Heat Pipe Thermally Enhanced Phase Change Material

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Abdulmajed A. A. Khalifa

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Publications from This Research

Conference Papers:


Journal Papers:


Manuscripts in Preparation:

1. KHALIFA, A., TAN, L., DATE, A. & AKBARZADEH, Heat pipe thermally enhanced phase change material composites. (Will be submitted to *Applied Thermal Engineering*).
Abstract

Concentrating solar power (CSP) is increasingly considered because it facilitates harvesting a large amount of solar thermal energy. In order for CSP plants to be operationally cost competitive and efficient, they must be able to harvest and store the maximum solar thermal energy during the period of solar availability and to utilize the stored heat for continuous electrical power generation during times when the sun is not available. Hence, efficient and high-density thermal energy storage is an essential aspect of CSP system development. Among the three types of thermal energy storage (sensible, latent and thermochemical energy storage systems), latent heat thermal energy storage (LHTES) has the advantage over the other types of using a high energy-density storage medium for storing solar thermal energy. Furthermore, LHTES is capable of minimizing the temperature rise during heat charging and discharging which facilitates high Rankine cycle efficiency. Consequently, using LHTES can directly reduce the development costs of CSP plants, in terms of smaller installation space and reduced heat storage material requirements. However, the low thermal conductivities of phase change materials (PCMs) (typically ~0.5W/m.K) have limited their thermal potentials.

This research focuses on increasing the overall thermal conductance of PCMs used in LHTES units through using heat pipes (HPs). Specifically, three new designs of LHTES units, which utilize HPs to enhance the energy transfer, were numerically and experimentally investigated. Three different numerical models along with three different experimental configurations were developed to assess the thermal performance of the proposed units. Also, a preliminary system sizing was conducted to estimate the size of each unit required for 50MW electrical power output. After calculating the size of each unit, the units were economically assessed.

Firstly, the advantages of utilising axially finned HPs in LHTES units were numerically and experimentally quantified. The experimental measurements were conducted on a bare heat...
pipe and on an identical heat pipe with four axial fins. The numerical predictions and the experimental measurements were found to be in good agreement. The results have shown that the energy extracted from the PCM increased by 86 % and the heat pipes effectiveness increased by 24 %. Secondly, finned heat pipes were held, in suspension, adjacent to the heat transfer channel in order to increase the overall heat conductance of the PCM and act as effective heat spreaders. The results have shown that the performance was significantly improved by adding suspended heat pipes, especially in the later stage of PCM solidification. It was found that the effectiveness of the twelve-heat pipe configuration reached 2.4 after 5h of simulated operation. Thirdly, the thermal performance of a micro heat pipe-phase change material (MHP-PCM) composite for CSP applications was studied. A 3D numerical model was introduced to firstly predict the effective thermal conductivity of MHPs-PCM composites, and secondly to simulate heat transfer and phase change processes in a high-temperature LHTES unit for CSP applications. The model takes into consideration the effects of the MHP orientation as well as the MHP volume fraction ($v_{hp}$). The results have shown that the thermal conductivity increased by approximately 35 times at a MHP volume fraction of 10%. Finally, the last portion of this research focused on size estimation of each of the designs proposed in Chapters 4 and 5. Economic assessments of each design are then presented (The design presented in Chapter 3 is excluded from the economic assessment as it will be discussed later). Compared with the existing two-tank sensible heat thermal energy storage (SHTES) system, the LHTES unit presented in Chapter 4 is found to be economically competitive. Specifically, its capital cost was found to be 8% lesser than that of the two-tank SHTES system. In contrast, the LHTES unit presented in Chapter 5 is found to be significantly more expensive than the two-tank SHTES system. That is, its total cost was found to be 5.6 times higher than the two-tank SHTES system.
CHAPTER 1

Introduction

1.1 Background

Worldwide energy demand has been increasing over recent years, and it will continue to increase as a result of the fast growth of the world population and of advanced technologies. Fossil fuels (coal, oil and natural gas) are supplying much of the worldwide energy needs. These sources are non-renewable and will not meet the rapidly growing energy demand form future generations. Additional sources should be utilised to avoid an energy crisis in the future. Therefore, efforts have been dedicated to evolving green energy technologies including solar, wind, hydropower and geothermal energies. According to Figure 1, the renewables’ share of world marketed energy grew slightly reaching almost 11% of global energy use over the period between 1990 and 2010. Moreover, it is projected to reach 15% in 2040 (IEA, International Energy Outlook 2013).

Although the renewables’ share of global energy use is expected to increase, fossil fuels (liquids, coal and natural gas) are also expected to remain the largest source of energy. The continued heavy reliance on these non-renewable fuels will lead to a 46% increase of carbon dioxide (CO₂) emissions from 31.2 billion metric tons in 2010 to 45.5 billion metric tons in 2040 (Figure 2). Therefore, research into improving the competitiveness of clean energy technologies in the wholesale electricity market can be a significant contribution to reducing the reliance on fossil fuels and consequently to reducing the carbon footprint.
1.2 Concentrating Solar Power Plants

The energy received daily from the Sun is as much as 26 times the annual world energy consumption. The Earth receives daily 14,100 quadrillion Btu (4.2 kWh per day per square meter) of solar energy compared to an annual world’s consumption in 2010 of 524 quadrillion
Btu (552 Exajoules). This makes solar energy a high potential source of renewable energy. Concentrating solar power (CSP) plants are devices which convert energy from the sun’s rays into electricity. In contrast to photovoltaics (PV), CSP plants cannot produce electricity using direct conversion of the sun’s rays. Rather, a working fluid is usually heated to high temperatures. This thermal energy is firstly converted into mechanical energy using turbines and then into electricity. CSP has probably the strongest potential of known renewable sources to meet the world’s energy demand and to mitigate climate change (Mills, 2004). CSP can be categorised into four main families according to the means of focusing and receiving the sun’s rays (IEA, International Energy Outlook 2013). The four main categories of CSP technology are shown in Table 1 and Figure 3. Although the first commercial CSP plant was constructed in California in 1984, fossil fuel price drop led to delay in the advancement of CSP. In 2006, Spain and the United States re-entered the market of CSP as a result of government plans obliging utilities to obtain some power from renewable sources. Thus, the global stock of CSP plants reached approximately 4 GW capacity in 2014, and is expected to represent approximately 11% of global electricity production by 2050 (IEA, Technology roadmap CSP 2014).

**Table 1: Four main technologies of CSP brought from (IEA, Technology roadmap CSP 2010).**

<table>
<thead>
<tr>
<th>Focus type</th>
<th>Line focus</th>
<th>Point focus</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Receiver type</strong></td>
<td>Collectors track the sun along a single axis and focus irradiance on a linear receiver. This makes tracking the sun simpler.</td>
<td>Collectors track the sun along two axes and focus irradiance at a single point receiver. This allows for higher temperatures.</td>
</tr>
<tr>
<td><strong>Fixed</strong></td>
<td>Linear Fresnel Reflectors</td>
<td>Towers (CRS)</td>
</tr>
<tr>
<td>Fixed receivers are stationary devices that remain independent of the plant’s focusing device. This eases the transport of collected heat to the power block.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mobile</strong></td>
<td>Parabolic Troughs</td>
<td>Parabolic Dishes</td>
</tr>
<tr>
<td>Mobile receivers move together with the focusing device. In both line focus and point focus designs, mobile receivers collect more energy.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Through combination with thermal energy storage (TES) units, CSP plants can store some thermal energy for later use. This makes CSP plants able to continually produce electricity even during cloudy periods or at night time. In addition, storage reduces the irregularity of the power generated arising from the sun’s intermittent availability. Collectively, CSP technology has promising characteristics to be a clean, flexible, reliable power source for future generations.

![Four main technologies of CSP](image)

**Figure 3: Four main technologies of CSP brought from (IEA, Technology roadmap CSP 2010).**

### 1.3 Statement of the Problem

Modern CSP plants have been equipped with TES units in an attempt to reduce the costs of electrical power produced from these plants and to address the problem of the intermittent nature of the generated energy (Mills, 2004). There are three types of TES that can be utilized in CSP plants. They are sensible heat thermal energy storage (SHTES), latent heat thermal energy storage (LHTES) and thermochemical energy storage (TCES). Currently, only SHTES technique is used in large-scale CSP plants (Robak et al., 2011a). The SHTES can be
subcategorised into liquid SHTES and solid SHTES. The two-tank molten salt system, in which molten salt is used as storage material, is a liquid SHTES system. It consists of two insulated storage tanks where the first is used to store molten salt at approximately 290°C (cold tank) and the other for storing molten salt at approximately 390°C (hot tank). In contrast, the solid SHTES system consists of a tubular heat exchanger embedded in a high-performance concrete (Bai and Xu, 2011, Jian et al., 2015). One of the major drawbacks of the SHTES systems is the requirement for large quantities of storage material (Vernon, 2011). For example, the storage material required to drive a 50-megawatt turbine for nine hours is approximately 42,288 tons of salt for the liquid SHTES (Herrmann et al., 2004), while it is 50,000 m³ of concrete for the solid SHTES (Laing et al., 2012). Although the two-tank molten salt system reduces the capital cost and increases the reliability of the CSP plants, it is one of the less developed systems (Gil et al., 2010). To illustrate, extra energy is required to pump the salt between the hot and the cold tanks during the charging and discharging processes and to keep the salt temperature above its freezing point (~200°C). This energy is considered as an energy loss and consequently reduces the plant’s power output. Figure 4 shows a schematic diagram of a CSP plant equipped with a SHTES system.

According to a SunShot Initiative, by 2020 the storage systems should have a second law of thermodynamics efficiency of more than 95% and a storage cost of less than $15/kWh, with minimum storage capacity of 14 h (SunShot Vision Study, 2012). Under these conditions, CSP plants can compete in the wholesale electricity market by a Levelized Cost of Electricity (LCE) of 6¢/kWh instead of the 2010 cost of 21¢/kWh. Among all thermal storage techniques no cost-effective compact storage technology is available (IEA, Task 42/24). Thus, of all components of CSP plants, thermal storage is a key one.
In an attempt to develop cost-effective compact storage technology, researchers around the world have been investigating the practicability of LHTES systems as an alternative storage technology. LHTES systems utilise PCMs to store thermal energy in the form of latent heat of fusion. PCMs are attractive because of their high energy-storage density at a constant temperature. For illustration, the energy that can be stored by raising the temperature of 99 kilograms of potassium nitrate (KNO$_3$) by 1K (sensible heat) can be stored in just one kilogram of the same material by making the KNO$_3$ undergo phase change (latent heat). Therefore, less storage material is required compared to a SHTES which leads to reducing the capital and construction costs (Robak et al., 2011a). In CSP-LHTES plants, the working fluid (referred as the heat transfer fluid (HTF) in this thesis) is pumped to charge and discharge thermal energy to and from the PCM respectively. However, the heat transfer rates between the HTF and the PCM are limited by the low thermal conductivity of the PCM (0.1-0.6 W/m.K). As a result, the deployment of LHTES systems into commercial large-scale...

Figure 4: CSP plant equipped with two-tank storage system (SolarMillennium, cited in IEA, Technology roadmap CSP 2010).
applications has not yet been achieved. In order to reduce the thermal resistance of the PCMs, several techniques have been developed including extended surfaces, multiple PCM’s, thermal conductivity enhancement and micro-encapsulation of the PCM (Jegadeeswaran and Pohekar, 2009). Of all of the techniques proposed to enhance the thermal performance of LHTES systems, using heat pipes may be the best solution. That conclusion is based on an economic study of LHTES equipped with bare heat pipes, where the reduction in capital cost was found to be 15% compared with that of the prevailing technique of energy storage (two-tank system) (Robak et al., 2011a).

This study focuses on increasing the overall thermal conductance of PCMs used in LHTES units through using HPs. Specifically, three new designs of LHTES units were numerically and experimentally investigated. In order to assess the thermal performance of the proposed units, three different numerical models along with three different experimental arrangements were developed. Furthermore, a preliminary system sizing was conducted in order to estimate, for each unit, the size which would be required to drive a 50MWₐ turbine. The proposed units are introduced in chapters 3 to 5 and economically assessed in chapter 6.

### 1.5 Research Objectives

The objective of this study was to assess the thermal performance of the three new proposed designs of LHTES unit. This assessment was based on the following:

a. Develop three different numerical models to investigate the thermal performance of each design of high-temperature LHTES unit.

b. Conduct an experimental investigation in order to verify the three developed numerical models.

c. Calculate the required size of each design required to drive a 50MWₑ turbine.

d. Conduct a preliminary economic assessment of each design.
1.6 Research Scope

It has been showed numerically by Shabgard et al. (2010) and experimentally by Robak et al. (2011b) that the solidified PCM on the cool surfaces of the LHTES unit, acts as a thermal insulation making the discharging stage take much longer than the charging stage. As a result, any design which works well during the discharging process (solidification), will work appropriately during the charging process (melting). Therefore, this study focuses on the discharging process (solidification).

1.7 Thesis Structure

The thesis is structured in the following chapters:

Chapter 2: Literature Reviews – presents a detailed background and literature review on LHTES systems, with emphasis on methods for conductivity enhancement. The reviewed studies provide a basis for understanding the existing challenges of commercial deployment of LHTES units in CSP plants.

Chapter 3: Latent Heat Thermal Energy Storage System with Embedded Finned Heat Pipes – (the first proposed design) – presents a numerical analysis of the thermal performance of a LHTES system equipped with axially finned HPs for CSP applications. Experimental verification of the numerical model is also presented in this chapter.

Chapter 4: Performance of Suspended Finned Heat Pipes in High-temperature Latent Heat Thermal Energy Storage – (the second proposed design) – presents a thermal network model to assess heat transfer enhancement in a high temperature LHTES unit by incorporating finned heat pipes kept in suspension adjacent to the HTF channels. Experimental verification of the numerical model is also presented in this chapter.

Chapter 5: Micro Heat Pipe-Phase Change Material (MHP-PCM) Composite – (the third proposed design) – presents a numerical model to predict the effective thermal
conductivity of MHPs-PCM composites. The model takes into consideration the effects of the MHP orientation as well as the MHP volume fraction ($v_{hp}$). Experimental verification of the numerical model is also presented in this chapter.

**Chapter 6: System Sizing and Economic Assessment** – estimates the size of each proposed design as discussed in Chapters 4 and 5 for large scale CSP applications. Economic assessments of each design are also presented in this chapter.

**Chapter 7: Conclusions and Recommendations for Future Work** – presents conclusions from the whole study and recommendations for future work.
CHAPTER 2

Literature Review

2.1 Introduction

The aim of this study was to improve the thermal performance of high-temperature LHTES units for CSP plants through enhancing the overall thermal conductance of the storage medium (the PCM) by utilising HPs. This chapter firstly introduces the TES types and some PCMs candidates which could be used in high-temperature applications (around 350°C). The chapter then, reviews performance enhancement techniques used in LHTES systems, followed by a brief introduction to heat pipes. Finally, an introduction to some proposed state-of-the-art high-temperature LHTES units for CSP applications is presented.

2.2 Thermal energy storage

Thermal energy storage is a crucial part of modern CSP plants. It allows the CSP plants to store the harvested energy during sunny times for power production during the night or cloudy periods. The classifications and types of TES are presented in Figure 5 (Sharma et al., 2009). Each type of TES has advantages and disadvantages in terms of the thermal performance and applications. The following subsections briefly discuss the TES which are sub-categorised into sensible heat storage and latent heat storage. Emphasis is placed here on latent heat storage, and the PCM used as a storage medium. Heat storage systems other than those using latent heat are not considered in detail in this thesis.
2.2.1 Sensible heat storage

In sensible heat storage, the specific heat capacity and the temperature difference provide the basis for storing thermal energy. The storage material does not undergo phase change but remains as a solid or liquid phase. The heat stored or released can be determined by multiplying the specific heat by the temperature change, and by the mass of the storage material. An expression for the sensible heat that can be stored in a material of mass $m$, specific heat $c_p$ and over a temperature range of $\Delta T$ is written as (Dincer and Rosen, 2011):

$$Q_{stored} = m c_p(\Delta T)$$  \hspace{1cm} (2-1)

One of the major drawbacks of the SHTES systems is the requirement for large quantities of storage material (Vernon, 2011). For example, the storage material required to drive a 50-megawatt turbine for nine hours is approximately 42,288 tons of salt for the liquid SHTES
(Herrmann et al., 2004), while it is 50,000 m$^3$ of concrete for the solid SHTES (Laing et al., 2012).

### 2.2.2 Latent heat storage

In latent heat storage, both sensible and latent heat are used to store the thermal energy. The storage material undergoes phase change during the processes of energy storage and release. From (Dincer and Rosen, 2011), the stored energy can be expressed as:

$$Q_{\text{stored}} = m [c_s(T_m - T_s) + h_{sl} + c_l(T_l - T_m)]$$  \hspace{1cm} (2-2)

The latent heat component is usually much higher than the sensible heat component as a result of the high heat of fusion of the PCM used. A comparison between sensible and latent heat materials is shown in Table 2 (TAN, 2013). As can be seen, latent heat storage is more attractive than sensible heat storage in terms of the required mass and volume for storing the same amount of energy. For example, the required mass of an organic PCM is less than half of the required mass of water.

**Table 2: Comparison of sensible and latent heat storage materials (TAN, 2013).**

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Rock</th>
<th>Inorganic PCM</th>
<th>Organic PCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg,m$^{-3}$)</td>
<td>1,000</td>
<td>2,240</td>
<td>1,600</td>
<td>800</td>
</tr>
<tr>
<td>Specific heat (J,kg$^{-1}$,K$^{-1}$)</td>
<td>4,200</td>
<td>1,000</td>
<td>2,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Latent heat (J,kg$^{-1}$)</td>
<td>-</td>
<td>-</td>
<td>232,000</td>
<td>190,000</td>
</tr>
<tr>
<td>Storage mass for $10^6$ J (kg)</td>
<td>16$^a$</td>
<td>67$^a$</td>
<td>4.35</td>
<td>5.3</td>
</tr>
<tr>
<td>Storage volume for $10^6$ J (m$^3$)</td>
<td>0.016</td>
<td>0.03</td>
<td>0.0027</td>
<td>0.0066</td>
</tr>
<tr>
<td>Relative storage mass</td>
<td>4</td>
<td>15</td>
<td>1</td>
<td>1.25</td>
</tr>
<tr>
<td>Relative storage volume</td>
<td>6</td>
<td>11</td>
<td>1</td>
<td>2.5</td>
</tr>
</tbody>
</table>

* Based on a temperature difference of 15°C.
### 2.3 Phase change material desired for CSP applications

Physical and chemical characteristics of PCMs play a crucial role in the design and operation of latent heat thermal storage systems. For a given application, the higher the latent heat of PCM, the smaller the required storage volume (Regin et al., 2008). Consequently, using PCM leads to a smaller, and cheaper, heat exchanger or a greater heat storage capacity for the same overall volume. Hasnain (1998) reported that not all available data on technical grade PCMs can be used reliably for designing an effective latent heat storage system. The thermo-physical properties vary between manufacturers as a result of the different level of impurities of PCMs. There are over 160,000 commercially available PCMs with a high heat of fusion covering almost any required temperature range (Khare et al., 2012). Nevertheless, the use of a particular PCM as a heat storage medium depends on its physical, thermal and chemical characteristics as well as economic factors. Table 3 lists the main required PCMs characteristics (Regin et al., 2008).

<table>
<thead>
<tr>
<th>Thermal properties</th>
<th>Temperature range suitable for the desired application.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High latent heat.</td>
</tr>
<tr>
<td></td>
<td>High specific heat.</td>
</tr>
<tr>
<td></td>
<td>High thermal conductivity for both solid and liquid states.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>High density.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small density variation during phase change process.</td>
</tr>
<tr>
<td></td>
<td>Slight or no supercooling during freezing.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chemical properties</th>
<th>Chemically stable.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Negligible or no chemical decomposition.</td>
</tr>
<tr>
<td></td>
<td>Compatible with container materials.</td>
</tr>
<tr>
<td></td>
<td>Non-toxic, fire-resistant and non-explosive.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Economic factors</th>
<th>Available in large quantities.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low-cost.</td>
</tr>
</tbody>
</table>

**Table 3: PCMs desirable characteristics (Regin et al., 2008).**
2.3.1 High-temperature PCMs

In general, PCMs are classified as summarized in Figure 6 (Sharma et al., 2009). However, the PCMs desired for CSP applications should have melting temperature above 300°C, high latent capacity and minimal volumetric change during phase change processes. Specifically, the PCMs used for this research should have a melting temperature approximately midway between 280°C and 390°C. These temperatures match the HTF temperatures coming from the superheated steam generator and from the solar field during discharging and charging processes respectively (Kelly and Kearney, 2006). In this particular temperature range, salts, salt eutectics, metals and metal eutectics are strong candidates.

2.3.1.1 Salts and Salt eutectics

Salts are strong potential high-temperature PCMs. Table 4 presents pure inorganic salts as well as salt eutectics that are within the desired range. The thermo-physical properties of the listed salts and salt eutectics are presented in the table including the melting point, the latent heat of fusion and the density. Salts and Salt eutectics have a high heat of fusion and low cost especially those based on chlorides and fluorides. As a result, they have been proposed by various authors as potential PCMs for CSP applications (Cárdenas and León, 2013).

Figure 6: classifications of PCMs (Sharma et al., 2009)
### 2.3.1.2 Metals and metal alloys

Metallic materials possess high thermal conductivity, high heat of fusion, low specific heat and relatively low vapour pressure (Sharma et al., 2009). The very good thermal conductivity of metals and metal alloys eliminates the need to add any heat conduction enhancement mechanism within the container of the LHTES unit. Despite this advantage, they have not been seriously considered as PCMs because of their high weight and price. However, some researchers have analysed the possibility of using metals and metal alloys as PCMs including (Farkas and Birchenall, 1985, Gasanalieva and Gamataeva, 2000) and recently (Kotzé, 2014). Table 5 presents some metallic materials that are within the desired temperature range (280°C to 390°C).

#### Table 4: Thermo-physical properties of some salts and salt eutectics.

<table>
<thead>
<tr>
<th>Material</th>
<th>Melting point (°C)</th>
<th>Heat of fusion (kJ.kg⁻¹)</th>
<th>Density (kg.m⁻³)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inorganic salts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaNO₃</td>
<td>306</td>
<td>182</td>
<td>2260</td>
<td>1908</td>
</tr>
<tr>
<td>RbNO₃</td>
<td>312</td>
<td>31</td>
<td>3685</td>
<td>2820</td>
</tr>
<tr>
<td>KNO₃</td>
<td>334</td>
<td>266</td>
<td>2109</td>
<td>n.a</td>
</tr>
<tr>
<td>KOH</td>
<td>380</td>
<td>149.7</td>
<td>2044</td>
<td>1470</td>
</tr>
<tr>
<td>Salt eutectics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KOH/LiOH 60/40 (wt%)</td>
<td>314</td>
<td>341</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>LiCl/KCl/BaCl₂ 54.2/39.4/6.4 (mol%)</td>
<td>320</td>
<td>170</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>KNO₃/KBr/KC 80/10/10 (wt%)</td>
<td>342</td>
<td>140</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>LiCl/NaCl/KCl 43/33/24 (wt%)</td>
<td>346</td>
<td>281</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>LiCl/KCl 58.5/41.5 (mol%)</td>
<td>355</td>
<td>234.6</td>
<td>n.a</td>
<td>1631</td>
</tr>
<tr>
<td>MnCl₂/KCl/NaCl 45/28.7/26.3 (mol%)</td>
<td>350</td>
<td>215</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>Li₂MoO₃/LiVO₃/LiCl/Li₂SO₄/LiF 27.1/24.8/23.4/17.3/6.1 (wt%)</td>
<td>360</td>
<td>278</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>NaOH/NaC 80/20 (wt%)</td>
<td>370</td>
<td>370</td>
<td>n.a</td>
<td>n.a</td>
</tr>
<tr>
<td>MgCl₂/KCl/NaCl 60/20/19.6 (wt%)</td>
<td>380</td>
<td>400</td>
<td>n.a</td>
<td>n.a</td>
</tr>
</tbody>
</table>

Table 5: Thermo-physical properties of some metals and metal alloys.

<table>
<thead>
<tr>
<th>Material</th>
<th>Melting point (°C)</th>
<th>Heat of fusion (kJ·kg⁻¹)</th>
<th>Density (kg·m⁻³)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zn/Mg, 52/48 (wt%)</td>
<td>340</td>
<td>180</td>
<td>n.a</td>
<td>a</td>
</tr>
<tr>
<td>Zn/Mg, 53.7/46.3 (wt%)</td>
<td>340</td>
<td>185</td>
<td>4600</td>
<td>b</td>
</tr>
<tr>
<td>Zn/Al, 96/4 (wt%)</td>
<td>381</td>
<td>138</td>
<td>6630</td>
<td>b</td>
</tr>
</tbody>
</table>

*(Farkas and Birchenall, 1985) and *(Gasanaliev and Gamataeva, 2000).

More details of a broad range of potential PCMs can be found in the following references:

![Figure 7: Comparison of high temperature PCMs found in literature (Kotzé, 2014)](image-url)
As seen in Tables 4 and 5, the metallic PCMs which fall in the desired temperature range have a lower heat of fusion than those of salts and salt eutectics. This is more obvious in Figure 7 which compares some PCMs in terms of the heat of fusion against the melting temperature. By considering the circled area in Figure 7, it is apparent that chloride eutectics have an exceptionally high heat of fusion. Therefore, salts and salt eutectics are adopted in this research as potential PCMs.

2.4 Performance Enhancement techniques in latent heat storage units

LHTES systems are desired as a result of their high energy storage capacity, charging and discharging of energy at a constant temperature, and small volume for a required storage capacity (Jegadheeswaran and Pohekar, 2009). Potential applications of LHTES systems include satellite/spacecraft thermal control, solar power, waste heat recovery, electronic system cooling, air conditioning systems, building applications, solar water heating and food industry (Zalba et al., 2003, Sharma and Sagara, 2005, Zhou et al., 2012, El Qarnia, 2009, Farid et al., 2004). However, the PCMs used as storage media in these systems are characterised as poor heat conductors. Hence, researchers have been investigating several methods to enhance the heat transfer rates and to maximise the stored/extracted energy in/from these systems. The following are some methods studied in the literature.

2.4.1 Using fins

The utilisation of extended surfaces or fins is the most studied technique for promoting heat transfer through PCMs. Stritih (2004) experimentally studied the melting and solidification of paraffin (RT 30) in a rectangular thermal storage system with and without fins (see Figure 8). The unit was charged and discharged using a heat exchanger attached to the unit wall. The results showed that adding fins reduced the solidification time by 40% with fin effectiveness of 3.06. However, the fins reduced the natural convection that dominates during melting, resulting in a fin effectiveness of less than one. It was concluded that during the charging
process fins lessen the heat transfer rate by partially inhibiting the natural convection currents. In contrast, the discharge process could be significantly enhanced by using fins. By analysing the measurements, two correlations for Nusselt number were presented associated with melting and solidification processes. The Nusselt number in the case of melting was found to be ten times that in the case of solidification because of natural convection effects.

Using a cylindrical coordinate system, Guo and Zhang (2008) conducted a numerical study of solidification in a shell and tube LHTES unit with PCM (KNO₃-NaNO₃) enclosed between radial Aluminium foils. The unit was proposed for direct steam generation in CSP plants in which water was used as HTF. Superheated steam is generated in the solar field which is then used to drive a turbine and to charge the storage unit. The stored heat is used to produce superheated steam during the night or cloudy periods. The HTF flow inside the storage unit is two-phase flow (steam/water). Therefore, the temperature of the HTF tube was assumed to be constant at the saturation temperature corresponding to the pressure of the HTF. It was found that addition of Aluminium foils enhances solidification not only in the radial direction but
also in the axial direction. Also it was noted that the time for complete solidification decreased linearly with the number of the Aluminium foils. This conclusion was also reached in earlier research conducted by Choi and Kim (1992a) where the effect of attaching radial fins to a cylindrical storage system was investigated experimentally (see Figure 9). The finned system had a larger temperature gradient at any radial location compared to the unfinned system. Moreover, the axial temperature distribution in the solid region is more uniform in the finned system as the fins enhance heat conduction in the axial direction. In contrast, the axial temperature difference in the liquid region is smaller for the finned system since the fins partially obstruct natural convection.

![Figure 9: Schematic diagram of the apparatus presented by Choi and Kim (1992a)](image)

Fins can be installed not just on the PCM side but also on the HTF side. Zhang and Faghri (1996) numerically investigated the enhancement of the melting process in an annular space filled with PCM associated with finning the inner cylinder in which the HTF flows (see Figure 10). The assumptions made included constant thermo-physical properties of the PCM.
and the HTF, isotropic PCM, axisymmetric melting around the tube with zero wall thickness, negligible natural convection and turbulent fully developed flow. A temperature transforming model was used to simulate PCM melting, whereas a finite difference method was used to simulate conductive heat transfer in the fins. It was concluded that increasing the height, thickness and number of the fins is an efficient way to enhance the melting process especially for HTFs with low thermal conductivities. Also, it was pointed out that the effect of adding internal fins is much more significant if the PCM is initially subcooled.

