7-30.

7.8.3. Stress at Point C

Peak shear stress is increased, from 13.7 MPa in the 45° chamfer model to 19.5 MPa for the 22.5° chamfered joint.

Figure 7-30: Comparison of over-laminate stress 45 and 22.5 degree models
T-Joint

7.8.4. Stress at Point D

No significant difference in stress between the 45° and 22.5° chamfered joints occurs at this location.

7.8.5. Stress at Point E

The shear stress comparison between the 45° and 22.5° chamfer joints is minimal, however the through thickness compressive stress is approximately double that of the 45° chamfered joint. Increasing from -10.4 MPa to -20.6 MPa for phase 1, 2, and from 17.3 MPa to 33.8 MPa for phase 3.

7.9. Discussion

7.9.1. General

The joint loading was considered to occur in three distinct phases: initial shock, reflected shock, and the bending phase. Each loading phase develops characteristic stress regimes that are affected by the span of the hull plate and the joints geometry. The initial phase is associated with the migration of the shock wave into the joint from the near side of the hull. The second phase occurs on the arrival of the shock wave from the far side of the hull. Both these phases are near identical with the second phase displaced in time. Both phases 1 and 2 are independent of the hull span and are the result of locally induced shock loads.

7.9.2. Bulkhead Stress at Point A

The axial stress in the bulkhead at point A is not significantly affected by the joint geometries considered in this research. The peak stress for phases 1 and 2 was approximately 40 MPa for all joint geometries. The phase 3 stress was significantly affected by hull span, raising the hull span from 1 m to 4 m resulted in the stress increasing from 13.5 MPa to 25.1 MPa. The maximum stresses at point A for all the joints considered are as per Table 7-1 and Table 7-2.
### Table 7-1: Peak average stress at point A for phases 1 and 2

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{11}$ (MPa)</th>
<th>$\sigma_{33}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>1</td>
<td>-40.3</td>
<td>+1.8</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-39.1</td>
<td>+0.7</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-38.4</td>
<td>+1.1</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-41.9</td>
<td>+1.2</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-39.1</td>
<td>+0.9</td>
</tr>
<tr>
<td>40mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-39.2</td>
<td>+1.0</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-40.1</td>
<td>+0.9</td>
</tr>
</tbody>
</table>

### Table 7-2: Peak average stress at point A for phase 3

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{11}$ (MPa)</th>
<th>$\sigma_{33}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>1</td>
<td>-13.5</td>
<td>+0.3</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-25.1</td>
<td>+0.2</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-24.8</td>
<td>+0.2</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-25.0</td>
<td>+0.4</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-25.1</td>
<td>+0.2</td>
</tr>
<tr>
<td>40mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-24.5</td>
<td>+0.4</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-25.6</td>
<td>+0.4</td>
</tr>
</tbody>
</table>

7.9.3. Over-Laminate Stress at Point B

The magnitude of stress in the over-laminate was significantly affected by the chamfer angle. Decreasing the chamfer angle from 45° to 22.5° increased the compressive stress from 109.2 MPa to 162.2 MPa, an approximate increase of 50%. This large increase in stress is attributed to the shear load attracted to the stiffest component, in this case the diagonal compressive strut of the over-laminate. This can be demonstrated by considering the joints geometry as a statically determinate pin ended compressive strut taking a percentage of the total hull plate shear as per Figure 7-31. Using this approach predicts an approximate stress increase of 85% as per Eqn (7-3).
The maximum stresses at point B for all the joints considered are as per Table 7-3 and Table 7-4.