![Figure 10: Schematic of PCM energy storage system with internally finned tube (Zhang and Faghri, 1996).](image)

Axial fins were also used on the side of the PCM. For instance, A numerical and experimental investigation of solidification around a vertical axially finned isothermal tube immersed in a PCM has been presented by Ismail et al. (2001b). The tube was targeted to be used in thermal storage systems. Pure conduction was assumed to be the dominant heat transfer mechanism. The latent heat of solidification was included through the enthalpy formulation approach. The influences of the fin parameters, the aspect ratio of the annular spacing and the temperature difference between the freezing point and the wall temperature of the tube were investigated. Whereas the fin thickness slightly influenced the solidification time, the fin length, the number of the fins, the aspect ratio and the temperature difference had a strong effect on the time for complete solidification. Interested readers can find more details of a broad range of

2.4.2 Using multiple PCMs

Multiple PCMs LHTES (or cascaded LHTES) systems have been investigated as a performance enhancement technique. In this technique, multiple PCMs with different melting points are arranged in decreasing order, according to their melting temperature. As a result, nearly constant heat transfer rate between the PCM and the HTF is achieved along the length of the unit. The flow direction is from the highest PCM melting point to the lowest during the charging process and the reverse during the discharging process. On the other hand, in non-cascaded LHTES systems, the temperature difference between the PCM and the HTF decreases as the HTF flows down the unit which leads to a drop in the heat transfer rate and reduced thermal performance. Figure 11 shows the concept of multiple PCMs LHTES unit (Jegadheeswaran and Pohekar, 2009).

![Diagram of multiple PCMs LHTES unit](https://via.placeholder.com/150)

Figure 11: Multiple PCMs LHTES unit (Jegadheeswaran and Pohekar, 2009).

Farid and Kanzawa (1989) developed a numerical model to predict the thermal performance of a storage unit consisting of vertical cylinders arranged in the manner of in-line tube bundles
The cylinders were filled with three different PCMs in order to achieve a constant heat flux to/from the cylinders during the charging/discharging processes. Air was used as HTF. The model presented in that study takes into account the radial temperature variations within the solid and liquid PCM and the effect of natural convection within the liquid PCM. Because of utilising multiple PCMs, the phase change process begins almost at the same time in all the cylinders leading to an improvement of around 10% in the heat transfer rates during both charging and discharging processes.

Michels and Pitz-Paal (2007) experimentally and numerically investigated the benefit of using multiple PCMs in a vertical shell and tube heat exchanger (see Figure 13). In their study, they used realistic operational parameters with three real PCMs suitable for parabolic trough power plants. The shell side was filled with sodium nitrate ($\text{NaNO}_3$), an eutectic mixture of potassium nitrate and potassium chloride ($\text{KNO}_3/\text{KCl}$) and potassium nitrate ($\text{KNO}_3$), with melting temperatures of 306°C, 320°C and 335°C respectively. Synthetic oil was used as HTF, and it flowed in the inner tube. It was claimed that using real operational parameters along with real PCMs and HTF results in a better understanding of the behaviour of such systems under real conditions. It was shown that the cascaded unit performed better than a
single PCM unit as a larger percentage of PCM underwent phase change during the charging/discharging processes. Furthermore, NaNO₃ was found to be an appropriate PCM considering its heat of fusion and corrosiveness. A paramount conclusion Michels and Pitz-Paal (2007) have made is that if the thermal conductivity of the PCMs is increased to 2 W/m.K, a significant improvement in the performance would be obtained. They also highlighted the sensitivity of cascaded systems to the inlet conditions; that is, the performance of these systems can be worse compared to non-cascaded ones if the inlet temperature or the flow rate of the HTF is inappropriately adjusted. This was highlighted by several authors, for example (Aceves et al., 1998, Gong and Mujumdar, 1995). Recently, Shabgard et al. (2012) carried out exergy analysis of a cascaded LHTES system with three different PCMs for CSP applications. It was found that 10% extra exergy was recovered by the cascaded LHTES unit during a 24 h charging–discharging cycle.

Figure 13: Test facility used in (Michels and Pitz-Paal, 2007)
2.4.3 Thermal conductivity enhancement

The heat transfer in LHTES systems is strongly affected by the thermal conductivity of the PCMs used as storage media. PCMs, apart from metallic PCMs, have poor thermal conductivities (0.1 to 0.6 W/m.K). For a comprehensive list of thermal conductivities of organic, inorganic and eutectics PCMs see the review articles by Abhat (1983), Sharma and Sagara (2005) and Liu et al. (2012). Researchers around the world have been examining ways to enhance the thermal conductivity of PCMs. The following provides a summary of some proposed techniques.

2.4.3.1 Using porous material

Mesalhy et al. (2005) have developed a numerical model to simulate the melting process in a horizontal cylindrical annulus occupied by a metallic matrix saturated with PCM. The inner cylinder was maintained at a constant temperature greater than the melting point of the PCM, whereas the outer cylinder was highly insulated. They have considered the volume averaged versions of the mass, momentum and energy conservation equations. Effects of the natural convection and phase change processes within the porous matrix were included in the model. Due to the significant difference between the thermal properties of the PCM and the metal matrix, two energy equations were applied to obtain the temperature distribution in the metallic matrix and the PCM. Furthermore, they carried out a parametric study to investigate the effects of porosity and thermal conductivity of the matrix on the melting process. The results showed that increasing the thermal conductivity of the porous matrix accelerates the melting process, especially at the bottom of the annulus space. Also, it was stated that the melting rate was enhanced by decreasing the porosity of the metallic matrix. However, as the porosity further decreases, the matrix permeability decreases which leads to damping the motion of the liquid phase. An analytical expression for the effective thermal conductivity was obtained in the same work. Zhao and Wu (2011) and Wu and Zhao (2011) experimentally
studied the benefits of utilising metal foam and expanded graphite inserted in the NaNO₃ to increase the heat transfer rates. They found out that the heat transfer rate was increased during both charging and discharging processes because of using such porous materials. Even though the heat rate through the solid phase of NaNO₃ was improved by a factor of 2.5, it deteriorated in the liquid phase as the porous structures hamper the natural convection currents.

2.4.3.2 Mixing of high conductivity particles with the PCM

As early as 1977, Siegel (1977) investigated the benefits of dispersing high conductivity particles in the PCM to improve the thermal conductivity of the solidified PCM. He dispersed highly conductive fine particles (steel, iron, aluminium and copper particles) in molten salts. He overlooked the phenomenon of particles settling over long periods of time assuming uniform distribution of particles during melting and freezing cycles. The results showed that the energy extraction rates increased by approximately 50% for particle volume fractions of about 0.8. Mettawee and Assassa carried out an experimental study to examine the enhancement of the thermal conductivity of paraffin wax resulted from adding aluminium powder. A solar collector thermal storage unit with PCM-aluminium mixture was used in the experiments. Figure 14 shows the experimental apparatus. The unit was charged by solar radiation, whereas cold water was used to extract the energy during the discharging process. By adding aluminium powder to the wax, the charging time reduced by 60% compared with pure paraffin wax, also the useful heat gained was increased. Moreover, the mean daily efficiency of the unit, which assesses the overall benefit of adding aluminium particles to the wax, was improved by 94%. The mass fraction used in the experiments was 0.5 and any value above this was found not to yield to any significant improvement. Although particle addition has improved the performance, the authors have overlooked the effect of that on the storage capacity. This is an important omission as there must be an optimum mass fraction that maximises the efficiency and minimises the charging/discharging time.
2.4.3.3 Insertion of metal structures

Velraj et al. (1999) experimentally investigated the benefits of placing lessing rings of 1cm diameter in a vertical cylindrical storage tube containing paraffin wax. Figure 15 shows the apparatus used including the lessing rings. The effective thermal conductivity was enhanced ten times, and the time of complete solidification was reduced to one-ninth compared with the tube filled with pure wax. Nevertheless, this was on the expense of losing 20% of the storage capacity displaced by the lessing rings. Other than lesser rings, there are other types of metal structure that can be used for the same purpose. In a double pipe arrangement, Ettouney et al. (2004) incorporated stainless steel balls joined with stainless steel screens within the PCM. They revealed that increasing the diameter and number of balls leads to an enhancement factor of up to 3.
2.4.3.4 Using carbon fibers

Using metal structure/powder does enhance the thermal performance of the LHTES systems. However, metals have high densities which may result in settlement of these metals to the bottom of the container. Additionally, material compatibility is another important issue of using metals with PCMs. As a consequence, materials characterised by high thermal conductivity, low density and resistant to corrosion, are necessary. Carbon fibers meet all of these requirements as their thermal conductivity is almost equal to that of aluminium, and they possess low density and high corrosion resistance.

Fukai et al. (2000) presented two ways of enhancing the thermal conductivity of paraffin wax via using carbon fibers. The first technique is to randomly distribute the fibers in the paraffin, and the other is to use a fiber brush in which the fibers have the same direction as the heat flow. They experimentally recorded the transient temperature response and the effective thermal conductivity of the two types of fiber-paraffin composites which were packed in cylindrical steel capsules each of 50 mm diameter and 130 mm height. The capsules were provided with thermocouples for temperature measurements as shown in Figure 16. A temperature controlled bath was used to charge and discharge the capsules. The results revealed that the effective thermal conductivity was increased in both types of composite. However, the composite with fiber brush was found to possess effective thermal conductivity
three times higher than that of the composite with random fibers. In the melting stage, it was found that the randomly distributed fibers suppress natural convection currents which lead to longer melting time than that of pure paraffin. Nevertheless, this is valid only for low mass fractions of fibers. As the mass fraction of the fibers reaches two percent, the increase in the effective thermal conductivity compensates for the loss of convective heat transfer. This effect on the natural convection currents was found a negligibly small in the case of composites provided with fiber brush. It can be concluded from this study that the fibers should be oriented in the direction of heat flow.

Figure 16: Configurations of the carbon fibers within a cylindrical capsule. In Figure (b), the open circle indicates the thermocouples locations (Fukai et al., 2000)

Elgafy and Lafdi (2005) experimentally studied the thermal performance of nanocomposites made by mixing carbon nanofibers of 100 nm diameter and 20 μm length with paraffin wax. A shear mixing and melting method was used to distribute the fibers evenly into the wax. They used four composite samples with fiber mass fractions of 1%, 2%, 3% and 4% as well as a sample of pure wax for comparison purposes. The solidification process was enhanced
significantly as the fiber mass fraction increased. Specifically, the thermal conductivity of the composite increases almost linearly with the mass fraction of the fibers. Also in the same study, an analytical model was developed to estimate the effective thermal conductivity of the specified nanocomposite. The model was validated against experimental finding and against published studies, and showed good agreement with them. Finally, they compared the thermal performance of two composites with the same fiber mass fraction but with different surface characteristics. The study showed that the heat transfer phenomenon at nanoscale is surface dependent.

2.4.4 Microencapsulation of PCM

Microencapsulation of PCM is the process of encapsulate the PCM in a solid shell. During the melting/solidification process, the PCM undergoes phase change within the solid shell. As a result, the volume change can be neglected which is an attractive advantage of this method. A variety of shell materials, as well as methods of production, can be found in (Hawlader et al., 2003, Chen et al., 2008, Alkan et al., 2009). Microencapsulated PCMs are commonly used to improve the thermal conductivity of some building materials (Schossig et al., 2005, Cabeza et al., 2007) and thermal fluids (Chen et al., 2008, Rao et al., 2007, Zhang and Faghri, 1995). Microencapsulated PCMs have large heat transfer area per unit volume of the solid shell; thus, they present higher transfer rate than pure PCMs. Hawlader et al. (2003) and Hawlader et al. (2000) reported that having higher transfer rate leads to larger energy storage density and therefore they recommended microencapsulated PCMs for solar energy storage applications. Nevertheless, Ozonur et al. (2006) revealed that microencapsulated fatty acid showed a higher heat transfer rate during charging. In contrast, in the discharging process, pure PCM showed a higher heat transfer rate.
2.5 High-temperature latent heat storage systems with incorporated heat pipes

Utilising heat pipes (HPs) in LHTES systems for high-temperature applications is the state-of-the-art technique. HPs are passive heat transfer devices which can transfer large amounts of heat rapidly from one point to another using a small temperature drop. They are often considered as "superconductors" of heat as they possess effective thermal conductivity several hundred times than that of copper (Faghri, 1995). This section firstly presents a brief introduction to the heat pipe and then reviews the state-of-the-art of LHTES designs which utilize heat pipes to enhance the thermal performance.

2.5.1 Brief introduction to heat pipes

2.5.1.1 Heat pipe historical development

The first device which was closest to the present HP was patented by Jacob Perkins (1836). It was a sealed cylinder containing a small amount of water functioning as a two-phase thermosyphon. Later in 1944, Gaugler (1944) patented a device to transfer heat from the interior of a refrigerator compartment to lower pot filled with crushed ice. This device was very similar to the modern HP; that is, the fluid inside the tube absorbs heat at one location and transforms it to a vapour. The vapour flows downwards where it recondenses and releases its latent heat to the crushed ice pot. The liquid then travels back to the top of the tube through a capillary structure (sintered iron wick) to restart the process. The actual development of HPs took place in 1964 when Grover et al. (1964) and later Grover (1966) fabricated and experimentally tested several HP prototypes. They first used water as working fluid, and later used high-temperature HPs (1370°C) with sodium as working fluid. Since then, several new HP designs and applications have emerged. Applications such as electronic cooling and energy systems have brought about a need for mass production of HPs. During the last
decade, several million HPs have been manufactured for cooling modern laptop computers. Moreover, other types of HPs such as loop heat pipes, micro and miniature heat pipes, and pulsating heat pipes, have been presented for use in different applications (Faghri, 2014).

2.5.1.2 Working principle of the HP

A heat pipe can be defined as an evacuated sealed cavity partially filled with a working fluid. HPs can have any size or shape, but cylindrical HPs are the most common ones. Figure 17 shows a conventional heat pipe (Faghri, 2014). It consists of a sealed cavity (pipe wall and end caps), a wick structure, and a working fluid (10-25% of the internal volume). The desired operating temperature range identifies the appropriate working fluid. For example, water is used for a temperature range of 30-280°C, acetone for a range of 0-120°C, methanol for a range of 10-130°C, and sodium for a range of 600-1200°C (Faghri, 2014). Usually, an HP can be divided into three sections: the evaporator section where the heat is absorbed, the adiabatic section and the condenser section where the heat is released (see Figure 17). The heat is applied to the external surface of the evaporator wall which is then conducted through the HP wall and the wick structure in order to vaporize the working fluid. The vapour pressure increases due to the conversion of the working fluid from a liquid to a vapour. As a result, the vapour travels to the condenser section via the core of the adiabatic section. When the vapour reaches the condenser section, it releases its latent heat of vaporization to a heat sink, by conducting the heat through the wick and the HP wall. At this point, the liquid needs to be returned to the evaporator section so that the whole process can start again. This is accomplished by utilising capillary pressure created by the menisci within the wick. The process of transporting the latent heat of vaporization between the evaporator and the condenser sections will continue as long as there is an enough capillary force to pump the condensate back to the evaporator. A consequence of transforming the latent heat of vaporization is that HPs have effective thermal conductivity significantly beyond that of any
known metal. It could be as much as several hundred times the thermal conductivity of copper (Faghri, 1995).

![Diagram of a cylindrical heat pipe showing the vapour and the liquid flows (Faghri, 2014).](image)

Figure 17: Schematic of a cylindrical heat pipe showing the vapour and the liquid flows (Faghri, 2014).

Both hydrodynamic and heat transfer processes are used in the heat pipe theory. The fluid mechanics theory describes the axial liquid pressure drop within the wick, the maximum capillary pumping head and the vapour flow in the HP core. In contrast, the heat transfer theory describes the heat transferred into and from the heat pipe. The HP theory involves several phenomena; for instance, conjugate conduction heat transfer in the wall and wick, evaporation and condensation at the liquid-vapour interface, and forced convection heat transfer in the vapour channel and wick. Figure 18 shows the thermal resistances network in a typical heat pipe (Faghri, 2014). A simplified version of the thermal resistances network illustrated in figure 18 is adopted in this thesis. Comprehensive details about heat and mass transfer in heat pipes including analytical and numerical modelling of a variety of heat pipe types are presented in (Faghri, 1995).
2.5.2 State-of-the-art designs of latent heat storage system with heat pipes for concentrating solar power applications

The heat transfer improvement approach of integrating HPs into PCMs has been patented by Faghri (1990). The aim was to reduce thermal resistances between the HTF and the PCMs. Later in 1999, Horbaniuc et al. (1999) developed a mathematical model to investigate the two-dimensional solidification of a PCM around an axially finned heat pipe. The effect of fin number on the duration of complete solidification was reported. Liu et al. (2006) carried on the work of Horbaniuc et al. (1999) investigating the effect of both HTF inlet temperature and mass flow on the solidification rate of paraffin wax around circumferentially finned thermosyphon. Tardy and Sami (2009) studied numerically and experimentally the enhancements of the melting process of low-temperature PCM resulted from using heat pipes.

All of the aforementioned studies focused only on low-temperature PCM applications that are less than 100°C. The first design of a high-temperature (> 300°C) LHTES unit with integrated HPs was presented by Shabgard et al. (2010). They developed a thermal network model which was used to predict the transient response of a high-temperature LHTES system for solar thermal electricity generation. As can be seen from Figure 19, they considered two
storage configurations; one with PCM around a tube that conveys the heat transfer fluid, and the second with the PCM enclosed within a tube over which the heat transfer fluid flows. The results demonstrated that adding HPs enhances thermal performance of the LHTES system. A year later, Nithyanandam and Pitchumani (2011) used a similar thermal network model which was presented by Shabgard et al. (2010) combined with a numerical optimization scheme in an attempt to maximize the transferred energy, the effectiveness, and the energy transfer rate. They concluded that the effectiveness of the HPs decreases if the HTF mass flow rate, module length, and tube radius are increased; in contrast, it increased if the length of the condenser section, the length of the evaporator section, and the vapour core radius are increased.

Figure 19: Two storage configurations: (a) the PCM is around the HTF tubes, (b) the PCM is inserted inside the HTF tubes and the HTF passes over them, (c) Module 1 and (d) Module 2. (Shabgard et al., 2010).

Robak et al. (2011a) introduced a new design for a commercial-scale high-temperature LHTES system with embedded gravity-assisted wickless HPs (thermosyphons) to reduce the
size and amount of materials needed for TES in CSP (see Figure 20). A reduction of 15% in the capital costs was predicted comparing with a design using current SHTES technology. In related work, Shabgard et al. (2012) conducted an exergy analysis of a cascaded LHTES unit with gravity-assisted HPs for CSP applications. It was found that 10% extra exergy was recovered by the cascaded unit during a 24 h charging–discharging cycle.

Recently, the benefits of using a combination of high porosity metal foams and heat pipes were presented by Allen et al. (2014). In their configuration as shown in Figure 21, the HP transfers heat deeper into the PCM, whereas the metal foam increases the thermal conductance of the PCM. As a result, the melting and solidification rates have increased approximately ten times compared to the configuration of HP without metal foam.

![Cross-sectional views of the storage unit with inserted thermosyphons during discharging: (a) lengthwise view and (b) widthwise view (Robak et al., 2011a).](image)

Figure 20: Cross-sectional views of the storage unit with inserted thermosyphons during discharging: (a) lengthwise view and (b) widthwise view (Robak et al., 2011a).
The use of heat pipes for CSP applications is not restricted to the energy storage units but also can be used for the thermal control of the concentrated thermoelectric power generator. Tan et al. (2011) introduced a passive cooling system that uses a latent heat thermal energy storage unit equipped with a two-phase closed thermosyphon (wickless heat pipe) for operating a concentrated thermoelectric generator (CTEG). As shown in Figure 22, The primary thermosyphon was used as an efficient heat bridge for transferring excess heat from the cold side of the thermoelectric module to the PCM storage tank, whereas the secondary thermosyphon dissipated heat from the melted PCM to the night cold air. The thermal performance and electrical power output of the system have been evaluated. They concluded that the system could maintain a temperature difference of 152 °C and produce 9.5 W of thermoelectric power.
Table 6 contains a summary of key works that involve phase change modelling and measuring. The numerical and experimental work which will be presented in the next three chapters is based on some of these researches.

**Table 6: Summary of key researches that involve phase change modelling and measuring.**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Nature of work EXP/NUM</th>
<th>Geometry</th>
<th>Mathematical scheme</th>
<th>Mesh type</th>
<th>Process M/S</th>
<th>Enhancement technique</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>EXP</td>
<td>Rectangular</td>
<td>-</td>
<td>-</td>
<td>M, S</td>
<td>longitudinal fin</td>
<td>Thermal storage</td>
</tr>
<tr>
<td>b</td>
<td>NUM (r, φ)</td>
<td>Shell and tube</td>
<td>Enthalpy formulation</td>
<td>Fixed non-uniform grid</td>
<td>S</td>
<td>Aluminium foils</td>
<td>CSP plants</td>
</tr>
<tr>
<td>c</td>
<td>EXP</td>
<td>Cylindrical</td>
<td>-</td>
<td>-</td>
<td>S</td>
<td>Radial fins</td>
<td>Thermal storage</td>
</tr>
<tr>
<td>d</td>
<td>NUM (r, φ)</td>
<td>Cylindrical</td>
<td>Temperature transforming method &amp; Finite difference scheme</td>
<td>Fixed grid</td>
<td>M</td>
<td>Internal fins (fluid side)</td>
<td>Heat exchanger</td>
</tr>
<tr>
<td>e</td>
<td>EXP, NUM (r, φ)</td>
<td>Cylindrical</td>
<td>Effective heat capacity formulation</td>
<td>Fixed grid</td>
<td>S</td>
<td>Axial fins</td>
<td>Thermal storage</td>
</tr>
<tr>
<td>f</td>
<td>NUM (r)</td>
<td>Tube bundles</td>
<td>Heat conduction in radial coordinate based on finite difference</td>
<td>Fixed grid</td>
<td>M, S</td>
<td>Cascaded (3 PCMs)</td>
<td>Thermal storage</td>
</tr>
</tbody>
</table>
Table 6 (continued)

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Nature of work</th>
<th>Geometry</th>
<th>Mathematical scheme</th>
<th>Mesh type</th>
<th>Process M/S</th>
<th>Enhancement technique</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>EXP, NUM (x)</td>
<td>Vertical shell and tube</td>
<td>Package (Dymola–Modelica)</td>
<td>Fixed grid</td>
<td>M, S</td>
<td>Cascaded (3 PCMs)</td>
<td>CSP plants</td>
</tr>
<tr>
<td>h</td>
<td>NUM (r, φ)</td>
<td>Horizontal cylindrical annulus</td>
<td>Volume averaged method, (melting) in the porous medium</td>
<td>Transformed domain (ξ, η)</td>
<td>M, S</td>
<td>High porous metallic matrix</td>
<td>Thermal storage</td>
</tr>
<tr>
<td>i</td>
<td>EXP</td>
<td>Rectangular solar collector</td>
<td>-</td>
<td>-</td>
<td>M, S</td>
<td>Mixing of Aluminium particles</td>
<td>Thermal storage (low temperature)</td>
</tr>
<tr>
<td>j</td>
<td>EXP</td>
<td>Vertical cylinder</td>
<td>-</td>
<td>-</td>
<td>S</td>
<td>Lessing rings</td>
<td>Thermal storage</td>
</tr>
<tr>
<td>k</td>
<td>EXP</td>
<td>Cylindrical</td>
<td>-</td>
<td>-</td>
<td>M, S</td>
<td>Paraffin-carbon fibers composite</td>
<td>Enhancing thermal conductivity of the PCMs</td>
</tr>
<tr>
<td>l</td>
<td>EXP</td>
<td>Cylindrical</td>
<td>-</td>
<td>-</td>
<td>S</td>
<td>Paraffin-carbon nanofibers composite</td>
<td>Enhancing thermal conductivity of PCMs</td>
</tr>
<tr>
<td>m</td>
<td>NUM (r)</td>
<td>Shell and tube</td>
<td>Thermal resistance network</td>
<td>Uniform moving grid</td>
<td>M, S</td>
<td>Embedded Heat pipes</td>
<td>CSP plants</td>
</tr>
</tbody>
</table>


### 2.6 Summary of the chapter

This chapter has introduced and reviewed the current technologies for heat transfer enhancement in LHTES systems. LHTES systems have a major problem of low thermal conductivity of the PCMs used as a storage medium. Thus, LHTES systems have not yet been implemented in high solar concentrating applications. Much attention has been devoted to incorporating heat pipes for their superior thermal conductivity in order to resolve this issue.

The present study focused on increasing the overall thermal conductance of PCMs used in LHTES units through using HPs. Specifically, the benefits of using axially finned HPs, suspended HPs and micro HPs have been numerically and experimentally investigated. In addition, a preliminary system sizing and economic assessment has been conducted.
The next three chapters present detailed descriptions including numerical modelling and experimental validation of three new approaches to improving the thermal performance of LHTES systems by using embedded heat pipes.
CHAPTER 3


3.1 Introduction

In CSP plants that equipped with LHTES systems, the HTF is pumped to charge and discharge the thermal energy to and from the PCM respectively. However, the heat transfer rates between the HTF and the PCM are limited by the low thermal conductivity of the PCM. In order to reduce the thermal resistance of the PCMs, Shabgard et al. (2010) suggested to stud the HTF tubes with bare HPs. The results demonstrated that adding bare HPs enhances thermal performance of the LHTES system. To the author’s knowledge, the benefits that could be gained from using HPs along with fins (finned HPs) in high-temperature LHTES systems have not been studied. The combination of HPs and fins has the potential to improve the performance and reduce the capital cost, this is said based on the fact that the HPs outperform fins in terms of heat transfer (Robak et al., 2011b), but fins are expected to be cost-competitive especially comparing with high-temperature HPs. The objective of this chapter was to develop a numerical model to investigate the effect of adding finned HPs to a high-temperature LHTES unit. The model was developed using a 2-D control volume method where the thermal resistances of the HPs are incorporated into the mathematical model which representing the physical problem. In addition, an experimental investigation was conducted to ensure the reliability of the simulation.
3.2 Description of the problem

The shell and tube configuration is very common in heat storage systems. It is the most intensely investigated configuration in LHTES systems (Agyenim et al., 2010). This can be attributed to two factors: (i) the vast majority of engineering systems employ cylindrical pipes and (ii) shell and tube systems have minimal heat losses. Consequently, the shell and tube configuration was adopted by all of the previously mentioned studies of high-temperature LHTES systems. In this chapter, the design considered is the one introduced by Shabgard et al. (2010). The unit consists of a number of tubes, which carry the HTF, inserted in an insulated shell filled with PCM (Figure 23a). In the top of the shell, an air gap is left to account for the expansion of the PCM during melting. Each tube is penetrated by four HPs repeatedly every a fixed length of $L_m$ which is the model unit cell length considered in this chapter (Figure 23b). It has been demonstrated numerically by Shabgard et al. (2010) and experimentally by Robak et al. (2011b) that the solidified PCM on the cool surfaces acts as thermal insulation making the discharging stage much slower than the charging stage. Consequently, it is expected that any design that works well during the discharging process will work likewise during the charging process. Therefore, only the solidification process is considered in this chapter. For a single HTF tube, both latent and sensible energies may be extracted from two regions of solidification: one adjacent to the HTF tube and the other around the HPs. These two regions are shown in Figure 23b.

The numerical analysis involves the following heat transfer processes: (i) forced convection from the HTF into the condenser section of the HPs as well as into the HTF tube wall and (ii) pure conduction within the PCM and in the material used to construct the HTF tube, the fins, and the HPs. It is noted that the solidification process can be modelled satisfactorily as pure heat conduction especially if it is initiated at the freezing temperature of the PCM (Sparrow et al., 1981b, Stritih, 2004).
In the adopted design, thermal energy is exchanged between the HTF and the PCM via both the HPs and the HTF tube wall. Consequently, there will be a region of solidification associated with each path. A numerical model has been developed to simulate the pure conduction heat transfer processes related to these two regions of solidification where each region is considered independently of the other. The calculation is terminated when these two

3.1 Mathematical model

In the adopted design, thermal energy is exchanged between the HTF and the PCM via both the HPs and the HTF tube wall. Consequently, there will be a region of solidification associated with each path. A numerical model has been developed to simulate the pure conduction heat transfer processes related to these two regions of solidification where each region is considered independently of the other. The calculation is terminated when these two
regions merge. The control volume approach based upon the effective heat capacity method was used to solve the energy equation for the solid PCM, the liquid PCM, the mushy zone, and the fin regions as one domain. The thermal resistances related to the HTF tube and the HPs were incorporated into the boundary conditions of the PCM domain. This is discussed in Section 3.3.3.