Table 7-3: Peak average stress at point B for phases 1 and 2

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{11}$ (MPa)</th>
<th>$\sigma_{33}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
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<td>-74.8</td>
<td>-2.9</td>
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<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
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<td>-74.6</td>
<td>-2.8</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-83.3</td>
<td>-2.8</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-76.9</td>
<td>-2.4</td>
</tr>
<tr>
<td>40 mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-77.5</td>
<td>-7.3</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-100.1</td>
<td>-5.5</td>
</tr>
</tbody>
</table>
Table 7-4: Peak average 3 at point B stress for phase

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{11}$ (MPa)</th>
<th>$\sigma_{33}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-109.2</td>
<td>-3.2</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-112.9</td>
<td>-4.4</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-99.1</td>
<td>-4.2</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-118.0</td>
<td>-3.5</td>
</tr>
<tr>
<td>40 mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-116.0</td>
<td>-11.0</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-162.2</td>
<td>-2.4</td>
</tr>
</tbody>
</table>

7.9.4. Stresses at Point C

These stresses are directly affected by the fillet radius, increasing the filler radius from 25 mm to 40 mm decreases the stresses by approximately 20% for phases 1 and 2. Phase 3 is affected by both, fillet radius and chamfer angle, decreasing the fillet radius from 40 mm to 25 mm and reducing the chamfer angle from 45° to 22.5° doubles the joint stresses at this point. The maximum stresses at point C for all the joints considered are as per Table 7-5 and Table 7-6.

Table 7-5: Peak stress at point C for phases 1 and 2

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{33}$ (MPa)</th>
<th>$\sigma_{13}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-9.9</td>
<td>10.5</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-10.0</td>
<td>10.8</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-11.6</td>
<td>12.4</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-9.7</td>
<td>7.8</td>
</tr>
<tr>
<td>40 mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-8.3</td>
<td>8.2</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-12.6</td>
<td>13.2</td>
</tr>
</tbody>
</table>

Table 7-6: Peak stress for phase 3 at point C

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{33}$ (MPa)</th>
<th>$\sigma_{13}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-11.2</td>
<td>13.7</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-12.7</td>
<td>16.7</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-9.3</td>
<td>10.6</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-11.4</td>
<td>11.7</td>
</tr>
<tr>
<td>40 mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-7.4</td>
<td>10.1</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-14.7</td>
<td>19.5</td>
</tr>
</tbody>
</table>
7.9.5. **Stresses at Point D**

All the model stresses from this point are relatively constant, to within 10% of each other. The 22.5° chamfer model experiences the highest stress. The maximum stresses at point D for all the joints considered are as per Table 7-7 and Table 7-8.

### Table 7-7: Peak stress at point D for phases 1 and 2

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{11}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-194</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-194</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-196</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-207</td>
</tr>
<tr>
<td>40 mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-183</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-201</td>
</tr>
</tbody>
</table>

### Table 7-8: Peak stress at point D for phase 3

<table>
<thead>
<tr>
<th>Basic model</th>
<th>Model Features</th>
<th>Span (m)</th>
<th>$\sigma_{11}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-325</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Separation distance = 0 mm</td>
<td>4</td>
<td>-329</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>Isolation void</td>
<td>4</td>
<td>-332</td>
</tr>
<tr>
<td>45° chamfer</td>
<td>High strain rate</td>
<td>4</td>
<td>-340</td>
</tr>
<tr>
<td>40 mm radius</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-309</td>
</tr>
<tr>
<td>22.5° chamfer</td>
<td>Separation distance = 20 mm</td>
<td>4</td>
<td>-332</td>
</tr>
</tbody>
</table>
7.9.6. Stresses at Point E

These stresses in this joint are directly affected by both, fillet radius and chamfer angle, a large fillet reduces the shear stress while a small chamfer angle increases the through thickness compressive stress. The peak stress for all phases occurs in the 22.5° chamfer model for both the shear and thought thickness stress. The maximum stresses at point E for all the joints considered are as per Table 7-9 and Table 7-10.

<table>
<thead>
<tr>
<th>Table 7-9: Peak stress at point E for phases 1 and 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic model</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>40 mm radius</td>
</tr>
<tr>
<td>22.5° chamfer</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7-10: Peak stress at point E for phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic model</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>45° chamfer</td>
</tr>
<tr>
<td>40 mm radius</td>
</tr>
<tr>
<td>22.5° chamfer</td>
</tr>
</tbody>
</table>
8. Conclusion

This research successfully developed a set of closed-form equations for predicting the salient 2-D hull motions of PTV, PA, and GMA to within ±15% of the LS-Dyna simulations. Four basic hull shapes were considered with and without charge slant angles. An estimate of the error associated with the LS-Dyna simulations and compared to real hull motions increased the uncertainty by an additional ±11%, resulting in a total error between the tier one equations and real hull motions of ±26%. The two largest errors were considered to be the shock wave definition accounting for a ±4.5% and the cavitation error at ±2.5%. The equations covered the combinations and permutations for charge sizes between 50 kg and 1,000 kg, four basic hull shapes, displaced hull masses from 17 tonne/m to 400 tonne/m, and charge standoffs of up to 100 m with varying slant angles. The LS-Dyna methodology and modelling strategy was verified against a known solution obtained from the literature. This example was dominated by local cavitation effects.

In this thesis, the range of numerical processes available in LS-Dyna has been reviewed and it was concluded that, due to the limited mass transportation experienced in shock/structure interaction, the ALES process was the most suitable computational method for 2-D problems. This method is the simplest and most computationally efficient. For large problems to reduce meshing complexity, the coupled Lagrangian-Eulerian solution with the third party code USA interfacing with the FE code was the industry norm. It was found that the affects of hull rotation due to off centre loading was of little significance on the determination of average vertical motions of a hull. However, the peak motions for PTV could be 45% greater than the average motion. Equations for peak motion estimates were also included in the research.

The sensitivity of UNDEX to the water properties can be considered to be three fold:

- Developing an UNDEX shock boundary condition by modelling the charge detonation and the surrounding water is best achieved using a non-linear material for modelling the water. In this scenario the expanding charge compresses and densifies the surrounding water, retarding the charge expansion. This increases the burn rate within the charge, resulting in a high peak pressure with a reduced
Conclusion

duration. The most recognised constitutive definition for non-linear density and
stiffness in the literature is the Gruneisen EOS.

- Developing an UNDEX shock boundary condition for a far-field scenario by
  mapping the shock pressure onto the external boundary of the model requires
  rudimentary water parameters, such as a linear acoustic model. This approach is it
  not overly sensitive to the water properties of stiffness or density.

- Hulls of limited draft with non vertical hull segments near the surface (such as V
  hulls with large slant angles) are sensitive to bulk cavitation and surface cut-off
  effects. When considering these types of hulls, a variable density cavitation
  model provides a significant advantage due to its accuracy. The smallest
  rectangular hull (of draft 4m) covered in this thesis is not significantly affected by
  bulk cavitation and a constant density cavitation model is considered adequate.

Also reviewed was the propagation and mitigation of a shock wave through the typical
GRP composite ship T-joint subject to an UNDEX shock loading. The joint was
reviewed to determine its strength sensitivity to the attributes of joint geometry and
material strain rate effects. The joint loading was considered to occur in three distinct
phases: initial shock, reflected shock, and the bending phase. Each loading phase
develops characteristic stress regimes that are affected by the span of the hull plate and
the joints geometry. The initial phase is associated with the migration of the shock wave
into the joint from the near side of the hull. The second phase occurs on the arrival of the
shock wave from the far side of the hull. Both of these phases are near identical with the
second phase displaced in time. Both phases 1 and 2 are independent of the hull span and
are the result of locally induced shock loads.