To simplify the modelling, the following assumptions are made.

a. The thermal resistances of the HP associated with the vaporisation and condensation processes are ignored. These include the evaporator liquid-vapour interface, the vapour channel, and the condenser vapour-liquid interface resistances (Faghri, 1995).

b. Both the liquid and solid phases of the PCM are assumed isotropic and homogeneous.

3.1.1 The effective heat capacity method

The effective heat capacity method treats the temperature as the dependent variable and was presented by C. Bonacina et al. (1973) and used by several authors (Hsiao and Chung, 1986, Ismail et al., 2001b, Ismail et al., 2000) to simulate conduction heat transfer problems involving melting and solidification. Solidification around the HTF tube as well as adjacent to the bare HPs is treated as a one-dimensional problem in the radial direction, whereas it is a two-dimensional problem around the finned HPs in polar coordinates. Figure 24 shows the geometrical details of the two regions of solidification including the symmetry surfaces. Hence, the governing equations based on the effective capacity model can be written as (Ismail et al., 2001b):

\[
\dot{C} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \bar{k} \frac{\partial T}{\partial r} \right) + \frac{1}{r \sin \phi} \frac{\partial}{\partial \phi} \left( \frac{1}{r} \bar{k} \frac{\partial T}{\partial \phi} \right)
\]  

(3-1)

Where the thermal conductivity \( \bar{k} \) and the specific heat per unit volume \( \dot{C} \) for the PCM are:
\[
\bar{k} = \begin{cases} 
  k_s, & T \leq T_m - \Delta T \\
  k_l, & T \geq T_m + \Delta T \\
  k_s + \frac{k_l - k_s}{2\Delta T}[T - (T_m - \Delta T)], & T_m - \Delta T < T < T_m + \Delta T 
\end{cases}
\]

(3-2)

\[
\bar{c} = \begin{cases} 
  c_s, & T \leq T_m - \Delta T \\
  c_l, & T \geq T_m + \Delta T \\
  \rho_s h_{sl} + \frac{c_l + c_s}{2}, & T_m - \Delta T < T < T_m + \Delta T 
\end{cases}
\]

(3-3)

In the fins region, these quantities \( \bar{k} \) and \( \bar{c} \) are related to the fins material and should be considered in Eq. (3-1) as:

\[
\bar{c} = c_{fin} = \rho_{fin} c_{fin} 
\]

(3-4)

\[
\bar{k} = k_{fin} 
\]

(3-5)

The second term on the right-hand side of Eq. (3-1) should be ignored if the solidification of PCM is being considered in the regions adjacent to the HTF tube or adjacent to the bare HPs.

The corresponding boundary and initial conditions are:

\[
-\bar{k} \frac{\partial T}{\partial r} \bigg|_{r_{tube}} = q_r 
\]

(3-6a)

\[
\frac{\partial T(R, \phi, t)}{\partial r} = 0 
\]

(3-6b)

\[
\frac{\partial T(r, 0, t)}{\partial \phi} = 0 
\]

(3-6c)

\[
\frac{\partial T(r, \phi, t)}{\partial \phi} = 0 
\]

(3-6d)

\[
T(r, \phi, 0) = T_m 
\]

(3-6e)
3.1.2 Thermal resistances

As mentioned previously, the thermal resistances associated with the HTF tube and the HPs were incorporated into the boundary conditions of the PCM region. Specifically, they appear implicitly on the right-hand side of Eq. (3-6a) as a heat flux at the PCM-HPs (HTF tube) adjoining boundary. The heat flux is given by (Incropera et al., 2011b):

\[
q_r^* = \frac{T_{HTF} - T\left[r_{hp}(r_{tube}), \phi, \xi\right]}{R_t A}
\] (3-7)

where \( A \) is the outer surface area of the HTF tube or of the evaporator section of the HP, and \( T_{HTF} \) is the temperature of the HTF at the inlet of the model unit cell considered. The thermal resistance \( R_t \) could be related either to the HPs or to the HTF tube. By using heat transfer principles and referring to Figure 25, it can be concluded that for the HPs \( R_t \) is:
\[ R_t = \frac{1}{\left( \frac{1}{R_8} + \frac{1}{R_9} + \frac{1}{(R_1 + R_2 + R_3 + R_4)} \right)} + R_7 \]  

(3-8)

Expressions regarding \( R_{1-7} \) may be found in (Faghri, 1995, Shabgard et al., 2010, Zuo and Faghri, 1998). In case of the HTF tube, \( R_t \) is given by:

\[ R_t = R_8 + R_9 \]  

(3-9)

Figure 25: Thermal resistance network system at the HP (HTF tube)-PCM interface.

Attentions are drawn to the resistances \( R_8 \) and \( R_9 \). There are two possible cases: one when the presence of the HPs is accounted for and the other when the presence of the HPs is ignored in
order to calculate the heat extracted through the HTF tube without HPs. Thus, Eq. (3-9) can be written as:

\[
R_t = R_8 + R_9 = \frac{\ln \left( \frac{2r_{tube}L_m - Nr_{hp}^2}{2r_{in}L_m - Nr_{hp}^2} \right)}{2\pi L_m k_{tube}} + \frac{1}{h_{tube} \pi \left( 2r_{in}L_m - Nr_{hp}^2 \right)}
\] (3-10)

where \( N \) is the number of HPs used in the model unit cell (\( N = 0 \) in the case of no HPs), \( r_{tube} \) and \( r_{in} \) are the outer and inner radii of the HTF tube respectively, \( h_{tube} \) is the convection coefficient at the internal surface of the HTF tube, and \( r_{hp} \) is the outer radius of the HP.

3.1.3 The discretisation equations

As described by Patankar (1980b), the calculation domain is divided into a number of control volumes during the mesh generation process. A grid point is placed in the centre of each control volume as well as on the face of each “near-boundary” control volume. As a result of this sequence of mesh generation, each internal “grid-point” can communicate with four neighbouring grid points, whereas each boundary “grid-point” can communicate with just one internal neighbouring grid point. In the present chapter, the calculation domain was divided into 100×100 control volumes, and it was found that any further discretisation yields to no significant change. Further, the first and the second 10 control volumes near the fins’ surfaces, in the case of finned HP, were subdivided into 8 and 4 smaller control volumes respectively. This was made to capture precisely the temperature history around the fins. Eq. (3-1) is integrated over each control volume, over \( r, \phi \) and \( t \), to obtain the required discretisation equation which can be expressed as:

\[
a_pT_p = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b
\] (3-11)

The expressions for \( a_p, a_E, a_W, a_N, a_S \) and \( b \) are given in (Patankar, 1980b). Here the subscripts \( E, W, N \) and \( S \) represent the neighbouring grid points around the main grid point \( p \).
The thermal resistances, $R_t$, represented by Eqs. (3-8) and (3-10) can be incorporated into Eq. (3-11) as follows:

When Eq. (3-11) is used to represent the boundary points, it can be rewritten as:

$$a_B T_B = a_I T_I + b$$  \hspace{1cm} (3-12)

where the subscript $B$ and $I$ represent the boundary point and its closest neighbouring internal point respectively. The coefficients in Eq. (3-12) are given by Patankar (1980b):

$$a_I = \frac{\bar{k}}{(\delta r)_i}$$  \hspace{1cm} (3-13a)

$$a_B = a_I - S_p \Delta r$$  \hspace{1cm} (3-13b)

$$b = S_c \Delta r + q_r$$  \hspace{1cm} (3-13c)

where $(\delta r)_i$ is the distance between any boundary grid-point and the nearest internal point. The quantities $S_p$ and $S_c$ stand for the variable and constant source terms respectively. They are equal to zero for the case of effective heat capacity method used in this study. $\Delta r$ is the control volume thickness and $\bar{k}$ is the interfacial thermal conductivity. By substituting Eq. (3-7) into Eq. (3-13c), then substituting Eq. (3-13) into Eq. (3-12) then Eq. (3-13) can be simplified and rearranged as:

$$a_I = \frac{\bar{k}}{(\delta r)_i}$$  \hspace{1cm} (3-14a)

$$a_I^{'b} = a_I + \frac{1}{R_t A} - S_p \Delta r$$  \hspace{1cm} (3-14b)

$$b^{'c} = S_c \Delta r + \frac{T_{HTF}}{R_t A}$$  \hspace{1cm} (3-14c)

As can be seen from Eq. (3-14), the thermal resistance, $R_t$, is absorbed into the new definition of the coefficients $a_I^{'b}$ and $b^{'c}$ for the boundary grid points. Eq. (3-11) and Eq. (3-12) along with Eq. (3-14) can now be applied for the entire calculation domain. The resulting set of equations can be solved by any convenient method for solving a system of algebraic equations such as a
Line-by-line technique. A MATLAB code was developed to simulate the solidification process described by Eq. (3-1) through Eq. (3-14). The iterative solution converges when the changes in the temperature at each grid point falls below a convergence criterion of $10^{-6}$. It is noted that using the thermal resistance approach to model heat transfer in the HPs undoubtedly has a shortcoming; which is that the transient operation of the heat pipe cannot be simulated. However, this will not cause large error as the thermal conductivity of the PCM is significantly smaller than that of the HP.

### 3.2 Experimental design and procedure

#### 3.2.1 Phase change material used in the experiment

As implies in the title of this paper, a high temperature PCM such as potassium nitrate (KNO$_3$) with melting point of 335°C could have been chosen as a PCM for the experimental tests. However, for safety reasons, commercial paraffin RT82 with solidifying point of 83°C is used in this study for verification purposes. As stated in the product specification provided by the manufacturer (RUBITHERM$^R$), RT82 has a thermal conductivity of 0.2W.m$^{-1}$.K$^{-1}$ for both liquid and solid phases. The thermo-physical properties of RT82 are listed in Table 7 (rubitherm). After verifying the model, KNO$_3$ will then be considered as the storage material.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density solid/liquid 15/90 °C (kg.m$^{-3}$)</td>
<td>880/770</td>
</tr>
<tr>
<td>Thermal conductivity solid/liquid (W.m$^{-1}$.K$^{-1}$)</td>
<td>0.2/0.2</td>
</tr>
<tr>
<td>Specific heat solid/liquid (J. kg$^{-1}$.K$^{-1}$)</td>
<td>2000/2000</td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>82</td>
</tr>
<tr>
<td>Solidifying point (°C)</td>
<td>83</td>
</tr>
<tr>
<td>Volume expansion at $\Delta T = 8$ °C (%)</td>
<td>14</td>
</tr>
<tr>
<td>Latent heat of fusion (J. kg$^{-1}$)</td>
<td>176,000</td>
</tr>
<tr>
<td>Phase change temperature range, $\Delta T$, (K)</td>
<td>3</td>
</tr>
</tbody>
</table>
3.2.2 Experimental setup and procedure

An experimental facility was assembled to validate the numerical predictions of the solidification process around the HPs (the solidification process around the HTF tube was not involved in the test). As shown in Figure 26, the test facility consisted of a cylindrical stainless-steel vessel in which the paraffin was contained, cool and hot water tanks, and a heat pipe convectively cooled using a water jacket. The vessel was 20 cm in diameter and 20 cm high. The heat pipes used were copper-water-charged HPs, while the fins were made from aluminium (A1100). Both the finned and the bare HPs were provided by Fujikura Ltd. The fins were attached to the HPs using thermal epoxy to minimise interface thermal resistance.

![Figure 26: The experimental rig.](image)

The dimensions of the heat pipe and the fins are listed in Table 8. A schematic diagram of the rig is shown in Figure 27. The experimental procedure was as follow: Firstly, the vessel was
filled with solid paraffin and placed in a constant temperature hot water tank maintained at 95 °C until the paraffin was fully melted. The water temperature in the hot tank was then set to 83 ± 0.2 °C (the solidifying point of the paraffin). The heat pipe was then placed inside the container in such a way that the evaporator section of the heat pipe was fully submerged in the liquid paraffin. The whole system was allowed to reach a steady state. The vessel and the heat pipe assembly was removed from the hot water tank and quickly insulated. Finally, cool water commenced circulating in the water jacket, and the measurements were taken. The test was conducted using a bare heat pipe as well as using an otherwise identical finned heat pipe equipped with four axial fins (see Figure 28). The fin thickness was equal to 2 mm, with a radial length of twice the HP diameter. These dimensions were recommended by K. A. R. Ismail et al. (Ismail et al., 2001b) as a compromise between enhancement of thermal conductance and reduction of storage capacity. In addition, wicked HPs were used in the experiment to achieve uniform circumferential distribution and also to create necessary capillary force needed to return the condensate.

Table 8: Dimensions of the heat pipe and the fins used in the experiments.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat pipe</td>
</tr>
<tr>
<td>Outer diameter (m)</td>
<td>0.008</td>
</tr>
<tr>
<td>Condenser section length (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Evaporator section length (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Adiabatic section length (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>0.0004</td>
</tr>
<tr>
<td>Porosity (−)</td>
<td>-</td>
</tr>
<tr>
<td>Thermal conductivity (W. m⁻¹. K⁻¹)</td>
<td>391</td>
</tr>
<tr>
<td>Heat capacity (J. m⁻³. K⁻¹)</td>
<td>3,441,900</td>
</tr>
<tr>
<td>Radial length (mm)</td>
<td>-</td>
</tr>
<tr>
<td>Axial length (mm)</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 27: Schematic diagram of the rig.

Figure 28: Bare and finned heat pipes used in the experiment.
The test facility was equipped with T-type twisted pair thermocouples of approximately 1.0 mm overall diameter which were arranged in a single horizontal plane as shown in Figure 29. The accuracy of these thermocouples was ±0.1 °C. The test was repeated three times and the maximum experimental uncertainty was 6.5% (Coleman and Steele, 2009). The experimental error bar is shown in Figure 30a. The thermocouple wires were guided into the paraffin through plastic tubes of 3 mm inner diameter and 4.5 cm length which were considered to have negligible influence on the paraffin phase change behaviour. The temperature field and the cooling water inlet and exit temperatures were measured at 1 minute intervals.

Figure 29: vessel-heat pipe cross section and thermocouple locations for bare and finned heat pipe.
3.1 Results and discussion

Before embarking on discussion of the thermal enhancement of LHTES systems for solar thermal power generation utilising potassium nitrate (KNO$_3$) as PCM, the model was validated against the experimental measurements obtained in this chapter. The model is used to solve the two-dimensional solidification problem where a vertical heat pipe is immersed in a supposedly infinite liquid PCM. The PCM is initially at its freezing temperature. The solidification process starts when the condenser section of the heat pipe is cooled using a water jacket through which water flows steadily at a rate of $0.29 \, L.s^{-1}$ and at a temperature of $0 \pm 0.1 \, ^\circ C$. The temperature of the condenser section of the heat pipe is maintained near $0 \, ^\circ C$ under these conditions.

Figure 30 shows a comparison between the numerically predicted and experimental temperature variations within the PCM for bare and finned heat pipes at the locations of the thermocouples indicated in Figure 29. Overall, the agreement between the numerical solutions and the experimental predictions is good. Nevertheless, in the numerical results, the rate of temperature drop is faster near the HP but the experimental and numerical curves become closer further from the HP. This could be attributed to the possibility that phase change might occur outside the specified phase change temperature range. Consequently, latent heat transfer will continue even after reaching the lower limit of the phase change temperature range, causing the model predictions and the experimental data to deviate quite significantly from each other especially near the HP where the temperature dropped rapidly allowing no time to transfer this excess heat further from the HP. This behaviour was also noticed by Abhat (1983) and Velraj et al. (1999).
Figure 30: Comparison of numerically and experimentally obtained temperature variation for (a) bare HP and (b) finned HP.
The numerical model was also compared against the results presented by Shabgard et al. (2010). The code input parameters were set to be equal to that used by Shabgard et al. (2010), and the trend was found very similar as can be seen from Figure 33. However the present model seems to slightly under-predict the HP effectiveness.

After model validation, numerical calculations for the LHTES system for solar thermal power generation were conducted. Stainless steel-mercury-charged HPs and stainless steel HTF tubes are considered for the LHTES unit and the PCM and HTF are potassium nitrate (KNO$_3$) and Therminol, respectively (Shabgard et al., 2010). The dimensions of the LHTES components and the properties of the PCM and the HTF are listed in Table 9 and Table 10 respectively (Shabgard et al., 2010). The design configuration of the HTF tube is appropriate to the dimensions of large-scale solar thermal storage. In addition, Nithyanandam and Pitchumani (2011) found these dimensions to yield the maximum energy discharged with the exception of the length of the model unit cell, $L_m$, being set here to $0.12m$ instead of $0.2m$.

Table 9: Dimensions of the LHTES components extracted from (Shabgard et al., 2010).

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat pipe</td>
</tr>
<tr>
<td>Outer diameter ($m$)</td>
<td>0.018</td>
</tr>
<tr>
<td>Evaporator section length ($m$)</td>
<td>0.14</td>
</tr>
<tr>
<td>Condenser section length ($m$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Adiabatic section length ($m$)</td>
<td>0.06</td>
</tr>
<tr>
<td>Thickness ($m$)</td>
<td>0.001</td>
</tr>
<tr>
<td>Wick porosity</td>
<td>0.9</td>
</tr>
<tr>
<td>Thermal conductivity ($W.m^{-1}.K^{-1}$)</td>
<td>20.1</td>
</tr>
<tr>
<td>Heat capacity ($J.m^{-3}.K^{-1}$)</td>
<td>4,423,210</td>
</tr>
<tr>
<td>Radial length (mm)</td>
<td>-</td>
</tr>
<tr>
<td>Axial length (mm)</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 10: Thermo-physical properties of the PCM (KNO₃) and HTF (Therminol® VP-1) used in the numerical simulation extracted from (Shabgard et al., 2010).

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>HTF (Therminol® VP-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg m⁻³)</td>
<td>2109</td>
<td>709</td>
</tr>
<tr>
<td>Thermal conductivity (W m⁻¹ K⁻¹)</td>
<td>0.425 (liquid)</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>0.5 (solid)</td>
<td></td>
</tr>
<tr>
<td>Specific heat (J kg⁻¹ K⁻¹)</td>
<td>953</td>
<td>2,588</td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>335</td>
<td>-</td>
</tr>
<tr>
<td>Thermal expansion coefficient (K⁻¹)</td>
<td>200×10⁻⁶</td>
<td>-</td>
</tr>
<tr>
<td>Latent heat of fusion (J kg⁻¹)</td>
<td>95×10³</td>
<td>-</td>
</tr>
</tbody>
</table>

Although the fact that HPs with different orientations have different performances, the effect of HPs orientation was neglected for simplicity. For the comparison purpose with the results presented by Shabgard et al. (2010), mercury was adopted as HPs working fluid even though it is difficult to be used (poisonous and does not readily wet wicks). Stainless steel-naphthalene-charged HPs would be a better choice for this temperature range (Mantelli et al., 2010, Anderson, 2007). The interface between the condenser and the adiabatic sections of each HP is placed at the inner diameter of the HTF tube. The fins material adopted in the numerical simulation is stainless steel to resist corrosion; alternatively, fins could be made from graphite foil which provides higher thermal conductivity, lower density and good corrosion resistivity against nitrate and nitrite salts (Liu et al., 2012).

Initially, the PCM was considered as a liquid salt at its freezing temperature ($T_m$). The HTF enters the HTF tubes at a rate of 3.95 L s⁻¹ and at a temperature of 295 °C. The energy transferred to the HTF from the PCM is used to raise the HTF temperature which will be used to produce superheated steam to drive a conventional turbine. For the specified configuration and conditions, turbulent flow $Re_D > 30000$, the Dittus–Boelter correlation (Incropera et al., 2011b, p. 544) was adopted to calculate the convection coefficient at the internal surface of
the HTF tube to be $h_{tube} = 35 \ W.m^{-2}K^{-1}$. Also, the Zukauskas correlation (Incropera et al., 2011b, p. 458) for cross flow over a single tube was used to calculate the convection coefficient at the condenser section of the HP to be $h_{hp} = 180 \ W.m^{-2}K^{-1}$. The difference between the inlet and the outlet temperatures of the HTF is assumed to be negligible. In addition, a time step of 5 sec was selected for the computation process which was sufficient to obtain stable results and adequate numerical accuracy. The computational time taken for each time step was approximately 9 sec.

Figure 31a shows a comparison of the interface position for a bare HP and a finned HP, while Figure 31b presents the variation of the solidification front position with time for the HTF tube (measured from the centreline of the HTF tube). As can be seen from Figure 31a, finning a heat pipe with four axial fins has led to accelerating the process of solidification; that is, the frozen layer proceeds faster in the case of a finned heat pipe. For the same case, the frozen layer is always approximately 40 % thicker in the case of a finned HP than in the case of a bare HP. It is evident from Figure 31b that the frozen layer thickness increases rapidly at the beginning of the solidification process. Then as the frozen layer thickness increases, radial heat conduction reduces which leads to a decrease in the growth rate of the frozen layer.

The sensible and latent energies extracted from the frozen PCM adjacent to the finned HPs ($Q_{finned\ HP}$), the bare HPs ($Q_{bare\ HP}$) and the HTF tube ($Q_{tube}$) are determined from the following equation (Shabgard et al., 2010):

$$Q = \sum_{i=1}^{n} \{-\rho_{PCM} c_{p_{i}}(T_{i} - T_{m}) + \rho_{PCM} h_{sl}\} V_{i}$$  \hspace{1cm} (3-15)

where $n$ is the total number of control volumes in the frozen PCM domain and $V_{i}$ is the volume of each control volume within the frozen PCM which can be calculated as $V_{i} = r_{i} d\phi_{i} dr_{i} L$. Here $r_{i}$ is the radial location of the midpoint of a control volume, $d\phi_{i}$ is the enclosed angle, $dr_{i}$ is the thickness of a control volume in the $r$ direction, and $L$ is the control
volume length in the $z$ direction. Evaluation of $n$ and $V_i$ is made three times at every time step in the numerical solution correspondingly to the frozen PCM adjacent to the finned HPs, bare HPs, and HTF tube.

Figure 31: (a) Comparison of the interface position for bare HP and finned HP. (b) Variation of the solidification front position with time for the HTF tube.
The energy extracted from the PCM adjacent to: a finned HP, a bare HP and the HTF tube is given in Figure 32. As can be seen, the energy stored increases steadily at the early stage due to the small thermal resistance between the working fluid and the PCM. As more PCM is solidified, the thermal resistance increases causing a decrease in the heat transfer rate as reflected by the decreasing slopes of the curves. Also, evident in Figure 32 is that more energy can be extracted by utilising finned HPs rather than bare HPs, hence the thermal resistance between the PCM and the HTF is further decreased in the case of finned HPs. It was found that the energy extracted by a finned HP was 86% higher than that of a bare HP after 4 hours. Finally, it can be inferred from Figure 32 that finned HP has better thermal performance than bare HP and hence using finned HP would reduce the number of HPs required, which in turn will decrease the overall cost.

![Figure 32: Energy extracted from the PCM adjacent to a finned HP, a bare HP and the HTF tube.](image)

The thermal performance of the LHTES system equipped with HPs is judged on the basis of the enhancement in the energy extracted relative to the no-HPs case. The performance of the
LHTES system is expressed in terms of the HP effectiveness, $\varepsilon_{hp}$, defined as (Shabgard et al., 2010):

$$
\varepsilon_{hp,\text{finned}} = \frac{N \times Q_{\text{finned,HP}} + Q_{\text{HP,tube}}}{Q_{\text{tube}}} \tag{3-16a}
$$

$$
\varepsilon_{hp,\text{bare}} = \frac{N \times Q_{\text{bare,HP}} + Q_{\text{HP,tube}}}{Q_{\text{tube}}} \tag{3-16b}
$$

where $Q_{\text{finned,HP}}$, $Q_{\text{bare,HP}}$, and $Q_{\text{tube}}$ can be obtained by substituting the appropriate values of $n$ and $V_i$ in Eq. (3-15). $Q_{\text{HP,tube}}$ is the amount of energy extracted from the PCM adjacent to the HTF tube in the presence of HPs. $Q_{\text{HP,tube}}$ is less than $Q_{\text{tube}}$ since the HPs not only reduce the surface area of the HTF tube but also reduce the volume of the PCM. To compare the performance of the fins, fin effectiveness, $\varepsilon_{\text{fin}}$, is considered as the ratio of $Q_{\text{finned,HP}}$ to $Q_{\text{bare,HP}}$. This ratio is calculated at every time step.

Figure 33 shows the time history of $\varepsilon_{hp}$ and $\varepsilon_{\text{fin}}$ for finned and bare HPs associated with the case of $N = 4$. It can be seen that $\varepsilon_{hp}$ can be significantly increased by finning the HPs. Specifically, utilising finned HPs results in an increase of $\varepsilon_{hp}$ by an average of 24% for the period of 3.5 hours. Also evident from Figure 33, the heat pipe effectiveness increases with the number of fins. This is expected to be the case but up to a limit where the losses in storage capacity overcome the benefits of adding more fins. It is suggested that HPs with four or five fins are used as a compromise between enhancement of thermal conductance and reduction in the storage capacity (Ismail et al., 2001b). Finally, fin effectiveness of around 2.2 to 2.4 is also observed from Figure 33 suggesting that finning the HPs can lead to significant improvement of the energy extracted by the system.
The heat transfer enhancement of a high temperature LHTES system using finned HPs was experimentally and numerically studied. The numerical model was developed by incorporating the thermal resistance of the heat pipe into a new set of discretisation equations at the boundary grid points of the PCM domain. The model validation has shown close agreement between the experiment and numerical results, which supported the validity of the mathematical approach and the assumptions that were made. The simulated results show that an improvement of 86% on thermal energy storage was obtained by using finned HPs instead of bare HPs. It was seen that by adding four fins to a HP, the overall effectiveness was improved by around 24% as compared to a bare HP, and the associated reduction in the thermal storage capacity was 0.86%. Further, it was seen that by adding five fins to a HP, the overall effectiveness was improved by around 34% as compared to a bare HP with associated reduction in the thermal storage capacity of 1.07%.

3.2 Chapter summary

The heat transfer enhancement of a high temperature LHTES system using finned HPs was experimentally and numerically studied. The numerical model was developed by incorporating the thermal resistance of the heat pipe into a new set of discretisation equations at the boundary grid points of the PCM domain. The model validation has shown close agreement between the experiment and numerical results, which supported the validity of the mathematical approach and the assumptions that were made. The simulated results show that an improvement of 86% on thermal energy storage was obtained by using finned HPs instead of bare HPs. It was seen that by adding four fins to a HP, the overall effectiveness was improved by around 24% as compared to a bare HP, and the associated reduction in the thermal storage capacity was 0.86%. Further, it was seen that by adding five fins to a HP, the overall effectiveness was improved by around 34% as compared to a bare HP with associated reduction in the thermal storage capacity of 1.07%.
### 3.3 Nomenclature used in Chapter 3

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Surface area [m$^2$]</td>
</tr>
<tr>
<td>$\overline{C}$</td>
<td>Specific heat per unit volume including the phase change [J.m$^{-3}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$c$</td>
<td>Molecular specific heat per kilogram [J.kg$^{-1}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$C$</td>
<td>Molecular specific heat per unit volume [J.m$^{-3}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$h_{\text{tube}}$</td>
<td>Convection coefficient at the internal surface of the HTF tube [W.m$^{-2}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$h_{\text{hp}}$</td>
<td>Convection coefficient at the condenser section of the HP [W.m$^{-2}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$h_{\text{sl}}$</td>
<td>Latent heat of fusion [J.kg$^{-1}$]</td>
</tr>
<tr>
<td>$\overline{k}$</td>
<td>Thermal conductivity including the mushy zone [W.m$^{-1}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$k$</td>
<td>Molecular thermal conductivity [W.m$^{-1}$.K$^{-1}$]</td>
</tr>
<tr>
<td>$L_m$</td>
<td>Model unit cell length [m]</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of the HPs in the model unit cell</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Heat flux at the PCM-HPs (HTF tube) adjoining boundary [W.m$^{-2}$]</td>
</tr>
<tr>
<td>$Q$</td>
<td>Extracted energy [MJ]</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial coordinate [m]</td>
</tr>
<tr>
<td>$r_{\text{fin}}$</td>
<td>Outer radius of the fin [m]</td>
</tr>
<tr>
<td>$r_{\text{hp}}$</td>
<td>Outer radius of the heat pipe [m]</td>
</tr>
<tr>
<td>$r_{\text{tube}}$</td>
<td>Outer radius of the HTF tube [m]</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Inner radius of the HTF tube [m]</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Thermal resistance [K.W$^{-1}$]</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of the symmetry circle [m]</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature [K]</td>
</tr>
<tr>
<td>$t$</td>
<td>Time [Sec]</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Freezing temperature [K]</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Half of the phase change temperature range [K]</td>
</tr>
<tr>
<td>$T_{\text{HTF}}$</td>
<td>The temperature of the HTF at the inlet of the model unit cell considered [K]</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume [m$^3$]</td>
</tr>
</tbody>
</table>

**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>Effectiveness</td>
</tr>
</tbody>
</table>
\[ \rho \] Density \([\text{kg.m}^{-3}]\)

\[ \phi \] Angular coordinate \([\text{deg}]\)

**Subscripts**

- \(\text{fin}\) Fin
- \(\text{hp}\) Heat pipe
- \(l\) Liquid phase
- \(\text{PCM}\) Phase change material
- \(s\) Solid phase
- \(\text{tube}\) HTF tube

**Abbreviations**

- CSP Concentrating solar power
- HPs Heat pipes
- HTF Heat transfer fluid
- LHTES Latent heat thermal energy storage
- PCM Phase change material
- SHTES Sensible heat thermal energy storage
- TES Thermal energy storage

**Others associated with the control volume approach**

- \(a_p, a_E, a_W, a_N, a_s, a_E, a_W, a_N\) Coefficients associated with the discretised equations
- \(a'_p, a'_E, a'_W, a'_N\) and \(b\) Modified coefficients incorporating the thermal resistance \(R_t\)
- \((\delta r)_b\) The distance between any boundary point and the nearest internal point \([\text{m}]\)
- \(\Delta r\) The control volume thickness in radial direction \([\text{m}]\)
- \(S_p\) and \(S_c\) Source terms associated with the discretised equations
CHAPTER 4


4.1 Introduction

In this chapter, heat pipes (HPs) are implemented in a LHTES system for high temperature applications and are similarly applicable to CSP systems. Generally, Using HPs in LHTES is not a new technique for improving heat transfer performance and many researchers have presented proposals for improving the aforementioned heat transfer limitations.

In relation to the studies mentioned above in the literature review as well as in chapter 3, issues of structural integrity between the HTF and HPs regions under high temperature and corrosive environments (due to molten salt) have not been addressed. Consequently, the objective of the present chapter is to propose a new heat enhancement configuration using HPs for LHTES which can reduce potential structural failures such as leakage after prolonged usage. This heat enhancement proposal is a simple design which can improve the durability of the heat transfer structures and improve the CSP plant’s operational reliability. A thermal network model was developed to investigate numerically the performance of suspended finned HPs in a High-temperature LHTES unit.
4.2 System Design Description

In this new heat enhancement design of an LHTES unit, the HTF channels are not penetrated by any HP which differs from the design presented by Shabgard et al. (2010). Studding HPs into HTF channels is complex in fabrication and possesses risks of structural failure (leakage) because of repetitive heating and cooling under harsh condition. To avoid such undesired risks, the proposed heat enhancement technique is to position HPs in suspension arrangements adjacent to the outer surfaces of the HTF channels (see Figure 34a). A unit model cell of length $L_m$ has been defined for the purpose of numerical analysis, and is shown in Figure 34b. To enhance the heat transfer rate, the HPs are equipped with high thermal conductance fins for better heat spreading between the HPs and the PCM. The fins are kept vertical in order not to dampen the natural convection currents occurring during the charging process, whereas the HPs are kept horizontal in order to transfer the same heat during the charging and discharging processes. Figure 34 shows the proposed design

4.1 Mathematical Modelling

In the proposed design, there are two regions of solidification; one around the HPs and the second around the HTF channel wall (see Figure 35, top left and top right). A thermal network model has been developed to simultaneously simulate the solidification associated with these two regions. The numerical model developed firstly computes the solidification around the HTF channel based on the temperature distribution and the solidification front position. Then the solidification around the HPs is considered assuming that the condenser length of each HP is equal to the thickness of the solidified PCM around the HTF channel ($s_i$). For each time step, this procedure is repeated until the amount of energy transferred from the liquid PCM, through the HPs, matches that released into the solid PCM adjacent to the HTF channel (this is discussed in section 4.3.2).
To simplify modelling, the following assumptions are made.

a. The thermal resistances of the HP associated with the vaporisation and condensation processes (Faghri, 1995) are ignored.

b. Only pure conduction heat transfer is considered. This assumption is justified since the initial temperature is the freezing point, $T_m$, of the PCM (Sparrow et al., 1981a).

c. The PCM is assumed isotropic and homogeneous.

### 4.1.1 Thermal Resistance Network

The thermal resistance network for the proposed LHTES is shown in Figure 35 (bottom left). This approach treats the HPs, the HTF channel and the PCM as a network of thermal resistances representing thermal elements. Each element has a cross sectional area $A_i$, a
thickness $\lambda_i$ and temperatures $T_i$, $T_{i,1}$ and $T_{i,2}$ at the middle, the face 1 and the face 2 respectively. Heat is transferred from the solid–liquid interface (at $T_m$) to the HTF (at $T_{HTF}$). In contrast to the network diagram described by Shabgard et al. (2010), there is no direct contact between the HPs and the HTF. Instead, the HPs span between the deeper PCM and the PCM adjacent to the HTF channel. Thus, all the transmitted heat must eventually flow through the wall of the HTF channel. In Figure 35 (bottom left), $E_{n+1}$ represents the HTF channel wall and $E_1$ to $E_4$ represent the components of the HP. $E_5$ to $E_n$ and $E_{n+2}$ to $E_{n+m}$ represent PCM adjacent to the HPs and HTF channels respectively. The governing equation is written as follows (Shabgard et al., 2010, Zuo and Faghri, 1998):

![Figure 35: The proposed thermal resistance network (bottom left), the definition of the unit model cell (top left), the physical model after applying symmetry conditions (top right) and a section view of a finned HP showing the condenser length (bottom right). Dimensions are not to scale.](image-url)
\[
\frac{dT_i}{dt} = \frac{2\alpha_i}{\lambda_i} \left( T_{i,1} + T_{i,2} - 2T_i \right) + q^* V_i
\]

(4-1)

where the last term on the right hand side of Eq. (4-1) is applicable only for elements \( E_{n+2} \) through \( E_{n+m} \) in order to account for the volumetric heat rate released by the HPs (\( q^* \text{ W/m}^3 \)).

By performing an energy balance at each surface temperature node and following Eq. (4-1), expressions for each element temperature, \( T_i \), can be written as (Zuo and Faghri, 1998):

For \( E_1 \) (HPs condenser wall)

\[
\frac{dT_1}{dt} = \frac{2\alpha_1}{\lambda_1} \left[ (\xi_{1,2} - 1)T_1 + \xi_{2,1}T_2 \right] + \frac{\alpha_1 Q}{\lambda_1 k_1 A_1}
\]

(4-2)

For \( E_2 \) (HPs condenser wick)

\[
\frac{dT_2}{dt} = \frac{2\alpha_2}{\lambda_2} \left[ (\xi_{2,3} + \xi_{2,4} - 2)T_2 + \xi_{3,2}T_3 \right]
\]

(4-3)

For \( E_3 \) (HPs evaporator wick)

\[
\frac{dT_3}{dt} = \frac{2\alpha_3}{\lambda_3} \left[ (\xi_{3,2} + \xi_{3,4} - 2)T_3 + \xi_{4,3}T_4 \right]
\]

(4-4)

For \( E_4 \) (HPs evaporator wall)

\[
\frac{dT_4}{dt} = \frac{2\alpha_4}{\lambda_4} \left[ (\xi_{4,3} + \xi_{4,5} - 2)T_4 + \xi_{5,4}T_5 \right]
\]

(4-5)

For \( E_{n+1} \) (HTF channel wall)
\[
\frac{dT_{n+1}}{dt} = \frac{2\alpha_{n+1}}{\lambda_{n+1}^2} \left[ \frac{h_i S_T \lambda_{HTF}}{2} \left( \frac{h_i S_T}{2 + k_{n+1} A_{n+1}/\lambda_{n+1}} + \left( \frac{\xi_{n+1, n+2}}{h_i S_T/2 + k_{n+1} A_{n+1}/\lambda_{n+1}} - 2 \right) T_{n+1} + \xi_{n+2, n+1} T_{n+2} \right) \right]
\]  

(4-6)

where \( h_i \) and \( S_T \) are the convection coefficient and the area of the HTF channel inner surface respectively. In addition, the PCM adjacent to the HPs and the HTF channel are divided into \((n-4)\) and \((m-1)\) elements of uniform thicknesses respectively. In the present model, \( n \) is set to 14 and \( m \) is set to 16. The results were found to be nearly independent of the number of PCM elements. After each time step, the solidification front position is advanced by applying the surface energy balance for elements \( E_n \) and \( E_{n+m} \) which yields (Shabgard et al., 2010, Ozisik, 1993):

\[
k_{n,eff} \left( s_{hp} + R_{hp} - \lambda_n/2 \right) \frac{T_n - T_m}{\lambda_n/2} = \left( s_{hp} + R_{hp} \right) \rho_{pcm,eff} h_{sl,eff} \frac{ds_{hp}}{dt}
\]  

(4-7)

\[
k_{n+m} \frac{T_{n+m} - T_m}{\lambda_{n+m}/2} = -\rho_{pcm} h_{sl} \frac{ds_{i}}{dt}
\]  

(4-8)

where \( s_{hp} \) is the thickness of the solidified PCM around the HPs. The expression for elements \( E_5 \) to \( E_{n-1} \) (within the PCM around each HP) is:

\[
\frac{dT_i}{dt} = \frac{2\alpha_i}{\lambda_i^2} \left[ \xi_{i-1,i} T_{i-1} + \left( \xi_{i,i} + \xi_{i,i+1} - 2 \right) T_i + \xi_{i+1,i} T_{i+1} \right]
\]  

(4-9)

For \( E_{n+2} \) to \( E_{n+m-1} \) (within the PCM around the HTF channel)

\[
\frac{dT_i}{dt} = \frac{2\alpha_i}{\lambda_i^2} \left[ \xi_{i-1,i} T_{i-1} + \left( \xi_{i,i} + \xi_{i,i+1} - 2 \right) T_i + \xi_{i+1,i} T_{i+1} \right] + \frac{\alpha_i q V_i}{k_A A_h}
\]  

(4-10)

No additional equations are needed for elements \( E_n \) and \( E_{n+m} \) since their surface temperatures are equal to \( T_m \). Eqs. (4-2) to (4-6) and Eqs. (4-9) and (4-10) constitute a system of ordinary differential equations that can be solved by the Runge-Kutta method.
\( Q \) (in Eq. 4-2) is the heat rate transmitted by a single HP, while \( q' \) (in Eq. 4-10), is the volumetric heat rate released by the HPs within the solidified PCM layers near the HTF channel. By applying an energy balance at the outer surface of a single HP condenser section and with reference to Figure 35 (bottom right), the following expression can be written:

\[
Q = \frac{k_{\text{interface}} A_{vs} \sum_{i=E_{vs2}}^{E_{ns1}} [T_i/(m-1)] - T_1}{\delta x}
\]

\( (4-11) \)

where \( A_{vs} = 2\pi R_{hp} s_i \) is the surface area of a HP condenser section, and \( \delta x = \frac{L_m - 2(R_{hp} + \lambda_t)}{4} \) is the distance between the midpoint of the HP condenser wall (which is at \( T_1 \)) and the midpoint of the solid PCM layer formed around the HP condenser section. The term \( \sum_{i=E_{vs2}}^{E_{ns1}} [T_i/(m-1)] \) represents the average temperature of elements \( E_{ns1} \) to \( E_{ns2} \). Also, the interface thermal conductivity \( (k_{\text{interface}}) \) is calculated as (Patankar, 1980a):

\[
k_{\text{interface}} = \left( \frac{1 - f_e}{k_1} + \frac{f_e}{k_{pcm,eff}} \right)^{-1}
\]

\( (4-12) \)

where \( f_e \) is defined as (Patankar, 1980a):

\[
f_e = \left( \frac{L_m - 2R_{hp}}{L_m - 2R_{hp} + 2\lambda_t} \right)
\]

\( (4-13) \)

\( \xi_{i,j} \) is written as below (Shabgard et al., 2010):

\[
\xi_{i,j} = \frac{k_i A_i / \lambda_i}{k_i A_i / \lambda_i + k_j A_j / \lambda_j}
\]

\( (4-14) \)

In the case of thin and long fins, the fins-PCM mixture can be considered as a single material with effective properties (Bauer, 2011). Hence, effective thermo-physical properties are used
in Eqs. (4-7) and (4-9) for the PCM elements around the HPs. The effective properties are calculated from (Bauer, 2011):

\[
\rho_{pcm, eff} = v_{pcm} \rho_{pcm} + v_f \rho_f
\]  
(4-15)

\[
c_{pcm, eff} = \frac{\rho_{pcm}}{\rho_{pcm, eff}} v_{pcm} c_{pcm} + \frac{\rho_f}{\rho_{pcm, eff}} v_f c_f
\]  
(4-16)

\[
h_{sl, eff} = \frac{\rho_{pcm}}{\rho_{pcm, eff}} v_{pcm} h_{sl}
\]  
(4-17)

\[
k_{pcm, eff} = v_{pcm} k_{pcm} + v_f k_f
\]  
(4-18)

where \( v_{pcm} \) and \( v_f \) are the volume fractions of the PCM and the fins respectively. The volume fraction of the fin is set to 0.1 which is desired for heat storage applications (Bauer, 2011).

4.1.2 The Solution Procedure

Since the solidification processes around the HPs and HTF channel are interdependent, the following iteration procedure is used:

1. Start with an estimated value of \( q^* \) and initial conditions of \( T_i^0 = T_m \) \((i = 1 \text{ to } n + m)\), \( s_{hp}^0 = 10^{-6} \) and \( s_i^0 = 10^{-6} \).
2. Solve the solidification process around the HTF channel for \( T_i \) \((i = n + 2 \text{ to } n + m - 1)\) and \( s_i \).
3. Calculate the energy rate transmitted by a HP, \( Q \), from Eq. (4-11).
4. Solve for solidification around the evaporator section of a HP for \( T_i \) \((i = 5 \text{ to } n - 1)\) and \( s_{hp} \).
5. Calculate the energy extracted by the evaporator section of a single HP, \( Q_{hp} \), as (Shabgard et al., 2010):
\[ Q_{hp} = \sum_{i=3}^{n} \left[ -\rho_{pcm, eff} c_{pcm, eff} (T_i - T_m) + \rho_{pcm, eff} h_{sl, eff} \right] \times 2\pi \left( L_{hp} - s_i \right) \left[ R_{hp} + \lambda_i \left( i - \frac{9}{2} \right) \lambda_i \right] \quad (4-19) \]

where \( L_{hp} \) is the HP total length.

6. Calculate \( q^- \) from:

\[ q^- = \frac{\left( Q_{hp}^1 - Q_{hp}^0 \right) N}{\Delta t V_{total}} \quad (4-20) \]

where \( Q_{hp}^1 \) and \( Q_{hp}^0 \) are the energy extracted by a HP at time steps \( t + \Delta t \) and \( t \) respectively.

\( V_{total} \) is the total volume of the solidified PCM around the HTF channel (after subtracting the volume occupied by the HPs and the fins) and \( N \) is the number of HPs used in the unit model cell.

7. With the newly calculated value of \( q^- \), return to step 2 and repeat until there is no significant change in the value of \( q^- \).

8. Repeat all of the above steps with the newly obtained values of \( T_i \) \( (i = 1 \text{ to } n + m) \), \( s_{hp} \) and \( s_i \) as initial conditions for the next time step.

9. At each time step, the energy extracted by the HTF channel is calculated from (Shabgard et al., 2010):

\[ Q_i = \sum_{i=n+2}^{n+m} \left[ -\rho_{pcm} c_{pcm} (T_i - T_m) + \rho_{pcm} h_{sl} \right] \times 2 \left[ H_{chan} + W_{chan} + 4(i - n - 2) \lambda_i \right] L_n \lambda_i \quad (4-21) \]

Different time steps were tested in order to check the stability of the model. A time step of 5sec was found to provide adequate numerical accuracy.
4.2 Experimental design and procedure

4.2.1 Phase change material used in the experiment

For safety reasons, low temperature PCM (RT60) was used for validation of the model. The thermo-physical properties of RT60 are listed in Table 11 (rubitherm). After the model validation, high-temperature PCM (LiCl-KCl) will be adopted as a storage medium for the proposed LHTES unit.

<table>
<thead>
<tr>
<th>Table 11: Properties of the PCM (RT60), HP and fins used in the experiments.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermo-physical properties of RT60</td>
</tr>
<tr>
<td>Density solid/liquid 15/90 °C (kg.m⁻³)</td>
</tr>
<tr>
<td>Thermal conductivity solid/liquid (W. m⁻². K⁻¹)</td>
</tr>
<tr>
<td>Specific heat solid/liquid (J. kg⁻¹. K⁻¹)</td>
</tr>
<tr>
<td>Melting point (°C)</td>
</tr>
<tr>
<td>Solidifying point (°C)</td>
</tr>
<tr>
<td>Latent heat of fusion (J. kg⁻¹)</td>
</tr>
</tbody>
</table>

Heat pipe specifications

<table>
<thead>
<tr>
<th>HP wall</th>
<th>Wick</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter (m)</td>
<td>0.008</td>
</tr>
<tr>
<td>Total length (m)</td>
<td>0.3</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>0.0004</td>
</tr>
<tr>
<td>Porosity</td>
<td>-</td>
</tr>
<tr>
<td>Thermal conductivity (W. m⁻¹. K⁻¹)</td>
<td>391</td>
</tr>
<tr>
<td>Heat capacity (J.m⁻². K⁻¹)</td>
<td>3,441,900</td>
</tr>
</tbody>
</table>

Fins

| Thermal conductivity (W.m⁻¹.K⁻¹)                                        | 50ᵇ |
| Heat capacity (J.m⁻³.K⁻¹)                                               | 2,449,840 |

ᵇ Representing the effective thermal conductivity of fin material and the epoxy.

4.2.2 Experimental arrangement and procedure

An experimental validation was required for the developed numerical model to ensure its reliability and accuracy. An experimental facility was built for this purpose consisting of two plate-shaped heat exchangers (made from Acrylic and copper), a square acrylic container and
a finned HP-PCM mixture. The whole rig was insulated using insulation sheets of 10cm thickness. The copper plates were thermally maintained at different particular temperatures by the means of hot and cold water flows, provided from a heat bench. Figures 36 and 37 show the experimental setup. The dimension of each copper plate was 320mm×320mm×5mm, with fins machined into its water circulating side. The heat pipe used was a copper-water-charged HP provided by Fujikura Ltd, which was later retrofitted with aluminium fins (120mm×120mm×2mm). The fins were thermally bonded with the HP by means of thermal epoxy (~1.8W/m.K) to minimise the interface thermal resistance. The HP specifications are listed in Table 11.

Figure 36: The experimental test rig.
The experimental procedure was as follows: Firstly, components were assembled as shown in Figure 37a. Hot water at 61±0.2°C was then pumped through both of the heat exchangers. After the copper plates had reached a constant temperature, the liquid PCM was poured to fill the acrylic container through an open cap at the upper part of the rig. The whole system was allowed to reach its thermal steady state. Secondly, cooling water at 0.5±0.2°C commenced to circulate at a rate of 0.3 L/s in one of the heat exchangers. The flow into the other heat exchangers was maintained at 61±0.2°C in order to provide temperature uniformity within the liquid PCM at the freezing temperature ($T_m$). Finally, the recording of measurements commenced after all prerequisites were met.

Figure 37: (a) Schematic diagram of the rig, (b) thermocouple locations (dimensions are in mm).

The test facility was equipped with T-type thermocouples of 1.0mm overall diameter and accuracy of ±0.1°C. The thermocouples were attached to different locations inside the
container as shown in Figure 37b. The test was repeated until the maximum experimental uncertainty fell below 5% (Coleman and Steele, 2009). The error bars are shown in Figure 38a. The temperature fields, inlet temperature and exit temperature of the cooling water were measured at 1 minute intervals.

4.3 Results and Discussion

Before embarking on the numerical analysis of the proposed LHTES unit, the numerical model was validated against the experimental results obtained in this study. The geometry and the input data of the model were amended to simulate the solidification problem described in Section 4.4. The initial temperature for the whole system was set at the freezing temperature of the PCM (RT60). The copper plate was assumed to be at a constant average temperature of 1.8°C (experimentally obtained). Figure 38 shows a comparison between the numerical and experimental results for the PCM adjacent to the copper plate. The agreement is very good, especially at the interface position shown in Figure 38a. In spite of achieving close results in Figure 38b, the numerical curves indicate a longer time for the phase change process. After phase change was completed, the numerical curves dropped sharply and caught up with the experimental curves. At the later stage of phase change, the numerical curves showed a moderate rate of temperature drop. This process is smoother in the case of the experimental curves where there is no sharp drop. The numerical curves indicate longer time during the phase change because of the moving mesh technique adopted in the numerical model. In this technique, at each time step the calculation domain is restricted to the solid region of the PCM, whereas the liquid region is assumed to be always at a uniform temperature of $T_m$. Then the front position is advanced by applying Eq. (4-7) and (4-8). Consequently, the effect of the conduction between the liquid and the solid PCM is ignored which leads to a longer phase change period compared to the actual case.
Figure 38: Comparison of numerically and experimentally obtained results: (a) interface position near the copper plate, (b) temperature history of points T15 to T18 (see Figure 37b).
The model was also validated against experimental results obtained by Ismail et al. (2014) in which they investigated the freezing of water around a horizontal copper tube cooled with ethanol. The same conditions and geometry (HTF tube of circular cross section) used by Ismail et al. (2014) were adopted in the present model (with $N = 0$). The comparison with their work is showing in Figure 39. As can be seen, the model predictions are in good agreement with the results obtained by Ismail et al. (2014).

![Figure 39: Comparison of the present numerical prediction with the results of Ismail et al. (2014).](image)

The solidification process around the HP is presented photographically in Figure 40. The figure shows 12 photographs which are taken sequentially at run times of 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5h. It is observed that at any instant after 1h, the frozen layer grows at an approximately constant rate over the length of the HP apart from the dendrite structure formed on the surface of the fins. The frozen layer thickness reached approximately 30mm at 5h after commencement of the test. The average temperature difference between the HP surface and the surface of the fins ($T_4 - T_2$) was 2.9°C which indicates that there was imperfect contact between the fins and the HP. Consequently, if a similar HP with perfectly attached fins had been used in the experiments, the layer around the HP would have been
much thicker. In fact, by ignoring the interface thermal resistance between the fins and the HP, the numerical model shows that the frozen layer is doubled at any run time. Figure 40 also compares the numerical and experimental variations of the HP average temperature with time. Both of the curves show the same trend but with a difference of 4%. The small drop at the HP surface temperature is attributed to the phase change which occurs throughout the length of the HP surface.

![Figure 40: Comparison of numerical and experimental average temperature of the HP, and time history evolution of the frozen layer around the HP.](image)

Once the model was validated, numerical analysis of large scale LHTES unit for CSP plants has been carried out. In the proposed unit, stainless steel naphthalene-charged HPs were adopted for their appropriate temperature range and their insensitivity to corrosion (Mantelli et al., 2010, Anderson et al., 2007). In addition, Anderson (2007) reviewed several heat pipe life tests conducted over 40 years. He reported that in a 5520h life test of stainless steel-naphthalene charged HP at 350°C, naphthalene did not degrade, and he concluded that
naphthalene and stainless steel are compatible. Stainless steel was also selected for the HTF channel, whereas Anodized Aluminium was selected for the fins. Lithium chloride-potassium chloride eutectic (LiCl-KCl) and Therminol® VP-1 were chosen as PCM and HTF, respectively (Robak et al., 2011a). Table 12 presents the geometrical parameters of the unit, while Table 13 lists the properties of the LiCl-KCl and the Therminol® VP-1 (Robak et al., 2011a, Shabgard et al., 2010). The thermal resistances of the HP components and the PCM were recalculated based on these newly adopted materials. By using the recommended dimensions listed in table 12, the LHTES system can be cost competitive with the two-tank system (Robak et al., 2011a).

Table 12: Dimensions of the LHTES components.

<table>
<thead>
<tr>
<th>Description</th>
<th>HP wall</th>
<th>Wick</th>
<th>HTF channel</th>
<th>Fin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical dimensions ((m))</td>
<td>0.025 (diameter)</td>
<td>-</td>
<td>0.08 (height)</td>
<td>-</td>
</tr>
<tr>
<td>Total length ((m))</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Thickness ((m))</td>
<td>0.001</td>
<td>0.001</td>
<td>0.005</td>
<td>-</td>
</tr>
<tr>
<td>Wick porosity</td>
<td>-</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Thermal conductivity ((W.m^{-1}.K^{-1}))</td>
<td>20.1</td>
<td>8.1(^a)</td>
<td>20.1</td>
<td>220</td>
</tr>
<tr>
<td>Heat capacity ((J.m^{-3}.K^{-1}))</td>
<td>4,423,210</td>
<td>1.7218\times10^6</td>
<td>4,423,210</td>
<td>2,449,840</td>
</tr>
</tbody>
</table>

\(^a\) Based on stainless steel sintered wick with naphthalene as working fluid, and wick porosity of 0.5 (Faghri, 1995).

Table 13: Thermo-physical properties of the PCM and the HTF (Williams, 2006, Robak et al., 2011a).

<table>
<thead>
<tr>
<th>Description</th>
<th>PCM 44.0 wt.% LiCl/56.0 wt.% KCl</th>
<th>HTF (Therminol® VP-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ((kg/m^3))</td>
<td>1600</td>
<td>709</td>
</tr>
<tr>
<td>Thermal conductivity ((W/m.K))</td>
<td>0.42 (liquid)</td>
<td>0.078</td>
</tr>
<tr>
<td>Specific heat ((J/kg.K))</td>
<td>1201.84</td>
<td>2,588</td>
</tr>
<tr>
<td>Melting point ((^\circ C))</td>
<td>348</td>
<td>-</td>
</tr>
<tr>
<td>Latent heat of fusion ((J/kg))</td>
<td>170\times10^3</td>
<td>-</td>
</tr>
</tbody>
</table>
Initially, LiCl-KCl was considered as a liquid salt at its freezing temperature ($T_m$). Then the inlet conditions of the HTF flow were set to 2 $kg/s$ at 280°C (Shabgard et al., 2010). The chosen inlet temperature matches the exiting temperature of the HTF from the superheated steam generator (Kelly and Kearney, 2006). Under these flow conditions ($Re_D > 30,000$) and based on the hydraulic diameter, the Dittus–Boelter correlation (Incropera et al., 2011a) was used to calculate the convection coefficient ($h_t$) between the HTF and the HTF channel which was found to be $74.7\, W/m^2K$. The model length $L_m$ was set to 0.162m, and the difference between the inlet and the outlet temperatures of the HTF through the unit model cell was assumed to be negligible.

Figure 41 shows the temperature profiles of the HP and the HTF channel as well as the energy transported through a HP. As the heat extraction starts from the molten PCM to the HTF at the freezing point of the PCM, the temperature profiles decrease at different rates. It can be seen that the temperature of the HTF channel decreases rapidly at the initial stage of solidification. In contrast, the temperature of the HP remains constant at $T_m$ until a point at approximately 1h. As the solidification front around the HTF channel advances, the temperature of the HP starts to drop slightly. The temperature difference between the HTF channel and the HPs reached 55°C at the end of the simulation. The HP behaviour can be further explained by observing the energy transported through a HP. The HP operating point occurs soon after three minutes when the transported energy starts to grow steadily, showing a saturating trend and peak at $74\, W$. 

82
The thermal performance of the investigated configuration was quantified by introducing the HP effectiveness, $\varepsilon_{hp}$, defined as (Shabgard et al., 2010):

$$\varepsilon_{hp} = \frac{N \times Q_{hp} + Q_i}{Q_{baseline}}$$  \hspace{1cm} (4-22)

where $Q_{baseline}$ is the energy extracted from the baseline case (HTF channel without any HPs).

Figure 42 presents the $\varepsilon_{hp}$ histories for the cases with 6, 8, 10, and 12 finned HPs. At the early stage, the values of $\varepsilon_{hp}$ are less than one, indicating that the HPs are slowing down the heat transfer because of reductions in both heat transfer area and storage volume. As the solidification front moves out from the HTF channel surface, the HPs become operative and are able to transfer heat deeper from the far hotter PCM to the surrounding cooler PCM. As a result, the $\varepsilon_{hp}$ values start to grow showing a clear upward trend. Eventually, all the curves intersect at a single point corresponding to $\varepsilon_{hp} = 1$ which indicates that the losses resulted
from reductions in heat transfer area and the storage volume are compensated by the enhancement gained from the HPs. Beyond this time, the HPs enhance the heat transfer rate. It is also evident in Figure 42 that the $\varepsilon_{hp}$ values grow steadily at an approximately constant rate corresponding to the number of the HPs used in the model.