Clearly the 22.5° chamfered joint was the least capable joint. Under all phases and
locations, this joint experienced stresses up to 60% greater than the other joints. For
phase 1 and 2 loadings, it is considered beneficial for all joints, to provide a filler material
with the highest possible stiffness and the least area of voids. This allowed the filler to
act as a more competent load path, providing a reduction in the load experienced by the
over-laminate and its joint.
References


References


References


References


References


Appendix A: Differential Solution for Exponential Loading

The solution for a mass (M) loaded with a transient exponential shock pressure pulse, the retardation force \( F_{\text{RET}} \) being a function of the effective area \( A_{\text{EFF}} \), instantaneous velocity \( V \), density \( \rho \) and the speed of sound \( C \). A representation of the system is shown in Figure A-1 and the transient force is as per Figure A-2. The closed form differential solution for this loading is as per Taylor [1].

\[
F_{T} = F_{0} \cdot e^{-\frac{t}{\sigma}}
\]

\[
F_{\text{RET}} = \rho \cdot C \cdot V \cdot A_{\text{EFF, R}}
\]

Letting
\[
\sigma = \left( \frac{\rho \cdot C \cdot A_{\text{EFF, R}}}{M} \right)
\]
Appendix A

\[ \text{and } \Omega = \left( \frac{F_0}{M} \right) \]  \hspace{1cm} (A-4)

\[ V' + \vec{\sigma} \cdot V = \Omega \cdot e^{\frac{-t}{\vec{\sigma}}} \]  \hspace{1cm} (A-5)

Determine the integrating Constant \( I(t) \) and multiply through by \( I(t) \)

\[ I = e^{\int \vec{\sigma} \, dt} = e^{\vec{\sigma} \cdot t} \]  \hspace{1cm} (A-6)

\[ V' \cdot e^{\vec{\sigma} \cdot t} + V \cdot e^{\vec{\sigma} \cdot t} = \Omega \cdot e^{\left( \frac{-t}{\vec{\sigma}} \right)} \]  \hspace{1cm} (A-7)

Rewriting the equation

\[ \frac{d}{dt} V \cdot e^{\vec{\sigma} \cdot t} = \Omega \cdot e^{\left( \frac{-t}{\vec{\sigma}} \right)} \]  \hspace{1cm} (A-8)

Integrate both sides

\[ V \cdot e^{\vec{\sigma} \cdot t} = \Omega \int e^{\left( \frac{-t}{\vec{\sigma}} \right)} \cdot dt \]  \hspace{1cm} (A-9)

\[ V \cdot e^{\vec{\sigma} \cdot t} = \left( \frac{\Omega}{\omega - \frac{1}{\vec{\sigma}}} \right) \cdot e^{\left( \frac{-t}{\vec{\sigma}} \right)} + C \]  \hspace{1cm} (A-10)

Divide through by \( I(t) \)

\[ V = \left( \frac{\Omega}{\omega - \frac{1}{\vec{\sigma}}} \right) \cdot e^{\left( \frac{-t}{\vec{\sigma}} \right)} + C \cdot e^{\vec{\sigma} \cdot t} \]  \hspace{1cm} (A-11)

Solve for initial conditions at \( V(0) = 0 \) at \( t=0 \)

\[ 0 = \left( \frac{\Omega}{\omega - \frac{1}{\vec{\sigma}}} \right) \cdot 1 + \frac{C}{1} \]  \hspace{1cm} (A-12)

\[ \Rightarrow C = \left( \frac{-\Omega}{\omega - \frac{1}{\vec{\sigma}}} \right) \]  \hspace{1cm} (A-13)
Appendix A

\[
V = \left( \frac{\Omega}{\omega - \frac{1}{\vartheta}} \right) e^{\frac{\vartheta}{\theta}} - \left( \frac{\Omega}{\omega - \frac{1}{\vartheta}} \right) e^{\omega t}
\]  
(A-14)

\[
V = \frac{\Omega \left( e^{\omega t} - e^{\vartheta} \right)}{\left( \frac{1}{\vartheta} - \omega \right)}
\]  
(A-15)
Appendix B: Differential Solution for Complex loading

The solution for a mass (M) loaded with a transient exponential shock pressure pulse, the retardation force ($F_{RET}$) being a function of the effective area ($A_{EFF}$), instantaneous velocity ($V$), density ($\rho$) and the speed of sound (C). A representation of the system is shown Figure B-1 and the transient force is as per Figure B-2.