![Figure 42: The $\varepsilon_{hp}$ histories for the model with 8, 10, 12, 14 and 16 finned HPs.](image)

Fin effectiveness ($\varepsilon_f$) is another aspect that can be considered in order to quantify the system performance. It is defined as the ratio of the actual heat transferred to that which would transfer without fins. The influence on the fin effectiveness as a function of the product of the fin factor ($F_f = v_f k_f / v_{pcm} k_{pcm}$) and Stefan number ($Ste = C_p (T_m - T_{HTF}) / h_{st}$) is shown in Figure 43. The fin factor is the ratio of the heat flow in the fins to that within the PCM (Bauer, 2011). The product ($F_f Ste$) takes into account the fin volume fraction, the sensible heat and the latent heat. Figure 43 implies that the use of fins is justified when $F_f Ste \geq 20$, as the fin effectiveness is recommended to be greater than 2 (Incropera et al., 2011a).
Based on the numerical results, a correlation for prediction of $t_{\varepsilon_f=1}$ can be expressed as:

$$t_{\varepsilon_f=1} = 15.128 \times \left[ F_f \times \frac{(H_{\text{chan}} + W_{\text{chan}})}{\pi R_{hp}} \right]^{-0.409}$$

(4-23)

To obtain Eq. (4-23), 90 geometries have been modelled with three different fin volume fractions and four fin thermal conductivities. In total 1,080 simulations have been performed. However, the thermal conductivity of the PCM and the Stefan number were fixed at 0.5 W/m.K and 0.4 respectively. The importance of equation (4-23) consists in the fact that the system response is low in the first minutes because the HPs become operative just when the solidification front moves out from the HTF channel surface. Hence, a better design should have as low as possible $t_{\varepsilon_f=1}$. In other words, the less the $t_{\varepsilon_f=1}$ is, the faster the response become.
4.4 Chapter Summary

A new thermal enhancement technique for LHTES using embedded HPs in a suspension arrangement was numerically analysed. The model was validated through comparison with experimental measurements conducted in this chapter as well as with experimental results published in the literature. The results have shown that for the conditions considered the heat transfer performance of the finned HPs-PCM composite has improved significantly after 45 min in spite of the poor thermal response at the initial stage. In addition, utilising more HPs in a suspension arrangement has further improved the performance at the later stage of PCM solidification. The effectiveness of the 12-HP configuration reached 2.4 after 5h of simulation. This implies that the energy extracted has increased by 140% comparing with the baseline configuration. A correlation was also made for predicting the time when $\varepsilon_{hp} = 1$, based on the product of two dimensionless parameters.

4.5 Nomenclature used in Chapter 4

\[ A_{cs} \] Surface area of a HP condenser section [m$^2$]
\[ A_i \] Cross sectional area of element $i$ [m$^2$]
\[ c \] Specific heat per kilogram [J.kg$^{-1}$.K$^{-1}$]
\[ C \] Specific heat per unit volume [J.m$^{-3}$.K$^{-1}$]
\[ f_e \] Interpolation factor (Eq. 13)
\[ F_f \] Fin factor, $F_f = v_f k_f / \rho_{med} k_{pcm}$
\[ H_{chan} \] HTF channel height [m]
\[ h_{sl} \] Latent heat of fusion [J.kg$^{-1}$]
\[ h_t \] Convection coefficient at the internal surface of the HTF channel [W.m$^{-2}$.K$^{-1}$]
\[ k \] Thermal conductivity [W.m$^{-1}$.K$^{-1}$]
\[ k_{interface} \] Interface thermal conductivity [W.m$^{-1}$.K$^{-1}$]
\[ L_{hp} \] Total length of a HP [m]
\[ L_m \] Unit model cell length [m]
\( N \)  
Number of the HPs in the unit model cell

\( q^* \)  
Heat generation rate \([\text{W.m}^{-3}]\)

\( Q \)  
Heat rate transmitted by a single HP \([\text{W}]\)

\( Q_{\text{baseline}} \)  
Amount of energy extracted from the HTF channel without HPs

\( Q_{\text{hp}} \)  
Amount of energy extracted by the evaporator section of a single HP \([\text{J}]\)

\( Q_t \)  
Amount of energy extracted by the HTF channel \([\text{J}]\)

\( R_{\text{hp}} \)  
Outer radius of the heat pipe \([\text{m}]\)

\( s_{\text{hp}} \)  
Solid layer thickness around a HP \([\text{m}]\)

\( s_t \)  
Solid layer thickness around the HTF channel \([\text{m}]\)

\( S_t \)  
Internal surface area of the HTF channel \([\text{m}^2]\)

\( T \)  
Temperature \([\text{K}]\)

\( t \)  
Time \([\text{S}]\)

\( T_m \)  
Freezing temperature \([\text{K}]\)

\( T_{\text{HTF}} \)  
The temperature of the HTF at the inlet of the unit model cell considered \([\text{K}]\)

\( V \)  
Volume \([\text{m}^3]\)

\( v \)  
Volume fraction

\( W_{\text{chan}} \)  
HTF channel width \([\text{m}]\)

**Greek symbols**

\( \alpha \)  
Thermal diffusivity \([\text{m}^2.\text{s}]\)

\( \Delta t \)  
Time step \([\text{s}]\)

\( \delta r \)  
Distance (Eq. 11) \([\text{m}]\)

\( \varepsilon \)  
Effectiveness

\( \lambda_i \)  
Thickness of element \(i\) \([\text{m}]\)

\( \rho \)  
Density \([\text{kg.m}^{-3}]\)

**Subscripts**

\( \text{eff} \)  
Effective property of PCM and fin

\( f \)  
Fin

\( \text{hp} \)  
Heat pipe

\( \text{HTF} \)  
Heat transfer fluid

\( i \)  
Element \(i\)

\( l \)  
Liquid phase

\( \text{pcm} \)  
Phase change material
$s$  Solid phase

$chan$  HTF channel

Superscript

$0$  Denotes quantities at time step $t$

$1$  Denotes quantities at time step $t + \Delta t$

Abbreviations

CSP  Concentrating solar power

HPs  Heat pipes

HTF  Heat transfer fluid

LHTES  Latent heat thermal energy storage

PCM  Phase change material

SHTES  Sensible heat thermal energy storage

TCES  Thermochemical energy storage

Other

$E$  Thermal element
CHAPTER 5

Micro Heat Pipe-Phase Change Material (MHP-PCM) Composite

5.1 Introduction

In this chapter, the mixture of PCM and micro heat pipes (MHPs) is addressed as a “composite material”. MHPs are wickless, noncircular channels with hydraulic diameters in the range of 10 to 500μm, and lengths of several centimetres (10-60mm) (Longtin et al., 1994, Babin et al., 1990, Cao and Faghri, 1994). Figure 44 shows a schematic diagram of a triangular MHP as well as the cross-sectional geometry at different positions along the length of the MHP (Peterson, 1992). The effective thermal conductivity of an individual MHP depends on the wall material, the working fluid and the operating temperature. It varies from 3,500W/m.K to 7,000W/m.K (Chang and Hung, 2014, Rahmat and Hubert, 2010). It is noted that one needs to differentiate between the effective thermal conductivity of an individual MHP and that of an array of MHPs integrated into a semiconductor wafer. The array has much lower effective thermal conductivity of just around 180% of that of the plain wafer (Peterson et al., 1993).

At present, no published work has addressed the enhancement resulting from the dispersion of MHPs in PCMs. For that reason, the objective of this chapter is to examine the thermal performance of micro heat pipe-phase change material (MHP-PCM) composites for CSP applications. A numerical model is introduced to predict the effective thermal conductivity of MHP-PCM composites, and to simulate heat transfer and phase change processes in a high-
temperature LHTES unit for CSP applications. The model takes into consideration the effects of the MHP orientation as well as the MHP volume fraction ($v_{hp}$).

![Diagram of MHP](image)

*Figure 44: Schematic diagram of a MHP (Peterson, 1992).*

### 5.2 PCM selection

In modern CSP plants, the temperature of the HTF entering to the storage unit is planned to be 280°C during the discharging process and 390°C during the charging process (Kelly and Kearney, 2006). These temperatures match the HTF temperatures coming from the superheated steam generator and from the solar field during discharging and charging processes respectively. There are over 160,000 commercially available PCM that can be used as a storage medium (Khare et al., 2012). A similar eutectic mixture to that used in Chapter 4 was selected as PCM for this Chapter. However, it has a slightly different percentage composition of 58.5% Lithium Chloride (LiCl) and 41.5% Potassium Chloride (KCl) (in mol basis). This resulted in an increase of the melting temperature to 355°C which is approximately midway between 280°C and 390°C, and it is 15°C above the desired LHTES
HTF exit temperature (340°C) during the discharging process (Robak et al., 2011a). This will lead to better thermodynamic efficiency of the Rankine cycle when using the storage unit as the heat source. The properties of the candidate PCM and HTF are listed in Table 14.

<table>
<thead>
<tr>
<th>Description</th>
<th>PCM 58.5 mol% LiCl/41.5 mol% KCl at 355 °C</th>
<th>HTF (Therminol® VP-1) at 390 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>1631ᵃ (liquid)</td>
<td>709ᵇ</td>
</tr>
<tr>
<td>Thermal conductivity ( W/m.K )</td>
<td>0.69ᵃ (liquid) 1.01ᵃ (solid)</td>
<td>0.078ᵇ</td>
</tr>
<tr>
<td>Specific heat (J/kg.K)</td>
<td>1305ᵃ</td>
<td>2,588ᵇ</td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>355ᵃ</td>
<td>-</td>
</tr>
<tr>
<td>Latent heat of fusion (J/kg)</td>
<td>(234.6 ×10³)ᵃ</td>
<td>-</td>
</tr>
<tr>
<td>Prandtl number, ( Pr = \frac{C_p \mu}{k} )</td>
<td>5.863ᵃ</td>
<td>-</td>
</tr>
<tr>
<td>Dynamic viscosity (Pa.s)</td>
<td>(3.1 ×10⁻¹)ᵃ</td>
<td>-</td>
</tr>
<tr>
<td>Thermal expansion coefficient (K⁻¹)</td>
<td>(2.93 ×10⁻⁴)ᶜ</td>
<td>-</td>
</tr>
</tbody>
</table>

ᵃ (Janz et al., 1979),ᵇ (Solutia Inc. [Accessed 12 Feb 2015]) andᶜ (Bengtson et al., 2014).

### 5.3 Micro heat pipe selection and characterization

A MHP with an equilateral triangular cross section is adopted in this chapter. The selection of the MHP involves three criteria: (i) selection of the wall material for compatibility with the PCM and MHP’s working fluid (ii) selection of the MHP’s working fluid for suitability with the temperature range (iii) selection of the MHP’s material and dimensions for having an average density equal to that of the liquid PCM.

With regard to MHP wall material, any material chosen should have relatively low density (in order to stay suspended in the liquid PCM) and have high corrosion resistance. The first lightweight material which may be considered is aluminium because of its low cost, ease of
manufacture, high thermal conductivity and high corrosion resistance (Yang et al., 2012). Thus, aluminium seems to satisfy the requirements imposed on the MHP wall material. However, it must also be compatible with the MHP’s working fluid at the temperature range (280-390°C) existing in the proposed storage unit. For this range, Mercury, Sulphur, Dowtherm, Toluene and Naphthalene are good candidate working fluids (Faghri, 2014, Anderson et al., 2012). Unfortunately, limited data are available for compatibility of aluminium with these working fluids, especially at high temperatures (> 300°C). Indeed, Sulphur was found to be incompatible with aluminium (Anderson et al., 2007), and Mercury is not preferable (toxic and it does not wet the wall) (Reay et al., 2013). The other three are organic fluids which naturally break down when the temperature reaches the critical value forming non-condensable gases (Reay et al., 2013). Consequently, aluminium cannot be used for the present application.

Lately in 2012, Anderson et al. (2012) published a study on intermediate-temperature HPs life test. They revealed that Titanium (Ti) and Titanium tetrabromide (TiBr₄) are compatible at 380°C based on a 5.7 years life test. Thus, this combination seems to be suitable for the MHPs adopted in the present work. Nevertheless, the only disadvantage of the Ti/TiBr₄ combination is the high cost of titanium (11 times the cost of aluminium (Yang et al., 2012)). This, of course, is a serious drawback when the storage unit is economically assessed. Despite the aforementioned drawback, the Ti/TiBr₄ combination is adopted for the purpose of the present chapter in assessing the thermal performance of such storage units. The author encourages future researchers to investigate the feasibility of using other combinations of wall materials and working fluids which may lead to economically competitive storage units. For example, adding a titanium coating layer inside an aluminium HP creating a cheaper envelope for the TiBr₄ working fluid.

The last criterion for MHP selection and characterization is the dimensions of the MHP. In order for the MHP to remain suspended in the liquid PCM and for flows with natural
convection currents, the average density should be equal to the density of the liquid PCM (Yunus and Cimbala, 2006). The geometric parameters of the selected MHP are firstly extracted from (Hung and Tio, 2010, Chang and Hung, 2014), and then slightly amended so that the average density matches that of the liquid PCM (1622 kg/m$^3$, at average temperature between 355°C and 390°C). Table 15 lists the geometrical and thermo-physical parameters of the adopted MHP. The MHP was assumed to have been charged with 3.2 mg of TiBr$_4$. This mass of working fluid was found to be the optimal charge level for approximately similar MHP (Babin et al., 1990, Chang and Hung, 2014).

5.4 Effective thermal conductivity of MHP-PCM composites

A numerical approach has been developed for predicting the effective thermal conductivity of the MHP-PCM composite. The approach is described below:

5.4.1 The numerical procedure

Consider a 3D cubical body of pure PCM with a side length of several times the MHP length. This body of PCM is firstly divided into cubical cells each with a side length equal to the MHP length. Secondly, a single MHP is incorporated into each cubical cell and randomly oriented taking the centre of mass of each MHP as the origin (see Figure 45a). The MHP orientation is defined by a pair of angles ($\theta, \phi$) shown in Figure 45b. Thirdly, each cell is treated as a unidirectional fibrous composite consisting of pure PCM and a single MHP. Each cell’s effective thermal conductivity can then be calculated, as discussed below, based on the conductivities of the PCM ($K_{pcm}$) and the MHP ($K_{hp}$). Next, the thermal conductivity of the PCM is replaced with the effective thermal conductivity obtained from the previous incorporation step. That is, for the next incorporation of the MHPs, the PCM and the original incorporated MHPs are considered as a composite material with effective properties ($K_{pcm} = K_{pcm}^{eff}$ for the second incorporation step and beyond). Finally, a single MHP is

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incorporated again into each cell and the effective thermal conductivity is calculated based on the effective conductivity of the PCM ($K_{pcm}^{eff}$) and the conductivity of the MHP ($K_{hp}$). The incorporation process of the MHP is repeated until the desired MHP volume fraction is reached.

Table 15: Geometrical and thermo-physical parameters of the adopted MHP

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHP total length, $L_{hp}$ (m)</td>
<td>0.05&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>MHP side width, $w$ (m)</td>
<td>0.00147&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>MHP wall thickness, $t_w$ (m)</td>
<td>0.00014&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Wall material density (kg/m&lt;sup&gt;3&lt;/sup&gt;)</td>
<td>4500&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Wall material thermal conductivity (W/m.K)</td>
<td>17&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Working fluid density (kg/m&lt;sup&gt;3&lt;/sup&gt;)</td>
<td>3370&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

* Amended in order to have an average density equal to that of the liquid PCM.
<sup>a</sup>(Chang and Hung, 2014), <sup>b</sup>(MatWeb, 2015a) and <sup>c</sup>(MatWeb, 2015b).

Figure 45: (a) Cubic PCM body with a single MHP incorporated into each cell, (b) Fiber orientation in 3D space and (c) Definitions of $\theta_x$, $\theta_y$, and $\theta_z$. 

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5.4.2 Effective thermal conductivity calculation

There follows an explanation of how the effective thermal conductivity of a single cell was calculated. The effective thermal conductivity calculation was conducted separately and independently for each cell in the PCM cubic body.

In 2D unidirectional fibrous composites, the effective thermal conductivity can be calculated from (Feliciani and Takai, 2014):

\[ K_{eff,i} = K_1 \cos^2 \theta + K_2 \sin^2 \theta \quad (5-1) \]

where \( \theta \) is the angle that the fibers (the MHPs in the present chapter) make with the “i” axis, and \( K_{eff,i} \) represents the effective thermal conductivity along the “i” axis. \( K_1 \) and \( K_2 \), in Eq. (5-1), are the thermal conductivities parallel and perpendicular to the direction of the fiber (MHP) respectively, as given from (Feliciani and Takai, 2014):

\[ K_1 = \frac{1 + 2 \alpha \mu_i v_{hp}}{1 - \mu_i v_{hp}} K_{pcm} \quad (5-2) \]

\[ K_2 = \frac{1 + 0.5 \mu_2 v_{hp}}{1 - \mu_2 v_{hp}} K_{pcm} \quad (5-3) \]

where \( \alpha \) is the ratio of the MHP length to its hydraulic diameter. \( v_{hp} \) (which replaced \( v_f \) in Feliciani and Takai (2014)) is the MHP volume fraction, whereas \( K_{pcm} \) (which replaced \( K_m \) in Feliciani and Takai (2014)) is the thermal conductivity of the PCM. \( \mu_i \) and \( \mu_2 \) are given from:

\[ \mu_i = \frac{K_{hp}/K_{pcm} - 1}{K_{hp}/K_{pcm} + 2\alpha} \quad (5-4) \]
\[ \mu_2 = \frac{K_{hp}/K_{pcm} - 1}{K_{hp}/K_{pcm} + 0.5} \]  

(5-5)

where \( K_{hp} \) replaced \( K_f \) in Feliciani and Takai (2014). As reported by Fu et al. (1999), this model can be extrapolated to handle 3D fibrous (MHPs-PCM) composites by replacing the MHP with its projections onto the three planes xy, yz and xz (see Figure 45c). The 3D form of Eq. (5-1) is:

\[ K_{eff,x} = K_1 \cos^2 \theta_x + K_2 \sin^2 \theta_x \]  

(5-6a)

\[ K_{eff,y} = K_1 \cos^2 \theta_y + K_2 \sin^2 \theta_y \]  

(5-6b)

\[ K_{eff,z} = K_1 \cos^2 \theta_z + K_2 \sin^2 \theta_z \]  

(5-6c)

\( K_1 \) and \( K_2 \) are the same as above; however, \( \theta_x \), \( \theta_y \) and \( \theta_z \) are the angles that the projections of the MHP make with the x, y and z axes respectively (see Figure 45c). It is noted that the MHP projection lengths should be used instead of the MHP true length. The projection lengths can be different from plane to plane. Moreover, in order to account for the interaction between the MHPs, the effective volume fraction of the MHP should be used rather than the regular volume fraction \( (V_{hp}/V_{cell}) \) (Fu and Mai, 2003). The effective volume fraction of a single MHP, in the present chapter, is written as:

\[ V_{hp,i} = \frac{V_{hp}}{V_{cell} - (N-i)V_{hp}} \quad \text{where} \quad i = 1 \text{ to } N \]  

(5-7)

where \( N \) is the number of MHPs in each cell required to achieve the desired volume fraction, while \( i \) represents the number of the incorporation step. \( V_{hp} \) and \( V_{cell} \) are the volumes of a single MHP and the cubic cell respectively. Figure 46 shows a single cubic cell with different MHP volume fractions where the lines represent the centreline of the MHPs.
The calculation process can be summarized as follows:

1. After dividing the PCM into cells, incorporate a single MHP into each cubic cell, then randomly orient each MHP by assigning the orientation angles $(\theta, \phi)$. The orientation angles $(\theta, \phi)$ can be any values between 0 and $\pi$.

2. Replace each MHP with its projections in the three planes. Specifically, calculate the angles $\theta_x$, $\theta_y$ and $\theta_z$ as well as the projections lengths $L_{p,x}$, $L_{p,y}$ and $L_{p,z}$ for each MHP based on $\theta$ and $\phi$. The following approximations of the angles and lengths of the projections have been used:

\[
L_{p,x} = L_{hp} \sin\phi
\]  

(5-8a)
\[ L_{p,x} = \begin{cases} L_{hp} \cos \phi & 0 \leq \phi \leq \pi/2 \\ L_{hp} \cos (\pi - \phi) & \pi/2 < \phi \leq \pi \end{cases} \] (5-8b)

\[ L_{p,z} = L_{hp} \sin \theta \] (5-8c)

\[ \theta_x = \frac{\pi}{2} - \theta \] (5-9a)

\[ \theta_y = \begin{cases} \theta & 0 \leq \theta \leq \pi/2 \\ \pi - \theta & \pi/2 < \theta \leq \pi \end{cases} \] (5-9b)

\[ \theta_z = \begin{cases} \phi & 0 \leq \phi \leq \pi/2 \\ \pi - \phi & \pi/2 < \phi \leq \pi \end{cases} \] (5-9c)

3. For each cubic cell, calculate the effective thermal conductivities in the directions of x, y and z axes using Eq. (5-6). Note that Eq. (5-7) should be used to determine \( v_{hp,i} \) (\( i \), in the subscript, is equal to 1 for the first incorporation step and 2 for the second, etc.).

4. For the second incorporation step and beyond, update the thermal conductivity of the PCM in each cubic cell by replacing \( K_{pcm} \) with \( K_{pcm}^{eff} \) which varies from direction to direction and from cell to cell. \( K_{pcm}^{eff} \) is equal to \( K_{eff,x}^{pcm} \), \( K_{eff,y}^{pcm} \) and \( K_{eff,z}^{pcm} \) in the directions of x, y and z axes respectively.

5. Perform new incorporation step by adding a single MHP into each cubic cell, and assigning the orientation angles \( \theta \) and \( \phi \).

6. Repeat the steps above from 2 to 5 till the desired MHP volume fraction is reached.

7. The resulting set of \( K_{eff,x} \), \( K_{eff,y} \) and \( K_{eff,z} \) represents the effective thermal conductivities of each cell in the cubic PCM body.

8. The average effective thermal conductivity of a single cubic cell can be calculated from (Aboudi et al., 2012):
\[ K_{\text{eff}} = \frac{(K_{\text{eff,x}} + K_{\text{eff,y}} + K_{\text{eff,z}})}{3} \]  

(5-10)

The above procedure is applied independently and separately for each cell which results in an approximation of the effective thermal conductivity of a misaligned MHP-PCM composite.

In the present model, the following simplifications were made:

- The MHPs are uniformly distributed in the PCM.
- All the MHPs have the same dimensions.
- The effective thermal conductivity of the MHP is equal to 7,000 W/m.K as reported in Chang and Hung (2014).
- The presence of the MHPs has no effect on the PCM latent heat of fusion and specific heat as the adopted volume fraction is rather low (~0.07).

5.5 Effective thermal conductivity verification: model vs available data

Although there are neither numerical results nor experimental data regarding MHP-PCM composites, the data available on the topic of misaligned short-fiber composites can still be used for verification purposes. The approach developed was employed to predict the effective thermal conductivity of 3D random short-fiber composites. The properties used were adopted from Hatta and Taya (1985) which include \( \alpha = 100 \) and \( K_f/K_m = 20 \). The comparison between the predicted results and those reported in Hatta and Taya (1985) and Chou and Nomura (1981) are shown in Figure 47. Comparing with Hatta and Taya (1985) model, the present approach appears to over predict the effective thermal conductivity of misaligned short-fiber composites. However, the present predictions are still within the bounded solution of Chou and Nomura (1981) and close to the higher bound solution. It is believed that the omission of the thermal contact resistance (Chen et al., 2005) between the matrix and the fibers may have led to a slight overestimation of the effective thermal conductivity.
The model was also validated against experimental results obtained by Frusteri et al. (2005) in which they evaluated the influence of three different types of carbon fiber on thermal conductivity enhancement of an inorganic PCM. The same fibers and PCM used in Frusteri et al. (2005) were adopted in the present model. The comparison of the predicted thermal conductivity enhancement with their work is shown in Figure 48. As was emphasised by Frusteri et al. (2005), “the efficiency of the heat diffusion” is “a function of the homogeneity grade of the composite (PCM-fiber)”. That is, the present model predicted an enhancement of thermal conductivity of 20 times as compared to pure PCM at fiber loading of 10 vol%. However, the results reported in Frusteri et al. (2005) suggested an enhancement of approximately 8 times. The difference between the present predictions and the reported experimental results is attributed to the fact that fibers form agglomerates of different diameters resulting in less homogeneity of the composite. When the non-homogeneity of the composite is considered, the present model agrees well with the experimental data reported in Frusteri et al. (2005). The non-homogeneity was simulated in the model by assuming that the
fibers form agglomerates of an average diameter of 0.12mm. The present model is useful for estimating the upper limit of the thermal conductivity of homogenous fibrous (MHP-PCM) composites. In addition, the adopted approach considers separately the contribution of each single fiber, not the total number of the fibers as a whole, to the composite effective thermal conductivity. This feature cannot be found in the other available models such as that presented by Hatta and Taya (1985), Fu and Mai (2003) and more recently by Wang et al. (2009).

![Graph showing predicted thermal conductivity enhancement](image)

**Figure 48:** The predicted thermal conductivity enhancement of misaligned fiber-PCM composite compared to available experimental data. The properties used are $K_{pcm} = 0.47 \text{ W/m.K}$, $K_f = 180 \text{ W/m.K}$, $d_f = 6 \mu\text{m}$ and $L_f = 6 \text{ mm}$.

### 5.6 Effective thermal conductivity verification: model vs measured data

#### 5.6.1 PCM and HPs used in the experiments

An experimental validation was conducted to ensure the reliability of the model. Similar to Chapter 4, RT60 (low temperature PCM) was used for model validation. The thermo-physical properties of RT60 are as listed in Table 11 (rubitherm). Although the targeted HPs to use in
the MHP-PCM composite are micro HPs (see Table 15), miniature copper-water-charged HPs of 2 mm diameter and 100 mm long were used in the experiments. The miniature HPs were designed and manufactured especially for this work by Fujikura Ltd. The miniature HPs are shown in Figure 49, while their specifications are listed in Table 16.

![Figure 49: The miniature heat pipes used in the experiments.](image)

**Table 16: The specifications of the miniature HPs used in the experiments.**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter (m)</td>
<td>0.002</td>
</tr>
<tr>
<td>Total length (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Thermal conductivity (W/m.K)</td>
<td>9,000</td>
</tr>
</tbody>
</table>

**5.6.2 Experimental design and procedure**

An experimental facility was built for validation purposes consisting of an electrically-heated aluminium plate, water-cooled aluminium plate, a square acrylic container and MHP-PCM mixture. The entire rig was insulated using insulation sheets of 10cm thickness. Figures 50 and 51 show the experimental arrangement. The dimensions of each plate were 100mm×100mm×20mm. The experimental procedure was as follows: Firstly, components were assembled as shown in Figure 51. The hot plate was then heated to approximately 70°C through applying electrical power of 50W to electrical heater cartridges which were
embedded in the hot plate. After the hot plate had reached a constant temperature, the liquid PCM was poured in to fill the acrylic container. Secondly, different amounts of miniature HPs were added to the PCM. Extra care was taken in achieving as uniform as possible distribution of the miniature HPs. Next, the electric power was turned off and the whole system was allowed to cool down gradually whilst topping up of the liquid PCM to offset the volume shrinkage. Next, the water-cooled aluminium plate was placed on a set of stops attached to the acrylic container. Then, an electrical power of 25W was applied to the electrical heater cartridges and cooling water of 0.25 $L/s$ and 0.5±0.2°C commenced to circulating though the top Aluminium plate. Under these conditions, the hot plate was maintained at approximately 55°C, whereas the cold plate was maintained at approximately 1°C. Finally, recording of measurements commenced after all prerequisites were met. Copper based thermal paste was used to minimize the thermal resistance between the solidified PCM and the Aluminium plates.

![The experimental test rig.](image)

**Figure 50: The experimental test rig.**
The test facility was equipped with T-type thermocouples of 1.0mm overall diameter and accuracy of ±0.1°C. The thermocouples were attached to locations as shown in Figure 52. The test was repeated until the maximum experimental uncertainty fell below 5% (Coleman and Steele, 2009). The error bars are shown in Figure 53. The temperature fields, power input, inlet temperature and exit temperature of the cooling water were measured at 5 minutes intervals.

Figure 51: Schematic diagram of the test rig.
5.6.3 Experimental results and discussion

The effective thermal conductivities of several samples of MHP-PCM mixtures were measured. Particularly, mixtures containing 1%, 2% and 3% in volume of MHP were prepared. Nine embedded thermocouples were used to measure the temperature profile of each sample as well as the temperatures of the water inlet and outlet, the hot plate and the cold plate. When steady conditions were achieved, the temperature drop, $\Delta T$, across the MHP-PCM mixture was recorded. As the apparatus was well insulated, one-dimensional heat conduction through the MHP-PCM mixture can be assumed. Hence, the total heat transfer rate through each sample was assumed to be equal to the electrical power drawn by the electrical heater cartridges. By using Fourier’s law of heat conduction, the thermal conductivity of each sample can be calculated as:

$$K_{\text{eff}} = \frac{W a}{\Delta T A}$$

(5-11)
where $\dot{W}$ is the electrical power supplied to the hot plate, $a$ is the distance between any two thermocouples, $\Delta T$ is the temperature drop across the length $a$, and $A$ is the cross section area.

The geometry and the input data of the model were amended according to the PCM and the HPs described in Section 5.6. Figure 53 shows the experimental measurements and the numerical predictions. The numerical predictions are a little higher than the experimental measurements, which may have resulted from the thermal contact resistance between the PCM and the HPs not being counted in the present model. However, the numerical predictions are still comparable with the experimental measurements.

![Figure 53: Comparison between the experimental measurements and the numerical predictions.](image)

### 5.7 Results and discussion

Once the model was validated, numerical predictions of the MHP-PCM composite effective thermal conductivity were carried out. The PCM and MHPs adopted are the ones described previously in sections 5.2 and 5.3. The thermal conductivity of the MHPs was assumed to be
7,000 W/m.K as described in Chang and Hung (2014). Figure 54 presents the thermal conductivity enhancement of MHP-PCM composites as a function of the volume fraction of the MHP. As the MHPs volume fraction increased, the thermal conductivity enhancement increased to approximately 320 at a volume fraction of 20%. It can be seen that the thermal conductivity enhancement increased gradually up to a volume fraction of close to 0.05 and then it rose sharply. The sharp increase of the thermal conductivity enhancement at volume fraction of approximately 0.05 indicates that the thermal percolation threshold has been achieved. The thermal percolation threshold can be defined as the minimum MHP volume fraction at which a continuous connected MHPs network is formed for the transport of heat through the percolation pathway.

![Graph](image)

**Figure 54:** Thermal conductivity enhancement of MHP-PCM composite as a function of MHP volume fraction.

The results reported in Figure 54 along with a thermal network model which is similar to that presented in chapter 4 were adopted and used to estimate the required length of the HTF tube. As mentioned early in section 5.2, the HTF temperatures at the inlet and the outlet of the unit were assumed to be 280°C and 340°C respectively with LiCl/KCl mixture as PCM (Kelly and
Kearney, 2006, Robak et al., 2011a). Also, the unit capacity was set to 9h and the power output to 50MWe. The shell and tube configuration was adopted both because of its simplicity and popularity. A MHP volume fraction of 0.1 was assumed which is desired for heat storage applications (Bauer, 2011). The required mass flow rate of the HTF was assumed to be approximately of 0.7 kg/s per a HTF tube that of 0.125m outer diameter. Figure 55a shows the history of growth of solid PCM (with $v_{hp} = 0.1$) on a single HTF tube, whereas Figure 55b shows the required length of HTF tube to reach the desired outlet temperature of 340°C in cases of $v_{hp} = 0$ and $v_{hp} = 0.1$. As can be seen, using MHP-PCM composite with 0.1 MHP volume fraction has led to a reduction of 77% of the required HTF tube length.

![Figure 55: A single HTF tube for 50MW and 9h unit: (a) PCM build-up on a single HTF tube as a function of time (with $v_{hp} = 0.1$) and (b) HTF tube length in cases of $v_{hp} = 0$ and $v_{hp} = 0.1$.](image-url)
5.8 Chapter Summary

A new thermal enhancement method for LHTES using a MHP-PCM mixture was numerically analysed. The model was validated through comparison with experimental measurements conducted in this chapter as well as with experimental and mathematical data published in the literature. The results have shown that using MHP-PCM mixture has significantly reduced the required HTF tube length, as well as the total length of the LHTES unit, by 77%. As lightweight materials are desired for manufacturing the MHPs shell, the only combination of materials that found to satisfy the imposed MHP design criteria are Titanium as shell material and Titanium tetrabromide as working fluid. Consequently, it is expected that the current design will not be economically competitive unless other relatively cheap materials are used for manufacturing the MHPs. This will be further discussed in the following chapter where sizing and economic assessment of the LHTES units presented in Chapters 4 and 5 are conducted.

5.9 Nomenclature used in Chapter 5

\[ A \quad \text{Cross section area [m}^2\text{]} \]
\[ a \quad \text{Distance between two thermocouples [m]} \]
\[ K \quad \text{Thermal conductivity [W.m}^{-1}.\text{K}^{-1}\text{]} \]
\[ K_1 \quad \text{Thermal conductivity parallel to the MHP direction [W.m}^{-1}.\text{K}^{-1}\text{]} \]
\[ K_2 \quad \text{Thermal conductivity perpendicular to the MHP direction [W.m}^{-1}.\text{K}^{-1}\text{]} \]
\[ L_{hp} \quad \text{Total length of a MHP [m]} \]
\[ L_p \quad \text{MHP projection length [m]} \]
\[ N \quad \text{Number of MHPs in each cell that required to achieve the desired volume fraction} \]
\[ t_w \quad \text{MHP wall thickness [m]} \]
\[ \Delta T \quad \text{The temperature drop across the distance a [K]} \]
\[ V \quad \text{Volume [m}^3\text{]} \]
\[ v \quad \text{Volume fraction} \]
\( w \) MHP side length [m]

\( W \) Electric power [W]

Greek symbols

\( \alpha \) Aspect ratio (Eq. 2 and 3), \( \alpha = \frac{L_{hp}}{w \sqrt{3/3}} \)

\( \theta \) and \( \phi \) Orientation angles (Figure 1b) [deg]

\( \theta_x \) The angle between the projection of a MHP on \( xy \) plane and \( x \) axis direction [deg]

\( \theta_y \) The angle between the projection of a MHP on \( yz \) plane and \( y \) axis direction [deg]

\( \theta_z \) The angle between the projection of a MHP on \( zx \) plane and \( z \) axis direction [deg]

Subscripts

\( \text{eff} \) Effective

\( f \) Fiber

\( hp \) Heat pipe

\( i \) Incorporation step number \( i \)

\( m \) Matrix

\( pcm \) Phase change material

\( x \) In \( x \) axis direction

\( y \) In \( y \) axis direction

\( z \) In \( z \) axis direction

Abbreviations

CSP Concentrating solar power

MHPs Micro Heat pipes

HTF Heat transfer fluid

LHTES Latent heat thermal energy storage

PCM Phase change material

SHTES Sensible heat thermal energy storage

TCES Thermochemical energy storage
CHAPTER 6

System Sizing and Economic Assessment

6.1 Introduction

Based on numerical predictions and experimentation, this chapter includes estimates of the sizes and the costs of the LHTES units introduced in Chapters 4 and 5 for commercial CSP applications. The design of the LHTES unit presented in Chapter 3 is not included here as it is essentially an upgrade of a previous design which was presented by Shabgard et al. (2010) and economically assessed by Robak et al. (2011a). The desired LHTES unit should have sufficient capacity for 9 hours of thermal energy supply for generating the superheated steam required to operate a 50MW_e steam turbine continuously. The use of the numerical models included estimation of the size and the number of the HPs and the HTF tubes (or channels) as well as the overall size of the LHTES units and the required mass of the PCM. Additionally, economic evaluations of the systems were carried out.

6.2 System sizing

In modern CSP plants, the temperature of the HTF entering the LHTES unit is planned to be 280°C during the discharging process (Kelly and Kearney, 2006) with the overall HTF flow rate of approximately 3×10^6 kg/h (Luz_International_Ltd. et al., 1989). This temperature and the flow rate match the HTF temperature returning from the superheated steam generator and the flow rate in the 50MWe steam power plant, respectively. The HTF needs then to be
reheated to 340°C before being sent back again to the superheated steam generator system. Under these conditions, Robak et al. (2011a) have determined the thermal efficiency of a CSP plant, with an integrated LHTES unit, to be 0.353. In order to facilitate comparison with the storage units presented in Chapters 4 and 5, the same PCM will be adopted here. Specifically, through this chapter, a mixture of LiCl/KCl, of which the properties are listed in Table 14, is selected as PCM. Figure 56 shows a schematic diagram of a CSP plant with an integrated LHTES unit during night time operating conditions (discharging). The physical parameters of the LHTES units introduced in chapters 4 and 5 were based on preliminary optimization of the dimensions of the HTF tubes (or channels) as well as the size and spacing of the HPs for maximum storage capacity. Therefore, the parameters listed in Tables 12 and 15 (with HTF tubes of 0.125m outer diameter in the case of the design presented in chapter 5) are adopted in this chapter and considered as design parameters.

In order to estimate the size required for each LHTES unit, the numerical models presented in chapters 4 and 5 were used. Firstly, the known physical and operational parameters were
specified including the temperatures of the HTF at the inlet and outlet of the LHTES unit, the HTF mass flow rate per HTF tube, the properties of the HTF and the PCM, the dimensions of the HTF tubes and HPs, the targeted storage operation time, and the thermal efficiency of the Rankine cycle. Note that in order to calculate the HTF flow rate for a single HTF tube, the required number of HTF tubes (or channels) was initially assumed which would then be iteratively corrected. Next, the overall dimensions of the LHTES units (length, width and height), the number of the HTF tubes (or channels), and the number of HPs were determined and the capital cost was estimated.

By applying the symmetry conditions, unit model cells for example as shown in Figure 57 can be defined. These cells are assumed to be located in the centres of the LHTES units. Consequently, it is expected that the HTF enters these cells at an average temperature of 310°C, which is the average between the HTF temperatures at the inlets and the outlets of the LHTES units.

At the start of the discharging process, the PCM is assumed to be fully liquefied and at its solidification temperature. The total energy extracted from the PCM over the targeted storage operation time (9h) is recorded. Once the total extracted energy is determined, the increase of
the HTF temperature though each model cell can be calculated (as the HTF mass flow rate and the HTF properties are specified). Starting from the HTF inlet temperature, 280°C and considering the increase of the HTF temperature, the length of the HTF tube (or channel) required to achieve the outlet temperature, 340°C, can be found. When the HTF length has been identified the energy stored by a single HTF tube (or channel) can be readily determined. Subsequently, the total required energy is calculated by multiplying the net energy output of the plant (50MW) by the operation time (9h), divided by the thermal efficiency (0.353). The required total energy was found to be approximately 920,000 MJ. Once the total energy and the energy stored by a single HTF channel are known, the required number of HTF tubes (or channels) can be found.

The overall size of each LHTES unit can now be estimated since the required number of HTF tubes (or channels) and the length of the HTF tube (or channel) have been calculated. In addition, the required number of HPs can be found from knowing the total number of model cells (or the MHPs volume fraction for the unit presented in chapter 5). Tables 17 and 18 list the required physical parameters of the LHTES units introduced in chapters 4 and 5 respectively.

<table>
<thead>
<tr>
<th>HTF channel cross section (m)</th>
<th>HP diameter (m)</th>
<th>HP length (m)</th>
<th>Fin volume fraction</th>
<th>Unit model cell length, L_{m} (m)</th>
<th>HP number in each model cell</th>
<th>Mass flow per HTF tube (kg.s^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08 (height) 1 (width)</td>
<td>0.025</td>
<td>0.5</td>
<td>0.1</td>
<td>0.162</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>Electrical power output (MW)</td>
<td>Unit length (m)</td>
<td>Unit width (m)</td>
<td>Unit height (m)</td>
<td>Number of finned HP</td>
<td>number of HTF channel</td>
<td>PCM Mass (ton)</td>
</tr>
<tr>
<td>50</td>
<td>55</td>
<td>46</td>
<td>10^{a}</td>
<td>1,840,000</td>
<td>456</td>
<td>20,605</td>
</tr>
</tbody>
</table>

* Including an air gap of 14% of the total height to account for the expansion of the PCM during melting.
Table 18: Physical parameters of the LHTES unit presented in Chapter 5, for 9 h storage capacity.

<table>
<thead>
<tr>
<th>Electrical power output (MW)</th>
<th>Unit length (m)</th>
<th>Unit width (m)</th>
<th>Unit height (m)</th>
<th>Number of MHPs</th>
<th>number of HTF tubes</th>
<th>PCM Mass (ton)</th>
<th>HTF tube diameter (m)</th>
<th>MHPs volume fraction</th>
<th>Mass flow per HTF tube (kg.s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>61</td>
<td>61</td>
<td>7ᵃ</td>
<td>34×10⁹</td>
<td>1305</td>
<td>26,175</td>
<td>0.125</td>
<td>0.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>

ᵃ Including an air gap of 14% of the total height to account for the expansion of the PCM during melting.

6.3 Cost evaluations

As stated by Robak et al. (2011a), the capital costs of LHTES systems are associated with: (1) the exterior insulated container, (2) the storage medium (PCM), and (3) the internal heat transfer equipment including the HPs, the HTF tubes (or channels) and the fins. Furthermore, construction and assembly costs are included in the assessment along with addition of 10% of the total capital cost as an overhead percentage for the additional costs associated with necessary equipment, piping, wiring, valves, and insulation.

The adopted storage container is the same as that used in two-tank SHTES systems. It is well documented and its cost is available in the literature. The walls of the storage container are made of carbon steel and have an average thickness of 20mm (6mm at the top and 38mm at the bottom of the tank). The container is insulated with calcium silicate of 380 mm thickness. The total cost of the container involves the wall material and construction cost of US$4.40/kg, the foundation cost of US$688/m², and the insulation cost of US$200/m² (Kelly and Kearney, 2006).

The HP and the MHP costs are associated with the metallic container (wall and wick), the internal working fluid, the external fins (in case of the design presented in chapter 4), and the manufacturing costs. Considering the costs of: (1) the stainless steel (321 SS) wall and wick of the HPs at US$2.00/kg (Alibaba.com, 2015), (2) the working fluid (naphthalene) at
US$0.36/kg (Ganapathi and Wirz, 2012), (3) the Aluminium fins at US$2.425/kg (Yang et al., 2012) and (4) the estimated manufacturing cost of 5% of the material costs, a finned HP will cost approximately US$10.50. Correspondingly, a MHP will cost approximately US$0.0065 based on US$27.25/kg for the Titanium wall (Yang et al., 2012), and US$1320.00/kg for the working fluid (TiBr$_4$) (Scientific, 2015). These values representing the lower price limit of the HPs/MHPs as other factors were not included, especially the technical difficulty associated with fabrication of MHPs.

In the modelling described in this chapter, carbon steel HTF tubes (or channels) are adopted in order to reduce the capital cost. The cost of the HTF tubes (or channels) was calculated based on a thickness of 3mm and the dimensions listed in Tables 17 and 18. The unit price of carbon steel is US$0.8/kg (Robak et al., 2011a). Finally, the PCM (LiCl/KCl) is priced at US$0.50/kg (Robak et al., 2011a).

6.4 Results and discussion

The aim of this section was to give an overview of the capital cost of the LHTES units proposed in Chapters 4 and 5. The cost breakdowns for the LHTES units presented in Chapters 4 and 5 are shown in Table 19. Also listed in Table 19 are the cost breakdowns of the LHTES unit introduced by Robak et al. (2011a) as well as the two-tank SHTES system reported in Robak et al. (2011a) and Herrmann et al. (2004). It is noted that all the listed costs are subject to change as PCM, HPs, MHPs, stainless steel, carbon steel as well as labour costs are expected to change.

Compared with the existing two-tank SHTES system, the LHTES unit presented in Chapter 4 is found to be economically competitive and feasible for implementation in CSP plants as the future storage system. Specifically, its capital cost is 8% lesser than that of the two-tank SHTES system. Also from Table 19, the finned HPs were found to be the highest cost component for the LHTES unit representing 44.9% of the total LHTES unit cost. In addition,
and compared with the LHTES unit presented by Robak et al. (2011a), the unit presented in Chapter 4 was found to be 8.9% more expensive. However, the present design is still competitive as it addresses the potential issue of HTF leakage after prolonged usage associated with the design presented in Robak et al. (2011a).

Table 19: Capital costs for LHTES units presented in Chapters 4 and 5, LHTES unit introduced by Robak et al. (2011a) and two-tank SHTES system, for 9 h storage capacity.

<table>
<thead>
<tr>
<th>Equipment</th>
<th>LHTES(^a) cost (US$ MM)</th>
<th>LHTES(^b) cost (US$ MM)</th>
<th>LHTES(^c) cost (US$ MM)</th>
<th>SHTES(^d) cost (US$ MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy storage material</td>
<td>10.3(^f)</td>
<td>13.0</td>
<td>14.9</td>
<td>21.0</td>
</tr>
<tr>
<td>Storage container</td>
<td>6.58</td>
<td>8.94</td>
<td>4.8</td>
<td>10.4</td>
</tr>
<tr>
<td>HPs/MHPs</td>
<td>17.46</td>
<td>212.9</td>
<td>11.1</td>
<td>-</td>
</tr>
<tr>
<td>HTF tubes (or channels)</td>
<td>1.01</td>
<td>0.58</td>
<td>1.6</td>
<td>-</td>
</tr>
<tr>
<td>Molten salt heat exchanger</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.4</td>
</tr>
<tr>
<td>Molten salt pump</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
</tr>
<tr>
<td>Overhead (10%)</td>
<td>3.54</td>
<td>2.25(^g)</td>
<td>3.3</td>
<td>3.9</td>
</tr>
<tr>
<td>Total</td>
<td>38.89</td>
<td>237.67</td>
<td>35.7</td>
<td>42.3</td>
</tr>
</tbody>
</table>

\(^a\) LHTES unit introduced in chapter 4.
\(^b\) LHTES unit introduced in chapter 5.
\(^c\) LHTES unit introduced by Robak et al. (2011a).
\(^d\) Two-tank SHTES unit reported in (Robak et al., 2011a) and (Herrmann et al., 2004).
\(^f\) The required PCM quantity is less than that reported in Robak et al. (2011a) even though the overall size of the unit is larger. This was attributed to the higher effective thermal conductivity of the PCM resulted from the use of the Aluminium fins.
\(^g\) Excluding the MHPs cost.

In contrast, the LHTES unit presented in Chapter 5 is significantly more expensive than the two-tank SHTES system. That is, the total cost for this LHTES unit is 5.6 times higher than the two-tank SHTES system introduced in Herrmann et al. (2004). As can be seen from Table 19, the MHP cost is US$212.9 MM, which is 89.5% of the total capital cost of the LHTES unit. As a result, and in order for this design to be economically competitive, the MHPs must be fabricated from cheaper materials. For example, if the MHP had been made using ammonia as working fluid and aluminium as shell material, the capital cost would have been significantly lower. In this case, the capital cost would be reduced to just US$31.5 MM, which is 25.5% lesser than that of the two-tank SHTES system. Unfortunately, Aluminium
ammonia-charged HPs are not suitable for the targeted temperature range (> 300°C). Hence, researchers are encouraged to investigate other cheap potential combinations of shell material and working fluid in order to make feasible the use of MHPs-PCM mixture in CSP applications.

6.5 Chapter Summary

Two new designs of LHTES system were sized, economically assessed and compared against the two-tank SHTES system currently used in modern CSP plants. The comparative investigation was based on a storage capacity of 9h and power output of 50MWₑ. The economic analysis revealed that the LHTES unit presented in Chapter 4 is 8% cheaper than the two-tank molten salt system. This shows that the unit described in Chapter 4 has the potential to be implemented in future generation CSP plants. In contrast, the design described in Chapter 5 is 5.6 times more expensive than the two-tank SHTES system. This is due to the high cost of the MHPs which represented 89.5% of the total capital cost of the LHTES unit. Using cheaper materials for manufacturing MHPs can reduce the capital cost by over 80%. However, and based on the literature review conducted during this project, the options for suitable lightweight materials at the desired temperature range (280 to 390°C) are limited.
CHAPTER 7

Conclusions and Recommendations for Future Work

7.1 Conclusions

In this research program, three different approaches to improving the thermal performance of LHTES units for CSP applications were numerically and experimentally investigated. Furthermore, two of the introduced LHTES units were sized and economically assessed. The aim was to improve the overall thermal conductivity of the PCM by utilising HPs as heat transfer promoters. Overall, the HPs were found capable of significantly improving the thermal performance of the presented LHTES units. Moreover, they were found to be the most expensive component in the LHTES units and HP costs dominate the capital costs of the presented LHTES units. Therefore, it was concluded that using competitively priced HPs will lead to economically competitive LHTES system and vice versa.

7.1.1 LHTES system with embedded axially finned HPs

In Chapter 3, the heat transfer improvement of high temperature LHTES units resulting from utilising axially finned HPs was studied experimentally and numerically. The thermal resistances of the HPs were integrated into a new set of discretisation equations which involve the PCM domain. The model validation has shown close agreement between the experiment and numerical results, which supports the validity of the mathematical approach and the assumptions that were made. An enhancement of 86% of thermal energy storage was
achieved through utilising finned HPs instead of bare HPs. Moreover, an improvement of 24% and 34% in the overall HPs effectiveness was achieved as a result of using HPs equipped with four and five fins, respectively. The associated reductions in the thermal storage capacity were found to be 0.86% for the former and 1.07% for the latter case.

As this design involves studding HPs into HTF channels, it is complex in fabrication and possesses risks of structural failure (leakage) because of repetitive heating and cooling under harsh conditions which reduces the reliability of the system. In particular, if just one HP from the 3 million HPs caused leakage, the whole system would need to be shut down for leakage maintenance. In addition, the HTF and the PCM would mix together adding another maintenance issue. As a result, this technique is not recommended for use for storage units in CSP plants.

7.1.2 Performance of suspended finned HPs in high-temperature LHTES

In the case of using suspended finned HPs as described in Chapter 4, the results have shown that the thermal performance of the LHTES unit with suspended finned HPs was improved significantly after 45 min regardless of the poor thermal response at the initial stage. That is, the effectiveness of the 12-HP configuration reached 2.4 after 5h of simulation which is equivalent to an energy extraction increase of 140% comparing with the baseline configuration. The required quantity of storage material to drive a 50MW_e turbine for 9h has decreased by 51% and 81% comparing with the two-tank molten salt and concrete storage systems, respectively. Furthermore, the numerical model, which was validated against experimental measurements obtained in the present project as well as against experimental results published in the literature, was used to find a correlation for predicting the time the system spends in the start-up condition (the time required to reach $\varepsilon_{hp} = 1$).
Regarding the deficiency of possible HTF leakage associated with the LHTES as presented in Chapter 3, the present design has fully addressed this issue as the HPs are kept in suspension adjacent to the HTF channel rather than penetrating it.

### 7.1.3 MHP-PCM composite system

The benefits of utilising a MHPs-PCM mixture to enhance the thermal performance of LHTES systems were numerically and experimentally investigated as described in Chapter 5 of this thesis. A numerical model was developed to predict the effective thermal conductivity of the MHPs-PCM mixture as a function of the MHPs volume fraction. Compared with experimental measurements conducted in this project as well as with experimental and mathematical data published in the literature, the present numerical predictions were found to be slightly over predicted. This was attributed to the fact that the present numerical model omits the surface thermal resistance between the MHPs and the PCM. However, the numerical results are still satisfactory in relation to the conducted and published experimental measurements. The present numerical model along with the model presented in Chapter 4 was used to predict the required HTF tube length to satisfy the desired HTF outlet conditions during the discharging process. The results have shown that using MHP-PCM mixture has significantly reduced the required HTF tube length, and therefore the total length of the LHTES unit, by 77%.

### 7.1.4 System sizing and economic assessment

The last chapter of the thesis presents an economic assessment of the LHTES systems presented in Chapters 4 and 5. The two systems were sized, economically assessed and compared against the two-tank SHTES system currently used in modern CSP plants. The comparative investigation was based on a storage capacity of 9h and power output of 50MWₑ. The economic analysis revealed that the capital costs of the LHTES units presented in Chapters 4 and 5 are 8% less and 460% higher respectively than that of the two-tank molten
salt system. The substantial difference between the capital costs of the two analysed LHTES units is attributed to the extremely high cost of the MHPs used in the LHTES unit introduced in Chapter 5 which represents 89.5% of the capital cost. Using cheaper materials for manufacturing MHPs can reduce the capital cost by over 80%. However, based on the literature review conducted during this thesis, the options of appropriate lightweight materials which are also compatible with the known HP’s working fluid at the temperature range between 280 and 390°C, are very limited. Consequently, it is concluded that the unit described in Chapter 4 has the potential to be implemented in future generation CSP plants. On the other hand, the unit presented in Chapter 5 is found to be economically uncompetitive unless another combination of materials is used instead of Titanium and Titanium tetrabromide. This matter is still under investigation and will be a subject of future work. However, the MHPs-PCM mixture has a major potential benefit for other applications. For example, much cheaper lightweight compatible combinations of materials such as Aluminium and Ammonia are available for moderate temperature ranges (<300°C). Furthermore, even heavy materials such as stainless steel with naphthalene as working fluid can be freely used in the field of satellite thermal management where there is no any gravity force.

Regarding the LHTES unit presented in Chapter 3, which may be regarded as an upgraded design of a previous LHTES unit which has been economically assessed by Robak et al. (2011a), it can be concluded that the use of finned HPs, instead of bare HPs, has reduced the number of required HPs by 30%.

The uncertainties associated with the experimental works conducted in chapters 3, 4 and 5 were found to be 6.5%, 5% and 5% respectively.

7.2 Recommendations:

The positive outcomes from this research work encourage further work to be conducted on LHTES with embedded HPs. The recommended future work can be summarised as follows:
Future study should focus on the promising finned HP configurations investigated in this research. That is, using different fin materials can lead to more enhancements beyond that reported in this research; for example the fins could be made from graphite foil which provides high thermal conductivity, low density and good corrosion resistance against nitrate and nitrite salts. Furthermore, extending the study to different HP working fluids, PCMs and wall materials, can decrease (or increase) the capital cost of the LHTES units. In addition, quantifying the significance of the fins as well as the impact of the convection currents during melting are warranted.

The numerical models developed in this work could be extended to assess the effects of PCM volume change during melting and freezing cycles. In addition, and if more time and funding had permitted, a prototype LHTES unit could have aided the research, and provided valuable real data on the thermal performance of such units under repetitive melting and freezing cycles.

The proposed LHTES units were sized based on a preliminary optimization. Therefore, a detailed optimization scheme to augment the thermal performance of such units is highly recommended, and it will be the subject of future research.

An investigation on the potential benefits of using the proposed heat pipe technique in other engineering applications is highly recommended, and it will be the subject of future research.

Finally, and as related to the extremely high cost of the MHPs reported in Chapter 5 and economically assessed in Chapter 6, further investigation of potential combinations of lightweight materials for the desired temperature range (280°C to 400°C) is highly encouraged. However, the price of the proposed MHPs is expected to be relatively high as new technologies are usually initially very expensive, but the costs tend to decrease considerably when these technologies become fully established.
References


Appendixes

Appendix A: Pictures and descriptions of test facilities used during models verification.

Figure 58: The plate shaped heat exchanger: the Copper plate (left), the Acrylic cover (middle) and the heat exchanger assembly (right).

Figure 59: The test facility used in chapter 4 and 5.
Figure 60: The assembly steps during the preliminary test of HP-PCM mixture (HPs of 300mm length were firstly used to prove the concept of HP-PCM mixture).
Appendix B: The Matlab source codes developed in this research.

Here just the main Matlab codes are provided. The user-defined functions developed by the author are not provided. Also, the flow chart of the global calculation procedure is depicted below.