\[ F_T = F_0 \cdot \left( e^{-\frac{t}{\kappa \cdot \theta}} - e^{-\frac{t}{\kappa \cdot (0.6) \cdot \theta}} \right) \]

\[ V' + \frac{F_{RET}}{M} = \frac{F_T}{M} \quad (B-1) \]

\[ F_{RET} = \rho \cdot C \cdot V \cdot A_{EFF \cdot R} \quad (B-2) \]

Figure B-1: SDOF structure

Figure B-2: Force verses time
Letting

$$\sigma = \left( \frac{\rho \cdot C \cdot A_{\text{EFF}} \cdot R}{M} \right)$$

and $$\Omega = \left( \frac{F_\theta}{M} \right)$$

$$V' + \sigma \cdot V = \Omega \left[ e^{\kappa \theta} - e^{-(\kappa(0.6)\theta)} \right]$$

(B-3)

(B-4)

(B-5)

Determine the integrating Constant $$I(t)$$ and multiply through by $$I(t)$$

$$I_t = e^{[\sigma \cdot dt} = e^{\sigma \cdot t}$$

(B-6)

$$V' \cdot e^{\sigma \cdot t} + V \cdot e^{\sigma \cdot t} = \Omega \cdot \left[ e^{t \left( \frac{1}{\kappa \theta} \right)} - e^{t \left( \frac{1}{\kappa(0.6)\theta} \right)} \right]$$

(B-7)

Rewriting the equation

$$\frac{d}{dt} V \cdot e^{\sigma \cdot t} = \Omega \cdot \left[ e^{t \left( \frac{1}{\kappa \theta} \right)} - e^{t \left( \frac{1}{\kappa(0.6)\theta} \right)} \right]$$

(B-8)

Integrating both sides

$$V \cdot e^{\sigma \cdot t} = \Omega \cdot \left[ e^{t \left( \frac{1}{\kappa \theta} \right)} - e^{t \left( \frac{1}{\kappa(0.6)\theta} \right)} \right]$$

(B-9)

$$V \cdot e^{\sigma \cdot t} = \frac{\Omega \cdot e^{t \left( \frac{1}{\kappa \theta} \right)}}{\sigma - \frac{1}{\kappa \cdot \theta}} - \frac{\Omega \cdot e^{t \left( \frac{1}{\kappa(0.6)\theta} \right)}}{\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}} + C$$

(B-10)

Dividing through by $$I(t)$$ and rewriting equation

$$V = \frac{\Omega \cdot e^{t \sigma}}{e^{\sigma \cdot \left( \frac{1}{\kappa \cdot \theta} \right)} \cdot e^{\kappa \theta}} - \frac{\Omega \cdot e^{t \sigma}}{e^{\sigma \cdot \left( \frac{1}{\kappa \cdot (0.6) \cdot \theta} \right)} \cdot e^{\kappa(0.6)\theta}} + C$$

(B-11)
Appendix B

\[ V = \frac{\Omega}{\left(\sigma - \frac{1}{\kappa \cdot \theta}\right) \cdot e^{\kappa \theta}} - \frac{\Omega}{\left(\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}\right) \cdot e^{\kappa(0.6)\theta}} + \frac{C}{e^{\sigma t}} \]  

(B-12)

Solve for initial conditions at \( V(0) = 0 \) at \( t=0 \)

\[ 0 = \frac{\Omega}{\left(\sigma - \frac{1}{\kappa \cdot \theta}\right) \cdot 1} - \frac{\Omega}{\left(\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}\right) \cdot 1} + \frac{C}{1} \]  

(B-13)

\[ \Rightarrow C = \frac{-\Omega}{\left(\sigma - \frac{1}{\kappa \cdot \theta}\right)} + \frac{\Omega}{\left(\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}\right)} \]  

(B-14)

\[ V = \frac{\Omega \cdot e^{\kappa \theta}}{\left(\sigma - \frac{1}{\kappa \cdot \theta}\right)} - \frac{\Omega \cdot e^{\kappa(0.6)\theta}}{\left(\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}\right)} + \frac{-\Omega \cdot e^{-\sigma t}}{\left(\sigma - \frac{1}{\kappa \cdot \theta}\right)} + \frac{\Omega \cdot e^{-\sigma t}}{\left(\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}\right)} \]  

(B-15)

\[ V = \Omega \cdot \left[ \frac{\left( e^{\kappa \theta} - e^{-\sigma t} \right)}{\left(\sigma - \frac{1}{\kappa \cdot \theta}\right)} - \frac{\left( e^{\kappa(0.6)\theta} - e^{-\sigma t} \right)}{\left(\sigma - \frac{1}{\kappa \cdot (0.6) \cdot \theta}\right)} \right] \]  

(B-16)
Appendix C: Tabulated Bi-linear Water Model

From the water properties from Table 4-2 the following bi-linear density pressure curve was constructed using a tabulated EOS as per Hallquist [8].

At $\rho=999.8$ kg/m$^3$ and $C=1423.4$ m/s the bulk modulus is $2.03 \times 10^9$ Pa as per Eqn (C-1)

$$B_{MOD} = C^2 \cdot \rho = (1423.4)^2 \cdot (999.8) = 2.03 \times 10^9 \text{ Pa}$$  \hspace{1cm} (C-1)

The arbitrary reference pressure was taken as 70 MPa resulting in the reference volume $(VO)$ of 1.034482 and density $(RO)$ of 1034.276 kg/m$^3$ as per Eqn (C-2)

$$P = \left( \frac{RO}{\rho_{ATMOSPHERE}} - 1 \right) \cdot B_{MOD} = 70 \times 10^6 \left( \frac{RO}{999.8} - 1 \right) \cdot 2.03 \times 10^9$$

$$=> RO = 1034.276 \text{ kg/m}^3$$  \hspace{1cm} (C-2)

$$=> VO = \frac{RO}{\rho_{ATMOSPHERE}} = \frac{1034.276}{999.8} = 1.034482$$

As entropy is not considered the equation that relates pressure to volumetric strain simplifies to Eqn (C-3)

$$P_i = C_i \cdot \varepsilon_{vi}$$

Where

$P_i = Resulting \ pressure$

$C_i = Tabulated \ constant$

$\varepsilon_{vi} = Tabulated \ volumetric \ strain$
Appendix C

Table C 1:  Tabulated bi-linear EOS

<table>
<thead>
<tr>
<th>i\textsuperscript{th} point</th>
<th>Pressure (Pa)</th>
<th>Density (kg/m\textsuperscript{3})</th>
<th>Volumetric Strain</th>
<th>Natural Log of Volumetric Strain EVi</th>
<th>Tabulated Constant Ci</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.1</td>
<td>10,342.76</td>
<td>9.244042</td>
<td>0.000387</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>1</td>
<td>1,034.276</td>
<td>6.941457</td>
<td>0.029006</td>
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<tr>
<td>3</td>
<td>12,015</td>
<td>999.756</td>
<td>1.034528</td>
<td>0.033946</td>
<td>11,614.12</td>
</tr>
<tr>
<td>4</td>
<td>41,115,556</td>
<td>1,020</td>
<td>1.013996</td>
<td>0.013899</td>
<td>40,548,042</td>
</tr>
</tbody>
</table>

The constitutive water model is as per Table 4-2.

```
*MAT_NULL
$WATER
    $ MID 0 0 0 1.034276 -1E-10 0.0 0.0 0.0 0.0

$EOS_TABULATED
    $ EOSID GAMA EO VO
    2 0.0 0 1.034932
    9.244042, 6.941457, 0.033946, 0.013899
    0.000387, 0.029006, 11,614.12, 40,548,042
```