![Flow chart of the global calculation procedure](image)

Figure 61: Flow chart of the global calculation procedure
1. The numerical source code presented in chapter 3.

```matlab
clear all;
close all;

% ------------------- Input data -------------------
Nf = 4;  % number of the fins in a HP
Theta = 360/Nf;  % [degrees] the angle between two fins
Theta = Theta/2;
Ntheta = 100;  % number of control volumes in Theta direction
N = 4;  % number of the HPs in the model unit cell

% ------------------- Input data -------------------
hhp = 180;  % heat coefficient between the HP and the HTF
THTF_Ch = 391;  % [C] HTF temperature Charging
THTF_Dis = 295;  % [C] HTF temperature Discharging

% ------------------- Input data -------------------
stop = 4*60*60;  % [Sec] the end of time range=>
dt = 5;  % time step
Ndt = 40;  % After how many time step do u want to print results

underr = [0.78 0.05];  % under relaxation factor
underr1 = [1 0.6];  % under relaxation factor2
convv = [-4 -1];
num_iter = 100000;  % number of iteration sufficient to reach convergence

fin=fopen('Input_finnedHP_SaltKNO3.txt');% open the input file named Inputs.txt
c=textscan(fin,'%s %s %f', 'delimiter', ','); % scan the file
fclose(fin);  % close the file

% =============== Assign values to variables from the input file =========
T1 = property ('PCM melting point',c) ; % [C] Melting point of PCM
T2 = property ('PCM solidifing point',c) ; % [C] Melting point of PCM
Rho_PCM_l = property ('Liquid PCM density',c) ;  % (kg/m3) Rho = Liquid PCM Density
Rho_PCM_s = property ('Solid PCM density',c) ;  % (kg/m3) Rho = Solid PCM Density
Rho_fin = property ('Fins density',c) ;  % (kg/m3) Rho = Fins Density
Cp_PCM_l = property ('Liquid PCM specific heat',c) ; % (J/(kg K)) Liquid PCM specific heat
Cp_PCM_s = property ('Solid PCM specific heat',c) ;  % (J/(kg K)) Solid PCM specific heat
Cp_f = property ('Material specific heat',c) ;  % (J/(kg K)) HTF specific heat
Cp_hp = property ('HP wall specific heat',c) ;  % (J/(kg K)) HP wall specific heat
Lm = property ('Module Length',c) ;  % [m] length of the module
Le = property ('Evaporator section length',c) ;  % [m] Heat pipe section in contact with HTF
Lc = property ('Condenser section length',c) ;  % [m] Heat pipe section in contact with PCM
Ld = property ('Adiabatic section length',c) ;  % [m] Adiabatic section length

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\[ dT = \text{property (Melting range,} c\text{)}; \quad \%[\text{k}] \text{Melting range} \]
\[ M_{\text{PCM}} = \text{property (PCM dynamic viscosity,} c\text{)}; \quad \%[\text{Pa.s}] \text{PCM dynamic viscosity} \]
\[ M_f = \text{property (HTF dynamic viscosity,} c\text{)}; \quad \%[\text{Pa.s}] \text{HTF dynamic viscosity} \]
\[ Pr_{\text{PCM}} = \text{property (PCM Prandtl number,} c\text{)}; \quad \% \text{PCM Prandtl number} \]
\[ Pr_{\text{HTF}} = \text{property (HTF Prandtl number,} c\text{)}; \quad \% \text{HTF Prandtl number} \]
\[ B = \text{property (PCM thermal expansion coefficient,} c\text{)}; \quad \%[\text{K}] \text{PCM thermal expansion coefficient} \]

%======================================================================
%------
Fin details
-----------------------
\[ L_f = 2*2*R_{\text{hp out}}; \quad \% \text{fin length (twice the hp diameter)} \]
\[ X_f = W_{\text{thick}}+\text{Thick}_{\text{hp}}; \quad \% \text{fin thickness (equal to haet pipe wall thickness + wick thickness)} \]

\[ \text{ThFin} = \text{ThFin}(2*(R_{\text{hp out}+1})); \quad \% \text{calculate theta for the fin} \]

\[ N_{\text{theta}} = \text{ceil(ThFin*(Ntheta)/deg2rad(Theta))}; \quad \% \text{number of control volumes in Theta direction in fin region} \]
\[ N_{r} = \text{ceil(Lf*(Nr)/(Lr-R_{\text{hp out}}))}; \quad \% \text{number of control volumes in r direction in fin region} \]

\[ t = 0; \quad \% \text{end program} = 0; \quad \% \text{counter1} = 0; \quad \% \text{counter2} = 0; \quad \% \text{iResults} = 0; \quad \% \text{n} = 0; \quad \%
\[ nR = \text{stop(Ndt*dt)}; \quad \% \text{number of results will be printed} \]
\[ \text{under} = \text{underr}(1); \quad \% \text{under1} = \text{underr1}(1); \quad \% \text{cov} = \text{conv}(1); \quad \%
\[ \text{Process} = \text{input(' press M for melting or S for solidification } \Rightarrow \text{ ',} \text{',}'\text{');} \quad \%
\[ \text{if Process} == \text{'}m\text{'} || \text{Process} == \text{'}M\text{'} \]
\[ \text{THTF} = \text{THTF}_{\text{Ch}}; \quad \% \text{Thermal Conductivity of a control volume (heat capacity)} \]
\[ \text{Tm} = \text{T1}; \quad \%
\[ \text{elseif Process} == \text{'}s\text{'} || \text{Process} == \text{'}S\text{'} \]
\[ \text{THTF} = \text{THTF}_{\text{Dis}}; \quad \% \text{Thermal Conductivity of a control volume (heat capacity)} \]
\[ \text{Tm} = \text{T2}; \quad \% \text{else} \quad \%
\[ \text{exit}; \quad \%
end
\%
--------
Matrices declaration
---------------------------
\[ xu = \text{ones(Nr,Ntheta+1)}; \quad \%
\[ x = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ dx = \text{ones(1,Ntheta)}; \quad \%
\[ yu = \text{ones(1,Nr+1)}; \quad \%
\[ y = \text{ones(1,Nr+2)}; \quad \%
\[ dy = \text{ones(1,Nr)}; \quad \%
\[ K = \text{ones(Nr,Ntheta)}; \quad \% \text{Thermal conductivity of a control volume (heat capacity)} \]
\[ C = \text{ones(Nr,Ntheta)}; \quad \% \text{[J/m3.K]} \text{Specific heat per unit volume (heat capacity)} \]
\[ Kx = \text{ones(Nr,Ntheta+1)}; \quad \% \text{Thermal conductivity at the interfaces along x axis} \]
\[ ky = \text{ones(Nr+1,Ntheta)}; \quad \% \text{Thermal conductivity at the interfaces along y axis} \]
\[ sp = \text{ones(Nr,Ntheta)}; \quad \% \text{Temperature dependant source term} \]
\[ sc = \text{ones(Nr,Ntheta)}; \quad \% \text{Constant source term} \]
\[ a = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ b = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ c = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ d = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ T = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ f = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ g = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ P = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ Q = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ T0 = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ TT = \text{ones(Nr+2,Ntheta+2)}; \quad \%
\[ \text{TiMe} = \text{ones(1,NRe)}; \quad \% \text{array contains the time corresponding to the results printed} \]
\%
--------
Initial condition
------------------------
\[ \text{TT}(:,:) = \text{Tm}; \quad \%
\%
---
fidP1 = \text{fopen('Result1_exp.m', 'w')}; \quad \%
\text{fidP2} = \text{fopen('Result2_exp.m', 'w')}; \quad \%
\[ \text{[dx xu x dy y u r Theta]} = \text{xu_x_yu}(dx,xu,x,dy,yu,y,Theta,Ntheta,Lr,Nr,R_{\text{hp out}},N_{\text{theta}},N_{r},Lf,\text{ThFin}); \quad \% \text{call function for dividing} \]
\[ \text{[R_total]} = \text{Resistances(Thick_{hp},K_{hp},R_{\text{hp out}},Lc,L,e,Ld,W_{\text{thick}},W_{\text{khp}})}; \quad \% \text{Call function for calculate heat pipe elements' resistance} \]
while t < stop;
    tic
    n=n+1;
    disp (' New time step');
    t=t+dt;
    starter = 0;
    counter1 = counter1 +dt;
    T(:,:,)=TT(:,:,);
    rj=0; % a counter for the loop of nonlinearity
    Max1 = 1;

    % pause
    while Max1 > 10^cov
        disp (' **** nonlinearity ***** ');
        rj=rj+1;
        starter = starter +1;
        if starter <= 50
            under = underr(1);
            under1 = underr1(1);
            cov = convv(1);
        elseif starter > 50
            under = underr(2);
            under1 = underr1(2);
            cov = convv(2);
        end
        To(:,:,)= TT(:,:,);
        [K kk ky C] = K_k(TT,Tm,dT,K,kx,ky,yu,y,K_l,K_s,K_fin,Rho_PCM_l,Rho_PCM_s,Cp_PCM_l,Cp_PCM_s,Rho_fin,Cp_fin,C,hsl,Ntheta,Nr,Nf_theta,Nf_r,under1); % call function for calculate K_faces
        [sp sc] = source(sp,sc,Ntheta,Nr); % call source function
        [a b c d f g] = a_b_c_d_boun(TT,kx,x,ky,y,a,b,c,d,f,g,Ntheta,Nr,R_total,THTF,Rhp_out,Lc,r); % call boun(a)(b)(c)(d)s.function
        [a b c d f g] = a_b_c_d_int(kx,x,dx,ky,y,dy,sp,sc,T,a,b,c,d,f,g,dt,Ntheta,Nr,r,yu,C);% call (a)(b)(c)(d)(f)(g).s function
        %[TT endprgram] = solve1(TT,a,b,c,d,f,g,num_iter,Ntheta,Nr,endprgram,under); % call solve function
        %[TT endprgram] = solve2fin(TT,a,b,c,d,f,g,num_iter,Ntheta,Nr,endprgram,under,cov);
        %TT(1:20,1:20)
        Max1=max(max(abs(To-TT))); % Test converge
        if rj == num_iter+1
            disp (' the iteration process has diverged ');
            disp (' The internal loop has been broken ');
            endprgram = 1;
            break;
        end
        if endprgram ==1;
            break;
        end
        if counter1 >= Ndt*dt
            counter2 = counter2+1;
            iResults = iResults+1;
            % call print function
            [dx] = printResult(dx,xu,x,dy,yu,y,K,kx,ky,sp,sc,a,b,c,d,f,g,P,Q,TT,t,C,Ntheta,Nr,fidP1,fidP2,nRe,counter2,iResults);
            TiMe(iResults)=t;
            % bnb = input('bnb ==>   ');
            counter1 = 0;
        end
    end
    timeMeasure(n)= toc
end
fclose(fidP1);
fclose(fidP2);
%------------------------------------------------------------------------
%======================== Input data ===================================
%------------------------------------------------------------------------
clear all

global n m Lmda Apha Eta Eta2 Sig Xi THTF Tm x St ht K A Q y Qe Lm Rt_out

Npcm = 10; %number of PCM elements adjacent to HP
Mpcm = 15; % number of PCM elements adjacent to HTF tube

TimE = 3; % how many operation hours do you want to simulate
Dt = 5; % [sec] Time stip

THTF_Ch = 391; % [C] HTF temperature Charging
THTF_Dis = 295; % [C] HTF temperature Discharging

N = 4; % Number of heat pipes in a module

ht = 35000; % heat coefficient between the HTF tube and the HTF
hhp = 180; % heat coefficient between the HP and the HTF

g = 9.80665; % [m/s^2]

Timer = TimE*Dt/60;

%========================================================================

Process = input(' press M for melting or S for solidification ===> ', 's');
n = Npcm+6; % calculate the number of last element in the PCM adjacent to HP
m = Mpcm+1; % calculate the number of last element in the PCM adjacent to HTF tube

%========================================================================

% open the file which contains HP, HTF tube, HTF, and PCM properties
fin=fopen('Inputs_kno3.txt');% open the input file named Inputs.txt
c=textscan(fin,'%s %s %f', 'delimiter', ','); % scan the file
fclose(fin);

% =============== Assign values to variables from the input file ==========
Tm = property ('PCM melting point',c) ; % [C] Melting point of PCM
Rho_PCM = property ('PCM density',c) ;  % (kg/m3) Rho =  Solid PCM Density
Cp_t = property ('HTF tube specific heat',c) ; % (J/(kg K)) Solid PCM specific heat
Cp_f = property ('HTF specific heat',c) ; % (J/(kg K)) HTF tube specific heat
Cp_hp = property ('HP wall specific heat',c) ; % (J/(kg K)) HP wall specific heat
Lm = property ('Module Length',c) ; % [m] length of the module
Le = property ( 'Evaporator section length',c) ; % [m] Evaporator section length
Lc = property ( 'Condenser section length',c) ; % [m] Condenser section length
Le = property ( 'Adiabatic section length',c) ; % [m] Adiabatic section length
Thick_t = property ('HTF tube wall thickness',c) ; % [m] HTF tube wall thickness
Thick_hp = property ('HP wall thickness',c) ; % [m] HP wall thickness

hl = property ('PCM latent heat of fusion',c) ; %[J/kg] Latent heat of fusion of the PCM
Rho_HTF = property ('HTF density',c) ;  % (kg/m3) Rho =  HTF Density
Rho_HP = property ( 'HP wall density',c) ; % (kg/m3) Rho = HP wall density

W_poros = property ( 'Wick porosity',c) ; % Wick porosity
W_thick = property ( 'Wick thickness',c) ; %[m] Wick thickness

M_PCM = property ( 'PCM dynamic viscosity',c) ; %[Pa.s] PCM dynamic viscosity
M_f = property ( 'HTF dynamic viscosity',c) ; %[Pa.s] HTF dynamic viscosity
Pr_PCM = property ( 'PCM Prandtl number',c) ; % PCM Prandtl number
Pr_HTF = property ( 'HTF Prandtl number',c) ; % HTF Prandtl number
B = property ( 'PCM thermal expansion coefficient',c) ; %[K-1] PCM thermal expansion coefficient

Mas_flow = property ( 'HTF Mass Flow',c) ; % [kg/s] HTF Mass Flow
k_fin = property ( 'Fins thermal conductivity',c) ; %[w/m.k] Fins thermal conductivity
Rho_fin = property ( 'Fins density',c) ; %[kg/m^3] Fins density

cp_fin = property ( 'Fins specific heat',c) ; %[J/kg,k] Fins specific heat

%========================================================================
calculate the effective properties

Time_of_one_effectiveness = ones(1,7);
bad_config= ones(1,4);
i_time_one = 0;
i_time_two = 0;

\[ v_f_C = [0.01, 0.05, 0.1]; \]
\[ k_{fin}_C = [200, 2000, 400]; \]
\[ D_{hp}_C = [0.006, 0.008, 0.01, 0.012, 0.015, 0.018, 0.02, 0.022, 0.025]; \]
\[ D_{t}_C = [0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6]; \]

for i_vf = 1:length(vf_C)
    for i_kfin = 1:length(kfin_C)
        for i_Dhp = 1:length(Dhp_C)
            for i_Dt = 1:length(Dt_C)

                \[ v_f = v_f_C(i_vf); \]
                \[ v_s = 1-v_f; \]
                \[ k_{fin} = k_{fin}_C(i_kfin); \]
                \[ R_{hp\_out} = D_{hp}_C(i_Dhp)/2; \]
                \[ R_{t\_out} = D_{t}_C(i_Dt)/2; \]
                \[ D_{hp}_C(i_Dhp); \]
                \[ D_{t}_C(i_Dt); \]

                if Process == 'm' || Process == 'M'
                    \[ n_\_ = 0.3; \]
                    n_\_ is constant for Nusselt number equation
                elseif Process == 's' || Process == 'S'
                    \[ n_\_ = 0.4; \]
                end

                \[ V_{mean} = (M_{as\_flow})/(R_{hTF}\_base\_diameter)^2; \]
                \[ Re_{h} = V_{mean}\cdot(R_{t\_out}\_base\_diameter)/R_{hTF}/\rho_{HTF}; \]
                \[ h_{t} = \frac{K_{f}}{2}\cdot\frac{(R_{t\_out}-\text{Thick\_t})^2}{0.023}\cdot Re_{h}^{0.8}\cdot Pr_{HTF}\cdot n_{\_}; \]
                \[ \text{heat coefficient between the HTF tube and the HTF} \]

                \[ \text{fin\_factor} = \frac{v_f k_{fin}}{(v_s K_{s})}; \]
                \[ \text{Factor}_C = \text{fin\_factor} \cdot (R_{t\_out}/R_{hp\_out}); \]

                \[ L_{fin} = \frac{L_{m}}{2\cdot R_{hp\_out}}; \]
                \[ Rho_{eff} = \frac{v_s R_{PCM}+v_f R_{hTF}}{Rho_{fin}}; \]
                \[ cp_{eff} = \frac{Rho_{PCM}/Rho_{eff}}{v_s \cdot cp_{PCM}+Rho_{fin}/Rho_{eff}}; \]
                \[ hsl_{eff} = \frac{Rho_{PCM}/Rho_{eff}}{v_s \cdot hsl}; \]
                \[ K_s_{eff} = \frac{v_s K_s + v_f k_{fin}}{v_f k_{fin}}; \]
                \[ K_l_{eff} = \frac{v_s K_l + v_f k_{fin}}{v_f k_{fin}}; \]

                \[ T_{c} = \text{ones(Timer, n+m); \ [C] temperature of elements Conduction dominated} \]
                \[ T_{v} = \text{ones(Timer, n+m); \ [C] temperature of elements Convection dominated} \]
                \[ Eta = \text{ones(2, n+m); \ Eta = parameter Equation (12) Note first row for conduction dominated} \]
                \[ t = \text{ones(1, Timer); \ [% sec] time} \]
                \[ tt = \text{ones(1, Timer); \ [% sec] time} \]
                \[ Xi_{c} = \text{ones(2, n+m); \ Xi = parameter Equation (15) conduction dominated} \]
                \[ Xi_{v} = \text{ones(2, n+m); \ Xi = parameter Equation (15) convetion dominated} \]
                \[ L_{mda} = \text{ones(2, n+m); \ Lambda = characteristic length for conduction (element thicknesses)} \]
                \[ Apha = \text{ones(2, n+m); \ Alpha = thermal diffusivity} \]
                \[ Sig = \text{ones(2, n+m); \ Sigma variant = parameter Equation (14)} \]
                \[ Qt = \text{zeros(2, Timer); \ [%] stored energy due to HTF tube without HPs} \]
                \[ Qtt = \text{zeros(2, Timer); \ [%] stored energy due to HTF tube without HPs} \]

                \[ q = \text{ones(n+m, 2); \ % damy matrix} \]
                \[ qt = \text{ones(n+m, 2); \ % damy matrix} \]

                \[ Qhp = \text{zeros(2, Timer); \ [%] stored energy due to HP} \]
                \[ Qt_{hp} = \text{zeros(2, Timer); \ [%] stored energy due to HTF tube with HPs} \]

                \[ V = \text{ones(2, n+m); \ [%] volume of PCM elements} \]
                \[ st = \text{ones(2, Timer); \ [%] solid–liquid interface location adjacent to HTF tube} \]
                \[ slp = \text{ones(2, Timer); \ [%] solid–liquid interface location adjacent to HP} \]
                \[ A = \text{ones(2, n+m); \ [%] average cross-sectional area of the element perpendicular to heat transfer direction} \]

                \[ E = \text{ones(1, n+m); \ Epsilon = heat pipe effectiveness} \]
                \[ EG = \text{ones(1, n+m); \ Eg = generalized heat pipe effectiveness} \]
                \[ rt = \text{ones(2, n+m); \ % element distances for HTF tube and adjacent PCM} \]
                \[ rhp = \text{ones(2, n+m); \ % element distances for HP and adjacent PCM} \]
                \[ r_c = \text{ones(2, n+m); \ % element distances for all the system Conduction dominated} \]
                \[ r_v = \text{ones(2, n+m); \ % element distances for all the system Convection dominated} \]

                \[ L = \text{ones(2, n+m); \ % element length} \]
                \[ K = \text{ones(2, n+m); \ % Thermal conductivity} \]

                \[ power_{throughput} = \text{zeros(1, Timer); \ % effective power throughput by a HP} \]
for i = 1:Timer;
    t(i)=i*Dt;
end

Rt_in= Rt_out - Thick_t; % [m] inner diameter of HTF tube
Rhp_in= Rhp_out - Thick_hp; % [m] inner diameter of HP wall
rhp(:,1) = Rhp_in - W_thick;
rhp(:,2) = Rhp_in;
rhp(:,3) = Rhp_out;
rt(:,1) = Rt_in;
rt(:,2) = Rt_out;
c(1,1)= rhp(1,2);
c(2,1)= rhp(1,3);
c(1,2)= rhp(1,1);
c(2,2)= rhp(1,1);
c(1,3)= rhp(1,1);
c(2,3)= rhp(1,2);
c(1,4)= rhp(1,2);
c(2,4)= rhp(1,3);
c(1,5)= rhp(1,1);
c(2,5)= rhp(1,2);
c(1,6)= rhp(1,2);
c(2,6)= rhp(1,3);
c(1,n+1)= rt(1,1);
c(2,n+1)= rt(1,2);

%-----------------

r_v(1,1)= rhp(2,2);
r_v(2,1)= rhp(2,3);
r_v(1,2)= rhp(2,1);
r_v(2,2)= rhp(2,2);
r_v(1,3)= rhp(2,1);
r_v(2,3)= rhp(2,2);
r_v(1,4)= rhp(2,2);
r_v(2,4)= rhp(2,3);
r_v(1,5)= rhp(2,1);
r_v(2,5)= rhp(2,2);
r_v(1,6)= rhp(2,2);
r_v(2,6)= rhp(2,3);
r_v(1,n+1)= rt(2,1);
r_v(2,n+1)= rt(2,2);

K (:,1)= K_hp;
K (:,2)= W_k;
K (:,4)= K_hp;
K (:,3)= W_k;
K (:,6)= K_hp;
K (:,5)= W_k;
K (:,n+1)= K_t;

Apha (:,1)= K_hp/Rho_hp/Cp_hp;
Apha (:,2)= W_k/W_HC;
Apha (:,4)= K_hp/Rho_hp/Cp_hp;
Apha (:,3)= W_k/W_HC;
Apha (:,6)= K_hp/Rho_hp/Cp_hp;
Apha (:,5)= W_k/W_HC;
Apha (:,n+1)= K_t/Rho_t/Cp_t;

st(:,1)=(10*10^(-6)+Rt_out);
shp(:,1)=(10*10^(-6)+Rhp_out);

T_c(:,1)=Tm+0.000001; % Initial temperature of the system is equal to Tm
T_v(:,1)=Tm+0.000001; % Initial temperature of the system is equal to Tm

L(:,n+1:n+m)=Lm;
L(:,5:6)= 0.000001;
Lt = Le+Lc+Ld; % total length of the heat pipe

Q = 0.0; % energy input for the HP
St = 2*pi*Rt_in*Lm; % Surface area for convection HTF tube without HPs
S2 = (2*pi*Rt_in*Lm)-(N*pi*(Rhp_out^2)); % Surface area for convection HTF tube in exist of HPs

Qhp_start1 = (Rho_eff * cp_eff *(T_c(1,1)-Tm)+ Rho_eff*hsl_eff)*...
            ((pi*(shp(1,1)^2-Rhp_out^2))/(Lt-(abs((st(1,1)-Rt_out)-L(1.5)))));

Qhp_start2 = (Rho_eff * cp_eff *(T_v(1,1)-Tm)+ Rho_eff*hsl_eff)*...
            ((pi*(shp(2,1)^2-Rhp_out^2))/(Lt-(abs((st(2,1)-Rt_out)-L(2.5)))));
for i =1:Timer-1;
    dst1 = abs((st(1,i)-Rt_out))/Mpcm; % PCM element thickness conduction dominated near tube
    dshp1 = abs((shp(1,i)-Rhp_out))/Npcm; % PCM element thickness convection dominated near HP
    dst2 = abs((st(2,i)-Rt_out))/Mpcm; % PCM element thickness conduction dominated near tube
    dshp2 = abs((shp(2,i)-Rhp_out))/Npcm; % PCM element thickness convection dominated near HP

if abs((st(1,i)-Rt_out)) <= Lt
    L(1,1:2)= abs((st(1,i)-Rt_out));
    L(1,3:4)= Lt -(abs((st(1,i)-Rt_out)-L(1,5)));
    Se1 = 2*pi*Rhp_out*abs((st(1,i)-Rt_out)); % Surface area for evaporator section of HP conduction dominated
end

if abs((st(2,i)-Rt_out)) <= Lt
    L(2,1:2)= abs((st(2,i)-Rt_out));
    L(2,3:4)= Lt -(abs((st(2,i)-Rt_out)-L(2,5)));
    Se2 = 2*pi*Rhp_out*abs((st(2,i)-Rt_out)); % Surface area for evaporator section of HP convection dominated
end

[rt, rhp]= grid_gen (dst1,dshp1,dst2,dshp2,Npcm,Mpcm,rt,rhp);
% disp(' ********************* !!')

for j= 7:n;
    r_c(1,j)=rhp(1,j-4);
    r_c(2,j)=rhp(1,j-3);
    r_v(1,j)=rhp(2,j-4);
    r_v(2,j)=rhp(2,j-3);
end

for j= n+2:n+m;
    r_c(1,j)=rt(1,j-n);
    r_c(2,j)=rt(1,j-n+1);
    r_v(1,j)=rt(2,j-n);
    r_v(2,j)=rt(2,j-n+1);
end

for j = 1:(n+m);
    if j==5 || j==6
        Lmda(1,j)=L(1,j);
        Lmda(2,j)=L(2,j);
        A(1,j)= pi*((r_c(2,j))^2-(r_c(1,j))^2);
        A(2,j)= pi*((r_v(2,j))^2-(r_v(1,j))^2);
    else
        Lmda(1,j)=r_c(2,j)-r_c(1,j);
        Lmda(2,j)=r_v(2,j)-r_v(1,j);
        A(1,j)= pi*(r_c(2,j)+r_c(1,j))*L(1,j);
        A(2,j)= pi*(r_v(2,j)+r_v(1,j))*L(2,j);
    end
end

fe = (Lm-2*Rhp_out)/(Lm-2*Rhp_out+2*Lmda(1,1)); % Interpolation factor used to calculate K at the interface

Gr_t = (g*B*(abs(T_v(i,n+1)-Tm))*(abs((st(2,i)-Rt_out)))^3)/(M_PCM/Rho_PCM)^2; % Grashof number near the tube convection dominated
Gr_hp = (g*B*(abs(T_v(i,4)-Tm))*(abs((shp(2,i)-Rhp_out)))^3)/(M_PCM/Rho_PCM)^2; % Grashof number near the HP convection dominated
Ra_t = Gr_t*Pr_PCM; % Rayleigh number near the tube
Ra_hp = Gr_hp*Pr_PCM; % Rayleigh number near the HP

if Process == 'm' || Process == 'M'
    C = 0.073;% C, nn, and mm are constants used for nusselt number equation
    nn = 1/3;
    mm = -1/9;
    for j = 7:n;
        if (r_c(1,j)-Rho_out) > L_fin
            K(1,j) = K_l;
        else
            K(1,j) = K_l_eff;
        end
    end

    if (r_v(1,j)-Rho_out) > L_fin
        K(2,j) = (K_l*C*(Ra_hp)^nn)*((L(2,j)/abs((shp(2,j)-Rhp_out))))^mm ;
    else
        K(2,j) = (K_l_eff*C*(Ra_hp)^nn)*((L(2,j)/abs((shp(2,j)-Rhp_out))))^mm ;
    end
end

140
if K(2,j) < K(1,j);
    K(2,j) = K(1,j);
end
end

C = 0.11;
nn = 0.29;
mm = 0.0;

for j = n+2:n+m;
    K(1,j) = K_1;
    K(2,j) = (K_1**C*(Ra_t)**nn*(L(2,j)/abs((st(2,i) - R_out)))**mm;
    if K(2,j) < K(1,j);
        K(2,j) = K(1,j);
    end
end

elseif Process == 's' || Process == 'S'
    for j = 7:n;
        if (r_c(1,j) - R_out) > L_fin
            K(1,j) = K_s;
        else
            K(1,j) = K_s_eff;
        end
    end
    for j = n+2:n+m;
        K(1,j) = K_s;
    end
else
    disp(' Invalid input !!')
    disp('  Please run the program again and input m or s')
end

[Eta, Eta2, Sig, Xi_c, Xi_v] = paraCalculate(K,A,Lmda,hhp,Se1,Se2,Eta,Eta2,Sig,Xi_c,Xi_v,n,m);
for j = 7:n;
    if (r_c(1,j) - R_out) > L_fin
        Apha(1,j) = K(1,j)/Rho_PCM/Cp_PCM;
    else
        Apha(1,j) = K(1,j)/Rho_eff/cp_eff;
    end
    if (r_v(1,j) - R_out) > L_fin
        Apha(2,j) = K(2,j)/Rho_PCM/Cp_PCM;
    else
        Apha(2,j) = K(2,j)/Rho_eff/cp_eff;
    end
end
for j = n+2:n+m;
    Apha(1,j) = K(1,j)/Rho_PCM/Cp_PCM;
    Apha(2,j) = K(2,j)/Rho_PCM/Cp_PCM;
end

%=========================Runge-Kutta method ================================
if Process == 'm' || Process == 'M'
    THTF = THTF_Ch;
    x = 1;
    y = -1;
    Xi = Xi_c;
end

Qel1 = 0; % energy added/extracted from single PCM element by the HP (conduction)
Qel1_old = 1;
iter_Tc = 0;
iter_Tv = 0;
while (abs(Qel1_old-Qel1) > (10^-6))
    Qel1_old = Qel1;
    Q_old = Q;
    Qe = Qel1;
    iter_Tc = iter_Tc + 1;
    tspan = [t(i),t(i+1)];
    T0 = T_c(i,a+1:n+1);
    [-,T] = ode15s('per2',tspan,T0);
    T_c(i+1,a+1:n+1) = T(end,);
\[ T_c(i+1,m+n) = \left( \frac{-Lmda(x,(m+n) - 1)/2}{-Lmda(x,(m+n) - 1)/2 - Lmda(x,(m+n) - 1)/2} \right) \times \left( T_{\text{end},\text{end}} + \frac{Lmda(x,(m+n) - 1)}{Lmda(x,(m+n) - 1) + Lmda(x,(m+n) - 1)/2} \times Tm \right) \];

\[ st(1,i+1) = \frac{Dt}{Rho_PCM \times st(1,i) \times hsl} \times \left( K(1,n) \times \left( st(1,i) - \frac{Lmda(1,n)}{2} \right) \times \left( T_c(i+1,m+n) - Tm \right) / \left( \frac{Lmda(1,n)}{2} \right) \right) + st(1,i) \];

\[ \text{Tmean} = \text{mean} \left( T_c(i+1,n+2:m+n) \right) \];

\[ K_{\text{interface}} = \frac{1}{(1 - fe) / K(1,1) + fe / K_{\text{pcm}}} \];

\[ Q = K_{\text{interface}} \times \text{Sel} \times \left( \text{Tmean} - T_c(i+1,1) \right) / \left( \text{Lm} / 4 - \text{Rhp}_{\text{out}} + Lmda(1,1)/2 \right) \];

\[ T0 = T_c(i,1:n-1) \];

\[ \left[ -T, T \right] = \text{ode}15s \left( 'per1', tspan, T0 \right) \];

\[ TT = \left( -Lmda(x,n - 1)/2 \right) \times \left( T_{\text{end},\text{end}} + \frac{Lmda(x,n-1)}{Lmda(x,n-1) + Lmda(x,n-1)/2} \times Tm \right) \];

\[ \text{shp}(1,i+1) = \frac{Dt}{Rho_{\text{pcm}} \times \text{shp}(1,i) \times hsl} \times \left( K(1,n) \times \left( \text{shp}(1,i) - \frac{Lmda(1,n)}{2} \right) \times \left( TT - Tm \right) / \left( \frac{Lmda(1,n)}{2} \right) \right) + \text{shp}(1,i) \];

\[ T_{\text{c_old}} = T_c(i+1,1:n) \];

\[ \text{if } (r_c(1,n) - \text{Rhp}_{\text{out}}) > L_{\text{fin}} \]

\[ \text{q}(j,1) = \frac{(Rho_{\text{pcm}} \times \text{Cp}_{\text{pcm}} \times (T_c(i+1,j) - Tm) + \text{Rho}_{\text{pcm}} \times hsl) \times \pi \left( r_c(2,j)^2 - r_c(1,j)^2 \right) \times L_t \times \left( \text{abs}(st(1,i) - \text{Rt}_{\text{out}}) - L(1,5) \right)}{\pi \times N \times \left( \text{Rhp}_{\text{out}} \right)^2 \times \left( \text{st}(1,i+1) - \text{Rt}_{\text{out}} \right)} \];

\[ \text{Qhp}(1,i+1) = \text{sum}(q(7:n,1)) - \text{Qhp}_{\text{start}1} \];

\[ \text{if } \text{Qhp}(1,i+1) < 0 \]

\[ \text{T}_{c}(i+1,1:n) = T_{c\text{old}} ;\]

\[ \text{Qhp}(1,i+1) = 0 ;\]

\[ \text{break} ;\]

\[ \text{end} \]

\[ \text{Qel1} = N \times \left( \text{Qhp}(1,i+1) - \text{Qhp}(1,i) \right) / \left( \pi \times Lm \times \left( st(1,i)^2 - \text{Rhp}_{\text{out}}^2 \right) / Dt \right) ;\]

\[ \% \quad \text{Qhp1} = \text{Qhp}(1,i+1) \]

\[ \% \quad \text{Q_input1} = \text{Q} \times \text{Dt} \]

\[ \% \quad \text{pause} \]

\[ \% \quad \text{break} ;\]

\[ \text{end} \]

\[ \% \quad \text{while} \]

\[ \% \quad \text{pause} \]

\[ \% \quad \text{for } j = n+2:n+m ;\]

\[ \% \quad \text{q}(j,1) = \frac{(Rho_{\text{pcm}} \times \text{Cp}_{\text{pcm}} \times (T_c(i+1,j) - Tm) + \text{Rho}_{\text{pcm}} \times hsl) \times \pi \left( r_c(2,j)^2 - r_c(1,j)^2 \right) \times Lm \times \pi \times N \times \left( \text{Rhp}_{\text{out}}^2 \right) \times \left( \text{st}(1,i+1) - \text{Rt}_{\text{out}} \right) / \left( \pi \times N \times \left( \text{Rhp}_{\text{out}} \right)^2 \times \left( \text{st}(1,i+1) - \text{Rt}_{\text{out}} \right) \right)}{\pi \times Lm \times \left( \text{Rt}_{\text{out}} + \text{Lmda}(1,j)^{(j-n-3)/2} \right) \times Lmda(1,j)} ;\]

\[ \% \quad \text{Qt}_{hp}(1,i+1) = \text{sum}(q(n+2:n+m,1)) ;\]

\[ \% \quad \text{for } j = n+2:n+m ;\]

\[ \% \quad \text{q}(j,1) = \frac{(Rho_{\text{pcm}} \times \text{Cp}_{\text{pcm}} \times (T_c(i+1,j) - Tm) + \text{Rho}_{\text{pcm}} \times hsl) \times \pi \left( r_c(2,j)^2 - r_c(1,j)^2 \right) \times Lm \times \pi \times N \times \left( \text{Rhp}_{\text{out}}^2 \right) \times \left( \text{st}(1,i+1) - \text{Rt}_{\text{out}} \right) / \left( \pi \times N \times \left( \text{Rhp}_{\text{out}} \right)^2 \times \left( \text{st}(1,i+1) - \text{Rt}_{\text{out}} \right) \right)}{\pi \times Lm \times \left( \text{Rt}_{\text{out}} + \text{Lmda}(1,j)^{(j-n-3)/2} \right) \times Lmda(1,j)} ;\]

\[ \% \quad \text{end} \]

\[ \% \quad \text{Qt}(1,i+1) = \text{sum}(q(n+2:n+m,1)) ;\]

\[ \% \quad \text{pause} \]
x = 2;
Xi = Xi_v;
Qel2 = 0; % energy added/extracted from single PCM element by the HP (convection)
Qel2_old = Qel2;
Qc = Qel1;
iter_Tv = iter_Tv + 1;
tspan = [t(i),t(i+1)];
T0 = T_v(i+1:m:n-1);
[-T] = ode15s ('per2',tspan,T0);
T_v(i+1:m:n-1) = T(end,:);
T_v(i+1+m:n) = (-Lmda(x,m+n-1)/2-Lmda(x,m+n)/2)*T(end,end)+Lmda(x,m+n-1)/(Lmda(x,m+n-1)+Lmda(x,m+n)/2)*Tm;

Qel2 = N*(Qhp(2,i+1)-Qhp(2,i))/(pi*Lm*(st(2,i)^2-Rhp_out^2))/Dt;

%=================================================================================================
for j = 7:n;
if (r_v(1,j)-Rhp_out)> L_fin
    q(j,2)= (Rho_PCM * Cp_PCM *(T_v(i+1,j)-Tm)+ Rho_PCM*hsl)*(pi*(r_v(2,j)^2-r_v(1,j)^2)*Lm-(abs((st(1,i)-Rt_out)-L(1,5))))(j-n-3/2)*Lmda(2,j);
else
    q(j,2)= (Rho_eff * cp_eff *(T_v(i+1,j)-Tm)+ Rho_eff*hsl_eff)*(pi*(r_v(2,j)^2-r_v(1,j)^2)*Lm-(abs((st(1,i)-Rt_out)-L(1,5))))(j-n-3/2)*Lmda(2,j);
end
end
Qhp(2,i+1)=sum(q(7:n,2))-Qhp_start2;
if Qhp(2,i+1)< 0
    T_c(i+1,1:n) = Tc_old;
    Qhp(2,i+1) = 0;
    break;
end
Qo2 = N*(Qhp(2,i+1)-Qhp(2,i))/(pi*Lm*(st(2,i+1)^2-Rhp_out^2))/Dt;
% Qhp2 = Qhp(2,i+1)
% Q_input2 = Q*Dt
% pause
else
    break;
end
end % while
for j =n+2:n+m;
q(j,2)= (Rho_PCM * Cp_PCM *(T_v(i+1,j)-Tm)+ Rho_PCM*hsl)/(pi*(r_v(2,j)^2-r_v(1,j)^2)*Lm-
\[ \pi \cdot N \cdot ((R_{hp_{out}})^2) \cdot (st(2,i+1) - Rt_{out})) \times 2\pi \cdot L_m \times (R_{hp_{out}} + L_m) \times (j-n-3/2) \times L_m \cdot (2,j) \] 

end

\[ Q_{hp}(2,i+1) = \sum(q(n+2:n+m,2)) \]
% for j = n+2:n+m;
% \[ q(j,2) = ((Rho_{PCM} \cdot Cp_{PCM} \cdot (T_v(i+1,j) - T_m) + Rho_{PCM} \cdot hsl) \times (\pi \cdot (r_v(2,j)^2 - h(1,j)^2) \times L_m) \times (\pi \cdot L_m) \times (R_{hp_{out}} + L_m) \times (j-n-3/2) \times L_m \cdot (2,j)) \]
% end

\[ Q_{hp}(2,i+1) = \sum(q(n+2:n+m,2)) \]

elseif Process == 's' || Process == 'S'
THTF = THTF_Dis;
x = 1;
y = 1;
Xi = Xi_c;
% **************************************
Qel1 = 0; % energy added/extructed from single PCM element by the HP (conduction)
Qel1_old = 1;
iter_Tc = 0;
iter_Tv = 0;
while (abs(Qel1_old - Qel1) > (10^-6))
Qel1_old = Qel1;
Q_old = Q;
Qe = Qel1;
iter_Tc = iter_Tc + 1;
tspan = [t(i),t(i+1)];
T0 = T_c(i+1,m+n-1);
[T, T] = ode15s ('per2', tspan, T0);
T_c(i+1, m+n) = (Lmda(x,m+n-1)/2) / (Lmda(x,m+n-1)/2 + Tm) ;

st(1,i+1) = -(Dr / (Rho_{PCM} * st(1,i) * hsl) * (K(1,m+n) * st(1,i) - Lmda(1,m+n)/2) / (Lmda(1,m+n)/2) + st(1,i));

Tmean = mean(T_c(i+1,n+2:m+n-5));
K_{pcm} = vs * mean(K(1,n+2:n+m)) + v*K_fin;
K_{interface} = 1 / (1 - (K(1,1) + K(1,1)/2) / K_{pcm});
Q = K_{interface} * Scl * (Tmean - T_c(i,1)) / (Lm/4 - R_{hp_{out}}/2 + Lmda(1,1)/2);
power_throughput(i+1) = Q;

T0 = T_c(i+1,n-1);
[T, T] = ode15s ('per1', tspan, T0);
TT = (Lmda(x,n-1)/2) / (Lmda(x,n-1)/2) / (T_c(i,1,n-1)/2 + Lmda(x,n-1)/2) / Tm ;

if (r_c(1,n) - R_{hp_{out}} > L_fin)
shp(i+1) = -(Dr / (Rho_{eff} * shp(1,i) * hsl_eff) * (K(1,n) * shp(1,i) - Lmda(1,n)/2) / (TT) / (Lmda(1,n)/2) + shp(1,i));
else
shp(i+1) = -(Dr / (Rho_{eff} * shp(1,i) * hsl_eff) * (K(1,n) * shp(1,i) - Lmda(1,n)/2) / (TT) / (Lmda(1,n)/2) + shp(1,i));
end

T_c_old = T_c(i+1,1:n);
if shp(i+1) > shp(i)
T_c(i+1, n-1) = T(end,);
T_c(i+1, n) = (Lmda(x,n-1)/2) / (Lmda(x,n-1)/2) / (T(end, end) + Lmda(x,n-1)/2) / Tm ;

for j = 7:n;
if (r_c(1,j) - R_{hp_{out}} > L_fin)
q(j,1) = -(Rho_{PCM} * Cp_{PCM} * (T_c(i+1,j) - T_m) + Rho_{PCM} * hsl) / (pi * (r_c(2,j)^2 - r_c(1,j)^2) / (Lt - (abs(st(1,j) - R_{hp_{out}}) / L_m)) / (pi * (r_c(2,j)^2 - r_c(1,j)^2) / (Lt - (abs(st(1,j) - R_{hp_{out}}) / L_m))));
else
q(j,1) = -(Rho_{eff} * Cp_{eff} * (T_c(i+1,j) - T_m) + Rho_{eff} * hsl_eff) / (pi * (r_c(2,j)^2 - r_c(1,j)^2) / (Lt - (abs(st(1,j) - R_{hp_{out}}) / L_m))));
end

end

Qhp(1,i+1)=sum(q(7:n,1))-Qhp_start1;

if Qhp(1,i+1)< 0
  T_c(i+1,1:n) = Tc_old;
  Qhp(1,i+1) = 0;
  break;
end

Qel1 = N*(Qhp(1,i+1)-Qhp(1,i))/(pi*Lm*(st(1,i+1)^2-Rhp_out^2))/Dt;
Qhp1 = Qhp(1,i+1);
Q_input1 = Q*Dt;

for j =n+2:n+m;
  q(j,1)= (-Rho_PCM * Cp_PCM *(T_c(i+1,j)-Tm)+Rho_PCM*hsl)*(pi*(r_c(2,j)^2-r_c(1,j)^2)*Lm-
pi*N*(Rhp_out*2)*(st(i+1,j)-Rt_out))/((pi*N*(Rt_out+Lmda(1,j))*(j-n)^2))/Lmda(1,j);
end
Qt_hp(1,i+1)=sum(q(n+2:n+m,1));
if Qt_hp(1,i+1)< 0
  Qt_hp(1,i+1) = 0;
end

disp(' Invalid input !!')
disp('  Please run the program again and input m or s')
break
rhp(:,2) = Rhp_in;  
rhp(:,3) = Rhp_out;  
rt(:,1) = Rt_in;    
rt(:,2) = Rt_out;    
r_c(1,1)= rhp(1,2);  
r_c(2,1)= rhp(1,3);  
r_c(1,2)= rhp(1,1);  
r_c(2,2)= rhp(1,2);  
r_c(1,3)= rhp(1,1);  
r_c(2,3)= rhp(1,2);  
r_c(1,4)= rhp(1,2);  
r_c(2,4)= rhp(1,3);  
r_c(1,5)= rhp(1,1);  
r_c(2,5)= rhp(1,2);  
r_c(1,6)= rhp(1,2);  
r_c(2,6)= rhp(1,3);  
r_c(1,n+1)= rt(1,1);  
r_c(2,n+1)= rt(1,2);  

% ------------------  

r_v(1,1)= rhp(2,2);  
r_v(2,1)= rhp(2,3);  
r_v(1,2)= rhp(2,1);  
r_v(2,2)= rhp(2,2);  
r_v(1,3)= rhp(2,1);  
r_v(2,3)= rhp(2,2);  
r_v(1,4)= rhp(2,2);  
r_v(2,4)= rhp(2,3);  
r_v(1,5)= rhp(2,1);  
r_v(2,5)= rhp(2,2);  
r_v(1,6)= rhp(2,2);  
r_v(2,6)= rhp(2,3);  
r_v(1,n+1)= rt(2,1);  
r_v(2,n+1)= rt(2,2);  

st(:,1)=(10*10^(-6)+Rt_out;  
shp(:,1)=(10*10^(-6)+Rhp_out;  
L(1:2)= Le;  
L(3:4)= Lc;  
L(5:6)= Ld;  
L(7:m)= Le;  
L(n+1:n+m)=Lm;  
T_c(:,:,1)=Tm+0.000001; % Initial temperature of the system is equal to Tm  
T_v(:,:,1)=Tm+0.000001; % Initial temperature of the system is equal to Tm  
K (:,1)= K_hp;  
K (:,2)= W_k;  
K (:,4)= K_hp;  
K (:,3)= W_k;  
K (:,6)= K_hp;  
K (:,5)= W_k;  
K (:,n+1)= K_t;  

Apha (:,1)= K_hp/Rho_hp/Cp_hp;  
Apha (:,2)= W_k/W_HC;  
Apha (:,4)= K_hp/Rho_hp/Cp_hp;  
Apha (:,3)= W_k/W_HC;  
Apha (:,6)= K_hp/Rho_hp/Cp_hp;  
Apha (:,5)= W_k/W_HC;  
Apha (:,n+1)= K_t/Rho_hp/Cp_t;  

Se = 2*pi*Rhp_out*Lc; % Surface area for convection HP  
St = 2*pi*Rt_in*Lm; % Surface area for convection HTF tube without HPs  
St2 = (2*pi*Rt_in*Lm)-(N*pi*(Rhp_out)^2); % Surface area for convection HTF tube in exist of HPs  
for i =1:Timer-1;  
dst1 = abs((st(1,i)-Rt_out))/Mpcm; % PCM element thickness conduction dominated near tube  
dshp1 = abs((shp(1,i)-Rhp_out))/Npcm; % PCM element thickness conduction dominated near HP  
dst2 = abs((st(2,i)-Rt_out))/Npcm; % PCM element thickness convection dominated near HP  
dshp2 = abs((shp(2,i)-Rhp_out))/Npcm; % PCM element thickness convection dominated near HP  
[rt rhp]=grid_gen2 (dst1,dshp1,dst2,dshp2,Npcm,Mpcm,rt,rhp);  
end;  
for j= n+1:n+m;
for j = 1:(n+m);
    if j==5 || j==6
        Lmda(:,j)=L(j);
        A(1,j)= pi*((r_c(2,j))^2-(r_c(1,j))^2);
        A(2,j)= pi*((r_v(2,j))^2-(r_v(1,j))^2);
    else
        Lmda(1,j)=r_c(2,j)-r_c(1,j);
        A(1,j)= pi*(r_c(2,j)+r_c(1,j))*L(j);
        Lmda(2,j)=r_v(2,j)-r_v(1,j);
        A(2,j)= pi*(r_v(2,j)+r_v(1,j))*L(j);
    end
end

Gr_t = (g*B*(abs(T_v(i,n+1)-Tm))*(abs((st(2,i)-Rt_out)))^3)/(M_PCM/Rho_PCM)^2; % Grashof number near the tube
convection dominated
Gr_hp = (g*B*(abs(T_v(i,4)-Tm))*(abs((shp(2,i)-Rhp_out)))^3)/(M_PCM/Rho_PCM)^2; % Grashof number near the Hp
convection dominated
Ra_t = Gr_t*Pr_PCM; % Rayleigh number near the tube
Ra_hp = Gr_hp*Pr_PCM; % Rayleigh number near the HP

if Process == 'm' || Process == 'M'
    C = 0.073;% C, nn, and mm are constants used for nusselt number equation
    nn = 1/3;
    mm = -1/9;
    for j = 7:n;
        K(1,j) = K_l;
        K(2,j) = (K_l*C*(Ra_hp)^nn)*(L(j)/abs((shp(2,i)-Rhp_out)))^mm;
        if K(2,j)< K(1,j);
            K(2,j)= K(1,j);
        end
    end
    C = 0.11;
    nn = 0.29;
    mm = 0.0;
    for j = n+2:n+m;
        K(1,j) = K_s;
    end
    for j = n+2:n+m;
        K(1,j) = K_s;
    end
    elseif Process == 's' || Process == 'S'
    for j = 7:n;
        K(1,j) = K_s;
    end
    for j = n+2:n+m;
        K(1,j) = K_s;
    end
    else
        disp(' Invalid input !!')
        disp(' Please run the program again and input m or s')
        exit
    end
[Eta Eta2 Sig Xi_c Xi_v] = paraCalculate2 (K,A,Lmda,hhp,Se,Eta,Eta2,Sig,Xi_c,Xi_v,n,m);
for j = 7:n;
    Apha(1,j) = K(1,j)/Rho_PCM/Cp_PCM;
    Apha(2,j) = K(2,j)/Rho_PCM/Cp_PCM;
end
for j = n+2:n+m;
    Apha(1,j) = K(1,j)/Rho_PCM/Cp_PCM;
    Apha(2,j) = K(2,j)/Rho_PCM/Cp_PCM;
end
%================================ Runge-Kutta method ===============================
if Process == 'm' || Process == 'M'
    THTF = THTF_Ch;
    x = 1;
    Xi = Xi_c;
    tspan = [tt(i),tt(i+1)];
    T0 = T_c(i,n+1:m+n-1);
```plaintext
[~,T] = ode15s ('per22',tspan,T0);
T_c(i+1,n+1:m+n-1) = T(end,:);
T_c(i+1,m+n) = -Lmda(x,(m+n)-1)/2/Lmda(x,(m+n)-1)*Tm;