Figure C-1: LS-Dyna keywords for tabulated EOS bi-linear water model
# Appendix D: LS-Dyna Files Names and Descriptions

<table>
<thead>
<tr>
<th>File Name</th>
<th>File Description</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthotropic_speed_ls.k</td>
<td>The speed of sound $C_{11}$, $C_{22}$ and $C_{33}$ for the orthotropic T_joint materials at Low strain rates</td>
<td>3.10</td>
</tr>
<tr>
<td>Orthotropic_speed_hs.k</td>
<td>The speed of sound $C_{11}$, $C_{22}$ and $C_{33}$ for the orthotropic T_joint materials at High strain rates</td>
<td>3.10</td>
</tr>
<tr>
<td>AS_EOS_TNT.k</td>
<td>Axi-Symmetric model with TNT represented as an EOS, considering pressure profile</td>
<td>4.2.1</td>
</tr>
<tr>
<td>Water_prop_density_Bmod.k</td>
<td>2-D solid model determining the affects of water stiffness and density</td>
<td>4.2.2</td>
</tr>
<tr>
<td>Water_prop_viscosity.k</td>
<td>2-D solid model determining the affects of viscosity</td>
<td>4.2.3</td>
</tr>
<tr>
<td>ST_Tran_mat_001.k</td>
<td>Shock tube model investigating the Tran and marco boundary conditions</td>
<td>4.3.1</td>
</tr>
<tr>
<td>ST_Tran_EOS.k</td>
<td>Axi-Symmetric model considering bulk cavitation affects at zero slant</td>
<td>4.4.2</td>
</tr>
<tr>
<td>Shin_SDC_2.k</td>
<td>Axi-Symmetric model with V hull at zero slant</td>
<td>4.4.3</td>
</tr>
<tr>
<td>2d_c.k</td>
<td>Axi-Symmetric model considering bulk cavitation affects with V hull at 45° slant</td>
<td>4.4.4</td>
</tr>
<tr>
<td>AS_far.k</td>
<td>Axi-Symmetric model considering bulk cavitation affects with V hull at 45° slant</td>
<td>4.4.5</td>
</tr>
<tr>
<td>Bleich_mat_001.k</td>
<td>1-D cavitation model with constant density</td>
<td>4.4.5</td>
</tr>
<tr>
<td>Bleich_Bi_EOS_001.k</td>
<td>1-D cavitation model with non constant bi-linear density</td>
<td>4.4.5</td>
</tr>
<tr>
<td>Bleich_Tri_EOS_001.k</td>
<td>1-D cavitation model with non constant bi-linear density</td>
<td>4.4.5</td>
</tr>
<tr>
<td>90V_cav_0.k</td>
<td>90° V hull cavitation model with 0 kPa cavitation pressure and constant density</td>
<td>4.4.6</td>
</tr>
<tr>
<td>90V_cav_2000.k</td>
<td>90° V hull cavitation model with 2000 kPa cavitation pressure and constant density</td>
<td>4.4.6</td>
</tr>
<tr>
<td>90V_EOS_0.k</td>
<td>90° V hull cavitation model with 0 kPa cavitation pressure and non constant density</td>
<td>4.4.6</td>
</tr>
<tr>
<td>Element_density_flat.k</td>
<td>Successive approximation for flat bottom hull, element sizes 12.5 mm to 1000 mm</td>
<td>4.7</td>
</tr>
<tr>
<td>Element_density_45_1.k</td>
<td>Successive approximation for 45° V hull, element sizes 117 mm to 1114 mm</td>
<td>4.7</td>
</tr>
<tr>
<td>Element_density_45_2.k</td>
<td>Successive approximation for 45° V hull, element size 58mm</td>
<td>4.7</td>
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Table D-2: LS-Dyna and EXCEL files circular hulls with zero slant

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## Table D-3: LS-Dyna and EXCEL files for 90° V Hulls with zero slant