%-------------------------------------------------------------------------------------------------

x = 2;
Xi = Xi_v;
tspan = [tt(i),tt(i+1)];
T0 = T_v(i+1,m+n-1);
[-T] = ode15s ('per22',tspan,T0);
T_v(i+1,m+n-1) = T(end,:);
T_v(i+1,m+n) = -Lmda(x,(m+n)-1)/2/Lmda(x,(m+n)-1)*Tm;

%-------------------------------------------------------------------------------------------------

for Q of tube without HPs ------

for j =n+2:n+m;
qt(j,1)= 1/Rho_PCM * Cp_PCM *(T_c(i+1,j)-Tm)/pi*(r_c(2,j)^2-r_c(1,j)^2)*Lm; 
qt(j,2)= 1/Rho_PCM * Cp_PCM *(T_v(i+1,j)-Tm)/pi*(r_v(2,j)^2-r_v(1,j)^2)*Lm; 
end
Qtt(1,i+1)=sum(qt(n+2:n+m,1));
Qtt(2,i+1)=sum(qt(n+2:n+m,2));

elseif Process == 's' || Process == 'S'
THTF = THTF_Dis;

x = 1;
Xi = Xi_c;
tspan = [tt(i),tt(i+1)];
[-T] = ode15s ('per22',tspan,T0);
T_c(i+1,n+1:m+n-1) = T(end,:);
T_c(i+1,m+n) = -Lmda(x,(m+n)-1)/2/Lmda(x,(m+n)-1)*Tm;

%-------------------------------------------------------------------------------------------------

end

%======================================================================

end % i_vf
end % i_kfin
```

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end % i_Dhp
end % i_Dt

% =============== Drawing graph ============================
if Process == 'm' || Process == 'M'

% =============== plot a graph ============================
hl1 = line(t/60/60,T_c(:,17),'Color','r','LineStyle','--','LineWidth',1.5);
hold on
hl2 = line(t/60/60,T_v(:,17),'Color','r','LineWidth',1.5);
ylabel('Tt (C)');
xlabel('t (h)');

ax1 = gca;
set(ax1,'Position',get(ax1,'Position'),
    'XAxisLocation','top',
    'YAxisLocation','right',
    'Color','none',
    'XColor','r','YColor','r');

ax2 = axes('Position',get(ax1,'Position'),...
    'XAxisLocation','top',
    'YAxisLocation','right',
    'Color','none',
    'XColor','k','YColor','k');

hl3 = line(t/60/60,st(1,:)-Rt_out,'LineStyle','--','LineWidth',1.5,'Color','k','Parent',ax2);
hl4 = line(t/60/60,st(2,:)-Rt_out,'Color','k','LineWidth',1.5,'Parent',ax2);

else

figure
plot(t/60/60,power_throughput(1,:),'Color','k','LineStyle','--','LineWidth',1.5);
ylabel('Power Throughput by a Heat Pipe (W)');
xlabel('Time (h)');
grid on
end

3. The numerical source code presented in chapter 5.

% --------------- Input data -----------------------------
kpcm = 0.7; % The thermal conductivity of the PCM

khp = 7000; % The thermal conductivity of the HP
Lhp = 0.05; % [m] length of the HP
w = 0.00147; % [m] HP side width
HP_angle = 60; % [deg] HP corner angle
vhp = (0:0.01:0.3); % total volume fraction of the HP

% ----------------- Matrices declaration -------------------
xu = ones(1,Nx+1);
x = ones(1,Nx+2);
dx = ones(1,Nx);
yu = ones(1,Ny+1);
\[
y = \text{ones}(1,N_y+2);
d_y = \text{ones}(1,N_y);
zu = \text{ones}(1,N_z+1);
z = \text{ones}(1,N_z+2);
dz = \text{ones}(1,N_z);
Kc_x = \text{zeros}(1,\text{length}(vhp)); \quad \% \text{Composite thermal conductivity}
Kc_y = \text{zeros}(1,\text{length}(vhp)); \quad \% \text{Composite thermal conductivity}
Kc_z = \text{zeros}(1,\text{length}(vhp)); \quad \% \text{Composite thermal conductivity}
\]
\[
\text{theta} = \text{ones}(N_y,N_x,N_z); \% 
\text{phi} = \text{ones}(N_y,N_x,N_z); \% 
\text{th}_x = \text{ones}(N_y,N_x,N_z); \% 
\text{th}_y = \text{ones}(N_y,N_x,N_z); \% 
\text{th}_z = \text{ones}(N_y,N_x,N_z); \%
\]
\[
l_x = \text{ones}(N_y,N_x,N_z); \%
l_y = \text{ones}(N_y,N_x,N_z); \%
l_z = \text{ones}(N_y,N_x,N_z); \%
k_x = \text{ones}(N_y,N_x,N_z+1); \quad \% \text{Thermal conductivity at the interfaces along z axis}
k_y = \text{ones}(N_y+1,N_x,N_z); \quad \% \text{Thermal conductivity at the interfaces along y axis}
k_z = \text{ones}(N_y,N_x+1,N_z); \quad \% \text{Thermal conductivity at the interfaces along x axis}
\]
\[
\text{sp} = \text{ones}(N_y,N_x,N_z); \quad \% \text{Temperature dependant source term}
\text{sc} = \text{ones}(N_y,N_x,N_z); \quad \% \text{Constant source term}
\]
\[
a = \text{ones}(N_y+2,N_x+2,N_z+2);
b = \text{ones}(N_y+2,N_x+2,N_z+2);
c = \text{ones}(N_y+2,N_x+2,N_z+2);
d = \text{ones}(N_y+2,N_x+2,N_z+2);
T = \text{ones}(N_y+2,N_x+2,N_z+2);
To = \text{ones}(N_y+2,N_x+2,N_z+2);
T_{\text{mid}} = \text{ones}(1,2);
\]
\[
[dx, xu, x, dy, yu, y, dz, zu, z] = \text{xu_x_yu}(dx, xu, x, dy, yu, y, dz, zu, L_x, L_y, L_z, \text{N}; \text{N}); \quad \% \text{call function for dividing}
\]
\[
\text{for } j = 1:\text{length}(vhp)\]
\[
\text{Kxx} = \text{ones}(N_y,N_x,N_z) * \text{kpcm}; \quad \% \text{Thermal conductivity of a control volume in x direction}
\text{Kyy} = \text{ones}(N_y,N_x,N_z) * \text{kpcm}; \quad \% \text{Thermal conductivity of a control volume in y direction}
\text{Kzz} = \text{ones}(N_y,N_x,N_z) * \text{kpcm}; \quad \% \text{Thermal conductivity of a control volume in z direction}
\text{Kxx0} = \text{zeros}(N_y,N_x,N_z); \%
\text{Kyy0} = \text{zeros}(N_y,N_x,N_z); \%
\text{Kzz0} = \text{zeros}(N_y,N_x,N_z); \%
\text{Kxx0} = \text{Kxx};
\text{Kyy0} = \text{Kyy};
\text{Kzz0} = \text{Kzz};
\]
\[
[\text{Kxx}, \text{Kyy}, \text{Kzz}, \text{Kx}, \text{Ky}, \text{Kz}] = \text{K_k}(\text{Kxx}, \text{Kyy}, \text{Kzz}, \text{Kx}, \text{Ky}, \text{Kz}, \text{w}, \text{vhp}\_\text{single}, \text{khp}); \quad \% \text{call function for calculate K_faces}
\]
\[
\text{Kc}_x(j) = \text{mean(mean(mean}(\text{Kxx})));
\text{Kc}_y(j) = \text{mean(mean(mean}(\text{Kyy})));
\text{Kc}_z(j) = \text{mean(mean(mean}(\text{Kzz})));
\]
\end
\[ K_{\text{eff}} = (K_{c_x} + K_{c_y} + K_{c_z})/3 \]

dsetuu=1;