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### Table D-5: LS-Dyna and EXCEL files for rectangular hulls with slant

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## Table D-6: LS-Dyna and EXCEL files for circular hulls with slant

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<td>34.44</td>
<td>0.3912</td>
<td>&quot;</td>
<td>&quot;</td>
<td>V45S_20m_34MPa_0_39ms.k</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>all</td>
<td>&quot;</td>
<td>0.9</td>
<td>&quot;</td>
<td>&quot;</td>
<td>V45S_20m_34MPa_0_9ms.k</td>
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</tr>
<tr>
<td>15</td>
<td>all</td>
<td>&quot;</td>
<td>1.4086</td>
<td>&quot;</td>
<td>&quot;</td>
<td>V45S_20m_34MPa_1_4ms.k</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>all</td>
<td>67.82</td>
<td>0.9</td>
<td>&quot;</td>
<td>&quot;</td>
<td>V45S_20m_67MPa_0_9ms.k</td>
<td></td>
</tr>
</tbody>
</table>

Table D-9: LS-Dyna and EXCEL files for modified hull forms

<table>
<thead>
<tr>
<th>File name</th>
<th>Excel summary file</th>
<th>File description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified_hull.xls</td>
<td>Summary of LS-Dyna model outputs</td>
<td></td>
</tr>
<tr>
<td>C_6m_34MPa_0_9ms_1_0M.k</td>
<td>Average circular hull with zero slant and 1.0 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>C_6m_34MPa_0_9ms_1_5M.k</td>
<td>Average circular hull with zero slant and 1.5 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>C_6m_34MPa_0_9ms_2_0M.k</td>
<td>Average circular hull with zero slant and 2.0 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>V90_6m_34MPa_0_9ms_1_0M.k</td>
<td>Average 90° V hull with zero slant and 1.0 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>V90_6m_34MPa_0_9ms_1_5M.k</td>
<td>Average 90° V hull with zero slant and 1.5 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>V90_6m_34MPa_0_9ms_2_0M.k</td>
<td>Average 90° V hull with zero slant and 2.0 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>VS90_6m_34MPa_0_9ms_1_0M.k</td>
<td>Average 90° V hull with slant and 1.0 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>VS90_6m_34MPa_0_9ms_1_5M.k</td>
<td>Average 90° V hull with slant and 1.5 times the hull mass</td>
<td></td>
</tr>
<tr>
<td>VS90_6m_34MPa_0_9ms_2_0M.k</td>
<td>Average 90° V hull with slant and 2.0 times the hull mass</td>
<td></td>
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</table>
Table D-10: LS-Dyna model files for T-Joint

<table>
<thead>
<tr>
<th>File name designation</th>
<th>File description</th>
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<tbody>
<tr>
<td>Local_z.k</td>
<td>Unit cell models to check correct orientation of material directions and stress output</td>
</tr>
<tr>
<td>Global_z.k</td>
<td>Unit cell models to check correct orientation of material directions and stress output</td>
</tr>
<tr>
<td>T_joint_45_4m.k</td>
<td>T-joint with 45° chamfer with 4m span</td>
</tr>
<tr>
<td>T_joint_45_4m_void.k</td>
<td>T-joint with 45° chamfer with isolation joint with 4m span</td>
</tr>
<tr>
<td>T_joint_45_4m_zero_sep.k</td>
<td>T-joint with 45° chamfer with zero separation with 4m span</td>
</tr>
<tr>
<td>T_joint_45_4m_high_sr.k</td>
<td>T-joint with 45° chamfer with high strain rate properties with 4m span</td>
</tr>
<tr>
<td>T_joint_40R_4m.k</td>
<td>T-joint with 40 mm fillet and 4m span</td>
</tr>
<tr>
<td>T_joint_22_5_4m.k</td>
<td>T-joint with 22.5° chamfer with 4m span</td>
</tr>
</tbody>
</table>